

## Microwave-induced phase escape in a Josephson tunnel junction

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We perform both theoretical and experimental investigations on the phase escape of a current-biased Josephson tunnel junction under microwave irradiation. The switching current distributions exhibit abundant nonlinear behaviors depending on the power and frequency of the applied microwave. We present a model to describe the behavior of the primary peak in the switching current distribution, which is confirmed by our experimental results. The obtained features can be used to characterize the damping parameter of Josephson junctions.

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Josephson junctions are very important nonlinear systems for both fundamental researches and practical applications. Driving these nonlinear systems with microwave produce enormous interesting phenomena such as Shapiro steps and photon-assisted tunneling.<sup>1,2</sup> So far, many useful applications based on the microwave driven junctions have already been developed. Moreover, recent investigation on the quantum computing in Josephson junction devices has opened another vigorous area for this topic. Many kinds of superconducting Josephson qubits<sup>3</sup> have been proposed and demonstrated. In those superconducting qubits, Josephson junctions are the key elements and microwave irradiation is crucial to perform the qubits' control and readout.

Therefore, it is mandatory to thoroughly understand both the classical and quantum dynamics of the microwave driven Josephson junctions. Theoretical and experimental works<sup>4-7</sup> have been conducted, in which a lot of attractive characteristics have been reported. In this paper, we present here some phenomena of the phase escape in a microwave driven Josephson tunnel junction. The switching current distribution  $P(I_s)$  under microwave irradiation shows a qualitative difference when microwave power and frequency vary. We develop a quantitative model based on the classical dynamics to understand the experimental results. Our model also shows the effect of the damping parameter on the junction's phase escape. Therefore, using this model, we can experimentally determine the junction damping parameter, which is an important parameter while difficult to obtain.

The equation of motion of a current-biased Josephson tunnel junction is identical to that of a particle moving in a washboard potential [shown in Fig. 1(a)],

$$\frac{\hbar C}{2e} \ddot{\theta}(t) + \frac{\hbar}{2eR} \dot{\theta}(t) + I_0 \sin \theta = I_b, \quad (1)$$

where  $\theta$  is the phase difference across the junction and corresponds to the coordinate of the phase particle.  $C$  and  $R$  are the shunt capacitance and resistance, respectively.  $I_b$  is the bias current and  $I_0$  is the critical current. For  $I_b$ , slightly

smaller than  $I_0$ , the potential has a series of metastable wells with barrier height given by  $\Delta U = 2E_J(\sqrt{1-(i_{dc})^2} - i_{dc} \cos^{-1}(i_{dc}))$ . Here,  $E_J = I_0 \Phi_0 / 2\pi$ ,  $i_{dc} = I_b / I_0$ , and  $\Phi_0 \equiv h/2e$ . A particle initially trapped in a potential well (corresponding to a zero voltage drop across the junction) oscillates at the bottom of the well with the plasma frequency  $\omega_p = \omega_{p0}[1 - (i_{dc})^2]^{1/4} \equiv (2\pi I_0 / C \Phi_0)^{1/2} [1 - (i_{dc})^2]^{1/4}$ . Thermal or quantum fluctuation can kick the particle out of the potential well to a voltage state. The current where the junction switches from zero to finite voltage is called switching current ( $I_s$ ). Since junction switching is a random process, the

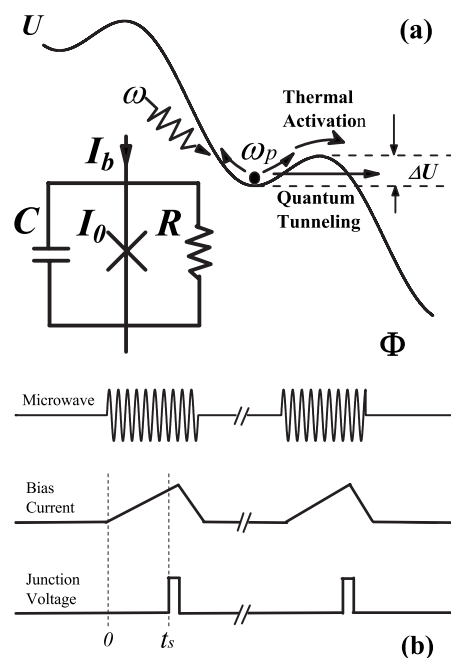


FIG. 1. (a) Washboard potential and equivalent circuit (inset) of a current-biased Josephson tunnel junction with  $I_b < I_0$ . (b) Schematic time profile for measuring the switching current distribution  $P(I_s)$  of a Josephson junction with or without microwave irradiation.

