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What Are We Doing When We Translate from Quantitative Models?

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Although quantitative analysis (in which behavior principles are defined in terms of equations) has become common in basic behavior analysis, translational efforts often examine everyday events through the lens of narrative versions of laboratory-derived principles. This approach to translation, although useful, is incomplete because equations may convey concepts that are difficult to capture in words. To support this point, we provide a nontechnical introduction to selected aspects of quantitative analysis; consider some issues that translational investigators (and, potentially, practitioners) confront when attempting to translate from quantitative models; and discuss examples of relevant translational studies. We conclude that, where behavior-science translation is concerned, the *quantitative* features of quantitative models cannot be ignored without sacrificing conceptual precision, scientific and practical insights, and the capacity of the basic and applied wings of behavior analysis to communicate effectively.

Key words: quantitative analysis, quantitative models, curve fitting, equations, translational research

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A signature notion in Skinner's (e.g., 1953, 1957) "translational" writings was that, in behavior analysis, the same empirical and conceptual tools apply equally to all levels of analysis, from laboratory to field. Such continuity is evident, for instance, when applied behavior-analytic research and practice are conceived explicitly as an extension of laboratory-based principles (e.g., Critchfield & Reed, 2004; Lerman, 2003; Mace, 1996; Wacker, 2000). This point was highlighted effectively in an essay by Waltz and Follette (2009), who showed how concepts that originate in the laboratory can advance clinical assessment and treatment.

A robust historical tradition of translation in behavior analysis would be jeopardized if basic and applied personnel ever ceased to speak the same language, and there are signs that this is now occurring. Basic behavioral science increasingly emphasizes quantitative methods (Nevin, 2008) in which data are organized and theoretical principles are delineated through equations. Yet few potential translators (i.e., those who focus on behavior outside the laboratory) are well equipped to understand this work.[1] The Waltz and Follette (2009) article epitomized the contemporary tension concerning quantitative analysis. On the one hand, it featured three conceptual advances—matching, delay discount-

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[1] We can cite no actuarial data to support this point, but instead rely on extensive anecdotal evidence. This includes reactions of applied reviewers and convention audiences to manuscripts and presentations, respectively, that feature quantitative analyses. We also note that graduate training standards in behavior analysis (Behavior Analysis Certification Board®, Association for Behavior Analysis International) do not require much attention to quantitative analysis.

ing, and behavioral momentum—that all derive from quantitative research. This suggests that a laboratory shift toward quantitative methods has not squelched the belief that "basic research still has much to offer" (Waltz & Follette, p. 65) toward revealing functional relations of practical importance. On the other hand, Waltz and Follette presented equations for only one of the three conceptual advances, and in that case (matching) described models that, while relatively easy to explain, have been largely supplanted by better models in basic research. This suggests that Waltz and Follette share our concern about the quantitative skills of behavior analysts who seek to translate from basic principles.

We hope that, by describing clinical advances, Waltz and Follette (2009) succeeded in whetting reader appetites for the fruits of the quantitative analysis of behavior. We worry, however, that a mostly narrative (word-based) approach to translating from quantitative models both obscures the important conceptual role that equations play in behavioral science and suggests that nothing is lost when equations are ignored. The present essay addresses this perspective in the context of a more fundamental question: What, exactly, are we doing when we seek to translate[2] from quantitative models? Surprisingly, little has been written for general consumption on this topic. To be sure, many valuable resources on quantitative analysis are available, including treatises on the role of quantitative analysis in basic science (e.g., Marr, 1989; Shull, 1991) and reviews that focus on specific quantitative models (e.g., Baum, 1979; Green & Myerson, 2004; Nevin, 1988; Rachlin, 2006). These, however, often lack a translational perspective. Also available are a number of translational essays and reviews that invoke quantitative concepts (e.g., McDowell, 1988; Nevin, 2005, 2008; Pierce & Epling, 1995; Vuchinich, 1995). Often these either assume technical expertise that behavior analysts may lack or, like Waltz and Follette, present narrative insights from quantitative research to solve particular applied problems while saying little about quantitative analysis per se.

The present essay seeks to provide a beginner's peek at two issues. The first issue concerns how quantitative models are to be understood as the launching pad for translational efforts. For behavior analysts with limited quantitative training (most of us, we believe), understanding equations is no mean feat. We will, therefore, comment very globally on what equations are, how they are read, and what they say that is difficult to express in words. The second issue concerns the challenges inherent in understanding everyday problems from the perspective of quantitative analysis. Among these challenges are identifying models from which to translate, deciding whether to use quantitative or nonquantitative methods to explore insights from quantitative models, and deciding which aspects of a given model will be the focus of a particular translational investigation. In discussing these challenges, we will describe examples of translational inquiries that reflect varying degrees of emphasis on equation-based principles and analyses. Although we do not dispute the value of narrative extensions of quantitative principles, like those emphasized by Waltz and Follette (2009), we will argue that translation potentially is richer and more nuanced when the underlying equations are considered.

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[2] Here we take the most general view of translation as exploring the generality of laboratory-based principles. Better applied practice is a valuable by-product of this exploration but unlike in Waltz and Follette (2009) is not our explicit focus, for two reasons. First, we are content to leave the challenge of tailoring interventions to the peculiar challenges of each practice domain to domain experts. Second, insights about everyday behavior need not spawn immediate interventions to be useful (for elaboration on this point, see Critchfield et al., 2003).

INTRODUCTION TO QUANTITATIVE ANALYSIS

Why Quantitative Models?

If we are right about the training backgrounds of many behavior analysts, probably the first question to arise when equations surface (outside the fairly small community of quantitatively inclined basic researchers) concerns what all of the fuss is about. Why *has* quantitative analysis come to dominate basic behavior analysis? A cynic might suggest that some basic researchers simply love equations qua equations, and might offer as supporting evidence comments like those of Nevin (2008): "I must confess to feeling a genuine thrill when I entered the summed-exponential model into an Excel spreadsheet and saw the fitted line snap into place on top of the data points" (p. 123). Taken out of context, this statement appears to place greater emphasis on curve fitting than on behavior.

But Nevin's (2008) comment should *not* be taken out of context. His essay (which we recommend to anyone concerned with the everyday relevance of quantitative analysis) briefly traces the evolution of experimental behavior analysis as portrayed in the *Journal of the Experimental Analysis of Behavior*. What began as an exploratory search for the simplest of momentary cause–effect relations has become a more mature science grounded in higher order principles that can be understood only by observing behavior over time and under many different conditions. To put it another way, early experimental analyses often focused on how behavior is controlled in a given situation. The quantitative approach attempts to understand relations among various situations in which behavior is controlled. From this general perspective, quantitative analysis should appeal to those who care about the everyday world, in which no two problems arise under identical circumstances. Tackling each everyday problem as a unique case is inefficient. Quantitative analyses unite a whole continuum of circumstances under which problems may arise. To wit: In the preceding quote, Nevin's "thrill" derived from an equation that revealed commonality among armed conflicts, dating back to 1816, that historians often regard as unique in cause and scope.

For an example relevant to Waltz and Follette (2009), consider problems of interest in research on delay discounting. In many situations, people must choose between impulsive actions that lead to small, immediate reinforcers and self-controlled actions that lead to larger, delayed reinforcers. Each of us, therefore, decides whether to eat nutritious or nonnutritious foods; to be active or sedentary; to pursue a pleasurable hobby or an important work project; and so on. It is important to note that nobody is impulsive or self-controlled all of the time. The central contribution of discounting research is to define a continuum of impulsiveness, that is, to identify factors (and interactions among these factors) that push an individual toward impulsive or self-controlled actions. Within this continuum it is possible to see many apparently unrelated everyday behaviors (e.g., gambling, eating, and drug use) as related, to predict whether impulsivity or self-control is expected in a given situation and, if the result is undesirable, to devise interventions that should reduce impulsive behavior.

