Cognitive Limitations and Investment "Myopia"

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Cognitive Limitations and Investment "Myopia"*

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ABSTRACT

Optimization of investment decisions in an uncertain and dynamically evolving environment is difficult due to the limitations of the decision maker's cognitive capacity. Thus, actual investment decisions may deviate from the dynamically optimal decision rule. This paper investigates how a potential investment rule bias affects the expected payoff from a project that has an uncertain development time and an uncertain completion cost. The result shows that the presence of a potential bias in the adopted decision rule dissipates project value and that the dissipating effect is greater for a longer-term project if the completion cost is an increasing function of the time to completion.

Subject Areas: Cognitive Bias, Investment Decision Making, Project Management, and Stochastic Models.

INTRODUCTION

Do U.S. businesses suffer from myopia in their investment decisions? Despite the widely shared belief in the affirmative among practitioners, the answers given by academics are much less in agreement. The rationale often given for the existence of investment myopia in an efficient capital market rests on the presence of imperfections in the managerial labor market. Narayanan (1985) argues that managers who have private information about the firm's investment opportunities will have an incentive to choose projects that pay back faster because early realization of earnings leads to early enhancement of their reputation in the managerial labor market. This rationale, however, has recently been questioned by several authors. Darrough and Stoughton (1993), for instance, suggest that the existence of managerial impatience may turn out to be a blessing for stockholders when managers enjoy an information advantage regarding the firm's investment opportunities. Specifically, they show that stockholders can potentially take advantage of the managerial impatience through a compensation scheme and appropriate some of the rent that the managers would otherwise get due to their information advantage. Moreover, Thakor (1990) points out that the funding of investment projects through new stock issues will be more costly than internal financing when the firm's existing stockholders have an information advantage over potential new stockholders. Under this form of information imperfection in the capital market, existing stockholders will also prefer projects that pay back faster because earlier payback enhances the firm's ability to finance projects internally, thus saving the extra cost associated with external financing. Although such behavior has the appearance of investment myopia, its consequence is welfare-maximizing for the firm's existing stockholders.

The intent of this paper is to shift the focus from various forms of market imperfections to imperfections in the human cognitive process and suggest a rationale for favoring short-term over long-term projects in the presence of potential human errors and biases in the investment process. The paper concentrates on projects that take time to develop and have an uncertain cost to completion. The most prominent feature of such a project is that the progression of its development process may sequentially yield new information that can be used to update the estimate for the additional investment needed to bring the project to completion. A revised estimate for the investment cost in turn may entail a reevaluation of the project as to whether its continuation is economically justified. The correct investment rule under these conditions should take into account the potential benefit from learning more about the project's economic viability in its development process (see Pindyck, 1993; and Roberts & Weitzman, 1981). However, because investment decision making under the prevailing management technology must still rely on at least some subjective judgments, the effective investment rule under which actual investment decisions are made is inevitably subject to the possible biases of the decision maker.

The paper uses a continuous-time model to investigate how the presence of a potential bias in the decision maker's investment rule will affect the expected value of a project whose cost to completion is uncertain. As expected, we find the presence of the potential bias to dissipate the value of the project. What initially surprised us, however, is the finding that the

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extent of the value dissipation is an increasing function of the project's time to completion when its cost to completion is assumed to be proportional to its time to completion. In other words, given two projects that require different lengths of development time and have the same expected value if managed according to the correct decision rule, the long-term project will be worth less than the short-term project if they are managed under a biased decision rule. Intuitively, this suggests that human errors and biases in the management of an uncertain investment project tend to result in greater losses when the project requires a longer gestation period and a higher development cost. Hence, if one takes into account the cost of potential human errors and biases in investment decision making, the economically rational decision rule for the screening and management of uncertain projects can be expected to have a myopic appearance.

The outline of the paper is as follows. The next section sets up a stochastic optimization model and derives the unbiased decision rule. The third section presents the rationale for treating the decision maker's investment rule as subject to a potential bias. The fourth section investigates how the presence of such a potential bias may impact the expected value of a project and how the project's time to completion may affect the size of the impact. The last section summarizes and concludes the paper.

THE BASIC MODEL

We use a continuous-time model whose basic structure is similar to those of Pindyck (1993) and Roberts and Weitzman (1981). Suppose the project in question takes time to develop and has no salvage value if it is terminated before completion. Assume that the cost of development is incurred at an instantaneous rate $c \in (0,\infty)$. One can consider this cost as the opportunity cost of personnel, equipment and other resources devoted to the development of the project. Further assume the project is expected to yield a perpetual income at an instantaneous rate $p \in (0,\infty)$ after completion. Suppose both p and c are known parameters and the only uncertainty involves the time it takes to complete the project. This uncertainty represents what Pindyck (1993) refers to as technical uncertainty and is manifested as the potential for gaining new information about the likely date of completion during the development process. Let τ_t denote the time to completion according to the estimation made as of $t \geq 0$. In order to capture the dynamic evolvement of the project's development status, we assume that τ_t follows a Brownian motion with negative drift,

$$d\tau = -dt + \sigma(dz), \tag{1}$$

where $dz = \varepsilon\sqrt{dt}$ and $\varepsilon \sim N(0,1)$. This assumption means that, along with anticipated progress $-dt$, each time increment during development may also bring about an unanticipated revision to the project's estimated date of completion, $\sigma(dz)$, due to the receipt of new information in that time increment. Note that the parameter σ reflects the degree of uncertainty about the technical difficulty of developing the project since it determines the extent of the potential surprises about the time needed to finish the project. Finally, assuming this uncertainty to be uncorrelated with the uncertainty about the general economy, we can take the risk-free rate of return, $\lambda \in (0,\infty)$, as the exogenously determined discount rate (see Dixit & Pindyck, 1994).

