

Self Competition Versus Group or Individual  
Competition in Learning the Four Fundamentals  
of Arithmetic, With Special Emphasis Given to  
Speed and Accuracy

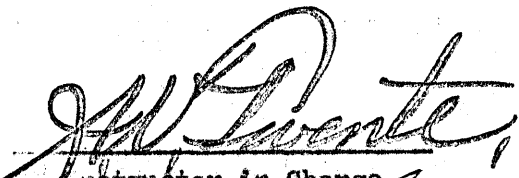
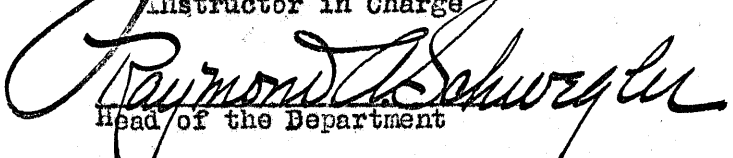
by

Roy E. Taylor

Bachelor of Science, Kansas State Teachers  
College, Pittsburg, Kansas  
1922.

Submitted to the Department of  
Education and the Faculty of the  
Graduate School of the University  
of Kansas in partial fulfillment  
of the requirements for the degree  
of Master of Science in Education.

Approved by

  
Instructor in Charge  
  
Head of the Department

June, 1927

## TABLE OF CONTENTS

	Page
Chapter I Introduction	1
Chapter II Related Studies	5
1. Lillian Shenk's Study	6
2. S. A. Curtis's Study	7
3. W. J. Osburn's Study	8
4. J. C. Brown's Study	8
5. H. C. Child's Study	9
6. W. L. Uhl's Study	9
7. Batson's and Combillicks Study	9
8. F. B. Knight's Study	10
Chapter III Specific Statement of the Problem, -- Method Used and Questions to be Answered.	11
1. Grades used.	12
2. Types of Tests Used	13
3. Experimental Method	15
4. Drill Methods Used	14
5. Character of Town and School Population	18
Chapter IV. Presentation and Interpretation of Data	20
1. Scores made Before and After Each Drill Period Beginning in March 1926	21
2. Scores Made Before and After Each Drill Period Beginning in October, 1926	23

3.	Total Gain Resulting from Drill in EF' (G.C.)	27
4.	Total Gain <sup>R</sup> esulting from Drill in EF'' (S.C.)	28
5.	Comparison of Gains in each EF	31
6.	Summaries for Comparison in Achievement in each EF	35
7.	Reliability of Differences	36
Chapter V. Summaries and Conclusions		55

## INTRODUCTION

Any plan which has as its objective an attempt to evaluate teaching methods in terms of pupil accomplishment is eminently worth while. The problem as here set forth has purposely been limited in scope, because the four fundamental operations of arithmetic can be handled conveniently, in an experiment such as is proposed, in a satisfactory manner.

It is practicable not only to measure the general conditions of arithmetic teaching throughout a school, and growth in ability and efficiency from year to year or grade to grade, the defects and needs of any one grade or grades, but the effects of changes in method or procedure as well. By a series of tests throughout a number of years it ought to be possible to build up a real science of teaching and to determine by strictly experimental method the truth or falsity of any teaching hypothesis.

The evaluation of teaching methods is not applicable to mathematics alone. Comparative teaching methods have lent themselves and will continue to lend themselves to other subjects. When one feels certain that a method is being used which has been proved valid (measured in terms of results secured) the element of uncertainty will disappear, and confidence together

with the right method will enable one to secure better results. The way in which a thing is presented to the mind of the learner is considered vital in the learning process.

Today we hear a great deal said about this or that method of teaching, but little is being done to determine by experimental methods the relative merits of the different methods. Much talk is current concerning individualized instruction. Carleton Washburn, Superintendent of Schools at Winnetka, Illinois, believes in individualized instruction, and has organized his school on this basis. The Gary Schools are also commended much upon and praised by educators for the fine work they are doing. The plan of instruction here as in Winnetka is largely individual. The Dalton plan is largely one of individual instruction in the secondary school division. There are many other schemes being evolved all over the country, but it seems that in all the major emphasis is being placed upon individualized instruction which allows each child to proceed at his own rate, that is, a rate where achievement is comparable with ability. The question to be asked here is whether each child gets as much out of his work when he is setting his own pace or does he get more out of school work when the pace is set by a fellow classmate?

As a result of the emphasis which has been given to

individualized instruction, many work books dealing with different subjects have been placed upon the market by the different publishing houses. These books make use largely of the element of self-competition. They come off the press highly advertised, and their sales are pushed by high pressure salesmen. For arithmetic many work books are represented to remedy all the defects in the children's work in a very short period of time. There is no doubt but that the material found in the work books is good, much of it at least, but how much use should be made of it is the question of paramount issue. For example, will these books serve as a good substitute for much of the work which has been carried on in a different way? Can formal drill work be carried on as effectively when these books are used as can be carried on without them? How can the fundamental skills of arithmetic be best fixed, by drill where the individual competes with himself, or by drill where competition is with others? There is no question but that remedial work can be carried on as effectively one way as the other. In the light of Osburn's (6) recent findings, the relative difficulties of numbers and their combinations are known. This material is now made available in a good many courses of study in arithmetic,

especially the St. Louis and Denver courses.

If it is discovered that better results can be secured by means of work books in arithmetic, then no expense should be spared to provide them for the children. On the other hand, if it is discovered that as good results may be secured without them, this money may be saved and the funds devoted to more worthy and profitable causes.

In conclusion it may be said that any work which is carried on in the right way and which attempts to evaluate teaching procedure is worth while. Thousands of books are leaving the press each year which are represented as the panacea for many of the existing shortcomings of school room instruction. A scientific evaluation of this material is needed. Individual instruction is now receiving a great impetus in many school systems, but whether this method of instruction will secure the desired results in formal drill in the fundamentals of arithmetic is questioned in this study.

## CHAPTER II

Related Studies

In looking over the literature which is related to the problem of this thesis, very little was found which related itself directly to the evaluation of drill methods in arithmetic.

Many different methods have been used in teaching the fundamental operations in arithmetic, but little has been done to evaluate the different methods. Good results have been secured by the use of many of these methods, but little or no scientific means have been used to check the results secured in one method with those secured in others. Often teachers become vitally interested in doing things by one particular method and quite often the results are counted good. Is it not probable that a different method pushed as enthusiastically would have secured results as good and perhaps better?

It is very often true that a novel plan for doing things is suggested and teachers adopt it with great eagerness. Sometimes the method is a good one and secures good results, sometimes it brings about good results largely because of the enthusiasm of the teacher, and often it fails in



spite of the interest with which the teacher receives it.

As was stated above, very little has been done in the scientific evaluation of different methods for handling drill work in arithmetic. A brief sketch of the literature which has been found helpful in working out this problem is given below. (The numbers in parenthesis after the names of those quoted corresponds with numbers given for the study in the bibliography.)

Miss Lillian Shenk (1) in her Master's Thesis attempted to prove the relative value of three types of drill work in arithmetic. She set to work to find out the vailidity of the particular kind of drill work in the Studebaker, Curtis, and dictated drill types. The dictated drill type as defined by Miss Shenk is drill given by the conventional method. The teacher dictated the problems and the pupils worked them. No rivalry was stimulated save that which naturally would exist between certain individuals in the room. The Curtis and Studebaker material probably needs no explanation, since it has been quite widely adopted and used. Suffice it to say that in using this material the children use small work books and solve graded exercises in the simple operations of arithmetic. The exercises are all timed and the individual scores recorded. The authors

provided cards upon which is found the drill material. Miss Shenk tried out each drill type in a different grade building in the city of Lawrence, Kansas. The experiment ran for 20 days. The Curtis Research Test was given as the initial and final test. On the basis of results secured, Miss Shenk concluded that the Practice Sets were more effective in teaching the fundamentals of arithmetic in grades five and six, and that dictated drill is more effective in grades four, seven and eight. Miss Shenk did no more to equate the teachers who taught the different drill types than to say that they were equally good and competent. The children were not equated on any other basis save buildings as far as could be discovered in reading her study. The reliability of the differences were not shown.

S. A. Curtis (2) has experimented extensively with his tests. His experiment performed in the Detroit Schools is published in "The Annual Accounting Series". Mr. Curtis claims his tests to be superior for three reasons, namely, (1) the material is standardized, (2) the material makes provision for supervisory test and reports, and (3) makes a direct appeal to the instincts of the child. Mr. Curtis reports the use of his practice material in many of the large city schools. He also reports great gain in

accomplishment where the Courtis Practice material is used. Nothing is said about giving an equal amount of time in drill to those children who did not use the Courtis Practice material.

In "The School Efficiency Monograph" (7) an account is given of a comparative study made in the schools of Cincinnati, Ohio. The results secured from the Courtis Practice Material and from other kinds of practice materials are reported. The pupils who used the Courtis material always excelled those who did not. At lowest they averaged 85% better in division, and at the highest they averaged 500% better in subtraction. Nothing is said in the Monograph as to just how the other drills were carried on, or how much time was given to them.

J. C. Brown (4) tested seventh and eighth grade pupils with Stone's Fundamental Test. Some children were used as control groups and given no drill at all. The gain as registered by the final test greatly favored the children who were taking the drill.

