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Top partners and Higgs boson production

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The Higgs boson is produced at the LHC through gluon fusion at roughly the Standard Model rate. New colored fermions, which can contribute to $gg \to h$, must have vectorlike interactions in order not to be in conflict with the experimentally measured rate. We examine the size of the corrections to single and double Higgs production from heavy vectorlike fermions in $SU(2)_L$ singlets and doublets and search for regions of parameter space where double Higgs production is enhanced relative to the Standard Model prediction. We compare production rates and distributions for double Higgs production from gluon fusion using an exact calculation, the low energy theorem (LET), where the top quark and the heavy vectorlike fermions are taken to be infinitely massive, and an effective theory (EFT) where top mass effects are included exactly and the effects of the heavy fermions are included to $\mathcal{O}(\frac{1}{M_X^2})$. Unlike the LET, the EFT gives an extremely accurate description of the kinematic distributions for double Higgs production.

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I. INTRODUCTION

Having discovered a particle with the generic properties of the Standard Model Higgs boson, the next important step is to determine what, if any, deviations from the standard picture are allowed by the data. The observed production and decay modes of the Higgs boson are within ~20% of the expectation for a weakly coupled Higgs particle [1,2] and so the possibilities for new physics in the Higgs sector are highly constrained [3]. A convenient framework to examine possible new high scale physics is the language of effective field theories, where the theory is constructed to reduce to the Standard Model at the electroweak scale, but new interactions are allowed at higher scales. We study an extension of the Standard Model where there are new massive quarks which are allowed to interact with the Standard Model particles, and thus potentially modify Higgs production and decay rates. Heavy fermions occur in many beyond the Standard Model (BSM) scenarios, in particular, little Higgs models [4–7] and composite Higgs models [8–12] in which the Higgs is strongly interacting at high scales. Direct searches for the heavy fermions have been extensively studied in the literature [13–26]. We consider models with both charge $\frac{2}{3}$ and $-\frac{1}{3}$ heavy quarks, but where there are no additional Higgs bosons beyond the Standard Model $SU(2)_L$ doublet.

New heavy colored fermions which couple to the Higgs boson cannot occur in chiral multiplets since they would give large contributions to the rate for Higgs production from gluon fusion [27,28]. A single $SU(2)_L$ heavy quark doublet with corresponding right-handed heavy quark singlets would increase the gluon fusion Higgs production rate by a factor of ~9, which is definitively excluded. Vectorlike quarks, on the other hand, decouple at high energy and can be accommodated both by precision electroweak data and by Higgs production measurements. Models with a single multiplet of new vectorlike fermions have been studied extensively in the context of single and double Higgs production from gluon fusion [4,29–36]. The rates for both single and double Higgs production in this class of models are close to those of the Standard Model, and the gluon fusion processes are insensitive to the top partner masses and couplings. This general feature is a result of the structure of the quark mass matrix and can be proven using the Higgs low energy theorems (LETs) [12,37,38].

We study more complicated models with several multiplets of vectorlike quarks in both $SU(2)_L$ doublet and singlet representations [39], which are allowed to mix with the Standard Model quarks and with each other. Higgs production from gluon fusion can be significantly altered from the Standard Model prediction when this mixing is allowed [32,35,38]. We explore the possibility of having the double Higgs production rate be strongly enhanced or suppressed relative to the Standard Model, while keeping the single Higgs rate close to that of the Standard Model. Models with multiple representations of vectorlike fermions have also been considered in the context of flavor, where they have been used to generate a hierarchy of masses for the Standard Model fermions [40,41].

Effective field theory (EFT) techniques can be used to integrate out the effects of heavy fermions. Low energy physics is then described by an effective Lagrangian,

$$L_{\text{eff}} = L_{\text{SM}} + \sum_i \frac{f_i O_i}{\Lambda^2} + \cdots, \tag{1}$$

where O_i are the dimension-6 operators corresponding to new physics at the scale Λ. These operators have been catalogued under various assumptions [11,42,43], and in this paper, we consider only those operators affecting the gluon fusion production of Higgs bosons. We calculate the contributions to the f_i obtained by integrating out heavy vectorlike quarks in $SU(2)_L$ singlet and doublet representations, using the equations of motion. The new physics arising from the heavy vectorlike quarks yields corrections to the Standard Model $SU(2)_L \times U(1)$ gauge couplings and to the Yukawa couplings of the light fermions.

For arbitrary fermion mass matrices, we compute both single and double Higgs production from gluon fusion. As a by-product of our calculation, we compare rates found by diagonalizing the mass matrices exactly, from the effective theory of Eq. (1) which contains terms of $\mathcal{O}(\frac{m_t^2}{\Lambda^2})$, and from the low energy theorems, where $m_t \to \infty$ along with the new vectorlike quarks, in order to establish the numerical accuracy of the various approximations.

Section II contains a brief description of the class of models studied here. A description of single and double Higgs production using the LET description and the EFT with top and bottom quark mass effects included is given in Sec. III. Analytic results in an example with small mixing between the Standard Model third generation quarks and the heavy quarks are given in Sec. IV in order to give an intuitive understanding of the new physics resulting from integrating out the heavy vectorlike fermions, while Sec. V summarizes limits from precision electroweak measurements. Our major results are contained in Sec. VI, where total rates and distributions for double Higgs production are given in the full theory, the LET, and the EFT. Finally, some conclusions are given in Sec. VII.

II. THE MODEL

We consider models where in addition to the Standard Model field content, there are two vectorlike $SU(2)_L$ singlets, U and D, and one vectorlike $SU(2)_L$ doublet, Q, with hypercharges $Y = 4/3$, $-2/3$, and $1/3$, respectively. We only allow mixing between the new fermions and the third generation Standard Model quarks since the interactions of the two light generations of quarks are highly constrained. The Standard Model third generation fermions are

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, t_R, b_R, \tag{2}$$

and the heavy vectorlike fermions are

$$Q = \begin{pmatrix} T \\ B \end{pmatrix}, U, D, \tag{3}$$

where the left- and right-handed components have identical transformation properties under $SU(2)_L \times U(1)$, allowing for Dirac mass terms. Finally, the Higgs doublet takes its usual form in unitary gauge after electroweak symmetry breaking,

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \tag{4}$$

where $v = 246$ GeV is the Higgs vacuum expectation value and h is the Higgs boson. The Standard Model Lagrangian involving the third generation fermions and the Higgs boson is

$$L_{\rm SM} = i\bar{q}_L \not{D} q_L + i\bar{t}_R \not{D} t_R + i\bar{b}_R \not{D} b_R - (\lambda_t \bar{q}_L \tilde{H} t_R + \lambda_b \bar{q}_L H b_R + \text{H.c.}) + |D_\mu H|^2 - V(H), \tag{5}$$

where $\tilde{H} \equiv i\sigma_2 H^*$, $V(H)$ is the Higgs potential, $D_\mu = (\partial_\mu - i\frac{g}{2} T \cdot W_\mu - i\frac{g'}{2} Y B_\mu - i g_s t \cdot G_\mu)$, $T^a = \sigma^a$ for $SU(2)_L$ doublets, $T^a = 0$ for $SU(2)_L$ singlets, σ^a are the Pauli matrices, for the quarks t are the $SU(3)_C$ fundamental representation matrices, for the Higgs $t = 0$, and $Q = (Y + T^3)/2$ is the electric charge operator. The classical equations of motion corresponding to Eq. (5) are [42]

$$\begin{aligned} i\not{D} q_L &= \lambda_t \tilde{H} t_R + \lambda_b H b_R \\ i\not{D} t_R &= \lambda_t \tilde{H}^\dagger q_L \\ i\not{D} b_R &= \lambda_b H^\dagger q_L. \end{aligned} \tag{6}$$

The most general Lagrangian coupling the third generation quarks and the new fermions is L_{NP},

$$\begin{aligned} L_{NP} &\equiv L'_M + L'_{\rm KE} + L'_Y \\ L'_M &= -M\bar{Q}Q - M_U \bar{U}U - M_D \bar{D}D \\ L'_{\rm KE} &= \bar{Q}(i\not{D})Q + \bar{U}(i\not{D})U + \bar{D}(i\not{D})D \\ L'_Y &= -\{\lambda_1 \bar{Q}_L \tilde{H} U_R + \lambda_2 \bar{Q}_L H D_R + \lambda_3 \bar{Q}_R \tilde{H} U_L \\ &\quad + M_4 \bar{q}_L Q_R + M_5 \bar{U}_L t_R + M_6 \bar{D}_L b_R \\ &\quad + \lambda_7 \bar{q}_L \tilde{H} U_R + \lambda_8 \bar{q}_L H D_R + \lambda_9 \bar{Q}_L \tilde{H} t_R \\ &\quad + \lambda_{10} \bar{Q}_L H b_R + \lambda_{11} \bar{Q}_R H D_L + \text{H.c.}\}. \end{aligned} \tag{7}$$

The much studied cases where the Standard Model top quark mixes with only a singlet or doublet vectorlike fermion [29–31,33,44–48] can be obtained from this study, as can the composite model case where the Standard Model quarks do not couple to the Higgs doublet ($\lambda_t = \lambda_b = \lambda_7 = \lambda_9 = \lambda_{10} = 0$). We will consider various mass hierarchies in the following sections.

The mass and Yukawa interactions can be written as

$$-L_{Y'} = \bar{\chi}_L^t M^{(t)}(h) \chi_R^t + \bar{\chi}_L^b M^{(b)}(h) \chi_R^b + \text{H.c.}, \tag{8}$$

where $\chi_{L,R}^t \equiv (t, T, U)_{L,R}$, $\chi_{L,R}^b \equiv (b, B, D)_{L,R}$, and the Higgs-dependent fermion mass matrices are

$$M^{(t)}(h) = \begin{pmatrix} \lambda_t(\frac{h+v}{\sqrt{2}}) & M_4 & \lambda_7(\frac{h+v}{\sqrt{2}}) \\ \lambda_9(\frac{h+v}{\sqrt{2}}) & M & \lambda_1(\frac{h+v}{\sqrt{2}}) \\ M_5 & \lambda_3(\frac{h+v}{\sqrt{2}}) & M_U \end{pmatrix},$$

$$M^{(b)}(h) = \begin{pmatrix} \lambda_b(\frac{h+v}{\sqrt{2}}) & M_4 & \lambda_8(\frac{h+v}{\sqrt{2}}) \\ \lambda_{10}(\frac{h+v}{\sqrt{2}}) & M & \lambda_2(\frac{h+v}{\sqrt{2}}) \\ M_6 & \lambda_{11}(\frac{h+v}{\sqrt{2}}) & M_D \end{pmatrix}, \quad (9)$$

where typically $\lambda_i \sim \mathcal{O}(1)$. The mass eigenstate fields, $\psi^t \equiv (T_1, T_2, T_3)$ and $\psi^b \equiv (B_1, B_2, B_3)$, are found by means of bi-unitary transformations,

$$\begin{aligned} -L_{Y'} &= \bar{\chi}^t_L (V^{t\dagger}_L V^t_L) M^{(t)}(h) (V^{t\dagger}_R V^t_R) \chi^t_R \\ &\quad + \bar{\chi}^b_L (V^{b\dagger}_L V^b_L) M^{(b)}(h) (V^{b\dagger}_R V^b_R) \chi^b_R + \text{H.c.} \\ &= \bar{\psi}^t_L M^t_{\text{diag}} \psi^t_R + \bar{\psi}^b_L M^b_{\text{diag}} \psi^b_R + \bar{\psi}^t_L \mathcal{Y}^t \psi^t_R h \\ &\quad + \bar{\psi}^b_L \mathcal{Y}^b \psi^b_R h + \text{H.c.}, \end{aligned} \quad (10)$$

and (T_1, B_1) are the Standard Model third generation quarks. The diagonal mass matrices can be written

$$\begin{aligned} M^t_{\text{diag}} &= V^t_L M^{(t)}(0) V^{t\dagger}_R \\ (M^t_{\text{diag}})^2 &= V^t_L M^{(t)}(0) M^{(t)}(0)^\dagger V^{t\dagger}_L \\ &= V^t_R M^{(t)}(0)^\dagger M^{(t)}(0) V^{t\dagger}_R, \end{aligned} \quad (11)$$

where we have set $h = 0$ and the Yukawa matrix is

$$\mathcal{Y}^t h = V^t_L (M^{(t)}(h) - M^{(t)}(0)) V^{t\dagger}_R \quad (12)$$

and similarly in the b sector.

