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Text of paper:

JOINT MAIL-IN REBATE DECISIONS IN SUPPLY CHAINS UNDER DEMAND UNCERTAINTY

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JOINT MAIL-IN REBATE DECISIONS IN SUPPLY CHAINS UNDER DEMAND UNCERTAINTY

Abstract

We study the joint decisions of offering Mail-in rebates (MIR) in a single-manufacturer-single-retailer supply chain using a game theoretic framework. Either party can offer an MIR to the end consumer if it is in his best interest. The consumer demand is stochastic and depends on the product price and the amount of MIRs. When the retail price is exogenous, we show the existence of a unique Nash equilibrium under both additive and multiplicative demand functions and characterize it completely. We show that any of the following four scenarios can be the equilibrium: both parties offer MIR, only one party offers MIR, none offers MIR. When the retail price is a decision variable for the retailer and that the rebate redemption rate increases with the amount of MIR, we once again prove the existence of a unique Nash equilibrium where both the retailer and the manufacture offer MIRs. Using a numerical study, we show that the average post-purchase price of the product is higher not only than the perceived pre-purchase price but also than the newsvendor optimal price without an MIR. This implies that an MIR makes a product look cheaper while the consumers actually pay more on an average.

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1. INTRODUCTION

Mail-in rebate (henceforth abbreviated as MIR and/or called rebate) is a common promotional tool used in marketing consumer products and/or services. An MIR offers a delayed incentive to a consumer by offering cash (or gift cards) upon the purchase of a product, a bundle of products (e.g. a desktop computer along with a monitor and printer), or an upgrade. MIRs are common in consumer products ranging from software and electronics to home appliances and cosmetics. Millman (2003) notes that $10 billion worth of consumer rebates were offered in the United States in 2002. Further, the amount of unpaid rebates in the personal computer industry alone was estimated to have reached $10 billion by the year 2005 (Tugend, 2006). Young America Corporation, a rebates clearinghouse, handles almost 60 million rebates every year on behalf of its clients (Source: http://www.young-america.com/promotions_rebates.html, retrieved on 3/30/2009). A search of the website of the popular electronics retailer J & R Music & Computer World by the authors found 521 products with MIRs on 3/30/2009.

MIRs usually require consumers to follow certain rules to redeem cash, such as collecting the paperwork, filling out the forms, cutting UPC codes, and sending out the rebate request within the correct time frame. MIR redemption rate rarely reaches 100 percent. Some consumers do not apply, while some others apply but are not paid (Lisante, 2006). Despite proliferation of MIRs, consumer complaints about MIRs have soared. Consumers suspect that companies design the rules to keep redemption rates down. According to an article in *Business Week* (Grow, 2005), "what rebates do is get consumers to focus on the discounted price of a product, then buy it at full price." However, "processors and companies offering rebates insist that there is no intentional effort to deny them… the rules are aimed at stopping fraud." The academic research establishes that MIRs serve multiple purposes from the perspectives of manufacturer and/or retailer including increasing the sales of a new/upgraded product (Banks and Moorthy, 1999), disposing excess inventory (Kumar et al., 1998), encouraging brand switching (Ali et al., 1994) and improving profit (Soman, 1998).

The objective of this paper is to analyze the joint mail-in rebate decisions of a retailer and a manufacturer in a supply chain under demand uncertainty. We consider a supply chain consisting of a single manufacturer and a single retailer. The manufacturer produces a single product and sells it exclusively through the retailer. Either party can, however, offer an MIR to the end consumer if it is in its best interest. The consumer demand is stochastic and is affected by the amount of rebates. Under such circumstances, we use game theoretic models to answer the following questions. Why is MIR offered? How to characterize Nash equilibrium of the game when both manufacturer and the retailer consider offering MIRs? When will the MIRs be offered by both parties versus by only one party? How are optimal MIR decisions and profits affected by consumers' valuation of MIRs and by the rebate redemption rate? Our analyses also show how answers to such questions might differ across additive and multiplicative demand functions. We also compare our work with the literature.

Manufacturer's MIR is common in various product categories such as home appliance, consumer electronics, and software. Electronics retailer Best Buy supports MIRs on its products offered by Best Buy as well as various manufacturers.[1] Similar information can also be found on the websites of Staples and Office Depot. Sears, on the other hand, notes on its website that a vast majority of the MIRs on its products are directly supported by Sears[2]. On a recent purchase of a Motorola cellular phone at Staples, one of the authors received two concurrent MIRs on it, one from Motorola and another from Staples. During the fourth quarter of 2008, GE was offering MIRs of varying amounts on home appliances sold through Lowe's. Lowe's, concurrently, was offering an MIR of an amount equal to local delivery charges for any appliance purchase over $397. Finally, many products are sold without any MIR. These examples illustrate that each of the following four scenarios is common in practice: both the manufacturer and the retailer offer MIRs concurrently, only the manufacturer offers the MIR, only the retailer offers the MIR, and no party offers MIR.

Our paper shows that when the retail price is exogenous, there exists a unique Nash equilibrium to the MIR game under both additive and multiplicative demand functions. Depending upon the problem parameters, all four possible scenarios of MIR described above can be the equilibrium. We characterize sufficient conditions for each of the four MIR scenarios to be the Nash equilibrium. Our numerical study suggests that the expected profits of both retailer and manufacturer increase with the customers' valuation for MIR and decrease with the rebate

[1] Source: http://www.bestbuy.com/site//olspage.jsp?id=cat12098&entryURLType=&categoryId=cat10007&type=page&entryURLID=&contentId=1087340679900, retrieved on 3/26/2009

[2] Source: https://www.mysearsrebate.com/faqs.aspx, FAQ # 12, retrieved on 3/30/2009

redemption rate. We also consider a more general case where the retail price is a decision variable for the retailer and that the rebate redemption rate increases with the amount of MIR. We once again prove the existence of a unique Nash equilibrium, where both the retailer and the manufacture offer MIRs. Using a numerical study, we show that the average post-purchase price of the product is higher not only than the perceived pre-purchase price but also than the newsvendor optimal price without an MIR. This implies that an MIR makes a product look cheaper while the consumers actually pay more on an average.

Our work makes the following contributions to the operations management literature. The literature typically considers MIR offered *exclusively* by the manufacturer. While the amount of MIR is still a decision variable, the literature exogenously assumes that the retailer will not offer MIR. The only work that we are aware of where the decision to offer an MIR is endogenous to the model is by Cho et al. (2009). They consider the case where either the manufacturer or the retailer (or both) can offer an MIR. However, they model the consumer demand using a simple deterministic demand function. We provide a generalization of these works by considering stochastic demand and *simultaneous* and *endogenous* decision making where both parties consider offering MIR. A newsvendor framework is used to model demand uncertainty. In our model, the decision to offer an MIR and its amount are determined endogenously by the Nash equilibrium outcome of the game between the retailer and the manufacturer. To the best of our knowledge, our work is the first to study simultaneous MIRs with endogenous decisions under demand uncertainty. We compare and contrast the results of our simultaneous game with those from exclusive games where either the manufacturer or the retailer alone offers MIR. When both rebate and retail price are decision variables, we show that the manufacturer offers a lower MIR in a simultaneous game compared to an exclusive game with manufacturer MIR. We also show that the expected profits of both the retailer and the manufacturer are higher in the simultaneous game compared to an exclusive game with retailer MIR. Our numerical studies suggest that a similar relationship about the expected profits might hold between the simultaneous game and the exclusive game with manufacturer MIR.

The remainder of this paper is organized as follows. We review the appropriate literature in the next Section. Section 3 describes our models and results when the retail price is

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exogenous. We consider a more general case with endogenous retail price and rebate dependent redemption rate in Section 4. Section 5 summarizes and concludes the paper.

2. LITERATURE REVIEW

Rebates have been studied extensively in both operations and marketing literature. The marketing literature typically looks at the pricing and consumer choice issues under rebates, while the operations literature typically focuses on the over-all supply chain dynamics including inventory and profit implication of rebates.

Rebates, in marketing literature, are widely considered as a form of price discrimination between high- and low-reservation price consumers (Narasimhan, 1984; Gerstner and Holthausen, 1986; Tirole, 1989; Gerstner et al., 1994). Indeed, Blattberg and Neslin (1990) have described rebates as the "durable goods analog" of coupons. Gerstner and Hess (1991a, 1991b, 1995) compare a push strategy where a manufacturer offers a trade discount to the retailer and a pull strategy where the manufacturer offers rebates directly to consumers. The market consists of the high and the low consumers. They find that MIR can be profitable even if all consumers redeem the rebate and price discrimination does not occur. Citing the post-purchase delay associated with the redemption of an MIR, Chen, Moorthy, and Zhang (2005) argue that MIRs present a seller with an opportunity of price discrimination within a consumer, giving discounts when they are most valuable, and withholding it when they matter least. They argue that this might increase a consumer's upfront willingness to pay. Using a game theoretic framework, Khouja and Zhou (2009) examine manufacturer's MIR in a single-manufacturer-single-retailer supply chain where the manufacturer is the Stackelberg leader. They find that rebates are profitable for the manufacturer if consumers are inconsistent in the sense that their valuation of rebate when they make purchase decisions is independent of redemption probabilities.

The operations management literature considers two types of rebates, the *sales* rebate that goes from a manufacturer to a retailer when certain conditions for sales are met; and the *consumer* rebate that goes directly to a consumer. The focus of our work is on consumer rebate. As a result, we choose not to review the literature on sales rebate contracts. Aydin et al. (2008) provide a recent review of this literature in their paper. The literature on consumer rebate often uses the newsvendor model as a building block to study MIRs in a single-manufacturer-single-

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retailer supply chain. Arcelus et al. (2005) consider the joint pricing and ordering policies of such a retailer. Both additive and multiplicative demand functions are considered. The manufacturer provides a direct price discount with zero rebates or a rebate with zero price discounts. Arcelus et al. (2007) extends their earlier work by incorporating stochastic rebate-redemption rate that depends upon the rebate value itself. Their main finding is that the introduction of uncertainty in the redemption rate leads the rebate policy to dominate its price-discount counterpart. Further extensions of these works include incorporation of risk averseness (Arcelus et al., 2006) and information asymmetry (Arcelus et al., 2008). Chen et al. (2007) consider a game where the manufacturer makes decisions on wholesale price and MIR while the retailer makes decisions on retail price as a Stackelberg follower. Consumers are divided into a rebate-sensitive segment and a rebate-insensitive segment. Their key result is that unless all of the customers redeem the rebate, it is in the manufacturer's best interest to offer rebate and that the instant rebate does not necessarily benefit the manufacturer. Aydin et al. (2008) also consider manufacturer MIR under exogenous wholesale price. The manufacturer in their model sets the MIR while the retailer sets the retail price and the order quantity under a multiplicative demand function. They find that the retailer gets a fixed fraction of the supply chain profit. Cho et al. (2009) consider a single-manufacturer-single-retailer supply chain where both parties strategically consider offering MIRs. The MIR expands the consumer demand; however, there is a fixed cost associated with administering the program which reduces the profit of the party offering it. Using a deterministic demand model they determine the equilibrium of the game and characterize the conditions under which a firm should offer rebates at equilibrium.

Finally, there is a stream of literature in operations management that studies advance booking discount (ABD) using the newsvendor setting. Unlike an MIR which is a delayed discount, an ABD is an *early* purchase discount for the consumers. Under the ABD scheme a retailer allows the consumers to purchase a product at a discounted price before the selling season. The consumers under the ABD program are guaranteed delivery during the selling season, while the consumers not under the scheme pay a higher price during the season and are not guaranteed availability. Tang et al. (2004) study the dynamics of the ABD program using a newsvendor setting and determine the optimal discount rate. They show that ABD program generates additional sales for the retailer. A similar scheme has also been considered by Weng and Parler (1999), while McCardle, Rajaram and Tang (2004) consider a competitive version of the ABD scheme.

