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Fracture propagation as means of rapidly transferring surface meltwater to the base of glaciers

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[1] Propagation of water-filled crevasses through glaciers is investigated based on the linear elastic fracture mechanics approach. A crevasse will penetrate to the depth where the stress intensity factor at the crevasse tip equals the fracture toughness of glacier ice. A crevasse subjected to inflow of water will continue to propagate downward with the propagation speed controlled primarily by the rate of water injection. While the far-field tensile stress and fracture toughness determine where crevasses can form, once initiated, the rate of water-driven crevasse propagation is nearly independent of these two parameters. Thus, rapid transfer of surface meltwater to the bed of a cold glacier requires abundant ponding at the surface to initiate and sustain full thickness fracturing before refreezing occurs.
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1. Introduction

[2] *Zwally et al.* [2002] observed increased speeds at a location in the equilibrium zone of the west-central Greenland Ice Sheet following periods of summer melting. Similar speed-up events have been observed on tidewater glaciers (e.g. Columbia Glacier, Alaska [*Meier et al.*, 1994]) as well as on mountain glaciers [e.g., *Iken et al.*, 1983; *Iken and Bindschadler*, 1986], but the observations of *Zwally et al.* [2002] are somewhat surprising, however. While increased discharge following summer melt has been observed on Greenland outlet glaciers [*Joughin et al.*, 1996; *Mohr et al.*, 1998], previous velocity measurements inland of Jakobshavn Isbræ indicated a lack of seasonal variation [*Echelmeyer and Harrison*, 1990]. Further, at the location of the measurements reported by *Zwally et al.* [2002] the ice thickness is ~1 km and at sub-freezing temperatures over most of the thickness. Thus, a drainage connection between the surface and the glacier bed must be established sufficiently rapid to prevent meltwater from refreezing at depth. Usual mechanisms by which drainage conduits develop [*Röthlisberger*, 1972; *Shreve*, 1972] are likely to be too slow to satisfy this requirement and propagation of water-filled fractures appears to be the only viable possibility.

[3] Observations on John Evans Glacier, a predominantly cold valley glacier located on the east coast of Ellesmere Island, Arctic Canada, suggest that hydrologically-driven propagation of fractures may be the mechanism by which a drainage connection between the surface and the subglacial drainage system is established [*Boon and Sharp*, 2003]. Water levels measured in a supraglacial stream entering the glacier via a crevasse indicated that multiple, relatively abrupt, drainage events occurred over a period of about one week during which the crevasse was water-filled or overfilled. During these events, drainage was not sustained, likely because of refreezing of surface meltwater penetrating the initial fractures exceeded water inflow. However, after eight such events the surface pond drained completely within one hour, suggesting a passage through the entire 150 m ice thickness had been established. *Boon and Sharp* [2003] suggest that warming of the englacial ice due to refreezing during preceding drainage events allowed the establishment of a permanent surface-to-bed connection, developed through fracturing driven by a large volume of ponded surface water. A survey of 48 holes drilled into Storglaciaren, Sweden, indicated that englacial drainage is dominantly accommodated by fracture-derived drainage passages, rather than through conduits [*Fountain et al.*, 2005]. These authors argue that conduits controlled by the balance between inward creep of ice and outward melting resulting from frictional heating from moving water [*Röthlisberger*, 1972; *Shreve*, 1972] may be special cases of the englacial flow system, and that the seasonal development of the drainage system may be controlled by fracturing, with surface crevasses providing access for surface meltwater.

[4] Theoretical analyses based on fracture mechanics show that water-filled crevasses can penetrate the full ice thickness of glaciers subject to tension [*Weertman*, 1973; *van der Veen*, 1998]. Because the density of water is slightly greater than that of ice, provided the crevasse remains water-filled, the weight of the water can overcome the lithostatic stress in the ice, thus allowing the crevasse to reach the glacier bed. This water-driven fracturing has been proposed as the mechanism responsible for the rapid collapse of ice shelves in the Antarctic Peninsula [*Scambos et al.*, 2000]. However, the rate at which fractures propagate through glaciers has received little attention so far. *Alley et al.* [2005] investigate crack propagation using the model developed by *Rubin* [1995] for propagation of magma-filled fractures through brittle crustal rocks. They conclude that downward fracture propagation through cold ice requires a large inflow of water to maintain water pressure and offset water loss from refreezing. Given the number of assumptions involved, *Alley et al.* [2005] do not place great faith in the numerical results and, consequently, it is not obvious from their study how rapidly fractures propagate downward.

