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RESUMO/ABSTRACT

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Key-words: location, allocation, coverage, heuristic, regret, scenarios.

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Abstract

This article addresses issues related to location and allocation problems. Herein, we intend to demonstrate the influence of congestion, through the random number generation, of such systems in final solutions. An algorithm is presented which, in addition to the GRASP, incorporates the Regret with the *p-minmax* method to evaluate the heuristic solution obtained with regard to its robustness for different scenarios. Taking as our point of departure the Facility Location Problem proposed by Balinski [27], an alternative perspective is added associating regret values to particular solutions.

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1. Introduction

Given the vital importance of providing services in contexts of ever increasing demand and the costs of setting up such allocation systems, location problems are of utmost importance both in our daily lives as well as in scientific circles. Typically, the performance of such services is evaluated by the number of customers in the queue and the waiting time ensuing from the arrival of a request at the center. Overall, what can be concluded is that these indicators are highly correlated with the number of centers providing services and their specific location. Examples of such services are medical systems, police operations, firefighters, roadside assistance services, amongst others.

The problem that researchers strive to resolve is related to the location of service centers and the respective allocation of demand to these centers. In attempting to characterize such systems and approximate them to reality, the location models have become so complex that obtaining results by complete enumeration has became more difficult, in great part as a result of the exponential growth of computing time.

The various models heretofore developed have been affected by the problem of complexity that hampers the process of finding a solution. Formulations have been constrained by simplifying assumptions, yet planners are often faced with deviations from reality that occur both in the public and private sectors. Technological advances have allowed the gradual development of more realistic formulations, given the possibility of finding solutions to complex models within acceptable computing times.

Despite the different formulations that have been proposed over time for the various problems related to location and allocation, it is commonly assumed that regions are represented by networks, a continuous space or a set of discrete nodal points.

This paper presents an algorithm which, besides the known GRASP - Greedy Randomized Adaptive Search Procedure - incorporates the *p-minmax* Regret method to evaluate the heuristic solution obtained with regard to its robustness for different scenarios. The use of these processes is undertaken in accordance with previous seminal works, such as that of Daskin et al [2]. It aims to integrate these new findings in order to explore new methodologies that improve or are better adapted to the circumstances of the cases studied.

We found that by varying the limits imposed on waiting times, maximum distance, demand processing capacity and size of network, it is possible to ascertain significant changes in the final solutions. The problem under study is the well-known Maximum Cover Location Problem developed by Church and Revelle [1] that incorporates an alternative model in which the server choice behavior does not only depend on the elapsed time from the node to the center, but includes also the waiting time for the service.

Our model and its various examples were tested using the random number generation model. In most cases different results were obtained by making it possible to confirm that the proposed formulation produces significant differences in terms of results. In general, the "tighter" the systems are, i.e., when the related distance limit between demand and center values are lower, the number of service centers to locate or the related waiting time limit, location decisions are generally more sensitive to pre-defined parameters for the model.

The classification of the location problems that concern us may be undertaken in accordance with the grouping suggested by Current et al. [3]: maximum distance models, "p-dispersion" problems and average or overall distance models. We can characterize this model as a maximum distance model given that maximum distances are explicitly considered as a pertinent factor as well as considerations relating to the maximum distance within which a facility must be located to provide the relevant service as well as the time limit within which the service can be provided. This is usually the case of schools, hospitals or police station locations where people generally expect to have access to a facility within acceptable limits outside of the original demand area.

A consumer is considered to be covered by a server – which is considered fixed and presents unlimited capacity concerning the possibility of processing the relevant service needs - if you have an installation within the pre-established distance limit. Where a service is provided by a facility located below this maximum, the service is considered appropriate or acceptable.

The formulation used in this work is based on previous studies, such as the works of Marianov and Serra [4] and those of Silva and Serra [5], particularly those relating to "Maximum Coverage Models" and the additional incorporation of results from the Queuing Theory.

The complexity associated with the model requires sophistication in our attempts to represent more elements or aspects of reality and the use of heuristic procedures in the search for solutions. Thus, in addition to GRASP, the algorithm contains a Regret component, based on the work of Daskin [2] that has produced acceptable results, both in terms of computing speed and, perhaps more importantly, in terms of approximation to the optimal solution.

This work highlights the importance, corroborated by real-life events, of considering the various forms of system congestion as a vital factor in terms of location and allocation decision-making.

2. Related Literature

Location Models have been studied for decades and have produced implementation solutions for the public and private sectors. Classic examples of such problems are those that were explored and pioneered by Hakimi [6, 7]. He postulated a network without imposed direction on arcs, where consumers are located only at the nodes (or demand point). Each demand point presents a certain percentage of demand or need for care. The 1-median problem that is herein considered is based on the location of a facility on a network. Its goal is that helping consumers minimize the average travel distance between the facility and the demand node. Hakimi showed that, among all of the nodes, there was at least an optimal location for the server, in this manner reducing a continuous search by a finite one. A similar result is also applied to (p-median) multi-median problem, in which various facilities could be located so as to minimize the average distance from the nearest facility to the consumers.

