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*Emphasis on Adolescents and Young Adults*

# FORMAL REASONING ABILITIES OF LEARNING DISABLED ADOLESCENTS: IMPLICATIONS FOR MATHEMATICS INSTRUCTION

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Research Report #7

January, 1980

The University of Kansas Institute for Research in Learning Disabilities is supported by a contract (#300-77-0494) with the Bureau of Education for the Handicapped, Department of Health, Education, and Welfare, U. S. Office of Education, through Title VI-G of Public Law 91-230. The University of Kansas Institute, a joint research effort involving the Department of Special Education and the Bureau of Child Research, has specified the learning disabled adolescent and young adult as the target population. The major responsibility of the Institute is to develop effective means of identifying learning disabled populations at the secondary level and to construct interventions that will have an effect upon school performance and life adjustment. Many areas of research have been designed to study the problems of LD adolescents and young adults in both school and non-school settings (e.g., employment, juvenile justice, military, etc.)

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## Cooperating Agencies

Were it not for the cooperation of many agencies in the public and private sector, the research efforts of The University of Kansas Institute for Research in Learning Disabilities could not be conducted. The Institute has maintained an on-going dialogue with participating school districts and agencies to give focus to the research questions and issues that we address as an Institute. We see this dialogue as a means of reducing the gap between research and practice. This communication also allows us to design procedures that: (a) protect the LD adolescent or young adult, (b) disrupt the on-going program as little as possible, and (c) provide appropriate research data.

The majority of our research to this time has been conducted in public school settings in both Kansas and Missouri. School districts in Kansas which are participating in various studies include: United School District (USD) 384, Blue Valley; USD 500, Kansas City; USD 469, Lansing; USD 497, Lawrence; USD 453, Leavenworth; USD 233, Olathe; USD 305, Salina; USD 450, Shawnee Heights; USD 512, Shawnee Mission, USD 464, Tonganoxie; USD 202, Turner; and USD 501, Topeka. Studies are also being conducted in Center School District and the New School for Human Education, Kansas City, Missouri; the School District of St. Joseph, St. Joseph, Missouri; Delta County, Colorado School District; Montrose County, Colorado School District; Elkhart Community Schools, Elkhart, Indiana; and Beaverton School District, Beaverton, Oregon. Many Child Service Demonstration Centers throughout the country have also contributed to our efforts.

Agencies currently participating in research in the juvenile justice system are the Overland Park, Kansas Youth Diversion Project and the Douglas, Johnson, and Leavenworth County, Kansas Juvenile Courts. Other agencies have participated in out-of-school studies--Achievement Place and Penn House of Lawrence, Kansas, Kansas State Industrial Reformatory, Hutchinson, Kansas; the U.S. Military; and the Job Corps. Numerous employers in the public and private sector have also aided us with studies in employment.

While the agencies mentioned above allowed us to contact individuals and supported our efforts, the cooperation of those individuals--LD adolescents and young adults; parents; professionals in education, the criminal justice system, the business community, and the military--have provided the valuable data for our research. This information will assist us in our research endeavors that have the potential of yielding greatest payoff for interventions with the LD adolescent and young adult.

## ABSTRACT

Modern mathematics education relies heavily upon the cognitive theories of Jean Piaget and Jerome Bruner. These theories provide the basis for explanations of levels of development as well as direction for instructional procedures. Research related to cognitive abilites in learning disabled adolescents, specifically in mathematics, are virtually nonexistent. The present investigation sought to determine the level of formal reasoning in mathematics of LD adolescents. The results of the study suggest that LD junior high school students are functioning at the concrete operations stage of Piaget's developmental sequence. The need for mathematics interventions which use enactive and iconic, as well as verbal/symbolic, representations is stressed.

# FORMAL REASONING ABILITIES OF LEARNING DISABLED ADOLESCENTS: IMPLICATIONS FOR MATHEMATICS INSTRUCTION

While it is generally accepted that research and programmatic considerations related to the learning disabled adolescent are limited, the problems associated with mathematics instruction (e.g., learner characteristics, interventions, curricula) for this population are even more acute. In their review of theoretical and programmatic considerations relative to methods of learning disorders, Myers and Hammill (1969) reported very little attention given to mathematical disabilities. Cawley (1978) stated that similar attention to mathematical disabilities at the upper grade levels has been almost completely ignored. Mathematics education literature is similarly lacking in regard to disabilities in mathematics. Again, the problem is more acute at the secondary level. For example, the 1972 Yearbook of the National Council of Teachers of Mathematics (NCTM, 1972) while devoted entirely to the slow learner in mathematics, did not provide significant insights into mathematics disabilities at the secondary level (Cawley, 1978).

Providing appropriate mathematical interventions for learning disabled adolescents is further complicated by the fact that very little empirical evidence exists regarding the learner characteristics of this population which preclude successful mathematics performance. While a limited amount of literature does exist regarding the mathematical characteristics of learning disabled children (e.g., Johnson and Myklebust, 1967; Lerner, 1976), Deshler (1978) warned against directly applying characteristics of elementary age children to adolescents with learning disabilities.