switching current distribution  $P(I_s)$  contains all useful information and can be used to investigate classical and quantum dynamics. To build  $P(I_s)$  experimentally, we sweep the bias current from zero to  $I_0$  and repeatedly measure the switching. A peak with mean switching current value slightly smaller than  $I_0$  can be observed in  $P(I_s)$ . We hereafter name this peak as the primary peak, which is caused by either the thermal activation or quantum tunneling. If we drive the junction using a continuous microwave with appropriate frequency and power, there will be an enhancement of the escape from the potential well, which manifests itself by a resonant peak besides the primary peak in  $P(I_s)$ . This is known as the microwave induced double-peak structure.<sup>4-7</sup> The positions of the primary peak and the resonant peak are denoted as  $I_p$  and  $I_r$ , which are the current values of two maxima, respectively.

The resonant peak is predicted in both classical and quantum dynamics. Qualitatively speaking, at high temperature, the junction is in the classical regime. When the driving frequency equals to the plasma frequency, the classical resonance occurs. Therefore, the particle has the largest oscillation amplitude in the potential well and the escape is enhanced. At low temperature, especially lower than the crossover temperature  $T^*$  ( $T^* = \hbar\omega_{p0}/2\pi k_B$ , where  $k_B$  is Boltzmann's constant), the energy levels in the potential well become quantized. The quantum tunneling dominates over the thermal activation. When microwave frequency matches the energy difference between the energy levels, it will pronouncedly increase the population of the excited state, in which the tunneling from the well is much easier.<sup>8-11</sup> Consequently, the resonant peak appears in the switching current distribution. In cubic approximation of the potential, the energy differences between the lowest two energy levels are determined by<sup>12</sup>  $\omega_{01} = \omega_p[1 - (5/36)(\hbar\omega_p/\Delta U)]$ . Similar to that in the microscopic quantum systems, such a process is also applicable to the multiphoton absorption for  $n\omega = \omega_{01}$ ; here  $n$  is the number of photons absorbed in the transitions. Some interesting phenomena will appear in the behavior of the resonant peak when the defects in the tunnel barrier couple to the energy levels of the qubits, which have been reported by other groups.<sup>13-15</sup>

Our work focuses on the behavior of the primary peak under microwave irradiation. For a weak microwave power, the potential of the junction is almost unperturbed by microwave irradiation. It is very easy to enhance the population in the excited state. Therefore, only single photon absorption is observed in this process and the position of the primary peak exhibits almost no change compared to that without microwave irradiation. For a strong microwave power, multiphoton absorption occurs. Under such high power irradiation, the potential is strongly perturbed and suppressed. The phase of the junction escapes from the well much more easily, which means the position of the primary peak will move to a lower value along the bias current axis.

A semiquantitative model based on the classical equation of motion can be used to describe the behavior of the primary peak. With microwave irradiation presence, we add  $I_{mw} \sin \Omega t$  on the right hand side of Eq. (1), where  $\Omega$  is microwave frequency and  $I_{mw}$  represents amplitude of microwave-induced alternating current flowing through the junction. For convenience, we rewrite the equation of motion in a normalized form as

$$\ddot{\theta}(\tau) + \alpha\dot{\theta}(\tau) + \sin \theta = i_{dc} + i_{mw} \sin \omega\tau, \quad (2)$$

where  $\alpha = \hbar\omega_{p0}/2eRI_0$  is the damping parameter. Other re-scaled parameters are  $\tau = t/\omega_{p0}^{-1}$ ,  $i_{dc} = I_b/I_0$ ,  $i_{mw} = I_{mw}/I_0$ , and  $\omega = \Omega/\omega_{p0}$ . For a weak damping system and small microwave amplitude,  $\alpha \ll 1$  and  $i_{mw} \ll 1$ .

