GEOPHYSICAL RESEARCH LETTERS, VOL. 34, L12501, doi:10.1029/2007GL030046, 2007

Subglacial topography and geothermal heat flux: Potential interactions with drainage of the Greenland ice sheet

C. J. van der Veen,[1,2] T. Leftwich,[3] R. von Frese,[3] B. M. Csatho,[4] and J. Li[2]

Received 15 March 2007; revised 4 May 2007; accepted 23 May 2007; published 21 June 2007.

[1] Many of the outlet glaciers in Greenland overlie deep and narrow trenches cut into the bedrock. It is well known that pronounced topography intensifies the geothermal heat flux in deep valleys and attenuates this flux on mountains. Here we investigate the magnitude of this effect for two subglacial trenches in Greenland. Heat flux variations are estimated for idealized geometries using solutions for plane slopes derived by Lachenbruch (1968). It is found that for channels such as the one under Jakobshavn Isbræ, topographic effects may increase the local geothermal heat flux by as much as 100%. **Citation:** van der Veen, C. J., T. Leftwich, R. von Frese, B. M. Csatho, and J. Li (2007), Subglacial topography and geothermal heat flux: Potential interactions with drainage of the Greenland ice sheet, *Geophys. Res. Lett.*, *34*, L12501, doi:10.1029/2007GL030046.

1. Introduction

[2] The flow of glaciers has a bi-modal character. Where the temperature at the interface between ice and underlying substrate is below the pressure-melting point, ice flow primarily results from internal deformation associated with the viscous properties of glacial ice. Depending on the geometry of the glacier, velocities associated with this mode of flow are typically on the order of 1–100 m/yr. Glacier discharge can increase ten-fold or more where the basal ice reaches the pressure-melting point and a lubricating water layer forms, allowing the ice to slide column-wise over the substrate at speeds that can reach several km/yr. Consequently, for modeling drainage from glaciers and ice caps, one of the important processes to consider is the thermal regime at the base of the ice.

[3] Glacier ice is an excellent thermal insulator and heat generated at the glacier bed is primarily used to warm the basal ice. The two main heat sources are viscous heat dissipation due to vertical shear and friction at the glacier base, and geothermal heat originating from within the Earth. While viscous dissipation can be adequately incorporated into numerical ice-flow models, the magnitude of geothermal heat remains poorly constrained, not in the least because of the inaccessibility of the glacier bed and consequent lack of reliable measurements. The usual approach is to adopt a value appropriate for the type of basement rock and apply the same value over the entire model domain. While this may be a reasonable approach when modeling smaller ice masses, it remains questionable how well this assumption of constant geothermal heat flow applies to the large polar ice sheets in Greenland and Antarctica.

[4] The distribution of present-day basal melt under the Greenland ice sheet has been predicted using a high-resolution, three-dimensional thermomechanical ice-sheet model [e.g., *Huybrechts*, 1996]. Assuming a geothermal heat flux of 42 mW/m^2 – typical for Precambrian shields – *Huybrechts* [1996] found that most of the ice sheet is frozen to the bedrock and melting is confined to lower elevations in the northeast and western regions. However, there is a growing body of evidence suggesting more extensive basal melting in the central region of northern Greenland. *Fahnestock et al.* [2001a] determined age-depth relationships and basal melt rate from ice-penetrating radar in northern Greenland and detected high basal melt, in places up to 0.2 m/yr, under the onset region of the Northeast Ice Stream and its southern tributaries. This basal melt requires a geothermal heat flux much greater than the estimated continental background of 57 mW/m^2 [*Sclater et al.*, 1980] and *Fahnestock et al.* [2001a] speculate that the inferred large heat flow may be of volcanic origin. Similarly, at the base of the NGRIP deep ice core, drilled on the ice divide 170 km northwest of the onset region of the Northeast Ice Stream, the observed basal temperature is at the pressure-melting point [*Anderson et al.*, 2004; *Dahl-Jensen et al.*, 2003]. The basal melt rate at NGRIP reaches 7.5 mm ice per year, and the modeled geothermal heat flux is between 90 and 160 mW/m^2 along the flow line originating 50 km upstream of the drill site [*Dahl-Jensen et al.*, 2003]. Again, the origin of the large geothermal heat flux remains unidentified. Ice-penetrating radar profiles show bright bed reflections in many locations in northern Greenland, indicating the presence of lubricating meltwater at the glacier base.