Although a paucity of research addressing strategies for teaching adolescents with learning disabilities in mathematics is clearly evident, literature does exist which may provide direction for curricular and instructional programming for the adolescent learning disabled population in the area of mathematics. The following review presents relevant literature from the areas of learning disabilities in mathematics, cognitive and mediational processes, and mathematics education.

## Learning Disabilities in Mathematics

Some commonalities exist in descriptions of characteristics of mathematics disorders in learning disabled children. Johnson and Myklebust (1967) enumerated the following seven characteristics in reference to elementary-age learning disabled children:

1. Problems in visual spatial organization and nonverbal integration
2. Above average auditory receptive abilities
3. Above average ability in reading skills and vocabulary
4. Disturbances of body image
5. Spatial disorientation
6. Problems with social perception
7. High verbal abilities and low non-verbal abilities

Lerner (1976) also included disturbances of spatial relationships and visual perception, visual motor association problems, body image problems, spatial disorientation, and perseveration as characteristics of the child with an arithmetic disability. Lerner as well as Johnson and Myklebust theorized that visual perceptual problems, spatial problems, and body image disturbances may relate to the child's lack of sensory motor experiences considered by Piaget (Copeland, 1974) as prerequisite to mathematics learning. Further, these authors recommended an instructional approach which provides many experiences with concrete devices in combination with extensive instruction in the form of auditory input from the teacher.

Another hypothesis regarding the cause of arithmetic disorders has been proposed by Cohn (1971). He stressed the symbolic nature of mathematics and its relationship with verbal and nonverbal thought processes and concluded that arithmetic disability is a type of language disability.

Cognitive/Mediational Processes

Theoretical foundations of modern mathematics education are most heavily grounded in the works of Jean Piaget and Jerome Bruner. The theories of cognitive development proposed by Piaget and Bruner, while addressing the nature of one's development in general, have important implications for the development of mathematical abilites.

Jean Piaget's theory of cognitive development and its implications for mathematics learning and instruction is well known and accepted in mathematics education. Bell (1980) described Piaget's stages of development as follows.

| | Stage | Actions | Approximate Age |
|---|---|---|---|
| 1. | Sensori-motor | Sense and motor actions upon things | (0-1½) |
| 2. | Pre-conceptual | Preoperational actions upon things | (1½-4) |
| 3. | Intuitive | Intuitive operations with things | (4-7) |
| 4. | Concrete operations | Concrete operations with things | (7-12) |
| 5. | Formal operations | Comtemplation about things | (12-15) |

The order in which the stages occur remains constant, however, rate of development varies for individuals. In the concrete operations stage (7 years to 12 or 13 years) learners have difficulty understanding and applying verbal abstractions. While they are capable of performing complex operations using concrete objects, they may not be able to carry out these operations with verbal symbols. During the formal operations stage (12 years to 15 or 16 years old), however, adolescents begin to think abstractly and to reason symbolically. Piaget's contention that formal operation (abstract) thought is requisite to mathematical thinking (Ginsburg & Opper, 1969; Copeland, 1971) is accompanied by the belief among mathematics educators that successful problem solving in mathematics is accomplished at an abstract level and that adolescents typically have achieved the level of formal operational thought necessary for abstract reasoning.

Bruner's (1966) theory of cognitive development tempered the Piagetian notion somewhat. He suggested that enactive, iconic, and symbolic representations may be requisite stages in any learning process regardless of the age of the learner. Bruner theorized that enactive (concrete) and iconic (pictorial) representations of experience involve internal thought processes that are used to complete some tasks throughout life. He also suggested that symbolic learning may depend upon enactive experience in certain instances (e.g., a football player's enactive experience with playing football is prerequisite to his comprehension of a verbal explanation of how to play the game). On the other hand, Piaget and Vygotsky (1962) suggested that inner language composes thought. Bruner's examples of tasks such as hammering, football playing, etc,. suggested that thought can take the form of iconic representations of enactive experience.

Kendler (1972) hypothesized a system of mediational or inner thought that can be either enactive, iconic, or verbal/symbolic. Kendler's model for mediation consists of a stimulus event that triggers an internal mediating response. The internal mediating response, in turn, precipitates an internal response which signals the overt behavioral response. Kendler found that children at kindergarten and first grade levels make little use of verbal external stimulus events in formulating responses. However, by second grade, verbal events assume more meaning, but overt response errors persist, albeit with less frequency than in earlier grades. Kendler hypothesized that the overt response errors of young children are due to production deficiencies; that is, the external stiumlus event fails to provoke the internal response. The overt response errors of older children and adults, according to Kendler, occur because of control deficiences; that is, the internal response to the original stimulus is not used to provide the feedback at the internal stage to produce a correct overt response. Kendler based his hypothesis on the fact that errors in overt motor responses (point to the correct picture) increase significantly among adults when overt labels are not attached to a stimulus picture. However, children in kindergarten and first grade do not show significant differences in overt response with or without a verbal label for the stimulus picture.