Our paper also considers consumer rebates in a single-manufacturer-single-retailer supply chain under a newsvendor framework. Unlike ABD, we study MIR, which is a delayed incentive for consumers. We allow both retailer and manufacturer to offer MIRs and that the decision to offer an MIR is determined by the Nash equilibrium of the non-cooperative game between them. Our modeling framework, thus, is similar to that of Cho et al. (2009). There are, however, important differences between our work and theirs. Unlike Cho et al., we consider a stochastic consumer demand and use a newsvendor framework to study MIR. To the best of our knowledge, ours is the only paper to consider joint rebate decisions in a newsvendor framework. Further, the fixed cost of offering MIR is not the focus of our analysis. As a result, we differ extensively from Cho et al. (2009) in terms of model formulation, analyses, and the resulting insights. We compare and contrast our results with those from literature where appropriate.

3. THE MODEL AND ANALYSES

Consider a supply chain involving a single manufacturer selling a single product over a single time period to a single retailer at a constant wholesale price w. Each unit of the product incurs a production cost $c > 0$. The retailer resells the product to the end consumers at a retail price p. We will let Q denote the retailer's order quantity. The demand is uncertain and is affected by the amount of MIR. We will consider both additive and multiplicative demand functions. Both the manufacturer and the retailer consider offering MIR. The amount of MIR is a decision variable for any party offering the MIR. We will assume that the wholesale price is exogenously fixed. While this assumption is mainly for analytical tractability, it is also an approximation of the environment where the rebate offers constitute a further stage of decision making in a supply chain with a well-established wholesale price. The examples of such supply chains are consumer electronics, home appliance, etc. Furthermore, such assumption is standard in literature (Aydin et al., 2008; Arcelus et al., 2005, 2007). We will, for this section, further assume that the product retail price p is also exogenous to our model. We relax this assumption in the next Section. The rationale behind the exogenous retail price assumption is as follows. An MIR is a temporary and delayed incentive for consumers that allows a retailer to maintain the

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current price point. Thus, in practice, the product retail price rarely changes because of the introduction/expiration of an MIR. In line with this observation, this Section treats the product retail price to be exogenous. Section 4 considers the case where the retail price is a decision variable.

We model the scenario in which both the manufacturer and the retailer decides on MIR. Note that the decision to offer an MIR is endogenous to our model and is determined by the Nash equilibrium outcome of the game between the manufacturer and the retailer. The MIR is non-negative. Hence a zero MIR at equilibrium simply means that it is optimal for a party to not offer any MIR. We will let $r_R \geq 0$ and $r_M \geq 0$ denote the MIRs of the retailer and the manufacturer respectively while r_R^* and r_M^* will denote values of the corresponding quantities at equilibrium. We assume that the consumers treat \$1 MIR as the equivalent of a \$$\alpha$ price deduction. The quantity α represents the effective fraction of MIR that the consumer values. Such modeling approach is standard in the literature on MIR (Aydin et al., 2008). We call α to be the rebate *sensitivity* parameter. We assume that consumers are homogenous in the sense that they treat the manufacturer and the retailer MIRs equally. Therefore, when the product retail price is p and both the manufacturer and the retailer offer MIRs, the effective price perceived by the consumers at the time of purchase is $p-\alpha(r_R+r_M)$, $0 \leq \alpha \leq 1$. We further assume that a constant fraction β, $0 \leq \beta \leq 1$, of the consumers can successfully redeem the rebate and that the consumers who redeem an MIR successfully from the manufacturer/retailer will also redeem it successfully from the retailer/manufacturer. Such an assumption, once again, is standard in literature (Arcelus et al., 2006; Aydin et. al., 2008), and its evidence is well-documented in the popular business press (Bulkeley, 1998). We will relax the assumption of constant redemption rate in Section 4.

The timing of the events is as follows. The wholesale and the retail prices are exogenous. In the first stage of the game, the manufacturer and the retailer simultaneously decide on the amount of MIRs. After observing the MIRs, the retailer places his order Q with the manufacturer. Finally, the consumer demand is realized. The remainder of this section is organized as follows. We will consider both additive and multiplicative demand functions to model the consumer demand. Sections 3.1 and 3.2, respectively, will model and analyze these two demand functions.

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3.1. Additive Demand Function

We will use the demand function $D = N - b(p - \alpha r_R - \alpha r_M) + \varepsilon$ to model the consumer demand, where, N and b are constants, and D, p, r_R , and r_M respectively are the product demand, retail price, and MIRs offered by the retailer and the manufacturer respectively. The randomness in demand is modeled through the random variable ε, defined on [A, B] with $B > A \geq 0$. In order to assure that positive product demand is possible for some range of the retail price p, we will assume $N - bp + A \geq 0$. The product retail price is exogenous. We will let f(.), F(.), and μ, respectively, denote the probability density function, the cumulative distribution function, and the mean of the random variable ε. We will assume that ε exhibits increasing failure rate (IFR). Many commonly used distributions exhibit IFR, such as, the normal distribution, power distribution, uniform distribution, Beta distribution with both parameters greater than or equal to one, Gamma distribution with shape parameter greater than or equal to one, etc. The IFR assumption is widely used in operations management literature (Lariviere and Porteus, 2001).

The analyses of our paper will follow the standard techniques used in the price setting newsvendor literature where the stocking quantity and the price are set simultaneously. A reader is referred to Petruzzi and Dada (1999) for a complete description and analytical treatment of the problem. Following the standard approach, we define a stocking factor, z, given by $z \equiv Q - \{N - b(p - \alpha r_R - \alpha r_M)\}$ and write the profit functions in terms of z. The variable z is a proxy for service level, the probability that consumers do not encounter a stock out. It also represents the number of standard deviations that stocking quantity deviates from expected demand (Silver and Peterson 1985).

3.1.1. Retailer's Optimal Response

We solve the game using the standard technique of working backwards beginning with the final stage of the game. Given two rebates r_M and r_R from the manufacturer and the retailer, the retailer decides the order quantity Q to maximize his expected profits. The retailer's expected profit function is given by:

$$\pi_R(r_M, r_R, Q)$$
$$= pE\min(D,Q) - \beta r_R E\min(D,Q) - wQ$$
$$= (p - \beta r_R - w)(N - b(p - \alpha r_R - \alpha r_M) + z - \Lambda(z)) - w\Lambda(z) = \pi_R(r_M, r_R, z), \quad (1)$$

where $\Lambda(z) = \int_A^z (z-x) f(x)dx$ is the expected leftover factor. Taking the first order derivative with respect to z and noting that the second order condition for maximization of (1) is satisfied, we get,

$$\partial\pi_R(r_M, r_R, z)/\partial z = (p - \beta r_R)(1 - F(z)) - w. \quad (2)$$

By setting (2) to zero, we have

$$z(r_R) = F^{-1}(1 - w/(p - \beta r_R)). \quad (3)$$

Define z_0 to be the retailer's optimal stocking factor when he does not offer a rebate, i.e.,

$$z_0 \equiv F^{-1}(1 - w/p). \quad (4)$$

It is easy to see from (3) and (4) that, $z(r_R) \leq z_0$ if and only if $r_R \geq 0$. Thus, the non-negativity of the MIR can also be expressed as a constraint $z(r_R) \leq z_0$. The retailer solves for the optimal rebate decisions next. Using the chain rule of differentiation, the first order necessary condition with respect to the retailer's MIR can be written as follows.

$$\frac{\partial\pi_R(r_M, r_R, z(r_R))}{\partial r_R} = \frac{\partial\pi_R(r_M, r_R, z)}{\partial r_R} + \frac{\partial\pi_R(r_M, r_R, z)}{\partial z}\frac{dz(r_R)}{dr_R}.$$

Note that the second term in the right hand side of the above equation is zero as $\partial\pi_R(r_M, r_R, z)/\partial z = 0$. Setting the above expression to zero and simplifying we get,

$$r_R(r_M, z) = \frac{\alpha b(p - w) - \beta(N - bp + z - \Lambda(z))}{2\alpha b\beta} - \frac{r_M}{2}. \quad (5)$$

Combining equations (2) and (5) we get

$$\frac{\partial\pi_R(r_M, r_R(r_M, z), z)}{\partial z} = \frac{(1 - F(z))}{2\alpha b}[\beta(N - bp + z - \Lambda(z)) - \frac{2\alpha bw}{(1 - F(z))} + \alpha b(p + w + \beta r_M)]. \quad (6)$$

Lemma 1. *Given a non-negative MIR from the manufacturer,*

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(a) the retailer's profit function $\pi_R(r_M, r_R(r_M, z), z)$ *is quasi-concave in the stocking factor z, and that it has a unique maximizer* $z(r_M)$ *that satisfies* $\partial\pi_R(r_M, r_R(r_M, z), z)/\partial z = 0$.

(b) the retailer's best response stocking factor and best response MIR, respectively, are given by $z^*(r_M) = \min\{z(r_M), z_0\}$ *and* $r_R^*(r_M) = [p - w/(1 - F(z^*(r_M)))]/\beta$.

Proofs for all results are included in the Appendix. Lemma 1(a) indicates that for a non-negative MIR from the manufacturer, there is a unique stocking factor $z(r_M)$ that maximizes the retailer's expected profit. However, when the manufacturer's rebate is very large we might have a scenario where $z(r_M) > z_0$. This implies, from (3) and (4), a negative MIR from the retailer. Thus, we impose the non-negativity constraint $z(r_M) \le z_0$ into retailer's maximization problem. With such a constraint, due to quasi-concavity of the retailer's profit function $\pi_R(r_M, r_R(r_M, z), z)$, the optimal stocking factor $z^*(r_M)$ is the minimum of $z(r_M)$ and z_0 as formally stated in Lemma 1(b). Once the best response function $z^*(r_M)$ is known, the corresponding MIR can be derived from setting equation (2) zero. We further have the following result.

Proposition 1: *Retailer's best response stocking factor increases in the manufacturer's MIR and that retailer's best response MIR decreases in manufacturer's MIR.*

The findings in Proposition 1 are intuitive. All else being equal, as the manufacturer's MIR increases, the acquisition cost of the consumer goes down. Under this scenario, the manufacturer bears the extra cost of the rebate while the retailer does not incur any additional cost. As a result, the retailer reacts by increasing his stocking factor which is a proxy for the retailer's service level. The intuition behind the retailer's best response MIR is similar. As the MIR from the manufacturer increases, the product demand increases. The rational retailer, as a result, decreases his own MIR which allows him to take advantage of the manufacturer's rebate without incurring any additional cost.

3.1.2. Manufacturer's Optimal Response

Given a non-negative MIR from the retailer ($r_R \ge 0$), the manufacturer can infer the

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retailers stocking factor $z(r_R)$ from (3) and incorporate this information to make his own decision about the MIR r_M. The manufacturer's expected profit function is given by

$$\pi_M(r_M, r_R, Q) = (w-c)Q - \beta r_M E\min(D,Q).$$

Using the definition of stocking factor *z*, we can rewrite the manufacturer's profit as

$$\begin{aligned}&\pi_M(r_M, r_R, z(r_R))\\ &= (w-c-\beta r_M)[N - b(p-\alpha r_R - \alpha r_M) + z(r_R) - \Lambda(z(r_R))] + (w-c)\Lambda(z)\end{aligned} \quad (7)$$

Taking first derivative with respect to r_M and setting it to zero yields

$$r_M(r_R) = \frac{\alpha b(w-c) - \beta(N - bp + z(r_R) - \Lambda(z(r_R)))}{2\alpha b\beta} - \frac{r_R}{2}. \quad (8)$$

It is analytically impractical to solve r_M in terms of r_R from (8). So we invert $z = z(r_R)$ to $r_R = r_R(z)$ and work with the stocking factor z. Such a technique is common in literature; for example, see Lariviere and Porteus (2001), and Song et al. (2008). Substituting (3) into (8) yields

$$r_M(z) = \frac{\alpha b(w-c) - \beta(N - bp + z - \Lambda(z)) - \alpha b(p - w/(1-F(z))}{2\alpha b\beta}. \quad (9)$$

The rebate $r_M(z)$ derived from (9) is not guaranteed to be non-negative. However, it is easy to verify that $\partial^2\pi_M(r_M, r_R(z), z)/\partial r_M{}^2 \le 0$, implying that for a given *z*, the manufacturer's profit function is concave in his rebate r_M. This allows us to characterize the manufacturer's best response MIR for non-negative MIRs. The following Lemma states our result.