Given that the project's development cost is by assumption an increasing function of its time to completion, there must exist a threshold time to completion above which the anticipated future income from the completed project no longer justifies the expected cost of development. The task that the decision maker faces in project screening and management, then, is to find the threshold time to completion above which the project should be rejected or abandoned for lack of chance to afford a profit. Let $b \in (0,\infty)$ represent the control threshold employed by the decision maker, such that the project is rejected or abandoned if its estimated time to completion exceeds this threshold (i.e., if $\tau_t > b$). Under such a control policy, the project will terminate under either of two conditions: completion (when $\tau_t = 0$) or abandonment (when $\tau_t > b$). Let T be the time it takes for either condition to occur. Then, the project's expected payoff in present value terms as of $t = 0$, conditioned on the initial estimate of its time to completion $x = \tau_0$, can be expressed as

$$h(b,x) = E_x\left(\int_T^\infty p \cdot e^{-\lambda t} dt; \tau_T = 0\right) - E_x\left(\int_0^T c \cdot e^{-\lambda t} dt\right) \tag{2}$$

where $\tau_T = 0$ after the semicolon in the first term denotes that the potential income stream from the project only accrues in the event of completion. The optimal decision rule for the project can be derived by finding a control threshold *b* such that the expectation given in (2) is maximized. The derivation may utilize two alternative approaches. One approach entails first solving for the exact expression of the expectation directly by probability means and then computing the optimal control threshold based on the expression derived. The other approach makes use of Ito's formula to first express the optimality condition as a differential equation and then solve the differential equation for the optimal control threshold. Although the problem of finding the optimal control threshold can be easily solved using the method of differential equations, the solution obtained under this approach does not tell us how the adoption of a suboptimal threshold affects the value of the expected payoff from the project. Because an extension of our model later in the paper requires this information, we will derive the solution by probability means.

Based on the results obtained by Harrison (1985, pp. 38-44), it is straightforward to show that the expectation given in (2) has the following functional form:

$$h(b,x) = \frac{(p+c)\left[e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} - e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}} e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right] + c\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}} - e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right] e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}}{\lambda\left(1 - e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)} - \frac{c}{\lambda}, \tag{3}$$

where $\sqrt{\Delta} = (1+2\sigma^2\lambda)^{1/2}$. As can be seen from (3), the project's expected payoff is not a function of time *t*, so the value and shape of the function *h*(*x*,*b*) remains the same regardless of how time is denoted. Given this "stationary" property of our problem, we can simply treat the current time as $t = 0$ and take the currently estimated time to completion as the initial estimate $x = \tau_0$ in our analysis of the problem (see Bensoussan & Lion, 1982).

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Optimality Condition

With the expected payoff function given in (3), we can solve the problem of finding the optimal control threshold by standard optimization procedures. As derived in the Appendix, this optimal control threshold is characterized by the following first-order condition:

$$\frac{2(p+c)\sqrt{\Delta}}{c\left(\sqrt{\Delta}+1\right)}e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}=0. \qquad (4)$$

Noting $\sqrt{\Delta}=(1+2\sigma^2\lambda)^{1/2}$, we can easily see from (4) that the optimal control threshold is a function of four parameters: the rate of income after completion p, the cost of carrying the project c, the variability of the project's time to completion σ, and the discount rate λ. As shown in the Appendix, there is a unique value of $b\in(0,\infty)$ that solves (4) and maximizes (3) for given values of these parameters. For convenience in later discussions, we will use $\bar{b}$ to denote the optimal control threshold and use $h(\bar{b},x)$ to denote the maximized expected payoff function.

Other Features of the Expected Payoff Function

It is helpful to learn more about the features of the expected payoff function before we examine the determinants of the optimal threshold $\bar{b}$. Figure 1 shows how the expected payoff function $h(b,x)$ varies with the adopted control threshold b for three hypothetical projects. As noted earlier, the optimal threshold is determined by the values of four parameters: p, c, σ and λ. Since the values of these parameters are assumed to be the same for the three projects, the value of the optimal control threshold for those projects is also identical ($\bar{b}=10$). The only difference among the three projects lies in their estimated time to completion x. As illustrated by the plot for Project C, the maximum expected payoff from the project falls to zero when its estimated time to completion becomes as large as the optimal threshold for abandonment, that is, $x=\bar{b}$. In addition, the plots in Figure 1 also show that the maximized expected payoff of a project is a decreasing function of its time to completion, that is, $dh(\bar{b},x)/dx<0$. This result is algebraically proved in the second part of the Appendix, as is the result that the expected payoff function increases with the project's rate of the income after completion, that is, $dh(\bar{b},x)/dp>0$.

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Place Figure 1 here

It can also been seen from Figure 1 that the expected payoff function is quasi-concave in b and reaches its maximum when the adopted control threshold equals the optimum (i.e., $b = \bar{b}$). Since a project is necessarily rejected or abandoned if its estimated time to completion exceeds the adopted control threshold (i.e., $x > b$), the part of the function to the left of the point $b = x$ is censored and forced to have a value of zero in the graph. Another interesting feature of the function is that its value eventually falls toward a limit as the adopted threshold b approaches infinity. As shown in Figure 1, the value of this limit can be positive or negative, depending on whether the estimated time to completion x is sufficiently shorter than the optimal threshold for abandonment $\bar{b}$. The sign of this limit indicates whether the expected value of the project is positive or negative if it is managed under a policy whereby abandonment never occurs no matter how much time may be needed to finish it. The reason for the sign of this limit to stay positive for a project's whose estimated time to completion is far below the optimal threshold for abandonment is that the chance for such a project to not reach a profitable conclusion is small.

Determinants of the Optimal Decision Rule

We now examine how the optimal control threshold $\bar{b}$ varies with the values of the four parameters p, c, σ and λ. By inspecting (4), we can see that p increases and c decreases the first and only positive term in this first-order optimality condition. Since neither p nor c enters the other two terms in the equation, we must have $d^2h/dbdp > 0$ and $d^2h/dbdc < 0$. In addition, by the second-order condition for the existence of a unique maximum, we also have $d^2h/db^2 < 0$ for any control threshold that is in the neighborhood of the optimum $\bar{b}$ (see the Appendix). Then, by the Envelope theorem, we obtain

$$\frac{d\bar{b}}{dp} = -\frac{d^2h/dbdp}{d^2h/db^2} > 0 \text{ and } \frac{d\bar{b}}{dc} = -\frac{d^2h/dbdc}{d^2h/db^2} < 0.$$

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The intuition behind these results is straightforward: the decision maker should wait longer as the income from the completed project p is higher or as the cost of carrying the project c is lower.