[5] Growth of a water-filled crevasse may be modeled using theories developed for hydraulic fractures. Since its development during the last half of the 1940s, hydraulic fracturing has been one of the primary engineering tools

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for improving the productivity of oil wells [*Smith and Shlyapobersky*, 2000]. Starting with the pioneering work of *Sneddon* [1946] and *Sneddon and Elliot* [1946], an extensive body of theoretical knowledge has developed (for recent reviews, see: *Ben-Naceur* [1989], *Valkó and Economides* [1995], and *Mack et al.* [2000]). In essence, the fluid pressure within the fracture is related to the fluid flow rate and size or shape of the fracture (Poiseuille's equation), while crack propagation is evaluated based on linear elastic fracture mechanics, subject to the condition that inflow of water equals crack volume growth. Where refreezing takes place (as in magma intrusions or meltwater penetration into cold glaciers), conservation of energy needs to be considered as well. The results are often complex involving many assumptions and poorly constrained material parameters, rendering predictions rather uncertain. Given these reservations, an heuristic approach based on the fracture toughness criterion is adopted in this study to investigate the effect of water inflow on crevasse propagation.

2. Model Formulation

[6] The approach followed in this study to investigate fracture propagation is based on linear elastic fracture mechanics (LEFM) in which the stress intensity factor is used to describe the elastic stresses near the tip of the fracture. This factor provides a measure for the depth to which crevasses will penetrate into the ice; this depth being where the stress intensity factor equals the fracture toughness of ice. Only Mode I opening is considered here, with the crevasse propagating vertically downward and perpendicular to the applied tensile stress (Figure 1). Crevasse opening is due to a far field tensile normal stress acting perpendicular to the plane of the crevasse, while crevasse closure is caused by the weight of the ice. Following *van der Veen* [1998], these effects are separated by equating the normal stress associated with crevasse opening to the resistive stress, R_{xx}, defined as the full or total stress minus the weight-induced lithostatic stress [*van der Veen and Whillans*, 1989]. This resistive stress is associated with glacier flow and analogous to the tectonic stress commonly used in geodynamics [*Turcotte and Schubert*, 2002, p. 77]. For ease of calculation, this resistive stress is taken independent of depth. Crevasse closure results from the weight of the ice and the corresponding closing stress is the lithostatic stress. If the crevasse is partially filled with water, the water pressure compensates all or part of the lithostatic stress in the ice, thus allowing the crevasse to penetrate deeper than when water is absent.

[7] *van der Veen* [1998] provides expressions for the stress intensity factors associated with the resistive tensile stress, the lithostatic stress in the ice, and water pressure. Because single-mode fracturing is considered here, the net stress intensity factor is given by the sum of these three contributions [*Broek*, 1986, p. 84]. To abbreviate the calculations presented here, the simpler expression applied first by *Smith* [1976] to study crevasse penetration is used here rather than the complete expression given by *van der Veen* [1998]. That is, the net stress intensity factor is estimated from

$$K_I = 1.12 R_{xx}\sqrt{\pi d} - 0.683 \rho_i g d^{1.5} + 0.683 \rho_w g b^{1.5} \qquad (1)$$

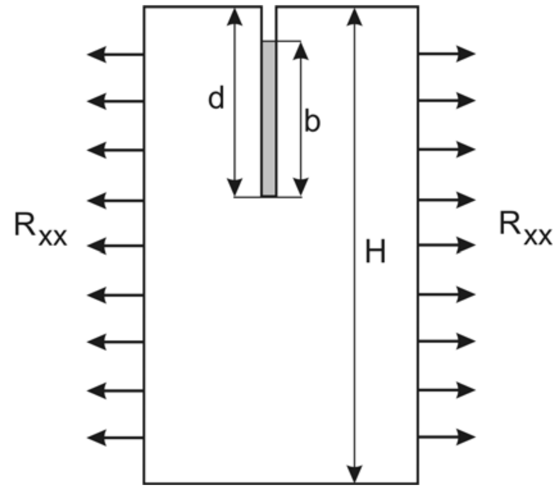

Figure 1. Model geometry. A crevasse of depth d extends vertically downward into a glacier, and is filled with water to a level b above the bottom of the crevasse. The far-field tensile stress, R_{xx}, is assumed constant with depth.

The first term on the right-hand side corresponds to the stress intensity factor associated with a tensile stress; the second term corresponds to the lithostatic stress in ice with density ρ_i, while the third term corresponds to the water pressure effect. The second term is negative because the weight of the ice tends to close the crevasse. By setting the stress intensity factor, K_I, equal to the fracture toughness of ice, K_{IC}, the crevasse depth, d, can be calculated for given tensile stress, R_{xx}, and water height, b, in the crevasse.

