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INVITED REVIEW

Decision making under ambiguity: a belief-function perspective

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In this article, we discuss problems with probability theory in representing uncertainties encountered in the "real world" and show how belief functions can overcome these difficulties. Also, we discuss an expected utility approach of decision making under ambiguity using the belief function framework. In particular, we develop a proposition for decision making under ambiguity using the expected utility theory. This proposition is based on Strat's approach of resolving ambiguity in the problem using belief functions. We use the proposition to explain the Ellsberg paradox and model the decision making behavior under ambiguity. We use the empirical data of Einhorn and Hogarth to validate the proposition. Also, we use the proposition to predict several decision making behaviors under ambiguity for special conditions. Furthermore, we discuss the general condition under which the "switching" behavior, as observed by Einhorn and Hogarth, will occur using the concept of "precision measure" in the expected utility theory.

1. Introduction

This article has four objectives. As the first objective, we discuss problems with probabilities in representing uncertainties encountered on real decisions and then show how belief functions (Shafer, [13]) provide a better framework than probabilities to represent such uncertainties. As the second objective, we develop a proposition for decision making under ambiguity, represented in terms of belief functions, using Strat's [20, 21] approach. As the third objective, we explain Ellsberg's paradox [6] using the proposition and validate it using the empirical data of Einhorn and Hogarth ([5], hereafter called EH).

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It is a well known fact that probability theory is unable to distinguish between a situation of complete ignorance and a situation where we have complete knowledge. In fact, as discussed in Section 2.2, Ellsberg's paradox stems from this inability of probability theory. There are many real world situations such as auditing or medical diagnosis where the decision maker is not able to easily represent uncertainties using probabilities. We show in this article how belief functions provide a better framework to represent such uncertainties than probabilities.

In general, the expected utility approach under the belief-function framework gives an interval for the expected value instead of a single estimate. In fact, this has been the major hurdle of using the traditional decision theory in the belief-function framework. Recently, several approaches to decision making under the belief-function framework have been proposed (see, i.e., Jaffray, [7, 8]); Nguyen and Walker, [11]; Smets, [15, 16]; Strat, [20, 21]; and Yager, [24] for details). We use[1] Strat's approach [20, 21] to develop a proposition for decision making under ambiguity. In essence, this proposition suggests that the decision maker (hereafter called DM) will take a conservative approach and make the decision based on the most unfavorable resolution of ambiguity. Under the above scheme, if there is no clear choice, then he or she will make the decision based on the most favorable resolution of ambiguity. This rule is equivalent to "minimax" rule in a two-person zero-sum game. It is interesting to note that the proposition developed in the article not only explains Ellsberg's paradox but also models correctly all the behaviors observed by EH.

The remaining article is divided into six sections. Section 2 discusses the issues related to Ellsberg's paradox, and shows the problems with probability theory in modeling uncertainties encountered in the real world. Section 3 presents an introduction to belief functions. Section 4 presents the expected utility approach to decision making under probabilities, and belief functions. A proposition for decision making under ambiguity is developed in Section 5. Also, the "switching" behavior, as observed by EH, is discussed in Section 5. Section 6 validates the proposition using the EH empirical results. Finally, Section 7 presents summary and conclusions of the study.

2. Ellsberg paradox and problems with probability

2.1. Ellsberg paradox

Einhorn and Hogarth [5] conducted an experiment where the subjects were asked to imagine two urns. Urn 1 contained 100 balls with unknown proportions of black and red. Urn 2 contained 50 red balls and 50 black balls. The subjects were asked to play a gamble and win \$100 if they picked a ball of specified color (red or black) from one of the urns. If a wrong ball was picked then the payoff was \$0. Most of the subjects preferred to pick from Urn 2 (non-ambiguous). If one tries to explain the above experimental result using

[1] Strat's approach is the only approach that explains Ellsberg's paradox as shown in this article.

probability theory he or she runs into a paradox. Such a paradox is known as the Ellsberg paradox [6]. Here is an argument provided by EH [5, pp. S227-228] using probability theory to illustrate the paradox.

In case of Urn 1, we have a completely unknown situation, i.e., we are completely ignorant about how many red or black balls there are. In such an ignorant situation, the probability theory tells us to assign equal probability to all the possible mutually exclusive and exhaustive states. Under such an assumption, we find that the probability of picking a red ball from Urn 1 is $P(R_1) = 0.5$ and so is the probability of picking a black ball, i.e., $P(B_1) = 0.5$. In case of Urn 2, we know that it contains 50 red and 50 black balls which implies that the probabilities of picking a red or a black ball, respectively, are: $P(R_2) = 0.5$ and $P(B_2) = 0.5$. As EH discuss, the preference of Urn 2 over Urn 1 for picking a ball of either color would mean that

$$P(R_2) > P(R_1) = 0.5, \quad P(B_2) > P(B_1) = 0.5,$$

or

$$P(R_2) = 0.5 > P(R_1), P(B_2) = 0.5 > P(B_1).$$

The first condition implies that the probabilities for Urn 2 add to more than one and the second condition implies that the probabilities for Urn 1 add to less than one. This is the paradox. EH define these conditions, respectively, as "superadditivity" and "subadditivity." They further comment that (Einhorn and Hogarth [5, p. S228]):

> ... either urn 2 has complementary probabilities that sum to more than one, or urn 1 has complementary probabilities that sum to less than one. As we will show, the nonadditivity of complementary probabilities is central to judgments under ambiguity.

However, we show that no such super- or sub-additivity is needed to explain the decision maker's behavior if we use a belief functions treatment of ambiguity. The paradox stems from the difficulties we face in representing ignorance using probabilities. Under probability theory, the above two situations are treated in exactly the same manner. However, decision makers clearly perceive the situations to be different. We show in this article that really there is no paradox if we model the uncertainties properly, i.e., using the belief-function framework.

2.2. Problems with probability

We will not present a philosophical discussion about the concept of probability and objective versus subjective probabilities. This has been done several times and it has been an ongoing process (see, e.g., Schoemaker, [12]; and Shafer, [13]). Instead, we give some real world situations where probability concepts are not adequate to model the feelings of the DMs about uncertainties.

Let us first consider the studies in behavioral decision theory. There are several studies that have demonstrated the problems of representing human judgment about uncertainties using probabilities (see, e.g., Einhorn and Hogarth, [4, 5]; Kahneman, Slovic, and Tversky, [9]; Kahneman and Tversky, [10]; and Tversky and Kahneman, [22]). Just to reemphasize the point, we reconsider the famous Ellsberg paradox presented earlier. It is clear that we are not able to distinguish between a state of complete ignorance (Urn 1 with unknown proportions of red and black balls) and a state of complete knowledge (Urn 2 with 50 red balls and 50 black balls) when we use probability theory to model uncertainties involved in the two cases.