W. H. Osburn (6) points out the lack of adequate content in the Courtis Practice Sets. He determined by studying the relative difficulties of number combinations and certain arithmetical processes, that Courtis had not provided for enough drill on certain combinations while too

much was probably given to others. Osburn's major criticism of the Courtis Practice Pads was that the contents were not presented according to the relative difficulty of the drill material. He calls attention to many combinations which are necessary for a child to learn in order to become skilled in handling the fundamentals of arithmetic, but which we have been wholly unconscious of in our teaching.

H. G. Childs (5) experimented with the Courtis Practice Pads. He discovered that there was little or no transfer of training from one operation to another.

W. L. Uhls (5) points out the value of certain standard tests in arithmetic for the purpose of diagnosing pupils' difficulty. It seems that a great many errors in work may be pointed out and corrections applied by the intelligent use of practice sets.

Batson and Combellick (13) collaborated in making a study of the relative number difficulties, by studying the reaction time of pupils to number combinations. They point out that in many of the standard text books in arithmetic drill material does not appear in frequency according to its difficulty.

F. B. Knight (12) recorded on the basis of research the importance of building drill material according to exact specifications. Knight points out the careless haphazard manner in which drill work is usually given and suggests this as the reason for children failing to master the fundamentals as they should. He attempts to present drill in his own work books according to the needs of the children and relative difficulty of test material.

After reviewing the above material, there still appears to be a need for scientific evaluation of many of the methods used in teaching drill work in arithmetic.

The question which this study proposes to answer, especially for the School at Herculaneum, Missouri, is one that has not been satisfactorily answered in the mind of the writer, that is, which of the two types of drill mentioned in this study will excel in the grades at Herculaneum.

## CHAPTER III

Specific Statement of the Problem.Questions to be Answered and Method Used.

Do the pupils in grades four to eight inclusive of the schools of Herculaneum, Missouri, achieve more in the number of problems worked right and in accuracy when drill in the four fundamental operations of arithmetic (addition, subtraction, multiplication, and division) is given so as to encourage group or individual competition, or do these same pupils show greater improvement in the number of problems worked right and in accuracy, when drill is given so as to stress self-competition? Accuracy as here used means the ratio of problems worked right to those attempted or tried. For example, pupil A works 4 problems right out of 10 attempted or tried on an addition test. After 30 days of drill the same pupil works 6 problems right out of ten attempted. Pupil A made an accuracy score of 40% on the first test and 60% on the second test. He gains 20% in accuracy and 2 (6-4) in Rights.

An attempt will be made to answer these questions in connection with the study: What type of drill and in what

operations do the grades show the greatest improvement for accuracy? What type of drill and in what operations do the grades show the greatest improvements for Rights?

At the outset of the experiment it was planned to use grades three to eight inclusive, but it was found necessary to carry the work over a period of two years, and the eighth grade for the first year was lost through graduation. The grades actually used in this study are, then, Grades three to seven inclusive for the school year 1925-26 and grades four to eight inclusive for the year 1926-27.

In the beginning, an initial test (IT), the Curtis Research Test in Arithmetic, Series B, Form 2, was given to all grades above the third. The material found in the Curtis Research Tests was too difficult for the third grade, so multiplication and division were eliminated and instead of using the nine addends which appear in the Curtis addition problems, only the lower five were used. In subtraction the first four digits in the minuend were taken and the first three in the subtrahend ("first" here means beginning at the left of the problem). This was done in order to make the test material comparable.

After the problems were selected for the third grade, copies were mimeographed for each pupil. The third

grade took all the initial and final tests taken by the other grades only in the abbreviated form outlined above.

In giving all of the initial and final tests, the instructions printed on the Curtis Research Tests were carefully followed.

The following represents the procedure followed in working out the problem:

A ----- IT' - EF' - FT' - EF'' - FT'' ----- 25 - 26

B ----- IT' - EF' - FT' - EF'' - FT'' ----- 26 - 27

Where IT' represents the initial test -- EF' six weeks of dictated drill for six minutes per day; FT' represents the final test for the first six week drill period; EF'' is the second experimental factor, or the drill method using self-competition. FT'' is the final test coming after the six weeks of drill in which self-competition is used.

"A" above was carried on during the school year 1925 and 1926. "B" is identical with "A" but was carried on during the first part of the school year 1926-27.

IT' opposite "A" was given the first Monday in February, 1926. FT' was given in May. Twelve weeks of time elapsed between IT' and FT''.



During the school year 1926-27, as suggested above, the same experiment in the same order was repeated beginning in October. October was set as the best time to give the initial test because this allowed three weeks of schooling and time to wear off the rust accumulated during the summer. The children did enough drill work during this three week period to restrengthen the bonds which had weakened during the summer vacation. The results of the initial test in October showed the children had made little or no progress since the last test in May.

The methods were rotated in order to eliminate as much transfer as possible. If one method had directly followed another and the experiment stopped there, the growth in ability to do the fundamentals that took place during the second drill period might have been unduly influenced by the first drill method.

While the pupils who were being given drill EF'' might have been helped some by the results of drill EF' the very fact that EF' followed EF'' the next year should have cared for any transfer that might have taken place.

The sum of the gains made by the different classes in Rights and Accuracy in each operation for the two six week drill periods where the method of self-competition was

used, compared with the sum of the gains made in each operation for the two six week periods in which competitions between members of the class resigned, should give some insight into the relative merits of the two systems.

In order to carry on the work in EF" (Self Competition) the Courtis Practice Sets were used as basic material in all the grades taking part in the experiment. Six minutes of drill were given at the beginning of each arithmetic period on the fundamentals of arithmetic, addition, subtraction, division, and multiplication.

For EF' drill work was carried on by the teacher dictating problems in the four fundamentals and allowing the pupils to compete with one another individually or in groups. The members of the class worked with this drill material either at the blackboard or at their seats. They were all kept busy. This drill was carried on largely in the conventional manner. The idea stressed to motivate this type of drill was, work to excel the other pupil or pupils. Competition was not limited to certain members of the class, but every member competed with all. No records at all were kept of individual or class scores.

The drill EF" (Self-Competition) followed drill EF' (Group or Individual Competition). The final test for EF' also served as the initial test for EF". Directions

outlined by the author of the Courtis Practice Sets were followed, except for the time limit. Six minutes of formal drill were given each day for 30 days. The pupils in Herculaneum were much below the average pupil in handling the fundamentals of arithmetic for their particular grades, so sufficient material was provided in the Practice Sets for each assignment to keep all the pupils busy. Each drill exercise was done and individual scores recorded either on graphs provided by the author of the Practice Sets or on specially prepared graph sheets made by the pupils under the supervision of the instructor. During this drill period the pupils recorded their scores in such manner that each knew from day to day and week to week just what progress he was making in his work. Much of the test material found in the Practice Sets is comparable, making it possible for a pupil to express graphically his accomplishment over a period of time. During this entire drill period nothing was said or done, purposefully on the part of the teacher, to call attention to any pupil the scores of others. No scores were advertised. Each pupil worked so far as the situation could be controlled, to better his own previous performance without thought of excelling the others.

During the entire run of the test period the arithmetic of grades three to six inclusive was taught by one teacher. The arithmetic for the seventh and eighth grades was taught by another. Both teachers were well trained to teach arithmetic. Each had had several years of teaching experience and entered into the experiment with enthusiasm. The drill work was first gone over carefully with the teachers and then printed instructions concerning procedure were given to each. Both teachers were strongly urged to be as enthusiastic in giving one drill as the other. Neither expressed a mind set in favor of either EF. They seemed very anxious to do what could be done to evaluate the two drill types for the Herculaneum School in terms of results secured in Rights and Accuracy. The Superintendent of Schools who had initiated the problem visited the classes often during the experiment and gave it most careful supervision. All of the initial and final tests were given by the Superintendent, the papers graded by the teachers and then carefully checked in the office.

Several pupils who were in school when the experiment began dropped out of school before its completion. No scores were tabulated nor used in connection with this work unless the pupil was present to take all the tests. It might be stated here that a check was made to determine relative attendance for the two EF's. The per cent of

attendance for all the pupils during the two drill periods of E. F' was 97.5, and the per cent of attendance during the two drill periods of EF'' was 97.6. There was no grade showing a difference of more than 1.5% in favor of attendance for either drill period. It is therefore seen that the factor of attendance played little or no part in favor of either drill.

Up until the school year 1925-25 very little if any formal drill work had been given in the school for several years. All the drill which the pupils had received in the fundamentals of arithmetic was received in connection with the regular work in arithmetic, that is, no regular drill period was given over to the mastery of the fundamental skills in arithmetic. The school rooms had been badly crowded, the tenure of teachers not more than two years and most of the time just one.

The town has a population of about 2500. It is strictly industrial in nature. Lead smelting is the principal industry. The population consists largely of unskilled workmen. There is much illiteracy among the parents of the school children. On the basis of I. Q. determined by means of the National Intelligence Test, the children here are below the mean of the children in the average community. There is a possibility that home training and environment as well as heredity influenced

these scores. The character of the people, the past conditions of the school plant, and the general nature of the town all contributed to a lack of adequate learning on the part of the pupils. The first initial test in arithmetic pointed out very clearly the need of more emphasis on drill in the fundamentals. Progress in the development of these skills naturally has been slow. The problem to solve, of course, was the evaluation of the drill methods. The status of the children at the beginning of the experiment doesn't matter much. The above conditions are pointed out merely to indicate the general situation, so that a little more light might be shed on the results herein tabulated.