Parsimonious shorthand. One advantage of equations, therefore, is parsimony. Figure 1 shows how selected features of a continuum of impulsiveness are succinctly expressed in an equation that has been used in discounting theory and research (Mazur, 1987). The equation predicts the subjective value of a delayed consequence (its capacity to control current actions that compete with impulsive

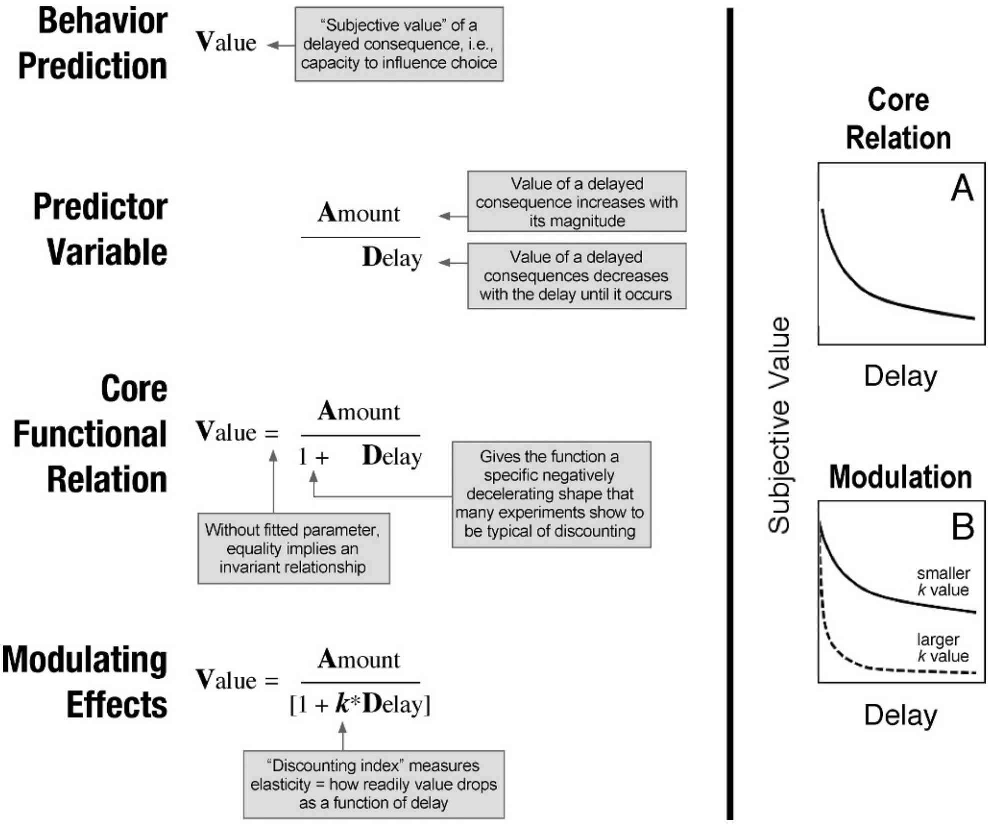

Figure 1. Left: components of Mazur's (1987) quantitative model of delay discounting. Right: Top panel shows the general hyperbolic shape of the function that the model anticipates. As delay to an outcome increases along the abscissa, the subjective value of the outcome deceases on the ordinate. Bottom panel shows modulation of the core hyperbolic function, as measured by the fitted parameter k.

actions that yield small, immediate reinforcers). In far fewer characters than were required to construct this paragraph, the equation says that subjective value increases with reinforcer magnitude (amount) and decreases with delay. In this regard, an equation may be thought of simply as a narrative principle expressed in convenient shorthand.

Higher order relations. In addition, however, equations often incorporate ideas that are hard to adequately express in words. Figure 1, for example, indicates precisely how amount and delay trade off against one another to determine subjective value. Extensive basic research shows that, when amount is held constant, subjective value decreases with delay according to a negatively decelerating function that approximates a hyperbola (Panel A); this dictates the mathematical form of the equation. In descriptive terms, the hyperbolic equation says that subjective value decreases dramatically when only a small amount of delay is introduced; additional delays have diminishing impact on value. An equation can readily express what the present words do not: precisely how much subjective value changes with a given amount of delay. Research also shows that the degree of negative deceleration in subjective value is amount dependent, that is, larger delayed reinforcers are discounted more steeply than smaller ones (e.g., Green & Myerson, 2004). This difference in details of the discounting curves for different-sized reinforcers, in turn, defines the circumstances under which individuals are expected to be impulsive versus self-controlled. If the basis of this conclusion is not evident to you, then our point about equations conveying nonnarrative information has been illustrated. In this case, everything depends on discounting curves taking roughly a hyperbolic shape, rather than, say, an exponential shape. For a reasonably nontechnical explanation of why, see Critchfield and Madden (2006), but for present purposes it is sufficient to say that equations and the data functions they describe are easier to appreciate on their own terms rather than in narrative descriptions, a point to which we will return in our final section.

Understanding Equations

Four Features of Equations

The example of discounting. If translation involves exploring the relevance of laboratory-derived principles to the everyday world, then for quantitative theories a preliminary challenge lies in deciphering those principles. Toward this end, many quantitative models may be thought of as having four features that are depicted in Figure 1 for delay discounting. First, an equation identifies a key behavioral phenomenon that theorists hope to predict. In Figure 1 (discounting), this was subjective value. Second, an equation predicts behavior by reference to selected environmental events. In Figure 1, reinforcer amount and delay were the predictor variables. Third, an equation organizes the predicted behavior and predictor variables into a core functional relation that is the theoretical model's backbone. Various mathematical conventions are employed to put this core relation into a form that follows the available data (and, often, that is relatively easy to evaluate in calculations or graphic display). Recall that in Figure 1 the delay-discounting function was represented mathematically as a hyperbola (Panel A). Fourth, an equation often acknowledges that details of the core relation can vary due to factors that the equation does not directly invoke. Most equations include one or more fitted parameters that may be thought of as higher order dependent variables that measure the modulation of the core relation. In mathematical terms, fitted parameters are unknowns in an equation whose values can only be determined (solved for) by examining the full core relation. In visual terms, a fitted parameter measures how functions that fit the model differ. In Figure 1, the parameter k describes the elasticity of the hyperbolic discounting function. Panel B shows two discounting functions, with the lower curve reflecting a larger k value, or greater sensitivity of subjective value to delay.

We indicated earlier that equations sometimes communicate more effectively than words. This applies particularly to modulating effects that are measured via fitted parameters. Imagine, for example, an intervention that seeks to increase self-control (reduce impulsiveness). Discounting theory says that every individual's capacity to forgo an immediate reinforcer depends on the duration of the wait for a larger alternative reinforcer. Thus, pretreatment impulsiveness is best characterized in terms of a complete discounting function in which the appeal of a given consequence is known for many levels of delay (e.g., Figure 1, Panel B, lower function). If it is difficult in words to say exactly how fast value decreases as a function of delay in this single instance, then the problem is magnified substantially when we compare two discounting functions (Panel B), as would be the case when evaluating the intervention's effectiveness (for