[8] Compared to the more complete solution given by *van der Veen* [1998], equation (1) involves several simplifying assumptions. First, the ice is assumed to be a semi-infinite plane or, equivalently, the ratio of crevasse depth to ice thickness is assumed to be small. For crevasses that penetrate a significant fraction of the ice thickness, this assumption severely underestimates the first term on the right-hand side [cf. *van der Veen*, 1998]. Second, the ice density is taken constant so that the compressive overburden stress increases linearly with depth below the surface. Accounting for the lower-density firn layer reduces this overburden stress and thus allows deeper penetration [*van der Veen*, 1998]. However, because this study considers penetration of water-filled crevasses in regions with extensive surface melting, the firn layer likely is absent and a constant density may be assumed. Third, in calculating the stress intensity factor associated with water pressure, the crevasse depth above the water level is neglected which may greatly underestimate the corresponding stress intensity factor (third term on the right-hand side of equation (1)) [*van der Veen*, 1998]. However, for water-filled crevasses to penetrate deep into the ice, the water level must remain near the ice surface (that is, b $\approx$ d), so this simplification is not overly restrictive. Lastly, equation (1) applies to single crevasses only. In a field of closely-spaced crevasses, the tensile stress, R_{xx}, is reduced due to the blunting effect of neighboring crevasses, thus reducing the first term on the right-hand side of equation (1) [*van der Veen*, 1998]. However, for water-filled crevasses extending to a significant depth, this term is small compared to the other two

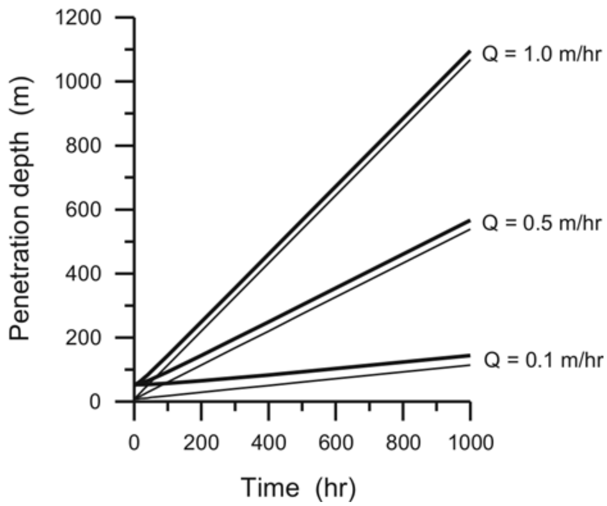

Figure 2. Growth of a water-filled crevasse for different values of the filling rate, Q. Heavy curves correspond to calculations with a prescribed tensile stress of 300 kPa and light curves to a tensile stress of 75 kPa.

terms. Thus, ignoring crevasse interactions is not a severe limitation of the present model.

[9] To investigate crevasse penetration, we start with a water-free crevasse (b = 0) and solve equation (1) for crevasse depth for prescribed tensile stress, R_{xx}, and ice fracture toughness, K_{IC}. Next, the crevasse is filled with water at a constant rate, Q, such that the water level increases linearly with time:

$$b = Q\ t \tag{2}$$

Other filling rates could be adopted, accounting for the changing width and shape of the fracture as it penetrates deeper, but this has little effect on the essential properties of the model. As the crevasse fills with water, the stress intensity factor increases, allowing the crevasse to penetrate to a greater depth which can be found by solving equation (1) for $K_I = K_{IC}$ at select times (every 10 hr). Note that while the height of the water in the crevasse increases with time, so does the crevasse depth and in all calculations, the water level remained 10–20 m below the ice surface.

3. Results

[10] Pertinent results are summarized in Figure 2 and indicate that the single most important factor controlling downward crevasse propagation is the filling rate, Q. Within a few percent, the penetration velocity equals the rate at which the crevasse is filled, independent of the value of the far field tensile stress, R_{xx} (compare light and heavy curves) and of the fracture toughness. Results shown in Figure 2 refer to solutions obtained for a fracture toughness K_{IC} = 100 kPa $m^{1/2}$, but calculations using K_{IC} = 400 kPa $m^{1/2}$ yielded essentially the same results.

