RELIABILITY-BASED STRENGTH REDUCTION FACTOR FOR BOND

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ABSTRACT

The formulation and calculation of a reliability-based strength-reduction (ϕ) factor for developed and spliced bars is described. Conventional and high relative rib area bars, both with and without confining reinforcement, are considered. The ϕ-factor is determined using statistically-based expressions for development/splice strength and Monte Carlo simulations of a range of beams.

A strength-reduction factor of 0.9 is obtained for the design expressions for development/splice length, based on a probability of failure in bond equal to about one-fifth of the probability of failure in bending or combined bending and compression. $\phi = 0.9$ is incorporated into two expressions for development/splice length in a manner that is transparent to the user. A major advantage of each of the final expressions is that they provide identical values for development and splice length, removing the need to multiply development length by 1.3 or 1.7 to obtain the length of most splices.

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INTRODUCTION

Recent work to improve the development characteristics of reinforcing bars by modifying bar deformation patterns (Darwin and Graham 1993a, 1993b, Darwin, Tholen, Idun and Zuo 1995a) has included a reevaluation of existing development and splice tests and the formulation of an expression to represent the bond force of bottom-cast bars at development/splice failure (Darwin, Zuo, Tholen, and Idun 1995b).

Based on this analysis, the best-fit equation for the ultimate bond force, T_b, is

$$\frac{T_b}{f_c'^{1/4}} = \frac{A_b f_s}{f_c'^{1/4}} = [63\, l_d(c_m + 0.5\, d_b) + 2130\, A_b]\left(0.1\, \frac{c_M}{c_m} + 0.9\right)$$

$$+ 2226\, t_r t_d\, \frac{NA_{tr}}{n} + 66 \tag{1}$$

in which T_b = force in bar at development or splice failure, in lb

A_b = bar area, in in.2

d_b = nominal bar diameter, in in.

f_s = steel stress at failure, in psi

f_c' = concrete compressive strength, in psi; $f_c'^{1/4}$ in psi

l_d = development or splice length, in in.

c_m, c_M = minimum or maximum value of c_s or c_b ($c_M/c_m \le 3.5$), in in.

c_s = min (c_{si} + 0.25 in., c_{so}), in in.

c_{si} = one-half of clear spacing between bars, in in.

c_{so}, c_b = side cover or bottom cover of reinforcing bars, in in.

N = number of transverse reinforcing bars (stirrup or ties) crossing l_d

A_{tr} = area of transverse reinforcement crossing the potential plane of splitting adjacent to the reinforcement being developed, in in.2

n = number of bars being developed or spliced along the plane of splitting

$$t_r \quad = 9.6\, R_r + 0.28$$

$$t_d \quad = 0.72\, d_b + 0.28$$

$R_r \quad$ = ratio of projected rib area normal to bar axis to the product of the nominal bar perimeter and the center-to-center rib spacing

The final term in Eq. 1, 66, is used only if the member has confining transverse reinforcement.

Eq. 1 is based on the analysis of 133 development and splice tests of bottom-cast bars without confining reinforcement and 166 tests with confining reinforcement (Chinn et al. 1955, Chamberlin 1956, 1958, Mathey and Watstein 1961, Ferguson and Thompson 1965, Ferguson and Breen 1965, Thompson et al. 1975, Zekany et al. 1981, Choi et al. 1990, 1991, DeVries et al. 1991, Hester et al. 1991, 1993, Rezansoff et al. 1991, 1993, Azizinamini et al. 1993, 1995, Darwin et al. 1995a). The data base includes specimens with concrete strengths, f'_c, between 1820 and 15,760 psi (13 and 109 MPa) and bars with relative rib areas, R_r, between 0.056 and 0.140; the relative rib area has been shown to significantly affect the contribution of transverse reinforcement to bond strength (Darwin and Graham 1993a, 1993b, Darwin et al. 1995a). The effect of R_r is reflected in the expression for t_r. R_r averages 0.0727 for conventional reinforcement and 0.1275 for newly proposed high relative rib area bars (Darwin et al. 1995a, 1995b).

Eq. 1 produces a mean test/prediction ratio of 1.00, with a coefficient of variation, $V_{T/P}$, of 0.107 for beams in which the bars are not confined by transverse reinforcement and a mean test/prediction ratio of 1.01, with $V_{T/P} = 0.125$, for beams in which the bars are confined by transverse reinforcement.

Eq. 1 can be used to calculate development/splice length, l_d, by dropping the final term, 66, and setting $N = l_d/s$, in which s = spacing of transverse reinforcement, in in.

$$\frac{l_d}{d_b} = \frac{\dfrac{f_s}{f_c'^{1/4}} - 2130 \left(0.1\, \dfrac{c_M}{c_m} + 0.9 \right)}{80.2 \left(\dfrac{c + K_{tr}}{d_b} \right)} \tag{2}$$

in which $c = (c_m + 0.5\ d_b)(0.1\ c_M/c_m + 0.9)$, $K_{tr} = 35.3\ t_r t_d A_{tr}/sn$, and $(c + K_{tr})/d_b \leq 4.0$

Eq. 2 can be further simplified by setting $c_M/c_m = 1$ and dropping 0.25 in. from the definition of c_s.

$$\frac{l_d}{d_b} = \frac{\dfrac{f_s}{f_c'^{\,1/4}} - 2130}{80.2 \left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{3}$$

in which $c = (c_m + 0.5\ d_b)$.

Converting Eqs. 2 and 3 back to a form that can be used to predict $T_b = A_b f_s$ gives, respectively,

$$T_b = A_b f_s = f_c'^{\,1/4} \left\{ [63\ l_d(c_m + 0.5\ d_b) + 2130\ A_b]\left(0.1\ \frac{c_M}{c_m} + 0.9\right) \right.$$
$$\left. + 2226\ t_r t_d \frac{l_d A_{tr}}{sn} \right\} \tag{4}$$

$$T_b = A_b f_s = f_c'^{\,1/4} \left\{ [63\ l_d(c_m + 0.5\ d_b) + 2130\ A_b] + 2226\ t_r t_d \frac{l_d A_{tr}}{sn} \right\} \tag{5}$$

in which $t_r = 0.98$ for conventional bars and 1.50 for high relative rib area bars, and c_s (used to determine c_m), is defined appropriately in the two expressions.

Eq. 4 (and Eq. 2) represents, very nearly, the best-fit equation for the full data base, Eq. 1. Therefore, 50 percent of the development/splice designs based on this expression will be weaker than predicted by Eq. 1 – a situation that presents unacceptable safety risks. Eq. 5 (and Eq. 3) is, in general, more conservative than Eq. 4, but will provide the same value of l_d as Eq. 4 when $c_m = c_b = c_{so}$.

The level of safety can be improved by reducing the usable bond force, $T_b = A_b f_s$, by multiplying the right side of Eqs. 4 and 5 by a suitable strength reduction (ϕ) factor. A longer development length is then required to provide the desired value of T_b.

This report describes the calculation of a reliability-based ϕ-factor for developed and spliced bars with relative rib areas of 0.0727 and 0.1275 (for conventional and high relative rib area bars, respectively). Bars both with and without confining transverse reinforcement are considered. The ϕ-factor is used in conjunction with Eqs. 4 and 5 to formulate design expressions for l_d that are similar in format to Eqs. 2 and 3. The overall approach is described first, followed by the details of the calculation. As will be demonstrated, a major advantage of the final expressions is that they provide identical values for development and splice length.

CALCULATION OF STRENGTH REDUCTION FACTORS

Overall Approach

The capacity reduction factor, ϕ_b, must be selected to insure an acceptably low probability of bond failure. Considering the brittle nature of bond failures, that probability should be lower than the probability of failure under a main load-carrying mechanism, such as bending or combined bending and compression. This can be achieved by using the concepts of structural reliability.

Limiting consideration to "statically" applied load for the purpose of this analysis (i.e., not seismic or shock loading), it is recognized that the bar force, $A_b f_s$, that appears on the left side of Eqs. 4 and 5 has already been increased by a factor of $1/\phi$, in which ϕ = strength reduction factor for the main loading, before development/splice design is undertaken. So as not to double-count ϕ-factors, the resistance to which ϕ_b is applied corresponds to $\phi A_b f_s$ (equivalent to the factored load). That is,

$$\phi A_b f_s \geq \phi_b \text{[Right side of Eq. 4 or 5]} \tag{6}$$

Therefore, the effective ϕ-factor for use in calculating development/splice length becomes $\phi_d = \phi_b/\phi$, although the overall ϕ-factor against bond failure remains ϕ_b.

$$A_b f_s \geq \phi_d \text{ [Right side of Eq. 4 or 5]} \tag{7}$$

Determining the value of ϕ_b (and ultimately ϕ_d) requires the selection of the desired level of reliability, which can be represented by the reliability index, β (Ellingwood, Galambos, Mac-Gregor, and Cornell 1980). For a resistance, R, and a loading, Q, failure will not occur if $R/Q \geq 1$. Using the formulation shown in Fig. 1 and the small-variance approximations (Ellingwood et al. 1980), $\overline{\ln(R/Q)} \approx \ln(\overline{R}/\overline{Q})$ and $\sigma_{\ln(R/Q)} \approx (V_R^2 + V_Q^2)^{1/2}$, in which the overbar represents the average, σ = standard deviation, and V = coefficient of variation,

$$\beta = \frac{\overline{\ln(R/Q)}}{\sigma_{\ln(R/Q)}} \approx \frac{\ln(\overline{R}/\overline{Q})}{(V_R^2 + V_Q^2)^{1/2}} \tag{8}$$

Under typical conditions of loading, $\beta \approx 3.0$ for reinforced concrete beams and columns (Ellingwood et al. 1980). A higher value of β is needed to insure that the probability of a bond failure is lower than the probability of a failure in bending for beams or in combined bending and compression for columns. Therefore, $\beta = 3.5$ is used in the calculation of development/splice length, producing a probability of failure equal to approximately one-fifth of that obtained with $\beta = 3.0$.

Eq. 8 can be used to calculate ϕ_b, but to do so requires knowledge of R and Q, both of which are random variables. This knowledge can be obtained through the application of Monte Carlo analysis, used in conjunction with data obtained from field measurements and test results. The derivation that follows parallels techniques used by Ellingwood et al. (1980), Mirza and MacGregor (1986), and Lundberg (1993):

R = random variable for resistance, which is represented as

$$R = X(1)\, R_p \tag{9}$$

in which $X(1)$ = test-to-predicted load capacity random variable

R_p = predicted capacity random variable, dependent on material and geometric properties of member, which are also random variables

$$Q = \Sigma \text{ loads} \tag{10}$$

For dead load and live load,

$$Q = Q_D + Q_L \tag{11}$$

$$Q = \left(\frac{Q_D}{Q_{Dn}} + \frac{Q_L}{Q_{Dn}} \right) Q_{Dn} \tag{12}$$

in which Q_D and Q_L = random variables representing dead and live load effects

Q_{Dn} = nominal dead load

$$\frac{Q_D}{Q_{Dn}} = X(2) \tag{13}$$

$$\frac{Q_L}{Q_{Dn}} = \frac{Q_L}{Q_{Ln}} \frac{Q_{Ln}}{Q_{Dn}} = X(3) \left(\frac{Q_L}{Q_D} \right)_n \tag{14}$$

in which Q_{Ln} = nominal live load

$X(2), X(3)$ = actual-to-nominal dead and live load random variables

$\left(\dfrac{Q_L}{Q_D} \right)_n$ = nominal ratio of live load to dead load

In design,

$$\phi_c R_n = \gamma_D Q_{Dn} + \gamma_L Q_{Ln} \tag{15}$$

in which ϕ_c = "composite" strength reduction factor (for this derivation, $\phi_c = \phi_b$)

R_n = nominal resistance

γ_D, γ_L = load factors for dead and live loads

Factoring out Q_{Dn} on the right side of Eq. 15 and setting $Q_{Ln}/Q_{Dn} = (Q_L/Q_D)_n$ gives

$$\phi_c R_n = Q_{Dn}\left[\gamma_D + \gamma_L\left(\frac{Q_L}{Q_D}\right)_n\right] \tag{16}$$

Solving Eq. 16 for Q_{Dn} gives

$$Q_{Dn} = \frac{\phi_c R_n}{\gamma_D + \gamma_L\left(\dfrac{Q_L}{Q_D}\right)_n} \tag{17}$$

The total load, Q, is obtained by substituting Eqs. 13, 14, and 17 into Eq. 12.

$$Q = \frac{\left[X(2) + X(3)\left(\dfrac{Q_L}{Q_D}\right)_n\right]\phi_c R_n}{\gamma_D + \gamma_L\left(\dfrac{Q_L}{Q_D}\right)_n} \tag{18}$$

Letting

$$q = \frac{\left[X(2) + X(3)\left(\dfrac{Q_L}{Q_D}\right)_n\right]}{\gamma_D + \gamma_L\left(\dfrac{Q_L}{Q_D}\right)_n} \tag{19}$$

$$Q = \phi_c \, q \, R_n \tag{20}$$

$$\text{Defining } r = \frac{R}{R_n} = \frac{X(1) \, R_p}{R_n} \tag{21a}$$

$$R = X(1) \, R_p = r \, R_n \tag{21b}$$

From Eq. 8,

$$\beta = \frac{\overline{\ln(R/Q)}}{\sigma_{\ln(R/Q)}} = \frac{\overline{\ln(r \, R_n / \phi_c \, q \, R_n)}}{\sigma_{\ln(r \, R_n / \phi_c \, q \, R_n)}} = \frac{\overline{\ln(r / \phi_c q)}}{\sigma_{\ln(r / \phi_c q)}}$$

$$= \frac{\ln(\overline{r} / \phi_c \overline{q})}{(V_r^2 + V_{\phi q}^2)^{1/2}} = \frac{\ln(\overline{r} / \phi_c \overline{q})}{(V_r^2 + V_{\phi q}^2)^{1/2}} \tag{22}$$

in which

$$\overline{r} = \overline{\left(\frac{X(1) \, R_p}{R_n}\right)} \tag{23}$$

$$V_r = \frac{\sigma_r}{\overline{r}} \tag{24}$$

$$\overline{q} = \overline{\left[\frac{X(2) + X(3)\left(\dfrac{Q_L}{Q_D}\right)_n}{\gamma_D + \gamma_L \left(\dfrac{Q_L}{Q_D}\right)_n}\right]} \tag{25}$$

$$V_{\phi q} = \frac{\phi_c \sigma_q}{\phi_c \overline{q}} = \frac{\sigma_q}{\overline{q}} = \frac{\left\{ \left[\overline{X(2)} \ V_{Q_D} \right]^2 + \left[\overline{X(3)} \left(\frac{Q_L}{Q_D} \right)_n V_{Q_L} \right]^2 \right\}^{1/2}}{\overline{X(2)} + \overline{X(3)} \left(\frac{Q_L}{Q_D} \right)_n} \tag{26}$$