The couplings to the W contain both left- and right-handed contributions,

$$\begin{aligned} L_W &= \frac{g}{\sqrt{2}} \left(\bar{\chi}^{t,2}_R \gamma_\mu \chi^{b,2}_R + \sum_{i=1,2} \bar{\chi}^{t,i}_L \gamma_\mu \chi^{b,i}_L \right) W^{+\mu} + \text{H.c.} \\ &= \frac{g}{\sqrt{2}} \sum_{j,k=1,2,3} (\bar{\psi}^{t,j}_L (U_L)_{jk} \gamma_\mu \psi^{b,k}_L + \bar{\psi}^{t,j}_R (U_R)_{jk} \gamma_\mu \psi^{b,k}_R) W^{+\mu} \\ &\quad + \text{H.c.}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} (U_L)_{jk} &= \sum_{i=1,2} (V^t_L)_{ji} (V^{b\dagger}_L)_{ik} \\ (U_R)_{jk} &= (V^t_R)_{j2} (V^{b\dagger}_R)_{2k}. \end{aligned} \quad (14)$$

Finally, the couplings to the Z are

$$\begin{aligned} L_Z &= \frac{g}{2c_W} \sum_{j,k=1,2,3} \{ \bar{\psi}^t_{L,j} (X^t_L)_{jk} \gamma_\mu \psi^t_{L,k} + \bar{\psi}^t_{R,j} (X^t_R)_{jk} \gamma_\mu \psi^t_{R,k} \\ &\quad - \bar{\psi}^b_{L,j} (X^b_L)_{jk} \gamma_\mu \psi^b_{L,k} - \bar{\psi}^b_{R,j} (X^b_R)_{jk} \gamma_\mu \psi^b_{R,k} \} Z^\mu \\ &\quad - \frac{g}{2c_W} (2 s^2_W) J^\mu_{\text{EM}} Z_\mu, \end{aligned} \quad (15)$$

where $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, θ_W is the weak mixing angle,

$$\begin{aligned} (X^t_L)_{jk} &= \sum_{i=1,2} (V^t_L)_{ji} (V^{t\dagger}_L)_{ik} \\ (X^t_R)_{jk} &= (V^t_R)_{j2} (V^{t\dagger}_R)_{2k} \\ (X^b_L)_{jk} &= \sum_{i=1,2} (V^b_L)_{ji} (V^{b\dagger}_L)_{ik} \\ (X^b_R)_{jk} &= (V^b_R)_{j2} (V^{b\dagger}_R)_{2k}, \end{aligned} \quad (16)$$

and J^μ_{EM} is the usual electromagnetic current,

$$J^\mu_{\text{EM}} = Q_t [\bar{\psi}^t_L \gamma^\mu \psi^t_L + \bar{\psi}^t_R \gamma^\mu \psi^t_R] + Q_b [\bar{\psi}^b_L \gamma^\mu \psi^b_L + \bar{\psi}^b_R \gamma^\mu \psi^b_R]. \quad (17)$$

The Z couplings contain flavor nondiagonal contributions due to the off diagonal terms in $X^{t,b}_{L,R}$. It is straightforward to apply the results of Eqs. (13) and (15) to find the gauge boson couplings in a specific model.

III. EFFECTIVE THEORY RESULTS

In this section, we consider single and double Higgs production from gluon fusion in the general model described in the previous section. We begin with the results using the LET, in which the top quark and all top partners are taken infinitely massive. We next include the top quark and bottom quark masses exactly and compute to $\mathcal{O}(\frac{1}{M_X^2})$, where M_X is a generic heavy vector fermion mass. These results (EFT) are then matched to an effective Lagrangian to determine the coefficients of the dimension-6 operators. We are interested in comparing the numerical accuracy of the two approximations with the exact calculations for the gluon fusion rates.

A. Effective theory from low energy theorems

The low energy theorems can be used to integrate out the effect of the charge $\frac{2}{3}$ massive particles, including the top quark. In the limit in which fermion masses $(M_{T_1}, M_{T_2}, M_{T_3})$ are much heavier than the Higgs mass, the hgg coupling can be found from the low energy effective interaction of a colored Dirac fermion with the gluon field strength [49],

$$L_{hgg}^{(t)} = \frac{\alpha_s}{24\pi} h \left(\frac{\partial}{\partial h} \ln[\det(M^{(t)}(h)^{\dagger} M^{(t)}(h))] \right)_{h=0} G^{A,\mu\nu} G_{\mu\nu}^{A}, \tag{18}$$

where $G_{\mu\nu}^{A}$ is the gluon field-strength tensor. With no approximation on the relative size of the parameters in $M^{(t)}$, the LET gives, for the contributions from the top sector alone,

$$L_{hgg}^{(t)} = \frac{\alpha_s}{12\pi} \frac{h}{v} \left[1 + 2\lambda_3 v^2 \left(\frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{X} \right) \right] G^{A,\mu\nu} G_{\mu\nu}^{A}, \tag{19}$$

where

$$\begin{aligned} X &\equiv -\frac{v}{2\sqrt{2}} \det M^{(t)}(0) \\ &= v^2 \lambda_3 (\lambda_1 \lambda_t - \lambda_7 \lambda_9) \\ &\quad + 2[-\lambda_1 M_4 M_5 + M M_5 \lambda_7 + M_U M_4 \lambda_9 - \lambda_t M M_U]. \end{aligned} \tag{20}$$

Having nonzero λ_3, the coupling between the doublet and singlet vectorlike quarks and Higgs boson is critical for achieving a result which is different from the LET for the Standard Model:

$$L_{h^n gg}^{\rm SM} = \frac{\alpha_s}{12\pi} \left[\frac{h}{v} - \frac{h^2}{2v^2} + \cdots \right] G^{A,\mu\nu} G_{\mu\nu}^{A}. \tag{21}$$

This can be understood by noting that when the mass matrix factorizes,

$$\det(M^{(t)}(h)) = F\left(\frac{h}{v}\right) G(\lambda_i, M_X, m_t), \tag{22}$$

the LET has no dependence on the heavy mass scales and Yukawa couplings as in Eq. (21) [12,37,38]. In the limit $\lambda_3 \to 0$, we have

$$\det M^{(t)}(h)|_{\lambda_3=0} = -\frac{h+v}{2\sqrt{2}} X|_{\lambda_3=0}, \tag{23}$$

and the LET reduces to the Standard Model result.

In the limit $M, M_U \gg M_5, M_4, v$ and all the Yukawa couplings λ_i are $\mathcal{O}(1)$,

$$L_{hgg}^{(t)} \to \frac{\alpha_s}{12\pi} \frac{h}{v} \left[1 - \lambda_3 v^2 \left(\frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{M M_U \lambda_t} \right) \right] G^{\mu\nu,A} G_{\mu\nu}^{A}. \tag{24}$$

If, motivated by composite models [12], we assume that there are no couplings of the Standard Model quarks to the Higgs, then $\lambda_t = \lambda_b = \lambda_7 = \lambda_9 = \lambda_{10} = 0$, and with no assumption about the relative sizes of the remaining terms,

$$L_{hgg}^{(t)} \to \frac{\alpha_s}{12\pi} \frac{h}{v} G^{A,\mu\nu} G_{\mu\nu}^{A}, \tag{25}$$

and the Standard Model result is recovered. Similarly to the above, in this limit the determinant of the mass matrix factorizes.

Double Higgs production can also be found using the LET [29,32,49],

$$L_{hhgg}^{(t)} = \frac{\alpha_s}{48\pi} h^2 \left(\frac{\partial^2}{\partial h^2} \ln[\det M^{(t)}(h)^{\dagger} M^{(t)}(h)] \right)_{h=0} G^{A,\mu\nu} G_{\mu\nu}^{A}, \tag{26}$$

and we obtain

$$L_{hhgg}^{(t)} = -\frac{\alpha_s}{24\pi} \frac{h^2}{v^2} \left\{ 1 - 2\lambda_3 v^2 (\lambda_1 \lambda_t - \lambda_7 \lambda_9) \left[\frac{1}{X} - \frac{2\lambda_3 v^2 (\lambda_1 \lambda_t - \lambda_7 \lambda_9)}{X^2} \right] \right\} G^{A,\mu\nu} G_{\mu\nu}^{A}. \tag{27}$$

In the limit $M, M_U \gg M_4, M_5, v$,

$$L_{hhgg}^{(t)} = -\frac{\alpha_s}{24\pi} \frac{h^2}{v^2} \left\{ 1 + \lambda_3 v^2 \left(\frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{M M_U \lambda_t} \right) \right\} G^{A,\mu\nu} G_{\mu\nu}^{A}. \tag{28}$$

Since the b quark is not a heavy fermion, the effective ggh Lagrangian in the charge $-\frac{1}{3}$ sector requires more care and the LET cannot be naively applied. In the next section, we formally integrate out the heavy T_2, T_3 and B_2, B_3 fields, while retaining all mass dependence from the light Standard Model-like quarks, T_1, B_1.

B. Effective theory with top and bottom quark masses

The effects of finite top and bottom quark masses can be included by using the classical equations of motion to integrate out the heavy fields T_2, T_3, B_2 and B_3 [41,42,44,45,50]. We assume that M, M_U and M_D are of similar magnitude and are much larger than v and that the Yukawa couplings λ_i are of $\mathcal{O}(1)$, and expand to $\mathcal{O}(1/M_X^2)$,

$$
\begin{aligned}
U_L &= \left(-\frac{\lambda_7}{M_U}+\frac{\lambda_1 M_4}{MM_U}-\frac{\lambda_t M_5}{M_U^2}\right)(\tilde{H}^\dagger q_L)+\mathcal{O}\left(\frac{1}{M_X^3}\right)\\
U_R &= \left[-\frac{M_5}{M_U}+\left(\frac{\lambda_3\lambda_9}{MM_U}-\frac{\lambda_t\lambda_7}{M_U^2}\right)(H^\dagger H)\right]t_R-\frac{\lambda_7}{M_U^2}i(D_\mu\tilde{H})^\dagger\gamma^\mu q_L+\mathcal{O}\left(\frac{1}{M_X^3}\right)\\
D_L &= \left(-\frac{\lambda_8}{M_D}+\frac{\lambda_2 M_4}{MM_D}-\frac{\lambda_b M_6}{M_D^2}\right)(H^\dagger q_L)+\mathcal{O}\left(\frac{1}{M_X^3}\right)\\
D_R &= \left[-\frac{M_6}{M_D}+\left(\frac{\lambda_{10}\lambda_{11}}{MM_D}-\frac{\lambda_b\lambda_8}{M_D^2}\right)(H^\dagger H)\right]b_R-\frac{\lambda_8}{M_D^2}i(D_\mu H)^\dagger\gamma^\mu q_L+\mathcal{O}\left(\frac{1}{M_X^3}\right)\\
Q_L &= \left[-\frac{M_4}{M}+\left(\frac{\lambda_8\lambda_{11}}{MM_D}-\frac{\lambda_b\lambda_{10}}{M^2}\right)(HH^\dagger)+\left(\frac{\lambda_3\lambda_7}{MM_U}-\frac{\lambda_t\lambda_9}{M^2}\right)\tilde{H}\tilde{H}^\dagger\right]q_L\\
&\quad -\frac{\lambda_9}{M^2}(i\not{D}\tilde{H})t_R-\frac{\lambda_{10}}{M^2}(i\not{D}H)b_R+\mathcal{O}\left(\frac{1}{M_X^3}\right)\\
Q_R &= \left(-\frac{\lambda_9}{M}+\frac{\lambda_1 M_5}{MM_U}-\frac{\lambda_t M_4}{M^2}\right)\tilde{H}t_R+\left(-\frac{\lambda_{10}}{M}+\frac{\lambda_2 M_6}{MM_D}-\frac{\lambda_b M_4}{M^2}\right)Hb_R+\mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned}
\tag{29}
$$