Let us consider another situation, an example in auditing. Auditors have to deal with uncertain items of evidence on every engagement. But the uncertainties associated with such items of evidence and the auditor's feeling or judgment about these uncertainties cannot be modeled easily using the concepts of probability. For example, auditors use analytical procedures such as ratios, projections, time series analysis, and linear regression to compare the numbers thus computed with the recorded account balances in the company's books. If the computed figure is reasonably close to the recorded account balance, then the auditor considers the recorded balance to be fairly stated. But he or she cannot put a high level of assurance that the balance is fairly stated just on the basis of projections and ratio analyses, because there could be several other reasons that the balance is close to the calculated figure when in reality it contains errors.

Suppose the auditor has a positive feeling about the recorded balance based on the analytical procedures, since the projection is close to the recorded balance. Suppose also that the economic environment has been almost the same as in the previous periods. That is, there is no reason to expect that the recorded balance should be any different from the projected one. The auditor wants to assign a positive, but low, level of support from this item of evidence that the account is fairly stated. Let us say that the auditor assigns a low level of support, say 0.3 on a scale of 0-1, to the state that the account balance is fairly stated based on this evidence alone. The question is, what is this number 0.3 and what happens to the remaining 0.7? If we interpret these numbers as probabilities, then we at once conclude that the auditor is saying he or she has 0.7 degree of confidence that the account is not fairly stated. But this is not what the auditor has in mind. Shafer and Srivastava [14], Srivastava [17], and Srivastava and Shafer [19] have argued that belief-function theory provides a better framework for audit decisions.

Moreover, there are several studies in AI (Artificial Intelligence) that focus on modeling human decision making and have found probabilities to be inadequate in modeling uncertainties encountered on real decisions. Davis, Buchanan, and Shortliffe [1] developed MYCIN to diagnose the common cold. They were unable to use probability framework to model uncertainties encountered in making this real decision. They used instead a certainty factor with ad hoc rules to combine them in their system but acknowledged the need for a normative theory to deal with their situation. They also acknowledged that the Dempster-Shafer theory of belief functions seems to have some promise in managing the kinds of uncertainties they were facing.

There are several frameworks that have been proposed to model uncertainties of the kind discussed above: Zadeh's fuzzy set theory [25], Dubois and Prade's possibility and necessity theory [2, 3], and Dempster-Shafer theory of belief functions (Shafer, [13]). We focus on the Dempster-Shafer theory of belief functions in the current article.

3. Belief functions and ambiguity

The basic concepts of belief functions[2] have appeared in several places. Of course, A Mathematical Theory of Evidence by Shafer [13] provides the most comprehensive coverage on the subject. The current form of the belief-function formalism, known as Dempster-Shafer theory of belief functions, is the works of Dempster in the 1960's and of Shafer in the 1970's. In this article, we will give only the basics of belief functions. Since the present paper does not deal with the combination of evidence, we will not give Dempster's rule of combination. Interested readers should see Shafer [13] for details.

The basic difference between probability theory and the belief-function formalism is in the assignment of uncertainties to a set of mutually exclusive and exhaustive states or assertions under consideration (we will call this set a frame and represent it by the symbol Θ). In probability theory, we assign uncertainty to each individual element of the frame and call it the probability of occurrence of the element. The sum of all these probabilities equals one.

Let us consider an auditing example. The accounts receivable balance is not materially misstated (ar) and it is materially misstated $(\sim ar)$ are the two assertions representing a mutually exclusive and exhaustive set. Here the frame consists of the two elements[3]: $\Theta = \{ar, \sim ar\}$. In probability theory, we will assign probability to each element of the frame, i.e., $P(ar) \geqslant 0$, and $P(\sim ar) \geqslant 0$. Also, we know that $P(ar) + P(\sim ar) = 1$. In the belief-function framework, uncertainty is not only assigned to the single elements of the frame but also to all other proper subsets of the frame and to the entire frame. We call these uncertainties m-values or the basic probability assignment function.

3.1. m-values (The Basic Probability Assignment Function)

Similar to probabilities, all these m-values add to one. For the example considered above[4], we will have $m(ar) \geqslant 0$, $m(\sim ar) \geqslant 0$, $m(\{ar, \sim ar\}) \geqslant 0$, and $m(ar) + m(\sim ar) + m(\{ar, \sim ar\}) = 1$. Let us assume that the auditor has performed analytical procedures, as discussed in the introduction, relevant to the accounts receivable balance and finds no

[2]A major portion of this section has been taken from Srivastava [17].

[3]In the case of n elements in the frame, we will have $P(a_i) \geqslant 0$, and $\sum_{i=1}^{n} P(a_i) = 1$, where *ai* represents the ith element of the frame.

[4]For a frame of n elements, we will have, in general, m-values for each individual elements, each set of two elements, each set of three elements, and so on, to the m-value for the entire frame. All such m-values add to one, i.e.,$\sum_{A \subseteq \Theta} m(A) = 1$, where A represents all the proper subsets of the frame Θ. The m-value for the empty set is zero.

significant difference between the recorded value and the predicted value. Based on this finding, he or she feels that the recorded value appears reasonable and is not materially misstated. However, he or she does not want to put too much weight on this evidence. He/she feels he/she can assign a small level of assurance, say 0.3 on a scale of 0-1, that the account is not materially misstated. We can express this feeling in terms of m-values as: $m(ar) = 0.3$, $m(\sim ar) = 0$, and $m(\{ar, \sim ar\}) = 0.7$. The belief function interpretation of these m-values is that the auditor has 0.3 level of support to ar, no support to $\sim ar$, and 0.7 level of support remains uncommitted which represents ignorance.

However, if we had to express the above feelings in terms of probabilities, then we get into problems, because we will assign $P(ar) = 0.3$ and $P(\sim ar) = 0.7$ which implies that there is a 70 percent chance that the account is materially misstated, but we know that this is not what the auditor is trying to say. The auditor has no reason to believe that the account is materially misstated. Thus, we can use m-values to express the basic judgment about the level of support or assurance the auditor obtains from an item of evidence for an assertion. An example of a negative item of evidence which will have a direct support for $\sim ar$ would be the following set of inherent factors: (1) in the prior years the account has had major problems, and (2) there are economic reasons for management to misstate the account. In such a case we can express the auditor's feelings as $m(ar) = 0$, $m(\sim ar) = 0.2$, and $m(\{ar, \sim ar\}) = 0.8$, assuming that the auditor feels a low, say 0.2, level of support for $\sim ar$.