[5] Several factors may affect the magnitude of the geothermal heat flux, including age and type of basement rock, current and past volcanic activity, crustal thickness, and the presence of radiogenic sources in the crust [*McLaren et al.*, 2003]. Here we investigate topographic effects which have long been known to influence the local heat flux. The motivation for this study is the observation that many of the major drainage routes in the Greenland ice sheet are associated with subglacial valleys. Many of these valleys are narrow and not well-captured on the continental-scale bed-elevation map, but can be readily seen on ice-penetrating radar profiles, or inferred from seismic traverses. In northern Greenland, airborne radar profiling has identified a number of narrow channels such as the one under Petermann Glacier shown in Figure 1. *Ekholm et al.*

---

[1] Department of Geography, University of Kansas, Lawrence, Kansas, USA.

[2] Center for Remote Sensing of Ice Sheets, University of Kansas, Lawrence, Kansas, USA.

[3] School of Earth Sciences, Ohio State University, Columbus, Ohio, USA.

[4] Department of Geology, State University of New York at Buffalo, New York, USA.

0094-8276/07/2007GL030046

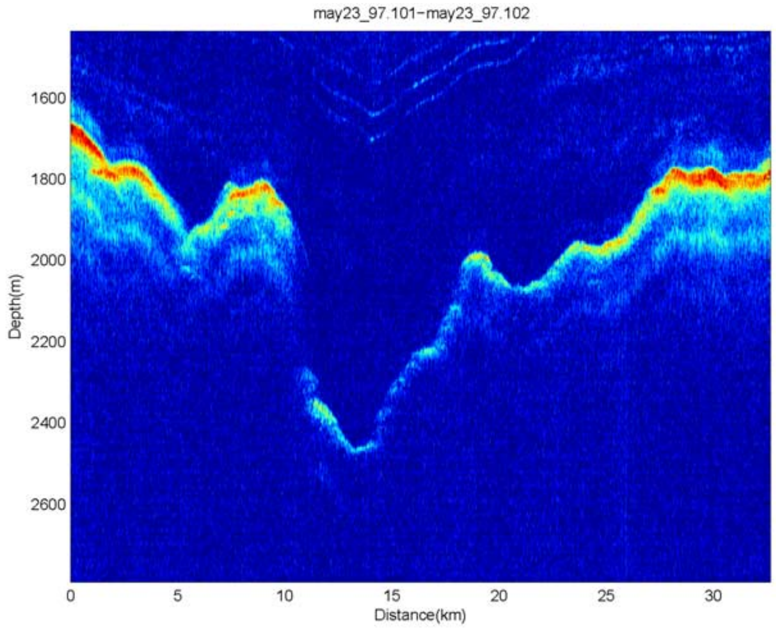

Figure 1. Ice-penetrating radar profile across a subglacial trench under Petermann Glacier, northwest Greenland. This trench, with a depth of 500 m, is overlain by 2 km thick ice.

[1998] detected increased IPR reflections over this bedrock channel which they interpreted as an indicator for water transportation in the channel. On the surface of the ice sheet, this channel is marked by a pronounced depression. Similar channels were detected in east Greenland under the Kangerlugssuaq and Helheim glaciers. Seismic profiling indicated that Jakobshavns Isbræ on the west coast overlies a deep U-shaped trough incised up to 1500 m into bedrock and approximately 6 km wide at the top [*Clarke and Echelmeyer*, 1996]. Recent airborne radar-echo sounding has shown the channel under this glacier to be much deeper and narrower than previously inferred from seismic profiling (Figure 2) [*Gogineni et al.*, 2006; *Lohoefener et al.*, 2006]. Similarly, the ice stream in northeast Greenland is associated with a basal valley. This ice stream is of particular interest because onset of enhanced flow occurs at low flow speeds and close to the present ice divide, suggesting localized heating at the glacier bed [*Fahnestock et al.*, 2001b]. We propose that such localized heating is partly associated with pronounced basal topography.