For a slowly varying bias current, it is a good approximation to separate  $\theta$  in Eq. (2) into two terms: an equilibrium position term  $\varphi_0$  ( $i_{dc} = \sin \varphi_0$ ) and a quickly oscillating term  $\varphi(\tau)$  around  $\varphi_0$ . Inserting  $\theta = \varphi_0 + \varphi(\tau)$  into Eq. (2) yields the dynamic equation for  $\varphi(\tau)$ , which can be Taylor expanded in powers of  $\varphi(\tau)$  as long as the oscillation amplitude of  $\varphi(\tau)$  is small. Up to order of  $\varphi^2(\tau)$ , we obtain the nonlinear dynamic equation,

$$\ddot{\varphi}(\tau) + \alpha\dot{\varphi}(\tau) + \sqrt{1 - i_{dc}^2}\varphi(\tau) - \frac{i_{dc}}{2}\varphi^2(\tau) = i_{mw} \sin \omega\tau. \quad (3)$$

The standard relationship between the normalized resonance frequency and bias current reads  $\omega_r = (1/n)(\omega_p/\omega_{p0}) = (1/n)(1 - i_{dc}^2)^{1/4}$  ( $n = 1, 2, 3, \dots$ ). To observe both the primary peak and the resonant peak, the amplitude of microwave should be tuned to a very narrow regime, which is defined as  $i_{c_{rmw}}(\omega_r)$  in this paper. From the classical point of view,  $i_{c_{rmw}}(\omega_r)$  is actually the critical microwave amplitude, which can provide a suitable energy necessary for the phase particle to resonantly escape from the potential well. Above (below)  $i_{c_{rmw}}(\omega_r)$ , only the resonant peak (the primary peak) shows up in the switching distribution [i.e., no double-peak  $P(I_s)$  can be observed when  $i_{mw} \neq i_{c_{rmw}}(\omega_r)$ ].

In the following section, we calculate  $i_{c_{rmw}}(\omega_r)$  for the harmonic ( $n=1$ ) and  $n$ th-subharmonic resonances ( $n = 2, 3, 4$ ).

$n=1$ . By retaining only the linear term of  $\varphi(\tau)$  in Eq. (3), we obtain

$$\ddot{\varphi}(\tau) + \alpha\dot{\varphi}(\tau) + \sqrt{1 - i_{dc}^2}\varphi(\tau) = i_{mw} \sin \omega\tau. \quad (4)$$

The stationary solution to this linearly forced oscillation equation is given by

$$\varphi^{(1)}(\tau) = A \cos(\omega\tau + \delta_1),$$

where  $\delta_1$  is a phase constant and  $A = i_{mw}/\sqrt{(\sqrt{1 - i_{dc}^2} - \omega^2)^2 + \alpha^2\omega^2}$  represents the steady oscillation amplitude. In the case of resonance,  $\omega_r = (1 - i_{dc}^2)^{1/4}$ , the resonance amplitude  $A_r = i_{mw}/\alpha\omega_r$ . By equating  $A_r$  to the phase distance between the stable phase position  $\varphi_0$  and the location of the primary peak  $i_p$  ( $i_p = I_p/I_0$ ), we are able to obtain  $i_{c_{rmw}}^{(1)}(\omega_r) = \alpha\omega_r(\arcsin i_p - \arcsin i_{dc})$ . It is evident that the critical microwave amplitude for  $n=1$  can be quite small provided that the damping is weak, which is the case in our experiment.

