BUCKLING LENGTH OF UNBRACED FRAME COLUMNS

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Structural Engineering and Engineering Materials
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ABSTRACT

In the design of steel columns in unbraced frames, the current AISC specification commentaries from both LRFD and ASD contain an alignment chart to determine the K factor for a particular column. The K factor is based on the effective length concept where K factors are used to equate the strength of a compression member of length L to an equivalent pin-ended member of length KL subjected to axial load only. The unbraced frame alignment chart is a graphical representation of a transcendental equation of a buckling solution of a subassemblage. This solution involves several assumptions limiting the use of the alignment chart to idealized cases not necessarily satisfying a particular practical situation.

The aim of this study is to 1) compare K factor values from frame instability analysis using structural software with values from the alignment chart in situations where the assumptions of the alignment chart are violated and 2) suggest application of appropriate known solutions to particular situations in which violations of the assumptions occur. Situations investigated are: variations in bay width, variations in column moment of inertia, variations in loading, and variations in column height.

The nomograph performance was found to be relatively insensitive to bay width variation. Variations in column moment of inertia and column loading lead to large inaccuracies in the nomograph K factor values but Lui's method handled these cases well. The nomograph performance was found to be most sensitive to column height variation. Configurations with large variation in column height require system stability analysis to obtain accurate K factors.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1
Introduction

1.1 Problem Statement.

In the design of steel columns in unbraced frames, the current American Institute of Steel Construction (AISC), commentaries for both the Load and Resistance Design Factor (LRFD) Specification and [AISC 1993] and the Allowable Stress Design (ASD) Specification [AISC 1990] contain an alignment chart to determine the *K* factor of a particular column [Lui 1992]. The *K* factor is based on the effective length concept where *K* factors are used to equate the strength of a compression member of length *L* to an equivalent pin-ended member of length *KL* subjected to axial load only. The alignment chart is deemed to satisfy the "analysis" requirement of LRFD-C2.2 to get "adequate" *K* factor values of columns to determine their effective lengths [Salmon and Johnson 1990]. The LRFD [AISC 1993] and ASD [AISC 1990] commentaries recommend its use instead of frame buckling analysis to compute *K* factors.

The alignment chart is widely used because of its straight forward method of obtaining the effective length of a column [Shanmugam and Chen 1995]. The unbraced frame alignment chart is a graphical representation of a transcendental equation of a buckling solution of a subassemblage. This solution involves several assumptions limiting the use of the alignment chart to idealized cases not necessarily satisfying a particular practical situation. The aim of this study to observe the accuracy of the *K* values obtained from the alignment chart when the assumptions are violated (see article 1.4 for discussion of assumptions considered in study). The study will also find solutions to particular situations that are applicable when the violations of the assumptions underlying the alignment chart occur.

1.2 The Effective Length Method.

The effective length approach is an approximate method of second order analysis used for column stability evaluation. It examines an individual member instead of the framed structure as a whole. This is necessary in order to simplify the analysis to a level that is practical for use in routine design.

A compression member in a frame interacts not only with other members which are adjacent to it (horizontally) but it also interacts with members in other stories (vertically). In order to adequately represent these interactions the analytical process can become very complex and would require a full system instability analysis.

Estimation of the interaction effects of the total frame on an individual compression member is the essence of the effective length concept. The K factor is used to equate the strength of an individual framed compression member of length L to an equivalent pin-ended compression member of length KL subjected to axial load only. Although it is completely valid only for ideal structures, the effective length concept is the only tool available capable of handling cases which occur in practically all structures and is also an essential part in many analysis procedures [AISC 1993]. Although it is well known that the effective length approach introduces inaccuracies into the process, the simplicity of examining an individual member is likely to make the approach an important part of framed column design in the foreseeable future [Hellesland and Bjorhovde 1996].

1.2.1 The Alignment Chart.

The alignment chart is recommended by the LRFD [AISC 1993] specification for the computation of K factors. The chart is based on the buckling of the subassemblage as shown in Fig. 1.1 [Shanmugam and Chen 1995]. The resulting transcendental equation of the buckling solution for the unbraced subassemblage is of the form,

$$\frac{G_A G_B (\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{\pi/K}{\tan(\pi/K)} = 0$$

where G_A and G_B are the column to beam stiffness ratios at the two column ends defined as,

$$G_A = \frac{\sum_A (EI/L)_{column}}{\sum_A (EI/L)_{beam}}$$

$$G_B = \frac{\sum_B (EI/L)_{column}}{\sum_B (EI/L)_{beam}}$$

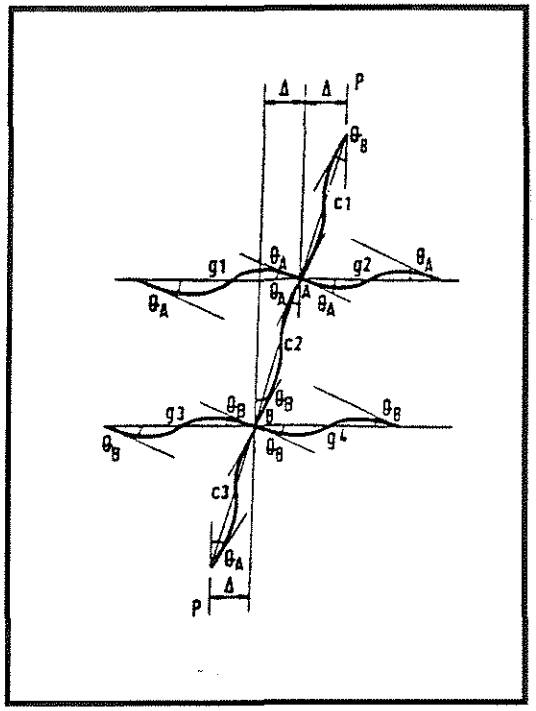


Fig. 1.1
Subassemblage of an unbraced frame used in the development of the alignment chart

The graphical representation of the solution of the transcendental equation is the sidesway permitted alignment chart shown in figure C-C2.2 in the AISC manual [AISC 1993] (Figure 1. 2).

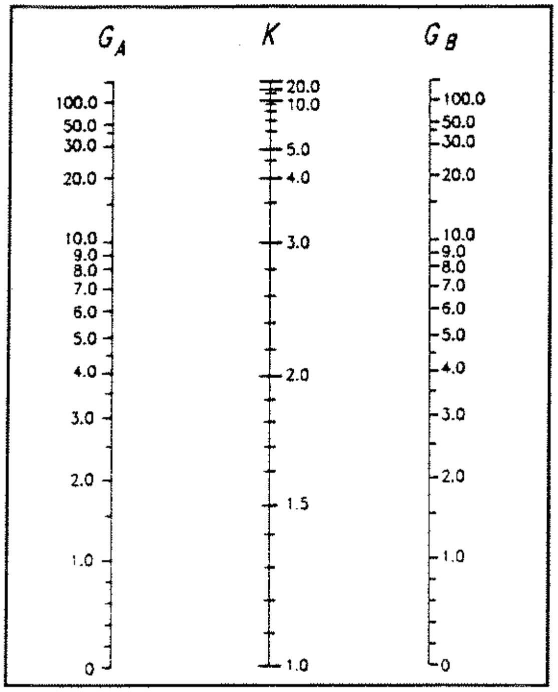


Fig 1.2

The alignment chart

The alignment chart is the most widely accepted method of obtaining the K factor to be used in the design process. However it should be realized that in obtaining the alignment chart various simplifications and assumptions were used. Violation of these simplifications and assumptions can lead to inaccurate K factors. Cases investigated in this study where the violations occur are listed and discussed in article 2.3 of Chapter 2.

1.2.2 Assumptions and Simplification.

The alignment chart as developed by O. J. Julian and L. S. Lawrence is presented in detail by Kavanaugh [1962]. They prepared the currently used chart with the following assumptions [AISC 1993]:

1. Behavior is purely elastic.
2. All members have constant cross section.
3. All joints are rigid.
4. For braced frames, rotations at opposite ends of restraining beams are equal in magnitude and opposite in sign, producing single curvature bending.
5. For unbraced frames, rotations at the far ends of the restraining members are equal in magnitude and opposite in sign, producing reverse-curvature bending.

6. The column stiffness parameter $\phi = L\sqrt{P/EI}$ must be identical for all columns.
7. Joint restraint is distributed to the column above and below the joint in proportion to I/L of the two columns.
8. All columns buckle simultaneously.
9. No significant axial compression force exists in the girders.

The LRFD Commentary [AISC 1993] section C-C2 on frame stability notes that these assumptions and simplifications are based on idealized conditions, rarely existing in practice. It goes on to state that when the assumptions are violated, unrealistic design may result.

Guide to Structural Stability Design Criteria for Metal Structures [Galambos 1988] suggests that the chart is applicable to symmetrical frames, symmetrically loaded, and gives reliable results for frames where the stiffness is approximately proportional to the loading.

1.3 Alignment Chart Performance.

Various researchers have dealt with the problem of the performance of the alignment chart when the assumptions on which it is based are violated. Most studies presented situations in which the alignment chart gives unrealistic K values and presents solutions to them. Some [Duan and Chen 1989, Bridge and Fraser 1987, Yura 1971] modify the stiffness ratio, i.e. the G factor, some [LeMessurier 1977, Chu and Chow 1969] introduce corrections to values from the alignment chart, some [Aristazabal-Ochoa 1994, Lui 1992] give new equations to obtaining the K value, and some [Cheong-Siat-Moy 1986] go so far as to propose the elimination of K factor use in the design process.

Yura [1971] attributed much of the misunderstanding of the effective length concept to the direct use of the alignment chart in situations where the basic assumptions in deriving the chart are violated. Two basic assumptions; 1) elastic action and 2) simultaneous buckling of all columns in a story; are identified as being inaccurate in practical situations and producing overly conservative K factors.

In the case of elastic action, Yura [1971] suggests that inelastic action starts at about $0.5F_y$ where the column slenderness ratio is less than the critical slenderness ratio. In the inelastic range the stiffness ratio $G_{inelastic}$ is defined by the equation: $\frac{E_T}{E}.G_{elastic}$, where E_t is the tangential modulus of the column in the inelastic range. The reduced G computed as $G_{inelastic}$ is then used to find K values from the alignment chart. This approach can produce significant reductions in K factors. In the current LRFD Manual of Steel Construction [AISC 1993] stiffness reduction factors are used in this manner to account for columns in the inelastic range.

For columns in a single story which buckle simultaneously under proportionate sharing of total gravity load, Yura [1971] suggests that design based on the alignment chart is reasonably accurate. However, there are situations where an individual member can have excess buckling strength. This can occur when different loading conditions exist within a story and thus different columns will have different buckling loads and will not buckle at the same time. The overall frame will not buckle until buckling loads for all columns have been reached. The columns with excessive buckling loads will increase their buckling loads and thus decrease their effective length. Yura [1971] states that this can cause the effective length of some of the columns to be less than 1.0 (which is allowable in the current LRFD Manual of Steel

Construction [AISC 1993]) even with no bracing. Sidesway buckling is a total story phenomena and a single individual column will not fail without all columns in the story participating in the overall buckling in the sway mode. Yura [1971] presented a simple design approach which considers the potential bracing capacity of columns in a story.

LeMessurier [1977] suggests correction to the alignment chart assuming that stronger columns within a story will brace weaker columns during sidesway buckling. His approach accounts accurately for the fact that all columns in a story buckles simultaneously. Initial G and K values from the sway uninhibited alignment chart are used to introduce factor that account for the reduction in column stiffness due to the presence of axial load and column end restraints. These factors are then used in an expression to determine the K factor. A chart to determine the factors is also given and simple solutions to certain cases are suggested.

Lui [1992] in his paper suggests that in the development of the alignment charts certain situations are unrealistic in practice which results in inaccurate K factor values. Variations in the value of the column stiffness parameter $\phi = L\sqrt{P/EI}$ across a story in frames due to unequal distribution of column axial force, moment of inertia, and frames with leaner columns can cause significant errors. Lui explicitly accounts for the effect of member instability and frame instability effects in his approach towards a more accurate K value. The equation introduced contains terms that represent member instability and frame instability effects. K factors for columns in unbraced frames with unequal distribution of lateral stiffness and gravity loads and frames with leaner columns can be predicted with sufficient accuracy using his formula.

There are researchers that suggest eliminating the use of *K* factor values in column design. Cheong-Siat-Moy [1986] suggests that the use of the K factor presents a paradox to the designer where member instability analysis and frame instability analysis give different values. An example is leaning columns, which in practice would be given a K factor of 1.0, while a buckling analysis would give values of greater or even less than 1.0. He also states how columns with semi-rigid and fully rigid connections are also subject to the same "paradox". Elimination of these conflicts can be done via a column interaction formula without the consideration of the K factor. It has been found that such interaction equations that do not take the K factor into account may lead to unconservative results [Lieu et al. 1991]. Thus, this study does not take this approach.

1.4 Research Objectives.

The aim of this study is to :-

1. Compare *K* factor values from frame instability analysis using structural software with values from the alignment chart in situations where the assumptions of the alignment chart are violated.
2. Suggest application of appropriate known solutions to particular situations in which violations of the assumptions occur.

Objective 1 shall be achieved by using an unbraced frame to determine values of the K factor using both the analysis software and the alignment chart. Properties such as the bay width, members moment of inertia, loading, and column height shall be varied with the aim of violating the assumptions that the alignment chart was based upon. Robot V6 [Manual 1996] is the structural analysis program to be used in analyzing the buckling length of columns. It is a structural analysis and design program that is capable of performing linear or nonlinear buckling analysis. Values from the alignment chart shall be calculated by solving the transcendental equation that is used to develop the alignment chart instead of a visual inspection of the chart itself. The comparison shall be done in the form of graphs of K values against the parameter studied.

Various methods suggested by researchers in the literature shall be used in achieving objective 2. Solution/solutions shall be suggested for situations where the basic assumptions are violated with the aim of being a guide for designers when they are confronted with such situations.

Chapter 2
Parametric Study

2.1 Objectives.

The parametric study is carried out with the following objectives.

- To observe the difference between K factor values obtained from structural analysis software and values obtained from the alignment chart.
- Give solution/solutions to the designer when faced with such situations.

2.1.2 Scope of Study.

The study covers unbraced frames, i.e., frames where sidesway is uninhibited. The study attempts to create situations when certain assumptions of the alignment chart are violated (see Item 2.3). Analysis by using both the alignment chart and structural analysis software (Robot V6) are carried out and the results compared. Based on the results, recommendations are made to designers when faced with such situations.

The study only covers buckling in the elastic range. Elastic buckling analysis is used when running problems through Robot V6. When the alignment chart is used to evaluate K factors it is implicitly assumed that elastic buckling controls [Salmon and Johnson 1990]. The problem of inelasticity is well covered by Yura [1971] and Salmon and Johnson [1990] .

The study mostly revolves around the assumptions that the column stiffness parameter, $\phi = L\sqrt{P/EI}$, is identical for all the columns in a particular story and that all the columns within a story buckle simultaneously. In practical situations, column dimensions and loadings may vary thus varying the column stiffness parameters. In order for all the columns within a story to buckle simultaneously the column stiffness parameters of all the columns within the story must be the same which, in practical situation, is often not the case.

2.2 *K* Factor - System Buckling Analysis Versus Alignment Chart.

In order to compare the difference of the *K* factor values between a total system buckling analysis and the alignment chart, a structural analysis software package ROBOT V6 [Metrosoft 1996] was used. System buckling analysis is the most accurate method of obtaining the effective length factor in a framed structure [Shanmugam and Chen 1995]. Due to its complexity it is rarely used in practice and instead an approximate effective length method is used. Due to its assumptions and simplifications results obtained are often inaccurate. This study attempts to address the inaccuracies and find solutions to correct them.

2.2.1 ROBOT V6.

ROBOT V6 [Metrosoft 1996] is an integrated structural analysis and design software package. It has graphical input and output capabilities. It is comprised of a set of integrated modules running in a common system. It is capable of performing linear and nonlinear static, buckling, modal and dynamic analyses. This study uses the linear buckling analysis capabilities of the package. The buckling analysis produces results of critical buckling loads and coefficients and effective lengths. The effective lengths from the analysis are divided by the actual column length and used to obtain the *K* factor values from the system buckling analysis.

2.2.1.1 Verification of Robot V6.

In order to verify the results obtained from Robot V6, a trial run was carried out using an example single-bay three-story frame with even column loads [Shanmugam and Chen 1995]. Bay width is 25 ft. Story height is 13 ft. Loads are 28 kip per column applied at the top of the frame. Column sizes are W8x48 for the first story (elements 1 and 2), W8x35 for the second story (elements 4 and 5), and W8x35 for the third story (elements 7 and 8). Girder sizes are W21x44 for the first and second floor and W14x30 for the third floor. The results from both Robot V6 and system buckling analysis from Shanmugam and Chen [1995] are the same (see Table 2.1).

Elem.	K-factor (Robot V6)	K-factor by system buckling analysis from Shanmugam and Chen
1	1.14	1.14
2	1.14	1.14
4	1.14	1.14
5	1.14	1.14
7	1.52	1.52
8	1.52	1.52

Results from ROBOT V6 and System Buckling Analysis from Shanmugam and Chen

Table 2.1

The results validates the use of Robot V6 as a means of obtaining the total system buckling analysis for this study.

2.2.2 Values from Alignment Chart

K values from the alignment chart are obtained from solving the transcendental equation that the alignment chart is based upon. This is to avoid any errors that can occur due to errors in reading from the alignment chart. The equation [Shanmugam and Chen 1995] to be used is,

$$\frac{G_A G_B (\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{\pi/K}{\tan(\pi/K)} = 0$$

where,

$$G_A = \frac{\sum_A (EI/L)_{column}}{\sum_A (EI/L)_{beam}}$$

$$G_B = \frac{\sum_B (EI/L)_{column}}{\sum_B (EI/L)_{beam}}$$

2.3 Parametric Study.

The parameters to be studied are:

- the bay width
- column moment of inertia
- loading and loading configuration
- column height.

The column moment of inertia and the column height affect the stiffness of the framed columns and thus the buckling strength of the columns. The bay width sets the girder length affecting the stiffness of the girder with respect to the column and thus the column buckling strength. The loading affects column buckling strength both by material nonlinearity introduced by inelasticity due to the actual load and also by the leaning effects from other columns in the story. Symmetric and unsymmetric loading are considered.

This study uses as a baseline structure a three story two bay unbraced frame, fixed at the column footings (see Fig. 2.1). The structure is taken from an article by Shanmugam and Chen [1995]. From the reference structure the parameters to be studied are varied. The variations are run through Robot V6 and their buckling lengths and *K* factor values are obtained. The *K* factor values from the alignment chart are then obtained by solving the transcendental equation.

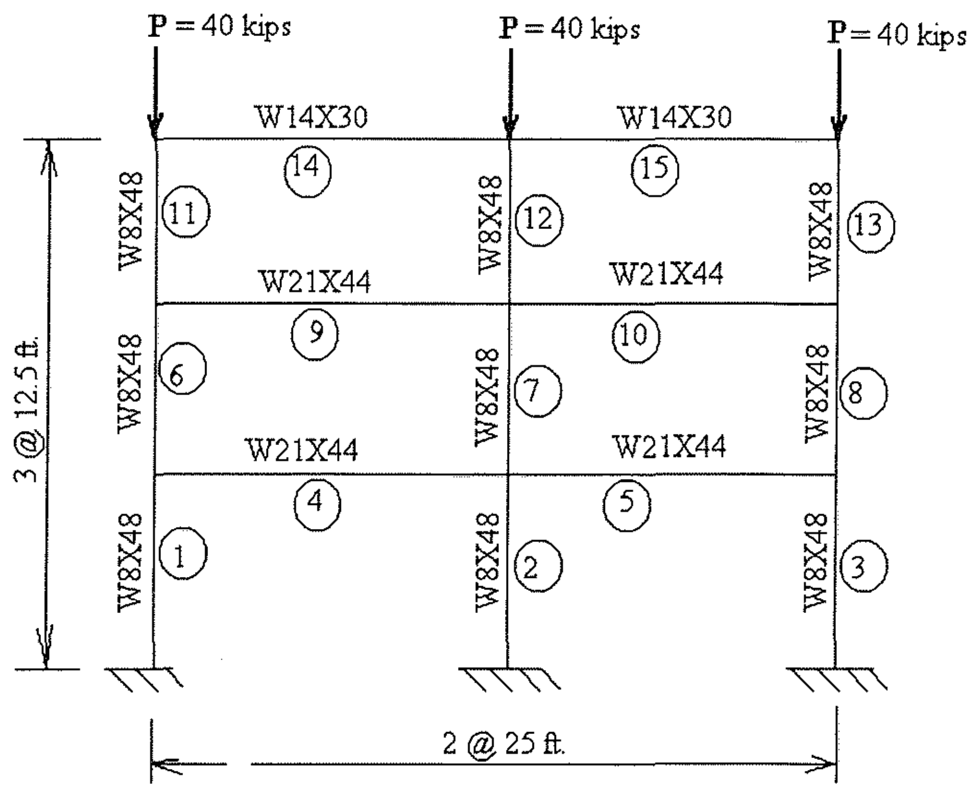


Fig 2.1 Baseline unbraced frame for the study.

