History of Elementary Algebra in the United States

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Submitted to the Faculty

Graduate School

University of Kansas

in

Partial Fulfillment of the

Requirements for the

Degree of Master of Arts

May , 1914.

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The history of the teaching of elementary algebra in the United States necessarily needs to be based on the texts that have been used, the descriptions of the contents of texts examined by others, the history of the earliest colleges, the memoirs of people who were educated in the earliest schools of the United States, and the writers on education throughout the time covered by this work.

It is necessary to draw from all of these sources because only a few copies of the earliest texts used in the United States are in existence, and not all of them are accessible. As far as it is possible to examine the different texts the methods of presentation are most worthy of notice.

To gain a comprehensive understanding of the early teaching of algebra in the United States it seems best to give a brief survey of the teaching of arithmetic during the same time. The material is more abundant for examining early methods used in teaching arithmetic. The two subjects, arithmetic and algebra when the latter started, were very closely associated in the United States.

Algebra was not taught as early as arithmetic . Arithmetic had its beginning in the district schools, among the common people, while algebra began in the first colleges established in the Colonies. But algebra did not appear in the curriculum of the early colleges until nearly a century after the establishment of the earliest one, Harvard, in 1636.

The development of the subject-matter of algebra as well as arithmetic did not begin in the United States but was brought from England. The transfer of arithmetic and algebra from England to the Colonies was the natural thing to have occured.

The earlier educated people in the Colonies came from England and there sons were sent there for their college education. The first school masters and the first instructors in the colleges were educated in England. Their ideas of how to teach were based on their experiences of how they had been taught, and on the textbooks, used in the United States, which were the reprints of or paterned after English texts.

A brief survey of the attention given to mathematics from the first, in the district schools, town schools, and the colleges, will show the environment surrounding the growth and development of mathematics in the United States. In this environment we find that algebra had its beginning in the curriculums of the earliest colleges.

The earliest record bearing on the history of the rise of mathematics at Harvard is a tract entitled "New England's First Fruits". It was originally published in 1643, five years after the college had opened, and contained the curriculum of the studies pursued. The courses then in the colleges could not be compared with the courses in our colleges now, but would compare favorably with courses in average high schools of today. A student applying for admission to Harvard in 1643 was not required to takeany examination in mathematics. Mathematics began in the senior year and consisted of arithmetic and geometry during the first three-quarters of the year, and astronomy during the last quarter. Algebra was then an unknown science in the New World. It is interesting that in the original curriculum, the attention of each class was concentrated for a whole day upon only one or two subjects. Thus: Mondays, and Tuesdays were devoted by the third year students exclusively to mathematics or astronomy. The importance attached to mathematical studies as compared with other branches of discipline may be inferred from the fact that ten hours per week were devoted to philosophy, seven to Greek, six to rhetoric, four to oriental languages, but only two to mathematics, i.e. only two out of 29 school hours per week.

This slight attention given to mathematics was in strong contrast with the attention given other subjects. But we must remember that this course was laid out for students who were supposed to choose the clerical profession.[I]

In the middle of the seventeenth sentury rapid progress was made in the mathematical sciences in the English universities. At Cambridge, England algebra seems not to have been known to any of the instructors in 1636, but about 1654 it was mastered by some of the students. It was not until the last

(I). Old South Leaflets, vol.3, pp.1-16.

half of the seventeenth century that mathematical studies at Old Cambridge rose into prominence. Impelled by the genius of Sir Isaac Newton Old Cambridge advanced with rapid strides.(I)

The term mathematics as used in the United States prior to I726 included arithmetic, geometry, andastronomy, but not algebra. Physics is mentioned separately, and was speculative rather than experimental, and not mathematical. Until I655 the college course at Harvard extended through three years. At this time it was lengthened to four years. Arithmetic, geometry, and astronomy were in the last year of the course when the course was three years and also when it was changed to four years.(2) Little or no change is shown in the course of study at Harvard in I726.(3) Natural philosophy and physics continued to be taught before arithmetic, geometry, and astronomy. In I700 algebra had not yet become a college study in the United States, but it was not an unknown science in the United States at this time.(4)

John Winthrop of Boston was graduated at Harvard in I732 and was only twenty-six years old when he was chosen professor of mathematics and natural philosophy at Harvard in I738.(5) He filled this chair for over forty years, until I779. There seems to be little known about the history of his teaching at Harvard. We know only that during the early part of his

(I). Cajori, p.20. (2). Merriwether, p.5I. (3). Quincey, vol. I, page 44I. (4) Cajori, p.22. (5). International vol.20, p.58I.

career as professor and probably many years before the text-books were the following: Ward's Mathematics,Gravesande's Philosophy,and Euclid's Geometry;besides these lectures were delivered by the professors of divinity and mathematics.[I]

From this it seems that sometime between the years I726 and I738,Ward's Mathematics had been introduced at Harvard which continued for a long time to be a favorite textbook. It is probable[2] that with the introduction of this text for mathematics,algebra began to be studied at Harvard. The second part of Ward's Young Mathematician's Guide consists of a rudimentary treatise on the subject.[3] It is XXX possible then that the teaching of algebra at Cambridge, Mass. may have been begun some time between I726 and I738. But no direct evidence shows that algebra actually was in the Harvard college curriculum previous to I786. Ward's Mathematics were used not only at Harvard,but also at Yale, Brown,and Dartmouth,and as a reference book at the University of Pennsylvania[4].

The second part of Ward's Young Mathematician's Guide, containing I40 pages,is devoted to algebra. Ward had published a small book on algebra in I698,but that he says,was only a compendium of that which he treats more fully in his Young Mathematician's Guide. This name is appropriate,in

(I)Cajori,p.24. (2)Merriwether,p.I70.
(3)Cajori,p.26. (4)Merriwether,p.I80.

as much as he does not (at least at the beginning) recognize the existence of XXX negative quantities but speaks of the minus sign always as meaning subtraction,as in arithmetic. A little farther on,however,he brings in "affirmative" and "negative" quantities. The knowledge of algebra to be gotten from this book is exceedingly meagre. Factoring is not touched upon. The rule of signs in multiplication is proved, bur further on all rules are given without proof. The author developed a rule showing how binomials can be raised "to what height you please without the trouble of continued involution. Ward thought he was the first to discover this method, and states that he worked out the method without knowing that it had been discovered before by Isaac Newton. The subject of"interest" is taught algebraically,by the use of simple equations.(I)

In I786 algebra was still in the Senior year at Harvard. In I787 the course of study at Harvard was revised with the purpose of raising the standard of learning. Previously the classics formed the principal study during the first three college years

The elementary mathematics were now studied in the first half of the college course instead of the second half. According to Judge Story ,Saunderson's Elements of Algebra was used in I795 and I796.Which possibly was the third edition, London,I77I,published in one volume.

(1) Cajori, p. 26.

In 1818 algebra was studied in the Freshman year and and about half of the Sophomore year, at Harvard. In 1820 the Freshmen studied Lacroix's Algebra; in 1830 Legendre's Plane Geometry; XXXXXXXXXXXXXX ,Algebra, and Solid Geometry; in 1836 and 1837 Smyth's Algebra; in 1838 Pierce's Geometry and Algebra. In 1869 algebra was required of only Freshmen. Sometime between 1869 and 1889 the whole of elementary algebra was required for entrance to Harvard. This requirement appears in the catalog of 1888-89[I].

The requirements for admission to Harvard did not include mathematics in 1643. From that time to the close of the century very little can be learned about the requirements for admission. Since the curriculum remained almost unchanged to the close of the eighteenth century, it is probable that the entrance requirements remained about the same during that time. In 1802 the standard for admission to Harvard college was raised. In mathematics, a knowledge of arithmetic to the Rule of Three was required. After 1816 the whole of arithmetic was required for admission. In 1819, a trifling amount of algebra was added. In 1825, the catalog specifies all of arithmetic, and algebra to the end of simple equations. The book used in the examination was the cambridge edition of Euler's Algebra. In 1841, Euler's Algebra or The First Lessons in Algebra were required. No other changes were made until 1843. Then there is mention-

(I) Cajori, 130-142.

ed in the catalog for admission ,Davies' First Lessons in Algebra. No other subjects were added until 1866-67,though there were some change in textbooks. In 1859 Euler's Algebra, or Davies' First Lessons,or Sherwin's Common School Algebra are mentioned;in 1865 Sherwin's Algebra.

In addition to these statements taken from catalogs it will be interesting to add the following account given by Prof.William F.Allen of the Harvard class in 1851,in a letter to Mr.Cajori Nov.6,1888. "The requirements for admission were not much above a common school.That is,I got my arithmetic and algebra in a country district school,well taught, Geometry,I picked up for myself in a very small quantity".

