FLOW OVER WEIRS WITH THIN EDGES AND FULL CONTRACTIONS.

by

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A thesis submitted to the Department of Civil Engineering and the Faculty of the Graduate School in partial fulfillment of the requirements for the degree of Civil Engineer.

1915.

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Department of Civil Engineering.

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A THESIS SUBMITTED TO THE FACULTY OF

THE SCHOOL OF ENGINEERING OF

THE UNIVERSITY OF KANSAS

FOR

THE DEGREE OF CIVIL ENGINEER

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1915

PREFACE

The work upon which this thesis is based was done in the hydraulic laboratory at Fort Collins, Colorado, during the years 1913-14, under a co-operative agreement between the Colorado Experiment Station, and the U. S. Department of Agriculture. The experimental data has been prepared for publication by the U. S. Department of Agriculture, essentially in the form offered in this thesis.

The hydraulic laboratory at Fort Collins, Colorado, was designed and constructed under the direction of the author during the summer of 1912. Except for the severe winter months, the laboratory has been in constant operation since the spring of 1913, during which time about 3,000 experiments have been made with devices for the measurements of water flowing in open channels. Although the laboratory has its limitations we feel that only a fair start has been made on the hydraulic research work which may be well done in it and for which there is a decided need.

V. M. C.

Fort Collins, Colorado

April, 1915.

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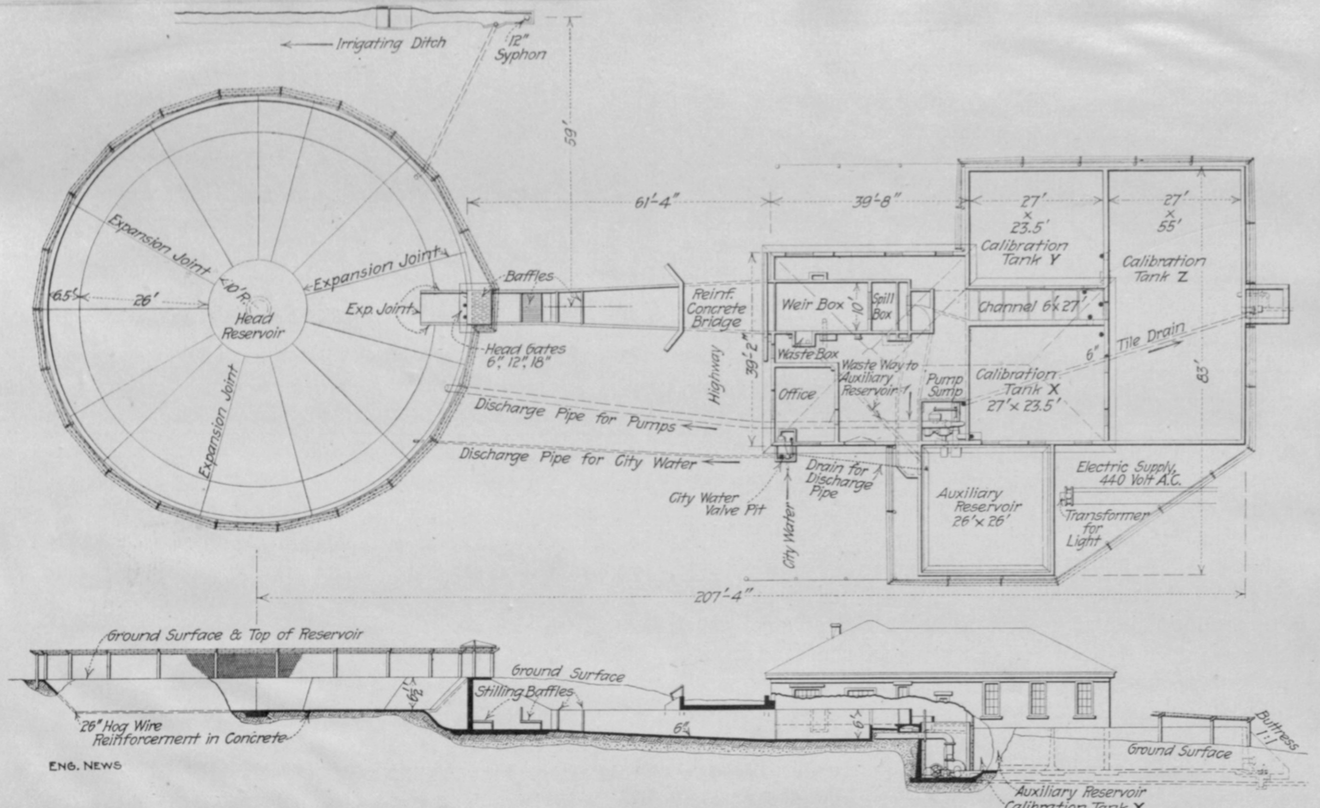
ILLUSTRATIONS

Hydraulic Laboratory
Fort Collins, Colorado.



PLAN AND SECTIONAL ELEVATION OF HYDRAULIC LABORATORY FOR IRRIGATION INVESTIGATIONS. FORT COLLINS, COLO.

FLOW OVER WEIRS WITH THIN EDGES AND FULL CONTRACTIONS

The development of irrigated agriculture in the arid West has caused many changes to be made in the methods of delivering water to the canals and to the individual irrigator. The value of water increases with the increase of irrigated acreage, and the long accepted practice of fixing the charge for water on the acre per annum basis is rapidly losing ground in favor of a charge per volume used. When the irrigator pays for the amount of water he uses there is every incentive for him to study the water requirement of his crop and use the least amount he judges to be necessary, which leads to a proper economy, permits of a greater acreage being irrigated with the available water supply, and conserves the land.

This transition from a flat rate to an actual water consumption basis is calling for a better knowledge of the accuracy and practicability of existing measuring devices, as well as the development of new devices. It is generally considered that a weir is the most accurate device for measuring flowing water, and this is doubtlessly true when the weir is properly installed and a correct formula is used for determining the discharge. Weirs constitute a large proportion of the measuring devices in use at the present time, being principally of the rectangular or Francis type, and of the trapesoidal or Cippoletti type. The greater number of these weirs are small, having crest lengths of

4 feet or less, such as are adapted to the delivery of water to the farm unit, and unfortunately various standards of dimensions have been used in their construction. This lack of uniformity which results in erroneous measurement of water, has been due to the confusion of statements contained in the literature on weirs. The basic experiments with weirs having thin edges and complete contractions were made by James B. Francis from 1848 to 1852, and subsequently several experimenters and mathematicians have amplified certain phases of his work.

Francis [(1)] made three series of experiments with rectangular weirs, but the discharges were measured directly in only one series, [(2)] while in the others an equal flow of water was made to pass over different lengths of weirs, the crest length and head being noted. In the experiments where the discharges were calibrated volumetrically, [(3)] only weirs of approximately 8 feet and 10 feet crest lengths were used, and the heads ranged from 7 inches to 19 inches. The greater part of the experiments were made with the 10-ft. weir, for it must be remembered that the experiments were to be directly applied to the measurement of water for power purposes. Francis stated [(4)] that the formula which he derived would apply to heads ranging from 6 to 24 inches, but in no case should it be used for H exceeding L/3, nor for very small heads. According to the limits imposed by Francis, therefore, the use of the formula was automatically eliminated in con-

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(1) Francis, Jas. B. Lowell, Hydraulic Experiments, 5th Edition
(2) " " " " " " " " pp. 75-76
(3) " " " " " " " " " 122-125
(4) " " " " " " " " " 133

nection with weirs having crest lengths of less than 1.5 feet, would apply to the single head of 0.5 foot on the 1.5 foot weir, and regardless of the length of weir the head should not exceed 2.0 feet. Horton states [1] that Francis data and formula will hold for heads of from 0.5 foot to 4.0 feet.

Francis' experiments were very carefully and conscientiously made, but were with longer weirs and greater volumes of water than are demanded for the delivery of water to the irrigator. Subsequent use of Francis formula has been made without regard for the limits which he imposed upon it, and it is not uncommon to see weir discharge tables computed from that formula for heads as low as .01 foot, as high as 1.0 foot for a 1-ft. weir, and for lengths varying from 0.5 to 20 feet.

The most popular weir has been the trapezoidal type with side slopes of 1 to 4, as designed by the Italian engineer, Caesar Cippoletti, [2] to meet the conditions of automatically eliminating the correction for end contractions as found by Francis and used in his formula, producing a discharge proportional to the length of the weir, and being free from error in excess of one half percent from any single cause. The shape and slope of the trapezoidal weir with full contractions and free fall, he derived by a mathematical modification of the Francis formula for the rectangular weir, and obtained the values for the coefficient and exponent by an examination of Francis experimental data, and somewhat arbitrarily

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(1) Horton, Robt. E., U.S.G.S. Water Supply Paper #200, p. 39-46
(2) Cippoletti, Caesar, Canale Villoresi; Module per le Dispense delle Acque. Milan, 1886

increased Francis coefficient value by 1 percent. Cippoletti also made a few experiments, but the formula proposed by him was stated to be subject to the limitations imposed by Francis, and the subsequent extension of range of application of the formulas was an excursion into unexplored territory. Furthermore, Cippoletti designed the weir for a minimum discharge of 150 liters (5.3 cu.ft.) per second, and a maximum discharge of 300 liters (10.6 cu. ft.) per second, which, together with Francis' limits, restrict the use of Cippoletti's formula to crest lengths of not less than 3 feet nor more than 8 feet.

Since there is a practical need for small weirs and for measuring small depths of water over weirs, it was considered necessary to secure data upon which to base formulas which would meet those conditions accurately. If the old weir dimensions, formulas and resulting discharges were wrong, those errors had been incorporated in the calibration of many other forms of measuring devices which had been calibrated by being hitched in tandem with a weir. For these reasons a series of experiments was made on weirs with thin edges and full contractions, in the hydraulic laboratory at Fort Collins, Colorado, during 1913-14, and the results of these tests are given herein.

Experimental Equipment and Accuracy. The laboratory (1) was designed for research work in hydraulics, especially for gravity flow work. It was constructed almost entirely of concrete and metal for rigidity, permanency, and water-tightness. All water faces of concrete are covered with a 3 to 1 cement plaster coat 3/8" thick and tests

(1) Described in Engineering News, Vol. 70, No. 14, P 662, Oct. 2, 1913.

have shown the seepage losses to be negligible.

The general plan of the laboratory is as follows; The water supply is obtained from the city mains. A circular storage reservoir, with side slopes of 1 to 1, 6½ feet deep and 87 feet top diameter, is connected by three circular headgates 8", 12" and 18" in diameter, with a concrete channel 4 feet deep and 6 feet wide. Immediately below these headgates is a series of two horizontal and two vertical baffles. The channel is gradually enlarged to a depth of 6 feet and a width of 10 feet, at a distance of approximately 60 feet from the headgates, and an additional length of 20 feet with parallel sides and level bottom, constitutes the weir box. The weirs are placed in the end of this box, while on one side of the weir box are waste ways, or by-passes, and on the opposite side is the hook gage still box. The water flows over the weir into a concrete basin or tail box, 4 feet deep, 10 feet wide and 9 feet long, which is connected with an auxilliary or waste reservoir by one channel, and with the calibrated tanks by another channel. The water passes into these channels through two circular openings, 22" in diameter, which are separated only by a steel plate. A single disk on a lever arm makes a double shear gate of these openings. A twelve-inch and a five-inch horizontal centrifugal pump, electrically driven, return the water to the storage reservoir. The difference in elevation between the floor of the calibrated or auxilliary reservoirs and the coping of the storage reservoir is 19 feet.

Some of the means employed for securing accuracy in observations are,--The water has a high velocity when it leaves the storage reservoir headgates, but the series of baffles breaks up the eddy currents and reduces pulsation and wave action to such an extent that by the time the water enters the weir box it is in a quiet or pond-like condition.

On one side of the weir box is installed an over-pour spillway which resembles a door 2 feet high and 3 feet long, hinged at the bottom. The top of this spillway, when in an upright position is slightly below the top of the weir box. This spillway has an apron of oil-canvas attached to the side of the weir box and the face of the door in such a manner as to permit of no leakage and compel the water to pass over the crest. A 4-inch gate valve placed at the side of the spillway permits of a finer regulation than can be secured by the over-pour spillway. Both by-passes discharge into a concrete box having a tile connection with the auxiliary reservoir. The hook-gage observer on the opposite side of the weir box operated the by-passes by means of screw controls and hand wheels placed on the end of long rods. By always running an excess of water over the spillways it was possible to keep the head upon the weir at a constant height throughout the duration of the experiment, which was from 20 to 40 minutes depending upon the volume of water being run.

The elevation of the water in the weir box was observed in a concrete still box, having inside dimensions of 1' by 2' and 4' deep, which was built outside of the wall of the weir box and connected with the weir box by 4 one-inch pipes 6 inches long. The still box is 10 feet upstream from the plane of the weir. It is equipped with a Boyden type hook gage anchored in the concrete, and an electric drop light permits of careful readings of the water level to the one-thousandth of a foot.

All weir plates were constructed either entirely of brass or of steel with brass edges. The crests and sides were dressed to true angles and straight lines and by use of a micrometer caliper were calibrated to an allowable diversion of two-thousandths of an inch from a straight line. The triangular notches were dressed to templates. A heavy T-iron frame about 3 feet high and 6 feet long was placed in the concrete end wall of the weir box, which was 6 inches thick. This frame was surfaced, bored for 3/8" bolts and so arranged that all steel and brass plates which formed the weirs, or to which the weirs were attached, could be adjusted accurately, and the joints between the T-frame and such plates were made perfectly water tight by flat rubber gaskets. These plates were placed in a vertical position, with crests accurately leveled with a 12" steel-frame level, in which a variation of a bubble division produced an error of .0004 of a foot for a length of one foot. Triangular notches were similarly placed, except that a vertical line bisected the angle formed by the sides of the notch. The inner face of the bulkhead was flush with the crest.

