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RESUMO/ABSTRACT

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Competitive Location Models seek the positions which maximize the market captured by an entrant firm from previously positioned competitors. Nevertheless, strategic location decisions may have a significant impact on inventory and shipment costs in the future affecting the firm's competitive advantages. In this work we describe a model for the joint replenishment competitive location problem which considers both market capture and replenishment costs in order to choose the firm's locations. We also present an metaherusitic method to solve it based on the Viswanathan's (1996) algorithm to solve the Replenishment Problem and an Iterative Local Search Procedure to solve the Location Problem.

Keywords: Market's capture, joint replenishment, Iterated Local Search.

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Abstract

Competitive Location Models seek the positions which maximize the market captured by an entrant firm from previously positioned competitors. Nevertheless, strategic location decisions may have a significant impact on inventory and shipment costs in the future affecting the firm's competitive advantages. In this work we describe a model for the joint replenishment competitive location problem which considers both market capture and replenishment costs in order to choose the firm's locations. We also present an metaherusitic method to solve it based on the Viswanathan's (1996) algorithm to solve the Replenishment Problem and an Iterative Local Search Procedure to solve the Location Problem.

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JE: C61, L80.

1. Introduction.

Competitive location models consider explicitly the fact that when an entrant facility is going to choose its location there are already other facilities in the market and that the entrant facility will have to compete with them for its market share. Customers will patronize the most attracting facility, and distance between the facility and the customer often plays an important role in this attraction. (see Plastria, 2001).

ReVelle's Maximum Capture Problem (1986) initiated a series of studies on the location of retail facilities in discrete space (see Serra and ReVelle, 1995). The MAXCAP model makes the following assumptions: (1) the product sold is homogeneous, (2) the consumer's decision on patronizing the store is based on distance and (3) unit costs are the same in all stores regardless of ownership. Examples of services that best fit on these three assumptions can be found mainly in the fast food sector, in convenience stores and in the banking sector.

When locating retail facilities a major topic of concern besides market share is the inventory decision that will be associated to each location scenario, given the fact that inventory replenishment costs are an important component on retail stores total costs. Some firms may be willing to sacrifice some market share in order to have a more convenient location for their inventory replenishment.

In this paper we formulate a model which considers both location and inventory decisions for an entrant firm. When entering in the market the new firm will decide the location of the facilities, the market to capture and the replenishment policy, including the replenishment frequency,

In chapter 1 we will revise some literature on competitive spatial modeling. In chapter 2 we describe a model, which incorporates explicitly waiting time, and in chapter 3 we propose a metaheuristic to solve the model. Some results of our computational experiments are described in chapters 4 and 5.

2. Literature Review.

The MaxCap (maximum capture) model introduced by ReVelle (1986) finds the optimal location on a network considering that each demand point will patronize the closest facility. Several authors have expanded ReVelle's formulation: Eiselt and Laporte (1989) generalize ReVelle's findings in two directions: they allow differential weights for the facilities and they leave a parameter of the cost function variable so as to facilitate sensitivity analysis, Serra and ReVelle (1993) introduce in the model facilities that are hierarchical in nature and where there is competition at each level of the hierarchy, the same authors, Serra and ReVelle (1994), account the possible reaction from competitors to the entering firm in the preemptive location problem, on which the leader wishes to preempt the entering firm in its bid to capture market share to the maximum extend possible. Serra, Ratick and ReVelle (1996) offer a modification of the MaxCap problem on which they consider that a firm wants to locate a fixed number of servers so as to maximize market capture in a region where competitors are already located but where there is uncertainty. The authors consider different future scenarios with respect to demand and/or the location of competitors.

Most competitive location problems were at first developed under the hypothesis that different firms provide the same indistinguishable product and that all customers have the same preferences, i.e., the same deterministic utility function. Some literature refers to the topic of dropping the hypothesis of the homogeneity of the product.