The system of elective studies at Harvard can be traced back to 1824. In 1838 students were permitted to discontinue their mathematics at the end of the Freshman year. It was announced in the catalog of 1839 that every student who has completed,during the Freshman year,the studies of geometry,algebra,plane trigonometry,and spherical trigonometry, and passed satisfactory examinations in each may discontinue the study of mathematics at the end of the Freshman year. In 1850 the elective system was abandoned almost completely. Mathematical studies were elective only in the Junior and Senior years. In 1867 the elective system was again adopted on a most liberal scale. Sophomore mathematics were no longer required. In 1869 mathematics was a required study only for Freshmen. In 1888-89, algebra through quadratic equations was required for admission. Then all mathematical courses were elective.[I]

The mathematical teaching at Yale University during the first years of its existence was even more scanty than the early years at Harvard. In 1714, little more than the rudiments, not including algebra, of mathematics was recited and studied. As at Harvard so at Yale, the mathematics were studied, in 1714, during the last year of the college course, and after physics had been completed. But in 1720 the mathematical courses at Yale was identical with the Harvard course of 1726.[2]

In 1742 mathematics at Yale was removed from its position as a senior study to the beginning of the course, and algebra was studied in the first year. The exact date of this change is not certain but such was the curriculum in 1742. From which it is evident that algebra was studied at Yale at this time.[3] The texts used in mathematics at Yale at the close of Pres. Clapp's administration, in 1777, were: Freshman class, Ward's Arithmetic; Sophomore class, Hammond's Algebra, Ward's Geometry (Saturdays), Ward's Mathematics; Junior class, Ward's Trigonometry, Atkinson and Wilson's trigonometry.[4]

(I) Cajori, pp. 134-150
(2) Merriwether, p. 59.
(3) Cajori, p. 31 (Yale Biographies and Annals, 1701-14, p. 724)
(4) Cajori, p. 32. (Diary of Dr. Styles)

The Young Mathematician's Guide, by John Ward, including arithmetic, algebra, geometry, conic sections, etc. was used by the Freshmen at Yale in I778; in I774-78, Hammond's Elements of Algebra was used by the Sophomores.(I) In I824 Day's Algebra was begun and completed the first year.(I)

At about the beginning of the nineteenth century the mathematical courses at Yale were: Freshmen, Webber's Mathematics; Sophomores, Webber's Mathematics and Euclid's Elements; Juniors, Enfield's Natural Philosophy and Astronomy, and Vince's Fluxions; Seniors, Natural Philosophy and Astronomy. In the mathematicscourse for I824 Day's Algebra was in the first two thirds of the Freshman year; in I848, Freshmen, Day's Algebra; in I850 Freshmen, Loomis' Algebra; in I855, Freshmen, Beebe's Graphic Algebra; in I887, the whole of elementary algebra was required for admission.(2)

As has been stated, nothing definite is known about the early entrance requirements in mathematics at Yale. In I824 arithmetic was required; in I833, Barnard's or Adam's Arithmetic; in I845 arithmetic and Day's Algebra to quadratics; in I852 and I855, no change for algebra requirement; in I870, Loomis' Algebra to quadratics; in I885, algebra as far as logarithms in Loomis' Algebra (this included quadratics); in I887 the same. This, now, included all of elementary algebra for entrance to Yale.(3)

(I) Snow, pp. 9I, I28.

(2) Cajori, p. I52, I58.

(3) Cajori, p. I52, I58.

William and Mary College is next to Harvard the oldest of American colleges. From 1688, the year of its organization at Williamsburg, Virginia until the inaugeration of the University of Virginia, it was the leading educational institution of the South. Owing to repeated destruction by fires of the college buildings and records, very little is known of its early history. The early courses were in all probability much the same as the contemporaneous courses at Harvard.

Five professorships were provided for by the charter, one of which was mathematics. The earliest professor of mathematics whose name has come down to us is Rev. Hugh Jones. The college had a professor of mathematics from its beginning, and at a date when the mathematical teaching at Harvard was still in the hands of tutors. The names of the predecessors of Hugh Jones are not known. He was the earliest professor of mathematics in America whose name has been handed down to us. He became professor of mathematics at William and Mary about the year 1700. He was a man of broad scholarly attainments and endeared himself to the students of history quite as much as to the mathematician, by writing his books on that present State of Virginia (1724). Dr. Herbert Adams states that this history is acknowledged to be one of the best sources of information respecting Virginia in the early part of the eighteenth century. The following quotation from it(p.44) is interesting: "They (the Virginians) are more inclinable to read men by business and conversation than to dive into books,and are for the most part only desirous of learning what is absolutely necessary in the shortest and best method. Having this knowledge of their capacities and inclinations from sufficient experience,I have compiled on purpose some short treatises adapted with my best judgment to a course of education for the gentlemen of the plantations, consisting in a short English Grammar;an Accidence of Christianity;an Accidence to the Mathematick in all its parts, Algebra,Geometry,Surveying of land,and Navigation".[I] These theses existed,mostly,only in manuscript copies. From the above it appears that about I724 the mathematical course at William and Mary was quite equal to that of the two New England Colleges[I].

The University of Virginia was opened to students in March,I825. It then had eight distinct schools.

Charles Bonneycastle ,son of John Bonneycastle,became professor of mathematics in I827. He continued to fill this position until his death in I840. The textbooks which he used in connection with his lectures,were the Arithmetic, Algebra,and Differential Calculus of Lacroix. In pure mathematics there were for a time three classes: the First Junior,Second Junior,and Senior. Of these the First

(I)Cajori,p.33.

Junior began with arithmetic. In teaching the rules for adding and subtracting, in algebra, they were compared with the corresponding rules in arithmetic, and the agreement and diversity were noticed and explained. Both algebra and geometry were begun in the First Junior class (catalog, I836), and then continued to the Second Junior Class. In I845 algebra was taught to the Junior Class. During the Civil War the Institution barely subsisted, but as soon as the war was closeed the institution prepared to enlarge its capacity. The method of instruction had not changed much. The course in mathematics as stated in a catalog of I887-88 has elementary algebra in the Junior Class only.[I]

The College of New Jersey, now Princeton, first opened at Elizabethtown in I746. Soon after, it was transferred to Newark, and in I756 to Princeton.

The traditions and practices of Yale were transmitted to the College of New Jersey. The course of study had its origin in Yale's course of study at that time. There is good evidence that algebra was studied during the Freshman year in I750. In I772 the whole branches of Mathematics and Philosophy were taught by one professor.[2]

In I803 vulgar arithmetic was required for entrance to the Freshman class; in I830, arithmetic[3]; in I850, elements of algebra through simple equations; in I852, algebra through

(I) Cajori, pp. I93-6, 20I ; (2) Snow, 38-9, 76; (3) Snow, II8-20.

through quadratics; in I888-89, the whole of elementary algebra.[I]

In I882 the curriculum contained algebra for the entire first and second years; in I830 Bonneycastle's Algebra, in Freshman year[2]; in I850 and I856, Freshmen, Hackley's Algebra; in I88I, Freshmen, Ray's University Algebra; in I888-89, Freshmen, Well's University Algebra.[3]

Kings College, now Columbia University, was established in I754. The course of study was four years. In I755 Mathematics, and the Mathematical Experimental Philosophy in all the several branches of it were taught during the freshman year. In I757 there was eatablished the chair of Mathematics and Natural History, the first created in the institution. Soon after I763, arithmetic (vulgar and decimal), algebra, and the first books of Euclid were in the first year; in the second year, algebra, Euclid, astronomy, chronology, navigation and other most useful branches.[4] In I785 an understanding of the first four rules of arithmetic, with the Rule of Three, were required for entrance. Algebra as far as quadratic equations, was taught to the Freshmen, and the higher branches of algebra to the Sophomores; in I789, to the Freshmen, algebra as far as cubic equations; to Juniors, higher parts of algebra and applications of algebra to geometry; in I792, to Freshmen, as far as quadratic equations; to the Junior class, higher branches of

(I) Cajori, I6I-4 (2) Snow, p. II8-20 (3) Cajori, p. I6I-4.
(4) Snow, p. 56-65.

algebra,and applications to geometry.(I)

The courses of study at Columbia College for the following years show algebra as follows: I8IO,in the second year; I8II,in the second year;I82I,in the first and second years; in I82I,in the first and second years. The mathematical courses remained practically unchanged until after the Civil War(2).

The University of Pennsylvania,chartered in I755,was known before the revolution as the College,Academy,and Charitable School of Philidelphia. Dr.William Smith,D.D.,was the first provost until I779. He was very fond of mathematical studies and gave lectures on mathematics,natural philosophy,astronomy,and rhetoric. Associated with him at the college as professor of mathematics from I760 to I763 was Hugh Williams(3).

In I758 algebra was taught through simple and quadratic equations during the first year of their three years course. In I795 algebra was included in the third and fourth years. There is not anything to show how much of these two years was devoted to algebra.

The entrance requirement of the college in I8II included common arithmetic,vulgar and decimal arithmetic. The curriculum included algebra in the second year of their three years course. In I8I7 algebra was studied half of the first

(I) Snow,pp.94-99. (2) Snow,pp.96-I02. (3) Cajori,p.35.

year; in I820,half of the second year; in I826,in the first year of the four years course through quadratic equations,in the second year,the elements of algebra completed and applications of algebra to geometry,in the third year,higher algebra.[I]

The later history of the University of Pennsylvania is not easily traced or very well known.[2]

The College of Rhode Island ,now Brown University,was established in I764. In the course of study for I783,vulgar fractions were required for entrance,and Hammond's Algebra was studied in the third year; in I793 Hammond's Algebra was taught in the second year; in I803,rules of vulgar arithmetic were required for entrance,and Hammond'sAlgebra was studied in the second year. The later entrance requirements and courses of study correspond very closely to those of Princeton. In I827the rules of arithmetic were required for entrance,and algebra was taught during the first half of the second year.[3]

Dartmouth College at Hanover,N.H. was chartered in I769. The course of study was four years. It is probable that the mathematical studies of the first three years were; arithmetic,vulgar and decimal: trigonometry,altimetry,longimetry, navigation,surveying,dialing,gauging,and astronomy. In I790 Hammond's Algebra was used,in either the first or second

(I)&(2),Snow,pp.I29-40. (3) Snow,pp.I09-24.

year; in 1824, the Freshmen studied algebra during the third (last) term of the first year; in 1828, the college course was the same but algebra to the end of equations was added to the requirements for admission(1). The catalog of 1834 shows that after 1828 a remodeling of the course had taken place. The Freshmen in the mathematical and physical department studied Playfair's Euclid, reviewed Adams's Arithmetic, and commenced Day's Algebra during the first term; continued Day's Algebra during the second term; and completed Euclid in the third. The Sophomores continued Day's Algebra, devoting their attention to applications to geometry and logarithms. In 1839 Bourdon's Algebra displaced Day's Algebra(2).