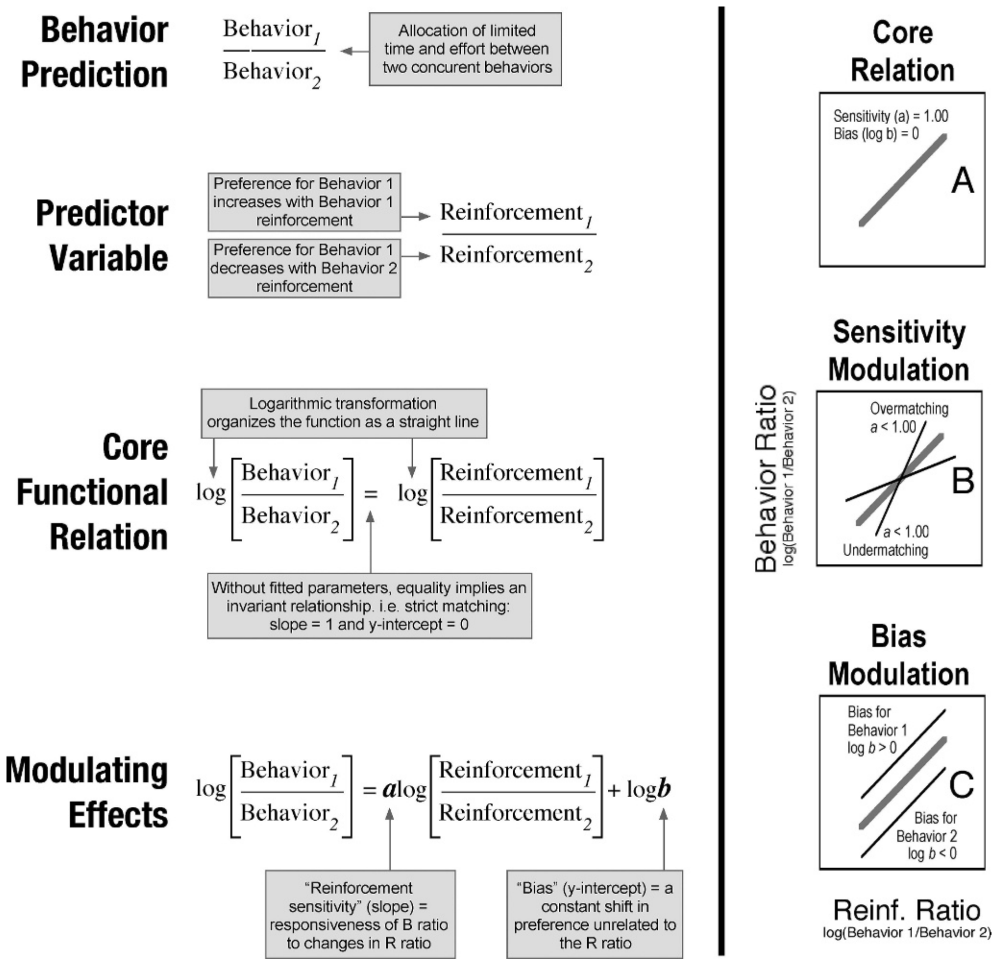

Figure 2. Left: components of Baum's (1974) quantitative model of operant choice. Right: Top panel shows the general linear shape of the function that the model anticipates. As relative reinforcement rate for a target behavior increases along the abscissa, relative investment of time and effort in that behavior increases along the ordinate. Bottom panels show modulation of the core linear function, as measured by the fitted parameters *a* and log *b*.

examples of this approach, see Bickel & Odum, 1999; Yi et al., 2008).

Operant choice. For the sake of demonstrating generality, it may be helpful to briefly examine how the four features of equations apply to a second topic addressed by Waltz and Follette (2009). Figure 2 shows the most widely used equation in the study of operant choice, the generalized matching law (Baum, 1974). Matching theory attempts to predict how the allocation of limited time and effort between two concurrent behaviors varies as a function of differences in reinforcement contingent on those two behaviors. Thus, in Figure 2 the predicted variable is the ratio of frequencies of two behaviors, and the predictor variable is a ratio that shows the relative reinforcement (usually, but not always, measured in rate) produced by the two behaviors. When the behavior and reinforcement ratios are logarithmically transformed, the core relation becomes a straight line (Panel A). This is helpful in part because most people know the equation of a line, and as a result the two fitted parameters of the generalized matching law are expressed in a familiar mathematical context.

The *a* parameter measures the slope of the linear function, which is considered to be a measure of sensitivity to reinforcement, or how behavior allocation changes with relative reinforcement. When the behavior and reinforcement ratios vary proportionally, $a = 1$ (Panel A); in laboratory research the norm is undermatching, $a < 1$, in which the behavior ratio changes less than the reinforcement ratio, and overmatching ($a > 1$) is fairly rare (Panel B). Considerable research has focused on why this is so (e.g., Davison & Nevin, 1999), in the process revealing factors that reliably modulate sensitivity. For example, sensitivity tends to vary negatively with the cost of switching between behaviors and positively with the quality of discriminative stimuli associated with the component behavior–reinforcer contingencies (Davison & Nevin).

The log *b* parameter is the *y* intercept of the overall matching function and a measure of bias for one behavior or reinforcement source. When bias occurs, the behavior ratio still covaries with the reinforcement ratio, but a constant degree of preference exists for one behavior that the reinforcement ratio does not explain. Graphically, this yields an upward or downward shift in the matching function (Figure 2, Panel C). Bias tends to be modulated by factors that differ for the concurrent behaviors but are constant across situations. For example, preference tends to be biased toward behaviors that produce especially large or high-quality reinforcers (e.g., Landon, Davison, & Elliffe, 2003; Miller, 1976).

Curvilinear Functions and Hypothetical Constructs

One take-home message of the preceding section is that an equation is shorthand, not for a narrative behavior principle, but rather for a particular kind of graphic curve[3] that empirically defines a principle. To those not trained in quantitative analysis, such curve-based concepts can seem quite abstract. For instance, to speak of reinforcement sensitivity as a feature of a line on logarithmic coordinates (Figure 2) seems far removed from the direct inspection of momentary behavior fluctuations that Skinner (e.g., 1956) often emphasized in his discussions of method. This does not imply, however, that quantitative analysis is unusually obtuse. Rather, quantitative analysis is just one reflection of a general trend for maturing sciences to develop higher order concepts that subsume many individual observations.

Consider a familiar nonquantitative example. Contemporary behavior analysis relies heavily on single-subject experimental designs such as A-B-A-B to identify an effect consisting of a systematic relation between behavior and an independent variable. What happens when we evaluate an effect? First, several different body movements are jointly considered to determine whether a particular response occurred at a given instant. Several responses that occur within a defined time period may then be jointly considered to determine a response rate. Several response rates, from different days or sessions, may be jointly considered to determine a condition mean or trend. Finally, means or trends from several conditions are jointly considered to evaluate the effect. By the time we speak of an effect we have engaged in many rounds of data aggregation and thus traveled far from momentary behavior fluctuations. Yet the connection between such fluctuations and the derived concept of an effect is, on close inspection, completely transparent.

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[3] *Curve* is used generically to mean both linear and nonlinear functions.

So it is with quantitative analysis. Consider the generalized matching law (Figure 2) as a convenient example. A collection of movements constitutes a response. A collection of responses over time defines rate, which, when compared with the rate of a different behavior (itself a collection of movements sampled over time) defines the generalized matching law's behavior ratio. Parallel aggregation of reinforcers defines the reinforcement ratio. When behavior ratio and reinforcement ratio are jointly considered for several different conditions (e.g., combined in a scatter plot), a linear matching function with a particular slope and intercept is defined. By the time we speak of matching, we have traveled far from momentary behavior fluctuations, but the connection between such fluctuations and the derived concept of matching is completely transparent.

Curve Fitting

Many readers will readily appreciate the general goal of predicting behavior based on environmental variables (which lies at the heart of all empirical psychology) but find equations to be challenging because training in behavior analysis often includes limited attention to mathematics generally and to curve fitting specifically. This underscores that there is no such thing as a casual foray into quantitative analysis, which should be neither surprising nor especially daunting. Foundational skills also are needed to design single-subject experiments (Sidman, 1960), arrange laboratory reinforcement schedules (Lattal, 1991), conduct functional analyses (Hanley, Iwata, & McCord, 2003), devise performance management systems (Daniels, 1994), indeed, to do everything of value in behavior analysis. Professionals acquire the skills they need to function effectively in their respective professions; this is what separates them from laypersons.

Of particular use in conducting or consuming quantitative analyses is familiarity with general classes of curves and the equations that describe them, which greatly eases the task of interpreting core relations. For this we refer the reader to other sources (e.g., Kazdan & Flanigan, 1990; see also Shull, 1991). A related issue concerns the regression techniques used to fit equations to empirical data. This, too, is best addressed by other sources (e.g., Lunneborg, 1994; Motulsky & Christopoulis, 2006), although a brief comment is warranted. In regression, the process of measuring correspondence between an equation's predictions and empirical observations begins by identifying a best fitting function. Recall that most equations anticipate a range of functions; for example, the delay-discounting equation in Figure 1 is compatible with many curves that are hyperbolic but differ in the extent to which subjective value decreases with delay (i.e., k; Panel B). From among all possible curves that are hyperbolic, regression techniques can determine which one best describes a set of empirical observations.