Substituting Eq. (29) into $L'_M + L'_Y$, we get

$$
\begin{aligned}
L'_M + L'_Y &\equiv L_{\text{eff}}^{(a)}\\
&\to -\left\{\left(-\frac{M_5\lambda_7}{M_U}-\frac{M_4\lambda_9}{M}+\frac{\lambda_1 M_4 M_5}{MM_U}+\frac{\lambda_3\lambda_7\lambda_9}{MM_U}(H^\dagger H)\right)\bar{q}_L\tilde{H}t_R\right.\\
&\quad\left.+\left(-\frac{M_6\lambda_8}{M_D}-\frac{M_4\lambda_{10}}{M}+\frac{\lambda_2 M_4 M_6}{MM_D}+\frac{\lambda_8\lambda_{10}\lambda_{11}}{MM_D}(H^\dagger H)\right)\bar{q}_L H b_R+\text{H.c.}\right\}+\delta L_{\text{eff}}^h+\mathcal{O}\left(\frac{1}{M_X^3}\right),
\end{aligned}
\tag{30}
$$

where δL_{eff}^h collects the contributions from the terms in Eq. (29) containing derivatives of the Higgs field [45],

$$
\begin{aligned}
\delta L_{\text{eff}}^h &= \left\{\frac{1}{4}\left(\frac{\lambda_7^2}{M_U^2}-\frac{\lambda_8^2}{M_D^2}\right)(H^\dagger iD_\mu H)(\bar{q}_L\gamma^\mu q_L)-\frac{1}{4}\left(\frac{\lambda_7^2}{M_U^2}+\frac{\lambda_8^2}{M_D^2}\right)(H^\dagger\sigma^a iD_\mu H)(\bar{q}_L\gamma^\mu\sigma^a q_L)\right.\\
&\quad -\frac{\lambda_9^2}{2M^2}[(H^\dagger iD_\mu H)(\bar{t}_R\gamma^\mu t_R)]+\frac{\lambda_{10}^2}{2M^2}[(H^\dagger iD_\mu H)(\bar{b}_R\gamma^\mu b_R)]\\
&\quad\left.+\frac{\lambda_9\lambda_{10}}{M^2}[(\tilde{H}^\dagger iD_\mu H)(\bar{t}_R\gamma^\mu b_R)]\right\}+\text{H.c.}+\mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned}
\tag{31}
$$

Equation (31) corresponds to ΔL_{F_1} of Refs. [43,51].

Similarly, substituting Eq. (29) into the kinetic energy terms of L'_{KE}, we get

$$
\begin{aligned}
L'_{\text{KE}} &\equiv L_{\text{eff}}^{(b)}\\
&\to \frac{1}{2}\left\{\lambda_t\bar{q}_L\tilde{H}t_R\left(\frac{M_4^2}{M^2}+\frac{M_5^2}{M_U^2}+(H^\dagger H)\left[\frac{\lambda_7^2}{M_U^2}+\frac{\lambda_9^2}{M^2}\right]\right)\right.\\
&\quad\left.+\lambda_b\bar{q}_L H b_R\left(\frac{M_4^2}{M^2}+\frac{M_6^2}{M_D^2}+(H^\dagger H)\left[\frac{\lambda_8^2}{M_D^2}+\frac{\lambda_{10}^2}{M^2}\right]\right)\right\}+\text{H.c.}+\mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned}
\tag{32}
$$

The effective low energy Lagrangian after electroweak symmetry breaking contains only Standard Model fields, but non-Standard Model coefficients and operators have been generated by integrating out the heavy fields. The procedure of integrating out by the equations of motion occurs at tree level. However, at loop level, integrating out heavy colored particles will generate operators of the form $G^{A,\mu\nu}G^A_{\mu\nu}h^2$ and $G^{A,\mu\nu}G^A_{\mu\nu}h$, which need to be included in the effective Lagrangian:

$$
\begin{aligned}
L_{\rm eff} &= L_{\rm SM} + L_{\rm eff}^{(a)} + L_{\rm eff}^{(b)} + \delta L_{\rm eff}^{h} + \frac{c_g \alpha_s}{12\pi v} G^{A,\mu\nu} G_A^{\mu\nu} h - \frac{c_{gg}\alpha_s}{24\pi v^2} G^{A,\mu\nu} G_A^{\mu\nu} h^2 \\
&= i\bar{q}_L \not{D} q_L + i\bar{t}_R \not{D} t_R + i\bar{b}_R \not{D} b_R + |D_\mu H|^2 - V(H) - m_t \bar{t}t - Y_t \bar{t}th + c_{2h}^{(t)} \bar{t}th^2 \\
&\quad - m_b \bar{b}b - Y_b \bar{b}bh + c_{2h}^{(b)} \bar{b}bh^2 + \frac{c_g \alpha_s}{12\pi v} G^{A,\mu\nu} G_A^{\mu\nu} h - \frac{c_{gg}\alpha_s}{24\pi v^2} G^{A,\mu\nu} G_A^{\mu\nu} h^2 \\
&\quad + \frac{g}{\sqrt{2}} \{ [\delta g_L \bar{t}_L \gamma^\mu b_L + \delta g_R \bar{t}_R \gamma^\mu b_R] W_\mu^+ + \text{H.c.} \} \\
&\quad + \frac{g}{c_W} \{ \bar{t}_L \gamma_\mu t_L \delta Z_L^t + \bar{t}_R \gamma_\mu t_R \delta Z_R^t + \bar{b}_L \gamma_\mu b_L \delta Z_L^b + \bar{b}_R \gamma_\mu b_R \delta Z_R^b \} Z^\mu + \delta L_{\rm eff}^{h'} + \mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned}
\tag{33}
$$

The non-Standard Model-like gauge boson coupling in lines 4 and 5 in the above equation originates from $\delta L_{\rm eff}^{h}$, and $\delta L_{\rm eff}^{h'}$ is defined to be $\delta L_{\rm eff}^{h}$ with these terms removed. In Fig. 1 we show representative diagrams illustrating the generation of the (a) $\bar{t}th^2$, (b) $G^{A,\mu\nu} G_{\mu\nu}^A h^2$, and (c) $G^{A,\mu\nu} G_{\mu\nu}^A h$ effective operators. To $\mathcal{O}(\frac{1}{M_X^2})$, the Yukawa couplings are shifted from their Standard Model values,

$$
\begin{aligned}
\sqrt{2} Y_t &= \lambda_t \left\{ 1 - \left[\frac{M_4^2}{2M^2} + \frac{M_5^2}{2M_U^2} + \frac{3v^2}{4}\left(\frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right) \right] \right\} - \frac{M_5 \lambda_7}{M_U} - \frac{M_4 \lambda_9}{M} + \frac{\lambda_1 M_4 M_5}{M M_U} + \frac{3v^2}{2 M M_U} \lambda_3 \lambda_7 \lambda_9 \\
&= \sqrt{2} \frac{m_t}{v} + \frac{v^2}{M M_U} \lambda_3 \lambda_7 \lambda_9 - \lambda_t \frac{v^2}{2} \left(\frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right) \\
\frac{Y_t v}{m_t} &\equiv 1 + \delta_t \\
\sqrt{2} Y_b &= \lambda_b \left\{ 1 - \left[\frac{M_4^2}{2M^2} + \frac{M_6^2}{2M_D^2} + \frac{3v^2}{4}\left(\frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right) \right] \right\} - \frac{M_6 \lambda_8}{M_D} - \frac{M_4 \lambda_{10}}{M} + \frac{\lambda_2 M_4 M_6}{M M_D} + \frac{3v^2}{2 M M_D} \lambda_8 \lambda_{10} \lambda_{11} \\
&= \sqrt{2} \frac{m_b}{v} + \frac{v^2}{M M_D} \lambda_8 \lambda_{10} \lambda_{11} - \lambda_b \frac{v^2}{2} \left(\frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right) \\
\frac{Y_b v}{m_b} &\equiv 1 + \delta_b.
\end{aligned}
\tag{34}
$$

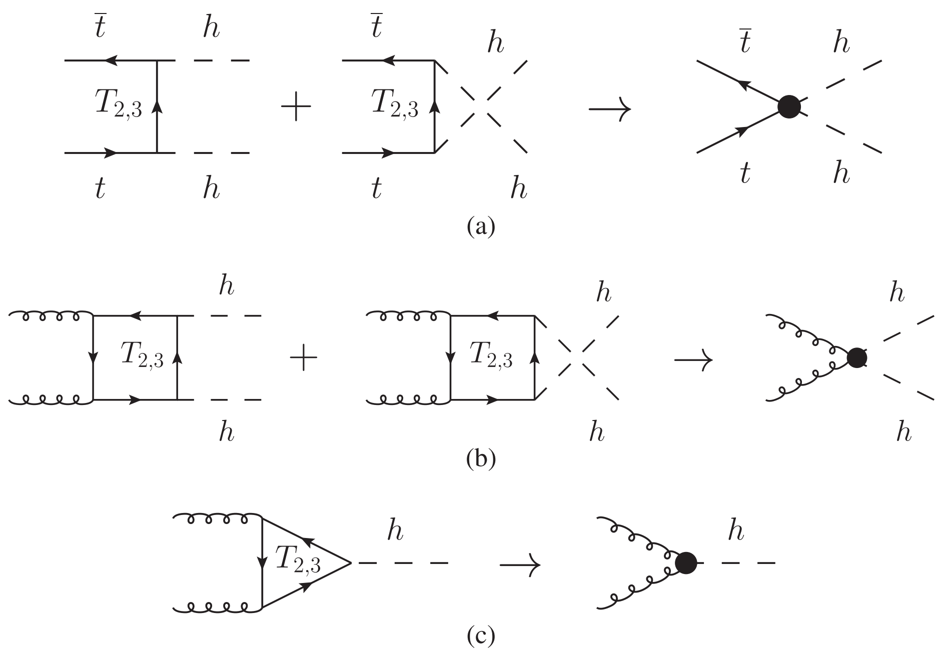


FIG. 1. Representative diagrams corresponding to integrating out heavy fields and generating the (a) $\bar{t}thh$, (b) $G^{A,\mu\nu} G_{\mu\nu}^A h^2$, and (c) $G^{A,\mu\nu} G_{\mu\nu}^A h$ operators in Eq. (33).