The instruction in the Freshman and Sophomore years being done by tutors was very poor according to a graduate in 1840. In 1849, Chase's Algebra began to be used; in 1865, Robinson's Algebra was used; in 1868, Loomis's Algebra was used; in 1870, Olney's and also Quimby's Algebras; in 1888-89, the courses in algebra were above the elementary algebra.(3)

The terms for the admission to the college were: in 1828, arithmetic, algebra through simple equations; in 1841, the same; in 1864, the same with the addition of two books of Loomis's geometry; in 1886 and for some years previous all of plane geometry; in 1888, arithmetic, algebra to quadratics, and plane geometry(3).

(1) Cajori, p. 73-75. (2) Cajori, p.166-8; (3) Cajori, p.166-8

About 1785, was opened what, in 1865, became a part of the University of Kentucky. The records, Sept. 16, 1789, show that mathematics was taught; Oct. 18, the same year, Saunderson's Algebra, in the Freshman year; in 1803, algebra was taught; in 1816 algebra as far as affected equations, in the Junior year; In 1817, Webber's Mathematics; in 1829, Colburn's algebra, in the Freshman year; in 1884, Davies' Bourdon, in the Freshman year; In 1848, Loomis's Algebra in the Freshman year; in 1878, the requirement was algebra through equations (1).

The University of North Carolina was first opened to students in 1795. It was organized after the model of Princeton College. For admission the elements of arithmetic were required from the beginning in 1795 to 1835; in 1800, arithmetic as far as the Rule of Three; in 1834, arithmetic to square root. In 1795, algebra and the applications of algebra to geometry were offered, probably in the latter part of the first year or in the first part of the second year. The first text used in algebra was probably Thomas Simpson's. It was studied in 1803 and in 1615, and perhaps as late as 1826. In 1818 algebra was begun in the Freshman year and completed in the Sophomore year. It is probable that Simpson's Algebra was used at this time. In 1835 it seems that a little algebra, "Young's Algebra to simple equations", was required for entrancebut after three years algebra was withdrawn, and was not again required until 1855 when algebra through equations

(1) Cajori, pp. 84, 225-6

of the first degree was required. No more alterations in entrance requirements were made until 1868. In 1835 algebra was completed in the Freshman year; from 1857 to 1868, algebra and geometry. Ryan's Algebra was used in 1827; Young's algebra was introduced in 1836; Pierce's was studied from 1844 to 1868. Since 1875, arithmetic and algebra to quadratic equations have been required for admission, and algebra and geometry were taught to the Freshmen. All studies of the first and second years were required of all graduates. Robinson's Uinversity Algebra was used from 1869 to 1871, and since 1875 Schuyler's, Venable's, Newcomb's, and Wells' -- Newcomb's most.(1)

When Bowdin College was first organized in 1802, an acquaintance with the fundamental rules of arithmetic was required for entrance. Later, the expression "Well versed in Arithmetic" is used. The first definite increase in these entrance requirements did not occur until 1834 when part of algebra was added. The course of study at the beginning and for twenty years afterwards had algebra in the second year. Algebra was gradually forced back to the Freshman year, but a part of the first term of the first year was given to arithmetic. Webber's Mathematics was used from Freshman to Senior year, in 1820, according to Rev. T.T. Stone, a graduate of Bowdin in 1820. In 1834 the requirements for admission

(1) Cajori, p. 79, 205, 207.

was increased to include six sections of Smyth's Algebra. These six sections included nearly the entire Algebra, logarirhms and the binomial theorem being excluded. In 1867 the entrance requirement included the first eight sections of Smyth's New Elementary Algebra to equations of the second degree.

Mathematics have never been taught at Bowdin by lectures, though the instruction has been frequently supplemented by lectures. Since 1880 all mathematics have been elective after the Sophomore year; since 1886, all after the Freshman year. In 1882-83 the Freshmen studied Loomis's Algebra. In 1888-89 there was no difference for algebra, except Wentworth's Algebra took the place of Loomis's.(1)

The United States Military Academy at West Point was established by Congress in 1802. The dates and texts for the instruction in algebra are: in 1808 algebra was taught; in 1816, Hutton's Mathematics, the best to be had at the time; in 1828, a poor translation of Lacroix's Algebra; in 1841, Davies' Bourdon in the first year; in 1888, Davies' Elements of Algebra in the first year(2).

The South Carolina College, University of South Carolina after 1865, was opened to students in January, 1805. The requirements at the beginning for admission were arithmetic and proportion. This most probably did not include fractions.

(1) Cajori, p.76, 171-3. (2) Cajori, pp.84, 86, 115, 119, 126.

In I836 the terms of admission were: arithmetic,including fractions and the extraction of roots. It is not certain that the early curriculum included algebra. In a stronger course of I8II the Sophomores had lectures on algebra. In I848,algebra to equations of the first degree was required for entrance. In I85I Davies' Bourdon was the algebra used; in I853,the whole of Davies'Bourdon was required for entrance. in I859,this was reduced in Bourdon's algebra to chapter IX (thus omitting the general theory of equations and Sturm's Theorem),or Loomis's Algebra to section XVII (omitting purmutations,combinations,series,logarithms,and general theory of equations). In I836,the course of study included in the Freshman year:Bourdon's Algebra to the equations of the third degree,ratios and proportions,summation of infinite series, nature and structure of logarithms. In I838,the Freshmen finished the whole,both of algebra and geometry. After entrance requirements were raised Bourdon's Algebra was omitted from the first year. In I858 the Freshmen reviewed algebra (the applications of algebra to geometry); in I86I, the same is mentioned in the catalog.

The institution was closed in I863 . When it was opened in I866 it was reorganized on the plan of the University of Virginia. The only entrance requirement was regarding age. Afterwards,algebra through equations of the second degree was required for admission into the schools of mathematics, and civil and military engineering and construction. From 1867 to 1872 the terms for admission were the same as the above. In 1872, for admission to the college of literature, science, and arts, the requirements were, in addition for the classical course, algebra as far as equations of the second degree, for the scientific course, algebra up to radical quantities. In 1876, algebra as far as equations of the second degree was required for entrance. The course of study for mathematics contained in 1866-67 algebra from equations of the second degree to the general theory of equations, and logarithms, in the first year, Loomis's Algebra being used as a text. In 1870, there were no changes affecting algebra. In 1872, Robinson's University Algebra was studied in the first year. Later on Ficklin's Algebra was introduced.

Under the Reconstruction Period, the University was closed from 1876 to 1880. The terms of admission on the reopening of the institution included algebra through equations of the first degree; radicals were added in 1883; in 1884, to equations of the second degree; no more additional entrance requirements were made before 1890. In 1882, Newcomb's Algebra was used in the first year; in 1888, Todhunter's Algebra for beginners.(1)

The University of Alabama was opened in 1831. The terms of admission were: 1833-56, arithmetic; 1857-59, arith-

(1) Cajori, p.83, 209-14.

metic, and algebra through equations of the second degree; I860-62, arithmetic, and algebra to equations of the first degree. In I833 the Freshman class completed algebra (Colburn, Lacroix), and commenced geometry. In I845, Pierce's Algebra was introduced, but after two years it was displaced by Davies'.

Much interruption occured during the Civil war, and the buildings were burned in I865, and the institution was not opened again until I869.

From I869 to I87I, only the elements of arithmetic were required for admission; during the next two years, algebra to equations of the second degree was added; in I873, the requirements were reduced to arithmetic alone; no change was made until I878, when algebra through equations of the second degree was required; then gradual changes were made each year; in I887-88 the whole of algebra was required.

In I88I, algebra was completed in the second year in the classical, scientific, and engineering courses, later, in the engineering course higher algebra and geometry were taught to the Freshmen, and the theory of equations to the Sophomores.

In I87I, Davies' Algebra was used; in I872 and I873, Robinson's Algebra; in I889, Well's Algebra.

As a rule mathematics was required of every student for graduation, from I83I to I865. After the reorganization, in I869, mathematics was also required until I875, when the elective system was adopted; it was entirely optional with the student then until I880, when every student was required to take this subject through analytic geometry. In I889, there were no electives, and all mathematics in each course was required for a degree in that course.[I]

The University of Mississippi was organized in I848. The mathematical requirement for admission was: at first, a knowledge of arithmetic; in I857-58, the whole of arithmetic; in I859-60, algebra, as far as simple equations was added. In I854, the Freshmen studied Davies' University Arithmetic and Davies' Bouddon, and the Sophomores continued Davies' Bourdon part of the year; in I857-58, pure mathematics commenced with the beginning of the Freshman year, and continued till the close of the Sophomore year. Much emphasis was placed on original mathematical work in every study.

The exercises of the institution were suspended from I86I to I865. The mathematical requirement for entering were, in I886, arithmetic, and algebra including equations of the first degree. Algebra was completed during the Freshman year. In I870, the terms of admission to the A.B. and B.S. courses were arithmetic and Davies' Elementary Algebra through equations of the second degree; for entrance to the B.Ph. and civil engineering courses the whole of Davies' Elementary Algebra was required. There was no change in

(I) Cajori, pp. 2I6-2I9

these requirements in I888. In I872, the first year's mathematical work included Davies' Bourdon's Algebra; in I888, Davies' textbooks were used in Algebra.[I]

It has not been possible to find the exact datethat algebra began at any of the earliest American Universities. Evidence indicates very XXXXXXX conclusively that it was taught at Harvard during the period I726 to I738 and afterwards. At Yale, algebra was in the first year of their three year course in I742. At William and Mary, it is very probable that algebra was taught as early as I724. In a course of study at the University of Pennsylvania in I758, it is shown that algebra was taught in the first year through simple quadratic equations.