In order to refer the elevation of the weir crest to a reading of the hook-gage in the still box to the nearest one-thousandth of a foot, an instrument was devised as shown in Fig. 1. The length of the legs and hook were adjusted to make the distance from the top of the plate to the groove in the legs exactly equal to the distance from the top of the plate to the point of the hook. By resting these

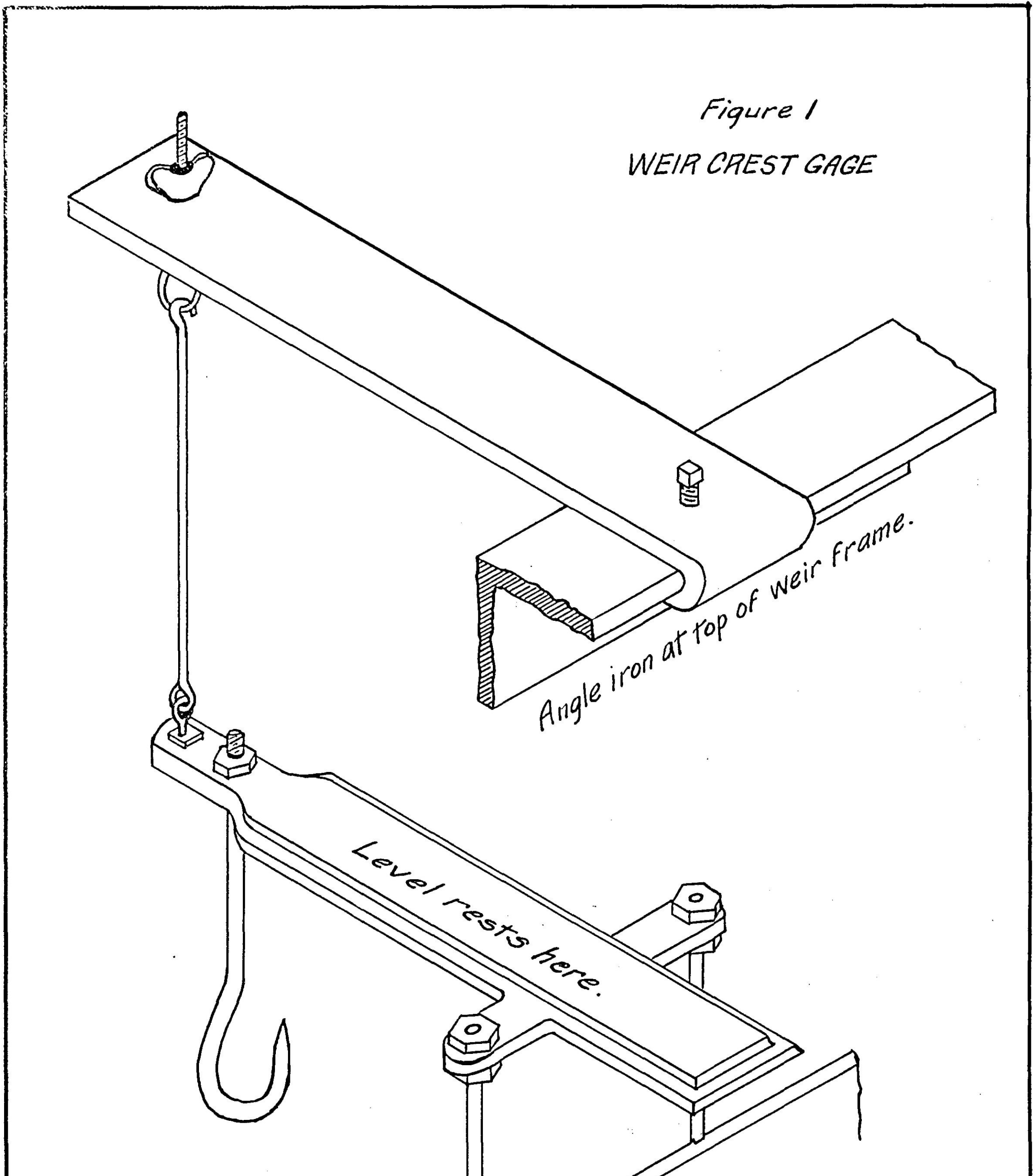

Figure 1

WEIR CREST GAGE

notched legs on the crest and adjusting the plate to a horizontal position as determined by a sensitive level, the point of the hook was brought to the same elevation as the weir crest. Water was run into the weir box and the surface of the water adjusted to the point of the crest hook-gage. Since it was possible to maintain this water level quite accurately, the hook gage reading in the weir still-box was taken to correspond to the crest elevation of the weir. Repeated determinations of this nature indicated a nice accuracy.

In order to avoid the fluctuating conditions of flow obtained during starting and stopping tests, means had to be provided for quickly turning the flow into the calibrated tanks, and when the desired conditions for the test has been obtained, this was accomplished by means of a double shear gate, having two 22-inch circular openings operated by a circular disk on an 8 foot lever arm. The disc was seated by means of steel shear springs, and was positive and practically instantaneous in action. It was never necessary to have both openings closed at the same time because the purpose of the gate was to direct the flow to the auxiliary reservoir or to the calibrated tanks, as the case might be. When the gate handle reached the mid-point of its swing, it struck a gong, which was the signal to the hook-gage observer to start or stop the stop-watch, thus recording the duration of the experiment. The error in time in operating the shear gate and stop-watch was a small fraction of a second.

The auxiliary or waste reservoir received the water from the waste ways or by-passes, and also received the full flow over the weir while regulations were being made previous to beginning an experiment, and at the close of an experiment until the headgates could be closed. The dimensions of this reservoir were 26' by 26' and $8\frac{1}{2}$ feet deep.

The calibrated tanks cover an area 55 feet square, which area is divided by vertical sided concrete walls 12 inches thick, to form one tank 27' by 55', a channel 6' by 27', and two tanks each $23\frac{1}{2}$' by 27'. Water delivered to the calibrated tanks drops into the channel and is let into either or all of the tanks by 14" circular headgates placed on the floor line. These tanks are $8\frac{1}{2}$ feet deep, with all floors at the same elevation, and have a combined capacity in excess of 22,000 cubic feet available for experimental purposes. The capacity of each tank was carefully determined and tables prepared from which the capacity at each 1/1000 foot in elevation could be taken.

A round brass rod 1 inch in diameter and 9 feet long was placed in a vertical position in each calibrated tank, being held out from the wall about 6 inches by iron brackets set in the concrete Fig.--2---. At intervals of about 18 inches on the rod are holes which serve as data points. A heavy brass clamp fixed to the back of a hook-gage is provided with a pin which snugly fits the hole in the rod. A steel ladder was placed adjacent to the brass standard rod and anchored to the concrete and provided with a 20" by 24" platform, which can be lowered close to the water surface and secured to the ladder by means of hooks. A funnel-shaped arrangement having a half inch hole in the bottom, was attached to the platform and adjusted to

Figure 2 Observing water-level in calibrated tank.

form a stilling basin for the hook when there was wave action in the tank. The water level could therefore be determined to the one-thousandth of a foot, and by taking the water levels at the beginning and close close of each experiment, the volume run during the experiment was quite accurately determined.

Unless otherwise stated all original weir experiments recorded herein were made with weirs having sharp crests and sides, and were placed in a concrete box 6 feet deep and 10 feet wide, and the crest of the weir was approximately 4½ feet above the bottom of the weir box in every case. From thirty to forty tests were made upon each weir, the experimental variable being the head. Intervals of head of 0.05 feet were used, and duplicate tests were run for heads of tenths of a foot. An arbitrary rule was followed which called for an agreement of the data from duplicate tests within one-half of one per-cent, or repeating the tests until such an agreement was obtained. Of course the rule did not assure the accuracy of the result of the individual tests, but it lead to the detection of irregularities in the working conditions, and increased the probability of accuracy. Comparatively few tests had to be re-run, which indicates the stability of the experimental conditions and the nice control of head made possible by the waste-ways or by-passes.

GENERAL POINTS CONCERNING WEIRS.

The literature on the use of weirs contains many statements which do not agree, largely because of limited experimental date on some points, and also because the several sets of experiments were made under conditions which are not entirely comparable. The weir experiments recorded herein gave light on some of the matters which have been variously stated.

SHARP CRESTS. The impression prevails among many that the term "sharp crest" when applied to weirs, means a crest with a knife edge. The crest or side of a weir notch is "sharp" when the inner corner is distinctly angular, and this angle should be 90 degrees or less. When this condition is met, the allowable thickness of the crest to prevent water from adhering will depend upon the head. It was found that with a thickness of crest of $\frac{1}{4}$" the water would adhere for a head of 0.15 foot, but with a head of 0.2 foot the water would flow clear of the crest after it had left the crest along the inner or upstream edge. However, the angle was very accurately made, and since this precision would not be obtained in the field it would be safer to allow a thickness of weir crest of not more than 1/8" for a head of 0.2 foot. For a minimum head of 1.0 foot the crest thickness could probably be $\frac{3}{4}$" without causing adherence.

Since weir crests should be straight, true and rigid, it would be better to make the crest and side of angle iron or similar material which may be securely fastened to the weir bulk-head, rather than to use thin sheet metal as is commonly done. The thin metal buckles and bends easily, and wood warps and splinters with exposure to water and weather. Whatever the material of which the crest is made, it is more permanent and reliable if the crest be left as thick as possible and still insure free flow, rather than to have it beveled to a knife edge, but in any event the inner corner must be definitely angular.

Measurement of Head on Weirs. The accuracy of weir measurements where proper principles of construction have been observed, varies with the degree of precision with which the head is determined.

In connection with the experiments on sharp-crested and full-contracted weirs, measurements were made to determine the transverse and longitudinal curves of water surface upstream and laterally from the weir for several sizes of weirs and several depths of water flowing over those weirs. From plots of these data it was determined that the measurements of head should be made upstream from the weir a distance of at least 4 H, or sidewise from the end of the weir crest a distance of at least 2 H. This distance, 2 H, would be used where the gage would be placed on the upstream face of the weir bulk-head, and 4 H where the gage is placed upstream from the weir. The extent of the drawdown curve backward and to the side of the weir proper, depends upon the head and the length of the weir, but the distances above stated insure avoiding the effect of drawdown in taking head measurements.

Deduction of Weir Formulas. The general type of formulas heretofore used for flow over weirs is $Q = cLH^n$, in which (c) and (n) are constant, and which expressed logarithmically is $\log Q = \log c + \log$

L + n log H. The logarithmic plot is a straight line, the slope and intercept of which represent the exponent (n), and coefficient (c) and length (L) respectively. It is therefore a simple matter to determine the correct values for such an equation. When the weir data obtained in the Fort Collins hydraulic laboratory were plotted logarithmically, it was found that the triangular notches gave straight lines but the rectangular and trapezoidal weirs gave curves which proved to be represented by rather complex formulas.

The failure of these logarithmic curves to hold to straight lines is shown in table 1, giving discharge, and exponent and coefficient values for Cippoletti weirs, or trapezoidal weirs having side slopes of 1 horizontal to 4 vertical. The values of the exponent (n) and the coefficient (c) increase with the head (H) and decrease with the length of the weir (L), but the values of (c) decrease for heads greater than approximately 1 foot. These facts also hold for rectangular weirs, but are not so pronounced as for the Cippoletti weirs, owing to the rectangular weir curves being flatter. This table also serves to indicate the accuracy of the experimental data, for in the columns headed "Observed Q" are given the experimental values at greates variance with the curve. The conditions represented by the curves for the three types of weirs were dissimilar and made it necessary to use different methods in deriving the formulas. It must be remembered that many factors that have considerable effect upon the flow over small weirs, are gradually eliminated as the size of the weir is increased. Some of the terms in the new formulas which follow become negligible for large weirs.

Table I — Data regarding discharges through Cippoletti notches and exponents and coefficients necessary in using the Cippoletti formula

½-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve cubic feet per second	(n)	(c)
0.20	0.152	0.149	1.530	3.492
0.30	0.287	0.284	1.565	3.733
0.40	0.453	0.452	1.600	3.911
0.50	0.653	0.655	1.636	4.066
0.60	0.890	0.890	1.671	4.174
0.70	1.158	1.159	1.706	4.254
0.80	1.455	1.455	1.742	4.284
0.90	1.784	1.786	1.778	4.303
1.00	2.149	2.155		
1.10				
1.20				
1.30				

1-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve Cubic feet per second	(n)	(c)
0.20	0.300	0.300	1.498	3.327
0.30	0.554	0.554	1.517	3.424
0.40	0.866	0.865	1.536	3.516
0.50	1.216	1.217	1.555	3.558
0.60	1.626	1.622	1.574	3.608
0.70	2.079	2.072	1.593	3.639
0.80	2.565	2.562	1.612	3.653
0.90	3.109	3.111	1.631	3.676
1.00	3.693	3.696	1.650	3.678
1.10	4.332	4.340		
1.20				
1.30				

1½-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve Cubic feet per second	(n)	(c)
0.20	0.459	0.455	1.486	3.309
0.30	0.829	0.829	1.499	3.353
0.40	1.289	1.282	1.513	3.413
0.50	1.801	1.798	1.526	3.446
0.60	2.378	2.375	1.539	3.469
0.70	3.018	3.013	1.553	3.488
0.80	3.718	3.708	1.566	3.500
0.90	4.458	4.463	1.579	3.508
1.00	5.262	5.262	1.592	3.502
1.10	6.138	6.134	1.605	3.503
1.20	7.062	7.060	1.618	3.498
1.30	8.022	8.035	1.632	3.484

2-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve Cubic feet per second	(n)	(c)
0.20	0.603	0.602	1.480	3.255
0.30	1.100	1.100	1.490	3.307
0.40	1.693	1.695	1.501	3.353
0.50	2.323	2.373	1.511	3.381
0.60	3.146	3.142	1.521	3.416
0.70	3.952	3.952	1.532	3.412
0.80	4.836	4.845	1.542	3.717
0.90	5.815	5.814	1.553	3.424
1.00	7.792	6.844	1.563	3.422
1.10	7.943	7.941	1.573	3.417
1.20	9.116	9.107	1.584	3.411
1.30				

3-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve Cubic feet per second	(n)	(c)
0.20	0.909	.902	1.473	3.217
0.30	1.633	1.648	1.480	3.263
0.40	2.538	2.538	1.487	3.303
0.50	3.528	3.528	1.495	3.314
0.60	4.663	4.634	1.502	3.326
0.70	5.890	5.870	1.509	3.350
0.80	7.173	7.190	1.516	3.360
0.90	8.574	8.574	1.523	3.354
1.00	10.081	10.080	1.530	3.359
1.10	11.655	11.655	1.538	3.354
1.20	13.347	13.355	1.545	3.357
1.30				

4-foot notch

Head in Feet	Discharge: Observed Cubic feet per second	Discharge: Curve Cubic feet per second	(n)	(c)
0.20	1.200	1.206	1.470	3.205
0.30	2.200	2.191	1.476	3.232
0.40	3.342	3.365	1.481	3.261
0.50	4.700	4.700	1.487	3.287
0.60	6.172	6.180	1.493	3.305
0.70	7.807	7.795	1.499	3.319
0.80	9.532	9.532	1.504	3.326
0.90	11.342	11.390	1.510	3.331
1.00	13.376	13.376	1.515	3.337
1.10	15.467	15.425	1.521	3.329
1.20				
1.30				

EFFECT OF SUPPRESSING END AND BOTTOM CONTRACTIONS

End and bottom contractions with Rectangular and Cippoletti Weirs. In order to determine the effect on the discharge over rectangular and Cippoletti weirs and 90° triangular notch caused by placing the bottom of the weir box at various distances below the crest,(called bottom contractions) and the sides at various distances out from the ends of crest,(called end contractions) 353 experiments were made. Crest lengths of 1.0' and 3.0', and heads of 0.2',0.6' and 1.0' were used, and for end and bottom contractions, distance of from o.5' to 3.0' by increments of 0.5' for each type of weir used. However, a small error in the experimental determination of the discharge caused by a 0.2' ~~by increments of 0.5'~~ head gave such a large percentage error as to make them unreliable for use in this connection. The discharges obtained under those conditions were compared with the discharges for the same weirs when placed in the standard weir box of the following dimensions; width = 10.0', depth = 6.0', and distance from floor to weir crest = 4.5'. The percentage of error in the discharge, and the velocity of approach produced by different end and bottom contractions and for different heads of water and lengths of weir crests are given in tables 2 and 3. The curves for these data all have the general form $e = a\ (v+b)^n$ in which (e) is the percentage increase in the discharge due to the average velocity of approach (v), (b) is a numerical quantity which may be plus, minus, or zero, and (n) is the power of (v+b).