We see that Y_t and Y_b are no longer proportional to $m_t = M_{T_1}$ and $m_b = M_{B_1}$. Non-Standard Model couplings of the fermions to Higgs pairs are also generated, as are Higgs-gluon effective couplings,

$$\begin{aligned}
\frac{c_{2h}^{(t)}}{v} &= \frac{3}{2\sqrt{2}}\left\{-\frac{\lambda_3\lambda_7\lambda_9}{MM_U}+\frac{1}{2}\lambda_t\left(\frac{\lambda_7^2}{M_U^2}+\frac{\lambda_9^2}{M^2}\right)\right\}\\
&= -\frac{3}{2v^2}\left(Y_t-\frac{m_t}{v}\right)\\
&= -\frac{3}{2}\frac{m_t\delta_t}{v^3}\\
\frac{c_{2h}^{(b)}}{v} &= \frac{3}{2\sqrt{2}}\left\{-\frac{\lambda_8\lambda_{10}\lambda_{11}}{MM_D}+\frac{1}{2}\lambda_b\left(\frac{\lambda_8^2}{M_D^2}+\frac{\lambda_{10}^2}{M^2}\right)\right\}\\
&= -\frac{3}{2}\frac{m_b\delta_b}{v^3}\\
c_g &= v^2\left[-\frac{\lambda_1\lambda_3}{MM_U}-\frac{\lambda_2\lambda_{11}}{MM_D}+\frac{1}{2}\left(\frac{\lambda_7^2}{M_U^2}+\frac{\lambda_8^2}{M_D^2}+\frac{\lambda_9^2+\lambda_{10}^2}{M^2}\right)\right]\\
&= -c_{gg}. \qquad (35)
\end{aligned}$$

The top and bottom quark couplings to ggh and $gghh$ are not included in c_g and c_{gg}, but can be calculated at one loop using the effective interactions of Eq. (33). The effective Lagrangian depends on only three new parameters: c_g, δ_t, and δ_b, along with the physical masses, $m_t = M_{T_1}$ and $m_b = M_{B_1}$, and v. It is important to note that within the context of this model, the coefficients of the effective Lagrangian cannot all be independently varied. This feature can also arise in composite Higgs models [35].

The non-Standard Model couplings to the W and Z are given to $\mathcal{O}(1/M_X^2)$ by

$$\begin{aligned}
\delta g_L &= -\frac{v^2}{4}\left(\frac{\lambda_7^2}{M_U^2}+\frac{\lambda_8^2}{M_D^2}\right) \qquad \delta g_R = \frac{v^2\lambda_9\lambda_{10}}{2M^2}\\
\delta Z_L^t &= -\frac{v^2\lambda_7^2}{4M_U^2} \qquad \delta Z_R^t = \frac{v^2\lambda_9^2}{4M^2}\\
\delta Z_L^b &= \frac{v^2\lambda_8^2}{4M_D^2} \qquad \delta Z_R^b = -\frac{v^2\lambda_{10}^2}{4M^2}. \qquad (36)
\end{aligned}$$

IV. UNDERSTANDING THE FULL THEORY

A. Hierarchy 1

In order to understand some general features of the mass matrices, we consider a hierarchy where the mixing angles are small,

$$\theta \sim \frac{\lambda_i v}{M_4} \sim \frac{\lambda_i v}{M_5} \sim \frac{M_4}{M} \sim \frac{M_5}{M} \quad \text{and} \quad \theta^2 \sim \frac{\lambda_i v}{M}. \qquad (37)$$

This maintains the hierarchy $\lambda_i v \ll M_4, M_5 \ll M, M_U, M_D$, keeping the off-diagonal elements of the mass matrices small. In this limit the matrices which diagonalize the top quark mass matrix can be written as

$$\begin{aligned}
V_L^t &= \begin{pmatrix} 1-\frac{1}{2}\theta_L^{D2} & -\theta_L^D & -\theta_L^{S2}\\ \theta_L^D & 1-\frac{1}{2}\theta_L^{D2} & \theta_L^{H2}\\ \theta_L^{S2} & -\theta_L^{H2} & 1\end{pmatrix}\\
V_R^t &= \begin{pmatrix} 1-\theta_R^{S2} & -\theta_R^{D2} & -\theta_R^S\\ \theta_R^{D2} & 1 & -\theta_R^{H2}\\ \theta_R^S & \theta_R^{H2} & 1-\frac{\theta_R^{S2}}{2}\end{pmatrix}, \qquad (38)
\end{aligned}$$

where the matrices of Eq. (38) are unitary to $\mathcal{O}(\theta^3)$.[1] The angles θ^D (θ^S) can be thought of as the doublet (singlet) vector fermion mixing with the Standard Model-like top quark, and θ^H as the doublet-singlet vector fermion mixing. All angles are assumed to scale as Eq. (37).

In the small angle limit of Eq. (38), we can then solve for the parameters of the Lagrangian:

$$\begin{aligned}
\mathcal{M}^{(t)}(0) &= \begin{pmatrix} \lambda_t\frac{v}{\sqrt{2}} & M_4 & \lambda_7\frac{v}{\sqrt{2}}\\ \lambda_9\frac{v}{\sqrt{2}} & M & \lambda_1\frac{v}{\sqrt{2}}\\ M_5 & \lambda_3\frac{v}{\sqrt{2}} & M_U\end{pmatrix}\\
&= \begin{pmatrix} M_{T_1}\left(1-\frac{\theta_L^{D2}}{2}-\frac{\theta_R^{S2}}{2}\right) & M_{T_2}\theta_L^D - M_{T_1}\theta_R^{D2} & M_{T_3}\theta_L^{S2} - M_{T_1}\theta_R^S\\ M_{T_2}\theta_R^{D2} - M_{T_1}\theta_L^D & M_{T_2}\left(1-\frac{\theta_L^{D2}}{2}\right) & -M_{T_3}\theta_L^{H2} - M_{T_2}\theta_R^{H2} + M_{T_1}\theta_L^D\theta_R^S\\ M_{T_3}\theta_R^S - M_{T_1}\theta_L^{S2} & M_{T_2}\theta_L^{H2} + M_{T_3}\theta_R^{H2} & M_{T_3}\left(1-\frac{\theta_R^{S2}}{2}\right)\end{pmatrix}.
\end{aligned}$$

[1] Note that the hierarchy determines the leading behavior of the θ expansion of the mixing matrices. Higher orders of this expansion are determined by unitarity.

As can be seen, according to the θ scaling behavior, this obeys the structure that we want ($\lambda_i v \ll \lambda_{4,5} \ll M$). In the fermion mass-eigenstate basis,

$$L \sim -\sum_{i,j=1,2,3} \bar{\psi}_i^t \mathcal{Y}_{ij}^t \psi_j^t h, \tag{39}$$

the small angle approximation to the charge $\frac{2}{3}$ Yukawa interactions is

$$v \times \mathcal{Y}^t = \begin{pmatrix} M_{T_1} & M_{T_1}\theta_R^{D2} & M_{T_3}\theta_L^{S2} \\ M_{T_2}\theta_R^{D2} & 0 & -M_{T_3}\theta_L^{H2} - M_{T_2}\theta_R^{H2} \\ M_{T_1}\theta_L^{S2} & M_{T_2}\theta_L^{H2} + M_{T_3}\theta_R^{H2} & 0 \end{pmatrix}. \tag{40}$$

The mass matrix in the b quark sector can be parametrized in an identical fashion to the above discussion.

The W interactions defined in Eqs. (13) and (14), in the small angle approximation of Eq. (38), are

$$U_L = \begin{pmatrix} 1-\frac{1}{2}(\theta_L^{Db}-\theta_L^{Dt})^2 & \theta_L^{Db}-\theta_L^{Dt} & \theta_L^{Sb2} \\ \theta_L^{Dt}-\theta_L^{Db} & 1-\frac{1}{2}(\theta_L^{Db}-\theta_L^{Dt})^2 & -\theta_L^{Hb2} \\ \theta_L^{St2} & -\theta_L^{Ht2} & 0 \end{pmatrix}, \tag{41}$$

$$U_R = \begin{pmatrix} 0 & -\theta_R^{Dt2} & 0 \\ -\theta_R^{Db2} & 1 & \theta_R^{Hb2} \\ 0 & \theta_R^{Ht2} & 0 \end{pmatrix}, \tag{42}$$

where we have added the superscripts b, t to indicate mixing in the bottom and top sectors, respectively. The Z-fermion interactions defined in Eqs. (15) and (16), in the small angle approximation, are

$$X_L^t = \begin{pmatrix} 1 & 0 & \theta_L^{St2} \\ 0 & 1 & -\theta_L^{Ht2} \\ \theta_L^{St2} & -\theta_L^{Ht2} & 0 \end{pmatrix}$$
$$X_R^t = \begin{pmatrix} 0 & -\theta_R^{Dt2} & 0 \\ -\theta_R^{Dt2} & 1 & \theta_R^{Ht2} \\ 0 & \theta_R^{Ht2} & 0 \end{pmatrix}. \tag{43}$$

The results for the bottom sector can be found by the replacement $t \to b$.

Comparing to the EFT of Eq. (33) in the small angle approximation described above, we obtain

$$Y_t = \frac{M_{T_1}}{v}$$
$$Y_b = \frac{M_{B_1}}{v}$$
$$c_{2h}^{(t)} = c_{2h}^{(b)} = 0$$
$$c_g = -c_{gg} = 0. \tag{44}$$

In the EFT, this hierarchy reduces to the Standard Model and thus does not produce large deviations in Higgs production rates.

B. Hierarchy 2

Hierarchy 1 appears to give small $\lambda_7, \lambda_9, \lambda_3, \lambda_8, \lambda_{10}, \lambda_{11}$, which are the parameters that give deviations from the Standard Model. We now describe a different hierarchy with $M_{4,5} \ll \lambda_i v \ll M$:

$$\theta \sim \frac{M_{4,5}}{\lambda_i v} \sim \frac{\lambda_i v}{M} \quad \text{and} \quad \theta^2 \sim \frac{M_{4,5}}{M}. \tag{45}$$

The diagonalization matrices can be parametrized in both the t sectors as[2]

$$V_L^t = \begin{pmatrix} 1-\frac{1}{2}\theta_L^{S2} & -\theta_L^{D2} & -\theta_L^S \\ \theta_L^{D2}+\theta_L^H\theta_L^S & 1-\frac{1}{2}\theta_L^{H2} & \theta_L^H \\ \theta_L^S & -\theta_L^H & 1-\frac{1}{2}(\theta_L^{S2}+\theta_L^{H2}) \end{pmatrix}$$
$$V_R^t = \begin{pmatrix} 1-\frac{1}{2}\theta_R^{D2} & -\theta_R^D & -\theta_R^{S2} \\ \theta_R^D & 1-\frac{1}{2}(\theta_R^{D2}+\theta_R^{H2}) & -\theta_R^H \\ \theta_R^D\theta_R^H+\theta_R^{S2} & \theta_R^H & 1-\frac{1}{2}\theta_R^{H2} \end{pmatrix}. \tag{46}$$

The parameters of the original top mass matrix, $M^{(t)}(0)$ from Eq. (9), can be solved for, to $\mathcal{O}(\theta^2)$,

[2] We omit the superscripts t and b on the mixing angles where it is obvious.

$$
\begin{aligned}
\lambda_t \frac{v}{\sqrt{2}} &= M_{T_1}\left(1 - \frac{\theta_L^{S2}}{2} - \frac{\theta_R^{D2}}{2}\right) \\
M_4 &= M_{T_2}(\theta_L^{D2} + \theta_L^S \theta_L^H) + M_{T_3}\theta_L^S \theta_R^H - M_{T_1}\theta_R^D \\
\lambda_7 \frac{v}{\sqrt{2}} &= M_{T_3}\theta_L^S - M_{T_1}\theta_R^{S2} \\
\lambda_9 \frac{v}{\sqrt{2}} &= M_{T_2}\theta_R^D - M_{T_1}\theta_L^{D2} \\
M &= M_{T_2}\left[1 - \frac{1}{2}(\theta_L^{H2} + \theta_R^{D2} + \theta_R^{H2})\right] - M_{T_3}\theta_L^H \theta_R^H \\
\lambda_1 \frac{v}{\sqrt{2}} &= -M_{T_3}\theta_L^H - M_{T_2}\theta_R^H \\
M_5 &= M_{T_3}(\theta_R^D \theta_R^H + \theta_R^{S2}) + M_{T_2}\theta_R^D \theta_L^H - M_{T_1}\theta_L^S \\
\lambda_3 \frac{v}{\sqrt{2}} &= M_{T_3}\theta_R^H + M_{T_2}\theta_L^H + M_{T_1}\theta_L^S \theta_R^D \\
M_U &= M_{T_3}\left[1 - \frac{1}{2}(\theta_L^{S2} + \theta_L^{H2} + \theta_R^{H2})\right] - M_{T_2}\theta_L^H \theta_R^H.
\end{aligned}
\tag{47}
$$

Finally, the Higgs couplings to the charge $\frac{2}{3}$ fermions can be written as in Eq. (39),

$$
v \times \mathcal{Y}^t = \begin{pmatrix}
M_{T_1}(1-\theta_L^{S2}-\theta_R^{D2}) & M_{T_1}\theta_R^D - M_{T_2}\theta_L^H\theta_L^S - 2M_{T_3}\theta_L^S\theta_R^H & M_{T_1}\theta_R^D\theta_R^H + M_{T_3}\theta_L^S \\
M_{T_1}\theta_L^H\theta_L^S + M_{T_2}\theta_R^D & M_{T_2}(\theta_L^{H2}+\theta_R^{D2}+\theta_R^{H2}) + 2M_{T_3}\theta_L^H\theta_R^H & -M_{T_3}\theta_L^H - M_{T_2}\theta_R^H \\
M_{T_1}\theta_L^S - 2M_{T_2}\theta_L^H\theta_R^D - M_{T_3}\theta_R^D\theta_R^H & 2M_{T_1}\theta_L^S\theta_R^D + M_{T_2}\theta_L^H + M_{T_3}\theta_R^H & M_{T_3}(\theta_L^{H2}+\theta_L^{S2}+\theta_R^{H2}) + 2M_{T_2}\theta_L^H\theta_R^H
\end{pmatrix}.
\tag{48}
$$

Again, the b sector mass matrix and mixing can be parametrized in a similar fashion as above.