Ward's Mathematics were adopted at Harvard, Dartmouth, Brown, and the University of Pennsylvania. Since Ward's Young Mathematician's Guide was used at all of the colleges, and the second of the five parts into which the text was divided, treated of algebra, the study of the subject must have been started about the same time, also holding in mind that the dates do not differ very much, when it is shown that algebra was taught in those colleges.

The algebra which was taught must have been very meager. The courses changed very little before the revolution. The extent of the instruction was: the meaning of positive and

(I) Cajori, pp. 2I9-25.

negative numbers,addition,subtraction,multiplication,division, binomial theorem,simple equations,and,at the University of Pennsylvania quadratic equations was in the course.

The method of instruction was mainly by rules,and the mechanical solution of exercises in the processes of addition, subtraction,multiplication,division,binomial expression to any power,and very simple equations. Some effort was made to introduce every-day practical problems,such as interest, adapted to the simple equations.

Of all of the arithmetics used before the Revolution but one work(I) in the English language was written by an American author. The second book devoted exclusively to arithmetic compiled by an American author,and printed in the English language,was the New and Complete System of Arithmetic by Nicholas Pike,Newberryport,I788. It was a very extensive and complete book fob the time. A large portion of the rules were given without demonstration,while some were proven algebraically. In addition to the subjects ordinarily found in arithmetics,it contained logarithms,algebra,and conic sections,but these subjects were so briefly treated as to possess little value. After the appearance of Webber's,Day's, and Farrar's mathematics for colleges,they were finally omittedin the fourth edition of Pike's Arithmetic in I832(2). The second edition of Pike's Arithmetic was enlarged,revised,

(I) Greenwood's Arithmetic,I729. (2)Cajori,p.46.

and corrected by Ebenezer Adams, printed at Worcester, Mass. in 1797. Pages 473 to 505 were devoted to an Introduction to Algebra designed for the XXXXXXX use of academies.

The chapter devoted to algebra, begins with general definitions: of the subject, then of the axioms, without any illustrations of the definitions or of the axioms. The topics of algebra presented and their order are: addition, subtraction, multiplication, division; then the same four operations for fractions; involution, evolution; infinite series; progressions; equations: simple linear, simultaneous linear, and quadratic.

The rules are given for the procedure, and the examples worked out illustrating the application of the rules. No additional problems are given which are not solved by the author. The presentation of the entire part devoted to algebra is by rule. The treatment is very brief and only the simpler applications are touched upon.(1)

The short treatise on algebra coming in the latter part of Pike's Arithmetic would practically place it beyond the part of the text reached by most of the classes, because the proportionate time given to mathematics was smaller then, and algebra did not apply to practical life like the parts of ~~all~~ arithmetic which preceded the algebra in Pike's extensive text.

Consider and John Sterry published The American Youth being a new and Complete Course of Introductory Mathematics,

(1) Pike's Arithmetic, 2d edition, 1797.

Providence, Bennet Wheeler, I790.

The first part treats of arithmetic, in 239 pages, and the remainder of I47 pages is devoted to Algebra[I].

At the beginning of the nineteenth century there were three great arithmeticians in America, namley, Nicholas Pike, Nathan Daball, and Daniel Adams. Between I800 and I820, a large number of arithmetics sprung into existence. Most of them enjoyed a short XXXXXXXXX popularity[2].

In I800 John Gough published a Practical Arithmetic in Four Books with an appendix of algebra by William Atkinson, of Belfast, Dublin Printed, Wilmington reprinted[3].

The Key To The Tutor's Guide by Charles Vyse, ninth edition, printed by Joseph Crushbank, Philadelphia, I806, contained forty-eight pages devoted to the elements of algebra. The Tutor's Guide was a complete system of Arithmetic in six parts with various branches in Mathematics. It was an American reprint of an English work.[4]

An English mathematician, whose works found their way across the ocean, was John Bonneycastle, a professor of mathematics at the Royal Military Academy at Woolrich, England. His introduction to algebra (London, I782) was revised and edited in this country by James Ryan in I822. Bonneycastle was a teacher of rules rather than principles.

An English author well known in this country was Thomas

(I) Greenwood, p.809. (2) Cajori, p.49. (3) Greenwood, p.8I4.
(4) Greenwood, p.8I8.

Simpson. His treatise on Algebra was published in Philadelphia in 1809. The second from the eighth English edition, revised by David McClure, teacher of mathematics, came out in Philadelphia in 1821. As was frequently the case in those days, all demonstrations are given by themselves in the manner of notes placed below a horizontal line on the page. They could be taken or omitted by the teacher or pupil at pleasure, and were generally omitted. The author's demonstrations and explanations wanted simplicity, and we need not wonder that they were looked upon, by some, as rather tending to throw new difficulties in the way of the learner than to facilitate the process(1).

At the beginning of the nineteenth century the great want of the country in the department of pure mathematics was adaquate text-books. Prof. Webber of Harvard was the first who attempted to supply this want. In those colleges in which a single system of mathematics had been adopted preference was generally given to the Mathematics of Webber.(2)

In 1801 he published in two volumes his mathematics compiled from the best authors. These works were for a time almost exclusively used in New England colleges. Within the two volumes, each of 460 pages and in large print, 124 pages were given to algebra, while Newcomb's algebra numbered 545 pages(3).

(1) Cajori, p.56. (2) Cajori, p.63. (3) Cajori, p.60.

Charles Hutton was an English author. The sixth edition of his two volume Course in Mathematics for The Use of Academies was published in London in 1810. The first volume of this set has 384 pages: General Preliminary Principles, 3 pp.; Arithmetic, 142 pp.; Logarithms, 17 pp. plus 18 pp. of Tables; Algebra, 100 pp.; Geometry, 104 pp.

Hutton's Mathematics were used once at the United States Military Academy at West Point but soon became replaced by better works from the French. This was possibly the best course of Mathematics by the English at that time but was far behind the French(1).

The table of contents for Algebra of the copy examined is: definition and notation; addition; subtraction; multiplication; division; fractions; involution; evolution; surds; infinite series; arithmetical progression; piles of shot or shells; geometrical proportion; simple equations of the first degree; quadratic equations; cubic and equations of higher powers; simple interest. In the chapter just following, devoted to geometry, ten pages consist of applications of algebra to geometry.

The method of presentation is by stating the rule and then solving some type problems. Problems requiring the formation of the statements in any form are not introduced until equations are taken up, near the close of the part

(1) Cajori, p.70.

devoted to algebra. Seven rules are given for solving simple first degree equations.. The explanations for quadratic equations are long but clear. Most explanations are at the bottom of the page below a horizontal line. This treatment is not extensive, and might well be classed as the rudiments of algebra briefly treated.

The complilation by Webber was not satisfactory to the needs of the times. Accordingly, Jeremiah Day elected Prof. of mathematics at Yale, I80I, commenced to write a series of books which would supply more adaquately the needs of the American colleges. His algebra appeared in I8I4. It passed through numerous editions, the latest of which appeared in 1852, by himself and Prof. Stanley(I). It is likely the content and arrangement of the first edition remained unchanged. The 30th and 42d editions, published in I838 and I84I respectively, which I examined do not differ at all. Then, soon after publishing his algebra he was elected president of Yale, and did not devote any more time to text-books.

The following is the table of contents: notation, positive and negative quantities; addition; subtraction, multiplication, division; algebraic fractions; reduction of equations; involution; evolution; reduction of equations by involution and evolution; affected quadratic equations; solution of problems which contain two or more unknown quantities; ratio and

(I)Cajori, p.63.

proportion; variation and general proportion;progressions; infinity; division of compound divisiors; involution of compound quantities by the binomial theorem; evolution of compound quantities; infinite series; composition and resolution of higher equations; application of algebra to geometry; equations of curves; six pages of explanatory notes applying throughout the book.

As any topic is presented,the particular situation is stated,then the rule of precedure for solution,then the problems are solved,and if necessary an additional comment is made in way of explanation. The statement of the rules is plain. Explanations are clear and carefully applied.

Between 1815 and 1820 a reform in the mathematical teaching was inaugerated in this country. Foremost among the leaders was John Farrar of Harvard, who translated into English for the use of colleges a number of French works. The French books at that time were far in advance of the English. The reform in the teaching of the more advanced mathematics was accompanied by a similar reform in the arithmetical teaching.(1)

John D.Williams,private teacher of mathematics and natural philosophy etc.,wrote a Key to Daball's Arithmetic, preface date 1818,published by H.and S.Raynor,76 Bowry,New-York,1839. On the title page of this is mentioned an Elementary Treatise on Algebra by the author(2).

(1) Cajori,p.49. (2) Greenwood,p.22.

In I8I8 appeared Farrar's Introduction to the Elements of Algebra selected from the Algebra of Euler. It was a very elementary book,and was intended for students preparing to enter college. It differed from the English texts in that it taught pupils to reason instead of to memorize without understanding.

In the same year appeared also Farrar's translation of Lacroix,which was first published in France about twenty years previously. Lacroix was one of the most celebrated and successful teachers and writers of mathematical textbooks in France. Farrar was professor of mathematics at Harvard from I807 t6 I836[I].

John Bonneycastle was an English author. His Introduction to Algebra,second New York edition,revised,corrected ,and enlarged,by James Ryan,came on the market in I832. This date is shown on the advertisement page concerning the 2d New York edition by James Ryan,Jan.I,I822. This advertising note appearing in the copyright edition of I845 without any other advertisements would indicate that the copyright of I845 was the same tex t contents as the I822 revision.

In this I845 copyright edition of 288 pages the table of contents is: definitions; addition; subtraction; multiplication; division; algebraic fractions;involution;evolution; ~~evolution~~; surds; progressions;equations; the resolution of simple equations; miscellaneous ~~XXXXXXXX~~ questions; quadratic

(I)Cajori,p. I28.

equations; cubic equations; roots of equations by approximation; roots of exponental equations; binomial theorem; indeterminate analysis; Diophantine analysis; summation and interpolation of series; logarithms and their application; collection of miscellaneous questions; applications of algebra to geometry.