[11] The insensitivity of the propagation speed to the value of the tensile stress and of the fracture toughness can be understood by considering equation (1) for the net stress intensity factor. The three terms on the right-hand side are shown in Figure 3, using R_{xx} = 300 kPa and a filling rate Q = 1 m/hr. As the fracture grows, the second and third terms become dominant and, in good approximation,

$$d = \left(\frac{\rho_w}{\rho_i}\right)^{\frac{2}{3}} Q\ t \tag{3}$$

Thus, the propagation speed is essentially determined by the rate at which water is supplied to the crevasse. The tensile stress R_{xx} and fracture toughness are important to allow small fractures to develop into crevasses, but their respective values have little effect once water-driven propagation has started.

4. Applicability of the LEFM Approach

[12] *Valkó and Economides* [1995, p. 243] argue that the LEFM approach becomes more and more unrealistic as the fracture propagates and yields results that contradict common sense. To bolster their argument, they present the "Injection Rate Dependence Paradox." For the geometry under consideration, the stress intensity factor is proportional to the net pressure in the fracture, multiplied with the square root of the fracture length. That is,

$$K_I \sim P_n \sqrt{d} \tag{4}$$

[e.g., *Broek*, 1986, p. 10]. During fracture propagation, K_I remains equal to the fracture toughness, thus the net

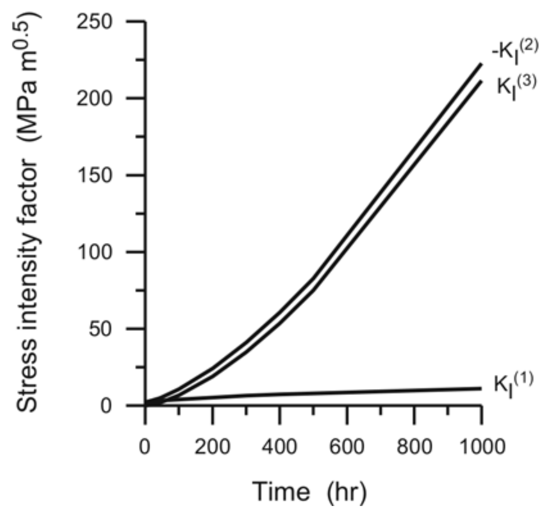

Figure 3. Evolution of the three contributions to the net stress intensity factor for a filling rate Q = 1 m/hr and a tensile stress R_{xx} = 300 kPa. $K_I^{(1)}$ represents the stress intensity factor corresponding to the tensile stress (first term on the right-hand side of equation (1)); $K_I^{(2)}$ represent the effect of weight-induced lithostatic stress causing the crevasse to close (note that this corresponds to the second term on the right-hand side of equation (1) without the minus sign); $K_I^{(3)}$ corresponds to crevasse opening due to water in the crevasse (third term on the right-hand side of equation (1)). During crevasse propagation, the net stress intensity factor equals the fracture toughness for glacier ice, K_{IC} = 100 kPa $m^{1/2}$.

pressure must decrease as the fracture length increases. Because the propagation speed is proportional to the fluid injection rate, this implies that the pressure is inversely proportional to the injection rate. The difficulty with this argument is that equation (4) applies to a fracture with the same pressure, P_n, along its length. However, in the case of water-filled crevasses in glacier ice, both the lithostatic stress and the weight of the water in the crevasse increase with depth and equation (1) should be used to estimate the stress intensity factor. As the crevasse grows deeper, the first term on the right-hand side of this expression increases because the far-field tensile stress, R_{xx}, is kept constant. This increase, combined with the increase in water pressure (third term on the right-hand side) is countered by the increased lithostatic stress (second term) such that the stress intensity factor remains equal to the fracture toughness.

[13] By adopting the fracture toughness criterion, dissipation of energy in the form of heat is neglected. However, as the fracture grows, a zone of microcracks develops at the fracture tip and energy consumed by this process becomes increasingly important and may slow fracture propagation. Thus, the present approach could overestimate the propagation speed [*Valkó and Economides*, 1995, p. 244]. On the other hand, the current model does not account for the finite thickness of the ice which greatly underestimates the first term on the right-hand side of equation (1) for crevasses deeper than about one third of the ice thickness [*van der Veen*, 1998]. Incorporating this effect would increase the propagation speed of deeper crevasses.

[14] Other modifications to the LEFM approach have been suggested, including introducing a fluid lag region to account for the unwetted zone near the crack tip, allowing for dilatancy just behind the fracture tip, and adopting the cumulative damage approach to model near-tip behavior [e.g., *Mack et al.*, 2000]. Without adequate observations against which refined models can be tested, it remains doubtful whether a more sophisticated model will greatly improve basic understanding of fracture propagation in glaciers.

