On the submultiplicative constant of an algebra

Khang V. Nguyen*

July 16, 2018

Abstract

If $\mathbb{A}$ is a finite dimensional commutative associative real algebra and $\|\|$ denotes a given norm on $\mathbb{A}$ then it was shown in [1] that there exists a constant m such that $\|xy\| \leq m\|x\|\|y\|$. However, the m given in [1] is not sharp. We prove there exists an optimal smallest choice for m which we denote as $m_{\mathbb{A}}$. Furthermore, we prove if $\mathbb{A}$ is the real group algebra of the cyclic group of order n given the usual Euclidean norm then $m_{\mathbb{A}} = \sqrt{n}$. We also find the submulticative constant for the complicated numbers $\mathbb{A} = \{x_o + \cdots + x_{n-1}i^{n-1} \mid x_i \in \mathbb{R},\ i^n = -1\}$ with the Eucildean 2-norm. Additional results concerning numbers generated from nilpotent elements are also discussed. Applications of our theorem to the study of power series in $\mathbb{A}$-variables are briefly discussed.

References

[1] J. S. Cook, *Introduction to $\mathcal{A}$-Calculus*, https://arxiv.org/abs/1708.04135.

*Liberty University