The auditor can express a mixed-type of evidence in terms of m-values without any problems. For example, consider that the auditor has accumulated several environmental factors, such as: management's attitude, integrity, and style; and economic conditions under which the business is operating. Some of these factors may be in support of and some against the assertion that the accounts receivable balance is not materially misstated. He/she assesses that there is a moderate, say 0.4, level of support in favor of the assertion and a low level of support, say 0.1, for its negation, and feels that he/she cannot assign the remaining 0.5 level of support to any particular state. We can express this feeling as: $m(ar) = 0.4$, $m(\sim ar) = 0.1$, and $m(\{ar, \sim ar\}) = 0.5$. In probability theory, we cannot express such a feeling.

3.2. Belief functions

The belief in A, $\mathrm{Bel}(A)$, for a subset A of elements of the frame, represents the total belief in A. This belief will be more than $m(A)$. Actually, $\mathrm{Bel}(A)$ is equal to $m(A)$ plus sum of all the m-values for the set of elements that are contained in A. In terms of symbols:

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B).$$

By definition, belief in the empty set is zero.

Let us consider an example to illustrate the definition of belief functions. Suppose you have a friend who lives on the East Coast in the New Jersey area. The only contact you have with him or her is through greeting cards that he/she sends you periodically with no return address. You want to find the belief that your friend lives in New Jersey. After looking through all the cards you have received over the years, you can identify the following post-office seals marked on the cards: 10% of the cards are marked North Brunswick, 15% East Brunswick, 10% Philadelphia, and 12% Newark. Thirty percent of the cards have only the Brunswick part legible which means you cannot determine from what part of Brunswick the card was mailed. For the remaining 23%, nothing is legible on the seals. These numbers can be interpreted as non-zero m-values for different subsets of the frame that your friend lives somewhere on the East Coast near New Jersey. Based on just this evidence, you wish to form your total belief that the friend lives in New Jersey. This belief will be the sum of the m-values that he/she lives in North Brunswick, East Brunswick, Brunswick, and Newark. For this example, the belief is 0.67. Similarly, the belief that your friend lives in Brunswick, which includes North Brunswick and East Brunswick, will be 0.55 (10% North Brunswick, 15% East Brunswick, 30% Brunswick).

Going back to our first auditing example of analytical procedures, the auditor's assessment of the level of support in terms of m-values was: $m(ar) = 0.3$, $m(\sim ar) = 0$, and $m(\{ar, \sim ar\}) = 0.7$. Based on analytical procedures alone, the belief that the account is not materially misstated is 0.3 (i.e., $\mathrm{Bel}(ar) = 0.3$) and no support that the account is materially misstated ($\mathrm{Bel}(\sim ar) = 0$). In general, a zero belief in the belief-function formalism means that there is no evidence to support the proposition. In other words, a zero belief in a proposition represents lack of evidence. In contrast, a zero probability in probability theory means that the proposition cannot be true which represents an impossibility. Also, one finds that beliefs for ar and $\sim ar$ do not necessarily add to one, i.e., $\mathrm{Bel}(ar) + Bel(\sim ar) \leqslant 1$, whereas in probability, it is always true that $P(ar) + P(\sim ar) = 1$.

Belief functions differ from probabilities in representing ignorance. In probability theory, we represent ignorance by assigning equal probability to all the outcomes or elements of the frame. In the belief-function framework, we represent ignorance by assigning an m-value of one to the entire frame and an m-value of zero to all its proper subsets. The belief-function formalism becomes the Bayesian formalism when non-zero m-values exist only for single elements of the frame. In such a case, m-values become probabilities, i.e., $m(a_i) = P(a_i)$, and Dempster's rule in the belief-function formalism becomes Bayes' rule in the probability theory (Shafer, [13]).

3.3. Plausibility functions

By definition, the plausibility of A is equal to one minus the belief in $\sim A$, i.e., $\mathrm{Pl}(A) = 1 - \mathrm{Bel}(\sim A)$ where $\sim A$ represents the set of elements that are not in A. Intuitively, the plausibility of A is the degree to which A is plausible given the evidence. In other words, $\mathrm{Pl}(A)$ is the degree to which we do not assign belief to its negation $\sim A$.

In our example of analytical procedures, we have $\mathrm{Bel}(ar) = 0.3$, $\mathrm{Bel}(\sim ar) = 0$. These values yield the following plausibility values: $\mathrm{Pl}(ar) = 1$, and $\mathrm{Pl}(\sim ar) = 0.7$.

$\mathrm{Pl}(ar) = 1$ indicates that ar is maximally plausible since we have no evidence against it. However, $\mathrm{Pl}(\sim ar) = 0.7$ indicates that if we had no other items of evidence to consider then the maximum possible assurance that the account is materially misstated would be 0.7, even though we have no evidence that the account is materially misstated ($\mathrm{Bel}(\sim ar) = 0$).

3.4. The measure of ambiguity

The measure of ambiguity in a proposition in probability theory is not easy to define. However, in the belief-function framework the measure is straightforward. It is the difference between plausibility and the belief in the proposition (Wong and Wang, [23]). The belief in a proposition, A, represents the direct support for A and the plausibility represents the maximum possible support that could be assigned to A if we were able collect further evidence that were all in support of A. The difference then represents the unassigned belief that could be assigned to A. This unassigned belief represents an ambiguity in A. Thus, by definition ambiguity in A is:

$$\text{Ambiguity in } A = \mathrm{Pl}(A) - \mathrm{Bel}(A).$$

4. Decision making under uncertainty

The utility maximization approach has been used to make decisions under uncertainty, especially when uncertainty is represented by probabilities. However, the traditional approach does not work when uncertainties are not represented by probabilities. In this paper, we illustrate Strat's approach [20, 21] of decision making when uncertainties are represented in terms of belief functions. In order to illustrate the process, we first discuss the example given by Strat in probability framework and change the situation and describe how the decision can be made using belief functions.

4.1. Decision making using probabilities

Consider Strat's example of Carnival Wheel #1(Strat, [21]). This wheel has ten equal sectors. Each sector is labeled with a dollar amount as follows. Four sectors are labeled \$1, three sectors \$5, two \$10, and one \$20. The player gets to spin the wheel for a \$6 fee and receives the amount shown in the sector that stops at the top. The question is would you spin the wheel?

In this example, we have four outcomes (\$1, \$5, \$10, \$20) and the related uncertainties are represented by the following probability distribution:

$$P(\$1) = 0.4, P(\$5) = 0.3, P(\$10) = 0.2, \text{ and } P(\$20) = 0.1.$$

The expected value of the game is:

$$E(x) = \sum xP(x) = 0.4(\$1) + 0.3(\$5) + 0.2(\$10) + 0.1(\$20) = \$5.90$$

The expected values of utility is:

$$E(U(x)) = \sum P(x)U(x) = 0.4U(\$1) + 0.3U(\$5) + 0.2U(\$10) + 0.1U(\$20).$$

If one had to make a decision based on the expected value, then one would not play the game since the expected value of the game ($5.90) is smaller that the ticket price ($6). Although not presented here, we can argue that a rational individual with a risk averse attitude will reach the same conclusion if he or she had to make the decision based on the utility maximization rule (e.g., $E(U(-\$6)) < U(\$0)$).

