SOME FLEXURE FORMULAS AND DIAGRAMS FOR REINFORCED CONCRETE BEAMS

BY

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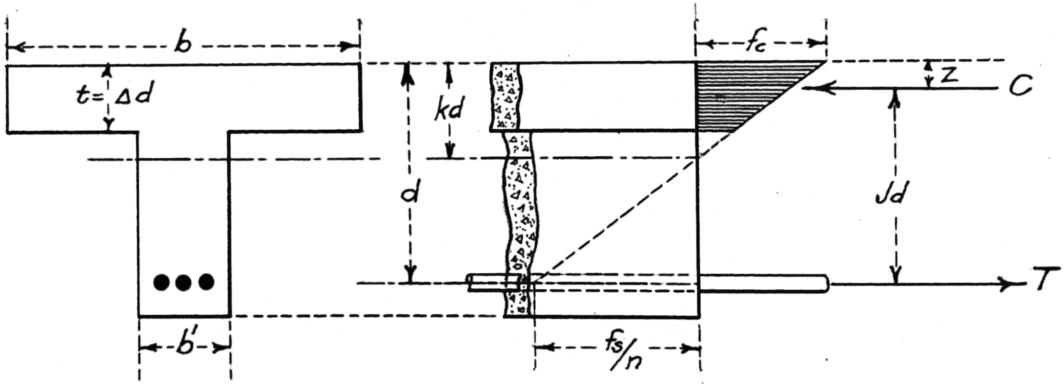
SOME FLEXURE FORMULAS AND DIAGRAMS FOR REINFORCED CONCRETE BEAMS.

The writer has been striving for some time to minimize the labor of designing tee and double reinforced concrete beams and at the same time to eliminate the cut and try methods that have been necessary to arrive at correct results. This thesis is the original publication of his new flexure formulas for both types of beams and of new flexure diagrams for designing simple slabs and tee-beams. The diagrams have been developed jointly with Mr. Don B. Schuler, who is studying architecture at the University of Illinois.

The nomenclature used throughout this work is that of the Joint Concrete Committee representing the American Society of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering and Maintenance of Way Association, and the Association of the American Portland Cement Manufacturers. In addition



b
t= Δd
b'
kd
d
f_c
z
C
jd
T
f_s/n

$\Delta=\frac{t}{d}$ and $Q=\frac{d'}{d}$ are used in the formulas for T- and double reinforced beams respectively.

1. T-Beam Formulas.

When the neutral axis of a t-beam is contained within its web, the simple beam formulas no longer hold. The following t-beam flexure formulas which cover this case are based on the customary assumptions that the aid of the concrete stresses in the web is negligible and that plane sections remain planes after bending:

$$(1.)\quad M=\left\{\frac{f_c}{\Delta}(3-3\Delta+\Delta^2)-\frac{f_s}{n}(\tfrac{3}{2}-\Delta)\right\}\frac{bt^2}{3}$$

$$(2a.)\quad Ms= f_s pjbd^2$$

$$(2b.)\quad Ms=\frac{f_s}{\Delta(2-\Delta)}\left\{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2pn}\right\}\frac{pbt^2}{3\Delta}$$

$$(3a.)\quad Mc=\frac{f_c j\Delta(2k-\Delta)}{2k}bd^2$$

$$(3b.)\quad Mc=\frac{nf_c}{2pn+\Delta^2}\left\{6-6\Delta+\Delta^2+\frac{\Delta^3}{2pn}\right\}\frac{pbt^2}{3\Delta}$$

$$(4.)\quad \Delta=\frac{Y-\sqrt{Y^2-12f_cX}}{2X}$$

in which $X=f_c+\frac{f_s}{n}$

and $Y=3\left\{\frac{M}{bt^2}+f_c+\frac{f_s}{2n}\right\}$

$$(5.)\quad P=\frac{\Delta}{f_s}(f_c-\frac{\Delta X}{2})$$

$$(6.)\quad f_c=\frac{\frac{2p}{\Delta}+\frac{\Delta}{n}}{2-\Delta}\cdot f_s$$

(7a.) $k = \dfrac{pn + \frac{1}{2}\Delta^2}{pn + \Delta}$

(7b.) $k = \dfrac{1}{1 + f_s / nf_c}$

(8.) $Z = \dfrac{3k - 2\Delta}{2k - \Delta} \cdot \dfrac{t}{3}$

(9.) $Jd = d - z$

(10.) $J = \dfrac{6 - 6\Delta + 2\Delta^2 + \Delta^3 / 2pn}{6 - 3\Delta}$

Formulas 1, 2b, 3b, 4, 5, and 6 of this group are original.

USE OF THE T-BEAM FORMULAS.

In the majority of cases the t-beam is designed as part of a floor system in which the thickness of the flange, t, has been predetermined by the floor slab moments or shears as the case may be. As an illustrative example of this case assume that a simply supported t-beam has the following factors predetermined:

l = 40 ft. w/ft. = 4000 lbs. = 1500 lbs. dead + 2500 lbs. live load.

f_c= 600 lbs. f_s= 15000 lbs. t=8 in. b=64 in.

Find A_s and d.

$M = \dfrac{4000 \times 40 \times 40 \times 12}{8} = 9{,}600{,}000$ in. lbs.

Then in formula 4, which should be solved by exact methods as the slide-rule does not give satisfactory results in this one operation.

$$Y = 3\left\{\frac{9,600,000}{64 \times 8^2} + 600 + \frac{15000}{2\times 15}\right\} = 19332$$

$$X = 600 + \frac{15000}{15} = 1600$$

$$\Delta = \frac{10332 - \sqrt{(10332)^2 - 12\times 600\times 1600}}{2 \times 1600} = \frac{10332-9759}{3200} = 0.1791$$

$$d = \frac{t}{\Delta} = \frac{8}{0.1791} = 44.6"$$

From formula 9

$$p = \frac{0.1791}{15000}\left(600 - \frac{0.1791\times 1600}{2}\right) = 0.00545$$

$$As = pbd = 0.00545 \times 64 \times 44.6 = 15.57 \text{ sq. ins.}$$

If the depth of the beam is predetermined by shear, headroom, or any other consideration, formula 5 gives the proper percentage of steel for whatever working stresses it is desired to use.

Formulas 6 and 1 make it possible to determine how much moment a given section will resist without exceeding certain working limits. The above example will now be checked backward by the use of these two formulas.

From 6

$$f_c = f_s \frac{\frac{2\times .00545}{0.1791} + \frac{0.1791}{15}}{2 - 0.1791} = 0.04\ f_s$$

Therefore if f_s = 15000, f_c must equal 600. From formula

1

$$M=\left[\frac{600}{0.1791}(3-3\times.1791+0.1791^2)+\frac{15000}{15}(0.1791-\frac{3}{2})\right]\frac{64\times8^2}{3}=9,570,000\text{"\#}$$

Now suppose that the bending up of 4 rods reduces the steel ratio, p, to 0.00358.

From formula 6

$$\frac{f_c}{f_s}=\frac{\frac{2\times.00358}{0.1791}+\frac{0.1791}{15}}{2-0.1791}=0.0286$$

Therefore if $f_c = 600$, $f_s = \frac{600}{0.0286} = 20980$ lbs. As this is in excess of the working stress in the steel it is evident that the stress in the steel is the limiting condition. If $f_s = 15000$ $f_c = 0.0286 \times 15000 = 429$ lbs. Then the maximum safe resisting moment of the section is

$$M=\left[\frac{429}{0.1791}(3-3\times0.1791+0.1791^2)-\frac{15000}{15}(\frac{3}{2}-0.1791)\right]\frac{64\times8^2}{3}$$

$= 6,360,000$ in. lbs.

It should be noticed that f_c is the only factor in this formula that varies with successive reductions of the steel area.