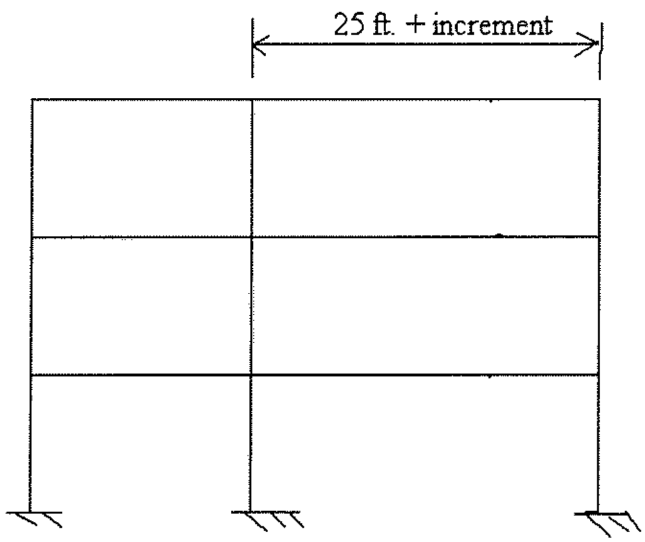
The increment and ranges of the parameters used in this study are given in Table 2.2.

Parameter	Baseline	Increment	Range
Bay width	25 ft	5 ft	25-50 ft.
Col Mom Inertia	184 in^4	~184 in^4	184-1900 in^4
Loading	40 kip	40 kip	40-200 kip
Col Height	12.5 ft	2.5 ft	12.5-25 ft

Table 2.2
The increment and ranges of the parameters used in this study

2.3.1 Bay Width.

The bay width is varied in increments of 5 feet from the baseline width of 25 feet until the width is doubled to 50 ft (see Table 2.2 and Figure 2.2). Only the right bay of the structure is varied, with the left bay held constant at 25 ft.



Variation in bay width.

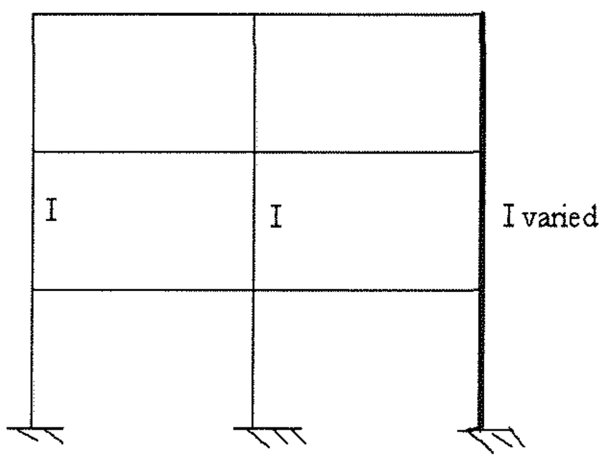
Figure 2.2

Based on the alignment chart alone it would be expected that the widening of the bay should have an effect of reducing the stiffness of the girder and thus increasing the G value of a particular joint which should increase the K factor value of the connected columns. K values for the left hand column tier (columns 1, 6, and 11) from the alignment chart will not be effected since the chart uses a local approach considering only those members directly joined to the column. This increase in right bay width will violate the alignment chart assumption that the structure is symmetric.

The results from the Robot V6 analysis and the alignment chart is presented in Table 3.1, Fig. 3.1a , Fig. 3.1b, and Fig. 3.1c. Results are discussed in Chapter 3.

2.3.2 Column Moment of Inertia.

The moment of inertia of the right hand column tier (columns 3, 8, and 13) of the structure is increased to about 10 times the ratio of the baseline moment of inertia (see Table 2.2 and Figure 2.3).



Variation in moment of inertia

Figure 2.3

The increments are done by picking an existing rolled section that has a moment of inertia value close to the desired value. For example the baseline moment of inertia is 184 in^4. Twice the value would be 368 in^4. But the closest value tabulated for a rolled section is 341 in^4. Due to the use of rolled table section in Robot V6, a value of 341 in^4 is used for ease of running the variations through the program (see Table 2.3). Results from Robot V6 and the alignment chart are shown in Table 3.2, Fig. 3.2a, Fig. 3.2b, and Fig. 3.2c. Results are discussed in Chapter 3.

Ratio	Section Used	Ix value(in^4)
I	W8X48	184
2I	W10x60	341
3I	W10x88	534
4I	W12x87	740
5I	W12x106	933
6I	W14x99	1110
7I	W14x109	1240
8I	W14x132	1530
9I	W14x145	1710
10I	W14x159	1900

Column sections used in moment of inertia variation
Table 2.3

Based on the alignment chart alone, it would be expected that the increase in the column moment of inertia should have the effect of reducing the *G* of the respective joints and thus reducing the *K* factor of the connected columns. As with bay width changes, the *K* factor values for the left-hand column tier (columns 1, 6, and 11) from the alignment chart will not be effected by the moment of inertia changes. The variation in moment of inertia of right column tier (columns 3, 8, and 13) will violate the assumption that the column stiffness parameter $\phi = L\sqrt{P/EI}$ for all the columns within the story is the same. This is also contrary to the guideline that the alignment chart be used when the structure is symmetric.

2.3.3 Loading and Loading Configuration.

Loading and loading configuration will also be varied and changed to observe the effect of violating the guideline that the alignment chart should be used only when the structure is symmetrically loaded. Again the assumption that the column stiffness parameter $\phi = L\sqrt{P/EI}$ for all the columns within a story is the same is violated. Alignment chart *K* factor values will not change with loading distribution and configuration since these are not accounted for in the alignment chart equation.

The baseline column load of 40 kip is increased to 200 kip in 40 kip increments. First, the right hand column tier is incrementally loaded while the left and middle column tier loads are held constant at 40 kip. Loading the right columns gives both asymmetry of loading and violation of the uniform column stiffness parameter. Next, the middle column tier is incrementally loaded while the outside column loads are held constant at 40 kip. Varying the load on the middle column tier will maintain symmetry of loading but violate the uniform story column stiffness parameter assumption (see Table 2.2 and Figure 2.4).

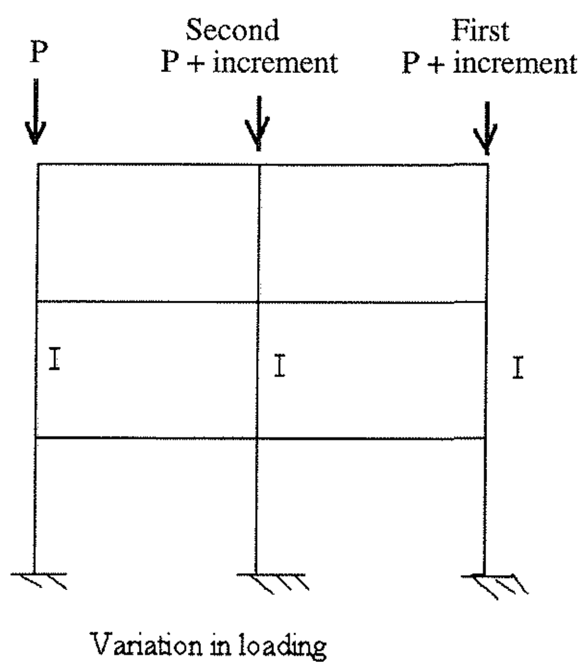


Figure 2.4

Results from both Robot V6 and alignment chart are given in the Tables 3.3a, Table 3.3b, Fig. 3.3a, Fig. 3.3b, Fig. 3.3c, Fig. 3.3d, Fig. 3.3e, and Fig. 3.3f. Results are discussed in Chapter 3.

2.3.4 Column Height.

Column height of the bottom and top stories are varied separately. Column height is varied from 12.5 ft. to 25 ft. in 2.5 ft. increments (see Table 2.2 and Figure 2.5).

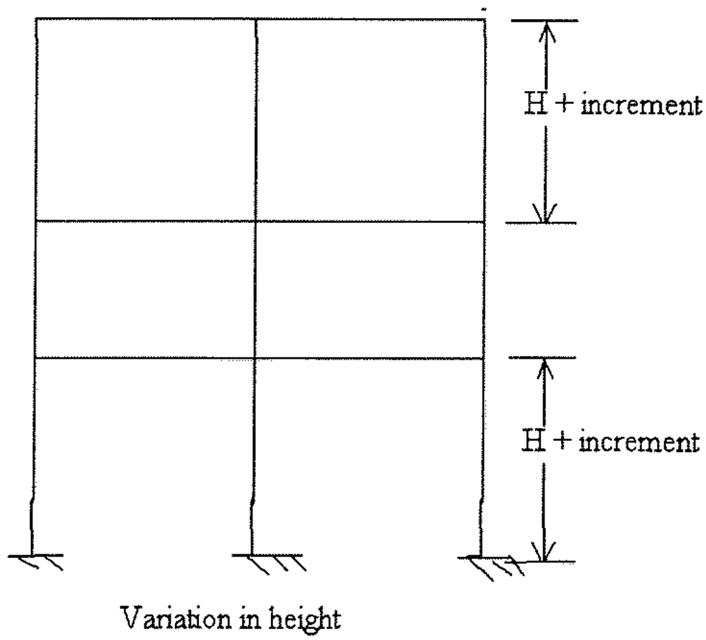


Figure 2.5

Based on the alignment chart alone, it would be expected that the change in column height would decrease the *G* value of one end of the column and thus decrease the value of the *K* factor value of the column considered. The *K* factor for the top story will not be effected by the column height changes to the columns in the bottom floor since the chart uses a local approach considering only those members directly joined to the column. The same should also be true if the situation was reversed.

To further observe the effect of the variation in column height, the topic least covered in literature, a simpler baseline structure is also investigated, a two story one bay unbraced frame with fixed footings. The bottom column height was varied from 2H times to 0.25H times the height of the top column, H (see figure 2.6). System buckling analysis, the nomograph, and Lui's method were used to find the *K* factor for the structure.

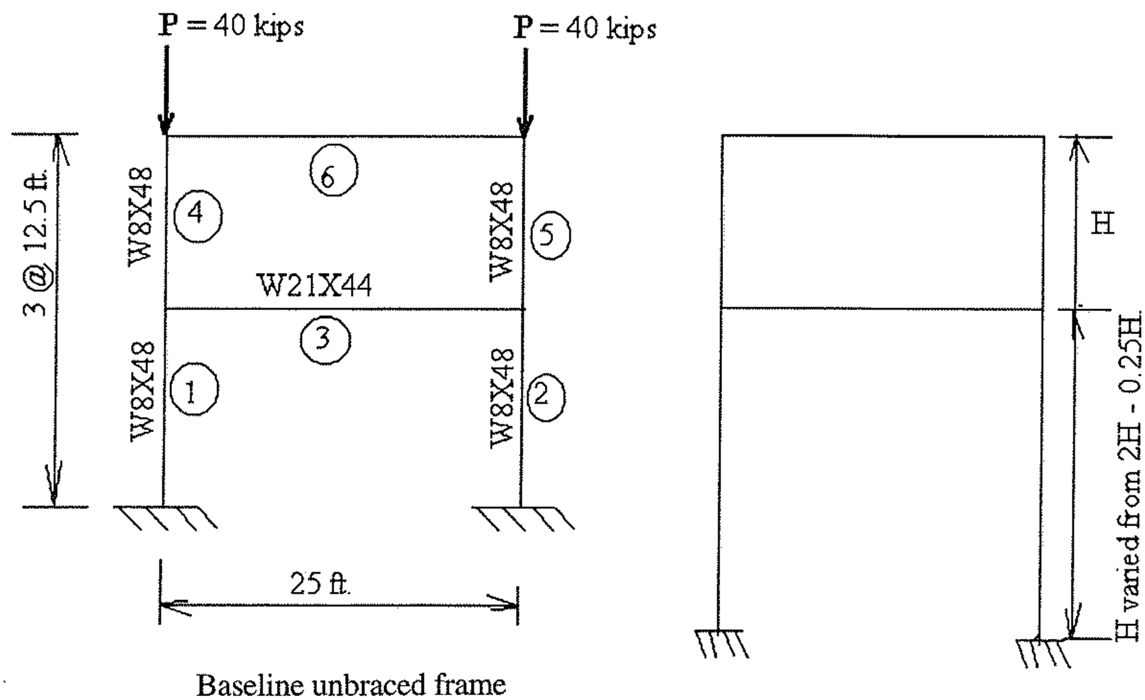


Figure 2.6 Simpler baseline structure for column height variation investigation.

The results from the above analyses are tabulated in Table 3.4 for the 3 story frame, and Table 3.5 for the two story frame. Fig. 3.4a, Fig. 3.4b, and Fig. 3.4c shows the results from the 3 story frame with the bottom columns varied, while Fig. 3.4d, Fig. 3.4e, and Fig. 3.4f show results when the top columns are varied. Results for the two story frame are presented graphically in Fig. 3.5a and Fig. 3.5b. Results are discussed in Chapter 3.

2.4 Selection of Recommendation.

Recommendations are made based on various methods of determining K factor values that are available from reviewed literature [Lui 1992, Duan and Chen 1989, LeMessurier 1977, Aristizabal-Ochoa 1994, Cheong-Siat-Moy 1986]. Lui's method is chosen based on its accuracy and simplicity of use. This method is also recommended by others [Shanmugam and Chen 1995, Hajjar and White 1994].

2.4.1 Lui's Method.

Lui [1992] proposes the following equation to estimate the K factor for an unbraced frame structure:

$$K_i = \sqrt{\left(\frac{\pi^2 EI_i}{P_i L_i^2}\right)\left[\left(\Sigma\frac{P}{L}\right)\left(\frac{1}{5\Sigma\eta}+\frac{\Delta_{oh}}{\Sigma H}\right)\right]}$$

where

P_i = compressive axial force in member I

$\Sigma\frac{P}{L}$ = sum of the axial force to the length ratio of all the members in the story

ΣH = sum of the story lateral forces at and above the story under consideration

Δ_{oh} = inter-story deflection i.e. relative displacement between adjacent stories

η = $\frac{(3+4.8m+4.2m^2)EI}{L^3}$

m = M_A/M_b

M_A, M_b = member end moments with $M_A < M_b$

$\Sigma\eta$ = sum of η of all members in the story being considered.

This method only requires a first order frame analysis to determine the horizontal deflection at every story level. A straight forward application of the formula then yields the K factor. Lui's method was also used to find the K factor for the simpler baseline structure along with Robot V6 and the alignment chart.

Chapter 3
Results and Discussion.

3.1 Presentation and Discussion of Results.

Results from both Robot V6 and the alignment chart (transcendental equation) are examined and discussed in their own context, in relation to each other in terms of differences and degree of error of the transcendental equation, and any recommendation/recommendations that can be given.

3.2 Variation in Bay Width.

3.2.1 Nomograph *K* Factor Values.

K factor values for variation in bay width are shown in Table 3 1.

		K factor from Nomograph.								
Span	K factor (RV6)	Column 1	Column 2	Column 3	Column 6	Column 7	Column 8	Column 11	Column 12	Column 13
25	1.256	1.300	1.230	1.300	1.280	1.140	1.280	1.340	1.180	1.340
30	1.276	1.300	1.240	1.320	1.280	1.156	1.330	1.340	1.190	1.390
35	1.294	1.300	1.245	1.350	1.280	1.170	1.380	1.340	1.200	1.460
40	1.310	1.300	1.250	1.370	1.280	1.180	1.430	1.340	1.210	1.510
45	1.325	1.300	1.250	1.400	1.280	1.187	1.480	1.340	1.220	1.570
50	1.337	1.300	1.250	1.420	1.280	1.190	1.520	1.340	1.230	1.620

Table 3.1a. *K* factor values for variation in bay width

Fig 3.la shows the result from the alignment chart for the left hand tier columns. The *K* factor value of those columns do not change with the widening of the right bay. Column 1 has a *K* factor of 1.30, column 6 a value of 1.28, and column 11 a value of 1.34 throughout the variation. This is because the alignment chart is a localized phenomena and any changes to the unbraced frame members that are not directly joined to the individual column considered will not have any effect on its *K* factor value.

The variation of the *K* factor values for the middle column tier (column 2, 7, and 12) is shown in Fig.3.1b. Column 2 shows values ranging from 1.23 to 1.25, column 7 shows values ranging from 1.14 to 1.19, and column 12 values ranging from 1.18 to 1.23. *K* factors for the columns in the middle tier increases in value as the bay width increases. This is expected since widening the bay reduce the stiffness of the girder and thus increasing the G value of the two ends of the column which increases the *K* factor value of the connected column. The increases in *K* factor though are relatively minor compared to doubling of the bay width. An increase of about 4% is experienced by columns 7 and 12 and less than 2% for column 2.

The variation of the *K* factor value for the right hand column tier (column 3, 8, and 13) is shown in Fig. 3.1c. Column 3 shows value ranging from 1.30 to 1.42, column 8 values ranging from 1.28 to 1.52, and column 13 values ranging from 1.34 to 1.62. *K* factors for the right column tier increases as the bay width increases. An increase of about 21% is experienced by column 13, and about 9% for columns 3 and about 19% for column 8. The increase in *K* values for the right column tier is more compared to the middle column tier since only one girder frames into the ends of the columns and thus the G factor for the ends of the columns are more sensitive to the changes in girder stiffness.

3.2.2 System Buckling *K* Factor Value.

For all of the columns in the unbraced frame structure the *K* factor values are identical throughout the variation of the bay width. *K* factor values for all the column increases from 1.256 to 1.337 (less than 7%) as the right bay width increases. The widening of the bay reduces the overall structure stiffness and thus increasing the *K* factor value for the whole structure. Since all the columns have the same stiffness parameter and the stories have the same stiffness parameter, the *K* factor for all the columns are the same throughout the bay width variation.

3.2.3 Difference in *K* Factor values.

For the left hand column tier, *K* values from the alignment chart does not seem to show any effect from the change in bay width (Fig. 3.1a). In comparison to the system buckling *K* factor values the *K* values for column 1 is conservative up till a bay width of about 37 ft. Then it gives unconservative values of *K* values of up to about 3% at bay width of 50ft. Column 6 gives conservative values up till a bay width of 32ft. Then it gives unconservative values of about 4% at bay width of 50ft. Column 11 gives conservative values all the way through the variation with a maximum value of about 7%.

For the middle column tier, *K* values from the alignment chart does to a certain degree show the effect of bay width variation. Columns 2, 7, and 12 all give unconservative values throughout the variation. The most unconservative is column 7 giving unconservative *K* values of more than 11%, with column 2 a value of 6.5%, and column 12 a value of 8% (Fig. 3.1b).

For the right column tier, all the columns gives conservative values when compared to values from system buckling. Column 13 is the most conservative (about 21%), column 3 about 6% and column 8 about 14% (Fig. 3.1c).

3.2.4 Application of Lui's Method.

Since column 7 gave the most conservative *K* factor values when compared to system buckling values, Lui's method was also used to find its *K* factor values and the results are compared in Table 3.1b and shown graphically in Fig. 3.1c.

Span	K factor (RV6)	Nomo 7	Lui's-7
25	1.256	1.140	1.262
30	1.276	1.156	1.282
35	1.294	1.170	1.300
40	1.310	1.180	1.316
45	1.325	1.187	1.330
50	1.337	1.190	1.342

Table 3.1b. *K* factor from Lui's method compared to RV6 and nomograph for column 7

From the values and graph obtained it can be seen that Lui's method does to a much better extent agree with the values from buckling analysis and in this case as an alternative method in obtaining *K* factors.

3.2.5 Limitation of Nomograph Use.

From the results obtained by comparing the *K* factor values from the nomograph and system buckling analysis, the most unconservative value obtained is about 11% when the span is doubled. The nomograph in the case of span variation should be able to give sufficiently accurate values for practical purposes.