In Drezner (1994) customers base facility choice on a utility function that incorporates a facility's attributes and the distance to the facility. Although customers are no longer assumed to patronize the closest facility, customers at a certain demand point apply the same utility function.

Drezner and Drezner (1996) assume the utility function to change from one consumer to another for customers located at the same demand point. Using this assumption the "all or nothing" property disappears.

Serra, Eiselt, Laporte and ReVelle (1999) developed two models allowing different customer choice rules. One model assumes that customers consider the closest facility of each firm and then patronize the two facilities in proportion to the customer-facility distance. The other model assumes that the demand captured by a facility is affected by the existence and location of all facilities of both firms.

Other improvements over the initial maximum capture model refer to minimum market shares that firms need to capture in order to survive. Carreras and Serra (1998) present a model that locates the maximum number of services that can coexist in a given region without having losses, taking into account that they need a minimum demand level in order to survive.

Serra, ReVelle and Rosing (1999) considered the problem of locating several facilities such that each facility attracts a minimum threshold of customers. Drezner and Eiselt (2002) consider a minimum market share threshold to be captured, below which the firm cannot survive and propose the objective of minimizing the probability that revenues fall short of the threshold necessary for survival.

3. The model for the Joint Replensishment Competitive Location Problem.

Competitive Location Problems seek the location of a fixed number of stores belonging to a firm in a spatial market where there are other stores from other firms already competing for clients. The objective of the entering firm is to maximize its profits. Whenever the prices charged at the different facilities are equal and there are no location-specific costs, the profit-maximizing objective reduces to maximization of sales (market capture).

A customer is an individual or a group of such with a unique and identifiable location and behavior. Since a customer has a location and issues demand, the term demand point is also used. The expression “point demand” as defined by Plastria (2001) refers to discrete demand concentrated in a finite set of points.

We consider a discrete location space in the sense that there is only a finite list of candidate sites and the market is characterized by point demand.

Each customer feels some attraction towards each of the competing facilities, that’s what is usually referred as “patronizing behavior”. The “attraction function” describes how a customer’s attraction, also called utility, towards a facility is obtained. In our model the attraction function is determined by distance to the store location.

Let us assume an entering firm (firm A) that wants to locate p new outlets when there are q other outlets from another firm (firm B) already competing at the market place.

$$Max\quad Z_1 = \sum_{i \in I} \sum_{j \in J^A} a_i X_{ij} \qquad (1)$$

s.t.

$$\sum_{j \in J} X_{ij} = 1 \qquad \forall i \in I \qquad (2)$$

$$X_{ij} \geq Y_j - \sum_{l \in S_{ij}} Y_l \qquad S_{ij} = \{l | d_{il} < d_{ij}\} \quad \forall i \in I\ \forall j \in J \qquad (3)$$

$$\sum_{j \in J} Y_j = p \qquad (4)$$

$$X_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \qquad (5)$$

In order to solve the problem we consider that the entering firm wants to maximize its market share, and that each demand point will patronize the store and the closest location. This problem can be written as

Where,

X_{ij} =1 if demand point i patronizes a store at j

=0 otherwise

i,I index and set of demand points

j,J index and set of potential locations

J^A set of firm's A (entrant firm) store locations

a_i demand at node i

d_{ij} is the distance from node i to node j

Constraints (2) limit the allocation of one demand point to only one store. Constraints (3) state that a demand point will always patronize the closest outlet location. Originally introduced by Rojeski and ReVelle (1970) this constrains establish that if *j* is an open outlet and no closer outlet is open, then demand *i* must be assigned to *j*. If *j* is open but a closer outlet is also open, then this relation does not constrain assignment in any way. Constraint (4) fixes the number of outlet to locate to be *p*. Constraints (5) are the binary constraints.

We also consider the replenishment policy as an important factor conditioning the location of the outlets. Replenishment decisions will result from a Joint Replenishment Problem- JRP. The JRP applies to a variety of situations. Nilsson et al. (2005), expose real life situations of different nature where different articles are ordered by a single client to a supplier; where several products share the same transportation or when an item is produced to be packaged in different packages. In our case we consider a same product sharing the same transportation to a set of stores where the product is sold to the customer.