Table 4 shows the variation of percentage of error in discharge with the ratio of the cross-sectional area of the weir box (A) to the area of the weir notch (a) for different end and bottom contractions, for a rectangular weir having a crest length of 1.0 foot and heads of 0.6 foot and 1.0 foot. From this table it will be seen that changing

TABLE 2

VELOCITY OF APPROACH AND PERCENTAGE OF ERROR CAUSED
BY DIFFERENT END AND BOTTOM CONTRACTIONS
FOR RECTANGULAR WEIRS.

B Distance Bottom Below Crest	A Distance Sides From end of Crest	L Equals 1 Ft.				L Equals 1.5 Ft.				L Equals 2 Ft.				L Equals 3 Ft.				L Equals 4 Ft.			
		H.=.6'		H.=1'		H.=.6'		H.=1'		H.=.6'		H=1 ft.		H.=.6'		H.=1'		H.=.6'		H.=1'	
		Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel- of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel- of App.	Per-cent Error
3.0	2.5			.132	.77																
3.0	2.0			.157	.81			.213	.82			.269	.84			.342	0.83			.402	.87
3.0	1.5			.196	.99			.260	1.08			.317	1.14			.399	1.22			.460	1.29
3.0	1.0			.260	1.40			.337	1.63			.398	1.81			.484	2.06			.543	2.22
3.0	.5			.40	2.94			.477	3.22			.540	3.44			.616	3.72			.661	3.88
2.5	2.5			.150	.74																
2.5	2.0			.178	.82			.242	.88			.302	.94			.391	1.04			.460	1.11
2.5	1.5			.224	1.05			.297	1.21			.362	1.34			.461	1.57			.528	1.69
2.5	1.0			.299	1.58			.385	1.89			.457	2.14			.553	2.50			.623	2.76
2.5	.5			.462	3.42			.549	3.73			.625	3.99			.704	4.25			.760	4.48
2.0	2.5	0.94	0.17	.175	0.73																
2.0	2.0	.115	.26	.209	.84			.284	.97			.352	1.11			.458	1.30			.539	1.42
2.0	1.5	.148	.39	.261	1.13			.348	1.42			.424	1.67			.538	2.01			.620	2.28
2.0	1.0	.188	.66	.353	1.83			.450	2.28			.535	2.63			.648	3.14			.733	3.52
2.0	.5	.288	2.05	.538	4.01			.646	4.46			.728	4.80			.829	5.17			.895	5.47
1.5	2.5	.119	.17	.208	.74																
1.5	2.0	.141	.30	.252	.94	.191	0.53	.341	1.18	.239	0.74	.424	1.41	.308	1.07	.539	1.71	.365	1.33	.638	1.98
1.5	1.5	.175	.40	.314	1.31	.234	.73	.418	1.74	.286	1.01	.512	2.12	.363	1.44	.648	2.65	.416	1.74	.750	3.07
1.5	1.0	.234	.76	.424	2.24	.304	1.24	.544	2.87	.361	1.62	.646	3.40	.435	2.12	.790	4.14	.489	2.49	.889	4.68
1.5	.5	.355	2.26	.648	4.80			.784	5.53			.885	6.09	.552	3.41	1.013	6.77			1.091	7.20
1.0	2.5	.154	.19	.260	0.82																
1.0	2.0	.209	.36	.314	1.12			.427	1.57			.532	2.00			.694	2.69			.810	3.15
1.0	1.5	.229	.50	.385	1.59	.311	1.09	.528	2.37	.377	1.55	.645	2.99	.478	2.25	.820	3.91	.552	2.79	.952	4.60
1.0	1.0	.308	1.01	.525	2.83	.400	1.77	.688	3.86	.476	2.39	.825	4.73	.577	3.22	.999	5.87	.650	3.83	1.135	6.77
1.0	.5	.469	2.84	.822	6.00	.573	3.74	.994	7.29	.646	4.38	1.129	8.29	.735	5.15	1.298	9.55	.794	5.64	1.405	10.27
.5	2.5	.221	.25	.350	1.11																
.5	2.0	.265	.50	.417	1.45	.368	1.30	.575	2.40	.460	2.05	.720	3.27	.624	3.34	.943	4.62	.711	4.05	1.120	5.65
.5	1.5	.337	.94	.530	2.20	.453	1.94	.710	3.53	.555	2.84	.875	4.33	.705	4.17	1.119	6.50	.818	5.15	1.308	7.88
.5	1.0	.450	1.84	.716	3.83	.588	3.22	.930	5.65	.704	4.35	1.118	7.23	.862	5.92	1.380	9.40	.975	7.01	1.576	11.2
.5	.5	.695	4.63	1.120	8.25	.852	6.43	1.37	11.0	.970	7.79	1.58	13.3	1.112	9.40	1.83	16.01	1.208	10.50	2.01	18.0

TABLE 3

VELOCITY OF APPROACH AND PERCENTAGE OF ERROR CAUSED BY DIFFERENT END AND BOTTOM CONTRACTIONS FOR CIPPOLETTI WEIRS.

B Distance Bottom Below Crest	A Distance Sides From end of Crest	L Equals 1 Ft.				L Equals 1.5 Ft.				L Equals 2 Ft.				L Equals 3 Ft.				L Equals 4 Ft.			
		H.=.6'		H.=1'		H.=.6'		H.=1'		H.=.6'		H.=1'		H.=.6'		H.=1'		H.=.6'		H.=1'	
		Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error.	Vel. of. App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel- of App.	Per-cent Error	Vel. of App.	Per-cent Error	Vel- of App.	Per-cent Error
2.0	2.0			.250	1.19			.322	1.22			.386	1.24			.488	1.28			.561	1.30
2.0	1.5			.314	1.52			.397	1.70			.467	1.84			.575	2.08			.648	2.22
2.0	1.0			.422	2.40			.514	2.80			.590	3.15			.698	3.62			.769	3.92
2.0	.5			.655	6.16			.746	6.41			.813	6.61			.896	6.88			.951	7.01
1.5	2.5																				
1.5	2.0	.158	0.84	.300	1.34	.207	1.02	.388	1.49	.251	1.21	.465	1.61	.321	1.45	.590	1.82	.373	1.61	.680	1.98
1.5	1.5	.196	1.11	.378	1.78	.255	1.38	.477	2.10	.304	1.60	.562	2.40	.377	1.95	.693	2.85	.429	2.19	.785	3.17
1.5	1.0	.260	1.70	.508	2.89	.329	2.08	.622	3.53	.381	2.36	.714	4.06	.454	2.77	.844	4.79	.504	3.02	.937	5.31
1.5	.5	.400	3.32	.795	7.29	.469	3.83	.906	7.79	.518	4.20	.989	8.18	.580	4.66	1.094	8.64	.617	4.93	1.163	8.95
1.0	2.5																				
1.0	2.0	.205	0.09	.374	1.60	.274	1.25	.489	2.06	.331	1.55	.586	2.44	.425	2.05	.758	3.13	.492	2.39	.864	3.55
1.0	1.5	.257	1.20	.471	2.20	.335	1.71	.601	2.92	.400	1.55	.710	3.55	.500	2.82	.888	4.53	.569	3.30	1.003	5.19
1.0	1.0	.344	1.84	.643	3.76	.434	2.60	.787	4.83	.501	3.17	.908	5.73	.607	4.06	1.083	7.07	.671	4.60	1.200	7.92
1.0	.5	.529	4.00	1.010	9.20	.622	4.92	1.159	10.28	.690	5.61	1.271	11.08	.770	6.41	1.410	12.09	.826	6.98	1.503	12.72
0.5	2.5			.64	3.3			.818	4.8			.968	6.07			1.21	8.07			1.391	9.56
0.5	2.0	.300	1.11	.508	2.30	.399	1.81	.660	3.64	.487	2.42	.799	4.87	.625	3.40	1.013	6.73	.725	4.09	1.202	8.39
0.5	1.5	.377	1.51	.640	3.30	.492	2.55	.818	4.80	.589	3.44	.969	6.09	.737	4.79	1.210	8.08	.847	5.80	1.391	9.56
0.5	1.0	.505	2.39	.864	5.40	.636	3.93	1.077	7.58	.750	5.30	1.258	9.43	.908	7.18	1.505	11.95	1.013	8.39	1.688	1.38
0.5	.5	.782	6.03	1. 390	11.89	.932	8.02	1.605	14.63	1.037	9.43	1.782	16.85	1.173	11.28	2.015	19.80	1.263	12.48		

Table 4 showing variation of Percentage of Error in discharge with Ratio of Cross-sectional Area of weir box to Area of Weir Notch for different end and bottom contractions for 1.0 rectangular weir with a head of 0.6', and also 1.0'.

Head = 0.6' Length = 1.0'				Head = 1.0' Length = 1.0'		
Side Out	Bottom Down	Ratio A/a	Percentage Increase in Q	Bottom Down	Ratio A/a	Percentage Increase In Q
0.5	.5	3.65	4.62	.5	2.98	8.25
	1.0	5.32	2.85	1.0	3.99	6.00
	1.5	6.98	2.25	1.5	5.11	4.80
	2.0	8.64	2.06	2.0	5.99	4.02
				2.5	6.98	3.44
				3.0	7.98	2.94
1.0	.5	5.48	1.84	.5	4.48	3.84
	1.0	7.96	1.03	1.0	5.98	2.83
	1.5	10.47	.74	1.5	7.46	2.22
	2.0	12.95	.67	2.0	8.96	1.82
				2.5	10.46	1.58
				3.0	11.96	1.40
1.5	.5	7.38	.95	.5	5.99	2.20
	1.0	10.63	.50	1.0	7.96	1.58
	1.5	13.94	.40	1.5	9.96	1.31
				2.0	11.95	1.13
				2.5	13.94	1.05
2.0	.5	9.14	.50	.5	7.47	1.45
	1.0	13.28	.35	1.0	9.96	1.12
				1.5	12.45	.94
				2.0	14.94	.82
2.5	.5	10.95	.26	.5	8.95	1.12
	1.0	15.93	.18	1.0	11.95	.83
				1.5	14.94	.72

the position of the sides of the weir box when the bottom is in a fixed position, has a greater effect on the discharge than when the sides are fixed and the bottom is moved. This indicates the effect of end contraction to be greater than the effect of bottom contraction. End contraction equal to 2 H and bottom contractions equal to 3 H, or end contractions equal to 3 H and bottom contraction equal to 2. H, will not give a discharge agreeing with the formulas or tables for medium to high heads closer than approximately 1 percent, because these dimensions cause a mean velocity of approach of about 1/3 ft. per second. These data indicate a mean velocity of approach of 1/3 ft. per second to be allowable for a 1 percent error in discharge. A distance equal to 2 H, therefore seems to be necessary to fulfill the conditions of complete contractions, proper, but an additional distance is necessary to increase the cross-sectional area of the weir box and thus reduce the velocity of approach.

By superimposing the curves showing the effect of suppression of end and bottom contraction upon the discharge over the rectangular weir and the same curves for the Cippoletti weir, it will be seen that the end contraction distance for Cippoletti weirs should be taken from about the mid-point of the side of the notch instead of from the end of the weir crest, in order to make the results of the two types of weirs comparable.

Since the error in discharge for any certain size of weir box increases with the head, it is essential that the weir box be made large enough to keep the discharge for the highest heads within the allowable limit of error.

Francis stated [1] "In order that the contraction may be complete, the sill and sides of the weir must be so far removed from the bottom and lateral sides of the reservoir (weir box), that they may produce no more effect upon the discharge, than if they were removed a distance infinitely great." He concluded from his experiments that an end contraction of 1 H and a bottom contraction of 2 H would practically provide complete contractions; Smith [2] gave the necessary end contraction as 3 H; and Cippoletti[3] specified 2 H for end, and 3 H for bottom contractions. As has been suggested by Smith [4] the effect of contractions shouldnot be confused with the effect of velocity of approach, but the ordinary conception of the term "complete contraction" includes both actions.