4.2. Decision making using belief functions

Let us consider a situation where uncertainties related to the random events in a decision problem are not expressible in terms of probabilities but in terms of belief functions. As an example of such a situation, we consider Carnival Wheel #2 of Strat [21]. Carnival Wheel #2 is divided into ten equal sectors, each labeled by either $1, $5, $10, or $20. Four sectors are labeled $1, two sectors $5, two $10, one $20, and for one sector the label is hidden from the view. You have to pay a $6 fee to play the game. Will you play the game?

Before we discuss how to make decision under such a situation let us first express the uncertainties in the problem by m-values in the belief-function framework:

$$m(\$1) = 0.4, m(\$5) = 0.2, m(\$10) = 0.2, m(\$20) = 0.1, \text{ and } m(\{\$1, \$5, \$10, \$20\}) = 0.1.$$

This simply means that we have direct evidence that $1 appears in four sectors out of ten on the wheel, $5 appears in two sectors out of ten, and so on. $m(\{\$1, \$5, \$10, \$20\}) = 0.1$ represents the basic probability assignment to the sector with the label hidden from the view; it may contain any one of the four labels: $\{\$1, \$5, \$10, \$20\}$. The corresponding beliefs in the four outcomes are:

$$\text{Bel}(\$1) = 0.4,\ \text{Bel}(\$5) = 0.2,\ \text{Bel}(\$10) = 0.2,\ \text{Bel}(\$20) = 0.1.$$

The unassigned probability mass can be assigned to any one of the dollar amounts. Thus, plausibilities for various outcomes are given as (see Section 3.3 for the definition):

$$\text{Pl}(\$1) = 0.5,\ \text{Pl}(\$5) = 0.3,\ \text{Pl}(\$10) = 0.3,\ \text{Pl}(\$20) = 0.2.$$

That is, for each dollar amount on the wheel we have 0.1 degree of ambiguity ($\text{Pl}(A) - \text{Bel}(A)$).

If one has to compute the expected value of the outcome or of the utility, one would get an interval for the expected value instead of a single value. This is because one needs to assign the unassigned part of probability mass in all possible alternative ways. In the present case, we have four alternatives: either assign 0.1 to $1, to $5, $10, or $20. Each alternative will give us a different expected value and thus yield an interval for the expected value. The lower end of the expected value interval[5] for the outcomes of the game in Wheel #2 is:

$$E(x)_* = 0.5(\$1) + 0.2(\$5) + 0.2(\$10) + 0.1(\$20) = \$5.50$$

and the upper end is

$$E(x)_* = 0.4(\$1) + 0.2(\$5) + 0.2(\$10) + 0.2(\$20) = \$7.40.$$

If you had to make a decision based on the expected value, then you are in a difficult situation. The lower end of the expected value is lower than the fee of $6 and the upper end is greater than the fee. Of course, if you were allowed to gather more evidence then you will eliminate the ambiguity by simply seeing the label on the hidden sector. But that is not allowed and you still need to make a decision. What will you do? How will you make a decision under such a situation?

As mentioned earlier, there have been several approaches for decision making using belief functions (see, e.g., Jaffray, [7,8]; Nguyen and Walker, [11]; Smets, [15,16]; Strat [20, 21]; and Yager, [24]). We describe Strat's approach here because it has a strong support from the empirical data as shown in this article.

Strat calculates a single value for the expected value for the outcomes of the game by resolving ambiguity in the problem through the choice of a parameter, ρ, that defines the probability that ambiguity will be resolved as favorably as possible. Which means $(1-\rho)$ represents the probability that ambiguity will be resolved as unfavorably as possible. Under this consideration, the probability for each outcome will be:

$$P(\$1) = 0.4 + 0.1(1-\rho),\ P(\$5) = 0.2,\ P(\$10) = 0.2,\ P(\$20) = 0.1 + 0.1\rho$$

and the expected value will be:

$$E(x) = \$5.5 + 1.90\rho.$$

To decide whether to play the game or not, you need to only estimate ρ. Given that the labels were put by the carnival hawker, you will be more in favor of choosing $\rho = 0$. This will yield $E(x) = \$5.50$ which is lower than the fee and thus you will not be interested in playing the game. A similar approach can be used for determining the expected utility of the DM.

[5] 5.The expected value interval, $[E_*(x), E^*(x)]$, is given by (see, e.g., Strat [20]):

$$E_*(x) = \sum_{A_i \subseteq \Theta} \inf(A_i) m(A_i)$$

and

$$E^*(x) = \sum_{A_i \subset \Theta} \sup(A_i) m(A_i)$$

where $\inf(A_i)$ represents the smallest element in the set $A_i \subseteq \Theta$ and $\sup(A_i)$ represents the largest element in the set $A_i \subseteq \Theta$.

5. Belief-function analysis of the Einhorn and Hogarth type experiment

In this section, we analyze the decision making behavior under ambiguity using the belief-function framework. First, we make a proposition based on Strat's approach [21] to decision making using belief functions. Next, we use the proposition to predict various observable decision making behaviors in a general case of the EH type experiment with two urns. In the next section, we show that Ellsberg's result and the EH results are special cases of our general results which follow directly from Proposition 1.

PROPOSITION 1. Under ambiguity, a rational decision maker with risk averse attitude will choose an alternative that yields the highest expected utility under the most unfavorable resolution of ambiguity. If the decision maker is indifferent among the alternatives based on this rule then he/she will pick the alternative with the highest expected utility under the most favorable resolution of ambiguity.

Proposition 1 suggests that the DM will take a conservative approach, i.e., he or she will pick a worst scenario case and make the decision based on the best alternative in terms of the expected utility. This decision rule is equivalent to "minimax" criterion in a two-person zero-sum game. However, if the DM is indifferent among the alternatives under this rule then he or she will prefer the alternative that gives the highest expected utility under the most favorable resolution of ambiguity. We will validate Proposition 1 in Section 6 using the empirical data of EH [5].

5.1. General case under positive payoff

Here, we consider an experiment with two urns similar to what EH used but make the example more flexible by introducing various levels of ambiguity. In particular, we analyze the following two cases: (1) one ambiguous and one non-ambiguous urn, and (2) both ambiguous urns.

5.1.1. One ambiguous and one non-ambiguous urn

We assume that there are total N balls of red and black color in each of the two urns. Also, we assume that there are at least Nb_1 red balls in Urn 1 and there are exactly Nb_2 red balls in Urn 2 where $1 \geqslant b_1 \geqslant 0$, and $1 \geqslant b_2 \geqslant 0$. We obtain the same situation as EH's example if we set $N = 100$, $b_1 = 0$, and $b_2 = 0.5$. We consider the following two alternatives with positive payoff for our discussion:

> Win \$X if you pick a red ball from Urn 1 and win \$0 if you pick a black ball (ambiguous case).
>
> Win \$X if you pick a red ball from Urn 2 and win \$0 if you pick a black ball (non-ambiguous).