Lemma 2: *Given a non-negative MIR from the retailer, the manufacturer's best response MIR,* $r_M^*(z)$, *is given by* $r_M^*(z) = \max\{r_M(z), 0\}$, *where* $r_M(z)$ *is defined by (9) and is convex in z.*

Lemma 2 characterizes the manufacture's best response in terms of the retailer's stocking factor. From its convexity, the function $r_M(z)$ is either increasing in z or is decreasing in z before increasing in it. This implies that unlike the retailer's best response MIR, the manufacturer's best response MIR can be either decreasing or increasing in retailer's MIR. The manufacturer's profit is directly affected by the retailer's order quantity. The retailer's order quantity, in turn, is determined by the random demand and the desired service level. The consumer demand increases in response to a higher MIR from the retailer. However, because of the shrinking profit margin, the retailer's service level goes down as well (per equation 4). If the demand effect dominates the service level effect, retailer orders more and the manufacturer takes the opportunity to reduce his MIR for better profit margin. On the other hand, when the service level effect dominates the demand effect, retailer may order less. In such a scenario, the manufacturer should provide more MIR in an effort to increase demand and thereby improving order quantity. We are now ready to describe the Nash equilibrium of the game.

3.1.3. Nash Equilibrium

Lemma 1 and Lemma 2 characterize the best response functions of the retailer and the manufacturer respectively. The non-negativity of MIRs makes the best response functions non-differentiable. To facilitate the characterization of the Nash equilibrium, we temporarily ignore the non-negativity constraints. In such a scenario, Nash equilibrium can be found by setting equation (6) equal to zero and solving the resulting equation simultaneously with (9). This yields:

$$\alpha b(w-c) - 2\alpha b(p-w) + \beta(N - bp + z - \Lambda(z)) + 3\alpha b(p - w/[1-F(z)]) = 0\,. \qquad (10)$$

The following lemma describes the solution of (10).

Lemma 3:

(a) There exists a unique solution z_s *to equation* (10).

(b) z_s *decreases in the rebate sensitivity parameter* α *and increases in the redemption rate* β, *while* $r_R(z_s)$ *increases in* α *and decreases in* β.

Lemma 3(a) states that ignoring the non-negativity of the MIRs, the best response functions of the retailer and the manufacturer can be solved uniquely. Lemma 3(b) further describes the properties of z_s, the solution to equation (10). Thus, z_s represents the Nash equilibrium when none of the two non-negativity constraints on MIRs are binding. Under such a scenario, per Proposition 3(b), as the rebate sensitivity parameter increases, the retailer's MIR increases while his stocking factor decreases. The result is intuitive. An increase in the rebate sensitivity parameter indicates that consumers perceive an MIR to be closer to a direct price reduction. The

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retailer responds to this by offering a higher MIR which further expands the product demand which may result in higher expected sales. However, a higher MIR reduces the margin of the retailer as well, which in turn results in a lower stocking factor. On the other hand, as the redemption rate increases, the retailer's MIR decreases while his stocking factor increases. A higher redemption rate negatively affects the profit margin of the retailer without expanding the consumer demand. As a result, the retailer's MIR goes down with the redemption rate. Once the non-negativity constraints of the MIRs are reintroduced, the best response functions described in Lemmas 1 and 2 become non-differentiable. Fortunately, there still exists a unique Nash equilibrium as shown below.

Proposition 2: *Depending upon the problem parameters, any of the following four scenarios can be the unique Nash equilibrium of the MIR game between the manufacturer and the retailer: no party offers MIR, only the retailer offers the MIR, only the manufacturer offers the MIR, both parties offer MIR. Specifically, the equilibrium can be characterized as follows.*

(a) If $z_s < z_0$ *and* $r_M(z_s) > 0$, *then* $z^* = z_s$, *and* $r_M{}^* = r_M(z^*)$.

(b) If $z_s < z_0$ *and* $r_M(z_s) \le 0$, *then* $r_M{}^* = 0$, *and* $z^* = \min\{z(0), z_0\}$.

(c) If $z_s \ge z_0$, *then* $z^* = z_0$, *and* $r_M{}^* = \max\{r_M(z_0), 0\}$.

Proposition 2 characterizes the unique Nash equilibrium of the MIR game in terms of the manufacturer's rebate and the retailer's stocking factor. Once the retailer's equilibrium stocking factor z^* is known, his equilibrium MIR is automatically determined from equation (3). Recall that z_s, in Proposition 2, is the solution to equation (10); while z_0 is the retailer's optimal stocking factor when no party offers MIR, and $z(0)$ is the retailer's optimal stocking factor if the manufacturer does not offer MIR. Figures 1(a)-1(d) schematically describe the four possible equilibrium scenarios. In each of these figures we have plotted $r_M(z)$ against $z(r_M)$. When the non-negativity constraints for the two MIRs are not binding, per Lemmas 1 and 2, these two functions respectively represent the best response functions of the manufacturer and the retailer (shown in solid lines in the figures). Once a non-negativity constraint becomes binding, the best response function no longer follows the original curve (shown in dotted lines), but is given by the solid vertical line. Proposition 2(a) describes the scenario where both the retailer and the

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manufacturer offer an MIR, as illustrated by Figure 1(a). Proposition 2(b) describes the scenario where the manufacturer does not offer MIR while the retailer may or may not offer it. Figure 1(b) shows a scenario where the retailer offers an MIR and the manufacturer does not. Note from this figure that $r_M(z_s)$ is negative while the retailer's best response stocking factor is positive and is given by $z(0)$. Thus, only the retailer offers MIR at equilibrium. Proposition 2(c) describes the scenario where the retailer does not offer MIR while the manufacturer may or may not offer an MIR. Per Lemma 2, the manufacturer's best response MIR is given by $r_M^*(z) = \max\{r_M(z), 0\}$. Thus, the condition $r_M(z_s) > 0$ does not necessarily imply that the manufacturer will offer an MIR. Figures 1(c) and 1(d) respectively describe the situations where the manufacturer does and does not offer MIRs.

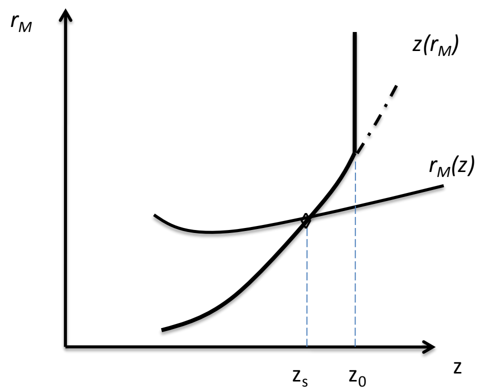

Figure 1(a): Illustration of Equilibrium

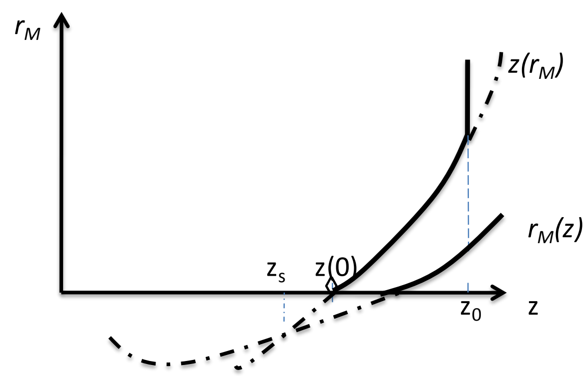

Figure 1(b): Illustration of Equilibrium

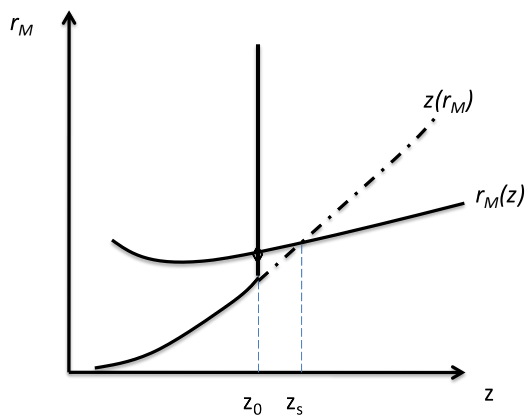

Figure 1(c): Illustration of Equilibrium

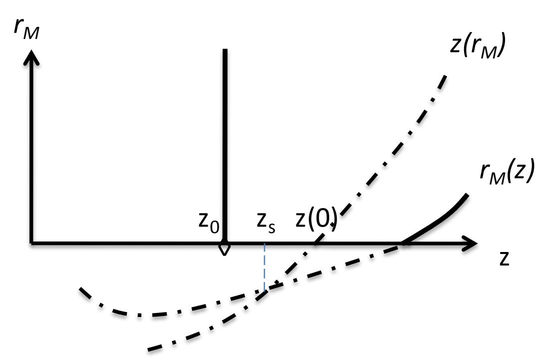

Figure 1(d): Illustration of Equilibrium

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Proposition 2 provides complete technical characterization of the equilibrium. We explore the properties of the Nash equilibrium and develop interesting insights in Proposition 3 below and the numerical study following it. For the ease of exposition, define $\Delta = N - bp + F^{-1}(1 - w/p) - \Lambda(F^{-1}(1 - w/p))$, which represents the retailer's expected sales when no party offers MIR.

Proposition 3:

(a) The retailer offers a mail-in-rebate at equilibrium, (i.e., $r_R^* > 0$) if and only if $p - w > \Delta\beta/(\alpha b)$, and $2p - 3w + c > \Delta\beta/(\alpha b)$.

(b) The manufacturer offers a mail-in-rebate at equilibrium (i.e., $r_M^* > 0$) if $2w - c - p - \beta(N - bp + \mu)/(\alpha b) > 0$.

Proposition 3(a) gives the necessary and sufficient condition for the retailer to offer MIR. The stated conditions are likely to be satisfied when the manufacturer's wholesale price is low. We found several products with retailer's MIR at Staples' rebate center website (www.stapleseasyrebates.com) on 6/1/2009. Interestingly, many of these MIRs were for refurbished/remanufactured printers/fax machines and Staples-branded products. This is consistent with the findings of our model as the wholesale prices for such products are likely to be low. The conditions in Proposition 3(a) are also likely to be satisfied when the retail price and the rebate sensitivity parameters are high and the rebate redemption rate is low. This implies that a high retailer margin is, once again, conductive for offering an MIR. Proposition 3(b) provides a sufficient condition for the manufacturer to offer MIRs at equilibrium. The condition is likely to be satisfied when the rebate sensitivity parameter α and the manufacturer's margin $w - c$ is relatively high, while the redemption rate β is relatively low. A higher margin allows the manufacturer to offer the MIR which lowers the acquisition cost of the consumer and results in a higher demand. The higher consumer demand, in turn, increases the retailer's order quantity. Packaged consumer software (e.g. multimedia software) is a product category that demonstrates such characteristics. This perhaps explains the wide-spread use of manufacturer MIR in that category. A higher value of the rebate sensitivity parameter makes an MIR more valuable to a consumer, resulting in an MIR from the manufacturer. Combining the results of Propositions

3(a) and 3(b), we see that both the retailer and the manufacturer will offer MIRs for moderate values of wholesale prices, high rebate sensitivity parameter and low values of redemption rate.