The types of problems addressed by Discrete Location Models are generally formulated with integer linear programming and can be solved using the algorithm known as "branch and bound". However, when applying heuristic processes, even the most basic location problems are classified as "NP-Hard" and require unacceptable computation times to find patterns of employment associated with more realistic situations.

The maximum coverage model proposed by Church and Revelle [1] seeks to limit the number of facilities to be located. The goal then becomes to locate a predetermined or budgeted number of facilities so that the requests directed to a certain server installation are maximized. This model does not require that all demand is covered.

However, problems of location have been studied for some time and the uncertainty that such allocation systems may exhibit lead researchers to explore alternative positions with regard to modeling demand, travel time or even costs associated with the chosen locations. Four basic approaches were formulated: approximations through a deterministic replacement, finding deterministic equivalents, probabilistic models with restraints, queuing systems spatially distributed and scenario planning.

Concerning this approach and its inclusion of allocation scenarios we highlight the seminal contribution of Sheppard [8] in 1974. His work demonstrated the importance of taking into account uncertain environments in location and allocation decision-making. These uncertain contexts are the object of this study inasmuch as they take into account randomness in the frequency of demand for certain services as a manifestation of uncertainty, the most important source of congestion of the systems in question.

Other researchers, such as Hogan and Revelle [9,10] working within a deterministic paradigm are addressing system congestion with alternative formulations for models that address the location and coverage with redundant coverage. Daskin [11, 12] suggest that probabilistic extensions can be added to the "Maximal Covering" problem, assuming that servers are busy according to a given probability. In this case the goal is to maximize the demand covered by the other servers that are not busy.

In such studies, an attempt is made to incorporate interactions occurring in queues with location models. There have been some interesting advances in recent decades, particularly with regard to emergency services. The pioneering work in this field of research was Larson´s [13] – “hypercube queuing” model. Its author proposes a system of stochastic service spatially distributed, with mobile servers scattered. Batta et al [14] used this model to demonstrate that the implicit assumption of independence of the server, as assumed by Daskin [11],[12], is often erroneous.

Fixed servers can "face" congestion. This is the case of health services and, in general, public services of any kind availing servers fixed. See it as an example Marianov and Serra [15].

Berman, Larson and Chiu [16] produced the first work that synthesized location theories and queuing theories. This work was expanded by Hakimi´s [6] 1-median problem, incorporating it in the context of research on queuing. In order to decide where to locate service centers due consideration must be granted to service times, travel times and delays resulting from queuing. Larson´s work [13] Hypercube Queuing Model has also served as a basis for such explorations.

In this context, the following objectives are proposed. Batta [17] considers the problem of locating a single service center in a network that operates as an M/G/1 queue where waiting calls are answered according to a class of queuing disciplines that rely solely on the information about the expected service time.

Brandeau and Chiu [18], also dedicated to congested systems, developed the model "Stochastic Queue Center Location" that aims to minimize the maximum response time to any consumer. For these authors the expected response time takes into account not only the waiting time until the server is on but also the travel time to the contact center.

In turn, the Revelle and Hogan [9] model previously mentioned, deals with congestion presenting a probabilistic version of the location problem. It is also in this line of reasoning that Marianov and ReVelle [19] presented the "Probabilistic Covering Problem with Queues". With an identical formulation as the one adopted in the "Probabilistic Covering Problem", a simple modification of Maximum Coverage Location Problem of Church and ReVelle [1], Marianov and Serra [4] introduce the "Maximum Coverage Location-Allocation Model with Queues" where the goal is to locate p service centers and affect these users so that the maximum population is covered.

3. The Location Problem with Limited Capacity Facilities

This section seeks to present an employed notation (3.1) and a proposed formulation (3.2), the Facility Location Problem and the application and measurement of a heuristic model is applied is order to produce the desired outcome.

3.1. Notation.

The following notation is defined as:

- $I = \{1,...,m\}$ represents a set of costumers with demand f_i , $i \in I$;
- $J = \{1,...,n\}$ is a subset of the network nodes which have operating facilities that will serve demand;
- For each location $j \in J$, the fix cost of operating a facility in j is f_j ;

- For each location $j \in J$, the respective capacity limit is C_j ;
- The cost, in terms of traveled distance, of allocating facility *j* to costumer *i* is d_{ij} .

3.2. Formulation.

The Location Problem with Limited Capacity Facilities is a transformation of the Facility Location Problem first introduced by Balinski [27]. The original model didn´t considered the capacity limit of delivering a service and now, with the inclusion of such feature, the minimum cost location pattern might not be capable of attending all demand.