The effect of a change in σ or λ on the optimal threshold $\bar{b}$, however, is not so easy to ascertain. Because the sign of the cross derivative of $h(b,x)$ with respect to b and σ or λ can not be unambiguously determined, we have to evaluate the effect using numerical methods. By varying the value of σ or λ while fixing the values of the other parameters, we were able to uncover a consistent pattern of change in $\bar{b}$ as σ or λ changes. We experimented with a wide range of values for each parameter, and the results of all the experiments show that a rise in σ or a fall in λ increases $\bar{b}$ (see Table 1).

Place Table 1 here

The negative relationship between the discount rate λ and the optimal control threshold $\bar{b}$ is straightforward, as it says that a higher discount rate makes waiting more costly. The intuition behind the positive relationship between σ and $\bar{b}$, however, requires some explanation. Since σ measures the variability in the length of time needed to complete the project, a rise in σ means a symmetric increase in the variability on both the upside and the downside. But in the presence of the option to abandon the project in the face of unanticipated difficulty and save the investment cost that has not yet been incurred, the decision maker is able to limit the effect of the increase in the downside risk while fully benefiting from the effect of the increase in the upside potential. In other words, the effect of an increase in the variability of the project's time to completion is asymmetric when abandonment in the face of unanticipated difficulty can save the part of the development cost that has not yet been spent. Hence, it pays for the decision maker to exploit this asymmetry by waiting a little longer when there is an increase in the variability of the project's time to completion. This result reflects what we referred to in the introduction as the potential benefit from learning more about the project's economic viability in its development process and is consistent with the results that Pindyck (1993) and Roberts and Weitzman (1981) obtained earlier.

A REEXAMINATION OF SOME BEHAVIORAL ASSUMPTIONS

In this section, we will step back from the theoretical model that we presented above and reexamine some of the behavioral assumptions that we have made either explicitly or implicitly in the construction of the model. Specifically, we have made a number of explicit assumptions about what factual information the decision maker possesses about the project and also implicitly assumed that the decision maker is able to figure out the optimal policy based on the available information. We will assess these behavioral assumptions in some detail below. Our purpose in doing so is to lay the groundwork for an extension of the model in the next section of the paper. We will make three related arguments in this section. The first is that the evaluation of an uncertain investment project under the present management technology must still rely in a large part on the subjective judgments of the decision maker. The second is that the decision maker's judgments are susceptible to potential errors and biases due to limitations of the human cognitive capacity. The third is that the potential errors and biases in the decision maker's judgments can result in the adoption of a biased decision rule.

Role of Subjective Judgments in Investment Decision making

In order to keep the model focused and manageable, we modeled the problem in the last section as a one-dimensional stochastic process involving only one randomly evolving variable (τ_t) and assumed the other exogenous variables (p, c and λ) as known parameters. Needless to say, the correctness of the decision rule derived under these assumptions rests on the validity of the assumptions. For an actual investment project, however, the values of these parameters are unlikely to be known with certainty and may well be subject to revisions due to the arrival of new information in the project's development process. If this is the case, these parameters should also be treated as stochastically evolving variables in order to correctly model the project. As pointed out by Ingersoll and Ross (1992), failure to take into account any such information dynamics will subject the derived decision rule to a bias and result in suboptimal decisions. Yet,

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the modeling of a multi-dimensional stochastic process poses a very complex mathematical problem that is often difficult even for a trained specialist of operations research to solve. So the cost of building such a model is likely to be rather high. In addition, even if a highly sophisticated model is built to assess whether to adopt the project in the beginning, information gathered in its early stages of development may necessitate some unexpected changes in the basic characteristics of the project, thus causing the initial model to lose relevance. Then, the construction of a new model to account for the changes in the attributes of the project would give rise to even greater modeling expenses.

In short, although there may exist a multitude of dynamically evolving factors that bear significantly on the value of an uncertain project, inclusion of all those factors in a model of the project is likely to make the model intractable or the modeling effort too costly. In the meantime, if a much simplified model is employed to assess the project, the decision maker is unlikely to place a high level of confidence in the result of the model due to its omission of factors that are perceived to be important. So the decision maker is still likely to supplement or even supplant the model with his/her subjective judgments in an attempt to take into account the effects of factors left out in the model. In fact, surveys of capital budgeting practices have consistently found that firms in the U.S. and elsewhere still rely on such traditional techniques as internal rate of return and payback period in project evaluation (see Gitman & Forrester, 1977; Hendricks, 1983; and Ross, 1986). We may interpret the results of these surveys as evidence that practitioners still perceive the benefit from building a stochastic optimization model as too limited to justify the effort in most managerial situations at the present time. The persistent reliance of practitioners on those traditional techniques of project evaluation is clearly reminiscent of what Simon (1957) termed as “satisficing” behavior in the face of high information costs.

Field studies also suggest that practitioners do often supplement or even supplant their financial analysis with subjective judgments in investment decision making (for example, see Bower, 1970). The justification that practitioners typically give for this type of behavior is the

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presence of "non-quantifiable" factors, which presumably include various dynamically evolving factors that are difficult to quantify. Although the use of subjective judgments to account for factors missing in the traditional financial analysis is easier to implement than the development of a complicated mathematical model, this mode of decision making is inevitably subject to the limitations of human intuition. In the next subsection, we will review some experimental studies on the human cognitive process and the susceptibility of human judgments to errors and biases.

Susceptibility of Subjective Judgments to Errors and Biases

Lab experiments in the study of cognitive psychology have documented the existence of not only random but also systematic errors (or biases) in human judgments under uncertainty. According to Tversky and Kahneman (1974, p.1124), the susceptibility of human judgments to errors and biases can be attributed to the limitations of the human cognitive capacity, that is, the capacity to store, retrieve and process information. They theorize that the human mind may have developed certain heuristic principles to reduce complex tasks to simpler judgmental operations in order to deal with situations where its capacity is exceeded. Although the employment of these heuristics enables the human mind to analyze very complex problems, they sometimes can also lead to severe and systematic errors or biases. Deducing from their findings in a series of lab experiments, they suggest that three heuristic principles are particularly operative in human judgments under uncertainty: representativeness, availability, and adjustment and anchoring.