This curve-fitting process yields two useful kinds of information. The first is a statistic that tells how closely the data mirror the best fitting function. One familiar statistic, R^2, indicates the proportion of variance in behavior for which an equation accounts.[4] R^2 ranges from 0 to 1, with

[4] There are many problems with R^2 as a measure of goodness of fit (e.g., Shull, 1991), including that it is easily skewed by outlier observations and that it may covary with fitted parameter estimates, which creates interpretative ambiguities that are beyond the scope of the present discussion. We mention R^2 here mainly because it is familiar; better measures exist (e.g., Shull, 1991). Suffice it to say that in curve fitting, as in everything else in science, the devil is in the details, hence our point about the importance of proper training.

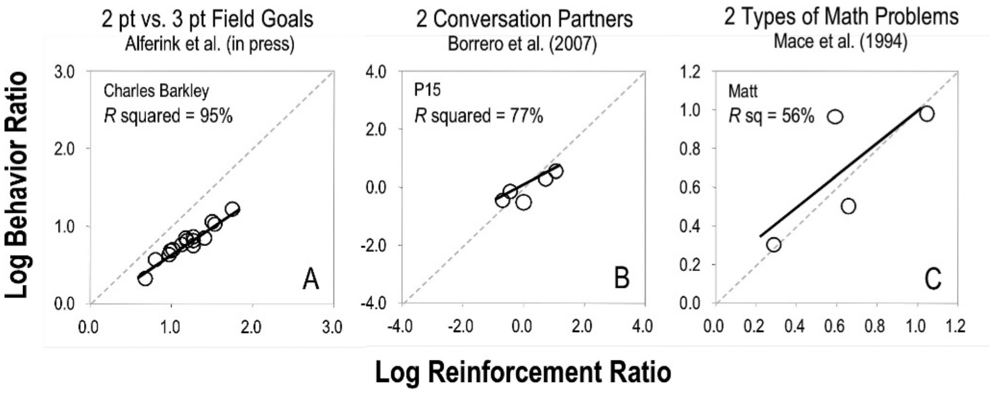

Figure 3. Three translational applications in which the generalized matching law accounted for different amounts of variance in behaviors of everyday interest. (See text for further explanation.) Note that on each logarithmic scale, zero marks the point of equality (here, in terms of either behavior ratio or reinforcement ratio) between two response options. Positive values measure the degree of imbalance in favor of one response option; negative values measure the degree of imbalance in favor of the other response option. All versions of the matching law predict that values on the behavior and reinforcement scales are positively correlated.

values often multiplied by 100 to yield a percentage (i.e., 0% to 100% of variance accounted for). When R^2 is 100%, every empirical observation is exactly what the best fitting function expects; the behavior of interest thus can be fully explained without reference to anything but the equation's predictor variables. When empirical observations deviate from model predictions, R^2 is pushed toward 0%. In practice, R^2 virtually always is less than 100%. Figure 3 shows examples of better and worse fits from translational investigations involving the generalized matching law.[5]

Although goodness-of-fit measure indicate how well behavior conforms to the best fitting function that an equation accommodates, they do not describe the specifics of that function. This is the job of fitted parameters. As noted above, the delay-discounting equation in Figure 1 is compatible with many hyperbolic curves, in all of which subjective value decreases as a function of delay. The fitted parameter k describes the extent of this decrease for a given data set, as described by the best fitting curve. As the tendency toward impulsivity increases (i.e., behavior becomes less governed by large, delayed reinforcers), k grows larger, meaning that the discounting curve becomes steeper. In plain language, fitted parameter estimates[6] are important because they show situational variations in an equation's core functional relation. If k is large in one situation and small in another, for example, the logical practical or scientific mission is to understand why. Later, we will distinguish between translational investigations that focus on core relations and those that focus on situational factors that modulate core functional relations.

[5] *Better* and *worse* are relative terms. In some research areas (e.g., personality theory) investigators may get excited about a model that accounts for only a small fraction of variance in everyday behavior, and therefore might envy even the worst fit shown in Figure 3.

[6] In statistical terms, regression models are said to estimate the value of fitted parameters; this language reflects the standard caution that research can sample only a small portion of the population of available behavior.

TRANSLATIONAL QUANTITATIVE ANALYSIS

Finding a Model to Inspire Translation

Standard Equations

For translation to proceed from a quantitative model, a worthwhile model must be available in the first place. What equations should be the focus of translational efforts? Answering this question requires two forms of hard work. The first involves learning enough about an applied domain to identify the core functional relations that it subsumes. The second involves learning enough about how equations express theory to select a quantitative model with the potential to account for those domain-specific functional relations. We assume that most potential translational investigators in behavior analysis will find the latter task more challenging, and not just because equations may seem difficult to decipher. Those who create quantitative models confront many thorny issues that are beyond both the scope of the present discussion and, we suspect, of the skills of many potential translational investigators. These issues include what style of theory (e.g., cognitive vs. behavioral, molar vs. molecular) should underpin the model; what type of mathematical function (e.g., line, hyperbola, sigmoid, etc.) best fits the relevant behavioral observations; and how complex the model should be, that is, how many empirical variables and fitted parameters are needed to provide the best compromise between parsimony of expression and conceptual thoroughness (Lunneborg, 1994; Motulsky & Christopoulis, 2006).

Fortunately, in one respect, selecting a model on which to base translational research is easy. In areas like those addressed by Waltz and Follette (2009), years of concerted effort by basic scientists have yielded one or more standard equations that receive most of the contemporary attention in basic research. For example, a translational investigator who knows that his or her domain of application involves choice (allocation of limited effort between concurrent behaviors) can inspect the basic literature and find frequent laboratory applications of the generalized matching law (Figure 2; Baum, 1974). This history of attention from basic researchers confers two benefits to the translational investigator. First, it transfers some of the responsibility for model selection to others, because an equation becomes standard only when some consensus exists regarding its proper theoretical underpinnings, a suitable mathematical form, and the number of variables that it should encompass. Second, the process of vetting a model in the laboratory creates a rich empirical literature that not only illustrates how to apply the model but also shows its relevance to a large number of behavior-controlling variables that may be of interest in an applied domain. The latter point matters because translation is in part a test of generality of laboratory-based principles. In devising translational investigations, therefore, it makes sense to focus on equations with a track record of success in accounting for laboratory behavior; in such cases there exists something to generalize *from*.

Correct and Incorrect Equations

Although translational research should focus on good models, it is a mistake to obsess too much about identifying the correct model. Equations are theories, and every theory is wrong in the sense that research ultimately will force it to be supplanted by a new or updated theory. A theory is called successful if it explains selected phenomena and suggests useful research, not if it endures indefinitely (e.g., Gribbin, 2002). The translational investigator who awaits the ultimate equation will never get around to translating.[7]

A standard equation, then, may be viewed as a "good enough" equation, one that, although sure to be imperfect, provides sufficient raw material for translation. Still, it is reasonable to wonder what becomes of translational research when the model on which it is based is supplanted in basic science. Must the insights of translational research be cast aside with the model? Probably not, for two reasons. First, a theory that achieves broad success usually is subsumed into successor theories. In physics, for instance, quantum theory does not invalidate Newtonian phenomena but rather explains them along with much more (Gribbin, 2002). Second, successor models usually add precision to the models they supplant, but the level of precision required to solve applied problems often is less than that required to support theory in basic research. Whatever the contributions of quantum theory, for example, it has changed little about how the typical structural engineer constructs a bridge or house foundation, because translations of earlier (less precise) physical principles are easier to work with and serve just fine for this purpose. The general point is that, once a standard equation becomes available, the time for translational research is now. If a better equation comes along later, it is unlikely to invalidate the contributions of current translational research.

Sometimes There Is No Standard Equation

In some research areas, basic scientists are rapidly developing and testing new equations. This creates a challenge for the translational investigator who seeks a standard equation. Rapid evolution of equations shows a quantitative theory to be in its infancy. Often this means that a core functional relation has been identified that excites basic scientists, but that no consensus has been reached about how to represent this relation in an equation. In such cases, equation-focused translation is premature. Among the three quantitative areas cited by Waltz and Follette (2009), behavioral momentum theory provides a possible example. Many different momentum equations have been proposed (e.g., Brown & White, 2009; Nevin, 1988, 2003; Nevin, Davison, & Shahan, 2005; Nevin & Grace, 2000; Reid, 2009), and it is difficult to find two investigations that employ precisely the same equation. Perhaps as a result, we are aware of no translational investigation in which a momentum equation was presented or featured in the analyses.