[6] The realization that the pattern of heat flow from the Earth's deep interior is distorted near the surface by topographic relief dates back to the early twentieth century, based on observations in alpine tunnels that showed intensification of heat flux in valleys and attenuation by ridges [e.g., *Lees*, 1910; *Beardsmore and Cull*, 2001]. This topographic effect arises because of the distortion of isotherms beneath the valley. Relatively elevated isotherms on the sides of the valley plunge deeper beneath the valley floor, but the isotherms are much closer together beneath the valley – especially at the valley corners – and further apart beneath the brink. Consequently, valleys experience an increased isotherm gradient, and hence increased heat flux. *Lees* [1910] derived an analytical solution for an idealized mountain range which can be inverted to obtain heat-flux perturbations in a valley. In the absence of radio-active heat generation in the crust, the increase in heat flux for a valley with a depth-to-width ratio of 0.5 is approximately 50% according to Lees' solution. Subsequent studies have refined calculations and extended solutions to more general topographies. Yet, within the glaciological community these variations have not received much attention despite the potentially important feedback between the basal thermal regime and ice discharge. *Nobles and Weertman* [1971] discussed briefly how variations in heat flux associated with subglacial topography may affect the rate of sediment deposition by in situ melting of debris-rich basal ice, but did not provide a quantitative estimate of the magnitude of heat-flux variations.

2. Heat Flux Anomalies in Subglacial Valleys

[7] To evaluate the effect of pronounced local subglacial topographic relief on geothermal heat supplied to the base of the Greenland ice sheet, we conducted calculations of the heat flux for idealized channel geometries approximating those shown in Figures 1 and 2. Our model is based on the solutions for heat flow through a plane slope derived by *Lachenbruch* [1968]. The subglacial valley is approximated as a channel with a horizontal floor and two sides with constant slope incised into flat bedrock. Heat-flux anomalies are obtained by adding the anomalies associated with each valley wall. As Lachenbruch noted, this provides a lower estimate for the heat-flux anomaly, as it ignores interactions between the two valley walls. The presence of a large slope on the left modifies heat flow through the slope on the right, and vice versa, and this interaction further modifies heat flow through each valley wall, etc. While it is possible to bracket the range of the combined heat flux anomaly, such refinements are not warranted for this exploratory study. The motivation for adopting the Lachenbruch model is that this allows different angles to be prescribed for the valley walls, whereas *Lees* [1910] solu-

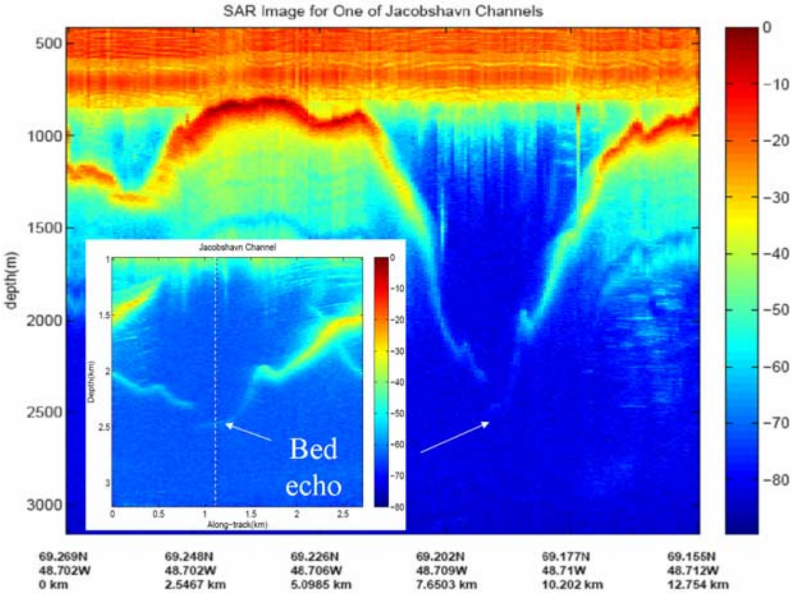

Figure 2. Ice-penetrating radar profile across the subglacial trench under Jakobshavn Isbræ, west Greenland. Inset shows the bed echo from the deepest part in greater detail.

tion applies to a smooth cross-sectional profile with only the height-to-width ratio as variable parameter.