Representativeness is a heuristic method that involves search and compare. When looking for the answer to an unfamiliar problem, a person who employs this heuristic will search through more familiar problems and compare which one is most representative of the unfamiliar problem. Then, the answer to the more familiar problem that the person considers to be the most similar to the unfamiliar problem is adopted as the most likely answer. Although this method can be very useful in problem solving under uncertainty, its employment does have the tendency to introduce biases in decision making. In their lab experiments, Tversky and Kahneman (1974, pp. 1124-1127) found that, in employing this heuristic, people tend to underutilize other relevant information and become overconfident about the strength of the relationship between the

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familiar problem and the unfamiliar problem. In the context of investment decision making, the use of this heuristic can be expected to result in a potential bias in the decision on a new project toward what would have been the correct decision on some past project that the decision maker believes to be similar to the new project.

If the representative heuristic can be considered a method of utilizing the best of what is stored in one's memory, the availability heuristic may be considered a method of utilizing what is accessible in one's memory at the time of decision making. Tversky and Kahneman (1974, pp. 1127-1128) found that people also have a tendency to put excessive emphasis on the more salient and more easily imaginable events in analyzing a problem. In the context of investment decision making, the use of this heuristic can be expected to result in a potential bias in the decision on a new project toward what would have been the correct decision on some past project that is more recent or has had more profound consequences.

The heuristic of adjustment and anchoring is essentially the method of trial and error. The use of this heuristic should not give rise to any bias in decision making if the processing of new information gathered through trial and error is unbiased. However, experiments by Tversky and Kahneman (1974, pp. 1129-1130) suggest that the initial judgments of people on an previously unfamiliar phenomenon tend to become a filter to bias their processing of new information on the phenomenon. One manifestation of this bias is the persistence of an erroneous perception despite repeated evidence to the contrary. Excessive reliance on the method of trial and error can cause people to overlook alternative sources of information or to focus too much on the gathering of new signals and fail to carefully analyze the signals already received. In the context of investment decision making, the presence of such a bias can be expected to result in the failure or slowness to correct an initial error in, for instance, estimating a project's time to completion.

The cognitive limitations discussed above can all be considered as contributing elements in what Simon (1957) calls bounded rationality, as they all impose constraints on the rationality of an intendedly rational decision maker. If we slightly expand the common definition of bounded rationality, we may also include under this rubric certain behavior that borders on irrationality. Specifically, we want to include the behavior exhibited by a person who intends to be rational but whose rationality is sometimes tempered by emotions. In investment situations, the decision on whether to continue or abandon a project is potentially influenced by such emotions as frustration and obsession. Given that the evaluation of an uncertain project generally must rely on at least some subjective judgments by the decision maker, the interference of various emotions in those subjective judgments can be difficult for the intendedly rational decision maker to detect.

This type of limitations to rationality has been the focus of study in what is known as the escalation of commitment research in social psychology. Although we are not convinced of the escalation study’s main claim that people tend to unduly delay the abandonment of a bad project, we do believe that the lab experiments conducted in this stream of research have produced two interesting findings about investment behavior under uncertainty. The first is that people tend to make different decisions on whether to abandon an uncertain investment project when they are given the same ambiguous information about the project without being told what is the correct decision rule. Significant variations in the decisions of the subjects were found in virtually all the experiments in the escalation research (for a review, see Brockner, 1992). The variations can be partially attributed to the use of the representativeness and availability heuristics, as they suggest that people make different judgments due to their idiosyncratic past experiences in an ambiguous situation. The second of these findings is that people’s investment decisions are subject to the influence of emotional factors. A study that most convincingly demonstrates this point is that of Strube and Lott (1984), who found that people with different personality characteristics differ in their tendency toward continuing a floundering project. Specifically, their experimental results show that people of Type A personality (who are more hard-driving and more easily aroused) tend to persist longer than people of Type B personality when the investment in the project is set to equal the time that has elapsed after commencement.

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Influences of Cognitive Biases on Investment Decision Making

In this subsection, we will use two examples to illustrate how the cognitive limitations of decision makers might suboptimize their investment decisions. Our first example is based on a case study that details the evolvement of an R&D project in a consumer electronics manufacturer (see Wheelwright & Clark, 1995, pp. 8-11]. The purpose of the project was to develop a compact disk system that would incorporate new technology in both the product and its manufacturing process. Based on the company's recent experiences in developing compact disk systems, the initial development plan anticipated completion by September of the following year—exactly one year after start—to allow volume production for the next Christmas season. The project team were very excited about their plan and developed a strong commitment to the project early on. But problems started to arise as the development proceeded. Because the project involved new technology, almost every development stage required more time than initially anticipated. It took six weeks longer to settle on the specification of the product's technical features and eight weeks longer to finish its design. Furthermore, technical difficulties and other unanticipated events delayed prototyping by more than three months. Despite the repeated delays in development, the managers of the project did not change the September launch date until there was less than a month left and they were still building the prototype. They simply reduced the projected time for each subsequent development stage to keep the planned launch date unchanged, even though unexpected technical difficulties in an early stage would normally suggest more difficulties and further delays ahead. As it turned out, the project was not completed until seven months after the original launch date and showed a poor financial result due to the extensive delay and large accumulation of its development cost.

This example illustrates several decision errors that are attributable to the cognitive biases of the decision makers. First, although the product to be developed in the project involved new technology and thus had some different characteristics than those that the firm introduced in the recent past, the decision makers did not carefully consider the possibility that it might require more time to develop. Instead, their original estimate for the project's development time was

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based on their recent experiences and turned out to be low by a large margin. This initial error, therefore, seems to be a reflection of the decision makers' excessive reliance on the heuristics of representativeness and availability discussed earlier. Their initial underestimation could have been corrected much sooner if the decision makers had treated the difficulties in the project's early stages as indications of more difficulties ahead. But instead of incorporating these early signals in their updated estimates for the lengths of time needed to finish the subsequent development stages, they elected to adjust downward the projected build time for later stages to avoid changing the planned launching date. This pattern of behavior indicates the existence of biased information processing in the sense that data suggesting an unpleasant conclusion were ignored and only the most optimistic estimates were used in decision making (see Schwenk, 1984). Their biased information processing can perhaps be traced to the project team's strong commitment to the project's success, which was thought to depend significantly on being able to catch the demand of the Christmas season. If the managers of the project had reached a more realistic assessment much sooner about the time and cost needed to develop the product, the firm might have been able to stop the project in an early stage and save much of the inefficiently spent development cost.

