

Essays on Broad Divisia Monetary Aggregates:

Admissibility and Practice

By

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Abstract

The assumption of weak separability of goods and services in the utility function is ubiquitous in macroeconomic modeling. If the goods and services are weakly separable, they can be combined into an “admissible” aggregate. This project tests the assumption of weak separability (or “admissibility”) for broad Divisia monetary aggregates for the United States provided by the Center for Financial Stability; Divisia M4, Divisia M4-, and Divisia M3. These broad monetary aggregates measure the service flow of money in the macroeconomy through a share weighted index method developed by Barnett (1980), and already established as superior to simple sum aggregates in the literature collected in Barnett and Serletis (2000). The problem to be addressed is the determination of how broad a monetary aggregate should be: is the Divisia M2 level sufficient for aggregation or should other like commercial paper, overnight repurchase agreements, short term securities and large denomination time deposits be included? These components contained in broad aggregates are subject to risk that is not accounted for in the traditional user cost estimate of Barnett (1980), but can be adjusted through methods proposed in Barnett and Wu (2005). The performance of these risk adjusted aggregates is tested alongside with the risk neutral case to determine admissibility.

Using microeconomic foundations in the non-parametric weak separability test literature of Varian (1982) the aggregates are examined for evidence of admissibility. Since Varian (1982) tends to over-reject weak separability, we implement methods from Barnett and de Peretti (2009) to avoid over rejection from noise in the data. Furthermore a necessary and sufficient weak separability condition is used based on the marginal rate of substitution between two goods, instead of an only sufficient condition. The results provide evidence for the use of Divisia M4 as an admissible aggregate. In the risk adjusted case several violations found in the risk neutral case dissolve, and the Divisia M4- gains support as admissible. There is less evidence to support the

use of Divisia M3 as an admissible aggregate as it passes the necessary but not sufficient conditions for admissibility.

Dedication

For Arely, Eva, and Eli. Te amo.

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Table of Contents

Abstract	iii
Dedication	v
Acknowledgements	vi
Chapter 1: Introduction	1
The CFS Advances in Monetary and Financial Measurement Data Set	3
Chapter 2: Testing Admissibility in the Risk Neutral Case.	10
Summary	10
Introduction	11
The User Cost of Money	12
Weak Separability Testing	14
The Center for Financial Stability and Data Sources	24
Results	27
Conclusion.....	36
Chapter 3: Deriving the Risk Adjusted User Cost of Money.	38
Summary	38
Introduction	39
Risk Adjusted User Cost of Money	39
Conditional CAPM Price Kernel.....	43
Data, Forecasts, and Adjustments	47

User Cost Adjustment Results	49
Conclusions	75
Chapter 4: Testing Admissibility in the Risk Adjusted Case.	77
Summary	77
Introduction	78
Risk Adjusted User Costs in GARP	79
Weak Separability Test Revisited	81
GARP and Weak Separability Results	83
Conclusion.....	94
Chapter 5: Conclusion.....	95
Selected Code Used	98
GARP Test: Two Step Iterative Procedure from Barnett and de Peretti (2009)	98
Instrumental Variables Regression Test Used on Risk Neutral Case from Barnett and de Peretti (2009).	103
Kalman Filter from Barnett and de Peretti (2009)	104
Multivariate Independence Test from Barnett and de Peretti (2009).....	106
Quarterly Adjustment.....	107
Seasonal Adjustment	108
Bibliography	109

List of Tables

Table 1 Divisia M4, Divisia M4-, Divisia M3 Components.....	27
Table 2 Admissibility Test Results	31
Table 3 GARP 1974 Results	31
Table 4 GARP 1982 Results	32
Table 5 GARP 1991 Results	33
Table 6 GARP 2006 Results	34
Table 7 Weak Separability Test	35
Table 8 Risk Adjusted Admissibility Results	87
Table 9 Risk Adjusted GARP 1974.....	87
Table 10 Risk Adjusted GARP 1982	88
Table 11 Risk Adjusted GARP 1991	89
Table 12 Risk Adjusted GARP 2006.....	90
Table 13 Risk Adjusted Weak Separability	91

List of Figures

Figure 1 Expenditure Shares on Monetary and Financial Assets	52
Figure 2 Expenditure Shares on Monetary Assets	53
Figure 3 Expected Real Rate of Return on Asset Portfolio	54
Figure 4 Risk Adjusted User Cost for Currency, Traveler's Checks, and Demand Deposits	55
Figure 5 Risk Adjusted User Cost Other Checkable Deposits	56
Figure 6 Risk Adjusted User Cost Savings (without Money Market Demand Accounts)	57
Figure 7 Risk Adjusted User Cost Money Market Demand Accounts	58
Figure 8 Risk Adjusted User Cost Savings Accounts (with Money Market Demand Accounts)	59
Figure 9 Risk Adjusted User Cost Money Market Mutual Funds (Retail and Institutional)	60
Figure 10 Risk Adjusted User Cost Small Denomination Time Deposits	61
Figure 11 Risk Adjusted User Cost Large Denomination Time Deposits and Three Month Treasury Bills	62
Figure 12 Risk Adjusted User Cost Commercial Paper and Overnight Repurchase Agreements	63
Figure 13 Portfolio User Cost, Certainty Equivalent and Estimated Risk Adjusted.	64
Figure 14 Estimated Risk Price Adjustment	65
Figure 15 Risk Adjustment for Currency, Travelers Checks, and Demand Deposits	66
Figure 16 Risk Adjustment for Other Checkable Deposits	67
Figure 17 Risk Adjustment for Savings Accounts (Without Money Market Demand Accounts)	68
Figure 18 Risk Adjustment Money Market Demand Accounts	69
Figure 19 Risk Adjustment for Savings Accounts (With Money Market Demand Accounts)	70
Figure 20 Risk Adjustment for Money Market Mutual Funds	71
Figure 21 Risk Adjustment for Small Denomination Time Deposits	72

Figure 22 Risk Adjustment for Large Denomination Time Deposits, Overnight Repurchase Agreements, Commercial Paper, and Three Month Treasury Bills.....	73
Figure 23 Risk Adjusted and Risk Neutral Divisia M4 Year-over-Year Growth Rate	74
Figure 24 1980 to 1982 GARP Violations.....	92
Figure 25 September 2008 Violations	93

Chapter 1: Introduction

In the study of macroeconomics a representative agent is constructed to reduce the multiple dimensions associated with an entire population of individual agents with their own preferences and resources. The agent is assumed to consume all goods and services available in some amount; consumption goods, financial assets, and services to monetary assets for example. These components are assumed to be weakly separable in order to aggregate to macroeconomic indicators such as personal consumption expenditures, monetary aggregates, and financial assets. Separability of these components is often assumed without further investigation: currency and travelers checks can be aggregated as “money”, apples and pears as “fruit”, or haircuts and restaurants as “services”. There exists a broad literature testing for weak separability of goods to support the assumption, part of which relies on an assumed functional form of the utility function (the parametric approach) and part on a non-parametric estimation of the data which does not assume a functional form. The use of monetary aggregates relies on its weak separability from consumer goods and financial assets; “money” supplies a service over and above its gross return and acts as a means of exchange providing liquidity service to agents. If weak separability does not hold, there is no utility maximization of the representative agent and therefore no aggregation admissible by economic theory.

The implicit assumption of the weak separability of components in the new Center for Financial Stability’s (CFS) broad Divisia aggregates is tested using recent developments in non-parametric weak separability testing and user cost risk adjustment. The CFS constructs broad Divisia monetary aggregates Divisia M4, Divisia M4-, and Divisia M3 (see

Table 1 Divisia M4, Divisia M4-, Divisia M3 Components) according to the methodology in Barnett, Liu, Mattson and Van den Noort (2013), without adjusting for risk. A more detailed description of this new data set is provided later in this introductory chapter. Chapter 2 examines these components according to the theory developed in Barnett and de Peretti (2009) using the test originally developed in Varian (1982, 1983) and adjusted for stochastic noise so that it does not over-reject¹ weak separability due to measurement error or other non-structural factors. The test is advantageous in that it does not require the justification of any form of the utility function and instead relies on the microeconomic theory of Revealed Preference. An adjustment to the sufficient weak separability condition in Barnett and de Peretti (2009) is also useful in that it provides a necessary and sufficient condition for admissibility. No narrow aggregates show evidence of admissibility while the broad aggregates pass necessary conditions. In the case of Divisia M4 the necessary and sufficient condition is passed.

Chapter 3 adjusts the user costs of the CFS' broad Divisia aggregates for risk inherent in the added components: commercial paper, large time deposits, overnight repurchase agreements, and short term treasury bills. The risk adjustment uses a simple conditional capital asset pricing model (CAPM) familiar to the financial literature. The risk adjusted Divisia M4 aggregate is shown to be similar in many time periods to the risk neutral (see Figure 23 Risk Adjusted and Risk Neutral Divisia M4 Year-over-Year Growth Rate), however the user cost prices show relatively large differences. Barnett and Wu (2005) predicted the larger adjustment relative to previous estimations using an unconditional CAPM, such as Barnett, Liu, and Jensen (1997), due to the risk price and risk adjustment in the conditional CAPM. The larger user cost adjustment affects the admissibility testing as it depends on expenditures on components.

¹ See Barnett and Choi (1989) on this problem.

The results of Chapter 4 suggest that risk adjusted user costs improve the performance of Divisia M4- and maintain support for Divisia M4 in admissibility tests with the exception of the period from 1974 to 1982, specifically the months between March 1980 and November 1982. Test are displayed in the tables and figures and reviewed in the text. Narrower aggregates still fail admissibility in all time periods. The cases for the failure of Divisia M4 in the 1974 to 1982 are considered in light of changes in Federal Reserve policy during the time period of the most significant violations and the large revision in survey methodology that introduced two new types of savings accounts to the monetary aggregates. While the risk adjusted Divisia M4 does not pass the necessary conditions as set out by the Generalized Axiom of Revealed Preference for early 1980s, it does for each time period after. The necessary and sufficient admissible result holds in the risk neutral and risk adjusted case for Divisia M4 from 1982 to the present with the exception of a unique and strong violation in the turbulent months of the financial crisis of 2008 in the risk adjusted case. The suggestion for the macro econometrician then is to use a broad Divisia monetary aggregate in lieu of narrow aggregates which do not show evidence of admissibility and to adjust user costs for risk.

The CFS Advances in Monetary and Financial Measurement Data Set

The following section is a detailed review of the material printed in Barnett, Liu, Mattson and van den Noort (2013) and linked to the CFS' Advances in Monetary and Financial Measurement broad Divisia monetary aggregate data set. The CFS produces broad Divisia monetary aggregates in the interest of encouraging monetary economics research, providing alternative monetary measures in a zero-interest rate environment, and allowing for transparent discourse on construction of aggregates and aggregation methods. Data that can be provided on the CFS website is available while data that cannot be directly uploaded to the website is available through other sources such as Wrightson ICAP, Bloomberg, the St.Louis Federal

Reserve Banks Federal Reserve Economic Data (FRED), and Bankrate.com. To further encourage transparency, replication, and open discourse the CFS makes available a document describing all data sources and transformations on its website².

The construction of broad Divisia monetary aggregates begins with the seminal work by Barnett (1980) and collected in Barnett and Serletis (2000). The first step is to determine the user cost price of holding some monetary asset as opposed to holding on to another. This user cost is the familiar opportunity cost from introductory economics (“you give up something in order to get something”) measuring the liquidity service provided by holding some other less liquid monetary asset with higher returns. It is imperative to understand that the user cost of a monetary asset is not the interest rate paid to that monetary asset. While interest is the user cost of a financial asset that provides no liquidity, the monetary asset acts not only as a store of value but as a means of exchange; the latter not being taken into account in a simple interest rate. Interest rates, however, are necessary for construction of the user cost as they determine the return on the asset. The definition of the user cost of a monetary asset, in terms of the gross real return defined as $R_{i,t} = (1 + i_{i,t})$ where $i_{i,t}$ is the real interest rate on some monetary asset i observed at time t is $\pi_{i,t} = \frac{R_t^b - R_{i,t}}{R_t^b}$ for some benchmark rate R_t^b . To re-state, the user cost is defined as the opportunity cost for holding some monetary asset that provides a liquidity service over some benchmark asset that provides less liquidity and higher return.

The choosing of the benchmark return is not trivial. Anderson and Jones (2011) use a maximum envelope method, where the highest possible return among a collection of monetary assets and comparable illiquid assets. In some cases the benchmark asset is chosen from among the monetary assets and a zero user cost is produced, which would disrupt the calculation of the Divisia index. To adjust for the case of a zero user cost, 100 basis points are added to the

² This methodology can also be found in Barnett, Liu, Mattson, and van den Noort (2013).

benchmark return to ensure a non-zero and positive user cost. Alternatively a bank loan rate could be considered in the maximum envelope set. Since a bank never gives more in interest than the rate that it charges for loans, the benchmark return is assured to be above the user cost of all monetary assets considered. This method follows the research of Offenbacher and Schachar (2011) at the Bank of Israel in their calculation of Divisia monetary aggregates. The CFS adopts the use of the bank loan rate, more specifically the Weighted Average Effective Loan Rate for Commercial and Industrial Loans, Low Risk provided by the E.2 Survey of Terms of Business Lending. The bank loan rate is quarterly and paired with monthly data, so an expansion method must be used to pair the data³. There has only been one month in the CFS data set where the bank loan rate (when available) has not been the maximum, and it was surpassed by a secondary market large denomination certificate of deposit return, not paired with any of the monetary assets under consideration⁴. Given the component returns and the return on the benchmark asset (whether it is an actual asset or not) the user cost is easily derived.

After calculation, the user costs are paired with the level of the corresponding monetary asset. The aggregate component levels are defined as $m_{i,t}$ and include monetary assets such as currency, traveler's checks, demand deposits, savings accounts, small denomination time deposits, and money market mutual funds. The Federal Reserve Board provides simple sum monetary aggregates that include all of these components under the designation M2, and the St. Louis Federal Reserve Bank provides a Monetary Services Index of M2 which is a Divisia share-weighted index. The broader Divisia aggregates provided by the CFS include large denomination time deposits and overnight repurchases that were all once available in the Federal Reserve Board's M3 simple sum monetary aggregate. Since the M3 component survey is no longer

³ Alternatively one could aggregate the monthly data into quarterly data.

⁴ It is also most likely an artifact of the quarterly to monthly transformation of the loan rate while the secondary CD rates are all updated monthly.

provided, the assets are collected through other surveys: the Federal Reserve Board's H.8 Assets and Liabilities of Commercial Banks and the New York Federal Reserve Bank's Primary Dealers Survey (overnight repurchase agreements). The CFS collects data from these surveys in order to construct their broad aggregates; however the CFS takes the aggregate a little further than the traditional M3 construction, including commercial paper short term Treasury bills in its most broad aggregates. Commercial paper comes from the Federal Reserve Board's Survey of Commercial Paper and the Treasury bill amounts and rates are taken from the Monthly Statement of Public Debt. The return and level data are now collected and the actual calculation of the Divisia monetary aggregate can proceed in a few last steps.

The Divisia monetary aggregate is a share weighted index, where the weights on each component are determined based on the total expenditure on that asset. With the user costs and levels, the total expenditure is derived and the share of that expenditure on a particular asset in a given time period is solved using a simple share formula $s_{i,t} = \frac{\pi_{i,t}m_{i,t}}{\sum_{l=1}^L \pi_{l,t}m_{l,t}}$. To get the Tornqvist-Theil approximation of Divisia, the average share of the previous and current time period is the weight imputed to each component which can be solved given the previous time periods share $\bar{s}_{i,t} = \frac{s_{i,t} + s_{i,t-1}}{2}$. Given the user costs, the share weights, and the levels the derivation of the Divisia index using the Tornqvist-Theil approximation is the solution of the difference equation:

$$\log(\mathbf{M}_t) - \log(\mathbf{M}_{t-1}) = \sum_i \bar{s}_{i,t} (\log(m_{i,t}^*) - \log(m_{i,t-1}^*)).$$

The series must begin with a normalization of the first period to a value of 100, and the most important information comes from the growth of the Divisia monetary aggregate, not the level. The growth of the Divisia aggregate for the CFS is calculated on a year-over-year basis to avoid seasonality and provide less volatile but still informative analysis. If the growth of the Divisia monetary aggregate is above zero, then the supply of monetary services in the economy is

growing. If the aggregate growth falls below zero, the monetary services in the economy are contracting. Numerous macroeconomic implications from these two simple analyses are drawn regarding growth of GDP, inflation, unemployment, liquidity, and so on. A monthly analysis of recent trends is provided by the CFS, though the primary goal is to provide the data for open analysis and discourse by researchers, economists, central bankers, and anyone interested in monetary economics.

As in most data base construction projects a certain amount of data transformation occurs to account for seasonality, the termination and beginning of new surveys, and the entry and exit of assets. Those components not already seasonally adjusted by the Federal Reserve Board such as overnight repurchase agreements, commercial paper, and short term treasury bills are adjusted using an X-12 ARIMA process in SAS (see the Code section). The change between surveys of commercial paper is accounted for by using the growth rate trends in the current survey and back-casting those values over Commercial Paper collected in the previous survey. The method is similar for overnight repurchase agreements and large denomination time deposits which were discontinued in the surveys constructing the M3, but not discontinued in other individual surveys with differing methodologies. In order to estimate missing values for interest rates, other comparable interest rates in the current time period are found a simple linear regression is estimated to back estimate the missing data. The CFS uses these methodologies to derive broad Divisia monetary aggregates back to 1967.

In addition to the usual long term survey issues, the presence of Retail Sweeps severely biases the narrow aggregates. Retail Sweeps are an accounting trick used by banks to misrepresent the amount of checking balances on their books by “sweeping” those funds into money market demand accounts and counting them as “savings”. The miscalculation of demand deposits allows the banks to hold less legally required reserves. The data survey for M1 is taken

after these sweeps have taken place, and therefore the M1 is biased downward by a catastrophic amount: Barnett (2012) estimates the bias at approximately half the level of M1. The survey taken by the St. Louis Federal Reserve which allows for a sweeps adjustment of narrow aggregates was oddly discontinued in May of 2012, so an estimation procedure based on the interest rate now paid on required reserves is used. While this procedure is now used for the CFS broad Divisia data set, its implementation followed the publication of the data and methodologies paper; it is unfortunately not available in the publication Barnett, Liu, Mattson, and van den Noort (2013). Therefore we now outline the procedure for estimation here. Let $s_t = \frac{S_t}{T_t}$ be the ratio of swept deposits over total deposits (being the sum of deposits not swept and deposits swept, $T_t = S_t + NS_t$). Let π_t^{RR} be the opportunity cost of required reserves derived from the difference in the benchmark return and required reserve return. The ratio of sweeps to total deposits is estimated and calibrated (the constant c) by simple linear regression of the opportunity cost of required reserved and the level of sweeps recorded by the St. Louis Fed before its discontinuation from November 2008 to March 2012, for s before the discontinuation and t for afterwards:

$$s_s = \rho \pi_s^{RR} + \varepsilon_s,$$

$$\hat{s}_t = \hat{\rho} \pi_t^{RR} + c.$$

The estimation of sweeps is solved according to the formula from April 2012 and onward, using:

$$\frac{(\hat{s}_t * NS_t)}{(1 - \hat{s}_t)} = \hat{S}_t.$$

For more details on Retail Sweeps programs see Anderson and Rasche (2001). Unfortunately as the data provided by the St. Louis Fed is no longer available, sweeps estimation must be used to account for the tremendous bias inherent in narrow aggregates that do separate checking accounts from savings accounts like M1. Such distortions in narrow aggregates serve as another

motivation to avoid use of the narrow aggregates above and beyond the analysis of Chapters 2 and 4 on admissibility of aggregates to “go broad”.

A more in-depth analysis of the CFS broad Divisia aggregates is available both on their website and in Barentt, Liu, Mattson, and van den Noort (2013). The grouping of the broad aggregates however into sub-aggregates is given more of an intuitive justification than a rigorous economic justification; these are the clusters that have come before and they are familiar to the typical introduction to economics alumnus. This project seeks to lay a foundation for those clusters through the use non-parametric statistical tests based on the economic theory of Revealed Preference. Furthermore the user costs employed by the CFS are not adjusted for the risk inherent in the broad aggregate components such as overnight repurchase agreements and commercial paper. We adjust those user costs for risk using asset pricing models as developed in Barnett and Wu (2005). Finally, the project concludes with determining the admissibility of clusters in a risk adjusted user cost case. The project builds on the new, foundational data set for broad Divisia monetary aggregates, which provides fertile ground for future research in monetary economics.

Chapter 2: Testing Admissibility in the Risk Neutral Case.

Summary

Chapter 2 defines the theory underlying the development and construction of broad Divisia aggregates for the United States as provided recently by the Center for Financial Stability. The stochastic semi-nonparametric test for weak separability of clusters by Barnett and de Peretti (2009), based on the Varian (1982) is applied to the new broad Divisia aggregates described in Barnett et al (2012). The test supports the use of the broadest possible cluster of monetary components, Divisia M4 and its sibling Divisia M4- (excluding Treasury bills), but does not support the narrower Divisia M3 cluster. Forms of Divisia M2 are rejected as well, including variations on Divisia M4 and Divisia M4-, with the exception of a Divisia M4 aggregate that excludes Retail Money Market funds in the most recent time period.

Introduction

Chapter 2 identifies admissible clusters of monetary aggregates for the United States using the nonparametric, necessary and sufficient, stochastic method developed in Barnett and de Peretti (2009). The definition of “admissible” comes from Barnett (1982), in that an aggregate is “admissible as an indicator” if composed of a weakly separable cluster of components. For the relevant theoretical background on the necessity of weak separability for the existence of aggregates see Barnett (1980, 1982)⁵ and the functional structure literature expositied Barnett and Binner (2004).

The test is a recent branching of the nonparametric revealed preference literature. This literature begins in earnest with the three seminal papers Varian (1982, 1983, 1985) which develop a test for satisfaction of revealed preference relations in observed data sets, namely the satisfaction of the Generalized Axiom of Revealed Preference (GARP). Swofford and Whitney (1987) and Fisher and Fleissig (1997) pushed the literature forward theoretically and empirically while Barnett and Choi (1989) pointed to the tendency for over-rejection in the presence of insignificant stochastic errors. A stochastic adaptation of the original Varian (1982, 1983) procedure was necessary. Several approaches developed such as Fleissig and Whitney (2003), Jones et al (2005), Swofford and Whitney (1994), and de Peretti (2005, 2007). The Barnett and de Peretti (2009) test builds on this literature, and then reaches outside the Afriat inequality index method, for a weak separability test based on the equality of the marginal rate of substitution and price ratio of goods. The price ratio weak separability test is advantageous as it is both a necessary and sufficient condition for weak separability if GARP holds.

In 2012 the Center for Financial Stability (CFS) began to provide broad Divisia monetary aggregates and their quantities and user costs of their components to the public. The CFS

⁵ Reprinted as Chapter 2 and Chapter 7 in Barnett and Serletis (2000).

program, Advances in Monetary and Financial Measurement (AMFM) provides three broad aggregates, Divisia M4, Divisia M4-, and Divisia M3, made up of fifteen separate components⁶ including currency, travelers checks, demand deposits, interest bearing deposits, savings accounts, small and large time deposits, retail and institutional money market mutual funds, commercial paper, overnight repurchase agreements, and short term treasury bills from 1967 to 2012⁷. The construction methods and data sources for these aggregates are described in Barnett et al (2013). After defining and developing the definition and construction of broad Divisia aggregates in a risk neutral context, the weak separability test in this chapter examines Divisia M3, Divisia M4- and Divisia M4 provided by the CFS and some of the narrower aggregates to determine if they are indeed “admissible” as defined by the theoretical literature. We find evidence for the admissibility of Divisia M4, the broadest aggregate when compared with real personal consumption expenditures; however narrower aggregates such as Divisia M2 and Divisia M3 do not pass these tests.

The User Cost of Money

The Divisia monetary aggregate literature begins with Barnett (1978, 1980) which is reprinted with a near exhaustive amount of complementary papers and research in Barnett and Serletis (2000) regarding the theoretical foundations and empirical evidence of its superiority over other monetary aggregates. Despite the overwhelming evidence of the utility and sense in using an aggregate that does not assume all monetary components are perfect substitutes many central banks insist on using the severely flawed simple sum aggregate, which has been known as far back as Fisher (1922) to be “the worst” aggregate to use in nearly every case. The key difference between the worst and best aggregate being the user cost of money, derived by Barnett

⁶ The chapter combines currency and travelers checks into one component.

⁷ Certain components are not available at certain time periods, for example money market mutual funds do not become available until 1973.

(1978), which is the opportunity cost of holding one form of money, say currency, instead of retail money market funds; two clearly different components which should in no case be treated as perfect substitutes.

The user cost (price) is derived in a straightforward manner using the return of the monetary asset and a benchmark rate “given up” in favor of the asset in question. With the user cost price now available, each asset can be weighted in terms of expenditure for a given time period. The expenditure share weights are then used to determine the Tornqvist-Theil approximation of the Divisia monetary aggregate⁸. For some asset $i=1, \dots, L$ and time observation $t=1, \dots, T$ the gross rate of return⁹ $R_{i,t}$, quantity $m_{i,t}$, benchmark gross rate of return R_t^b , expenditure share weight $s_{i,t}$ and $\bar{s}_{i,t} = (s_{i,t} - s_{i,t-1})/2$, and aggregate \mathbf{M}_t the Divisia aggregate can be constructed by solving for the user cost, the share weight, and the Tornqvist-Theil approximation respectively:

$$\pi_{i,t} = \frac{R_t^b - R_{i,t}}{R_t^b},$$

$$s_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{\sum_{l=1}^L \pi_{l,t} m_{l,t}^*},$$

$$\log(\mathbf{M}_t) - \log(\mathbf{M}_{t-1}) = \sum_i \bar{s}_{i,t} (\log(m_{i,t}^*) - \log(m_{i,t-1}^*)).$$

With the Divisia aggregate solved, one can then determine the dual aggregate user cost for the cluster, Π_t , using Fisher’s Factor Reversal.

$$\Pi_t \mathbf{M}_t = \boldsymbol{\pi}_t' \mathbf{m}_t,$$

$$\Pi_t = \frac{R_t^b - R_{i,t}}{R_t^b}.$$

⁸ For a more rigorous theoretical treatment, the reader is referred to Barnett (1980) re-printed in Barnett and Serletis (2000), or for an empirical approach on US data Barnett, Liu, Mattson, and van den Noort (2013). A wealth of literature on international applications of Divisia can be found on the Center for Financial Stability’s website: http://www.centerforfinancialstability.org/amfm_data.php

⁹ Gross rate of return is defined as $1 + i_{i,t}$ for $i_{i,t}$ the return interest on asset i at time t .

Using this method broad Divisia monetary aggregates are constructed and made available on by the Center for Financial Stability's program Advances in Monetary and Financial Measures (CFS). The aggregates and their components are freely available on the CFS website. For a more complete coverage of data sources and methodology, see Barnett, Liu, Mattson and van den Noort (2013).

Weak Separability Testing

Proper clustering of monetary components within an aggregate relies on the Generalized Axiom of Revealed Preference and the condition of weak separability. Let i be the observations of L monetary components. Define the following for the observation at time $t = 1, \dots, T$ and component asset $i = 1, \dots, L$:

$\mathbf{M} = (TxL)$ matrix of observed real per capita monetary quantities,

$\mathbf{m}_t = (m_{1,t}, m_{2,t}, \dots, m_{L,t})'$ the t th row of \mathbf{M} ; a vector of real per capita monetary assets,

$\mathbf{\Pi} =$ the (TxL) matrix of the user cost of holding monetary asset i at t ,

$\boldsymbol{\pi}_t = (\pi_{1,t}, \pi_{2,t}, \dots, \pi_{L,t})'$ be the i th row of $\mathbf{\Pi}$; the corresponding user costs of i at t ¹⁰.