The presentation of the topics is by the definition and rules, then explanations of type problems which are worked out. These explanations are too long and envolved. The essentials do not stand out as they should.

The opportunities for learning about the early methods of teaching algebra is very meager, but the method of presentation, developing memory instead of reasoning, would very probably not have been different in algebra than in arithmetic. Many of the authors of mathematical textbooks published arithmetics, and algebras, and other advanced texts in mathematics. For this reason, in the early history of the teaching of algebra in the United States, when the material for making the history of algebra continuous is lacking it is well to fill the gap by conclusions based on the methods of presenting arithmetic during the corresponding time.

Enoch Lewis, an American, edited an algebra of his own, about 1824.[I]

Warren Colburn's Introduction to Algebra upon the Induct-

(I) Greenwood, p.826.

ive Method of Instruction was copyrighted in 1825 and published the next year, The Arithmetics by Colburn marked a great educational epoch in the world's teaching. He seemed to have grasped the logical idea in teaching,more firmly than any other of his age(I). Judging from the success of his arithmetics his algebra,which was written on the same plan, must have been received successfully.

Colburn makes special effort throughout the text to develop the reasoning power of the student. His aim in presenting new topics at the beginning of the algebra is to make the transition from arithmetic as gradual as possible.

The text commences with practical questions in simple equations. By means of these questions which are problems for solution the pupil is lead to put down statements which lead to the formation of simple equations of the first degree. Then by arithmetical reasoning the solution of the equation is developed. Some few problems are worked out,developed very carefully to build up the pupil's reasoning on the knowledge learned in arithmetic. Pages II to 66,just following the author's introduction,are devoted to these practical questions for simple equations. Then follows: addition; subtraction; multiplication of simple algebraic quantities; the same three processes for compound algebraic quantities, then division,pp.66-82; fractions,pp.83-102; treatment of equations,one,two,or more unknown quantities,arising from

(I) Greenwood,p.824.

problems,with negative numbers explained arising from solutions.pp.162-131; questions producing equations of the second degree,questions producing pure equations of the third degree, extraction of second and third roots or any roots of simple and compound quantities,pp.131-197; fractional exponents and irrational quantities,binomial theorem,summation of series by differences,progressions,logarithms; compound interest, annuities,miscellaneous examples,pp.197-276.

A Key to Warren Colburn's Algebra was copyrighted in 1827 by the author. The one examined was published in 1831. This book of 50 pages contains mostly the answers only. Some of the more difficult problems in equations,and other difficult problems are solved carefully.

James Ryan,an American,wrote an Elementary Treatise on Algebra published in 1827.(1)

An Elementary Treatise on Algebra to which is added Exponential Equations and Logarithms was copyrighted in 1837 by Benjamin Pierce while he was professor of mathematics at Harvard(2). The presentation is by rule with only an occasional problem worked out to show the application of the rule.

The first chapter includes: definitions and notations, addition,subtraction,multiplication,and division; II,fractions and proportions; III&IV,equations of the first degree; V, powers and roots; VI,equations of the second degree; VII,

(1) Greenwood,p.829. (2) Cajori,p.34.

progressions; VIII, general theory of equations; IX, continued fractions, at close exponential equations and logarirhms.

Cajori states[I] that these texts of Pierce never became widely popular. The text would be difficult for the student to progress in unless considerable aid was given.

Another English Algebra reprinted in America was that of B.Bridge, fellow of St.Peters College, Cambridge (second American edition from the eleventh London, Philadelphia, I839). This book was introduced into many of the best schools of the United States. The subject was treated purely synthetically as had been done in previous texts. Analytic methods which had proved so powerful in the hands of the mathematicians in the Continent were still underrated in England. The exclusive adherence to the synthetic method was due to an excessive worship for the views of Newton who favored synthesis and had employed it throughout his Principia[2]. Charles Davies translated from the French of M.Bourdon the Elements of Algebra Adapted to the Course of Mathematical Instruction in the United States.

The explanations and reasoning are given carefel attention, but the processes are long and elaborate, and not well adapted to beginners.

Davies first translated Bourdon's Algebra in I834. The editions examined have the copyright dates I844, I853, I858.

(I) Cajori, p.34. (2) Cajori, p.55-56.

In 1839 appeared his Elementary Algebra Embracing the First Principles of the Science. The text examined has 1845 as the copyright date. This text was published with the intention of forming a connecting link between arithmetic and algebra, and to furnish an introduction to the author's translation, from the French, of the algebra of M. Bourdon. His elementary algebra was planned like his translation from M. Bourdon which he regarded as too difficult for beginners, He wished also to bring his Elementary algebra within the scope of the common schools. I do not think the text would have been useful in the commonschools, except in classes which had completed the usual common school work and were taking advance work. This would really class them in the grade of high schools. This may have been the author's idea.

Davies' texts became popular (I) and were extensively used.

The table of contents is as follows: introduction of eight pages which consists of problems to be solved by simple equations. This is a very helpful treatment in equations, for the beginner, for the purpose of transition from arithmetic to algebra. The presentation is good, and explanations together with questions bearing on the problems and the solutions are plentiful; after these 8 pages are the chapters: definitions, explanations of signs, similar terms, addition, subtraction, multiplication, division; algebraic fractions;

(I) Cajori, p. I20.

equations of the first degree; involution; evolution; radicals; equations of the second degree; progressions ; theory of logarithms. Throughout this text there are questions, at the bottom of eachpage, bearing upon the work of that page. The questions would certainly been helpful to the student.

Loomis published his Treatise on Algebra soon after 1844. The text examined was copyrighted in 1847, being the eighth edition and containing the preface to the second edition. The second edition was a revision mostly in details. In this preface the author states that the first edition had been adopted as a text in half a dozen colleged besides numerous academies and schools, and that most flattering testimonials were received from every part of the country. The presentation is clear and concise, and well adapted to beginners.

The contents are: definition and notation; addition; subtraction; multiplication; division; fractions; simple equations; equations of two or more unknown quantities; discussion of the equation of the first degree; involution and powers; evolution and radical quantities; equations of the second deg gree; ratio and proportion; progressions; greatest common divisor, continued fractions, permutations and combinations; involution of binomials; evolution of polynomials; infinite series; general theory of equations; solution of numerical equations; logarithms; miscellaneous examples.

Joseph Ray published in 1848 his revised edition of

Elementary Algebra, Part I, on the Analytic and Inductive Methods of Instruction with Numerous Exercises designed for the common schools and academies. The introduction consists of 24 pages of intellectual exercises. These are problems to be solved orally by analysis, then again by use of an unknown quantity in forming the equation and solving the equation. Following the introduction are: fundamental rules including definitions and principles, terms and signs, addition, subtraction, multiplication, division; special rules for multiplication and factoring including G.C.D. and L.C.M.; algebraic fractions; equations of the first degree; supplement to the equations of the first degree; powers, roots, radicals; equations of the second degree; progressions and proportion.

The explanations are clear and readily understood. The text would be suited to pupils of high school grade but not to common school grade unless advanced classes were formed.

In the revision of 1866 of the above text the essential changes are: omission of the 24 pages of intellectual exercises; reducing the number of examples; abridging some rules and demonstrations.

A Practical Treatise on Algebra designed for the use of Students in the High Schools and Academies and Advanced Classes in the Common Schools was copyrighted by Benjamin Greenleaf in 1852 and 1853. The 1853 copyright is the "37th Improved Edition."

Judging from the author's comment in the preface of another revision in 1862 his texts in algebra must have been popular and extensively used.

His algebras were intended to develop the power of reasoning. The presentation is by rules followed by illustrative examples carefully worked out, but no explanations of the different steps follow.

The main changes in the revisions of 1852 and 1862 are changes in detail, and possibly corrections. There appears very little difference in the topics treated and methods of presentation. In the table of contents are shown; definition and notation; addition, subtraction, multiplication, division (multiplication theorem, factoring, G.C.D., L.C.M., in the 1862 edition); fractions; simple equations; discusion of problems; involution; evolution; radicals; quadratic equations; ratio and proportion; series; miscellaneous examples. The 1853 edition had several more topics, than mentioned above, dealing with special topics applying mostly to practical life.

The United States Bureau of Education Bulletin, 1911, Whole Number 463, is a report of the International Commission on the Teaching of Mathematics. It is the American Report of Committees III & IV.(1)

The report deals with Mathematics in Public and Private Secondary Schools in the United States.

(1) Pages 41-55 of this thesis are based on the above report.

The Curriculum

Directive influences. --- The curriculum in mathematics is determined in general by the admission requirements of colleges. That is, of course, confessedly so in the States in which there is a complete and State-wide organization of education, with the university recognized as the final stage. It is also true in other States, and even in smaller communities where few, if any of the pupils may be planning to go to college, and where the local school committe disclaim the intention of following university guidance; and it is true of the mathematical curriculum even where it is not true of other subjects of study. The reason for this nearly universal dependence on college definitions is that mathematics is not otherwise defined by any authority that the schools feel willing to accept.

The Subject Matter of Algebra

Elementary algebra. --- Every regular high school in the United States offers algebra for at least one year. Half of them give algebra for an extra half year; less than 20% give algebra for two years and a half. Advanced algebra, so called, including certain special topics listed below, for a half year, is given in some of the larger high schools or in some of the smaller ones that definitely prepare for college.

The fact that few of the high school teachers of mathematics are thoroughly trained in their subject and that the subject matter is settled for the most part without their iniative indicates that the content of the curriculum will be closely defined by the textbooks used

The kind of textbook in general use up to I2 to I5 years ago, and the kind most widely used today will make possible to define the subject matter and the methods of algebra as presented to more than 75% of the high schools pupils of the United States. Hereon the committee based the following outline(I).