Comparing to the EFT in Eq. (33) (counting $M_{T_1}/M_{T_{2,3}} \sim M_{B_1}/M_{B_{2,3}} \sim \theta$), this hierarchy yields small deviations from the Standard Model,

$$
\begin{aligned}
Y_t &= \frac{M_{T_1}}{v}(1 - \theta_R^{Dt2} - \theta_L^{St2}) \\
Y_b &= \frac{M_{B_1}}{v}(1 - \theta_R^{Db2} - \theta_L^{Sb2}) \\
c_{2h}^{(t)} &= \frac{3M_{T_1}}{2v^2}(\theta_R^{Dt2} + \theta_L^{St2}) \\
c_{2h}^{(b)} &= \frac{3M_{B_1}}{2v^2}(\theta_R^{Db2} + \theta_L^{Sb2}) \\
c_g = -c_{gg} &= (2\theta_L^{Ht2} + \theta_L^{St2}) + (2\theta_R^{Ht2} + \theta_R^{Dt2}) + 2\frac{M_{T_2}^2 + M_{T_3}^2}{M_{T_2}M_{T_3}}\theta_L^{Ht}\theta_R^{Ht} \\
&\quad + (2\theta_L^{Hb2} + \theta_L^{Sb2}) + (2\theta_R^{Hb2} + \theta_R^{Db2}) + 2\frac{M_{B_2}^2 + M_{B_3}^2}{M_{B_2}M_{B_3}}\theta_L^{Hb}\theta_R^{Hb}.
\end{aligned}
\tag{49}
$$

Again, the superscripts t and b indicate mixing angles in the top and bottom sectors, respectively. Now we want to match onto the LET, i.e, integrate out the top quark (T_1), along with the heavier fermions T_2, T_3, B_2, and B_3. The effective Higgs-gluon interactions are

$$
\mathcal{L}_{\rm LET} = \frac{\alpha_s}{12\pi}\left[(1 + c_g^{\rm LET})\frac{h}{v} - \frac{1 + c_{gg}^{\rm LET}}{2}\frac{h^2}{v^2}\right]G^{A,\mu\nu}G_{\mu\nu}^A.
\tag{50}
$$

To obtain $c^{\rm LET}$ we use the full LET for the top quark sector in Eqs. (24) and (28) and then add in the effect of integrating out the heavy bottom quark partners, that is, the heavy down-type quark contributions to c_g and c_{gg} in Eq. (35). To $\mathcal{O}(\theta^2)$, this yields,

$$\begin{aligned} c_g^{\rm LET} &= -c_{gg}^{\rm LET} \\ &= 2(\theta_L^{Hb2} + \theta_L^{Ht2} + \theta_R^{Hb2} + \theta_R^{Ht2}) + \theta_L^{Sb2} + \theta_R^{Db2} + 2\frac{M_{T_2}^2 + M_{T_3}^2}{M_{T_2}M_{T_3}}\theta_L^{Ht}\theta_R^{Ht} + 2\frac{M_{B_2}^2 + M_{B_3}^2}{M_{B_2}M_{B_3}}\theta_L^{Hb}\theta_R^{Hb}. \end{aligned} \tag{51}$$

For degenerate heavy fermions, c_g is positive definite and so the contribution to double Higgs production from c_{gg} always decreases the rate. Additionally, to increase the double Higgs contribution from c_{gg}, θ_L^H and θ_R^H should have opposite signs.

The mixing matrices for the W interactions are [Eqs. (13) and (14)]

$$\begin{aligned} U_L &= \begin{pmatrix} 1-\frac{1}{2}(\theta_L^{Sb2}+\theta_L^{St2}) & \theta_L^{Db2}-\theta_L^{Dt2}+\theta_L^{Hb}\theta_L^{Sb} & \theta_L^{Sb} \\ \theta_L^{Dt2}-\theta_L^{Db2}+\theta_L^{Ht}\theta_L^{St} & 1-\frac{1}{2}(\theta_L^{Hb2}+\theta_L^{Ht2}) & -\theta_L^{Hb} \\ \theta_L^{St} & -\theta_L^{Ht} & \theta_L^{Hb}\theta_L^{Ht}+\theta_L^{Sb}\theta_L^{St} \end{pmatrix} \\ U_R &= \begin{pmatrix} \theta_R^{Db}\theta_R^{Dt} & -\theta_R^{Dt} & -\theta_R^{Hb}\theta_R^{Dt} \\ -\theta_R^{Db} & 1-\frac{1}{2}(\theta_R^{Db2}+\theta_R^{Hb2}+\theta_R^{Dt2}+\theta_R^{Ht2}) & \theta_R^{Hb} \\ -\theta_R^{Db}\theta_R^{Ht} & \theta_R^{Ht} & \theta_R^{Hb}\theta_R^{Ht} \end{pmatrix}. \end{aligned} \tag{52}$$

The mixing matrices for Z interactions [Eqs. (15) and (16)] in the charge $\frac{2}{3}$ sector are

$$\begin{aligned} X_L^t &= \begin{pmatrix} 1-\theta_L^{St2} & \theta_L^{Ht}\theta_L^{St} & \theta_L^{St} \\ \theta_L^{Ht}\theta_L^{St} & 1-\theta_L^{Ht2} & -\theta_L^{Ht} \\ \theta_L^{St} & -\theta_L^{Ht} & \theta_L^{Ht2}+\theta_L^{St2} \end{pmatrix} \\ X_R^t &= \begin{pmatrix} \theta_R^{Dt2} & -\theta_R^{Dt} & -\theta_R^{Ht2} \\ -\theta_R^{Dt} & 1-\theta_R^{Dt2}-\theta_R^{Ht2} & \theta_R^{Ht} \\ -\theta_R^{Dt}\theta_R^{Ht} & \theta_R^{Ht} & \theta_R^{Ht2} \end{pmatrix}. \end{aligned} \tag{53}$$

The Z couplings in the bottom sector are found from Eq. (53) with the replacement $t \to b$.

V. LIMITS FROM PRECISION MEASUREMENTS

New heavy quarks which couple to the Standard Model gauge bosons are restricted by the oblique parameters [52]. In addition, the couplings of charge $-\frac{1}{3}$ quarks are significantly limited by the measurements of $Z \to b\bar{b}$. These limits typically require small mixing parameters.

General formulas for the contributions of the fermion sector to ΔS and ΔT are given in Appendix A. It is useful to consider several special cases here. For the case with only a top partner singlet (T_3) with a mass $M_{T_3} \gg M_{T_1}$, the only nonzero entries of the left-handed mixing matrices are

$$\begin{aligned} V_{L,11}^t &= V_{L,33}^t = c_L \\ V_{L,31}^t &= -V_{L,13}^t = -s_L \\ V_{L,11}^b &= 1, \end{aligned} \tag{54}$$

while $V_R^{t,b}$ can be set to the unit matrix, $c_L \equiv \cos\theta_L$, $s_L \equiv \sin\theta_L$, and

$$\tan(2\theta_L) = \frac{\sqrt{2}v}{M_U}\left(\frac{\lambda_7}{1-(\lambda_7^2+\lambda_t^2)\frac{v^2}{2M_U^2}}\right). \tag{55}$$

The result for large top partner masses is (after subtracting the Standard Model top and bottom contributions)

$$\begin{aligned} [\Delta T]_{\rm top\ singlet} &= \frac{N_c}{16\pi s_W^2 M_W^2} s_L^2\left(-(1+c_L^2)M_{T_1}^2 - 2c_L^2 M_{T_1}^2 \ln\left(\frac{M_{T_1}^2}{M_{T_3}^2}\right) + s_L^2 M_{T_3}^2\right) \\ [\Delta S]_{\rm top\ singlet} &= -\frac{N_c}{18\pi} s_L^2\left(5c_L^2 + (1-3c_L^2)\ln\left(\frac{M_{T_1}^2}{M_{T_3}^2}\right)\right), \end{aligned} \tag{56}$$

where $N_C = 3$, in agreement with Ref. [29], which found that fits to the oblique parameters require $s_L \lesssim 0.16$ for $M_{T_3} \sim 1$ TeV at 95% confidence level. For fixed values of the Yukawa couplings, λ_i, the mixing angle scales for large M_{T_3} as

$$s_L \sim \frac{v\lambda_i}{M_{T_3}}, \tag{57}$$

and the contributions to the oblique parameters from the top partner decouple,

$$[\Delta T]_{\text{top singlet}} \sim [\Delta S]_{\text{top singlet}} \sim \frac{\lambda_i^2 v^2}{M_{T_3}^2}. \tag{58}$$

The limit on the angle s_L in the above example arises because of the mixing with the Standard Model top quark. Reference [29] contains an example where there is a heavy vectorlike $SU(2)_L$ doublet, Q, along with vectorlike charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks, U and D, which are not allowed to mix with the Standard Model fermions. This corresponds to $M_4 = M_5 = M_6 = \lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$ in Eq. (7). In this case, limits from the oblique parameters require that the heavy fermions be approximately degenerate, $M_{T_2} \simeq M_{T_3} \simeq M_{B_2} \simeq M_{B_3}$, while one combination of mixing angles is unconstrained.