Pairing these quantities and user cost prices constructs the monetary data set of levels and users costs represented by the sequence $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$. Let \mathbf{M} be partitioned into $\mathbf{M}^{(1)}$ a (Txa) matrix with and $\mathbf{M}^{(2)}$ a $(Tx(L-a))$ matrix with the corresponding partitions of the user cost matrix $\mathbf{\Pi}^{(1)}$ and $\mathbf{\Pi}^{(2)}$. $\mathbf{M}^{(1)}$ can then be defined as weakly separable. If there exists an overall utility function $U(\cdot)$, a strictly increasing macro function $V(\cdot)$, and a sub-utility function $f(\cdot)$

¹⁰ The theoretical procedure outlined in Barnett and de Peretti (2009) works for any real per capita quantity data, and is represented by a matrix of real per capita expenditures \mathbf{X} and the corresponding price matrix \mathbf{P} . Since this paper deals specifically with monetary asset components and their user cost, the notation is changed to \mathbf{M} and $\mathbf{\Pi}$ to reflect the use of monetary assets and their associated user costs in the Divisia literature collected in Barnett and Serletis (2000).

which admits the following rewriting of the overall utility function then $\mathbf{M}^{(1)}$ is weakly separable from $\mathbf{M}^{(2)}$ if:

$$(1) \quad U_t = U(\mathbf{m}_t), \quad t = 1, \dots, T,$$

$$(2) \quad U_t = V(\mathbf{m}_t^{(2)}, f(\mathbf{m}_t^{(1)})), \quad t = 1, \dots, T.$$

Furthermore the weak separability of the component vectors $\mathbf{m}_t^{(1)}$ implies that the marginal rate of substitutions between any components j and l within the separable group is independent of changes in those goods outside the group, that is $\mathbf{m}_t^{(2)}$. More formally, for $j, l = 1, \dots, a$, for $j \neq l$, $m = a + 1, \dots, L$

$$(3) \quad \partial \left(\frac{\partial U(\mathbf{m}_t) / \partial m_{tj}}{\partial U(\mathbf{m}_t) / \partial m_{tl}} \right) / \partial m_{tm} = 0.$$

The nonparametric weak separability literature, beginning with Varian (1982) and Varian (1983) disregards the marginal rate of substitution identity by checking for the existence of the overall utility, the macrofunction, and the sub-utility using the Generalized Axiom of Revealed Preference (GARP). If the overall utility and sub-utility are to exist within the dataset $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$ they must satisfy GARP, i.e. they must behave in a utility maximizing way. The Varian (1982) procedure begins with testing the observed data for satisfaction of GARP. To properly define GARP, we need to define the standard binary relations from microeconomic theory. Consider as before the bundle of real per capita monetary assets at two observations t and s . Direct revealed preference of \mathbf{m}_t to \mathbf{m}_s , $\mathbf{m}_t P^0 \mathbf{m}_s$, means the expenditure at t on its asset bundle is strictly greater than the expenditure on any other bundle using prices at s ; $\boldsymbol{\pi}_t \cdot \mathbf{m}_t > \boldsymbol{\pi}_t \cdot \mathbf{m}_s$. An asset bundle at observation t , \mathbf{m}_t , is directly revealed preferred to \mathbf{m}_s , $\mathbf{m}_t R^0 \mathbf{m}_s$, if $\boldsymbol{\pi}_t \cdot \mathbf{m}_t \geq \boldsymbol{\pi}_t \cdot \mathbf{m}_s$. Finally, \mathbf{m}_t is revealed preferred to \mathbf{m}_s , $\mathbf{m}_t R \mathbf{m}_s$, if a transitive closure

$\mathbf{m}_t R^0 \mathbf{m}_u, \mathbf{m}_v R^0 \mathbf{m}_w, \dots, \mathbf{m}_r R^0 \mathbf{m}_s$ exists between the observations or $\pi_t \cdot \mathbf{m}_t \geq \pi_t \cdot \mathbf{m}_u, \pi_u \cdot \mathbf{m}_u \geq \pi_u \cdot \mathbf{m}_v, \dots, \pi_r \cdot \mathbf{m}_r \geq \pi_r \cdot \mathbf{m}_s$. GARP is then formally defined for the data set as:

DEFINITION GARP. *Data set $\{(\mathbf{m}_t, \pi_t)\}_{t=1}^T$ satisfies the general axiom of revealed preference if for observations $t \in (1, \dots, T)$ and $s \in (1, \dots, T)$ the revealed preference of t over s , $\mathbf{m}_t R \mathbf{m}_s$, implies that the other is not strict directly revealed preferred to the former; “not $\mathbf{m}_s P^0 \mathbf{m}_t$ ”.*

Alternatively, using the inequality of expenditure definitions above, GARP can be described as satisfied for some data set $\{(\mathbf{m}_t, \pi_t)\}_{t=1}^T$ if for all observations, $\pi_t \cdot \mathbf{m}_t \geq \pi_t \cdot \mathbf{m}_u, \pi_u \cdot \mathbf{m}_u \geq \pi_u \cdot \mathbf{m}_v, \dots, \pi_r \cdot \mathbf{m}_r \geq \pi_r \cdot \mathbf{m}_s$ implies $\pi_s \cdot \mathbf{m}_s \leq \pi_s \cdot \mathbf{m}_t$. With GARP well defined, as in Varian (1982), the key theorem for non-parametrically testing weak separability is proposed as:

THEOREM Varian (1982). *For the observed data set $\{(\mathbf{m}_t, \pi_t)\}_{t=1}^T$, the following are equivalent: (i) An overall utility function $U(\cdot)$ exists, is locally non-satiated, and rationalizes the data, (ii) there exist marginal income indexes λ_t and strictly positive indexes U_t that satisfy the Afriat inequalities, $U_t \leq U_s + \lambda_t(\pi_s \cdot \mathbf{m}_t - \pi_s \cdot \mathbf{m}_t)$, for any $t, s = 1, \dots, T, t \neq s$, and (iii) the generalized axiom of revealed preference holds for $\{(\mathbf{m}_t, \pi_t)\}_{t=1}^T$.*

Based on Theorem 1, checking for weak separability involves the determining if the data satisfies all of the following three conditions:

CONDITION 1. $\{(\mathbf{m}_t, \pi_t)\}_{t=1}^T$ satisfies GARP ($U(\cdot)$ exists).

CONDITION 2. $\left\{(\mathbf{m}_t^{(1)}, \pi_t^{(1)})\right\}_{t=1}^T$ satisfies GARP ($f(\cdot)$ exists).

CONDITION 3. $\left\{ \left(\mathbf{m}_t^{(2)}, U_t \right), \left(\boldsymbol{\pi}_t^{(2)}, \lambda_t^{-1} \right) \right\}_{t=1}^T$ satisfies GARP, where U_t and λ_t are strictly positive indexes satisfying the Afriat inequalities $U_t \leq U_s + \lambda_t(\boldsymbol{\pi}_s \cdot \mathbf{m}_t - \boldsymbol{\pi}_s \cdot \mathbf{m}_s)$, implying $\left\{ \left(\mathbf{m}_t^{(1)}, \boldsymbol{\pi}_t^{(1)} \right) \right\}_{t=1}^T$ is weakly separable in $U(\cdot)$.

As described in Barnett and de Peretti (2009) and Fleissig and Whitney (2003) Condition 3 suffers the disadvantage of being a necessary but not sufficient condition for weak separability and the test depends on the method of computing the Afriat inequalities. Adding to the problems associated with Condition 3, Barnett and Choi (1989) prove through Monte Carlo simulations from a pre-determined, admissible functional form of the utility that any single violation of GARP leads to rejection of weak separability, even if the violation is caused by measurement error or stochastic factors. Starting with a modification of Condition 3 that is necessary and sufficient, Barnett and de Peretti (2009) outline a method to resolve these problems, as well as deal with over rejection in the Varian (1982) test.

Assume for now that $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$ and $\left\{ \left(\mathbf{m}_t^{(1)}, \boldsymbol{\pi}_t^{(1)} \right) \right\}_{t=1}^T$ are consistent with GARP (that Condition 1 and Condition 2 hold). Recall from Equation (3) that the independence condition of the marginal rate of substitution between two assets of a separable group and any assets existing outside that group is necessary and sufficient for weak separability to hold. The unknown form of the utility function is a drawback to this procedure, however if GARP does hold for $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$, then there is a sub-utility that rationalizes the data by Theorem 1, $f(\mathbf{m}^{(1)})$. That is, $f(\mathbf{m}^{(1)})$ is behaving as a utility maximizing function, and by microeconomic analysis the ratio of the prices gives the marginal rate of substitution between assets.

$$(5) \quad \frac{\pi_{i,t}^{(1)}}{\pi_{j,t}^{(1)}} = \frac{\partial f(\mathbf{m}_t^{(1)}) / m_{i,t}^{(1)}}{\partial f(\mathbf{m}_t^{(1)}) / m_{j,t}^{(1)}}.$$

See Barnett and de Peretti (2009) Lemma 1 for a formal treatment of this condition. Given a maximizing sub-utility function as assumed from the satisfaction of GARP by $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$ and $\{(\mathbf{m}_t^{(1)}, \boldsymbol{\pi}_t^{(1)})\}_{t=1}^T$, at first-order conditions the ratio of prices between two assets i and j are equal to the corresponding marginal rate of substitution between those two assets. A necessary and sufficient Condition 3 is a test of independence between the price ratios of two goods within the cluster to be checked with the level quantities of those assets outside the cluster.

A regression set up is required, following the method in Barnett and de Peretti (2009). Stack the price ratios of the clustered group into a newly defined vector \mathbf{Y} , a $(T \cdot \sum_{i=1}^{a-1} (a - i) \times 1)$ for $\boldsymbol{\pi}_{i,\cdot} = (\pi_{i,1}, \pi_{i,2}, \dots, \pi_{i,T})$ the i th column of the clustered group's prices matrix $\boldsymbol{\Pi}^{(1)}$ and $i = 1, \dots, a$.

$$\mathbf{Y} = \begin{bmatrix} \log(\boldsymbol{\pi}_{1,\cdot}/\boldsymbol{\pi}_{2,\cdot}) \\ \vdots \\ \log(\boldsymbol{\pi}_{1,\cdot}/\boldsymbol{\pi}_{a,\cdot}) \\ \log(\boldsymbol{\pi}_{2,\cdot}/\boldsymbol{\pi}_{3,\cdot}) \\ \vdots \\ \log(\boldsymbol{\pi}_{2,\cdot}/\boldsymbol{\pi}_{a,\cdot}) \\ \vdots \\ \log(\boldsymbol{\pi}_{(a-1),\cdot}/\boldsymbol{\pi}_{a,\cdot}) \end{bmatrix}.$$

Define the matrix of quantities $\mathbf{M}^{(3)} = [\mathbf{1} \log(\mathbf{M})]$. Note that $\mathbf{M}^{(3)}$ can be re-written given the clustered group of assets $\mathbf{M}^{(1)}$, $\mathbf{M}^{(3)} = [\mathbf{1} \log(\mathbf{M}^{(1)}) \log(\mathbf{M}^{(2)})]$. Let ε be the $(T \cdot \sum_{i=1}^{a-1} (a - i) \times 1)$ vector of residuals, and $\beta_i = [\beta_i^{(0)} \beta_i^{(1)} \beta_i^{(2)}]$ be the corresponding $((k + 1) \times 1)$ vector of parameters. The test for weak separability would then consist of running the estimation on the following regression

$$(6) \quad \mathbf{Y} = \begin{bmatrix} \mathbf{M}^{(3)} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{(3)} & \dots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{M}^{(3)} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{\sum_{i=1}^{a-1}(a-i)} \end{bmatrix} + \varepsilon.$$

then checking if the parameters of the assets outside the cluster are all equal to zero:

$$(7) \quad \beta_1^{(2)} = \beta_2^{(2)} = \dots \beta_{\sum_{i=1}^{a-1}(a-i)}^{(2)} = 0.$$

The three conditions based on the Varian (1982) theorem are then re-written formally as:

CONDITION 1. $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$ satisfies GARP ($U(\cdot)$ exists).

CONDITION 2. $\{(\mathbf{m}_t^{(1)}, \boldsymbol{\pi}_t^{(1)})\}_{t=1}^T$ satisfies GARP ($f(\cdot)$ exists).

CONDITION 3a. $\beta_1^{(2)} = \beta_2^{(2)} = \dots \beta_{\sum_{i=1}^{a-1}(a-i)}^{(2)} = 0$, meaning $\mathbf{M}^{(1)}$ is weakly separable in $U(\cdot)$.

Having established a necessary and sufficient weak separability condition using the price ratios of the components within the potential cluster, the problem of stochastic error leading to over rejection in the Varian (1982) test must now be addressed. To properly deal with stochasticity in the data, begin by setting aside the assumption that data are perfectly observed. Data is now be subject to two assumptions regarding the true data generating process observed with noise, and an assumption regarding variance and stability properties of the distribution of that noise. Let \mathbf{M} be the matrix of observed monetary quantity data, such that the elements of the matrix are $m_{i,t}$ with observations $t = 1, \dots, T$ and components $i = 1, \dots, L$. Define \mathbf{M}^* as the true component level generated by a weakly separable utility function under the null hypothesis there exists a weakly separable data generating function. The first assumption dictates that the observations \mathbf{M} are produced by the addition of the true null matrix \mathbf{M}^* with an independently and identically distributed, mean zero error term $\psi_{i,t}$:

$$(8) \quad m_{i,t} = m_{i,t}^* + \psi_{i,t}.$$

The second assumption is that for component $i = 1, \dots, L$ the error term $\psi_{i,t}$ is independent and identically distributed with $\mu_{\psi_i} = 0$ and variance $\sigma_{\psi_i}^2$ and has a distribution which is max-stable and min-stable; that is all a linear combination of two independent draws of its extreme values has the same location and scale parameters of the original distribution.

Less formally these assumptions dictate that violations of GARP found when testing the subutility and macro utility could be due to stochastic factors in the data; i.e. error in measurement. If these violations are not significant than the Condition 3a weak separability test can follow using the regression model (4) with smoothed quantity level instruments. However if these violations are not significant the cluster must be rejected. Advantageously, the stochastic version of the test does not over-reject as noted in Barnett and Choi (1989) if noise is identified correctly.

The two step iterative procedure developed in de Peretti (2005, 2007), and Barnett and de Peretti (2009) follows a clear logical path in dealing with the stochasticity of the data: first find the minimal adjustment that is need for the data to satisfy GARP in the sub and macro utility functions, then test if the adjustment is significant at an acceptable level. The process is begun by solving for z_{ij} , the minimal adjustment, in the quadratic program subject to the constraints determined by the definition of GARP:

$$(9) \quad obj = \min_{z_{i,t}} \sum_{t=1}^T \sum_{i=1}^L (m_{i,t} - z_{i,t})^2$$

for every $i = 1, \dots, T$; $j = 1, \dots, L$ and $\mathbf{z}_i^{(1)} = (z_{1,t}, z_{2,t}, \dots, z_{a,T})'$ subject to

$$(C.1) \quad \mathbf{z}_t R \mathbf{z}_s \Rightarrow \boldsymbol{\pi}_s \cdot \mathbf{z}_s \leq \boldsymbol{\pi}_s \cdot \mathbf{z}_t,$$

$$(C.2) \quad \mathbf{z}_t^{(1)} R \mathbf{z}_s^{(1)} \Rightarrow \boldsymbol{\pi}_s^{(1)} \cdot \mathbf{z}_s^{(1)} \leq \boldsymbol{\pi}_s^{(1)} \cdot \mathbf{z}_t^{(1)}.$$

Given the solution matrix to the optimization of (9), $\hat{\mathbf{Z}}$, the matrix of theoretical residuals $\hat{\mathbf{\Omega}} = \mathbf{M} - \hat{\mathbf{Z}}$, is the minimal adjustment need to satisfy GARP for the Condition 1 and Condition 2 restraints; C.1 and C.2¹¹. The theoretical residuals matrix $\hat{\mathbf{\Omega}}$ should then be compared to the true measurement error as defined by the difference of the observed sequence of monetary asset quantities and the true sequence of monetary asset quantities: $\hat{\mathbf{\Psi}} = \mathbf{M} - \mathbf{M}^*$. Both residual matrices are (TxL) . Only those bundles that violate GARP are adjusted, meaning only a few can be tested for significance. As this is the case the procedure is to test if the extremes of the theoretical residuals are consistent with the noise created by measurement error. If the extremes of $\hat{\mathbf{\Omega}}$ are significantly outside the possible extremes of $\hat{\mathbf{\Psi}}$ then the GARP violation is significant and the cluster is rejected.

Define the extreme values of the theoretical and true residuals as follows. For $i = 1 \dots, L$ and $(\hat{\omega}_{i,1}, \hat{\omega}_{i,2}, \dots, \hat{\omega}_{i,T})$, the i th column of $\hat{\mathbf{\Omega}}$, let $\widehat{Max}_i = \max(\hat{\omega}_{i,1}, \hat{\omega}_{i,2}, \dots, \hat{\omega}_{i,T})$ and $\widehat{Min}_i = \min(\hat{\omega}_{i,1}, \hat{\omega}_{i,2}, \dots, \hat{\omega}_{i,T})$ be the assumed stable maximum and minimum for the theoretical residuals. Similarly for the i th column of the true residual matrix $\hat{\mathbf{\Psi}}$ to be $(\psi_{i,1}, \psi_{i,2}, \dots, \psi_{i,T})$ and the stable maximum to be $Max_i = \max(\psi_{i,1}, \psi_{i,2}, \dots, \psi_{i,T})$, and the minimum is simply $Min_i = -\max(\psi_{i,1}, \psi_{i,1}, \dots, \psi_{i,T})$. If Assumption 2 holds and these minima and maxima are indeed stable, then the Fisher-Tippett theorem holds:

THEOREM Fisher-Tippett. *For $\Psi_{i,\cdot} = (\psi_{i,1}, \psi_{i,2}, \dots, \psi_{i,T})$ an independently and identically distributed sequence, and existing norming constants $a_i \in R$ and $b_i > 0$ and a nondegenerate distribution function G such that*

$$(10) \quad (Max_i - a_i)b_i^{-1} \xrightarrow{L} G ,$$

¹¹ See the Appendix 1 in Barnett and de Peretti (2009), or de Peretti (2005,2007) for the computational details of the solution procedure.

then G belongs to one of the three laws:

$$(10.III) \text{ Frechet (type III): for } \alpha_i > 0, G_3(x) = \begin{cases} 0, & x \leq 0 \\ -\exp(-x^{-\alpha_i}), & x > 0 \end{cases}$$

$$(10.II) \text{ Weibull (type II): for } \alpha_i > 0, G_2(x) = \begin{cases} \exp[-(-x^{-\alpha_i})], & x \leq 0 \\ 1, & x > 0 \end{cases},$$

$$(10.I) \text{ Gumbel (type I): for } x \in \mathbb{R}, \alpha_i = 0, G_1(x) = \exp[-\exp(-x)].$$

If there is some prior information regarding the true distributions of errors then using the Fisher-Tippet Theorem, one could determine the law of extremes and choose between the Frechet, Weibull, or Gumbel types. For simplification, we assume the errors behave according to a Guassian distribution, leading to the use of the Gumbel type law¹². Using the Gumbel type law, the tail areas for the significance test can be computed according to the p-value for the maxima:

$$(11) \quad 1 - \exp[-\exp(-y_{i,max})],$$

and the p-value for the minima:

$$(12) \quad 1 - \exp[-\exp(y_{i,min})],$$

Where $y_{i,max} = (\widehat{Max}_i - a_{i,max})b_{i,max}^{-1}$ and $y_{i,min} = (\widehat{Min}_i - a_{i,min})b_{i,min}^{-1}$. What is left is to calculate the location parameters $a_{i,max}$ and $a_{i,min}$ as well as the scale parameters $b_{i,max}$ and $b_{i,min}$ for the distribution $\Psi_{i,\cdot}$. In practice however, the true residual distribution is not observable except in a small minority of cases. An estimation of $\Psi_{i,\cdot}$ is required to continue with the procedure to determine the location and scale parameters and test the extreme values.

Two methods are proposed in Barnett and de Peretti (2009) regarding the estimation of the true residuals, both involving the use of state-space models. Equation (6) from Assumption 1 is just one form of a time invariant state-space model of the form:

¹² Another more general method using the Generalized Extreme Value distribution is suggested in Barnett and de Peretti (2009), equation 19 for the form, and Hosking, Wallis, and Wood (1985) and Hosking (1985) for parameter estimation methods.

$$(13) \quad m_{i,t} = m_{i,t}^* + \psi_{i,t}$$

$$m_{i,(t+1)}^* = Fm_{i,t}^* + c_i + \xi_{i,t}$$

In this case m_{ij}^* is the unobserved level of monetary assets, $\psi_{i,t}$ and $\xi_{i,t}$ are uncorrelated errors.

The variances $\sigma_{\psi_i}^2$ and $\sigma_{\xi_i}^2$, c_i , and F are the hyperparameters. If $F = 1$ and $c_i = 0$ then the model is linear with a trend. If the residuals are assumed to be drawn from a Gaussian distribution, and $\hat{\sigma}_{\psi_i}^2$ is the maximum likelihood estimator of the variance of $\psi_{i,t}$ then the theoretical residuals can be estimated using the minimum adjustment of residuals $\hat{\Omega}$, the location and scale parameters, and aforementioned maximum likelihood estimator of the variance of the true residuals. Using Geugan (2003) if we assume to follow a Gumbel type law, the maximum of a series of T draws has location and scale parameters defined as:

$$(14) \quad a_i = (2\ln(T))^{1/2} - \left[\frac{\ln(\ln(T)) + \ln(4\pi)}{2(2\ln(T))^{1/2}} \right]$$

$$(15) \quad b_i = (2\ln(T))^{-1/2}$$

As assumed earlier the distributions of the residuals are assumed to be min and max stable, meaning the extreme value distributions has identical location and scale parameters; $a_{i,max} = a_{i,min} = a_i$ and $b_{i,max} = b_{i,min} = b_i$. Max_i and Min_i are estimated by

$$\widehat{Max}_j = \widehat{Max}R_j = \max(\hat{\omega}_{1j}\hat{\sigma}_{\psi_j}^{-1}, \hat{\omega}_{2j}\hat{\sigma}_{\psi_j}^{-1}, \dots, \hat{\omega}_{Tj}\hat{\sigma}_{\psi_j}^{-1}),$$

$$\widehat{Min}_i = \widehat{Min}R_i = \min(\hat{\omega}_{i,1}\hat{\sigma}_{\psi_i}^{-1}, \hat{\omega}_{i,2}\hat{\sigma}_{\psi_i}^{-1}, \dots, \hat{\omega}_{i,T}\hat{\sigma}_{\psi_i}^{-1})$$

The Condition 1 and Condition 2 test are prepared with the tools described in this section.

After running a Kalman filter on the series of monetary asset quantities, the estimation of variance for the true residuals as well as a smoothed series of asset levels is retrieved. The estimated variance is then used to find the tail areas for the significance test as described above,

and the smoothed values of the levels can be used as instruments for the Condition 3 regression test.

The theoretical background of this test is grounded in the Varian (1982, 1983) nonparametric weak separability test. Building on the literature of de Peretti (2005, 2007), and Barnett and de Peretti (2009), a stochastic element is introduced to the procedure as well as a necessary and sufficient test for the final condition of weak separability, determined by the dependence of user cost price ratios within the cluster to changes in quantity levels of components outside the cluster. The following section describes the data used and the results that followed.

The Center for Financial Stability and Data Sources

User cost and quantity data for all monetary components come from the openly accessible Advances in Monetary and Financial Measures website from the Center for Financial Stability (CFS). User costs are derived as described in Barnett (1980), using the quarterly weighted low risk average effective loan rate from banks to commercial and industrial clients as a benchmark rate, and the own rates of return on the individual monetary assets. These data come from various sources, including the Federal Reserve Board's statistical surveys, Bankrate.com, iMoney.net, and the US Department of the Treasury. Short term treasury bills and repurchase agreements are seasonally adjusted by the CFS using an X-12 ARIMA procedure. See

Table 1 for an organizational chart of components into the CFS broad Divisia clusters. For a more exhaustive description of the data, see Barnett et al (2012).

To compensate for the addition and subtraction of monetary components from aggregates, such as the introduction of money market mutual funds to monetary measures in 1973, the data is first split into smaller periods, the most current period starting 1991, at which point money market savings accounts and savings accounts are grouped together under “money market accounts and savings” by the Federal Reserve Board. For time periods before 1991 these two components are treated separately and given different user costs.

Traveler’s checks and currency were combined into one monetary asset as the user costs are identical and the rare use of travelers’ checks has made them a small share of the monetary assets available. Demand deposits are kept as a separate asset despite having the same user cost price as currency and travelers’ checks.

In total there were fourteen monetary components considered for the aggregate clusters, as listed in

Table 1. Currency and travelers checks, demand deposits, and other checkable deposits are traditionally considered the most liquid. Savings, small time deposits, and money market mutual funds are considered less liquid, and included in M2. The Federal Reserve Board, having discontinued M3 in 2006, no longer considers the broader aggregate components. The CFS has included these components in a larger cluster including those components discontinued with M3, large time deposits, overnight repurchase agreements, and large time deposits, in addition to the inclusion of short term treasury bills as another source of possible liquidity.

Table 1 Divisia M4, Divisia M4-, Divisia M3 Components

LIST OF COMPONENTS AND BROAD CFS AGGREGATE CLUSTERS (1991 TO 2012)

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	X	X	X
Demand Deposits	X	X	X
Other Checkable Deposits at Commercial Banks	X	X	X
Other Checkable Deposits at Thrift Institutions	X	X	X
Savings and Money Market Accounts at Commercial	X	X	X
Savings and Money Market Accounts at Thrift	X	X	X
Small Time Deposits at Commercial Banks	X	X	X
Small Time Deposits at Thrift Institutions	X	X	X
Retail Money Market Funds	X	X	X
Institutional Money Market Funds	X	X	X
Large Time Deposits	X	X	X
Overnight Repurchase Agreements	X	X	X
Commercial Paper		X	X
3 Month Treasury Bills			X

Source: The Center for Financial Stability Advances in Monetary and Financial Measures.

Notes: X denotes inclusion of the component in the aggregate cluster.

Results

The broadest aggregate Divisia M4 was first tested for weak separability from personal consumption expenditures¹³. For this time period the minimum adjustment value suggests an acceptance of GARP for the DM4 components from consumption expenditures. The maximum adjustment value however weakly rejects weak separability only for other checkable deposits at thrift institutions. Indeed this large adjustment occurs in the latest part of the subsample; the last periods for 1982. These violations of GARP coincide with two important issues: the addition of two monetary assets not before used in the clustering of the aggregates with their own quantities and user costs, and the Savings and Loan crisis. The GARP test could be resolved by changing the sample periods to avoid such violations (as is sometime done in the parametric literature), or

¹³ In this the sequence of observed data can be represented at $\{(x_t, p_t), (m_t, \pi_t)\}_{t=1}^T$ for x_t the observed personal consumption expenditure at each observation t and the corresponding price index p_t . The test for separability of monetary assets from consumption expenditures follows the work of Binner, Bissondecal, Elger, Jones, and Mullineux (2008) on the construction of Divisia monetary aggregates for the Euro area.

by adjusting for user cost and determining if the volatility of the time period is not captured by a risk neutral user cost derivation.

Aside from interest bearing checking accounts creating violations for the maximum adjustment that satisfies GARP, all other components in DM3, DM4-, and DM4 failed to reject revealed preference. Indeed the only strong rejection for the sub period of 1974 to 1982 comes with the DM3 aggregate, where interest bearing thrift accounts reject GARP with a 0.0033 tail area.

From the end of 1982 to 1991 the absence of GARP is strongly rejected for all components within and outside DM4 aggregate. All components for DM4 and the personal consumption expenditures aggregate clearly fail to reject the axiom. Both DM3 and DM4- are rejected as satisfying GARP with violations that are too significant in their extreme values.

After August of 1991 the Federal Reserve Board stopped measuring money market deposit accounts and savings deposits as separate components. This subperiod consists of the most major revisions to the data series in the changes in how savings deposits are measured. The evidence for revealed preference is strong for this period as all three clusters pass the Conditions 1 and 2 GARP tests, and there is weak evidence for rejection only in DM3. Both the minimum and maximum adjustment tail areas for interest checking at thrifts stay close to a 5% rejection area (.0460 and 0.609 respectively), however all other components point toward admissible clustering. This is the largest sub-period in terms of observations with T=175 monthly data points.

In July of 2006 the Federal Reserve Board discontinued its M3 simple sum aggregate as well as its component data series for Commercial Paper, Overnight and Term Repurchase Agreements, and Large Time Deposits. Repos are now tracked by the New York Fed in their Primary Dealer Survey, Large Time Deposits in the H.8 Survey of Commercial Banks, and

Commercial Paper is accounted for in its own survey by the Federal Reserve Board. Due to this change in data and survey methodology of the components this contemporaneous subsample remains separate from the major revision in August of 1991. It is the shortest sub period with $T=75$ monthly observations.

The broadest clusters maintain satisfaction of the GARP test. There is evidence of DM3 rejecting revealed preference again in interest checking accounts available from thrifts. The tail area for large time deposits also dropped precipitously in this sub period (0.0936 for the maximum adjustment and 0.1407 for the minimum adjustment).

Several alternatives to the traditionally clustered Divisia aggregates were tested. The most successful alternatives clusters included variations of inclusion with retail or institutional money market funds. As seen in

Table 1, for the first and second sub periods the an altered Divisia M4 that excluded retail money market funds instead of excluding Treasury bills satisfied the Condition 1 and Condition 2 GARP tests when the DM4- cluster provided by the CFS actually failed both GARP and weak separability from 1982 to 1991. In fact the Divisia M4 excluding retail money market funds satisfies GARP and the weak separability test with similar tail areas as Divisia M3 seems to improve once retail or institutional money market funds are removed from the measure as they satisfy GARP conditions, but still fail the weak separability test.

More narrowly constructed Divisia aggregates using the CFS' methodology and benchmark rate in forms similar to MZM, and M2-ALL produced by the Federal Reserve Board in simple sum format. For most sub periods Divisia MZM and Divisia M2-ALL did not satisfy the Condition 1 and Condition 2 GARP tests. When they did they were strongly rejected by the Condition 3 weak separability test. As a comparison the narrower aggregates did not satisfy the requirements of admissibility.

Table 2 Admissibility Test Results

RESULTS OF THE WEAK SEPARABILITY TEST ON BROAD AND NARROW CLUSTERS

	1974-1982	1982-1991	1991-2006	2006-2012
	G**/WS*	G*/WS**	G*/WS**	G*/WS*
Divisia M4	G**	G*	G*	G*/WS**
Divisia M4 Less RMF	G**	G*	G*	X
Divisia M4 Less MMF	G**	X	G*	G*
Divisia M4-	G**	X	G*	G*
Divisia M3	X	X	G**	G**
Divisia M3 Less RMF	G**	G*	G**	G*
Divisia M3 Less MMF	G**	G*	G**	G*
Divisia M2 ALL	G**	X	X	G**
Divisia MZM	G**	X	X	G**

Notes: G denotes failure to reject Conditions 1 and 2, but rejection of Condition 3. G/WS denotes failure to reject all three conditions. X denotes a rejection of all conditions.

Source: Author calculations.

** Significant at the 1 percent level

* Significant at the 5 percent level.