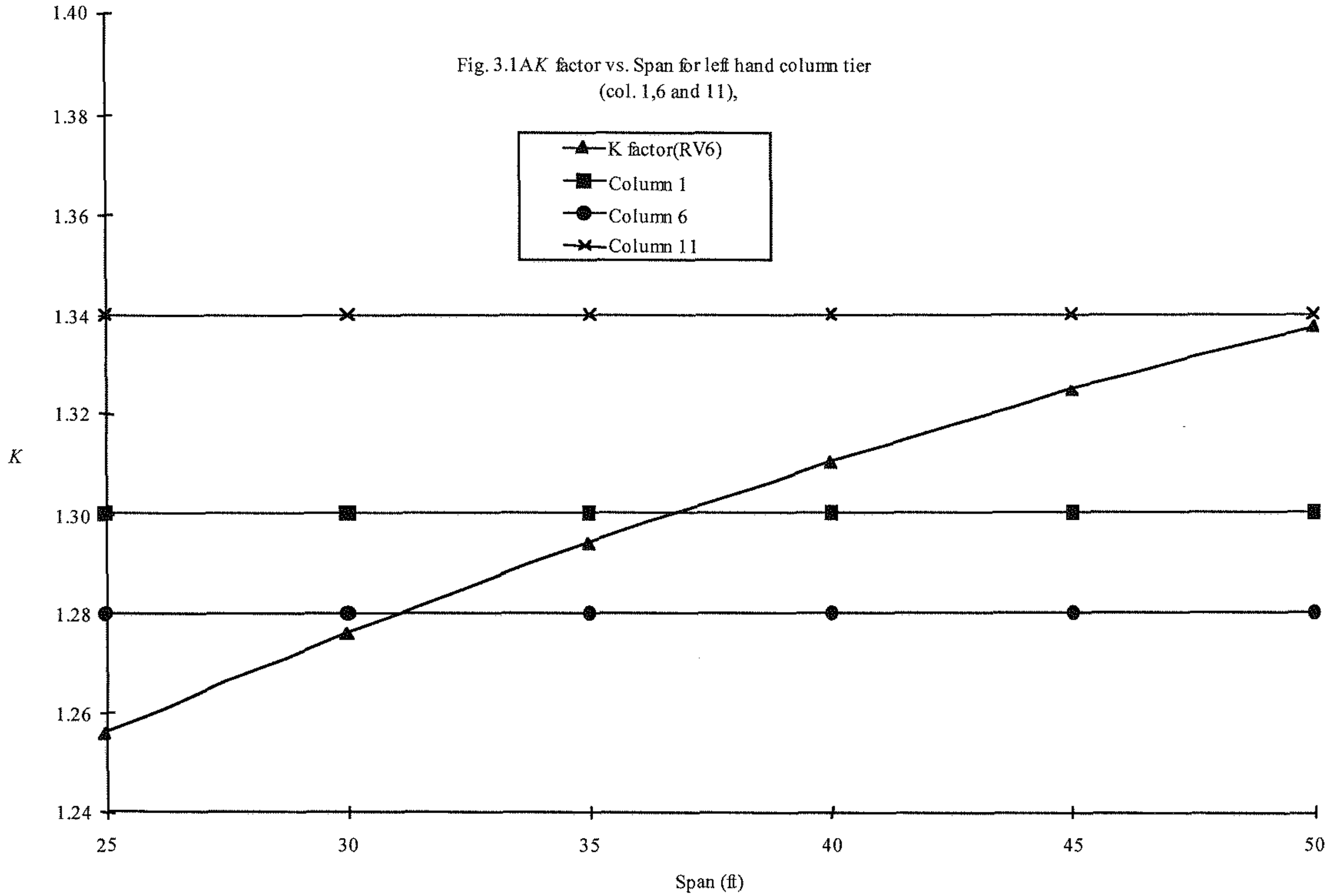


Fig. 3.1A K factor vs. Span for left hand column tier
(col. 1,6 and 11),
K factor(RV6)
Column 1
Column 6
Column 11
1.40
1.38
1.36
1.34
1.32
1.30
1.28
1.26
1.24
K
25
30
35
40
45
50
Span (ft)

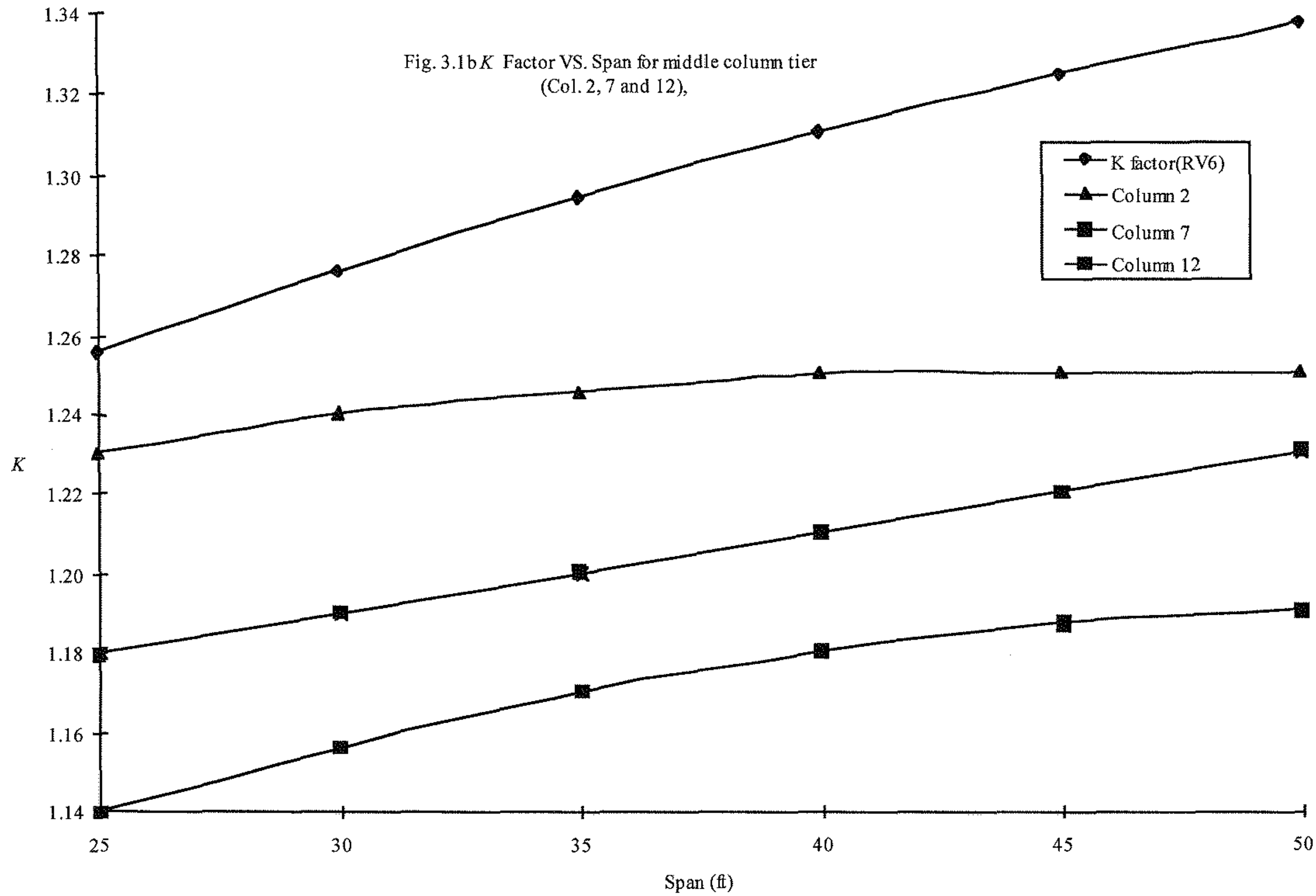


Fig. 3.1b K Factor VS. Span for middle column tier
(Col. 2, 7 and 12),
K factor(RV6)
Column 2
Column 7
Column 12
1.34
1.32
1.30
1.28
1.26
1.24
K
1.22
1.20
1.18
1.16
1.14
25
30
35
40
45
50
Span (ft)

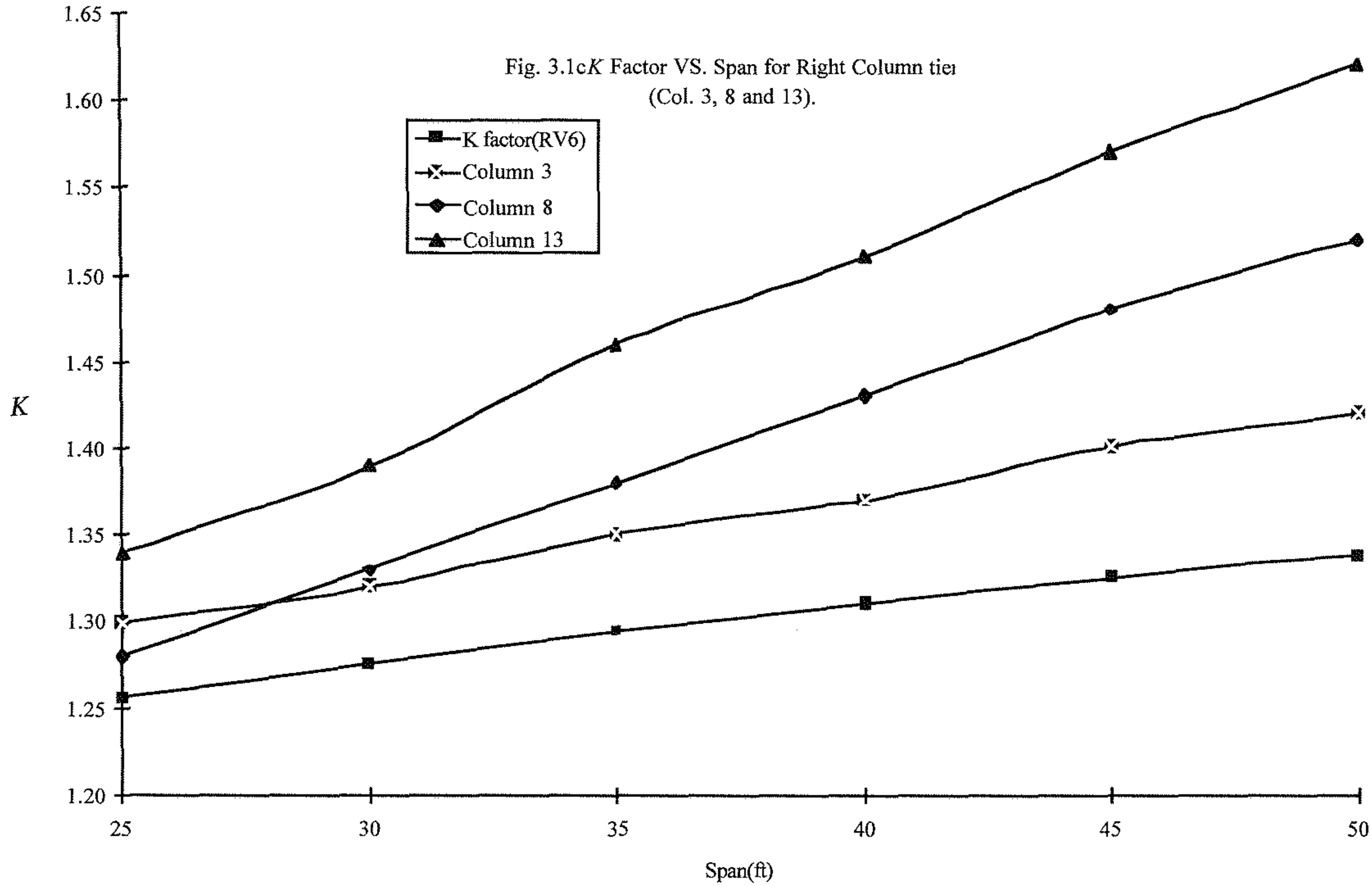
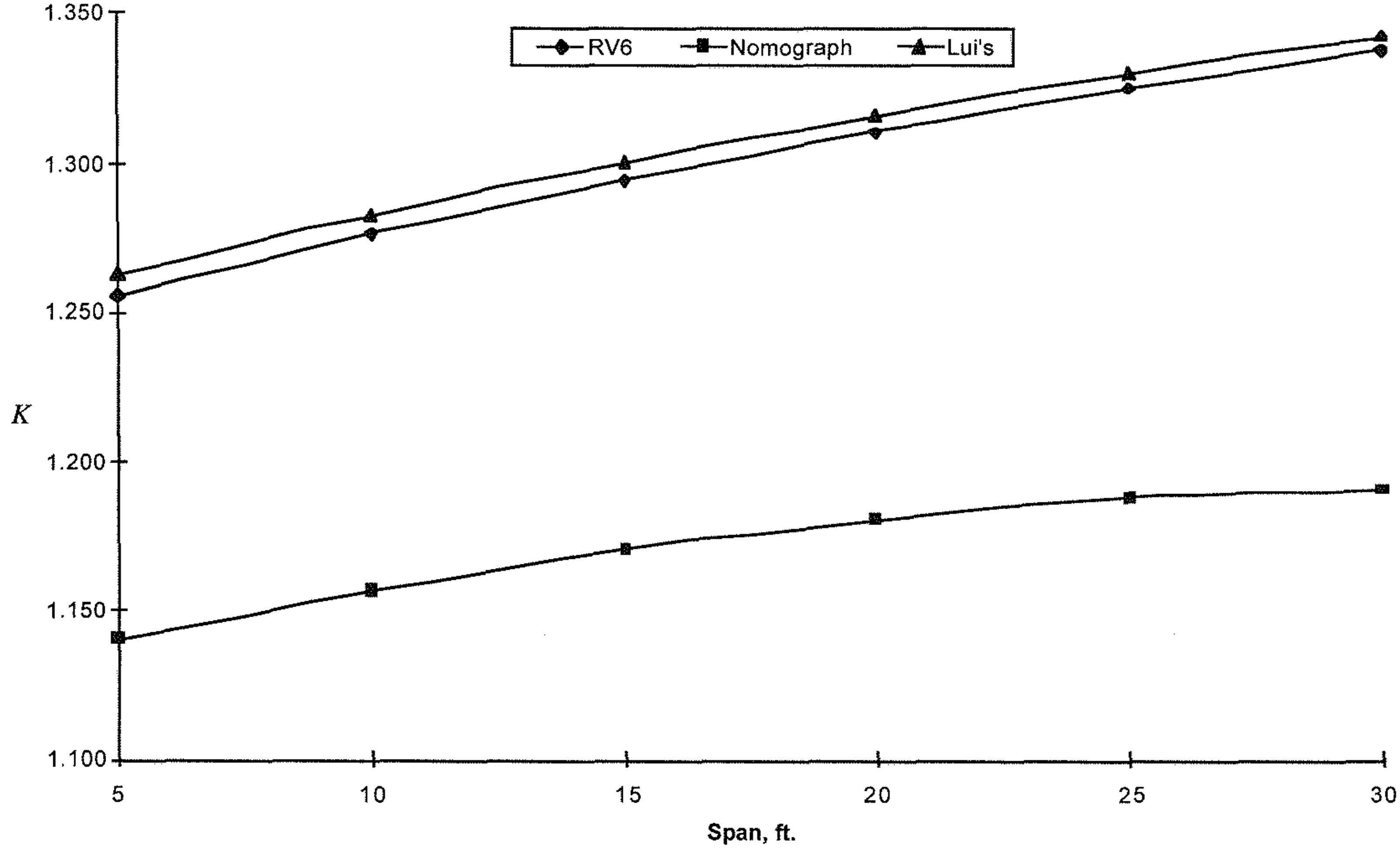


Fig. 3.1c*K* Factor VS. Span for Right Column tie
(Col. 3, 8 and 13).

Fig 3.1d *K* Factor VS. Span For Col. 7



3.3 Variation in Moment of Inertia.

Results from both the alignment chart and system buckling analysis are shown in Table 3.2.

Table 3.2 K Factor Values for Ix variation.

	Column 1		Column 2		Column 3		Column 6		Column 7		Column 8		Column 11		Column 12		Column 13	
Ix of 3,8,13	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
184	1.26	1.30	1.26	1.23	1.26	1.30	1.26	1.28	1.26	1.14	1.26	1.28	1.26	1.34	1.26	1.18	1.26	1.34
341	1.20	1.30	1.21	1.23	1.64	1.40	1.20	1.28	1.20	1.14	1.64	1.49	1.20	1.34	1.20	1.18	1.64	1.58
534	1.17	1.30	1.18	1.23	1.99	1.51	1.17	1.28	1.18	1.14	2.00	1.72	1.17	1.34	1.17	1.18	2.00	1.84
740	1.15	1.30	1.16	1.23	2.31	1.60	1.15	1.28	1.16	1.14	2.31	1.94	1.15	1.34	1.15	1.18	2.31	2.08
933	1.14	1.30	1.14	1.23	2.56	1.66	1.14	1.28	1.14	1.14	2.56	2.12	1.14	1.34	1.14	1.18	2.56	2.28
1110	1.13	1.30	1.13	1.23	2.77	1.72	1.13	1.28	1.13	1.14	2.77	2.28	1.13	1.34	1.13	1.18	2.77	2.46
1240	1.12	1.30	1.13	1.23	2.91	1.75	1.12	1.28	1.12	1.14	2.91	2.38	1.12	1.34	1.12	1.18	2.91	2.57
1530	1.11	1.30	1.11	1.23	3.19	1.81	1.11	1.28	1.11	1.14	3.19	2.61	1.11	1.34	1.11	1.18	3.19	2.82
1710	1.10	1.30	1.10	1.23	3.35	1.84	1.10	1.28	1.10	1.14	3.35	2.74	1.10	1.34	1.10	1.18	3.35	2.97
1900	1.09	1.30	1.10	1.23	3.50	1.87	1.09	1.28	1.10	1.14	3.51	2.87	1.09	1.34	1.09	1.18	3.51	3.11

3.3.1 Nomograph *K* Factor Values.

From Fig. 3.2a, for the columns in the right hand tier, their *K* factor values increase through the variation. Column 3 has *K* values ranging from 1.30 to 1.87 (about 44%), column 8 values range from 1.28 to 2.87 (about 124%), and column 13 range from 1.34 to 3.11 (about 132%).

Columns in the middle column tier (Fig.3.2b) maintain constant values throughout the variation. Column 2 has a constant value of 1.23, column 7 a value of 1.14 and column 12 a value of 1.18. Constant *K* values were also maintained by columns in the left column tier (Fig.3.2a). Column 1 maintains a value of 1.30, column 6 a value of 1.28, and column 11 a value of 1.34. This is again due to the localized effect of the alignment chart failing to capture the overall effect of the whole structure.

3.3.2 System Buckling *K* Factor Value.

From Fig.3.2a, 3.2b,and 3.2c, it can be seen that as the moment of inertia of the right column tier increases the *K* factor values of the columns in the left column tier and middle column tier decrease from a *K* factor value of 1.26 to about 1.10 (about 13%), while the values of the columns in the right column tier increase from 1.26 to about 3.51 (about 178%). This is due to the interaction effect between the various member of the unbraced frame. The columns in the right column tier becomes stronger as their moment of inertia is increased. This will enable them to brace the weaker columns (left and middle tier) increasing the *K* factor value of the stronger column and decreasing the *K* factor value of the weaker columns. This phenomena is also observed by Hajjar and White [1994] and Lui [1992]. *K* factor values of less then 1.0 will result if the moment of inertia value were further increased. *K* factor values of less than 1.0 can now be used to design columns according to the LRFD Specification [AISC 93].

3.3.3 Differences in *K* Factor values.

Due to the failure of the alignment chart to capture the full effect of the interaction between components of an unbraced frame, differences between *K* factor values from the alignment chart and system buckling ranges from being overly conservative to being overly unconservative. For columns in the left column tier, values from the alignment chart are conservative, with column 11 having the highest value of about 23% conservative. For the middle column tier, most values obtained are also conservative with column 2 having a conservative value of about 12%. Columns in the right column tier on the other hand gives mostly unconservative values of up to about 47% for column 3.

3.3.4 Application of Lui's Method.

From the comparison of *K* factor values from the nomograph and the system buckling analysis, column 3 was giving the most unconservative values. Lui's method was also used to find *K* factor values for column 3 and the results are compared to system buckling analysis and the nomograph in Table 3.2b and Fig 3.2d.