Numerical Examples

We now turn to a numerical study to provide examples of the equilibrium and to develop additional insights. Our computations are based on the following data: $N = 100$, $b = 2$, $\varepsilon \sim N[100,30]$, $c = 10$, $w = 43$ and $p = 67$. Figure 2(a) plots the retailer's and the manufacturer's equilibrium MIRs as a function of the rebate sensitivity parameter α for a fixed value of the redemption rate ($\beta = 0.3$). The plot shows that neither party offers MIR for small values of the rebate sensitivity parameter ($\alpha < 0.2$). As the sensitivity parameter increases ($0.2 \leq \alpha < 0.6$), the manufacturer offering MIR becomes the equilibrium. Finally, as α increases further, both the manufacturer and the retailer offering MIRs becomes the Nash equilibrium. Figure 2(b) plots the retailer's and the manufacturer's equilibrium MIRs as a function of the redemption rate β for a fixed value of the rebate sensitivity parameter ($\alpha = 0.6$). It shows that both parties offer MIR when the redemption rate is low ($\beta < 0.3$). As the redemption increases ($0.3 \leq \beta < 0.7$), the manufacturer offering MIR becomes the equilibrium. Finally, as β increases further, no party offers MIR at equilibrium. Figure 2(b) has an interesting implication. The number of rebate redemptions seen by the retailer is *always* a fixed fraction β of his actual sales. However, the number of rebate redemptions seen by the manufacturer is *at most* a fraction β of his actual sales. In fact, when the retailer has leftovers, the number of rebate redemptions seen by the manufacturer is strictly less than the fraction β of his actual sales. This fact makes it feasible for the manufacturer to offer MIR at such a value of β when it is no longer feasible for the retailer to offer an MIR ($0.3 \leq \beta < 0.7$, in Figure 2b).

Figure 2(c) plots the retailer's and the manufacturer's equilibrium MIRs as a function of the wholesale price w with $\alpha = 0.6$, $\beta = 0.4$ and $p = 67$. It shows that only the retailer offers MIR when the wholesale price is low ($w < 35$). As the wholesale price increases ($35 \leq \omega < 40$), both the manufacturer and the retailer offering MIR becomes the equilibrium. Finally, as w increases further, only the manufacturer offering MIRs becomes the Nash equilibrium. Figure 2(d) plots the retailer's and the manufacturer's equilibrium MIRs as a function of the retail price

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p with $\alpha = 0.6, \beta = 0.4$ and $w = 43$. It shows that only the manufacturer offers MIR when the retail price is low ($p < 65$). As the retail price increases ($p \geq 65$), both the manufacturer and the retailer offering MIR becomes the equilibrium. In summary, our numerical study complements our analytical findings in Propositions 2 and 3 by demonstrating how the nature of the equilibrium changes with changes in problem parameters. We were able to obtain three of the four possible equilibriums by changing a single parameter in Figure 2(a)-2(c). We, however, were unable to find an example where all four equilibriums can be obtained by changing the value of a single parameter.

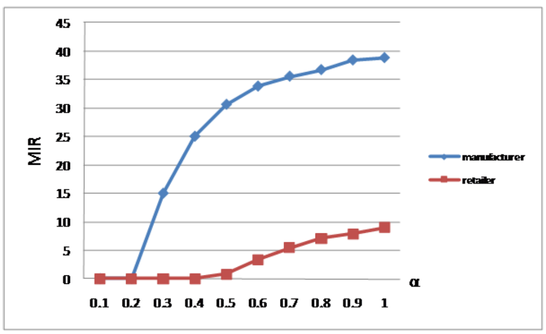

Figure 2(a): MIR as a function of α

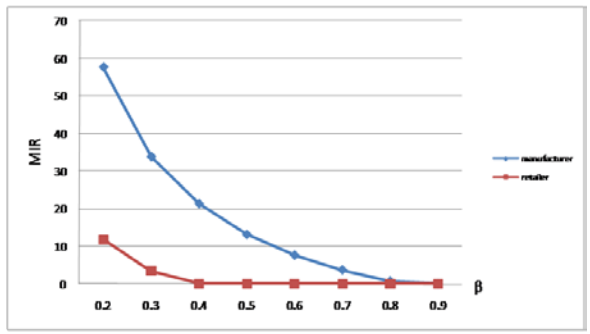

Figure 2(b): MIR as a function of β

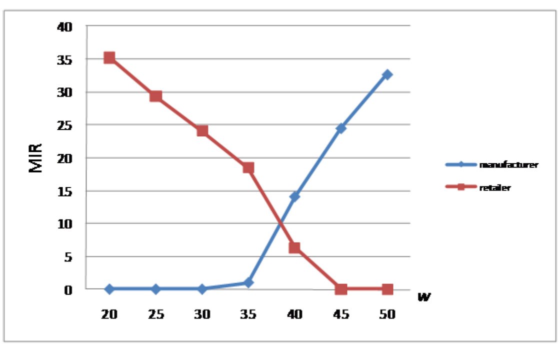

Figure 2(c): MIR as a function of *w*

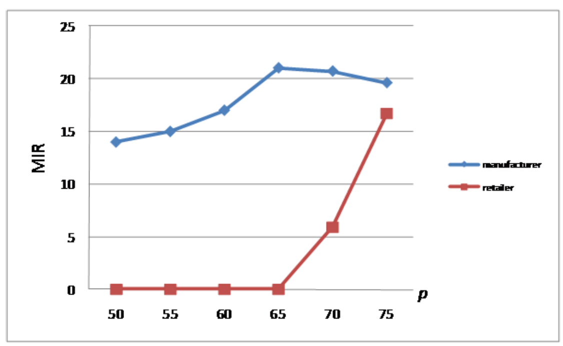

Figure 2(d): MIR as a function of *p*

How do the parameters α and β affect the expected profits? While the effects are hard to establish analytically, our extensive numerical experimentation indicates that the retailer's profit, the manufacturer's profit, and hence, the supply chain's profit increase in α and decrease in β. Figure 3, based on the data for Figure 2(a), illustrates this. As the rebate sensitivity parameter increases, the consumers perceive an MIR to be closer to a direct price reduction, and

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the demand for the product increases. This may result in higher expected sales for both the retailer and the manufacturer at no extra cost. As a result, the expected profits go up. On the other hand, the redemption rate of an MIR directly affects the profitability of any party offering it. Thus, as the redemption rate increases, the expected profits decline.

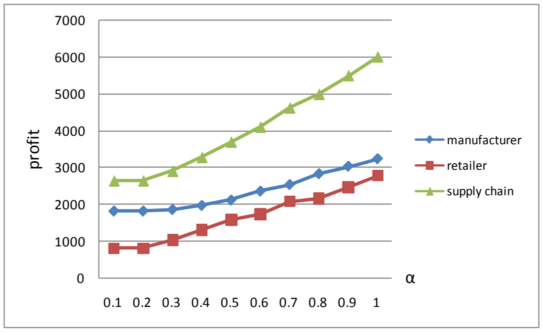

Figure 3: Expected profit as a function of α

3.1.4 Comparison with Literature

We mentioned in Section 2 that the literature considers the scenario where the MIR is offered by a single party, typically by the manufacturer. Thus, these papers will *exogenously* assume that the retailer will *not* offer a rebate, while the amount of rebate to be offered is still a decision variable for the manufacturer. Our work is a generalization of the literature in the sense that we let both the retailer and the manufacturer offer mail-in rebates. Our modeling framework can easily be adapted to the special cases considered in literature by substituting z_0 from equation (4) into Lemma 2, i.e., by forcing the retailer's MIR to zero. To facilitate the comparison of our work with the literature, we will call the framework of our paper as *simultaneous* game while that of the literature as *exclusive* game with manufacturer MIR. A reader will immediately notice that a third scenario, while not studied explicitly in literature, is possible where only the retailer considers offering MIR. We will call this as the as *exclusive* game with retailer MIR. This game, once again, is a special case of our simultaneous game and can easily be solved from Lemma 1 by forcing the manufacturer's MIR to zero. We will next examine the effect of joint mail-in rebate decisions in our newsvendor supply chain by comparing the simultaneous game with the two exclusive games. The following corollary describes our result.

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Corollary 1: *The retailer offers a lower MIR, a higher stocking factor, and derives a higher expected profit in a simultaneous game compared to an exclusive game with retailer MIR.*

Corollary 1 is intuitive. Both parties share the burden of offering MIRs in a simultaneous game. This forces the manufacture to share at least some of the demand risks in the supply chain. As a result, the retailer is able to offer a lesser MIR and a higher service level. How does the manufacturer's MIR compare under the simultaneous and exclusive games? Per Lemma 2, the manufacturer's MIR response function is not guaranteed to be decreasing in retailer's MIR. Note that the retailer's optimal stocking factor in an exclusive game with manufacturer MIR is given by z_0 defined in equation (4). The retailer's optimal stocking factor in a simultaneous game is less than z_0. Thus, the manufacturer will offer a lower MIR in a simultaneous game when the condition $r_M(z_0) \geq r_M(z)$ holds for any $z < z_0$. A sufficient condition to ensure such a scenario is $p - w \geq (\mu - A)\beta/(\alpha b)$. This condition is likely to hold when the profit margin for the retailer is high and/or the ratio β/α is low. A higher retailer margin, a lower rebate redemption rate, and a higher rebate sensitivity represent a favorable environment for the retailer. As a result, the strategic manufacturer offers a lower rebate in a simultaneous game compared to an exclusive game with manufacturer MIR. We further illustrate the differences between the simultaneous game and the exclusive games using a numerical study.

Numerical Examples

Table 1 below provides two illustrative numerical examples based on the following data: $N = 100$, $b = 2$, $\varepsilon \sim N[100,30]$, $c = 10$. We have used the notations π_M^* and π_R^* to denote the equilibrium expected profits of the manufacturer and the retailer respectively in the Table. In the first example, both the manufacturer and the retailer offer MIRs at equilibrium under a simultaneous game. They also offer positive MIRs under exclusive games. Comparing the magnitudes of the equilibrium rebates we see that both parties offer less rebates in a simultaneous game. The consumers, however, enjoy a higher total rebate under a simultaneous game. As indicated by the numbers in bold, the equilibrium expected total supply chain profit in the simultaneous game is at least as large as (strictly higher in the first example) that in the two exclusive games. Interestingly, however, each of the two players prefers an exclusive game where the other player offers the MIR. The second example is instructive. Only the manufacturer

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offers MIR in the simultaneous game and that the outcome of this game is identical to that of an exclusive game with manufacturer MIR. The dynamics of the two games, however, are fundamentally different. In the exclusive game with manufacturer MIR, it is exogenously determined that the retailer will not offer MIR. In the simultaneous game, it is optimal for the retailer not to offer any MIR. The second example once again underscores the fact that the expected total supply chain profit in the simultaneous game is at least as large as those in the two exclusive games. Are the consumers better off in the simultaneous game compared to the exclusive games? Our examples in the current Section assume that the product retail price is exogenous. Under this assumption, whether the consumers are better off with a simultaneous or an exclusive game is determined solely by the magnitudes of the rebates. As can be seen from Table 1, consumers in the simultaneous game enjoy an aggregate rebate that is at least as large as the rebates received in an exclusive game. This implies that both the providers (the manufacturer and the retailer) and the consumers are (weakly) better off in a simultaneous game compared to the exclusive games.

Table 1: Simultaneous Vs. Exclusive MIR Games

Parameters & Games		Equilibrium MIR	z^*	π_M^*	π_R^*	$\pi_R^* + \pi_M^*$
$w = 43$, $p = 67$, $\alpha = 0.9$, $\beta = 0.4$	Simultaneous game (Our work)	$(r_M^*, r_R^*) = (26.9, 3.5)$	88	2492	2002	**4494**
	Exclusive game with manufacturer MIR	28	89	2382	**2041**	4423
	Exclusive game with retailer MIR	18	83	**2679**	1048	3727
$w = 42$, $p = 68$, $\alpha = 0.8$, $\beta = 0.8$	Simultaneous game (Our work)	$(r_M^*, r_R^*) =(3.6, 0)$	89	**1833**	**953**	**2786**
	Exclusive game with manufacturer MIR	3.6	89	**1833**	**953**	**2786**
	Exclusive game with retailer MIR	1.8	90	1818	889	2707

3.2 Multiplicative Demand Function

We consider the following iso-elastic demand function in our analyses for the current section: $D = (p - \alpha r_R - \alpha r_M)^{-b} \varepsilon$. The parameter b in the demand function represents the price elasticity. Such demand function is common in literature (see for example, Petruzzi and Dada, 1999). For the reasons of analytical tractability, we assume that $b \geq 2$. This assumption is purely technical in nature and ensures the uniqueness of the Nash equilibrium. Furthermore, such assumption is standard in literature (Petruzzi and Dada, 1999; Boyaci and Ozer, 2009). It is a reasonable assumption when the consumers are highly sensitive to price. Section 1 of the paper describes several examples of MIRs involving consumer electronics, home appliances, cell phones, etc. The easy availability of comparison shopping over the internet makes consumers of such products highly price sensitive. As in the previous section, we will, once again, write the retailer's order quantity in terms of a stocking factor *z* defined by $Q = (p - \alpha r_R - \alpha r_M)^{-b} z$. Our results and analyses in this section are similar to those in Section 3.1. As a result, we omit the details of the analyses and simply highlight the differences in results between the additive and multiplicative demand functions.