Table 3 GARP 1974 Results

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1974:1982, T=100

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.3664 / .2034	.3668 / .2229	.4043 / .3085
Demand Deposits	.5936 / .7928	.7011 / .6189	.7182 / .6717
Other Checkable Deposits Comm.	.3387 / .6932	.3357 / .6808	.3516 / .6859
Other Checkable Deposits Thrift	.0033 / .0921	.0284 / .3362	.0329 / .3460
Savings Less MMDA Comm.	.7159 / .8148	.7273 / .8160	.7364 / .8151
Savings Less MMDA Thrift	.6232 / .5809	.4244 / .6702	.4479 / .6754
Retail Money Market Funds	.8386 / .7888	.6986 / .6666	.6952 / .6385
Small Time Deposits Comm.	.8464 / .8052	.8472 / .8071	.8466 / .7722
Small Time Deposits Thrift	.7111 / .6819	.8433 / .7990	.8427 / .7604
Institutional Money Market Funds	.8647 / .8430	.8046 / .7909	.8031 / .7786
Large Time Deposits	.7269 / .6818	.8515 / .8161	.8510 / .7853
Overnight and Term Repos	.7043 / .5824	.8176 / .7835	.8367 / .7985
Commercial Paper	(.8296 / .8070)	.6576 / .6109	.7070 / .6543
Short Term Treasury Bills	(.8618 / .8746)	(.8870 / .8870)	.8191 / .8021
Personal Consumption Expenditures	NA	NA	(.8870 / .8870)

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 4 GARP 1982 Results

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1982:1991, T=105

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.5722 / .7053	.6251 / .6375	.7183 / .7983
Demand Deposits	.7831 / .8268	.8010 / .8050	.8310 / .8357
Other Checkable Deposits Comm.	.2499 / .6234	.3110 / .7612	.7036 / .7359
Other Checkable Deposits Thrift	.0063 / .2387	.0129 / .5197	.3736 / .4750
MMDA Commercial	.8388 / .8091	.8196 / .8499	.8405 / .8566
MMDA Thrift	.8183 / .7741	.7897 / .8340	.8209 / .8434
Savings Less MMDA Comm.	.8558 / .8209	.8477 / .8651	.8558 / .8606
Savings Less MMDA Thrift	.7946 / .6477	.7673 / .8221	.7950 / .8381
Retail Money Market Funds	.7057 / .7483	.7981 / .8466	.7985 / .7560
Small Time Deposits Comm.	.8049 / .8270	.8414 / .7609	.8423 / .8302
Small Time Deposits Thrift	.7965 / .8210	.8369 / .7478	.8324 / .8245
Institutional Money Market Funds	.8079 / .8260	.8498 / .8677	.8493 / .8293
Large Time Deposits	.8142 / .8337	.8464 / .7754	.8217 / .8365
Overnight and Term Repos	.7482 / .8235	.8228 / .7754	.8675 / .8096
Commercial Paper	(.8835 / .8757)	.8235 / .7718	.8283 / .8230
Short Term Treasury Bills	(.8833 / .8864)	(.8875 / .8875)	.8783 / .8603
Personal Consumption Expenditures	NA	NA	(.8875 / .8875)

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 5 GARP 1991 Results

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1991:2006, T=175

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.5272 / .6526	.5757 / .6760	.4899 / .8618
Demand Deposits	.7707 / .8132	.7877 / .8207	.6815 / .8807
Other Checkable Deposits Comm.	.7558 / .6190	.7806 / .7368	.4533 / .8581
Other Checkable Deposits Thrift	.0609 / .0460	.1607 / .1178	.2195 / .5829
Savings and MMDA Comm.	.8679 / .8361	.8731 / .8576	.7554 / .8820
Savings and MMDA Thrift	.8258 / .8400	.8457 / .8455	.7616 / .8776
Retail Money Market Funds	.7689 / .8046	.8078 / .8286	.7584 / .8417
Small Time Deposits Comm.	.8383 / .7989	.8592 / .8322	.7902 / .8779
Small Time Deposits Thrift	.8309 / .8563	.8453 / .8603	.7804 / .8694
Institutional Money Market Funds	.8399 / .8341	.8561 / .8499	.7906 / .8652
Large Time Deposits	.7979 / .7533	.8374 / .8024	.7669 / .8634
Overnight and Term Repos	.7957 / .7396	.8380 / .8143	.7784 / .8648
Commercial Paper	(.8891 / .8897)	.8139 / .7707	.7201 / .8655
Short Term Treasury Bills	(.8909 / .8908)	(.8915 / .8915)	.7716 / .8800
Personal Consumption Expenditures	NA	NA	(.8915 / .8915)

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 6 GARP 2006 Results

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 2006:2012, T=75

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.4712 / .2766	.4682 / .6865	.3928 / .7409
Demand Deposits	.7434 / .6512	.7422 / .8184	.7106 / .8359
Other Checkable Deposits Comm.	.5822 / .2737	.5994 / .6674	.4480 / .6891
Other Checkable Deposits Thrift	.0211 / .0879	.2521 / .0975	.1784 / .5809
Savings and MMDA Comm.	.7494 / .8198	.7487 / .7681	.7239 / .8273
Savings and MMDA Thrift	.6369 / .5137	.7209 / .6597	.6679 / .6687
Retail Money Market Funds	.4971 / .4361	.5640 / .4357	.5868 / .3987
Small Time Deposits Comm.	.7025 / .6669	.7941 / .7957	.7725 / .8020
Small Time Deposits Thrift	.1928 / .5868	.5521 / .5120	.7410 / .7210
Institutional Money Market Funds	.7261 / .6974	.7562 / .6972	.7649 / .6780
Large Time Deposits	.0936 / .1407	.1430 / .3408	.3995 / .3933
Overnight and Term Repos	.1360 / .5749	.5571 / .5898	.5545 / .4260
Commercial Paper	(.8544 / .6932)	.6438 / .5427	.8524 / .5434
Short Term Treasury Bills	(.8262 / .8796)	(.8843 / .8843)	.7944 / .8400
Personal Consumption Expenditures	NA	NA	(.8843 / .8843)

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 7 Weak Separability Test

TAIL AREA RESULTS OF CONDITION 3 WEAKLY SEPARABLE TEST

Cluster	Time Period	Sample Size	MinTail Area
DM4	1971:1982	100	0.1164
	1982:1991	105	0.0231
	1991:2006	175	0.0131
	2006:2012	75	0.0678
DM4-	1971:1982	100	0.0056
	1982:1991	105	0.0000
	1991:2006	175	0.0060
	2006:2012	75	0.0041
DM4 Less RMF	1971:1982	100	0.0027
	1982:1991	105	0.0000
	1991:2006	175	0.0013
	2006:2012	75	0.0401

Notes: The smallest minimum tail area of the price ratios is reported. Tail areas below .01 are immediately rejected while those between .01 and .10 are reported.

Source: Author calculations.

Conclusion

The empirical application of a nonstochastic semi nonparametric test for weak separability as outlined in Barnett and de Peretti (2009) points to the admissibility of broad Divisia monetary aggregates, in favor of narrower aggregates. The evidence is weak for the admissibility of narrow monetary components.

The test used in this chapter provides a new branch in the varied literature on non-parametric weak separability tests, which stretches back to Varian (1982)'s use of revealed preference as a basis. Stochastic factors are taken into account using quadratic programming and the Kalman filter in order to determine when violations of utility maximizing behavior are significant. The new stochastic version of the GARP test performs well in the empirical context, identifying clusters that are appropriate according to revealed preference. The Condition 3 weak separability procedure examines the relationship between the user cost price ratios of the monetary components within the cluster and the quantities of those components outside the cluster based on the relationship of the marginal rate of substitution and price ratios. The rejection of most narrow aggregates, in favor of the broadest aggregate is informative for the use of monetary aggregates in policy and research.

Natural extensions of the research include a test to determine if bonds and securities should be included with in the "monetary cluster", as well as extensions of Barnett and de Peretti's (2009) theory to test other possibly admissible clusters of goods. The next important step would be to carry out an analysis of the recent time series of Divisia aggregates would produce interesting results especially during the time period of the 2008 financial crisis and succeeding recession in the spirit of Barnett, Offenbacher, and Spindt (1982). One could also explore the alternative approaches to the Condition 1 and 2 test using different aspects of extreme value theory to test the minimum and maximum adjustments for significance.

Furthermore it should be recognized that the risk environment in the US economy has fluctuated over several periods. An incorporation of the risk-adjusted user cost as proposed in Barnett and Wu (2005) and Barnett, Liu and Jensen (1997) would alter the user costs associated with the components and reveal more information on the structure of monetary assets. In fact most violations of GARP exist during periods of high uncertainty such as the Savings and Loan Crisis and the change in regulation for savings and loan (thrift) institutions, and the recent financial crisis. Finally an incorporation of these admissible aggregates would resolve many of the problems in macroeconomic models that have so far only used the inferior simple sum measures, often leading to the erroneous conclusion that only interest rates are important to policy and “money does not matter”.

Chapter 3: Deriving the Risk Adjusted User Cost of Money.

Summary

Barnett, Liu, Mattson, and Van den Noort (2012) construct a new data set of broad Divisia monetary aggregates freely available to the public by the Center for Financial Stability. Chapter 2 uses a state of the art methodology developed in in Barnett and de Peretti (2009) to find strong support for the admissibility of the broadest possible aggregate, Divisia M4, over narrower clusters. However these aggregates are not adjusted for risk despite the strong effects of risk on components such as commercial paper and overnight repurchase agreements. This paper details the construction of the broad risk adjusted Divisia M4 and components such as the risk adjusted user cost, the risk price of the representative agent's asset portfolio, and the risk adjustment for each component asset used in the following chapter. While the risk adjusted Divisia M4 aggregate follows closely with the risk neutral aggregate, the user costs of each component generally become larger and smoother than their risk neutral counterparts.

Introduction

Chapter 3 reviews in detail the literature on risk adjustment for Divisia monetary aggregates with a focus on the methods introduced in Barnett and Wu (2005). The estimation methods and data used in calculating the risk adjusted Divisia M4 monetary aggregate is developed in detail from the theoretical foundations. The user costs available from the CFS are not adjusted for risk, although the Divisia M4 contains inherently risk assets such as commercial paper, overnight repurchases, and money market mutual funds. It is necessary to use the data from the CFS as well as other data collected on consumption growth, equities, treasury notes, and treasury bonds in order to estimate the asset portfolio return and user cost. Given the return and user cost of the asset portfolio and the risk adjustment and risk price estimation the risk adjusted user cost can be constructed. Given the risk adjusted user cost, the construction of Divisia M4, Divisia M4-, and Divisia M3 just follows from the same routine outline in Barnett, Liu, Mattson, and van den Noort (2013). Chapter 4 focuses on the construction of Divisia M4 as its evidence for admissibility is strongest of the three in both cases¹⁴.

The estimation of the risk price by a conditional CAPM model is described and the instruments used are explained. The data and other methodologies are covered in the following section. The second section reviews the theory and construction behind the risk neutral Divisia monetary aggregate, and the third section continues with an in-depth review of the theory and construction of the risk adjusted aggregate.

Risk Adjusted User Cost of Money

In the methodologies outlined in the previous chapter on the risk neutral Divisia calculation, there is no significant treatment of the presence of risk in the US financial market affecting

¹⁴ Shown in Chapter 2 and Chapter 4.

monetary assets. It is clear that certain monetary asset components included in Divisia are subject to risk; overnight repurchase agreements, commercial paper, and money market mutual funds provide higher returns but should be considered volatile and risky in their payoff. Previous literature on Divisia assumes that agents who hold money are paid the interest quoted in time period t for asset i with certainty, or in other words that the bond or mutual fund they hold today pay off exactly as quoted when it was acquired. Risk in the market can have a positive or negative effect on the user cost if the return is more or less risky, and therefore should be accounted for when determining the user cost.

As detailed in Barnett, Liu, and Jensen (1997) and Barnett and Wu (2005), relaxing the risk neutral assumption in the derivation of user cost is not trivial, and requires some re-working of the macroeconomic theory behind the initial Barnett (1978) construction of user cost. Begin with an intertemporally non-separable utility function for a representative consumer at observations $t=1, \dots, T$

$$U(\mathbf{m}_t, c_t, c_{t-1}, \dots, c_{t-s}).$$

The consumer holds a vector of non-monetary assets $\mathbf{k}_t = (k_{1,t}, k_{2,t}, \dots, k_{K,t})'$ which do not produce services like \mathbf{m}_t , but pay out a rate of return. For now assume that the monetary assets in the overall utility function satisfy weak separability and therefore there exists an aggregate function $M(\cdot)$ that allows a rewriting with macro and sub utility functions:

$$U(\mathbf{m}_t, c_t, c_{t-1}, \dots, c_{t-s}) = V(M(\mathbf{m}_t)c_t, c_{t-1}, \dots, c_{t-s}).$$

Now suppose that consumption is subject to the budget constraints on the current and next period, for the subjective discount factor $\beta_{i,t} \in (0,1)$, the cost-of-living index p_t^* , and the real value of the consumer's asset portfolio $A_t = \sum_{i=1}^L m_{i,t} + \sum_{j=1}^K k_{j,t}$:

$$W_t = p_t^* c_t + \sum_{i=1}^L p_t^* m_{i,t} + \sum_{j=1}^K p_t^* k_{j,t} = p_t^* c_t + p_t^* A_t,$$

$$W_{t+1} = \sum_{i=1}^L R_{i,t+1} p_t^* m_{i,t} + \sum_{j=1}^K \tilde{R}_{j,t+1} p_t^* k_{j,t} + Y_{t+1}.$$

$\tilde{R}_{j,t+1}$ is the expected gross return on non-monetary assets between t and $t+1$ while $R_{i,t+1}$ is the expected gross return on assets which also provide monetary services. The consumer divides his wealth between consumption, monetary assets, and non-monetary assets at t . Income from other sources, Y_{t+1} , is received at the beginning of $t+1$. The consumer also faces the transversality condition $\lim_{s \rightarrow \infty} \beta^s p_t^* A_{t+s} = 0$. A Hamiltonian function can now be set up and solved according to the Bellman equation:

$$H_t = \max_{(c_t, m_t, k_t)} E_t \{ U(m_t, c_t, c_{t-1}, \dots, c_{t-n}) + \beta H_{t+1} \}.$$

subject to the budget constraints defined above. The first order conditions defining the expected present value of the marginal utility of consumption $\lambda_t = E_t(\frac{\partial U_t}{\partial c_t} + \frac{\partial U_{t+1}}{\partial c_t} + \dots + \frac{\partial U_{t+s}}{\partial c_t})$:

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} \tilde{R}_{j,t+1} p_t^*}{p_{t+1}^*} \right),$$

$$\frac{\partial U_t}{\partial m_{i,t}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1} R_{i,t+1} p_t^*}{p_{t+1}^*} \right).$$

So the contemporaneous user cost is then the ratio of the marginal utility of the monetary asset to the marginal utility of consumption.

$$\pi_{i,t} = \frac{\partial U_t / \partial m_{i,t}}{\lambda_t}.$$

Note that while this is not the same as the risk neutral user cost derived in Barnett (1978), many of the conclusions regarding the Divisia aggregate shown in Barnett (1980) still hold as if under the perfect certainty case. Specifically given the second of the first three Divisia equations from Chapter 2 hold, reprinted here:

$$s_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{\sum_{l=1}^L \pi_{l,t} m_{l,t}^*},$$

$$\log(\mathbf{M}_t) - \log(\mathbf{M}_{t-1}) = \sum_i \bar{s}_{i,t} (\log(m_{i,t}^*) - \log(m_{i,t-1}^*)).$$

The user cost aggregate dual can be derived from Fisher's Factor Reversal and maintains the form in the risk neutral case:

$$\Pi_t = \frac{\pi'_t m_t}{M_t},$$

and can be tracked by the Divisia user cost price index

$$\log(\Pi_t) - \log(\Pi_{t-1}) = \sum_i s_{i,t} (\log(\pi_{i,t}) - \log(\pi_{i,t-1})).$$

Now define a pricing kernel for the assets held to be:

$$Q_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{\partial U(m_{t+1}, c_{t+1}) / \delta c_{t+1}}{\partial U(m_t, c_t) / \delta c_t}.$$

such that the pricing kernel measures the positive growth of marginal utility between periods. If the price kernel is approximated as in CAPM then it can be expressed as a linear function of the real rates of return on the assets held by the consumer.

$$\begin{aligned} 0 &= 1 - E_t(Q_{t+1} \tilde{r}_{j,t+1}), \\ \pi_{i,t} &= 1 - E_t(Q_{t+1} r_{i,t+1}). \end{aligned}$$

For the excess real return on the non-monetary asset $\tilde{r}_{j,t+1} = \tilde{R}_{j,t+1} \left(\frac{p_t^*}{p_{t+1}^*} \right)$, and excess real return on the monetary asset $r_{i,t+1} = R_{i,t+1} \left(\frac{p_t^*}{p_{t+1}^*} \right)$. From these equations the theoretical risk adjusted use cost is derived.

PROPOSITION Barnett and Wu (2005). Given the excess real rates of return on a monetary asset and some arbitrary non-monetary asset available in the portfolio, and defining the covariance of the price kernel and real returns as $\omega_{i,t} = -Cov_t(Q_{t+1}, r_{i,t+1})$ and $\omega_{j,t} = -Cov_t(Q_{t+1}, \tilde{r}_{j,t+1})$, the risk adjusted real user cost of monetary asset services is

$$\pi_{i,t} = \frac{(1 + \omega_{j,t}) E_t[\tilde{r}_{j,t+1}] - (1 + \omega_{i,t}) E_t[r_{i,t+1}]}{E_t[\tilde{r}_{j,t+1}]}.$$

Note that any j non-monetary asset is useful as a benchmark asset, and is not necessarily risk free. This allows a useful generalization of the conclusions in Barnett, Liu, and Jensen (1997): a

proxy for a risk-free rate chosen from any of the non-monetary assets would function, however the econometrician should be careful to avoid biased estimations of the covariance $\omega_{i,t}$ as this would bias the user cost as well. If risk neutrality is imposed, the user cost simplifies to the certainty equivalent user cost for some risk free real return r_t^f :

$$\pi_{i,t}^e = \frac{r_t^f - E_t[r_{i,t+1}]}{r_t^f}.$$

It is then known that the the risk-free asset, which is a non-monetary asset within the portfolio has a first order condition $1 = E_t[Q_{t+1}r_t^f]$, which is also in the first order condition for the monetary asset, yielding:

$$E_t[Q_{t+1}] = \frac{1}{r_t^f},$$

and

$$\pi_{i,t} = \frac{r_t^f - E_t[r_{i,t+1}]}{r_t^f} + \omega_{i,t} = \pi_{i,t}^e + \omega_{i,t}.$$

So if the return to the monetary asset is positively correlated with the pricing kernel, the user cost is adjusted down from the certainty-equivalent user cost reflecting less risk in the monetary asset. If there exists a negative correlation the use cost would be adjusted upwards, reflecting a riskier asset.

Given the established theoretical literature on user cost adjustment for risk, what remains is an acceptable estimation of the pricing kernel to determine the change from the certainty-equivalent case.

Conditional CAPM Price Kernel

Barnett and Wu (2005) make the assumption that the price kernel is a linear function of the rate of return on the assets held by the representative agent's portfolio in the presence of intertemporal nonseparability (what was consumed in the past matters today). Other

specifications of the pricing kernel could work as well or better, but we limit our focus to the linear case for simplicity.

If we consider the share weighted rate of return on the portfolio $r_{A,t+1}$, which includes monetary and non-monetary assets then the linear approximation of the pricing kernel is $Q_{t+1} = a_t - b_t r_{A,t+1}$. This is a straightforward linear estimation of the marginal rate of substitution of consumption between time periods. From the definitions of A_t and the vectors of non-monetary and monetary assets held by the representative agent define the shares of expenditure on the two types as:

$$\begin{aligned}\varphi_{i,t} &= \frac{m_{i,t}}{A_t}, \\ \varphi_{j,t} &= \frac{k_{j,t}}{A_t}.\end{aligned}$$

Since the shares sum to one over the assets, the rate of the return for the portfolio is then

$r_{A,t+1} = \sum_{i=1}^L \varphi_{i,t} r_{i,t+1} + \sum_{j=1}^K \varphi_{j,t} \tilde{r}_{j,t+1}$. The Euler equations for the pricing kernel can be transformed by multiplying the share weights to their respective equation:

$$\begin{aligned}0 &= \varphi_{j,t} - E_t[Q_{t+1} \tilde{r}_{j,t+1} \varphi_{j,t}], \\ \pi_{i,t} \varphi_{i,t} &= \varphi_{i,t} - E_t[Q_{t+1} r_{i,t+1} \varphi_{i,t}].\end{aligned}$$

If we define the user cost of the wealth portfolio of the representative agent to be the sum of the expenditure shares of each component by its corresponding user cost, and recalling that the user cost of non-monetary assets is zero as it is providing a rate of return and not a monetary service in the utility function, then there is a simple term for the user cost dual of A_t ,

$$\Pi_{A,t} = \sum_{i=1}^L \pi_{i,t} \varphi_{i,t}.$$

Summing up the second Euler equation above yields the identity:

$$\Pi_{A,t} = \sum_{i=1}^L \pi_{i,t} \varphi_{i,t} = 1 - E_t[Q_{t+1} r_{A,t+1}].$$

If there is an acceptable estimation for the pricing kernel, then the user cost dual for the wealth portfolio is solvable with the data. Furthermore if we have some locally risk-free non-monetary asset with return r_t^f , the certainty equivalent user cost becomes:

$$\Pi_{A,t}^e = \frac{r_t^f - E_t[r_{A,t+1}]}{r_t^f}.$$

Given the certainty equivalent user cost of the portfolio and the estimation of the user cost of the portfolio with risk through the price kernel, and the covariance of the observed real rates of return, one can determine the risk adjusted user cost of an individual asset.

PROPOSITION Barnett and Wu (2005). If there is an arbitrary locally risk free non-monetary asset with real interest rate r_t^f , if the pricing kernel is linearly approximated using the real rate of return on the wealth portfolio A_t , $r_{A,t+1}$, such that $Q_{t+1} = a_t - b_t r_{A,t+1}$, and if we define the user costs and certainty equivalent user costs of the monetary asset and the wealth portfolio to be $\pi_{i,t}$, $\pi_{i,t}^e = \frac{r_t^f - E_t[r_{i,t+1}]}{r_t^f}$, $\Pi_{A,t}$, and $\Pi_{A,t}^e = \frac{r_t^f - E_t[r_{A,t+1}]}{r_t^f}$, then for $\beta_{i,t} = \frac{Cov_t(r_{A,t+1}, r_{i,t+1})}{Var_t(r_{A,t+1})}$ the risk adjusted user cost of asset i at time observation t is:

$$\pi_{i,t} - \pi_{i,t}^e = \beta_{i,t} (\Pi_{A,t} - \Pi_{A,t}^e).$$

$\beta_{i,t}$ in this case can be thought of as the risk exposure of the asset portfolio at the observed time period. As $\beta_{i,t}$ gets larger through more risk exposure, the risk adjustment to the user cost must also necessarily increase. Similarly if $\beta_{i,t}$ is very small at the observation for the component then the risk adjustment is be relatively small.

In order to estimate $\beta_{i,t}$ above, the growth rate of consumption is used as an estimator in the conditional CAPM model $Q_{t+1} = a_t - b_t r_{A,t+1}$, where the parameters a and b are time varying.

Following the methods in Cochrane (2005) and Lettau and Ludvigson (2001) an instrument must be chosen to estimate the time varying parameters. The parameters a and b cannot be estimated using unconditional expectations as is done with unconditional CAPM models as the unconditional model does not necessarily simplify to a conditional CAPM. We therefore cannot count on estimating Q_{t+1} just using expectations of the given time period. If the parameters a and b are determined by another dynamic vector of variables then a and b can be estimated as they rely on changes over time. The conditional CAPM model then becomes $Q_{t+1} = a(z_t) - b(z_t)r_{A,t+1}$. In the CAPM literature employs such variables as price to earnings ratios and book to market values. We restrict our analysis to one time varying estimator, the growth rate of consumption and a constant so that the vector $z_t = [1 \quad \Delta c_t]$ for Δc_t the observed growth rate of consumption from the previous period $(t - 1)$ to the present t .

The justification for the growth rate of consumption as a parameter estimator derives from the nature of the use of monetary assets as described in the representative agent problem. Recall that the Q_{t+1} is interpreted as the marginal utility growth of consumption; it is not too unreasonable to consider the changes in levels of consumption over time as an instrument for the time varying parameters of the linear approximation. Monetary assets are used to facilitate consumption in the present and future time periods through their liquidity service. Alternatively the forecast expectations of the consumer's consumption growth could be used, but for simplicity we assume consumer's determine how much consumption they need for the next period is based on how their consumption grows from the previous period. Advantageously the unconditional CAPM could account for the inclusion of other instruments as z_t is a vector. Capital investment or productivity or changes in income proposed in Lettau and Ludvigson (2001) are alternative instruments to be used in estimation.

Using the conditional CAPM estimated of the parameter $b(z_t)$, the aggregate user cost $\Pi_{A,t}$ is estimated given the relationship $\Pi_{A,t} = \Pi_{A,t}^e + b(z_t)Var_t(r_{A,t+1})$ without yet needing all the individual user costs. Taking the covariance of the pricing kernel estimation yields $Cov_t(Q_{t+1}, r_{A,t+1}) = -b(z_t)Var_t(r_{A,t+1})$ giving us the last piece of information necessary to carry out the risk adjustment characterized in Proposition 3. In more direct terms the calculation for the risk adjusted user cost then becomes:

$$\hat{\pi}_{i,t} = \pi_{i,t}^e + \beta_{i,t}(\hat{\Pi}_{A,t} - \Pi_{A,t}^e).$$

Once the risk adjusted user costs are determined from the estimation the broad Divisia aggregates can be produced in a manner similar to Barnett et al (2013). In order to determine the benchmark rate a collection of monetary, non-monetary, and other returns are included in an envelope method. The non-monetary asset rates of return are included in the determination of the benchmark rate (see Corollary 1 to Proposition 2 in Barnett and Wu (2005)). Since the aggregates are now constructed taking risk into account the moderately risky return to bank loan rate was included, consistent with the method in Offenbacher and Shachar (2011). While the risk neutral US broad Divisia aggregates used the low risk bank rate, we use the moderate risk bank rate to reflect new environment for the user costs. With the estimation methodology provided, the data must now be determined and collected.

Data, Forecasts, and Adjustments

All levels are converted to real per capita levels using a cost of living index and population measure. The CPI is used to deflate the monetary level and the total civilian population of the United States is used to determine the representative agent holdings of monetary and non-monetary components. Real per capita levels must be used in compliance with

the representative agent theory supporting the risk adjustment and the weak separability tests as seen in the lifetime utility function and budget constraints in the previous section.

With the theoretical foundations reviewed in the previous sections it remains to take the model to the data. The data source for user cost and quantity components of monetary aggregates is the CFS Advances in Monetary and Financial Measurement web site, which provides not just the risk neutral broad Divisia aggregates, but their component user costs, rates of return, and quantities. For the non-monetary assets held in the portfolio this is the only needed source. For a detailed treatment of the construction of this data set and its sources see Barnett, Liu, Mattson, and van den Noort (2013).

For non-monetary assets a different source is required. Data for treasury bonds and treasury notes were collected through the Monthly Survey of Public Debt which included both the levels and interest rate returns on the assets. The levels of bonds and notes were seasonally adjusted using the X-12 ARIMA procedure available through the US Census Bureau website and SAS. Equity levels are available from the Z.1 Flow of Funds survey under “Corporate Equities Held by Households and Nonprofits”. The levels were seasonally adjusted using the X-12 ARIMA process and expanded to monthly data from quarterly data. The rate of return is considered to be the six month return on the S&P 500 stock index. The six month return was chosen in order to compare the rate of return on holding equities compared to the six month rate of returns on interest checking, savings deposits, and certificates of deposits.

Other interest data are considered for the benchmark rate, as suggested in Offenbacher and Shachar (2011). The bank loan rate is a useful inclusion in that banks do not offer better interest on their loans to consumers and firms than they would offer to pay on their deposits. Therefore the inclusion of a bank loan rate helps maintain positive user costs to be paired with assets. In the risk neutral case as developed in Barnett, Liu, Mattson and van den Noort (2013),

for broad US aggregates there exist only rare exceptions to the loan rate acting as the benchmark. Therefore an adjustment procedure ensuring a non-zero user cost of adding a fixed amount of basis points to the benchmark is not necessary given the loan rate always exceeds the returns of bank interest offers (see Anderson and Jones (2012)). However, given the volatile and often high returns to equities we found it necessary to include the adjustment, adding 100 basis points to the benchmark return as the returns to equities on some observations did in fact exceed the loan rate by banks. The bank loan rate is the quarterly weighted average effective loan rate for moderately risky commercial and industrial loans. The data were converted from quarterly to monthly data.

Excess rates of return and the price index are forecast using a trend smoothing procedure, while equities are forecast with a simple AR(2) process. The AR(2) lag length was chosen through the Akaike information criterion which is not uncommon for stock forecasts. The representative agent is then basing his expectations on the past smoothed values of the excess rate of return and price index levels, and the previous two periods of equity returns. These forecast data are the returns used to calculate the current period user cost, and are paired with current period levels as shown in the budget constraints.

All data, transformations, and forecasts used are easily accessible. Level and interest data that cannot be collected from the Center for Financial Stability's website, such as the i-Repo index is available through the Wrightson i-Cap website after a brief and free registration process. The forecasts, quarterly to monthly transformations and seasonal adjustments were completed in SAS while the excess return shares, real per capita adjustments, variance and covariance calculations, and other estimations related to the database are computed using Excel.

User Cost Adjustment Results

The risk adjustment itself is relatively larger than that used in the unconditional CAPM adjustment literature drawing from Barnett, Liu, and Jensen (1997). In fact Binner, de Peretti,

and Elger (2002) find small estimated covariances using unconditional CAPM estimation for bonds, shares, and unit trusts in their broad Divisia aggregate of the United Kingdom; .00012687, 0.00017035 and 0.00019376 respectively. The following tables display the expenditure shares of monetary and non-monetary financial assets used in calculating the asset portfolio gross rate of return, the gross rate of return of the asset portfolio, the user cost risk adjustments for all monetary assets, the certainty equivalent and risk adjusted user cost for the asset portfolio Π_t^e and $\hat{\Pi}_t$. The risk price \hat{b}_t is also not relatively small, and given its use in determining the aggregate user cost of the portfolio is not trivial compared to previous literature. The user cost adjustment increases and smooths the user costs of components, as seen in the figures below. For the broad aggregate components such as large denomination time deposits, short term treasury bills, and commercial paper there still exists volatility with risk adjustment sometimes increasing user cost by a large magnitude as in Figure 11 Risk Adjusted User Cost Large Denomination Time Deposits and Three Month Treasury Bills and Figure 12 Risk Adjusted User Cost Commercial Paper and Overnight Repurchase Agreements.

The risk adjustment results of the component return and portfolio return reveal differences in the narrowest monetary instruments and broadest. Currency, traveler's checks, demand deposits, and the recent combined measure of savings accounts and money market demand accounts yield negative risk adjustments while the broader aggregate components from small denomination time deposits to short term treasury bills have positive risk adjustments. This is in line with the expectation of Barnett and Wu (2005) that assets with more risk face an increase in user cost price based on their risk exposure, as measured by the $\beta_{i,t}$. These risk adjustments are not small for broader aggregates as shown in Figure 22 Risk Adjustment for Large Denomination Time Deposits, Overnight Repurchase Agreements, Commercial Paper, and Three Month Treasury Bills. Interestingly in the periods from 1974 to 1982 the risk adjustments

for money market demand accounts and savings accounts without money market demand accounts switches from a negative risk adjustment to a positive; see Figure 7 Risk Adjusted User Cost Money Market Demand Accounts and Figure 6 Risk Adjusted User Cost Savings (without Money Market Demand Accounts). This flip in risk adjustment could be leading to the violations in the risk adjusted GARP test to be described in the chapter. After 1991 the savings accounts with money market demand accounts included in the measure return to a negative risk adjustment reflecting the relative non-riskiness of that monetary instrument.

Interest bearing checkable deposits yield positive risk adjustments relative to the overall asset portfolio. The returns to interest bearing checking accounts are the smallest among the set of portfolio returns, and are fairly close to savings account returns. The return on savings and other monetary assets then outweighs the return on the more liquid and return yielding interest checking leading to a decreased adjustment in savings and an increased adjustment in interest checking. The interest checking adjustment remains small for periods after 1982, and it should be noted that historical data before the 1980s on interest checking is taken from a constant measure used in Anderson and Jones (2011). The large user cost adjustment in the 1974 to 1982 period of interest checking therefore could be an artifact of the estimated data set. However the conclusion still remains in general that the risk adjustments are smaller relative to the broad components for narrower monetary assets.