ϕ_c is calculated using Eq. 22. Starting with $\beta = [\ln (\overline{r} / \phi_c \overline{q})] / (V_r^2 + V_{\phi q}^2)^{1/2}$,

$$(V_r^2 + V_{\phi q}^2)^{1/2} \beta = \ln \left(\frac{\overline{r}}{\phi_c \overline{q}} \right) \tag{27}$$

$$e^{(V_r^2 + V_{\phi q}^2)^{1/2} \beta} = \frac{\overline{r}}{\phi_c \overline{q}} \tag{28}$$

$$\phi_c = \frac{\overline{r}}{\overline{q}} e^{-(V_r^2 + V_{\phi q}^2)^{1/2} \beta} \tag{29}$$

Random Variables

The mean values, $\overline{r}$ and $\overline{q}$, and the coefficients of variation, V_r and $V_{\phi q}$, are needed to calculate ϕ_c using Eq. 29. The values associated with the resistance random variable r, $\overline{r}$ and V_r, are obtained first, followed by the values associated with the load random variable q, $\overline{q}$ and $V_{\phi q}$.

Resistance Random Variable–Resistance random variable r is obtained using Eq. 21a.

$$r = \frac{R}{R_n} = \frac{X(1) R_p}{R_n} \tag{21a}$$

Test-to-predicted load random variable, X(1). The test-to-predicted load random variable, X(1), is based on a comparison of test results with Eq. 1. X(1) is treated as a normal random

variable with a mean equal to the mean test/prediction ratio. $\overline{X(1)}$ = 1.00 and 1.01 for members without and with confining transverse reinforcement, respectively (Darwin et al. 1995b). The coefficient of variation $V_{X(1)}$ is equal to the coefficient of variation associated with the predictive equation (or model) itself, V_m, as separate from uncertainties in the measured loads and differences in the actual material and geometric properties of the specimens from values used to calculate the predicted strength, represented by V_{ts}. The total coefficient of variation in the test/prediction ratio, $V_{T/P}$, is equal to $(V_m^2 + V_{ts}^2)^{1/2}$ (Grant, Mirza, and MacGregor 1978). Therefore, $V_m = (V_{T/P}^2 - V_{ts}^2)^{1/2}$.

For reinforced concrete, $V_{ts} \approx 0.07$ (Grant et al. 1978). For beams without confining reinforcement, $V_m = (V_{T/P}^2 - V_{ts}^2)^{1/2} = (0.107^2 - 0.07^2)^{1/2} = 0.081$. For beams with confining reinforcement, additional uncertainty occurs because the relative rib area, R_r, is not known for 34 of the beams used to establish Eq. 1. This is handled with $V_{R_r} = 0.02$, giving $V_m = (V_{T/P}^2 - V_{ts}^2 - V_{R_r}^2)^{1/2} = (0.125^2 - 0.07^2 - 0.02^2)^{1/2} = 0.102$.

Predicted Capacity Random Variable, R_p. The individual values of the predicted capacity random variable, R_p, are obtained for hypothetical beams using the Monte Carlo method. The random variables used to calculate R_p are the concrete strength, f'_c (adjusted for the rate of loading), the development/splice length, l_d, the member width, b, the cover, c_b, the side cover, c_{so}, and the relative rib area of the developed/spliced bar, R_r. The predicted capacity, R_p, is calculated by solving Eq. 1 for $A_b f_s$.

$$R_p = A_b f_s = f_c'^{1/4} \left\{ [63\, l_d(c_m + 0.5\, d_b) + 2130\, A_b] \left(0.1\, \frac{c_M}{c_m} + 0.9 \right) \right.$$
$$\left. + 2226\, t_r t_d\, \frac{NA_{tr}}{n} + 66 \right\} \tag{30}$$

Individual values of R_p are calculated by substituting values for each of the variables that are determined based on the nominal value and statistical properties of that variable. Beams with

spliced bars are used as the physical model in this study.

Concrete strength, f'_c [X(4)]. The random variable for concrete strength, X(4), must take into consideration the strength and variability of concrete, as used in practice, and the effect of the actual load rate in the structure, as opposed to the load rate used in standard tests (Mirza, Hatzinikolas, and MacGregor 1979). The latter point is considered first.

A relation proposed by Jones and Richart (1936) is used to take into account the fact that, under practical conditions, loading rates will be different than the average value of 35 psi/sec (0.24 MPa/sec) used in a standard compression test (ASTM C 39).

$$f'_{c\dot{f}} = 0.89\ f'_{c35}(1\ +\ 0.08\ \log \dot{f}) \tag{31}$$

in which 0.1 psi/sec $\leq \dot{f} \leq$ 10,000 psi/sec

$f'_{c\dot{f}}$ = compressive strength of concrete at stress rate $\dot{f}$

f'_{c35} = compressive strength of concrete at $\dot{f}$ = 35 psi/sec (0.24 MPa/sec)

It is assumed that, in practice, the load rate will be such that failure will occur in one hour, resulting in a lower effective compressive strength than would be obtained in a standard test. The stress rate, $\dot{f}$, corresponding to compressive failure in one hour is

$$\dot{f} = \frac{f'_{c\dot{f}}}{3600} \tag{32}$$

The values of $\dot{f}$ and $f'_{c\dot{f}}$ are obtained by iteration using Eqs. 31 and 32.

The selection of the value of f'_{c35}, which should be representative of concrete strength in the field, is affected by two considerations: 1) Splice tests are calibrated against the compressive strength of standard cylinders that are cured in the same manner as the splice test specimens, not on the actual strength of the concrete in the splice specimens. The closest thing in concrete construc-

tion is the use of field-cured specimens. 2) In practice, concrete must be proportioned to produce a higher strength than used to design the structure to insure that the strength of most of the concrete will exceed the specified value of f'_c.

The two considerations have opposite effects on the value of f'_{c35} used in the analysis, since field-cured cylinders usually produce a lower strength than standard laboratory-cured specimens (the basis upon which f'_c is measured), while the average strength of concrete produced in the field, as measured using standard specimens, exceeds f'_c by a considerable amount. These opposing effects largely cancel each other out. Therefore, the specified value of f'_c is used as the mean value of the concrete strength for use in determining R_p.

$$f'_{c35} = f'_c \tag{33}$$

f'_c in Eq. 30 is replaced by the normally distributed random variable $X(4)$ with a mean value $\overline{X(4)} = f'_{cf}$ [Eqs. 31 and 32]. For $f'_c = 4000$ psi (28 MPa), $\overline{X(4)} = f'_{cf} = 3559$ psi (24.54 MPa). The standard deviation $\sigma_{X(4)} = V_c f'_{cf}$ is based on 1) an assumed standard deviation for standard laboratory cylinders, $\sigma_{ccyl} = 550$ psi (3.8 MPa), representative of good job-site quality control, and 2) an assumed variability for in-place concrete, expressed as $V_c = (V_{ccyl}^2 + 0.0084)^{1/2}$ (Mirza et al. 1979), in which $V_{ccyl} = \sigma_{ccyl}/f'_{cr}$ and f'_{cr} = required average compressive strength of concrete = $f'_c + 2.33\ \sigma_{ccyl} - 500$ psi (Eq. 5-2 of ACI 318-89). For $f'_c = 4000$ psi (28 MPa), $V_c = 0.147$ and $\sigma_{X(4)} = 523$ psi (3.6 MPa).

Geometric Properties. The balance of the random variables used to calculate R_p are the geometric properties of the structural member and the reinforcement. The tolerances in ACI 117-90 are used as the basis for establishing the variability of the geometric properties of concrete sections. All geometric properties are represented using normal distributions.

The splice length, l_d, is represented by the random variable $X(5)$, with a mean equal to the

specified value of l_d. The tolerance for the embedded length of bars and the length of bar laps in ACI 117-90 is − 1 in. (25 mm) for No. 3 through No. 11 (9.5 through 36 mm) bars. It is assumed that 95 percent of all bars will meet this criterion. For the normal distribution X(5), this means that 1.645 $\sigma_{X(5)}$ = 1 in. (25 mm), or $\sigma_{X(5)}$ = 0.61 in. (16 mm). [The values of $\sigma_{X(i)}$ are shown rounded to two significant figures. No rounding, however, is used in the calculation of ϕ.]

Concrete cover, c_b, is represented by random variable X(6), with a mean equal to the specified cover. The tolerance on cover in ACI 117-90 is − 3/8 in. (9.5 mm) for members less than or equal to 12 in. (305 mm) in size and − 1/2 in. (13 mm) for members greater than 12 in. (305 mm) in size. Again, assuming that 95 percent of all members will meet these criteria, 1.645 $\sigma_{X(6)}$ = 0.375 in. (9.5 mm), or $\sigma_{X(6)}$ = 0.23 in. (6 mm) for members ≤ 12 in. (305 mm) in size, and 1.645 $\sigma_{X(6)}$ = 0.5 in. (13 mm), or $\sigma_{X(6)}$ = 0.30 in. (8 mm) for members > 12 in. (305 mm) in size.

Side cover, c_{so}, is represented by random variable X(7), with a mean equal to the specified value of c_{so}. In this case, the tolerances on placement of reinforcement in ACI 117-90 are ± 3/8 in. (9.5 mm) for members between 4 and 12 in. (102 and 305 mm) in size and ± 1/2 in. (13 mm) for members greater than 12 in. (305 mm) in size. Since c_{so} is bounded on two sides, if 95 percent of all bar placements meet these criteria, the tolerances are equal to 1.96 $\sigma_{X(7)}$. Using procedures similar to those used for c_b and l_d, $\sigma_{X(7)}$ = 0.19 in. for members between 4 and 12 in. (102 and 305 mm) in size and $\sigma_{X(7)}$ = 0.26 in. (7 mm) for members > 12 in. (305 mm) in size.

One-half of the clear spacing between bars, c_{si}, is calculated as

$$c_{si} = \frac{b - 2n_b d_b - 2c_{so}}{2(n_b - 1)} \tag{34}$$

in which n_b = number of bars. In this expression, in addition to c_{so}, beam width, b, is a random variable, represented by X(8).

The tolerances on cross-sectional dimensions in ACI 117-90 are + 3/8 in. and − 1/4 in.

(+9.5 mm and –6.5 mm) for members with dimensions of 12 in. (305 mm) or less and + $\frac{1}{2}$ in. and – $\frac{3}{8}$ in. (+ 13 mm and – 9.5 mm) for members with dimensions greater than 12 in. (305 mm), but less than 3 ft (914 mm). ACI 117-90 also provides criteria for members over 3 ft (914 mm) in dimension, but these are not used in the current Monte Carlo analysis. The mean value of beam width, $\overline{X(8)}$, is taken as the nominal beam width plus the average of the tolerances = b + 0.0625 in. (1.6 mm) for members in both size categories. The standard deviations are selected such that 95 percent of all members have dimensions between the tolerances, giving $\sigma_{X(8)} = 0.16$ in. (4 mm) for members with b ≤ 12 in. (305 mm) and $\sigma_{X(8)} = 0.22$ in. (6 mm) for members 12 < b ≤ 36 in. (305 < b ≤ 914 mm).

The term representing the effect of relative rib area on the effectiveness of transverse reinforcement on bond strength, $t_r = 9.6\,R_r + 0.28$, depends on the random variable representing $R_r = X(9)$. $\overline{R}_r = \overline{X(9)} = 0.0727$ for conventional reinforcement and 0.1275 for high relative rib area reinforcement. Conservatively, the standard deviations are $\sigma_{X(9)} = 0.0090$ for conventional reinforcement and 0.0045 for high relative rib area reinforcement (Darwin et al. 1995b).

In Eqs. 4 and 5, the number of stirrups crossing the splice, N, (Eqs. 1 and 30) has been replaced by l_d/s. N, of course, must have an integer value, although l_d/s is the value used in Eqs. 2 and 3 to calculate development and splice length. As an example, if $l_d/s = 3.6$, the development/ splice length would be crossed by four stirrups 60 percent of the time and three stirrups 40 percent of the time, for an average of 3.6 stirrups. Thus, the average strength can be based on 3.6 stirrups. However, using 3.6 stirrups does not account for the variability in strength that occurs because some splices are crossed by 3 stirrups, while others are crossed by 4. This variability is accounted for in the Monte Carlo simulation by applying the appropriate weights to the calculated strengths for the two integer values for the number of stirrups. This results in a lower ϕ-factor than if N = l_d/s were used to calculate R_p.

Nominal Strength, R_n. The nominal strength, R_n, is calculated using Eq. 4 or Eq. 5 with

the specified concrete strength, f'_c, and the nominal dimensions of the member.