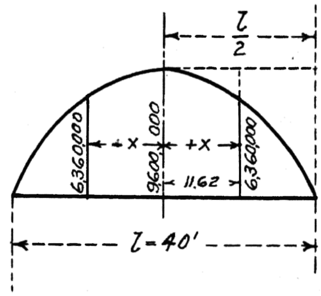
The point on the t-beam at which the moment equals 6,360,000 in. lbs. may be found by solving the general moment equation covering this condition, which is

$M=\frac{1}{2}wlx-\frac{1}{2}wx^2$ in which x is the distance from the left support, or the distance from the center to this section may be found by taking advantage of the fact that this moment curve is a parabola.

$$\frac{x^2}{\left(\frac{l}{2}\right)^2} = \frac{9,600,000-6,360,000}{9,600,000}$$

$$x^2 = \frac{400 \times 3,240,000}{9,600,000}$$

$$x = \pm 11.62$$



MOMENT DIAGRAM.

Now suppose that the section worked out in the first problem is subjected to a moment of 12,500,000 in. lbs. What are the corresponding fiber stresses in the steel and the concrete? From formulas 2b and 3b

$$12,500,000 = \frac{f_s}{0.1791(2-0.1791)}\left(6-6\times0.1791+2\times0.1791^2+\frac{0.1791^3}{2\times0.00545\times15}\right) \times \frac{0.00545\times64\times8^2}{3\times0.1791}$$

$$12,500,000 = \frac{15\ f_c}{2\times0.00545\times15+0.1791^2}\left(\quad '' \quad\right) \quad ''$$

f_s = 19520 lbs.
f_c = 781 lbs.

These stresses are directly proportional to the moment as is evident from an inspection of formulas 2b and 3b, page 2. Therefore they can be derived directly from the first problem.

$$f_s = 15000 \times \frac{12,500,000}{9,600,000} = 19510 \text{ lbs.}$$

$$f_c = 600 \times \frac{12,500,000}{9,600,000} = 781 \text{ lbs.}$$

DERIVATION OF THE T-BEAM FORMULAS.

From the assumption that plane sections remain planes after bending, it follows that unit deformations of the fibers of the steel and the concrete vary as their distance from the neutral axis. Then, since under working conditions the stress equals the strain times the modulus of elasticity,

(1.) $\frac{f_s}{1-k} = \frac{nf_c}{k}$

From which

(2.) $k = \frac{nf_c}{f_s+nf_c} = \frac{1}{1+f_s/nf_c}$ This equation is formula 7b.

For the same reasons the concrete stress at the underside of the flange $= f_c \frac{kd-\Delta d}{kd} = f_c \frac{k-\Delta}{k}$.

From the statical law that $\Sigma H=0$, the total tension on any section must equal the total compression on that section.

Therefore

(3.) $f_s pbd = \frac{1}{2}\left[f_c + f_c\frac{k-\Delta}{k}\right] b\Delta d = \frac{f_c \Delta bd}{2k}(2k-\Delta)$

(4.) $\frac{f_s}{f_c} = \frac{\Delta bd}{pbd}\left(\frac{2k-\Delta}{2k}\right) = \frac{\Delta}{2pk}(2k-\Delta)$

From equation 1,

(5.) $\frac{f_s}{f_c} = \frac{n-kn}{k}$

Equating 4 znd 5.

(6.) $\frac{\Delta}{2p}(2k-\Delta) = n-kn$

(7.) $2k\Delta-\Delta^2 = 2pn-2kpn$

(8.) $k\Delta+kpn = pn+\frac{1}{2}\Delta^2$

(9.) $k = \frac{pn+\frac{1}{2}\Delta^2}{pn+\Delta}$ This equation is formula 7a.

To determine the distance, z, from the center of compression to the top of the beam, call kd-z, y and take moments abomt the neutral axis. The moment of the force trapezoid must equal the difference between the moments of the triangles whose vertices lie in the neutral axis and whose bases are the top and bottom of the flange respectively. Then

(10) $(f_c+f_c\frac{k-\Delta}{k})\frac{\Delta d}{2}\times Y=\frac{1}{2}f_c kd\times\frac{2}{3}kd-\frac{1}{2}f_c\frac{k-\Delta}{k}(kd-\Delta d)\times\frac{2}{3}(kd-\Delta d)$

(11) $\frac{f_c\Delta y}{2k}(2k-\Delta)=\frac{f_c d}{3k}\left\{k^3-(k-\Delta)^3\right\}$

(12) $\frac{\Delta y}{2}(2k-\Delta)=\frac{d}{3}(k^3-k^3+3k^2\Delta-3k\Delta^2+\Delta^3)=\frac{\Delta d}{3}(3k^2-3k\Delta+\Delta^2)$

(13) $Y=\frac{2}{3}d\frac{3k^2-3k\Delta+\Delta^2}{2k-\Delta}=\frac{6k^2d-6k\Delta d+2\Delta^2 d}{6k-3\Delta}$

(14) $z=kd-y=\frac{6k^2d-3k\Delta d-6k^2d+6k\Delta d-2\Delta^2 d}{6k-3\Delta}=\frac{3k\Delta d-2\Delta^2 d}{6k-3\Delta}$

(15) $z=\frac{\Delta d}{3}(\frac{3k-2\Delta}{2k-\Delta})=\frac{3k-2\Delta}{2k-\Delta}\cdot\frac{t}{3}$ This is formula 8.

(16) $Jd=d-z$

(17) $J = 1-\frac{z}{d}=1-\frac{3k\Delta-2\Delta^2}{6k-3\Delta}$

Substituting for k its value given in equation 9,

(18) $$J = 1-\frac{3\Delta(\frac{pn+\frac{\Delta^2}{2}}{pn+\Delta})-2\Delta^2}{6(\frac{pn+\frac{\Delta^2}{2}}{pn+\Delta})-3\Delta}$$

(19) $$J = 1-\frac{3pn\Delta+\frac{3}{2}\Delta^3-2pn\Delta^2-2\Delta^3}{6pn+3\Delta^2-3pn\Delta-3\Delta^2}=1-\frac{3\Delta-2\Delta^2-\frac{\Delta^3}{2pn}}{6-3\Delta}$$

(20) $$J=\frac{6-3\Delta-3\Delta+2\Delta^2+\frac{\Delta^3}{2pn}}{6-3\Delta}$$

(21.) $$J = \frac{6 - 6\Delta + 2\Delta^2 + \frac{\Delta^3}{2Pn}}{6 - 3\Delta}$$ This equation is formula 10.

From equation 6,

(22.) $$p = \frac{\Delta(2k-\Delta)}{2n(1-k)}$$

Substituting for k its value given in equation 2,

(23.) $$p = \frac{\Delta}{2n}\left[\frac{\frac{2nf_c}{f_s+nf_c} - \Delta}{1 - \frac{nf_c}{f_s+nf_c}}\right] = \frac{\Delta}{2n}\left(\frac{2nf_c - \Delta f_s - \Delta nf_c}{f_s + nf_c - nf_c}\right)$$

(24.) $$p = \frac{\Delta}{2nf_s}(2nf_c - \Delta f_s - \Delta nf_c) = \Delta\frac{f_c}{f_s}\left(1-\frac{\Delta}{2}\right) - \frac{\Delta^2}{2n}$$

This equation is one form of formula 5.

Formula 6 is found by solving equation 24 for $\frac{f_c}{f_s}$

(25.) $$\frac{f_c}{f_s} = \frac{P + \frac{\Delta^2}{2n}}{\Delta\left(1-\frac{\Delta}{2}\right)} = \frac{\frac{2p}{\Delta} + \frac{\Delta}{n}}{2-\Delta} = \frac{2pn+\Delta^2}{\Delta n(2-\Delta)}$$

Formula 2a is found by taking moments about the center of compression.