The above example represents a case of excessive persistence where investment decisions are biased toward continuation of a project that should be abandoned. Although agency problems might also have played a role in the outcome of such a case (see Boot, 1992), the project team's over-commitment was likely to have had an important impact. This kind of psychological bias has often been used to explain Lyndon Johnson's prolonging of the Vietnam War. In the rest of this subsection, we will discuss an example that illustrates the opposite type of bias in investment decisions making—excessive conservatism—the bias toward abandonment of a project that still warrants further experimentation. This example is also based on a case study involving an R&D project (see Kerzner, 1989, pp. 565-574). The project was undertaken by a manufacturer of rubber components to develop a new rubber material for one of its clients. The contract between the firm and its client called for a representative from the client to be stationed at the firm to facilitate the project. The individual representing the client was described as rather neurotic about the project but extremely arrogant toward the other members of the project team. The first set of tests based on the firm's initial design was not fully satisfactory but showed the experiments to be on the right track according to the firm's scientists. The client's representative, however, regarded the results as indications of a failure and demanded a radical change in the design. After considerable arguments, he agreed to wait for the results of further experiments. Although later tests did show improvements, he still believed the project to be headed in the wrong direction and, by threatening project cancellation, forced the project manager to change the basic design of the experiments. As it turned out, the radical and often erratic changes he proscribed did not produce any conclusive results and led to severe conflicts within the project team and the eventual cancellation of the project. Although it is not certain that further experiments based on the original design would have produced a fully satisfactory outcome, this example does illustrate how the personality traits and prejudices of an influential manager might dispose a project to the fate of premature abandonment.

A Revised Set of Behavioral Assumptions

We can summarize our discussion in this section in a revised set of assumptions about the decision maker's behavior in managing a project of uncertain time to completion. First, under the prevailing technology of project evaluation, the decision maker finds it infeasible or too costly to build a stochastic optimization model that incorporates every factor that he/she considers to bear significantly on the economic prospect of the project. Second, although he/she may employ some mathematical calculation (such as sensitivity analysis of prospective cash flows) in evaluating the project, the decision maker also makes use of heuristic-based judgments in order to assess factors that are not adequately accounted for in the model. Finally, the judgments of the decision maker are subject to errors and biases due to the limitations of his/her cognitive capacity (including, but not limited to, the possible interference of human emotions). Consequently, the investment rule that the decision maker effectively follows may deviate from the optimal investment rule derived from the correct stochastic optimization model.

This set of behavioral assumptions forms the basis for an extension of our model in the next section that explores how the presence of a potential bias in the decision maker's investment rule might affect the expected value of the project that he/she manages. Of particular interest to our investigation is how the effect of such a potential bias might relate to the project's time to completion. Our investigation will be conducted in the context of the model constructed in the second section of the paper. Specifically, we will assume the project in question to be exactly like the one scrutinized in that earlier section and examine how the expected value of the project might be affected by a deviation of the adopted decision rule from the optimal decision rule.

IMPACT OF A POTENTIAL DECISION BIAS ON PROJECT VALUE

We continue to use $\overline{b}$ to denote the optimal control threshold derived from the stochastic model developed earlier and use b to denote the control threshold the decision maker effectively follows. In the preceding section, we have distinguished between two types of biases: excessive conservatism and excessive persistence. Excessive conservatism amounts to the adoption of a biased control threshold that is smaller than the optimum (i.e., $b < \overline{b}$), since such a bias results in the rejection/abandonment of a project that still warrants acceptance/continuation. Excessive persistence, on the other hand, amounts to the adoption of a biased control threshold that is larger than the optimum (i.e., $b > \overline{b}$), since such a bias results in the acceptance/continuation of a project that should be rejected/abandoned.

Figure 1 graphically shows how the value of the adopted control threshold b influences the expected value of a project $h(b,x)$. Note that the optimal control threshold is the same for all three hypothetical projects depicted in the graph, that is, $\overline{b} = 10$. So, a control policy of $b < 10$ contains a bias of excessive conservatism, and a control policy of $b > 10$ contains a bias of excessive persistence. One can easily see from the graph that the presence of either type of bias depreciates a project's *expected value* and thus results in an *expected loss*. The amount of this *expected loss* is measured by the difference between the project's value under the optimal decision rule and its value under the biased decision rule, that is, $h(\overline{b},x) - h(b,x)$ for $b \neq \overline{b}$.

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Which Type of Bias is More Damaging?

First, note that under unbiased decision making the expected value of a project is positive if and only if its estimated time to completion x falls below the optimal control threshold $\bar{b}$. So, by this standard, any project characterized by $x < \bar{b}$ is a *good* one and any project characterized by $x \geq \bar{b}$ is a *bad* one. Among the three projects depicted in Figure 1, Project A is clearly very good, Project B is only marginally good and Project C is undoubtedly bad. By comparing the plots of these three hypothetical projects, we can see that the amount of damage a given type of decision bias (excessive conservatism or excessive persistence) could do to a project depends on how good the project is. Since a project would be rejected or abandoned and thus provide no payoff if its estimated time to completion exceeds the decision maker's control threshold (i.e., $x > b$), the worst damage that excessive conservatism can do *ex ante* is the complete loss of a project's otherwise positive value $h(\bar{b}, x)$. In this sense, we can say that the potential loss from the bias of excessive conservatism becomes greater as the project is better (see Figure 1).

In contrast, the potential loss from the bias of excessive persistence is inversely related to the quality of a project as measured by the distance between its estimated time to completion and the optimal control threshold, $\bar{b} - x$, which is non-positive for a bad project. We can see from Figure 1 that two factors are responsible for this inverse relationship. First, as explained earlier, the expected payoff function $h(b,x)$ approaches a limit as the adopted control threshold b goes to infinity, and the value of this limit is positive for a very good project such as Project A and negative for a marginal or bad project such as Project B or C. Note that the policy of having an infinitely large threshold b implies that the project will be continued till completion no matter how long it may take to finish. So, under this extreme policy of no control, a very good project is still expected to offer a positive payoff, but a marginal or bad project can have a highly negative payoff. In addition, Figure 1 also shows that a given deviation of the adopted control threshold b from the optimal $\bar{b}$ engenders a smaller expected loss as the project is better (i.e., with a shorter time to completion). As can be seen, the fall in the expected payoff function due to a given bias of excessive persistence is smaller for Project A than for Projects B and C. This numerical result

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(which can also be proved analytically) suggests that a better project is less vulnerable to the bias of excessive persistence not only in terms of the potential loss but also in terms of the expected loss from a given deviation of the adopted decision rule from the optimal.