Vygotsky, like Piaget and Kendler, noted the superiority of symbolic thinking in the form of language for retrieving events from memory, considering more than one feature of a problem simultaneously, and developing strategies for response in an efficient form that is not dependent on trial and error learning or serial processing of visual images. However, adapting Bruner's theory that enactive and iconic forms of mediation or inner thought persist into adulthood in performance of certain acts, one might conjecture that application of enactive or iconic strategies to mathematics problems would be more efficient for students who have difficulty interpreting or using the abstract symbols of mathematical language. Indeed, Johnson and Myklebust's description of the learning disabled child who exhibits disorders of arithmetic would suggest that a combination of enactive, graphic, and verbal approaches to mathematical problems would maximize the mediational thought processes which the learning disabled student can use in formulating an accurate response.

## Mathematics Education

Modern mathematics education draws heavily on the work of Piaget and Bruner (Reisman, 1978; Grossnickle & Reckzeh, 1973) for justification of the sequence of instructional experiences provided to children in elementary mathematics. Elementary mathematics methods texts (e.g., Grossnickle & Rechzeh, 1973; Jerman and Beardslee, 1978; Underhill, 1977) recommend instructional sequences that include: (a) experiences with concrete manipulative objects; (b) experiences with pictorial representations of the problem; and (c) abstract representations of mathematics using only numerical symbols. These instructional sequences are recommended for most mathematical concepts presented in grades K-6. The concrete, pictorial, and symbolic sequence is directly analogous to Bruner's enactive, iconic, and symbolic stages of representation. While the goal of such an approach is always the symbolic use of mathematical concepts and skills, the content presented and the instructional mode used are ordered according to whether students have achieved concrete operational or formal operational thought.

Current trends in mathematics education provide instructional alternatives through mathematics laboratory approaches which emphasize discovery learning using concrete and pictorial representations of abstract quantitative situations. There is no all-inclusive definition of the laboratory or activities approach (Barson, 1977), but significant characteristics can be described.

The mathematics laboratory approach is activity centered in that it involves the student in problem-solving situations. Mathematics skills and concepts are developed through the use of various instructional representations including concrete mainipulative devices, pictorial representations, and films and tapes in combination with symbolic representations of quantitative situations. Concrete (i.e., enactive or manipulative) devices are objects or things the student is able to feel, touch, handle, and move which are characterized by a physical involvement of students in an active learning situation (Reys, 1977). Iconic (i.e., pictorial) aids are actual photographs of real objects of graphic representations of them.

## Manipulative Aids

Studies of the use of manipulative aids across grade levels one to six have shown relatively positive results when compared to traditional teaching methods. Cuisenaire (Gattegno, 1962) materials were found superior to traditional teaching methods at the first grade level by Aurick (1963), Hollis (1964), and Crowder (1965). The use of other concrete devices were, likewise, found superior at the first grade level by Lucas (1966). In grades two through six, Nasca (1966) found cuisenaire methodology more effective than traditional methods. For grades three through six, Ekman (1966) found better retention among students who had used concrete aids for addition and subtraction algorithms. Dawson and Ruddell (1953) reported greater gains from the use of concrete models to teach division of whole numbers to fourth graders. Norman (1955) demonstrated better retention among third graders using concrete and pictorial models to teach division of whole numbers.

Pictorial Aids

Gibson (1977) reported greater success using slides and overhead transparencies to teach numeral recognition to first graders than by using concrete, manipulative materials. Kulm, Omari, Lewis, and Cook (1974) found that, given a verbal presentation of a word problem, a pictorial representation of the problem, and the student's interpretation of the problem, secondary students with high IQ scores performed significantly better with verbal and pictorial presentations, while secondary students with low IQ scores (92-109 range) performed significantly better with their own interpretation of the problem. Sherrill (1970) found that prose together with an accurate picture produced performance significantly and positively correlated with IQ, reading score, and grade average.

Eastman and Carry (1975) conducted an aptitude-treatment-interaction study involving spatial visualization and verbally presented general reasoning treatments to teach quadratic inequalities at the high school level. Eastman and Carry's findings indicated that the spatial visualization group predicted success on graphic measures and that the general reasoning group predicted success on verbal reasoning measures. As reported, these findings appear to suggest some promise for the use of graphic (iconic) representations as substitutes for, and supplements to, verbal (general reasoning) mediation in problem solving.

All of the preceding studies should be interpreted with caution with regard to the current study since the classification of students in these studies was based primarily on IQ or achievement scores only. The reported findings may, however, have implications for the preferred problem solving strategies of learning disabled adolescents, particularly in light of the previously cited characteristics, including visual spatial problems, nonverbal integration, and memory problems.

Clearly, the development of mathematical abilities is dependent on one's level of cognitive development. In addition, mathematics education has stressed the importance of matching the student's instruction to their level of cognitive development. Based on the characteristics of LD students cited previously, one may hypothesize that mathematics interventions for this population should involve concrete or pictorial, in addition to symbolic, representations of mathematical problems. However, the limited research available on characteristics of LD adolescents, particularly those related to cognitive abilities in mathematics, necessitated an investigation of the formal reasoning abilities of LD adolescents. Information from such an investigation will be used to provide direction for the selection and development of instructional strategies for mathematics interventions with LD adolescents.

