Design of composite beams with web openings

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Summary

Design techniques for composite beams with web openings have been under development for well over 30 years. During the past decade, these efforts have reached a level of maturity that allows for an accurate assessment of strength and the development of economical designs. This paper describes the behavior of steel–concrete composite beams with web openings and summarizes the key aspects of strength design and deflection calculation.

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Web openings provide an economical means for reducing the depth of floor systems in steel framed buildings. In the vast majority of these structures, the concrete slab is designed to act compositely with the steel. Until recently, the design of regions around web openings has been approached as four separate problems, involving unreinforced or reinforced openings, with the beam treated as composite in positive/sagging moment regions and non-composite in negative/hogging moment regions. During the past decade, however, techniques for the four problems have been combined based on the observed similarity in behavior of steel and composite sections with or without opening reinforcement. Using current procedures, unreinforced openings and non-composite sections are treated as special cases of the more general composite member with a reinforced opening.

Design techniques for openings in composite members have reached a level of maturity such that major changes have not been made in recent years. A number of design techniques[1, 2**, 3*, 4**, 5, 6*, 7] are available, some with easy to use design aids and software support[4**, 8].

Placing a penetration in the web of a member has the effect of reducing both flexural and shear strength. Early design techniques were often highly conservative[9–11], requiring most openings to be reinforced. Designers now have procedures available that provide a more realistic assessment of the effect of web penetrations on strength and, thus, require significantly less reinforcement than used only

Abbreviations

ASCE = American Society of Civil Engineers
AISC = American Institute of Steel Construction
CIRIA = Construction Industry Research and Information Association
SCI = Steel Construction Institute
SEI = Structural Engineering Institute

Terminology

a_o = length of opening
d_h, d_ℓ = distance from top of steel section to centroid of concrete force at high and low moment ends of the opening, respectively
d_r = distance from outside edge of flange to centroid of reinforcement
Δ_b = maximum bending deflection for unperforated beam
$\Delta_{m,b}$ = maximum bending deflection for a member with an opening
F_y = yield stress of steel
$\bar{F}_y$ = reduced axial strength of steel in web due to combined axial stress and shear stress
f_c' = compressive strength of concrete
h_o = length of opening
I_g = gross moment of inertia of unperforated member
I_{wo} = moment of inertia at web opening
L_s = span length
M = moment
M_m = maximum nominal flexural capacity at opening
M_n = nominal bending capacity
M_{th}, M_{tl} = secondary moments at high- and low-moment ends of opening, respectively
M_u = factored bending moment
μ = dimensionless ratio relating secondary bending moment contributions of concrete and opening reinforcement to product of plastic capacity of a tee and the depth of the tee
ν = aspect ratio of a tee = a_o/s_t
P_{ch}, P_{cl} = concrete forces at high- and low-moment ends of opening, respectively
P_r = force in reinforcement along edge of opening
s_t = depth of tee
t_w = thickness of web
V = shear
V_m = maximum shear capacity at location of opening
V_{mt} = maximum shear capacity of a tee
V_n = nominal shear capacity
V_{pt} = plastic shear capacity of web of tee
V_u = factored shear

a decade ago. With current techniques, many openings remain unreinforced.

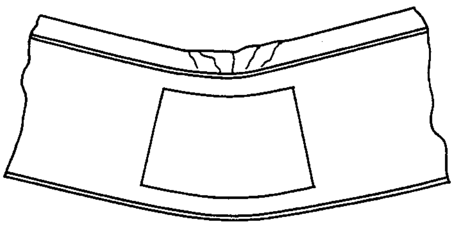
Web penetrations also reduce the local stiffness at an opening, which can have a significant effect on deflection. However, in most cases, the deflection of members with web openings differs little from that of members without openings.

This article provides a brief overview of the behavior, strength design, and deflection calculation techniques for composite beams with web openings.