This problem seeks to minimize the total cost of locating service delivery facilities, as well the minimization of all costs related to the transportations given the distances traveled. Its formulation is as follows:

$$Min \quad \sum_{j \in J} f_j X_j + \sum_{i \in I} \sum_{j \in J} d_{ij} Y_{ij} \qquad (3.2.1)$$

$$s.\quad t.$$

$$\sum_{j \in J} Y_{ij} = 1 \qquad \forall i \in I$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i \in I, \forall j \in J \qquad (3.2.2)$$

$$\sum_{i \in I} a_i Y_{ij} \leq C_j \qquad \forall j \in J \qquad (3.2.3$$

$$X_j \in \{0,1\}, \; Y_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J$$

4. Heuristic Procedure

We proceed to formulate, with Reeves [22], a heuristic method that attempts to find good solutions (i.e. near-optimal) in reasonable computing time. Therefore it is assumed that the solutions found by these heuristics methods do not always ensure an optimal outcome and may not present possible solutions.

The implemented algorithm adopts, at some point, the "Greedy Randomized Adaptive Search Procedure" (GRASP) developed by Feo and Resende [23] which comes up as an unfolding of one of the heuristics used to solve the 1st location models designed by Teitz and Bart [24]. This is interpreted as an approach through "exchange" or "replacement", moving the servers from their current positions to other non-used positions whilst keeping this new position whenever the objective value is improved. For additional information on the subject see Festa and Resende [25].

This methodology is also used in the proposed algorithm the heuristic *p-minmax* Regret that was developed by Daskin et al [2]. To understand the importance of this method and the issue it addresses in the attempt to deal with the unpredictability of demand, the problem is solved for different scenarios by random generation (where these represent different population levels) and different frequencies of demand for service.

The term Regret is associated with the notion of deviation or difference and it provides us with an "opportunity cost" when implementing the chosen location. Given the design of the service delivery system that we considered optimal for a given scenario, the Regret, based on population characteristics of the remaining scenarios and respective objective function value (covered population), returns the difference in case the locations were kept.

According to the author, there may be many objectives for this indicator. What is intended in the present work is to select the lowest from the largest differences and, by successive adaptations, minimize it. We can interpret this process as finding a location solution that, in the worst possible scenario (the one that presents the greatest Regret given the optimal solution) presents the lower deviation or "opportunity cost".

We now present the algorithm implemented in C + + in which we introduce the two heuristics discussed above - GRASP and *p-minmax* Regret. Beforehand, we indicate the notation used:

- j index of possible locations;
- i index of demand nodes;
- D_j list of potential location points for services ordered according to the total population;
- S solution
- $\overline{S}$ complementary solution
- C candidate set of points
- p number of services to locate
- n number of demand nodes
- *inc_j* overall rate of calls to potential service location j
- D_{ij} list of demand nodes within the distance limit counting from the potential location for service j

Thus, the allocation process starts by reading the distance file identifying and recognizing at this early stage the network of demand (nodal) points - all of which are potential locations – and the distances between them - our associated cost measured in terms of time units.

For each node representative of a population centre, the respective population is generated according to a Uniform distribution. Since this is a user-defined parameter, in order to subject the model to different conditions, the demand is then estimated based on a percentage of the population previously determined. These values – population and demand- are generated for each one of the (ns) scenarios and will be as many as the number of scenarios with which we intend to work.

When the network is characterized, i.e., the number of nodes, the distance between each node and the respective populations and demand, using the CPLEX optimization software, the model begins by solving the maximum coverage location problem for each scenario.

The optimal value of our objective function for each scenario is thus obtained, acting as a future reference when compared to the results obtained for other levels of population and when maintaining the optimum locations obtained. This solution includes: which nodes are located service centers, the allocation of demand points to the respective centers and the value for the objective function concerning the covered population.

Having simulated the scenarios, a matrix entitled Regret 1 (ns, ns) is activated. The diagonal of the matrices Regret 1 indicate the optimal values obtained in each of the simulated scenarios. Values outside this diagonal, i.e., adjacent to the optimal solution values, are obtained by calculating the objective function considering the characteristics of the other scenarios (populations and demands) but maintaining the location allocation patterns given to us by the CPLEX optimization software.

197182	201432	198357	196556	195265	194325	198562	196663	197701	194003
161050	190007	181250	182356	192123	179125	183459	168971	179157	189457
170892	167845	157853	164887	169741	190187	178112	156746	192454	189451
171313	175949	194848	164073	169787	199454	185464	177989	184188	188774
169787	191717	188772	169745	180869	183556	197010	191141	186311	188745
196787	192776	193741	183797	181579	171896	181235	182454	192478	179878
197121	195798	177131	193457	189743	172656	164681	165888	178432	182747
177336	181656	189743	187141	191778	194331	184556	175624	185655	189774
179487	173998	167131	189477	183454	199466	193473	195471	164635	167979
189741	185731	159887	169741	200874	190157	182486	189478	168635	182916

Figure 1. 10 scenarios Regret 1 matrix.