The ratio of cross-sectional area of weir box to cross-sectional area of weir notch for complete contraction conditions [5]

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(1) Francis, James B. Lowell Hydraulic Experiments, 5th Edition pp 72 and 134.
(2) Smith, Hamilton, Jr. Hydraulics, p. 120.
(3) Cippoletti, Caesar, Canale Villoresi, Milan 1886. p. 23.
Cippoletti accepted the results of Francis' experiments for end and bottom contractions. He also quotes a rule deduced by Lesbros from the results of his (Lebros) experiments, thatboth contractions should be at least 2.70 times the depth of the nappe and from the experiments of Francis, Cippoletti deduced the following; (a) When the end contraction is equal to 2 H and thebottom contraction equals 3 H, thebottom and walls have no longer an appreciable influence on the discharge of the weir. This condition may cause an increase of about 0.15 percent. (b) With an end contraction of 1.5 H and a bottom contraction of 2.5 H, theincrease in discharge would be about 0.5 percent. (c) If the end contraction is 1 H and the bottom contraction 2 H, the discharge will be increased about 1 percent. He also takes account of thefact that the velocity of approach shall not exceed a certain limit.
(4) Smith, Hamilton, Jr., Hydraulics, p.122.
(5) The coefficient using this expression of ratios, was proposed by J. Weissbach in 1845 andhas been elaborated upon by a great many. See Forchheimer, Phillipp Hydraulik, Leipsig, 1914, p.312

has been given[1] as 7, but Table 4 shows how the percentage of error of discharge for each ration of A:a, indicating that no fixed value of the ratio A:a can give a constant percentage of error, and that the value should be greater than 7 in any case, and probably 15 would meet average conditions.

Suppressed Bottom Contraction with 90° Triangular Notch.

In order to throw further light upon the question of the effect bottom contraction has upon the discharge over a triangular notch[2], experiments were made with the 90° triangular notch having the floor of the weir box level with the vertex. In this case the width of the weir box was 10 feet, the same as in the standard tests with complete contractions, but in the standard tests the floor was about 4.5 feet below the vertex.

The discharge over the 90° triangular notch with bottom contractions entirely suppressed, was found to be represented by the formula $Q = 2.53\ H^{2.496}$, which is, peculiarly, practically the same as Thompson's formula for the flow over the 90° triangular notch having complete contractions. It is probable that some part of the increased discharge obtained when the floor was placed at the level of the vertex, was due to increased velocity of approach. This increase in discharge amounted to 1.6 percent for a head of 1.0 feet, and gradually diminished as the head decreased. The percentage increase is represented by the formula $E = 101.6\ H^{.016} - 100$, which does not hold below a head equal to 0.3 feet.

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(1) Carpenter, L. G. Colorado Experiment Station, Bulletin 150, p.29
(2) Parker, A. Morley, Control of Water p. 114-115.

TRIANGULAR NOTCHES.

Little has been known concerning the flow through triangular weirs. General theoretical formulas have been given [1] and Thompson[2] experimented with the 90° triangular notch. No other experiments are known to have been recorded.

In the Fort Collins Hydraulic Laboratory ninety-eight tests were made with heads ranging from 0.2 feet to 1.35 feet, on triangular notch weirs of the following sizes; 120°, 90°, 60°, 30°, 28° 4'. The data for the last named notch were used in connection with derivation of the Cippoletti formula as given on page 38 .

Logarithmic plots, of the heads and corresponding discharges were made and found to be straight lines represented by the following equations;

120°	Triangular	Notch.	Slope =	1.732	$Q = 4.40\ H^{2.487}$
90°	"	"	"	1.0	$Q = 2.487\ H^{2.4805}$
60°	"	"	"	0.577	$Q = 1.446\ H^{2.4705}$
30°	"	"	"	0.268	$Q = 0.6848\ H^{2.4476}$
1 to 4 Slope	"	"(28°4)		0.25	$Q = 0.6405\ H^{2.4448}$

The discharging stream had a free fall for all the triangular notches with the exception of the 120° notch. The upper portion of the discharging stream over the 120° notch adhered to the "crest" for a distance of approximately 0.1 foot, along the "crest." This action was quite uniform for all heads. The sides of the notch were formed of

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(1) Horton U. S. G. S. W.S. Paper #200 p.46.
Merriman, Treatise on Hydraulics, 9th Edition p.168

(2) British Association Report 1858 p. 133.

brass $\frac{1}{4}$ inch thick, and dressed at an angle of about 45^{0} to make a "crest" thickness of about 1/32". The amount of adherence of nappe for the 120^{0} triangular notch will depend upon the thickness of the "crest" which makes its use impractical and because of the error due to adherence of nappe for triangular notch weirs with such flat sides, it is probable that no notch should be used with a slope greater than about 1.4.

Excluding the data for the 120^{0} notch, thegeneral formula for the discharge over triangular notch weirs was found to be;-

$$Q = (.025 + 2.462\ S)\ H^{\left(2.5\ -\ \frac{.0195}{S^{.75}}\right)}$$

where (S) is the slope of each side, and (H) is the head in feet.

It was found that the individual equation for the 120^{0} triangular notch should have been;-

$$Q = 4.289\ H^{2.4871}$$

to conform to the general formula stated above.

There is much in favor of the use of the 90^{0} triangular notch, especially. It is simple in construction, and requires no greater precautionary measures than other types of weirs. It is especially well adapted to the measurement of small flows but has a comparatively large range, delivering 4.33 second feet with a head of 1.25 feet and approximately 14 second feet with a head of 2.0 feet. It will permit of the accurate measurement of a stream too small to pass over a 0.5 trapezoidal or rectangular weir without adhering to the crest, and because of the apparently complicated conditions of flow produced in these weirs, the 90^{0} notch is much to be preferred. Since the 90^{0} notch is the practical size for general conditions, its individual formula may be taken as $Q = 2.49\ H^{2.48}$ which gives discharge values agreeing very closely with those obtained by the general formula found in Table 5.

TABLE 5

DISCHARGE TABLE FOR TRIANGULAR NOTCHES.

Computed From the Formula. $Q = (.025 + 2.462\ S)\ H^{\left(2.5 - \frac{.0195}{S^{.75}}\right)}$

Head in ft.	Head in feet & inches	$\frac{1}{4}$ Slope 28° 4'	30°	60°	90°
.20	0-2-3/8	0.012	0.013	0.027	0.046
.21	2-1/2	0.014	0.015	0.031	0.052
.22	2-5/8	0.016	0.017	0.034	0.058
.23	2-3/4	0.018	0.019	0.038	0.065
.24	2-7/8	0.020	0.021	0.043	0.072
.25	0-3	0.022	0.023	0.047	0.080
.26	3-1/8	0.024	0.025	0.052	0.088
.27	3-1/4	0.026	0.028	0.057	0.096
.28	3-3/8	0.029	0.030	0.062	0.105
.29	3-1/2	0.031	0.033	0.068	0.115
.30	0-3-5/8	0.034	0.036	0.074	0.125
.31	3-3/4	0.037	0.039	0.080	0.136
.32	3-13/16	0.040	0.042	0.087	0.147
.33	3-15/16	0.043	0.045	0.094	0.159
.34	4-1/16	0.046	0.049	0.101	0.171
.35	0-4-3/16	0.049	0.052	0.108	0.184
.36	4-5/16	0.053	0.056	0.116	0.197
.37	4-7/16	0.056	0.060	0.124	0.211
.38	4-9/16	0.060	0.064	0.132	0.225
.39	4-11/16	0.064	0.068	0.141	0.240
.40	0-4-13/16	0.068	0.073	0.150	0.256
.41	4-15/16	0.072	0.077	0.160	0.272
.42	5-1/16	0.077	0.082	0.170	0.289
.43	5-3/16	0.081	0.087	0.180	0.306
.44	5-1/4	0.086	0.092	0.190	0.324

Triangular Notches -2-

.45	0-5-3/8	0.091	0.097	0.201	0.343
.46	5-1/2	0.096	0.102	0.212	0.362
.47	5-5/8	0.101	0.108	0.224	0.382
.48	5-3/4	0.106	0.114	0.236	0.403
.49	5-7/8	0.112	0.120	0.248	0.424
.50	0-6	0.118	0.126	0.261	0.445
.51	6-1/8	0.123	0.132	0.274	0.468
.52	6-1/4	0.129	0.138	0.287	0.491
.53	6-3/8	0.136	0.145	0.301	0.515
.54	6-1/2	0.142	0.152	0.315	0.539
.55	0-6-5/8	0.148	0.159	0.330	0.564
.56	6-3/4	0.155	0.166	0.345	0.590
.57	6-13/16	0.162	0.173	0.360	0.617
.58	6-15/16	0.169	0.181	0.376	0.644
.59	7-1/16	0.176	0.188	0.392	0.672
.60	0-7-3/16	0.184	0.196	0.409	0.700
.61	7-5/16	0.191	0.204	0.426	0.730
.62	7-7/16	0.199	0.212	0.444	0.760
.63	7-9/16	0.207	0.221	0.462	0.790
.64	7-11/16	0.215	0.230	0.480	0.822
.65	0-7-13/16	0.223	0.239	0.499	0.854
.66	7-15/16	0.232	0.248	0.518	0.887
.67	8-1/16	0.241	0.257	0.537	0.921
.68	8-3/16	0.250	0.266	0.557	0.955
.69	8-1/4	0.259	0.276	0.578	0.991
.70	0-8-3/8	0.268	0.286	0.599	1.03
.71	8-1/2	0.277	0.296	0.620	1.06
.72	8-5/8	0.287	0.306	0.642	1.10
.73	8-3/4	0.297	0.317	0.664	1.14
.74	8-7/8	0.307	0.328	0.687	1.18
.75	0-9	0.317	0.339	0.710	1.22
.76	9-1/8	0.327	0.350	0.734	1.26
.77	9-1/4	0.338	0.361	0.758	1.30
.78	9-3/8	0.349	0.373	0.782	1.34
.79	9-1/2	0.360	0.385	0.807	1.39
.80	0-9-5/8	0.371	0.397	0.833	1.43
.81	9-3/4	0.383	0.409	0.859	1.48
.82	9-13/16	0.394	0.421	0.885	1.52
.83	9-15/16	0.406	0.434	0.912	1.57
.84	10-1/16	0.418	0.447	0.940	1.61

Triangular Notches -3-

.85	0-10-3/16	0.430	0.460	0.968	1.66
.86	10-5/16	0.443	0.473	0.996	1.71
.87	10-7/16	0.456	0.487	1.02	1.76
.88	10-9/16	0.469	0.501	1.05	1.81
.89	10-11/16	0.482	0.515	1.08	1.86
.90	10-13/16	0.495	0.529	1.11	1.92
.91	10-15/16	0.509	0.544	1.15	1.97
.92	11-1/16	0.522	0.558	1.18	2.02
.93	11-3/16	0.536	0.573	1.21	2.08
.94	11-1/4	0.551	0.589	1.24	2.13
.95	11-3/8	0.565	0.604	1.27	2.19
.96	11-1/2	0.580	0.620	1.31	2.25
.97	11-5/8	0.595	0.636	1.34	2.31
.98	11-3/4	0.610	0.652	1.38	2.37
.99	11-7/8	0.625	0.668	1.41	2.43
1.00	1-	0.641	0.685	1.45	2.49
1.01	0-1/8	0.656	0.702	1.48	2.55
1.02	0-1/4	0.672	0.719	1.52	2.61
1.03	0-3/8	0.688	0.736	1.56	2.68
1.04	0-1/2	0.705	0.754	1.59	2.74
1.05	1-0-5/8	0.722	0.772	1.63	2.81
1.06	0-3/4	0.739	0.790	1.67	2.87
1.07	0-13/16	0.756	0.808	1.71	2.94
1.08	0-15/16	0.773	0.827	1.75	3.01
1.09	1-1/16	0.791	0.846	1.79	3.08
1.10	1-1-3/16	0.809	0.865	1.83	3.15
1.11	1-5/16	0.827	0.884	1.87	3.22
1.12	1-7/16	0.845	0.904	1.91	3.30
1.13	1-9/16	0.864	0.924	1.96	3.37
1.14	1-11/16	0.882	0.944	2.00	3.44
1.15	1-1-13/16	0.901	0.964	2.04	3.52
1.16	1-15/16	0.921	0.985	2.09	3.59
1.17	2-1/16	0.940	1.01	2.13	3.67
1.18	2-3/16	0.960	1.03	2.18	3.75
1.19	2-1/4	0.980	1.05	2.22	3.83
1.20	1-2-3/8	1.00	1.07	2.27	3.91
1.21	2-1/2	1.02	1.09	2.32	3.99
1.22	2-5/8	1.04	1.11	2.36	4.07
1.23	2-3/4	1.06	1.14	2.41	4.16
1.24	2-7/8	1.08	1.16	2.46	4.24
1.25	1-3	1.11	1.19	2.51	4.33

Comparison of the 90° Triangular Notch Formulas:- The discharge indicated by the old andnew formulas for the 90° triangular notch are shown in Table 6 . These indicate the old values to be too great.

Since no general formulas with experimental values of (c) had been used for the various sizes of notches, no comparison can be made with past practics.

TABLE 6

COMPARISON OF OLD AND NEW FORMULAS (1) for 90° TRIANGULAR NOTCH

$$Q = 2.49\ H^{2.48}\ \text{(new)}$$

$$Q = 2.53\ H^{2.50}\ \text{(old)}$$

Head in Feet	New Q	Old Q	Old %
.20	.046	.045	97.8
.33	.159	.158	99.4
.50	.445	.447	100.4
.67	.921	.930	101.0
.85	1.66	1.69	101.8
1.00	2.49	2.53	101.6
1.25	4.33	4.42	102.1

(1) Relation of values for old and new formulas is shown by percentage, taking values of new formula to be 100%.

RECTANGULAR WEIRS.

The actual crest lengths of the weirs used in these experiments were 0.50721 foot, 1.0055 feet, 1.5026 feet, 2.0057 feet, 2.9970 feet, and 4.0056 feet, these lengths being used in all computations connected with the derivation of the formula.

The heads and corresponding discharges found by experiment for the various lengths of weir crests were plotted on a large scale to permit values to be taken from the curve to the third decimal place. The discharge values taken from this curve for 0.05 feet, increments of head were used in the following deductions because experimental irregularities were thereby largely eliminated.