The main objective of the problem of joint replenishment will consists on finding a situation of balance between the fixed costs of ordering and the holding costs for the different stores locations through the adjustment of the frequencies of replenishment (k_j) at each location. The problem will involve the calculation of the base time period (T) that corresponds to the policy of cycle of joint replenishment and its multiple positive integers (k_j´s) for the frequency of replenishment of each item. The objective is going to find the values of T and of the k_j´s that drive the relevant total costs for period of time (C_{tr}) to its minimum value. For the JRP we need the following additional notation:

n – Number of stores in joint replenishment

A – Fixed cost of ordering by joined, independently of the number of stores (major set-up cost).

c_j - Variable Cost of including store j in the joint order, with $j = 1, 2, ... , n$.

h_j – Holding Cost (maintenance) of a unit of the item in warehouse at store location *j* by unit of time.

a_j – Demand for the item at store location *j* by unit of time, constant and known.

T – Joint replenishment cycle time, i.e., period of time that elapses between each revision of the stocks (*Basic Cycle Time*).

t_j - Period of time that elapses between each replenishment at store location *j*, with $j = 1, 2,..., n$.

V – Mean number of joint replenishment orders by unit of time.

v_i – Mean number of replenishment orders for item at store location *j* by unit of time ($v_j = 1/t_j$), with $j = 1, 2,... , n$.

k_j – Frequency of replenishment at store location *j*, assuming discrete values which are multiple of T.

C_{tr} – relevant average total costs of the joint replenishment system by unit of time.

In the literature, as is an example Silver (1976), Silver et al (1998), Andres et al. (1975) and Goyal et al (1989), it is usual to find the same group of assumptions for the JRP with continuous revision and deterministic demand: the demand is known and constant; the stock replenishment admits non integer quantities of the items; the costs of output and/or the prices of acquisition do not depend on the quantities ordered; the stock replenishment is immediate, assuming an infinite quantity available of each item; stock shortage is not admitted; the waiting time of supply is zero; there is no limitation for the space of storage; the operation of the system of storage admits an infinite time horizon; the joint replenishment of the orders, requires that, at least, one of the items to be always ordered, i.e., to have T as periodicity of order (restricted cycle policy).

The relevant costs associated to the problem of joint replenishment of stocks in accordance with the model assumptions are classified as ordering costs and holding costs. The ordering costs are subdivided into a fixed component A (major set-up cost), that is incurred whenever an order occurs, independently of the number of stores in replenishment, and in a variable costs component (c_i), that is related with each store integrated in the order (minor set-up cost).

The holding costs by unit of time h_j, result from the maintenance of each unit of the item in the warehouse at store location *j*, while waiting for its commercialization.

Grouping all items object of a joint replenishment we will be able to identify the equation of the medium relevant total costs by unit of time (see as an example Viswanathan, 1996):

$$C_{tr}(T,k)=\frac{A+\sum_{j=1}^{n}\frac{c_j}{k_j}}{T}+\sum_{j=1}^{n}\frac{h_j.a_j.k_j.T}{2} \tag{5}$$

Additionally, consider a_j as the demand for the item at store *j* by unit of time, then

$$a_j=\sum_{i=1}^{m}a_i X_{ij}$$

Rewriting Equation (5) and calling it Z_2 we obtain

$$Z_2=\frac{A+\sum_{j=1}^{n}\frac{c_j}{k_j}}{T}+\sum_{j=1}^{n}\frac{h_j.k_j.T\sum_{i=1}^{m}a_i X_{ij}}{2} \tag{6}$$

The final model we want to solve is now given by the following

$$
\begin{aligned}
& Max\ \ Z_1 \\
& Min\ \ Z_2 \\
& s.t. \\
& \sum_{j\in J} X_{ij}=1 && \forall i\in I \\
& X_{ij}\geq \mathrm{Y_j} - \sum_{\mathrm{l}\in \mathrm{C_{ij}}} Y_l && \forall i\in I\ \forall j\in J \\
& \sum_{j\in J} Y_j = p \\
& X_{ij}\in\{0,1\} && \forall i\in I, \forall j\in J
\end{aligned}
$$

This is a bi-objective model where we want to find the set of firm A's outlet locations that maximize the market captured and minimize replenishment costs. In practice both objectives will hardly be optimized at the same time and we will have a tradeoff curve between the two objectives.