The m-value and the belief that a red ball is picked from Urn 1 is b_1 by construct since we know that we have at least Nb_1 red balls out of N total. When the red ball is picked the player wins \$X. Therefore, the belief that the DM wins if he/she picks a red ball from Urn 1 is also b_1. Moreover, we know in the case of Urn 1 that there could be a possibility that all the balls are red. In that situation the maximum belief that could be assigned to picking a red ball from Urn 1 is 1.0 which is the plausibility value. The m-value and the belief that a black ball is picked from Urn 1 is zero. Similar arguments can be presented for Urn 2. Thus, m-values, beliefs, and plausibilities for the two alternatives are given below:

$$\begin{aligned}\text{Urn 1}: & \ m(R_1) = b_1,\ m(B_1) = 0,\ \text{i.e.,Bel}(R_1) = b_1,\ \text{Bel}(B_1) = 0,\\ & \text{Pl}(R_1) = 1,\ \text{Pl}(B_1) = 1 - b_1,\\ & \text{Ambiguity in } R_1 = 1 - b_1, \text{ and Ambiguity in } B_1 = 1 - b_1.\end{aligned} \tag{1}$$

$$\begin{aligned}\text{Urn 2}: & \ m(R_2) = b_2,\ m(B_2) = 1 - b_2,\ \text{i.e., Bel}(R_2) = b_2,\ \text{Bel}(B_2) = 1 - b_2,\\ & \text{Pl}(R_2) = b_2,\ \text{Pl}(B_2) = 1 - b_2,\\ & \text{Ambiguity in } R_2 = 0, \text{ and Ambiguity in } B_2 = 0.\end{aligned} \tag{2}$$

According to Strat [21], the reassigned m-values for Urn 1 with ρ being the probability of resolving ambiguity as favorably as possible are given by:

$$m(R_1) = b_1 + (1 - b_1)\rho, m(B_1) = (1 - b_1)(1 - \rho). \tag{3}$$

The expected utility in the case of Urn 1 is:

$$\begin{aligned}E(U_1) &= m(R_1)U(\$X) + m(B_1)U(\$0),\\ &= [b_1 + (1 - b_1)\rho]U(\$X) + [(1 - b_1)(1 - \rho)]U(\$0).\end{aligned} \tag{4}$$

For the most unfavorable resolution of ambiguity ($\rho = 0$), we have:

$$E_*(U_1) = b_1 U(\$X) + (1 - b_1)U(\$0). \tag{5}$$

and under the most favorable resolution of ambiguity ($\rho = 1$), we get:

$$E^*(U_1) = U(\$X). \tag{6}$$

The expected utility in the case of Urn 2 is:

$$E(U_2) = m(R_2).U(\$X) + m(B_2)U(\$0) = b_2.U(\$X) + (1 - b_2)U(\$0). \tag{7}$$

According to Proposition 1, the general decision rule is that the DM will chose the alternative that yields the highest expected utility under the most unfavorable resolution of ambiguity (the worst scenario case). Thus, the DM will compare the expected utility in (5) with the expected utility in (7) and pick the alternative with the highest expected utility. When the DM is indifferent among the alternatives, then he or she will pick the alternative that yields the highest expected utility under the most favorable resolution of ambiguity by comparing (6) and (7). We consider the following special cases to discuss our results:

(i) $b_1 < b_2$:

When $b_1 < b_2$ the DM will chose Urn 2 (non-ambiguous) over Urn 1 (ambiguous) because $E(U_2) > E_*(U_1)$ (compare Equations 5 and 7). This is the situation in one of the cases of EH [5]) if we set $b_1 = 0$ and $b_2 = 0.5$ as discussed in Section 6. This behavior is called "ambiguity avoidance" behavior in the literature (EH [5]). However, in the present approach, such behavior is a result of the expected utility maximization rule. When $(b_2 - b_1)$ is very small, the preference may switch from Urn 2 to Urn 1 (see the discussion in Section 5.3 for details). Such switching behavior is observed by EH. We contend that the switching behavior is normal under the expected utility maximization rule when decisions are made under ambiguity (see the discussion in Section 5.3).

(ii) $b_1 = b_2$:

The condition when $b_1 = b_2$ is an interesting case. We find that the DM will always pick the ambiguous alternative (Urn 1) over the non-ambiguous alternative (Urn 2). The reason is that the DM is as well-off in the worst scenario case with the ambiguous urn as with the non-ambiguous one, i.e., $E_*(U_1) = E(U_2)$. However, if he/she picks Urn 1 he/she has a possibility of increasing his/her expected utility if resolution of ambiguity is in his/her favor, i.e., $E^*(U_1) > E(U_2)$ (compare Equations 5-7).

(iii) $b_1 > b_2$:

In the case of $b_1 > b_2$, the DM will always pick the ambiguous alternative (Urn 1) because the expected utility in the worst scenario case, $E_*(U_1)$, is always greater than $E(U_2)$ (see Equations 5 and 7).

5.1.2. Both ambiguous urns

For our present discussion we impose a condition on Urn 2 similar to Urn 1, that Urn 2 contains at least Nb_2 red balls. This means we do not know precisely the proportions of red and black balls in it, which makes Urn 2 ambiguous, too. The m-values, beliefs and plausibilities for this case are given as:

$$\begin{aligned}\text{Urn } 1: & \, m(R_1) = b_1,\ m(B_1) = 0, \text{ i.e., } \mathrm{Bel}(R_1) = b_1,\ \mathrm{Bel}(B_1) = 0, \\ & \mathrm{Pl}(R_1) = 1,\ \mathrm{Pl}(B_1) = 1 - b_1, \\ & \text{Ambiguity in } R_1 = 1 - b_1, \text{ and Ambiguity in } B_1 = 1 - b_1.\end{aligned} \tag{8}$$

$$\begin{aligned}\text{Urn } 2: & \, m(R_2) = b_2,\ m(B_2) = 0, \text{ i.e., } \mathrm{Bel}(R_2) = b_2,\ \mathrm{Bel}(B_2) = 0, \\ & \mathrm{Pl}(R_2) = 1,\ \mathrm{Pl}(B_2) = 1 - b_2, \\ & \text{Ambiguity in } R_2 = 1 - b_2, \text{ and Ambiguity in } B_2 = 1 - b_2.\end{aligned} \tag{9}$$