Graphs of expenditure shares, user costs, the risk price estimate, the risk adjustment, and the year-over-year growth of the Divisia M4 aggregate are printed below.

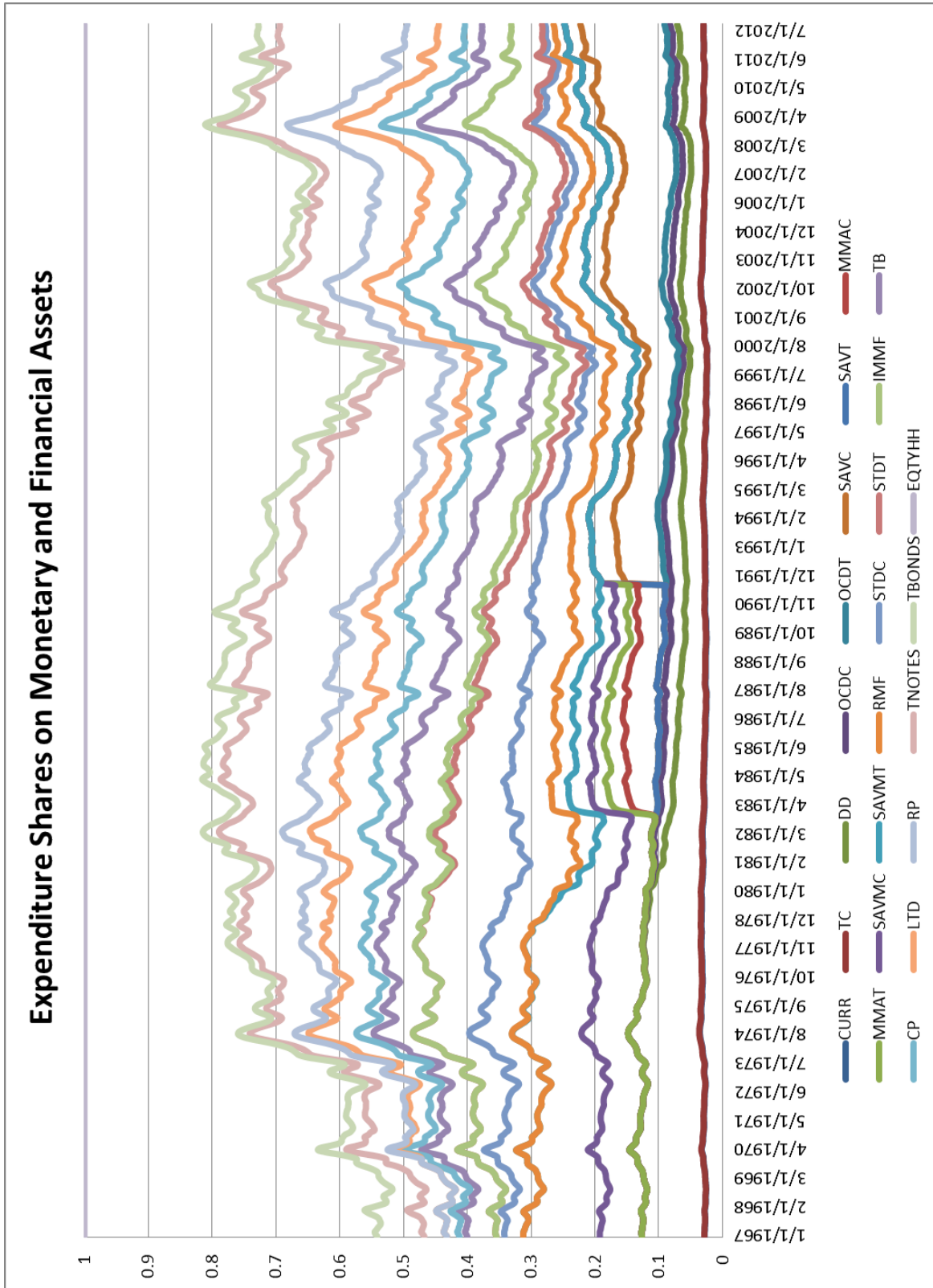


Figure 1 Expenditure Shares on Monetary and Financial Assets

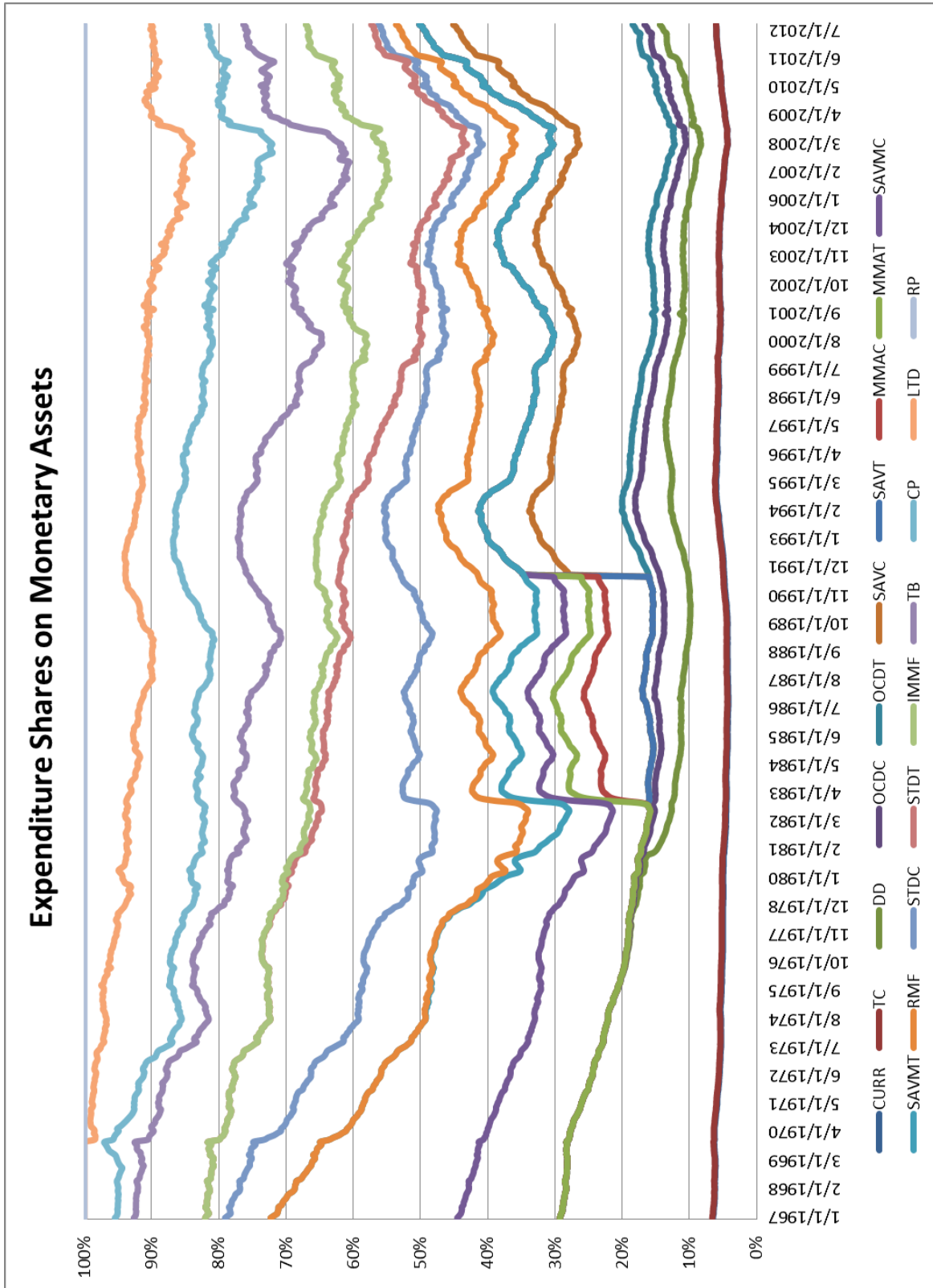


Figure 2 Expenditure Shares on Monetary Assets

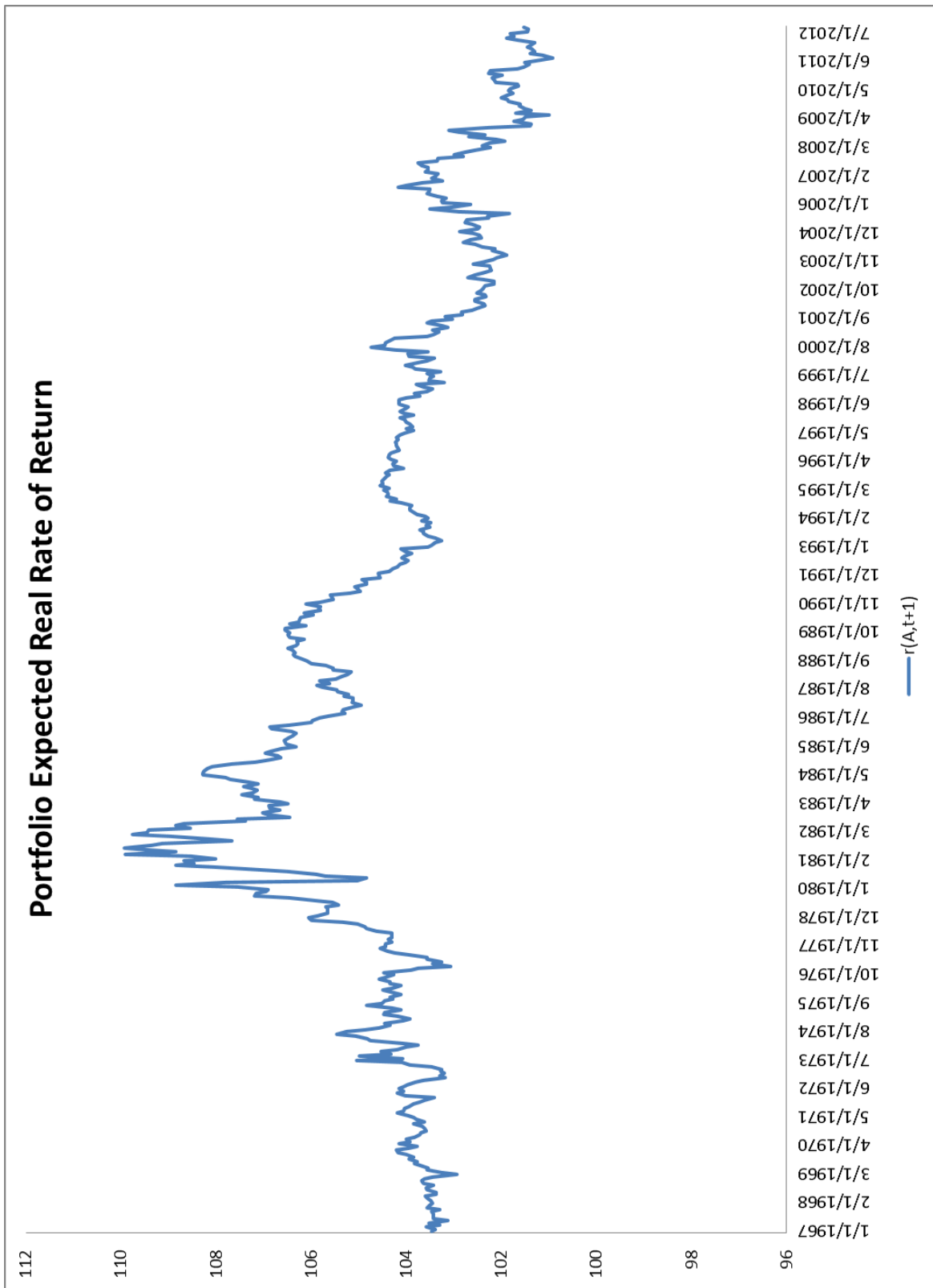


Figure 3 Expected Real Rate of Return on Asset Portfolio

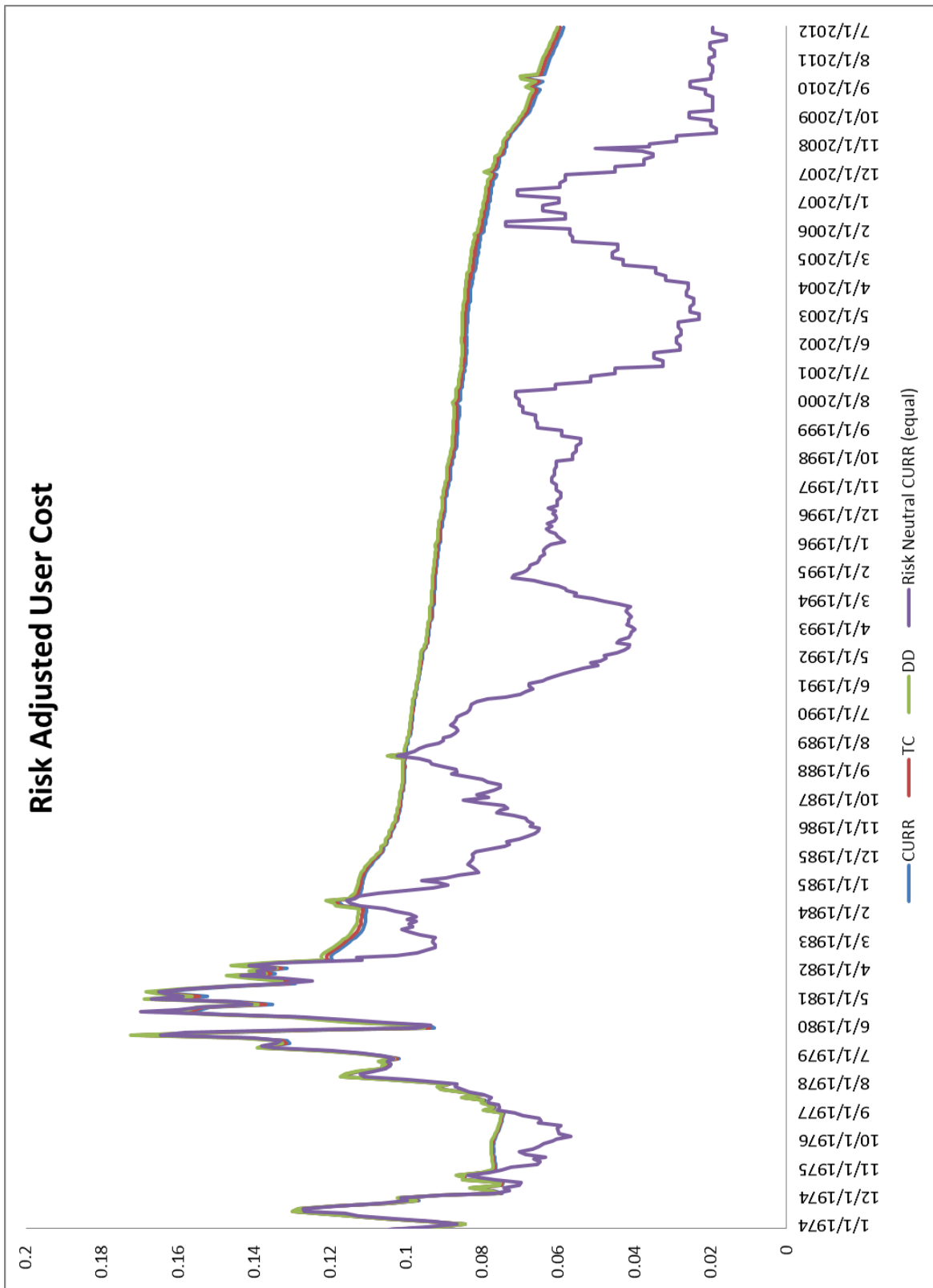


Figure 4 Risk Adjusted User Cost for Currency, Traveler's Checks, and Demand Deposits

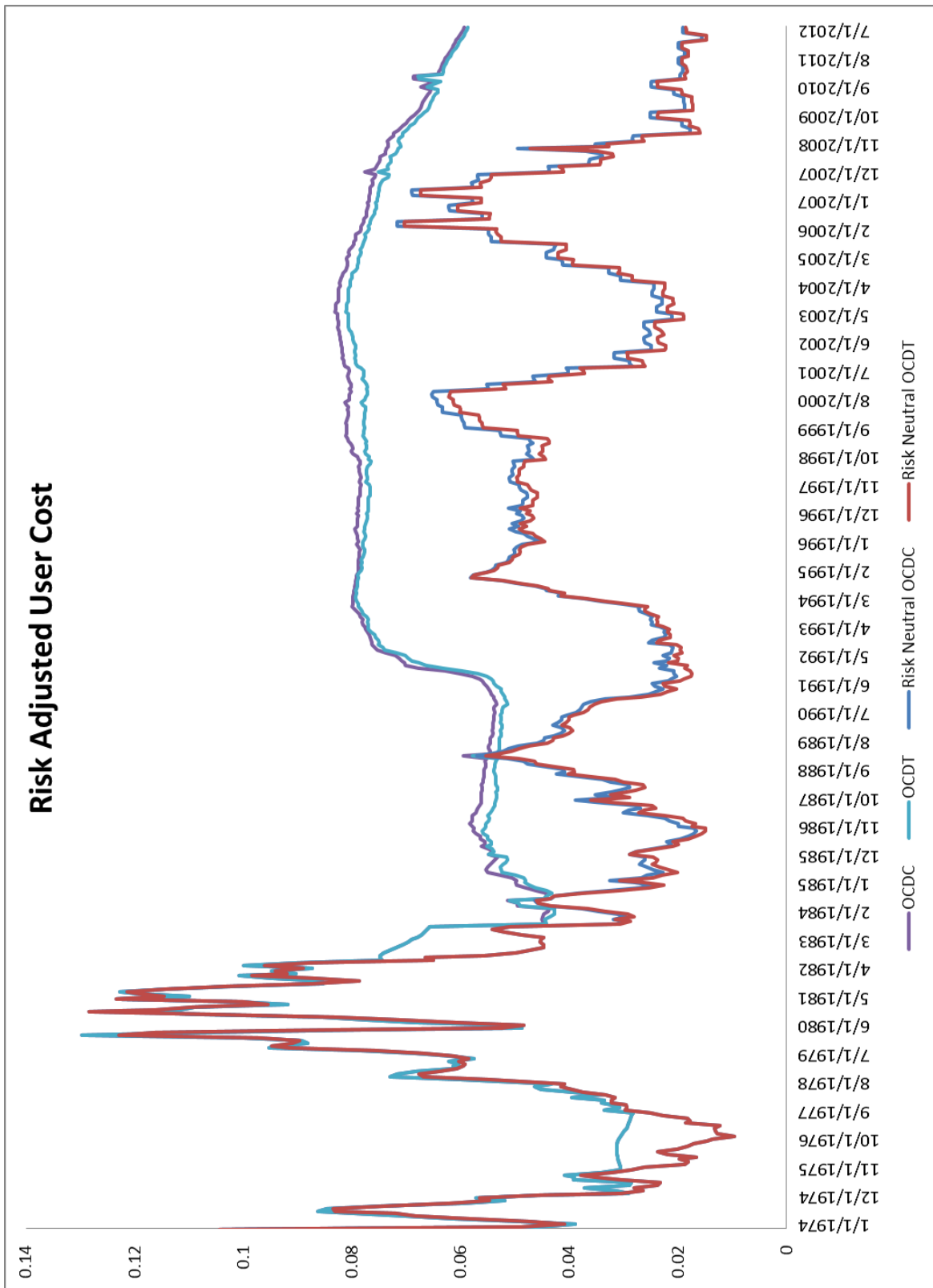


Figure 5 Risk Adjusted User Cost Other Checkable Deposits

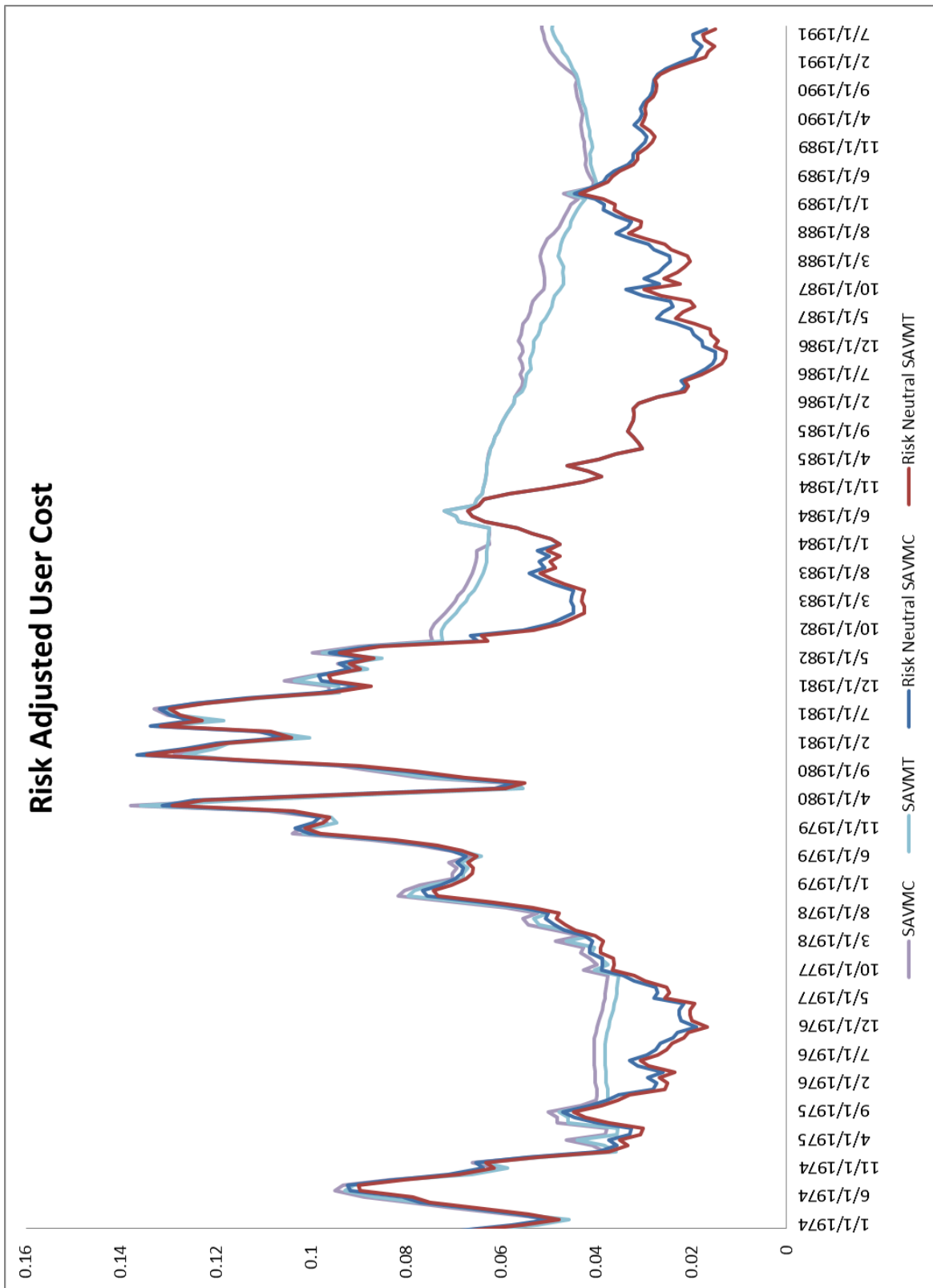


Figure 6 Risk Adjusted User Cost Savings (without Money Market Demand Accounts)

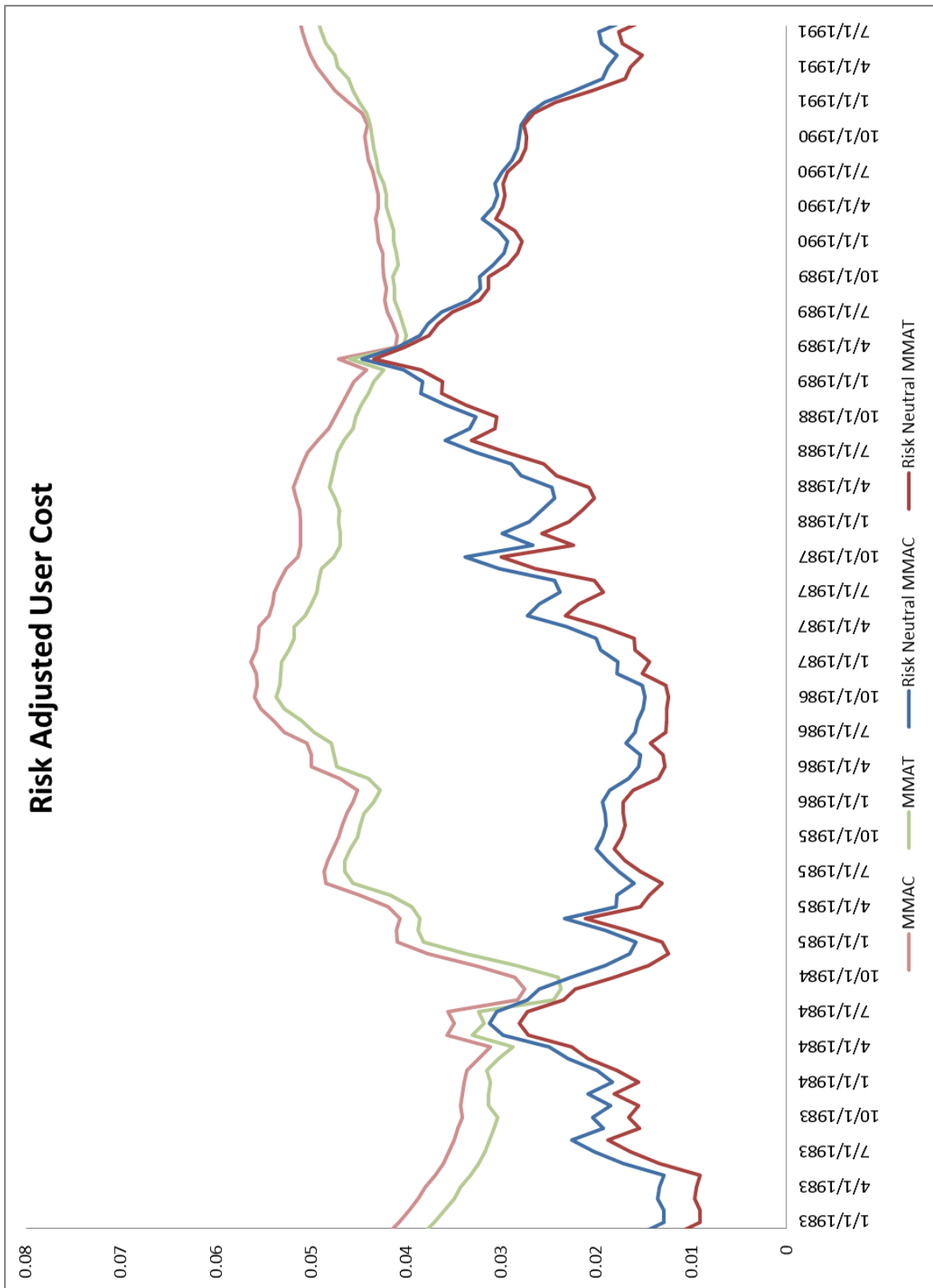


Figure 7 Risk Adjusted User Cost Money Market Demand Accounts

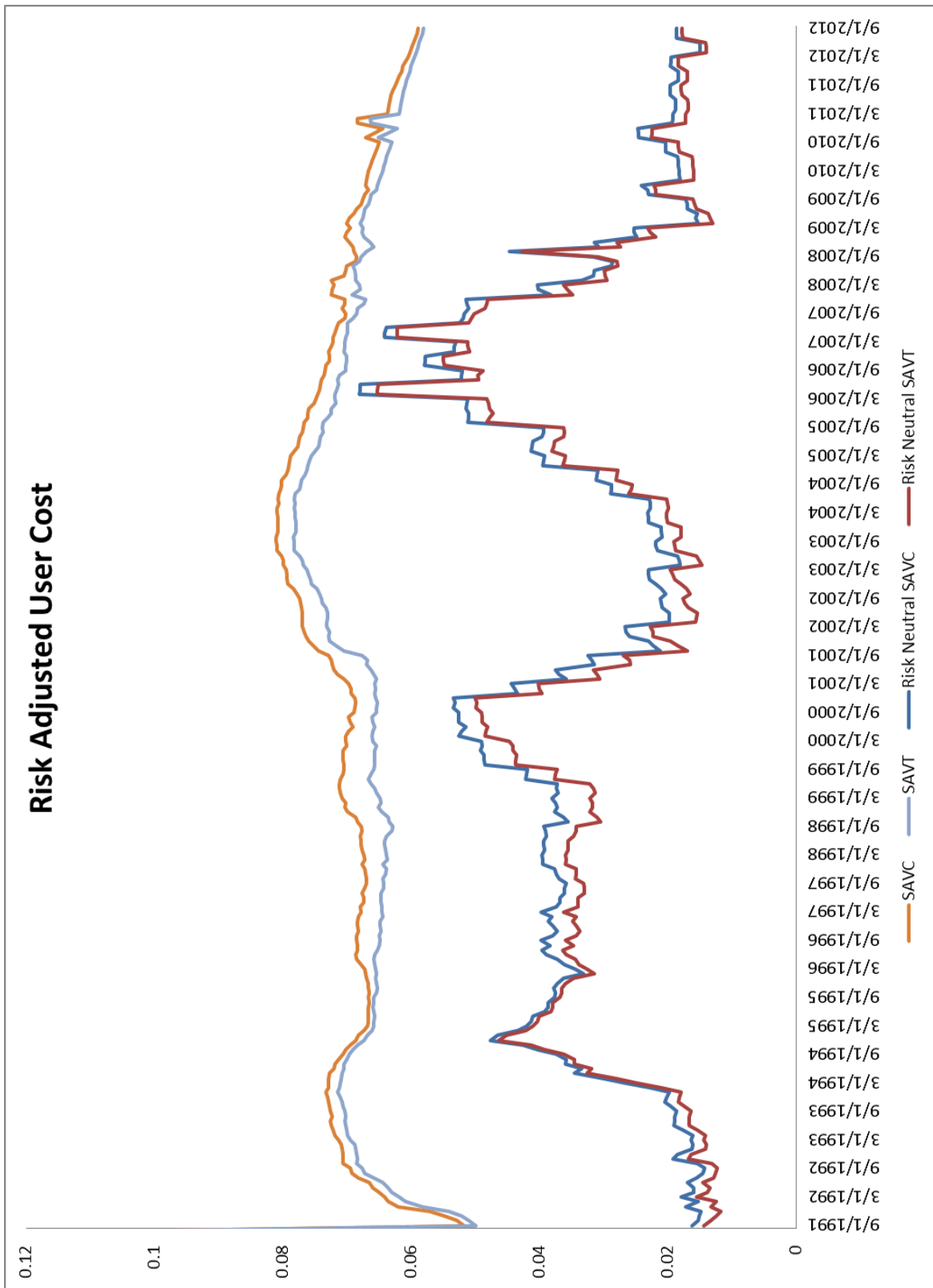


Figure 8 Risk Adjusted User Cost Savings Accounts (with Money Market Demand Accounts)