Monte Carlo Simulation. The values of $\bar{r}$ and V_r (Eq. 24) are obtained using Monte Carlo simulations of a selected number of beams. For each beam and simulation, values are selected for normally distributed random variables $X(1)$ and $X(4) - X(9)$. To do this for each variable, a random number between 0 and 1 is used with the cumulative distribution function to calculate the standard normal random variable, z ($-\infty < z < \infty$). For variable i, $X(i) = \overline{X(i)} + z\sigma_{X(i)}$. The values of $X(i)$ are used to calculate r (Eq. 21a) for the simulation. The results of multiple simulations are combined to obtain $\bar{r}$ and V_r.

Loading Random Variable–The term q, given in Eq. 19, depends on random variables $X(2)$ and $X(3)$, representing the actual-to-nominal ratios for dead and live load, respectively; load factors for dead and live load, γ_D and γ_L; and the nominal live load-to-dead load ratio, $(Q_L/Q_D)_n$. γ_D and γ_L are selected based on the load factors used in design, 1.4 and 1.7 for ACI 318-89, ACI 318-95, and AASHTO Highway (1992), and 1.2 and 1.6 for ASCE 7-93. Values of $(Q_L/Q_D)_n$ of 0.5, 1.0, and 1.5 are normally selected for evaluating the reliability of reinforced concrete structures, with a nominal live load-to-dead load ratio of 1.0 serving as the standard for calculating ϕ-factors or determining the reliability index, β.

For reinforced concrete structures, $\overline{X(2)} = \overline{Q}_D/Q_{Dn} = 1.03$ and $V_{Q_D} = 0.093$ (Ellingwood et al. 1980). $X(3) = Q_L/Q_{Ln}$ depends on the tributary area, A_T, and the influence area, A_I (Ellingwood et al. 1980). For $A_T = 400$ ft^2 (37 m^2) and $A_I = 800$ ft^2 (74 m^2) (representative values for a reinforced concrete flexural member),

$$\overline{Q}_L = L_o \left(0.25 + \frac{15}{\sqrt{A_I}} \right) = 0.780 \, L_o \tag{35}$$

$$Q_{Ln} = L_o \left\{ 1 - \min \left[0.0008 \, (A_T - 150), \, 0.6, \, 0.23 \left(1 + \left(\frac{Q_D}{Q_L} \right)_n \right) \right] \right\} = 0.8 \, L_o \tag{36}$$

in which L_o = basic (unreduced) live load and areas are in ft².

Thus, $\overline{X(3)} = \overline{Q}_L/Q_{Ln} = 0.975.$ $V_{Q_L} = 0.25$ (Ellingwood et al. 1980).

STRENGTH REDUCTION FACTORS

Strength reduction (ϕ) factors are calculated for Eqs. 4 and 5 using 1) nominal live load-to-dead load ratios of 0.5, 1.0, and 1.5; 2) two combinations of dead and live load factors — a) 1.4 and 1.7 (with ϕ for bending = 0.9), and b) 1.2 and 1.6 (with ϕ for bending = 0.8); 3) bars with relative rib areas of 0.0727 and 0.1275; and 4) members with and without confining transverse reinforcement.

The evaluations are based on splice lengths obtained from the respective equations calculated with a provisional value of $\phi_d = 0.9$. [Note: The calculated ϕ-factors are independent of the provisional value of ϕ_d.] Thirty-five beams in which the bars are not confined by transverse reinforcement and 140 beams (in four groups of 35 each) in which the bars are confined by transverse reinforcement are used in the calculations. The beams have widths of 8, 12, 18, or 24 in. (203, 305, 457, and 610 mm) and depths of 12 or 24 in. (305 and 610 mm). Concrete strengths of 3000, 4000, and 6000 psi (21, 28, and 41 MPa) are evaluated, and 2, 4, 6, or 8 bars are spliced at the same location. No. 6, No. 8, No. 10, and No. 11 (19, 25, 32, and 36 mm) bars are used. For bars with confining transverse reinforcement, No. 3 and No. 4 (9.5 and 12.5 mm) bar stirrups are spaced at values ranging from 4 to 10.8 in. (102 to 275 mm). A summary of the beams used for the analysis is presented in Appendix A.

For each of the 35 beams without transverse reinforcement, 1000 Monte Carlo simulations are carried out in which the predicted strengths are calculated using Eq. 30 and the material and geometric random variables described in this report. For each of the 140 beams with transverse reinforcement, 250 simulations are carried out. The programs used for the Monte Carlo simula-

tions are presented in Appendix B. The individual predicted strengths are used to calculate $\overline{r}$ (Eq. 23) and V_r (Eq. 24). The selected load factors and live load-to-dead load ratios are used to calculate $\overline{q}$ (Eq. 25) and $V_{\phi q}$ (Eq. 26). The results are combined with $\beta = 3.5$ to calculate $\phi_c = \phi_b$ (Eq. 29). The value of $\phi_d = \phi_b/\phi$ is then obtained.

The results of the Monte Carlo simulations are presented in Table 1.

Load factors 1.4 and 1.7–For Eq. 4, which is based on Eq. 2 (the more accurate of the two design equations), ϕ_d equals 0.94, 0.91, and 0.88 for bars without confining transverse reinforcement at live load-to-dead load ratios of 0.5, 1.0, and 1.5, respectively; and 0.93, 0.90, and 0.88 ($R_r = 0.0727$) and 0.92, 0.89, and 0.87 ($R_r = 0.1275$) for bars with confining transverse reinforcement at the same live load-to-dead load ratios.

For Eq. 5, which is based on Eq. 3 (the more simplified of the two expressions), ϕ_d equals 0.89, 0.87, and 0.85 for bars without confining transverse reinforcement at live load-to-dead load ratios of 0.5, 1.0, and 1.5, respectively; and 0.99, 0.97, and 0.95 ($R_r = 0.0727$) and 0.97, 0.95, and 0.93 ($R_r = 0.1275$) for bars with confining transverse reinforcement at the same live load-to-dead load ratios.

Load factors 1.2 and 1.6–For load factors of 1.2 and 1.6, the values of ϕ_d increase slightly compared to those obtained for load factors of 1.4 and 1.7. Using Eq. 4 and a live load-to-dead load ratio of 1.0, ϕ_d equals 0.93 for bars without transverse reinforcement and 0.92 ($R_r = 0.0727$) and 0.91 ($R_r = 0.1275$) for bars with transverse reinforcement. Using Eq. 5, the respective values are 0.89, 0.99, and 0.97.

$\phi_d = 0.9$ appears to be generally conservative and satisfactory for application with Eqs. 4 and 5 for both sets of load factors. The lower values of ϕ_d for bars without confining reinforcement obtained for Eq. 5 compared to Eq. 4 pose no safety problems, since l_d obtained with Eq. 5 is never shorter than l_d obtained with Eq. 4. The lower values of ϕ_d calculated for Eq. 5 are due to the greater scatter (higher V_r) obtained with Eq. 5, as shown in Table 1.

Table 1 demonstrates that an increase in the live load-to-dead load ratio results in a reduc-

tion in the ϕ-factor. This reduction is due to the increased variability, represented by $V_{\phi q}$, that results from the greater uncertainty in the live load.

Design Expressions–For ease in application, ϕ_d can be incorporated directly into the design expressions so that its value becomes transparent to the user. Multiplying the right side of Eqs. 4 and 5 by $\phi_d = 0.9$, setting $f_s = f_y$, and solving for l_d/d_b gives, respectively,

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f'_c{}^{1/4}} - 1900 \left(0.1 \dfrac{c_M}{c_m} + 0.9 \right)}{72 \left(\dfrac{c + K_{tr}}{d_b} \right)} \tag{37}$$

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f'_c{}^{1/4}} - 1900}{72 \left(\dfrac{c + K_{tr}}{d_b} \right)} \tag{38}$$

The development and splice lengths obtained with Eqs. 37 and 38 are compared with those obtained using the provisions of ACI 318-89 and ACI 318-95 for both conventional and high relative rib area bars by Darwin et al. (1995b).

The analysis described here provides an important advantage over current design procedures (ACI 318-89, ACI 318-95, AASHTO Highway 1992) in that Eqs. 37 and 38 apply directly to both development and splice lengths, since the equation calibration and ϕ-factor calculations are based on data that consists predominantly of splice tests in which all bars are spliced at the same location; over 90 percent of the specimens used to establish Eq. 1 contain Class B (ACI)/Class C (AASHTO) splices (Darwin et al. 1995b). Thus, following the procedures described here and by Darwin et al. (1995b) eliminates the need to multiply l_d by 1.3 (ACI) or 1.7 (AASHTO) to obtain the length of most splices.

SUMMARY AND CONCLUSIONS

The formulation and calculation of a reliability-based strength-reduction (ϕ) factor for developed and spliced bars is described. Conventional and high relative rib area bars, both with and without confining reinforcement, are considered. The ϕ-factor is determined using statistically-based expressions for development/splice strength and Monte Carlo simulations of a range of beams.

A strength-reduction factor of 0.9 is obtained for the design expressions for development/splice length, based on a probability of failure in bond equal to about one-fifth of the probability of failure in bending or combined bending and compression. $\phi = 0.9$ is incorporated into two expressions for development/splice length in a manner that is transparent to the user. A major advantage of each of the final expressions is that they provide identical values for development and splice length, removing the need to multiply development length by 1.3 or 1.7 to obtain the length of most splices.

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Table 1

Strength Reduction (ϕ) Factors for Bond
Eq. 4

$\gamma_D = 1.4$ $\gamma_L = 1.7$ ($\phi_{bending} = 0.9$)

	Without Stirrups			With Stirrups					
Average R_r	N/A			0.0727			0.1275		
$\bar{r}$	0.955			0.989			0.980		
V_r	0.106			0.125			0.125		
$(Q_D/Q_L)_n$	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
$\bar{q}$	0.675	0.647	0.631	0.675	0.647	0.631	0.675	0.647	0.631
$V_{\phi q}$	0.102	0.131	0.152	0.102	0.131	0.152	0.102	0.131	0.152
ϕ_b	0.846	0.819	0.792	0.833	0.812	0.788	0.826	0.805	0.780
ϕ_d	0.940	0.910	0.880	0.926	0.902	0.875	0.917	0.894	0.867

$\gamma_D = 1.2$ $\gamma_L = 1.6$ ($\phi_{bending} = 0.8$)

	Without Stirrups			With Stirrups					
Average R_r	N/A			0.0727			0.1275		
$\bar{r}$	0.955			0.989			0.980		
V_r	0.106			0.125			0.125		
$(Q_D/Q_L)_n$	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
$\bar{q}$	0.759	0.716	0.693	0.759	0.716	0.693	0.759	0.716	0.693
$V_{\phi q}$	0.102	0.131	0.152	0.102	0.131	0.152	0.102	0.131	0.152
ϕ_b	0.752	0.740	0.722	0.741	0.733	0.718	0.734	0.727	0.711
ϕ_d	0.940	0.925	0.902	0.926	0.917	0.897	0.917	0.908	0.889

Table 1

Strength Reduction (ϕ) Factors for Bond (continued)
Eq. 5

$$\gamma_D = 1.4 \qquad \gamma_L = 1.7 \qquad (\phi_{bending} = 0.9)$$

	Without Stirrups			With Stirrups					
Average R_r	N/A			0.0727			0.1275		
$\bar{r}$	1.046			1.136			1.101		
V_r	0.159			0.150			0.146		
$(Q_D/Q_L)_n$	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
$\bar{q}$	0.675	0.647	0.631	0.675	0.647	0.631	0.675	0.647	0.631
$V_{\phi q}$	0.102	0.131	0.152	0.102	0.131	0.152	0.102	0.131	0.152
ϕ_b	0.800	0.787	0.768	0.892	0.875	0.853	0.875	0.857	0.835
ϕ_d	0.889	0.874	0.853	0.991	0.973	0.948	0.972	0.953	0.928