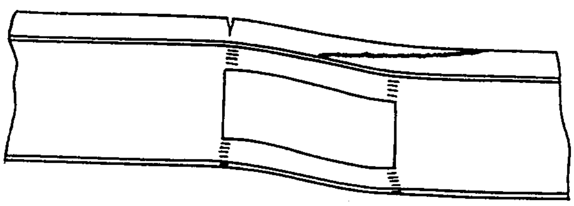
Behavior

The behavior of a member with a web opening depends on the ratio of moment to shear, M/V, at the opening[12–17]. In members subjected to pure bending (Fig. 1a), the concrete slab is placed in compression while the region below the opening is placed in tension. The steel section above the opening may be in tension or compression or both, depending on section dimensions.

As the M/V ratio decreases, the shear across the opening induces secondary bending or Vierendeel action, which adds to the effects of primary bending at the high moment end of the opening (right side of Fig. 1b) and counters the effects of primary bending at the low moment end of the opening (left side of Fig. 1b). Secondary bending often places the top of the concrete slab in tension at the low-moment end of the opening. The portions of the steel above and below the opening are referred to as the top and bottom tees, respectively. At the low-moment end of the opening, secondary bending places the top of the tees in tension and the bottom of the tees in compression. Stresses of opposite sign are induced at the high-moment end of the opening. It is not unusual in tests for the effects of secondary bending to dominate to the extent that the low-moment end of the opening actually rises at failure, even as a downward load is placed near the center of the beam.



(a)



(b)

Fig. 1 Failure modes at web openings for composite beam with solid slab (a) pure bending, (b) low-moment–shear ratio. Reproduced from Ref. [4**] by permission of AISC

Secondary bending can also result in strains past yield in the webs of the top and bottom tees when the member is under working loads. This premature yielding, however, has little measurable effect on deflection and no effect on the strength of a properly proportioned beam.

For opening regions subjected to a high bending moment (high M/V ratio), failure is governed by crushing of the concrete slab. For opening regions subjected to high shear (low M/V ratio), failure involves cracking in tension at the top of the slab at the low-moment end of the opening, accompanied by sliding of the slab from the high to the low-moment end of the opening, which places the bottom of the slab in compression at the low-moment end of the opening. At the high-moment end, the nature of the failure depends on the type of slab. Solid slabs usually undergo a diagonal tension failure, as indicated in Fig. 1b. In members with ribbed slabs with the ribs perpendicular to the beam, failure involves rib separation and failure of the concrete around the shear connectors. For members with ribs parallel to the steel section, failure often involves the formation of a (nearly) horizontal crack which separates the longitudinal rib from the slab. In many cases, for openings with low M/V ratios, the slab will exhibit separation from the steel section and appear to bridge between the low-moment end and a point past the high-moment end of the opening. However, tests indicate that, in these cases, strength is never lower than that calculated based on the formation of hinges at all four corners of the opening. In all cases, the concrete slab contributes significantly to shear strength at the opening.

Sliding of the concrete slab from the high-moment end to the low-moment end of the opening has the effect of mobilizing the shear connectors, not only over the opening, but between the opening and the support, even past an inflection point for members that undergo negative bending near the support. This can significantly increase the contribution of the concrete slab to bending strength. Overall, the failure of composite beams with web openings is quite ductile. The strains in the concrete slab remain low, even when steel strains are well past yield. Strength, however, is ultimately governed by failure of the slab.

In some test members, tearing has been observed at the corners of the top tee at the high-moment end and the bottom tee at the low-moment end of the opening. Tearing, however, occurs well after the peak load has been attained and has no discernible effect on the strength of the member.

A particularly fortuitous aspect of the behavior of beams with web openings is the very weak interaction between moment and shear capacity. That is, an opening subjected to both high moment and high shear, has bending and shear strengths that do not differ greatly from those of the opening subjected to pure bending or pure shear. While moment–shear interaction must be accounted for in design, the relatively weak interaction significantly reduces the need for reinforcement. Research has also demonstrated that the use of unshored construction, even for members subjected to construction loads as high as 60% of ultimate, does not effect the strength of composite members at openings[15].