The reassigned m-values for the two urns are given below where ρ_1 and ρ_2 represent the two probabilities of resolving ambiguities in Urn 1 and Urn 2, respectively, as favorably as possible:

$$m(R_1) = b_1 + (1 - b_1)\rho_1,\ m(B_1) = (1 - b_1)(1 - \rho_1), \tag{10}$$

$$m(R_2) = b_2 + (1 - b_2)\rho_2, m(B_2) = (1 - b_2).(1 - \rho_2). \tag{11}$$

The expected utilities for the two cases are given by:

$$E(U_1) = [b_1 + (1 - b_1)\rho_1].U(\$X) + [(1 - b_1).(1 - \rho_1)]U(\$0), \tag{12}$$

$$E(U_2) = [b_2 + (1 - b_2)\rho_2]U(\$X) + [(1 - b_2).(1 - \rho_2)].U(\$0). \tag{13}$$

Under the most unfavorable resolution of ambiguity ($\rho_1 = 0$, and $\rho_2 = 0$) we get:

$$E_*(U_1) = b_1.U(\$X) + (1 - b_1).U(\$0), \tag{14}$$

$$E_*(U_2) = b_2.U(\$X) + (1 - b_2).U(\$0). \tag{15}$$

And under the most favorable resolution of ambiguity ($\rho_1 = 1$, and $\rho_2 = 1$) we get:

$$E^*(U_1) = U(\$X), \tag{16}$$

$$E^*(U_2) = U(\$X). \tag{17}$$

Comparing Equations (14)–(17), the DM will prefer to pick a ball from Urn 1 when $b_1 > b_2$, and from Urn 2 when $b_2 > b_1$. For $b_1 = b_2$, the DM will be indifferent, since the expected utility in the two cases are the same under the most and least favorable resolution of ambiguities. One can consider a third case where we have a minimum number of black balls in the two urns with a different minimum for each. This requirement will change the ambiguity values, but the basic preference rule will remain the same.

5.2. General case under negative payoff

Let us consider again the example used earlier of two urns but with a negative payoff for the two special cases: (1) one ambiguous and one non-ambiguous urn, and (2) both ambiguous urns.

5.2.1. One ambiguous and one non-ambiguous urn

The two alternatives for this case are:

- Lose \$X if you pick a red ball from Urn 1 and lose \$0 if you pick a black ball (ambiguous case).
- Lose \$X if you pick a red ball from Urn 2 and lose \$0 if you pick a black ball (non-ambiguous case).

Since there is no change in the information about the urns used earlier in Section 5.1 for a similar case, we have the same values for m-values, beliefs, and plausibilities as given in (1) and (2):

According to Strat [21], the reassigned m-values for Urn 1 with ρ being the probability of resolving ambiguity as favorably as possible are given by:

$$m(R_1) = b_1 + (1 - b_1)(1 - \rho),\ m(B_1) = (1 - b_1)\rho.$$

Note the change in the allocation of ambiguity from (3) for a positive payoff to the m-values given above for a negative payoff.

For the present situation, the expected utility in the case of Urn 1 is:

$$\begin{aligned} E(U_1) &= m(R_1)U(-\$X) + m(B_1)U(\$0), \\ &= [b_1 + (1 - b_1)(1 - \rho)]U(-\$X) + [(1 - b_1)\rho]U(\$0). \end{aligned}$$

For the most unfavorable resolution of ambiguity ($\rho = 0$), we have:

$$E_*(U_1) = U(-\$X). \tag{18}$$

and under the most favorable resolution of ambiguity ($\rho = 1$), we have

$$E^*(U_1) = b_1U(-\$X) + (1 - b_1)U(\$0). \tag{19}$$

The expected utility in the case of Urn 2 is:

$$\begin{aligned} E(U_2) &= m(R_2)U(-\$X) + m(B_2)U(\$0) \\ &= b_2U(-\$X) + (1 - b_2)U(\$0). \end{aligned} \tag{20}$$

According to Proposition 1, irrespective of the relative values of b_1 and b_2, the DM will always choose Urn 2 because the expected utility in the case of Urn 2 is always higher than the expected utility in the case of Urn 1 under the most unfavorable resolution of ambiguity ($E(U_2) > E_*(U_1)$, compare Equations 18 and 20). However, the choice will switch from Urn 2 to Urn 1 if the ambiguity in Urn 1 is reduced such that the plausibility of R_1 is below the plausibility of R_2. For example, if we had a situation where Urn 1 contained at least 51 black balls out of total 100 and Urn 2 contained 50 red and 50 black balls, then the choice is naturally Urn 1. Even in the worst situation the DM has a higher expected utility with Urn 1 if the game is to lose \$100 if a red ball is picked from any of the urns and lose \$0 if a black ball is picked. It is not difficult to derive a general result for such a case. Interested readers can obtain a copy of the solution from the author.

5.2.2. Both ambiguous urns

We have discussed earlier the case of two ambiguous urns under positive payoff. Here we consider exactly the same situation but with a negative payoff and then analyze the preferences. The m-values, beliefs, and plausibilities for the urns remain the same as given in (8) and (9). For this case, the reassigned m-values for the two urns are given below where ρ_1 and ρ_2 represent the two probabilities of resolving ambiguities in Urn 1 and Urn 2, respectively, as favorably as possible (again note the change in the allocation of ambiguities from (10-11) to the m-values given below):

$$m(R_1) = b_1 + (1 - b_1)(1 - \rho_1),\ m(B_1) = (1 - b_1)\rho_1.$$
$$m(R_2) = b_2 + (1 - b_1)(1 - \rho_2),\ m(B_1) = (1 - b_1)\rho_2.$$

The expected utility for the two cases are:

$$E(U_1) = [b_1 + (1 - b_1)(1 - \rho_1)]U(-\$X) + [(1 - b_1)\rho_1]U(\$0),$$
$$E(U_2) = [b_2 + (1 - b_2)(1 - \rho_2)]U(-\$X) + [(1 - b_2)\rho_2]U(\$0).$$

Under the most unfavorable resolution of ambiguity ($\rho_1 = 0$, and $\rho_2 = 0$) we get:

$$E_*(U_1) = U(-\$X), \tag{21}$$
$$E_*(U_2) = U(-\$X). \tag{22}$$

And under the most favorable resolution of ambiguity ($\rho_1 = 1$, and $\rho_2 = 1$) we get:

$$E^*(U_1) = b_1U(-\$X) + (1 - b_1)U(\$0), \tag{23}$$
$$E^*(U_2) = b_2U(-\$X) + (1 - b_2)U(\$0). \tag{24}$$

According to Proposition 1, the DM is indifferent based on the most unfavorable resolution of ambiguity (worst scenario case, see Equation 21 and 22). The next step is to compare the expected utility under the most favorable scenario and chose the alternative with the highest expected utility. In the above case, we see from (23) and (24) that the DM will choose Urn 1 if $b_1 < b_2$, and choose Urn 2 if $b_2 < b_1$. For $b_1 = b_2$, the DM will be indifferent. One can consider a third case where we have a minimum number of black balls in the two urns with a different minimum for each. This requirement will change the plausibility values and ambiguities, but the basic preference rule will remain the same.