## Methodology

### Purpose

The purpose of this study was to: (a) describe the developmental level of formal reasoning of LD adolescents, (b) identify specific subcomponents of mathematics aptitude and achievement which represent deficiencies in the adolescent LD population, and (c) identify the relationship among mathematics achievement and aptitude, reading achievement, and level of formal reasoning of LD adolescents.

Specifically, this study was designed to answer the following research questions:

1. Is there a significant difference between the level of formal reasoning attained by learning disabled and non-learning disabled junior high school students?
2. What specific mathematics deficiencies are exhibited by learning disabled adolescents?
3. What contribution do mathematics achievement, mathematics aptitude and reading achievement, alone or in combination, make to the prediction of formal reasoning?

### Operational Definition of Variables

Developmental level of formal reasoning was defined as the students' score on the Classroom Test of Formal Reasoning (CTFR) (Lawson, 1978). This measure, dsecribed in detail on pages 15 through 17, classifies the respondent according to three levels of formal reasoning: (a) concrete operational stage, (b) transitional stage, and (c) formal operational stage.

All other variables were defined as student scores on selected subtests of the Woodcock-Johnson Psycho-Educational Battery (WJPB) (Woodcock & Johnson, 1977). The ten subtests which were administered are listed below. A complete description of the WJPB as well as the ten subtests can be found on pages 12 through 15.

1. Visual Matching
2. Antonyms-Synonyms
3. Analysis-Synthesis
4. Concept Formation
5. Quantitative Concepts
6. Calculation
7. Applied Problems
8. Letter-Work Identification
9. Word Attack
10. Passage Comprehension

Subjects

One hundred students [70 learning disabled (LD) and 30 non-learning disabled (NLD) students] participated in this study. The students were selected from seventh and eighth grade classes in the four middle-schools of the cooperating district.

LD Sample. The mean age of the LD sample was 164.3 months (SD = 8.1 months). There were 39 seventh grade students and 31 eighth grade students. Forty-nine of the LD students were male and 21 were female. The average full scale IQ on the WISC-R for the LD group was 93.6 (SD = 10.2), with IQs ranging from 70 to 121. The mean Reading Achievement cluster score for the LD group on the WJPB was 486.93 (SD = 17.36).

NLD Sample. The NLD subjects were matched with LD students for age, sex, and school. Mean age for this group was 166.5 months (SD = 7.7 months). There were 14 seventh and 16 eighth grade NLD subjects. Twenty were males and 10 females. IQs for the NLD sample were not available. The mean Reading Achievement cluster score for the NLD group on the WJPB was 520.93 (SD = 15.83).

Due to the loss of some subjects, two Chi-square tests were conducted on the proportions of male to female and seventh to eighth grade subjects to maintain the proportionality of the samples in terms of sex and age. The results of both tests were nonsignificant at alpha = .05.

Informed consent was obtained on the LD sample by mailing consent forms and an explanation of the study to the parents of all seventh and eighth grade LD students in the cooperating district. Eighty-seven per cent of the parents gave consent for their child's participation.

Informed consent for the NLD sample was obtained using the same procedures as for the LD sample, except that once LD consent was confirmed, random selection procedures were employed to select five times the number of NLD subjects necessary to conduct the study. This allowed for the possibility that parents of some NLD students would refuse to allow their children to participate. In all cases where more than enough NLDs consented to participate, the final selection was done randomly.

Instrumentation

Woodcock-Johnson Psycho-Educational Battery (WJPB). The WJPB is a wide-range set of achievement, aptitude and interest tests designed to be individually administered. It was normed on a representative national sample of 4,732 individuals. Norms are provided for children, adolescents, and adults. Evidence pertaining to the reliability and validity of the WJPB is provided by Woodcock (1978). Table 1 represents a summary of reliability coefficients provided by Woodcock for a sample of eighth grade students. Only subtests and clusters used in the present study are considered.

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Insert Table 1 about here

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Extensive information concerning the content validity of the WJPB is provided by Woodcock. Items were selected and subtests carefully constructed on the basis of input from outside experts. Latent-trait theory and the Raush model were used extensively in test construction. Woodcock also presented information concerning criterion-related and construct validity for the WJPB.

Documentation of criterion-related validity of the WJPB is provided relative to data collected on three samples by Woodcock: 82 fifth-grade normal students, 75 twelfth-grade normal students, and a sample of 20 learning-disabled adolescents being served in a private school for LD students. Woodcock reports correlation coeffficients of .77 and .71 between the WJPB Mathematics cluster and the Total Mathematics score of the Iowa Test of Educational Development (Lindquist & Feldt, 1972) for the fifth grade and twelfth grade samples, respectively. For the LD adolescent sample, Woodcock reports a correlation of .77 between the Key Math (Connolly, Nachtman, & Pritchett, 1971) and the Mathematics cluster scores of the WJPB. Cluster analysis is used by Woodcock to provide evidence in support of the organization of the WJPB subtest into clusters.

