E C O N O M I C S B U L L E T I N

A nonparametric adjustment for tests of changing mean

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Abstract

When testing for a change in mean of a time series, the null hypothesis is no change in mean. However, a change in mean causes a bias in the estimation of serial correlation parameters. This bias can cause nonmonotonic power to the point that if the change is big enough, power can go to zero. In this paper, we show that a nonparametric correction can restore power. The procedure is illustrated with a small Monte Carlo experiment.

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1. Introduction

There are a number of tests designed to test for a change in mean of a time series. Many of the tests can be applied to general regression models or single time series. Some of these tests include the CUSUM tests of Brown, Durbin, and Evans (1975) which were analyzed again in Ploberger and Kramer (1992). Related tests were developed by Gardner (1969) and MacNeill (1978) which are based on the square of cumulative sums of residuals. Andrews (1993) analyzed functions of a Wald test and Andrews and Ploberger (1994) proposed optimal tests. The tests in Elliott and Müller (2003) satisfy a different optimality property and are shown to behave in a similar manner to many of the existing tests.

The unit root literature is large, and, for a time, developed independently of the literature on testing for change in mean. However, Perron (1989) showed that if there is a change in the mean of an otherwise stationary time series, traditional unit root tests are unable to reject the unit root hypothesis. Perron (1990) provides an analysis of the coefficient bias in an autoregression when there is a change in mean. The results show that the estimated coefficient is biased toward one so that the estimator is closer to one the larger the change in mean. Based on these findings, a huge literature has developed which attempts to sort out the unit root versus break in deterministic trend hypothesis. Moreover, Kuan, Newbold, and Nunes (1995) show that the converse is also true; if there is a unit root, some tests for a change in mean will find one. In response to this interaction between changes in mean and the unit root hypothesis, Vogelsang (1998) developed a test for change in mean (or trend) that is robust to the unit root hypothesis.

Vogelsang (1997, 1999) shows that the confusion between a unit root and a change in mean has another adverse affect on inference for a change in mean. When testing for a change in mean, one must estimate some effect for serial correlation, either in a parametric or nonparametric fashion. If one is dealing with a dynamic time series model and there is a change in mean, the effect discovered by Perron (1990) causes the autoregressive coefficient to be biased toward one. Now the test for a change in mean interprets the change in mean as persistence and hence a larger variance for the process rather than a change in mean. Vogelsang (1999) shows that as the change in mean increases, some tests are *less likely* to detect the change. This effect causes the power function to initially increase but eventually decrease, hence the phenomenon is referred to as nonmonotonic power. Therefore, a change in mean causes a bias in the autoregressive parameters causes poor inference in both unit root tests *and* tests for a change in mean.

In this paper, we show that a nonparametric adjustment to some tests can alleviate nonmonotonic power while retaining the original asymptotic distribution for the test. The correction is based on a nonparametric estimation of the mean of the time series. We then use the residuals to construct an estimate of the dynamic parameters in the model. By using the nonparametric estimation technique, we are able to consistently estimate the dynamic parameters even if there is some form of change in the mean. The modified tests are compared to their original counterparts in a small Monte Carlo experiment where it is revealed that the nonparametric adjustment is effective in avoiding nonmonotonic power.

2. Model

We introduce a simple time series model given by

$$y_t = \theta\left(\frac{t}{T}\right) + u_t$$
$$u_t = \alpha u_{t-1} + \epsilon_t$$

where ϵ_t is a martingale difference sequence with mean zero, variance given by σ_ϵ^2, and possessing finite fourth moments. The mean of y_t is given by $\theta(t/T)$ and the variance of y_t is given by $\sigma_\epsilon^2/(1-\alpha^2)$.