A note of caution since the calculation of the values adjacent to the "optimality diagonal" requires a possibility test so that, in the continuation of the algorithm, a necessarily workable initial solution is obtained. The possibility test of a value adjacent to the optimal solution should take into account:

- The distance limit (ldist) from demand node *i* allocated to service center in *j* é is taken into account;
- The limit for the waiting time (wlim) in *j* is respected; and
- Whenever negative valued waiting times are associated to a solution, in the matrix Regret 1 a zero will be shown.

When the objective value calculated with the standard locations and allocations for the other scenarios is not possible, under the conditions of possibility described above, the value in the Regret 1 matrix will be zero. Thus the associated solution is omitted of the rest of the process as an initial starting solution for GRASP since this zero value will not be considered in the choice of maximum deviation that is performed in the *p-minmax* process.

After the possibility test, based on Regret 1 matrix, the Regret 2 and Regret 3 matrixes are build up, where each one being a transformation of the one preceding, as described below:

Regret 2: each value of this matrix will be obtained by the difference between the goal of a given scenario and its respective optimal (reciprocated by CPLEX and contained in the diagonal of the Regret 1 matrix). This way, imperatively the diagonal of matrix of Regret 2 will contain only zeros. This procedure allows us to obtain the values of an "Absolute Regret".

0	4250	1175	-626	-1917	-2857	1380	-519	519	-3179
-28957	0	-8757	-7651	2116	-10882	-6548	-21036	-10850	-550
13039	9992	0	7034	11888	32334	20259	-1107	34601	31598
7240	11876	30775	0	5714	35381	21391	13916	20115	24701
-11082	10848	7903	-11124	0	2687	16141	10272	5442	7876
24891	20880	21845	11901	9683	0	9339	10558	20582	7982
32440	31117	12450	28776	25062	7975	0	1207	13751	18066
1712	6032	14119	11517	16154	18707	8932	0	10031	14150
14852	9363	2496	24842	18819	34831	28838	30836	0	3344
6825	2815	-23029	-13175	17958	7241	-430	6562	-14281	0

Figure 2. 10 scenarios Regret 2 matrix

Regret 3: the difference obtained according to the calculations of the Regret 2 matrix are now divided by the optimal value of reference for the scenario in question. Hence, we ascertain the value of "Relative Regret" associated with each scenario.

0,00000	0,02155	0,00596	0,00317	0,00972	0,01449	0,00700	0,00263	0,00263	0,01612
0,15240	0,00000	0,04609	0,04027	0,01114	0,05727	0,03446	0,11071	0,05710	0,00289
0,08260	0,06330	0,00000	0,04456	0,07531	0,20484	0,12834	0,00701	0,21920	0,20017
0,04413	0,07238	0,18757	0,00000	0,03483	0,21564	0,13037	0,08482	0,12260	0,15055
0,06127	0,05998	0,04369	0,06150	0,00000	0,01486	0,08924	0,05679	0,03009	0,04355
0,14480	0,12147	0,12708	0,06923	0,05633	0,00000	0,05433	0,06142	0,11974	0,04644
0,19699	0,18895	0,07560	0,17474	0,15219	0,04843	0,00000	0,00733	0,08350	0,10970
0,00975	0,03435	0,08039	0,06558	0,09198	0,10652	0,05086	0,00000	0,05712	0,08057
0,09021	0,05687	0,01516	0,15089	0,11431	0,21156	0,17516	0,18730	0,00000	0,02031
0,03731	0,01539	0,12590	0,07203	0,09818	0,03959	0,00235	0,03587	0,07807	0,00000

Figure 3. 10 scenarios Regret 3 matrix.

Basing our judgments on the Regret 3 matrix, we continue the application of our algorithm i.e., *p-minmax* heuristic as suggested by Daskin et al. [2]. Before the application of the latter matrix, the procedures are as follows:

- From the Regret 3 matrix, observing the values in line, we choose the one that presents the higher relative regret, in other words, the possible solution that departs furthest in percentage terms from the optimum control solution contained in Regret 1 diagonal obtained using the CPLEX;

- Subsequently, within all these maximum percentage deviations (relative Regrets) the minor is picked up with the intention of using it as an initial solution in the local search that follows on the GRASP heuristic.

The GRASP consists of two phases – construction phase and local search phase – and is an iterative process with reliable solution built independently at each iteration. Described below is a pseudo-code for the GRASP.

```
Procedure GRASP (Max_iterations, Seed)
    For k = 1 to Max_iterations do
            S←Greedy_Randomized_Construction(Seed, γ);
            S←Local_Search(Solution);
            Update_Solution(Solution, Best_Solution)
    Enddo
    end GRASP
```

Pseudo-Code 1: Pseudo-Code GRASP

From a general point of view, the process developed in this heuristic, after the selected initial solution as described above, follows like this:

- From the locations contained in the initial solution previously obtained, randomly one is chosen to be removed and replaced by another which, necessarily, must be on the RCL – *Restricted Candidate List*;
- The potential locations belonging to the RCL must meet the requirements of acceptability with regard, not only the distance limits imposed but

presenting a priori a demand frequency greater than or equal to *gamma* percent of the search node with the highest demand frequency;

- If part of the RCL it is temporarily accepted to be considered in the iterative process and, when replacing the previous location, switches its position regarding its allocation to demand points;
- When the initial solution is improved, new location and allocation patterns are accepted;
- Otherwise, the initial solution persists.