The Orthodox Syllabus

I Algebra to Quadratics

Introductory. --- Definitions and "axioms", discussion of the negative quantities, brief practice in algebraic expression and interpretation, one or two lessons in the use of algebra for problems so simple that algebra adds to their difficulty.

The four operations. --- Addition, subtraction, multiplication, and division, completed sucessively in that order, with formal rules of manipulation (not necessarily stated in advance); literal and fractional coefficients and expon-

(I)For the purpose of the part (pp. 43-55)of this thesis based upon the following outline "numbers" will be understood as expressed without letters, and "problems" will be understood as "clothed" in words.

ents used; ingeniously involved parentheses, brackets, and braces; and expressions sometimes more complicated than most of the pupils will ever see again.

Factors. --- Factoring expressions, such as the difference of two squares, ax^2+bx+c, $x^n \pm y^n$ (often with demonstrations, as of the case where the sign is $+$ and n is odd); factoring "by parts"; forms like $x^4+x^2y^2+y^4$; expressions such as can be obtained from the simpler forms by substituting binomials for one or more of the letters. Usually no application is made of these feats except in the reduction of fractions, and in highest common factor and least common multiple, which follow as introductory to fractions.

Highest common factor and lowest common multiple. --- Highest common factor, first by factoring, then by the Euclidian method. A demonstration is usually given for this, but is hardly ever assimulated by the pupils. Not seldom the proof given is applicable only to numbers, that is, it will not hold for literal expressions in which, as is usual, some of the dividends have to be multiplied and some monomial factors have to be saved out. Lowest common multiple, generally by factoring only, with a perfunctory comment on the method which utilizes the highest common factor.

Fractions. --- Fractions with rules of transformation formally demonstrated, and the four operations each completed in its turn. Expressions of ingenious complexity are handled under each head.

Simple equations and problems. --- Simple equations, that is, equations of the first degree with one unknown letter, and abounding in parentheses and fractions; and, at last, problems to be solved by means of such equations, except that the equations needed for the purpose are really simple.

Linear elimination and problems. --- Two-letter linear equations, including fractional and literal equations; three-letter equations of the first degree, and ocassionally four or five letter sets. Equations solved for the reciprocals of the letters involved. Probably leading to equations of the first degree in two or more letters. Literal equations are scattered at random under this topic and the preceding one.

Inference equations. --- The model examples worked out in the textbook generally indicate the inference of an equation from the preceding work by means of phrases printed at the side, such as "transposing", "clear of fractions", "adding to eliminate the x terms", etc. No expectation is indicated that the pupil will use any substitute for these annotations.

Neglect of practice in devising equations. --- The number of problems given under this topic and the preceding one is generally insufficient to give real facility in algebraic expression, and their introduction seems isolated, an XXXXXXXXXXXXXX interruption in the progress of manipulation. Haste or neglect at this pointis explained by the fact that no topic is more difficult to test adaquately than the devising of equations,and where time is scant it will be devoted to topics that show.

Involution and evolution. --- Under the title "Involution" next are treated powers of numbers and of monomials,and squares and cubes of binomials; under the title "Evolution" are treated square roots and cube roots of polynomials. Some details of the theory of exponents are necessarily included under these heads.

Radicals and radical equations. --- Radicals,including the rationalization of binomial denominators,and the square root of a binomial surd (generally given without adaquate demonstration);radical equations,carefully selected or constructed so as to give,upon rationalization,equations of the first degree. Extraneous solutions,sometimes declared admissable because of the "ambiguous" sign of the square root. Problems again,few in number,leading with suitable choice of letters to radical equations.

Exponents. --- Theory of exponents,without any mention of logarithms; good correlation with the preceding topic.

II Quadratic Equations

Quadratics in one and two unknowns. --- Quadratics in one unknown,first without the second term (pure quadratics), and then complete quadratics. Linear-quadratic pairs,elimination by substitution; special cases of quadratic pairs solved by devices suited to each case; special emphasis on symmetrical equations solved by reducing to values for x+y and x+y. Very few problems. Literal equations at random under this topic.

Ratio and proportion. --- Ratio and proportion, including the traditional transformations of a proportion; examples of literal equations and problems to which proportion can be applied if one exists; no mention of its application in geometry, and no comment on the relation of this subject to fractional equations. This topic is not referred to under any other part of algebra.

The progressions. --- Arithmetical progressions; formulas for the nth term and for the sum of the terms, any three of the five constants being given, to find the other two.

Geometrical progressions; formulas for the nth term and for the sum of the terms, certain groups of three of the five constants being given to find the other two constants. Formula for the "sum of the series" when the ratio is less than one and the number of terms indefinitely great; recurring decimals.

Inserting means. --- Arithmetic and geometric mean; insertion of two or more arithmetic or geometric means between two given numbers.

Binomial theorem.--- The binomial theorem for positive integral exponents; proof of the same; application to powers of a binomial of which the terms may be complicated with fractions, with radical signs or with exponents that may or may not be positive or integral. Formula by which any term of any power may be written down.

VI Advanced Algebra(I)

Topics treated. --- Under this head are given various disconnected topics including: Theory of equations, with graphs of $y = ax^2 + bx + c$; solution of numerical equations of higher degree in one unknown, with graphical illustration; occasionally successive derivatives of algebraic polynomials, geometrically interpreted, are incidentally taken up as far as advisable in utilizing graphs for explanation; ocassionally trigonometric solutions are given for certain equations. Choice and chance. Determinants, with practice in reduction and evaluation (the multiplication theorem ommitted). Indeterminate coefficients.

Purpose. --- The sole purpose of this course seems to furnish information that may be useful in later mathematical study.

Confusion of title. --- In many schools the latter part of the course in elementary algebra described previously is styled "Advanced Algebra". The usage is confusing and should be avoided.

(I) The Roman numerals are the same as in the outline examined, and not consecutive for the thesis.

Growth of the College Requirement

Harvard College as an Example

Advance of the last 60 years. --- The definition which has just been completed of algebra covered in high school work represents a very considerable advance over the conditions 60 years ago. This change can well be traced by a study of the requirements for admission to Harvard.

Early textbooks. --- In 1845 Euler's Algebra, and Davies' "Algebra to the extent of Square Root" were required for admission; this was a larger requirement than was made at that time by any other New England college.

In 1866 Sherwin's Common School Algebra "to section XXXVIII" was required. The algebra requirement so defined omitted radicals and fractional exponents, proportions, and algebraic and geometric progressions. The next year the algebra requirement included "through quadratics". No reference was made to a particular textbook. Two years later logarithms (not the theory) was required.

Elective requirements. --- In 1870, while the old subjects of examination were retained for such as chose to take them, an alternative specification was made with a much reduced requirement in Latin and Greek and an increased requirement in mathematics. This included permutations, probability and determinants, in algebra.

Changes in subject matter. --- In the textbooks of algebra 50 years ago much more stress was placed on logical exposition than on solution of problems. The development of arithmetic, as followed in the textbooks of the elementary school, was faithfully imitated in algebra and various operations (some, like the square and cube root of polynomials, having no conceivable use, and others mere elaborations of methods that in simple form were well worth while) were laborously discussed and exploited before the use of equations in discussing problems entered upon.

The algebras of Todhunter and Hamlin Smith, in England, were followed in America by Wentworth's Algebra, published about 1881. The exercises in this book, which were widely commended by teachers of the day as "well chosen", numerous, and carefully graded", were the selling feature, and the book sprang into great popularity at once. The idea was to "learn by doing", and since that time the exercises have been of much greater importance in textbooks and in teaching than before.

Causes of change. --- The changes thus noted in the school work of algebra are the result of attempts to take subjects that were originally a part of the college curriculum and adapt them to the comprehension of high school pupils. Abstract discussions were to be replaced by means for practical drill; the teacher rather than the mathematical scholar, was the arbiter in choosing books. At first he rejected books like Hill's and Sherwin's, which were actually better adapted to his classes, for those which had been familiar to him in his work in college. From these the progress was gradual. Ten years ago a prospectus of the two books last referred to would have seemed quite up to date.

Recent Progress

The last ten years. --- In the last ten years changes in the details of high school mathematics have been radical and rapid. They are largely the result of the active interest taken in the work of high school teachers by university professors, and of the conference between high school men of different localities. Both of these influences have made themselves felt through teachers associations. So far as the changes and tendencies referred to have actually begun to affect teaching, they appear in recently published textbooks in good use. For this reason the committee examined some thirty of the recent high school textbooks on algebra, geometry, and trigonometry and presented the results of that examination.

The order of topics.--- In almost all these books geometry is supposed to follow a year's work in algebra, though there are one or two books whose success is still not completely assured, which essay a combination of the two subjects. A combination (or blending) of plane and solid geometry does not seem to have been seriously attempted.

In algebra the order of topics is only slightly varied from the following:

I. Introduction, negative numbers, etc.

2. The four operations.

3. Factors, H.C.F. and L.C.M. by factoring.

4. Fractions.

5. Simple equations and problems.

6. Elimination, linear system.

7. Powers and roots, exponents, radicals.

8. Quadratic equations.

9. Elimination of quadratics.

10. Literal equations, generalizations.

11. Proportion, the progressions, logarithms.

12. The binomial theorem.

Graphical methods of discussion. --- For some fifteen years there has been increasing pressure for the introduction of Cartesian coordinates as an instrument of study in elementary algebra. It began to appear in the school books about 1898. All but one of the books examined by the Commitee make use of this devise; sometime it is given in separate chapters, in one case in an appendix. In a few others it is made an effective part of the structure of the subject. The word "function" is sometimes used, but even without it the work generally begins by plotting curves in which y is a non-algebraic function of x, so that the student gets some insight into the functional relation.