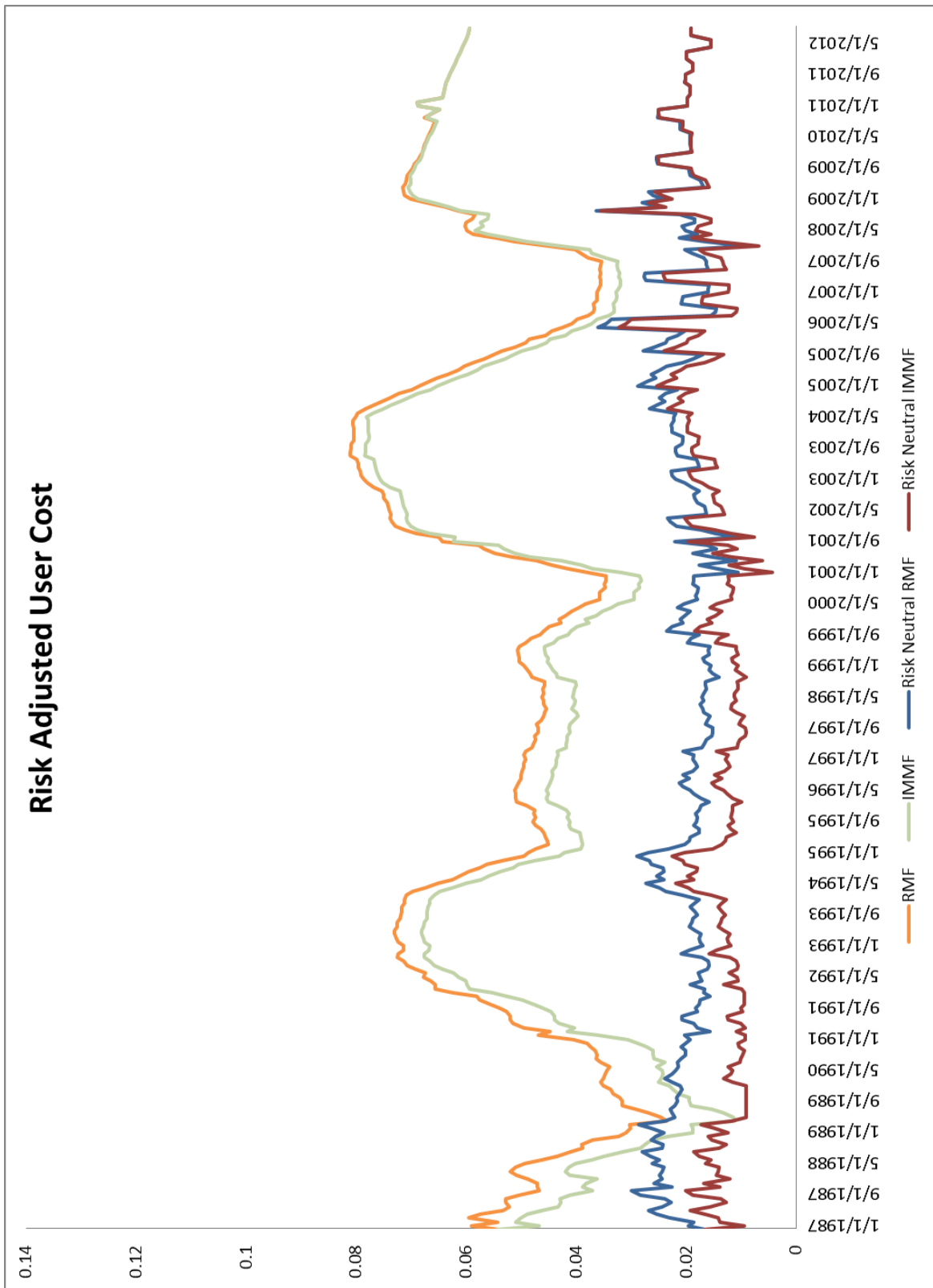


Figure 9 Risk Adjusted User Cost Money Market Mutual Funds (Retail and Institutional)

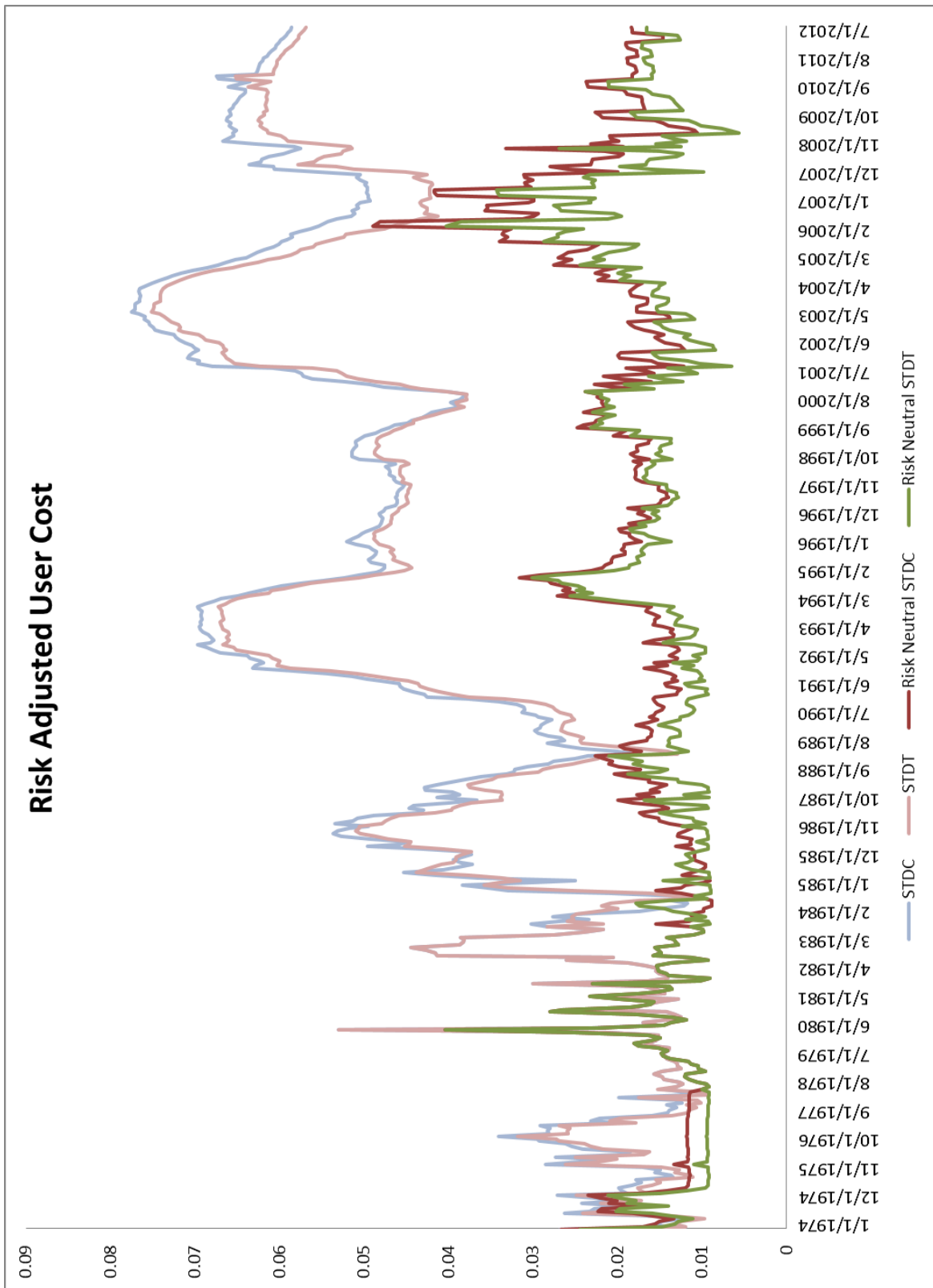


Figure 10 Risk Adjusted User Cost Small Denomination Time Deposits

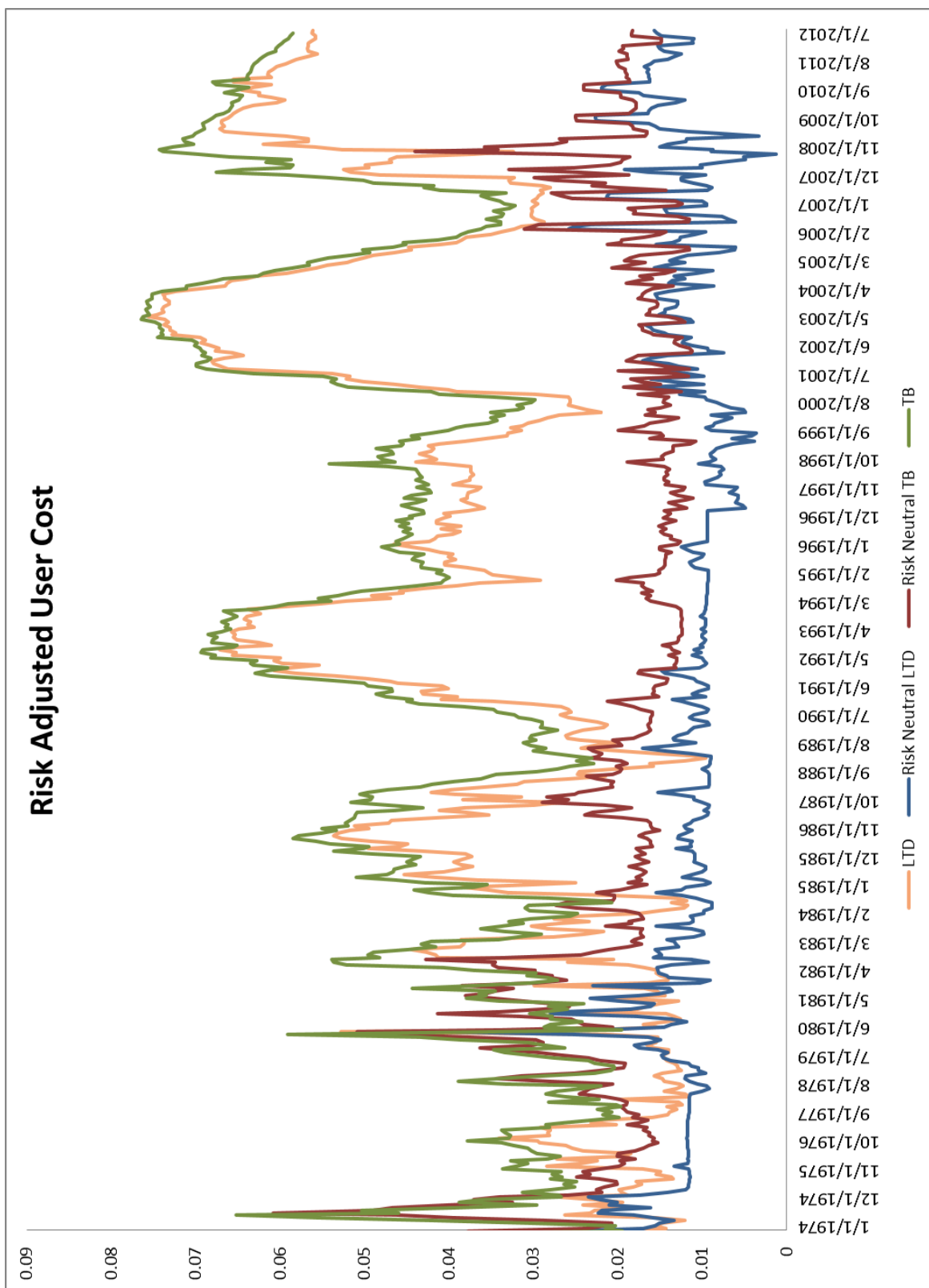


Figure 11 Risk Adjusted User Cost Large Denomination Time Deposits and Three Month Treasury Bills

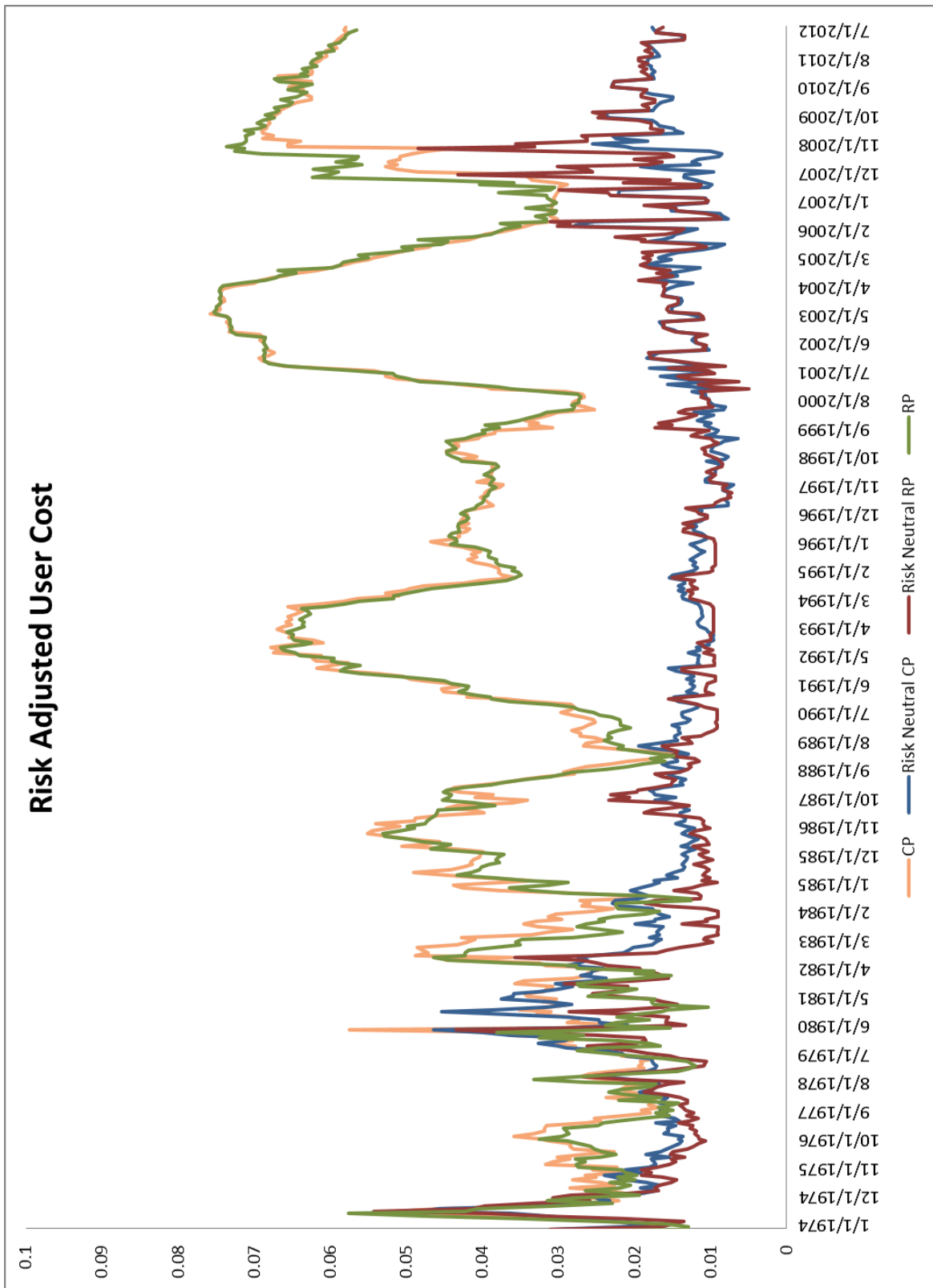


Figure 12 Risk Adjusted User Cost Commercial Paper and Overnight Repurchase Agreements

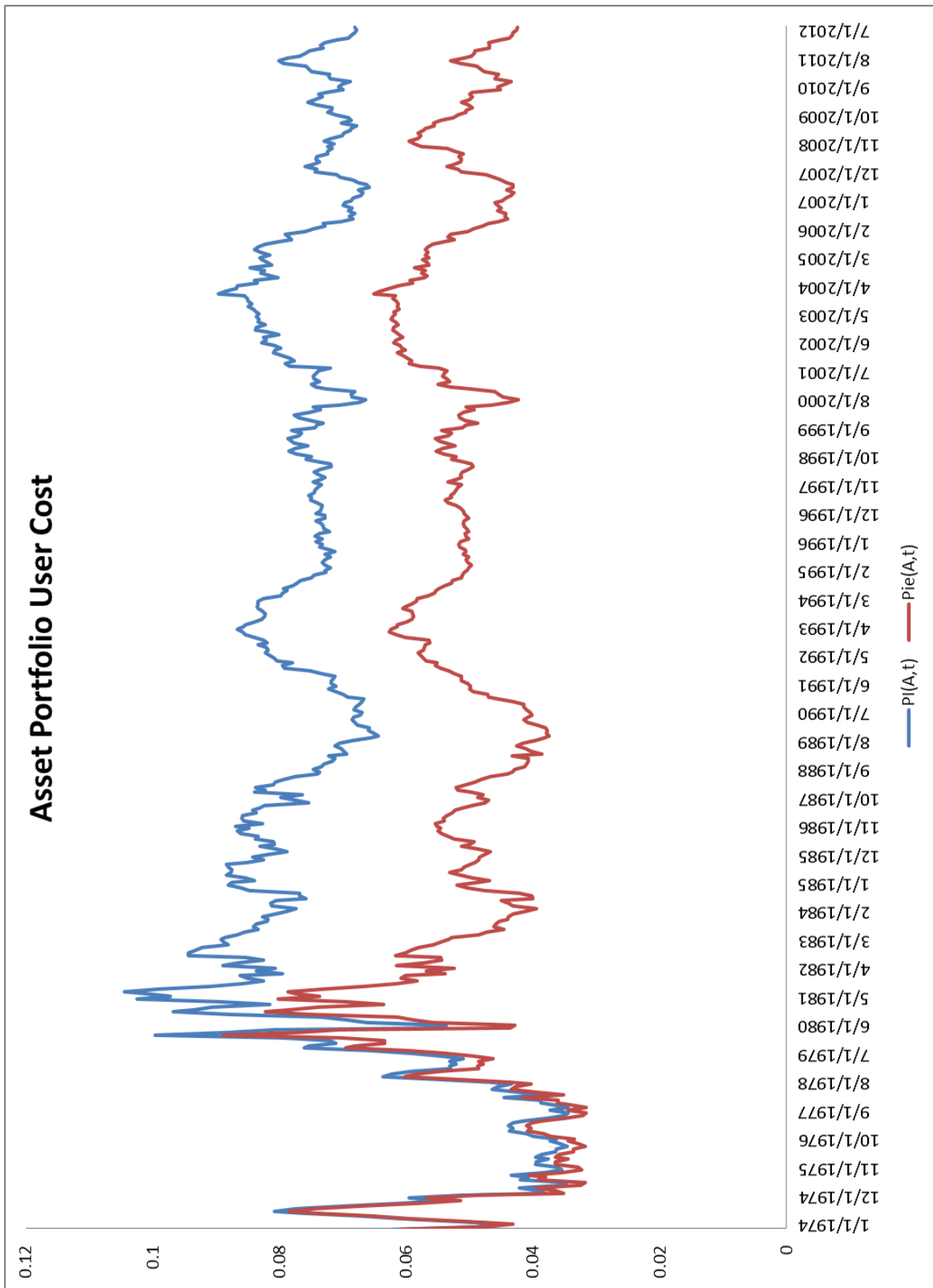


Figure 13 Portfolio User Cost, Certainty Equivalent and Estimated Risk Adjusted.