Strength design

Strength design is now used universally for regions at web openings because the stresses induced by secondary bending can be very high and lead to early yielding. If used as a basis for design, these high local stresses would result in highly conservative designs, with little regard for the actual strength of the section.

The design of regions around web openings is more complex than some other aspects in structural steel design. The need for the extra complexity, however, becomes apparent when it is considered that the placement of an opening within a member changes the entire nature of its response to load—addition of the opening has the effect of introducing a small frame into an otherwise monolithic flexural member. It is the design of this 'frame' that requires the extra effort. Over the years, a number of design procedures have been developed. Until about 10 years ago, separate procedures were used for composite and non-composite sections and for reinforced and unreinforced openings. The similarity in behavior of steel sections with and without reinforcement and with and without a composite slab, however, has allowed for the development of design techniques that apply to all four beam/opening combinations.

There are two general approaches used in design. One involves the direct calculation of the shear and moment capacity at the opening. In the CIRIA/SCI[2**] procedure, which is the prime example of this approach, the shear capacity of the bottom tee is neglected. The other approach involves the calculation of the maximum moment capacity (under zero shear) and the maximum shear capacity (under zero moment) at the opening. The capacity of the region at the opening is then checked using an interaction expression[1, 3*, 6*]. Of the design approaches currently in use, that developed by Darwin & Lucas[3*] remains the most accurate and is generally the easiest to apply. It has been adopted by the American Institute of Steel Construction (AISC) in the Steel Design Guide series[4**] and is the principal design technique in Structural Engineering Institute/American Society of Civil Engineers (SEI/ASCE) Standard 23–97[18**] and is discussed in greater detail next. The technique also appears in Refs[19*, 20*].

MOMENT CAPACITY

The design procedure in Refs[3*, 4**, 18**] involves the calculation of the moment capacity at the section based on zero applied shear, M_m. Standard design assumptions are made, with the possible exception that in determining the force in the concrete, the total shear connector capacity from the high moment end of the opening to the support is used, rather than the shear capacity to the point of zero moment (inflection point in a member with rigid or semirigid supports). The higher contribution of the shear connectors is due to the observed sliding of the concrete slab, which has the effect of mobilizing all of the shear connectors from the opening to the support at the time of failure of the region around the opening.

SHEAR CAPACITY

The shear capacity at the opening is calculated based on the model shown in Fig. 2. Here the contribution of the flanges to secondary bending is ignored. This results in a simple expression for the maximum shear capacity of a tee V_{mt}:

$$V_{mt} = \frac{\sqrt{6} + \mu}{\nu + \sqrt{3}} V_{pt} \leqslant V_{pt} \quad (1)$$

where V_{pt} $(= F_y t_w s_t/\sqrt{3})$ is the plastic shear capacity of web of the tee, ν $[(= a_0/s_t)$ the aspect ratio of the tee, t_w the thickness of web, s_t the depth of the tee, a_0 the length of the opening, and μ is given by

$$\mu = \frac{2P_r d_r + P_{ch} d_h - P_{cl} d_\ell}{V_{pt} s_t} \quad (2)$$

Here P_r is the force in reinforcement along the edge of the opening, d_r the distance from outside edge of flange to centroid of reinforcement, P_{ch} and P_{cl} are the concrete forces at high- and low-moment ends of the opening, and d_h and d_ℓ the distances from top of steel section to centroid of concrete force at high- and low-moment ends of opening, respectively. $P_{ch} = P_{cl} = 0$ for a bottom tee in a positive moment region and (conservatively) for both tees in a negative moment region. Based on tests in which the opening was located at a point of inflection, there is strong evidence that the slab will contribute to shear strength (P_{ch}, $P_{cl} \neq 0$), even for openings in negative moment regions. This is currently not done, however, since no verification tests have been run.