$n=2$ . We now consider the stationary solution of a  $2\omega$  mode to Eq. (3), namely,  $\varphi(\tau) = \varphi^{(1)}(\tau) + \varphi^{(2)}(\tau)$ , in which  $\varphi^{(2)}(\tau) = B \cos(2\omega\tau + \delta_2)$ , with  $\delta_2$  as a phase constant and  $B$  as the oscillation amplitude. Inserting  $\varphi(\tau)$  into Eq. (3) produces

$$\begin{aligned} & \ddot{\varphi}^{(2)}(\tau) + \alpha \dot{\varphi}^{(2)}(\tau) + \sqrt{1 - i_{dc}^2} \varphi^{(2)}(\tau) \\ & = \frac{i_{dc}}{4} A^2 \cos(2\omega\tau + 2\delta_2) + \text{other terms.} \end{aligned} \quad (5)$$

We are justified to neglect other terms since the first term dominates the dynamics of  $\varphi^{(2)}(\tau)$ . We can then easily obtain the oscillation amplitude for the  $2\omega$  mode,

$$B \cong \frac{A^2 i_{dc}/4}{\sqrt{[\sqrt{1 - i_{dc}^2} - (2\omega)^2]^2 + \alpha^2 (2\omega)^2}}.$$

For  $\omega_r = \frac{1}{2}(1 - i_{dc}^2)^{1/4}$ , the resonance amplitude  $B_r = [(i_{dc}/4)/2\alpha\omega_r][i_{crmw}^2/(\sqrt{1 - i_{dc}^2} - \omega_r^2)^2 + \alpha^2\omega_r^2]$ . We can then estimate  $i_{crmw}^{(2)}(\omega_r) \cong 6\omega_r^2[i_{crmw}^{(1)}(2\omega_r)/i_{dc}]^{1/2}$ , indicating that a larger microwave amplitude is required to trigger a  $2\omega$ -mode resonance compared to that for the  $\omega$ -mode resonance.

$n=3$  and  $n=4$ . Similar to the previous analysis, by writing the solution of Eq. (3) as a  $3\omega$ -mode solution and  $4\omega$ -mode solution, we obtain  $i_{crmw}^{(3)}(\omega_r) \cong 8(40\omega_r^8)^{1/3} \times [i_{crmw}^{(1)}(3\omega_r)/i_{dc}^2]^{1/3}$  with the resonance condition  $\omega_r = \frac{1}{3}(1 - i_{dc}^2)^{1/4}$  and  $i_{crmw}^{(4)}(\omega_r) \cong 30(84)^{1/4}\omega_r^3[i_{crmw}^{(1)}(4\omega_r)/i_{dc}^3]^{1/4}$  with the resonance condition  $\omega_r = \frac{1}{4}(1 - i_{dc}^2)^{1/4}$ , respectively.

Note that as we derive the above expressions, the thermal current noise is neglected. We should expect that the presence of thermal fluctuations will shift the critical amplitude of microwave to a slightly smaller value.

In the absence of microwave as well as thermal fluctuations, the potential energy of the Josephson junction is  $U_0(\varphi_0) = -E_J[(I_b/I_0)\varphi_0 + \cos\varphi_0]$  and  $I_p$  equals  $I_0$ . However, in the presence of microwave radiation, the potential energy of the Josephson junction is effectively suppressed as<sup>16</sup>

$$\begin{aligned} & U_{eff}(\varphi_0, i_{crmw}, \omega) \\ & = -E_J \left[ \frac{I_b}{I_0} \varphi_0 + \cos\varphi_0 \left( 1 - \frac{i_{crmw}^2/2}{(\cos\varphi_0 - \omega)^2 + \alpha^2\omega^2} \right) \right]. \end{aligned} \quad (6)$$

When the resonance condition is satisfied,  $i_p$  is subsequently shifted down to

$$i_p = 1 - \frac{i_{crmw}^2(\omega_r)/2}{(\sqrt{1 - i_p^2} - \omega_r^2)^2 + \alpha^2\omega_r^2}. \quad (7)$$