Our analysis in the preceding paragraphs shows that the bias of excessive conservatism tends to be more damaging for a good project and that the bias of excessive persistence tends to be more damaging for a marginal or bad project. If firms face differing project opportunities, we can expect those with superior opportunities (e.g., firms in a rapidly growing industry) to be more averse to excessive conservatism and less averse to excessive persistence than those with only limited opportunities (e.g., firms in a declining industry). This result may explain why firms in high-tech industries often regard persistence as a desirable trait while firms in mature industries often regard conservatism as a desirable trait for their managers.

Influence of Time to Completion

We now move on to examine in more detail how the size of the loss from a given bias relates to a project's time to completion. To facilitate the exposition that follows, let $\delta = b - \bar{b}$ denote the deviation of the decision maker's adopted control threshold from the optimum, so that the expected payoff of a project that is being managed under a potentially biased decision rule can be expressed as $h(\bar{b}+\delta, x)$ where $\delta \in [-\bar{b}+x, \infty)$. The reason for defining the domain of the potential bias as $\delta \in [-\bar{b}+x, \infty)$ is that any δ smaller than $-\bar{b}+x$ is irrelevant to our inquiry because the project is necessarily rejected or abandoned unless the adopted control threshold is greater than the project's estimated time to completion, that is, $b = \bar{b}+\delta \geq x$. The problem that we want to investigate here can be explained as follows. We know that the expected value of a project is necessarily lower if it is managed under a biased decision rule than under an unbiased decision rule, that is, $h(\bar{b}+\delta, x) < h(\bar{b}, x)$ for $\delta \neq 0$. Suppose there are two projects which differ in their time to completion but would have the same expected payoff if they were managed under the unbiased decision rule, that is, $h(\bar{b}_L, x_L) = h(\bar{b}_S, x_S)$ where $x_L > x_S$. What we are trying to find out is whether the expected payoffs from these two projects will still be the same if they are managed under a decision rule that is biased in the same direction and by the same magnitude, that is, whether $h(\overline{b}_L+\delta, x_L) = h(\overline{b}_S+\delta, x_S)$ is true for $\delta \neq 0$. The simplest way to get an answer to this question is to compare the expected payoffs from two such projects as described above. We will first construct a numerical comparison to illustrate the answer intuitively and then present the result of a mathematical proof that generalizes the numerical result.

As noted in the last paragraph, a proper comparison of how the employment of a biased decision rule affects the expected payoffs of projects with differing time to completion requires that the projects being compared have the same expected payoff if managed under the optimal decision rule. From the results derived in the second part of the Appendix, we know that the maximized expected payoff of a project is a decreasing function of its time to completion *x* and an increasing function of its rate of income after completion *p*. So, in order to maintain a constant expected payoff under the optimal decision rule, we will perform the numerical comparison by looking at two hypothetical projects that differ in their time to completion but have an offsetting difference in their rates of income after completion. Let the expected payoff functions of these two projects be denoted by $h_L(\delta) = h(\overline{b}_L + \delta, x_L, p_L)$ and $h_S(\delta) = h(\overline{b}_S + \delta, x_S, p_S)$, respectively, with $x_L > x_S$, $p_L > p_S$ and $h_L(0) = h_S(0)$. Then, by plotting $h_L(\delta)$ and $h_S(\delta)$ against δ in a graph, we can see whether these two functions will coincide or diverge as the possible bias δ deviates from zero. Figure 2 gives the result from such an experiment for a given set of parameter values.

Place Figure 2 here

In the graph, one can easily see the existence of a vertical gap between the two functions for values of $\delta \in (-\overline{b}+x, \infty)$ other than zero. The gap shows that the long-term project has a lower expected payoff than the short-term project in the presence of a bias (i.e., for $\delta \neq 0$). This result suggests that the loss due to any deviation of the investment rule from the optimum is greater for the long-term project than for the short-term project. In other words, an identical bias is expected to cost more to the long-term project than to the short-term project.

We can generalize the numerical result presented above by obtaining the derivative of the expected payoff function $h(b,x)$ with respect to the time to completion x subject to two constraints. One constraint requires that the value of the function at the optimal control threshold $\bar{b}$ remain the same as the time to completion changes, that is, $dh(\bar{b},x)/dx = 0$. The other constraint is that the deviation of the adopted threshold from the optimal threshold remain constant as the time to completion changes, that is, $d(b-\bar{b})/dx = 0$. The derivative thus obtained can reveal how a longer time to completion affects the value of the function under a given decision bias when the effect of the longer time to completion is fully compensated for by an increase in the project's rate of income after completion if the optimal decision rule is followed. As shown in the third part of the Appendix, this derivative can be expressed as:

$$\left.\frac{dh(b,x)}{dx}\right|_{\substack{\frac{dh(\bar{b},x)}{dx}=0,\\ \frac{d(b-\bar{b})}{dx}=0}} = -\frac{c(\sqrt{\Delta}+1)\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right)\left\{\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\left[e^{\frac{(b-\bar{b})(\sqrt{\Delta}+1)}{\sigma^2}} - 1\right] + \left[e^{-\frac{(b-\bar{b})(\sqrt{\Delta}-1)}{\sigma^2}} - 1\right]\right\}}{\lambda\sigma^2 e^{\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right)^2}, \quad (5)$$

and has a negative sign for $b \in (x, \bar{b})$ or $b \in (\bar{b}, \infty)$. The meaning of this result is that a given bias in the decision maker's investment rule is expected to dissipate more of the expected payoff from a project if the project is subject to a longer time to completion. Intuitively, the result suggests that a longer-term project is more susceptible to the risk that the decision maker may follow a biased investment rule. There is, therefore, an economic reason for treating a longer-term project less favorably when there is a chance that future decisions on whether to continue or abandon the project will be subject to potential biases due to the cognitive limitations of the decision maker.

SUMMARY AND CONCLUSION

In this paper, we provided a justification for favoring short-term over long-term projects based on the presence of imperfections in the human cognitive process. First, we presented a stochastic optimization model for evaluating a project whose time to completion is uncertain. The optimal decision rule derived from the model was shown to involve a threshold value of the estimated time to completion above which the project should be rejected or abandoned.