The total maximum shear capacity of the opening, V_m, is obtained by summing the values of V_{mt} for the top and bottom tees.

MOMENT–SHEAR INTERACTION

A cubic interaction curve (Fig. 3) is used to determine the nominal shear and moment capacities, V_n and M_n,

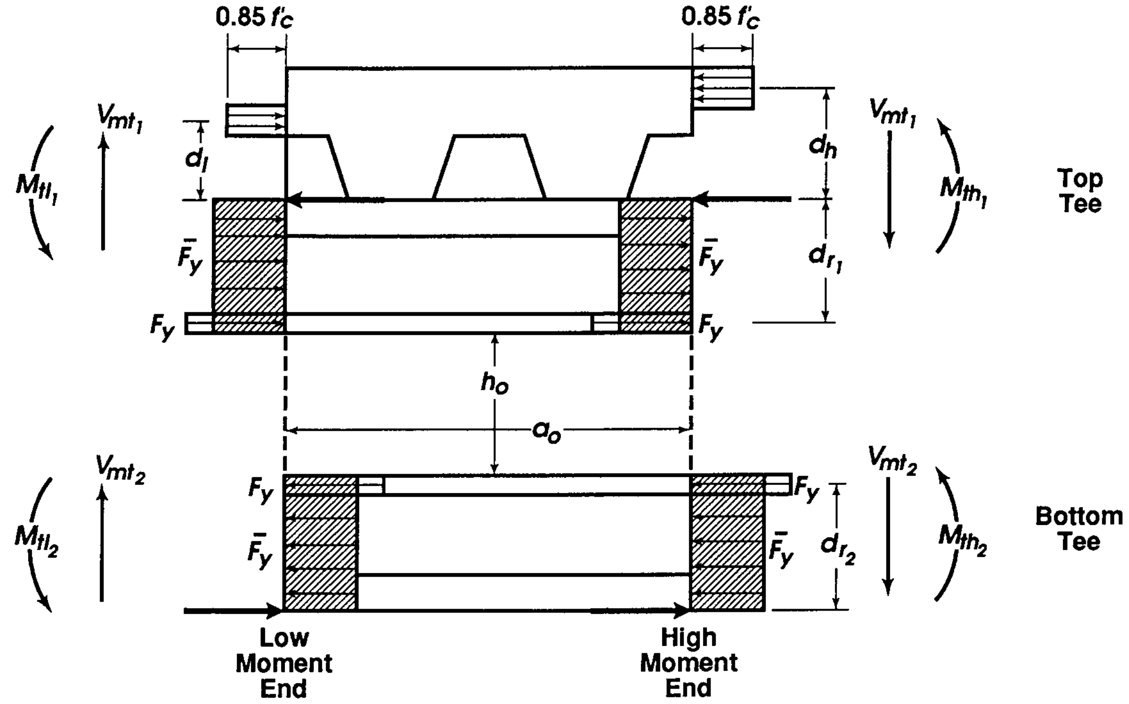


Fig. 2 Simplified axial stress distributions for opening at maximum shear; $\bar{F}_y$ is the reduced axial strength of steel in web due to combined axial stress and shear stress. Reproduced from Ref. [20] by permission of ASCE

respectively, for a particular M/V ratio:

$$V_n = V_m \left[\left(\frac{M_u V_m}{V_u M_m} \right)^3 + 1 \right]^{1/3} \tag{3a}$$

$$M_n = V_m \left(\frac{M_u}{V_u} \right) \tag{3b}$$

where V_u and M_u are the factored shear and moment, respectively, at the centerline of the opening.