Note here we have neglected the  $n$  dependence of  $i_{crmw}(\omega_r)$ . The presence of thermal fluctuations will simply decrease  $I_p$ , which can be written as<sup>4</sup>

$$i_p = 1 - \langle \delta i_p(T) \rangle - \frac{i_{crmw}^2(\omega_r)/2}{(\sqrt{1 - i_p^2} - \omega_r^2)^2 + \alpha^2\omega_r^2}, \quad (8)$$

where  $\langle \delta i_p(T) \rangle = [(k_B T/2E_J) \ln(\omega I_0/2\pi \dot{I}_b)]^{2/3}/I_0$  is the shift of  $i_p$  due to thermal fluctuations at temperature  $T$  and  $\dot{I}_b$  is the ramp rate of the bias current. As we have determined  $i_{crmw}(\omega_r)$  already, by solving transcendental Eq. (8) for  $i_p$ , we obtain  $I_p$ . By using this model, we calculated the position of the primary and resonant peaks in a typical  $P(I_s)$ , as shown in Fig. 2. The damping parameters are chosen to be  $\alpha = 0.002$  and  $\alpha = 0.2$ . The sample parameters we used are  $I_0 = 36.08 \mu\text{A}$ ,  $C = 5.8 \text{ pF}$ , and  $T = 80 \text{ mK}$ . As we can see, the

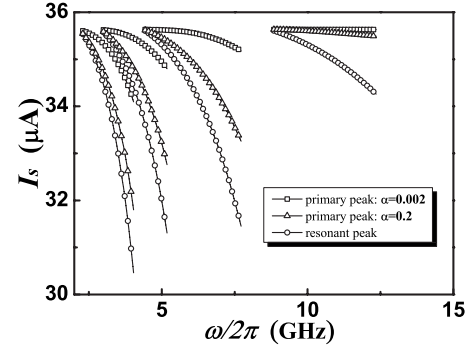


FIG. 2. Dependence of the location of the primary and resonant peaks (circles) on microwave frequency for  $\alpha=0.002$  (squares) and  $\alpha=0.2$  (triangles) based on the presented model in the text.

curvature of the primary peak is closely related to  $\alpha$  and  $n$ . In the case of weak damping ( $\alpha=0.002$ ),  $I_p$  for  $n=1$  is nearly a constant and just slightly suppressed for  $n=2, 3$ , and  $4$ . This is reasonable. Since the critical microwave power for a sub-harmonic resonance is larger than that for a harmonic resonance, consequently the phase particle “sees” a more suppressed potential barrier. In the case of stronger damping ( $\alpha=0.2$ ),  $I_p$  for  $n=1$  is also slightly lowered, while for  $n=2, 3$ , and  $4$ ,  $I_p$  is heavily suppressed. This indicates that a larger critical microwave power is required for a higher damping system, leading to a larger suppression of the potential barrier.

The experimental results quantitatively agree with the prediction of our model. The sample is a  $10 \times 10 \mu\text{m}^2$  Nb/ $\text{AlO}_x$ /Nb Josephson tunnel junction fabricated with a standard trilayer process. This junction has a critical current of  $I_0 = 36.08 \mu\text{A}$  and an effective capacitance of  $C = 5.8 \text{ pF}$ , determined from independent measurements.<sup>17</sup> The sample is mounted on a chip carrier enclosed in a superconducting aluminum sample cell. The assembly is put in the dilution refrigerator and magnetically shielded by three layers of Mu-metal cylinders surrounding the dewar at room temperature. All electrical leads that connect the junction to room temperature electronics are carefully filtered by RC filters and Cu-powder filters. Battery-powered low-noise preamplifiers are used for all measurements. Microwave can be injected via a separate coaxial cable, which is capacitively coupled to the junction. A 20 dB attenuator at the 1 K stage, a 10 dB attenuator, and a dc block at the 600 mK stage are used to reduce the external noise. In general,  $P(I_s)$  is obtained using the time-of-flight technique [shown in Fig. 1(b)]: For each escape event, with microwave irradiation always on, we start the cycle by ramping up the bias current linearly from zero to a value slightly higher than  $I_0$ . The junction is sure to escape from the potential well because when  $I_b > I_0$ , there is no potential well at all. This escape generates a zero-voltage to finite-voltage jump across the junction which triggers the timer to record a switching time  $t_s$ . Then, the bias current  $I_b$  is gradually reduced to zero, resetting the junction to the zero-voltage state. This process is repeated  $5.0 \times 10^3$  times to obtain an ensemble of  $t_s$ . Having known the ramping rate  $\dot{I}_b$ , we obtain an ensemble of  $I_s$  and thus  $P(I_s)$ . Microwave with frequency between 2.5 and 20 GHz is applied to the sample.