## CHAPTER IV

Presentation of DataExplanation of Table 1-A

This table is read as follows: The third grade worked 37 problems right in addition on the Curtis Research Test and attempted 97. This gave the class a per cent of accuracy of 38.1. The Rights divided by the attempts equals the accuracy. These results were secured for the initial test under EF<sup>v</sup>. On the final test for EF<sup>v</sup> (Group or Individual Competition) the class worked 78 problems correctly and attempted 159. The per cent of accuracy here for addition is 49.06. The results in this second test also served as the initial test for EF<sup>iv</sup> (Self-competition). It might be well to state in this connection again that the Curtis Research Tests were used in all cases for the initial and final tests. Different series were used each time to avoid any familiarity with the test.

For the third and final test after the class had been given drill on EF<sup>iv</sup> for six weeks, the grade worked 129 problems right and attempted 242 for an accuracy per cent of 53.47.

The numbers in parenthesis after each grade represents the number of pupils taking part in the experiment.

The scores for all grades in the three test <sup>are</sup> read the same as addition for the third grade.

The results above were secured during the latter part of the school year 1925-26.

TABLE I A  
 SCORES MADE BEFORE AND AFTER EACH  
 DRILL PERIOD BEGINNING IN MARCH  
 1926

GRADE	Initial Test for E.F.			Final Test For EF <sub>1</sub> and Initial Test for EF <sub>2</sub>			Final Test for EF <sub>2</sub>		
	R	A	ACC %	R	A	ACC %	R	A	ACC %
5rd Grade (24)									
Addition	37	97	38.1	78	159	49.06	129	242	53.47
Subtraction	15	60	25	50	111	45.04	68	185	56.76
4th Grade (55)									
Addition	15	91	16.5	40	116	35.4	56	174	32.1
Multiplication	1	38	26.3	64	127	50.4	176	307	57.3
Division	0	31	0	92	274	33.6	210	315	66.9
Subtraction	10	77	12.9	38	85	44.7	68	180	37.2
5th Grade (54)									
Addition	34	167	20.4	77	189	40.7	97	202	48.02
Multiplication	19	119	15.9	80	150	53.3	74	163	46.6
Division	9	111	8.2	17	103	16.5	57	129	21.7
Subtraction	42	145	28.9	76	189	40.2	124	212	58.5
6th Grade (23)									
Addition	60	167	35.93	101	184	54.35	135	216	62.5
Multiplication	52	120	43.38	88	152	57.9	103	176	59.31
Division	41	83	49.39	68	119	57.14	112	149	75.2
Subtraction	110	180	61.11	122	187	64.49	167	210	79.52
7th Grade (15)									
Addition	38	122	31.1	73	157	46.5	84	150	56
Multiplication	49	103	47.5	92	159	57.87	102	141	72.3
Division	36	73	49.31	75	114	65.79	69	101	68.3
Subtraction	75	129	59.7	108	161	67.1	123	162	75.93



Explanation of Table 2-A

This table is read as follows: The *Fourth* grade worked 138 problems right and attempted 194 with a per cent of accuracy of 71.2. For the final test for EF<sup>1</sup> and the initial test for EF<sup>2</sup>, which came after six weeks of drill in which the class competed with each other and with groups, the *fourth* grade worked 165 addition problems correctly and attempted 242, securing a per cent of accuracy of 68.19.

For the final test under EF<sup>2</sup>, coming after six weeks of drill in which self-competition was used, the *fourth* grade worked 290 problems right and attempted 356 for an accuracy per cent of 81.5.

The number after each grade in parenthesis equals the number of pupils participating in the work.

The initial test for EF<sup>1</sup> in table 2-A was given in October of the school year 1926-27. The final for EF<sup>1</sup> and the initial for EF<sup>2</sup> was given after 50 days of drill in group or individual competition. The final for EF<sup>2</sup> came after 30 days of drill in self-competition. Sixty days of drill elapsed for each grade from the time of the first test until the last.

The scores for the other grades and operations are read from table 2-A the same as addition of the *fourth* grade.

TABLE II A

SCORES MADE BEFORE AND AFTER EACH  
DRILL PERIOD BEGINNING OCTOBER  
1926

GRADE	Initial Test for EF			Final Test for EF <sub>1</sub> & Initial Test for EF <sub>2</sub>			Final Test for EF <sub>2</sub>		
	R	A	Acc %	R	A	Acc %	R	A	Acc%
4th Grade (24)									
Addition	138	194	71.2	165	242	68.19	290	356	81.5
Subtraction	92	153	60.12	127	212	55.2	162	245	66.1
5th Grade (33)									
Addition	57	169	53.7	77	177	43.5	103	202	50.9
Multiplication	35	117	30	63	156	40.4	84	173	49.6
Division	17	42	40.5	28	93	30.1	51	131	38.9
Subtraction	73	178	41.6	86	168	51.1	113	196	57.6
6th Grade (35)									
Addition	115	208	55.3	112	222	50.5	128	240	52.5
Multiplication	85	150	56.3	97	188	51.6	116	213	54.4
Division	57	110	47.3	68	136	43.6	81	144	56.2
Subtraction	147	237	62	156	207	75.3	168	261	64.4
7th Grade (13)									
Addition	106	173	59.55	141	219	64.43	145	220	65.9
Multiplication	87	135	64.4	118	177	66.67	135	233	57.9
Division	57	110	47.3	68	136	43.6	81	144	56.2
Subtraction	147	237	62	156	207	75.3	187	229	81.6
8th Grade (15)									
Addition	85	139	61.22	105	162	64.81	91	161	56.52
Multiplication	84	118	70.8	78	149	52.3	92	152	60.53
Division	62	85	72.9	103	136	68.4	96	122	79.51
Subtraction	129	145	88.96	116	150	77.33	115	160	71.87

Explanation of Tables 3-A and 4-A

The data for these tables are taken from tables 1A and 2A. Referring to table 1A, the third grade worked 57 problems right in addition on the initial test for EF' and at the conclusion of the 30-day drill period, the class worked 78 problems correctly. Seventy-eight is found under final test for EF'. The gain between the number right on the initial test and the number right on the final test (78-57) is 41. Forty-one is found in the first column of table 3A under Right. (R)

In attempts the third grade in addition gained 62. Referring again to table 1A, 97 problems were attempted on the initial test for EF' and 159 problems were attempted in the final test for EF'. The difference between the attempts here is 62. Sixty-two is found in table 3A under (a) attempts for EF'.

The difference between the accuracies for the initial test for EF' and the final test for EF' on table 1A is (49.06% - 38.1%) or 10.96%. This per cent gain in accuracy is found in table 3A under accuracy for EF'. The gains for the other grades in all the different operations have been secured in the same manner as those for the third grade and are read in the same way.

The gains in (R) rights, (A) attempts, and accuracy (acc.) in table 3A under the EF'' are taken from table 2A.

The fourth grade (the third grade the year before) worked 138 addition problems right in the (IT) initial test for EF' and 165 correct in the (FT) final test for EF'. Twenty-seven problems is equal to the gains in rights between the initial and final test for EF', in table 2A, for fourth grade addition. This score 27 is found under (R) for fourth grade addition and under 2nd EF' in table 3A.

The attempts and accuracies in this table are figured in the same way as are Rights.

The combined gain for table 3A is found by adding the gains in rights for both drills as well as the gains in attempts and accuracies. For example, 41 plus 27 is equal to the total gain resulting from the two drill periods in EF' (Group or Individual Competition). Sixty-two plus 48 is equal to 110, the gains in Attempts. These combined gains are for fourth grade addition.

Table 4A is read the same as table 3A. The gains which are tabulated on this table though are for EF'' (Self Competition). The difference between the scores made on the initial tests and the final tests for each EF'' in table 1A and 2A summated is equal to the gain resulting from the EF''s. Referring again to table 1A, the third grade worked 78 problems right on the initial test for EF'' and for the Final Test the class worked 129 problems correctly. The gain here is 51. Fifty-one is found in the first column of table 4A .

Table 4A is similar to 3A, except that it represents the gain in (R), (A) and (Acc.) which took place as a result of EF". The data for table 4A are taken from table 1A and 2A. Referring to table 1A, the initial tests score in rights for the third grade addition under EF" is found to be 78 and the final score 129. This is a gain of 51 in rights; the gain in attempts is 85, and the gain in accuracy is 4.41%. These gains are placed in the appropriate columns in table 4A.

From table 2A the "gains" for EF" are calculated in the same way as for EF'. The total gains are also shown in table 4A.