[8] Consider first the subglacial valley shown in Figure 1. The approximated cross-sectional profile is shown in Figure 3 (top) and consists of a 15° slope on the left and combined 9° (lower part) and 3° (upper part) slopes on the right. Calculated heat-flux perturbation is shown in the lower panel and is characterized by a strong negative anomaly at the left brink, where the bed starts sloping down, and concentration of geothermal heat at the toes of the sloping walls. Because of the gentler slopes on the right, the peaks are less pronounced on that side of the valley. These large localized anomalies are likely muted for Greenland subglacial trenches that tend to have smoother transitions from plateau to slope to valley floor. Within the main valley, the geothermal flux perturbation is about 25% of the background geothermal heat flux. As energy considerations require, the integrated heat-flux perturbation is zero and increased heat flow in the valley is compensated by decreased heat flux on the walls and the regions adjacent to the valley.

[9] For the valley in Figure 3, geothermal heat flux anomalies are relatively modest, primarily due to the gentle slopes of the valley walls. The second example, shown in Figure 4, refers to an adopted valley geometry based on cross-sectional transects from radar soundings on Jakobshavn

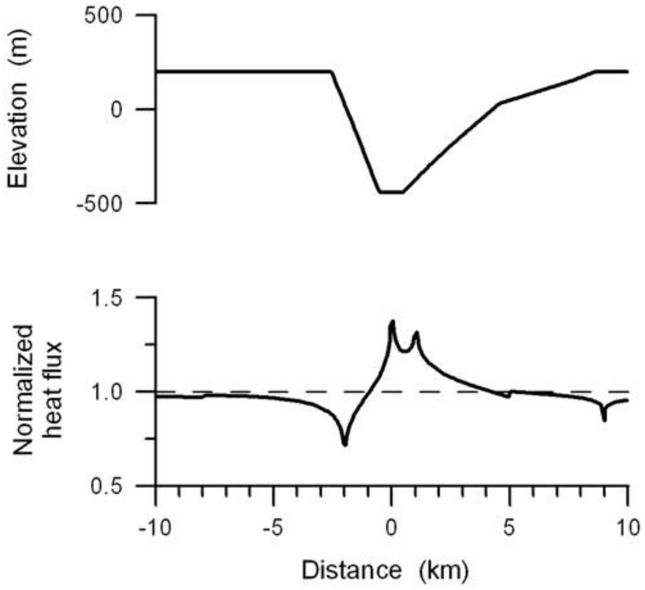

Figure 3. (top) Simplified geometry of the channel under Petermann Glacier and (bottom) estimated normalized heat-flux perturbations.

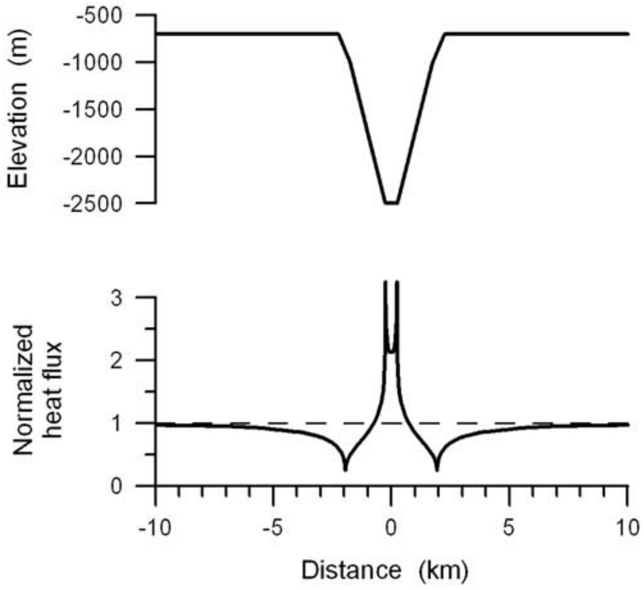

Figure 4. (top) Simplified geometry of the channel under Jakobshavn Isbræ and (bottom) estimated normalized heat-flux perturbations.