A total of ten subtests of the WJPB were administered to students in the present study. Four of the subtests comprise the Mathematics Aptitude Cluster: Visual Matching, Antonyms-Synonyms, Analysis-Synthesis, and Concept Formation. In the Visual Matching subtest, the student is required to identify and circle two identical numbers in a row of six numbers. In the Antonyms-Synonyms subtest, the student is required to provide a word whose meaning is either the same as or the opposite of a stimulus word. The Analysis-Synthesis subtest requires a student to "analyze the components of an equivalency statement and reintegrate them to determine the components of a novel equivalency statement" (Woodcock, 1978, p. 28). In the Concept Formation subtest, the student is asked to demonstrate knowledge of rules for a concept when presented with examples and counter-examples of that concept.

The Mathematics Achievement cluster is composed of two subtests: Calculation and Applied Problems. As the names imply, the Calculation subtest involves the use of basic mathematics operations, while the Applied Problems subtest presents word problems in which the student must identify the relevant information and then perform the correct algorithms.

The Quantitative Concepts subtest is not part of the Mathematics Aptitude or the Mathematics Achievement clusters but provides information supplementary to both clusters. In this subtest, the student is asked a number of questions related to quantitative concepts and vocabulary.

The Reading Achievement cluster consists of three subtests: Letter-Word Identification, Word Attack, and Passage Comprehension. In the Letter-Word Identification Subtest, the student is asked to identify the letters of the alphabet and to pronounce increasingly difficult words. The Word Attack subtest requires the student to pronounce nonsense words. The Passage Comprehension subtest utilizes a modified cloze procedure in which the student is required to identify a key word that is missing in a short passage.

Classroom Test of Formal Reasoning (CTFR). Formal operations have been defined to include reasoning processes that will guide an individual in an evaluation of evidence that will support a hypothesis he has made. For the CTFR, Lawson (1978) designed items to tap what he refers to as "combinatorial reasoning, probabilistic reasoning, and proportional reasoning" (p. 12). One item involving conservation of weight and one item involving displaced volume are also included. Traditionally, levels of reasoning have been measured using individually administered Piagetian tasks. Typically, this involves the use of special equipment to demonstrate a problem and the recording of a person's solution in an interview format. The CTFR provides similar assessment information more efficiently as the test can be administered to an entire class at one time.

There are fifteen items on the CTFR. Each item entails a demonstration by the examiner in front of the group of students using the actual physical materials. Each demonstration is carried out until the students respond with predictions of what will happen next. The students record their responses in individual test booklets. These booklets contain the question posed to the student followed by several possible answers. Following the list of possible answers to a question, the student is instructed to explain why he/she chose a particular response. Items are scored correct if the appropriate response is checked and an adequate explanation is given for the selection.

Information pertaining to the reliability and validity of the CTRF has been provided by Lawson. He reported normative data for small samples of eighth, ninth and tenth grade students of varying ability levels from middle class suburban communities. Most pertinent to the present study are the norms for the eighth grade sample. The sample of 145 students included 73 males and 72 females, with a mean age of 14.1 years. The mean CTFR score for this group was 5.68 with a standard deviation of 3.23. The scores ranged from one to 12.

Lawson reported a K-R 20 reliability coefficient for the 15-item test of .78 which he described as "adequate". Three types of validity information are provided. First, experts were in 100% agreement that the items appeared to require either concrete and/or formal reasoning. Second, Lawson reported a correlation of .76 between the total score on the CTFR and a score based on the presentation of traditional Piagetian tasks in the traditional testing format. Finally, a principal components analysis of the CTFR items and several traditional Piagetian tasks revealed that items seemed to measure one of three stages of reasoning: a concrete operations stage, a transitional stage, or a formal operations stage. Lawson reported that the majority of students scoring between zero and five on the CTFR would be classified as functioning at the concrete operational stage according to traditional Piagetian measures. He stated that 35.3% of the total norming sample scored in this range.

Procedures

Two separate tests were administered to all LD and NLD participants. First, selected subtests of the Woodcock-Johnson Psycho-Educational Battery (WJPB) were administered to all LD subjects in the resource rooms of the four middle schools of the participating district. The LD subject administration took place during the individual student's regularly scheduled time in the resource room. The WJPB was administered to the NLD students during prearranged class periods. These students left their regularly scheduled class to take the test.

All WJPB administrations were carried out by research assistants (RA) assigned to this study. Each RA had been trained in WJPB administration by the Core staff of the Institute. Administration procedures provided in the WJPB manual were followed. Administration time was 60 minutes per test. The WJPB scores were derived by using the guidelines and conversion procedures presented in the manual.

The second test, Classroom Test of Formal Reasoning (CTFR), was administered to the LD and NLD subjects in groups of 1012 students. A separate room was available in each middle-school for CTFR administration. Both LD and NLD students were released from regularly scheduled classes to participate in the test administration. The administration procedures for the CTFR recommended by Lawson (1978) were followed for both the LD and NLD groups. The LD and NLD groups were tested separately. Each of the 15 items were demonstrated by one RA while two additional RAs circulated among the group to repeat demonstrations and help with the spelling of words in the students' responses. RAs did not offer explanations, but merely repeated demonstrations and spelled words upon request. Students were instructed as to what the circulating RAs could offer in the way of assistance. The entire test took 60 minutes to administer using this format. Scoring the CTFR was accomplished by following the guidelines established by Lawson. The score recorded for each student represented the sum of the items answered correctly on the test. The RAs who administered the CTFR were trained in the administration of the CTFR.