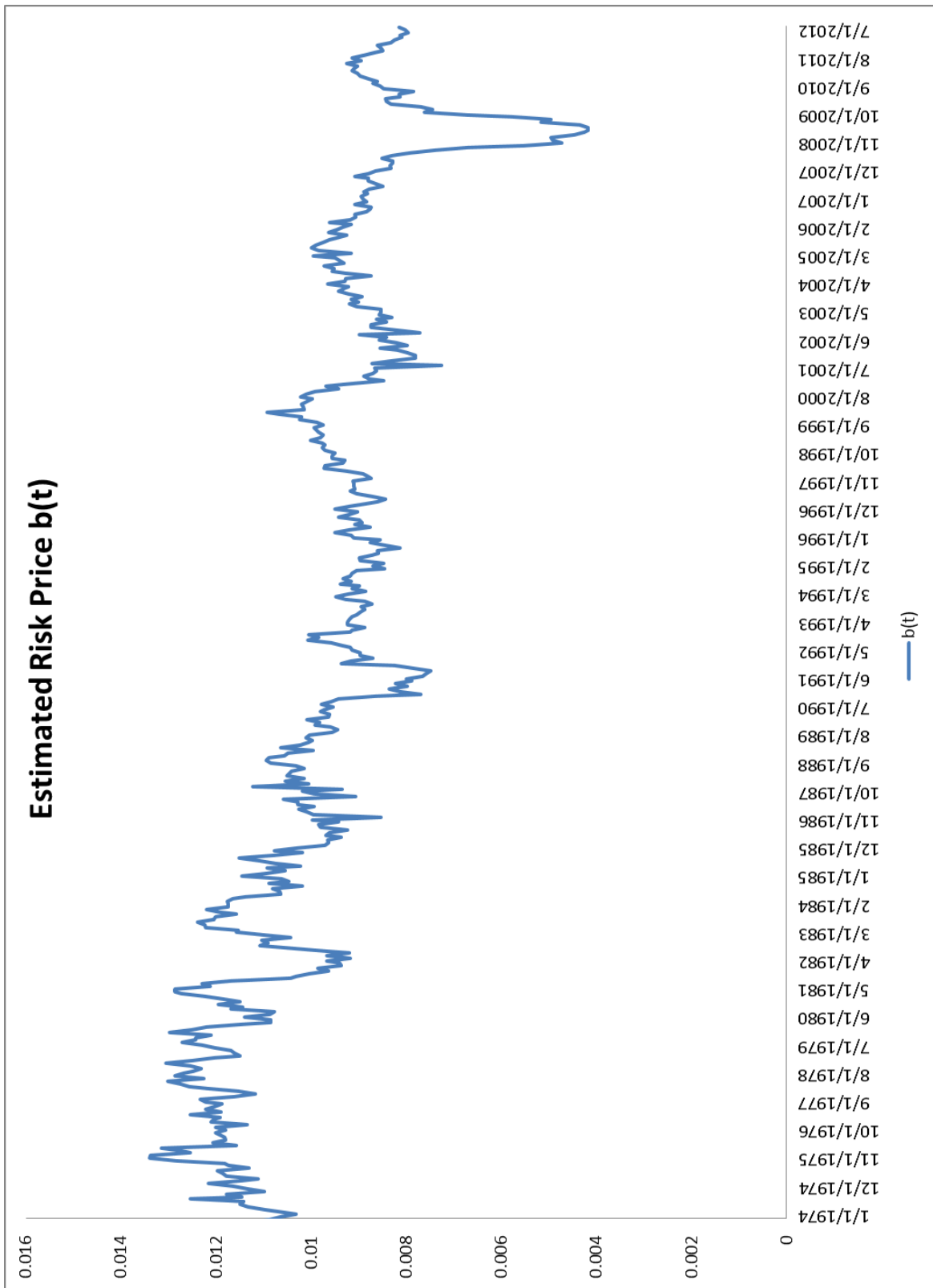


Figure 14 Estimated Risk Price Adjustment

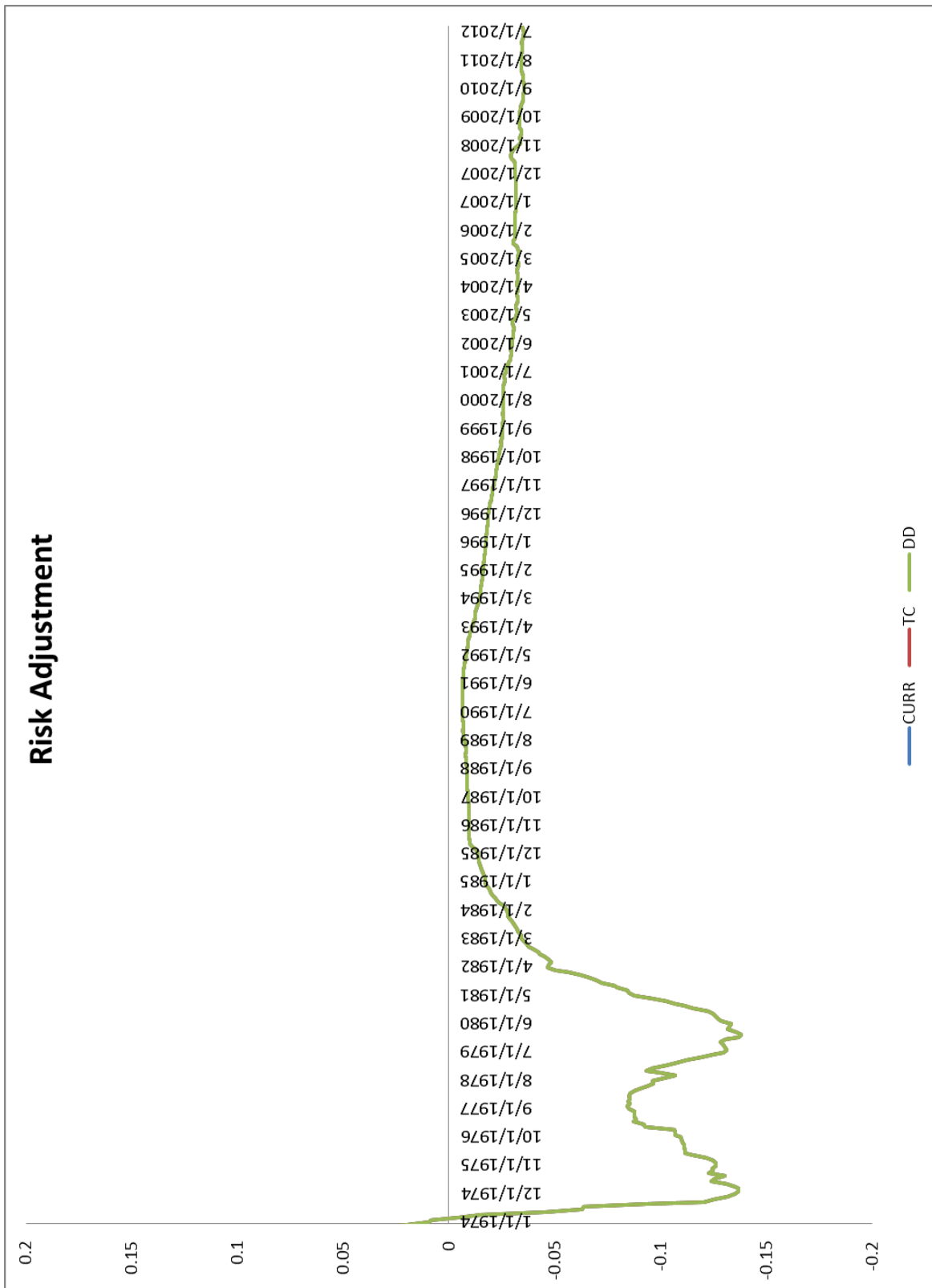


Figure 15 Risk Adjustment for Currency, Travelers Checks, and Demand Deposits

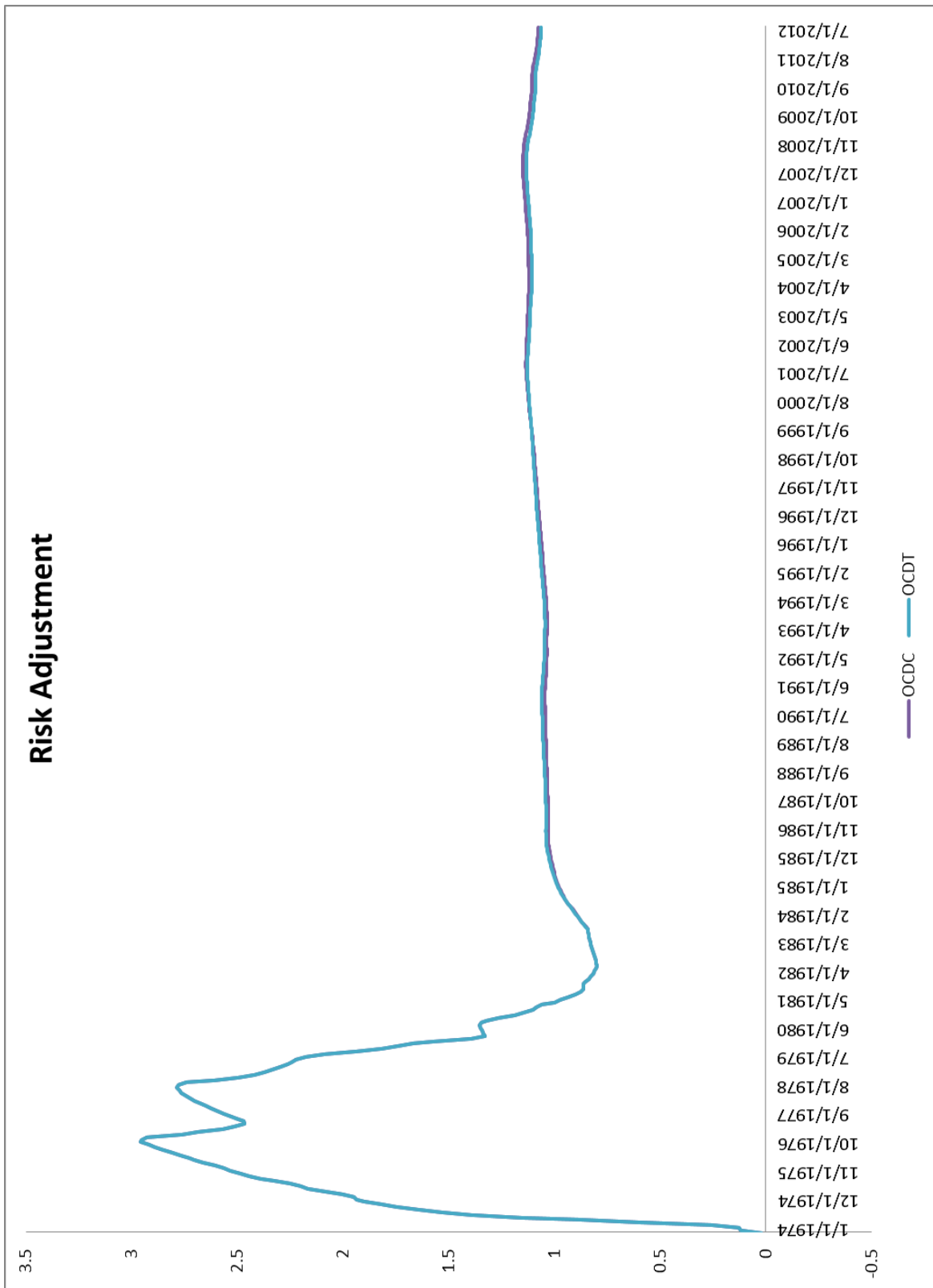


Figure 16 Risk Adjustment for Other Checkable Deposits

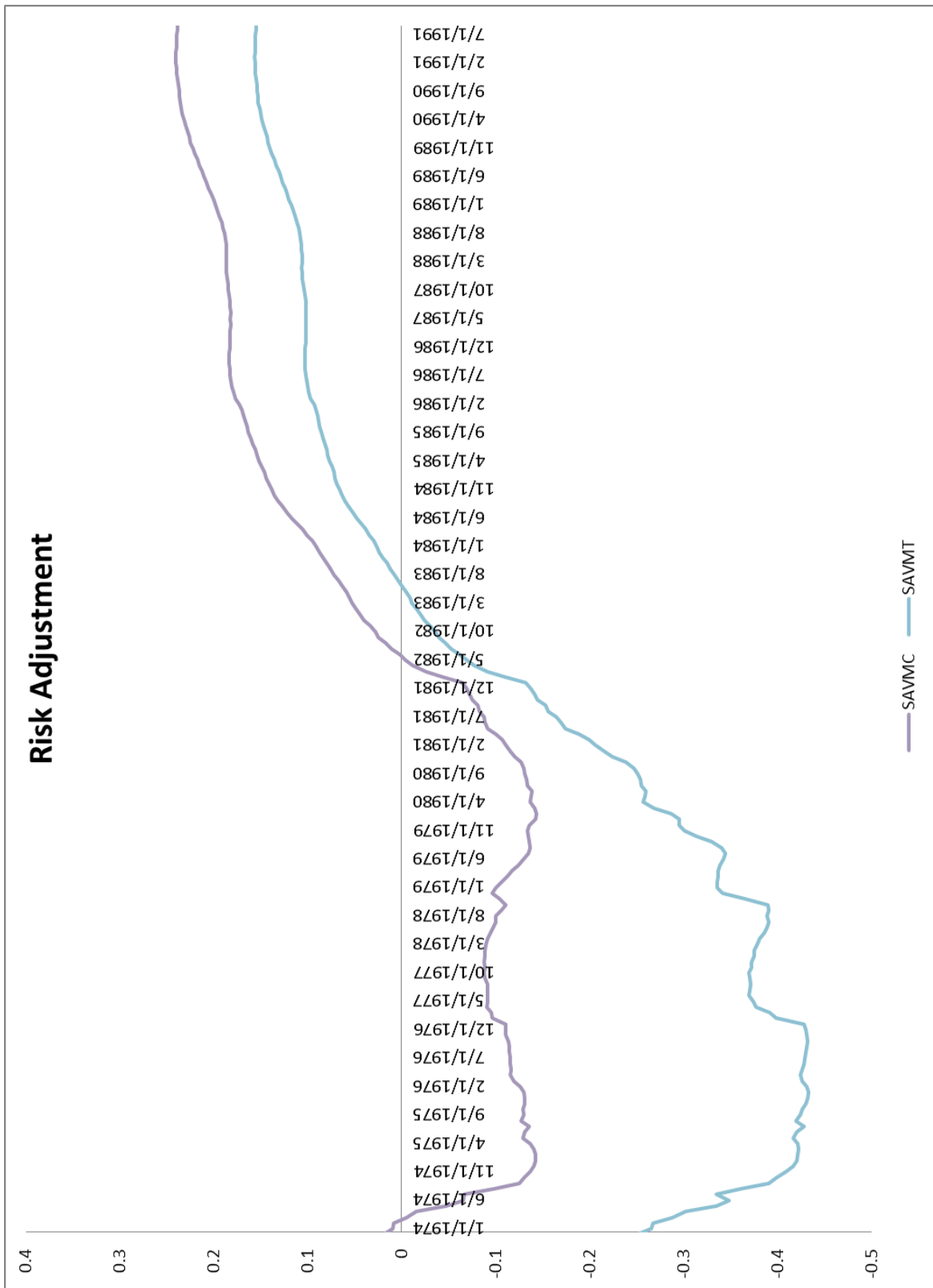


Figure 17 Risk Adjustment for Savings Accounts (Without Money Market Demand Accounts)

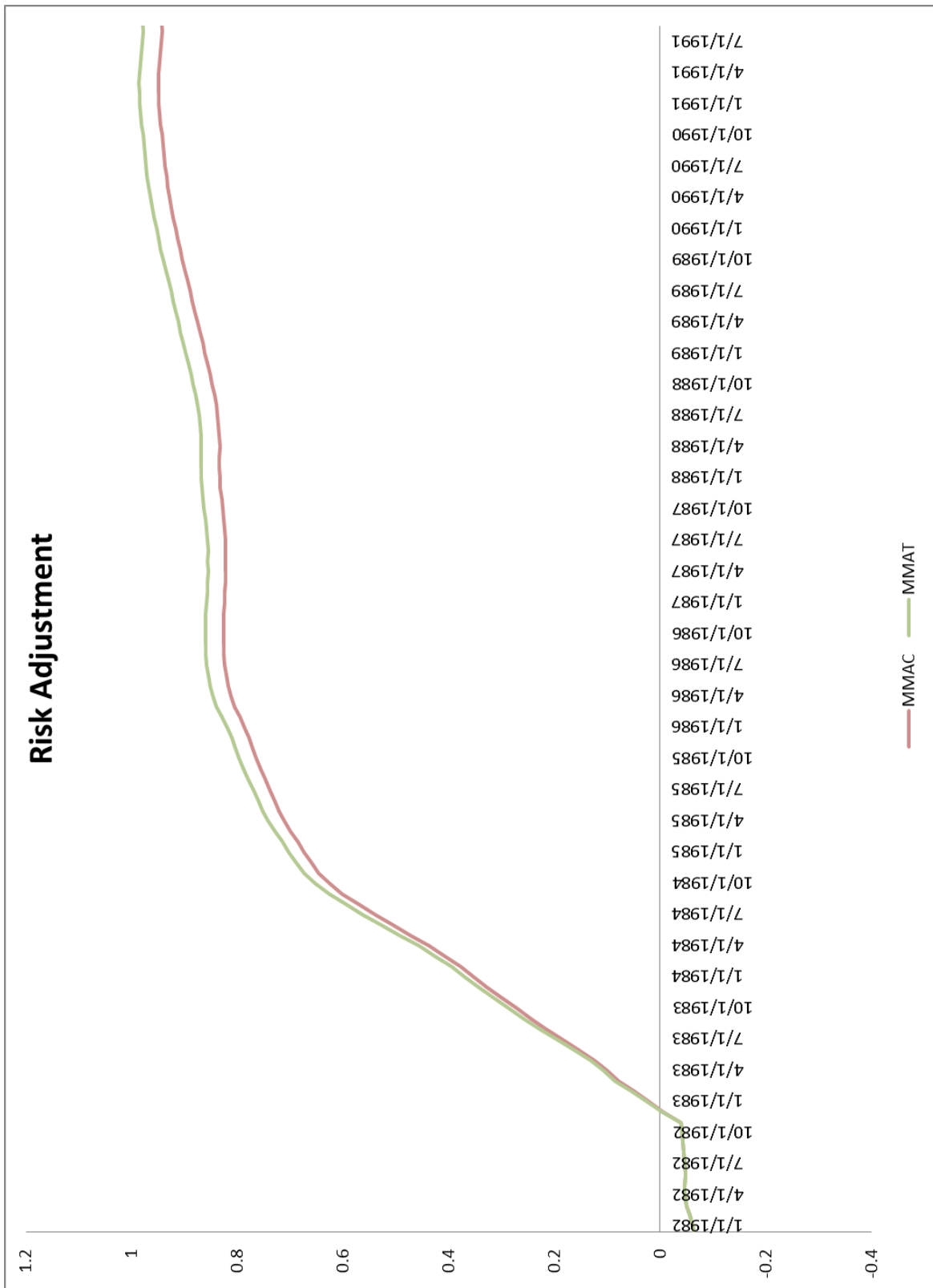


Figure 18 Risk Adjustment Money Market Demand Accounts

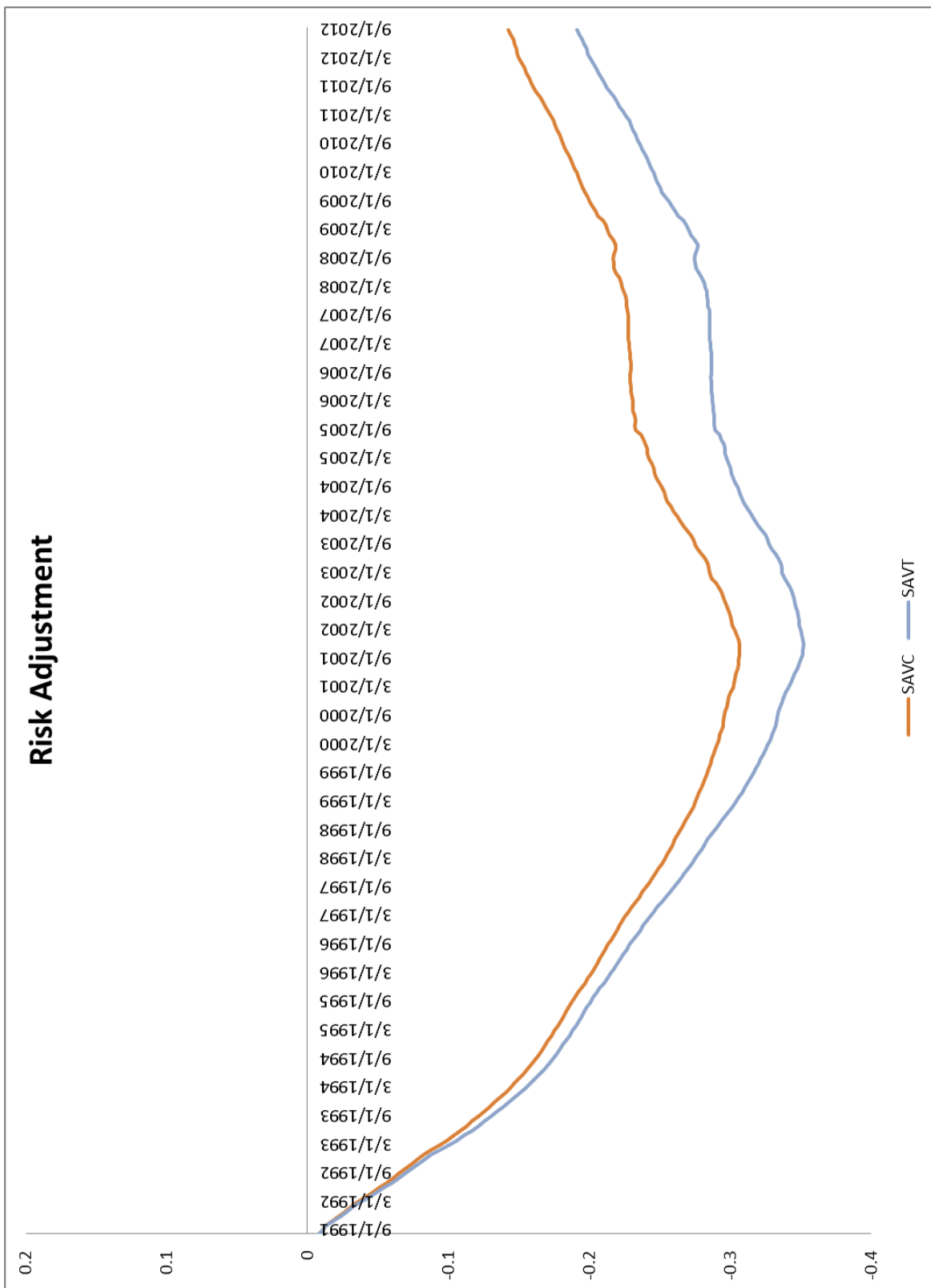


Figure 19 Risk Adjustment for Savings Accounts (With Money Market Demand Accounts)

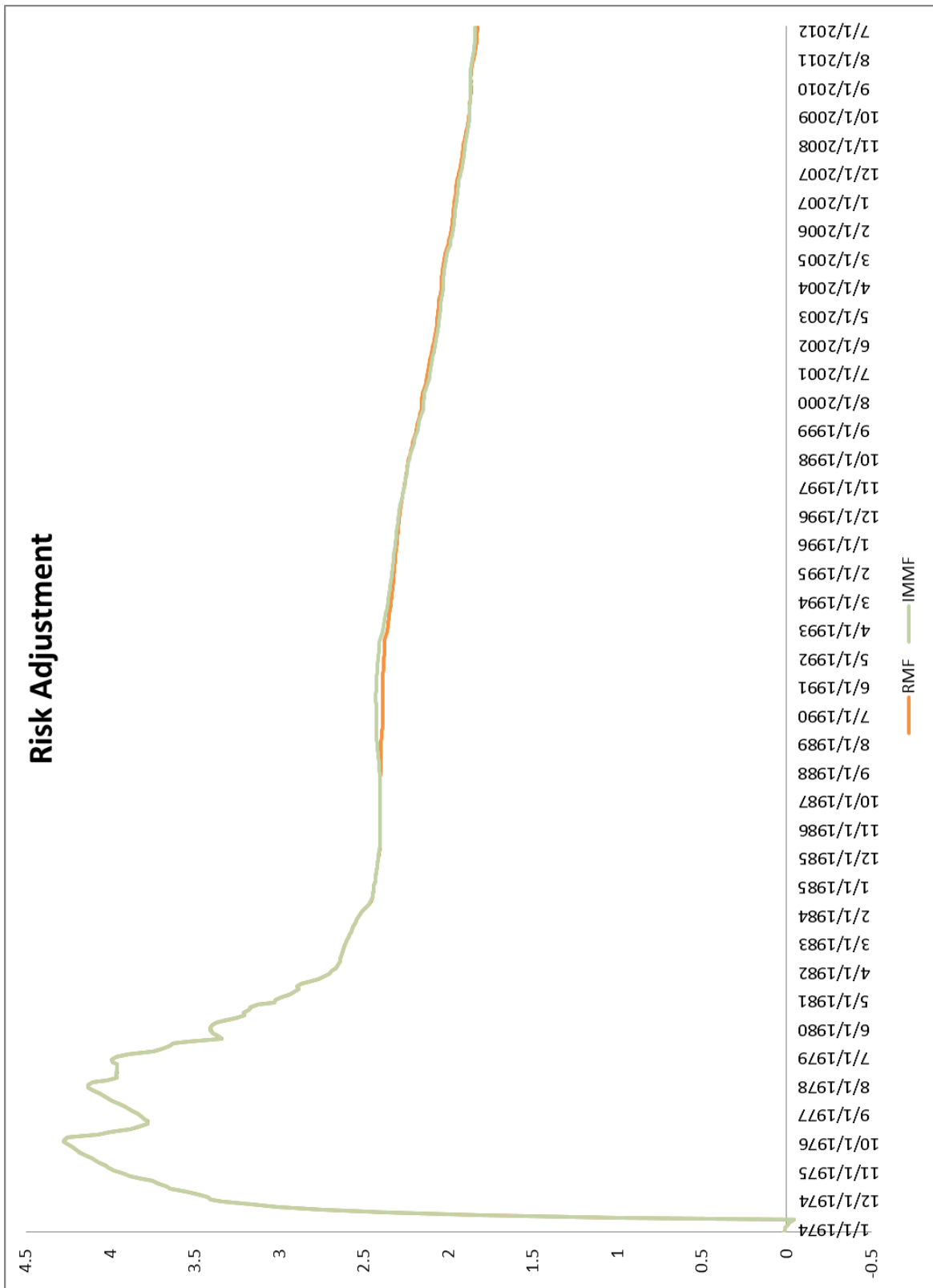


Figure 20 Risk Adjustment for Money Market Mutual Funds

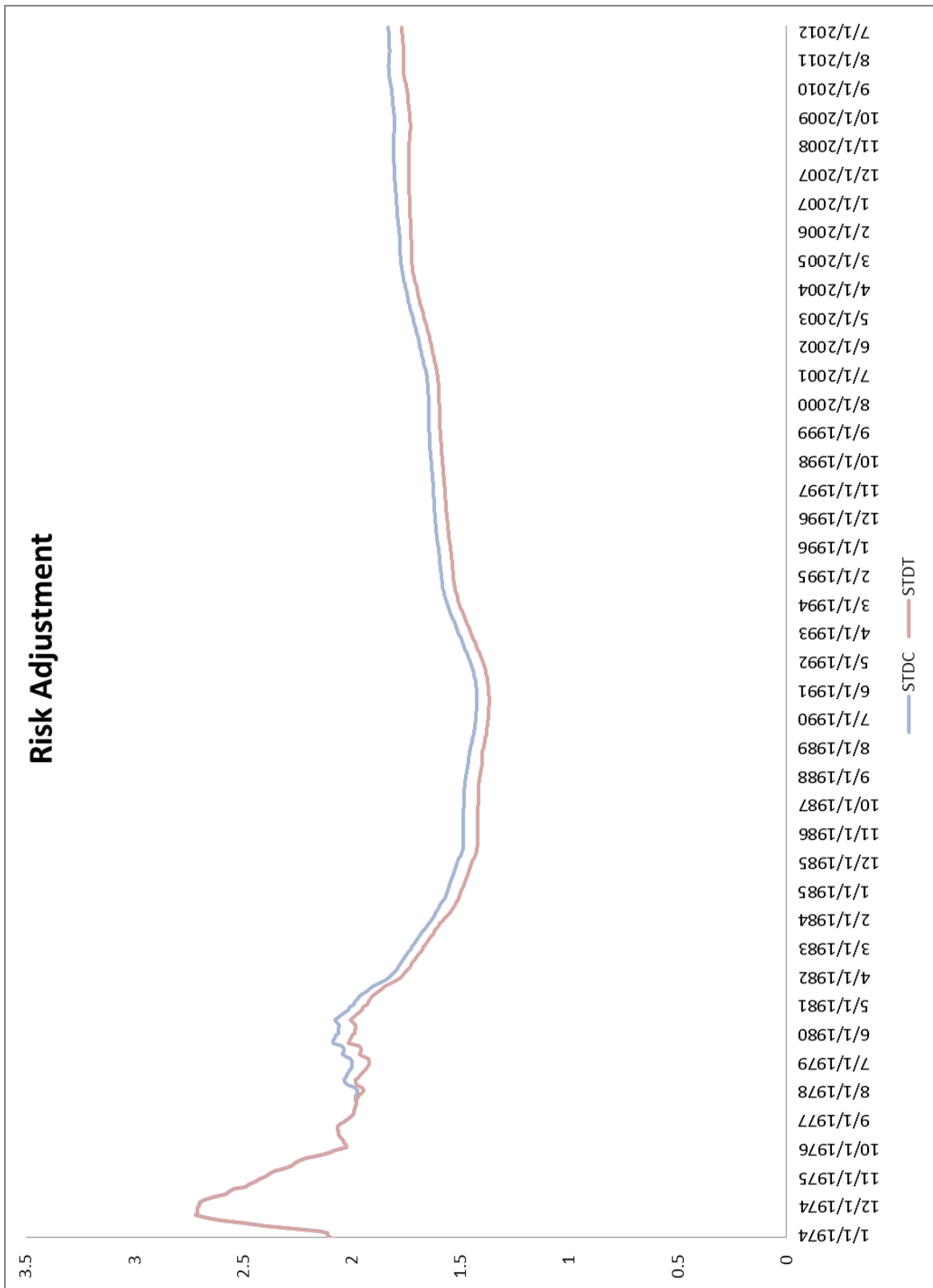


Figure 21 Risk Adjustment for Small Denomination Time Deposits

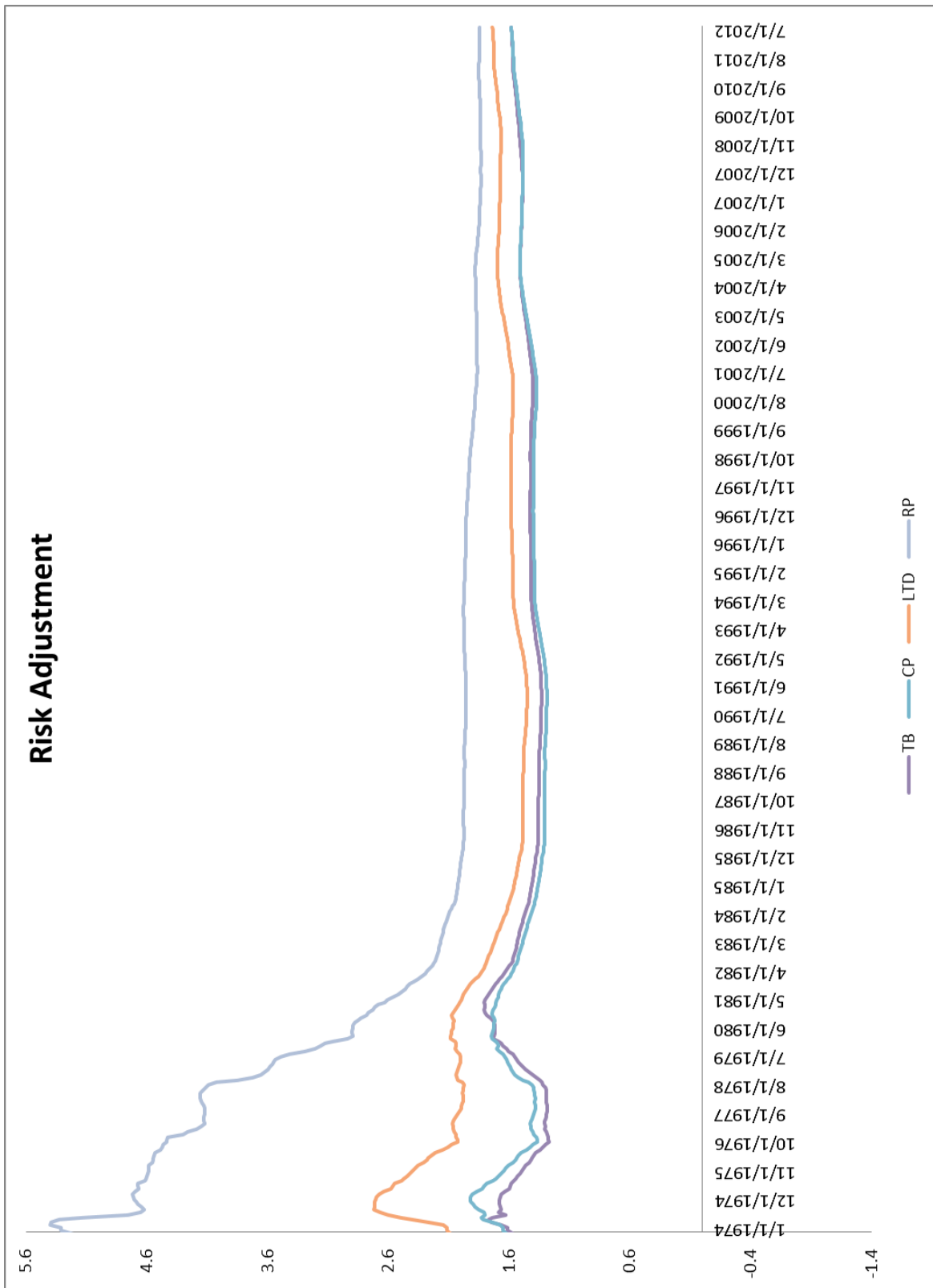


Figure 22 Risk Adjustment for Large Denomination Time Deposits, Overnight Repurchase Agreements, Commercial Paper, and Three Month Treasury Bills

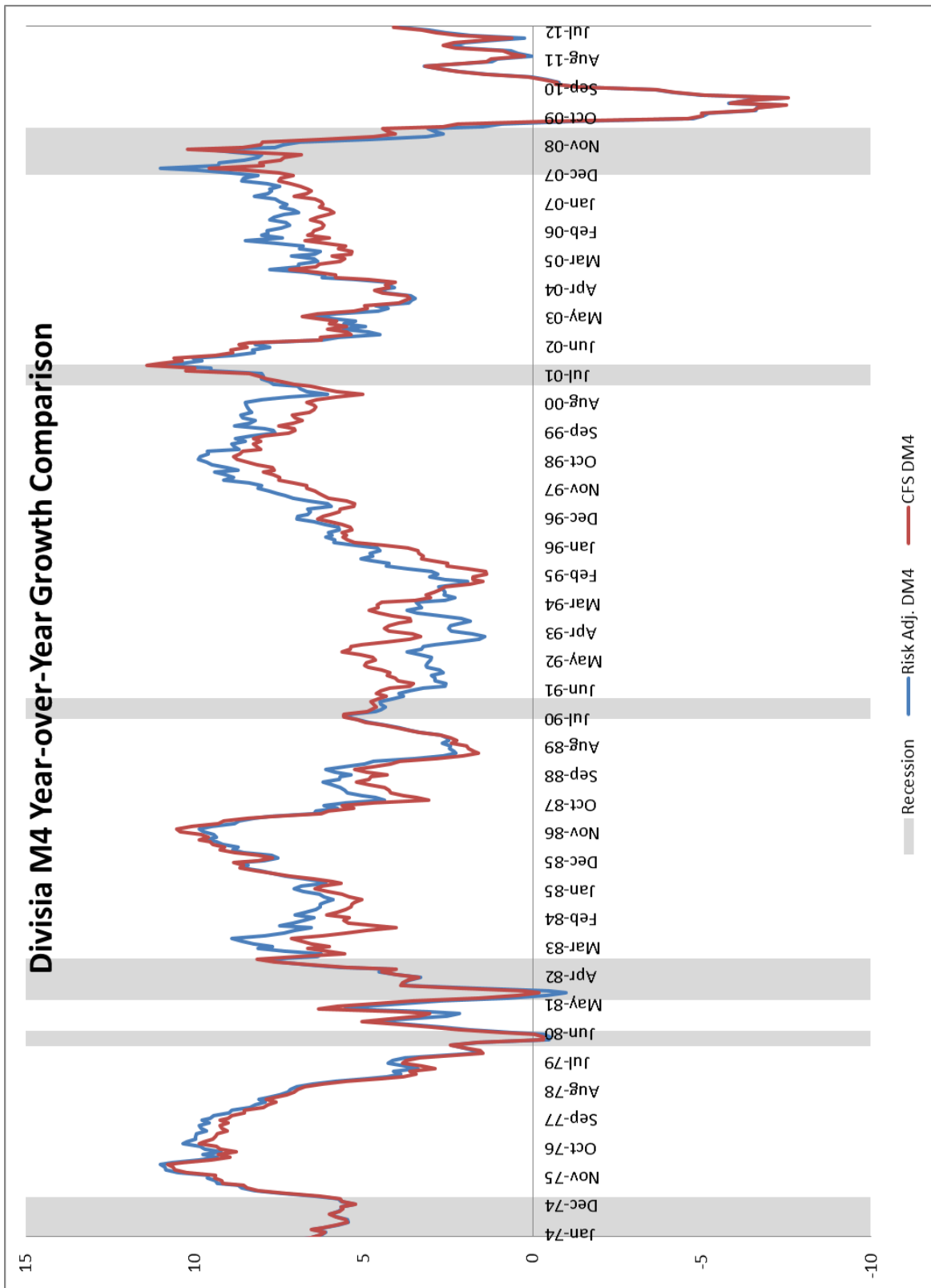


Figure 23 Risk Adjusted and Risk Neutral Divisia M4 Year-over-Year Growth Rate

Conclusions

The user cost risk adjustment for the Divisia monetary aggregate is necessary given the risky assets included in construction of the Divisia M4 cluster. Further when the asset portfolio's pricing kernel is estimated using an unconditional CAPM model with time varying parameters, the adjustment is not small as in the unconditional CAPM estimates. Notable differences occur between the user costs of the broad and narrow aggregates. Broader aggregates have a large adjustment upwards from the risk neutral user cost. Relatively "safe" assets like currency, demand deposits, and savings accounts face an upward adjustment of a much smaller magnitude. With risk adjustment the user costs are not identical to the risk neutral case; the representative agent in holding riskier assets should be taking into account the user cost effect of risk.

The effect of risk can be estimated using the methodologies presented in this paper. There is room for improvement in the risk adjustment as this paper focuses only on linear CAPM methods. Other methods and instruments could be used to determine the pricing kernel of the full portfolio that could lead to changes in the risk adjustment. Further research is required on risk adjustment, as well as better and more available data on non-monetary financial assets. For example while equity data is available from the Z.1 Flow of Funds survey, the kinds of equities held by households and firms are not distinguished; are there higher returns, are certain equities more risky, are others taxed more or less, are dividends paid? As equities become a higher share of expenditure on monetary assets as shown in Figure 1 Expenditure Shares on Monetary and Financial Assets and as more agents put money into equities to maintain wealth in a zero-interest environment, a more nuanced measure of equity holdings is necessary. Further work on risk adjustment for a multilateral aggregation over heterogeneous agents in the European Union would be a fruitful area for expanding the risk adjustment research. Relative riskiness of some European countries affects liquidity in other European countries, which are perceived to be less

risky. So far as the author knows, while the EU has a risk neutral Divisia index database, it has not yet been adjusted even in the current “risk abundant” environment.

Chapter 4: Testing Admissibility in the Risk Adjusted Case.