Isbræ (Figure 2). For simplicity, the valley is taken to be symmetrical with the same slope angle for both valley walls (45°). The width of the valley floor is 0.5 km, and the depth of the valley is 1800 m. The estimated minimum normalized heat flux perturbation is shown in Figure 4 (bottom). For this geometry, the flux anomaly in the center of the valley is about 100% of the background geothermal flux.

[10] The magnitude of heat flow anomalies is determined by the intensity of the topographic relief, that is, by the steepness of the valley walls, and by the width of the valley. For a vertical cliff, the heat flux at the brink all but disappears, while near the toe the heat flux may more than double. As the slope becomes less steep, heat flow anomalies decrease. Similarly, the geothermal anomaly in the valley decreases rapidly with distance from the toe and becomes negligible for distances that exceed two or three times the valley depth. Thus, topographic control on geothermal heat supplied to the glacier base is only of importance where the subglacial trough is deep and narrow and bounded by steep walls. For wider valleys bounded by steep walls, concentration of geothermal heat is restricted to the outboard regions of the valley close to the toes of the walls.

3. Implications for Basal Thermal Regime

[11] The results shown in Figure 4 suggest that geothermal heat flux under the fast-moving Jakobshavns Isbræ is ~100 % greater than the regional average flux. The continental background geothermal heat flux is *ca.* 57 $\mathrm{mW/m^2}$ [*Sclater et al.*, 1980], and this would be increased to approximately 115 $\mathrm{mW/m^2}$ in the subglacial trench, and greater near the trench walls. To evaluate how this increase may affect the thermal regime at the glacier base, the steady-state temperature profile derived by *Robin* [1955] is used.

[12] For the case of a frozen bed, the difference between the temperature at the glacier base and that at the surface is given by [*van der Veen*, 1999, p. 182]

$$\mathrm{T_b - T_s = \frac{G\sqrt{\pi}}{2\,Kq}\,erf(Hq)},$$

where G represents the geothermal heat flux, H the ice thickness, K = 2.10 W/m K is the thermal conductivity of ice, and

$$\mathrm{q = \left(\frac{M}{2\,kH}\right)^{1/2}}$$

with M the surface mass balance and the thermal diffusivity $\mathrm{k = 34.4\ m^2/yr}$. Keeping the surface temperature fixed, a change in geothermal heat flux ΔG results in a change in basal temperature given by

$$\mathrm{\Delta T_b = \frac{\Delta G\sqrt{\pi}}{2\,Kq}\,erf(Hq)}.$$

Adopting values applicable to Jakobshavn (H = 2500 m, M = 0.5 m/yr, $\Delta\mathrm{G} = 50\ \mathrm{mW/m^2}$) an increase in basal temperature of *ca.* 12°C is obtained. Of course, such a large warming likely will raise the basal temperature to the pressure-melting point, rendering the Robin temperature solution invalid.

[13] Where the basal ice is at the pressure-melting temperature, extra geothermal heat will be used to melt basal ice. The rate of basal melting can be estimated from

$$\mathrm{M_b} = \frac{\Delta G}{\mathrm{L_f}\rho},$$

where the specific latent heat of fusion is $\mathrm{L_f}$ = 333.5 kJ/kg, and the density of glacier ice is $\rho = 917\ \mathrm{kg/m^3}$. With $\Delta\mathrm{G} = 50\ \mathrm{mW/m^2}$ the melt rate is 5 mm/yr, or more than half the estimated melt rate at NGRIP [*Dahl-Jensen et al.*, 2003].

4. Discussion

[14] Potentially, an important feedback between increased heat flow in subglacial channels and deepening of the channel through erosion exists. Numerical model experiments on large-scale glacial erosion suggest that an initially modest basal depression may develop into a deep trench bounded by uplifted bedrock [*Oerlemans*, 1984]. This lateral uplift occurs as a result of the strength of the lithosphere and flexure associated with the greater ice load in the trough. As the trough deepens, more geothermal heat will be concentrated in the valley, thus facilitating glacier flow further. Additionally, glacial erosion typically produces U-shaped valleys with relatively steep walls [*Harbor et al.*, 1988], which would further concentrate geothermal heat into the valley. We posit that this positive feedback facilitated the development of the basal temperate layer observed on Jakobshavn Isbræ.