Proposition 4: *Under the multiplicative demand function:*

(a) The retailer's best response stocking factor decreases in manufacturer's MIR; and retailer's best response MIR increases in manufacturer's MIR.

(b) The manufacturer's best response MIR decreases in retailer's stocking factor and increases in retailer's MIR.

Comparing Proposition 4 with Proposition 1 and our numerical study, we see that the best response behavior of the retailer and the manufacturer differ under additive and multiplicative demand functions. Proposition 4(a) indicates that as the manufacturer increases his MIR, the retailer also increases his MIR and decreases service level. We provide the following intuitive explanation. In response to a higher MIR from the manufacturer, which expands demand, the retailer can either improve his margin by reducing his MIR or can further expand the demand by increasing his MIR. When the demand highly price sensitive ($b \geq 2$), the later effect dominates the former and the retailer increases his own MIR to improve upon the expected profit. Proposition 4(b) indicates that as the retailer increases his MIR, so does the manufacturer. The intuitive explanation for this behavior is similar to that in Proposition 4(a). Thus, the manufacturer and the retailer's MIRs are strategic complements under the multiplicative demand function. While the best response behaviors of the retailer and the manufacturer differ under additive and multiplicative demand functions, the following proposition shows that there still exists a unique Nash equilibrium of the MIR game under multiplicative demand function with mild assumptions.

Proposition 5: *There exists a unique equilibrium of the MIR game under multiplicative demand function when* $c \le w(b-2)/(b-1)$.

Proposition 5 establishes the existence of unique Nash equilibrium for $c \le w(b-2)(b-1)$. This condition is a technical one. It will hold when the manufacturer's profit margin is high and production cost is low. Who offers MIRs at equilibrium? Our extensive numerical experimentation suggests that all four scenarios (both parties offering MIR, only one party offering MIR, and none offering MIR) can once again be the unique Nash equilibrium depending upon the values of the problem parameters. For the reasons of brevity, we omit the numerical examples and conclude this section by comparing the simultaneous game with the exclusive games with retailer and manufacturer MIRs.

Corollary 2: *Under the multiplicative demand function, the retailer (the manufacturer) offers a higher MIR in a simultaneous game compared to an exclusive game with retailer (manufacturer) MIR. The retailer offers a lower stocking factor in the simultaneous game compared to an exclusive game with retailer MIR.*

Our discussion in this sub-section shows that some of the dynamics of the MIR game are different across the additive and multiplicative demand functions. Our main finding of the existence of unique Nash equilibrium and that of the validity of four MIR scenarios continue to hold across the two demand functions.

4. ENDOGENOUS RETAIL PRICE

The analyses in the previous section are based on the assumption that the retail price is exogenous. We relax this assumption in this section and explore the case where the retail price is a decision variable for the retailer. We limit our analyses to the additive demand function only

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and discuss the multiplicative demand function briefly at the end. The timing of the events is as follows. In the first stage of the game, the manufacturer and the retailer simultaneously decide on the amount of MIR. After observing MIRs, the retailer determines the retail price p and his order quantity Q. Finally, the consumer demand is realized. The retailer's maximization problem from equation (1) can be rewritten as:

$$\begin{aligned} \underset{p,r_R,z}{Max}\ \pi_R(r_M, r_R, z) &= (p-\beta r_R - w)[N + z - \Lambda(z) - b(p-\beta r_R) + \alpha b r_M + b r_R(\alpha-\beta)] - w\Lambda(z) \\ &= (\hat{p} - w)\,[N + z - \Lambda(z) - b\hat{p} + \alpha b r_M + b r_R(\alpha - \beta)] - w\Lambda(z), \qquad (11) \end{aligned}$$

where, $\hat{p} = p - \beta r_R$. The term $b r_R(\alpha-\beta)$ in (11) represents the retailer's revenue attributed to the difference between the rebate sensitivity parameter and the redemption rate. It is easy to see that when $\alpha > \beta$, the retailer should choose the retail price p and the rebate r_R so that the term $(\hat{p} - w)$ is positive and the term $b r_R(\alpha-\beta)$ is very large. Similarly, when $\alpha < \beta$, the rebate r_R should be as small as possible (i.e., $r_R^* = 0$). When $\alpha = \beta$, there will be multiple optimal solutions with different combinations of r_R^* and p^* that satisfy $p^* = \hat{p}^* + \beta r_R^*$, where the optimal $\hat{p}^*$ is unique and can be solved as in a standard price-setting newsvendor's problem (Petruzzi & Dada, 1999).

In order to get more meaningful insights, we will further assume that the redemption rate depends on the amount of MIR, i.e., $\beta_R = \beta(r_R)$ and $\beta_M = \beta(r_M)$, with $\beta'(.) \geq 0$ and $\beta''(.) \geq 0$. This assumption allows us a more general framework to study the mail-in rebates. It is also consistent with the intuition that consumers are increasingly likely to redeem a rebate successfully as the cash value of the rebate goes up. Having different rebate redemption functions for the manufacturer and the retailer (for example, $\beta_i = \beta_i(r_i), i = R, M$) does not yield additional insights. We also assume $\beta(0) = 0$, i.e., the redemption rate is zero when no MIR is offered. We assume that the rebate sensitivity parameter α to be same for manufacturer and retailer rebates. Our insights remain qualitatively the same if we relax this assumption. The following proposition describes our first result.

Proposition 6:

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(a) Retailer's equilibrium mail-in rebate r_R^* *is uniquely determined by the solution of the problem* $\underset{r_R}{Max}\{\alpha - \beta(r_R)\}r_R$ *and that the following condition must hold at equilibrium:*

$$\alpha > \beta(r_R^*) = \beta_R . \quad (12)$$

(b) The retailer's equilibrium rebate and equilibrium redemption rate $\beta(r_R^*)$ *are increasing in the rebate sensitivity parameter* α.

Per Proposition 6(a), the retailer's equilibrium MIR depends only on the rebate sensitivity parameter and the redemption rate function. In particular the equilibrium MIR of the retailer does not depend on the manufacturer's rebate r_M. Note that unlike Section 3.1, the retailer (or the manufacturer) in our current setting can control the redemption rate by changing the amount of the rebate offered. Thus, the condition $\alpha > \beta_R$ in (12) simply implies that the retailer should design his rebate such that the redemption rate is strictly less than the rebate sensitivity parameter. A similar condition can also be found in Khouja and Zhou (2009) who study manufacturer MIR. It is also worthwhile to mention that our model does not require any additional assumption about the relative magnitudes of α and β_M for its feasibility. Proposition 6(b) indicates that the retailer's equilibrium MIR (and hence the equilibrium redemption rate) is increasing in the rebate sensitivity parameter.

We next turn our attention to the derivation of the Nash equilibrium of the game. The manufacturer's expected profit function is given by

$$\pi_M(\hat{p}, r_M, r_R^*, z)$$
$$= (w - c - \beta(r_M)r_M)[N + z - \Lambda(z) - b(\hat{p} - (\alpha - \beta_R)r_R^* - \alpha r_M)] + (w - c)\Lambda(z) . \quad (13)$$

The Nash equilibrium of the game can be derived from equation (11) and (13) using standard techniques. The following proposition characterizes it.

Proposition 7: *There exists a unique Nash Equilibrium to the MIR game with endogenous retail price. At equilibrium, both the manufacturer and the retailer offer MIR.*

Proposition 7 confirms the existence and uniqueness of the Nash equilibrium of the game under endogenous retail price. However, unlike Proposition 2, both parties offer MIRs at equilibrium under endogenous retail price.

We explore the properties of the Nash equilibrium through a numerical study based on the following data: $c = 10$, $b = 2$, $N = 100$, $\varepsilon \sim N[100,30]$. We use the following redemption functions in our computations: $\beta_R = \beta(r_R) = 0.1r_R$ and $\beta_M = \beta(r_M) = 0.1r_M$. To facilitate our discussion, we will call the quantity $p^* - \alpha(r_R^* + r_M^*)$ to be the *perceived* price, while the quantity $p^* - \beta_R r_R^* - \beta_M r_M^*$ will be called the *redeemed* price. The former of the two represents the perceived price of the product before purchase while the latter denotes the average price a consumer actually paid for the product after purchase. Define p_0 to be the optimal retail price in a conventional price-setting newsvendor problem with identical parameters but without MIR. We call p_0 to be the *baseline* price. Figure 4(a) compares the equilibrium retail price to the perceived price, redeemed price, and baseline price for different values of the rebate sensitivity parameter, and for a constant wholesale price ($w = 36$). Figure 4(b) shows how these prices vary with the wholesale price for a fixed value of the rebate sensitivity parameter ($\alpha = 0.8$). We have also plotted the manufacturer's equilibrium MIR in the two figures. We find that the following relationship holds consistently in Figures 4(a) and 4(b) as well as in several additional computations:

$$p^* - \alpha(r_R^* + r_M^*) \le p_0 \le p^* - \beta_R r_R^* - \beta_M r_M^* \le p^*.$$

Thus, the redeemed price or the average post-purchase price of the product is not only higher than the perceived price, it is also higher than the baseline price. This implies that an MIR makes a product look cheaper while the consumer actually pays more on an average. In fact, the average post purchase price for the consumer is even higher than the newsvendor optimal price without an MIR. Interestingly, Soman (1998) used an experimental study involving university students to conclude that at the time of a product purchase, the consumers under-weigh future effort relative to future savings. Consequently, an incentive that appears attractive at the time of purchase may appear unattractive at the time of redemption. Our numerical study directly supports this conclusion.

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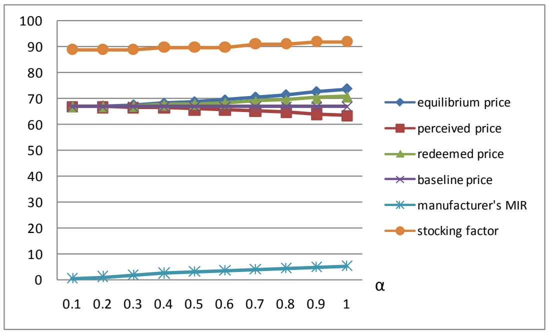

Figure 4(a): Variation of equilibrium prices and decisions with respect to α

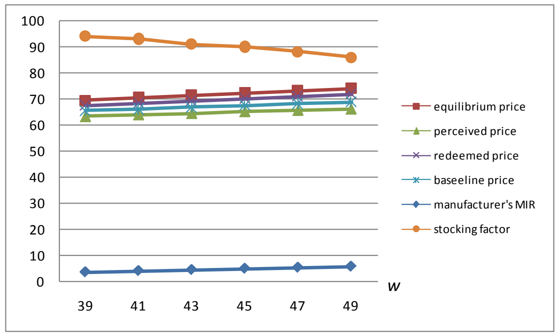

Figure 4(b): Variation of equilibrium prices and decisions with respect to w

We next turn our attention to the comparison of simultaneous and exclusive games under endogenous retail prices. The following proposition describes our result.

Proposition 8: *When the retail price is endogenous,*

(a) the retailer charges a higher retail price and offers a higher stocking factor in a simultaneous game compared to an exclusive game with retailer MIR. The retailer offers the same mail-in rebate under the two scenarios. Both the retailer and the manufacturer derive a higher expected profits in a simultaneous game compared to an exclusive game with retailer MIR.

(b) the retailer charges a higher retail price and offers a higher stocking factor in a simultaneous game compared to an exclusive game with manufacturer MIR. The manufacturer offers a lower MIR in a simultaneous game compared to an exclusive game with manufacturer MIR.