Kariv and Hakimi (1979) prove that the p-Median problem is a NP-Hard problem on a general graph. Besides that, notice that the objective Z_2 is non-linear and that we need to solve a p-median model for each firm *A* stores' possible locations. This explains the important role played by the metaheuristics in solving the model.

4. A Metaheuristic Solution Procedure

In the heuristic we used a Iterated Local Search Procedure (ILS). The algorithm comprises the following steps: construction of an initial solution, perturbation in the locations and perturbation in the weights. Figure 1 gives us the pseudo-code of the algorithm.

In order to implement first step it was necessary to define an evaluation function to construct the restricted candidate list - RCL. We chose the following one:

$$F(j) = w_1 F_1(j) - w_2 F_2(j) \qquad (7)$$

where,

$$F_1(j) = a_j \qquad (8)$$

$$F_2(j) = C_j + \frac{h_j . F_1(j)}{2} \qquad (9)$$

The Restricted Candidate List (RCL) will then contain all the candidate solutions within a given distance of the top candidate as a function of φ. The threshold value can be expressed as:

$$\gamma\varphi_{\max} \qquad 0 \leq \gamma \leq 1$$

Where $\varphi_{\max}$ is the maximum value of the function, and γ is a parameter defining which candidate nodes will be included in the RCL (e.g. with γ=0.7 we include in the RCL all the candidate nodes with a value for the greedy function higher than 70% of the maximum).

In the function the value assigned to each of the candidate locations is obtained by the population that would be allocated to a firm A store at this location. To obtain the initial solution we follow the steps:

1) Choose randomly *p* locations from the RCL.

2) Allocate the demand nodes to their closest facility location.

3) Compute location costs' objective considering the allocations obtained in 1).

4) Use Viswanathan (1996)'s algorithm in order to find the optimal joint replenishment policy and compute the replenishment costs objective. In this step we will obtain not only the value of the joint replenishment objective but also the optimal values for the frequencies k and the cycle time T.

5) Compute objective. In this step we use the following function:

$$F(X) = w_1 F_1(X) - w_2 F_2(X) \tag{10}$$

$$F_1(X) = \sum_{i \in I} \sum_{j \in J^A} a_i X_{ij} \tag{11}$$

$$F_2(X) = \frac{A + \sum_{j=1}^{n} \frac{a_j}{k_j}}{T} + \sum_{j=1}^{n} \frac{h_j . k_j . T \sum_{i=1}^{m} a_i X_{ij}}{2} \tag{12}$$

The pseudo-code for the initial solution procedure is given in figure 3.

Over the initial solution we make a local improvement. For each Distribution Center at a time we de-allocate the demands that were allocated to it and move it to all possible unused potential locations, repeating the following steps:

1) Always allocate a demand point to the closest potential facility location.

2) Compute location costs' objective.

3) Use Viswanathan (1996)'s algorithm in order to find the optimal joint replenishment policy and compute the replenishment costs objective.

4) Compute objective *F(X)* using expressions (10) to (12). If the objective improves keep the new location for the Distribution Center and the new allocations, otherwise keep the old locations and allocations.

The pseudo-code for the local improvement procedure is given in figure 4.

Following the algorithm we implement first a perturbation with fixed weights and then a perturbation in the weights. The first type of perturbations consists in the following:

1) Close a random number of outlets (between 2% and 30% of the total number of outlets to locate).

2) Measure the contribution to total objective of each of the individual locations in the final solution using expressions (7), (8) and (9).