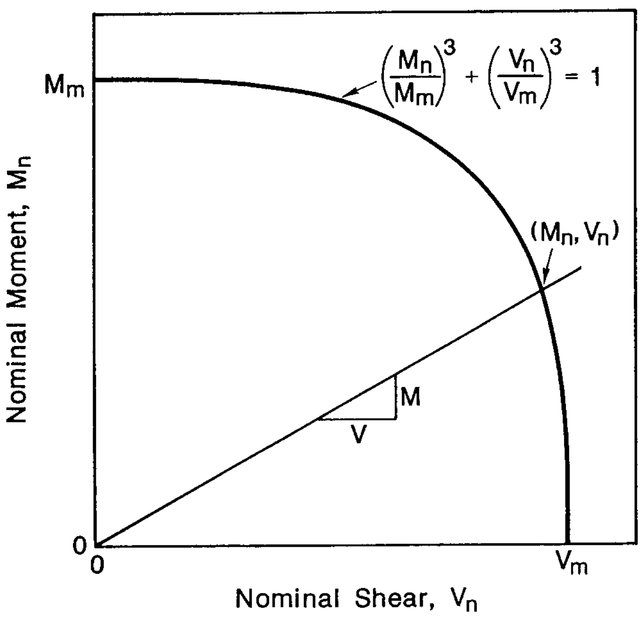


Fig. 3 Cubic moment–shear interaction diagram. Reproduced from Ref. [1] by permission of ASCE

In US practice, overall strength reduction factors of 0.85 and 0.90 are applied to V_n and M_n for composite and non-composite members, respectively.

Additional requirements

In addition to the basic strength calculation, design procedures apply addition restrictions to member and opening geometries and require additional checks to complete the design[2**, 3*, 4**, 6*, 18**].

Stability considerations

Nearly, all test specimens for composite members have been constructed using compact sections. Therefore, design procedures invariably limit application of the strength design equations to members with compact steel sections.

Stability considerations also include checks for

- local buckling of the compression flange or reinforcement,
- web buckling,
- buckling of the tee-shaped compression zone below the opening, and
- lateral buckling of the compression flange.

Guidelines, available since 1980[21*], have been generally adopted.

Local buckling of the compression flange or reinforcement is controlled based on the width-to-thickness ratio of the projecting portion of the flange or reinforcement.

Web buckling is limited by controlling the geometry of the opening. When the web qualifies as 'stocky', openings with length-to-height ratios as high as 3.0 can be used.

In negative/hogging moment regions, the bottom tee must be investigated as an axially loaded column. For lateral buckling in the same region, the general rule is that for members subject to lateral buckling of the compression flange, strength should not be governed by strength at the opening (calculated without regard to lateral buckling).

In current practice, stability criteria represent the area with which designers have the most difficulty, because these criteria often require modifications that are more costly than those required to satisfy the strength equations. This is especially true for projects involving the retrofit of existing structures.

Opening and tee dimensions

Opening depth is typically limited to 70% of the depth of the steel section. The depth of the top steel tee is usually limited to 15% of the depth of the steel section, while the depth of the bottom tee is not restricted, although when depths drop below 12–15%, it is strongly recommended that the contribution of the bottom tee to shear capacity be neglected[18**].

Restrictions are usually placed on corner radii for rectangular openings to limit effects on fatigue capacity.

Placement of openings and of concentrated loads

It is generally accepted that concentrated loads should not be placed above an opening and restrictions are usually placed on how closely concentrated loads can be placed to an opening without the need for bearing stiffeners. Without a special investigation, the strength design expressions are usually limited to cases in which the edge of the opening is not closer than the depth of the steel section to a support. Because of slab bridging, it is recommended that openings not be located closer to each other than two times the depth of steel section or the length of the opening, whichever is greater.

Circular openings

Regions around circular openings are significantly stronger than square openings with the same maximum dimensions. A a result, circular openings can be designed as equivalent rectangular openings with approximately the same height as the diameter of the opening, but with equivalent lengths equal to less than half of the diameter.