$$\gamma_D = 1.2 \qquad \gamma_L = 1.6 \qquad (\phi_{bending} = 0.8)$$

	Without Stirrups			With Stirrups					
Average R_r	N/A			0.0727			0.1275		
$\bar{r}$	1.046			1.136			1.101		
V_r	0.159			0.150			0.146		
$(Q_D/Q_L)_n$	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
$\bar{q}$	0.759	0.716	0.693	0.759	0.716	0.693	0.759	0.716	0.693
$V_{\phi q}$	0.102	0.131	0.152	0.102	0.131	0.152	0.102	0.131	0.152
ϕ_b	0.711	0.711	0.700	0.793	0.791	0.778	0.778	0.774	0.761
ϕ_d	0.889	0.888	0.875	0.991	0.988	0.972	0.972	0.968	0.951
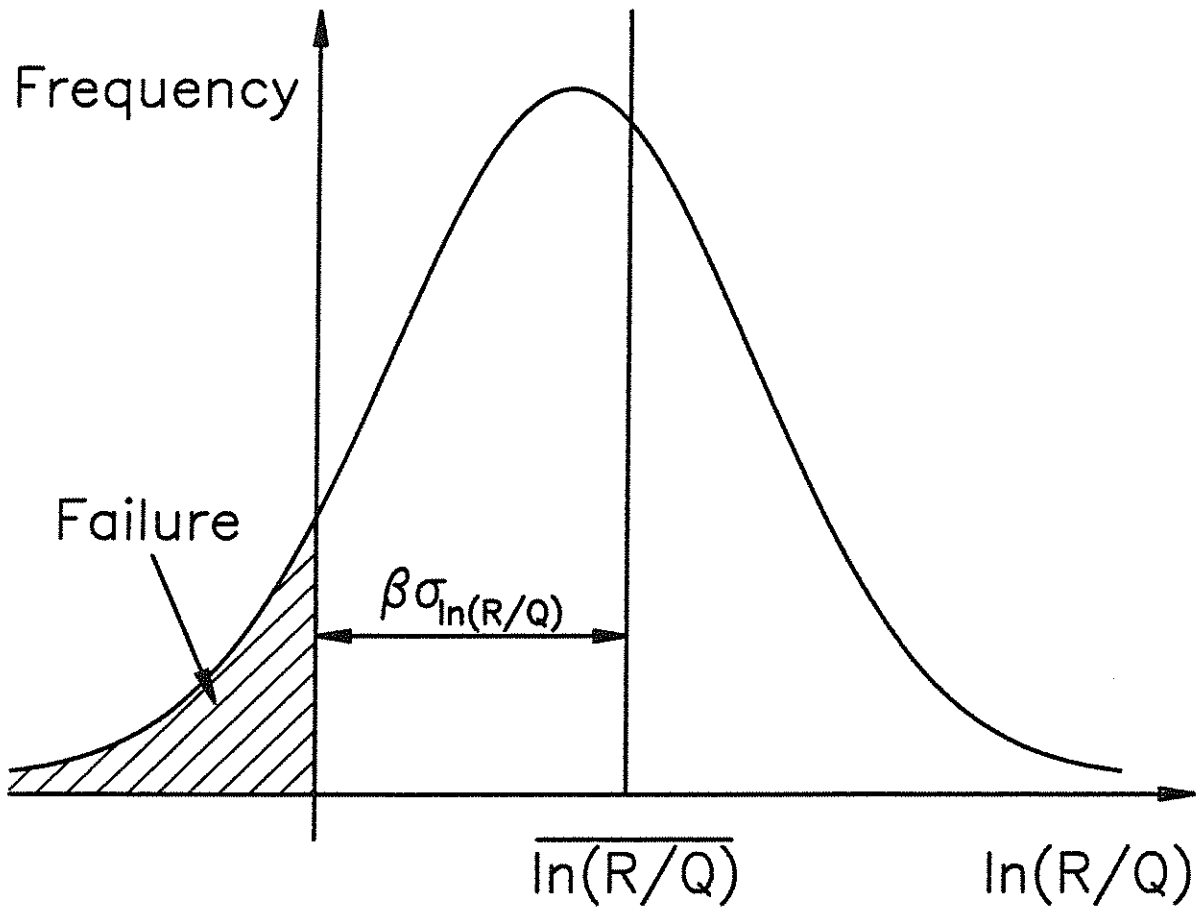
Fig. 1 Illustration of reliability index (after Ellingwood et al. 1980). β = number of standard deviations, $\sigma_{\ln(R/Q)}$, between $\ln(R/Q) = \overline{\ln(R/Q)}$ and $\ln(R/Q) = 0$.

Appendix A

Table A.1

Data for hypothetical beams used for the Monte Carlo Analyses

(a)
(without confining reinforcement)

Beam No.	n	l_d* (in.)	d_b (in.)	b (in.)	h (in.)	c_{so} (in.)	c_{si} (in.)	c_b (in.)	f'_c (psi)
1	2	31.38	0.75	8.00	12.00	2.00	0.50	2.00	4000
2	2	18.39	0.75	12.00	12.00	2.00	2.50	2.00	4000
3	2	31.36	1.00	12.00	12.00	2.00	2.00	2.00	4000
4	2	52.55	1.27	12.00	12.00	2.00	1.46	2.00	4000
5	2	68.79	1.41	12.00	12.00	2.00	1.18	2.00	4000
6	2	18.39	0.75	24.00	12.00	2.00	8.50	2.00	4000
7	4	18.39	0.75	24.00	12.00	2.00	2.33	2.00	4000
8	6	23.76	0.75	24.00	12.00	2.00	1.10	2.00	4000
9	8	30.36	0.75	24.00	12.00	2.00	0.57	2.00	4000
10	2	31.36	1.00	24.00	12.00	2.00	8.00	2.00	4000
11	4	31.36	1.00	24.00	12.00	2.00	2.00	2.00	4000
12	6	44.96	1.00	24.00	12.00	2.00	0.80	2.00	4000
13	2	47.84	1.27	24.00	12.00	2.00	7.46	2.00	4000
14	4	49.53	1.27	24.00	12.00	2.00	1.64	2.00	4000
15	2	57.24	1.41	24.00	12.00	2.00	7.18	2.00	4000
16	4	62.82	1.41	24.00	12.00	2.00	1.45	2.00	4000
17	2	20.23	0.75	12.00	24.00	2.00	2.50	2.00	3000
18	2	18.39	0.75	12.00	24.00	2.00	2.50	2.00	4000
19	2	16.01	0.75	12.00	24.00	2.00	2.50	2.00	6000
20	2	34.50	1.00	12.00	24.00	2.00	2.00	2.00	3000
21	2	31.36	1.00	12.00	24.00	2.00	2.00	2.00	4000
22	2	27.31	1.00	12.00	24.00	2.00	2.00	2.00	6000
23	2	57.84	1.27	12.00	24.00	2.00	1.46	2.00	3000
24	2	52.55	1.27	12.00	24.00	2.00	1.46	2.00	4000
25	2	45.72	1.27	12.00	24.00	2.00	1.46	2.00	6000
26	2	75.77	1.41	12.00	24.00	2.00	1.18	2.00	3000
27	2	68.79	1.41	12.00	24.00	2.00	1.18	2.00	4000
28	2	59.78	1.41	12.00	24.00	2.00	1.18	2.00	6000
29	4	21.53	0.75	18.00	24.00	2.00	1.33	2.00	4000
30	6	31.38	0.75	18.00	24.00	2.00	0.50	2.00	4000
31	2	31.36	1.00	18.00	24.00	2.00	5.00	2.00	4000
32	4	41.41	1.00	18.00	24.00	2.00	1.00	2.00	4000
33	2	47.84	1.27	18.00	24.00	2.00	4.46	2.00	4000
34	4	70.37	1.27	18.00	24.00	2.00	0.64	2.00	4000
35	2	57.24	1.41	18.00	24.00	2.00	4.18	2.00	4000

Table A.1

Data for hypothetical beams used for the Monte Carlo Analyses (continued)

(b)
(with confining reinforcement)

Beam No.	n	l_d^* $R_r=0.0727$ (in.)	l_d^* $R_r=0.1275$ (in.)	d_b (in.)	b (in.)	h (in.)	c_{so} (in.)	c_{si} (in.)	c_b (in.)	f_c (psi)	A_{tr} (in.^{2})	s (in.)
Group 1												
1	2	21.01	17.84	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.110	4.81
2	2	14.45	12.95	0.75	12.00	12.00	2.00	2.50	2.00	4000	0.110	4.81
3	2	23.76	21.02	1.00	12.00	12.00	2.00	2.00	2.00	4000	0.110	4.75
4	2	37.36	32.33	1.27	12.00	12.00	2.00	1.46	2.00	4000	0.110	4.68
5	2	46.57	39.68	1.41	12.00	12.00	2.00	1.18	2.00	4000	0.110	4.65
6	2	14.45	12.95	0.75	24.00	12.00	2.00	8.50	2.00	4000	0.110	4.81
7	4	16.18	15.20	0.75	24.00	12.00	2.00	2.33	2.00	4000	0.055	4.81
8	6	21.22	20.07	0.75	24.00	12.00	2.00	1.10	2.00	4000	0.037	4.81
9	8	27.15	25.69	0.75	24.00	12.00	2.00	0.57	2.00	4000	0.028	4.81
10	2	23.76	21.02	1.00	24.00	12.00	2.00	8.00	2.00	4000	0.110	4.75
11	4	27.04	25.17	1.00	24.00	12.00	2.00	2.00	2.00	4000	0.055	4.75
12	6	38.83	36.18	1.00	24.00	12.00	2.00	0.80	2.00	4000	0.037	4.75
13	2	34.97	30.55	1.27	24.00	12.00	2.00	7.46	2.00	4000	0.110	4.68
14	4	41.59	38.29	1.27	24.00	12.00	2.00	1.64	2.00	4000	0.055	4.68
15	2	41.13	35.72	1.41	24.00	12.00	2.00	7.18	2.00	4000	0.110	4.65
16	4	51.65	47.14	1.41	24.00	12.00	2.00	1.45	2.00	4000	0.055	4.65
17	2	18.04	17.04	0.75	12.00	24.00	2.00	2.50	2.00	3000	0.110	10.81
18	2	16.40	15.50	0.75	12.00	24.00	2.00	2.50	2.00	4000	0.110	10.81
19	2	14.28	13.49	0.75	12.00	24.00	2.00	2.50	2.00	6000	0.110	10.81
20	2	30.22	28.34	1.00	12.00	24.00	2.00	2.00	2.00	3000	0.110	10.75
21	2	27.48	25.76	1.00	12.00	24.00	2.00	2.00	2.00	4000	0.110	10.75
22	2	23.93	22.43	1.00	12.00	24.00	2.00	2.00	2.00	6000	0.110	10.75
23	2	49.09	45.39	1.27	12.00	24.00	2.00	1.46	2.00	3000	0.110	10.68
24	2	44.60	41.25	1.27	12.00	24.00	2.00	1.46	2.00	4000	0.110	10.68
25	2	38.81	35.89	1.27	12.00	24.00	2.00	1.46	2.00	6000	0.110	10.68
26	2	62.71	57.39	1.41	12.00	24.00	2.00	1.18	2.00	3000	0.110	10.65
27	2	56.94	52.11	1.41	12.00	24.00	2.00	1.18	2.00	4000	0.110	10.65
28	2	49.48	45.28	1.41	12.00	24.00	2.00	1.18	2.00	6000	0.110	10.65
29	4	19.05	17.94	0.75	18.00	24.00	2.00	1.33	2.00	4000	0.100	10.81
30	6	27.70	26.05	0.75	18.00	24.00	2.00	0.50	2.00	4000	0.067	10.81
31	2	24.95	22.48	1.00	18.00	24.00	2.00	5.00	2.00	4000	0.200	10.75
32	4	35.29	32.69	1.00	18.00	24.00	2.00	1.00	2.00	4000	0.100	10.75
33	2	36.99	32.97	1.27	18.00	24.00	2.00	4.46	2.00	4000	0.200	10.68
34	4	57.43	52.26	1.27	18.00	24.00	2.00	0.64	2.00	4000	0.100	10.68
35	2	43.67	38.73	1.41	18.00	24.00	2.00	4.18	2.00	4000	0.200	10.65

Table A.1

Data for hypothetical beams used for the Monte Carlo Analyses (continued)

(b)
(with confining reinforcement)

Beam No.	n	l_d R_r=0.0727 (in.)	l_d R_r=0.1275 (in.)	d_b (in.)	b (in.)	h (in.)	c_{so} (in.)	c_{si} (in.)	c_b (in.)	f'_c (psi)	A_{tr} (in.^{2})	s (in.)
Group 2												
1	2	17.17	14.23	0.75	8.50	12.00	2.00	0.75	2.00	4000	0.200	6.00
2	2	17.17	14.23	0.75	8.50	12.00	2.00	0.75	2.00	4000	0.200	6.00
3	2	25.54	21.18	1.00	10.00	12.00	2.00	1.00	2.00	4000	0.200	6.00
4	2	34.65	28.74	1.27	11.62	12.00	2.00	1.27	2.00	4000	0.200	6.00
5	2	39.37	32.66	1.41	12.46	12.00	2.00	1.41	2.00	4000	0.200	6.00
6	2	17.17	14.23	0.75	8.50	12.00	2.00	0.75	2.00	4000	0.200	6.00
7	4	21.25	18.84	0.75	14.50	12.00	2.00	0.75	2.00	4000	0.100	6.00
8	6	23.08	21.13	0.75	20.50	12.00	2.00	0.75	2.00	4000	0.067	6.00
9	8	24.12	22.49	0.75	26.50	12.00	2.00	0.75	2.00	4000	0.050	6.00
10	2	25.54	21.18	1.00	10.00	12.00	2.00	1.00	2.00	4000	0.200	6.00
11	4	31.60	28.02	1.00	18.00	12.00	2.00	1.00	2.00	4000	0.100	6.00
12	6	34.31	31.41	1.00	26.00	12.00	2.00	1.00	2.00	4000	0.067	6.00
13	2	34.65	28.74	1.27	11.62	12.00	2.00	1.27	2.00	4000	0.200	6.00
14	4	42.84	38.01	1.27	21.78	12.00	2.00	1.27	2.00	4000	0.100	6.00
15	2	39.37	32.66	1.41	12.46	12.00	2.00	1.41	2.00	4000	0.200	6.00
16	4	48.67	43.18	1.41	23.74	12.00	2.00	1.41	2.00	4000	0.100	6.00
17	2	18.95	15.70	0.75	8.50	24.00	2.00	0.75	2.00	3000	0.200	6.00
18	2	17.17	14.23	0.75	8.50	24.00	2.00	0.75	2.00	4000	0.200	6.00
19	2	14.87	12.32	0.75	8.50	24.00	2.00	0.75	2.00	6000	0.200	6.00
20	2	28.15	23.34	1.00	10.00	24.00	2.00	1.00	2.00	3000	0.200	6.00
21	2	25.54	21.18	1.00	10.00	24.00	2.00	1.00	2.00	4000	0.200	6.00
22	2	22.17	18.39	1.00	10.00	24.00	2.00	1.00	2.00	6000	0.200	6.00
23	2	38.15	31.64	1.27	11.62	24.00	2.00	1.27	2.00	3000	0.200	6.00
24	2	34.65	28.74	1.27	11.62	24.00	2.00	1.27	2.00	4000	0.200	6.00
25	2	30.12	24.98	1.27	11.62	24.00	2.00	1.27	2.00	6000	0.200	6.00
26	2	43.33	35.95	1.41	12.46	24.00	2.00	1.41	2.00	3000	0.200	6.00
27	2	39.37	32.66	1.41	12.46	24.00	2.00	1.41	2.00	4000	0.200	6.00
28	2	34.25	28.41	1.41	12.46	24.00	2.00	1.41	2.00	6000	0.200	6.00
29	4	21.25	18.84	0.75	14.50	24.00	2.00	0.75	2.00	4000	0.100	6.00
30	6	23.08	21.13	0.75	20.50	24.00	2.00	0.75	2.00	4000	0.067	6.00
31	2	25.54	21.18	1.00	10.00	24.00	2.00	1.00	2.00	4000	0.200	6.00
32	4	31.60	28.02	1.00	18.00	24.00	2.00	1.00	2.00	4000	0.100	6.00
33	2	34.65	28.74	1.27	11.62	24.00	2.00	1.27	2.00	4000	0.200	6.00
34	4	42.84	38.01	1.27	21.78	24.00	2.00	1.27	2.00	4000	0.100	6.00
35	2	39.37	32.66	1.41	12.46	24.00	2.00	1.41	2.00	4000	0.200	6.00