3) Close the facilities with the worse values of $F_j(X)$

4) Open the same number facilities as closed in 1).

4.1) For each potential facility location compute $F(j)$ using expression (7) to (9).

4.2) Open facilities in the locations with probability P_j, where

$$P_j = \frac{w_1 F_1(j) - w_2 F_2(j)}{\sum_j w_1 F_1(j) - w_2 F_2(j)}$$

4.3) Proceed to a local improvement procedure as in figure 4.

4.4) Update the non-dominated set.

The second type of perturbations consists in changing the weights given to both objectives, proceed to a local search and update the non-dominated set.

Finally, we proceed to a search in the PE: For each solution in set S at a time check if this solution is not dominated by any other solution in that set. Case it is a non-dominated solution keep the solution in set S, otherwise delete the solution from set S. Return S as the solution set for the problem.

5. Numerical Examples

In the numerical examples we started working with Swain's (1974) well-known 55-node network. We assume that firm B already has five stores operating in the nodes with the largest population and we want to locate five firms' A stores.

There is a total of 26 235 possible combinations for the locations of the three stores in the 55 node network of Swain (1974). Suppose that a competitor firm is already operating in the five nodes with the largest populations, Table 1 shows the results of the regression of market capture in replenishment costs. As expected we found a positive relationship between market capture and replenishment costs: the larger the market captured by the firm the larger will be the cost with replenishment of the outlets.

For all possible combinations of outlets it was possible to select only 18 non-dominated solutions (with no other solution with a lower replenishment cost and simultaneously a higher market capture). The results are described on table 2 and illustrated by figure 5. It is clear from figure 5 the tradeoff between larger market captures and lower replenishment costs.

6. Conclusions

In this work we describe a new problem: the Joint Replenishment Competitive Location problem and present a mathematical model and metaheuritics to solve it. Also, we present the results of a computational experiment that reveals important insights on this problem.

We introduced Viswanathan (1996) in an iterated local search procedure in order to find the non-dominated solutions of the problem of maximizing market capture and simultaneously minimizing replenishment costs. From the numerical experiments we conclude about the tradeoff among both objectives, which is in consistency with the theoretical model.

```
procedure ILS (Max_iterations, Seed)
    For k = 1 to Max_iterations do
    {Construct Initial Solution}
        {Initiate weights}
    W ← random_generator (Seed);
        {Generate an initial solution}
    S ← initial_Solution (Seed,γ);
    S ← local_Search (S,W);
        {Initiate PE}
  PE ← S;
   {Iterated Local Search}
    do until q_1 iterations without improvement:
        S ← perturbation (S,W);
        S ← local_search (S,W);
        PE ← update_PE(S,PE);
    enddo
   {Perturbation in weights}
   do until q_2 iterations without improvement:
        W ← perturbation (W);
        S ← local_Search (S,W);
        PE ← update_PE(S,PE);
  enddo
  enddo
  {Define nom-dominated set}
  NDS ← NDS(PE);
end ILS
```

Figure 1: ILS pseudo-code

procedure evaluate_objective (S, W)

Allocate each demand point to its closest center location;

$F_1 := 0;$

For all $j \in J^A$ *do*

    For all $i \in I$ *do*

      if $(i \text{ is allocated to } j)$

        $F_1 = F_1 + pop(i);$

enddo

$F_2 \leftarrow Viswanathan(S)$

$obj := W_1 * F_1 - W_2 * F_2;$

end evaluate_objective;

Figure 2: Objective evaluation pseudo-code

$procedure\ initial\_Solution(Seed, \gamma, W)$

$\{Initialize\ solution\ set\}$

$S: =\{\ \};$

$\{compute\ the\ value\ of\ the\ greedy\ function\ for\ all\ potential\ locations\}$