These are again activated in the Regret 1, Regret 2 and Regret 3 matrixes based on the values obtained in the local search now held. In this process, the first matrix goes again through the possibility test already described. This process is repeated for a predefined number of iterations.

We now explain the use of pseudo-code for the two phases of that process, the GRASP heuristic. The construction phase, which will return an initial solution at each iteration, is invoked and is a function from the root in the random number generator and of the gamma parameter that defines what solutions will be included in RCL - *Restricted Candidate List*, the list containing the best solutions.

The development of the $Greedy\_Randomized\_Construction(Seed)$ is now described:

procedure *Greedy Randomized Construction* (Seed,γ)
{sort candidate sites by decreasing order of population}
  $D_j \leftarrow Sort\_Candidate\_Sites(\ population\ );$
  {initialize solution set}
  $S := \{\ \};$
  $\overline{S} := C;$
  {while solution is not a complete solution}
  while $|S| \neq p$ **do**
    {loop over all candidate sites not in the solution list}
  For j=1 to $|\overline{S}|$ **do**
      {initialize parameters}
      {restrict demand points list to the standard covering distance to site j}

$D_{ij} \leftarrow \{i \in D, d_{ij} \le d\}$

{sort demand points by increasing distance to site j}

$D_{ij} \leftarrow Sort\_Demand\_Points(distance);$

{loop over demand points in set Dij}

For i=1 to $|D_{ij}|$ **do**

{sum frequencies at each demand point if waiting time limit is not reached}

If ($W\_j < \tau^{J}$ and ρ_j<1) **do**

inc_ j : = inc_ j+ f_i ;

actualize w_j;

actualize ρ_j*;*

Endif

Enddo

{construct the restricted candidate list}

$c^{max} := max\{inc\_j\};$

$RCL \leftarrow \{j \in \overline{S}, inc\_j \ge \gamma c^{max}\};$

{select randomly one site from the RCL}

$j^* \leftarrow Random\_Select(RCL);$

$S := S \cup \{j^*\};$

$\overline{S} := \overline{S} \setminus \{j^*\};$

{take the demand points allocated to j* out of the demand points list}

For i=1 to $|D_{ij^*}|$ **do**

D := D$\setminus\{i \in D_{ij^*}\};$

Enddo

Enddo

end *Greedy Randomized Construction*

Pseudo-Code 2: Construction Phase Pseudo-Code

The proposed algorithm starts by choosing candidate nodes according to their respective demands/populations**.** We postulated in our example that all demand nodes are also potential service location points. Another possibility would be to only consider a subset of demand nodes from the $D_{j.}$ list.

Thus, starting with the first node from the candidate list, the closest demand nodes are affected to it until de coverage limit is reached. Here the coverage limit can be determined through the utilization coefficient or the imposed limit for the waiting time.

Total demand affected to each of the potential sites *j* is called *incoming call rate*. The *incoming call rate* works as a "*greedy*" function of the algorithm and can be defined as a weighing of the demand nodes no yet covered but that will do if location *j* was chosen to have a server facility.

It is included on the RCL - Restricted Candidate List (sub-set of best solutions) the candidate nodes with a total *incoming call rate* greater to or equal to *gamma* per cent of the *incoming call rate* indexed to the potential location with higher value.

In the GRASP, the *gamma* parameter is established beforehand. (for instance, if *gamma* equals 0.8, we therefore mean that we include on the list containing the best solutions - *Restricted Candidate List* – all the potential locations with a total incoming call rate greater than 80% of the highest value between all incoming call rates).

Note that in the "*greedy*" heuristic, as suggested by Marianov and Serra [4], the choice would always be to locate a center at the node with the highest sum of incoming call rates, i.e., γ=1.