Limits can also be obtained from Z decays to $b\bar{b}$ by comparing the experimental result [53] for R_b with the recent Standard Model calculation [54],

$$\begin{aligned} R_b &\equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to b\bar{b})} \\ R_b^{\text{exp}} &= 0.21629 \pm 0.00066 \\ R_b^{\text{SM}} &= 0.2154940. \end{aligned} \tag{59}$$

R_b can be related to the anomalous couplings of the b quark to the Z given in Eq. (33),

$$\frac{R_b^{\text{exp}}}{R_b^{\text{SM}}} = 1 - 3.57\delta g_L^b + 0.65 g_R^b. \tag{60}$$

From Eqs. (36) and (60), we extract the 95% confidence level bound,

$$\left(\frac{M}{\lambda_8}\right)^2 \left(\frac{1}{1 + 0.224(1 - \frac{\lambda_{10} M}{\lambda_8 M_D})^2}\right) \gtrsim (2\ \text{TeV})^2. \tag{61}$$

The following discussion focuses on hierarchy 2 of Sec. IV, although it can be shown that the conclusions are quite generic. We start by counting the degrees of freedom. Naively, there are six masses,

$$M_{T_1}, M_{T_2}, M_{T_3}, M_{B_1}, M_{B_2}, M_{B_3} \tag{62}$$

and 12 angles,

$$\theta^{St}_{L,R}, \theta^{Dt}_{L,R}, \theta^{Ht}_{L,R}, \theta^{Sb}_{L,R}, \theta^{Db}_{L,R}, \theta^{Hb}_{L,R}. \tag{63}$$

However, M_4 and M are the same in the top and bottom sectors, leaving a total of 16 independent parameters. Considering Eqs. (44) and (53), we see that if we forbid mixing between particles with different quantum numbers, then flavor changing neutral currents involving the Z are eliminated. That is, θ_L^{St} mixes a component of the Standard Model $SU(2)_L$ doublet with an $SU(2)_L$ singlet, and θ_R^{Dt} mixes a Standard Model $SU(2)_L$ singlet with a component of a vector fermion doublet. We set these angles to zero to avoid restrictions from deviations in the third generation quark neutral current couplings, in particular, $Z \to b\bar{b}$:

$$\theta_L^{St} = \theta_R^{Dt} = \theta_L^{Sb} = \theta_R^{Db} = 0. \tag{64}$$

The angles $\theta^{Ht}_{L,R}$ and $\theta^{Hb}_{L,R}$ remain nonzero, since from Eq. (51) we see that these are intimately tied to deviations from Standard Model Higgs production rates. The Z couplings to the top quark and heavy up-type vector quarks are then

$$X_L^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \theta_L^{Ht2} & -\theta_L^{Ht} \\ 0 & -\theta_L^{Ht} & \theta_L^{Ht2} + \theta_L^{St2} \end{pmatrix}$$
$$X_R^t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - \theta_R^{Ht2} & \theta_R^{Ht} \\ 0 & \theta_R^{Ht} & \theta_R^{Ht2} \end{pmatrix}, \tag{65}$$

and the t and b quarks have Standard Model-like neutral current couplings.

The W-mixing matrices in hierarchy 2 are

$$U_L = \begin{pmatrix} 1 & \theta_L^{Db2} - \theta_L^{Dt2} & 0 \\ \theta_L^{Dt2} - \theta_L^{Db2} & 1 - \frac{1}{2}(\theta_L^{Hb2} + \theta_L^{Ht2}) & -\theta_L^{Hb} \\ 0 & -\theta_L^{Ht} & \theta_L^{Hb}\theta_L^{Ht} \end{pmatrix}$$
$$U_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - \frac{1}{2}(\theta_R^{Hb2} + \theta_R^{Ht2}) & \theta_R^{Hb} \\ 0 & \theta_R^{Ht} & \theta_R^{Hb}\theta_R^{Ht} \end{pmatrix}. \tag{66}$$

U_R only depends on $\theta^{Ht}_{L,R}$ and $\theta^{Hb}_{L,R}$, the mixing angles between the heavy vector fermions, while U_L still depends on the mixing between the heavy states with the Standard Model. Forcing the heavy-light mixing to be isospin conserving, $\theta_L^{Db} = \theta_L^{Dt}$, U_L becomes

$$U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{1}{2}(\theta_L^{Hb2} + \theta_L^{Ht2}) & -\theta_L^{Hb} \\ 0 & -\theta_L^{Ht} & \theta_L^{Hb}\theta_L^{Ht} \end{pmatrix} \tag{67}$$

and there are no gauge boson currents mixing the Standard Model top and bottom quarks with the new vector fermions.

To summarize, taking into consideration electroweak precision observables, it is reasonable to impose the constraints

$$\theta_L^{St} = \theta_R^{Dt} = \theta_L^{Sb} = \theta_R^{Db} = 0, \qquad \theta_L^{Db} = \theta_L^{Dt}. \quad (68)$$

Under this assumption, the nonzero mixing angles are

$$\theta_R^{St}, \theta_L^{Dt}, \theta_{L,R}^{Ht}, \theta_R^{Sb}, \theta_{L,R}^{Hb}. \quad (69)$$

There are two constraints from M_4 and M,

$$M_4 = M_{T_2}\theta_L^{Dt2} = M_{B_2}\theta_L^{Db2}$$
$$M = M_{T_2}\left(1 - \frac{1}{2}\theta_L^{Ht2} - \frac{1}{2}\theta_R^{Ht2}\right) - M_{T_3}\theta_L^{Ht}\theta_R^{Ht}$$
$$= M_{B_2}\left(1 - \frac{1}{2}\theta_L^{Hb2} - \frac{1}{2}\theta_R^{Hb2}\right) - M_{B_3}\theta_L^{Hb}\theta_R^{Hb}. \quad (70)$$

So, $\theta_L^{Dt} = \theta_L^{Db}$ is only consistent if $M_{T_2} = M_{B_2}$, which fully eliminates isospin violation in the mixing between the new heavy states and the third generation quarks. To make things simpler, we can also assume $M_{T_3} = M_{B_3}$, and then Eq. (70) is satisfied when $\theta_L^{Ht} = \theta_L^{Hb}$ and $\theta_R^{Ht} = \theta_R^{Hb}$. (There are other possible solutions not requiring $M_{T_3} = M_{B_3}$, but for simplicity we focus on this limit.)

Now we only have a few remaining degrees of freedom: four masses (two of which are known),

$$M_{T_1}, M_{B_1}, M_{T_2} = M_{B_2}, M_{T_3} = M_{B_3}, \quad (71)$$

and five angles,

$$\theta_R^{St}, \theta_R^{Sb}, \theta_L^{Dt} = \theta_L^{Db}, \theta_L^{Ht} = \theta_L^{Hb}, \theta_R^{Ht} = \theta_R^{Hb}. \quad (72)$$

At lowest order these angles are unconstrained by $Z \to b\bar{b}$, and the oblique parameters only constrain the mixing among the heavy quarks. These constraints can be found in Ref. [29]. Although this result can be shown generically without assuming that θ_L^D and θ_R^S are small, these angles will manifest themselves in the CKM matrix when considering mixing among the first three generations [34]. We therefore continue with the small angle approximation.

VI. RESULTS FOR HIGGS PRODUCTION

In this section, we compare the accuracy of the LET with the effective Lagrangian obtained by including the top and bottom quark mass effects (EFT), Eq. (33), as well as with predictions obtained using the full theory. We have two goals: the first is to understand the numerical limitations of the approximations to the full theory. Our second goal is to search for a regime where single Higgs production from gluon fusion occurs at approximately the Standard Model rate, while double Higgs production is significantly altered. Again, we focus on hierarchy 2 of Sec. IV B, since hierarchy 1 (Sec. IV A) does not lead to significant deviations from the Standard Model [Eq. (44)].

We normalize the predictions to the Standard Model rates,

$$R_h \equiv \frac{\sigma(gg \to h)}{\sigma(gg \to h)_{\rm SM}}$$
$$R_{hh} = \frac{\sigma(gg \to hh)}{\sigma(gg \to hh)_{\rm SM}}. \quad (73)$$

To $\mathcal{O}(\delta_{\rm LET})$, the low energy theorems of Eqs. (19) and (27), including only the up-type quarks, predict,

$$R_h \sim 1 + 2\delta_{\rm LET}$$
$$R_{hh} \sim 1 + 2\delta_{\rm LET} - \frac{4\delta_{\rm LET}}{F_0^{\rm SM}(M_{T_1} \to \infty)}, \quad (74)$$

and

$$F_0^{\rm SM}(M_{T_1} \to \infty) \equiv 1 - \frac{3M_h^2}{s - M_h^2}, \quad (75)$$

where $\delta_{\rm LET} = 2\lambda_3 v^2(\lambda_1\lambda_t - \lambda_7\lambda_9)/X$ is given in Eq. (19) and F_0 is defined in Eqs. (79), (83), (84). In the effective field theory language of Eq. (33), $\delta_{\rm LET} = c_g$. The presence of the λ_3 coupling does indeed allow single Higgs production to differ from the Standard Model prediction. However, once R_h is measured to be approximately 1, the deviations of R_{hh} from 1 are restricted to be small. Thus, in order for the double Higgs rate to be different from the Standard Model prediction, we need a region of parameter space where the low energy theorem is not valid.

The rate for single Higgs production in the effective theory including all top and bottom quark mass effects (EFT), but integrating out the heavy vectorlike fermions to $\mathcal{O}(\frac{1}{M_X^2})$ and assuming δ_b, δ_t and c_g are small, is given by

$$R_h \to \frac{|(1+\delta_t)F_{1/2}(\tau_{T_1}) + (1+\delta_b)F_{1/2}(\tau_{B_1}) + c_g F_{1/2}^\infty|^2}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2}$$
$$\sim 1 + 2\left[\frac{\delta_t|F_{1/2}(\tau_{T_1})|^2 + \delta_b|F_{1/2}(\tau_{B_1})|^2 + (\delta_t+\delta_b){\rm Re}(F_{1/2}(\tau_{T_1})F_{1/2}^*(\tau_{B_1}))}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2}\right]$$
$$+ 2\left[\frac{c_g F_{1/2}^\infty {\rm Re}(F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1}))}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2}\right], \quad (76)$$

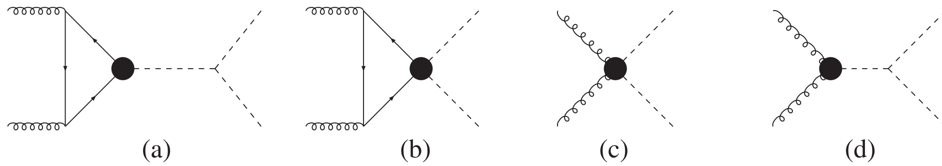


FIG. 2. Non-box contributions to the spin-0 component of $gg \to hh$. The dark circles represent the non-Standard Model contributions, while the solid lines are either t- or b quarks.

where $\tau_i \equiv 4M_i^2/M_h^2$,

$$F_{1/2}(\tau) = -2\tau[1 + (1-\tau)f(\tau)]$$
$$f(\tau) = \begin{cases} \left[\sin^{-1}\left(\frac{1}{\sqrt{\tau}}\right)\right]^2 & \text{if } \tau \geq 1 \\ -\frac{1}{4}\left[\ln\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right]^2 & \text{if } \tau < 1, \end{cases} \tag{77}$$

and $F_{1/2}^{\infty} = -\frac{4}{3}$ in the $M_{T_1} \to \infty$ limit of $F_{1/2}(\tau_{T_1})$. Neglecting the b contribution and noting that $F_{1/2}(\tau_{T_1})$ is well approximated by $F_{1/2}^{\infty}$,

$$R_h \sim 1 + 2(\delta_t + c_g). \tag{78}$$

The c_g contribution is in agreement with the LET result of Eq. (74).