A series of plots were made, with lengths of crest (L) as abcissas, and discharges (Q) as ordinates, having the head (H) constant for individual plottings. Straight lines were drawn tangent to the curves through the 3 and 4 foot crest lengths, and were of the form $Q' = aL - b$. The relation between the head (H) and (a) in the above formula was found from logarithmic plots, to be represented by the equation;

$$a = 3.25\ LH^{1.48}$$

The relation between the head (H) and (b) in the equation $Q' = aL - b$, was found by means of curves to be represented by the equation;

$$b = 0.283\ H^{1.9}$$

The offsets from the tangent lines to the curves were tabulated and an expression for them determined to be

$$\frac{0.283\ H^{1.9}}{1+2\ L^{1.8}}$$

This resulted in the following formula, which gives discharge values within a maximum of approximately $\frac{3}{4}$ of 1 percent of the values indicated on the curves plotted from the experimental data, but the average agreement is within $\frac{1}{2}$ of 1 percent.

$$Q = 3.247\ LH^{1.48} - 0.283\ H^{1.9} + \frac{0.283\ H^{1.9}}{1+2\ L^{1.8}}$$

Which reduces to,

$$Q = 3.247\ L\ H^{1.48} - \frac{0.586\ L^{1.8}}{1+2\ L^{1.8}}\ H^{1.9}$$

Discharge values computed from the above formula are given in Table 7 for exact crest lengths of 1.0, 1.5, 2.0, 3.0 and 4.0 feet. (Fig. 3)

It was found that the discharge for a rectangular weir having a crest length of 0.5 feet, did not follow the same law as for larger weirs, probably because of the greater effect of friction in the smaller weir and also because of the interference of end contraction filaments of flow crossing each other in the middle of the weir section. An individual formula was therefore devised for the 0.5 foot rectangular weir,

$$Q = 1.593\ H^{1.526} \left(1 \quad \frac{1}{800\ H^{2.3}}\right)$$

The discharge values for the 0.5 foot rectangular weir may also be represented by the logarithmic straight line formula

$$Q = 1.566\ H^{1.504}$$

which will give discharge values that will agree within approximately 1 percent of values indicated on the curve plotted from the experimental

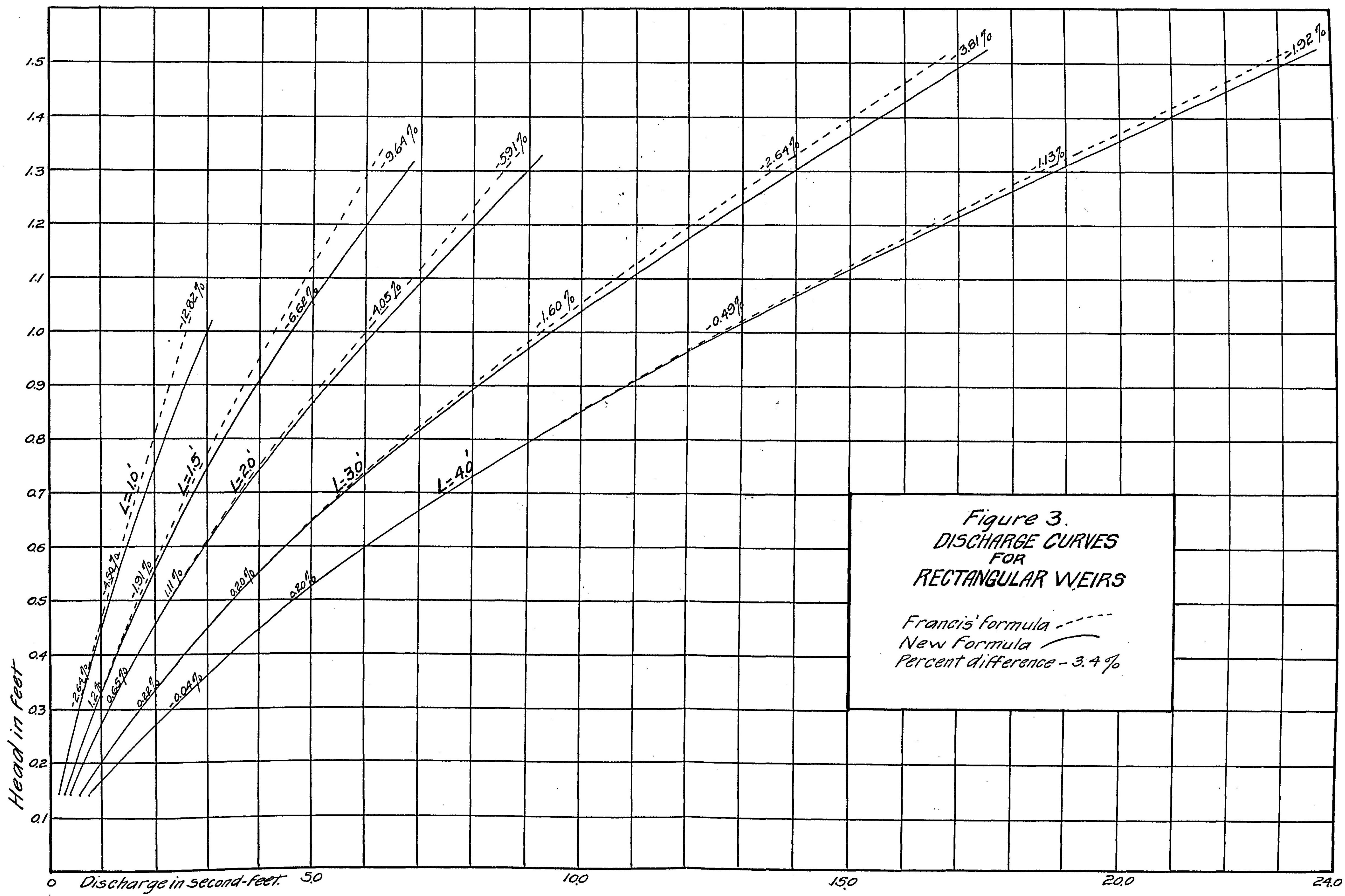


Figure 3.
DISCHARGE CURVES FOR RECTANGULAR WEIRS

TABLE 7

DISCHARGE TABLES FOR RECTANGULAR WEIRS

Computed From the Formula.

$$Q = 3.247\, L\, H^{1.48} - \frac{0.566\, L^{1.8}}{1 \quad 2\, L^{1.8}}\, H^{1.9}$$

Head in Feet	Head in Inches	Discharge in Cubic Feet per Second 1.0′	1.5′	2.0′	3.0′	4.0′
.20	2-3/8	.291	.439	.588	.887	1.19
.21	2-1/2	.312	.472	.632	.954	1.28
.22	2-5/8	.335	.505	.677	1.02	1.37
.23	2-3/4	.358	.539	.723	1.09	1.46
.24	2-7/8	.380	.574	.769	1.16	1.55
.25	3	.404	.609	.817	1.23	1.65
.26	3-1/8	.428	.646	.865	1.31	1.75
.27	3-1/4	.452	.682	.914	1.38	1.85
.28	3-3/8	.477	.720	.965	1.46	1.95
.29	3-1/2	.502	.758	1.02	1.53	2.05
.30	3-5/8	.527	.796	1.07	1.61	2.16
.31	3-3/4	.553	.836	1.12	1.69	2.27
.32	3-13/16	.580	.876	1.18	1.77	2.37
.33	3-15/16	.606	.916	1.23	1.86	2.48
.34	4-1/16	.634	.957	1.28	1.94	2.59
.35	4-3/16	.661	.999	1.34	2.02	2.71
.36	4-5/16	.688	1.04	1.40	2.11	2.82
.37	4-7/16	.717	1.08	1.45	2.20	2.94
.38	4-9/16	.745	1.13	1.51	2.28	3.06
.39	4-11/16	.774	1.17	1.57	2.37	3.18
.40	4-13/16	.804	1.22	1.63	2.46	3.30
.41	4-15/16	.833	1.26	1.69	2.55	3.42
.42	5-1/16	.863	1.31	1.75	2.65	3.54
.43	5-3/16	.893	1.35	1.81	2.74	3.67
.44	5-1/4	.924	1.40	1.88	2.83	3.80

Rectangular Weirs -2-

.45	-5-3/8	.955	1.44	1.94	2.93	3.93
.46	5-1/2	.986	1.49	2.00	3.03	4.05
.47	5-5/8	1.02	1.54	2.07	3.12	4.18
.48	5-3/4	1.05	1.59	2.13	3.22	4.32
.49	5-7/8	1.08	1.64	2.20	3.32	4.45
.50	-6	1.11	1.69	2.26	3.42	4.58
.51	6-1/8	1.15	1.73	2.33	3.52	4.72
.52	6-1/4	1.18	1.78	2.40	3.62	4.86
.53	6-3/8	1.21	1.84	2.46	3.73	4.99
.54	6-1/2	1.25	1.89	2.53	3.83	5.13
.55	-6-5/8	1.28	1.94	2.60	3.94	5.27
.56	6-3/4	1.31	1.99	2.67	4.04	5.42
.57	6-13/16	1.35	2.04	2.74	4.15	5.56
.58	6-15/16	1.38	2.09	2.81	4.26	5.70
.59	7-1/16	1.42	2.15	2.88	4.36	5.85
.60	-7-3/16	1.45	2.20	2.96	4.47	6.00
.61	7-5/16	1.49	2.25	3.03	4.58	6.14
.62	7-7/16	1.52	2.31	3.10	4.69	6.29
.63	7-9/16	1.56	2.36	3.17	4.81	6.44
.64	7-11/16	1.60	2.42	3.25	4.92	6.59
.65	-7-13/16	1.63	2.47	3.32	5.03	6.75
.66	7-15/16	1.67	2.53	3.40	5.15	6.90
.67	8-1/16	1.71	2.59	3.47	5.26	7.05
.68	8-3/16	1.74	2.64	3.56	5.38	7.21
.69	8-1/4	1.78	2.70	3.63	5.49	7.37
.70	8-3/8	1.82	2.76	3.71	5.61	7.52
.71	8-1/2	1.86	2.82	3.78	5.73	7.68
.72	8-5/8	1.90	2.87	3.86	5.85	7.84
.73	8-3/4	1.93	2.93	3.94	5.97	8.00
.74	8-7/8	1.97	2.99	4.02	6.09	8.17
.75	-9	2.01	3.05	4.10	6.21	8.33
.76	9-1/8	2.05	3.11	4.18	6.33	8.49
.77	9-1/4	2.09	3.17	4.26	6.46	8.66
.78	9-3/8	2.13	3.23	4.34	6.58	8.82
.79	9-1/2	2.17	3.29	4.42	6.70	8.99

Rectangular Weirs -3-

.80	0-9-5/8	2.21	3.35	4.51	6.83	9.16
.81	9-3/4	2.25	3.41	4.59	6.96	9.33
.82	9-13/16	2.29	3.47	4.67	7.08	9.50
.83	9-15/16	2.33	3.54	4.75	7.21	9.67
.84	10-1/16	2.37	3.60	4.84	7.34	9.84
.85	0-10-3/16	2.41	3.66	4.92	7.46	10.01
.86	10-5/16	2.45	3.73	5.01	7.59	10.19
.87	10-7/16	2.50	3.79	5.10	7.72	10.36
.88	10-9/16	2.54	3.85	5.18	7.85	10.54
.89	10-11/16	2.58	3.92	5.27	7.97	10.71
.90	0-10-13/16	2.62	3.98	5.35	8.12	10.89
.91	10-15/16	2.67	4.05	5.44	8.25	11.07
.92	11-1/16	2.71	4.11	5.53	8.38	11.25
.93	11-3/16	2.75	4.18	5.62	8.52	11.43
.94	11-1/4	2.80	4.24	5.71	8.65	11.61
.95	0-11-3/8	2.84	4.31	5.79	8.79	11.79
.96	11-1/2	2.88	4.37	5.89	8.93	11.98
.97	11-5/8	2.93	4.44	5.98	9.06	12.16
.98	11-3/4	2.97	4.51	6.07	9.20	12.34
.99	11-7/8	3.01	4.57	6.15	9.34	12.53
1.00	0-12	3.06	4.64	6.25	9.48	12.72
1.01	12-1/8		4.71	6.34	9.62	12.91
1.02	12-1/4		4.78	6.43	9.76	13.10
1.03	12-3/8		4.85	6.52	9.90	13.29
1.04	12-1/2		4.92	6.62	10.04	13.47
1.05	12-5/8		4.99	6.71	10.18	13.66
1.06	12-3/4		5.05	6.80	10.32	13.85
1.07	12-13/16		5.12	6.90	10.46	14.04
1.08	12-15/16		5.19	6.99	10.61	14.24
1.09	13-1/16		5.26	7.09	10.75	14.43
1.10	13-3/16		5.34	7.19	10.90	14.64
1.11	13-5/16		5.41	7.28	11.05	14.83
1.12	13-7/16		5.48	7.38	11.20	15.03
1.13	13-9/16		5.55	7.47	11.34	15.22
1.14	13-11/16		5.62	7.57	11.49	15.42

Rectangular Weirs -4-

1.15	13-13/16	5.69	7.66	11.64	15.62
1.16	13-15/16	5.77	7.76	11.79	15.82
1.17	14-1/16	5.84	7.86	11.94	16.02
1.18	14-3/16	5.91	7.96	12.09	16.23
1.19	14-1/4	5.99	8.06	12.24	16.43
1.20	14-3/8	6.06	8.16	12.39	16.63
1.21	14-1/2	6.13	8.26	12.54	16.83
1.22	14-5/8	6.20	8.35	12.69	17.03
1.23	14-3/4	6.28	8.46	12.85	17.25
1.24	14-7/8	6.35	8.56	13.00	17.45
1.25	15	6.43	8.66	13.15	17.65
1.26	15-1/8			13.30	17.87
1.27	15-1/4			13.45	18.07
1.28	15-3/8			13.61	18.28
1.29	15-1/2			13.77	18.50
1.30	15-5/8			13.93	18.71
1.31	15-3/4			14.09	18.92
1.32	15-13/16			14.24	19.13
1.33	15-15/16			14.40	19.34
1.34	16-1/16			14.56	19.55
1.35	16-3/16			14.72	19.77
1.36	16-5/16			14.88	19.98
1.37	16-7/16			15.04	20.20
1.38	16-9/16			15.20	20.42
1.39	16-11/16			15.36	20.64
1.40	16-13/16			15.53	20.86
1.41	16-15/16			15.69	21.08
1.42	17-1/16			15.85	21.29
1.43	17-3/16			16.02	21.52
1.44	17-1/4			16.19	21.74
1.45	17-3/8			16.34	21.96
1.46	17-1/2			16.51	22.18
1.47	17-5/8			16.68	22.41
1.48	17-3/4			16.85	22.64
1.49	17-7/8			17.01	22.85
1.50	18			17.18	23.08

data for this notch. This small weir gives a discharge curve consistent in itself, but since its range of application is very limited and it posses peculiarities of its own, there seems to be little practical reason for its use. The 90° triangular notch is at least as accurate and far more satisfactory.