At each iteration, we choose randomly from among the candidate locations with the highest incoming call rate (i.e., the ones included on *Restricted Candidate List*) the *p* locations for servers.

procedure *Local_Search* (Solution, Best_Solution)
obj_best : = *obj(S)*;
{loop over sites in the solution}

for all $j_1 \in S$ **do**
$S := S \setminus \{j_1\}$;
{loop over sites not in the solution}
for all $j_2 \in \overline{S}$ **do**
evaluate $obj(S \cup \{j_2\})$;
if *obj_best*< $obj(S \cup \{j_2\})$ **do**
$S := S \cup \{j_2\}$:
obj_best : = $obj(S \cup \{j_2\})$;
else
$S := S \cup \{j_1\}$:

```
            endif
            enddo
    Enddo
    end Local_Search
```

Pseudo-Code 3: Local Search Phase Pseudo-Code

At the local search phase, for each centre, we un-allocate its assigned demand and move it to all the potential locations not yet used, repeating at each time the steps 9 to 20 from the *Greedy Randomized Construction* procedure, aiming to evaluate the objective at hand. If any of the locations reciprocate a better objective value, we maintain the service centre at that node; otherwise, we keep it in the original location (see Pseudo-Code 3). We repeat the procedure until it is not possible to improve the initial solution or the limit of iterations is reached.

In a user-defined environment, the algorithm requires modification, both in its construction phase and in the local search in order to insure the closest possible allocation. The proposed algorithm penalizes the final objective whenever an unreliable solution is obtained. When a reliable solution is obtained, this set of locations are considered potential sites for the placement of service centers. Otherwise, we consider this set of locations an initial solution and not a potential service location penalizing the objective with a large negative value M. This will match the following objective evaluation procedure:

```
procedure evaluate_objective (S)

    Allocate each demand point to its closest center location;
    Evaluate W_j and ρ_j;
    obj(S):=0;
    If (W_j < τ and ρ_j<1) do

            For j=1 to p do
                For i=1 to n do
                    If (i is allocated to j) do
                        obj(S):=obj(S) + f_i;
                    endif;
                enddo;
            enddo;
```

```
Else
        obj(S):=M;
end evaluate_objective;
```

Pseudo-Code 4: Objective Evaluation Pseudo-Code

During the Local Search phase, for each center at a time, we un-allocate assigned demands and move them to unused potential locations. We always affect a demand node to the nearest potential location and check the possible waiting time limit. If a solution is not possible, the objective is penalized with a very high negative value M. Whenever new allocations are found and a more efficient objective is identified we maintain that center in that location. Otherwise, the starting location is kept. This procedure is repeated until, when comparing with the previous, no better solution is found.

5. Computational Experience

In order to observe the difference between the results of the heuristic solution and the initial results that have been obtained, which will serve as a starting point (and comparison) for the GRASP Local Search, randomly generated problematic situations in the demand network model are proposed. The size of this network will be variable and each center and a particular demand frequency is assigned (need for service / care). Furthermore, the characteristics of this process will also be amended with regard to the number of available nodes and demand centers.

The Location Problem with Limited Capacity Facilities in study, as well as the evaluated heuristic procedure, is based on a network of demand nodes that also stand for possible facilities location The size of this network will be variable and each node will have a demand frequency associated (need of attendance).

Networks of 25, 40, 50 nodes will be generated and to each of this nodes a demand frequency will also show accordingly to an Uniform distribution [800;1800]. Retrieving the population from this distribution, 1% is considered as demand frequency.

Bear in mind that for each scenario and specific network the distance between nodes is constant – changes only show on the size of the network and the verified demand. The distance between nodes is obtained using a distance matrix common to all scenarios and networks in use.

A summary of the characteristics and parameters of the worked data are presented below in table 5.1.

Cases	Number of Nodes	Capacity Limits	Number	
			Scenarios	Iterations
1	50	1000, 2000 e 3000	10	500, 1000 e 2000
2	40	1000, 2000 e 3000	10	500, 1000 e 2000
3	25	1000, 2000 e 3000	10	500, 1000 e 2000

Table 5.1. Characteristics and parameters of the worked data.

The algorithm in this study was implemented on a computer with 2.50 GHz Pentium Dual-Core processor with 1920 MB of memory and using the compiler C++ *Microsoft Visual Studio 2005* which integrates, for the resolution of the problems proposed the optimization software CPLEX *Optimization Studio 12.2*.

We attempted to analyze the results obtained whilst checking if the location-allocation patterns that were produced by the heuristics method showed any differences when it was compared with the location-allocation pattern from the initial solution in the GRASP Local Search phase. The average value of the percentage deviation associated with the solution is also obtained and, in a greedy fashion, we tried to low it as much as possible.

5.1. Changing the Network Size

Not having a predetermined parameter for the number of centers to locate, this value contained in the final solution that minimizes total cost is also a matter of discussion, besides all the other indicators shown on Table 5.2.

Before an increasing number of network nodes, it is possible to verify that the average processing time increases. The same behavior is also present when increasing the iterations number. If we only consider the network size, the increase in the processing time is exponential.

As we could expect, also with the increase in the size of the network used, the number of centers to locate increase and, generally, the same goes for the Minimum Relative Regret value. In this case, if we intend to compare the initial solution with the heuristics solution, we can point cases where the initial and final locations match, although with an erratic pattern.