TABLE III A  
TOTAL GAIN RESULTING FROM  
EACH EF<sub>1</sub>

GRADES	Group Competition Gain in 1st EF <sub>1</sub>			Group Competition Gain in 2nd EF <sub>1</sub>			Total Gain		
	R	A	Acc %	R	A	Acc %	R	A	Acc %
Grade 5									
Addition	41	62	10.96	27	48	-3.01	68	110	7.95
Subtraction	35	51	20.04	35	59	-4.92	70	86	15.20
Grade 4									
Addition	25	25	18.9	20	8	9.8	45	33	78.7
Multiplication	63	89	47.77	28	39	10.4	91	128	58.81
Division	92	243	53.6	9	51	-10.4	101	294	23.2
Subtraction	28	8	31.8	15	-10	9.5	41	-2	21.8
Grade 5									
Addition	43	22	20.3	-3	14	-4.8	40	34	15.5
Multiplicat.	61	31	28.8	12	38	-4.7	73	69	24.1
Division	8	-8	8.3	16	26	-3.7	24	18	4.6
Subtraction	34	44	11.3	9	-30	13.3	43	14	24.6
Grade 6									
Addition	41	17	18.42	35	41	4.88	76	58	23.30
Multiplication	36	52	14.57	31	42	2.27	67	74	16.84
Division	28	36	7.75	36	53	12.73	82	89	20.48
Subtraction	12	7	3.38	11	28	-4.85	23	35	-1.07
Grade 7									
Addition	35	35	15.40	20	23	3.59	55	58	18.99
Multiplic.	43	56	10.37	-6	31	-18.5	37	87	-8.13
Division	39	41	16.48	41	51	-4.5	80	92	11.98
Subtraction	33	32	7.4	-13	5	-11.63	20	37	-4.23

TABLE IV A  
TOTAL GAINS RESULTING FROM EACH EF<sub>2</sub>

GRADES	Gains for First EF <sub>2</sub>			Gains for Second EF <sub>2</sub>			Total for EF <sub>2</sub>		
	R	A	ACC %	R	A	ACC %	R	A	ACC %
Grade 3									
Addition	51	83	4.41	155	114	13.31	186	197	17.72
Subtraction	18	74	-8.28	55	33	10.9	53	107	2.62
Grade 4									
Addition	16	58	-5.3	26	25	7.4	42	83	4.1
Multiplic.	112	180	6.9	21	17	9.2	133	197	16.1
Division	118	40	33.3	23	38	8.8	141	78	42.1
Subtraction	30	95	-7.5	27	28	6.5	57	123	-1
Grade 5									
Addition	20	13	7.52	16	18	2.8	36	31	10.12
Multiplic.	14	15	1.9	19	25	-6.7	33	38	-4.8
Division	20	26	9.5	13	8	12.6	33	34	22.1
Subtraction	48	23	18.5	12	54	-10.9	60	67	7.4
Grade 6									
Addition	34	32	8.15	4	1	1.47	38	36	9.62
Multiplic.	15	24	1.41	17	56	-8.77	52	80	-7.36
Division	44	30	18.06	5	6	0	49	56	18.06
Subtraction	45	23	15.03	42	22	11.55	87	45	26.58
Grade 7									
Addition	11	-7	9.5	-14	-1	-8.29	-3	-8	1.21
Multiplication	10	-18	14.43	14	3	8.23	24	-15	22.66
Division	-6	-13	2.51	-7	-14	11.11	-15	-27	11.49
Subtraction	15	1	8.83	-1	10	-5.46	14	11	3.57

Explanation of Table 5A

The total gain for EF' and EF'' in the five classes and in each operation for Rights (R), Attempts (A), and Accuracy (Acc.) are shown in this table, and comparisons made between the gains in each EF' to determine which contributed to the greater gain.

The fourth grade made a total improvement of 68 in Rights, 110 in Attempts, and 7.95% in Accuracy for the two drill periods EF'. The same grade made a total gain of 176 in Rights, 197 in Attempts, and 17.7% in Accuracy during the drill periods EF''. All the above gains are taken from fourth grade addition. The improvements of all the grades in the different operations are read the same as that for fourth grade addition.

Under "Gains for EF' " and "Gain for EF'' " in the table is shown the difference in (R), (A), and (Acc.), favoring the EF. For example, using the scores made in Rights for fourth grade addition again, +68 represents the total gain in Rights for EF' and 176 the total gain in Rights for EF''; and the difference between these Rights is 108. This improvement favors EF'', so 108 is written under (R) for "Gains in EF''". Attempts and Accuracy are figured in the same way for all the operations in each grade.

This table indicates in a general way the drill which excelled in each grade for the different operations. No grade seems to be favored exclusively by either method.



The question which this table naturally raises is whether the differences in favor of EF' and EF'' are really significant differences.

Reliability of Difference Tables have been worked out for the different grades and in each operation to determine just how reliable the mean differences are.

TABLE V A  
COMPARISON OF GAINS IN EACH EF

GRADES	Total Gains in EF <sub>1</sub> G.C.			Total Gains in EF <sub>2</sub> S.C.			Gain favoring EF <sub>1</sub>			Gain Favor- ing EF <sub>2</sub>		
	R	A	Acc %	R	A	Acc %	R	A	Acc %	R	A	Acc
Grade 4												
Add.	68	110	7.95	106	197	17.72				118	67	9.7
Subtr.	70	86	15.2	53	107	2.62	17		12.58		21	
Grade 5												
Add.	45	35	28.7	42	83	4.1	3		24.6		50	
Mult.	91	128	58.81	133	197	16.1			42.71	41	69	
Div.	101	294	23.2	141	78	42.1		216		40		18.9
Sub.	41	-2	21.8	57	123	-1			22.8	16	125	
Grade 6												
Add.	40	54	15.5	36	31	10.12	4	3	5.58			
Mult.	73	69	24.1	23	38	-4.8	40	36	28.9			
Div.	24	18	4.6	53	24	22.1				9	16	17.5
Sub.	43	14	24.6	60	67	7.4			17.2	17	53	
Grade 7												
Add.	76	58	23.3	38	36	9.62	38	22	13.68			
Mult.	67	74	16.84	32	80	-7.36	35		24.20		6	
Div.	83	89	20.48	49	36	18.06	34	53	2.42			
Sub.	23	35	-1.07	87	45	26.58				64	15	26.
Grade 8												
Add.	55	58	38.4	-3	-8	1.21	58	66				17.78
Mult.	37	87	-8.13	24	-15	22.66	23	102				30
Div.	80	92	11.98	-13	-27	11.49	93	119	1.43			
Sub.	20	37	-4.23	14	11	3.37	6	26				7.6

Explanation and Interpretation of Table Fifteen B

In this table ~~is~~<sup>are</sup> summarized all the data appearing in table 1-B to 14-B, and also that from similar tables not appearing in this work.

The  $\sigma$  for the gains in rights in fourth grade addition is 4.24, the  $\sigma$  for the gains in the per cent of accuracy 31.8, the mean score for gains in accuracy 11.25%, and the mean score for gains in rights 2.83. The values above were attained while the grade was being given drill on EF' (Individual or Group Competition).

Under EF'' in the table is found 5.86, which is the  $\sigma$  for the distribution of gains in rights, 53.8 for the distribution of gains in accuracy, 31.25, the mean per cent gain in accuracy, and 7.33, the mean gains in rights. These figures are all taken from fourth grade addition in the table.

The formula  $\frac{D}{\sqrt{D}}$  is the reliability of difference formula which is used here to ascertain if the difference<sup>th</sup> score under each EF for rights and accuracy is a significant difference. The fourth grade class in addition made a mean score of 3.83 in Rights under EF' and a mean gain in Rights of 7.33 under the drill EF''. The question naturally arises as to the significance of the difference. The difference is seen to be 3.50 in favor of EF''. Would further testing of the two drill methods (EF' and EF'')

give comparable results, or is it probable that the results would be reduced to zero? This difference probably diverges from the true difference and to find out what the divergence is the reliability of the difference was calculated. The B series of tables will show how this was done.  $\sigma_{av}$ . (Sigma of the average) was determined from the Reliability Table. The  $\sigma_{av}$ 's were then substituted in the formula  $\sigma_{Diff} = \sqrt{(\sigma_{av}')^2 + (\sigma_{av}'')^2}$ . The sigma of the difference for 'rights' in fourth grade addition is thus found to be 1.17. This is also the Standard Error of the difference between the mean scores in 'rights' ( 7.33 - 363 ), 3.5.  $\sigma_{Diff}$  is interpreted to mean that in 68 times out of a 100 the true difference between the means in 'rights' does not vary from the obtained difference (3.5) by more than plus or minus 1.17 'rights'. The chances are 68 in 100 that the true difference lies within the limits, 3.5 plus or minus 1.17.

The  $\left( \frac{D}{\sigma_{Diff}} \right)$  between the mean score in 'rights for EF' (3.83) and the mean score in 'rights for EF'' (7.33) is 3.6 for fourth grade addition. This was obtained by dividing 3.5 by 1.17. The 3.5, as explained above, is the difference between the mean 'rights' for fourth grade addition, and the 1.17 is the Sigma Difference.

Translating 3.6 into chance by means of tables found in " Statistics in Education and Psychology " by Henry E. Garrett (7) it is discovered that in 99.9 times out of a hundred similar results will be secured from the two drill methods under the

same conditions. EF" proved to be superior to EF' in learning the fundamental skills needed to do addition in the fourth grade of the Herculaneum Schools.

The reliability of the difference in accuracy between the two drill methods was determined in the same way in which the reliability of the difference in Rights was determined. The results for each fundamental operation in arithmetic for each class were calculated in the same manner as explained above for fourth grade addition. The B series of tables will enable the reader to check and understand better the contents of table 15.

In columns 13, 14, 15, and 16 the  $\sigma$  of reliability differences in both rights and accuracy have been so arranged as to enable the reader to tell at a glance the particular EF" which the  $\sigma$  of reliability favors.



Explanation of Reliability of Difference Tables.

1. Accuracy Tables:--

All accuracy tables are read the same as Table 7B (Seventh Grade Subtraction).

The accuracy scores which make up the distribution found ~~found and~~ in all the accuracy tables were determined as follows: Pupil A made an accuracy score of 20% on the initial test for group competition and 90% on the final test. A's gain in percent of accuracy for this drill period is 70%. For drill on the same operation the next school year, pupil A makes an accuracy score of 50% on the initial test for group competition, and on the final test a score of 60%, thus showing a gain of 10% in the final test over the initial test. His total gain in per cent for both drill periods in group competition is 80% ( 70 + 10 ).

After the gains in percent of accuracy were calculated for every pupil and in each operation, distribution tables were made as 7B. These tables were used to secure certain definite information relative to the achievement in both drill methods. (EF' and EF'').