Comparison of Old and New Rectangular Weir Formulas: The discharge indicated for the old and new formulas for rectangular weirs are shown in graphic and tabular form in Fig. 3 and Table 8. It will be seen from these curves and the table, that the data obtained in these experiments agree fairly well with plottings from the Francis formula within the range of $H=\frac{L}{3}$. Furthermore these data support the statement of Francis that the formula proposed by him is correct within 2 percent when the limit of ratio of 1 to 3 of head to length is not exceeded. However, these data do not indicate any necessity for keeping within that limit if the proper formula is used for computing (Q); they even indicate a greater possible degree of accuracy for the higher heads; there is no sudden break or change of direction in the flow curve; and the limitation of (H) to (L)/3, is apparently necessary for the formula to which it was applied, but is not due to any peculiarity of the weir. In other words, the limit of use was imposed upon the formula, but the short-coming implied thereby is mathematical and not inherent in the weir. The use of the new formulas presented herein, not only provide a greater degree of accuracy but also extend the limits of use of weirs. The maximum limits of the ratio of head to crest length to which these formulas apply, is not known, but they hold for the data for the 1 foot weirs for heads of 1 foot, which was the greatest ratio of head to the length tried on weirs to which the general formulas apply, and for all weirs coming within the application of the new formula. A head of 1 foot was run, however, upon a weir with 0.5 foot crest length, but the flows for this weir follow a different formula. For all of the weirs experimented with the upper portions of the discharge curves are quite consistent. The new formula is more complicated than the old one, but since tables are generally consulted for determining the flow over weirs, especially when delivering water to the irrigator, the practical disadvantage of the new formula is largely overcome. When one is obliged to use a formula for computing the discharge in the field, an approximation is usually sufficient, in which case the old formula is sufficiently accurate, for any practical head over the weir. Although a weir table should be based upon the most accurate formula available, the computed discharge should not be expressed to a greater degree of exactness than that with which the head may be determined.

TABLE 8

COMPARISON OF OLD AND NEW FORMULAS.

Head in Feet	1 Foot Weir New Q	1 Foot Weir Old Q	1 Foot Weir Old %	1.5 Foot Weir New Q	1.5 Foot Weir Old Q	1.5 Foot Weir Old %	2 Foot Weir New Q	2 Foot Weir Old Q	2 Foot Weir Old %	3 Foot Weir New Q	3 Foot Weir Old Q	3 Foot Weir Old %	4 Foot Weir New Q	4 Foot Weir Old Q	4 Foot Weir Old %
RECTANGULAR WEIRS															
.15	.191	.188	98.4	.288	.284	98.6	.385	.381	99.0	.581	.575	99.0	.776	.768	99.0
.33	.606	.590	97.4	.916	.905	98.8	1.22	1.22	99.4	1.86	1.85	99.8	2.48	2.48	100.0
.50	1.11	1.06	95.5	1.68	1.65	98.1	2.26	2.24	98.9	3.42	3.41	99.8	4.58	4.59	100.2
.67	1.71	1.58	92.6	2.59	2.49	96.3	3.47	3.41	98.1	5.26	5.23	99.5	7.05	7.06	100.1
.85	2.41	2.17	89.8	3.66	3.47	94.8	4.92	4.78	97.0	7.46	7.38	99.0	10.01	9.99	99.9
1.00	3.06	2.67	87.2	4.64	4.33	93.4	6.24	5.99	96.0	9.48	9.32	98.4	12.72	12.65	99.5
1.25				6.43	5.81	90.4	8.65	8.14	94.1	13.14	12.80	97.4	17.65	17.45	98.9
1.33										14.40	13.96	97.0	19.34	19.07	98.6
1.50										17.17	16.52	96.2	23.08	22.64	98.1
1.75															
2.00															
CIPOLLETTI WEIRS															
.20	.302	.301	99.7	.450	.452	100.4	.599	.602	100.5	.898	.903	100.4	1.198	1.205	100.6
.33	.644	.638	99.1	.954	.957	100.3	1.27	1.28	100.8	1.89	1.91	101.1	2.52	2.55	101.2
.50	1.22	1.19	97.5	1.79	1.78	99.4	2.37	2.38	100.4	3.53	3.57	101.1	4.69	4.76	101.5
.67	1.93	1.85	95.9	2.81	2.77	98.6	3.70	3.69	99.7	5.49	5.54	100.9	7.28	7.38	101.4
.85	2.82	2.64	93.6	4.07	3.97	97.5	5.33	5.28	99.1	7.87	7.91	100.5	10.42	10.55	101.2
1.00	3.67	3.37	91.8	5.25	5.05	96.2	6.86	6.73	98.1	10.09	10.10	100.1	13.33	13.47	101.1
1.25				7.49	7.06	94.3	9.72	9.41	96.8	14.21	14.11	99.3	18.72	18.82	100.5
1.33										15.64	15.49	99.0	20.58	20.65	100.3
1.50										18.85	18.55	98.4	24.75	24.74	100.0

Relation of values for old and new formulas is shown by percentage, taking values of new formula to be 100%.

TRAPEZOIDAL WEIRS

Cippoletti Weirs; Trapezoidal weirs having side slopes of 1 to 4, as designed by the Italian engineer, Caesar Cippoletti, are very extensively used in the irrigated west, and were therefore experimented upon. The actual crest lengths were 0.50062 feet, 1.0050 feet, 1.5028 feet, 2.0002 feet, 3.0011 feet and 4.0058 feet, and these figures were used throughout the calculation, the nominal lengths being used merely for reference purposes.

Since the difference in the areas of Cippoletti and rectangular weir notches of equal crest lengths is represented by a triangular notch having 1 to 4 side slopes, or approximately a $28^{\circ}\ 4'$ angle, the discharges were determined experimentally for such a notch. It was found, however, that the discharge over this notch did not exactly equal the difference between the discharges for Cippoletti and rectangular weirs, which differences increase with the head for all lengths of weirs; there is no regular increase or decrease apparent with an increase in the crest length for heads up to approximately 0.8 feet, but for higher heads the differences in discharges decrease as the length increases; The comparison of these difference is very unreliable for heads as low as 0.2 and 0.3 feet; and the discharge over the 1 to 4 slope notch is greater than the difference between the discharges for Cippoletti and rectangular weirs, this percentage of excess of discharge decreases with an increase in head, and equals zero when the head equals approximately 2.5 feet.

From large scale plots of the experimental data the differences in discharge over rectangular and Cippoletti weirs for each 0.1 foot head, and each length of weir were taken. For each head the values of

~~the values of~~ these differences for the several lengths were averaged and plotted logarithmically against the head. Then from this curve the smoothed values of the differences were found to be represented by the equation

$$\text{Cip. Q.} - \text{Rect. Q} = .609\ H^{2.5}$$

Therefore, adding this term to the general formula for the discharge over rectangular weirs, will give the formula for the discharge over Cippoletti weirs, which is

$$Q = 3.247\ L\ H^{1.48} - \frac{0.566\ L^{1.8}}{1 + 2\ L^{1.8}}\ H^{1.9} + 0.609\ H^{2.5}$$

This formula gives discharge values that agree with in $\frac{1}{2}$ of 1 percent of the values indicated on the curves plotted from the experimental data for the 1, $1\frac{1}{2}$, 2, 3 and 4 foot notches, except for the 0.2 and 0.3 foot heads where the discrepancy is approximately 1 percent. The discrepancies are positive in some cases and negative in others.

The Cippoletti weir having a nominal crest length of 0.5 foot did not give a discharge following the same law as the larger weirs, possibly for the reasons noted on Page 30 for the 0.5 foot rectangular weir. Its use should be discouraged in favor of the 90° triangular notch. The following formula represented the flow over the 0.5 foot cippoletti weir, and is stated here for technical reasons only -

$$Q = 1.593\ H^{1.526} \left(1 + \frac{1}{800\ H^{2.3}}\right) + 0.587\ H^{2.53}$$

The discharge for the 0.5 foot Cippoletti Weir may also be represented by the equation,

$$Q = 1.566\ H^{1.504} + 0.56\ H^{2.55}$$

It will be noted that the last term of this equation represents the difference in the discharges over the rectangular and Cippoletti notches having 0.5 for the crest lengths.

The last term of the above formula represents the difference in discharge between the Cippoletti and rectangular weirs with 0.5 foot crest length.

Cippoletti weirs do not give a discharge proportional to their length, as is shown by the discussion on Page 47. (Relation of Length to Discharge).

TABLE 9

DISCHARGE TABLES FOR CIPOLLETTI WEIRS

Computed From the Formula.

$$Q = 3.247\ L\ H^{1.48} - \frac{0.566\ L^{1.8}}{1 + 2\ L^{1.8}}\ H^{1.9} + 0.609\ H^{2.5}$$

Head in Feet	Head in Feet & Inches	1.0	1.5	2.0	3.0	4.0
.20	0-2-3/8	0.30	0.45	0.60	0.90	1.20
.21	2-1/2	0.32	0.48	0.64	0.97	1.29
.22	2-5/8	0.35	0.52	0.69	1.04	1.38
.23	2-3/4	0.37	0.55	0.74	1.11	1.47
.24	2-7/8	0.39	0.59	0.79	1.18	1.57
.25	0-3	0.42	0.63	0.84	1.25	1.67
.26	3-1/8	0.45	0.67	0.89	1.33	1.77
.27	3-1/4	0.47	0.70	0.94	1.40	1.87
.28	3-3/8	0.50	0.74	0.99	1.48	1.97
.29	3-1/2	0.53	0.79	1.04	1.56	2.08
.30	0-3-5/8	0.56	0.83	1.10	1.64	2.19
.31	3-3/4	0.59	0.87	1.15	1.73	2.30
.32	3-13/16	0.61	0.91	1.21	1.80	2.41
.33	3-15/16	0.64	0.95	1.27	1.89	2.52
.34	4-1/16	0.67	1.00	1.32	1.98	2.64
.35	0-4-3/16	0.70	1.04	1.38	2.07	2.75
.36	4-5/16	0.73	1.09	1.44	2.16	2.87
.37	4-7/16	0.77	1.13	1.50	2.25	2.99
.38	4-9/16	0.80	1.18	1.57	2.34	3.11
.39	4-11/16	0.83	1.23	1.63	2.43	3.24
.40	0-4-13/16	0.87	1.28	1.69	2.53	3.36
.41	4-15/16	0.90	1.32	1.76	2.62	3.49
.42	5-1/16	0.93	1.37	1.82	2.72	3.61
.43	5-3/16	0.97	1.42	1.89	2.81	3.74
.44	5-1/4	1.00	1.47	1.95	2.91	3.87

Cipolletti Weirs -2-

.45	0-5-3/8	1.04	1.53	2.02	3.01	4.01
.46	5-1/2	1.07	1.58	2.09	3.11	4.14
.47	5-5/8	1.11	1.63	2.16	3.21	4.28
.48	5-3/4	1.15	1.68	2.23	3.32	4.41
.49	5-7/8	1.18	1.74	2.30	3.42	4.55
.50	0-6	1.22	1.79	2.37	3.53	4.69
.51	6-1/8	1.26	1.85	2.44	3.64	4.83
.52	6-1/4	1.30	1.90	2.51	3.74	4.97
.53	6-3/8	1.34	1.96	2.59	3.85	5.12
.54	6-1/2	1.38	2.02	2.66	3.96	5.26
.55	0-6-5/8	1.42	2.07	2.74	4.07	5.41
.56	6-3/4	1.46	2.13	2.81	4.18	5.56
.57	6-13/16	1.50	2.19	2.89	4.30	5.71
.58	6-15/16	1.54	2.25	2.97	4.41	5.86
.59	7-1/16	1.58	2.31	3.05	4.53	6.01
.60	0-7-3/16	1.62	2.37	3.13	4.64	6.17
.61	7-5/16	1.67	2.43	3.20	4.76	6.32
.62	7-7/16	1.71	2.49	3.28	4.88	6.47
.63	7-9/16	1.75	2.55	3.37	5.00	6.63
.64	7-11/16	1.80	2.62	3.45	5.12	6.79
.65	0-7-13/16	1.84	2.68	3.53	5.24	6.95
.66	7-15/16	1.89	2.75	3.61	5.36	7.11
.67	8-1/16	1.93	2.81	3.70	5.48	7.28
.68	8-3/16	1.98	2.87	3.79	5.61	7.44
.69	8-1/4	2.02	2.94	3.87	5.73	7.61
.70	0-8-3/8	2.07	3.01	3.95	5.86	7.77
.71	8-1/2	2.12	3.07	4.04	5.99	7.94
.72	8-5/8	2.16	3.14	4.13	6.12	8.11
.73	8-3/4	2.21	3.21	4.22	6.24	8.28
.74	8-7/8	2.26	3.28	4.31	6.38	8.45
.75	0-9	2.31	3.35	4.40	6.51	8.62
.76	9-1/8	2.36	3.42	4.49	6.64	8.80
.77	9-1/4	2.41	3.49	4.58	6.77	8.97
.78	9-3/8	2.46	3.56	4.67	6.90	9.15
.79	9-1/2	2.51	3.63	4.76	7.04	9.33

Cipolletti Weirs -3-

.80	0-9-5/8	2.56	3.70	4.85	7.18	9.51
.81	9-3/4	2.61	3.77	4.95	7.31	9.69
.82	9-13/16	2.66	3.84	5.04	7.45	9.87
.83	9-15/16	2.71	3.92	5.14	7.59	10.05
.84	10-1/16	2.77	3.99	5.23	7.73	10.23
.85	0-10-3/16	2.82	4.07	5.33	7.87	10.42
.86	0-10-5/16	2.87	4.14	5.43	8.01	10.60
.87	10-7/16	2.93	4.22	5.52	8.15	10.79
.88	10-9/16	2.98	4.29	5.62	8.30	10.98
.89	10-11/16	3.04	4.37	5.72	8.44	11.17
.90	0-10-13/16	3.09	4.45	5.82	8.59	11.36
.91	10-15/16	3.15	4.53	5.92	8.73	11.55
.92	11-1/16	3.20	4.60	6.02	8.88	11.74
.93	11-3/16	3.26	4.68	6.13	9.03	11.94
.94	11-1/4	3.32	4.76	6.23	9.17	12.13
.95	0-11-3/8	3.37	4.84	6.33	9.32	12.33
.96	11-1/2	3.43	4.92	6.44	9.48	12.53
.97	11-5/8	3.49	5.00	6.55	9.62	12.72
.98	11-3/4	3.55	5.09	6.64	9.78	12.92
.99	11-7/8	3.61	5.17	6.75	9.93	13.12
1.00	1-0	3.67	5.25	6.86	10.08	13.32
1.01	0-1/8		5.33	6.96	10.24	13.53
1.02	0-1/4		5.42	7.07	10.40	13.73
1.03	0-3/8		5.50	7.18	10.55	13.94
1.04	0-1/2		5.59	7.29	10.71	14.15
1.05	0-5/8		5.67	7.40	10.87	14.35
1.06	0-3/4		5.76	7.51	11.03	14.56
1.07	0-13/16		5.84	7.62	11.18	14.76
1.08	0-15/16		5.93	7.73	11.35	14.98
1.09	1-1/16		6.02	7.84	11.51	15.19
1.10	1-1-3/16		6.11	7.96	11.68	15.41
1.11	1-5/16		6.20	8.07	11.84	15.62
1.12	1-7/16		6.29	8.18	12.00	15.84
1.13	1-9/16		6.37	8.29	12.16	16.04
1.14	1-11/16		6.46	8.41	12.33	16.26