$for\ j = 1\ to\ N\ do$

$F[j] := W_1 * pop[j] - W_2 * (c[j] + h[j] * pop[j] / 2);$

$enddo$

$\{construct\ the\ restricted\ candidate\ list\}$

$F^{max} := max(F[j]);$

$RCL \leftarrow \{ j \in J\ | F[j] \geq \gamma * F^{\max} \};$

$while\ |S| \neq p\ do$

$j^* \leftarrow random\_Select\ (RCL);$

$S := S \cup \{ j^* \};$

$RCL := RCL / \{ j^* \};$

$enddo$

Figure 3: Initial Solution pseudo-code

```
procedure local_Search(S)
obj_best := obj(S);
for all j_1 ∈ S do
    S : S/{j_1};
        for all j_2 ∈ S̄/{j_1} do
            evaluate_obj(S ∪ {j_2});
            if obj_best < obj(S ∪ {j_2}) do
                S : = S ∪ j_2;
                obj_best := obj(S ∪ {j_2});
            else
                S := S ∪ j_1;
            endif
        enddo
enddo
end local_Search
```

Figure 4: Local Search Phase pseudo-code

Regression Statistics	
R square	0,435732979
Standard Error	55,98752822
Number of cases	26235

ANOVA

	df	*SS*	*MS*	*F*	*F significance*
Regressão	1	63498915,89	63498916	20257,4	0
Residual	26233	82230048,81	3134,603		
Total	26234	145728964,7			

	Coefficients	*Standard Error*	*t*	*P value*
Interceptar	129,2667759	1,312420964	98,4949	0
Variável X 1	0,11507904	0,000808545	142,3285	0

Table 1: regression of market capture in replenishment cost

Iteration	Market capture	Replenishment cost	Locations:		
7881	2870	349,324	7	9	32
7926	2520	250,855	7	10	32
8321	2320	247,783	7	20	32
10366	2670	300,019	9	18	32
10756	2960	390,064	9	31	32
10769	3010	471,815	9	31	45
11356	2320	184,134	10	18	32
11378	1820	133,508	10	18	54
11557	1840	169,553	10	24	32
11579	1340	119,593	10	24	54
11779	2000	174,175	10	32	42
11966	1500	121,659	10	42	54
11989	1270	101,774	10	44	54
12008	1230	94,2301	10	46	54
12033	730	68,1985	10	50	53
12034	1180	83,8006	10	50	54
18828	2540	295,264	19	32	41
23944	2740	343,385	31	32	41

Table 2: non-dominated solutions

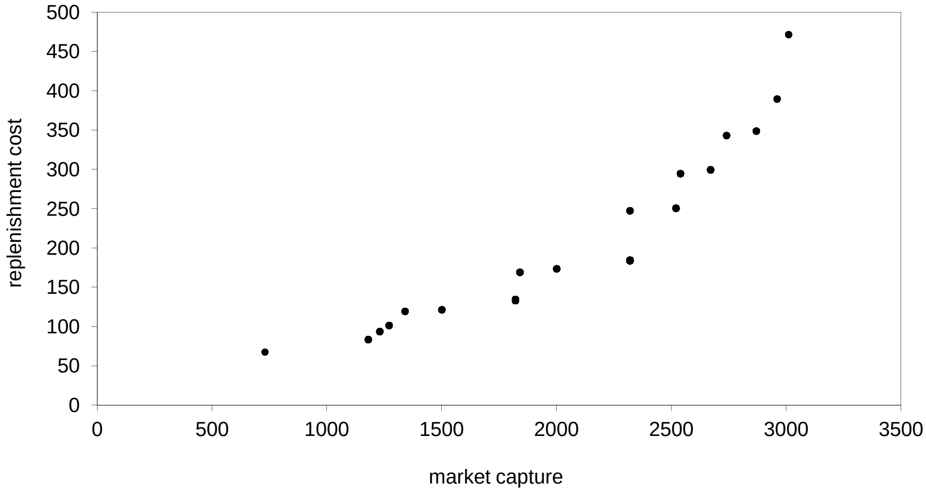

Figure 5: Non-dominated solutions

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