The graph of a two-letter linear equation in pointed out as a straight line. No proof is given, though one book remarks that the proof follows easily from the geometry of similar triangles. Elimination of linear pairs is illustrated, generally also linear-quadratic and quadratic pairs. No comment is made, as a general thing, on the limitation of this illustration to two-letter equations.

In about one half of the books the solution of a numerical quadratic equation by the use of a standard parabola ($y = x^2$) and a straight edge is mentioned, and its use recommended as a check on the solution obtained by the algebraic process.

Computation. --- In spite of the fact that John Perry's propositions for reform in mathematical teaching have been widely and on the whole favorably considered in this country, not one of the textbooks of algebra, and only one of them in geometry, makes any reference to the number of significant figures in a number as a criterion of the degree of approximation. None gives any directions for economical methods of computation having regard to the degree of accuracy warranted by the data, or gives problems with data appropriate for such practice.

Checks. --- The practice of checking the solution of an equation in algebra by substituting the roots, and checking an algebraic transformation by substituting arbitrary values, appears in almost all the algebra textbooks.

Notation. --- In algebra there is a tendency to break the monopoly that the letter x has had in representing numbers whose value is sought. The symbol ≠ for "is not equal to" and the symbol ≡ for algebraic identity have come into use.

Problems in algebra. --- Much more space is given to equations, and to problems giving rise to equations, as a response to the frequently repeated contention of teachers, uttered in magazine articles and in teachers' associations, that the equation should be the fundamental work at least for the first year. In most cases, however the preparatory study of transformations (multiplication and division, factoring, fractions, etc.) is carried to a degree far beyond what is necessary for the manipulation of any reasonably probable equations, certainly beyond what are given during first year work. The study presents, therefore a somewhat disconnected aspect --- first, transformations treated ina systematic and fairly complete fashion, with a few intrusive illustrations of the applications of them to equations that do not need much; then, equations treated as material for practice of a small part of this manipulation; and, finally problems, hopefully sought from newer quarries, designed to show how equations might arise that could be managed by this manipulative skill.

It would probably be easy for a young person of good judgment, engaged in reviewing his high school algebra, to learn that problems cause the invention of equations, and that transformations are necessary for the solutions. The prominence given, however, to this systematic development of what must be considered the mechanical side of algebra tends to weaken the interest of the pupil in that part of the subject that is of most value, not only to him whose education stops with the high school, but surely also with the future student of engineering or of pure mathematics; the study, that is, of expressing the conditions of actuality in mathematical form, and of interpreting mathematical results in terms of time, space, and things.

The problems in all the books are very plentiful; sometimes the teacher is warned that the work should not include the solution of all the problems and that their profusion is his opportunityto vary his work from class to class and specify fresh problems for review lessons. While the character of the problems in three of the books is strickly orthodox, the others take their data freely from geometry, physics, and even from engineering, and dietectics. There is no hesitation in utilizing the properties of similar triangles; or of the phenomena of falling bodies; but there is a chaste reluctance to do anything more than mention the existence of reasons at the back of the facts and formulas used. Our old friend, the clock problem, survives the dead and buried hare and hound, and the solution in odd elevenths of a second continues to pass without a challenge (I).

(I) See footnote ,page 43, 44

The American academy flourished from the close of the Revolutionary War to 1870. Then the High Schools began to supplant them.(1) The early development of the academies was almost entirely independent of the colleges. These academies were entirely a peoples college. They soon began to provide a college preparatory course for the benefit of the students intending to enter college. The mathematical curriculum found place by the side of the English, and classical courses(2).

The first stage in the introduction of natural science into the program of studies is seen in the laying of strong emphasis on mathematics, especially in algebra and geometry. Closely connected with these subjects was the study of astronomy.(3) The course of study in the earlier American academies was not clearly formulated. That part which looked to preparation for college was, however, fairly well defined in the tradition received from the grammar schools(4).

In 1820 algebra found a place in the curriculums of the acadenies and had strong emphasis laid on that branch because of its use in the sciences. At some of the academies it was in the first year and at some in the third year(5).

The curriculum of the Phillips Exeter Academy, for 1818, shows algebra in the third year and in the advanced class of the classical department. It was in the first year, through

(1) E.E.Brown, p.220. (2) J.F.Brown, p.20. (3) E.E.Brown, p.332.
(4) E.E.Brown, p.236. (5) E.E.Brown, p.238.

simple equations in the English department[I].

At the outset the academies were not intended as preparatory schools, and represented rather an independent movement. As tóme went on they came into close relations with the colleges. In the latter part of the 19th century when high schools had largely taken the place of academies as the ordinary agency of secondary education, the academies swung back toward the position of distinctly preparatory institutions. The reputation that some of them gained as among the foremost fitting schools for our foremost colleges, has obscured the fact that the fitting for college was a subordinate consideration in their original establishment. The new colleges growing up in the western and southern states where secondary schools were still few and weak, were generally under the necessity of maintaining preparatory departments. These came to be commonly known as academies[2].

An account of the Boston Latin School in 1820 shows that the course of study was five years long. The children were admitted at the age of nine, and must have mastered considerable skill in reading, writing, and use of the English grammar. Mathematics (certainly arithmetic) is mentioned in the first year; none, in the second or third years; the fourth and fifth years are listed together, with arithmetic, geometry, trigonometry, and algebra at the end of the list[3].

(I) E.E.Brown, p.238. (2) E.E.Brown, p.250. (3) E.E.Brown, p.277.

The English High School of Boston is regarded as the pioneer of the high school movement in the United States. A town meeting was held, Jan. I5, I82I, at which the plan, outlined by a committee in their report, was debated and finally adopted with only three dissenting votes. The course of study recommended by the committee consisted of a three years course. The qualifications for admission were: not less than twelve years of age, and be well acquainted with reading, writing, English grammar in all its XXXXX branches, and arithmetic as far as simple proportion. In the course of study, arithmetic was in the first year, algebra in the second, and mathematics in the third(I).

The Committee of Ten, in their report of I892, advised that it is desirable during the study of arithmetic to familiarize the pupil with the use of literal expressions and of algebraic language in general. Introduce the simple equation in the study of proportion, of the more difficult problems in analysis, and of percentage and its applications. The designation of positive integral powers by exponents may also be taught. Avoiding the introduction of negative numbers the pupils should be drilled in easy problems like the following:

If a stone weighs p pounds and another q pounds, what is the weight of both together?

If a yards of cloth cost b dollars what will c yards cost?

(I) E. E. Brown, p. 30I.

Such exercises should grow out of similar ones involving numerical data.

The average pupil should be prepared to undertake the study of formal algebra at the beginning of the fourteenth year. The time assigned to the study of algebra in the high school should be about the equivalent of five hours per week during the first year, and an average of two and one half hours per week during the following two years. This would afford ample time for the thorough mastery of algebra through quadratic equations and equations of the quadratic form.(I)

They also advise leaving out the proofs of certain propositions, the rigorous demonstrations of which are unintelligible to pupils at the time when those proposiyions are first encountered. Such is usually the case with the rule of the signs in multiplication, and with the binomial formula. In cases of this kind the proof should be at first omitted but introduced at a later period in school or college. When such omissions are made the pupil must be convinced of the truth of the propositions by illustration or induction. Oral exercises in algebra, similar to those in what is called mental arithmetic are recommended. Especial emphasis should be laid upon the fundamental nature of the equation(2).

In the I896-97 report of the Commissioner of Education of The United States, pp.457-6I3, on entrance requirements of

(I) Committee of Ten, p.III. (2) Committee of Ten, p.II2.

432 institutions having a course leading to the degree of A.B., 346 of these institutions specify arithmetic as an entrance requirement, the others probably regard it as implied in the requirement of algebra. Algebra is required in 412 institutions to the following amounts:

To quadratics -- ------------------- 37 institutions

Including quadratics ------------- 174 institutions

Amount not specified ------------ 201 institutions

The detailed statement of the requirements for each institution, shows that the better institutions require arithmetic (explicitly or implied), algebra including quadratics, and plane geometry (1).

The Committee on College Entrance Requirements recommended in 1889 that the course in mathematics required for all students be limited, roughly speaking, to the following topics: the four fundamental operations for integers, and common and decimal fractions; the most important weights and measures; percentage and its application to simple interest; and that it be completed in the sixth grade. Commercial arithmetic should be omitted from the prescribed course in mathematics, and if it be deemed necessary, an elective course in this subject may be offered at some convenient time during the high school period, and in connection with a course in bookkeeping.

(1) page 141, Report of Committee of Chicago Section of the American Mathematical Society, 1899.

The following arrange ment was suggested for the course in mathematics from the 7th to the I2th grades inclusive, assuming the length of the recitation period to be at least 45 minutes:

Seventh grade -- Concrete geometry and introductory algebra, --------- ---------------- 4 periods

Eighth grade---- Introductory demonstrative geometry and algebra --- ------ ----------- 4 "

Ninth and tenth grades -- Algebra and plane geometry ------------------------ 4 "

Eleventh grade -- Solid geometry and plane trigonometry ------------------- 4 "

Twelfth grade --- Advanced algebra and mathematical reviews ------------------- 4 "

The algebra of the 7th and 8th grades should at the outset be mere literal arithmetic. But they were of the opinion, by limiting the working material to very simple polynomials and fractional expressions, and to equations of the first degree with numerical coefficients, the four fundamental operations, rational expressions, simple factoring, and the solution of equations of the first degree in one and two unknown quantities may be taught effectively in the course of these two grades.

Young students enjoy reckoning, and elementary algebraic reckoning will interest them far more than the complexities of commercial arithmetic. The principles of the subject must, of course be presented concretely, and unnecessary generalizations should be carefully avoided. Simple problems which can be solved by aid of equations of the first degree should be introduced as early as possible. The sooner the pupil appreciates the power of algebraic methods, the sooner the subject-matter will attract him.