One hypothesis of interest is that the mean of y_t is not time-varying, so that the null hypothesis is given as

$$H_0 : \theta(t/T) = \theta = \text{constant}.$$

Following Vogelsang (1999), define the variable

$$DU_t = 1(t > T_b)$$

which is a simple indicator of whether the time period is greater than some reference point T_b. Suppose that σ_ϵ^2 and α are known but we wish to test the constancy of θ. First, note that the variance of $\sqrt{T}\bar{y}$ is given as $\omega^2 = \sigma_\epsilon^2/(1-\alpha)^2$. The tests of Andrews (1993) and and Andrews and Ploberger (1994) are based on the Wald test given as

$$W(T_b) = \left(\omega^2 \sum_{t=T_b+1}^{T} \hat{DU}_t^2\right)\left(\sum_{t=T_b+1}^{T} \hat{y}_t\right)^2, \tag{1}$$

where $\hat{DU}_t$ and $\hat{y}_t$ are the residuals from regressing DU_t and y_t respectively on a constant. Let Λ be a set of possible dates for a discrete shift in mean. The sup Wald, mean Wald, and exponential Wald statistics are defined as

$$\text{sup Wald} = \sup_{T_b \in \Lambda} W(T_b)$$
$$\text{mean Wald} = \frac{1}{T}\sum_{T_b \in \Lambda} W(T_b)$$
$$\text{exp Wald} = \log\left(\frac{1}{T}\sum_{T_b \in \Lambda} \exp\left(\frac{1}{2}W(T_b)\right)\right).$$

In applications, the parameter ω^2 is unknown and must be estimated in a parametric or nonparametric fashion.

As discussed in Vogelsang (1999), it is possible to calculate a dynamic version of the Wald statistic as

$$WD(T_b) = \left(s^2(T_b) \sum_{t=T_b+1}^{T} \tilde{DU}_t^2 \right) \left(\sum_{t=T_b+1}^{T} \tilde{y}_t \right)^2, \tag{2}$$

where $\tilde{DU}_t$ and $\tilde{y}_t$ are the residuals from regressing DU_t and y_t respectively on a constant, D_t, and y_{t-1} with $D_t = 1(t = T_b + 1)$. The term $s^2(T_b)$ is calculated from the residuals of regressing y_t on a constant, DU_t, D_t, and y_{t-1}.

Vogelsang (1999) provides a thorough anaylsis of the statistics calculated with nonparametric estimates of ω^2 and the statistics based on the dynamic version of the Wald tests. Both types of statistics may exhibit nonmonotonic power to the point that power can go to zero if the change in mean is large enough.

We propose a modification to the existing statistics based on a nonparametric estimator for ω^2. In a different test, this is the method used in Juhl and Xiao (2004) to modify a U-statistic for testing a constant mean. The procedure is as follows. First, obtain a nonparametric estimator for $u_t = y_t - \theta(t/T)$ by the quantity

$$\breve{u}_t = \frac{1}{Th} \sum_{s \neq t}^{T} K\left(\frac{t-s}{Th}\right)(y_t - y_s)$$

where $K(\cdot)$ is a kernel function and h is a bandwidth parameter that goes to zero at a prescribed rate. Then we regress $\breve{u}_t$ on $\breve{u}_{t-1}$ to get an estimate of ρ. The variance of ϵ_t is estimated using residuals of the regression of $\breve{u}_t$ on $\breve{u}_{t-1}$. The final estimator of ω^2 is defined as $\breve{\omega}^2$.

3. Asymptotic Distribution

We list the assumptions needed for $\breve{\omega}^2$ to be consistent for ω^2.

Assumption 1 *y_t is generated according to*

$$y_t = \theta\left(\frac{t}{T}\right) + u_t$$
$$u_t = \alpha u_{t-1} + \epsilon_t$$

where $|\alpha| < 1$.

Assumption 2 *Let* $\mathcal{F}_{t-1} = \sigma(\epsilon_t, \epsilon_{t-1}, \ldots)$. $E(\epsilon_t|\mathcal{F}_{t-1}) = 0$, $E(\epsilon_t^2|\mathcal{F}_{t-1}) = E(\epsilon_t^2) = \sigma^2 < \infty$, $E(\epsilon_t^4|\mathcal{F}_{t-1}) = E(\epsilon_t^4) = \mu_4 < \infty$.

Assumption 3 $g(\cdot)$ *belongs to the class* $\mathcal{G}_2^4$ *defined in Robinson (1988).*

Assumption 4 $k(\cdot) \in \mathcal{K}_2$*, a class of kernels defined in Robinson (1988).*

Assumption 2 are moment conditions, one of which requires ϵ_t to be a martingale difference. Assumptions 3 and 4 are a standard type of smoothness conditions and kernel restrictions such as those employed in Fan and Li (1999). We state the result below.