In table 7B are found the scores of 25 pupils.  $N = 25$ . The average 'gain in per cent for group competition', for EF', calculated from the table is 5% and for EF'' 37.61%.

$\sigma$  for EF' is 30.8 and for EF'' 40.2.

The  $\sigma_{av}$  for EF' is 6.42 and for EF'' 8.57.

The  $\sigma_{DFF}$  is 10.55

$\sqrt{\frac{D}{D_{FF}}}$  is 3.08  $\sigma$

The  $\sigma$  for EF' means that practically 68% of all the accuracy scores fall between 5% and  $\pm 30.8\%$ . This indicates that the scores are not grouped near the median, but are widely scattered.

The  $\sigma_{av}$  for EF' is 6.52. This means that the chances are 68 in 100 that the true mean does not vary from the obtained mean by more than 1, that is by more than  $5 \pm 6.42$ .

The  $\sigma_{Diff}$  means that the chances are 68 in 100 that the actual difference between the means ( $37.61 - 5$ ) does not vary from the true difference by more than  $32.61 \pm 10.55$ .

The  $D$   
 $\sqrt{Diff} (3.08)$  means that the chances are 99.9 in 100 that the mean gain in accuracy for EF' and EF'' will be greater than zero and in favor of EF'.

#### Right Tables :-

In these tables are shown the 'Gains' in the number of problems worked right for every pupil in both drill methods. For example:- Pupil A works 3 problems right in the initial test for EF' and 4 problems right in the final test. For this drill period a gain of 1 is shown in 'rights'. For the next drill period in EF', pupil A works 5 problems in the initial test and 7 in the final. A gain of 2 in 'Rights' is shown for this drill. The total gain for the two drill periods EF' is three in 'rights'. Accomplishment for every pupil in all the operations and for both drills (EF' and EF'') was determined as that of pupil A and then distribution tables made.



The findings of the tables showing the distribution of 'right' are interpreted the same as for the accuracy tables.

Note: Not all the tables are shown in this study. The summaries from all the tables are found in Table 15B, however.

TABLE I B

RELIABILITY OF DIFFERENCES IN ACCURACY  
FOURTH GRADE SUBTRACTION

GROUP COMPETITION					SELF COMPETITION				
RIGHTS ACC. %	F	D	FD	FD <sup>2</sup>	RIGHTS ACC. %	F	D	FD	FD <sup>2</sup>
90-100	1	8	8	64	90-100	3	8	24	192
80-90	0	7	0	0	80-90	2	7	14	98
70-80	1	6	6	36	70-80	1	6	6	36
60-70	1	5	5	25	60-70	0	5	0	0
50-80	1	4	4	16	50-60	1	4	4	16
40-50	0	5	0	0	40-50	1	3	3	9
30-40	6	2	12	24	30-40	1	2	2	4
20-30	0	1	0+35	0	20-30	1	1	1+54	1
10-20	2	0	0	0	10-20	5	0	0	0
0-10	4	-1	-4	4	0-10	2	-1	3	2
-10-0	2	-2	-4	8	-10-0	1	-2	-2	4
-20-10	1	-3	-3	9	-20-10	1	-3	-3	9
-30-20	0	-4	0	0	-30-20	0	-4	0	0
-40-30	2	-5	-10	50	-40-30	2	-5	-10	50
-50-40	1	-6	-6	36	-50-40	2	-6	-12	72
-60-50	2	-7	-14-41	98	-60-50	1	-7	-7-37	49
	N=24		-6	370		N=24		+17	533

$$C = \frac{6}{24} = .25 \quad C^2 = .0625$$

$$G.A. = 15$$

$$Av. = 15 + 10(-.25) = 12.5$$

$$\sigma = \sqrt{\frac{370}{24} - .0625} \times 10 = 39.2$$

$$\sigma_{Av.} = \frac{39.2}{\sqrt{24}} = 8.02$$

$$C = \frac{+17}{24} = +.708 \quad C^2 = .501$$

$$G.A. = 15$$

$$Av. = 15 - (10 \times .708) = 7.92$$

$$\sigma = \sqrt{\frac{533}{24} - .501} \times 10 = 46.6$$

$$\sigma_{Av.} = \frac{46.6}{\sqrt{24}} = 9.53$$

$$\sigma_D = \sqrt{(8.02)^2 + (9.53)^2} = 12.45$$

$$\text{Reliability of } D = \frac{12.5 - 7.92}{12.45} = .37$$

TABLE II B

RELIABILITY OF DIFFERENCES IN RATE  
FOURTH GRADE SUBTRACTION

GROUP COMPETITION					SELF COMPETITION				
SCORE RIGHTS	F	D	FD	FD <sup>2</sup>	SCORE RIGHTS	F	D	FD	FD <sup>2</sup>
-2	3	4	12	48	-4	2	6	12	72
-1	0	3	0	0	-3	2	5	10	50
0	4	2	8	16	-2	1	4	4	16
1	5	1	5+23	3	-1	0	3	0	0
2	1	0	0	0	0	2	2	4	8
3	3	-1	-3	3	1	2	1	2+32	2
4	3	-2	-6	12	2	2	0	0	0
5	4	-3	-12	36	3	4	-1	-4	4
6	1	-4	-4	16	4	1	-2	-2	4
7	1	-5	-5	25	5	3	-3	-9	27
8	0	-6	-6	0	6	4	-4	-16	64
9	1	-7	-7-37	49	7	1	-5	-5-36	25
	N=24		-14	208		N=24		-4	272

$$C = \frac{14}{24} = -.583$$

$$C^2 = .34$$

$$C = \frac{-4}{24} = -.167$$

$$C^2 = .028$$

$$D = \sqrt{\frac{208}{24} - .34} = 2.89$$

$$D = \sqrt{\frac{272}{24} - .028} = 3.36$$

$$\sqrt{AV_1} = \frac{289}{\sqrt{24}} = .591$$

$$M_1 = 2.92$$

$$\sqrt{AV_2} = \frac{3.36}{\sqrt{24}} = .687$$

$$M_2 = 2.21$$

$$D = \sqrt{(.591)^2 + (.687)^2} = .906$$

$$\text{Reliability of } D = \frac{2.92 - 2.21}{.906} = .78$$



TABLE IV B  
RELIABILITY OF DIFFERENCES IN RATE  
FIFTH GRADE MULTIPLICATION

GROUP COMPETITION					SELF COMPETITION				
SCORE	F	D	FD	FD <sup>2</sup>	SCORE	F	D	FD	FD <sup>2</sup>
-1	1	4	4	16	-2	1	6	6	36
0	3	3	9	27	-1	2	5	10	50
1	5	2	12	24	0	3	4	12	48
2	8	1	8+33	8	1	3	3	9	27
3	5	0	0	0	2	3	2	4	12
4	4	-1	-4	4	3	4	1	4+45	4
5	1	-2	-2	4	4	2	0	0	0
6	2	-3	-6	18	5	3	-1	-3	9
7	1	-4	-4	16	6	2	-2	-4	12
8	2	-5	-10-26	50	7	3	-3	-15	45
	H = 33		7	167	8	1	-4	-4	16
					9	1	-5	-5	25
					10	3	6	18-49	108
						N = 33		-4	306

$$C = \frac{-7}{33} = .212 \quad C^2 = .045$$

$$\sigma = \sqrt{\frac{167}{33} - .045} = 2.41$$

$$\sigma_{av1} = \frac{2.41}{\sqrt{33}} = .419$$

$$M_1 = 2.76$$

$$\sigma_D = \sqrt{(.419)^2 + (.577)^2} = .715$$

$$\text{Reliability of D} = \frac{4.03 - 2.76}{.715} = 1.78 \sigma$$

$$C = \frac{-4}{33} = .12 \quad C^2 = .0144$$

$$\sigma = \sqrt{\frac{306}{33} - .0144} = 5.53$$

$$M_2 = 4.03$$

$$\sigma_{av2} = \frac{5.53}{\sqrt{33}} = .577$$

TABLE V B  
 RELIABILITY OF DIFFERENCES IN ACCURACY  
 SIXTH GRADE DIVISION

GROUP COMPOSITION					SELF COMPETITION				
SCORES ACC. %	F	D	FD	FD <sup>2</sup>	SCORES ACC. %	F	D	BD	FD <sup>2</sup>
120-130	0	0	0	0	120-130	1	11	11	121
110-120	1	11	11	121	110-120	0	10	0	0
100-110	2	10	20	200	100-110	1	9	9	81
90-100	0	9	0	0	90-100	0	8	8	0
80-90	0	8	0	0	80-90	3	7	21	98
70-80	1	7	7	49	70-80	0	6	0	0
60-70	0	6	0	0	60-70	1	5	5	25
50-60	1	5	5	25	50-60	3	4	12	48
40-50	0	4	0	0	40-50	1	3	3	9
30-40	5	3	15	45	30-40	2	2	4	8
20-30	3	2	6	12	20-30	3	1	3+68	3
10-20	3	1	3+67	3	10-20	3	0	0	0
0-10	7	0	0	0	0-10	10	-1	-10	10
-10-0	3	-1	-3	3	-10-0	0	-2	0	0
-20-10	2	-2	-4	8	-20-10	2	-3	-6	18
-30-20	0	-3	-0	0	-30-20	1	-4	-4	16
-40-30	2	-4	-8	32	-40-30	0	-5	0	-0
-50-40	2	-5	-10	50	-50-40	0	-6	0	0
-60-50	2	-6	-12	72	-60-50	1	-7	-7	49
-70-60	0	-7	-0-57	0	-70-60	1	-8	-8	64
	N=34		50	620		N=34		24	631