Ix of 3,8,13	*K* Factor (RV6)	Ix of 3,8,14	Lui,s
184	1.260	1.300	1.163
341	1.638	1.400	1.460
534	1.995	1.510	1.726
740	2.308	1.600	1.946
933	2.560	1.660	2.116
1110	2.767	1.720	2.253
1240	2.906	1.750	2.343
1530	3.188	1.810	2.523
1710	3.347	1.840	2.623
1900	3.504	1.870	2.720

Table 3.2b

From the results obtained, Lui's method does compare better to values from buckling analysis when compared to values from the nomograph. At the end of the variation values from Lui's method are unconservative by about 20% as compared to about 45% for values from the nomograph.

3.3.5 Limitation of Nomograph Use.

In the case of moment of inertia variation, in order to keep values from being unconservative by more then 10%, the moment of inertia of frame column members should not vary by more then 1.5I. Using variation of up to 5I could lead to unconservative error of up to 35% and variation of 10I could lead to unconservative error of up to 50%.

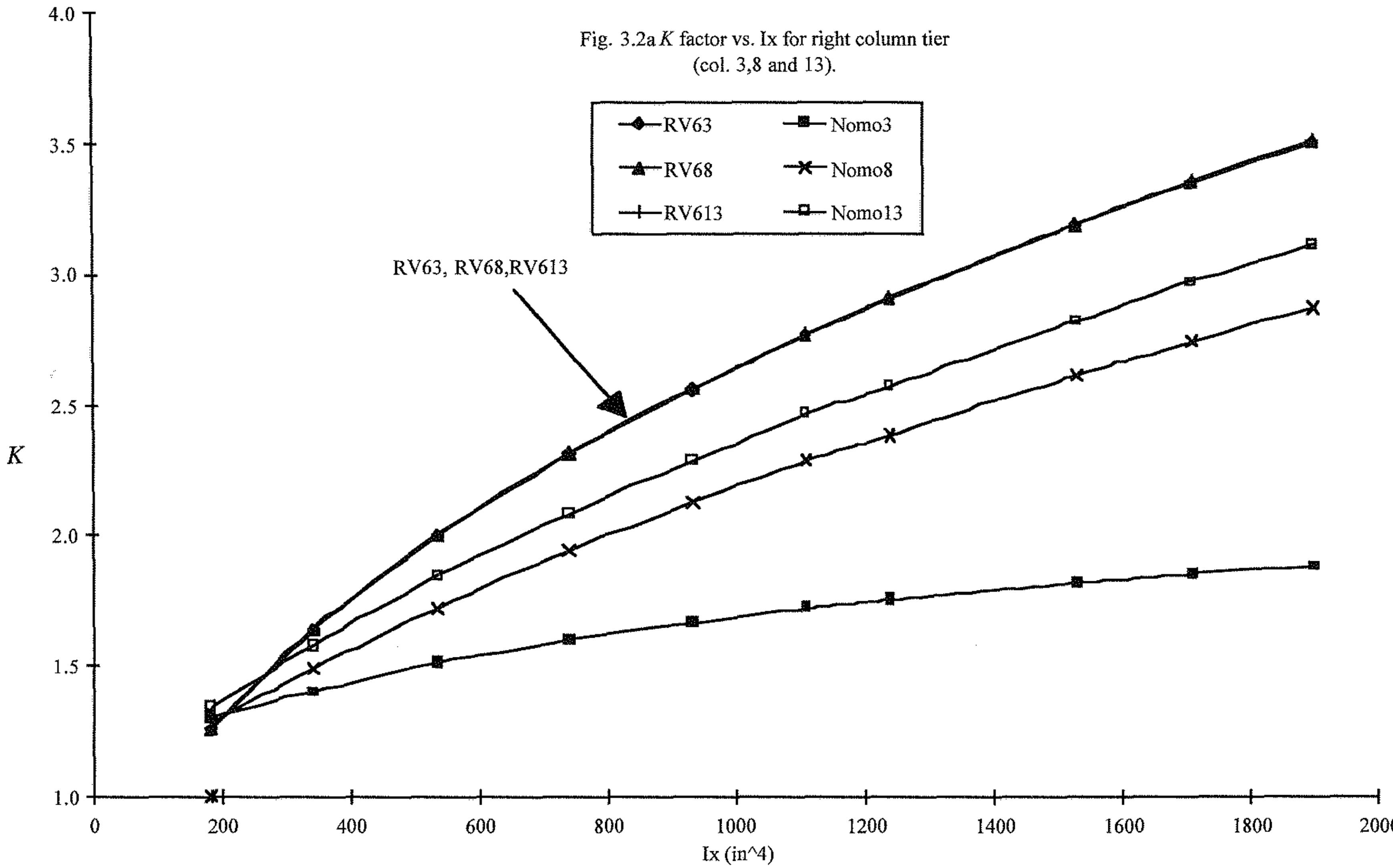


Fig. 3.2a K factor vs. Ix for right column tier
(col. 3,8 and 13).
RV63
Nomo3
RV68
Nomo8
RV613
Nomo13
RV63, RV68,RV613
4.0
3.5
3.0
2.5
2.0
1.5
1.0
K
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Ix (in^4)

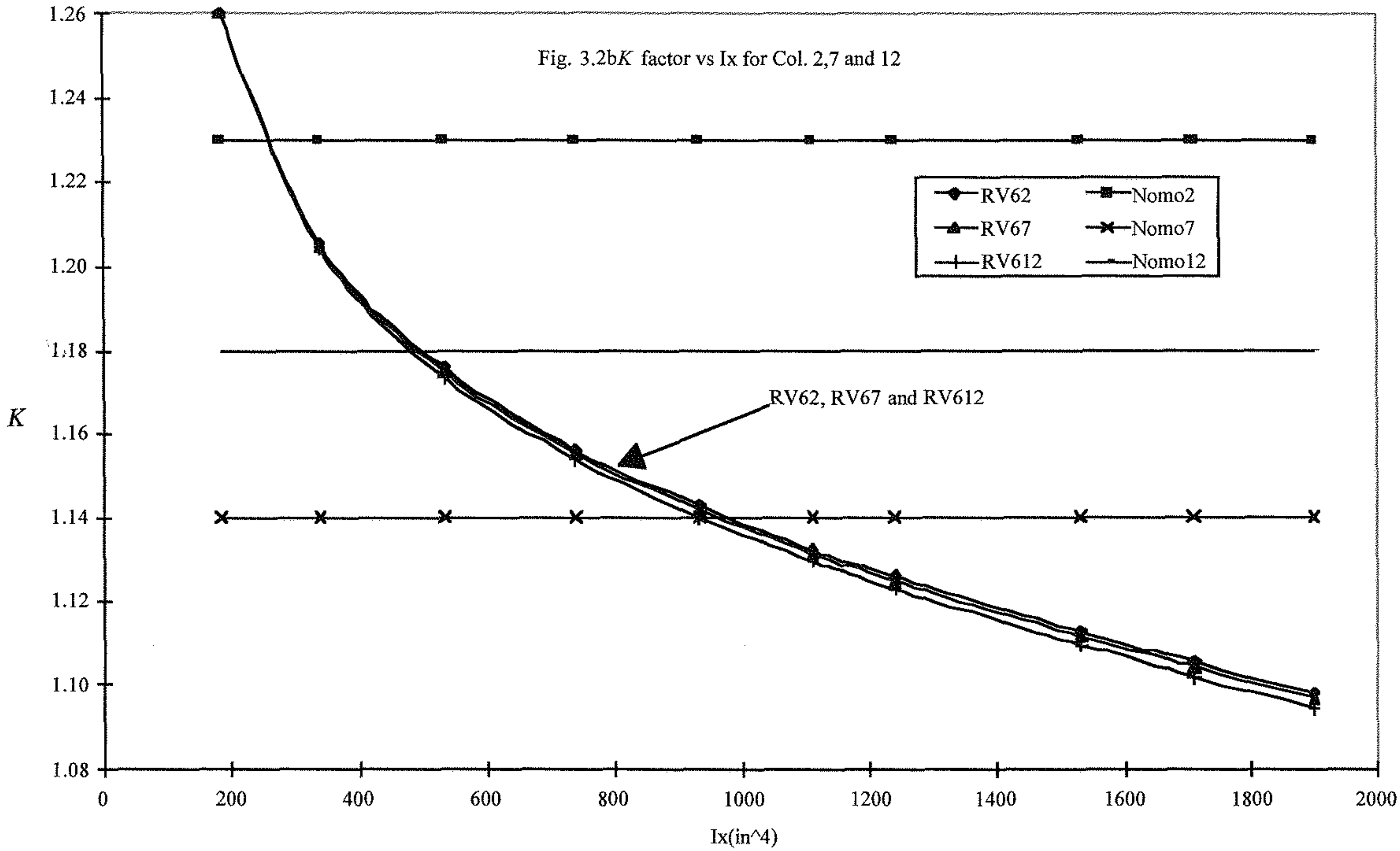


Fig. 3.2bK factor vs Ix for Col. 2,7 and 12
K
Ix(in^4)
RV62
Nomo2
RV67
Nomo7
RV612
Nomo12
RV62, RV67 and RV612
1.26
1.24
1.22
1.20
1.18
1.16
1.14
1.12
1.10
1.08
0
200
400
600
800
1000
1200
1400
1600
1800
2000

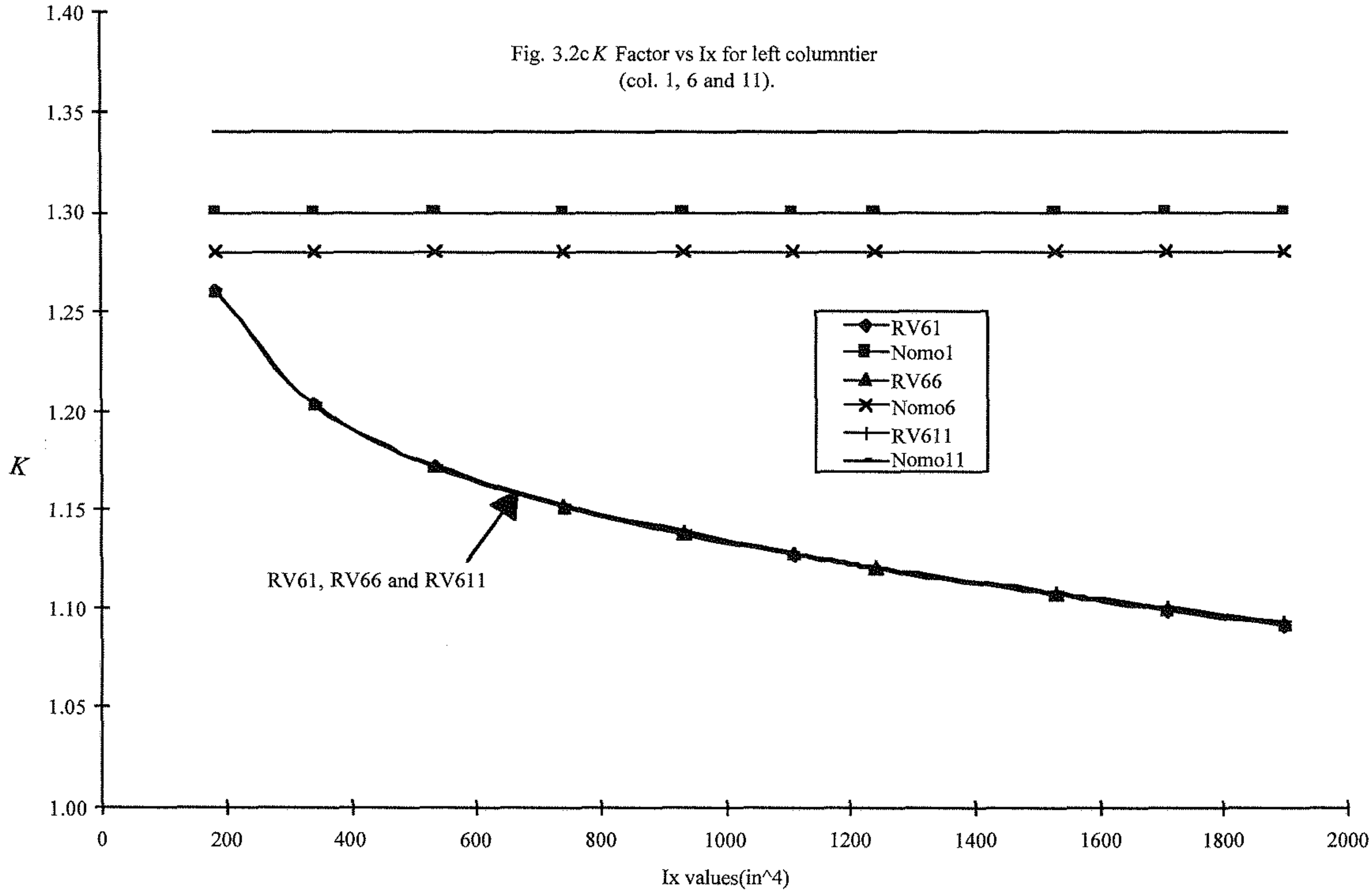


Fig. 3.2c K Factor vs Ix for left columntier
(col. 1, 6 and 11).
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
K
RV61
Nomo1
RV66
Nomo6
RV611
Nomo11
RV61, RV66 and RV611
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Ix values(in^4)

©

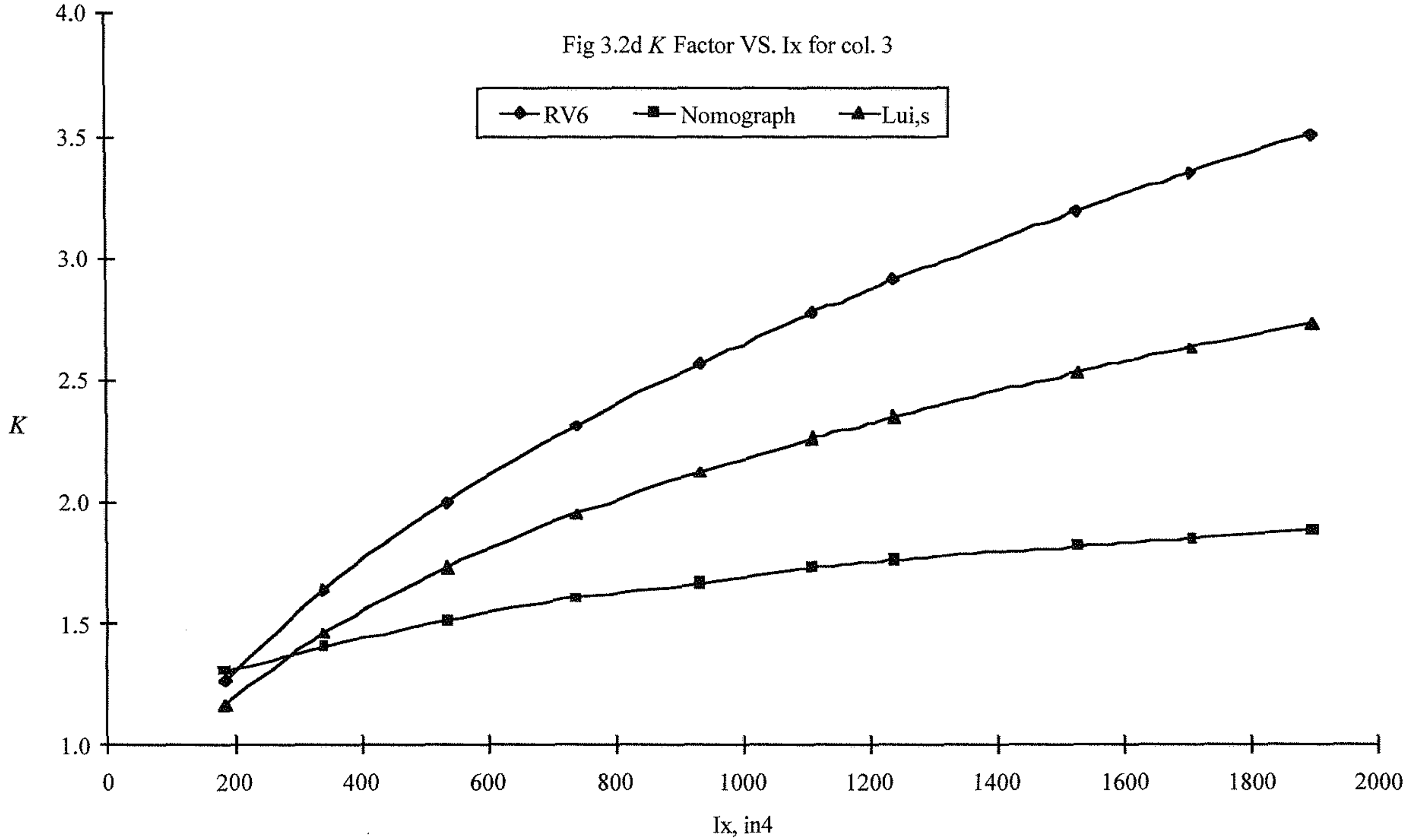


Fig 3.2d K Factor VS. Ix for col. 3
RV6
Nomograph
Lui,s
K
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Ix, in4

3.4 Variation in Loading and Loading Configuration.

Result from both alignment chart and system buckling are shown in Table 3.3a and Table 3.3b.

Table 3.3a. K Factor Values

	Column 1		Column 2		Column 3		Column 6		Column 7		Column 8		Column 11		Column 12		Column 13	
P value on 3,8,13	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
40.00	1.26	1.30	1.26	1.23	1.26	1.30	1.26	1.28	1.26	1.14	1.26	1.28	1.26	1.34	1.26	1.18	1.26	1.34
80.00	1.45	1.30	1.44	1.23	1.03	1.30	1.45	1.28	1.44	1.14	1.02	1.28	1.45	1.34	1.45	1.18	1.02	1.34
120.00	1.63	1.30	1.60	1.23	0.94	1.30	1.62	1.28	1.61	1.14	0.94	1.28	1.62	1.34	1.61	1.18	0.93	1.34
160.00	1.79	1.30	1.75	1.23	0.89	1.30	1.78	1.28	1.75	1.14	0.89	1.28	1.78	1.34	1.77	1.18	0.89	1.34
200.00	1.94	1.30	1.88	1.23	0.86	1.30	1.93	1.28	1.89	1.14	0.86	1.28	1.92	1.34	1.91	1.18	0.86	1.34

Table 3.3b. K Factor Values

	Column 1		Column 2		Column 3		Column 6		Column 7		Column 8		Column 11		Column 12		Column 13	
P value on 2,7,12	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
40.00	1.26	1.30	1.26	1.23	1.26	1.30	1.26	1.28	1.26	1.14	1.26	1.28	1.26	1.34	1.26	1.18	1.26	1.34
80.00	1.45	1.30	1.04	1.23	1.45	1.30	1.45	1.28	1.03	1.14	1.45	1.28	1.46	1.34	1.03	1.18	1.46	1.34
120.00	1.62	1.30	0.95	1.23	1.62	1.30	1.62	1.28	0.95	1.14	1.62	1.28	1.63	1.34	0.95	1.18	1.63	1.34
160.00	1.77	1.30	0.90	1.23	1.77	1.30	1.77	1.28	0.90	1.14	1.77	1.28	1.79	1.34	0.90	1.18	1.79	1.34
200.00	1.90	1.30	0.88	1.23	1.90	1.30	1.91	1.28	0.87	1.14	1.91	1.28	1.93	1.34	0.87	1.18	1.93	1.34

3.4.1 Alignment Chart *K* Factor Values.

Results show values of *K* factor values remaining constant throughout both loading variation for all the column tiers(see Fig. 3.3a to 3.3f) indicating no effect from loading variation. The effect of symmetry also did not effect the *K* factor values as values from both configurations of loading give the same values.

3.4.2 System Buckling *K* Factor Values.

When the right column tier was loaded, *K* factors for the columns in the right column tier decrease as the loading is increased (Fig. 3.3c). Columns 3, 8, and 13 shows values ranging from 1.26 decreasing to 0.86 (about 32%). While the values of the right column tier decrease, the values for the middle and left column tiers (Fig.3.3a and Fig. 3.3b) increase by about the same amount from about 1.26 to 1.9 (about 50%).

When the middle column tier was loaded, *K* factor values for the columns in the middle column tier (Fig. 3.3e) decrease. Columns 2, 7, and 12 shows values ranging from 1.26 to 0.87 (about 32%). While the values of the middle column tier decrease, the values for the right and left column tier (Fig. 3.3d and 3.3f) increase from about 1.26 to about 1.9 (about 50%).