Cipolletti Weirs -4-

1.15	1-1-13/16	6.56	8.53	12.50	16.48
1.16	1-15/16	6.65	8.65	12.67	16.70
1.17	2-1/16	6.74	8.76	12.84	16.93
1.18	2-3/16	6.83	8.88	13.01	17.15
1.19	2-1/4	6.93	9.10	13.18	17.37
1.20	1-2-3/8	7.02	9.12	13.35	17.59
1.21	2-1/2	7.11	9.24	13.52	17.81
1.22	2-5/8	7.20	9.36	13.69	18.03
1.23	2-3/4	7.30	9.48	13.87	18.27
1.24	2-7/8	7.40	9.60	14.04	18.49
1.25	1-3	7.49	9.72	14.21	18/71
1.26	3-1/8			14.39	18.95
1.27	3-1/4			14.56	19.17
1.28	3-3/8			14.74	19.41
1.29	3-1/2			14.92	19.65
1.30	1-3-5/8			15.11	19.88
1.31	3-3/4			15.29	20.12
1.32	3-13/16			15.46	20.34
1.33	3-15.16			15.64	20.58
1.34	4-1/16			15.82	20.82
1.35	1-4-3/16			16.01	21.06
1.36	4-5/16			16.19	21.29
1.37	4-7/16			16.37	21.53
1.38	4-9/16			16.57	21.78
1.39	4-11/16			16.75	22.02
1.40	1-4-13/16			16.94	22.27
1.41	4-15/16			17.13	22.51
1.42	5-1/16			17.31	22.75
1.43	5-3/16			17.51	23.01
1.44	5r1/4			17.70	23.26
1.45	1-5-3/8			17.89	23.50
1.46	5-1/2			18.08	23.75
1.47	5-5/8			18.28	24.01
1.48	5-3/4			18.47	24.26
1.49	5-7/8			18.66	24.50
1.50	1-6			18.85	24.75

Weirs With Odd Sided Slopes; Experiments were made with weirs having crest lengths of 2.0 foot and side slopes of 1 to 3, and 1 to 6, as well as 1 to 4 and rectangular, for the purpose of adding to our knowledge of the effect of side slopes on proportionality of discharge to crest length, which is discussed on Page 47. Since only the one length was used, a general equation was not secured for these odd slope weirs, but the discharge over them is shown graphically in Figure 4, which also includes the discharge over the 1 to 4 slope and rectangular weir for comparative purposes.

Comparison of Old and New Cippoletti Weir Formulas; The discharge indicated by the old and new formulas for Cippoletti weirs are shown in graphic and tabular form in Fig. 5 and Table 8. From which it will be seen that the error does not exceed 2% within the limit of the ratio of 1 to 3 of head to length.

The general statements concerning the application of the new formula and limitations of the old formula for rectangular weirs given on page 35 applies also to the Cippoletti weir.

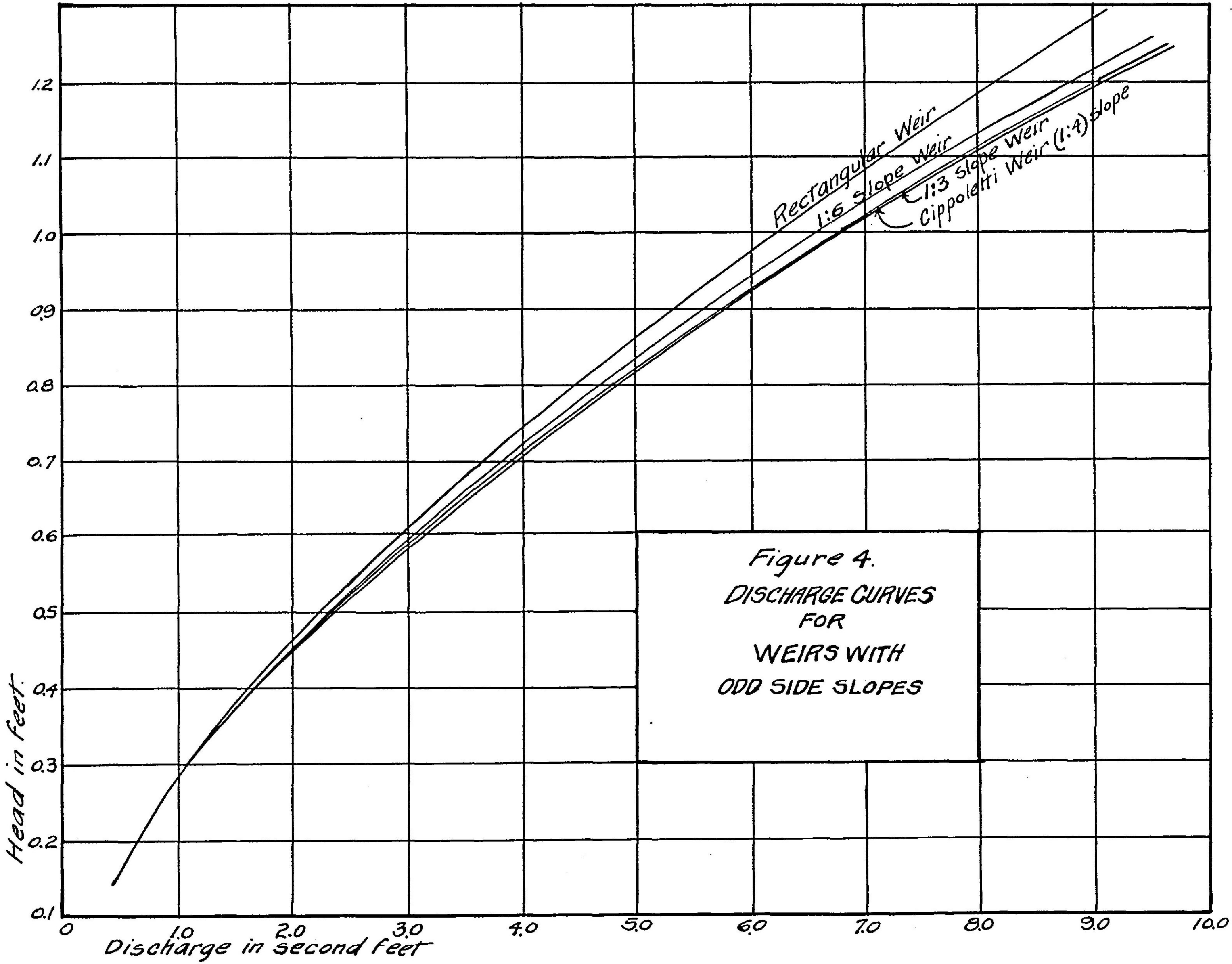


Figure 4.
DISCHARGE CURVES
FOR
WEIRS WITH
ODD SIDE SLOPES
Rectangular Weir
1:6 Slope Weir
1:3 Slope Weir
Cippoletti Weir (1:4) slope
Head in feet.
Discharge in second feet
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0

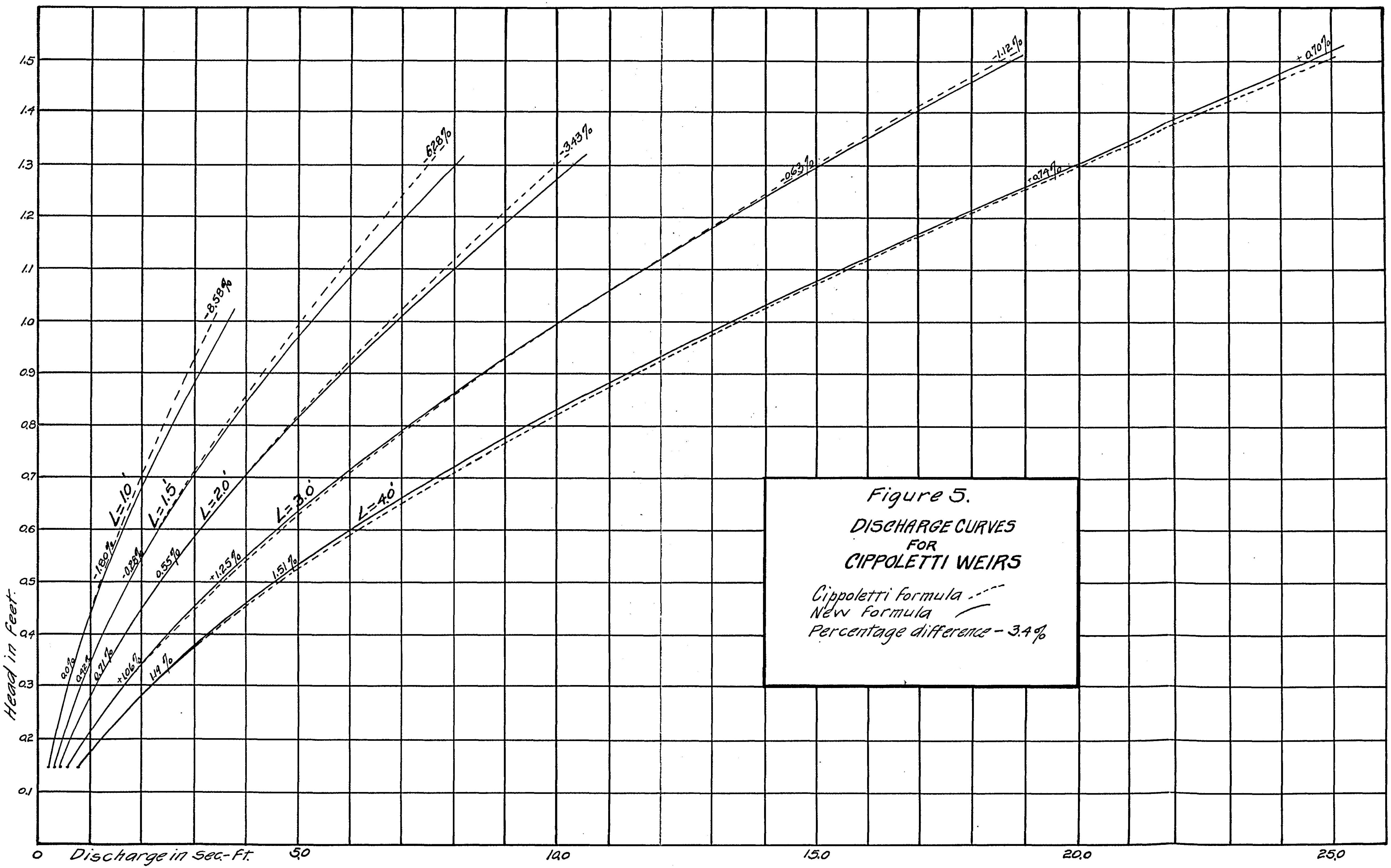


Figure 5.
DISCHARGE CURVES
FOR
CIPPOLETTI WEIRS
Cippoletti formula
New formula
Percentage difference - 3.4%
Head in feet.
Discharge in Sec.-Ft.
0
5.0
10.0
15.0
20.0
25.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
L=1.0'
L=1.5'
L=2.0'
L=3.0'
L=4.0'
-8.58%
-6.28%
-3.43%
-1.12%
-0.63%
+0.74%
+0.70%
-1.80%
-0.28%
0.55%
+1.25%
1.51%
0.0%
0.92%
0.71%
+1.06%
1.19%

CIRCULAR WEIRS

Apparently no experiments have even been made upon circular or semi-circular weirs in a vertical position with heads less than the diameter of the opening. In order to throw light upon the probable discharge through weirs of this shape, and for reference in connection with the flow through circular headgates when acting as a weir and not as an orifice, experiments were made on sharp crested circular weirs of 0.4995 feet and 1.0025 feet, diameters. The discharge data are plotted in Fig. 6, which also shows the agreement of the experimental data with the values computed from the formula.

An attempt was made to obtain a formula which would hold for the changing conditions from an orifice action to a weir action but the data were too limited. However an equation was obtained by a rather extensive series or ratio plots. The method of derivation is omitted because of the large scale plots necessary to a proper presentation.