Double Higgs production can be analyzed in a similar fashion. The diagrams shown in Fig. 2 contribute only to the spin-0 projection, while the box diagrams shown in Fig 3 have both spin-0 and spin-2 components. The amplitude for $g^{A,\mu}(p_1)g^{B,\nu}(p_2) \to h(p_3)h(p_4)$ is

$$A_{AB}^{\mu\nu} = \frac{\alpha_s}{3\pi v^2}\delta_{AB}\sum_i [P_1^{\mu\nu}(p_1,p_2)F_0^i(s,t,u,M_j) + P_2^{\mu\nu}(p_1,p_2,p_3)F_2^i(s,t,u,M_j)], \tag{79}$$

where the sum is over the diagrams, M_j denotes all relevant quark masses, P_1 and P_2 are the orthogonal projectors onto the spin-0 and spin-2 states, respectively,

$$P_1^{\mu\nu}(p_1,p_2) = p_1\cdot p_2 g^{\mu\nu} - p_1^{\nu}p_2^{\mu},$$
$$P_2^{\mu\nu}(p_1,p_2,p_3) = p_1\cdot p_2 g^{\mu\nu} + \frac{1}{p_T^2}(M_h^2 p_1^{\nu}p_2^{\mu} - 2p_1.p_3 p_2^{\mu}p_3^{\nu} - 2p_2.p_3 p_1^{\nu}p_3^{\mu} + s p_3^{\mu}p_3^{\nu}), \tag{80}$$

s, t, and u are the partonic Mandelstam variables,

$$s = (p_1+p_2)^2, \qquad t = (p_1-p_3)^2, \qquad u = (p_2-p_3)^2, \tag{81}$$

and p_T is the transverse momentum of the Higgs particle,

$$p_T^2 = \frac{ut - M_h^4}{s}. \tag{82}$$

The individual contributions from the diagrams of Figs. 2 and 3 to $\mathcal{O}(\frac{1}{M_X^2})$ are

$$F_0^{(a)} = \frac{9M_h^2}{4(s-M_h^2)}\left[(1+\delta_t)F_{1/2}\left(\frac{4M_{T_1}^2}{s}\right) + (1+\delta_b)F_{1/2}\left(\frac{4M_{B_1}^2}{s}\right)\right]$$
$$F_0^{(b)} = \frac{9}{4}\delta_t F_{1/2}\left(\frac{4M_{T_1}^2}{s}\right) + \frac{9}{4}\delta_b F_{1/2}\left(\frac{4M_{B_1}^2}{s}\right)$$
$$F_0^{(c)} = -c_{gg}$$
$$F_0^{(d)} = -c_g\frac{3M_h^2}{s-M_h^2}$$
$$F_0^{(\mathrm{box})} = (1+2\delta_t)F_0^{(\mathrm{box,SM})}(s,t,u,M_{T_1}) + (1+2\delta_b)F_0^{(\mathrm{box,SM})}(s,t,u,M_{B_1}), \tag{83}$$

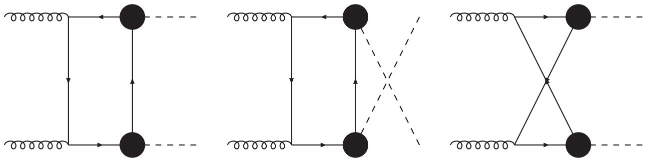


FIG. 3. Box contributions to $gg \to hh$. The dark circles represent the non-Standard Model contributions, while the solid lines are either t- or b quarks. The crossed diagrams from the initial state are not shown.

where $F_0^{\mathrm{box,SM}}(s,t,u,M_{T_1}) \to 1$ for $M_{T_1} \to \infty$ and $F_0^{(\mathrm{box,SM})}(s,t,u,M_j)$ contains the six box diagrams with a fermion of mass M_j in the loop. Analytic results can be found in Refs. [55,56].[3] In the effective theory, the spin-0 contribution is

$$\begin{aligned} F_0 &= F_0^{(a)} + F_0^{(b)} + F_0^{(c)} + F_0^{(b)} + F_0^{(\mathrm{box})} \\ &\to [1 - \delta_t - c_{gg}] - \frac{3M_h^2}{s - M_h^2}[1 + \delta_t + c_g], \end{aligned} \tag{84}$$

where the second line is found in the limit $M_{T_1}^2 \gg s$ and neglects the b contribution. Taking $c_{gg} = -c_g$,

$$F_0 \to [1 + \delta_t + c_g] F_0^{\mathrm{SM}}(M_{T_1} \to \infty) - 2(c_g + \delta_t). \tag{85}$$

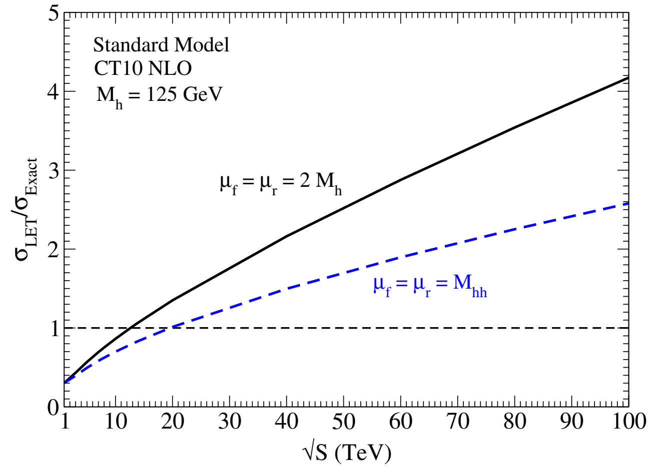
The c_g contribution is in agreement with the LET result of Eq. (74), while the δ_t contribution is no longer proportion to the Standard Model result.

The LET prediction for the total cross section for double Higgs production in the Standard Model normalized to the exact result is given in Fig. 4 as a function of center-of-mass energy. At $\sqrt{S} = 13$ TeV, the LET is a reasonable approximation to the total rate, while at higher energies the deviation from the exact result becomes large. We show this for two choices of factorization and renormalization scales, $\mu_f = \mu_r = 2M_h$ (solid line) and $\mu_f = \mu_r = M_{hh}$ (dashed line). The size of the deviation between the LET and exact calculation is very sensitive to the scale choices.

The divergence of the LET from the exact result can be understood by examining the partonic cross section for $gg \to hh$ shown in Fig. 5. For partonic subenergies above around 1 TeV, the LET and the exact results increasingly differ. The LET contains terms $\sim \frac{M_{hh}^2}{M_{T_1}^2}$, which are not present in the exact result.

The first hierarchy of small angles of Sec. IV reduces to the Standard Model, so we do not expect to gain insight from examining this limit. The second hierarchy (Sec. IV B), however, is more interesting. In Figs. 6 and 7 we show the total cross sections for $gg \to hh$ at $\sqrt{S} = 13$ TeV and 100 TeV as a function of the lightest top

[3]Our normalization is $\frac{3}{4}$ times that of Ref. [55] for the boxes.



(a)

FIG. 4 (color online). Standard Model rate for $pp \to hh$ from gluon fusion using the LET of Eq. (74) normalized to the exact cross section. This plot uses CT10NLO PDFs.

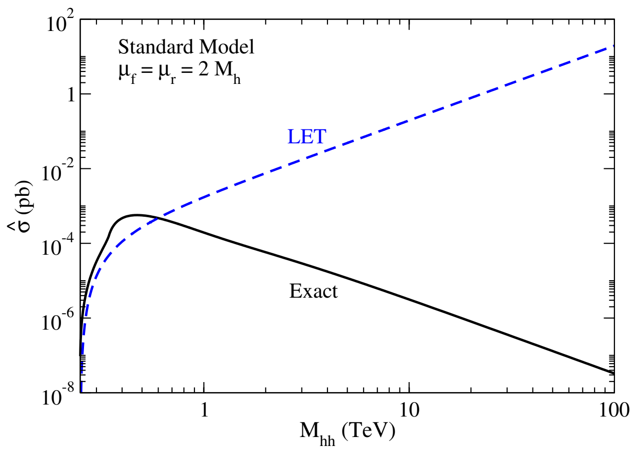


FIG. 5 (color online). Standard model partonic cross section for $gg \to hh$.

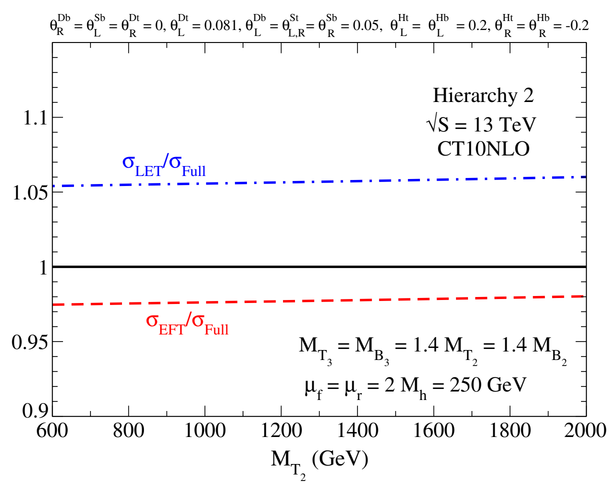


FIG. 6 (color online). Total cross section for $pp \to hh$ for a choice of small angles using the hierarchy of Sec. IV B. The EFT and LET results are normalized to the exact one-loop calculation.

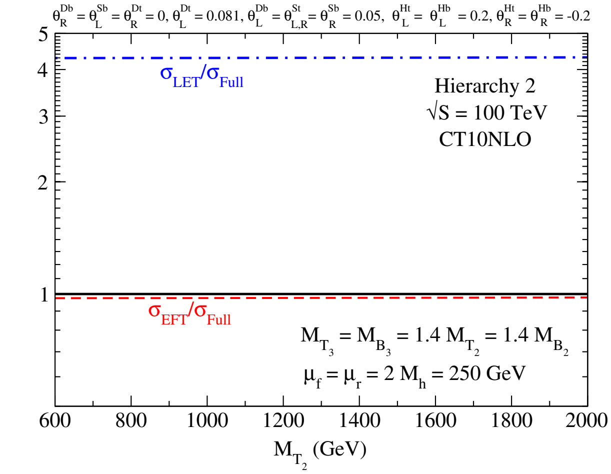


FIG. 7 (color online). Same as Fig. 6, except $\sqrt{S} = 100$ TeV.

partner mass, M_{T_2}, for a specific choice of small angles using the parametrization of Eq. (46). The LET significantly overestimates the rate at $\sqrt{S} = 100$ TeV, but is a reasonable approximation at $\sqrt{S} = 13$ TeV. The EFT, which contains the top and bottom quark contributions exactly, agrees within a few percent with the exact calculation. From Eqs. (49) and (51), we see that the EFT and LET depend on differences between the heavy vectorlike quark masses and not the overall mass scale. This result is confirmed in Figs. 6 and 7, which show all the results are insensitive to the heavy quark mass scale.

It is well known that the LET does not accurately reproduce distributions for double Higgs production [29,32,57]. For a choice of small angles and heavy quark

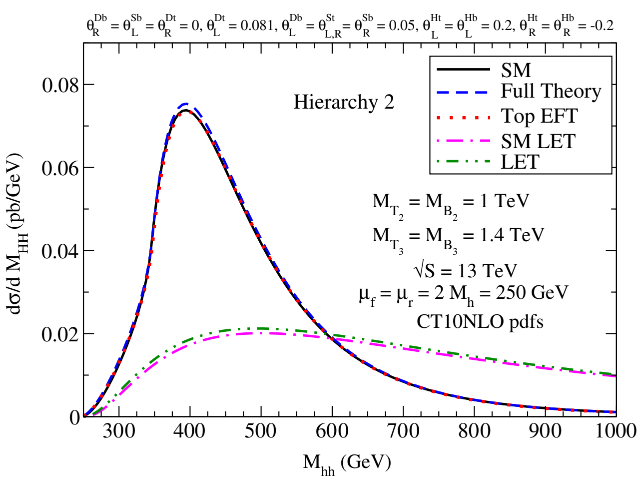


FIG. 8 (color online). Invariant mass distributions for $pp \to hh$ at the LHC. The SM and SM LET curves represent the exact Standard Model calculation, along with the LET limit. The curves labeled Full Theory, Top EFT, and LET are the top partner model in the small angle hierarchy of Sec. IV B, using the exact one-loop calculation, the EFT of Eq. (83), and the LET of Eq. (74).