[15] Modeling of englacial temperatures of Jakobshavn Isbræ suggests a basal layer of temperate ice under the lower 200 km of the ice stream [*Funk et al.*, 1994], but observations are lacking to confirm this result because the temperature profile measured on the ice stream did not reach the bed and extended to about two-thirds of the ice thickness below the surface [*Iken et al.*, 1993]. *Funk et al.* [1994] attribute the existence of this temperate basal layer to high deformational heating associated with the large driving stress. According to sensitivity experiments, decreasing the geothermal heat flux by 20% has an insignificant effect on the thickness of the temperate layer [*Fabri et al.*, 1992]. This may be because the basal ice is at the pressure-melting point throughout the temperate layer and no geothermal heat can be conducted upwards. Thus, geothermal heat causes melting from the glacier base but does not otherwise affect the temperate layer. It is not clear from the sensitivity study of *Fabri et al.* [1992] how basal melt rates are affected by a change in geothermal heat flux.

[16] The temperature modeling described by *Funk et al.* [1994] focuses on the current temperature distribution of Jakobshavn Isbræ and its environs, and comparison with profiles measured in boreholes. While that model compares favorably with the observations, it does not explain how the temperate layer formed. There is no evidence that such layers are widespread so a simple feedback involving strain heating may not fully explain the existence of the temperate layer. This suggests that the increased geothermal heat flux may have played an important role in the development of this basal layer. As the channel deepened, geothermal heating at the base gradually increased, thus raising the temperature of the basal ice. This, in turn, leads to enhanced

shearing and heat released by internal deformation, further increasing the basal temperature of the ice.

[17] Concentration of geothermal heat in valleys and troughs will enhance basal melting and evacuation of eroded materials through the subglacial drainage system. Countering this would be deposition of sediments by melting of debris-laden basal ice [*Nobles and Weertman*, 1971]. Where net deposition occurs, enhanced geothermal flux may provide enough heat to increase basal melt rates, allowing the sediments to weaken as they become water-saturated. This, in turn, could increase ice discharge as frictional resistance at the glacier base is lowered.

5. Concluding Remarks

[18] The calculations presented here indicate that small-scale pronounced basal topography can significantly alter the geothermal heat flux and thus influence the thermal regime at the base of glaciers. Concentrated heat flux in narrow and deep valleys can result in warming of the basal ice layers by several degrees or, where the basal ice is at the pressure-melting point, enhance the rate of basal melting.

[19] **Acknowledgments.** This research was supported by the National Science Foundation under grant 0424589.

References

Anderson, K. K., et al. (2004), High-resolution record of Northern Hemisphere climate extending into the last interglacial period, *Nature*, *431*, 147–151.

Beardsmore, G. R., and J. P. Cull (2001), *Crustal Heat Flow: A Guide to Measurement and Modeling*, 324 pp., Cambridge Univ. Press, New York.

Clarke, T. S., and K. Echelmeyer (1996), Seismic-reflection evidence for a deep subglacial trough beneath Jakobshavns Isbræ, west Greenland, *J. Glaciol.*, *43*, 219–232.

Dahl-Jensen, D., N. Gundestrup, S. P. Gogineni, and H. Miller (2003), Basal melt at NorthGRIP modeled from borehole, ice-core and radio-echo sounder observations, *Ann. Glaciol.*, *37*, 207–212.

Ekholm, S., K. Keller, J. L. Bamber, and S. P. Gogineni (1998), Unusual surface morphology from digital elevation models of the Greenland ice sheet, *Geophys. Res. Lett.*, *25*, 3623–3626.

Fabri, K., M. Funk, and A. Iken (1992), A sensitivity study for modeling ice temperatures in Jakobshavns Isbræ, Greenland, *Arbeitsheft 11*, Vers. für Wasserbau Hydrol. und Glaziol, Eidg. Tech. Hochsch., Zurich, Switzerland.

Fahnestock, M., W. Abdalati, I. Joughin, J. Brozena, and P. Gogineni (2001a), High geothermal heat flow, basal melt, and the origin of rapid ice flow in central Greenland, *Science*, *294*, 2338–2342.