5.3. Switching behavior

EH observed that, when the probability of picking a red ball from a non-ambiguous urn was very small (0.001), some of their subjects switched their choices from picking a ball from a non-ambiguous urn to an ambiguous one which is not in accordance with the expected utility theory. We investigate this behavior more closely using the concept of "precision measure" under our framework of decision making. Consider that the DM perceives the alternatives under the most unfavorable scenario to be indifferent, even if in principle, there is one alternative, say A_1, that has the highest expected utility. In such a situation, according to Proposition 1, the DM will look at the expected utility under the most favorable resolution of ambiguity and choose the alternative with the highest expected utility. Under certain conditions, this process results into picking an alternative that is different from A_1. Thus, the source of switching behavior is the immaterial difference between the expected utilities under different alternatives.

Theoretically, switching behavior is interesting. It brings in the concept of "materiality" or "precision measure" in decision theory or utility theory that has never been addressed. The "precision measure" is an important concept in science and engineering. Whenever we make a decision in those disciplines, we do take into consideration the precision measure or the uncertainty in the measured values. In auditing, when auditors make a decision about whether an account balance is fairly stated, they use the concept of "materiality" . If the audited account balance is not materially different from the recorded balance then the auditor considers the recorded balance to be fairly stated. Also, we use this concept in statistical tests or hypothesis testing.

Somehow we tend to ignore this concept of "precision measure" or "materiality" in the expected utility theory and expect DMs to measure their utilities precisely. If we bring this concept to utility theory then, even if, in principle, the expected values of a utility function were different under different alternatives, they would appear to be the same to a DM because of the immaterial difference between them. In the present context, this simply means that based on the expected utility under the most unfavorable resolution of ambiguity, the DM finds the alternatives to be indifferent and goes to the next step in Proposition 1 and finds the best alternative based on the most favorable resolution of ambiguity. This process causes the DM's choice to switch.

Let us develop the concept of "precision measure" or "materiality" in our context. Consider the general case discussed earlier of two urns each with a total of N balls, one being ambiguous (Urn 1) with at least Nb_1 red balls and the other being non-ambiguous (Urn 2) with exactly Nb_2 red balls where $1 \geqslant b_1 \geqslant 0$, and $1 \geqslant b_2 \geqslant 0$. The ambiguity in Urn 1 is $(1-b_1)$. We discussed earlier that if $b_1 \geqslant b_2$, the DM will always choose Urn 1 for a positive payoff. The case with $b_1 < b_2$ and a positive payoff is of special interest here because switching behavior will occur under such a condition for some DMs. From (5) and (7), the difference between the expected utility for Urn 2 and the expected utility for Urn 1 under the most unfavorable resolution of ambiguity is given by:

$$\begin{aligned}\Delta E_*(U_{2-1}) &= E(U_2) - E_*(U_1),\\ &= b_2.U(\$X) + (1-b_2).U(\$0) - b_1.U(\$X) - (1-b_1).U(\$0),\\ &= (b_2-b_1).[U(\$X) - U(\$0)].\end{aligned} \tag{25}$$

If the above difference is perceived to be insignificant or not "material" by the DM then he or she feels indifferent about the two alternatives. But how do we measure whether the difference is insignificant? Since we do not know the absolute values of the utility function at different wealth levels, it is better that we define the difference in (28) to be material on the basis of the relative value of the utility function. Thus, assume α_c to be the critical value such that if the difference in the expected utilities for the two alternatives is less than or equal to α_c times the relative value of the utility function then the DM is indifferent about the two alternatives, i.e., the difference is immaterial. In other words, for

$$DE_*(U_{2-1})/[U(\$X) - U(\$0)] = (b_2 - b_1) \leqslant \alpha_c, \tag{26}$$

the switching behavior will occur since the difference is not "material". But if the above ratio is greater than α_c, then the difference is material, and the DM is able to distinguish between the alternatives and hence there will be no switching.

One would like to know answers to such questions as: (1) What is the value of α_c? (2) What factors does it depend on? Well, we do not know the answers. To answer these questions we need to perform empirical tests. However, we can make some speculations. For example, a value of 0.01 for α_c would suggest that the DM is not able to distinguish between alternatives when the difference is 1 percent or less of the relative utility value. In response to the second question, we can argue that the initial wealth level and the risk attitude of the DM will influence the magnitude of α_c. For a wealthy individual, the value of α_c will be larger than for a person with not as much wealth. Also, for a risk averse individual the value will be smaller than for a person with risk seeking attitude. According to (26), the switching behavior will occur whenever the difference $(b_2 - b_1)$ is less than or equal to the critical value α_c (see Section 6 for further discussion).

6. Validation of proposition 1 using Einhorn and Hogarth data

In this section, we validate Proposition 1 by using the data of EH. As discussed in Section 2.1, Ellsberg's experiment was conducted with a positive payoff using two urns. EH repeated Ellsberg's experiment but for four different situations. In two situations, they used exactly the same set of urns as used by Ellsberg but considered both a positive and a negative payoff. In the other two situations, they used a different set of urns to make the probability of picking a winning or losing ball from the unambiguous urn to be 0.001. We will discuss these four cases below.

Case 1: Positive Payoff with probability 0.5

The first experiment of EH deals with a positive payoff and two urns. Urn 1 contains 100 red and black balls with unknown proportions. Urn 2 contains 50 red and 50 black balls. The subjects are offered \$100 if they pick a specific color ball (red or black) from one of the urns but get \$0 if they pick a wrong ball. The question they asked the participants is which urn would they prefer to pick from?

This experiment is equivalent to our general case with a positive payoff (Section 5.1) where $X = 100$, $N = 100$, $b_1 = 0$, and $b_2 = 0.5$. However, for simplicity of presentation, we assume that the participants will win \$100 if they pick a red ball from either urn to be able to use directly the results derived in (5-7). The treatment will be similar if the condition was based on picking a black ball. Thus, using (5-7) we obtain the following expected values of utility for this case. The expected utility in the case of Urn 1 for the most unfavorable resolution of ambiguity is:

$$E_*(U_1) = U(\$0),$$

and under the most favorable resolution of ambiguity is:

$$E^*(U_1) = U(\$100).$$

The expected utility in the case of Urn 2 is:

$$E(U_2) = 0.5U(\$100) + 0.5U(\$0).$$

According to Proposition 1, and knowing the fact that $E(U_2) > E_*(U_1)$ from the above values, the DM will prefer to pick a red ball from Urn 2. EH [5, see Table 1, p. S237, first row] data support this result. Forty-seven percent of the subjects picked non-ambiguous urn (Urn 2), 19 percent ambiguous urn (Urn 1), and 34 percent were indifferent.