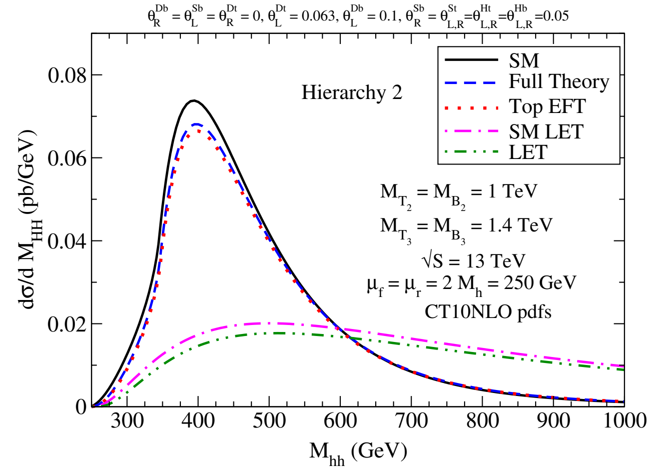


FIG. 9 (color online). Same as Fig. 8 with a different parameter point.

masses, we show the invariant mass distribution of the Higgs bosons, $\frac{d\sigma}{dM_{hh}}$, in Figs. 8, 9, 10, and 11 at the LHC with $\sqrt{S} = 13$ and 100 TeV. We include the Standard Model distributions for comparison. The LET does a poor job of reproducing the exact distributions, both in the Standard Model and in the top partner model. The curves labeled "SM" and "Full Theory" contain the exact one-loop calculations for the Standard Model and top partner model, respectively, while the curve labeled "Top EFT" is the top partner model calculation using the results of Eq. (33). The EFT reproduces the exact calculation quite accurately. We show this for two parameter points to illustrate the robustness of this conclusion. Both points reproduce the Standard Model single Higgs production rate to within ~10%. In a given model, therefore, the EFT can be used not only for the total rate, but also for distributions. The distributions in the top partner model are quite similar to the Standard Model. Scanning over small angles, we were not able to find an example with a large deviation from the Standard Model.

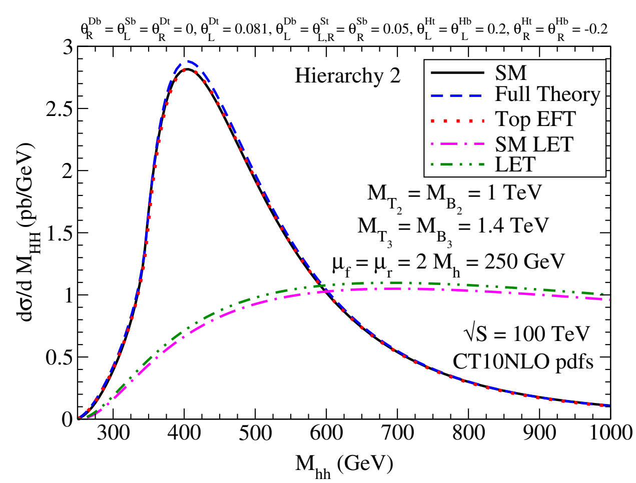


FIG. 10 (color online). Same as Fig. 8 with $\sqrt{S} = 100$ TeV.

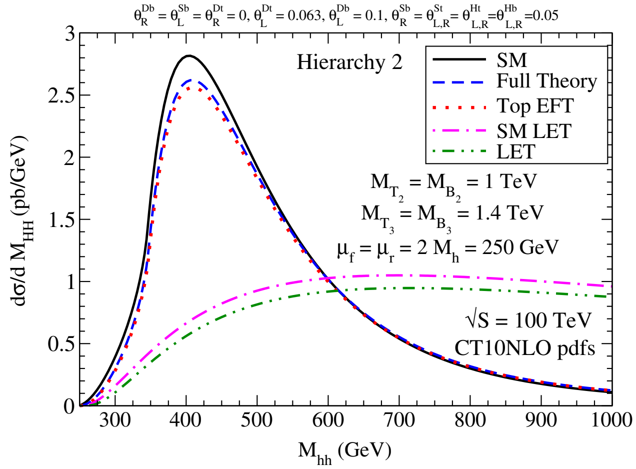


FIG. 11 (color online). Same as Fig. 9 with $\sqrt{S} = 100$ TeV.

VII. CONCLUSIONS

We considered a scenario with both $SU(2)_L$ singlet and doublet vectorlike fermions. Such a scenario could, in principle, have large deviations from the Standard Model predictions for single and double Higgs production. However, we were unable to find parameters consistent with electroweak precision measurements and the single Higgs production rate which gave a significant deviation from the Standard Model prediction for double Higgs production.

We constructed two versions of an effective theory. The well-known LET treats all fermions as infinitely massive. The total cross section for Higgs pair production is well approximated by the LET at $\sqrt{S} = 13$ TeV, but increasingly differs at higher energies. The LET cannot reproduce the invariant mass distribution of the hh pairs. In order to include top quark mass effects, we derived an effective Lagrangian (EFT) containing only light fermions, but with non-Standard Model coefficients, which we computed to $\mathcal{O}(\frac{1}{M_X^2})$. The EFT obtains accurate results for both total and differential double Higgs rates. Our results can be used to reliably compute the leading effects of models with heavy vectorlike fermions.

An important result is the observation that the coefficients of the effective Lagrangian of Eq. (33) are not free parameters, but are related to each other in any consistent model. Despite the proliferation of Yukawa couplings in Eq. (9), a consistent treatment yields an effective Lagrangian which depends on only three parameters, δ_b, δ_t, and c_g. This is similar to the case in composite Higgs models where deviations in Yukawa couplings and new effective operators relevant for double Higgs production are tightly correlated [35]. Hence, we expect that the EFT used to study Higgs production in composite Higgs models is a very good approximation to a complete calculation.

ACKNOWLEDGMENTS

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APPENDIX: OBLIQUE PARAMETERS

The limits on the parameters of the fermion sector arising from contributions to gauge boson two-point functions can be studied using the S, T, and U functions following the notation of Peskin and Takeuchi [52],

$$\alpha S = \left(\frac{4s_W^2c_W^2}{M_Z^2}\right)\left\{\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(M_Z^2) - \frac{c_W^2 - s_W^2}{c_W s_W}\Pi_{\gamma Z}(M_Z^2)\right\}$$
$$\alpha T = \left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right). \tag{A1}$$

In terms of the mixing angles and the mass eigenstates of the full theory, the contributions from heavy quarks, including the Standard Model top and bottom quarks, to ΔT and ΔS are [46,58][4]

$$\begin{aligned}
\Delta T = \frac{N_c}{16\pi s_W^2 M_W^2}&\{\Sigma_{i,j=1,2,3}[(|U_{L,ij}|^2 + |U_{R,ij}|^2)\theta_+(M_{T_i}, M_{B_j}) + 2U_{L,ij}U^\dagger_{R,ij}\theta_-(M_{T_i}, M_{B_j})] \\
&- \Sigma_{i<j=1,2,3}[(|X^t_{L,ij}|^2 + |X^t_{R,ij}|^2)\theta_+(M_{T_i}, M_{T_j}) + 2X^t_{L,ij}X^{t\dagger}_{R,ij}\theta_-(M_{T_i}, M_{T_j})] \\
&- \Sigma_{i<j=1,2,3}[(|X^b_{L,ij}|^2 + |X^b_{R,ij}|^2)\theta_+(M_{B_i}, M_{B_j}) + 2X^b_{L,ij}X^{b\dagger}_{R,ij}\theta_-(M_{B_i}, M_{B_j})]\} \\
\Delta S = \frac{N_c}{2\pi M_Z^2}&\{\Sigma_{i,j=1,2,3}[(|U_{L,ij}|^2 + |U_{R,ij}|^2)\psi_+(M_{T_i}, M_{B_j}) + 2U_{L,ij}U^\dagger_{R,ij}\psi_-(M_{T_i}, M_{B_j})] \\
&- \Sigma_{i<j=1,2,3}[(|X^t_{L,ij}|^2 + |X^t_{R,ij}|^2)\chi_+(M_{T_i}, M_{T_j}) + 2X^t_{L,ij}X^{t\dagger}_{R,ij}\chi_-(M_{T_i}, M_{T_j})] \\
&- \Sigma_{i<j=1,2,3}[(|X^b_{L,ij}|^2 + |X^b_{R,ij}|^2)\chi_+(M_{B_i}, M_{B_j}) + 2X^b_{L,ij}X^{b\dagger}_{R,ij}\chi_-(M_{B_i}, M_{B_j})]\},
\end{aligned} \tag{A2}$$

[4]We assume all entries in the mixing matrices are real.

where the functions $\theta_\pm, \chi_\pm$ are defined below and $N_c = 3$.

$$\begin{aligned}
\theta_+(m_1, m_2) &= m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \ln\left(\frac{m_1^2}{m_2^2}\right) \\
\theta_-(m_1, m_2) &= 2m_1 m_2 \left[\frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln\left(\frac{m_1^2}{m_2^2}\right) - 2\right] \\
\theta_+(m, m) &= 0 \\
\theta_-(m, m) &= 0
\end{aligned} \tag{A3}$$

and

$$\begin{aligned}
\psi_+(m_1, m_2) &= \frac{22m_1^2 + 14m_2^2}{9} - \frac{M_Z^2}{9} \log\left(\frac{m_1^2}{m_2^2}\right) + \frac{11m_1^2 + M_Z^2}{18} f(m_1, m_1) + \frac{7m_1^2 - M_Z^2}{18} f(m_2, m_2) \\
\psi_-(m_1, m_2) &= -|(m_1 m_2| \left[4 + \frac{1}{2}(f(m_1, m_1) + f(m_2, m_2))\right] \\
\chi_+(m_1, m_2) &= \frac{m_1^2 + m_2^2}{2} - \frac{(m_1^2 - m_2^2)^2}{3M_Z^2} + \left[\frac{(m_1^2 - m_2^2)^3}{6M_Z^4} - \left(\frac{M_Z^2}{2}\right) \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\right] \ln\left(\frac{m_1^2}{m_2^2}\right) + \frac{m_1^2 - M_Z^2}{6} f(m_1, m_2) \\
&\quad + \frac{m_2^2 - M_Z^2}{6} f(m_2, m_2) + \left[\frac{M_Z^2}{3} - \frac{m_1^2 + m_2^2}{6} - \frac{(m_1^2 - m_2^2)^2}{6M_Z^2}\right] f(m_1, m_2) \\
\chi_-(m_1, m_2) &= -|m_1 m_2| \left[2 + \left(\frac{m_1^2 - m_2^2}{M_Z^2} - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\right) \ln\left(\frac{m_1^2}{m_2^2}\right) + \frac{1}{2}(f(m_1, m_1) + f(m_2, m_2)) - f(m_1, m_2)\right] \\
\chi_+(m, m) &= 0 \\
\chi_-(m, m) &= 0
\end{aligned} \tag{A4}$$

and

$$\begin{aligned}
f(m_1, m_2) &= -\left(2\frac{\sqrt{\Delta}}{M_Z}\right) \left[\arctan\left(\frac{m_1^2 - m_2^2 + M_Z^2}{M_Z \sqrt{\Delta}}\right) - \arctan\left(\frac{m_1^2 - m_2^2 - M_Z^2}{M_Z \sqrt{\Delta}}\right)\right] \quad \text{if } \Delta > 0 \\
&= 0 \quad \text{if, } \Delta = 0 \\
&= \frac{1}{M_Z} \sqrt{-\Delta} \ln\left(\frac{m_1^2 + m_2^2 - M_Z^2 + M_Z\sqrt{-\Delta}}{m_1^2 + m_2^2 - M_Z^2 - M_Z\sqrt{-\Delta}}\right) \quad \text{if } \Delta < 0 \\
\Delta &= -M_Z^2 - \frac{m_1^4 + m_2^4}{M_Z^2} + 2m_1^2 + 2m_2^2 + \frac{2m_1^2 m_2^2}{M_Z^2} \\
&= -M_Z^2 \left(1 - \frac{m_1^2 + m_2^2}{M_Z^2}\right)^2 + \frac{4m_1^2 m_2^2}{M_Z^2}.
\end{aligned} \tag{A5}$$

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