$$C = \frac{50}{40} = .882 \quad C^2 = .777$$

$$G.A. = 5; Av = 5 + (10 \times .882) = 13.82$$

$$\sigma = \sqrt{\frac{620}{34} - .777} \times 10 = 41.7$$

$$\sigma_{Av_1} = \frac{41.7}{\sqrt{34}} = 7.16$$

$$C = \frac{24}{34} = .706 \quad C^2 = .498$$

$$G.A. = 15; Av = 15 + (10 \times .706) = 22.06$$

$$\sigma = \sqrt{\frac{631}{34} - .498} \times 10 = 42.5$$

$$\sigma_{Av_2} = \frac{42.5}{\sqrt{34}} = 7.30$$

$$\sigma_D = \sqrt{(7.16)^2 + (7.30)^2} = 10.27$$

$$\text{Reliability of D} = \frac{22.06 - 13.82}{10.27} = .79 \sigma$$

TABLE VI B

RELIABILITY OF DIFFERENCES IN RATE  
SIXTH GRADE DIVISION

GROUP COMPETITION					SELF COMPETITION				
EF <sup>1</sup>					EF <sup>2</sup>				
RIGHTS SCORE	F	D	FD	FD <sup>2</sup>	RIGHTS SCORE	F	D	FD	FD <sup>2</sup>
-2	3	3	9	27	-3	1	4	4	16
-1	4	2	8	16	-2	3	3	9	27
0	9	1	9+26	9	-1	3	2	6	12
1	10	0	0		0	11	1	11+30	11
2	5	-1	-5	25	1	6	0	0	0
3	1	-2	-2	4	2	3	1	-3	3
4	0	-3	-0	0	3	3	2	-6	12
5	1	-4	-4	16	4	1	3	-3	9
6	1	-5	-5 -16	25	5	1	4	-4	16
	N=54		10	122		N=34		-4	156

$$C = \frac{+10}{54} = +.284 \quad C^2 = .08$$

$$\sigma = \sqrt{\frac{122}{34} - .08} = 1.87$$

$$\sigma_{Av} = \frac{1.87}{\sqrt{34}} = .321$$

$$M_1 = .706$$

$$C = \frac{-.4}{34} = -.118 \quad C^2 = .014$$

$$\sigma = \sqrt{\frac{156}{34} - .014} = 2.14$$

$$\sigma_{Av} = \frac{2.14}{\sqrt{34}} = .368$$

$$M_2 = .971$$

$$\sigma_D = \sqrt{(.321)^2 + (.368)^2} = .488$$

$$\text{Reliability of D} = \frac{.971 - .706}{.488} = .54 \sigma$$

TABLE VII B

## RELIABILITY OF DIFFERENCE IN ACCURACY

## SEVENTH GRADE SUBTRACTION

GROUP COMPETITION					SELF COMPETITION				
SCORES ACC. %	F	D	FD	FD <sup>2</sup>	SCORES ACC. %	F	D	FD	FD <sup>2</sup>
70-80	1	7	7	49	140-150	1	11	11	121
60-70	0	6	0	0	130-140	0	10	0	0
50-60	1	5	5	25	120-130	0	9	0	0
40-50	0	4	0	0	110-120	0	8	0	0
30-40	3	3	9	27	100-110	1	7	7	49
20-30	2	2	4	8	90-100	0	6	0	0
10-20	2	1+28	2	2	80-90	0	5	0	0
0-10	5	0			70-80	2	4	8	32
-10-0	3	-1	3	3	60-70	2	3	6	18
-20--10	1	-2	2	4	50-60	1	2	2	4
-30--20	3	-3	9	27	40-50	4	1	4+36	4
-40--30	0	-4	0	0	30-40	4	0		
-50--40	1	-5	5	25	20-30	1	-1	-1	1
-60--50	0	-6	0	0	10-20	1	-2	-2	4
-70--60	1	-7-28	7	49	0-10	3	-3	-9	27
	N=23	0		219	-10-0	0	-4	-0	0
					-20--10	2	-5	-10	50
					-30--20	0	-6	-0	0
					-40--50	0	-7	-0	0
					-50--40	1	-8	-8-30	64
						N=23	6		374

$$C = 0$$

$$Av. = 5$$

$$\sigma = \sqrt{\frac{219}{23} - 0 \times 10} = 30.8$$

$$\sigma_{Av} = \frac{30.8}{\sqrt{23}} = 6.42$$

$$\sigma_D = \sqrt{(6.42)^2 + (8.37)^2} = 10.55$$

$$\text{Reliability of } D = \frac{37.61 - 5}{10.55} = 3.08 \sigma$$

$$C = \frac{6}{23} = .261 \quad C^2 = .068$$

$$G.A. = 35; Av = 35 + (10 \times .261) = 37.61$$

$$\sigma = \sqrt{\frac{374}{23} - .068 \times 10} = 40.2$$

$$\sigma_{Av} = \frac{40.2}{\sqrt{15}} = 8.37$$



TABLE VIII B

RELIABILITY OF DIFFERENCES IN RATE  
SEVENTH GRADE SUBTRACTION

GROUP COMPETITION					SELF COMPETITION				
SCORE RIGHT	F	D	FD	FD <sup>2</sup>	SCORE RIGHT	F	D	FD	FD <sup>2</sup>
-7-	-5	1	5	9	-5-	-3	1	4	4
-5-	-3	2	2	4	-3-	-1	2	3	6
-3-	-1	3	1	3+10	3	-1-	1	5	2
-1-	1	5	0	0	0	1-3	1	1	1+25
1-3	8	1	-8	8	0	3-5	7	0	0
3-5	2	2	-4	8	4	5-7	4	-1	-4
5-7	2	3	-6-18	18	2	7-9	2	-2	-4
					0	9-11	0	-3	-0
					1	11-15	1	-4	-4-12
	N=25		-8	54		N=25		11	83

$$C = \frac{-8}{25} = -.347 \quad C^2 = .119$$

$$C = \frac{11}{25} = .472 \quad C^2 = .247$$

$$\sigma = \sqrt{\frac{54}{25} - .347 \times 2} = 2.82$$

$$\sigma = \sqrt{\frac{83}{25} - .247 \times 2} = 3.66$$

$$\sigma_{Av1} = \frac{2.82}{\sqrt{25}} = .569$$

$$\sigma_{Av2} = \frac{3.66}{\sqrt{25}} = .763$$

$$M_1 = 1$$

$$M_2 = 3.8$$

$$\sigma_D = \sqrt{(.569)^2 + (.763)^2} = .96$$

$$\text{Reliability of D} = \frac{3.8 - 1}{.96} = 2.93 \sigma$$

TABLE IX B  
 RELIABILITY OF DIFFERENCES IN RATE  
 SEVENTH GRADE ADDITION

GROUP COMPETITION					SELF COMPETITION				
SCORE RIGHTS	F	D	FD	FD <sup>2</sup>	SCORE RIGHTS	F	D	FD	FD <sup>2</sup>
-9- -7	0	6	0	0	-5- -3	3	2	6	12
-7- -5	1	5	5	25	-3- -1	4	1	4+10	4
-5- -3	0	4	0	0	-1- 1	5	0	-3	
-3- -1	4	3	12	36	1-3	3	-1	-4	3
-1- 1	2	2	4	8	3-5	2	-2	-15-22	8
1-3	3	1	3+24	3	5-7	5	-3		45
3-5	10	0				N=23		-12	72
5-7	0	-1	0	0					
7-9	1	-2	-2	4					
9-11	1	-3	-3	9					
11-13	1	-4	-4-9	16					
	N=23		15	101					

$$C = \frac{15}{23} = .652 \quad C^2 = .425$$

$$C = \frac{-12}{23} = -.521 \quad C^2 = .271$$

$$\sigma = \sqrt{\frac{101}{23} - .425 \times 2} = 3.96$$

$$\sigma = \sqrt{\frac{72}{23} - .51 \times 2} = 3.22$$

$$\sigma_{Av.1} = \frac{3.96}{\sqrt{23}} = .825$$

$$\sigma_{Av.2} = \frac{3.22}{\sqrt{23}} = .671$$

$$M_1 = 5.30$$

$$M_2 = 1.652$$

$$\sigma_D = \sqrt{(.825)^2 + (.671)^2} = 1.06$$

$$\text{Reliability of } D = \frac{5.30 - 1.652}{1.06} = 1.55 \sigma$$

TABLE X B

RELIABILITY OF DIFFERENCES IN ACCURACY  
SEVENTH GRADE ADDITION

GROUP COMPETITION					SELF COMPETITION				
SCORES ACC. %	F	D	FD	FD <sup>2</sup>	Scores ACC. %	F	D	FD	FD <sup>2</sup>
100-110	2	8	16	128	80-90	1	8	8	64
90-100	0	7	0	0	70-80	0	7	0	0
80-90	1	6	6	36	60-70	0	6	0	0
70-80	0	5	0	0	50-60	2	5	10	50
60-70	1	4	8	16	40-50	3	4	12	48
50-60	3	3	9	27	30-40	0	5	0	0
40-50	6	2	6	24	20-30	0	2	0	0
30-40	0	1	0+45	0	10-20	0	1	0+36	6
20-30	0	0	0	0	9-10	7	0		
10-20	5	1	-3	3	-10-0	5	1	5	5
0-10	0	2	0	0	-20- -10	2	2	4	8
-10- -0	3	3	-9	27	-30- -20	3	3	9	27
-20- -10	2	4	-8	32	-40- -30	0	4	0	0
-30- -20	0	5	-0	0	-50- -40	1	5	5-25	25
-40- -30	1	6	-6	36					
-50- -40	0	7	-0	0					
-60- -50	0	8	-0	00					
-70- -60	0	9	-0	0					
-80- -70	0	10	-0	0					
-90- -80	0	11	-0	0					
-100- -90	0	12	-0	0					
-110- -100	1	13	-15-39	139					
	N=23		6	498		N=23		7	227