(26.) $$M_s = f_s pbd \times Jd = f_s pJbd^2$$

Since $t = \Delta d$

(27.) $$M_s = \frac{f_s pJ}{\Delta^2} \cdot bt^2$$

Substituting for J its value in equation 21,

(28.) $$M_s = \frac{f_s pbt^2}{3\Delta^2(2-\Delta)}\left(6 - 6\Delta + 2\Delta^2 + \frac{\Delta^3}{2pn}\right)$$ This is equation 2b.

Formula 1 will now be derived from equation 28 by expressing p in terms of f_c, f_s, and Δ in accordance with equation 24.

$$(29.)\quad M_s=\frac{f_s b t^2}{3\Delta^2(2-\Delta)}\left\{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2n\times\frac{\Delta}{2nf_s}(2nf_c-\Delta f_s-\Delta nf_c)}\right\}\frac{\Delta}{2nf_s}(2nf_c-\Delta f_s-\Delta nf_c).$$

$$(30.)\quad M_s=\frac{bt^2}{3\Delta n(2-\Delta)}\left\{3-3\Delta+\Delta^2+\frac{\frac{1}{2}\Delta^2 f_s}{2nf_c-\Delta f_s-\Delta nf_c}\right\}(2nf_c-\Delta f_s-\Delta nf_c)$$

$$(31.)\quad M_s=\frac{bt^2}{3\Delta n(2-\Delta)}\left\{6nf_c-3\Delta f_s-3\Delta nf_c-6\Delta nf_c+3\Delta^2 f_s+3\Delta^2 nf_c+2\Delta^2 nf_c-\Delta^3 f_s-\Delta^3 nf_c+\tfrac{1}{2}\Delta^2 f_s\right\}$$

$$(32.)\quad M_s=\left\{\frac{nf_c(6-9\Delta+5\Delta^2-\Delta^3)-\Delta f_s(3-3\tfrac{1}{2}\Delta+\Delta^2)}{3\Delta n(2-\Delta)}\right\}bt^2$$

$$(33.)\quad M_s=\left\{\frac{f_c}{\Delta}(3-3\Delta+\Delta^2)-\frac{f_s}{n}(\tfrac{3}{2}-\Delta)\right\}\frac{bt^2}{3}\qquad\text{(Formula 1.)}$$

Taking moments about the centre of tension.

$$(34.)\quad M_c=\frac{f_c\Delta bd}{2k}(2k-\Delta)\times Jd=\frac{f_c\Delta J}{2k}(2k-\Delta)bd^2\qquad\text{(See equation 3.)}$$

This equation is formula 3a.

Formula 3b is found by substituting the value of f_s in equation 25 in equation 28.

$$(35.)\quad M_c=\frac{f_c p b t^2}{3\Delta^2(2-\Delta)}(6-6\Delta+2\Delta^2+\frac{\Delta^3}{2pn})\frac{\Delta n(2-\Delta)}{2pn+\Delta^2}$$

$$(36.)\quad M_c=\frac{f_c p n b t^2}{3\Delta(2pn+\Delta^2)}(6-6\Delta+2\Delta^2+\frac{\Delta^3}{2pn})\qquad\text{(Formula 3b.)}$$

Formula 1 may also be derived from equation 34. First substitute for J from equation 21.

$$(37)\quad M_c = \frac{(2k-\Delta)(6-6\Delta+2\Delta^2+\Delta^3/2pn)}{6k\Delta(2-\Delta)} f_c b t^2$$

Now substitute for k and p their values in equations 2 and 24.

$$(38)\quad M_c = \frac{\frac{2nf_c}{f_s+nf_c}-\Delta}{\frac{6\Delta nf_c}{f_s+nf_c}(2-\Delta)}\left[6-6\Delta+2\Delta^2+\frac{\Delta^3}{2n\times\frac{\Delta}{2nf_s}(2nf_c-\Delta f_s-\Delta nf_c)}\right]f_c b t^2$$

$$(39)\quad M_c = \frac{2nf_c-\Delta f_s-\Delta nf_c}{6\Delta nf_c(2-\Delta)}\left\{6-6\Delta+2\Delta^2+\frac{\Delta^2 f_s}{2nf_c-\Delta f_s-\Delta nf_c}\right\} f_c b t^2$$

This equation is the same as equation 30, therefore,

$$(40)\quad M_c = \left\{\frac{f_c}{\Delta}(3-3\Delta+\Delta^2)-\frac{f_s}{n}(\tfrac{3}{2}-\Delta)\right\}\frac{bt^2}{3} = M_s \text{ (Formula 1.)}$$

From this equation,

$$(41)\quad \frac{3M}{bt^2} = 3\frac{f_c}{\Delta}-3f_c+f_c\Delta-\tfrac{3}{2}\frac{f_s}{n}+\Delta\frac{f_s}{n}$$

$$(42)\quad \Delta(f_c+\frac{f_s}{n})-3(\frac{M}{bt^2}+f_c+\frac{f_s}{2n})+3\frac{f_c}{\Delta}=0$$

$$(43)\quad \Delta^2(f_c+\frac{f_s}{n})-3\Delta(\frac{M}{bt^2}+f_c+\frac{f_s}{2n})+3f_c=0$$

$$(44)\quad \Delta = \frac{3(\frac{M}{bt^2}+f_c+\frac{f_s}{2n})-\sqrt{9(\frac{M}{bt^2}+f_c+\frac{f_s}{2n})^2-12f_c(f_c+f_s/n)}}{2(f_c+\frac{f_s}{n})}$$

This equation is equivalent to formula 4.

THE DOUBLE REINFORCED BEAM FORMULAS.

The easiest method for designing any rectangular or t-shaped double-reinforced concrete beam is to determine first what moment the beam can carry as a simple beam of the same section when stressed to the working limits, and then to provide enough compression and additional tension steel to carry the excess moment without changing the fiber stresses. Other problems which arise will be discussed later.

The fundamental fact on which this method is based is that for any particular values of f_c and f_s, k has exactly the same value regardless of the shape or type of beam, This single value for all beams is expressed by the formula

$$k = \frac{1}{1 + f_s/nf_c}$$

It is derived in all cases by the same process of reasoning as that followed in deriving equation 2 under the t-beam formulas. It follows from this that if steel is added to the section without changing the extreme fiber stresses, this tension and compression steel must form a balanced couple whose stresses conform to the stresses already in the section.

Use the following nomenclature:.

P_c = the steel ratio for the single reinforced beam with the same k.

P_s = the steel ratio to balance P'. ($P=P_c+P_s$.)

M'_c= the moment of the single reinforced beam.

M'_s= the moment of the steel couple. ($M=M'_c+M'_s$)

From the assumption that unit deformations of the fibers vary as their distances from the neutral axis, it follows that

(1) $\frac{f'_s}{f_s} = \frac{k-Q}{1-k}$ (See figure, page 18)

Since the balanced steel couple must meet the statical requirement $\Sigma H=0$

(2) $f'_s p' bd = f_s p_s bd$

Substituting for f'_s its value in equation 1,

(3) $f_s p' \frac{k-Q}{1-k} = f_s p_s$

(4) $P_s = P' \frac{k-Q}{1-k}$

(5) $P_c = P - P_s = P - P' \frac{k-Q}{1-k}$

Taking the moment of the steel couple about the compression steel,

(6) $M'_s = f_s P_s (1-Q) bd^2$

(7) $P_s = \frac{M'_s}{f_s bd^2 (1-Q)}$

These formulas apply equally well to any type of beam. When P and P' are predetermined, first find k from the formulas,

(8) $k = -n(P+P') + \sqrt{n^2(P+P')^2 + 2n(P+P'Q)}$

and

(9) $k = \frac{P+P'Q+\frac{\Delta^2}{2n}}{P+P'+\frac{\Delta}{n}}$ (See pages 14, 15, & 16.)

for the rectangular and t-beam respectively.