Qualitative Applications of Quantitative Models

Although no standard behavioral momentum equation exists, as Waltz and Follette (2009) noted, translation

[7] Indeed, for both standard equations in Figures 1 and 2, competitor models exist, including models that take the same basic form but differ in detail, and models that take a different mathematical form. In the current state of development of behavior analysis, no universally embraced model is likely to exist for any phenomenon. Moreover, by the time enough basic research has accumulated to qualify a model as standard according to our definition, it may be out of date from the cutting-edge perspective of model developers. This is why we recommend against searching for the perfect model on which to base translational work. Probably the most important distinction is between models that lack versus incorporate conceptually meaningful fitted parameters. For example, Waltz and Follette (2009) described an early version of the matching law (Herrnstein, 1961) that lacks the sensitivity and bias parameters shown in Figure 2. This nonparameterized equation neither anticipates clinical outcomes like those illustrated in Figure 4 nor reveals modulating effects like those illustrated in Figures 5 and 6. In general, a model without fitted parameters offers limited raw material for translation. This, we believe, is a major consideration in selecting a standard model from which to translate.

from momentum models *is* occurring. In many cases, the underlying inspiration is not an equation per se but rather the general idea, common to all momentum equations, that resistance to change is a function of recent reinforcement frequency (e.g., translational references). For example, in an analysis of college basketball competition, Mace, Lalli, Shea, and Nevin (1992) predicted, and found, that performances with a rich recent reinforcement history were more likely to persist after adversity than those with a lean recent reinforcement history. As Waltz and Follette detailed, many studies show that compliance with requests can be increased by boosting the history of reinforcement for complying; these techniques were inspired by momentum theory, although perhaps not a particular momentum equation. In these cases, the prediction of interest is qualitative.[8] Rich reinforcement is expected to increase persistence (not decrease it), although the size of the effect is left unspecified. Directional predictions also sometimes are derived from the matching law (e.g., Neef, Mace, Shea, & Shade, 1992) and from discounting theory (e.g., Tucker, Vuchinich, & Gladsjo, 1994). Indeed, most of the research that Waltz and Follette cited in support of clinical applications takes this qualitative form.

Translation from quantitative models also can occur in narrative, interpretative analyses of the sort that Skinner (e.g., 1953, 1957) popularized. For example, Redmon and Lockwood (1986), after reviewing principles of operant choice as described by a precursor to the equation in Figure 2, discussed the implications of operant choice theory for the business world. They described problems in which work behavior could be understood as a concurrent operant and discussed interventions that reflect this general interpretation. In one case, attendance problems of single-parent employees were explained in terms of competition between work and parenting; a successful intervention involved eliminating most of the competition by creating an on-site day care center at the workplace.

If ideas from quantitative models can be applied to everyday problems without the mathematical hassle of quantitative analysis, it might be asked whether, once basic researchers have done their jobs in developing and testing a model, equations may simply be checked at the door leading outside the laboratory. We believe that the answer is a qualified "no." We do not discount the value of interventions that arise from word-based translation. From the broader perspective of developing a fully translational science of behavior, however, quantitative applications contribute in ways that qualitative extensions cannot.

Unique Qualitative Contributions of Quantitative Models

The acid test for any theoretical model, quantitative or otherwise, involves whether it makes unique, testable predictions. In other disciplines, equations are appreciated partly because their relevance to everyday events is unquestioned, in no small measure because of empirical tests involving unique, model-specific predictions. Perhaps the most familiar example is the use of a 1919 solar eclipse to test a critical prediction of Einstein's quantitative model of relativity ($E = mc^2$) (Gribbin, 2002).

A careful reading of the translational literature in behavior analysis will reveal that putative extensions of quantitative models to the everyday world do not always involve unique predictions. For example, the matching law may be invoked to support the

[8] *Qualitative* is used in the sense of predicting a direction of effect rather than an exact level of effect. In some research traditions the term implies a descriptive approach that shares more with humanities inquiries than natural science methods. We imply no parallel between this approach and the empirical clinical studies cited by Waltz and Follette (2009).

general notion that behavior strength reflects the contingencies that govern alternative behaviors. Discounting models may be invoked to support the general notion that impulsiveness varies as a function of the amount and delay of larger, later reinforcers. Yet these principles were widely understood, in general form, long before matching and discounting equations were proposed. For the reader who is interested in this point, a useful exercise is to examine each of the empirical articles cited by Waltz and Follette (2009) to see how many of them test predictions that can be anticipated *only* with the help of a particular quantitative model. If unique, equation-based predictions are not involved, then it is possible (and perhaps, in order to connect with a broad audience, desirable) to dispense with equations in translational efforts.

In some hybrid cases, quantitative models generate unique predictions that need not be tested through quantitative methods. For example, Mace, Mauro, Boyajian, and Eckert (1997) conducted an investigation to determine whether the use of high-quality reinforcement improved the efficacy of an intervention, based on behavioral momentum theory, to improve compliance with requests. The study was prompted by theoretical work integrating momentum with models of operant choice (for a summary, see Nevin & Grace, 2000). The critical comparisons were between high- and low-quality reinforcers; the experimental manipulations were not parametric and no curve fitting was involved. Other translational research has capitalized on the unique qualitative prediction of momentum models that reinforcing alternative behavior can increase the persistence of problem behavior (for a synopsis of this research program, see Mace et al., 2009).

To our knowledge, the preceding examples are atypical. To date, few translational investigations have addressed unique, equation-derived predictions with nonquantitative methods. Because the implications of quantitative models are not always apparent without reference to equations, even investigators who seek to avoid using curve fitting in their analyses may need to think in terms of equations when developing programs of research.

Quantitative Applications of Quantitative Models

Applying Quantitative Models to Field Observations: Special Considerations

Because equation-based principles are foundational to the quantitative analysis of behavior, certain kinds of translational questions can be addressed only by considering these principles on their own terms, that is, through quantitative analyses (curve fitting). Consistent with the tradition that behavior analysts apply the same empirical and conceptual tools at all levels of analysis, the process of curve fitting is the same regardless of whether data are obtained in the laboratory or the field. Yet equation-based analyses of everyday behavior do confront two kinds of special challenges.

Methodological considerations. An equation can be applied to an everyday behavior only if it is possible to measure that behavior across a range of situations that vary in terms of environmental events that appear to reflect the equation's predictor variables. Parametric data are required to fit a curve, and the functional relation specified by a quantitative model is defined by the entire curve. Thinking in terms of equations, therefore, prompts the translational investigator to look for ways to obtain parametric data. This might be unfamiliar territory, because parametric designs have not been the primary focus in single-subject methods (e.g., Sidman, 1960). Moreover, for various practical reasons, parametric data cannot easily be obtained for some everyday behaviors, in which case it may be difficult to test the unique predictions of quantitative models.

Conceptual considerations. In the laboratory, quantitative models are developed to account for the findings of formal experiments that manipulate events for which good a priori reason exists to assume reinforcing effects (e.g., studies of deprivation as an establishing operation for food reinforcement). As a result, when regression techniques are used in laboratory curve fitting, firm cause–effect conclusions are supported.

For some everyday behaviors, it may be impossible to conduct experiments or unambiguously identify naturally occurring reinforcers (e.g., see Critchfield, Haley, Sabo, Colbert, & Macropoulis, 2003). This has two implications. First, considerable thought must be invested in deciding exactly which behaviors and environmental events will be examined in translational extensions of quantitative models, and how these events will be quantified. In published reports of translational investigations, this investment usually is reflected in clear operational definitions of the predicted and predictor variables. Second, when laboratory-derived equations are applied to descriptive (nonexperimental) field data, a good curve fit (e.g., high R^2) does not demonstrate causation. Regression, an application of mathematics to curve fitting, is indifferent to how translational data were obtained. When data are obtained experimentally, correlation implies causation. Otherwise, effects that are structurally similar to laboratory findings could be very different functionally (for a well-elaborated example, see St. Peter et al., 2005; see also Shull, 1991). The take-home message is that translational investigators and those who consume their work are responsible for applying common sense regarding cause–effect inferences. No precise script exists for how to evaluate the conceptual parallel that is implied when a laboratory-derived equation is fit to nonexperimental field data. In general, however, the more a translational investigator knows about both laboratory and field, the better the odds of correctly judging the connections between them.