Decrease of *K* factor values of the loaded columns can be attributed to the fact that as the columns are loaded, the columns becomes weaker. Columns where loadings are held constant becomes relatively stronger as compared to the loaded column. As a result the weaker columns reach their buckling load earlier and thus "lean" on the stronger columns. This results in an increase in *K* factor value of the columns whose loadings were held constant. Columns that were loaded drop their *K* factor value to less than 1.0 which is now allowed to be used in the design of unbraced frames [AISC 93]. These facts were also observed by Lui [1992] and Hajjar and White [1994].

3.4.3 Differences in *K* Factor Values.

Differences in the values between the alignment chart and the system buckling *K* factor values can be attributed to loading not being considered in the deduction of *K* factor in the alignment chart. In both cases for the column tiers where the loading was increased, the *K* factor values from the alignment chart are conservative as compared to values from system buckling. For the columns where their loading are held constant, values are unconservative. When the right column tier was loaded, column 13 gives a conservative value of about 56%. Other columns in the tier give almost the same value of conservatism. Among the columns whose loadings were held constant, column 7 gives the most unconservative value of about 40%. When the middle column tier was loaded, column 2 gives a conservative value of 40%. Other columns in the tier gives lesser conservative values but still they are more than 20%. Among the columns held constant, column 8 is unconservative by about 33%.

3.4.4 Application of Lui's Method.

From the comparison of *K* factor values from the nomograph and the system buckling analysis, column 7 with the load applied to the right column tier gave the most unconservative values. Lui's method was used to find *K* factor values for column 7 and the results are compared to system buckling analysis and the nomograph in Table 3.3c and Fig. 3.3g.

P value on 3,8,13	RV6	Nomograph	Lui's
40	1.256	1.140	1.282
80	1.442	1.140	1.458
120	1.606	1.140	1.630
160	1.753	1.140	1.785
200	1.886	1.140	1.928

Table 3.3c. *K* factor from Lui's method compared to RV6 and nomograph for column 7

From the results obtained, values from Lui's method is almost comparable to values from system buckling analysis. It captures the effect of load variation very well and should be a good alternative method in finding the *K* factor.

3.4.5 Limitation of Nomograph Use.

In order to limit the unconservative error due to load variation to 10%, load on columns should not vary by more than 1.5P. Variation of up to 3P would lead to unconservative error of about 20% while variation of up to 5P would lead to unconservative error of up to 35%.

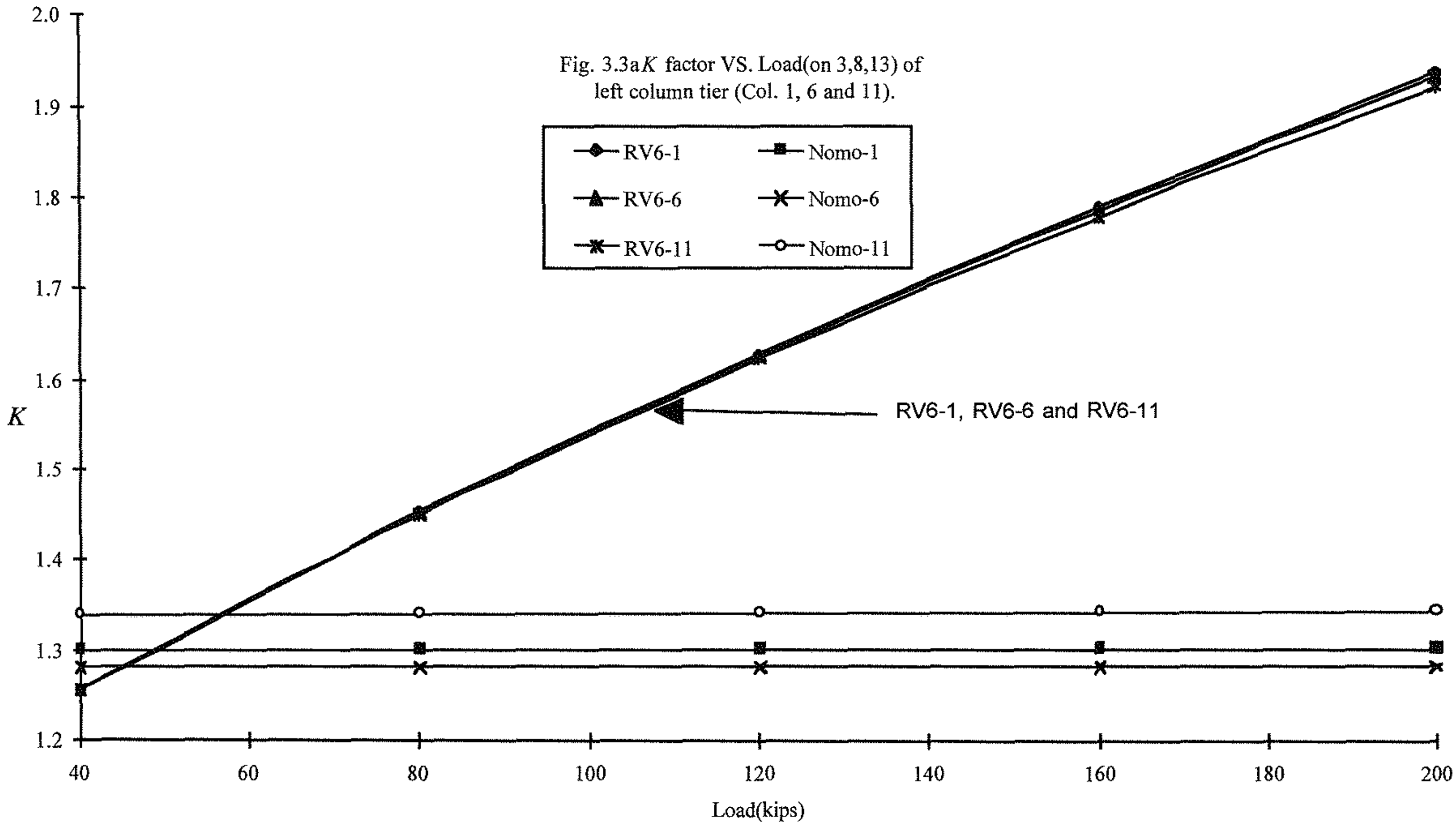


Fig. 3.3a K factor VS. Load(on 3,8,13) of
left column tier (Col. 1, 6 and 11).
RV6-1
Nomo-1
RV6-6
Nomo-6
RV6-11
Nomo-11
RV6-1, RV6-6 and RV6-11
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
K
40
60
80
100
120
140
160
180
200
Load(kips)

Fig. 3.3b K factor VS. Load(on 3, 8, 13) of middle column tier (col. 2, 7 and 12).

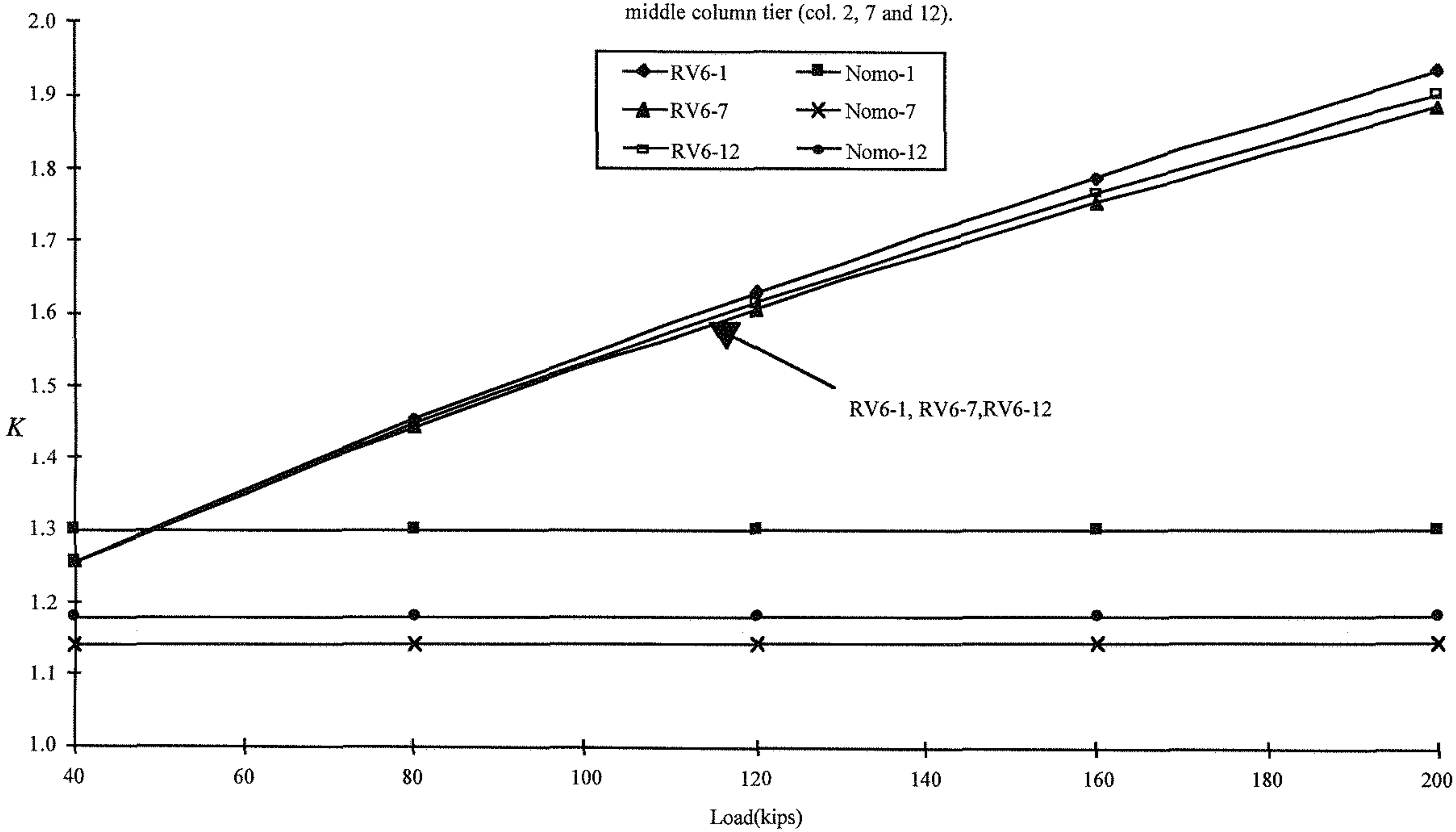


Fig. 3.3c K Factor VS. Load(on 3, 8, 13) for right column tier (Col. 3, 8 and 13).

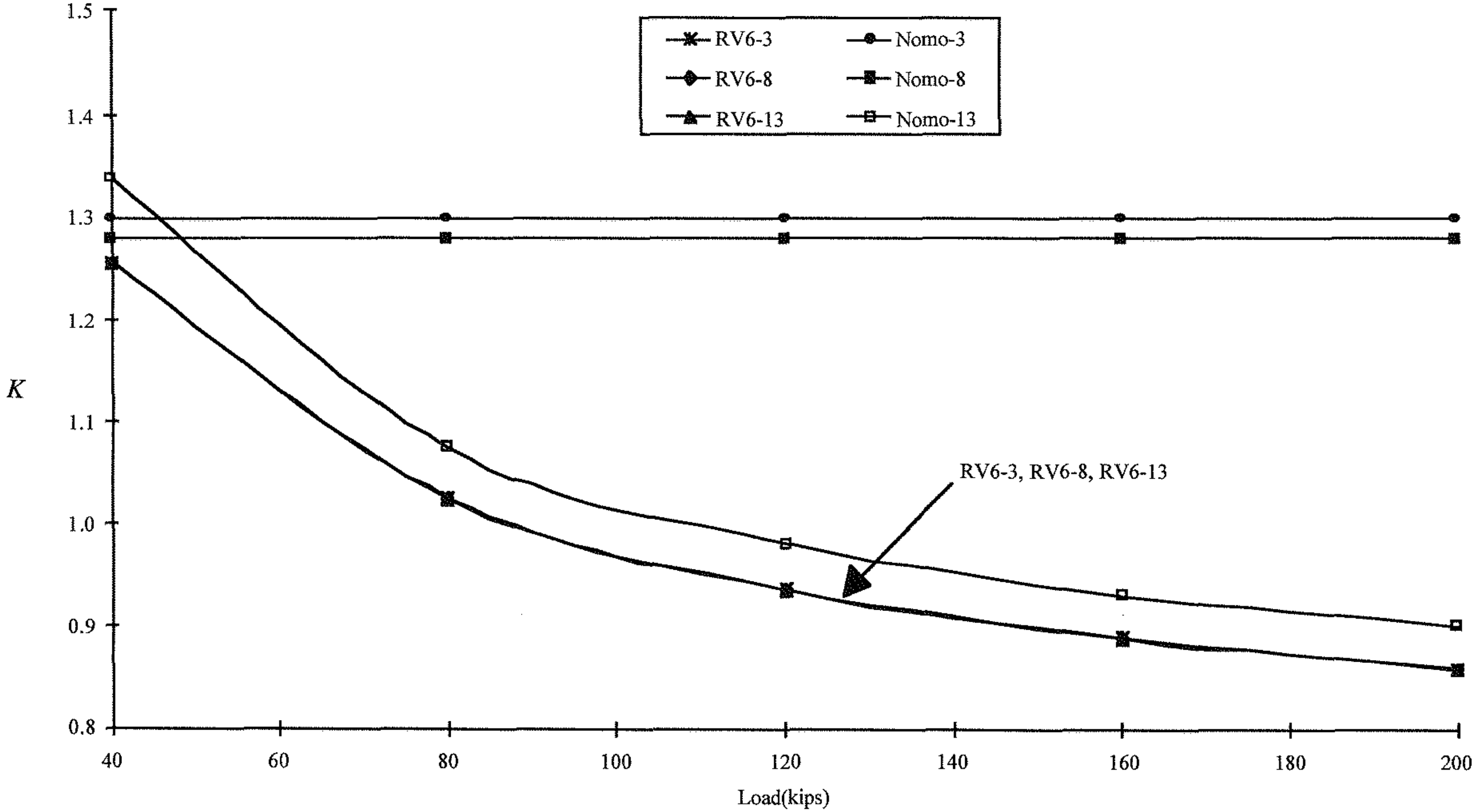


Fig 3.3d*K* Factor VS. Load(on 2, 7, 12)
of left column tier (Col. 1, 6 nad 11).

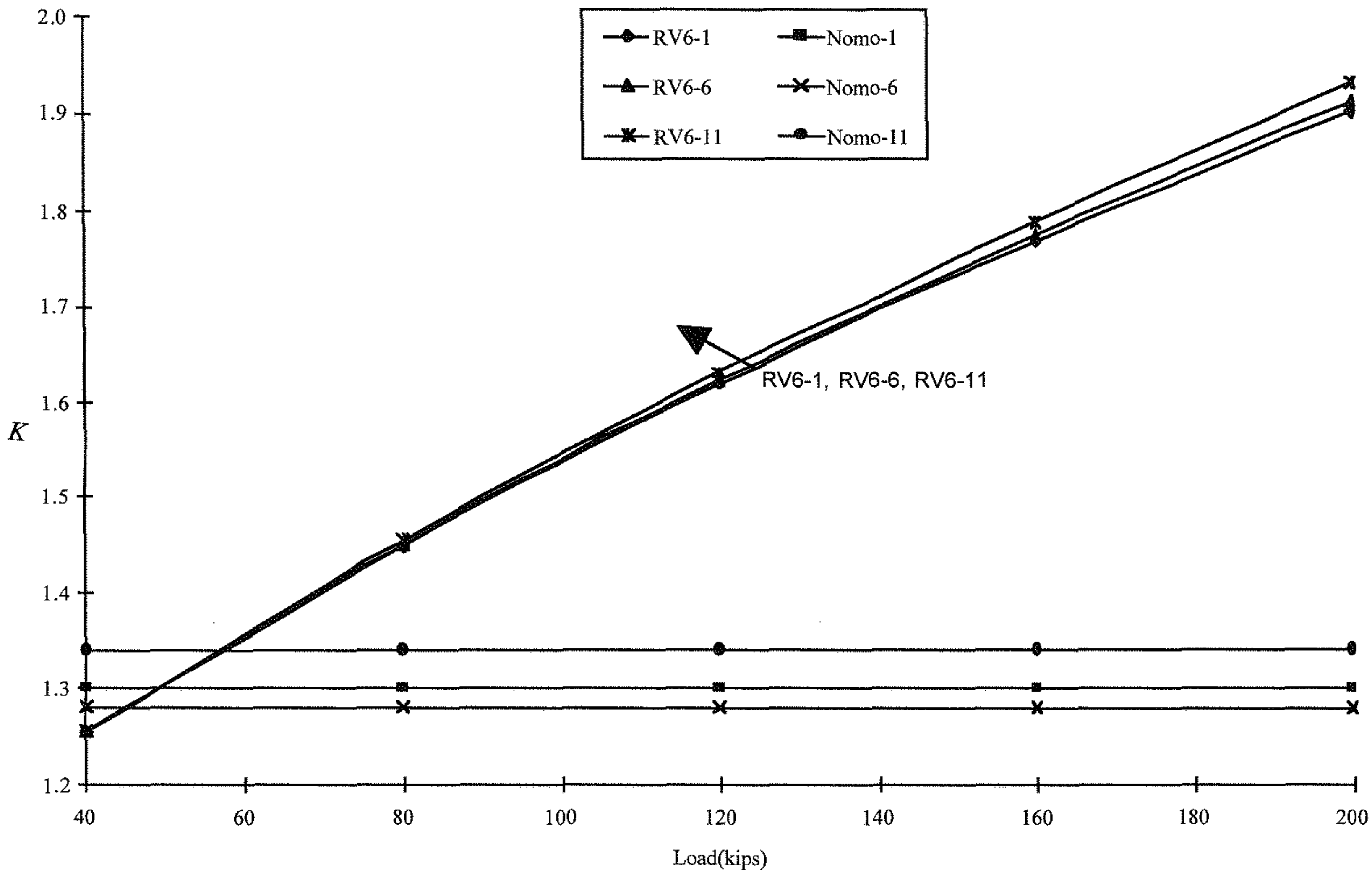


Fig 3.3e K Factor VS. Load(on 2, 7, 12)
of middle column tier (Col. 2, 7 and 12)