Fahnestock, M. A., I. Joughin, T. A. Scambos, R. Kwok, W. B. Krabill, and S. Gogineni (2001b), Ice-stream-related patterns of ice flow in the interior of northeast Greenland, *J. Geophys. Res.*, *106*, 34,035–34,045.

Funk, M., K. Echelmeyer, and A. Iken (1994), Mechanisms of fast flow in Jakobshavns Isbræ, west Greenland: part II. Modeling of englacial temperatures, *J. Glaciol.*, *40*, 569–585.

Gogineni, S., J. Paden, A. Torry, C. Allen, P. Kanagaratnam, D. Braaten, and K. Jezek (2006), Synthetic Aperture Radar imaging of ice-bed interface, *Eos Trans. AGU*, *87*(52), Fall Meet. Suppl., Abstract C14B-06.

Harbor, J. M., B. Hallet, and C. Raymond (1988), A numerical model of landform development by glacial erosion, *Nature*, *333*, 347–349.

Huybrechts, P. (1996), Basal temperature conditions of the Greenland ice sheet during the glacial cycles, *Ann. Glaciol.*, *23*, 226–236.

Iken, A., K. Echelmeyer, W. Harrison, and M. Funk (1993), Mechanisms of fast flow in Jakobshavns Isbræ, west Greenland: part I. Measurements of temperature and water level in deep boreholes, *J. Glaciol.*, *39*, 15–25.

Lachenbruch, A. H. (1968), Rapid estimation of the topographic disturbance to superficial thermal gradients, *Rev. Geophys.*, *6*, 365–400.

Lees, C. H. (1910), On the shapes of the isogeotherms under mountain ranges in radio-active districts, *Proc. R. Soc. London, Ser. A*, *83*, 339–346.

Lohoefener, A., C. Veeramachaneni, S. Raghunandan, V. Ramasami, J. Paden, T. Akins, P. Kanagaratnam, and P. Gogineni (2006), Radar sounding of Jakobshavn and other outlet glaciers, *Eos Tran. AGU*, *87*(52), Fall Meet. Suppl., Abstract C41A-0318.

McLaren, S., M. Sandiford, M. Hand, N. Neumann, L. Wyborn, I. Bastrakova (2003), The hot southern continent: Heat flow and heat production in Australian Proterozoic terranes, in *Evolution and Dynamics of the Australian Plate*, edited by R. R. Hillis and R. D. Muller, *Geol. Soc. of Aust. Spec. Publ.*, *22*, 151–161.

Nobles, L. H., and J. Weertman (1971), Influence of irregularities of the bed of an ice sheet on deposition rate of till, in *Till, A Symposium*, edited by R. P. Goldthwait, pp. 117–126, Ohio State Univ. Press, Columbus.

Oerlemans, J. (1984), Numerical experiments on large-scale erosion, *Z. Gletscherkd. Glazialgeol.*, *20*, 107–126.

Robin, G. de Q. (1955), Ice movement and temperature distribution in glaciers and ice sheets, *J. Glaciol.*, *2*, 523–532.

Sclater, J. G., C. Jaupart, and D. Galson (1980), The heat flow through oceanic and continental crust and the heat loss of the Earth, *Rev. Geophys. Space Phys.*, *18*, 269–311.

van der Veen, C. J. (1999), *Fundamentals of Glacier Dynamics*, 462 pp., A. A. Balkema, Rotterdam, Netherlands.

B. M. Csatho, Department of Geology, State University of New York at Buffalo, Buffalo, NY 14260, USA.

T. Leftwich and R. von Frese, School of Earth Sciences, Ohio State University, 275 Mendenhall Laboratory, 125 South Oval Mall, Columbus, OH 43210, USA.

J. Li, Center for Remote Sensing of Ice Sheets, University of Kansas, 316 Nichols Hall, 2335 Irving Hill Road, Lawrence, KS 66045, USA.

C. J. van der Veen, Department of Geography, University of Kansas, 213 Lindley Hall, 1475 Jayhawk Boulevard, Lawrence, KS 66045, USA. (cjvdv@ku.edu)