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Then, we reexamined some of the behavioral assumptions that are necessary in order for the prediction of the stochastic model to be an accurate depiction of investment behavior under uncertainty. Two particularly important assumptions were discussed. The first is that the model includes all the factors that the decision maker believes to be important in influencing the value of the project. The second is that the decision maker is able to figure out the correct decision rule based on the factual information that he/she possesses. We suggested that the first assumption is likely to fail in an actual managerial situation because under the present management technology it is generally infeasible for the decision maker to include all the important factors in a stochastic optimization model. Thus, in order to account for the effects of factors that can not be assessed economically via a stochastic optimization model, the decision maker will need to rely on at least some of his/her subjective judgments. Based on the results of experimental studies on the human cognitive process and field studies on project management, we argued that the decision maker's subjective judgments are subject to sometimes severe errors and biases that resemble the adoption of a suboptimal decision rule.

In the last part of the paper, we investigated the impact of a potential bias in the decision maker's investment rule on the expected payoff from a project. As expected, we found that the presence of the potential bias dissipates the value of the project and that the extent of the value dissipation is positively related to the degree of the bias. More interestingly, however, we also found that a given bias has a greater dampening effect on the value of the project as the project is subject to a longer time to completion. An intuitive explanation for this finding is that human errors and biases in the management of an uncertain investment project tend to pose a greater cost as the project has a longer gestation period. This suggests that, if the cost of potential human errors and biases in investment decision making is taken into account, the economically rational decision rule for the screening and management of uncertain projects can be expected to have a myopic appearance.

The growing body of research on investment behavior has pointed to various forms of market imperfections as the potential sources of the so-called myopia phenomenon. Although our model does not dispute any market-based explanations for the phenomenon, it does suggest that the practice of favoring short-term over long-term projects may still remain due to imperfections in the human cognitive process even if all markets were perfect. In addition, it also suggests that the practice may actually promote human welfare in a world where heuristic-based judgments are still widely used to supplement or even supplant mathematical modeling in investment decision making.

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APPENDIX

Derivation of the Optimality Condition

Differentiate the function defined in (3) with respect to b and we get the first-order condition for a maximum of the expected payoff function $h(b,x)$,

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$$\frac{dh}{db}=\frac{\left\{(p+c)e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\frac{2\sqrt{\Delta}}{\sigma^2}-c\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}\frac{\sqrt{\Delta}+1}{\sigma^2}\right\}\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)}{\lambda\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)^2}$$

$$-\frac{\left\{(p+c)\left[e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}-e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right]+c\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}\right\}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\frac{2\sqrt{\Delta}}{\sigma^2}}{\lambda\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)^2}=0. \tag{A1}$$

Dropping the common factor $\frac{1}{\lambda\sigma^2}\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)^{-2}$, we may simplify the rest as follows:

$$2(p+c)\sqrt{\Delta}e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}-2(p+c)\sqrt{\Delta}e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{4b\sqrt{\Delta}}{\sigma^2}}$$

$$-c(\sqrt{\Delta}+1)\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}+c(\sqrt{\Delta}+1)\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}$$

$$-2(p+c)\sqrt{\Delta}e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}+2(p+c)\sqrt{\Delta}e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{4b\sqrt{\Delta}}{\sigma^2}}-2c\sqrt{\Delta}\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}=0,$$

$$\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\left[2(p+c)\sqrt{\Delta}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}-c(\sqrt{\Delta}+1)e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-c(\sqrt{\Delta}-1)e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right]=0.$$

Note that the common factor involving x is a sinh function, which is positive given $x>0$,

$$e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}=e^{\frac{x}{\sigma^2}}\left(e^{\frac{x\sqrt{\Delta}}{\sigma^2}}-e^{-\frac{x\sqrt{\Delta}}{\sigma^2}}\right)=2e^{\frac{x}{\sigma^2}}\sinh\left(\frac{x\sqrt{\Delta}}{\sigma^2}\right)>0.$$

So we can also drop this common factor and have the first-order condition expressed as

$$2(p+c)\sqrt{\Delta}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}-c(\sqrt{\Delta}+1)e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-c(\sqrt{\Delta}-1)e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}=0.$$

Dividing it through by $c(\sqrt{\Delta}+1)e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}$, we can reduce the first-order condition to

$$\frac{2(p+c)\sqrt{\Delta}}{c\left(\sqrt{\Delta}+1\right)}e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}=0. \tag{A2}$$

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Note that we have dropped three common factors from the first-order derivative in the simplification process: $\frac{1}{\lambda\sigma^2}\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)^{-2}$, $\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]$ and $c(\sqrt{\Delta}+1)e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}$.

Put these factors back and we can write the full first-order derivative as:

$$\frac{dh(b,x)}{db}=\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\frac{c(\sqrt{\Delta}+1)e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}}{\lambda\sigma^2\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-1\right)^2}\left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\right] \quad \text{(A3)}$$

since $\frac{1}{\lambda\sigma^2}\left(1-e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}\right)^{-2}=\frac{1}{\lambda\sigma^2}e^{\frac{4b\sqrt{\Delta}}{\sigma^2}}\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-1\right)^{-2}$ and $e^{\frac{4b\sqrt{\Delta}}{\sigma^2}}e^{-\frac{2b\sqrt{\Delta}}{\sigma^2}}e^{-\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}=e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}$. As we will see in a moment, there is a unique value of $b\in(0,\infty)$ that solves (A2) and maximizes the function given in (3). Let the optimal value of b be denoted by $\bar{b}$. Substituting $\bar{b}$ back into (A3) and differentiating it with respect to b again, we get the second-order condition for a maximum,

$$\frac{d^2h}{db^2}=\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\frac{c(\sqrt{\Delta}+1)e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}}{\lambda\sigma^2\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-1\right)^2}\left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}\frac{(\sqrt{\Delta}+1)}{\sigma^2}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{2\sqrt{\Delta}}{\sigma^2}e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right]$$

$$=-2\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-1\right)^{-2}\frac{c(\sqrt{\Delta}+1)\sqrt{\Delta}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}}{\lambda\sigma^4}\left[e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}\right]. \quad \text{(A4)}$$