Table A.1

Data for hypothetical beams used for the Monte Carlo Analyses (continued)

(b)
(with confining reinforcement)

Beam No.	n	l_d^* R_r=0.0727 (in.)	l_d^* R_r=0.1275 (in.)	d_b (in.)	b (in.)	h (in.)	c_{so} (in.)	c_{si} (in.)	c_b (in.)	f'_c (psi)	A_{tr} (in.²)	s (in.)
Group 3												
1	2	20.38	17.15	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	8.00
2	2	20.38	17.15	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	8.00
3	2	31.85	26.54	1.00	9.00	12.00	2.00	0.50	2.00	4000	0.200	8.00
4	2	43.98	36.58	1.27	10.35	12.00	2.00	0.64	2.00	4000	0.200	8.00
5	2	50.35	41.84	1.41	11.05	12.00	2.00	0.71	2.00	4000	0.200	8.00
6	2	20.38	17.15	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	8.00
7	4	24.71	22.18	0.75	13.00	12.00	2.00	0.50	2.00	4000	0.100	8.00
8	6	26.60	24.58	0.75	18.00	12.00	2.00	0.50	2.00	4000	0.067	8.00
9	8	27.65	25.99	0.75	23.00	12.00	2.00	0.50	2.00	4000	0.050	8.00
10	2	31.85	26.54	1.00	9.00	12.00	2.00	0.50	2.00	4000	0.200	8.00
11	4	39.13	34.84	1.00	15.00	12.00	2.00	0.50	2.00	4000	0.100	8.00
12	6	42.35	38.90	1.00	21.00	12.00	2.00	0.50	2.00	4000	0.067	8.00
13	2	43.98	36.58	1.27	10.35	12.00	2.00	0.64	2.00	4000	0.200	8.00
14	4	54.16	48.16	1.27	17.97	12.00	2.00	0.64	2.00	4000	0.100	8.00
15	2	50.35	41.84	1.41	11.05	12.00	2.00	0.71	2.00	4000	0.200	8.00
16	4	62.08	55.17	1.41	19.51	12.00	2.00	0.71	2.00	4000	0.100	8.00
17	2	22.54	18.97	0.75	8.00	24.00	2.00	0.50	2.00	3000	0.200	8.00
18	2	20.38	17.15	0.75	8.00	24.00	2.00	0.50	2.00	4000	0.200	8.00
19	2	17.59	14.80	0.75	8.00	24.00	2.00	0.50	2.00	6000	0.200	8.00
20	2	35.22	29.35	1.00	9.00	24.00	2.00	0.50	2.00	3000	0.200	8.00
21	2	31.85	26.54	1.00	9.00	24.00	2.00	0.50	2.00	4000	0.200	8.00
22	2	27.48	22.90	1.00	9.00	24.00	2.00	0.50	2.00	6000	0.200	8.00
23	2	48.57	40.40	1.27	10.35	24.00	2.00	0.64	2.00	3000	0.200	8.00
24	2	43.98	36.58	1.27	10.35	24.00	2.00	0.64	2.00	4000	0.200	8.00
25	2	38.04	31.64	1.27	10.35	24.00	2.00	0.64	2.00	6000	0.200	8.00
26	2	55.58	46.19	1.41	11.05	24.00	2.00	0.71	2.00	3000	0.200	8.00
27	2	50.35	41.84	1.41	11.05	24.00	2.00	0.71	2.00	4000	0.200	8.00
28	2	43.59	36.23	1.41	11.05	24.00	2.00	0.71	2.00	6000	0.200	8.00
29	4	24.71	22.18	0.75	13.00	24.00	2.00	0.50	2.00	4000	0.100	8.00
30	6	26.60	24.58	0.75	18.00	24.00	2.00	0.50	2.00	4000	0.067	8.00
31	2	31.85	26.54	1.00	9.00	24.00	2.00	0.50	2.00	4000	0.200	8.00
32	4	39.13	34.84	1.00	15.00	24.00	2.00	0.50	2.00	4000	0.100	8.00
33	2	43.98	36.58	1.27	10.35	24.00	2.00	0.64	2.00	4000	0.200	8.00
34	4	54.16	48.16	1.27	17.97	24.00	2.00	0.64	2.00	4000	0.100	8.00
35	2	50.35	41.84	1.41	11.05	24.00	2.00	0.71	2.00	4000	0.200	8.00

Table A.1

Data for hypothetical beams used for the Monte Carlo Analyses (continued)

(b)
(with confining reinforcement)

Beam No.	n	l_d^* $R_r=0.0727$ (in.)	$R_r=0.1275$ (in.)	d_b (in.)	b (in.)	h (in.)	c_{so} (in.)	c_{si} (in.)	c_b (in.)	f'_c (psi)	A_{tr} (in.²)	s (in.)
Group 4												
1	2	15.09	11.80	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	4.00
2	2	15.09	11.80	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	4.00
3	2	23.21	17.97	1.00	9.00	12.00	2.00	0.50	2.00	4000	0.200	4.00
4	2	31.96	24.70	1.27	10.35	12.00	2.00	0.64	2.00	4000	0.200	4.00
5	2	36.54	28.22	1.41	11.05	12.00	2.00	0.71	2.00	4000	0.200	4.00
6	2	15.09	11.80	0.75	8.00	12.00	2.00	0.50	2.00	4000	0.200	4.00
7	4	20.38	17.15	0.75	13.00	12.00	2.00	0.50	2.00	4000	0.100	4.00
8	6	23.08	20.20	0.75	18.00	12.00	2.00	0.50	2.00	4000	0.067	4.00
9	8	24.71	22.18	0.75	23.00	12.00	2.00	0.50	2.00	4000	0.050	4.00
10	2	23.21	17.97	1.00	9.00	12.00	2.00	0.50	2.00	4000	0.200	4.00
11	4	31.85	26.54	1.00	15.00	12.00	2.00	0.50	2.00	4000	0.100	4.00
12	6	36.36	31.55	1.00	21.00	12.00	2.00	0.50	2.00	4000	0.067	4.00
13	2	31.96	24.70	1.27	10.35	12.00	2.00	0.64	2.00	4000	0.200	4.00
14	4	43.98	36.58	1.27	17.97	12.00	2.00	0.64	2.00	4000	0.100	4.00
15	2	36.54	28.22	1.41	11.05	12.00	2.00	0.71	2.00	4000	0.200	4.00
16	4	50.35	41.84	1.41	19.51	12.00	2.00	0.71	2.00	4000	0.100	4.00
17	2	16.69	13.05	0.75	8.00	24.00	2.00	0.50	2.00	3000	0.200	4.00
18	2	15.09	11.80	0.75	8.00	24.00	2.00	0.50	2.00	4000	0.200	4.00
19	2	13.03	10.18	0.75	8.00	24.00	2.00	0.50	2.00	6000	0.200	4.00
20	2	25.67	19.88	1.00	9.00	24.00	2.00	0.50	2.00	3000	0.200	4.00
21	2	23.21	17.97	1.00	9.00	24.00	2.00	0.50	2.00	4000	0.200	4.00
22	2	20.03	15.51	1.00	9.00	24.00	2.00	0.50	2.00	6000	0.200	4.00
23	2	35.30	27.28	1.27	10.35	24.00	2.00	0.64	2.00	3000	0.200	4.00
24	2	31.96	24.70	1.27	10.35	24.00	2.00	0.64	2.00	4000	0.200	4.00
25	2	27.64	21.36	1.27	10.35	24.00	2.00	0.64	2.00	6000	0.200	4.00
26	2	40.34	31.14	1.41	11.05	24.00	2.00	0.71	2.00	3000	0.200	4.00
27	2	36.54	28.22	1.41	11.05	24.00	2.00	0.71	2.00	4000	0.200	4.00
28	2	31.64	24.43	1.41	11.05	24.00	2.00	0.71	2.00	6000	0.200	4.00
29	4	20.38	17.15	0.75	13.00	24.00	2.00	0.50	2.00	4000	0.100	4.00
30	6	23.08	20.20	0.75	18.00	24.00	2.00	0.50	2.00	4000	0.067	4.00
31	2	23.21	17.97	1.00	9.00	24.00	2.00	0.50	2.00	4000	0.200	4.00
32	4	31.85	26.54	1.00	15.00	24.00	2.00	0.50	2.00	4000	0.100	4.00
33	2	31.96	24.70	1.27	10.35	24.00	2.00	0.64	2.00	4000	0.200	4.00
34	4	43.98	36.58	1.27	17.97	24.00	2.00	0.64	2.00	4000	0.100	4.00
35	2	36.54	28.22	1.41	11.05	24.00	2.00	0.71	2.00	4000	0.200	4.00

*   Predicted development/splice lengths based on Eq. 2, using $\phi_d = 0.9$ and $f_y = 60$ ksi
1 in. = 25.4 mm; 1 psi = 6.89 kPa; 1 ksi = 6.89 MPa

Appendix B

Programs Used For Monte Carlo Simulations

(a)

```
Program Bms;
{MONTE CARLO SIMULATION FOR BEAMS WITHOUT STIRRUPS}

Uses Crt, Dos;

{DECLARATION OF GLOBAL CONSTANTS FOR THE PROGRAM}

Const
 left = 0.0;  tol = 1.0e-6;

{DECLARATION OF GLOBAL VARIABLES FOR THE PROGRAM}

Var
 a, b, z, pz, Ls, Cb, Csi, fc, Db, Ab, Cmin, Cmax, fcm, Cbm, Cim, Lsm,
 E1m, E2m, R1, R2, h, tc1, tc2, Rc1, Rc2, a0, a1, a2, a3, a4, sd1, s1,
 sd2, r21, r22, Vc, mr1, mr2, ms1, ms2, mv1, mv2, s2, s3,
 result, errest, Cso, W, Cs, s4, e1, e2, CMm, fcR, Cmind : Real;
 Nb, Ns, err : Byte;
 n, l : Integer;
 k1, i, j, k, m, m0, hr, mn, sec, sec100, hr1, mn1, sec1, sec1001 : Word;
 fname : String[20];
 st : String;
 fin, fout, fl : Text;

{FUNCTION EVALUATES THE EXPRESSION e^-(0.5*z^2)}

Function F(z : Real) : Real;
Begin
 F:= exp(-z*z/2.0);
End;

{FUNCTION EVALUATES THE EXPRESSION FOR SIMPSON'S RULE}

Function Simpson(a,b,h : Real) : Real;
Var mid : Real;
begin
 mid := (b+a)/2.0;
 Simpson := h/6.0*(F(a)+4.0*F(mid)+F(b));
end;

{NUMERICAL METHOD TO DETERMINE THE CURRENT IMPROVED STANDARD}
{NORMAL VALUE, z[i+1], FROM A PREVIOUS VALUE OF z[i] DURING EACH CYCLE}
{OF THE ITERATIVE PROCESS}

Procedure Adap_Quad(left,z,tol:Real; var result:Real; var errest:Real);
Var h, I1, I2, mid, result1, result2, errest1, errest2 : Real;
Begin
 m := m+1;
 if m0 > 100 then
  begin
    window(20,23,50,24);
    writeln(pz:6:4,m0:9,m:9);
  end;
 h := z-left;
 I1 := Simpson(left,z,h);
 h := h/2.0;
 mid := (left+z)/2.0;
```

```
    I2 := Simpson(left,mid,h)+Simpson(mid,z,h);
    errest := abs((I2-I1)/15.0);
    if abs(errest) > tol then
      begin
        Adap_Quad(left,mid,tol/2,result1,errest1);
        Adap_Quad(mid,z,tol/2,result2,errest2);
        result := result1+result2;
        errest := errest1+errest2;
      end
    else
      result := I2-errest;
End;
```

{ITERATIVE PROCESS TO OBTAIN THE STANDARD NORMAL VALUE, z, FOR ANY}
{RANDOMLY GENERATED CUMULATIVE PROBABILITY, pz, USING THE}
{PROCEDURE Adap_Quad AS OFTEN AS IT IS REQUIRED}

```
Procedure Getz;
Begin
  pz := Random;
  a := 0.0;  z := 2.0;  b := 4.0;  m0 := 0;
  Repeat
    m0 := m0+1;  m := 0;
    Adap_quad(left,z,tol,result,errest);
    result := 0.5+result/sqrt(2.0*pi);
    if pz < 0.50 then
      begin
        result := 1.0-result;
        if result < pz then b := z else a := z;
      end
    else
      begin
        if result < pz then a := z else b := z;
      end;
    z := 0.5*(a+b);
    if m0 > 500 then Exit;
  Until abs(pz-result) < tol;
End;
```

{FUNCTION DETERMINES THE MINIMUM OF TWO VARIABLES}

```
Function Min(a,b :Real) : Real;
begin
  if a < b then Min := a
  else Min := b;
end;
```

{FUNCTION DETERMINES THE MAXIMUM OF TWO VARIABLES}

```
Function Max(a,b :Real) : Real;
begin
  if a > b then Max := a
  else Max := b;
end;
```

{FUNCTION DETERMINES PREDICTED BOND FORCE}

```
Function Eqn1 : Real;
Var Eq : Real;
Begin
  Eq := (63.0*Ls*(Cmin+0.5*Db)+2130.0*Ab)*(0.10*CMm+0.90);
  Eqn1 := Sqrt(Sqrt(fc))*Eq;
End;
```