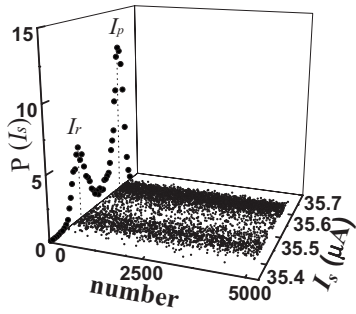


FIG. 3. Raw data of the switching current ( $I_s$ ) with microwave irradiation (nominal power:  $-22.20$  dBm and  $\omega/2\pi=9.300$  GHz) and the corresponding stochastic switching current distribution  $P(I_s)$ .  $I_p$  and  $I_r$  denote the location of the primary peak and the resonant peak, respectively.

For all the measurements, the temperature is 16 mK, which is significantly below the classical-to-quantum crossover temperature ( $T^*=80$  mK) of our junction.

In Fig. 3, on the  $x$ - $y$  plane we show the raw data of the switching current with microwave irradiation (nominal power:  $-22.20$  dBm and  $\omega/2\pi=9.300$  GHz). Shown on the  $x$ - $z$  plane is the corresponding stochastic distribution  $P(I_s)$ . The double-peak structure containing the primary peak and the resonant peak is observed as expected.

Figure 4 shows the three-dimensional (3D) plot of the selected distributions with the double-peak structure in the whole range of microwave frequency. Note that microwave power for each double-peak structure distribution is different. We have observed two typical types of double-peak structure in the experiments. The first one is shown in Fig. 5(a). Because the damping of our system is low, microwave power for harmonic resonance is low. As we can see, although the height of the primary peak decreases as microwave power increases, its position remains almost unchanged. Yet, the other one is shown in Fig. 5(b), in which both the position and height of the primary peak move to lower values with the increase of microwave power because higher microwave power is needed for subharmonic resonance as discussed above.

Inserting the experimentally determined parameters in the model, we calculate the dependence of  $I_p$  on microwave fre-

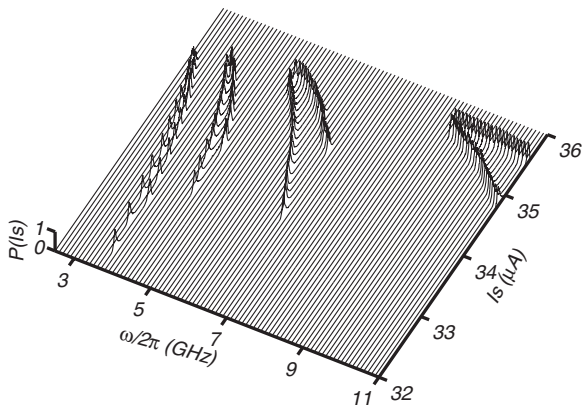


FIG. 4. 3D plot of the double-peak structure.