Proposition 8(a) suggests that the simultaneous game gives rise to higher expected profits for both the manufacturer and the retailer (and hence for the total supply chain) compared to an exclusive game with retailer MIR. It can also be shown analytically that the expected supply chain profit in the simultaneous game is higher than that when no MIR is offered (i.e., a price-setting newsvendor without MIR). Aydin et al. (2008) report a similar result by comparing the exclusive game with manufacturer MIR with the no MIR situation. Proposition 8(b) suggests that when retail price are endogenous, the manufacturer, unlike the retailer, offers a lower MIR in a simultaneous game compared to an exclusive game with manufacturer MIR. The retail price of

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the product is higher in the simultaneous game compared to either of the two exclusive games. The presence of two rebates allows the retailer to set a higher retail price in a simultaneous game without suffering a substantial reduction in demand. Given the exogenous wholesale price, a higher retail price implies a higher margin as well for the retailer. The higher margin allows the retailer to have a higher stocking factor in the simultaneous game compared to the exclusive games.

It is analytically hard to compare the equilibrium price and the expected supply chain profits between the simultaneous game and the exclusive game with manufacturer MIR. As a result, we once again, turn to a numerical study.

Numerical Study

Our objective in this numerical study is to compare the equilibrium expected profits of the simultaneous game and the exclusive game with manufacturer MIR. Figure 5(a) compares the expected profits of the retailer and the manufacturer (and hence the total supply chain profit) for the two games for different values of the rebate sensitivity parameter given a fixed wholesale price (w=43). Observe that the simultaneous game results in higher expected profits for both manufacturer and the retailer compared to the exclusive game with manufacturer MIR. Further, the expected profits under both games increase as the rebate sensitivity parameter increases. We had similar results in Section 3, when the product retail price was exogenous (Figure 3 and Table 1). Figure 5(b) compares the expected profits of the retailer and the manufacturer for the two games for different values of the wholesale price given a fixed value of the rebate sensitivity parameter ($\alpha = 0.8$). We once again observe that the expected profits in the simultaneous game are higher compared to the exclusive game. Under both the games, the retailer's and the supply chain's expected profits decrease in the wholesale price due to increasing double marginalization effect.

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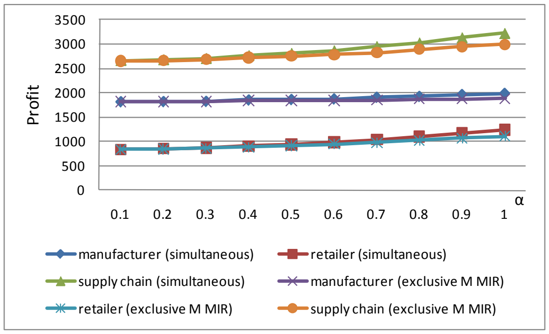

Figure 5(a): Change of expected profit wrt α

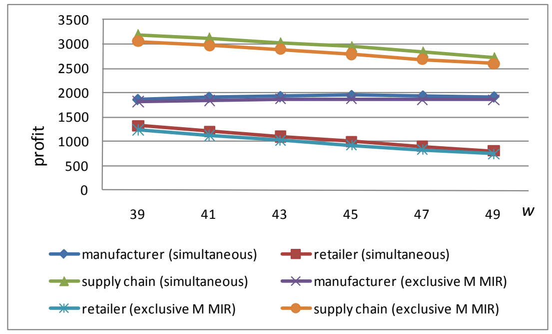

Figure 5(b). Change of expected profit wrt w

We next compare the two exclusive games with the simultaneous game with respect to the perceived and redeemed prices. These yield insights about how the consumers fare under the three games. Figures 6(a) and 6(b), respectively, plot the perceived and redeemed prices in the three games for different values of the rebate sensitivity parameter α for a fixed wholesale price (w=43). It is interesting to note from the two figures that the simultaneous game has the lowest perceived price, $p^* - \alpha(r_R^* + r_M^*)$, but the highest redeemed price, $p^* - \beta_R r_R^* - \beta_M r_M^*$. The baseline price (i.e., the optimal solution of a price-setting newsvendor under no MIR) on the other hand is higher than all other perceived prices but is lower than all other redeemed prices. The two exclusive games have intermediate values of the perceived and redeemed prices. The result is intuitive. Two rebates make a simultaneous game look attractive to a consumer. As a result, it has the lowest perceived price. However, as discussed in Section 1, rebate redemption rate rarely reaches hundred percent. This effect might be more pronounced in a simultaneous game in presence of multiple rebates. Thus, the redeemed price is highest in the simultaneous game indicating that the consumers on an average pay the highest price under this game and are worse off compared to two exclusive games. Figures 6(a) and 6(b) once again suggest that MIRs make a product look cheaper but consumers pay more on an average.

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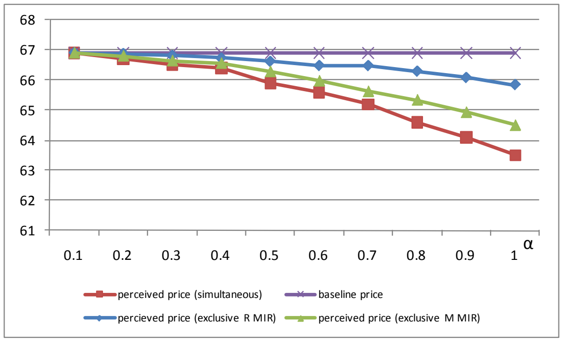

Fig. 6(a): Change of perceived prices wrt α

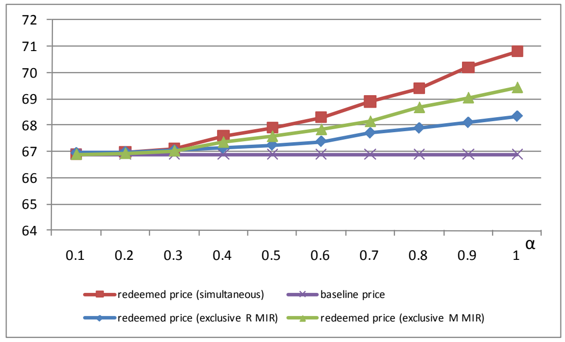

Fig. 6(b): Change of redeemed prices wrt α

Comparing our results from Section 3.1 with those from Section 4 we see that the equilibrium outcome under endogenous retail price might differ from that under exogenous retail price. The exogenous retail price assumption allows four possible equilibrium scenarios while the endogenous price assumption results in an equilibrium where both parties offer MIRs. The existence and the uniqueness of the equilibrium continue to hold under both exogenous and endogenous retail prices as does our other key insights regarding the properties of the equilibrium. Section 1 of the paper provides examples for exclusive MIRs by the manufacturer and the retailer, as well as both parties offering MIR simultaneously. These scenarios are consistent with our equilibrium outcome with exogenous retail price. In the authors' own experience, the retail prices rarely change with the introduction/expiration of MIRs. These facts suggest that exogenous retail price might be a reasonable assumption to explain the observed practices.

How do the analyses in Section 4 change under a multiplicative demand function? It can be shown that Proposition 6 continues to hold under a multiplicative demand function with endogenous retail price. Moreover, there exists at least one Nash equilibrium where both the manufacturer and the retailer offer MIR. However, it is analytically hard to establish the uniqueness of the equilibrium. It can be shown through a numerical study that simultaneous game result in higher expected profits for the manufacturer and the retailer compared to the two exclusive games.

5. SUMMARY AND CONCLUSIONS

Geng, Qin, and Suman Mallik. "Joint Mail-In Rebate Decisions in Supply Chains Under Demand Uncertainty." *Production and Operations Management* (2010): 581-602. Publisher's official version: http://dx.doi.org/10.1111/j.1937-5956.2010.01171.x. Open Access version: http://kuscholarworks.ku.edu/dspace/.

MIR is a common promotional tool used in marketing of consumer products. We study the joint decisions of offering MIRs in a one-manufacturer-one-retailer supply chain with demand uncertainty. Both the manufacturer and the retailer consider offering MIRs. The end consumer demand is stochastic and depends on the price and the amount of MIRs. Using a game theoretic framework we study the Nash equilibrium outcome of the game. Both additive and multiplicative demand functions are considered. Consistent with our observation that the product retail price rarely changes in practice because of the introduction or expiry of MIR, we first consider the case where the product retail price is exogenous. We next consider a more general case where the product retail price is a decision variable and that the rebate redemption rate increases with the amount of MIR. We also compare and contrast our work with the literature, which considers MIR offered exclusively by the manufacturer.

When the retail price is exogenous, we show the existence of a unique Nash equilibrium under both additive and multiplicative demand functions and characterize it completely. We show that depending upon the problem parameters, any of the following four scenarios can be the equilibrium: both parties offer MIR, only one party offers MIR, none offer MIR. The manufacturer, in general, prefers to offer MIR when the wholesale price is higher while the retailer prefers to offer MIR under lower wholesale prices. As described in the discussion following Proposition 3, this result seems to be consistent with MIR examples found in the website of the office supply retailer Staples. These insights can be valuable qualitative guiding tools for practicing managers. We discussed how the redemption rate and rebate sensitivity parameters affect the equilibrium decisions. We show that under additive (multiplicative) demand function, the retailer offers lower (higher) MIR under a simultaneous game compared to an exclusive game with retailer MIR. Our numerical studies demonstrate that the expected total supply chain profit in the simultaneous game is at least as large as that in a game with exclusive MIR from either the retailer or the manufacturer. This is also a valuable qualitative insight for a practitioner.

Under more general conditions, when the retail price is a decision variable for the retailer and that the rebate redemption rate increases with the amount of MIR, we once again prove the existence of a unique Nash equilibrium where both the retailer and the manufacturer offer MIRs. Using a numerical study, we show that the average post-purchase price of the product is not only

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higher than the perceived pre-purchase price; it is also higher than the newsvendor optimal price without an MIR. This implies that an MIR makes a product look cheaper while the consumers pay more on an average. An article in *Business Week* makes a similar argument (Grow 2005). Our work explains why the common practice of displaying after-rebate price prominently is beneficial to a retailer. We also show that the expected profits of both the retailer and the manufacturer are higher in the simultaneous game compared to an exclusive game with retailer MIR. Our numerical studies suggest that a similar relationship about the expected profits might hold between the simultaneous game and the exclusive game with manufacturer MIR.

Our work makes the following contribution to the operations management literature. First, we examine simultaneous MIR consideration by both the retailer and the manufacturer under demand uncertainty. To the best of our knowledge, our paper is the first to consider endogenous rebate decisions under stochastic demand. The literature typically considers the scenario where the demand is deterministic or the manufacturer offers MIR. Second, we characterize the conditions under which both parties offer MIR, only one party offers MIR, none offers MIR. As our examples in Section 1 demonstrate, all four situations are common in practice. Third, by comparing our results with the exclusive MIR scenarios, we gain valuable insights about expected profits and magnitude of rebates at equilibrium.

This paper provides several avenues for future research. We consider a single period model. Considering a multi-period model, while analytically challenging, will allow us to answer questions such as at what stage of a product lifecycle should an MIR be introduced and when should it be withdrawn. An MIR can perhaps also be used to eliminate excess/shortage in a supply chain and strategically match supply with demand under a multi-period setting. We consider a single manufacturer and a single retailer. Extending our framework to multiple retailers will allow us to capture the strategic interactions among the retailers in the presence of an MIR.

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APPENDIX: PROOFS OF RESULTS

Proof of Lemma 1

(a) Rewrite (6) as $\frac{\partial \pi_R(r_M, r_R(r_M, z), z)}{\partial z} = \frac{(1-F(z))}{2\alpha b} L(r_M, z)$,

where $L(r_M, z) \equiv \beta(N - bp + z - \Lambda(z)) - \frac{2\alpha bw}{(1-F(z))} + \alpha b(p + w + \beta r_M)$. (A1)

$$\partial L(r_M, z)/\partial z = \beta(1-F(z)) - 2\alpha bwf(z)/(1-F(z))^2 .$$

$$\partial^2 L(r_M, z)/\partial z^2 = -\beta f(z) - 2\alpha bw[(\frac{f(z)}{1-F(z)})' \frac{1}{(1-F(z))} + (\frac{1}{(1-F(z))})' \frac{f(z)}{(1-F(z))}]$$

≤ 0 due to IFR.