An example may partially illustrate the necessary process of conceptual bootstrapping. Several studies have applied the generalized matching law (Figure 2) to basketball shot selection, using shots attempted (two-point vs. three-point field goals) in the behavior ratio and shots made in the reinforcement ratio (e.g., Alferink, Critchfield, Hitt, & Higgins, 2009; Vollmer & Bourret, 2000). One reliable finding is that shooting is biased toward three-point attempts. This parallels the laboratory finding that reinforcer size modulates bias, and therefore encourages confidence about functional similarities between the laboratory and the basketball court. In the laboratory, however, matching has been evaluated most often in concurrent interval schedules. By contrast, a knowledgeable basketball observer might describe the naturally occurring reinforcement schedules that govern shooting as ratio-like (Vollmer & Bourret, 2000). This creates an interpretative conundrum, because it is widely thought that the incremental allocation of effort that characterizes matching does not occur under ratio schedules. Thus, although the generalized matching law fits basketball data well (for an example, see Figure 3, Panel A), it is reasonable to ask whether shot-selection matching should be expected at all given the nature of the underlying contingencies.

A particular frustration of translational research is that such questions may not be immediately answerable because the relevant laboratory frame of reference is incomplete. In the present case, for instance, careful reading of the basic literature reveals that debate is unresolved about whether or when ratio-like schedules contribute to matching (Green, Rachlin, &

Hanson, 1983; Herrnstein & Heyman, 1979; MacDonnall, 1988; Savastano & Fantino, 1994; Shimp, 1966; Shurtleff & Silberberg, 1990). Gaps in basic research limit the functional parallels that can be drawn when a laboratory-derived equation is fit to field data. These interpretive dead ends remind us that basic research has not anticipated every important question about everyday contingencies. Thus, although translation normally is thought to proceed from basic to applied, translational inquiries also can suggest fruitful avenues for future basic research (Mace, 1994; Stokes, 1997). An advantage of applying a laboratory-derived quantitative model (rather than a narrative version of its core relation) is that the equation makes this applied-to-basic connection explicit.

Quantitative Evaluation of Core Relations

In some quantitative analyses of everyday behavior, major emphasis is placed on exploring the generality of a model's core relation, including by posing the following questions.

In a given everyday domain, does the model unite cases that might otherwise be seen as disparate? In other words, do various events of interest in this domain fall along the continuum that the quantitative model's core relation defines? Consider offensive strategies in American-rules football, which football experts describe as differing qualitatively across teams. Each team must decide how much it will rely on running versus passing, and some teams are regarded as running teams and others are viewed as passing teams. Reed, Critchfield, and Martens (2006) highlighted the similarities among teams by using the generalized matching law (Figure 2), with frequencies of the two types used in the behavior ratio and yards gained from those plays used in the reinforcement ratio. They found that the 32 National Football League teams, although differing in preference for running versus passing plays, fell along a single linear function that accounted for about 75% of the variance in play selection. Put simply, this function suggested that on all teams, play selection approximately matches the relative success of running and passing plays in gaining yards.

Similarly, interventions based on noncontingent reinforcement (NCR) tend to weaken problem behavior, but the effects are somewhat variable across applications. Ecott and Critchfield (2004) conducted a laboratory simulation in which this variability was evident: For each participant, the extent to which NCR weakened a target behavior differed across conditions, sometimes, apparently, unsystematically. Ecott and Critchfield highlighted the similarities among these outcomes in an analysis using the generalized matching law (Figure 2). Rates of the target behavior and an alternative behavior were used in the behavior ratio, and the frequencies with which NCR followed these behaviors were used in the reinforcement ratio. Individual participant outcomes for the various conditions fell along a single linear function that, in most cases, accounted for more than 80% of the variance in behavior allocation. Put simply, this suggested that the success of NCR interventions depends partly on how often free reinforcers happen to follow problem behavior rather than some other behavior.

Does the model fit behavior in a variety of everyday domains? The beauty of curvilinear functions is that, once they are familiar, they may be digested at a glance. This allows easy evaluation of parallels across domains of everyday interest. Discounting research, for example, shows that subjective value weakens with delay in the same roughly hyperbolic fashion (Figure 1) regardless of whether the delayed outcome under consideration is money (Vuchinich & Simpson, 1998), a drug of abuse (Odum, Madden, Badger, & Bickel, 2000), or good health (Odum, Madden, & Bickel, 2002), thereby suggesting a common conceptual framework for a variety of impulsivity problems (e.g., Madden & Bickel, 2009).

Similarly, many everyday situations incorporate the challenge of allocating limited time and effort between concurrent behaviors. Applications of the generalized matching law (Figure 2) suggest that in many of these situations behavior allocation is a common function of relative reinforcement. Figure 3 shows the core matching relation as it describes conversation (Panel A; Borrero et al., 2007), basketball shot selection (Panel B; Alferink et al., 2009), and academic behavior (Panel C; Mace, Neef, Shade, & Mauro, 1994).

How robust is the core relation in a given everyday domain? Goodness-of-fit statistics can help to guide the conceptual bootstrapping process that curve fitting in the field demands. As noted above, there are limits to what can be concluded when an equation does a good job of describing everyday data. When the equation accounts for relatively little variance, however, something is clearly amiss, which raises interesting questions about the generality of laboratory-derived principles. Among the possibilities are that everyday behavior is strongly controlled by factors that the model does not consider and that the model guesses wrong about the mathematical form of a real-world functional relation (Lunneborg, 1994). A poor fit sets the occasion for further investigation.

Quantitative Evaluation of Modulating Effects

Importance of modulating effects. An equation is defined by its core relation, but the role of higher order concepts in quantitative analysis is most evident in fitted parameters, which describe ways in which the core relation can change. Translational investigations thus make the fullest use of quantitative models when they examine the relevance of fitted parameters to everyday circumstances. From a scientific perspective (the primary focus here), this matters because translation is an exploration of generality. A model is of limited interest if its fitted parameters show only effects that are peculiar to some laboratory procedure. The working assumption, therefore, should be that these parameters apply in meaningful ways to the world outside the laboratory. As will be illustrated shortly, translational research can determine whether this is the case by evaluating the relation between a model's fitted parameters and face-valid effects in an everyday domain.

First, however, a brief digression will show why research that delineates fitted parameter effects is of more than academic interest, because therapeutic outcomes may be better understood by reference to general, parameter-defined principles. Behavior-analytic interventions often increase reinforcement rate for desirable target behavior relative to that for some undesirable alternative behavior; when effective, they have a parallel effect on relative frequency of target versus alternative behavior. The top left panel of Figure 4 shows hypothetical pre- and posttreatment outcomes in the graphic format familiar in matching law (Figure 2) analyses. The black data point designated *b* indicates the baseline relation between relative reinforcement and relative behavior frequency. An intervention that adds reinforcement may yield the black data point *i*, which reflects an increase on both the horizontal axis (relative reinforcement rate) and the vertical axis (relative behavior frequency). What does this intervention accomplish? One obvious possibility is anticipated by narrative principles regarding the reinforcement-based trade-offs between concurrent behaviors: Perhaps the intervention harnesses a preexisting matching relation by shifting