{FUNCTION DETERMINES NOMINAL BOND FORCE}

```
Function Eqn2 : Real;
Var Eq : Real;
Begin
  Eq := (63.0*Ls*(Cmind+0.5*Db)+2130.0*Ab);
  Eqn2 := Sqrt(Sqrt(fc))*Eq;
End;
```

{FUNCTION DETERMINES THE APPROPRIATE VALUES FOR SOME PARAMETERS}
{AND FUNCTIONS USED IN THE DETERMINATION OF BOND FORCE}

```
Function Eqn(n : Byte) : Real;
Begin
  Csi := (0.5*W-Nb*Db-Cso)/(Nb-1.0);
  Cs := Min(Cso,Csi+0.25);
  Cmin := Min(Cs,Cb);
  Cmind :=min(min(Cso,Csi),Cb);
  Cmax := Max(Cs,Cb);
  CMm := Cmax/Cmin;
  if CMm > 3.5 then CMm := 3.5;
  if n = 1 then Eqn := Eqn1
  else Eqn := Eqn2;
End;
```

{PROCEDURE FOR READING, FROM THE DATA FILE, AND DISPLAYING, ON THE}
{SCREEN, THE NOMINAL VALUES FOR EACH BEAM}

```
Procedure InputData;
Begin
  readln(fin,W,h,fc,Cb,Cso,Ls,Nb,Db,Ab);
  n := 1+n;
  writeln('          CURRENT INPUT DATA FROM FILE ',fname+'.DAT');
  writeln('          ----------------------------------------');
  writeln('          Data for beam number ............ ',n:5);
  writeln('          Beam width (ins.) ............... ',W:5:2);
  writeln('          Beam depth (ins.) ............... ',h:5:2);
  writeln('          Concrete strength (psi) ......... ',fc:4:0);
  writeln('          Concrete cover (ins.) ........... ',Cb:5:3);
  writeln('          Concrete side cover (ins.) ...... ',Cso:5:3);
  writeln('          Splice length (ins.) ............ ',Ls:5:2);
  writeln('          Number of bars spliced .......... ',Nb:2);
  writeln('          Spliced bar diameter (ins.) ..... ',Db:5:3);
  writeln('          Spliced bar area (sq. ins.) ..... ',Ab:4:2);
End;
```

{PROCEDURE FOR WRITING THE NOTATION AND HEADING INFORMATION FOR}
{EACH BEAM INTO THE OUTPUT FILE FOR THE BEAM}

```
Procedure OutData;
Begin
  writeln(fout,'     RESULTS OUTPUT FOR BEAMS W/O STIRRUPS');
  writeln(fout,'     --------------------------------------');
  writeln(fout,'    n   = Number of iterations');
  writeln(fout,'    W   = Beam width (ins.)');
  writeln(fout,'    fc  = Concrete strength (psi)');
  writeln(fout,'    Cb  = Concrete cover (ins.)');
  writeln(fout,'    Cso = Concrete side cover (ins.)');
  writeln(fout,'    Csi = One-half clear bar spacing (ins.)');
  writeln(fout,'    Ls  = Splice length (ins.)');
  writeln(fout,'    Nb  = Number of bars spliced');
  writeln(fout,'    Db  = Spliced bar diameter (ins.)');
  writeln(fout,'    Ab  = Spliced bar area (sq. ins.)');
  writeln(fout,' ');
  write(fout,' n   W    fc   Cb   Cso  Csi  Ls  Nb  Db   Ab ');
  writeln(fout,' Eq1  Eq2   R1   R2   MR1   MR2   SD1   SD2   V1   V2');
End;
```

```
  write(fout,'------------------------------------------------------------');
  writeln(fout,'---------------------------------------------------------------');
End;
```

{ITERATIVE PROCESS FOR DETERMINING THE LONG-TERM IN-SITU}
{COMPRESSIVE STRENGTH OF CONCRETE}

```
Procedure fcstrR;
Var fc35 : Real;
Begin
  fc35 := fc;
  Repeat
    fcR := fc;
    fc := fc35*(0.89*(1.0+0.08*ln(fcR/3600)/ln(10.0)));
  until Abs(fcR-fc) < 1.0;
End;
```

{MAIN PROCEDURE FOR THE MONTE CARLO SIMULATIONS WHERE THE DATA}
{FOR EACH BEAM IS READ FROM THE DATA FILE; RUNS ALL THE PROCEDURES}
{REQUIRED FOR THE SIMULATIONS; COMPUTES THE MEANS, STANDARD}
{DEVIATIONS, AND COV FOR EACH BEAM; COMPUTES THE CUMULATIVE}
{MEANS, STANDARD DEVIATIONS, AND COV; AND WRITES THE RESULTS}
{INTO FILES}

```
Procedure Simulate;
Begin
```

{WRITES THE NOTATION AND HEADING INFORMATION FOR}
{MONTE CARLO SIMULATION OUTPUT RESULT FILE}

```
  writeln(fl,'    RESULTS OUTPUT FOR BEAMS W/O STIRRUPS');
  writeln(fl,'    --------------------------------------');
  writeln(fl,'    W   = Beam width (ins.)');
  writeln(fl,'    fc  = Concrete strength (psi)');
  writeln(fl,'    Cb  = Concrete cover (ins.)');
  writeln(fl,'    Cso = Concrete side cover (ins.)');
  writeln(fl,'    Csi = One-half clear bar spacing (ins.)');
  writeln(fl,'    Ls  = Splice length (ins.)');
  writeln(fl,'    Nb  = Number of bars spliced');
  writeln(fl,'    Db  = Spliced bar diameter (ins.)');
  writeln(fl,'    Ab  = Spliced bar area (sq. ins.)');
  writeln(fl,' ');
  write(fl,'Beam  W    fc   Cb   Cso   Csi   Ls   Nb  Db   Ab  ');
  write(fl,' E1    E2    R1    R2    S1    S2    V1  ');
  writeln(fl,' V2    MR1   MR2   MS1   MS2   MV1   MV2');
  write(fl,'------------------------------------------------------------');
  write(fl,'-------------------------------------------------');
  writeln(fl,'--------------------------------------------');
```

{READS THE FIRST LINE (HEADING) FROM THE INPUT FILE}

```
  readln(fin,st);
```

{INITIALIZES VARIABLES}

```
  mr1 := 0.0;  mr2 := 0.0;  ms1 := 0.0;  ms2 := 0.0;  mv1 := 0.0;  mv2 := 0.0;
  s1 := 0.0;  s2 := 0.0;  k1 := 0;  s3 := 0.0;  s4 := 0.0;
```

{ITERATION FOR READING AND PROCESSING THE DATA FOR EACH BEAM}

```
  While not Eof(fin) do
    begin
      Window(1,1,80,25);
      ClrScr;  InputData;  Str(n,st);
      Assign(fout,fname+'.'+st);  {$I-}Rewrite(fout);{$I+}
```

```
Chkfile('FILE FOR OUTPUT','DISK/DRIVE');
if err = 1 then Exit;

{INITIALIZES AND EVALUATES VARIABLES}

R1 := Eqn(1);  R2 := Eqn(2);
OutData;
write(fout,W:10:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2,Nb:3);
writeln(fout,Db:7:3,Ab:6:2,R1:7:0,R2:7:0);
a0 := Ls;  a1 := Cb;  a2 := Cso;  a3 := W;  a4 := fc;
Vc := 550.0/(fc+2.33*550.0-500.0);  Vc := sqrt(Vc*Vc+0.0084);
Rc1 := 0.0;  Rc2 := 0.0;  sd1 := 0.0;  sd2 := 0.0;  r21 := 0.0;
r22 := 0.0;  fcm := 0.0;  Cbm := 0.0;  Cim := 0.0;  Lsm := 0.0;
E1m := 0.0;  E2m := 0.0;

{ITERATION FOR PERFORMING THE MONTE CARLO SIMULATIONS k TIMES}
{FOR EACH BEAM}

for j := 1 to k do
  begin
    Randomize;
    Window(20,21,40,22);
    writeln('WORKING ON CYCLE ',j);
    Ls := a0;  Cb := a1;  Cso := a2;  W := a3;  fc := a4;
    fcstrR;

    {ITERATION FOR RANDOMLY GENERATING THE VARIABILITY ASSOCIATED}
    {WITH EACH OF THE VARIABLES FOR CALCULATING THE PREDICTED}
    {BOND FORCE}

    for i := 1 to 6 do
      begin
        Getz;
        if pz < 0.50 then z := -z;
        Case i of
          1 : Ls := Ls+0.6079*z;
          2 : if h > 12.0 then Cb := Cb+0.3040*z
              else Cb := Cb+0.2280*z;
          3 : if W > 12.0 then Cso := Cso+0.2551*z
              else Cso := Cso+0.1913*z;
          4 : if W > 12.0 then W := W+0.0625+0.2232*z
              else W := W+0.0625+0.1594*z;
          5 : fc := fcR*(1.0+Vc*z);
          6 : tc1 := 1.0000*(1.0+0.0809*z);
        end;
      end;

    {COMPUTES THE MEAN, STANDARD DEVIATION, AND COV FOR}
    {EACH BEAM AND CUMULATIVE MEAN, STANDARD DEVIATION}
    {AND COV INCLUDING PRECEDING BEAMS}

    e1 := tc1*Eqn(1);  e2 := tc2*Eqn(1);
    Rc1 := Rc1+e1/R1;  Rc2 := Rc2+e2/R2;  k1 := k1+1;
    r21 := r21+e1*e1/R1/R1;  r22 := r22+e2*e2/R2/R2;
    s3 := s3+e1/R1;  s4 := s4+e2/R2;
    s1 := s1+e1*e1/R1/R1;  s2 := s2+e2*e2/R2/R2;
    if j > 1 then
      begin
        sd1 := sqrt((r21-Rc1*Rc1/j)/j);
        sd2 := sqrt((r22-Rc2*Rc2/j)/j);
      end;
    if k1 > 1 then
      begin
        ms1 := sqrt((s1-s3*s3/k1)/k1);
        ms2 := sqrt((s2-s4*s4/k1)/k1);
```

```
       end;
   fcm := fcm+fc;  Cbm := Cbm+Cb;  Cim := Cim+Csi;  Lsm := Lsm+Ls;
   Elm := Elm+e1;  E2m := E2m+e2;

   {SECTION DISPLAYS THE CURRENT RESULTS ON THE SCREEN}

   Window(10,16,70,21);
   writeln(' CURRENT R1 = ',e1/R1:6:3,'  CURRENT R2 = ',e2/R2:6:3);
   writeln('  MEAN R1  = ',Rc1/j:6:3,'    MEAN R2  = ',Rc2/j:6:3);
   writeln(' STD DEV 1  = ',sd1:6:3,'  STD DEV 2  = ',sd2:6:3);
   writeln('  C.O.V. 1 = ',sd1/Rc1*j:6:3,'    C.O.V. 2 = ',sd2/Rc2*j:6:3);

    {WRITES THE CURRENT RESULTS INTO THE RESULT FILE FOR}
    {EACH BEAM}

   write(fout,j:3,W:7:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2,Nb:3,Db:7:3,Ab:6:2);
   write(fout,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3,Rc1/j:7:3,Rc2/j:7:3,sd1:7:3);
   writeln(fout,sd2:7:3,sd1/Rc1*j:7:3,sd2/Rc2*j:7:3);
  end;
 writeln(fout,' ');
 Close(fout);

  {COMPUTES THE AVERAGE VALUES FOR ALL VARIABLES FOR}
  {EACH OF THE BEAMS}

  fc := fcm/j;  Cb := Cbm/j;  Csi := Cim/j;  Ls := Lsm/j;  e1 := Elm/j;
  e2 := E2m/j;  mr1 := mr1+e1/R1;  mr2 := mr2+e2/R2;  mv1 := ms1/mr1*n;
  mv2 := ms2/mr2*n;

  {WRITES THE CURRENT BEAM RESULTS INTO THE RESULT FILE}
  {THAT CONTAINS THE SUMMARY OF ALL THE RESULTS FOR ALL BEAMS}

  write(fl,n:3,W:7:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2,Nb:3,Db:7:3);
  write(fl,Ab:6:2,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3);
  write(fl,sd1:7:3,sd2:7:3,sd1/Rc1*j:7:3,sd2/Rc2*j:7:3,mr1/n:7:3);
  writeln(fl,mr2/n:7:3,ms1:7:3,ms2:7:3,mv1:7:3,mv2:7:3);
 end;
End;

{CHECKS TO SEE IF A SPECIFIED FILE EXISTS OR WAS OPENED SUCCESSFULLY}

Procedure Chkfile(s1,s2 : String);
Begin
 if Ioresult <> 0 then
  begin
   Window(10,10,70,15);
   ClrScr;
   writeln('     CANNOT OPEN ',s1);
   writeln;
   writeln(' PRESS ANY KEY TO END AND CHECK ',s2);
   err := 1;
   st := Readkey;
   Exit;
  end;
End;

{START OF THE MAIN PROGRAM WHERE ALL INPUT IS MADE}

BEGIN
 GetTime(hr,mn,sec,sec100);
 Window(1,1,80,25);
 ClrScr;
 err := 0;
 n := 0;
 write('   ENTER NAME OF THE DATA FILE W/O EXTENSION : ');
```

```
            readln(fname);
            Assign(fin,fname+'.DAT');
            {$I-}Reset(fin);{$I+}
            Chkfile('DATA FILE ','DATA FILE');
            if err = 1 then Exit;
            Assign(fl,fname+'.RST');
            {$I-}Rewrite(fl);{$I+}
            Chkfile('FILE FOR OUTPUT','DISK/DRIVE');
            if err = 1 then Exit;
            writeln;
            write('   ENTER THE NUMBER OF CYCLES REQUIRED : ');  readln(k);
            Simulate;
            GetTime(hr1,mn1,sec1,sec1001);
            writeln(fl,'  ');
            writeln(fl,'STOPPING TIME : ',hr1,':',mn1,':',sec1,'.',sec1001);
            writeln(fl,'STARTING TIME : ',hr,':',mn,':',sec,'.',sec100);
            Close(fin);
            Close(fl);
          END.
```