Case 2: Negative payoff with probability 0.5

This case uses the same set of urns as used in Case 1, but the payoff is negative. The participants in the game are told that they will lose \$100 if they pick a specified color ball and lose \$0 if they pick a wrong ball. Let us assume again, for simplicity of discussion, that the participants are told that they will lose \$100 if they pick a red ball but lose nothing if they pick a black ball. This is a special case of our general case discussed in Section 5.2 with $X = 100$, $N = 100$, $b_1 = 0$, and $b_2 = 0.5$. Thus using (18-19), the expected utility in the case of Urn 1 for the most unfavorable resolution of ambiguity is:

$$E_*(U_1) = U(-\$100),$$

and under the most favorable resolution of ambiguity is

$$E^*(U_1) = U(\$0).$$

From (20), the expected utility in the case of Urn 2 is

$$E(U_2) = 0.5U(-\$100) + 0.5U(\$0).$$

Again according to Proposition 1, and knowing the fact that $E(U_2) > E_*(U_1)$ from the above values, the DM will prefer to pick a red ball from Urn 2. The EH [5, see Table 1, row 3] data support our result. Thirty percent chose Urn 2 (non-ambiguous), only 14 percent chose Urn 1 (ambiguous), and 56 percent were indifferent.

Case 3: Positive payoff with probability 0.001

In this experiment, EH asked the participants to imagine two urns with 1,000 balls. These balls are numbered. The participants are told that they will be paid \$1,000 if they pick the ball with number 687 from one of the urns, otherwise, they get \$0. Along with it, they are told that the balls in Urn 1 may all or none be marked 687. The balls in Urn 2 are sequentially marked from 1 to 1,000. The question is which urn would they prefer to pick from?

This situation is equivalent to our general case discussed in Section 5.1 with $X = 1{,}000$, $N = 1{,}000$, $b_1 = 0$, and $b_2 = 0.001$. Thus we can use (5-7) to determine the expected values of utility for different alternatives. For Urn 1, the expected utility for the most unfavorable resolution of ambiguity is:

$$E_*(U_1) = U(\$0),$$

and under the most favorable resolution of ambiguity it is:

$$E^*(U_1) = U(\$1{,}000).$$

The expected utility in the case of Urn 2 is:

$$E(U_2) = 0.001U(\$1{,}000) + 0.999U(\$0).$$

Based on these expected values of utility it seems the DM should pick Urn 2 according to Proposition 1 because $E(U_2) > E_*(U_1)$. However, the difference between $E(U_2)$ and $E_*(U_1)$ is so small that some DMs would be indifferent about the two alternatives. In such a situation, according to Proposition 1, the DM will make the decision based upon the most favorable resolution of ambiguity. In the present case, since $E^*(U_1) > E(U_2)$, the decision will be to pick a red ball from Urn 1. This is the switching behavior described by EH. As discussed in Section 5.3, the switching behavior will depend on the "measure of precision" or "materiality" , i.e., how material the difference is between the expected utilities perceived by the DM. Also, as mentioned in Section 5.3, this "precision measure" would depend on the initial wealth of the DM and the risk attitude. Thus, not all DMs will switch their preferences at the same time. EH data show this mixed pattern. Thirty-five percent of subjects picked Urn 1 (the ambiguous urn) in the present case compared to nineteen percent in the previous case where probability of picking a winning ball was 0.5 (rows 1 and 2 in Table 1 of EH, [5]. This result shows the switching behavior. However, we see that still a significant percentage (43 percent) of DMs picked Urn 2 over Urn 1. The above result also makes intuitive sense. The DM has 999 chances out of 1,000 to win if he or she picks the ball from Urn 1 (ambiguous) and only one chance out 1,000 if he/she picks from Urn 2 (non-ambiguous).

Case 4: Negative payoff with probability 0.001

This case is similar to Case 3 discussed above except the payoff is negative. The subjects are told that they will lose \$1,000 if they pick a ball marked 687. The question is which urn would they choose? This situation is equivalent to our general case discussed in Section 5.2 with $X = 1{,}000$, $N = 1{,}000$, $b_1 = 0$, and $b_2 = 0.001$. Thus we can use (18-20) directly to determine the expected values of utility for different alternatives. For Urn 1, the expected utility for the most unfavorable resolution of ambiguity is

$$E_*(U_1) = U(-\$1{,}000),$$

and under the most favorable resolution of ambiguity, it is:

$$E^*(U_1) = U(\$0).$$

The expected utility in the case of Urn 2 is:

$$E(U_2) = 0.001U(-\$1{,}000) + 0.999U(\$0).$$

Based on these expected utilities it seems the DM should pick Urn 2 according to Proposition 1 because $E(U_2) > E_*(U_1)$. There will be no switching behavior in this case because the two expected utilities ($E_*(U_1) = U(-\$1{,}000)$, and $E(U_2) = 0.001U(-\$1{,}000) + 0.999U(\$0)$) are very different. This case has very strong support from the EH data [5, see Table 1, row four]. Seventy-five percent of the subjects chose Urn 2 (non-ambiguous), only 5 percent Urn 1 (ambiguous), and 20 percent were indifferent. This result again makes intuitive sense. The DM has only one chance out of 1,000 to lose if he/she picks the ball from Urn 2 (non-ambiguous) but has 999 chances out of 1,000 if he/she picks from Urn 1 (ambiguous).

7. Summary and conclusion

In conclusion, we have shown how the belief-function framework can help us understand the human decision making process under ambiguity. In particular, we have developed a proposition for decision making under ambiguity using the expected utility theory under the belief-function framework. This proposition is based on Strat's approach [20, 21] of resolving ambiguity in the problem. The proposition suggests that the DM will take a conservative approach and use the most unfavorable resolution of ambiguity for making a decision. If he or she is indifferent among the alternatives under this scenario, then he/she will use the most favorable resolution of ambiguity and choose the alternative that has the maximum expected utility. Using this proposition, we have explained the Ellsberg paradox. Also, we have used the empirical data of EH [5] to validate Proposition 1.

Using Proposition 1, we have predicted several observable behaviors of decision making under ambiguity for the EH type experiment under four general situations: (1) positive payoff with one ambiguous and one non-ambiguous urns, (2) positive payoff with both ambiguous urns, (3) negative payoff with one ambiguous and one non-ambiguous urns, and (4) negative payoff with both ambiguous urns. Also, we have discussed the general condition under which the "switching" behavior will occur using the concept of "precision measure" in the expected utility theory. In the context of Proposition 1, this concept simply means that the DM is indifferent among the alternatives under the most unfavorable resolution of ambiguity, even though theoretically there is an alternative, say A_1, that has the highest expected utility. Consequently, the DM decides to pick an alternative based on the most favorable resolution of ambiguity which, under certain conditions, may be different from A_1.

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