When k has been determined, calculate P_s and P_c from formulas 4 and 5, and then find the working stresses from the formula,

(10) $\frac{f_c}{f_s} = \frac{2P_c}{k}$ for the rectangular beam

and

(11) $\frac{f_c}{f_s} = \frac{\frac{2P_c}{\Delta} + \frac{\Delta}{n}}{2 - \Delta}$ for the t-beam.

With the working stresses known equation 6 gives M'_s for all types of beams.

For the rectangular beam,

(12) $M'_c = \frac{f_c k}{2}(1-\frac{k}{3})bd^2 = f_s P_c (1-\frac{k}{3})bd^2$

Then since $M = M'_s + M'_c$,

(13) $M = f_s bd^2 \left\{ P_s(1-Q) + P_c(1-\frac{k}{3}) \right\}$ for rectangular beams.

For the t-beam.

(14) $M'_c = f_s P_c bd^2 \left(\frac{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2Pn}}{6-3\Delta} \right)$

Then

(15) $M = f_s bd^2 \left\{ P_s(1-Q) + P_c \frac{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2Pn}}{6-3\Delta} \right\}$ for t-beams.

If the beam is subjected to a smaller or larger stress than M, the safe resisting moment, the fiber stresses are decreased or increased in direct proportion to the moments.

k, z, and J for the Double Reinforced Rectangular Beam.

While there is nothing new in the derivation of these factors for the rectangular beams, this work is inserted to completely cover this flexure theory.

(1) $f_s pbd = \frac{1}{2} f_c kbd + f'_s p'bd$ $(\Sigma H = 0.)$

(2) $f_s p = \frac{1}{2} f_c k + f'_s p'$

(3) $f_s = nf_c(\frac{1-k}{k})$ (Unit deformations vary as their distance from neutral axis)

(4) $f'_s = nf_c(\frac{k-Q}{k})$ (Unit deformations vary as their distance from neutral axis.)

(5) $pnf_c(\frac{1-k}{k}) = \frac{1}{2}f_c k + p'nf_c(\frac{k-Q}{k})$ (Equations 2, 3, and 4.)

(6) $pn - pnk = \frac{1}{2}k^2 + p'nk - p'nQ$

(7) $k^2 + 2kn(p+p') - 2n(p+p'Q) = 0$

(8) $k = -n(p+p') + \sqrt{n^2(p+p')^2 + 2n(p+p'Q)}$

(9) $z(c+c') = \frac{1}{3}kdc + d'c'$ (Moments around top of beam).

(10) $z = \dfrac{\frac{1}{3}kdc + d'c'}{c + c'} = \dfrac{\frac{1}{3}kd + d'\frac{c'}{c}}{1 + \frac{c'}{c}}$

(11) $\dfrac{c'}{c} = \dfrac{f'_s p' bd}{\frac{1}{2}f_c kbd} = \dfrac{2f_s' p'}{f_c k}$

(12) $\dfrac{c'}{c} = \dfrac{2p'}{f_c k} \times nf_c(\dfrac{k-Q}{k}) = \dfrac{2p'n(k-Q)}{k^2}$ (Equation 4 and 11.)

(13) $z = \dfrac{\frac{1}{3}kd + \frac{2p'nd'}{k^2}(k-Q)}{1 + \frac{2p'n}{k^2}(k-Q)} = \dfrac{\frac{1}{3}k^3 d + 2p'nd'(k-Q)}{k^2 + 2p'n(k-Q)}$

(14) $J = 1 - \frac{z}{d} = \dfrac{k^2 + 2p'n(k-Q) - \frac{1}{3}k^3 - 2p'nQ(k-Q)}{k^2 + 2p'n(k-Q)}$

(15) $J = \dfrac{k^2(1-\frac{1}{3}k) + 2p'n(k-Q)(1-Q)}{k^2 + 2p'n(k-Q)}$

<u>k, z, and J for the Double Reinforced T-Beam.</u>

(1) $f_s pbd = f_s' p' bd + f_c \Delta(1 - \frac{\Delta}{2k})bd$

(2) $f_s p = f_s' p' + f_c \Delta(1 - \frac{\Delta}{2k})$

(3) $f_s = nf_c \frac{1-k}{k}$ (Unit deformations vary as their distance from neutral axis.)

(4) $f_s' = nf_c \frac{k-Q}{k}$ (" " " " " distance from neutral axis.)

(5) $pnf_c(\frac{1-k}{k}) = p'nf_c(\frac{k-Q}{k}) + f_c\Delta(1-\frac{\Delta}{2k})$ (Equations 2,3, and 4.)

(6) $pn(1-k) = p'n(k-Q) + f_c\Delta(k-\frac{\Delta}{2})$

(7) $p - pk = p'k - p'Q + \frac{\Delta k}{n} - \frac{\Delta^2}{2n}$

(8) $pk + p'k + \frac{\Delta k}{n} = p + p'Q + \frac{\Delta^2}{2n}$

(9) $k = \frac{p + p'Q + \frac{\Delta^2}{2n}}{p + p' + \frac{\Delta}{n}}$

(10) $z(c+c') = c(\frac{3k-2\Delta}{2k-\Delta})\frac{\Delta d}{3} + c'Qd$ (Momets around top of beam) (See equation 15, page 8.)

(11) $z = \frac{c(\frac{3k-2\Delta}{2k-\Delta})\frac{\Delta d}{3} + c'Qd}{c + c'} = \frac{(\frac{3k-2\Delta}{2k-\Delta})\frac{\Delta}{3} + \frac{c'}{c}Q}{1+\frac{c'}{c}}$

(12) $\frac{c'}{c} = \frac{f_s'p'bd}{f_c\Delta(1-\frac{\Delta}{2k})bd} = \frac{f_s'p'}{f_c\Delta(1-\frac{\Delta}{2k})}$

(13) $\frac{c'}{c} = \frac{p'nf_c(\frac{k-Q}{k})}{\Delta f_c(\frac{2k-\Delta}{2k})} = \frac{2p'n(k-Q)}{\Delta(2k-\Delta)}$

$$(14)\quad z = \frac{\left(\frac{3k-2\Delta}{2k-\Delta}\right)\frac{\Delta}{3} + \frac{2p'nQ(k-Q)}{\Delta(2k-\Delta)}}{1 + \frac{2p'n(k-Q)}{\Delta(2k-\Delta)}}$$

$$(15)\quad z = \frac{\frac{\Delta^2}{3}(3k-2\Delta)+2p'nQ(k-Q)}{\Delta(2k-\Delta)+2p'n(k-Q)}$$

$$(16)\quad J = 1-\frac{z}{d}=\frac{\Delta(2k-\Delta)+2p'n(k-Q)-\frac{\Delta^2}{3}(3k-2\Delta)-2p'nQ(k-Q)}{\Delta(2k-\Delta)+2p'n(k-Q)}$$

$$(17)\quad J = \frac{\Delta(2k-\Delta)-\frac{\Delta^2}{3}(3k-2\Delta)+2p'n(k-Q)(1-Q)}{\Delta(2k-\Delta)+2p'n(k-Q)}$$

CHECKS ON THE K FORMULAS FOR THE DOUBLE REINFORCED BEAMS.

(1) For the rectangular beam, $k=-n(p+p')+\sqrt{n^2(p+p')^2+2n(p+p'Q)}$. This
(2) is the solution of the equation $k^2+2kn(p+p')-2n(p+p'Q)=0$ (Eq. 7 P.

$$(3)\quad p+p'=p_c+p_s+p_s\left(\frac{1-k}{k-Q}\right)=p_c+p_s\left(\frac{k-Q+1-k}{k-Q}\right)=p_c+p_s\left(\frac{1-Q}{k-Q}\right)$$ (Formula 6, Pg.