Summary

The chapter reviews the theory of user cost risk adjustment and weak separability and extends the results of the semi nonparametric test for admissibility in Barnett and de Peretti (2009) to account for risk. User costs for the monetary components are adjusted for risk following the linear, conditional CAPM procedure described in Barnett and Wu (2005), and the new risk adjusted clusters are tested for weak separability. The test finds evidence of support for the admissibility of Divisia M4 and Divisia M4- but not narrower aggregates such as Divisia M3 or Divisia M2 ALL. The tests further reject the inclusion of longer term government securities and equities (given a six month return) held by households. Those these assets do enter into the risk adjustment as non-monetary financial assets and the determination of the overall portfolio return. While the results for 1974 to 1982 yield less evidence of weak separability in all aggregates than in the neutral case, these violations can be explained by changes in monetary policy and the lack of data collection on a monetary instrument that was available but not accounted for by the Federal Reserve Board surveys. A unique violation in September of 2008 draws attention to the effects of Federal Reserve policy and risk on the clustering of aggregates.

Introduction

User costs are adjusted by estimating a price kernel for a portfolio of monetary and non-monetary assets in Chapter 3, allowing for a re-examination of admissibility of the CFS aggregates, now adjusted for risk. The weak separability tests outlined in de Peretti (2005, 2007), Barnett and de Peretti (2009), are applied to determine any changes in the admissibility results of broad Divisia clusters determined in Chapter 2. It is hypothesized that adjusting for risk yields admissibility for narrower aggregates than Divisia M4, possibly Divisia M4- and Divisia M3. It is not expected that narrower aggregates clusters like Divisia M2 will pass admissibility.

As described before, the literature for nonparametric weak separability tests emerges from Varian (1982, 1983) with a focus on the generalized axiom of revealed preference (GARP). Varian's test is found by Barnett and Choi (1989) to have an over-rejection problem. Several branches of the literature attempt to compensate for this over-rejection with the most recent efforts being made by de Peretti (2005, 2007) and Barnett and de Peretti (2009) which measure the significance of the violations in the data. If the violations are found to be not significant then GARP is not rejected. If the violations are found to be significant either through a traditional distribution measure of the residuals of a "GARP satisfying adjustment" or through the use of extreme value theory to see if the violations are outside the bounds of a reasonable extreme. The risk neutral CFS aggregate Divisia M4 produced positive results in Chapter 2, providing evidence for admissibility of the broad cluster, while Divisia M4- and Divisia M3 did not.

The construction method for broad Divisia aggregates by the CFS does not take into account the risk environment of monetary and financial assets. Several of the broad Divisia components contain inherently risky assets and their user cost is approximated by a risk free calculation. Such calculations could bias the results of the weak separability test. Retail money market funds for example produce several violations of GARP in the risk free assets, however if

they are adjusted for risk to have a higher user cost then the violations might not be as significant or occur at all. The literature for user cost risk adjustment is summarized in Barnett, Jensen, and Liu (1997), who estimate the risk adjustment with an unconditional CAPM model and found the adjustment to be relatively small compared with the risk free Divisia aggregate. Recently however Barnett and Wu (2005) have re-visited the theory of the risk adjustment with a conditional CAPM model and posit that the risk adjustment might be relatively larger and create significant change in user costs. If true, the change in user costs would lead to a change in results for the weak separability tests.

The clusters that pass and fail the weak separability test are summarized and discussed after the relevant theory is revisited. The risk adjustment provides further support for the use Divisia M4 and Divisia M4- and finds justification for not including long term treasury securities and equities as monetary assets, although they are used in the determination of the representative's agent portfolio return and risk adjustment.

Risk Adjusted User Costs in GARP

As in Chapter 3, the risk adjusted user cost price of a monetary asset is determined by the estimation:

$$\hat{\pi}_{i,t} = \pi_{i,t}^e + \beta_{i,t}(\hat{\Pi}_{A,t} - \Pi_{A,t}^e)$$

Where $\beta_{i,t}$ is the risk adjustment for the monetary component i at t which is the ratio of the covariance of the expenditure share weighted asset portfolio return, $r_{A,t+1} = \sum_{i=1}^L \varphi_{i,t} r_{i,t+1} + \sum_{j=1}^K \varphi_{j,t} \tilde{r}_{j,t+1}$ and the return on individual monetary asset. The certainty equivalent user costs for the individual assets and the overall portfolio are determined using the user cost derivation of Barnett (1980) and adjusted in Barnett and Wu (2005):

$$\Pi_{A,t}^e = \frac{r_t^f - E_t[r_{A,t+1}]}{r_t^f},$$

$$\pi_{i,t}^e = \frac{r_t^f - E_t[r_{i,t+1}]}{r_t^f},$$

using an envelope method benchmark method that includes all monetary assets, financial assets, and a bank loan rate as suggested in Offenbacher and Shachar (2011). The overall user cost of the asset portfolio is estimated with a price kernel using a linear, conditional CAPM model to estimate risk price, $\hat{\Pi}_{A,t} = \Pi_{A,t}^e + \hat{b}(z_t)Var_t(r_{A,t+1})$.

The risk adjustment produced relatively large changes in magnitude that should not be neglected. Components of the broader Divisia M4 aggregate all had increases in their user cost, implying the representative agent face some positive cost for using these instruments relative to other less risky assets in the portfolio. Components of narrower aggregates such as currency and savings accounts received relatively smaller adjustments as well as yielded negative risk adjustments in relation to the asset portfolio, meaning any increase in their user cost due to risk was much smaller than the other more risky assets found in broad components. While the 1974 to 1982 aggregate tracked the risk-neutral Divisia M4 provided by the CFS, the two broad aggregates yielded different magnitudes of money supply growth in certain periods of the 1980s and 1990s; see Figure 23 Risk Adjusted and Risk Neutral Divisia M4 Year-over-Year Growth Rate.

While the tracking of the risk neutral aggregate is an interesting result, the purpose of this project is to determine the sub clusters that are admissible within the broader aggregate, and if the monetary assets are indeed weakly separable from other consumption expenditures. The change in user costs could provide similar aggregate patterns, but lead to different clustering conclusions based on the changes weights given to risky and less risky monetary assets. To determine the clusters the prices and levels of the “within-cluster” assets must satisfy the Generalized Axiom of Revealed Preference (GARP) and be independent from changes in outside

cluster levels. The Barnett and de Peretti (2009) procedure for testing GARP is described in detail in Chapter 2 and the main points are reviewed in the next section.

Weak Separability Test Revisited

The weak separability test relies on three conditions: satisfying of GARP for the overall grouping of all components, satisfying GARP for the group to be clustered, and the independence of the marginal rate of substitution of within-cluster components from changes in the outside cluster. In formal terms the conditions to be satisfied are summed up in de Peretti and Barnett (2009) as the original Varian (1982) conditions with an adjustment to the final condition of weak separability depending on the identity of the marginal rate of substitution and the price ratios of two components:

CONDITION 1. $\{(\mathbf{m}_t, \boldsymbol{\pi}_t)\}_{t=1}^T$ satisfies GARP ($U(\cdot)$ exists).

CONDITION 2. $\left\{(\mathbf{m}_t^{(1)}, \boldsymbol{\pi}_t^{(1)})\right\}_{t=1}^T$ satisfies GARP ($f(\cdot)$ exists).

CONDITION 3a. $\beta_1^{(2)} = \beta_2^{(2)} = \dots \beta_{\sum_{i=1}^{a-1}(a-i)}^{(2)} = 0$, meaning $\mathbf{M}^{(1)}$ is weakly separable in $U(\cdot)$.

In this case $\beta_i^{(2)}$ are the coefficients for the regression of the log of the price ratios on the asset component levels found outside the clustered group according to the linear regression equation from Chapter 2.

$$\begin{bmatrix} \log(\pi_{1,\cdot}/\pi_{2,\cdot}) \\ \vdots \\ \log(\pi_{1,\cdot}/\pi_{a,\cdot}) \\ \log(\pi_{2,\cdot}/\pi_{3,\cdot}) \\ \vdots \\ \log(\pi_{2,\cdot}/\pi_{a,\cdot}) \\ \vdots \\ \log(\pi_{(a-1),\cdot}/\pi_{a,\cdot}) \end{bmatrix} = Y = \begin{bmatrix} \mathbf{M}^{(3)} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{(3)} & \dots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{M}^{(3)} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{\sum_{i=1}^{a-1}(a-i)} \end{bmatrix} + \varepsilon,$$

$$\beta_i = [\beta_i^{(0)} \beta_i^{(1)} \beta_i^{(2)}],$$

$$\mathbf{M}^{(3)} = [\mathbf{1} \log(\mathbf{M}^{(1)}) \log(\mathbf{M}^{(2)})],$$

for $\mathbf{M}^{(3)}$ the overall matrix of components, $\mathbf{M}^{(1)}$ the within-cluster group of components, $\mathbf{M}^{(2)}$ the outside group of components and β_i the corresponding coefficients for each group and the constant.

The original Varian (1982) test is also adjusted for noise in order to compensate for the over-rejection of GARP found in Barnett and Choi (1989) using methods developed in de Peretti (2005, 2007) and Barnett and de Peretti (2009). For those violations of GARP found in the clusters, estimation is made of the minimum adjustment needed for the violation to satisfy GARP according to the proposed null hypothesis that the cluster does indeed satisfy GARP. The adjustments are then tested for significance using an extreme value theory method developed in Barnett and de Peretti (2009) which determines if the adjustments are larger than a threshold maximum or minimum; if the adjustments are within the noise threshold, they are not significant and GARP passes, if not then GARP is violated. Formally the adjustments are estimated by:

$$obj = \min_{z_{i,t}} \sum_{t=1}^T \sum_{i=1}^L (m_{i,t} - z_{i,t})^2$$

for every $i = 1, \dots, T$; $j = 1, \dots, L$ and $\mathbf{z}_i^{(1)} = (z_{1,t}, z_{2,t}, \dots, z_{a,T})'$ subject to

$$(C.1) \quad \mathbf{z}_t R \mathbf{z}_s \Rightarrow \boldsymbol{\pi}_s \cdot \mathbf{z}_s \leq \boldsymbol{\pi}_s \cdot \mathbf{z}_t,$$

$$(C.2) \quad \mathbf{z}_t^{(1)} R \mathbf{z}_s^{(1)} \Rightarrow \boldsymbol{\pi}_s^{(1)} \cdot \mathbf{z}_s^{(1)} \leq \boldsymbol{\pi}_s^{(1)} \cdot \mathbf{z}_t^{(1)}.$$

as described in Chapter 2 for the constraints being the satisfaction of GARP for the overall and within group cluster and the estimated adjustment $\hat{z}_{i,t}$.

No other adjustments to the procedure are necessary once the risk adjustment is made. The main adjustment to the admissibility test as outlined in Chapter 2 is simply the change in user costs. These are paired with the known levels, and the adjustments for satisfaction of GARP are calculated and tested for significance. Any large relative changes in user costs lead to different results in admissibility tests as GARP relies on the expenditure of components at particular time periods; if these user cost prices are adjusted the expenditure changes. If the user cost is not risk adjusted and they are lower than should be in a risk environment, then certain violations or satisfactions of GARP are not in that case correct. Periods of high risk not reflected in user costs could bias the GARP tests. This chapter compares the results produced by risk adjusted user costs to those in the risk neutral case of Chapter 2.

GARP and Weak Separability Results

The results for the GARP tests were strikingly different for the risk adjusted case. The period of 1974 to 1982, despite tracking closely to the risk neutral CFS Divisia M4, rejected all aggregates for GARP. Containing almost all of the 546 violations (significant or not) over the 100 month timeline, the period from March of 1980 to November of 1982 proved the most problematic for GARP. If the 1974 to February 1980 time period is considered on its own, there are fewer, not significant violations for the risk adjusted Divisia M4 and GARP is satisfied,

separating our monetary assets from personal consumption expenditures. The scale of the violations for all components is visible in Figure 24 1980 to 1982 GARP Violations.

There are two important considerations to be made when looking at these violations. The first is that money market demand accounts, while they were not measured by the Federal Reserve Board until the beginning of the 1982 period here to be tested, were still in existence. Any movement from regular savings or interest checking into these newer money market demand accounts would only be accounted for as a flow out of savings and checking with no flow into the money market demand accounts. Complete measurement is necessary in aggregation, and the lack of this data in the early 1980s could lead to problems. The sudden disappearance of violations in the 1982 to 1991 period when money market demand accounts are counted with their rate of return would imply this improvement in measurement. Secondly, the violation period overlaps with the change in Federal Reserve Policy in 1979 to target non-borrowed reserves (and thus money supply growth) as opposed to focusing on short term interest rates and a severe recession in 1981. For further discussion on this period and Divisia monetary aggregates see Barnett and Chauvet (2011) and Barnett and Serletis (2000). In this case the GARP test would suggest not aggregating for this period and focusing on more specific component growth. Despite this failure of GARP for the late 1970s and early 1980s, Divisia M4 remains a cluster that passes GARP from 1982 onwards.

The period extending from 1982 to 1991 which includes savings accounts and money market demand accounts as separate components sheds its GARP violations in both amount and significance for Divisia M4, Divisia M4-, and Divisia M3 clusters. These tables include the number of violations to emphasize the effect of the risk adjusted user cost on the GARP results. There are very few violations and none of them could be considered significant given the tail areas calculated by the method described in Chapter 2. When money market demand accounts

and savings accounts are combined in August 1991 and a new testing period begins, the results are similar. There are few violations for the 1990s into the 2000s and the tail areas for the GARP test confidently reject any significance of violations.

There is however a violation in the most recent time period of 2006 to 2012 focused on a single month. September of 2008 proved to be a risk intense month in the monetary history of the United States given the collapse of Lehman Brothers, the bail-out of AIG, and the formation of the Asset Backed Money Market Mutual Fund Liquidity Facility (AMLF) with the sole purpose of “providing funding to U.S. depository institutions and bank holding companies to finance their purchases of high-quality asset-backed commercial paper from money market mutual funds under certain conditions” according to the St. Louis Federal Reserve’s Timeline of the Financial Crisis. As the financial market began to sink, the Federal Reserve disbursed funds through non-recourse loans to banks to purchase asset-backed commercial paper, interestingly coinciding with the large violation of GARP in the commercial paper market in September of 2009. This single violation is large enough to fail GARP for the entire 2006 to 2012 time period. The magnitude of the commercial paper violation is visible in Figure 25 September 2008 Violations¹⁵. If this violation is removed from consideration, all broad aggregates including Divisia M3 which did not pass GARP in the risk neutral case, pass the GARP conditions.

The F-test on dependence of within-cluster price ratios and outside cluster component levels however remains the caveat to the admissibility tests. Divisia M4 and Divisia M4- still perform well with tail areas above the 1% and sometimes 5% level. Divisia M3 however still fails the third condition, despite satisfying GARP. See Table 13 for the results of the weak separability test. The conclusion to “go broad” remains when it comes to monetary aggregation, while it is also maintained that monetary assets can be sub-clustered out of consumption. The

¹⁵ Note the large violation in treasury bills is still not significant according to the estimation through extreme value theory. It still remains a relatively large violation compared to the small levels in previous months.

analysis of the third condition using non-linear regression could produce interesting results and left for further investigation, however it stands that in the risk neutral and risk adjusted case that Divisia M4 is the preferred aggregate.

It was considered that longer term treasury securities and equities could provide liquidity services above and beyond their rates of return so that they should be included in a broad monetary aggregate. However the GARP test so strongly rejected any addition of treasury notes, treasury bonds, and equities that the idea was abandoned. The Barnett and de Peretti (2009) GARP test found no evidence to support the inclusion of long term securities or equities as monetary components.

Table 8 Risk Adjusted Admissibility Results

RESULTS OF THE WEAK SEPARABILITY TEST ON RISK ADJUSTED CLUSTERS

	1974-1982	1982-1991	1991-2006	2006-2012
Divisia M4	X	G* / WS*	G* / WS**	G** / WS**
Divisia M4-	X	G* / WS*	G* / WS**	G** / WS**
Divisia M3	X	X	X	X
Divisia M4 Notes	X	X	X	X
Divisia M4 Notes, Bonds	X	X	X	X
Divisia M4 Notes, Bonds, Equity	X	X	X	X

Notes: G denotes failure to reject Conditions 1 and 2, but rejection of Condition 3. G/WS denotes failure to reject all three conditions. X denotes a rejection of all conditions.

Source: Author calculations.

** Significant at the 1 percent level or included due to September, 2008 violation.

* Significant at the 5 percent level.

Table 9 Risk Adjusted GARP 1974

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CLUSTERS 1974:1982, T=100

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.02 / .06	.02 / .05	.01 / .02
Demand Deposits	.27 / .39	.30 / .38	.22 / .25
Other Checkable Deposits Comm.	.20 / .07	.23 / .38	.16 / .00
Other Checkable Deposits Thrift	.01 / .00	.23 / .09	.69 / .67
Savings Less MMDA Comm.	.67 / .74	.01 / .00	.35 / .28
Savings Less MMDA Thrift	.32 / .43	.67 / .73	.03 / .11
Retail Money Market Funds	.11 / .11	.32 / .45	.30 / .60
Small Time Deposits Comm.	.36 / .50	.10 / .12	.26 / .57
Small Time Deposits Thrift	.32 / .47	.28 / .55	.26 / .40
Institutional Money Market Funds	.41 / .40	.24 / .52	.22 / .53
Large Time Deposits	.51 / .63	.40 / .42	.30 / .63
Overnight and Term Repos	.38 / .57	.32 / .46	.30 / .63
Commercial Paper	(.68 / .70)	.39 / .57	.29 / .59
Short Term Treasury Bills	(.74 / .84)	(.70 / .82)	.48 / .69
Personal Consumption Expenditures	(.67 / .59)	(.68 / .65)	(.89 / .89)
Sub Utility Violations	245	240	123
Overall Utility Violations	267	333	547

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 10 Risk Adjusted GARP 1982

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1982:1991, T=105

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.69 / .79	.78 / .80	.84 / .77
Demand Deposits	.82 / .85	.85 / .86	.87 / .85
Other Checkable Deposits Comm.	.74 / .84	.84 / .87	.86 / .84
Other Checkable Deposits Thrift	.44 / .68	.66 / .84	.82 / .81
MMDA Commercial	.89 / .86	.88 / .88	.89 / .88
MMDA Thrift	.88 / .82	.88 / .88	.88 / .87
Savings Less MMDA Comm.	.87 / .88	.87 / .88	.88 / .88
Savings Less MMDA Thrift	.85 / .88	.85 / .88	.88 / .88
Retail Money Market Funds	.86 / .80	.85 / .85	.82 / .83
Small Time Deposits Comm.	.73 / .83	.85 / .78	.87 / .86
Small Time Deposits Thrift	.83 / .86	.84 / .84	.79 / .82
Institutional Money Market Funds	.87 / .87	.87 / .86	.85 / .85
Large Time Deposits	.86 / .87	.87 / .87	.86 / .88
Overnight and Term Repos	.80 / .86	.86 / .83	.83 / .87
Commercial Paper	(.88 / .88)	.85 / .79	.81 / .85
Short Term Treasury Bills	(.88 / .88)	(.88 / .89)	.85 / .88
Personal Consumption Expenditures	(.89 / .89)	(.89 / .88)	(.89 / .89)
Sub Utility Violations	33	33	25
Overall Utility Violations	8	15	16

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 11 Risk Adjusted GARP 1991

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 1991:2006, T=175

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.83 / .88	.76 / .88	.88 / .86
Demand Deposits	.86 / .88	.84 / .88	.88 / .88
Other Checkable Deposits Comm.	.84 / .88	.80 / .88	.87 / .88
Other Checkable Deposits Thrift	.77 / .88	.64 / .64	.85 / .88
Savings and MMDA Comm.	.88 / .88	.87 / .86	.88 / .88
Savings and MMDA Thrift	.87 / .88	.87 / .86	.88 / .88
Retail Money Market Funds	.88 / .83	.88 / .78	.88 / .85
Small Time Deposits Comm.	.88 / .88	.87 / .87	.88 / .88
Small Time Deposits Thrift	.86 / .86	.88 / .83	.88 / .86
Institutional Money Market Funds	.88 / .86	.88 / .84	.88 / .87
Large Time Deposits	.87 / .88	.84 / .88	.88 / .88
Overnight and Term Repos	.88 / .86	.83 / .83	.86 / .86
Commercial Paper	(.89 / .84)	.82 / .87	.88 / .88
Short Term Treasury Bills	(.86 / .88)	(.89 / .89)	.84 / .88
Personal Consumption Expenditures	(.88 / .88)	(.89 / .89)	(.89 / .89)
Sub Utility Violations	4	2	2
Overall Utility Violations	4	4	4

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility).

Source: Author calculations.

Table 12 Risk Adjusted GARP 2006

TAIL AREA RESULTS OF THE GARP TEST ON BROAD CFS CLUSTERS 2006:2012, T=75

	Divisia M3	Divisia M4-	Divisia M4
Currency and Travelers Checks	.76 / .21	.25 / .08	.31 / .25
Demand Deposits	.84 / .60	.63 / .52	.67 / .72
Other Checkable Deposits Comm.	.80 / .37	.41 / .16	.45 / .51
Other Checkable Deposits Thrift	.67 / .07	.06 / .01	.11 / .25
Savings and MMDA Comm.	.75 / .80	.71 / .83	.72 / .83
Savings and MMDA Thrift	.87 / .65	.73 / .29	.79 / .79
Retail Money Market Funds	.33 / .67	.54 / .36	.55 / .49
Small Time Deposits Comm.	.48 / .88	.30 / .82	.46 / .78
Small Time Deposits Thrift	.36 / .84	.17 / .74	.36 / .65
Institutional Money Market Funds	.62 / .79	.71 / .65	.74 / .72
Large Time Deposits	.75 / .48	.72 / .17	.84 / .58
Overnight and Term Repos	.51 / .88	.34 / .70	.46 / .87
Commercial Paper	(.01 / .88)	.00 / .69	.04 / .84
Short Term Treasury Bills	(.88 / .37)	(.88 / .87)	.84 / .20
Personal Consumption Expenditures	(.85 / .88)	(.68 / .88)	(.88 / .88)
Sub Utility Violations	9	12	14
Overall Utility Violations	10	30	18

Notes: “Max Tail Area / Min Tail Area”. (Tail Area for Overall Utility Component not in Sub-utility). If the month of September, 2008, is not considered, all tail areas rise above the 10% level.

Source: Author calculations.

Table 13 Risk Adjusted Weak Separability

TAIL AREA RESULTS OF CONDITION 3 WEAKLY SEPARABLE TEST

	Cluster	Time Period	Sample Size	Tail Area
DM4		1974:1982	100	X
		1982:1991	105	0.0626
		1991:2006	175	0.0387
		2006:2012	78	0.0352
DM4-		1974:1982	100	X
		1982:1991	105	0.0662
		1991:2006	175	0.0412
		2006:2012	78	0.0335
DM3		1974:1982	100	X
		1982:1991	105	0.0058
		1991:2006	175	0.0022
		2006:2012	78	0.0012
DM2 ALL		2006:2012	78	0.0000

Notes: The tail area of the F-Test on all outside quantity coefficients is reported. X indicates failed GARP test.

Source: Author calculations.

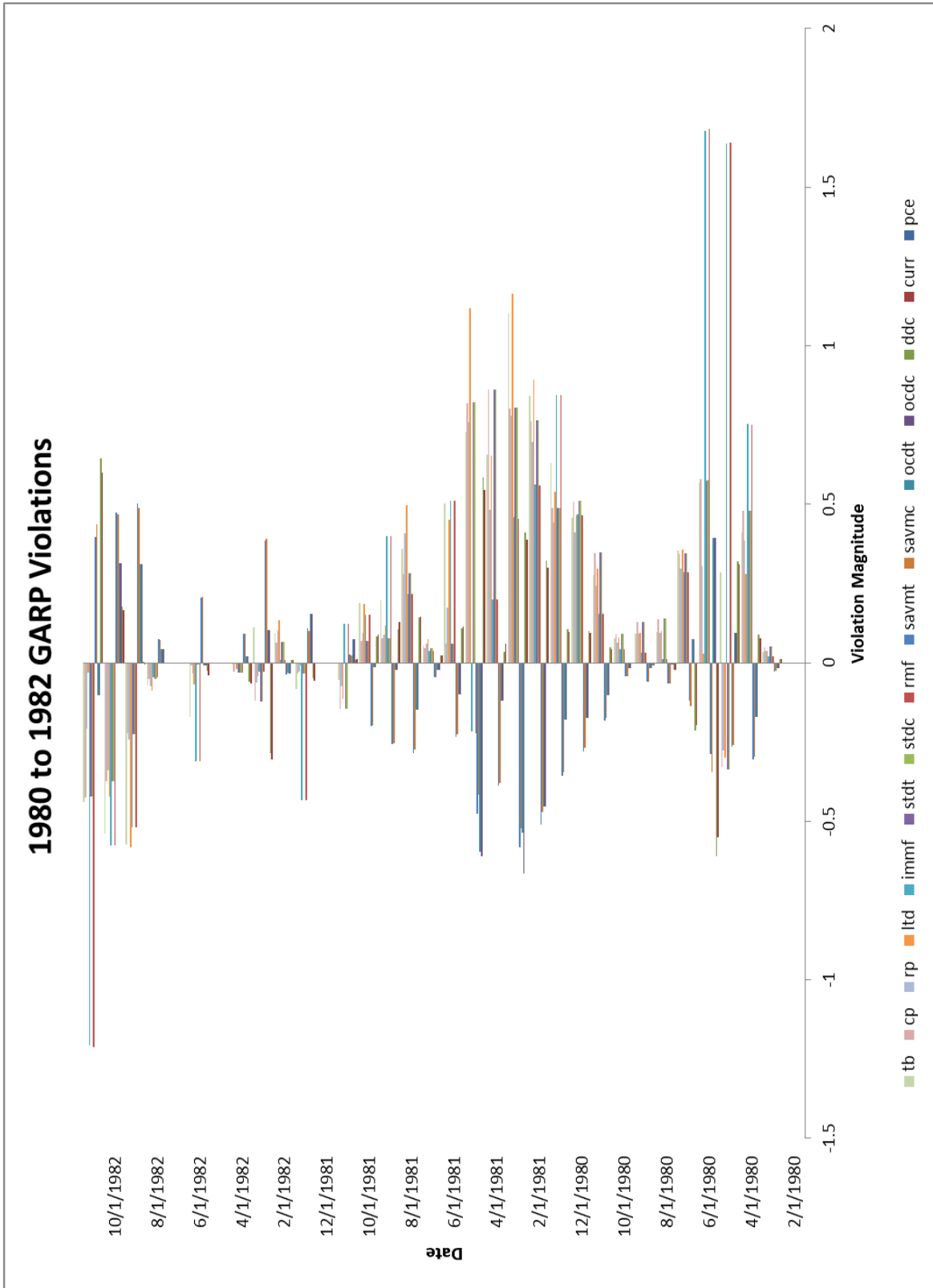


Figure 24 1980 to 1982 GARP Violations

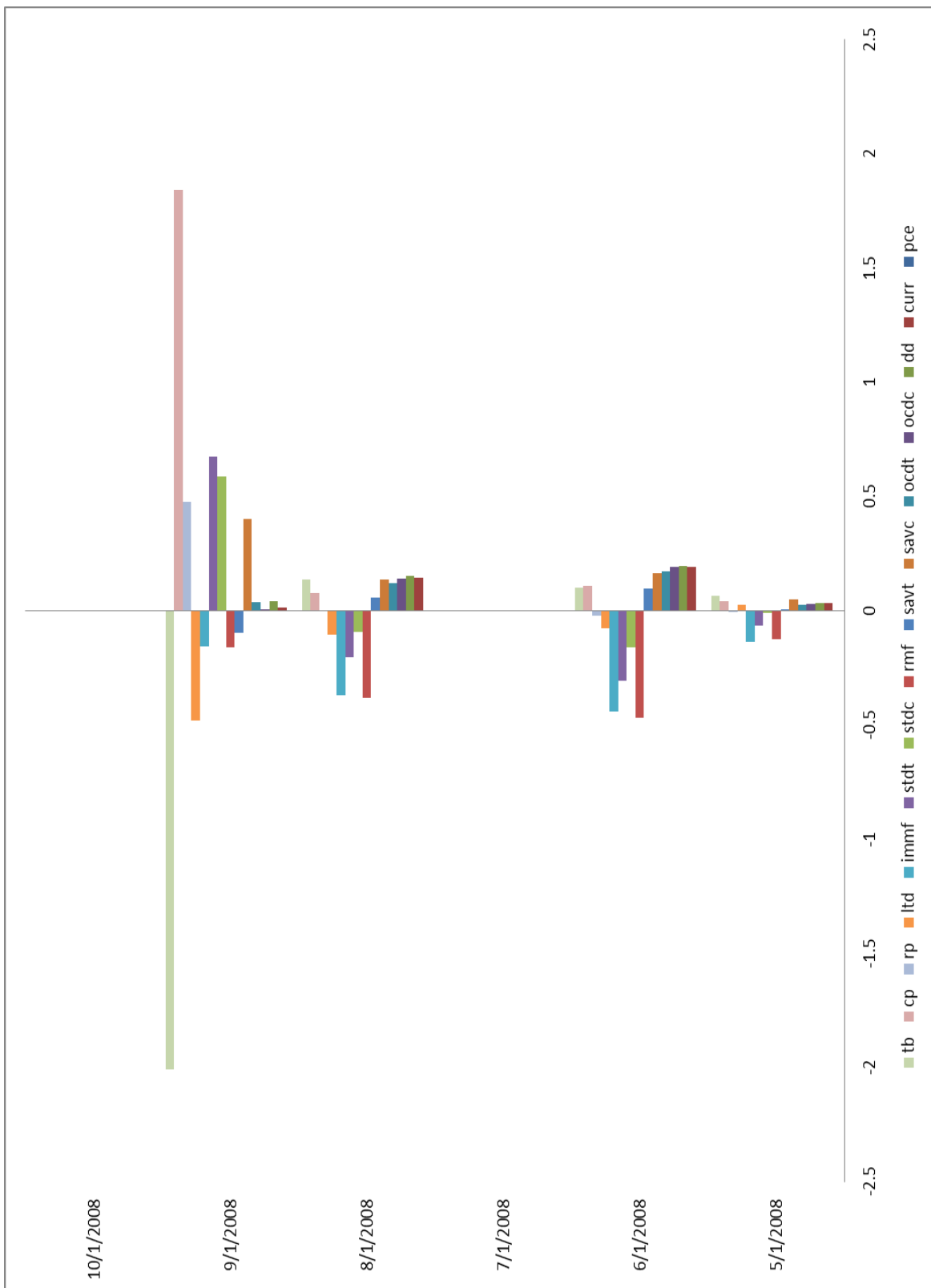


Figure 25 September 2008 Violations

Conclusion

After adjusting the user costs for risk the results of the admissibility test for broad Divisia monetary aggregates did indeed change. For the earliest time period the admissibility tests showed no support for clustering based on the violations found in the early 1980s. Such violations could be due to data availability on money market demand accounts and/or the change in Federal Reserve Board policy known as the “monetarist experiment”. Such conditions could result in preferable analysis of individual components as opposed to a broader aggregate. However the broad aggregates Divisia M4 and Divisia M4- pass GARP and weak separability conditions from 1982 onward. Divisia M3 is found to satisfy GARP when risk adjusted for more time periods, but still fails the last condition of weak separability.