So $L(r_M, z)$ is concave in z given r_M. Moreover, $L(r_M, A) = \beta(N - bp + A) + \alpha b(p - w + \beta r_M) \geq 0$ and $L(r_M, B) \leq 0$. Thus there must exist a unique solution $z(r_M)$ to $L(z) = 0$. Also,

$$\partial L(r_M, z)/\partial z \Big|_{z=z(r_M)} \leq 0, \quad \text{(A2)}$$

thus, $\frac{\partial^2 \pi_R(r_M, r_R(r_M, z), z)}{\partial z^2}\Big|_{\partial \pi_R/\partial z = 0} \leq 0$.

Therefore, $\pi_R(r_M, r_R(r_M, z), z)$ is quasi-concave in z, the maximizer is $z(r_M)$.

(b) To ensure $r_R \geq 0$, $z^*(r_M) = \min\{z(r_M), z_0\}$. If $z^*(r_M) = z(r_M)$, $r_R^*(r_M)$ satisfies both (3) and (5). If $z^*(r_M) = z_0$, it implies that $r_R^*(r_M) = 0$ from (3). □

Proof of Proposition 1

$z(r_M)$ is the solution to $L(r_M, z) = 0$. From implicit function theory,

$\frac{\partial L(r_M, z)}{\partial z} \frac{dz(r_M)}{dr_M} + \frac{\partial L}{\partial r_M} = 0$. Using equation (A2),

sign $(dz(r_M)/dr_M) =$ sign$(\partial L(r_M, z)/\partial r_M) = sign(\alpha b \beta) \geq 0$.

As $z^*(r_M) = \min(z(r_M), 0)$ and that z_0 is independent of r_M, $z^*(r_M)$ is increasing in r_M. From lemma 1(b), $r_R^*(r_M)$ increases in r_M. □

Proof of Lemma 2

From (9), $2\alpha b\beta \frac{dr_M(z)}{dz} = -\beta(1-F(z)) + \frac{\alpha bwf(z)}{\beta(1-F(z))^2}$, so $dr_M{}^2(z)/dz^2 \geq 0$ due to IFR. □

Proof of Lemma 3

(a) $L_s(z) \equiv \alpha b(w-c) - 2\alpha b(p-w) + \beta(N - bp + z - \Lambda(z)) + 3\alpha b(p - w/(1-F(z)))$ (A3)

$\partial L_s(z)/\partial z = \beta(1-F(z)) - 3\alpha bwf(z)/(1-F(z))^2$ and $\partial L_s{}^2(z)/\partial z^2 \leq 0$ due to IFR.

So $L_s(z)$ is concave. Moreover, $L_s(A) = \beta(N - bp + A) + \alpha b(p-c) \geq 0$ and $L_s(B) \leq 0$. Thus, there must exist a unique solution to $L_s(z) = 0$.

(b) From (A3) and implicit function theory,
sign $(\partial z_s / \partial\alpha)$ = sign($\partial L_s(z)/\partial\alpha$)=sign($b(w-c)-2b(p-w)+3b(p-w/(1-F(z_s))$)
=sign $(-(N-bp+z_s-\Lambda(z_s))\beta/\alpha) \le 0$, where the last equality follows from $L_s(z_s)=0$.
Similarly, sign $(\partial z_s/\partial\beta)$ = sign($\partial L_s(z)/\partial\beta$)=sign $(N+bp+z_s-\Lambda(z_s)) \ge 0$. Thus z_s decreases in α and increases in β. Rewrite (3) as

$$r_R(z) = (p - w/(1-F(z)))/\beta, \quad \text{(A4)}$$

and from which we have

$$\beta\frac{\partial r_R(z_s)}{\partial\alpha} = -\frac{f(z_s)}{(1-F(z_s))^2}\frac{\partial z_s}{\partial\alpha}w \ge 0.$$

Thus $\partial r_R(z_s)/\partial\alpha \ge 0$. Similarly, $\beta\frac{\partial r_R(z_s)}{\partial\beta} = -\frac{f(z_s)}{(1-F(z_s))^2}\frac{\partial z_s}{\partial\beta}w - r_R \le 0$. □

<u>Proof of Proposition 2</u>

Let $\tilde{r}_M(z)$ be the inverse function of $z(r_M)$. So $\tilde{r}_M(z)$ denotes the manufacturer's rebate at which the retailer responds with *z*. Then, $L_s(z) \equiv r_M(z) - \tilde{r}_M(z)$. Per concavity of $L_s(z)$, $L_s(z) \ge 0$ if and only if $z \le z_s$. That is, for $z > z_s$, $\tilde{r}_M(z) > r_M(z)$; for $z = z_s$, $\tilde{r}_M(z) = r_M(z)$; and for $z < z_s$, $\tilde{r}_M(z) < r_M(z)$. We will call this result **Lemma A1.** Now, consider the following scenarios.

(1) If $z_s < z_0$ and $r_M(z_s) > 0$, then from lemma 1 and Lemma 2, the equilibrium is $z^* = z_s$, and $r_M{}^* = r_M(z^*)$. For uniqueness, we show that $z = z_0$ cannot be an equilibrium for the following reason: given $z_s < z_0$, $\tilde{r}_M(z_0) > r_M(z_0)$ per lemma A1; and $z(r_M)$ increases in r_M from Proposition 1. Therefore, $z_0 = z(\tilde{r}_M(z_0)) > z(r_M(z_0))$. Similarly, $r_M = 0$ cannot be an equilibrium because, as $r_M(z_s) = \tilde{r}_M(z_s) > 0$, $z_s = z(\tilde{r}_M(z_s)) > z(0)$ from Proposition 1. Thus, given $r_M = 0$, $r_M(z(0)) > \tilde{r}_M(z(0)) = 0$ from lemma A1.

(2) If $z_s < z_0$ and $r_M(z_s) \le 0$, then $r_M{}^* = 0$ and $z^* = z^*(0) = \min(z(0), z_0)$ from lemma 1(b) is an equilibrium. To see this, as $\tilde{r}_M(z_s) = r_M(z_s) < 0$, $z_s = z(\tilde{r}_M(z_s)) < z(0)$ from Proposition 1. From lemma A1, $r_M(z(0)) \le \tilde{r}_M(z(0)) = 0$. Similarly, as $z_s < z_0$, $r_M(z_0) < \tilde{r}_M(z_0)$. If $z_0 < z(0)$, then $\tilde{r}_M(z_0) \le \tilde{r}_M(z(0)) = 0$ from Proposition 1. In sum, $r_M(\min(z(0), z_0)) = r_M(z^*(0)) \le 0$. From Lemma 2, $r_M{}^*(z^*(0)) = \max\{r_M(z^*(0)), 0\} = 0$.

For uniqueness, we only need to show that $r_M > 0$ cannot be in equilibrium. This holds as from Proposition 1, $z(r_M) > z(r_M(z_s)) = z_s$ when $r_M > 0$ and $r_M(z_s) < 0$. Therefore, $z^*(r_M) = \min(z(r_M), z_0) > z_s$. From lemma A1, $r_M = \tilde{r}_M(z(r_M)) > r_M(z(r_M))$.

(3) If $z_s \ge z_0$, then $z^* = z_0$ and $r_M{}^* = \max(r_M(z_0), 0)$ from Lemma 2 is an equilibrium. To see this, $\tilde{r}_M(z_0) < r_M(z_0)$ from lemma A1. From Proposition 1, $z_0 = z(\tilde{r}_M(z_0)) < z(r_M(z_0)) < z(\max(r_M(z_0), 0))$. Thus, $z^*(\max(r_M(z_0), 0)) = z_0$ from lemma 1(b). For uniqueness, we only need to show that $z < z_0$ cannot be in equilibrium, which is true because from lemma A1 and Proposition 1, $z = z(\tilde{r}_M(z)) < z(r_M(z)) \le z(\max(r_M(z), 0)) = z(r_M{}^*(z))$ when $z < z_0$. □

Proof of Proposition 3

(a) We first show that if $z(0) \geq z_0$, then $r_R^* = 0$. This is because if $z(0) \geq z_0$ and $\tilde{r}_M(z_s) > 0$, then $z_s = z(\tilde{r}_M(z_s)) > z(0) \geq z_0$ from Proposition 1, and thus $r_R^* = 0$ from Proposition 2(c). On the other hand, if $z(0) \geq z_0$ and $\tilde{r}_M(z_s) = r_M(z_s) < 0$, then $r_R^* = 0$ from Proposition 2(b) and 2(c). This result, together with Proposition 2, implies that $r_R^* > 0$ if and only if $z(0) < z_0$ and $z_s < z_0$. From concavity of $L(z)$ and $L_s(z)$, it means that $r_R^* > 0$ if and only if $L(z_0) < 0$ and $L_s(z_0) < 0$, which is satisfied by the specified condition after applying (4).

(b) From (9), $2\beta r_M(z) = (w-c) - (p - w/(1-F(z))) - \beta(N - bp + z - \Lambda(z))/(\alpha b)$
$\geq (w-c) - (p-w) - \beta(N - bp + \mu)/(\alpha b) \geq 0$. □

Proof of Corollary 1

The following table lists four exhaustive cases.

	Retailer's MIR	
	Simultaneous Game ($r_M \geq 0$)	Exclusive Game with Retailer MIR ($r_M = 0$)
Case I	0	0
Case II	0	>0
Case III	>0	>0
Case IV	>0	0

In Case I and Case II, the retailer provides less MIR in a simultaneous game. For case III, from Proposition 1, $r_R^*(r_M = 0) \geq r_R^*(r_M \geq 0)$, which also implies that Case IV cannot exist. From equation (3), a simultaneous game results in a higher service level. To compare expected profits, it is easy to see from (1) that $\partial\pi_R(r_M, r_R, z)/\partial r_M = \alpha b(p - \beta r_R - w) \geq 0$, thus a simultaneous game results in higher expected profits for the retailer. □

Proof of Proposition 4

(a) The retailer's expected profit function can be written as

$$\pi_R(r_R, r_M, z)$$
$$= (p - \alpha r_R - \alpha r_M)^{-b}[(p - \beta r_R)(z - \Lambda(z)) - wz].$$

Taking first order derivative with respect to z and set it to zero yields equation (3), i.e.,

$$(p - \beta r_R)(1 - F(z)) = w.$$

Taking first order derivative with respect to r_R and set it to zero yields

$$r_R(r_M, z) = \frac{(\alpha b - \beta)p - \alpha bwz/(z - \Lambda(z)) + \alpha\beta r_M}{(b-1)\alpha\beta}.$$

Combining the above two equations yields

$$\frac{bwz}{(b-1)(z-\Lambda(z))} - \frac{w}{(1-F(z))} - \frac{(\alpha - \beta)p}{(b-1)\alpha} - \frac{\beta r_M}{(b-1)} = 0. \quad \text{(A5)}$$

Lemma A2. $\boldsymbol{\frac{bz}{(b-1)(z-\Lambda(z))} - \frac{1}{(1-F(z))}}$ **decreases in *z* when *b*≥2.**

Proof: Per Petruzzi and Dada (1999), $\frac{bz}{(b-1)(z-\Lambda(z))}-\frac{1}{(1-F(z))}$ is quasi- concave with IFR distribution and if $b \geq 2$. Furthermore,

$$\frac{d}{dz}[\frac{bz}{(b-1)(z-\Lambda(z))}-\frac{1}{(1-F(z))}]|_{z=A}$$

$$=\frac{b(zF(z)-\Lambda(z))(1-F(z))^2-(b-1)f(z)(z-\Lambda(z))^2}{(b-1)(z-\Lambda(z))^2(1-F(z))^2}|_{z=A}=-f(A)/A^2 \leq 0,$$

which implies that $\frac{bz}{(b-1)(z-\Lambda(z))}-\frac{1}{(1-F(z))}$ decreases in z. □

From (A5), given r_M, $z(r_M)$ that satisfies (A5) decreases in r_M. From (3), $r_R(r_M)$ increases in r_M.