The WJPB administration for the total group was conducted over a four-week period from mid-April to mid-May, 1979. The CTFR administration period lasted two weeks, i.e., the last two weeks of May, 1979. The entire study was conducted over the three month period of March through May, 1979.

Research Design

This study can be characterized as correlational research. The first research question was answered by describing the extent of the relationship between group membership and level of formal reasoning. The groups were defined as LD students and NLD students. Level of formal reasoning was operationally defined as the student's score on the CTFR.

The second reasearch question was answered by describing the relationship between group membership and subtest scores obtained on the WJPB. The third research question involved the use of multiple regression techniques to explore the relationship between variability on the CTFR and variances associated with specific independent variables, including sex and subtest scores from the WJPB.

## Results

All of the WJPB variables were analyzed using part scores and cluster scores. The derived scores based on the Rausch model and their derivation are described in detail by Woodcock (1978). He recommended these scores for research purposes. A transformation of CTFR raw scores served as the dependent variable for reseach questions one and three. Because the scores on the CTFR were considerably right skewed, a logarithmic transformation of this variable was made. The transformed variable was related to raw scores on the CTFR as follows: LOGLAWS = Log (CTFR Raw Score + 1). LOGLAWS then served as the dependent variables for research questions one and three. All statistics were computed using the BMPD computer programs (Dixon, 1975).

In order to provide statistical tests relevant to research questions one and two, an approach combining multivariate and univariate $\underline{t}$-tests was used (Hummel & Sliglo, 1971). Each of seven WJPB subtests related to mathematics performance and the logarithmic transformation of the CTFR were used as dependent variables, with the independent variable being classification, i.e., LD and NLD. Hotelling's $T^2$ was equal to 67.25 and was statistically significant ($p < .001$). Subsequent to this over-all multivariate test, univariate ttests were conducted with alpha set equal to .01. The means, standards deviations, $\underline{t}$ values and $\underline{p}$ values for each of the eight dependent variables are provided in Table 2. With the exception of two variables, all of the differences were statistically significant. The two exceptions were the Analysis-Synthesis part score and the Concept Formation part score from the WJPB.

Insert Table 2 about here

Date representing raw scores on the CTFR, before the logarithmic transformation, can be summarized as follows: For the LD group, the mean was 2.43 (SD = 2.15) with a range of 0 to 8. The distribution for this group was very skewed, with lower scores being over-represented. For the NLD group, the mean was 4.76 (SD = 2.91) with a range of 0 to 12. The data from the current NLD sample corresponded quite well to the CTFR normative data for eighth graders reported earlier. Ninety-two per cent (56 of 61) of the LD sample scored in the range of 0 to 5 on the CTFR. For the NLD sample, 69% (20 of 29) scored within this range. Five of 61 LD students, or 8%, scored in the range of 6 to 10 correct. For the NLD group, 8 of 29 (28%) scored in the same range. One NLD student and no LD students scored above 10.

The third research question was concerned with the relationship between the logarithmic transformation of the CTFR (LOGLAW) (serving as the dependent variable) and the following independent variables: (a) SEX, (b) WJPB Mathematics Aptitude (MATHAPT), (c) WJPB Mathematics Achievement cluster score (MATHACH), (d) WJPB Reading Achievement cluster score (READACH), and (e) the interaction of the MATHACH and READACH scores (i.e., INTERACT). This relationship was analyzed in an exploratory way using multiple regression techniques. (The entire sample of 100 cases was used in the analysis. Twelve cases, with data missing on one or more variables, were not used.) Table 3 presents the intercorrelations of the dependent and independent variables and the classification variable (LD vs. NLD).

Insert Table 3 about here

At the first stage of analysis of research question three, a stepwise multiple regression analysis was conducted according to the default stepping method (method F) of BMDP2R (Dixon, 1975). In addition, a partial ordering of the independent variables was included such that MATHAPT was entered first at Level One. READACH and MATHACH were then allowed to enter at Level Two. Finally, INTERACT was allowed to enter last at Level Three. SEX was set at zero and not allowed to enter. Based on this procedure, only two variables MATHAPT and MATHACH entered the equation. The results of this analysis are presented in Table 4. The F-to-Enter value for

Insert Table 4 about here

INTERACT after the above two variables were in the equation was .757. Clearly, the addition of this variable would not have made a significant additional contribution to the prediction of LOGLAWS. Likewise, after MATHACH and MATHAPT were entered, the F-to-Enter value for READACH was .87. Although the variable SEX was not allowed to enter, it had an F- to-Enter value at the end of the analysis of 8.128. Thus, in subsequent analyses three independent variables were included: MATHAPT, MATHACH, and SEX.

At the second stage, subsequent to the initial stepwise procedure, a multiple regression analysis, which was not stepwise, was completed. Again, LOGLAWS served as the dependent variable while MATHAPT, MATHACH, and SEX served as independent variables. The analysis of variance associated with this analysis is presented in Table 5 (Multiple R = .7066 and Multiple $R^2$ = .4993). In addition, all three of the regression coefficients were tested for statistical significance. Each was significant beyond the .01 level. Thus, each of the three independent variables made a statistically significant contribution to the prediction of LOGLAWS scores.