Theorem 1 *Suppose that Assumptions 1-4 hold,* $h \to 0$, $Th^8 \to 0$, *and* $Th^2 \to \infty$. *Then*

$$\sqrt{T}(\breve{\alpha} - \alpha) = O_p(1),$$

and $\breve{\omega}^2 \xrightarrow{p} \omega^2$.

This result is proven in Juhl and Xiao (2004). Given the assumptions above, one can replace ω by its estimate in the Wald statistics. With the consistency of $\breve{\omega}^2$ in hand, it is obvious that under the null hypothesis, the functions of the Wald statistics will have the distributions found in Andrews (1993) and Andrews and Ploberger (1994).

4. Monte Carlo

This section provides an illustration of the perfmance of the modified test statistics in a small experiment. The model is given as

$$\begin{aligned} y_t &= \gamma DB_t + u_t \\ u_t &= \alpha u_{t-1} + \epsilon_t, \ t = 1, \ldots 200, \end{aligned}$$

where $DB_t = 1(75 < t < 150)$ and $\alpha = 0.5$. To examine size, we set $\gamma = 0$ and we count the percentage of rejections in 2000 replications. We compute the $supW$, $meanW$, and $expW$ statistics based on the dynamic regression model given in (2). The modified statistics are denoted $supW^*$, $meanW^*$, and $expW^*$ are based on (1). These modified statistics all require a bandwidth parameter to obtain the estimator $\breve{\omega}^2$. To examine the sensitivity of the results to the bandwidth choice and serial correlation in the data, we use three versions of the bandwidth, $h = c \times T^{-1/5}$, where $c = 1, 1.5, 2$ and $\alpha = 0.0, 0.5, 0.7$. The results appear in Table 1. Notice that all of the statistics have size very close to the nominal 5% and that the modified statistics are not particularly sensitive to the bandwidth.

To examine the power of the tests, we let γ increase up to 4.5. A nonzero γ causes the process to have two changes in mean, one at $t = 0.25T$ and another at $t = 0.75T$.

The power is calculated using size-adjusted critical values based on the size experiment. For the modified statistics, we use a bandwidth of $T^{-1/5}$. Power is shown in Figures 1-3. The nonmonotonic power of the original statistics based on the dynamic regression is apparent from the Figures. However, as expected, the modified statistics have excellent power throughout the entire range of γ.

5. Conclusion

We have illustrated how to avoid nonmonotonic power in tests for changing mean in this paper. The tests in this paper use a result from Juhl and Xiao (2004) on the consistency of long-run variance estimators under changing means. Our Monte Carlo experiment shows that the tests are not particularly sensitive to the bandwidth parameter needed in the nonparametric estimation of the mean function.

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Table 1.
Size

$\alpha = 0.0$	supW	meanW	expW
	0.026	0.037	0.028
	supW*	meanW*	expW*
$c = 1.0$	0.041	0.032	0.0275
$c = 1.5$	0.037	0.030	0.025
$c = 2.0$	0.035	0.029	0.024
$\alpha = 0.5$	supW	meanW	expW
	0.030	0.040	0.034
	supW*	meanW*	expW*
$c = 1.0$	0.031	0.041	0.034
$c = 1.5$	0.026	0.039	0.030
$c = 2.0$	0.022	0.035	0.027
$\alpha = 0.7$	supW	meanW	expW
	0.037	0.042	0.035
	supW*	meanW*	expW*
$c = 1.0$	0.041	0.058	0.0445
$c = 1.5$	0.030	0.048	0.038
$c = 2.0$	0.024	0.042	0.031