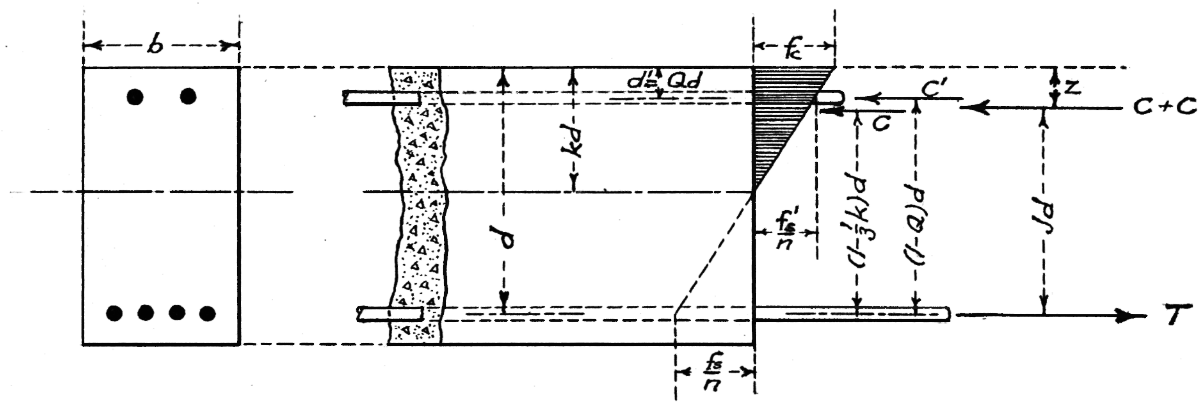
$$(4)\quad p+p'Q=p_c+p_s+p_sQ\frac{1-k}{k-Q}=p_c+p_s\left(\frac{k-Q+Q-kQ}{k-Q}\right) = p_c+kp_s\left(\frac{1-Q}{k-Q}\right)$$

$$(5)\quad k^2+2knp_c+2knp_s\left(\frac{1-Q}{k-Q}\right)-2np_c-2knp_s\left(\frac{1-Q}{k-Q}\right) = 0.$$

$$(6)\quad k = -\frac{2np_c}{2} + \tfrac{1}{2}\sqrt{4n^2p_c^2+8np_c}$$

$$(7)\quad k = -np_c+\sqrt{n^2p_c^2+2np_c}$$

Equation 7 is the k formula for the rectangular beam reinforced for tension only.



b
f_c
$d' = Qd$
kd
d
$\frac{f_s'}{n}$
$\frac{f_s}{n}$
C'
C
z
C+C'
$(1-\frac{1}{3}k)d$
$(1-Q)d$
jd
T

The k formula for the double reinforced t-beam may be transformed in the same way.

(1) $k = \dfrac{p + p'Q + \frac{\Delta^2}{2n}}{p + p' + \frac{\Delta}{n}} = \dfrac{p_c + p_s k(\frac{1-Q}{k-Q}) + \frac{\Delta^2}{2n}}{p_c + p_s(\frac{1-Q}{k-Q}) + \frac{\Delta}{n}}$ (See equations 3 and 4 above.

(2) $kp_c + kp_s\frac{1-Q}{k-Q} + \frac{k\Delta}{n} = p_c + p_s k\frac{1-Q}{k-Q} + \frac{\Delta^2}{2n}$

(3) $k = \dfrac{p_c + \frac{\Delta^2}{2n}}{p_c + \frac{\Delta}{n}}$

(4) $k = \dfrac{p_c n + \frac{\Delta^2}{2}}{p_c n + \Delta}$ This equation 4 is the k formula for the t-beam reinforced for tension only.

Both of these k formulas reduce directly to the k formulas for the single reinforced beams when p' = 0.

FORMULAS FOR DOUBLE REINFORCED RECTANGULAR BEAMS.

Formulas 1 to 7 are used to determine the steel ratios, p and p', which a beam must have to resist a given moment, and formulas 8 to 12 are used to determine the safe resisting moment that a given beam is able to exert.

(1) $k = \dfrac{1}{1 + \frac{f_s}{nf_c}}$

(2) $p_c = \dfrac{f_c k}{2f_s}$

(3) $M_c' = f_s p_c(1-\frac{k}{3})bd^2$

(4) $M_s' = M - M_c'$

(5) $p_s = \dfrac{M_s'}{f_s(1-Q)bd^2}$

(6) $p' = p_s \dfrac{1-k}{k-Q}$

(7) $p = p_c + p_s$

(8) $k = -n(p+p') + \sqrt{n^2(p+p')^2 + 2n(p+p'Q)}$

(9) $p_s = p' \dfrac{k-Q}{1-k}$

(10) $p_c = p - p_s$

(11) $\dfrac{f_c}{f_s} = \dfrac{2p_c}{k}$

(12) $M = f_s bd^2 \left\{ p_s(1-Q) + p_c\left(1-\dfrac{k}{3}\right) \right\}$

(13) $z = \dfrac{\frac{1}{3}k^3 d + 2p'nd'(k-Q)}{k^2 + 2p'n(k-Q)}$

(14) $J = \dfrac{k^2(1-\frac{1}{3}k) + 2p'n(k-Q)(1-Q)}{k^2 + 2p'n(k-Q)}$

(15) $f_s' = f_s \dfrac{k-Q}{1-k}$

FORMULAS FOR DOUBLE REINFORCED T-BEAMS.

(1) $k = \dfrac{1}{1 + {}^{f_s}\!/\!_{nf_c}}$

(2) $p_c = \Delta \dfrac{f_c}{f_s}\left(1-\dfrac{\Delta}{2}\right) - \dfrac{\Delta^2}{2n}$

(3) $M_c' = \left\{ \dfrac{f_c}{\Delta}(3-3\Delta+\Delta^2) - \dfrac{f_s}{n}\left(\dfrac{3}{2} - \Delta\right) \right\} \dfrac{bt^2}{3}$

(4) $M_s' = M - M_c'$

(5) $p_s = \dfrac{M_s'}{f_s(1-Q)bd^2}$

(6) $p' = p_s \dfrac{1-k}{k-Q}$

(7) $p = p_c + p_s$

(8) $k = \dfrac{p+p'Q+\dfrac{\Delta^2}{2n}}{p+p'+\dfrac{\Delta}{n}}$

(9) $p_s = p' \dfrac{k-Q}{1-k}$

(10) $p_c = p - p_s$

(11) $\dfrac{f_c}{f_s} = \dfrac{\dfrac{2p_c}{\Delta} + \dfrac{\Delta}{n}}{2 - \Delta} = \dfrac{k}{n(1-k)}$

(12) $M = f_s bd^2 \left\{ p_s(1-Q) + p_c \dfrac{6-6\Delta+2\Delta^2+\dfrac{\Delta^3}{2pn}}{6 - 3\Delta} \right\}$

(13) $z = \dfrac{\frac{1}{3}\Delta^2(3k-2\Delta)+2p'nQ(k-Q)}{\Delta(2k-\Delta)+2p'n(k-Q)} . d$

(14) $J = \dfrac{\Delta(2k-\Delta)-\dfrac{\Delta^2}{3}(3k-2\Delta)+2p'n(k-Q)(1-Q)}{\Delta(2k-\Delta)+2p'n(k-Q)}$

(15) $f_s' = f_s \dfrac{k-Q}{1-k}$

Formulas 1 to 7, and 8 to 12 cover the two typical cases described under the rectangular beam formulas. Seven of these formulas are the same as those for rectangular beams.

USE OF THE DOUBLE REINFORCED BEAM FORMULAS

Find the safe resisting moment of a rectangular double-reinforced concrete beam constructed to meet the following conditions:

$b = 12$ in. $d = 18$ in. $Q = \frac{1}{10}$ $p = 0.025$

$p' = 0.010$ $f_c = 600$ lbs. $f_s = 15000$ lbs.

This is the case covered by formulas 8 to 12, page 19.