Regarding the matching solutions, the increase in the iterations number does not allow also to generalize a behavioral pattern for this indicator. Still, we can verify some stability in the behavior of the Minimum Relative Regret. A call of attention for an exception that shows a decrease in this indicator before a "tighter" system with lower capacity limits and higher number of network nodes.

		25 Nodes Network			**40 Nodes Network**			**50 Nodes Network**		
	Capacity	***1000***	***2000***	***3000***	***1000***	***2000***	***3000***	***1000***	***2000***	***3000***
500	*Average Processing Time*	3.399	0,738	0.756	68.26	4.115	1.784	119.41	5.611	4.778

	% Matching Locations	0%	10%	10%	0%	0%	16%	0%	0%	2%
	Average Regret	1.417	4.970	0.831	1.563	1.201	1.088	1.399	1.112	1.290
	Located Centres	2	1	1	3	2	1	3	2	2
1000 iterations	*Average Processing Time*	4.101	1.042	1.001	72.379	5.471	2.443	137.42	7.138	6.532
	% Matching Locations	6%	2%	12%	0%	4%	16%	0%	4%	10%
	Average Regret	1.443	2.809	0.833	1.579	1.299	1.102	1.601	1.191	1.282
	Located Centres	2	1	1	3	2	2	3	2	2
2000 iterations	*Average Processing Time*	5.798	1.453	1.570	69.357	8.521	3.461	233.82	12.10	11,839
	% Matching Locations	0%	4%	8%	0%	6%	8%	0%	2%	6%
	Average Regret	1.423	0.832	0.844	1.565	1.117	1.043	1.599	1.219	1.259
	Located Centres	2	1	1	3	2	1	3	2	2

Table 5.2. Simulation results for 100 examples and 10 scenarios; average processing time measured in seconds.

5.2. Capacity Limits

Regarding this indicator, which is directly related with the facilities/servers ability or availability of providing the service, it is possible to conclude that its increase allows a less tight system. This idea is supported by the results on the number of centers to locate; this value is smaller when the capacity limits are higher.

As we would expect, also for the increase of the capacity limits, it´s possible to assume that the average processing time decreases.

For last, when assuming facilities with higher capacity limits, we can see that the Regret values are smaller but the percentage of matching locations (initial and final locations). We can see this as a sign that for simpler problems (less constrained systems) allow easily the heuristic to improve the initial solution.

5.3 Conclusions.

Increasing the limit for the waiting time (wlim) as well as increasing the distance limit (ldist) between demand node and service centers, lead us to believe that these heuristic produces solutions that improve the optimal solutions defined originally.

It is important here to analyze not only the percentage of matching locations, but also the values of Minimum Relative Regret. When decreasing, these indicate that possible solutions were found which, in turn, deviate less from the starting solution for the heuristic given by the Regret method for the sets of simulated scenarios.

It is patent that for the cases where the system is more constrained, although the increase noted on the processing time, the heuristic procedure produces solutions that are different from the initial solution but with higher values of Relative Regret.

The “tightness” caused on the system is mainly due to the facilities characteristics concerning the service delivery capacities.

6. General Conclusions

When analyzing the literature that addresses location and allocation problems we found that there is a trend of including in this type of models the effects of queues. This may happen for the following reason: when considering a certain demand for a service it appears that this demand is random and is one of the sources of system congestion.

This type of problem can arise both in the public or private sector, involving different types of formulations as maximum distance models and total/average distance models. The methodology associated with each specific problem should be carefully examined and the results that are obtained should be compared with others produced by other testing models.

In addition to the "*Greedy Randomized Adaptive Search Procedure*" (or GRASP), we also used the heuristic method developed the *p-minmax* Regret (Minimum Relative Regret) proposed by Daskin [2]. The use of these models processes is undertaken in accordance with previous research and aims at its integration into the current paradigm in order to explore new methodologies that enhance or better adapt to the circumstances of the cases studied. Thus, the developed models can be considered adequate to address the type of issue proposed in the current work. Varying limits, in terms of waiting times and maximum distance, limits of demand processing capabilities and network sizes, produces significant changes in the final solutions.

There are numerous real-life situations in which the waiting time is an important, oftentimes vital, factor when considering the duration of service rendered (time or distance traveled plus the waiting time). In such cases, taking into account the determination of a location pattern, the waiting time is to be regarded as absolutely essential in the respective modeling of the system. It may also interfere with the processing time of the number of centers to locate at a certain network scale, as well as the capacity of facilities in providing the sought service.

The proposed meta-heuristic reciprocates near optimal results demonstrating significant savings in computation time. Given the initial data, was with the use of simulation that in the present study the demand levels associated with each population data were obtained.

Regarding the application of Greedy heuristics to these formulations, these show acceptable behavior to the extent that the near-optimal solutions are sensitive to the worked examples and problematic situations proposed in each case.