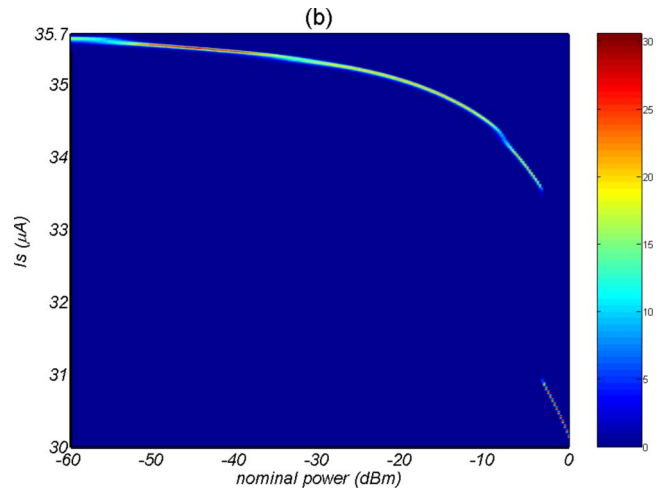
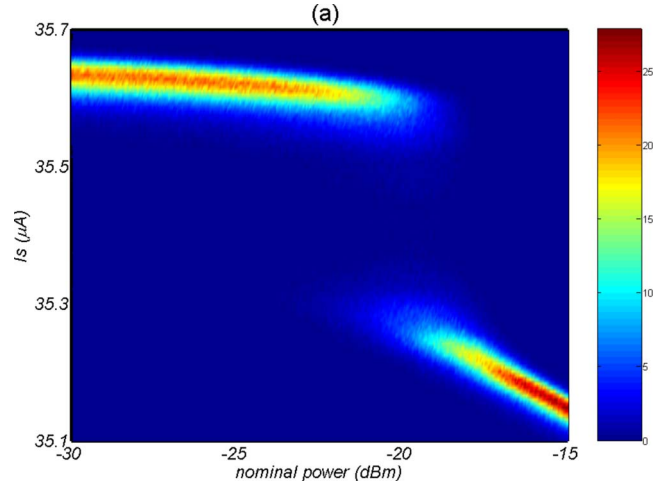


FIG. 5. (Color online) Density plot of  $P(I_s)$  versus the applied microwave power at 16 mK in two typical types of double-peak structure. (a)  $\omega/2\pi=9.950$  GHz and (b)  $\omega/2\pi=8.000$  GHz.

quency. From the best fitting to the experimental data, we get  $\alpha=0.08$ . In Fig. 6, we plot the dependence of  $I_p$  (circles) and  $I_r$  (squares) on microwave frequency in two dimensions. The theoretical  $I_r$  are plotted in red lines. The dependence of  $I_r$  on

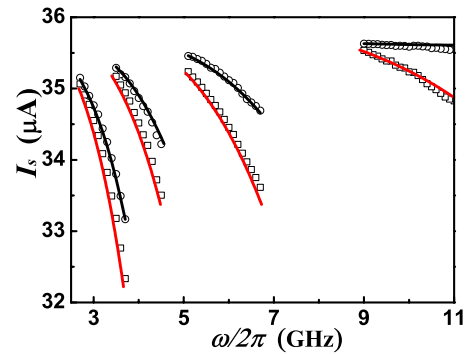


FIG. 6. (Color online) Dependence of the location of the primary and resonant peaks in  $P(I_s)$  on microwave frequency at 16 mK: red lines for the theoretical resonant peak, black lines for the theoretical primary peak, squares for the experimental resonant peak, and circles for the experimental primary peak.

the microwave frequency is in good accord with the experimental result. The theoretical  $I_p$  based on the above model are plotted in black lines, which also show good agreement with the experimental ones. The small difference between them is caused by the approximation in the model and/or the  $\alpha$ 's frequency dependence.<sup>9</sup>

A direct application of our analysis is that we can determine the damping parameter  $\alpha$  experimentally. The damping parameter is important for superconducting qubits because junctions are widely used as qubit element and readout devices. However, it is usually difficult to measure  $\alpha$  since it depends on many factors such as the qubit design, measurement setup, junction fabrication, and temperature.

In conclusion, we have carried out not only the experimental measurement but also theoretical analysis of the phase escape in a current-biased Josephson junction under microwave irradiation. The switching current distribution  $P(I_s)$  manifests itself by different behaviors depending on the

power and frequency of the applied microwave irradiation. Two typical types of double-peak structure are observed. Both the primary peak and the resonant peak are strongly related to the damping parameter, the microwave power, the microwave frequency, and the photon absorption process. We have successfully developed a model to explain the behavior of the primary peak in these double-peak structures. The model is useful to determine the damping parameter of the system.

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