[15] A concern with the LEFM approach to investigate fracturing in glaciers is that ice is a non-linear viscous material subject to stress-induced creep. Provided the creep-dominated process zone at the fracture tip is small relative to the fracture length, LEFM provides a valid approach [*Broek*, 1986, p. 14]. *Riedel and Rice* [1980] developed a model to estimate the size of the process zone for stationary fractures embedded within an isotropic non-linear viscous material. For a suddenly applied loading, they find that the size of the process zone increases linearly with time. Applying their results to glacier ice with a yield strength of 150 kPa, and using a rate factor corresponding to ice near the pressure-melting point, predicts a process zone of a few cm radius 1000 hr after initial loading. Based on numerical calculations, *Nanthikesan and Shyam Sunder* [1995a, 1995b], argued that transient creep may increase the size of the plastic zone by an order of magnitude compared to the analysis of *Riedel and Rice* [1980]. This would imply a plastic zone with a radius of a few meters, which is small compared to the penetration depth of water-filled crevasses. Thus, the LEFM approach is applicable to fracturing on glaciers.

5. Discussion

[16] The results presented here are based on a LEFM model in which crevasse propagation is dictated by the stress intensity factor as compared to the fracture toughness of ice. No rate-determining processes, such as energy concentration or release at the crevasse tip are considered as in more sophisticated models. Nevertheless, the main result, that the rate of crevasse penetration is dominantly controlled by the rate at which water is supplied to the crevasse, is broadly similar to results from related studies and field observations.

[17] An important criticism of the present model could be that refreezing of meltwater is not incorporated, especially considering that the initial question was how surface meltwater can penetrate the full ice thickness of a polar ice sheet with subfreezing temperatures without refreezing before reaching the glacier base. To address this criticism, compare the rate of meltwater freezing and the rate of crevasse propagation. For meltwater at or near the melting temperature, T_m, in contact with ice at initial temperature T_o, the thickness of the layer frozen on during a time t, is approximately given by [*Rubin*, 1995; *Alley et al.*, 2005]

$$w(t) = 2\frac{C(T_m - T_o)}{\sqrt{\pi}\,L}\sqrt{k\,t} \tag{5}$$

where $C = 2093\ J\ kg^{-1}\ K^{-1}$ is the specific heat, $k = 1.18 \times 10^{-6}\ m^2\ s^{-1}$ is the thermal diffusivity, and $L = 3.3 \times 10^5\ J\ kg^{-1}$ is the latent heat of ice. For a temperature difference, $T_m - T_o = 20$ K, the thickness of accreted ice is $\sim$30 cm after 1000 hr. For freezing occurring at both vertical faces of the crevasse, the decrease in crevasse width is $\sim$60 cm over this period. Similarly, the rate at which the tip freezes upward is of the same order of magnitude. Compared to the volume of water that must be supplied to continue downward propagation of the crevasse, and to the rate of crevasse propagation, the effects of meltwater freeze on are small. Further, as water flows into the crevasse, viscous dissipation will partly compensate for refreezing. Of course, if the meltwater source is insufficient to cause full thickness fracturing over periods of several days or so, the rate of crevasse penetration likely will be too small to prevent complete freeze up and arrested drainage to the bed.

[18] A crevasse filling rate of 1 m/hr as used in some of the calculations presented here, might seem to be perhaps unrealistically high. A simple estimate shows, however, that this may be achievable under the right circumstances. Assume a crevasse width of 10 m. Then, per unit length in the horizontal y-direction (coinciding with the crevasse orientation at the surface), 10 m^2/hr of water needs to be added to achieve a filling rate, Q = 1 m/hr. During melt events near the Greenland equilibrium line, surface melt rates can be on the order of 1 mm/hr (Jason Box, personal communication, 2006) so that surface meltwater originating from 5 km on both sides of the crevasse (or 10 km on one side) must drain into the crevasse. Alternatively, meltwater

may first collect in a supraglacial lake before draining into the crevasse. While such conditions may be somewhat unusual, it does not appear to present insurmountable difficulties.

6. Conclusions

[19] By adopting concepts based on linear elastic fracture mechanics, it is shown that water-filled crevasses can penetrate cold glaciers within periods of hours to days, depending on ice thickness and availability of ponding surface water. Once water-driven crevasse propagation is initiated, the growth rate is determined primarily by the amount of water flowing into the crevasse, and not by ice-mechanical properties (fracture toughness) or the magnitude of the remote tensile stress.

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