Reinforcement

Reinforcement can be placed above and/or below the opening and on one or both sides of the web. The reinforcement should consist of horizontal bars welded to the web (as close as feasible to the horizontal edge of the opening) and extending for a minimum distance past the opening to provide anchorage to insure full yield capacity is available to the reinforcement. Most design expressions (especially for M_m) are based on the use of equal reinforcement above and below the opening. However, there is nothing in the formulation of the design expressions that requires that the reinforcement be equal.

Flange reinforcement may be used to increase the flexural capacity of the member.

Slab reinforcement and shear connectors

Although not required by the design calculations, when possible, it is desirable to use a minimum amount of slab reinforcement equal to 0.0025 of the gross area (based on the full depth) of the slab within a distance of the opening equal to the larger of the depth of the steel section or the length of the opening. It is also recommended, but not required, that, to limit the effects of bridging, a minimum of six shear connectors per meter should be used for a distance equal to the larger of the depth of the steel section or the length of the opening from the high-moment end of the opening toward the direction of *increasing* moment. These shear connectors are not in addition to those used for flexural requirements.

Construction loads

If constructed without shoring, the strength of the section at the opening must be checked based on non-composite action.

Deflection Calculations

Single-web openings generally have little effect on the deflection of composite beams[14, 16, 22]. There are, however, cases where the effect can be significant, especially for openings located in regions of high shear. Over the years, a number of techniques have been developed to determine the effect of web openings on both total deflection and on deflection between the two sides of an opening[23–27, 28**]. The earliest of the techniques[23–25] were developed principally for steel beams, while the most recent apply to both steel and composite members[26, 27, 28**].

MATRIX MODEL

A recent study[28**] uses the stiffness method of matrix analysis to calculate deflections, producing a good match with both total deflection and deflection across the opening. In the model, unperforated sections of a beam are represented using uniform beam elements. Regions at web openings are represented by beam elements above and below the opening connected to the unperforated sections by rigid links. Moments of inertia for unperforated sections are calculated considering partial composite action. The best representation for total beam deflection is obtained by modeling the top tee at the opening as a composite member along its full length. The best representation for deflection through an opening is obtained by representing the top tee as a composite member for half the opening length, adjacent to the high moment end, and as a steel member for the other half. The 'lower bound moment of inertia' used in the AISC *Load and Resistance Factor Design Manual*[29] provides the best representation of the composite section properties. In the study, the effects of shear deformation were also considered and, in general, the effects of an opening and of shear deflections are of the same order.

MAXIMUM DEFLECTION

Based on a parameter study using the matrix model, it is clear that web openings have the greatest effect on total deflection when located near midspan. However, there is surprisingly little difference in the total deflection as a function of opening position. For a simply supported beam with a large opening, for example one with a depth equal to 70% of the steel section depth and a length equal to three times the opening depth, the increase in deflection compared to that obtained for an unperforated member is about 12% for an opening with a centerline located $\frac{1}{16}$ of the span from one support and about 15% when the opening is located at midspan. Openings with depths of 30% of the steel section result in a total increase in deflection of less than 4%.

DESIGN AID

Given that the maximum deflection for a uniformly loaded beam will always occur when the web opening is located at midspan allows for the development of a closed-form relationship to establish the upper bound on the effect of the web opening on deflection. That relationship is illustrated in the design aid shown in Fig. 4, which shows the ratio of the bending deflection of a member with an opening, $\Delta_{m,b}$, to the bending deflection of the unperforated member, Δ_b, as a function of the ratio of the gross moment of inertia of the unperforated member, I_g, to the moment of inertia at the web opening, I_{wo}, and the ratio of the opening length, a_o, to the span length, L_s.

Ref.[28**] also contains closed-form equations for deflections across the opening.