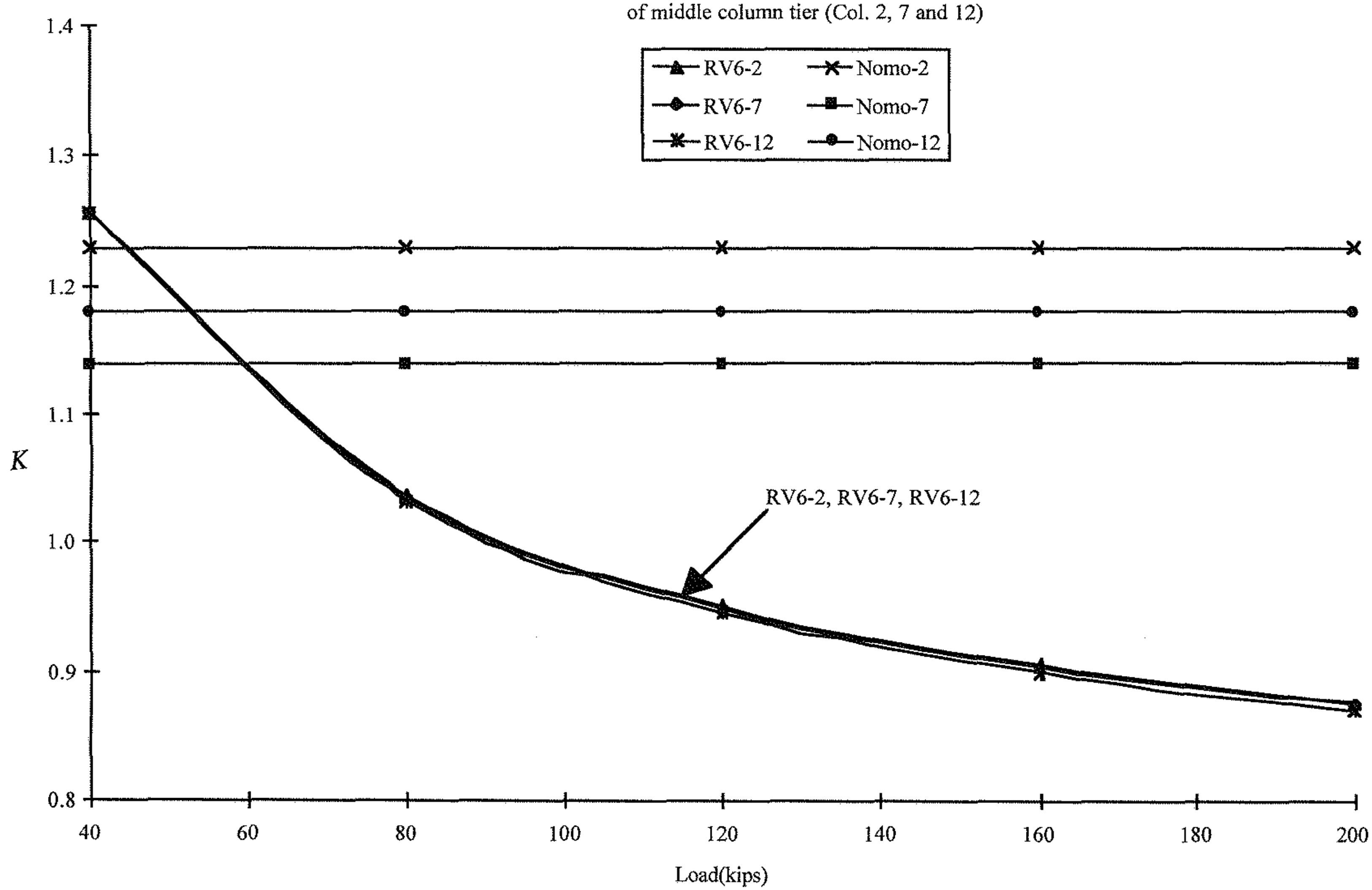
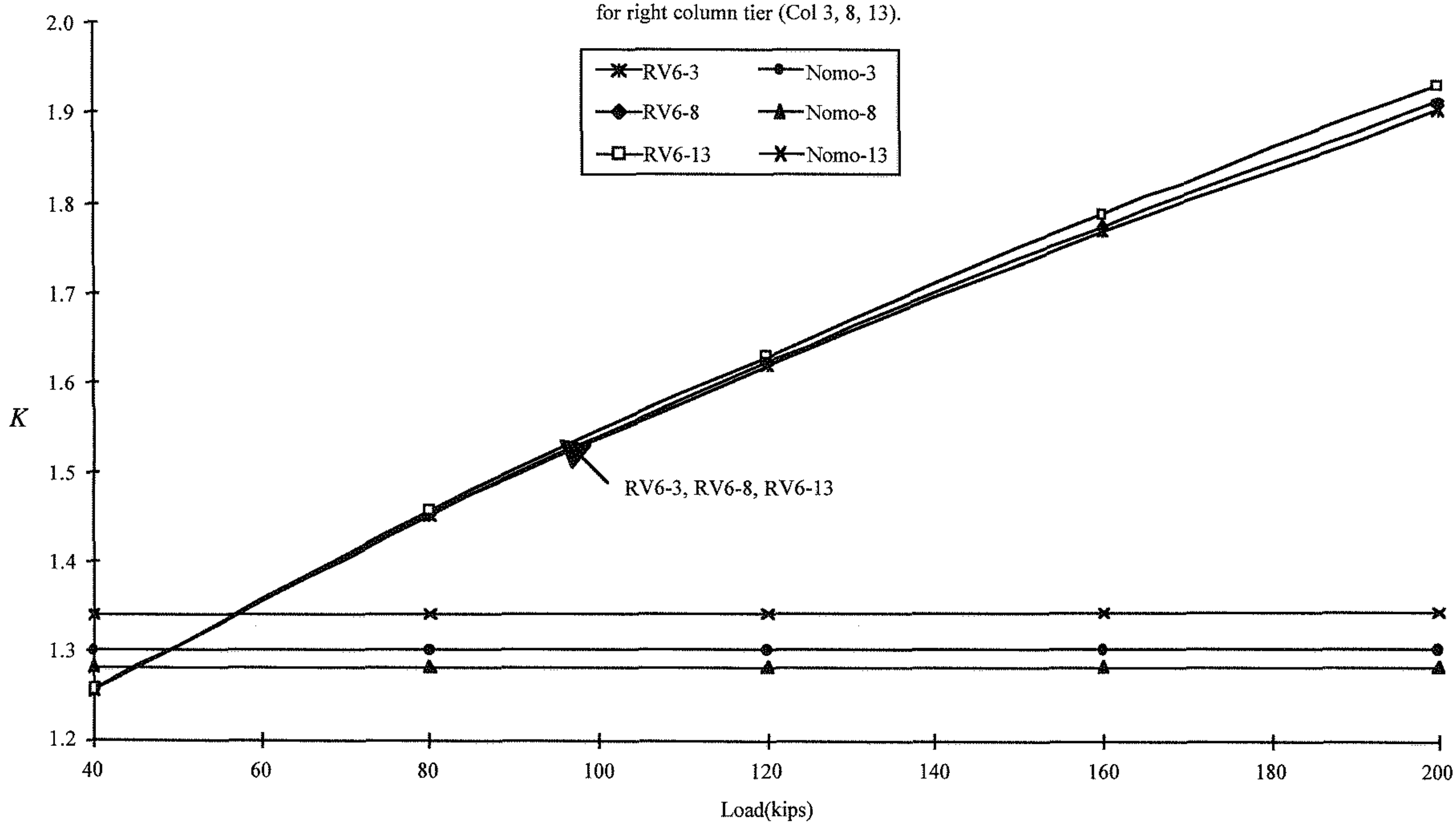


Fig 3.3 f K Factor VS. Load(on 2, 7, 12) for right column tier (Col 3, 8, 13).



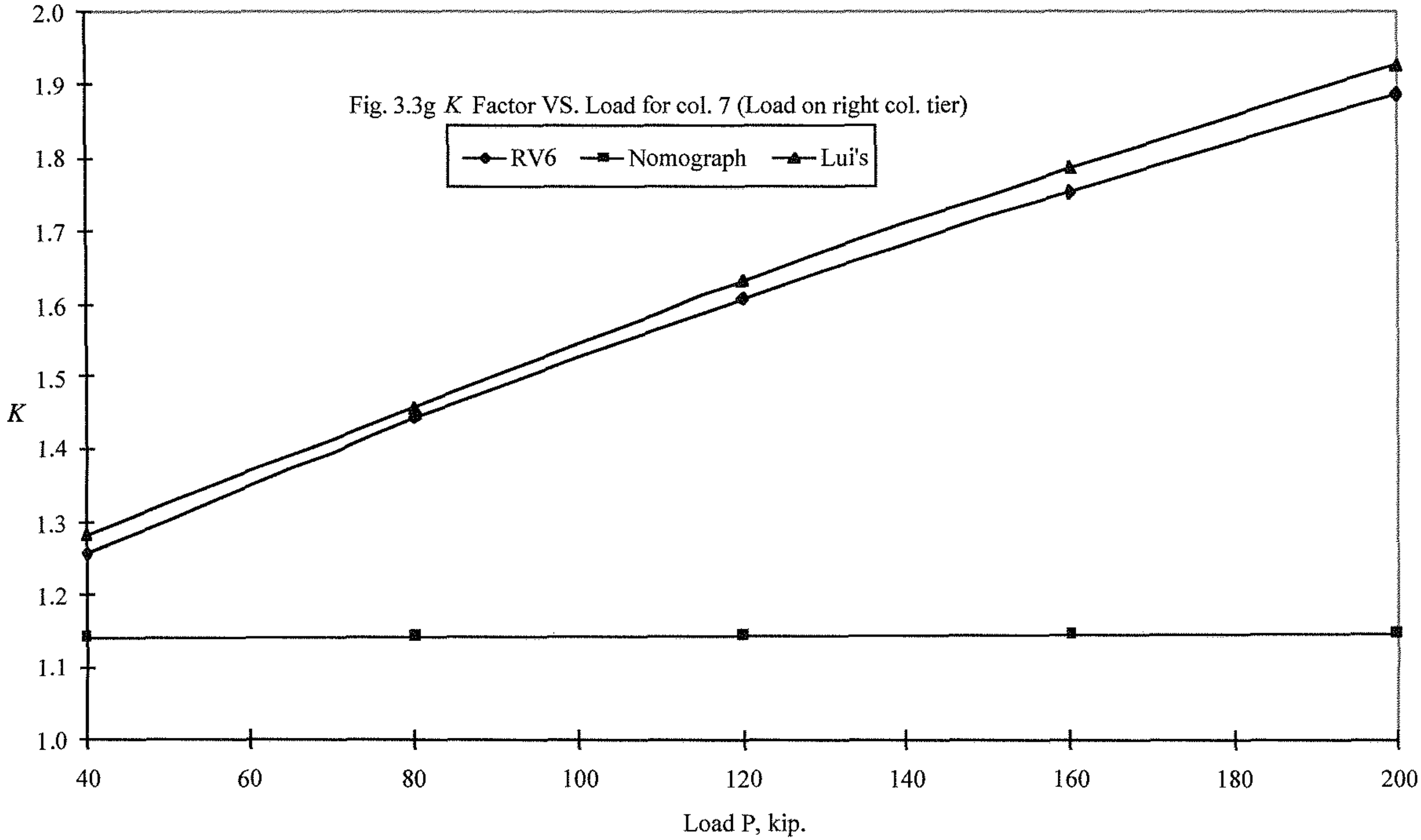


Fig. 3.3g *K* Factor VS. Load for col. 7 (Load on right col. tier)

3.5 Variation in Column Height.

Results from alignment chart and system buckling analysis are shown in Table 3.4a and 3.4b.

Table 3.4a. K Factor Values - 1st. floor varied.

	Column 1		Column 2		Column 3		Column 6		Column 7		Column 8		Column 11		Column 12		Column 13	
Height of 1st floor	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
2.50	6.24	1.52	6.24	1.36	6.24	1.52	1.25	1.49	1.25	1.27	1.25	1.49	1.25	1.34	1.25	1.18	1.25	1.34
5.00	3.13	1.39	3.13	1.28	3.13	1.39	1.25	1.37	1.25	1.19	1.25	1.37	1.25	1.34	1.25	1.18	1.25	1.34
7.50	2.09	1.34	2.09	1.25	2.09	1.34	1.25	1.32	1.25	1.17	1.25	1.32	1.25	1.34	1.25	1.18	1.25	1.34
10.00	1.57	1.31	1.57	1.24	1.57	1.31	1.25	1.3	1.25	1.15	1.25	1.3	1.25	1.34	1.25	1.18	1.25	1.34
12.50	1.26	1.30	1.26	1.23	1.26	1.30	1.26	1.28	1.26	1.14	1.26	1.28	1.26	1.34	1.26	1.18	1.26	1.34
15.00	1.08	1.29	1.08	1.22	1.08	1.29	1.29	1.27	1.29	1.14	1.29	1.27	1.29	1.34	1.29	1.18	1.29	1.34
17.50	1.05	1.28	1.05	1.22	1.05	1.28	1.47	1.26	1.47	1.13	1.47	1.26	1.47	1.34	1.47	1.18	1.47	1.34
20.00	1.04	1.27	1.04	1.22	1.04	1.27	1.66	1.25	1.66	1.13	1.66	1.25	1.66	1.34	1.66	1.18	1.66	1.34
22.50	1.03	1.27	1.03	1.21	1.03	1.27	1.86	1.25	1.86	1.13	1.86	1.25	1.86	1.34	1.86	1.18	1.86	1.34
25.00	1.03	1.26	1.03	1.21	1.03	1.26	2.05	1.25	2.05	1.13	2.05	1.25	2.05	1.34	2.05	1.18	2.05	1.34

Table 3.4b. K Factor Values - 3rd. floor varied.

	Column 1		Column 2		Column 3		Column 6		Column 7		Column 8		Column 11		Column 12		Column 13	
Height of 3rd floor	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
2.50	1.15	1.30	1.15	1.23	1.15	1.30	1.15	1.49	1.15	1.27	1.15	1.49	5.76	1.65	5.76	1.36	5.76	1.65
5.00	1.16	1.30	1.16	1.23	1.16	1.30	1.16	1.37	1.16	1.19	1.16	1.37	2.89	1.49	2.89	1.26	2.89	1.49
7.50	1.16	1.30	1.16	1.23	1.16	1.30	1.16	1.32	1.16	1.17	1.16	1.32	1.93	1.39	1.93	1.21	1.93	1.39
10.00	1.17	1.30	1.17	1.23	1.17	1.30	1.17	1.3	1.17	1.15	1.17	1.3	1.47	1.34	1.47	1.18	1.47	1.34
12.50	1.26	1.30	1.26	1.23	1.26	1.30	1.26	1.28	1.26	1.14	1.26	1.28	1.26	1.34	1.26	1.18	1.26	1.34
15.00	1.44	1.30	1.44	1.23	1.44	1.30	1.44	1.27	1.44	1.14	1.44	1.27	1.20	1.30	1.20	1.15	1.20	1.30
17.50	1.64	1.30	1.64	1.23	1.64	1.30	1.64	1.26	1.64	1.13	1.64	1.26	1.17	1.27	1.17	1.14	1.17	1.27
20.00	1.84	1.30	1.84	1.23	1.84	1.30	1.84	1.25	1.84	1.13	1.84	1.25	1.15	1.24	1.15	1.12	1.15	1.24
22.50	2.04	1.30	2.04	1.23	2.04	1.30	2.04	1.25	2.04	1.13	2.04	1.25	1.13	1.22	1.13	1.11	1.13	1.22
25.00	2.24	1.30	2.24	1.23	2.24	1.30	2.24	1.25	2.24	1.13	2.24	1.25	1.12	1.21	1.12	1.11	1.12	1.21

3.5.1 Alignment Chart *K* Factor Values.

When the height of the bottom story of the frame is increased from an initial height of 2.5 ft. , columns in the bottom and second stories experience minor reduction in *K* factor values even though the story height was doubled. For columns in the first story (Fig. 3.4a), values for columns 1 and 3 decrease from 1.30 to 1.26 (about 3%), and for column 2 from 1.23 to 1.21 (less than 2%). For the columns in the second story (Fig. 3.4b), values for columns 6 and 8 decrease from 1.28 to 1.25 (about 2%) and from 1.14 to 1.13 (about 1%) for column 7. For the top Story (Fig. 3.4c) the value remains the same throughout the variation with values of 1.34 for columns 1 and 13 and 1.18 for column 12.

When the top story was varied, a slight reduction in *K* factor values is observed at the second and top story. Values for columns in the bottom story (Fig. 3.4d) are constant with a value of 1.30 for columns 1 and 3 and a value of 1.23 for column 2. For the second story (Fig. 3.4e), values for columns 6 and 8 decrease from 1.28 to 1.25 (about 2%) and from 1.14 to 1.13 (about 1%) for column 7. For the top story (Fig. 3.4f), values for columns 11 and 13 decrease from 1.34 to 1.21 (about 10%) and from 1.18 to 1.11 (about 6%) for column 12.

3.5.2 System Buckling Values.

When the bottom story height was increased, the *K* factor for the columns in the bottom floor (Fig 3.4a) drops from a value of 1.26 to 1.03 (about 78%). The *K* factor for the upper stories (Fig. 3.4b and Fig. 3.4c) increases from a value of 1.26 up to a value of 2.05 (about 64%). The decrease in *K* factor values for the bottom story seems to flatten out at a story height of about 15.5ft. and maintains almost a constant value for the rest of the variation. The decrease in *K* factor values for the upper stories maintains almost a constant value until about a height of about 15.5ft. and then increases in value for the rest of the variation.

When the top story height was increased, the *K* factor values for the columns in the bottom and second stories (Fig. 3.4e and Fig. 3.4f) increase in values from 1.26 to 2.24 (about 78%). For the columns in the top story (Fig. 3.4d), the *K* factor values decrease in value from 1.26 to 1.12 (about 11%). The decrease in the *K* factor values for the top story seems to flatten out at a story height of about 15ft. and maintains almost a constant value for the rest of the variation.

The variation in column height also seems to exhibit the "leaning column" effect but it is more of an inter story phenomena. The columns that were given the increments are increasing in slenderness and thus getting weaker as compared to columns that are held constant. The weaker columns will then "lean" onto the columns that are stronger, thus increasing the *K* factor value of the stronger columns, i.e., columns in the stories that are held constant.

3.5.3 Differences in *K* Factor Values.

When the bottom story height was increased, the values from the alignment chart for the columns in the bottom story are initially about 4% conservative, then increase in conservativeness up to 23% at the end of the variation. For the columns in the second story, values from the alignment chart are mostly conservative (about 19% for columns 6 and 8 and about 2% for column 7) at the start of the variation and then decrease in conservativeness until about the baseline column height. Then the values are increasingly unconservative until the end of the variation (about 39% for columns 6 and 8 and 45% for column 7). For the columns in the top story, columns 11 and 13 give conservative values (about 7%) at the start of the variation and unconservative values (about 35%) at the end of the variation. Column 12 give unconservative values all the way through the variation with a value of about 43% at the end of the variation. For the columns in the top stories the difference in values are almost constant until a story height of about 15ft. and then start increasing in unconservativeness from then until the end of the variation.

When the top story height is increased, the alignment chart values for the columns 1 and 3 in the bottom story start off as conservative (about 4%). Column 2 starts off as being unconservative (about 2%). At the end of the variation all the values are unconservative (about 42%) for columns 1 and 3, and about 45% for column 2. Columns 6 and 8 in the second story also start with conservative values (about 2%) then change to unconservative values until the end of the variation (about 44%). For column 7 all values are unconservative with a value of about 50% at the end of the variation. For columns in the top story, columns 11 and 13 are conservative by less then 10% throughout the variation while values for column 12 are unconservative by less then 10%.

3.5.4 Application of Lui's Method.

K factor values for column 7 with the bottom columns varied were obtained using Lui's method. Nomograph values were about 45% unconservative when compared to system buckling values. Results are shown in Table 3.4c and Fig. 3.4g.

Height of 1st floor	RV6	Nomograph	Lui's
12.5	1.256	1.14	1.316
15.0	1.290	1.14	1.317
17.5	1.466	1.13	1.312
20.0	1.660	1.13	1.319
22.5	1.857	1.13	1.320
25.0	2.054	1.13	1.321

Table 3.4c.

From the results obtained, Lui's method gives better agreement with buckling analysis results but it still fails to capture the effect of story height variation.

3.5.5 Limitation of Nomograph Use.

In order to limit unconservative error to about 10%, height variation should not be more than 15ft. or 1.2H. Height variation of up to 2H could lead to unconservative error of up to 45%.