(8) $k = -15(0.025+0.010+\sqrt{15^2(0.025+0.010)^2+2\times15(0.025+0.01\times0.1)}$

$k = 0.502$

(9) $p_s = 0.01\dfrac{0.502 - 0.100}{1 - 0.502} = 0.00807$

(10) $p_c = 0.025 - 0.00807 = 0.01693$

If $f_c = 600$ lbs.

(11) $f_s = \dfrac{600\times0.502}{2\times0.01693} = 8900$ lbs.

(12) $M = 8900\times12\times18^2\left\{0.00807(1-0.1)+0.01693(1-\dfrac{0.502}{3})\right\}$

$M = 739{,}000$ in. lbs.

If this beam were subjected to a bending moment of 1,000,000 in. lbs., the new f_c and f_s can be determined by direct proportion from the above fiber stresses.

$$f_c = 600\times\frac{1{,}000{,}000}{739{,}000} = 812 \text{ lbs.}$$

$$f_s = 8900\times\frac{1{,}000{,}000}{739{,}000} = 12040 \text{ lbs.}$$

A beam having the same b, d, Q, and limiting fiber stresses, f_c **and** f_s as in the first problem will now be designed by the first method outlined on page 18 to carry the same bending moment of 739,000 in. lbs.

(1) $k = \dfrac{1}{1 + \dfrac{15000}{15\times600}} = 0.375$

(2) $p_c = \dfrac{600\times0.375}{2\times15000} = 0.0075$

(3) $M_c' = 15000\times0.0075(1-\dfrac{0.375}{3})\times12\times18^2 = 383000$ in. lbs.

(4) $M_s' = 739000 - 383000 = 356000$ in. lbs.

(5) $p_s = \dfrac{356000}{15000(1-0.1)\times12\times18^2} = 0.00678$

(6) $p' = 0.00678\,\dfrac{1 - 0.375}{0.375-0.1} = 0.0154$

Now compare these two sections both of which have the same concrete area.

1st section	2nd section
$f_c=600$ $f_s=8900$	$f_c=600$ $f_s=15000$
$p+p'=0.025+0.01$	$p_c+p_s+p'=0.0075$
$=0.0350$	0.00678
	0.0154
	0.0297

Since the second section uses 0.53 °/₀ less steel, it would appear from these examples that it always pays to stress both sides of the beam up to the working limits.

In the third problem under single reinforced T-beams on page 6, when the section in problem one was stressed to 12,500,000 in. lbs. the extreme fiber stresses were found to be $f_c=781$ lbs. and $f_s=19520$ lbs. We will now double reinforce this section to carry this moment with stresses of $f_c=600$ and $f_s=15000$. From problem one on pages 3 and 4, t=8 in., b=64 in., d=44.6 in., $p_c=0.00545$ and $M_c'=9,600,000$ in. lbs. Make Q=0.0673(3"imbedment)

From formula 4 on page 19

(4) $M_s' = 12,500,000 - 9,600,000 = 2,900,000$ in. lbs.

(4½) $$k = \frac{1}{1 + \frac{15000}{15 \times 600}} = 0.375$$

(5) $$p_s = \frac{2,900,000}{15000(1-0.0673) \times 64 \times 44.6^2} = 0.001628$$

(6) $$p' = 0.001628 \times \frac{1-0.375}{0.375-0.0673} = 0.00331$$

(7) $p = 0.00545 + 0.001628 = 0.00708$

k can also be determined by the use of formula 8.

(8) $$k = \frac{0.00708 + 0.00331 \times 0.0673 + \frac{0.1791^2}{2 \times 15}}{0.00708 + 0.00331 + \frac{0.1791}{15}} = 0.375$$

Now design the beam with the same concrete dimensions to carry this moment of 12,500,000 in. lbs. with limiting working stresses of f_c=600 and f_s=10000

(1) $$k = \frac{1}{1 + \frac{10,000}{15 \times 600}} = 0.474$$

(2) $$p_c = 0.1791 \times \frac{600}{10000}\left(1 - \frac{0.1791}{2}\right) - \frac{0.1791^2}{2 \times 15} = 0.00871$$

(3) $$M_c' = \left\{ \frac{600}{0.1791}(3 - 3 \times 0.1791 + 0.1791^2) - \frac{10000}{15}\left(\tfrac{3}{2} - 0.1791\right) \right\} \frac{64 \times 8^2}{3}$$

M_c' = 10,210,000 in. lbs.

(4) $M_s' = 12,500,000 - 10,210,000 = 2,290,000$

(5) $p_s = \dfrac{2,290,000}{10000(1-0.0673)\times 64\times 44.6^2} = 0.001929$

(6) $p' = 0.001929\,\dfrac{1-0.474}{0.474-0.0673} = 0.00249$

(7) $p = 0.001929 + 0.00871 = 0.01064$

The use of the higher steel stress in the first design saves 0.27 percent of steel over this second design.

CONSTRUCTION OF THE REINFORCED CONCRETE SLAB FLEXURE DIAGRAM.

The moment caused by a uniformly distributed load at any point on any beam which is fixed, partially restrained, or simply supported at the ends may be expressed in in. lbs. by the formula, $M = 12\,\frac{w}{\phi}\,l^2$, in which w is the uniform load in lbs. per ft., l is the span in feet, and ϕ is the moment denominator. ϕ is 8 for the moment at the center of a simply supported beam.

The formula for the resisting moment of a simple rectangular reinforced concrete beam is $M = Rbd^2$ in which $R = \frac{1}{2}f_c kJ$ for the compression couple and $f_s pJ$ for the tension couple. When the beam is supporting a uniformly distributed load the general formula may be expanded into the two forms,

$$12\,\frac{w}{\phi}\,l^2 = f_s pJbd^2$$

and $$12\,\frac{w}{\phi}\,l^2 = \tfrac{1}{2}f_c kJbd^2$$

In the upper left hand quadrant of the diagram, which is called the moment chart, the logarithmic abscissas and ordinates represent the spans in feet and moments in inch pounds respectively. Each sloping line on the diagram represents a particular value of $\frac{w}{\phi}$. With $\frac{w}{\phi}$ a constant the formula takes the form $M=Kl^2$ and when expressed in logarithms, the form $\log M = \log K + 2 \log l$. The moment chart is a graphical representation of this family of curves, which are parallel straight lines.

The upper right hand quadrant or the depth chart is the logarithmic plat of the equation $M=Rbd^2$. In this case also the logarithmic ordinates represent the moments, but the logarithmic abscissas represent values of R. Each sloping line represents a particular value of bd^2. b is taken as 12 inches in all cases and the line is designated by the corresponding value of d. This plat then represents the general equation $\log M = \log K' + \log R$. It is a series of parallel 45° lines.

The lower right hand quadrant or the stress chart is a logarithmic plat of the two families of curves

$$R = f_s pJ = f_s p \left\{1 - \tfrac{1}{3}\left(\sqrt{2pn+pn^2} - pn\right)\right\}$$

$$\text{and } R = \tfrac{1}{2} f_c kJ = \tfrac{1}{2} f_c \left(\sqrt{2pn+pn^2} - pn\right)\left\{1 - \tfrac{1}{3}\left(\sqrt{2pn+pn^2} - pn\right)\right\}$$

n = 15 in this chart, then for particular values of f_s and f_c these two equations take the form $R = K'' \Psi(p)$ or in logarithmic terms, the form, $\log R = \log K'' + \log \Psi_{(p)}$.

In the stress chart the logarithmic abscissas and ordinates represent values of R and p respectively, and the sloping lines represent particular values of f_c and f_s.

As, the total steel area, = pbd. In the lower left hand quadrant or the steel chart, the lines sloping upward to the right represent constant values of d and the abscissas and the ordinates, as numbered at the right hand side of the diagram, represent values of A_s and P respectively.