Conclusions

The placement of an opening in the web of a composite beam can significantly reduce the local strength and stiffness of the member. Under conditions of high

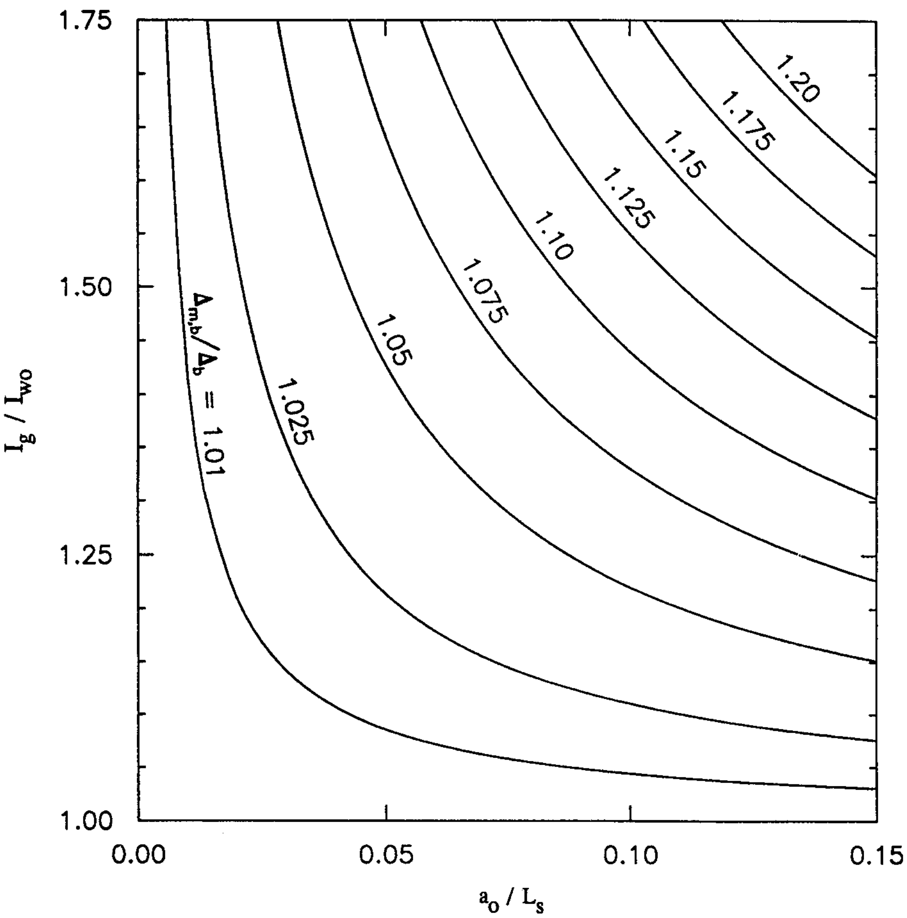


Fig. 4 Design aid for estimating maximum deflection in a beam with a web opening–deflection calculated for opening located at beam centerline [$\Delta_{m,b}$ is the maximum bending deflection for the perforated beam, Δ_b the maximum bending deflection for the non-perforated beam, a_o the opening length, L_s the span length, I_g the moment of inertia for non-perforated section, and I_{wo} the moment of inertia in region of web opening.] Reproduced from Ref. [28**] by permission of ASCE

bending moment and low shear, member behavior is generally similar to that of unperforated sections, with failure governed by crushing of the concrete and yielding of the steel. In regions of low moment and high shear, secondary bending moments induced by shear through the opening change the behavior from that of a beam to a member that behaves locally as a frame, in which failure is governed by the formation of hinges at the corners of the opening. Several design techniques exist. Some using simplified techniques allow direct calculation of capacity at the opening, others use an interaction procedure to account for the simultaneous effects of bending and shear at the opening. A widely adopted technique based on moment–shear interaction is summarized. A recently published procedure for calculating deflections resulting from web openings provides an easy-to-use design aid to check the upper-bound effect on deflection.

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** Exceptional interest

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