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Insert Table 5 about here

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At the third stage, communality analysis (Kerlinger & Pedhazer, 1973) was used to analyze the explained variance of LOGLAWS into the unique and combined contributions of the three independent variables of Stage Two. The explained variance of LOGLAWS ($R^2$ = .4993) was broken down into the proportions of variance explained uniquely by each of independent variables, the proportions explained by each of the three pairs of independent variables, and the proportion explained by all three variables operating in combination. These proportions are presented in Table 6. It is clear from these data that performance on LOGLAWS is largely related to the unique contribution of MATHAPT (.1381) and the contribution of MATHAPT and MATHACH operating in combination (.2627). Nearly all of the remaining explained variance is attributable to the unique contributions of MATHACH (.0524) and the unique contribution of SEX (.0439).

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Insert Table 6 about here

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## Discussion

### Research Questions One and Three

Two levels of Piaget's sequence of cognitive development are important in considering the results of the present study. In the concrete operations stage (typically 7 years to 12 or 13 years), learners are capable of performing complex operations using concrete objects; however, they may not be able to carry-out these operations with verbal symbols. By the formal operations stage (12 years to 15 or 16 years old), adolescents develop abilities to think abstractly and to reason symbolically. The order in which the stages occur remains constant; however, rate of development varies for individuals. Applying this information to the present study is complicated by the fact that junior high-school students (12-13 years old) typically are in a transitional stage between concrete and formal operations. They may think and act as children or adults at different times. Even if a student has reached the stage of formal operations, he/she will not use all the resultant intellectual abilities of that stage and may, at times, revert back to concrete operations.

Bruner (1966) adds to Piaget's developmental theory by identifying levels of representation of mathematical situations. His contention is that abstract learning is facilitated by matching the instructional representation of the mathematical situation to the learner's level of cognitive development. That is, learners who have not attained formal operations can learn mathematical abstractions if concrete, manipulative devices or models are used in instruction to demonstrate the abstraction.

The answer to the first research question in this study indicated that a significant difference exists between the level of formal reasoning attained by LD and NLD junior high school students. In addition, the third research question provided further evidence of the substantial relationship between level of formal reasoning and mathematics aptitude and achievement.

The contribution of SEX to the prediction of level of formal reasoning is consistent with other research (e.g., Anastasi & Foley, 1949; Stafford, 1972; Mullis, 1975) which demonstrated slight advantages for males in mathematics aptitude and achievement. The meaning of this relationship, however, is not clear and has been contested (Fennema & Sherman, 1977). The debate centers on whether the differences in mathematics ability between males and females are inherent or due to socialization.

The implications of the findings of this study are quite clear for both LD adolescents and children. At the secondary level, the efficiency of attempts to remediate mathematics deficiencies should be improved through the use of interventions which capitalize on the power of concrete and graphic representations of concepts, relationships, and operations. Given that these findings could be validated with younger LD populations, the clinical recommendations of Lerner (1976) and Johnson and Myklebust (1967) should be strictly adhered to. Concrete and graphic modes of developmental mathematics instruction may be essential to efficient learning for LD students across grade levels. It appears that LD youngsters, in general, would benefit from the extensive use of concrete learning experiences as well as from the use of these materials for longer periods of time.

The implications of a delay in cognitive development go beyond mathematics instruction and learning. Bell (1980) provided some insights into social learning and behavior associated with the individual's attained level of formal reasoning. For example, it is not until the concrete operations stage that children begin to understand jokes. The ability to understand hidden meanings in social messages is not well developed until the formal operations stage. A delay in cognitive development may be responsible, to some degree, for the characteristic of social imperception attributed to the LD population.

Reading comprehension may also be negatively affected by a delay in cognitive development. Problems in recognizing the deep meaning in printed material could be associated with delays in formal reasoning.

Research Question Two

The attempt to identify specific mathematics deficiencies in ability and performance in the LD sample produced somewhat mixed results. The LD group performed significantly less well than the NLD group on five of the seven mathematics subtests (Visual Matching, Antonyms-Synonyms, Quantitative Concepts, Calculation, and Application Problems) administered, while maintaining this trend toward lower performance on the other two subtests (Analysis-Synthesis and Concept Formation). Whether differences exist in the last two areas is difficult to say from these data and represents a need for further cross-validation. What is clear from these data is that mathematics performance appears to be a generalized deficit in cognitive development and achievement rather than the traditional uneven profile attributed to LD students.

## Need for Further Study

Further study should address a limitation of this study. Future studies should match LD and NLD students on the basis of mental age (MA). Although results are mixed (Cohn-Jones & Seim, 1978), MA has been implicated as an important variable in studies which attempt to compare cognitive development between groups. It was not possible to match students in this study due to the unavailability of IQ scores on the NLD sample. However, the fact that the mean IQ for the LD group was 93.6 may have implications for the mean level of formal reasoning attained by the LD sample.

The second recommendation for additional study is for the replication of this study with LD students at other age levels. The purpose of these replications should be to identify a developmental time-line specific to the LD population. Based on the results of this study, it is likely that a developmental schedule may result which is delayed in comparison to that posed for NLD students. If such a difference exists, it would hold serious implications for when and how skills and concepts were introduced to LD youngsters.