When the cross-sectional area of a round rod is α and the rod spacing is σ, $A_s = \frac{12\alpha}{\sigma}$. Each line sloping upward to the left in the steel chart is marked with the diameter of the rod whose area it represents. The abscissas represent the steel area and the ordinates, as marked at the left side of the diagram, represent the round rod spacing. The spacing for square rods is $\frac{4}{\pi}$ times the spacing of round rods of the same thickness. The k and J curves are also plotted in this steel chart.

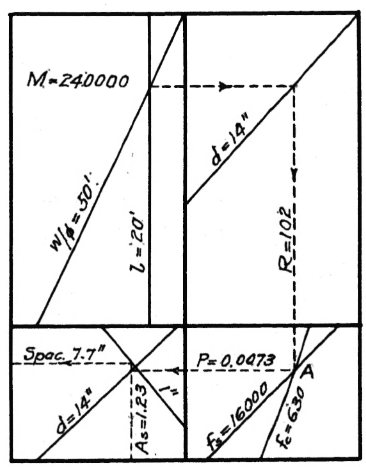
THE USE OF THE BEAM AND FLOOR SLAB CHART.

Suppose that it is required to design a slab to carry a total live and dead load of 400 lbs. per sq. ft. over a simple span of 20 ft. with limiting stresses of f_c=650 and f_s=16000 lbs.

Before entering the diagram the load per square foot, 400 lbs., must be divided by the moment denominator which is 8 in this case. $\frac{w}{\phi} = \frac{400}{8} = 50.$

In the stress-chart, at the intersection of the values f_c=650 and f_s=16000, as indicated at A in the first guide diagram, the value R = 107 satisfies both stress conditions. Any designer who uses a particular set of stresses constantly remembers the corresponding value of R, and omits this operation.

Now in the moment chart find the intersection of the sloping line $\frac{w}{\phi}$ = 50 with the line representing a span of 20 feet. The ordinate of this intersection corresponds to a moment of 240,000 inch pounds. Follow this ordinate into the depth chart to its intersection with R=107. At this intersection



GUIDE DIAGRAM NO.1.

d = 13.6 inches. It is decided to use a depth of 14 inches which corresponds to $R = 102$ for this moment. Follow the line $R=102$ into the stress chart. At the point where $f_c=650$, $f_s=17600$, at the point where $f_s=16000$, $f_c=630$. This second set are therefore the limiting working stresses. At this point $p = 0.0073$. Follow this abscissa into the steel chart to d=14". At this intersection $A_s=1.23$ sq. ins. Follow this A_s line to the 1" line. The required spacing for 1" round rods is 7.7 inches. Use a spacing of $7\frac{1}{2}$ inches. If it is required to use 1 inch square rods, the necessary spacing is, $\frac{4}{\pi} \times 7.7 = 9.8$ inches. Use $9\frac{3}{4}$ inch spacing.

If it is desired to find the resisting moment when every other rod is turned up, this process is reversed.

In the steel chart follow the rod spacing of 15 inches to the 1 inch line. $A_s=0.626$. Follow this A_s line to d=14"; $p=0.00372$. Follow this p line to $f_s=16000$, $f_c=439$, $R=54$. Follow the $R=54$ line to d=14 in the depth chart, $M=128000$ in. lbs. The location of the point on the beam where $M=128000$ in. lbs. can be found from the equation or form of the moment curve as on page 6..

If the moment curve is a continuous parabola a simple method for finding this point is derived as follows:. Call the moment at the center $M'=R'bd^2$ and the moment at the point distant X from the center, $M_x=R_xbd^2$. Find the value of R' and R_x from the stress chart.

From the law of the parabola (See figure on page 6.)

$$\frac{x^2}{\left(\frac{l}{2}\right)^2} = \frac{M'-M_x}{M'} = \frac{R'bd^2-R_xbd^2}{R'bd^2} = 1-\frac{R_x}{R'}$$

$$\text{Then } x = \pm \frac{l}{2}\sqrt{1-\frac{R_x}{R'}}$$

THE CONSTRUCTION OF THE REINFORCED CONCRETE T-BEAM CHART

The moment caused by a uniformly distributed load at any point on any beam may be expressed in inch pounds by the formula, $M=12\frac{w}{\phi}l^2$ as explained on page 24.

The resisting moment of the steel reinforcement in a t-beam is, from equation 27, page 9 ,

$$M_s=\frac{f_s pJ}{\Delta^2}bt^2=Rbt^2 \text{ when } R=\frac{f_s pJ}{\Delta^2}$$

Then when the t-beam is supporting a uniformly distributed load

$$M=12\frac{w}{\phi}l^2=Rbt^2$$

In order to make the diagram of more general application the moment is divided by b, the breadth of the beam in inches. The moment formula then takes the form

$$\frac{M}{b}=\frac{12\ w}{\phi b}l^2=\frac{w}{\phi B}l^2=Rt^2$$

B is used to express the width of the beam in feet, $\frac{w}{B}$ then expresses the uniformly distributed load in terms of live load per square foot of flange.

In the upper left-hand quadrant of the diagram the logarithmic abscissas and ordinates represent the spans in feet and moments in in. lbs. divided by the breadth of beam in inches respectively. Each sloping line represents a particular value of $\frac{w}{\phi B}$. With $\frac{w}{\phi B}$ a constant, the formula takes the form $\frac{M}{b}=Kl^2$. The moment chart is a logarithmic graphical representation of this family of curves.

The upper right hand quadrant or the slab chart is the logarithmic plat of the equation $\frac{M}{b} = Rt^2$. The abscissas represent values of R, and the ordinates, value of $\frac{M}{b}$. The sloping lines correspond to particular values of t.

The lower right hand quadrant or steel chart is a logarithmic plat of the equation $R=f_s\frac{pJ}{\Delta^2}$. In this steel chart the abscissas and ordinates represent values of R and $\frac{pJ}{\Delta^2}$ or Y respectively. The sloping lines represent particular values of f_s.

In the lower left hand quadrant or proportional chart, the lines sloping upward to the right are the logarithmic plat of the family of curves $Y=\frac{PJ}{\Delta^2}$. When J is expressed in terms of Δ, p, and n, this equation takes the form

$$Y = P\left\{\frac{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2pn}}{\Delta^2(6-3\Delta)}\right\}$$

For particular values of Δ, this equation takes the form $Y=K'''p+K^{IV}$. The abscissas and ordinates of the proportional chart represent values of P and Y respectively, and the sloping straight lines represent particular values of Δ. In this chart θ is used to represent the ratio $\frac{f_c}{f_s}$. From equation 24 on page 9 ,

$$P = \Delta\theta(1-\frac{\Delta}{2})-\frac{\Delta^2}{2n}$$

The curved lines on the proportional chart have been plotted from this formula by solving for the values of P corresponding to particular values of θ and Δ and then drawing the θ curves through the intersections of these values of P with the corresponding Δ lines in the proportional chart. The curve $\Delta=k$ is the line of division between the t-beam and the simple beam. From equation 22, page 9, its formula is $p=\frac{\Delta^2}{2n(1-\Delta)}$.

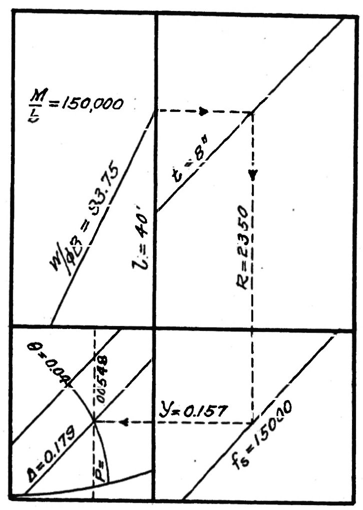
THE USE OF THE T-BEAM CHART:

The t-beam which was designed analytically on pages 3 & 4 will be checked by means of this chart to illustrate its use.