(b) The manufacturer's profit function can be written as

$$\pi_M(r_M,r_R,Q)=(w-c)Q-\beta r_M E\min(D,Q)$$

$$=(p-\alpha r_R-\alpha r_M)^{-b}[(w-c)z-\beta r_M(z-\Lambda(z))].$$

Taking first order derivative with respect to r_M and set it to zero yields

$$r_M(r_R,z)=\frac{\alpha b(w-c)z/(z-\Lambda(z))-\beta p+\alpha\beta r_R}{(b-1)\alpha\beta}.$$

Rewriting the manufacturer's best response in terms of z yields

$$r_M(z)=\frac{\alpha b(w-c)z/(z-\Lambda(z))+(\alpha-\beta)p-\alpha w/(1-F(z))}{(b-1)\alpha\beta}. \qquad \text{(A6)}$$

Hence, $\operatorname{sign}(\frac{\partial r_M(z)}{\partial z})=\operatorname{sign}\frac{d}{dz}[\frac{bz}{(b-1)(z-\Lambda(z))}\frac{(w-c)}{w}-\frac{1}{(1-F(z))}]$. Since $\frac{d}{dz}[\frac{bz}{(b-1)(z-\Lambda(z))}\frac{(w-c)}{w}-\frac{1}{(1-F(z))}] \leq \frac{d}{dz}[\frac{bz}{(b-1)(z-\Lambda(z))}-\frac{1}{(1-F(z))}] \leq 0$, $\partial r_M(z)/\partial z \leq 0$. As $\partial z/\partial r_R \leq 0$ from (3), $\frac{dr_M}{dr_R}=\frac{\partial r_M}{\partial z}\frac{\partial z}{\partial r_R} \geq 0$. □

Proof of Proposition 5

We can write $\tilde{r}_M(z)$ from (A5). Combining (A5) and (A6) and define

$$L_s(z) \equiv \frac{(b-1)\alpha\beta}{b}[r_M(z)-\tilde{r}_M(z)]$$

$$=-(b-2)\alpha w[(1+\frac{c}{w(b-2)})\frac{z}{(z-\Lambda(z))}-\frac{1}{(1-F(z))}]+(\alpha-\beta)p.$$

If $1+\frac{c}{w(b-2)} \leq \frac{b}{b-1}$, i.e., $c \leq w(b-2)/(b-1)$, then from lemma A2,

$$\frac{d}{dz}[(1+\frac{c}{w(b-2)})\frac{z}{(z-\Lambda(z))}-\frac{1}{(1-F(z))}] \leq \frac{d}{dz}[\frac{bz}{(b-1)(z-\Lambda(z))}-\frac{1}{(1-F(z))}] \leq 0,$$

which implies that $dL_s(z)/dz \geq 0$. If $(1-\beta/\alpha)p \leq c$, then $L_s(A) \leq 0$, together with $L_s(B) \geq 0$, there must exist a unique solution z_s to $L_s(z)=0$. This means that for $z > z_s$, $\tilde{r}_M(z) < r_M(z)$; for

$z = z_s$, $\tilde{r}_M(z) = r_M(z)$; and for $z < z_s$, $\tilde{r}_M(z) > r_M(z)$. The proof of equilibrium with non-negative constraints is similar to that in Proposition 2. □

Proof of Proposition 6

(a) $\partial \pi_R(\hat{p}, r_M, r_R, z)/\partial r_R = b(\hat{p} - w)[\alpha - \beta(r_R) - r_R\beta'(r_R)]$, which is positive at $r_R = 0$ and negative at $r_R = \infty$. So there must exist a solution to $\alpha - \beta(r_R) - r_R\beta'(r_R) = 0$.

(b) $\partial^2 \pi_R(\hat{p}, r_M, r_R, z)/\partial r_R{}^2 \Big|_{\partial \pi_R(\hat{p}, r_M, r_R, z)/\partial r_R = 0}$

$$= b(\hat{p} - w)[-2\beta'(r_R) - r_R\beta''(r_R)]\Big|_{\partial \pi_R(\hat{p}, r_M, r_R, z)/\partial r_R = 0} \le 0.$$

So $\pi_R(\hat{p}, r_M, r_R, z)$ is quasi-concave and r_R^* is unique.

$\partial r_R^*/\partial \alpha = 1/[2\beta'(r_R^*) + r_R^*\beta''(r_R^*)] \ge 0$. $\beta'(\alpha) = \beta'(r_R^*)\partial r_R^*(\alpha)/\partial \alpha \ge 0$. □

Proof of Proposition 7

From (11), $\partial \pi_R(\hat{p}, r_M, r_R, z)/\partial z = 0 \Rightarrow \hat{p}(1 - F(z)) = w$. (A7)

$$\partial \pi_R(\hat{p}, r_M, r_R, z)/\partial p = 0$$

$$\Rightarrow \hat{p} = \frac{N + z - \Lambda(z) + bw + br_R(\alpha - \beta(r_R)) + \alpha b r_M}{2b}, \quad \text{(A8)}$$

Substituting (A8) into (A7) yields

$$L_2(r_M, z) \equiv (1 - F(z))(N + z - \Lambda(z) + bw + br_R(\alpha - \beta(r_R)) + \alpha b r_M) - 2bw = 0. \quad \text{(A9)}$$

$L_2(r_M, A) \ge 0$, $L_1(r_M, B) \le 0$, $\partial L_2(r_M, z)/\partial z\Big|_{L_2(r_M, z) = 0} \le 0$, hence there is a unique solution to $L_2(r_M, z) = 0$ and

$$\frac{\partial z(r_M)}{\partial r_M} = \frac{-\partial L_2/\partial r_M}{\partial L_2/\partial z} \ge 0. \quad \text{(A10)}$$

From (13) we have,

$$\partial \pi_M(\hat{p}, r_M, r_R^*, z)/\partial r_M = -(\beta(r_M) + \beta'(r_M)r_M)[N + z - \Lambda(z) - b(\hat{p} - (\alpha - \beta_R)r_R^* - \alpha r_M)] + \alpha b(w - c - \beta(r_M)r_M)$$

Given $\hat{p}$, z and r_R^*, $\partial^2 \pi_M(\hat{p}, r_M, r_R^*, z)/\partial r_M{}^2 \le 0$. So $r_M(\hat{p}, z, r_R^*)$ is solved by setting the above first order condition to zero, that is,

$$\alpha b(w - c - \beta(r_M)r_M) - (\beta(r_M) + \beta'(r_M)r_M)[N + z - \Lambda(z) - b(\hat{p} - (\alpha - \beta_R)r_R^* - \alpha r_M)] = 0. \quad \text{(A11)}$$

We next show that $r_M(\hat{p}, r_R^*, z)$ satisfying (A7), (A8) and (A11) is unique. Substituting (A8) and (A7) into (A11) yields

$\alpha b(w - c - \beta(r_M)r_M) - (\beta(r_M) + \beta'(r_M)r_M)bw/(1 - F(z)) = 0$, from which, (A12)

$\partial r_M(z)/\partial z \le 0.$ (A13)

From (A10) and (A13), there must exist a unique equilibrium. From (13),

$$\partial \pi_M(\hat{p}, r_M, r_R^*, z)/\partial r_M\Big|_{r_M = 0} = \alpha b(w - c) > 0, \quad \text{(A14)}$$

so the manufacturer must offer MIR.

We next show the existence of equilibrium for multiplicative demand function. By Theorem 1.2 in Fudenberg and Tirole (1991), there exists a pure strategy of equilibrium if the payoff functions are

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continuous and quasi-concave with respect to each player's own strategy. We next show that the payoff functions of the manufacturer and the retailer are quasi-concave.

$$\pi_M(\hat{p}, r_M, r_R, z) = (\hat{p} - (\alpha - \beta_R)r_R - \alpha r_M)^{-b}[(w-c)z - \beta_M(r_M)r_M(z - \Lambda(z))],$$

$$\partial\pi_M(\hat{p}, r_M, r_R, z)/\partial r_M$$
$$= K^{-b-1}[\alpha b((w-c)z - \beta_M(r_M)r_M(z-\Lambda(z))) - (\beta_M'(r_M)r_M + \beta_M(r_M))(z-\Lambda(z))K],$$

where $K \equiv \hat{p} - (\alpha - \beta_R)r_R - \alpha r_M$.

$$\partial^2\pi_M(\hat{p}, r_M, r_R, z)/\partial r_M{}^2\Big|_{\partial\pi_M/\partial r_M=0}$$
$$= -K^{-b-1}(z-\Lambda(z))[\alpha(1-b)(\beta_M'(r_M)r_M + \beta(r_M)) + K(\beta_M''(r_M)r_M + 2\beta'(r_M))] \le 0.$$

From $\pi_R(\hat{p}, r_M, r_R, z) = (\hat{p} - (\alpha-\beta_R)r_R - \alpha r_M)^{-b}[\hat{p}(z-\Lambda(z)) - wz]$,

$$\partial\pi_R(\hat{p}, r_M, r_R, z)/\partial z = \hat{p}(1-F(z)) - w,$$

$$\partial\pi_R(\hat{p}, r_M, r_R, z)/\partial\hat{p} = 0 \Rightarrow -\hat{p}(b-1) + bwz(z-\Lambda(z)) - (\alpha-\beta_R)r_R - \alpha r_M = 0,$$ i.e.,

$$\partial\pi_R(\hat{p}(z), r_M, r_R, z)/\partial z = (1-F(z))[\frac{bwz}{(b-1)(z-\Lambda(z))} - \frac{w}{(1-F(z))} - \frac{(\alpha-\beta_R)r_R - \alpha r_M}{(b-1)}],$$ which

decreases in *z* from lemma A2. □

Proof of Proposition 8

(a) From Proposition 6(a), retailer's rebate is independent of r_M in the simultaneous game as well as in the exclusive game with retailer MIR.

From (A10), $z^* = z(r_M^*) \ge z(0)$. From (A8), $\hat{p}^* = \hat{p}(z(r_M^*)) \ge \hat{p}(z(0))$, thus,

$$p^* = \hat{p}^* + \beta r_R^* \ge \hat{p}(z(0)) + \beta r_R^*.$$

From (11), $\partial\pi_R(\hat{p}(z), r_R^*, r_M, z(r_M))/\partial r_M = \partial\pi_R(\hat{p}, r_R^*, r_M, z)/\partial r_M = \alpha b(\hat{p} - w) \ge 0$.

So $\pi_R(\hat{p}^*, r_R^*, r_M^*) \ge \pi_R(\hat{p}, r_R^*, 0) \ge \pi_R(\hat{p}(r_R = 0), 0, 0)$. From (13), $\partial\pi_M(\hat{p}(z), r_M, r_R^*, z)/\partial z\big|_{r_M=0} = (w-c)(N + z - b\hat{p}(z) + b(\alpha - \beta(r_R^*))$. From (A8), given $r_M = 0$, $N + z - b\hat{p}(z) + b(\alpha - \beta(r_R^*)) = b(\hat{p}(z) - w) + \Lambda(z)$, where the right hand side increases in z from (A7). This implies that given $r_M = 0$, $N + z - b\hat{p}(z) + b(\alpha - \beta(r_R^*))$ increases in z. From (A10) and A(14), $z^* = z(r_M^*) \ge z(0)$. Thus, $\pi_M(\hat{p}(z^*), r_M^*, r_R^*, z^*) \ge \pi_M(\hat{p}(z^*), 0, r_R^*, z^*) \ge \pi_M(\hat{p}(z(0)), 0, r_R^*, z(0))$.

(b) Let $r_M(0)$ denote the equilibrium manufacturer MIR. Suppose $r_M^* = r_M(r_R^*) > r_M(0)$, then from (A11), $z^* = z(r_M(r_R^*)) > z(r_M(0))$. Then from (A13), $r_M^* = r_M(z(r_M(r_R^*))) < r_M(z(r_M(0))) = r_M(0)$, contradiction. So it must be $r_M(r_R^*) \le r_M(0)$. From (A13), this implies that $z^* = z(r_M(r_R^*)) \ge z(r_M(0))$, which further implies that $\hat{p}^* \ge \hat{p}(z(r_M(0)))$ from (A7) and $p^* \ge p^*(z(r_M(0)))$. □