Chapter 5: Conclusion

This project has tested risk neutral Divisia monetary aggregates for weak separability, adjusted the user costs to account for risk, and re-tested these figures to determine the use of the risk adjustment in clustering. Risk adjustment as performed in Chapter 3 leads to different results for the admissibility tests performed in Chapter 2, as is evidenced with the results in Chapter 4. What the results agree on is the use of the broadest possible Divisia monetary aggregate, Divisia M4, without including long term government securities and equities. These broad monetary measures satisfy the economic theory necessary for aggregation in both the risk neutral and risk adjusted case.

The methods used included recently developed econometric techniques outline in Barnett and de Peretti (2009) in addition to established asset pricing techniques in the financial literature, as proposed by Barnett and Wu (2005). The violations of the Generalized Axiom of Revealed Preference (GARP) within a given data sequence are tested for significance given assumptions about the stability of their extreme values. If GARP is satisfied, a smoothed estimate of the “true values” of the outside group component levels are tested for correlation with the log ratio of the user cost prices of within group components. If correlation is significant, then despite the acceptance of GARP the cluster does not pass weak separability; the necessary condition of GARP is passed but the necessary and sufficient condition of independence of the within-cluster marginal rate of substitution to changes in the outside components. To adjust for user cost and re-test for admissibility a linear conditional CAPM model is assumed to estimate the risk price of a representative agent’s portfolio. From the risk price the overall user cost of the asset portfolio is estimated, and the risk adjustment is solved using the ratio of the covariance of the asset portfolio to a given asset and the variance of the asset portfolio. With these estimates the risk adjusted user cost can be solved, according to the theory developed in Barnett, Liu, and Jensen

(1997) and Barnett and Wu (2005). Once the user costs are adjusted for risk the admissibility test run again.

The clearest extensions for this research include using alternative risk adjustment methods for the user cost from those described in Chapter 2 and further investigation into the “Barnett Critique” of macroeconomic models. These methods are not the only ways to estimate asset prices in a portfolio, nor the only ways to test the significance of violation adjustments. Investigation using developing econometric techniques in non-linear methods could prove useful and practical. Aside from the estimation and data issues, the “Barnett Critique” as described by Chrystal and MacDonald (1994) proposes serious concerns with the behavior of macroeconomic models if the aggregator function used to produce an index and the aggregator function implied by the structural model using the index are inconsistent. For example there can be evidence of a structural shift in the economy that does not actually occur as in the supposed change in money velocity in the 1970s produced by simple sum figures but not reflected at all when Divisia monetary aggregates are used¹⁶. It is now well established that simple sum measures perform poorly when compared to Divisia monetary aggregates. Without good index numbers there can be no good macroeconomic estimation. Since so many macroeconomic models do begin with the assumption of the separability of goods (durables, non-durables, monetary, financial) it is important to establish that these components are indeed separable and can indeed be aggregated in line with the spirit of the Barnett Critique. While Divisia M2 is an improvement over the simple-sum in that it accounts for the returns on monetary assets and their expenditure share weights, the admissibility of Divisia M4 makes it the more appropriate measure; when using a cluster such as Divisia M4 the assumption of weak separability is supported and the macroeconometrician can continue his modeling without fear of that weakness in his results.

¹⁶ See Barnett and Chauvet (2011) and Barnett and Serletis (2000) for in depth discussions of the Barnett Critique and the money velocity episode of the 1970s.

More specifically, the risk adjusted Divisia clusters developed and tested in this chapter have uses in the analysis of exchange rate determination as in Barnett and Kwag (2006) as well as re-examinations of previous conclusions on the link of monetary policy and other macroeconomic indicators as performed by Hendrickson (forthcoming) with Divisia M2 aggregates. Risk adjustment for a multilateral aggregation should also be considered for cases such as the European Union, in which several heterogeneous countries with different risk conditions all fall under the same monetary system. Risk adjustment would allow for considering the cost of holding Greek versus German short term securities or Italian and French money market mutual funds (or similar monetary vehicle). The consideration of separability under risk for macroeconomic aggregation is a wide-open field for researchers interested in monetary and open economy macroeconomics.

Selected Code Used

The following codes were performed in SAS/IML. The codes for the GARP test, the Kalman Filter, and the F-Tests were provided by Philippe de Peretti and are from his work on Barnett and de Peretti (2009). These codes are available on Philippe de Peretti's website as well. The smoothing forecasts and AR forecast were performed using the SAS Time Series Forecasting System. The X-12 seasonal adjustment and the quarterly to monthly conversion of loan rates and equities used SAS manual programs.

GARP Test: Two Step Iterative Procedure from Barnett and de Peretti (2009)

```
proc iml;
reset log;
reset noname;

start tsip(X1,X2,P1,P2);
XX1=X1;XX2=X2;X=X1||X2;P=P1||P2;XX=X;

/*Sub-Routine 1: Quadratic Programming*/
start qp(noms,c,h,g,rel,b,teta);
reset noprint;
st=0.1;
if min(eigval(h))<0 then do;
print 'The Minimal Eigenvalue is Negative',
'H is not negative semidefinie',
'The Optimization Program Stops';
stop;
end;
nr=nrow(g);
nc=ncol(g);

rev=(rel='<=');
adj=(-1*rev)+^rev;
g=adj#g;
b=adj#b;
eq=( rel = '=' );
if max(eq)=1 then do;
g=g // -(diag(eq)*g)[loc(eq),];
b=b // -(diag(eq)*b)[loc(eq)];
end;

m=(h || -g`) // (g || j(nrow(g),nrow(g),0));
q=c // -b;

/*Solving*/
call lcp(rc,w,z,M,q);

/*report the solution*/
/*reset noname;
print ({
                                'Optimal Solution',
                                'No Solution',
                                '',
                                '',
                                '',
                                'Unstable Solution',
                                'Not enough memory',
```

```

                                'The number of iterations exceeds'}[rc+1]);*/
reset noname;
teta=z[1:nc];
cste=(c`/(-2))*(c/(-2));
objval=(c`*teta)+((teta`*H*teta)/2)+cste;

store teta objval;
finish qp;

                                /*Step 1 Sub-utility consistant Data*/

                                /*GARP for sub-utility*/

free sol;
SDRP=j(nrow(X2),nrow(X2),0);
DRP=j(nrow(X2),nrow(X2),0);
do i=1 to nrow(X2);
    do j=1 to nrow(X2);
        if (P2[i,]*X2[i,]`) > (P2[i,]*X2[j,]`) then SDRP[i,j]=1;else SDRP[i,j]=0;
        if (P2[i,]*X2[i,]`) >=(P2[i,]*X2[j,]`) then DRP[i,j]=1;else DRP[i,j]=0;
    end;
end;
GARP=DRP;
do k=1 to nrow(DRP);
    do i=1 to nrow(DRP);
        do j=1 to nrow(DRP);
            if GARP[i,k]=0 | GARP[k,j]=0 then t=1; else GARP[i,j]=1;
        end;
    end;
end;
/*# of violations*/
free couple;nvio=0;
do i=1 to nrow(X2);
    do j=1 to nrow(X2);
        if GARP[i,j]=1 & SDRP[j,i]=1 then do;
            nvio=nvio+1;
            couple=couple/(i||j);
        end;
    else t=1;
    end;
end;
nvioS=nvio;
if nvio^=0 then do;
S=1;
/*print couple;*/
end;
else S=0;
do while (nvio^=0);
/*searching for the maxima*/
free couple1;
do i=1 to nrow(couple);
    couple1=couple1/((couple[i,]||sum(GARP[couple[i,1],])));
end;
free couple1a;
m=max(couple1[,3]);
do i=1 to nrow(couple1);
    if couple1[i,3]=m then couple1a=couple1a/couple1[i,];
else t=1;
end;

/*simplification of couple1*/
free couple2;
couple2=couple1a[1,];
do i=2 to nrow(couple1a);
    if couple1a[i,1]^=couple1a[i-1,1] then couple2=couple2/couple1a[i,];
else t=1;
end;
free mat;
do i=1 to nrow(couple2);
    c=(-2*XX2[couple2[i,1],])`;
    H=I(ncol(XX2))#2;
    G=I(ncol(XX2))/P2[couple2[i,1],];

```

```

REL=j(ncol(XX2),1,'>')// '=';
B=j(ncol(XX2),1,0.01)/(P2[couple2[i,1],]*X2[couple2[i,1],]);
do j=1 to nrow(GARP);
  if GARP[couple2[i,1],j]=1 & (couple2[i,1]^=j) then do;
    G=G//P2[j,];
    rel=rel// '>=';
    B=B//P2[j,]*X2[j,]+0.000001;
  end;
  else t=1;
end;
  /*additional constraints for R=R* */
/*do j=1 to nrow(GARP);
  if GARP[couple2[i,1],j]=1 & (couple2[i,1]^=j) then do;
    G=G//P2[j,];
    rel=rel// '>=';
    B=B//(P1[j,]*X1[j,]+P2[j,]*X2[j,]-P1[j,]*X1[couple2[i,1],]+0.000001);
  end;
  else t=1;
end;*/
  run qp(noms,C,H,G,rel,B,teta);
load objval teta;
mat=mat/(objval|couple2[i,]|teta);
end;
mn=min(mat[,1]);
do i=1 to nrow(mat);
  if mat[i,1]=mn then do;
    X2[mat[i,2],]=mat[i,5:ncol(mat)];
    sol=sol//mat[i,];
  end;
  else t=1;
end;
/*GARP with adjusted bundles*/
SDRP=j(nrow(X2),nrow(X2),0);
DRP=j(nrow(X2),nrow(X2),0);
do i=1 to nrow(X2);
  do j=1 to nrow(X2);
    if (P2[i,]*X2[i,]) > (P2[i,]*X2[j,]) then SDRP[i,j]=1;else SDRP[i,j]=0;
    if (P2[i,]*X2[i,]) >=(P2[i,]*X2[j,]) then DRP[i,j]=1;else DRP[i,j]=0;
  end;
end;
GARP=DRP;
do k=1 to nrow(DRP);
  do i=1 to nrow(DRP);
    do j=1 to nrow(DRP);
      if GARP[i,k]=0 | GARP[k,j]=0 then t=1; else GARP[i,j]=1;
    end;
  end;
end;
  /*# of violations*/
free couple;nvio=0;
do i=1 to nrow(X2);
  do j=1 to nrow(X2);
    if GARP[i,j]=1 & SDRP[j,i]=1 then do;
      nvio=nvio+1;
      couple=couple/(i||j);
    end;
  else t=1;
end;
end;
end;
free adjS outS;
if nvioS^=0 then do;
  outS=sol;
  do i=1 to nrow(sol);
    adjS=adjS/(X2[sol[i,2],]-XX2[sol[i,2],]);
  end;
  store adjS outS;
end;
GARPA=GARP;

```

```

RX2=X2;
X=X1||X2;
P=P1||P2;

/*STEP 2 : OVERALL UTILITY & Sub-Utility*/

free sol;
/*1-a : GARP for the overall utility*/
SDRP=j(nrow(X),nrow(X),0);
DRP=j(nrow(X),nrow(X),0);
do i=1 to nrow(X);
    do j=1 to nrow(X);
        if (P[i,]*X[i,])` > (P[i,]*X[j,])` then SDRP[i,j]=1;else SDRP[i,j]=0;
        if (P[i,]*X[i,])` >=(P[i,]*X[j,])` then DRP[i,j]=1;else DRP[i,j]=0;
    end;
end;
GARP=DRP;
do k=1 to nrow(DRP);
    do i=1 to nrow(DRP);
        do j=1 to nrow(DRP);
            if GARP[i,k]=0 | GARP[k,j]=0 then t=1; else GARP[i,j]=1;
        end;
    end;
end;
/*# of violations*/
free couple;nvio=0;
do i=1 to nrow(X);
    do j=1 to nrow(X);
        if GARP[i,j]=1 & SDRP[j,i]=1 then do;
            nvio=nvio+1;
            couple=couple/(i||j);
        end;
    end;
else t=1;
end;
end;
nvioU=nvio;
/*if nvio^=0 then do;
print couple;
end;
else t=1;*/
do while (nvio^=0);
    /*searching for the maxima*/
    free couple1;
    do i=1 to nrow(couple);
        couple1=couple1/((couple[i,]||sum(GARP[couple[i,1],])));
    end;
    free couple1a;
    m=max(couple1[,3]);
    do i=1 to nrow(couple1);
        if couple1[i,3]=m then couple1a=couple1a/couple1[i,];
    end;
    else t=1;
end;

/*simplification of couple1*/
free couple2;
couple2=couple1a[1,];
do i=2 to nrow(couple1a);
    if couple1a[i,1]^=couple1a[i-1,1] then couple2=couple2/couple1a[i,];
else t=1;
end;
free mat;
do i=1 to nrow(couple2);
    c=(-2*XX[couple2[i,1],])`;
    H=I(ncol(XX))#2;
    G=I(ncol(XX))/P[couple2[i,1],];
    REL=j(ncol(XX),1,'>')/'=';
    B=j(ncol(XX),1,0.01)/(P[couple2[i,1],]*X[couple2[i,1],])`;
    do j=1 to nrow(GARP);
        if GARP[couple2[i,1],j]=1 & (couple2[i,1]^=j) then do;
            G=G/P[j,];
            rel=rel/'>=';
        end;
    end;
end;

```



```

        B=B//P[j,]*X[j,]^+0.000001;
        end;
        else t=1;
        end;
/*optional: additional constraint for sub-budget weakly separable*/
        G=G//((j(1,ncol(P1),0)||P2[couple2[i,1],]));
        B=B//((P2[couple2[i,1],]*X2[couple2[i,1],]^));
        rel=rel/'=';
/*additional constraint for sub-utility*/
        do j=1 to nrow(GARPA);
            if GARPA[couple2[i,1],j]=1 & (couple2[i,1]^=j) then do;
                G=G//((j(1,ncol(P1),0)||P2[j,]));
                rel=rel/'>=';
                B=B//P2[j,]*X2[j,]^+0.000001;
                end;
            else t=1;
            end;
        run qp(noms,C,H,G,rel,B,teta);
        load objval teta;
        mat=mat//((objval||couple2[i,]||teta`));
    end;
    mn=min(mat[,1]);
    do i=1 to nrow(mat);
        if mat[i,1]=mn then do;
            X[mat[i,2],]=mat[i,5:ncol(mat)];
            sol=sol//mat[i,];
            end;
        else t=1;
        end;
/*GARP with adjusted bundles*/
SDRP=j(nrow(X),nrow(X),0);
DRP=j(nrow(X),nrow(X),0);
do i=1 to nrow(X);
    do j=1 to nrow(X);
        if (P[i,]*X[i,]^) > (P[i,]*X[j,]^) then SDRP[i,j]=1;else SDRP[i,j]=0;
        if (P[i,]*X[i,]^) >= (P[i,]*X[j,]^) then DRP[i,j]=1;else DRP[i,j]=0;
    end;
end;
GARP=DRP;
do k=1 to nrow(DRP);
    do i=1 to nrow(DRP);
        do j=1 to nrow(DRP);
            if GARP[i,k]=0 | GARP[k,j]=0 then t=1; else GARP[i,j]=1;
            end;
        end;
    end;
/*# of violations*/
free couple;nvio=0;
do i=1 to nrow(X);
    do j=1 to nrow(X);
        if GARP[i,j]=1 & SDRP[j,i]=1 then do;
            nvio=nvio+1;
            couple=couple//(i||j);
            end;
        else t=1;
        end;
    end;
end;
end;
GARPB=GARP;
free adjU outU;
if nvioU^=0 then do;
    outU=sol;
    do i=1 to nrow(outU);
        adjU=adjU//((XX[outU[i,2],]-X[outU[i,2],]));
        X1=X[,1:ncol(XX1)];X2=X[,ncol(X1)+1:ncol(X)];
    end;

/*normalization*/
store outU adjU;
end;

```

```

adj=(X-XX);
print adj;
store adj;

/*Printings*/
adS=trace((RX2-XX2)`*(RX2-XX2));
adO=trace((X-XX)`*(X-XX));

RmS=sqrt(sum((RX2-XX2)##2)/(ncol(RX2)*nrow(RX2)));
RmO=sqrt(sum((X-XX)##2)/(ncol(X)*nrow(X)));

MAPES=(sum(abs(RX2-XX2))/(ncol(RX2)*nrow(RX2)));
MAPEO=(sum(abs(X-XX))/(ncol(X)*nrow(X)));

RmSP=sqrt(sum(((RX2-XX2)/XX2)##2)/(ncol(RX2)*nrow(RX2)));
RmOP=sqrt(sum(((X-XX)/XX)##2)/(ncol(X)*nrow(X)));

MAPESP=(sum(abs((RX2-XX2)/XX2))/(ncol(RX2)*nrow(RX2)));
MAPESPO=(sum(abs((X-XX)/XX))/(ncol(X)*nrow(X)));

resu=(nvioS//nvioU)||((adS//adO)||((RmS//rmO)||((RmSP//rmOP)||((MAPES//MAPEO)||((MAPESP//MAPESPO);
print ' _____ ',
,
      ' Adjustment Procedure in Order to Produce Data: ',
      'i) Consistant with the Sub-utility ',
      'ii) Consistant with the Overall Utility, given i)',
      'Objective function: SiSj[Zij-Xij]² ',
,
      ' _____ ';

print resu[rowname={'Sub-Utility Alone','Overall Utility'}
colname={'# of viol.','Adj.','RMSE','RMSE%','MAPE','MAPE%'} label='Summary Statistics'];

finish tsip;
store module=(tsip QP);

use sasuser.GARP06;

read all var {LTD RP CP TB} into X1;
read all var {UCLTD UCRP UCCP UCTB} into P1;

read all var {CURR DD OCDC OCDT SAVC SAVT RMF STDC STDT IMMF } into X2;
read all var {UCCTC UCDDC UCOCDC UCOCDT UCSAVC UCSAVT UCRMF UCSTDC UCSTDT UCIMF} into P2;

run tsip(X1,X2,P1,P2);

```

Instrumental Variables Regression Test Used on Risk Neutral Case from Barnett and de Peretti (2009).

```

start IVtest(y,X1,X2,Z);
/*Z : instruments, intercept added in the program
X1 : first set of explanatory variables
X2 : second set of explanatory variables, tested for nullity
y : dependent variable*/

X=X1||X2;

/*adding intercepts*/
Xa=j(nrow(Y),1,1)||X;
Za=j(nrow(Y),1,1)||Z;

/*stage 1*/
cX=Za*(inv(Za`*Za)*Za`*Xa);
/*stage 2*/
B2sls=inv(cX`*cX)*cX`*y;

```

```

/*F-Test for the nullity of the coefficient of X2 see Johnston p.158*/
/*restricted IV regression*/
Re=j(ncol(X1),ncol(X1)+1,0)||I(ncol(X2));
bs=B2sls+inv(cX`*cX)*Re`*inv(Re*inv(cX`*cX)*Re`)*(-Re*B2sls);
Er=y-cX*Bs;
/*unrestricted IV regression*/
Eur=y-cX*B2sls;
/*unrestricted OLS regression*/
Bols=inv(Xa`*Xa)*Xa`*y;
e=y-Xa*Bols;
/*test*/
F=((er`*er-eur`*eur)/ncol(X2)) / ((e`*e)/(nrow(y)-ncol(x)));
pF=1-probf(F,ncol(X2),(nrow(y)-ncol(xa)));
/*B-IV estimation*/
Pz=Za*inv(Za`*Za)*Za`;
Biv=inv(Xa`*Pz*Xa)*Xa`*Pz*y;

Siv=((y-Xa*Biv)`*(y-Xa*Biv))/(nrow(Y)-ncol(Xa));
/*Siv=((y-X*Biv)`*(y-X*Biv))/(nrow(Y)-ncol(X));given by softwares*/
varBiv=Siv*inv(Xa`*Pz*Xa);
/*Wald test for the nullity of the coefficient of X2*/
beta=Biv[nrow(Biv)-ncol(X2)+1:nrow(Biv)];
subVar=varBiv[nrow(Biv)-ncol(X2)+1:nrow(Biv),nrow(Biv)-ncol(X2)+1:nrow(Biv)];

Wald=beta`*inv(subVar)*beta;
PWald=1-probchi(Wald,ncol(X2));
std=sqrt(vecdiag(varbiv));
resu=Biv||std||(Biv/Std);
resu1=(wald)||PWald;
print resu[colname={'BETAiv','std','Student'} label='IV(2SLS) Estimation'];
print resu1[colname={'Statistic','Pr > ChiSq'} rowname={'Wald'} label='Testing the nullity of the
coefficient of X2'];
finish IVtest;
store module=IVtest;

use sasuser.wsrpc062012;

read all var {UCCTC} into UCCTC;
read all var {UCDDC} into UCDDC;
read all var {UCOCD C} into UCOCDC;
read all var {UCOCDT} into UCOCDT;
read all var {UCSAVC} into UCSAVC;
read all var {UCSAVT} into UCSAVT;
read all var {UCRMF} into UCRMF;
read all var {UCIMF} into UCIMF;
read all var {UCSTDC} into UCSTDC;
read all var {UCSTDT} into UCSTDT;
read all var {UCLTD} into UCLTD;
read all var {UCRP} into UCRP;
read all var {UCCP} into UCCP;
read all var {UCTB} into UCTB;

read all var {CTC DDC OCDC OCDT SAVC SAVT RMF STDC STDT IMF LTD RP CP} into X1;

read all var {TB} into X2;

read all var {sctc sddc socdc socdt ssavc ssavt srmf sstdc sstdt simf sltd srp scp stb} into Z;

y=log(UCCTC/UCDDC);
lx1=log(X1);
lx2=log(X2);
lz=log(Z);

run IVtest(y,lx1,lx2,lz);

```

Kalman Filter from Barnett and de Peretti (2009)

```
proc iml;
```

```

start kalman(Y);
reset noname;
store y;
start likeli(X);
/*X[1]=std(E(t))
X[2]=std(N(t))
X[3]=F
X[4]=A
X[5]=H
X[6]=B
*/
load Y;
cov=block(X[2],X[1]);
F=X[3];
A=X[4];
H=X[5];
B=X[6];
*call KALCVF(pred,vpred,filt,vfilt,y,0,a,f,b,H,cov);
call KALCVF(pred,vpred,filt,vfilt,y,0,A,F,B,H,cov);
dft=0;dft1=0;
do i=1 to nrow(Y);
et=y[i]-H*pred[i];
Ft=H*Vpred[i]*H`+X[1];
dft=dft+log(det(Ft));
dft1=dft1+(et*inv(Ft)*et`);
end;
ll=- (nrow(Y)*ncol(Y)*log(2*3.14116))/2-0.5*dft-0.5*dft1;
return(ll);
finish likeli;

/*optimization*/
load Y;
dY=Y[2:nrow(Y),]-Y[1:nrow(Y)-1,];
cov1=covlag(dY,2);
/*init[1]=Var(N(t)) init[2]=Var(E(t)) init[3]=F*/
init=j(6,1,0);
init[1]=abs(cov1[2]);
init[2]=cov1[1]-2*init[1];
init[3]=1;
if init[2]<0 then init[2]=0.1;
else init[2]=init[2];
init=sqrt(init);
init[5]=1;
print init;
cons={. . . 1 0,. . . 1 0,1 . . . ., . 1 . . . .}||{.,.,1,1}||{.,.,1e-6,1e-6};
optn={1 2};
*call nlpcc(rc,xres,"like",init,optn,cons);
*call nlptr(rc,xres,"likeli",init,optn,cons);
call nlpqn(rc,xres,"likeli",init,optn,cons);
*call nlpqua(rc,xres,"like",init,optn,cons);
cov=block(Xres[2],Xres[1]);
H=1;
F=Xres[3];
A=Xres[4];
H=Xres[5];
B=Xres[6];
*call KALCVF(pred,vpred,filt,vfilt,y,0,a,f,b,H,cov);
call KALCVF(pred,vpred,filt,vfilt,y,0,A,F,B,H,cov);
call KALCVS(sm,vsm,y,A,F,B,H,cov,pred,vpred);
store sm;
resu=y||pred||filt||sm||vfilt;
Xres1=Xres`;
print 'KALMAN FILTERING',
',
',
',
'TIME-INVARIANT MODEL:',
'Signal : Y(t)=B+HZ(t)+E(t)',
'State : Z(t)=A+FZ(t-1)+N(t)',
',
',

```

```

Xres1[rowname={'Var(E(t))','Var(N(t))','F','A','H','B'} colname={'Estimates'} label='Hyper-Parameters'],
,
resu[colname={'Observed','Pred. state','Filtered','Smoothed','Vfilt'}
label='Kalman Filter : Prediction, Filtering and Smoothing'];
obs=(1:nrow(Y));
final=obs||resu;
create final from final[colname={'date','series','Pred','Filtered','Smooth','vfilt'}];
append from final;
close final;
finish kalman;
store module=(kalman likeli);

use sasuser.kalmandata;

read all var {pce} into Y;

run kalman(Y);

```

Multivariate Independence Test from Barnett and de Peretti (2009)

```

proc iml;

*reset noname;
start Mtest(Ye,Xe,a);
*reset noname;
c=j(nrow(Ye),1,1);

/*Full Model*/
par=inv((c||Xe)`*(c||Xe))*(c||Xe)`*Ye;
r=Ye-(c||Xe)*par;
E_ome=Ye`*Ye-par`*((c||Xe)`*(c||Xe))*par;

/*Restricted Model*/
X1=Xe[,1:a];
par1=inv((c||X1)`*(c||X1))*(c||X1)`*Ye;
r1=Ye-(c||X1)*par1;
E_o=Ye`*Ye-par1`*((c||X1)`*(c||X1))*par1;

/*Global Significance of the Test*/
par2=inv(c`*c)*c`*Ye;
r2=Ye-c*par2;
E_g=Ye`*Ye-par2`*(c`*c)*par2;

/*Computing the tests*/
lambda=abs(det(E_ome)/det(E_o));
lambdaG=abs(det(E_ome)/det(E_g));

/*df*/
ve=nrow(Ye)-nrow(par);
vh=ncol(Xe)-ncol(X1);
p=ncol(Ye);
chi=-(ve-(p-vh+1)/2)*log(lambda);
if chi<=0 then chi=0.001;
else chi=chi;
pchi=1-probchi(chi,p*vh);
f=ve-(p-vh+1)/2;
if (p*p+vh*vh-5)>0 then d=sqrt((p*p*vh*vh-4)/(p*p+vh*vh-5));
else d=1;
l=(p*vh-2)/4;
Ft=((1-lambda##(1/d)) / lambda##(1/d)) * ((f*d-2*1)/(p*vh));
pf=1-probf(Ft,p*vh,f*d-2*1);
FStat= ((trace(r1`*r1)-trace(r`*r)) / (nrow(par)*ncol(par)-nrow(par1)*ncol(par))) /
(trace(r`*r)/(nrow(Xe)-nrow(par)*ncol(par)));
pfstat=1-probF(FStat,(nrow(par)-nrow(par1))*ncol(par),nrow(Xe)-ncol(par)*nrow(par));

FStatG= ((trace(r2`*r2)-trace(r`*r)) / ((nrow(par)-nrow(par2)*ncol(par)))) / (trace(r`*r)/(nrow(Xe)-
nrow(par)*ncol(par)));
pfstatG=1-probF(FStatG,(nrow(par)-nrow(par2))*ncol(par),nrow(Xe)-ncol(par)*nrow(par));

```

```

ind=(1:nrow(par)-1)`;
ind1=j(nrow(par)-1,1,'Q');
ind2='intercept'//concat(ind1,char(ind,2.0));
ind3=j(1,ncol(par),'equ. ');
ind4=(1:ncol(par));
ind5=concat(ind3,char(ind4,2.0));
resu=(Lambda||Chi||Ft)/(.||pchi||pf);
print '_____',
,
'Multivariate Test for Weak Separability',
'_____',
,
par[rowname=ind2 colname=ind5 label='A-Estimated parameters'],
'Significance of the model:', FstatG, 'pvalue', PFstatG,
'Separability Tests',
,
'H0: the first' a[format=2.0]' rows of X are separable',
resu[colname={'Lambda','Chi^2 approx','F approx'} rowname={'Statistic','P-Value'}
label='B-Statistical tests'];
if pchi>=0.05 then print 'Decision: H0 not rejected at 5%';
else print 'Decision: H0 is rejected, Data are not separable';
store pchi;
print PFstatG;
print Fstat;
print pfstat;
print Ft;
print ve;
print vh;
print pchi;
print pf;
finish Mtest;
store module=Mtest;

use work.LR82;

read all var {scurr sdd socdc socdt ssmac ssmat ssavmc ssavmt srmf sstdc sstdt simmf sltd srp scp stb}
into Xe;

read all var {
ctcddc ctcocdc ctcocdt ctcmmac ctcmmat ctcavmc ctcavmt ctcrmf ctcstdc ctcstdt ctcimf ctcldt
ctcrp
ddcocdc ddcocdt ddcmmac ddcmmat ddcsavmc ddcsavmt ddcrmf ddcstdc ddcstdt ddcimf ddcltd ddcrp
ocdcocdt odcmmac odcmmat ocacsavmc ocacsavmt ocacrmf ocacstdc ocacstdt ocacimf ocacldt ocacrp
ocdtmmac ocdtmmat ocotsavmc ocotsavmt ocotrmf ocdtstdc ocdtstdt ocdtimf ocdtldt ocdtrp
mmacmmat mmacsavmc mmacsavmt mmacrmf mmacstdc mmacstdt mmacimf mmacldt mmacr
mmatsavmc mmatsavmt mmatrmf mmastdc mmatstdt mmatimf mmatltd mmatr
savmcsavmt savmcrmf savmcstdc savmcstdt savmcimf savmcltd savmcrp
savmtrmf savmtstdc savmtstdt savmtimf savmtltd savmtrp
rmfstdc rmfstdt rmfimf rmfltd rmfrp
stdcstdt stdcimf stdcltd stdcrp
stdtimf stdtltd stdtrp
imfltd imfrp
ltdrp

} into Ye;

a=14;

run Mtest(Ye,Xe,a);

```

Quarterly Adjustment

```

proc expand data=sasuser.CAANDIMODERATE out=temp1 from=qtr to=month;
id date;

```

```
convert eeamnq;  
run;
```

Seasonal Adjustment

```
proc x12 data=tnotesnsa date=date;  
  var tnotes;  
  x11;  
  ods select d11;  
run;
```

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