(b)

```pascal
Program Bst;
{MONTE CARLO SIMULATION FOR BEAMS WITH STIRRUPS}

Uses Crt, Dos;

{DECLARATION OF GLOBAL CONSTANTS FOR THE PROGRAM}

Const
  left = 0.0;  tol = 1.0e-6;

{DECLARATION OF GLOBAL VARIABLES FOR THE PROGRAM}

Var
  a, b, z, pz, Ls, Cb, Csi, fc, Db, Ab, Cmin, Cmax, fcm, Cbm, Cim, Lsm,
  E1m, E2m, R1, R2, h, tc1, tc2, Rc1, Rc2, a0, a1, a2, a3, a4, sd1, s1,
  sd2, r21, r22, Vc, Av, Sv, Nv, mr1, mr2, ms1, ms2, mv1, mv2, s2, s3,
  result, errest, Cso, W, Cs, s4, Atr, Atm, e1, e2, e11, e12, e21, e22,
  CMm, fcR, Rr, a5, Vr, Rrm, Rm, Cmind, deno1, deno2, trtd: Real;
  Nb, Ns, err : Byte;
  n, l : Integer;
  k1, i, j, k, m, m0, hr, mn, sec, sec100, hr1, mn1, sec1, sec1001 : Word;
  fname : String[20];
  st : String;
  fin, fout, fl : Text;

{FUNCTION EVALUATES THE EXPRESSION e^-(0.5*z^2)}

Function F(z : Real) : Real;
Begin
  F := exp(-z*z/2.0);
End;

{FUNCTION EVALUATES THE EXPRESSION FOR SIMPSON'S RULE}

Function Simpson(a,b,h : Real) : Real;
Var mid : Real;
begin
  mid := (b+a)/2.0;
  Simpson := h/6.0*(F(a)+4.0*F(mid)+F(b));
end;

{NUMERICAL METHOD TO DETERMINE THE CURRENT IMPROVED STANDARD}
{NORMAL VALUE, z[i+1], FROM A PREVIOUS VALUE OF z[i] DURING EACH CYCLE}
{OF THE ITERATIVE PROCESS}

Procedure Adap_Quad(left,z,tol:Real; var result:Real; var errest:Real);
Var h, I1, I2, mid, result1, result2, errest1, errest2 : Real;
Begin
  m := m+1;
  h := z-left;
  I1 := Simpson(left,z,h);
  h := h/2.0;
  mid := (left+z)/2.0;
  I2 := Simpson(left,mid,h)+Smpson(mid,z,h);
  errest := abs((I2-I1)/15.0);
  if abs(errest) > tol then
    begin
      Adap_Quad(left,mid,tol/2,result1,errest1);
      Adap_Quad(mid,z,tol/2,result2,errest2);
      result := result1+result2;
      errest := errest1+errest2;
    end
```

```
   else
     result := I2-errest;
End;


{ITERATIVE PROCESS TO OBTAIN THE STANDARD NORMAL VALUE, z, FOR ANY}
{RANDOMLY GENERATED CUMULATIVE PROBABILITY, pz, USING THE}
{PROCEDURE Adap_Quad AS OFTEN AS IT IS REQUIRED}

Procedure Getz;
Begin
  pz := Random;
  a := 0.0;  z := 2.0;  b := 4.0;  m0 := 0;
  Repeat
    m0 := m0+1;  m := 0;
    Adap_Quad(left,z,tol,result,errest);
    result := 0.5+result/sqrt(2.0*pi);
    if pz < 0.50 then
      begin
        result := 1.0-result;
        if result < pz then b := z else a := z;
      end
    else
      begin
        if result < pz then a := z else b := z;
      end;
    z := 0.5*(a+b);
    if m0 > 500 then Exit;
  Until abs(pz-result) < tol;
End;

{FUNCTION DETERMINES THE MINIMUM OF TWO VARIABLES}

Function Min(a,b :Real) : Real;
begin
  if a < b then Min := a
  else Min := b;
end;

{FUNCTION DETERMINES THE MAXIMUM OF TWO VARIABLES}

Function Max(a,b :Real) : Real;
begin
  if a > b then Max := a
  else Max := b;
end;

{FUNCTION DETERMINES NOMINAL BOND FORCE WITH THE Cmax/Cmin TERM}

Function Eqn1 : Real;
Var Eq : Real;
Begin
  Eq := 80.2*Ls*Ab/Db*deno1+2130.0*Ab*(0.10*CMm+0.90);
  Eqn1 := Sqrt(Sqrt(fc))*Eq
End;

{FUNCTION DETERMINES NOMINAL BOND FORCE WITHOUT THE}
{Cmax/Cmin TERM}

Function Eqn2 : Real;
Var Eq : Real;
Begin
  Eq := 80.2*Ls*Ab/Db*deno2+2130.0*Ab;
  Eqn2 := Sqrt(Sqrt(fc))*Eq
End;
```

```pascal
{FUNCTION DETERMINES PREDICTED BOND FORCE}

Function EqnP : Real;
Var Eq : Real;
Begin
  Eq := 63.0*Ls*Db*deno1+2130.0*Ab*(0.1*CMm+0.9)+66.0;
  EqnP := Sqrt(Sqrt(fc))*Eq
End;


{FUNCTION DETERMINES THE APPROPRIATE VALUES FOR SOME PARAMETERS}
{AND FUNCTIONS USED IN THE DETERMINATION OF BOND FORCE}

Function Eqn(n : Byte) : Real;
Begin
  Csi := (0.5*W-Nb*Db-Cso)/(Nb-1.0);
  Cs := Min(Cso,Csi+0.25);
  if Cs < Cb then Atr := 2.0*Av/Nb
  else Atr := Av;
  Cmin := Min(Cs,Cb);
  Cmind :=Min(Min(Csi,Cso),Cb);
  Cmax := Max(Cs,Cb);
  CMm := Cmax/Cmin;
  if CMm > 3.5 then CMm := 3.5;
  trtd := (9.6*Rr+0.28)*(0.72*Db+0.28);
  deno1 := ((Cmin+0.5*Db)*(0.1*CMm+0.9)+35.3*trtd*Atr/(Ls/Nv))/Db;
  deno2 := ((Cmind+0.5*Db)+35.3*trtd*Atr/(Ls/Nv))/Db;
  if deno1 > 4.0 then deno1 := 4.0;
  if deno2 > 4.0 then deno2 := 4.0;
  case n of
    0 : Eqn := EqnP;
    1 : Eqn := Eqn1;
    2 : Eqn := Eqn2;
  end;
End;


{PROCEDURE FOR READING, FROM THE DATA FILE, AND DISPLAYING, ON THE}
{SCREEN, THE NOMINAL VALUES FOR EACH BEAM}

Procedure InputData;
Begin
  readln(fin,W,h,fc,Cb,Cso,Ls,Nb,Db,Ab,Av,Sv);
  n := 1+n;
  writeln('        CURRENT INPUT DATA FROM FILE ',fname+'.DAT');
  writeln('        ----------------------------------------');
  writeln('        Data for beam number ............ ',n:5);
  writeln('        Beam width (ins.) ................ ',W:5:2);
  writeln('        Beam depth (ins.) ............... ',h:5:2);
  writeln('        Concrete strength (psi) ......... ',fc:4:0);
  writeln('        Concrete cover (ins.) ........... ',Cb:5:3);
  writeln('        Concrete side cover (ins.) ...... ',Cso:5:3);
  writeln('        Splice length (ins.) ............ ',Ls:5:2);
  writeln('        Number of bars spliced .......... ',Nb:2);
  writeln('        Spliced bar diameter (ins.) ..... ',Db:5:3);
  writeln('        Spliced bar area (sq. ins.) ..... ',Ab:4:2);
  writeln('        Stirrup area (sq. ins.) ......... ',Av:4:2);
  writeln('        Stirrup spacing (ins.) .......... ',Sv:5:2);
  writeln('        Relative Rib Area, Rr ........... ',a5:5:3);
  writeln('        Coeff. of Variation for Rr ...... ',Vr:6:4);
End;


{PROCEDURE FOR WRITING THE NOTATION AND HEADING INFORMATION FOR}
{EACH BEAM INTO THE OUTPUT FILE FOR THE BEAM}

Procedure OutData;
Begin
```

```
writeln(fout,'    RESULTS OUTPUT FOR BEAMS WITH STIRRUPS');
writeln(fout,'    ----------------------------------------');
writeln(fout,'    n  = Number of iterations');
writeln(fout,'    W  = Beam width (ins.)');
writeln(fout,'    fc = Concrete strength (psi)');
writeln(fout,'    Cb = Concrete cover (ins.)');
writeln(fout,'    Cso = Concrete side cover (ins.)');
writeln(fout,'    Csi = One-half clear bar spacing (ins.)');
writeln(fout,'    Ls = Splice length (ins.)');
writeln(fout,'    Number of bars spliced ................',Nb:7);
writeln(fout,'    Spliced bar diameter (ins.) ...........',Db:7:3);
writeln(fout,'    Spliced bar area (sq. ins.) ...........',Ab:7:2);
writeln(fout,'    Stirrup effective area (sq. ins.) ....',Atr:7:2);
writeln(fout,'    Stirrup spacing (ins.) ...............',Sv:7:2);
writeln(fout,'    Relative Rib Area of Bar, Rr .........',a5:7:3);
writeln(fout,'    Coeff. of Variation for Rr ...........',Vr:7:4);
writeln(fout,' ');
write(fout,' n   W   fc   Cb   Cso   Csi   Ls   ');
writeln(fout,'Rr  Eq1  Eq2  R1  R2  MR1  MR2  SD1  SD2  V1   V2');
write(fout,'----------------------------------------------------');
writeln(fout,'----------------------------------------------------------------');
End;
```

{ITERATIVE PROCESS FOR DETERMINING THE LONG-TERM IN-SITU}
{COMPRESSIVE STRENGTH OF CONCRETE}

```
Procedure fcstrR;
Var fc35 : Real;
Begin
  fc35 := fc;
  Repeat
    fcR := fc;
    fc := fc35*(0.89*(1.0+0.08*ln(fcR/3600)/ln(10.0)));
  until Abs(fcR-fc) < 1.0;
End;
```

{MAIN PROCEDURE FOR THE MONTE CARLO SIMULATIONS WHERE THE DATA}
{FOR EACH BEAM IS READ FROM THE DATA FILE; RUNS ALL THE PROCEDURES}
{REQUIRED FOR THE SIMULATIONS; COMPUTES THE MEANS, STANDARD}
{DEVIATIONS, AND COV FOR EACH BEAM; COMPUTES THE CUMULATIVE}
{MEANS, STANDARD DEVIATIONS, AND COV; AND WRITES THE RESULTS}
{INTO FILES}

```
Procedure Simulate;
Begin
```

{WRITES THE NOTATION AND HEADING INFORMATION FOR}
{MONTE CARLO SIMULATION OUTPUT RESULT FILE}

```
writeln(fl,'    RESULTS OUTPUT FOR BEAMS WITH STIRRUPS');
writeln(fl,'    ----------------------------------------');
writeln(fl,'    W  = Beam width (ins.)');
writeln(fl,'    fc = Concrete strength (psi)');
writeln(fl,'    Cb = Concrete cover (ins.)');
writeln(fl,'    Cso = Concrete side cover (ins.)');
writeln(fl,'    Csi = One-half clear bar spacing (ins.)');
writeln(fl,'    Ls = Splice length (ins.)');
writeln(fl,'    Nb = Number of bars spliced');
writeln(fl,'    Db = Spliced bar diameter (ins.)');
writeln(fl,'    Ab = Spliced bar area (sq. ins.)');
writeln(fl,'    Atr = Stirrup effective area (sq. ins.)');
writeln(fl,'    Sv = Stirrup spacing (ins.)');
writeln(fl,'    Relative Rib Area, Rr =',a5:7:4,' ; COV for Rr =',Vr:7:4);
writeln(fl,' ');
write(fl,'Beam  W   fc   Cb   Cso   Csi   Ls   Nb   Db   Ab ');
```

```
write(fl,' Atr  Sv  Rr   E1   E2   R1   R2   S1   S2   V1  ');
writeln(fl,' V2   MR1  MR2  MS1  MS2  MV1  MV2');
write(fl,'----------------------------------------------------------');
write(fl,'----------------------------------------------------------');
writeln(fl,'-----------------------------------------------------');
```

{READS THE FIRST LINE, HEADING, FROM THE INPUT FILE}

```
readln(fin,st);
```

{INITIALIZES VARIABLES}

```
mr1 := 0.0;  mr2 := 0.0;  ms1 := 0.0;  ms2 := 0.0;  mv1 := 0.0;  mv2 := 0.0;
s1 := 0.0;  s2 := 0.0;  k1 := 0;  s3 := 0.0;  s4 := 0.0;
```

{ITERATION FOR READING AND PROCESSING THE DATA FOR EACH BEAM}

```
While not Eof(fin) do
 begin
   Window(1,1,80,25);
   ClrScr;  InputData;  Str(n,st);
   Assign(fout,fname+'.'+st);  {$I-}Rewrite(fout);{$I+}
   Chkfile('FILE FOR OUTPUT','DISK/DRIVE');
   if err = 1 then Exit;
```