Before entering the diagram the total live and dead load, 4000 lbs. per lineal foot must be divided by both the moment denominator, 8, and the breadth of the beam in feet, $5\frac{1}{3}$.

$$\frac{w}{\phi B} = \frac{4000}{8\times5\frac{1}{3}} = 93.75. \quad \theta = \frac{600}{15000} = 0.04.$$

Now perform the operations on the t-beam chart indicated in guide diagram, No.2. At the intersection of $\frac{w}{\phi B}=93.75$ with the line $l=40$ ft., the ordinate is $\frac{M}{b}=150000$, At the intersection of this ordinate with t=8 ", R=2350. At the intersection of the abscissa, R=2350 with with $f_s=15000$, Y=0.157. Now follow the ordinate Y=0.157 to its intersection withe the line $\theta=0.04$. At this point $\Delta=0.179$ and p=0.00548.



GUIDE DIAGRAM No. 2.

$$d = \frac{t}{\Delta} = \frac{8}{0.179} = 44.7"$$

$A_s=pbd=0.00548\times64\times44.7=15.67$ sq. ins.

The t-beam chart is worked in the reverse direction when it is desired to know what resisting moment a given section can exert. The problem on page 5 will be used to illustrate this operation. The same section is used as before but the bending up of 4 rods has reduced the steel ratio to 0.00358.

In the proportional chart follow the abscissa, p=0.00358 to its intersection with the sloping line Δ=0.1791. At this point θ=0.00285 and Y=0.103.

$$\text{If } f_c = 600, \quad \theta = \frac{600}{f_c} = 0.00285, \quad f_s = 21100 \text{ lbs.}$$

$$\text{If } f_s = 15000, \quad \theta = \frac{f_c}{15000} = 0.00285, \quad f_c = 428 \text{ lbs.}$$

This second group are the working stresses. Now follow the ordinate Y=0.103 to its intersection with the value f_c=15000. At this point R=1540. Trace this abscissa, R=1540, to its intersection with the line t=8". At this point $\frac{M}{b}$ = 98000. Then M=98000×64=6,270,000 inch pounds. The point on the beam at which this moment occurs may be found as on page 6 or by the formula on page 27.

The handling of problems on both of these diagrams is facilitated by the use of two pointers. The last value found is held by one pointer while the other is used to pick out the value determined by the next step in the problem. Any diagram can also be much more readily handled when it is mounted on an extra heavy paste-board mat with rounded corners which has been backed with cloth or passe-partout.

THE K AND J DIAGRAM FOR T-BEAMS.

The k and J diagram for t-beams represents graphically the two families of curves:

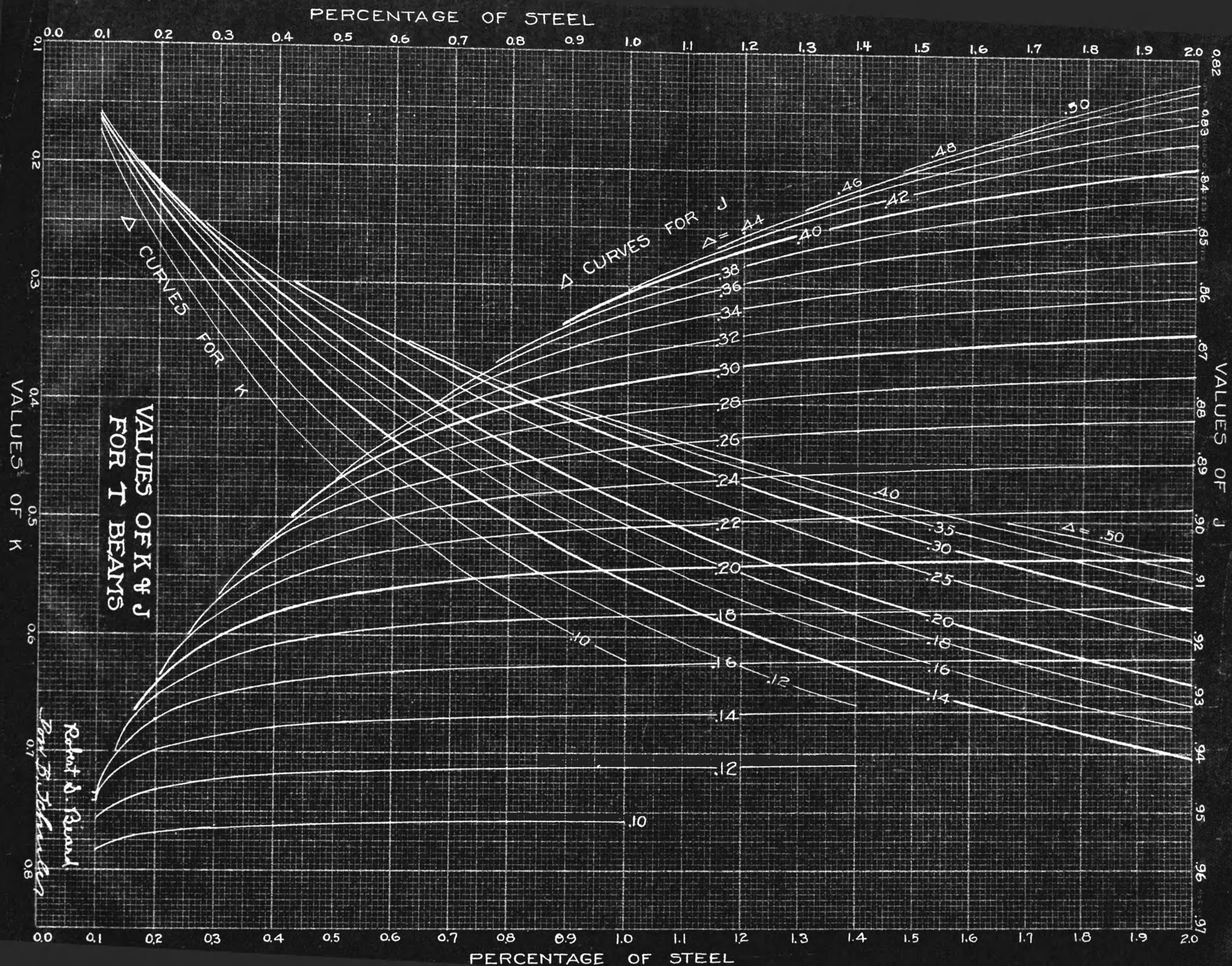
$$k = \frac{pn+\frac{1}{2}\Delta^2}{pn+\Delta}$$

$$\text{and } J = \frac{6-6\Delta+2\Delta^2+\frac{\Delta^3}{2pn}}{6-3\Delta}$$

The ordinates represent percentages of steel for both sets of curves and the abscissas as marked at the bottom of the diagram represent values of k, and, as marked at the top of the diagram values of J. The curves sloping upward to the right represent particular values of Δ in the k formula, and the curves sloping upward to the left represent particular values of Δ in the J formula.

In the first problem worked omt on the t-beam diagram $p=0.00548$ and $\Delta=0.179$. On the k and J diagram these values correspond to $k=0.376$ and $J=0.920$. This value of k is checked in two ways on page 23 by equations $4\frac{1}{2}$ and 8.

When $p=0.00871$ and $\Delta=0.1791$, $k=0.475$ and $J=0.917$. This value of k is checked by equation 1 on page 23.


VALUES OF K & J FOR T BEAMS
PERCENTAGE OF STEEL
VALUES OF K
VALUES OF J
Δ CURVES FOR K
Δ CURVES FOR J
Δ = .44
Δ = .50
.10
.12
.14
.16
.18
.20
.22
.24
.26
.28
.30
.32
.34
.36
.38
.40
.42
.46
.48
.50
.25
.35
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
0.82
.83
.84
.85
.86
.87
.88
.89
.90
.91
.92
.93
.94
.95
.96
.97
Robert S. Beard
Ray B. Fehrenbacher