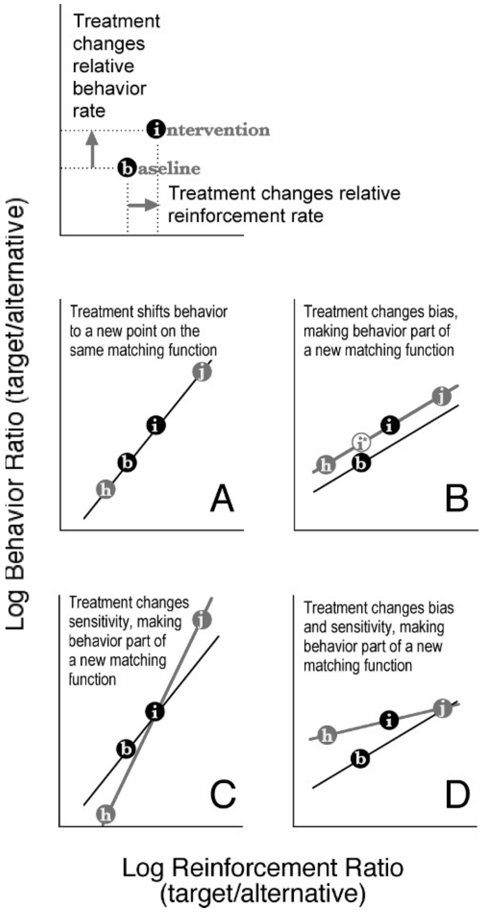

Figure 4. Four plausible interpretations of the same intervention based on the generalized matching law. Top panel: The intervention (*i*) both increases relative reinforcement rate and relative target behavior rate compared to baseline (*b*). Remaining panels: Some predictions (*h*, *i**, and *j*) based on whether the intervention harnesses an existing matching function (Panel A) or creates a new one (Panels, B, C, and D). See text for further explanation.

behavior along the linear function of which it was a part during baseline (Panel B). In descriptive terms, this means that every future observation would be expected to fall along the same behavior–reinforcement function (black diagonal). Thus, whether target behavior reinforcement is further enhanced (gray data point *j*) or restricted to below pretreatment levels (gray data point *h*), division of limited time and effort between target and alternative behaviors is anticipated by a single matching function.

It is also possible for an intervention to harness a new matching function (which would be expressed through the generalized matching law's fitted parameters). Panels B through D of Figure 4 illustrate some possible cases, with the original matching function shown as a black line and the new matching function shown as a gray line. One possibility is that the intervention affects bias (Panel B). Among the reasons why this could happen is that the intervention enhances establishing operations for target behavior reinforcement or adds target behavior reinforcers of especially high quality (Davison & McCarthy, 1988). The result is that no future outcome (e.g., *h* and *j*) is predictable from the baseline matching relation. A second possibility is that the intervention affects sensitivity (Panel C). Among the reasons why this could happen is that the intervention altered discriminative stimuli associated with the competing behaviors (Davison & McCarthy). A third possibility is that the intervention affects both bias and sensitivity (Panel D). In Panels C and D, some outcomes are compatible with the original matching function, but most are not.

In each of the four cases illustrated in Figure 4, the individual "got better" (i.e., relative behavior frequency increased on the vertical axis), but what this implies differs for the four cases. Matching occurs when relative target behavior frequency varies with relative reinforcement rate. In Panel A, this is the only insight of practical importance. In Panels B through D, however, there is more worth knowing. In these cases, some feature of the intervention besides reinforcement rate contributed to therapeutic gains, and the form of the new matching function must be considered in evaluating the intervention's efficacy. In Panel B, for example, because of a bias shift, the individual would have improved even without increasing relative reinforcement. This can be seen by imagining the outcome on the new matching function (data point *i**) with the same relative reinforcement rate as *b*. Note, too, that relative reinforcement rate could decrease somewhat from baseline levels without target behavior becoming worse than at baseline (*h*). In Panel C, heightened sensitivity means bigger changes in target behavior per unit of reinforcement change than would have occurred prior to treatment. This applies to both target behavior improvements associated with increased target behavior reinforcement (*j*) and target behavior deterioration associated with loss of target behavior reinforcement (*h*). In Panel D, the low sensitivity (flat slope) of the treatment-related matching function predicts limited clinical gains with further increases in relative reinforcement (*j*). This function also indicates that behavior would have improved even had relative reinforcement rate decreased somewhat compared to baseline (*h*). Such counterintuitive predictions illustrate how equation-based principles convey ideas that narrative accounts may not anticipate.[9]

Examples in discounting research. Let us return now to the task of scientifically analyzing modulating effects. In translational research, understanding everyday behavior in terms of a quantitative model's fitted parameters requires empirically mapping the model's core relation separately for at least two situations that are interesting in the everyday domain, as the following examples illustrate. In the study of delay discounting, one situation of interest has concerned membership in clinical versus nonclinical populations. For example, as Figure 5A illustrates, considerable research shows that the *k* parameter of the discounting equation (Figure 1) is larger (more impulsivity) for drug abusers than for nonabusers (e.g., Vuchinich & Simpson, 1998). The effect is especially pronounced when the delayed outcome of interest is a preferred drug rather than a generic reinforcer like money (Figure 5B; Odum et al., 2000). In addition, some evidence suggests that programs that reduce drug use also can shrink *k* (Figure 5C, Bickel & Odum, 1999). Other studies suggest that *k* changes as a function of developmental maturation (Green, Fry, & Myerson, 1994); as a function of economic inflation when the delayed outcome is money (Ostaszewski, Green, & Myerson, 1998); and in gamblers as a function of proximity to a gaming facility (Dixon, Jacobs, & Sanders, 2006).

Examples in matching law research. Translational work using the generalized matching law (Figure 2) also shows how everyday circumstances can modulate fitted parameters. Reinforcement sensitivity, for example, has been linked to group membership. Using a laboratory choice task, Kollins, Lane, and Shapiro (1997) found that sensitivity often was lower for children with attention deficit hyperactivity disorder than for typically developing controls (Figure 6A). Alferink et al. (2009) found that the sensitivity of basketball shot selection was higher for regular college players than for substitutes (Figure 6B).

Bias was linked to situational factors in a study by Reed and Martens (2008), in which children could complete math problems to earn tokens at either of two work stations. In general, more problems were completed at the station with the richer reinforcement schedule. When the problems at the two stations were of equal difficulty, little bias was evident, but when

[9] We do not wish to imply that it is possible, or desirable, to empirically determine matching relations in clinical practice. Rather, our point is consistent with one that Waltz and Follette (2009) emphasized: By knowing basic behavior principles it is possible to anticipate treatment effects more precisely than otherwise. We merely add that modulating effects, as reflected in fitted parameters and the variables that are known to affect them, are part of these principles, and can be fully appreciated and explained only by reference to equations and curve fitting.

problems were more difficult at one station, a bias for the other station emerged (Figure 6C). Stilling and Critchfield (in press; see also Reed et al., 2006) found that bias in football play selection varied as a function of the down on which a play was executed (Figure 6D); as part of the same investigation, Stilling (2008) found that bias varied with other situational factors such as the game score, the location on the field from which a play originated, and the amount of competition time remaining when a play began.

In Search of Unique Translational Predictions

In the preceding section we finessed a sensitive issue. Although earlier we stressed the capacity of quantitative models to generate unique predictions, many of the published studies in which quantitative analysis was applied to everyday problems have a distinct exploratory feel to them. For example, Reed et al. (2006) summarized their application of the generalized matching law (Figure 2) to football play selection by noting that, "The present findings are noteworthy in that a fairly simple quantitative model predicts behavior under the complex circumstances of elite sport competition" (p. 293). This conclusion illustrates a feature that is shared by many quantitative translational investigations to date, namely an interest in whether or not a model's core relation is relevant outside the laboratory. As Skinner (1956) noted, seeing order in behavior is (or should be!) a natural reinforcer, so it is indeed interesting when everyday behavior follows a curve of a particular shape that also happens to