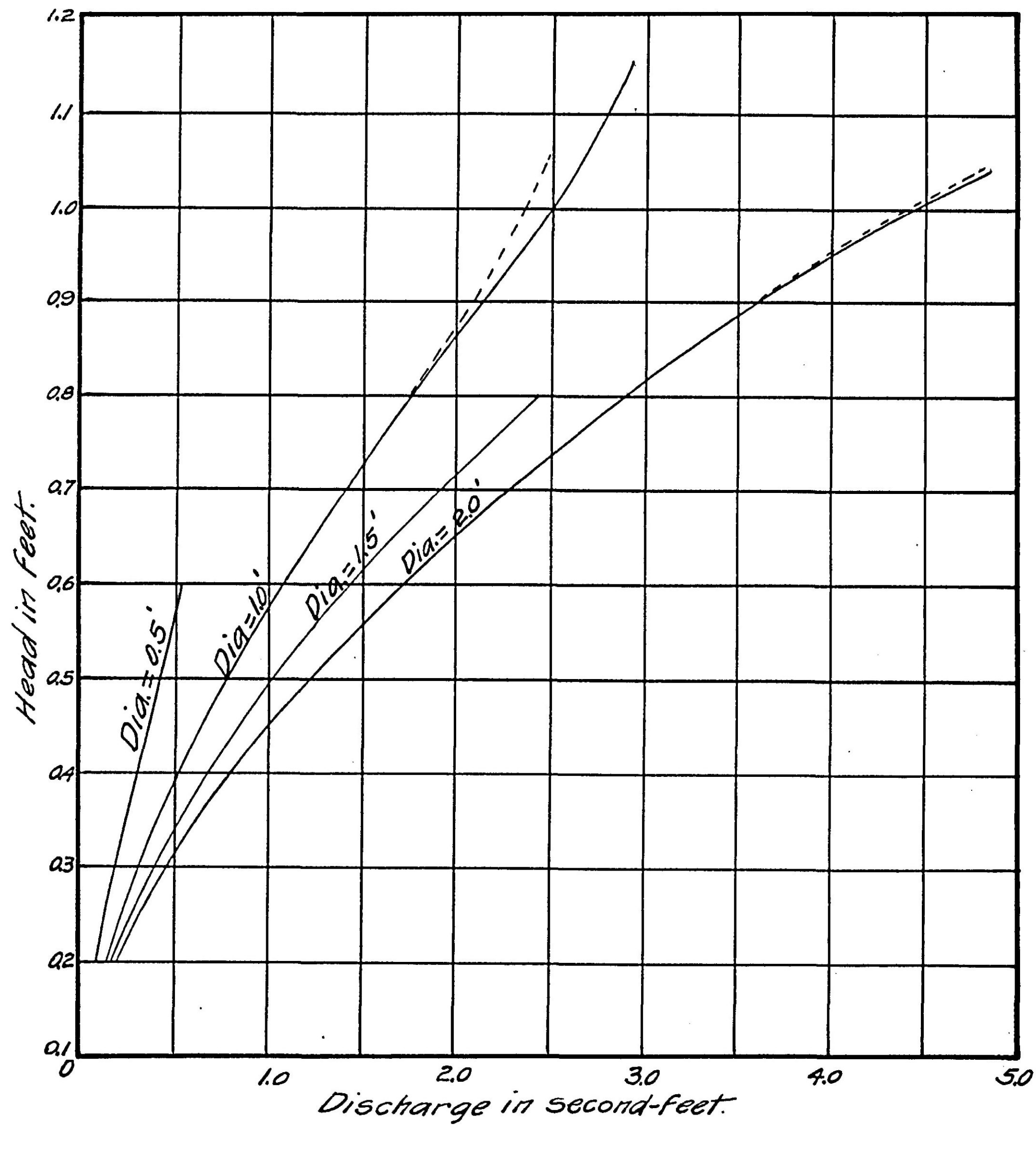
The resulting formula is,

$$Q = \left\{ 3.066\ D^{0.584} + 0.2 - \left(\frac{0.85}{D^{1.282}} + 0.03 \right) H \right\} H^{\left\{ 1.93 - 0.377 \left(\frac{H}{D} + 0.1 \right)^{2.57} \right\}}$$

Figure 6.

Discharge Curves For Circular Weirs.

Experimental values — Computed values - - -



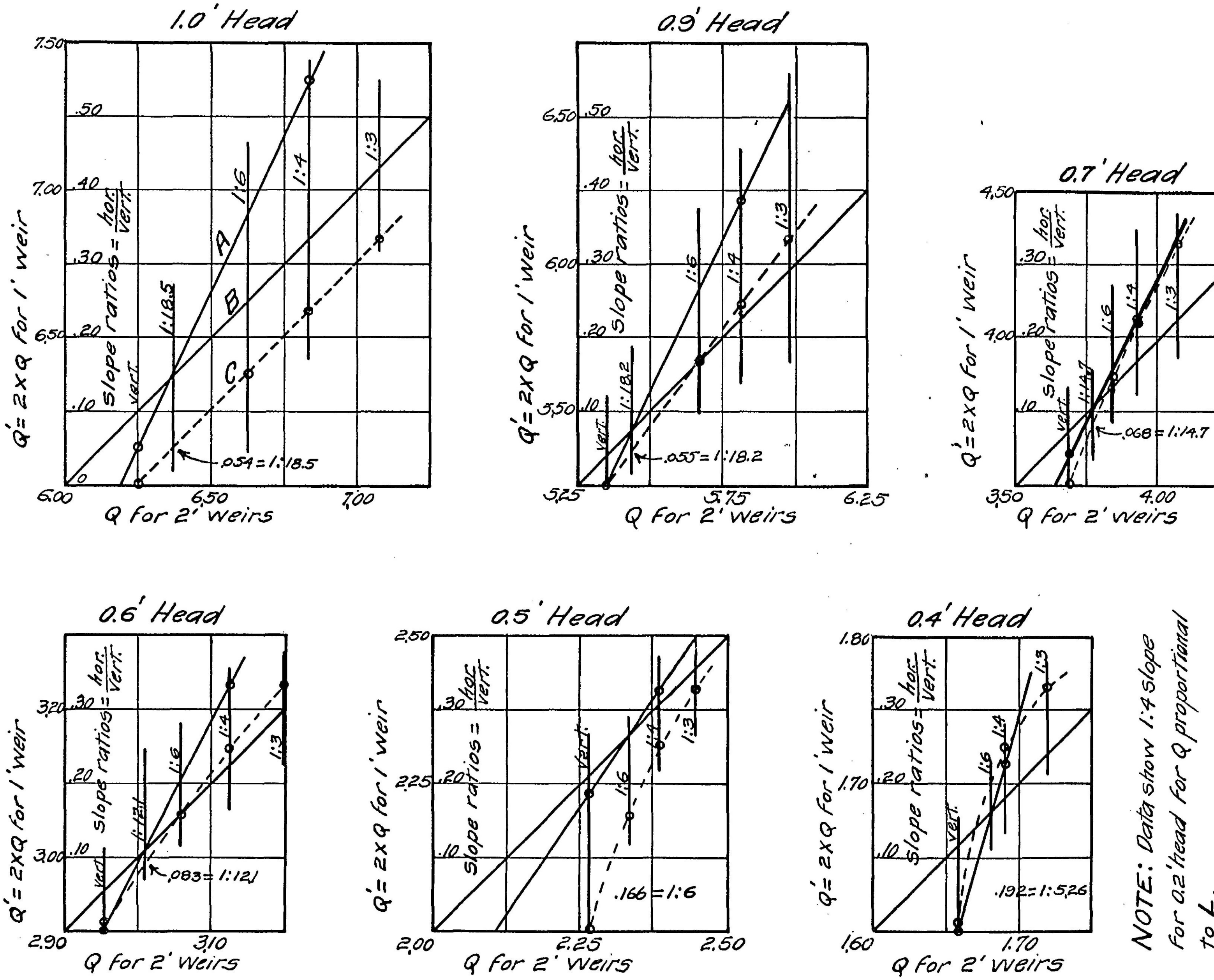


Figure 7.
Probable Shape Of Weir Sides To Give Discharge Proportional To Length Of Weir Crest.

RELATION OF LENGTH TO DISCHARGE

The principle advantage of the Cippoletti weir over other types was supposed to be that the discharge would be proportional to the crest length. , The fallacy of that theory is shown by Table 10, in which the discharges over the 1.0 foot weir are taken as a basis for comparison. It must be remembered that such comparisons are not in accord with the limitations put on the weir by Francis and Cippoletti, but they do conform to common practice in the field and are presented for that reason. The percentages represent the failure of the weir to give discharge proportional to length, and it will be seen that the Cippoletti weir is more in error than the rectangular weir. The error increases with the head andlength of crest, until the flow for a 1.0 foot head on a 4.0 Cippoletti weir is 9.2 percent less than four times the flow for a 1.0 foot head on a 1.0 foot weir of the same type; and similarconditions for rectangular weirs give a plus value of 3.96 percent. Side slopes 1 to 4 are, therefore, too flat and vertical sides are too steep, to give discharges proportional to length of crest.

For the purpose of throwing some light on theprobable shape of weir sides to meet theabove conditions, the few data obtained for weirs having different side slopes have been plotted as shown in figure 4. In addition to the rectangular and Cippoletti weir tests, a series of measurements was made of the discharges over weirs having side slopes of 1 to 3 and 1 to 6, but only with crest lengths of 2 feet. Each individual set of curves in Figure 7 is for a certain head, the actual discharge for the various types of weirs of 2.0 feet length is the abcissa common to all, twice the discharge for the 1.0 foot weir is theordinate

TABLE 9

RELATION OF LENGTH TO DISCHARGE OF WEIRS.

Head	1 foot weir Discharge in sec.ft	1.5 ft. weir. Discharge in sec.ft.				2 ft. weir Discharge in sec.ft.				3 ft. weir. Discharge in sec.ft.				4 ft. weir. Discharge in sec. ft.			
	L 1	L 1.5	L 1x 1.5	Diff	%*	L 2	L 1x 2	Diff.	%*	L 3	L 1x3	Diff.	%*	L 4	L 1x 4	Diff	%*
RECTANGULAR WEIRS																	
.20	.291	.439	.437	.002	.46	.588	.582	.006	1.03	.887	.873	.014	1.60	1.187	1.164	.023	1.98
.25	.404	.609	.606	.003	.50	.817	.808	.009	1.11	1.233	1.212	.021	1.73	1.650	1.616	.034	2.10
.30	.527	.796	.790	.006	.76	1.068	1.054	.014	1.33	1.612	1.581	.031	1.96	2.158	2.108	.050	2.37
.40	.804	1.214	1.206	.008	.66	1.630	1.606	.024	1.49	2.464	2.412	.052	2.16	3.299	3.216	.083	2.58
.50	1.113	1.684	1.670	.014	.84	2.262	2.226	.036	1.62	3.421	3.339	.082	2.46	4.583	4.452	.131	2.94
.60	1.453	2.201	2.180	.021	.96	2.956	2.906	.050	1.72	4.474	4.359	.115	2.64	5.996	5.812	.184	3.17
.70	1.819	2.756	2.729	.027	.99	3.705	3.638	.067	1.84	5.611	5.457	.154	2.82	7.522	7.276	.246	3.38
.80	2.210	3.351	3.315	.036	1.09	4.506	4.420	.086	1.95	6.828	6.630	.198	2.99	9.158	8.840	.318	3.60
.90	2.624	3.980	3.936	.044	1.12	5.354	5.248	.106	2.02	8.118	7.872	.246	3.13	10.891	10.496	.395	3.76
1.00	3.058	4.642	4.587	.055	1.20	6.247	6.116	.131	2.14	9.476	9.174	.302	3.29	12.716	12.232	.484	3.96
CIPOLLETTI WEIRS																	
.20	.302	.450	.453	-.003	-.7	.599	.604	-.005	-.8	.898	.906	-.008	-0.9	1.198	1.208	-.010	-.8
.25	.423	.628	.634	-.006	-.9	.836	.846	-.010	-1.2	1.252	1.269	-.017	-1.3	1.669	1.692	-.023	-1.4
.30	.557	.826	.835	-.009	-1.1	1.098	1.114	-.016	-1.4	1.642	1.671	-.029	-1.7	2.188	2.288	0.040	-1.8
.40	.866	1.277	1.299	-.022	-1.7	1.692	1.732	-.040	-2.3	2.526	2.598	-.072	-2.8	3.361	3.464	-.103	-3.0
.50	1.221	1.793	1.831	-.038	-2.1	2.370	2.442	-.072	-2.9	3.529	3.663	-.134	-3.7	4.691	4.884	-.193	-4.0
.60	1.623	2.371	2.434	-.063	-2.6	3.126	3.246	-.120	-3.7	4.644	4.869	-.225	-4.6	6.166	6.492	-.326	-5.0
.70	2.069	3.006	3.103	-.097	-3.1	3.955	4.138	-.183	-4.4	5.861	6.207	-.346	-5.6	7.772	8.276	-.504	-6.1
.80	2.559	3.700	3.838	-.138	-3.6	4.855	5.118	-.263	-5.1	7.177	7.677	-.500	-6.5	9.507	10.236	-.729	-7.1
.90	3.092	4.448	4.638	-.190	-4.1	5.822	6.184	-.362	-5.9	8.586	9.276	-.690	-7.4	11.359	12.368	-1.009	-8.2
1.00	3.667	5.251	5.500	-.249	-4.5	6.856	7.334	-.478	-6.5	10.085	11.001	-.916	-8.3	13.325	14.668	-1.343	-9.2

* Percentages represent failure of weir to give discharges proportinal to length, being the difference between the two discharges referred to the multiplied discharge for weirs of the length shown in the main heading.

for the curves drawn in solid lines, while the slopes of the weir side expressed decimally is the ordinate for the dotted curve. First taking the rectangular and Cippoletti data given in Table 10, the actual discharge for a head of 1.0 foot and a crest length of 2.0 feet, was plotted against twice the discharge value for 1.0 foot weir. The line marked "A" was drawn through these two points, and since data were lacking for 1.0 foot weirs with side slopes of 1 to 3, and 1 to 6, it was assumed that similar plottings for weirs with those slopes would lie on that straight line. Now weirs with proper side slopes to give flows proportional to thelength, must give values located on a straight line inclined at 45 degrees to the axes and passing through the origin, when plotted as stated above. Therefore, the inter-section of the 45 degrees line, marked "B", with the line marked "A", indicates the discharge value that would be given by a 2.0 foot weir with proper side slopes.

The curve marked "C" was constructed by plotting the actual discharge for 2.0 foot weirs having side slopes of 1 to 3, 1 to 4, 1 to 6, and vertical, against the decimal expression of these slopes. A vertical line was passed through the inter-section of the previously described curves, and its inter-section with curve "C" indicates the side slope at a 1.0 foot head on a weir necessary to give a discharge proportional to length. The other curves were obtained in a similar manner. The resulting values are; a slope of 1 to 18.5 for a 1.0 foot head, 1 to 18.2 for 0.9 foot head, 1 to 14.7 for a 0.7 foot head, 1 to 12.1 for a 0.6 foot head, 1 to 6 for 0.5 foot head, 1 to 5.26 for 0.4 foot head, and the tabular data shows 1 to 4 for 0.2 foot head. These figures indicate a curved side which would approach the vertical for high heads.

The above deductions are for 1.0 and 2.0 foot weirs, and it is very probable that a similar comparison for larger weirs would give different results, because of the appreciable change in effect of contraction. These deductions are not presented as a solution of the problem of the proper form of weir sides to give a discharge proportional to thelength, but are stated with the hope that they may lead to a discussion and finally a solution. However, such a complex side for a weir would beof little practical use in irrigation because it would be too complicated for construction on a farm.

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