On the other hand, the theory and the numerical examples obtained suggest that the solutions become less sensitive to the model parameters as the system becomes less busy. In the case that, for instance, the distance limit between the demand node and the service facility is smaller or when there are less service centers to locate, one can assume that henceforth there will be greater congestion associated with the model. These are cases where the heuristic has given results not identical to the initial solutions used as input in our algorithmic formulation.

Regarding the computational experiment conducted, a few final remarks. The tested models and their various examples were obtained using random number generation. In many cases, different results were obtained, but there are others where the proposed formulation does not produce significant differences in the results. As already mentioned, generally speaking, the "tightest" systems are those wherein the distance limit is smaller and the number of service centers to be located is smaller. In such cases where there are inferior processing capabilities, location decisions are more sensitive to pre-defined parameters for the model.

Concluding, in this paper we simulated populations and their respective demand frequencies. Furthermore, we demonstrated the real-life paramount importance of system congestion in its various forms as a determining factor in location and allocation decisions.

References.

1. Church, R. and C.R. VELLE, *The maximal covering location problem.* Papers in Regional Science, 1974. **32**(1): p. 101-118.
2. Daskin, M.S., S.M. Hesse, and C.S. Revelle, *α-reliable p-minimax regret: A new model for strategic facility location modeling.* Location Science, 1997. **5**(4): p. 227-246.
3. Current, J., M. Daskin, and D. Schilling, *3 D iscrete N etw ork Location M odels.* 2002.
4. Marianov, V. and D. Serra, *Probabilistic, maximal covering location-allocation models for congested systems.* Journal of Regional Science, 1998. **38**(3): p. 401-424.
5. Silva, F. and D. Serra, *Locating emergency services with different priorities: the priority queuing covering location problem.* Journal of the Operational Research Society, 2008. **59**(9): p. 1229-1238.
6. Hakimi, S.L., *Optimum locations of switching centers and the absolute centers and medians of a graph.* Operations Research, 1964: p. 450-459.
7. Hakimi, S., *Optimum distribution of switching centers in a communication network and some related graph theoretic problems.* Operations Research, 1965. **13**(3): p. 462-475.
8. Sheppard, E., *A conceptual framework for dynamic location-allocation analysis.* Environment and Planning A, 1974. **6**(5): p. 547-564.
9. ReVelle, C. and K. Hogan, *The maximum availability location problem.* Transportation Science, 1989. **23**(3): p. 192-200.
10. Hogan, K. and C. Revelle, *Concepts and applications of backup coverage.* Management Science, 1986. **32**(11): p. 1434-1444.
11. Daskin, M.S., *APPLICATION OF AN EXPECTED COVERING MODEL TO EMERGENCY MEDICAL SERVICE SYSTEM DESIGN*.* Decision Sciences, 1982. **13**(3): p. 416-439.
12. Daskin, M.S., *A Maximum Expected Covering Location Model - Formulation, Properties and Heuristic Solution.* Transportation Science, 1983. **17**(1): p. 48-70.
13. Larson, R.C., *A hypercube queuing model for facility location and redistricting in urban emergency services.* Computers & Operations Research, 1974. **1**(1): p. 67-95.
14. Batta, R., *A queueing-location model with expected service time dependent queueing disciplines.* European Journal of Operational Research, 1989. **39**(2): p. 192-205.
15. Marianov, V. and D. Serra, *4 Location Problems in the Public Sector.* 2002.
16. Chiu, S.S., O. Berman, and R.C. Larson, *Locating a mobile server queueing facility on a tree network.* Management Science, 1985: p. 764-772.
17. Batta, R. and O. Berman, *A Location Model for a Facility Operating as a M/G/K Queue.* Networks, 1989. **19**(6): p. 717-728.
18. Brandeau, M.L. and S.S. Chiu, *A center location problem with congestion.* Annals of Operations Research, 1992. **40**(1): p. 17-32.
19. Marianov, V. and C. Revelle, *The queuing probabilistic location set covering problem and some extensions.* Socio-Economic Planning Sciences, 1994. **28**(3): p. 167-178.
20. Kleinrock, L., *Queueing Systems. Volume 1: Theory.* 1975.

21. Berman, O. and D. Krass, *11 Facility Location Problems with Stochastic Demands and Congestion.* Facility location: Applications and theory, 2002: p. 329.
22. Rayward-Smith, V.J., et al., *Modern heuristic search methods*1996: Wiley New York.
23. Feo, T.A. and M.G.C. Resende, *Greedy randomized adaptive search procedures.* Journal of global optimization, 1995. **6**(2): p. 109-133.
24. Teitz, M.B. and P. Bart, *Heuristic methods for estimating the generalized vertex median of a weighted graph.* Operations Research, 1968. **16**(5): p. 955-961.
25. Festa, P. and M.G.C. Resende, *An annotated bibliography of GRASP.* Operations Research Letters, 2004. **8**: p. 67-71.
27. Balinski, M.L., *Integer programming: methods, uses, computations.* Management Science, 1965. **12**(3): p. 253-313.