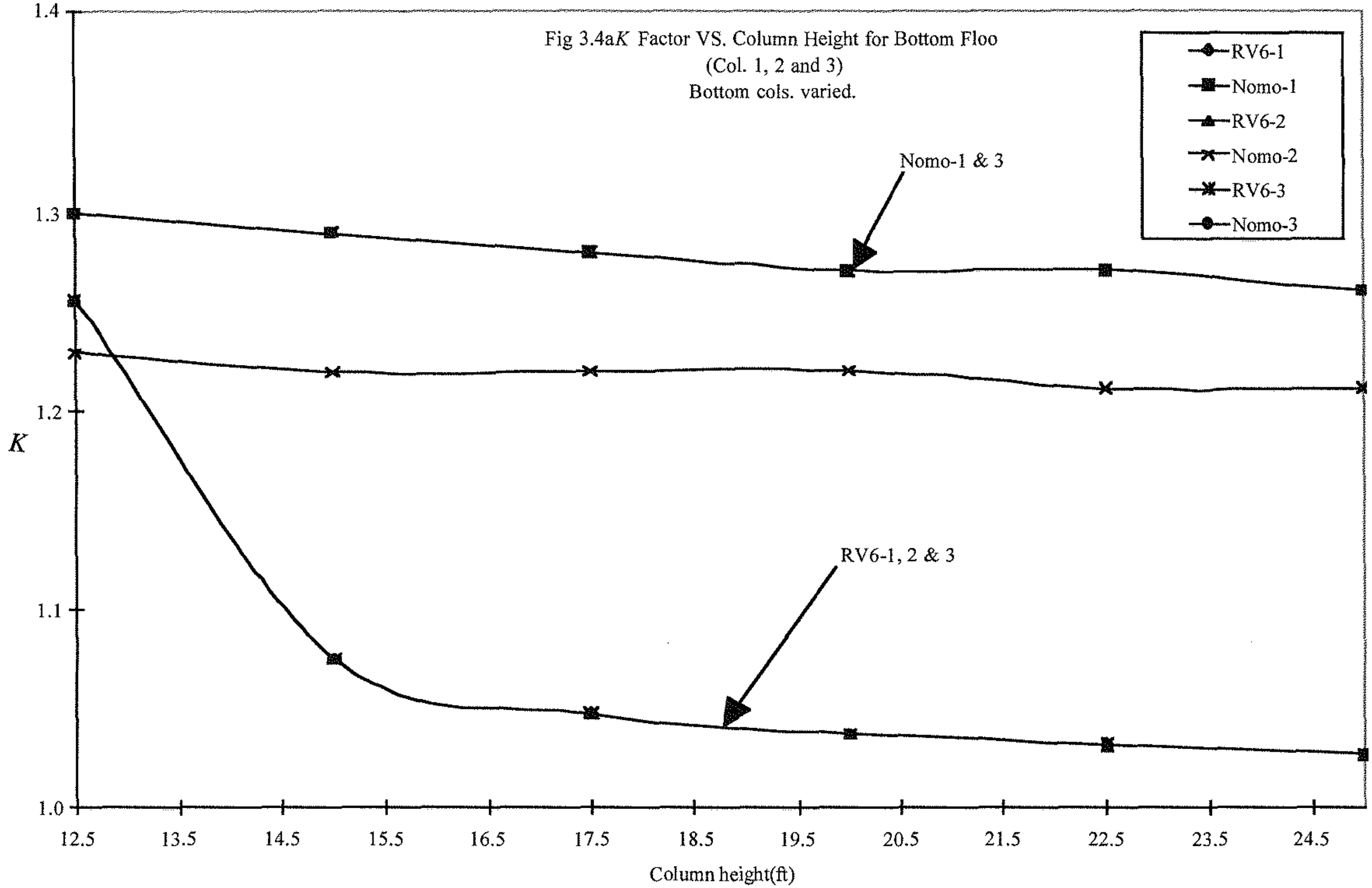


Fig 3.4aK Factor VS. Column Height for Bottom Floo
(Col. 1, 2 and 3)
Bottom cols. varied.
RV6-1
Nomo-1
RV6-2
Nomo-2
RV6-3
Nomo-3
Nomo-1 & 3
RV6-1, 2 & 3
K
1.4
1.3
1.2
1.1
1.0
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column height(ft)

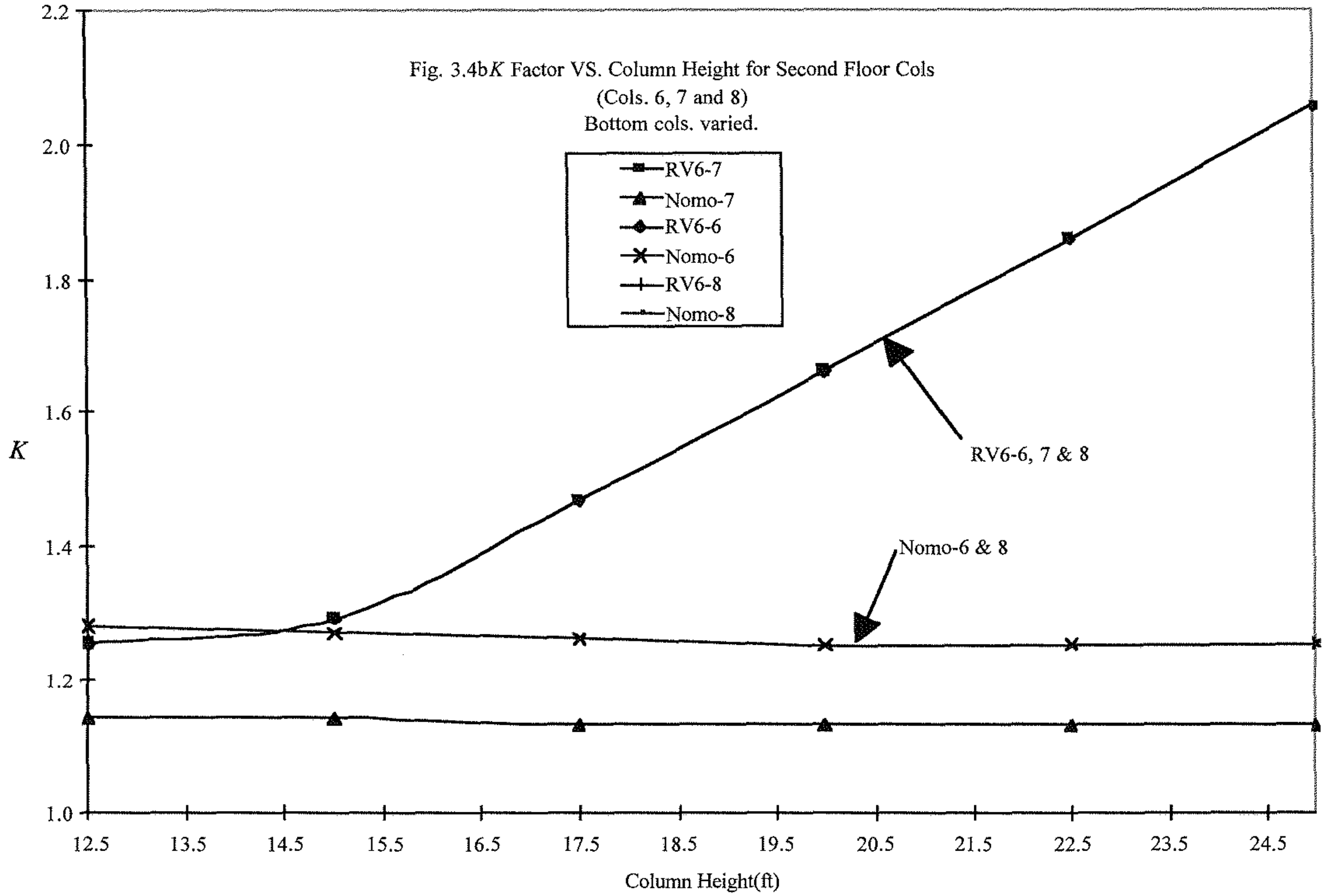


Fig. 3.4b*K* Factor VS. Column Height for Second Floor Cols
(Cols. 6, 7 and 8)
Bottom cols. varied.
RV6-7
Nomo-7
RV6-6
Nomo-6
RV6-8
Nomo-8
RV6-6, 7 & 8
Nomo-6 & 8
K
2.2
2.0
1.8
1.6
1.4
1.2
1.0
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column Height(ft)

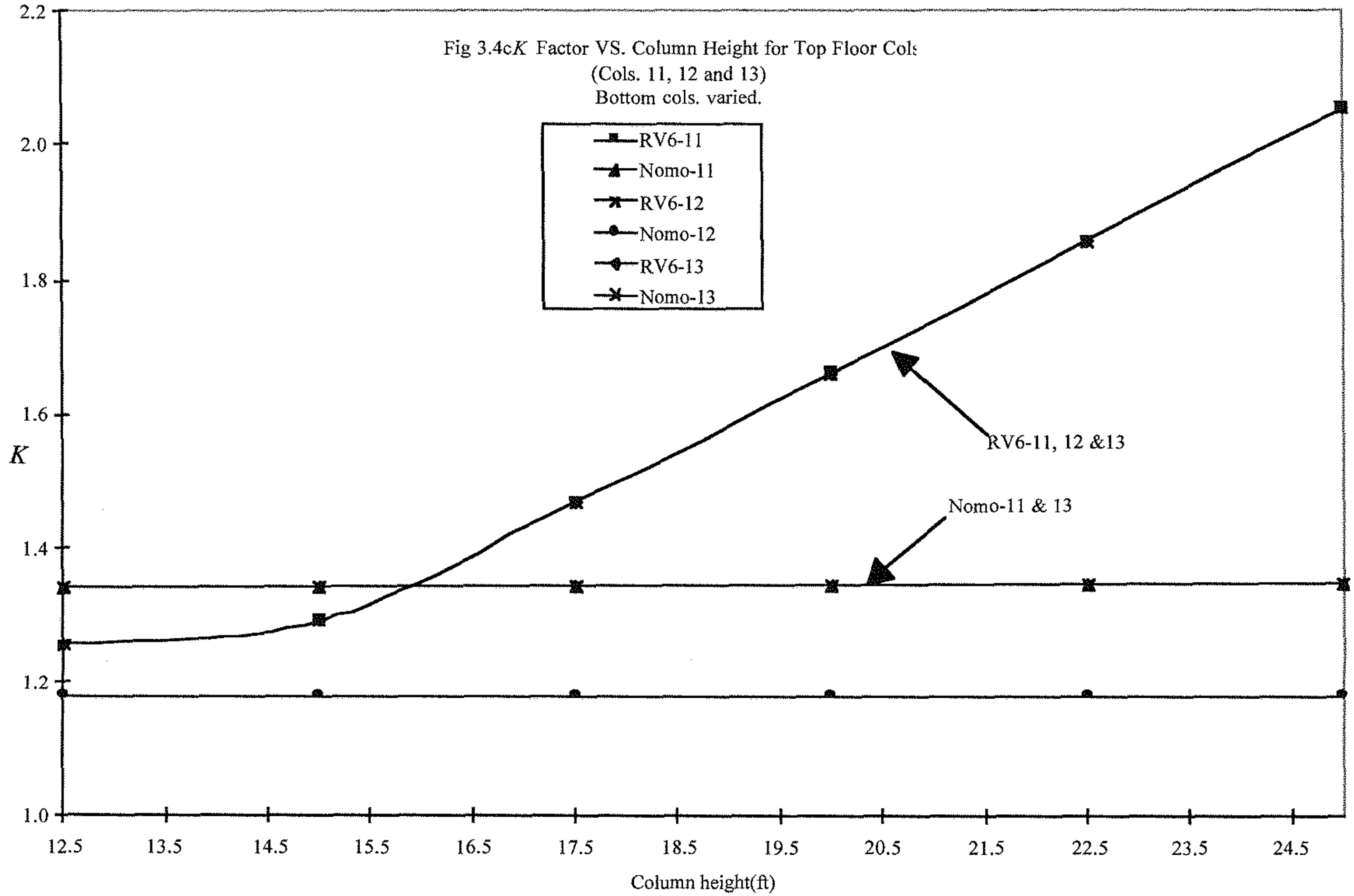


Fig 3.4cK Factor VS. Column Height for Top Floor Cols
(Cols. 11, 12 and 13)
Bottom cols. varied.
RV6-11
Nomo-11
RV6-12
Nomo-12
RV6-13
Nomo-13
RV6-11, 12 &13
Nomo-11 & 13
K
2.2
2.0
1.8
1.6
1.4
1.2
1.0
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column height(ft)

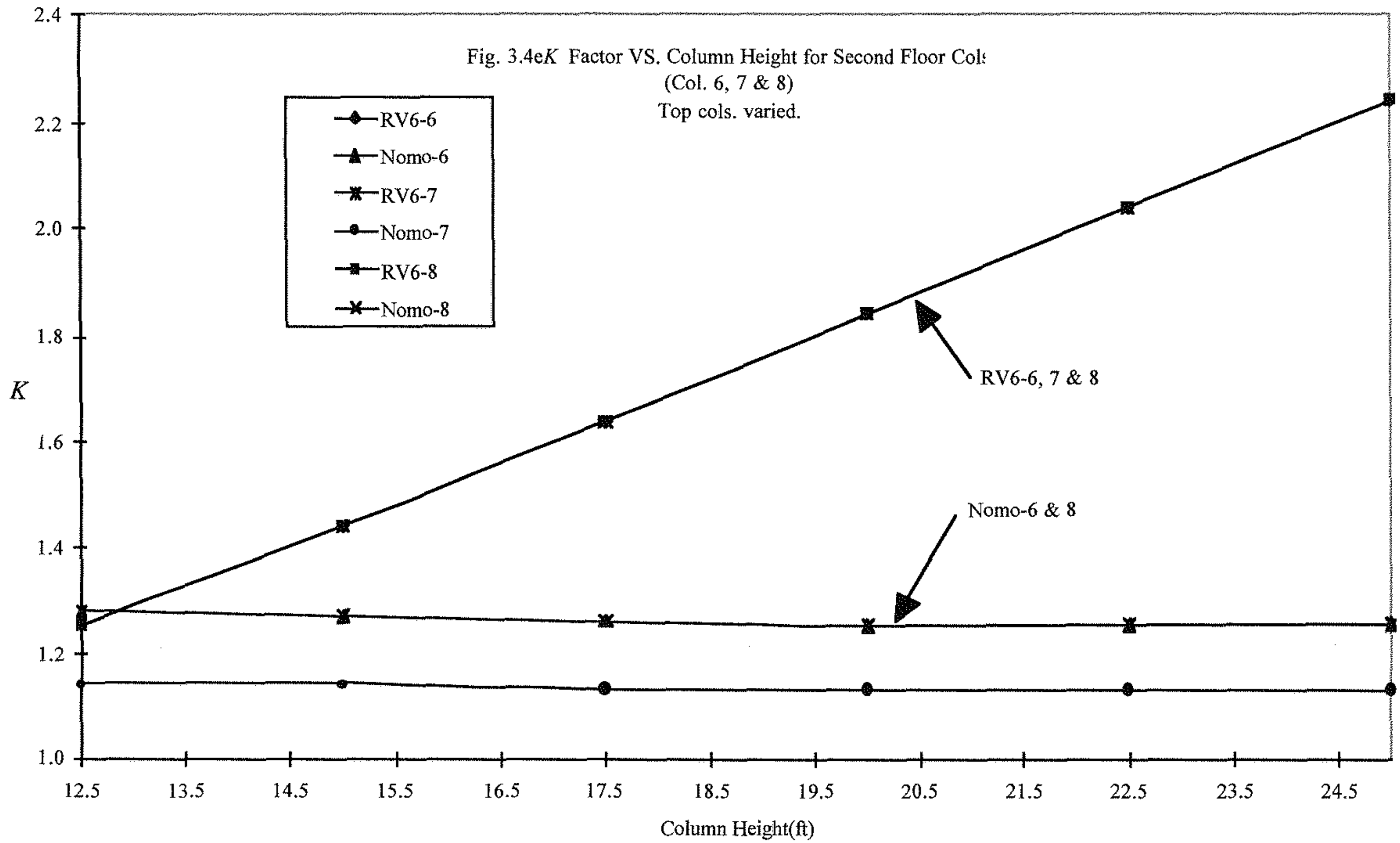


Fig. 3.4eK Factor VS. Column Height for Second Floor Col:
(Col. 6, 7 & 8)
Top cols. varied.
RV6-6
Nomo-6
RV6-7
Nomo-7
RV6-8
Nomo-8
RV6-6, 7 & 8
Nomo-6 & 8
K
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column Height(ft)

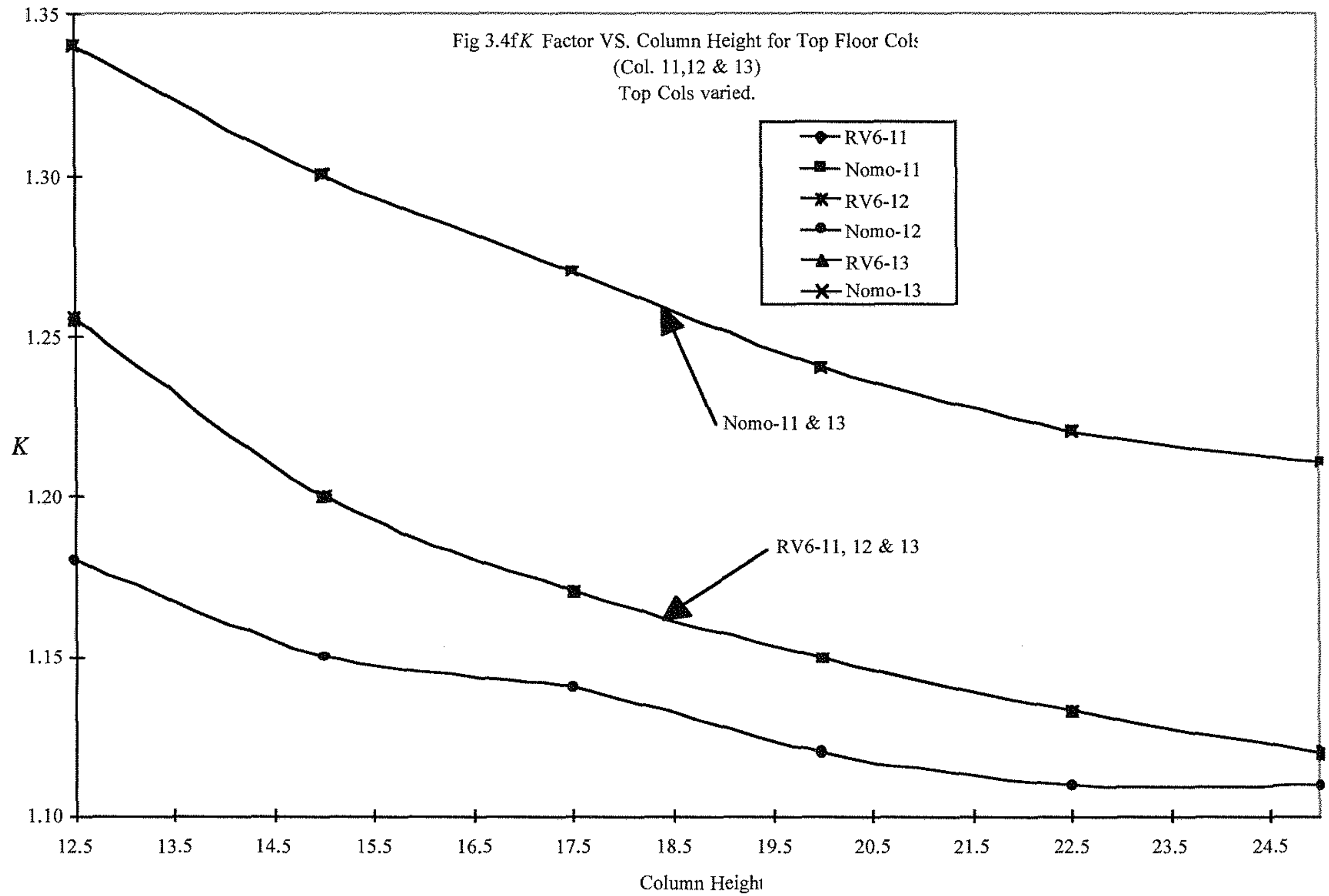


Fig 3.4f K Factor VS. Column Height for Top Floor Col:
(Col. 11,12 & 13)
Top Cols varied.
RV6-11
Nomo-11
RV6-12
Nomo-12
RV6-13
Nomo-13
Nomo-11 & 13
RV6-11, 12 & 13
K
1.35
1.30
1.25
1.20
1.15
1.10
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column Height

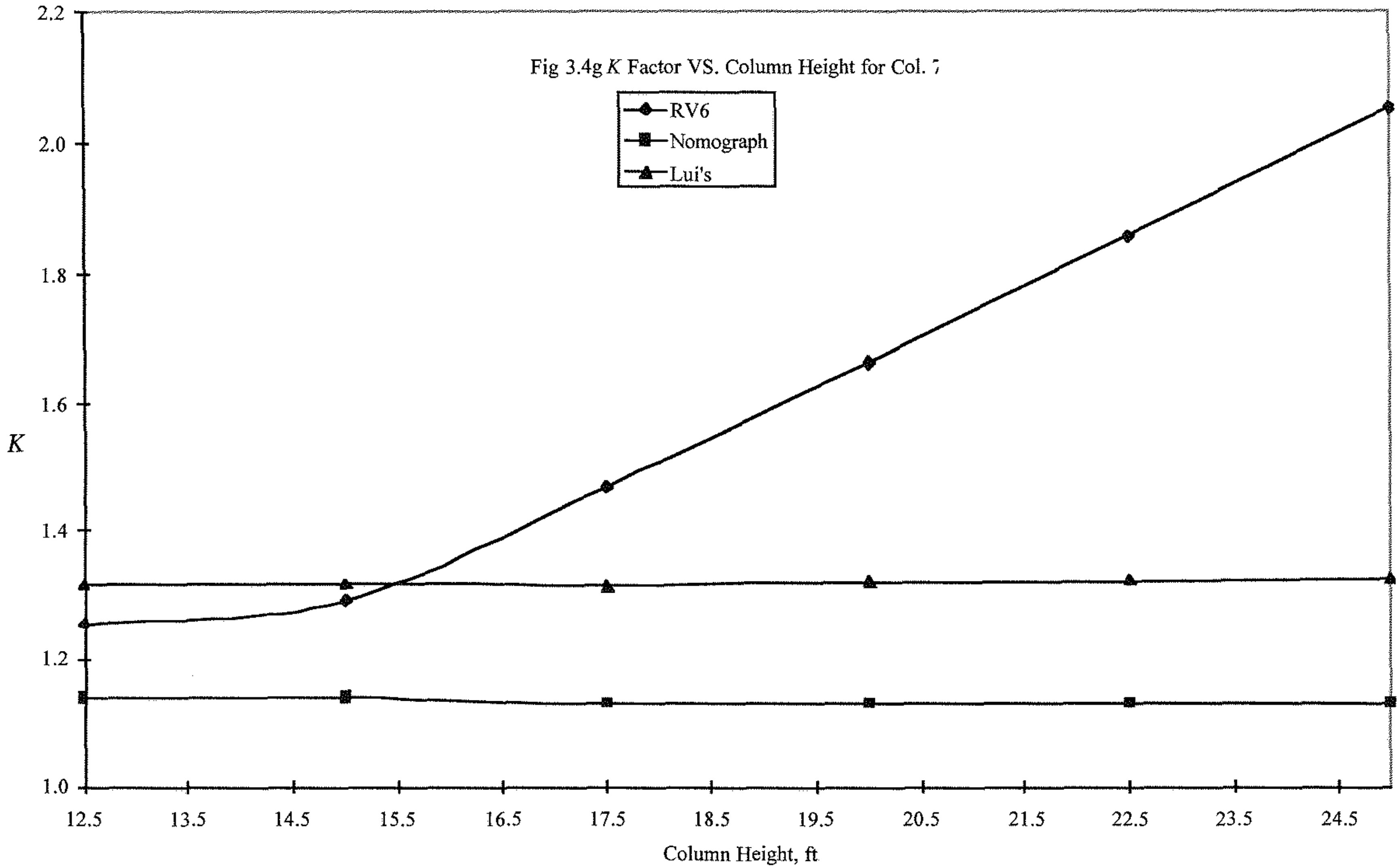


Fig 3.4g K Factor VS. Column Height for Col. 7
RV6
Nomograph
Lui's
K
2.2
2.0
1.8
1.6
1.4
1.2
1.0
12.5
13.5
14.5
15.5
16.5
17.5
18.5
19.5
20.5
21.5
22.5
23.5
24.5
Column Height, ft

3.6 Additional Column Height Problem.

Results from the simpler baseline structure are shown in Table 3.5.