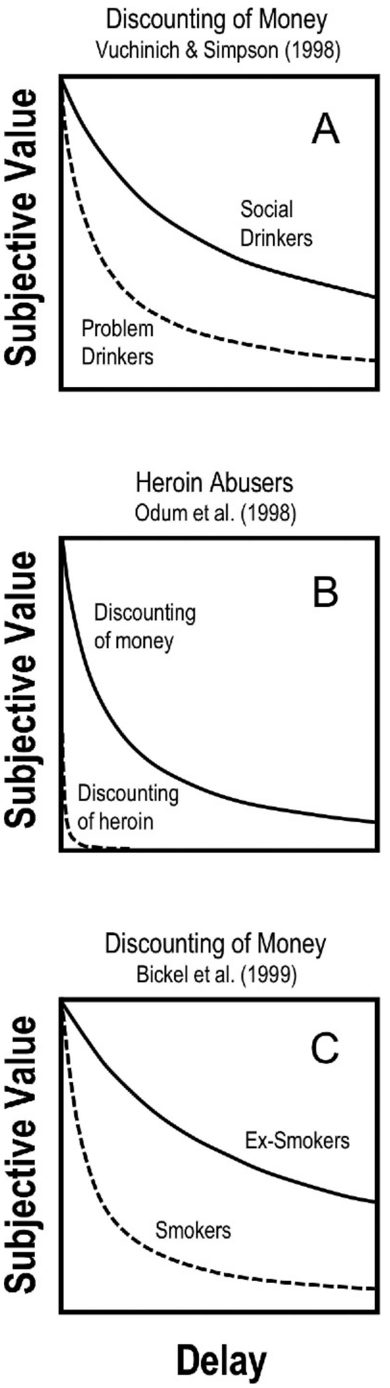

Figure 5. Three examples of modulating effects in translational delay-discounting research. In each case, the core discounting relation was evaluated twice, either in the same context for different types of individuals (Panels A and C) or in different contexts for the same type of individuals (Panel B), to

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support comparisons focusing on the elasticity (fitted parameter k) of the discounting functions. For ease of visual inspection, effects are shown as stylized best fitting functions; for raw data and fitted parameter estimates, see the original reports.

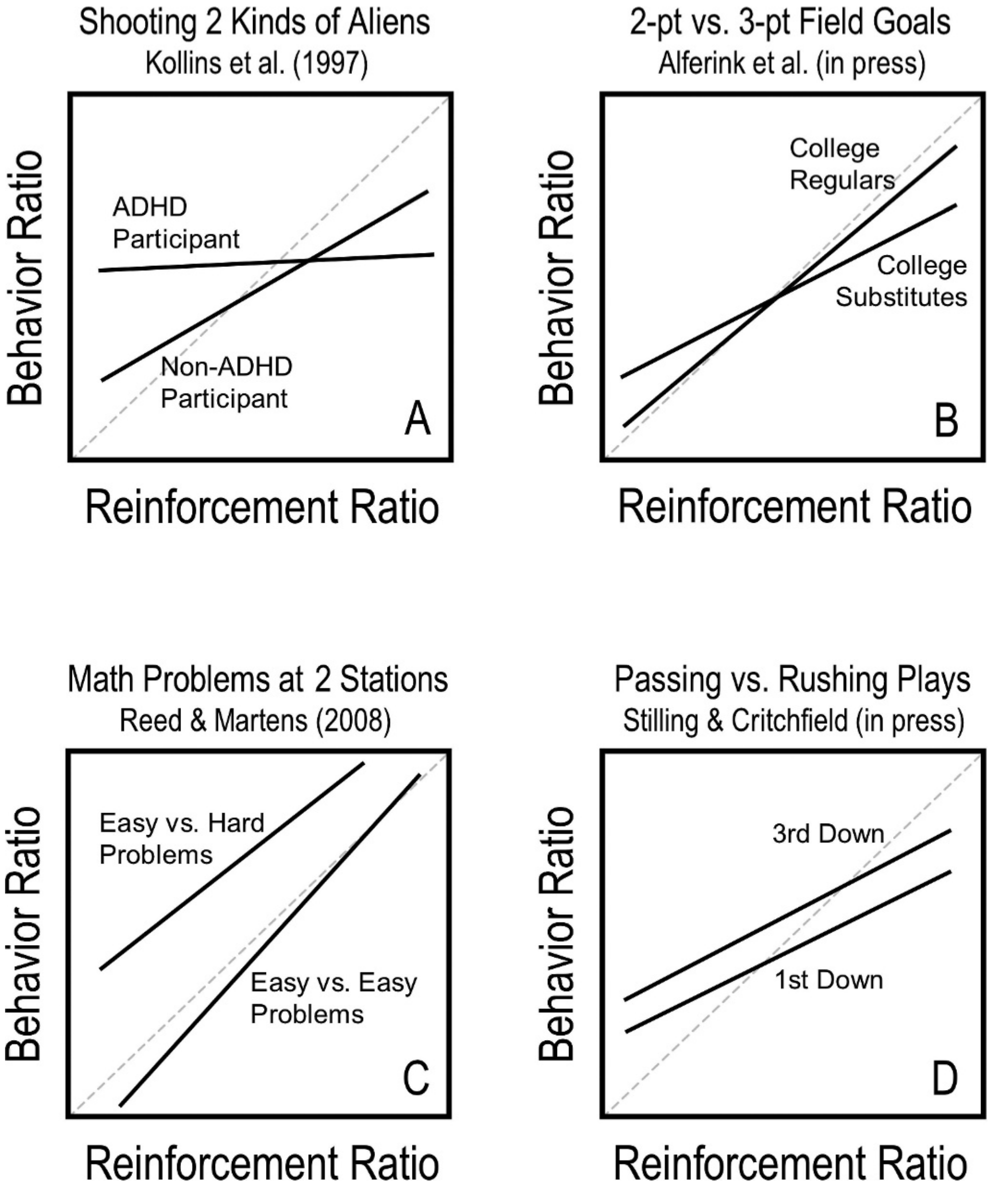

Figure 6. Four examples of modulating effects in translational matching law research. In each case, the core matching relation was evaluated twice, either in the same context for different types of individuals (Panels A and B) or in different contexts for the same type of individuals (Panels C and D), to support comparisons focusing on the slope (sensitivity; fitted parameter *a*) and *y* intercept (bias; fitted parameter log *b*) of the matching functions. For ease of visual inspection, effects are shown as stylized best fitting functions; for raw data and fitted parameter estimates, see the original reports.

describe laboratory behavior. As unique predictions go, however, this is a modest starting point.

Modulating effects are the bread and butter of quantitative models, and they deserve close attention in translational inquiries that employ quantitative models. Ideally, this attention will involve unique predictions that are derived from basic behavior principles, but some published studies fall short of this ideal.

Recall Stilling and Critchfield's (in press) application of the generalized matching law (Figure 2) to football play selection (Figure 6D). This study was inspired mainly by observations of football experts, who regard various game situations as conducive to passing or running plays. The biases that were revealed by a matching law analysis (running orientation on first down and passing orientation on third down; Figure 6D) were consistent with expert commentary. Nevertheless, although it is interesting that details of a matching function change across football situations, when these situations are defined strictly in football terms it is difficult to know what unique predictions the generalized matching law might contribute. For this modulating effect, face validity is more obvious than relevance to behavior principles. Further work is required to understand what behavior principles differentiate various football game situations.

The predictive, translational power of a quantitative model is maximized when everyday situations are linked to factors that are known to modulate the model's fitted parameters. For instance, Reed and Martens (2008) devised their study of matching in academic behavior (Figure 6C) explicitly to reflect research showing that response effort makes a behavior option less appealing; this research provided good reason to expect that the difficulty of academic problems would create bias. Vollmer and Bourret's (2000) application of the generalized matching law to basketball shot selection (two-point vs. three-point shots) was guided, in part, by laboratory research showing that differential reinforcer magnitude causes bias. In these cases, firm predictions were possible regarding the situation specificity of fitted parameter estimates because it was understood how everyday situations differed in ways that matter to behavior. Translation from quantitative models will have achieved a degree of maturity when investigators routinely focus on the modulation of core relations.

CONCLUSION

What *are* we doing when we translate from quantitative models? Ideally, we are searching for insights about everyday behavior that words alone cannot inspire. For readers whose interest was piqued by Waltz and Follette's (2009) review of clinical applications, the present essay is an attempt at an accessible introduction to the process of translating from quantitative models. For the uninitiated, we hope to have rendered quantitative analysis slightly less daunting than it otherwise would seem, while simultaneously communicating a basis for dissatisfaction with the current state of translation from quantitative models. In the broad translational mission of behavior analysis, the quantitative features of quantitative models cannot be ignored without sacrificing conceptual precision, practical and scientific insights, and the capacity of basic and applied wings of behavior analysis to speak the same language. Like it or not, quantitative analysis has become the mainstream of basic behavior analysis (Nevin, 2008). As a result, those who seek to advance the translation of basic science to everyday concerns have little choice but to develop the skills that are required to understand and apply quantitatively defined principles.

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