$$C = \frac{6}{23} = .261 \quad C^2 = .067$$

$$G.A. = 25; \quad Av. = 25 + (10 \times .261) = 27.61$$

$$\sigma = \sqrt{\frac{498}{23} - .067} \times 10 = 45.9$$

$$\sigma_{Av} = \frac{45.9}{\sqrt{23}} = 9.57$$

$$C = \frac{7}{23} = .304 \quad C^2 = .09$$

$$G.A. = 5 \quad Av. = 5 + (10 \times .304) = 8.04$$

$$\sigma = \sqrt{\frac{227}{23} - .09} \times 10 = 31.3$$

$$\sigma_{Av} = \frac{31.3}{\sqrt{23}} = 6.52$$

$$\sigma_D = \sqrt{(9.57)^2 + (6.52)^2} = 11.54$$

$$\text{Reliability of } D = \frac{27.61 - 8.08}{11.54} = 1.7 \sigma$$

TABLE XI B  
RELIABILITY OF DIFFERENCES IN RATE  
EIGHTH GRADE DIVISION

GROUP COMPETITION					SELF COMPETITION				
SCORE RIGHTS	F	D	FD	FD <sup>2</sup>	SCORE RIGHTS	F	D	FD	FD <sup>2</sup>
-9- -7	1	4	4	16	-5- -1	2	3	6	18
-7- -5	1	3	3	9	-1- 1	2	2	4	8
-5- -3	4	2	8	16	1-3	2	1	2	2
-3- -1	1	1	1+16	1	3-5	2	0	0	
-1- 1	4	0	0	0	5-7	3	1	-3	3
1-3	1	1	-1	1	7-9	1	2	-2	4
3-5	2	2	-4	8	9-11	0	3	0	0
5-7	0	3	-0	0	11-13	2	4	-8	32
7-9	0	4	-0	0	13-15	0	5	0	0
9-11	1	5	-5-10	25	15-17	1	6	-6-19	36
	N=15		6	76		N=15		-7	103

$$C = \frac{6}{15} = .4 \quad C^2 = .16$$

$$C = \frac{-7}{15} = -.467 \quad C^2 = .218$$

$$\sigma = \sqrt{\frac{76}{15} - .16 \times 2} = 4.38$$

$$\sigma = \sqrt{\frac{103}{15} - .218 \times 2} = 5.14$$

$$\sigma_{Av_1} = \frac{4.38}{\sqrt{15}} = 1.13$$

$$\sigma_{Av_2} = \frac{5.14}{\sqrt{15}} = 1.33$$

$$M_1 = 5.53$$

$$M_2 = .867$$

$$\sigma_D = \sqrt{(1.13)^2 + (1.33)^2} = 1.75$$

$$\text{Reliability of } D = \frac{5.53 - (-.867)}{1.75} = 3.5 \sigma$$

TABLE XII B  
 RELIABILITY OF DIFFERENCES IN ACCURACY  
 EIGHTH GRADE DIVISION

GROUP COMPETITION					SELF COMPETITION				
SCORES ACC. %	F	D	FD	FD <sup>2</sup>	SCORES ACC. %	F	D	FD	FD <sup>2</sup>
100-110	2	8	16	128	60-70	2	6	12	72
90-100	0	7	0	0	50-60	2	5	10	50
80-90	2	6	12	72	40-50	1	4	4	16
70-80	0	5	0	0	30-40	1	3	3	9
60-70	0	4	0	0	20-30	1	2	2	4
50-60	1	3	3	9	10-20	1	1	1+32	1
40-50	0	2	0	0	0-10	2	0		
30-40	3	1	3+34	3	-10-0	0	-1	0	0
20-50	0	0			-20- -10	1	-2	-2	4
10-20	2	1	-2	2	-30- -20	1	-3	-3	9
0-10	1	2	-2	4	-40- -30	0	-4	-0	0
-10-0	1	3	-3	9	-50- -40	1	-5	-5	25
-20- -10	1	4	-4	16	-60- -50	2	-6	-12-22	72
-30- -20	0	5	-0	0					
-40- -30	2	6	-12-23	72					
	N=15		11	315		N=15		10	262

$$C = \frac{11}{15} = .733 \quad C^2 = .537$$

$$C = \frac{10}{15} = .667 \quad C^2 = .448$$

$$G.A. = 25 \quad Av = 25 + (10 \times .733) = 32.53$$

$$G.A. = 5 \quad Av = 5 + (10 \times .667) = 11.67$$

$$\sigma = \sqrt{\frac{315}{15} - .733 \times 10} = 45.02$$

$$\sigma = \sqrt{\frac{262}{15} - .667 \times 10} = 41$$

$$\sigma_{Av} = \frac{45.02}{\sqrt{15}} = 11.62$$

$$\sigma_{Av} = \frac{41}{\sqrt{15}} = 10.58$$

$$\sigma_D = \sqrt{(11.62)^2 + (10.58)^2} = 15.7$$

$$\text{Reliability of } D = \frac{32.53 - 11.67}{15.7} = 1.51 \sigma$$

TABLE XIII B

## RELIABILITY OF DIFFERENCES IN ACCURACY

## EIGHTH GRADE MULTIPLICATION

GROUP COMPETITION					SELF COMPETITION				
SCORES ACC. %	F	D	FD	FD <sup>2</sup>	Scores ACC. %	F	D	FD	FD <sup>2</sup>
70-80	1	9	9	81	70-80	1	6	6	36
60-70	1	8	8	64	60-70	0	5	0	0
50-60	0	7	0	0	50-60	0	4	0	0
40-50	1	6	6	36	40-50	2	3	6	18
30-40	0	5	0	0	30-40	1	2	2	4
20-30	1	4	4	16	20-30	1	1	1+15	1
10-20	3	3	9	27	10-20	4	0		
0-10	1	2	2	4	0-10	1	0	-1	1
-10-0	0	1	0+38	0	-10-0	0	-1	0	0
-20--10	1	0			-20--10	1	-2	-3	9
-30--20	1	-1	-1	1	-30--20	2	-3	-8	32
-40--30	0	-2	0	0	-40--30	2	-4	-10-22	50
-50--40	1	-3	-3	9					
-60--50	0	-4	-0	0					
-70--60	0	-5	-0	0					
-80--70	2	-6	-12	72					
-90--80	1	-7	-7	49					
-100--90	1	-8	-8-31	64					
	<u>N=15</u>		<u>7</u>	<u>423</u>		<u>N=15</u>		<u>-7</u>	<u>151</u>

$$C = \frac{7}{15} = .467 \quad C^2 = .218$$

$$G.A. = -15 \quad Av = -15 + (10 \times .467) = -10.33$$

$$\sigma = \sqrt{\frac{423}{15} - .467 \times 10} = 52.9$$

$$\sigma_{Av} = \frac{52.9}{\sqrt{15}} = 13.67$$

$$C = \frac{-7}{15} = -.467 \quad C^2 = .218$$

$$G.A. = 15; \quad Av = 15 + (10 \times .218) = 17.18$$

$$\sigma = \sqrt{\frac{151}{15} - .218 \times 10} = 31.4$$

$$\sigma_{Av} = \frac{31.4}{\sqrt{15}} = 8.14$$

$$\sigma_D = \sqrt{(13.67)^2 + (8.14)^2} = 9.22$$

$$\text{Reliability of } D = \frac{17.18 - (-10.33)}{9.22} = 2.9 \sigma$$

TABLE XIV B

RELIABILITY OF DIFFERENCES IN RATE  
EIGHTH GRADE MULTIPLICATION

GROUP COMPETITION						SELF COMPETITION					
RIGHTS SCORE	F	D	FD	FD <sup>2</sup>		RIGHTS SCORE	F	D	FD	FD <sup>2</sup>	
-7-	-5	1	4	4	16	-5-	-3	1	3	3	9
-5-	-3	1	3	3	9	-3-	-1	3	2	6	12
-3-	-1	3	2	6	12	-1-	1	4	1	4+13	4
-1-	1	1	1	1+14	1	1-3	4	0	0	0	0
1-3	2	0	0	0	0	3-5	2	-1	-2	2	2
3-5	3	-1	-3	3	3	5-7	0	-2	0	0	0
5-7	2	-2	-4	8	8	7-9	0	-3	0	0	0
7-9	1	-3	-3	9	9	9-11	0	-4	0	0	0
9-11	1	-4	-4	14	16	11-13	1	-5	-5-7	25	25
	<u>N=15</u>		<u>0</u>	<u>74</u>			<u>N=15</u>		<u>6</u>	<u>52</u>	

$$C = 0$$

$$S = \sqrt{\frac{74}{15}}$$

$$S = 2.22$$

$$S_{Av} = \frac{2.22}{\sqrt{15}} = .574$$

$$M_1 = 2.47$$

$$C = \frac{6}{15} = .4 \quad C^2 = .16$$

$$S = \sqrt{\frac{52}{15} - .15 \times 2} = 3.64$$

$$S_{Av} = \frac{3.64}{\sqrt{15}} = .940$$

$$M_2 = 1.60$$

$$S_D = \sqrt{(.574)^2 + (.94)^2} = 1.09$$

$$\text{Reliability of D} = \frac{2.47 - 1.60}{1.09} = .80 \quad S$$

## CHAPTER V

Explanation and Interpretation of Table 16Summary and Conclusions

The Reliability of the Differences in this table was taken from Table 15 ( $\frac{D}{D}$ ). In addition, the fourth grades' reliability of difference ( $\frac{D}{D}$ ) between the gains in rights on EF' and EF'' is  $5\sigma$ . This is found under Rights for EF''. The class made a greater gain under EF'' for rights than under EF'. Three  $\sigma$  in this case expressed in terms of chance means that in 99.9 times out of 100 the difference between the average number of problems worked right, by the fourth grade in addition, where the two drill methods are used (EF' and EF'') the difference will be greater than zero and in favor of EF''. This, of course, presupposes a like situation.