{INITIALIZES AND EVALUATES VARIABLES}

```
   Rr := a5;  Nv := Ls/Sv;  R1 := Eqn(1);  R2 := Eqn(2);
   OutData;
   write(fout,W:11:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2);
   writeln(fout,Rr:7:4,R1:7:0,R2:7:0);
   a0 := Ls;  a1 := Cb;  a2 := Cso;  a3 := W;  a4 := fc;
   Vc := 550.0/(fc+2.33*550.0-500.0);  Vc := sqrt(Vc*Vc+0.0084);
   fcstrR;
   Rc1 := 0.0;  Rc2 := 0.0;  sd1 := 0.0;  sd2 := 0.0;  r21 := 0.0;
   r22 := 0.0;  fcm := 0.0;  Cbm := 0.0;  Cim := 0.0;  Lsm := 0.0;
   E1m := 0.0;  E2m := 0.0;  Atm := 0.0;  Rrm := 0.0;
```

{ITERATION FOR PERFORMING THE MONTE CARLO SIMULATIONS k TIMES}
{FOR EACH BEAM}

```
   for j := 1 to k do
     begin
       Randomize;
       Window(20,23,40,24);
       writeln('WORKING ON CYCLE ',j);
       Ls := a0;  Cb := a1;  Cso := a2;  W := a3;  fc := a4;  Rr := a5+Rm;
```

{ITERATION FOR RANDOMLY GENERATING THE VARIABILITY ASSOCIATED}
{WITH EACH OF THE VARIABLES FOR CALCULATING THE PREDICTED}
{BOND FORCE}

```
       for i := 1 to 7 do
         begin
           Getz;
           if pz < 0.50 then z := -z;
           Case i of
             1 : Ls := Ls+0.6079*z;
             2 : if h > 12.0 then Cb := Cb+0.3040*z
                 else Cb := Cb+0.2280*z;
             3 : if W > 12.0 then Cso := Cso+0.2551*z
                 else Cso := Cso+0.1913*z;
             4 : if W > 12.0 then W := W+0.0625+0.2232*z
                 else W := W+0.0625+0.1594*z;
             5 : fc := fcR*(1.0+Vc*z);
```

```
    6 : tc1 := 1.0082*(1.0+0.1022*z);
    7 : Rr := Rr*(1.0+Vr*z);
   end;
  end;
```

{EVALUATES THE PREDICTED BOND FORCE FOR THE TWO}
{INTEGER VALUES FOR THE NUMBER OF STIRRUPS}

```
for l := 0 to 1 do
  begin
    Nv := 1.0*(Trunc(Ls/Sv)+l);
    if l = 0 then
      begin
        e11 := tc1*Eqn(0)*(1.0-Frac(Ls/Sv));
        e12 := tc2*Eqn(0)*(1.0-Frac(Ls/Sv));
        e1 := tc1*Eqn(0);  e2 := tc2*Eqn(0);
        write(fout,j:3,'a',W:7:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3);
        writeln(fout,Ls:7:2,Rr:7:3,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3);
      end
    else
      begin
        e21 := tc1*Eqn(0)*Frac(Ls/Sv);
        e22 := tc2*Eqn(0)*Frac(Ls/Sv);
        e1 := tc1*Eqn(0);  e2 := tc2*Eqn(0);
        write(fout,j:3,'b',W:7:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3);
        writeln(fout,Ls:7:2,Rr:7:3,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3);
        e1 := e11+e21;
        e2 := e12+e22;
      end;
  end;
```

{COMPUTES THE MEAN, STANDARD DEVIATION, AND COV FOR }
{EACH BEAM AND CUMULATIVE MEAN, STANDARD DEVIATION,}
{AND COV INCLUDING PRECEDING BEAMS}

```
Rc1 := Rc1+(e11+e21)/R1;  Rc2 := Rc2+(e12+e22)/R2;  k1 := k1+1;
r21 := r21+(e11*e11/(1.0-Frac(Ls/Sv))+e21*e21/Frac(ls/Sv))/R1/R1;
r22 := r22+(e12*e12/(1.0-Frac(Ls/Sv))+e22*e22/Frac(ls/Sv))/R2/R2;
s3 := s3+(e11+e21)/R1;  s4 := s4+(e12+e22)/R2;
s1 := s1+(e11*e11/(1.0-Frac(Ls/Sv))+e21*e21/Frac(ls/Sv))/R1/R1;
s2 := s2+(e12*e12/(1.0-Frac(Ls/Sv))+e22*e22/Frac(ls/Sv))/R2/R2;
if j > 1 then
  begin
    sd1 := sqrt((r21-Rc1*Rc1/j)/j);
    sd2 := sqrt((r22-Rc2*Rc2/j)/j);
  end;
if k1 > 1 then
  begin
    ms1 := sqrt((s1-s3*s3/k1)/k1);
    ms2 := sqrt((s2-s4*s4/k1)/k1);
  end;
fcm := fcm+fc;  Cbm := Cbm+Cb;  Cim := Cim+Csi;  Lsm := Lsm+Ls;
E1m := E1m+e1;  E2m := E2m+e2;  Atm := Atm+Atr;  Rrm := Rrm+Rr;
```

{DISPLAYS THE CURRENT RESULTS ON THE SCREEN}

```
Window(10,18,70,23);
writeln(' CURRENT R1 = ',e1/R1:6:3,'  CURRENT R2 = ',e2/R2:6:3);
writeln('   MEAN R1  = ',Rc1/j:6:3,'    MEAN R2  = ',Rc2/j:6:3);
writeln(' STD DEV 1  = ',sd1:6:3,'  STD DEV 2  = ',sd2:6:3);
writeln('   C.O.V. 1 = ',sd1/Rc1*j:6:3,'    C.O.V. 2 = ',sd2/Rc2*j:6:3);
```

{WRITES THE CURRENT RESULTS INTO THE RESULT FILE FOR}
{EACH BEAM}

```
        write(fout,j:3,W:8:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2,Rr:7:3);
        write(fout,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3,Rc1/j:7:3,Rc2/j:7:3,sd1:7:3);
        writeln(fout,sd2:7:3,sd1/Rc1*j:7:3,sd2/Rc2*j:7:3);
      end;
    writeln(fout,' ');
    Close(fout);

    {COMPUTES THE AVERAGE VALUES FOR ALL VARIABLES FOR}
    {EACH OF THE BEAMS}

    fc := fcm/j;  Cb := Cbm/j;  Csi := Cim/j;  Ls := Lsm/j;  e1 := E1m/j;
    e2 := E2m/j;  mr1 := mr1+e1/R1;  mr2 := mr2+e2/R2;  mv1 := ms1/mr1*n;
    mv2 := ms2/mr2*n;  Atr := Atm/j;  Rr := Rrm/j;

    {WRITES THE CURRENT BEAM RESULTS INTO THE RESULT FILE}
    {THAT CONTAINS THE SUMMARY OF ALL THE RESULTS FOR ALL BEAMS}

    write(fl,n:3,W:7:2,fc:6:0,Cb:7:3,Cso:7:3,Csi:7:3,Ls:7:2,Nb:3,Db:7:3);
    write(fl,Ab:6:2,Atr:6:2,Sv:6:1,Rr:7:3,e1:7:0,e2:7:0,e1/R1:7:3,e2/R2:7:3);
    write(fl,sd1:7:3,sd2:7:3,sd1/Rc1*j:7:3,sd2/Rc2*j:7:3,mr1/n:7:3);
    writeln(fl,mr2/n:7:3,ms1:7:3,ms2:7:3,mv1:7:3,mv2:7:3);
  end;
End;

{CHECKS TO SEE IF A SPECIFIED FILE EXISTS OR WAS OPENED SUCCESSFULLY}

Procedure Chkfile(s1,s2 : String);
Begin
  if Ioresult <> 0 then
    begin
      Window(10,10,70,15);
      ClrScr;
      writeln('    CANNOT OPEN ',s1);
      writeln;
      writeln(' PRESS ANY KEY TO END AND CHECK ',s2);
      err := 1;
      st := Readkey;
      Exit;
    end;
End;

{START OF THE MAIN PROGRAM WHERE ALL INPUT IS MADE}

BEGIN
  GetTime(hr,mn,sec,sec100);
  Window(1,1,80,25);
  ClrScr;
  err := 0;
  n := 0;
  write('    ENTER NAME OF THE DATA FILE W/O EXTENSION : ');
  readln(fname);
  Assign(fin,fname+'.DAT');
  {$I-}Reset(fin);{$I+}
  Chkfile('DATA FILE ','DATA FILE');
  if err = 1 then Exit;
  Assign(fl,fname+'.RST');
  {$I-}Rewrite(fl);{$I+}
  Chkfile('FILE FOR OUTPUT','DISK/DRIVE');
  if err = 1 then Exit;
  writeln;
  write('    ENTER THE NUMBER OF CYCLES REQUIRED : '); readln(k);
  writeln;
  write('    ENTER THE RELATIVE RIB AREA OF BAR : '); readln(a5);
  writeln;
  write('    ENTER DEVIATION OF RELATIVE RIB AREA : '); readln(Rm);
```

```
    writeln;
    write('   ENTER  THE  COV  OF  RELATIVE  RIB  AREA : '); readln(Vr);
    Simulate;
    GetTime(hr1,mn1,sec1,sec1001);
    writeln(fl,'   ');
    writeln(fl,'STOPPING TIME : ',hr1,':',mn1,':',sec1,'.',sec1001);
    writeln(fl,'STARTING TIME : ',hr,':',mn,':',sec,'.',sec100);
    Close(fin);
    Close(fl);
END.
```

Appendix C

Notation

A_b	= bar area, in in.^{2}
A_I	= influence area, in ft^2
A_T	= tributary area, in ft^2
A_{tr}	= area of each stirrup or tie crossing the potential plane of splitting adjacent to the reinforcement being developed or spliced, in in.^{2}
b	= beam width, in in.
c	= $(c_m + 0.5\ d_b)(0.1\ c_M/c_m + 0.9)$ or $(c_m + 0.5\ d_b)$
c_b	= bottom cover of reinforcing bars, in in.
c_M	= maximum value of c_s or c_b ($c_M/c_m \le 3.5$), in in.
c_m	= minimum value of c_s or c_b ($c_M/c_m \le 3.5$), in in.
c_s	= min (c_{si} + 0.25 in., c_{so}) or min (c_{si}, c_{so}), in in.
c_{si}	= one-half of clear spacing between bars, in in.
c_{so}	= side cover of reinforcing bars, in in.
d_b	= nominal bar diameter, in in.
$\dot{f}$	= stress rate, in psi/sec
f'_c	= concrete compressive strength, in psi; $f'^{1/4}_c$ in psi
f'_{cf}	= concrete compressive strength at stress rate $\dot{f}$, in psi
f'_{cr}	= f'_c + 2.33 σ_{ccyl} - 500 psi, required average concrete compressive strength, in psi
f'_{c35}	= concrete compressive strength at $\dot{f}$ = 35 psi/sec, in psi
f_s	= steel stress at failure, in psi
f_y	= yield strength of bars being spliced or developed, in psi
h	= beam depth, in in.
K_{tr}	= 35.3 $t_r t_d A_{tr}/sn$
L_o	= basic (unreduced) live load
l_d	= development or splice length, in in.

N	= number of transverse reinforcing bars (stirrups or ties) crossing l_d
n	= number of bars being developed or spliced along the plane of splitting
n_b	= number of bars
Q	= total load
Q_D	= random variable representing dead load effects
Q_{Dn}	= nominal dead load
Q_L	= random variable representing live load effects
Q_{Ln}	= nominal live load
$(Q_L/Q_D)_n$	= nominal ratio of live to dead load
q	= random loading
R	= random variable for resistance
R_n	= nominal resistance
R_p	= predicted capacity random variable
R_r	= ratio of projected rib area normal to bar axis to the product of the nominal bar perimeter and the center-to-center rib spacing
r	= $R/R_n = X(1)R_p/R_n$
s	= spacing of transverse reinforcement, in in.
T_b	= total force in a bar at development or splice failure, in lb
T_s	= contribution of confining steel to total bar force at bond failure
t_d	= $0.72\,d_b + 0.28$, term representing the effect of bar size on T_s
t_r	= $9.6\,R_r + 0.28$, term representing the effect of relative rib area on T_s
V	= coefficient of variation
V_R	= coefficient of variation for random variable for resistance
V_Q	= coefficient of variation for random variable for total load
V_c	= $(V_{ccyl}^2 + 0.0084)^{1/2}$, assumed coefficient of variation for in-place concrete
V_{ccyl}	= σ_{ccyl}/f'_{cr}
V_m	= coefficient of variation associated with the predictive equation (or model) itself
V_{Q_D}	= coefficient of variation of random variable representing dead load effects

V_{Q_L} = coefficient of variation of random variable representing live load effects

V_{R_r} = coefficient of variation of relative rib area

V_r = coefficient of variation of resistance random variable r

$V_{T/P}$ = coefficient of variation of test/prediction ratio

V_{ts} = coefficient of variation of the predictive equation caused by uncertainties in the measured loads and differences in the actual material and geometric properties of the specimens from values used to calculate the predicted strength

$V_{X(i)}$ = coefficient of variation of random variable X(i)

$V_{\phi q}$ = coefficient of variation of loading random variable q

$X(1)$ = test-to-predicted load capacity random variable

$X(2)$ = actual-to-nominal dead load random variable

$X(3)$ = actual-to-nominal live load random variable

$X(4)$ = concrete strength, f'_c, random variable

$X(5)$ = splice length, l_d, random variable

$X(6)$ = concrete cover, c_b, random variable

$X(7)$ = side cover, c_{so}, random variable

$X(8)$ = beam width, b, random variable

$X(9)$ = relative rib area, R_r, random variable

β = reliability index

ϕ = strength reduction factor for the main loading

ϕ_b = overall strength reduction factor against bond failure

ϕ_c = "composite" strength reduction factor

ϕ_d = ϕ_b/ϕ, effective strength reduction factor for use in calculating development/splice length

γ_D = load factor for dead loads

γ_L = load factor for live loads

σ = standard deviation

σ_{ccyl} = standard deviation for standard laboratory cylinders

overbar represents average value of the variable