They recommended that the time alloted to mathematics in the ninth and tenth grades be divided equally between algebra and plane geometry, and that the course in algebra include:
(a) a more systematic and comprehensive study of the topics treated in the introductory course of the seventh and eighth grades with a thorough drill in factoring, highest common factor, lowest common multiple, and complex fractions;
(b) radicals and fractional exponents, and quadratic equations in one and two unknown quantities; (c) ratio and proportion, the progressions, the elementary treatment of permutations and combinations, the binomial theorem for positive integral exponents, and the use of logarithms. There is time enough in this course for the topics (c), and they seemed to the Committee to belong here rather than in the advanced algebra of the twelfth grade because of their elementary character and general interest.

By advanced algebra they mean the remaining topics which are to be found in the ordinary textbook of the college algebra, namely, elementary treatment of infinite series, undetermined coefficients, the binomial theorem for fractional and negative exponents, the theory of logarithms, determinants, and the elements of the theory of equations.

They recommended that the several mathematical subjects count toward satisfying the requirements for admission to college as follows:(I)

(a) Elementary algebra, as defined above,---------- I½ units

(b) Advanced, " " " " ---------I unit

(c) Plane geometry, (Which is defined in the report), I "

(d) Solid " " " " " " " ½ "

(e) Plane trigonometry " " " " " " ½ "

The Committee of the Chicago Section of the American Mathematical Society reported in I899 on "the scope, aim, and place of mathematical studies in the secondary schools and in preparation for college", page I4I.

While the committe did not recommend any radical alterations in the subject matter of algebra, as usually presented in our best schools, the committee desired to emphasize the following points:

I. Arithmetical side of algebra. --- Computations with numbers should be constantly introduced, problems with literal quantities being worked out or verified with numerical data also. The processes of arithmetic, both oral and written, should not be allowed to fall to disuse. At the same time,

(I) Committee of Ten on College Entrance Requirement, p. IO.

the pupil should understand the value of algebra in abridging or simplifying computation with numbers,or in proving the correctness of rules of computation,and should understand clearly that the devices of mathematics ,especially algebra,have the purpose of enabling us not to compute; and that actual comput tations are usually not to be made so long as they can be avoided; that cancellation is to be resorted to whenever possible; and that to obtain an expression in a factored form, ,or in any way in which the operations are indicated,is a distinct advantage not to be surrendered by needlesly performing the operations. Some of the topics omitted from arithmetic should be taken up at appropriate places in the work in algebra.

2. The equational side of algebra. --- The equation should be made from the very beginning. Very simple problems in words leading to equations can be given at the outset.

3. Algebraic translation. --- What has been said as to the value and necessity of translation in general applies with special force to algebra. Here the danger of mechanical,or even haphazard,manipulation of symbols is perhaps the greatest,and it must be especially guarded against by care that the meaning of the symbols,and the reason for the operations,be always clear in the pupil's mind. This can be done to a large extent by requiring the pupil to give

readily and clearly in words the meaning of formulas and equations.

4. Topics to be emphasized. --- The following topics require careful treatment:

The meaning and use of exponents, positive, negative and fractional; the handling of the simpler surds; the distinction between identical equations and equations of condition; the character of the roots of the quadratic equation as determined by inspection; the connection between the roots and the coefficients of the quadratic, the solution of the quadratic equations by factoring, and the making of algebraic statements for problems given in words.

5. Secondary school algebra and college algebra. --- It should be the aim of the secondary school to avoid taking up any of the topics which are customarily treated in college algebra, but rather to secure as thorough a mastery as possible of those topics which the college presupposes.

The progressions, arithmetical and geometrical (with applications to interest, simple and compound), theory and use of logarithms, might well, so far as the nature and difficulties of the subject are concerned, be included in the secondary school course, but as they are required for entrance by very few colleges and are accordingly taken up in connection with college algebra, the committe recommended that they be omitted from the secondary school course in the interest of economy and energy and to avoid duplication of work (I).

(1) Report of Committee on College Entrance Requirement, p. 141

It is probable that we shall long continue our present general plan of having a book of arithmetic, still another on geometry, thus creating a mechanical barrier between the sciences. We shall also doubtless combine in each book the theory and the exercise for practice because this is the English and the American custom, giving in our algebras a few pages of theory followed by a large number of exercises. The Continental, however, inclines decidedly toward the separation of the book on theory, thus allowing frequent changes of the former. It is doubtful, however, if the plan will find any favor in ~~the~~ America, its advantages being outweighed by certain undesirable features. There is, perhaps, more chance for the adoption of the plan of incorporating the necessary arithmetic, algebra, and geometry for two or three grades into a single book, a plan followed by Holzmüller with much success.[I]

J.W.A.Young favors the simultaneous teaching of algebra and geometry in secondary schools. He thinks algebra is more abstract than geometry, has fewer points of contact with real life and its reasoning is more difficult than that of the easier demonstrations of geometry. Also that the more difficult parts of either subject are harder than the easier parts of the other, and each can be made a valuable help in the development of the other. The ideas of both are assimulated slowly

(I) D.E.Smith, p.I73.

by the mind,requiring not so much daily use as occasional use in various ways (quite easy for some time) throughout a long period. To rush a pupil througha subject at high pressure with the exclussion of all other mathematics may train the pupil to pass examinations,but it will not develop the largest measure of thought power or be conducive to real assimulation of the subject.

He thinks that the conclusion follows naturally that algebra and geometry should be taught side by side; that if under pressure of circumstances either subjest must be taken up first it should be geometry. For many years the tanden order has been under question,and the weighty objections to it and the strong reasons in favor of another order have repeatedly been urged,though with little general effect.

The reasons for the failure to make the movement seem to be those of inertia,of conservation rather than a conviction that the change proposed is not good or that the customary order is better. "I know of no published defense of the traditional precedure,but still the old order,which finds no defenders in theory,persists in practice". He thinks there should not be any difference in the work in mathematics,in secondary schools,according to whether the pupil intends to go to college,to technical school,or into active life[I].

Thomas K .Mc.Kinney summarizes The Teaching of Algebra in Its Relation to The Present Educational Trend in the

(I) J.W.A.Young,pp.I84-I87.

N.E.A.Report for 1908,page 631. Only the parts are referred to which will apply definitely to algebra.

There is a strong and growing tendency to deveolp the concrete side of elementary algebra. This tendency is referred to in various phrases: such as "emphasizing the practical side of algebra";"correlating algebra with its principal applications in mechanics,physics,chemistry";"developing the subject in close touch with the affairs of daily life,enriching its content,multiplying its relations,and adding to its human interest". XXXXXXXXX He does not intend to say that these expressions refer to the same phase of this tendency but rather it is the one tendency viewed in several aspects that give rise to these several descriptions.

Much care should be taken that algebra shall grow easily and naturally out of arithmetic. Certain topics,which,because of their abstract character are difficult for beginners, or which,XXXXXXXX by their nature have little immediate application are omitted or postponed. Elementary texts are ~~pu~~ published which make these omissions. There is also evedence of a disposition to subordinate all other topics of elementary algebra to the equation. The graph is recognized as an important aid in elementary algebra both in the solution of problems and in illustrating and enforcing the idea of functional~~ity~~ relation. To what extent correlation of elementary mathematics and the physical sciences will be carried out is difficult to say. This is advocated strongly by the science teachers.

In most secondary schools in the United States algebra is studied in the ninth grade before geometry in the tenth grade, but lately this plan has been frequently assailed; sometimes it has been considered one of the chief causes of the inefficiency of mathematical teaching. While some reformers wish to place geometry before algebra most of them advocate a simultaneous teaching of the two subjects.

The three most widely advocated reforms of the mathematical study are:

I. The Use of Laboratory Methods.

2. The Teaching of Applied Problems.

3. Simultaneous teaching of Algebra and Geomerty.

The feasibility of the first (1) plan under certain conditions is proved by the experience of a number of schools in various countries. In regard to the second (1) plan it should be emphasized that it does not mean the teaching of three hours of algebra and two hours of geometry during the first two years of high school, but a complete merging of the two subjects. Any new method, whether it relates to geometry or algebra should, according to this plan, be studied whenever the necessity for it arises, and not before. Thus square roots should be taught in connection with the Pythagorean theorem, similar triangles with proportion, etc. All algebraic facts should as far as possible be illustrated geometrically and vice versa.

(1) Refer to lines 5-7 above.

This scheme, which has been advocated not only in the United States but also in other countries, especially in Italy and Germany, has undoubtedly a number of advantages. It may arouse more interest than the customary mode, it may at some stages of the work show the student the necessity of studying certain topics, it may train the student better to apply his knowledge, and it may prevent the rapid forgetting of algebra during the time when geometry is studied.

On the other hand there are weighty reasons against the introduction of simultaneous teaching of algebra and geometry in every secondary school. First of all, such a complete merging of the two subjects may not be at all possible. The courses of study of this new type, which have been proposed, are still in the experimental stage, and, as a rule lack detail. The textbooks that pretended to carry out this idea, usually do the merging for one or two topics, otherwise they simply alternate between algebraic and geometric topics.

Of course there can be no difference of opinion about the wisdom of introducing algebraic illustrations into geometry, and vice versa, whenever feasible. But there is a wide difference between such procedure and complete merging.

Moreover, there exists a large number of high schools in which the first year pupils are not capable of attacking geometry as successfully as algebra. Trained principally in mechanical modes of study, such students find the transition from arithmetic to geometry too difficult,while they attack ~~a~~ algebra,which resembles arithmetic, quite successfully.

Hence it seems doubtful whether the simultaneous teaching of algebra and geometry would produce sucha radical improvement as the advocates of this plan claim. Still the plan is worth trying,and schools that are in the position to make experiments should give this matter a thorough and impartial trial.(I)

(I) Report of N.E.A.,I908,p.63I.