	Bottom floor			Top floor		
Ratio of bottom to top floor	RV6	Nomograph	Lui's	RV6	Nomograph	Lui's
2.00	1.032	1.265	1.065	2.063	1.178	1.328
1.75	1.038	1.269	1.078	1.817	1.183	1.307
1.50	1.049	1.276	1.097	1.573	1.189	1.286
1.25	1.071	1.285	1.124	1.339	1.198	1.264
1.00	1.183	1.298	1.170	1.183	1.211	1.240
0.75	1.534	1.320	1.253	1.151	1.232	1.219
0.50	2.279	1.361	1.449	1.139	1.272	1.191
0.25	4.510	1.469	2.065	1.127	1.376	1.162

Table 3.5. *K* factor for simpler baseline structure with variation in column height

K factor values from the nomograph shows an increase of about 16% for the bottom floor columns, and about 17% for the top floor columns (Fig. 3.5a and 3.5b). Values from system buckling analysis shows an increase of about 300% for the first floor columns, and a decrease of about 45% for the top floor columns. When the nomograph *K* factor values for the columns in the bottom floor are compared to values from buckling analysis, they are conservative by about 20% at the beginning of the variation till about a height ratio of about 1.0. Then they are conservative up to about 67% at the end of the variation. Almost the reverse is true for the columns in the top floor. They start off as being unconservative by about 43% till a height ratio of about 1.0 and then conservative by about 22% at the end of the variation. Again for the simpler baseline structure the inter story "leaning effect" can be seen. At the start of the variation, columns in the top story are shorter and thus stronger when compared to columns in the bottom story resulting in higher *K* factor values for the columns in the top story. As the ratio is reduced the columns in the top story weaken and columns in the bottom story strengthen, resulting in decrease in K values for the columns in the top story and increase in values for columns in the bottom story. Values for the top story are less than values for the bottom story when the ratio of the stories is less than 1.0 since it is then weaker as compared to columns in the bottom story and it now “leans” onto the columns in the bottom story. From the results of the two story unbraced frame, the nomograph should only be used when the column considered is longer than columns that are on top or at the bottom of it.

K factor values obtained using Lui’s method as a whole gives better agreement with system buckling values when compared to values from the nomograph. For the columns in the bottom story, results are comparable to system buckling results when the ratio of the bottom to the top floor is greater then 1.0 (Fig. 3.5a). The reverse is true for the columns in the top story where the *K* factor value from Lui’s method are comparable when the height ratio is less then 1.0 (Fig. 3.5b). Lui’s method fails to capture the effect of the variation in the column height to it full extent. This may be due to the fact that Lui’s method can only predict with sufficient accuracy the *K* factor values for columns in unbraced frames with unequal distribution of lateral stiffness and gravity loads and for frames with leaner columns [Shanmugam and Chen 1995].

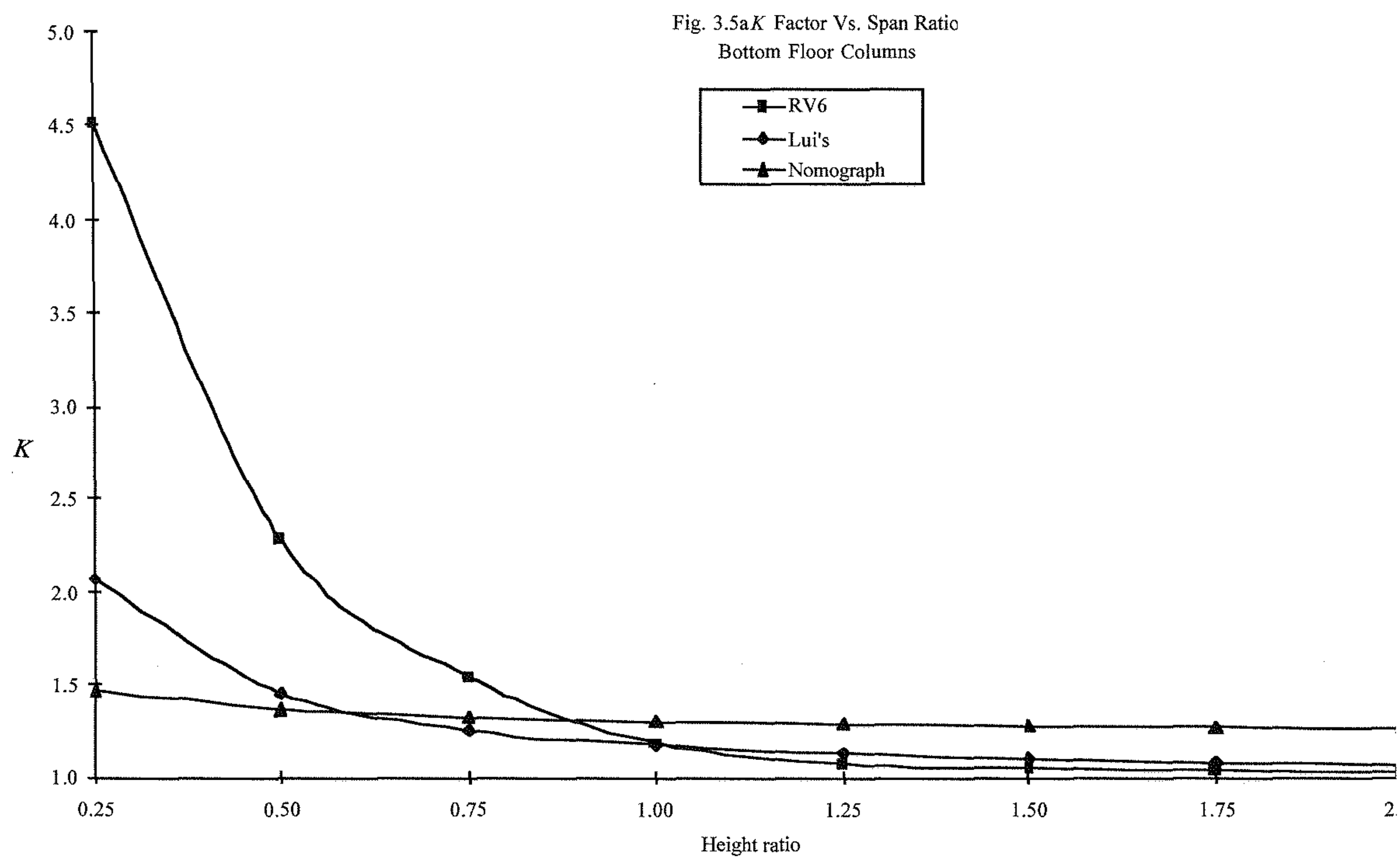


Fig. 3.5a*K* Factor Vs. Span Ratio
Bottom Floor Columns
RV6
Lui's
Nomograph
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
K
0.25
0.50
0.75
1.00
1.25
1.50
1.75
Height ratio

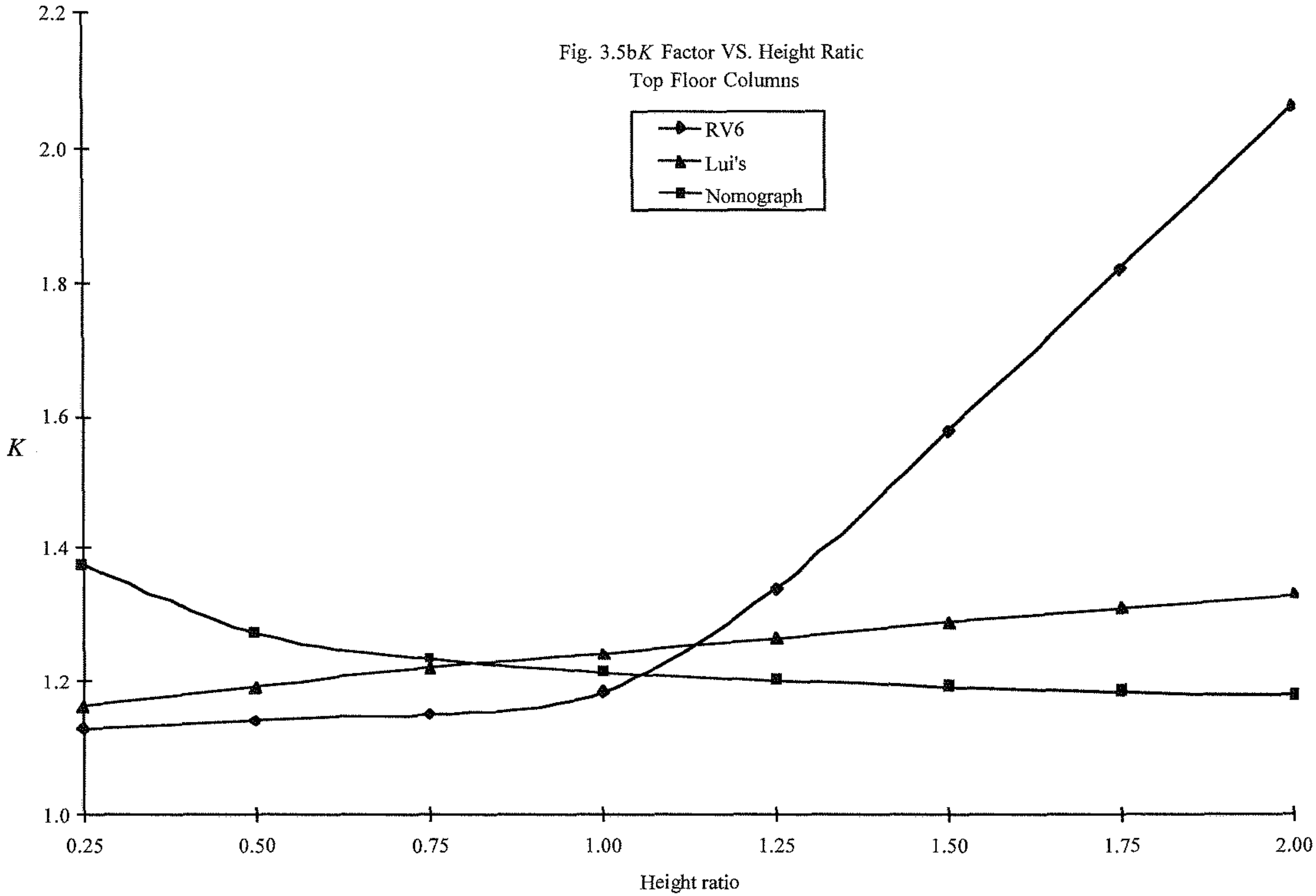


Fig. 3.5bK Factor VS. Height Ratic
Top Floor Columns
RV6
Lui's
Nomograph
K
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Height ratio

Chapter 4
Conclusions

This study was carried out with the aim of observing the performance of the nomograph when the assumptions underlying the development of the nomograph are violated. From the results of the study on the unbraced frame structure used in this study, the following observances and recommendations can be made.

4.1.1 Bay Width Variation.

From the comparison made between the *K* factors obtained by using the nomograph and system buckling analysis the most unconservative error of the nomograph value is about 11% even though the bay width was doubled. Therefore, the nomograph in the case of bay width variation should be able to give adequately accurate values for practical purposes or at least for preliminary design purposes. Use of Lui's method does give more accurate *K* factor values.

4.1.2 Moment of Inertia Variation.

From the comparison made between the *K* factors obtained by using the nomograph and system buckling analysis the most unconservative error of the nomograph values is about 45% through out the variation. This degree of unconservative error should not be acceptable in practical use. This is due to the fact that the nomograph fails to capture the full interaction, such as the "leaning" effect, of the components of the unbraced frame structure. In order to keep values from being unconservative by more than 10%, the moment of inertia of the framed column members should not vary by more than 1.5I. The use of Lui's method in this case gives more accurate *K* factor values. System buckling analysis should be used for extreme variations of moments of inertia.

4.1.3 Loading Variation

From the comparison made between the *K* factors obtained by using the nomograph and system buckling analysis the most unconservative error of the nomograph value is about 40% through out the variation. The nomograph also shows no effect of symmetry of loading as the *K* factor obtained from loading the middle and the left column tier are the same. Values from buckling analysis shows columns that were given loading increments decreased their *K* factor value while columns whose loadings were held constant increased their *K* factor values. This is again the "leaning column" phenomena which is not captured by the nomograph. In order to limit the unconservative error due to load variation to 10%, load on columns should not vary by more than 1.5P. Lui's method is a very good alternative in obtaining the *K* factor as results shows that it is almost comparable to values from system buckling analysis. System buckling analysis should be used for extreme variations of loading.

4.1.4 Variation in Column Height

From the comparison made between the *K* factors obtained by using the nomograph and system buckling analysis the most unconservative error of the nomograph values is about 45%. Limiting the variation to about 1.2H should give unconservative values of less than 10%. This is again due to the failure of the nomograph in capturing the interaction between the components of the unbraced frame member. The results from system buckling analysis show that there is also a "leaning column" effect in varying the column heights. But this effect is more of an inter story effect. Columns which are shorter become the stronger columns and the longer columns which are weaker "lean" on to the shorter columns causing the shorter columns to increase their *K* factor values. Results from the two story unbraced frame show that the nomograph should only be used when the column under consideration is longer than columns on top or at the bottom of it.

Lui's method does to some extent give better *K* factor values. But it also fails to capture the full effect of column height variation. This is may be due to the fact that Lui's method can only predict with sufficient accuracy the *K* factor values for columns in unbraced frames with unequal distribution of lateral stiffness and gravity loads and for frames with leaner columns.

4.2 Conclusions

The nomograph for unbraced frames produces inaccurate *K* factor values in cases where the underlying assumptions are violated. Unconservative errors of up to almost 50% were found. The nomograph should be used with the knowledge that it is an approximate method and that there are limits to its use. Knowing the limits would be helpful to practitioners using the nomograph.

In situations where the nomograph is impractical to use, there are other approximate methods, such as Lui's method, which produces *K* factors which are almost comparable to *K* factors obtained by buckling analysis. Again these methods are approximate and using them should be with the knowledge of their assumptions and limitations.

System buckling analysis is the most accurate method of obtaining *K* factors. The nomograph should be used only in preliminary design stages and finalized using system buckling analysis. The nomograph was developed to facilitate the determination of *K* factors as the use of system buckling analysis can be complicated and time consuming. But with the advent of computers and structural analysis software, such as Robot V6, capable of solving the most complicated of analyses, use of system buckling is essential in ensuring a safe and economic design.

REFERENCES

[AISC 90] American Institute of Steel Construction, *Specification for Structural Steel Buildings - Allowable Stress Design,* Chicago, 1990.

[AISC 93] American Institute of Steel Construction, *Load and Resistance Factor Design Specification for Steel Buildings,* Chicago, 1993.

[Aristizabal-Ochoa 94] Aristizabal-Ochoa, J. D., "*K*-factor for columns in any type of construction: Nonparadoxical approach," *J. Struct.. Engr.,* ASCE, 120(4), 1994, 1272-1290.

[Bridge and Fraser 87] Bridge, R. Q., and Fraser, D. J., "Improved *G*-factor Method for Evaluating Effective Lengths of Columns," *J. Struct. Engr.,* ASCE, 113(6), 1987, 1341-1356.

[Chu and Chow 69] Chu, K. -H., and Chow, H. L. , Effective Column Length in Unsymmetrical Frames," *Publ.,* Int. Assoc. Bridge Struct. Engr., Vol. 29(1), 1969.

[Cheong-Siat-Moy 86] Cheong-Siat-Moy, F., "K-factor paradox," *J. Struct. Engr.*, ASCE, 112(8), 1986, 1747-1760.

[Duan and Chen 89] Duan, L., and Chen, W. F. "Effective Length for Columns in Unbraced Frames," *J. Struct. Engr.,* ASCE, 115(1), January, 1989, 149-165.

[Galambos 88] Galambos, T. V. , ed. *Guide to Structural Stability Design Criteria for Metal Structures*, 4th. Ed. New York: John Wiley and Sons, 1988.

[Hellesland and Bjorhovde 96] Hellesland, J. and Bjorhovde, R., "Restraint Demand Factors an Effective Lengths of Braced Columns," *J. Struct.. Engr.,* ASCE, 122 (10), October, 1996, 1216-1224.

[Hajjar and White 94] Hajjar, J. F., and White, D. W., "The Accuracy of Column Stability Calculations in Unbraced Frames and the Influence of Columns with Effective Length Factors Less Than One," *Engineering Journal*, 3rd Quarter, 1994, 81-97.

[LeMessurier 77] LeMessurier, W. J., "A Practical Method of Second Order Analysis, Part 2-Rigid Frames," AISC, *Engineering Journal,* 2nd Qtr., 1977, 49-67.

[Liew, White, and Chen 91] Liew, J. Y. R., White, D. W., and Chen, W. F., Beam-Column Design in Steel Frameworks-Insight on Current Methods and Trends," *Journal of Constructional Steel Research,* 18, 1991, 269-308.

[Lui 92] Lui, E. M., "A Novel Approach for *K* Factor Determination," AISC, *Engineering Journal*, Vol. 19, No. 4, 4th. Quarter, 1992.

[Kavanaugh 62] Kavanaugh, T. C. "Effective Length of Framed Columns," *Transactions,* ASCE, 127 (1962) Part II, 81-101.

[Metrosoft 96] Metrosoft Structures, Inc., *Demonstration Manual - Robot V6 version 3.0, Structural Analysis and Design Software*, E. Rutherford, N.J., 1996

[Salmon and Johnson 90] Salmon, C. G. , and Johnson, J. E., *Steel Structures: Design and Behavior*, 3rd. Ed., Harper and Row, New York, N. Y., 1990.

[Shanmugam and Chen] Shanmugam, N. E. and Chen, W. F., "An Assessment of *K* Factor Formula.", *Engineering Journal*, 1st. Quarter, 1995, 3-11.

[Yura 71] Yura, J. A. , "The Effective Length of Columns in Unbraced Frames," *Engineering Journal,* Vol. 8, No. 2, 1971, 37-42.