Figure 1

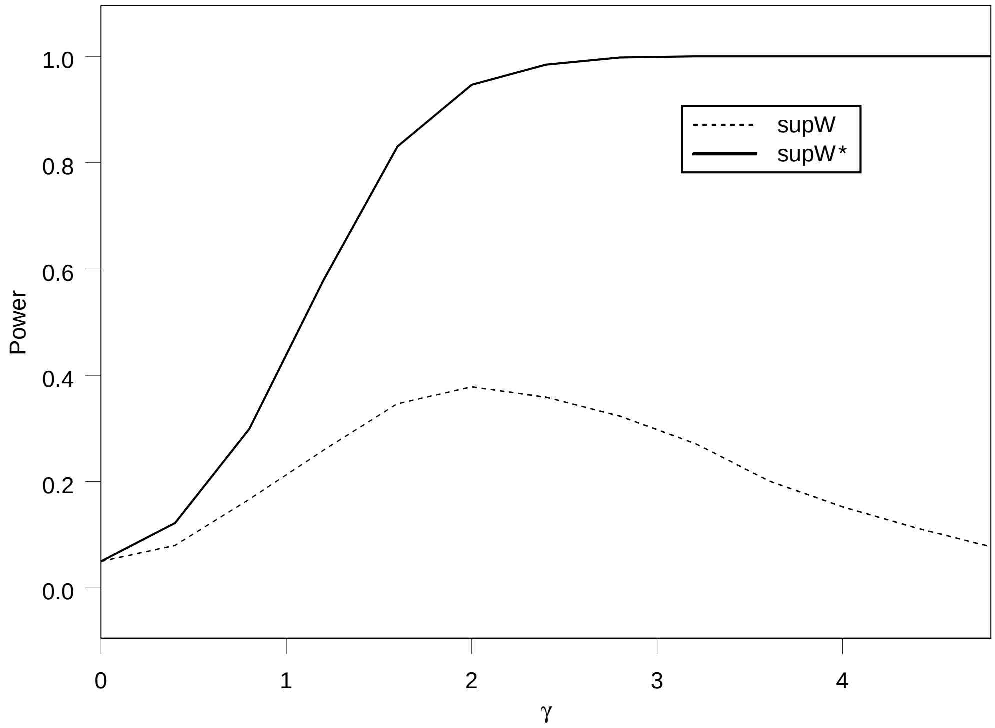

Figure 2

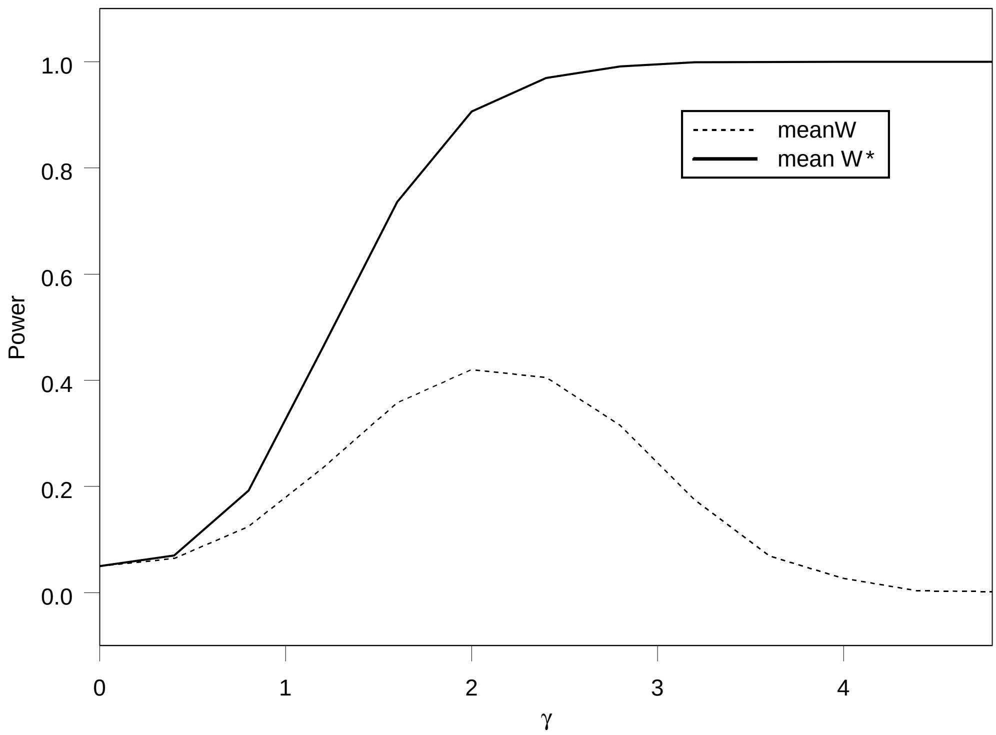

Figure 3