The reliability of difference for accuracy ( $\frac{D}{D}$ ) in fourth grade addition is  $2.18\sigma$  favoring EF''.  $2.18\sigma$  expressed in terms of chance means that in 98 times out of 100 the difference between the mean accuracies in addition for the two types of drill (EF' and EF'') will be greater than zero and in favor of EF''.

The reliability difference for the means in "Rights" and "Accuracies" for fourth grade addition greatly favors the drill EF'' (Self Competition). This is especially



TABLE XVI  
SUMMARIES AND DIFFERENCES IN CHANCE

EF <sup>I</sup> D/D		EF <sup>II</sup> D/D		Chances fav- oring EF <sub>1</sub>		Chances Fav- oring EF <sub>2</sub>	
R	Acc	R	Acc	R	Acc	R	Acc
GRADE 4							
Add.		5 <sup>σ</sup>	2.13 <sup>σ</sup>			99.9-100	98-100
Sub.	.78 <sup>σ</sup> .57 <sup>σ</sup>			78-100	65-100		
GRADE 5							
Add.	.29 <sup>σ</sup> 1.4 <sup>σ</sup>			62-100	92-100		
Mul.	2.7 <sup>σ</sup>	1.78 <sup>σ</sup>		991-1000	96-100		
Div.		1.88 <sup>σ</sup>	.87 <sup>σ</sup>			97-100	80-100
Sub.	.64 <sup>σ</sup> .79 <sup>σ</sup>			74-100	79-100		
GRADE 6							
Add.	.23 <sup>σ</sup> .9 <sup>σ</sup>			60-100	82-100		
Mul.	2 <sup>σ</sup> 2.11 <sup>σ</sup>			98-100	98-100		
Div.		.52 <sup>σ</sup>	.79 <sup>σ</sup>			71-100	79-100
Sub.	1.07 <sup>σ</sup> 2.4 <sup>σ</sup>			86-100	99.2-100		
GRADE 7							
Add.	1.55 <sup>σ</sup> 1.7 <sup>σ</sup>			95-100	96-100		
Mul.	1.6 <sup>σ</sup> 1.6 <sup>σ</sup>			94-100	94-100		
Div.	3.3 <sup>σ</sup> 1.06 <sup>σ</sup>			99.9-100	85-100		
Sub.		2.93 <sup>σ</sup> 5.56 <sup>σ</sup>				97.8-100	99.9-100
GRADE 8							
Add.	2.01 <sup>σ</sup>		2.01 <sup>σ</sup>	98-100			98-100
Mul.	2.9 <sup>σ</sup>		2.9 <sup>σ</sup>	99.8-100			99.8-100
Div.	3.5 <sup>σ</sup> 1.3 <sup>σ</sup>			99.9-100		90-100	
Sub.	.93 <sup>σ</sup>		.93 <sup>σ</sup>	83-100			83-100

true in the teaching situation which existed in the Her-  
 culaneum Schools during the experiment. For fourth grade  
 subtraction the reliability of the difference favors EF'  
 in both "gains in Rights and Accuracy." The sigmas here  
 are expressed in chance in the appropriate columns.

It will be seen by glancing at the table that  
 the achievements resulting from each drill method varied  
 greatly. The seventh grade did much better in the number  
 of problems worked right and in accuracy by means of drill  
 EF' in all the operations except subtraction.

According to the table, the eighth grade did  
 much better in "Rights" under method EF'. The chances of  
 the difference being greater than zero in favor of EF' in  
 addition, multiplication and division are much in evidence.  
 For accuracy in the eighth grade, the chances are in favor  
 of EF'' in every operation except division.

Taking the arithmetical operations for each grade  
 and summing them we have 18. Out of the 18 there are  
 seven reliability of differences ( $\frac{D}{7D}$ ) favoring EF'' in  
 Rights and eleven favoring EF'. In Accuracy the same  
 ratio holds true: 7 for EF'' and 11 for EF'.

In fourth grade addition, fifth grade division, sixth  
 grade division, and seventh grade subtraction, the chances  
 favor EF'' in both the number of problems worked right and  
 in accuracy. In fourth grade subtraction, fifth grade

addition, sixth grade addition and subtraction, seventh grade addition, multiplication and division, and eighth grade division, the chances favor EF<sup>1</sup> in both accuracy and in the number of problems worked right. This leaves six cases in which the gains in rights and accuracy are split, for example, the eighth grade did better in rights by means of drill EF<sup>1</sup>, in addition, multiplication and subtraction, but made a better gain in accuracy in these same operations by means of drill EF<sup>2</sup>. This simply means that the eighth grade worked more problems right in these three operations when drill was being given by means of individual or group competition, EF<sup>1</sup>, but the class worked with greater accuracy when drill in self competition was used, EF<sup>2</sup>. The ratio between Rights and Attempts was greater for EF<sup>1</sup>.

$3\sigma$  and above represents high reliability. The larger the sigma the greater the reliability and the smaller the sigma the less the reliability. The  $\frac{D}{\sqrt{D}}$  of .29 $\sigma$  in rights for fifth grade addition favoring EF<sup>1</sup> expressed in terms of chance is only equal to 62 to 100. 50 to 100 would be pure chance, so 62 to 100 does not express very high reliability.

In conclusion it may be said that drill in the fundamentals of arithmetic in general seems to favor the giving of drill so as to encourage individual or group competition, instead of self competition. Neither type of drill excelled exclusively in any one grade or in all the operations.

In the Herculaneum Schools drill given in EF' (Group or Individual Competition) secured better results in 61% of the operations for grades four to eight inclusive, in both number of problems worked right and in accuracy. In other words, Group competition is favored over Self Competition in eleven out of the eighteen cases in both Rights and in Accuracy. The reliability of differences are not high in all these cases, but neither are the reliability of differences all high in the seven cases in which EF'' seems to be favored.

From the results secured in some of the operations for the different grades, it appears that EF'' does possess some particular merit in learning the fundamental skills of arithmetic. It therefore would not be well to do away with the element altogether and neither would it be well to use it exclusively in giving all drill work.

As a result of the findings in Table 16, the Courtis Practice Set will be used <sup>not</sup> exclusively in the Herculaneum Schools next year. The major emphasis in drill work will be placed upon group and individual competition. The element

of self competition will, of course, be used some, but the drill material will be worked out by the teaching staff. In those operations in which EF<sup>11</sup> showed the greater gain more emphasis will be placed on the self competitive methods in mastering the four fundamentals of arithmetic. Work books may be used some, but not exclusively in any grade.

BIBLIOGRAPHY

- (1) Lillian Shenk, "Three Types of Drill in Arithmetic",  
A Thesis, University of Kansas, 1918.
- (2) S. A. Curtis, "Standard Practice in the Four Fundamentals",  
World Book Company.
- (3) Uhls, W. J., "The Use of Standard Materials in Arithmetic  
for Diagnosing Pupils Method of Work", Elementary  
School Journal, Nov., 1917.
- (4) Brown, J. C., "An Investigation of the Drill Work on the  
Fundamental Operations of Arithmetic", Journal  
of Educational Psychology, Vol. II Feb., 1911.
- (5) Childs, H. G., "A Half-Years Progress in the Achievement  
of One School System" (As measured by the Curtis  
Test) 15 Year Book Nat. Soc. for the Study of Ed.,  
Part 1, p. 83-90.
- (6) Osburn, W. J., "Corrective Arithmetic", Houghton Mifflin  
Co. Chicago.
- (7) Henry E. Garrett, "Statistics in Psychology and Education",  
Longman, Green and Co., N. Y.
- (8) "School Efficiency Monograph" by Mead, World Book Company,  
Chicago.
- (9) Haggerty, M. E., "Studies in Arithmetic", Indiana University  
Studies, no., 32 (Sept. 1916.)

- (10) Taylor, E. H., "Arithmetic Ability of Rural School Children", *Journal of Educational Psychology*, March, 1914.
- (11) Newcomb, E. H., "Effective Drill Exercises in Arithmetic", *Journal of Educational Psychology*, 15 (Feb. 1925) p. 129-31.
- (12) Knight, F. B., "A Note on the Organization of Drill Work" *Journal of Ed. Psychology*, 15 (Feb. 1925) p. 108-17.
- (13) Batson, William H. and Combellick, Olin E., "Relative Difficulty of Number Combinations in Addition and Subtraction", *Journal of Educational Psychology* 15 (October 1925) p. 467-81.
- (14) Hallegas, Milo B., "Teaching Number Fundamentals", J. B. Lippincott and Co. Phil. (A hand book of methods)