A final area for further study would include the experimental validation of both developmental and remedial interventions for LD students in the area of mathematics. A strong developmental intervention strategy which capitalizes on the power of manipulative devices may lessen the need for remedial interventions with older LD students in the future.

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TABLE 1

Reliability Coefficients for Eighth Grade Sample ($n \cong 470$) on Selected WJPB Subtests **

| | |
|---|---|
| Mathematics Aptitude | |
| Quantitive Concepts | r = .89 |
| Visual Matching | r = .63 (test-retest) |
| Antonyms-Synonyms | r = .90 |
| Analysis-Synthesis | r = .86 |
| Concept Formation | r = .87 |
| Mathematics Achievements | |
| Calculation | r = .84 |
| Applied Problems | r = .83 |
| Math Aptitude Cluster | r = .88 |
| Reading Cluster | r = .96 |
| Mathematics Cluster | r = .90 |

** All coefficients with the exception of that for Visual Matching are split-half coefficients corrected with the Spearman-Brown formula. (From Woodcock, 1978, pp. 178-180.)

TABLE 2

Means, Standard Deviations, *t*, and *p* Values for Eight Dependent Variables--LD and Non-LD Groups Compared

| Variable Name | Mean | SD | *t* | *p* |
|---|---|---|---|---|
| Visual Matching | | | | |
| LD | 131.57 | 8.26 | | |
| NLD | 136.87 | 5.82 | -3.18 | =.002 |
| Antonyms--Synonyms | | | | |
| LD | 198.76 | 6.41 | | |
| NLD | 205.37 | 6.30 | -4.75 | <.001 |
| Analysis--Synthesis | | | | |
| LD | 134.20 | 3.12 | | |
| NLD | 135.43 | 3.95 | -1.67 | =0.098 |
| Concept Formation | | | | |
| LD | 41.60 | 2.06 | | |
| NLD | 43.47 | 1.81 | -2.00 | =0.049 |
| Quantitative Concepts | | | | |
| LD | 240.44 | 9.16 | | |
| NLD | 253.07 | 8.23 | -6.42 | <0.001 |
| Calculation | | | | |
| LD | 259.57 | 8.47 | | |
| NLD | 271.30 | 5.66 | -6.94 | <0.001 |
| Application Problems | | | | |
| LD | 249.56 | 11.87 | | |
| NLD | 262.30 | 9.86 | -5.16 | <0.001 |
| CTFR (log transformation) | | | | |
| LD | 0.44 | 0.30 | | |
| NLD | 0.69 | 0.27 | -3.86 | <0.001 |

TABLE 3

Correlation Coefficients for Selected Variables **

| | SEX | MATHAPT | MATHACH | READACH | LOGLAWS | INTERACT | Classif. (LD vs. NLD) |
|---|---|---|---|---|---|---|---|
| SEX | 1.00 | .02 | -.05 | .12 | -.21 | .05 | .00 |
| MATHAPT | | 1.00 | .59 | .47 | .63 | .61 | .47 |
| MATHACH | | | 1.00 | .51 | .57 | .84 | .59 |
| READACH | | | | 1.00 | .43 | .89 | .68 |
| LOGLAWS | | | | | 1.00 | .57 | .43 |
| INTERACT | | | | | | 1.00 | .73 |
| Classif. (LD vs. NLD) | | | | | | | 1.00 |

** All of the above coefficients were based on 88 cases. Any coefficient with an absolute value greater than .27 (approximately) is significant at alpha = .01, using a two-tailed test. Problems of error rate are not taken into consideration.

TABLE 4

Summary of Analysis of Variance Based on a
Stepwise Multiple Regression Analysis of LOGLAWS

| SOURCE | SS | df | MS | F | p |
|---|---|---|---|---|---|
| Regression | 3.817 | 2 | 1.909 | 35.547 | .001 |
| Residual | 4.565 | 85 | .0537 | | |

Multiple R = .6748

Multiple $R^2$ = .4554

TABLE 5

Summary of Analysis of Variance for
LOGLAWS with MATHAPT, MATHACH, and SEX
as Independent Variables **

| Source | SS | df | MS | F | P |
|---|---|---|---|---|---|
| Regression | 4.185 | 3 | 1.395 | 27.919 | .001 |
| Residual | 4.197 | 84 | .05 | | |

Multiple R = .7066

Multiple $R^2$ = .4993

** Each of the regression coefficients was significant at alpha = .01, using a two-tailed test.

TABLE 6

Partition of the Explained Variances on LOGLAWS into Unique and Combined Contributions of Three Independent Variables

| Variables | Contributions - Proportion of Variance Explained |
|---|---|
| Unique of MATHAPT | .1381 |
| Unique of MATHACH | .0524 |
| Unique of SEX | .0439 |
| Common to MATHAPT and MATHACH | .2627 |
| Common to MATHAPT and SEX | -.0102 |
| Common to MATHAPT and SEX | .009 |
| Common to MATHAPT, MATHACH, and SEX | .0034 |
| Total Variance Explained | .4993 |