Note that the elements outside the last brackets in (A4) are all positive. Hence, the sign of this second-order derivative is determined by the sign of the elements inside the brackets. Factor $\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}$ out of the brackets and we are left with $\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}-1$. So one expression for the second-order condition is:

$$\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}>1. \quad \text{(A5)}$$

This expression will be useful in later derivations. By the first-order condition expressed in (A2), however, we have $e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}=\frac{2(p+c)\sqrt{\Delta}}{c\left(\sqrt{\Delta}+1\right)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}$. Then, the elements inside the last brackets in (A4) can also be written as:

$$\frac{2(p+c)\sqrt{\Delta}}{c\left(\sqrt{\Delta}+1\right)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}=\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}\left(\frac{2\sqrt{\Delta}}{\sqrt{\Delta}+1}-1\right)-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}$$

$$=\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\left[\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-1\right].$$

So an equivalent expression for the second-order condition is $\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}>1$. Given $\bar{b}>0$, the second-order condition is satisfied whenever the first-order condition is satisfied. In other words, the first-order condition expressed in (A2) is both necessary and sufficient for the existence of a unique $b\in(0,\infty)$ that maximizes $h(b,x)$. This proves that $h(b,x)$ is quasi-concave in b.

As a check on the model, we can solve the first-order equation (A2) for the optimal threshold $\bar{b}$ in the special case where the length of time it takes to finish the project is precisely known, that is, $\sigma=0$. Since $\sqrt{\Delta}=(1+2\lambda\sigma^2)^{1/2}$, we have $\frac{2\sqrt{\Delta}}{\sqrt{\Delta}+1}=1$ and $\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}=0$ when $\sigma=0$. Then, for $\sigma=0$, the first-order condition (A2) can simply be written as $e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}=\frac{p+c}{c}$ because $e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}\Big/e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}=e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}$. Taking log on both sides, we have $b\frac{(1+2\lambda\sigma^2)^{1/2}-1}{\sigma^2}=\ln\frac{p+c}{c}$. Since $\lim_{\sigma\to 0}\frac{(1+2\lambda\sigma^2)^{1/2}-1}{\sigma^2}=\lambda$, the optimal value of b given $\sigma=0$ is just $\frac{1}{\lambda}\ln\frac{p+c}{c}$. It is easy to check that this solution is the same as obtained under the following deterministic model that assumes the time to completion to be exactly known in its initial formulation,

$$h(x)=p\int_x^{\infty}e^{-\lambda t}dt-c\int_0^x e^{-\lambda t}dt=\frac{p+c}{\lambda}e^{-\lambda x}-\frac{c}{\lambda},$$

where x in the function denotes the *known* time to completion. The solution to $h(x)=0$ is exactly the same as the value of b that solves equation (A2) when $\sigma=0$.

Variation of $h(\bar{b},x)$ in p and x

Substituting the optimal control threshold $\bar{b}$ back into the function given in (3), we can write the maximized expected payoff function as:

$$h(\bar{b},x)=\frac{(p+c)\left[e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}-e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right]+c\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}}{\lambda\left(1-e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right)}-\frac{c}{\lambda}. \tag{A6}$$

By the Envelope theorem, the rate of the change in $h(\bar{b},x)$ with respect to a change in p is just

$$\frac{dh(\bar{b},x)}{dp}=\frac{e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}-e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}}{\lambda\left(1-e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right)}=\frac{e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}}{\lambda e^{\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-1\right)}. \tag{A7}$$

We have $dh(\bar{b},x)/dp>0$ because $\bar{b}>x$ for any project that is not already rejected or abandoned. This result is intuitive as it says that the expected payoff must rise with the net operating revenue from the completed project.

By the same token, the rate of the change in $h(\bar{b},x)$ with respect to a change in x is

$$\frac{dh(\bar{b},x)}{dx}=\frac{(p+c)\left[-\frac{\sqrt{\Delta}-1}{\sigma^2}e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}-\frac{\sqrt{\Delta}+1}{\sigma^2}e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right]+c\left[\frac{\sqrt{\Delta}+1}{\sigma^2}e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}+\frac{\sqrt{\Delta}-1}{\sigma^2}e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]e^{-\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}}{\lambda\left(1-e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right)}$$

$$=-\frac{c(\sqrt{\Delta}+1)\left\{\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\left[\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-1\right]+\left[\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right]e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right\}}{\lambda\sigma^2 e^{\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}\left(1-e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}\right)}. \tag{A8}$$

The sign of $dh(\bar{b},x)/dx$ depends on the sign of the elements inside the braces in (A8). These elements can be simplified as:

$$\gamma=\frac{(p+c)(\sqrt{\Delta}-1)}{c(\sqrt{\Delta}+1)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}+\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}$$

$$=\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}+e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}+\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}e^{-\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}$$

$$=\left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\right]+\frac{p+c}{c}e^{-\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right)\left[\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}-1\right]. \tag{A9}$$

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But by the first-order condition in (A2), the elements in the first brackets sum to zero. So we have:

$$\frac{dh(\bar{b},x)}{dx} = -\frac{(p+c)(\sqrt{\Delta}+1)\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right)\left[\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} - 1\right]}{\lambda\sigma^2 e^{\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}} - 1\right)}, \qquad \text{(A10)}$$

since $e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}} = e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}$. We can see that $dh(\bar{b},x)/dx < 0$ because $\left[c/(p+c)\right]e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} > 1$ by the second-order condition expressed in (A5). This result is also intuitive as it says that the longer the project's time to completion the lower its expected payoff.

The Constraint Derivative of *h*(*b*,*x*) with Respect to *x*

This part of the Appendix accomplishes two related tasks. First, it obtains the derivative of $h(b,x)$ with respect to x under two constraints: $dh(\bar{b},x)/dx = 0$ and $d(b-\bar{b})/dx = 0$. Second, it proves that sign of this derivative is negative for $b \in (x,\bar{b})$ or $b \in (\bar{b},\infty)$.

Based on the results obtained in the preceding part of the Appendix, we know that the maximized payoff is a decreasing function of the time to completion and an increasing function of the rate of income after completion, that is, $dh(\bar{b},x)/dx < 0$ and $dh(\bar{b},x)/dp > 0$. So, in order to satisfy the constraint that the maximized value of the function is held constant as the value of x changes, the value of another parameter such as p must be adjusted correspondingly. In addition, the results obtained in the first part of the Appendix show that a change in any of the parameters will alter the optimal control threshold $\bar{b}$. So, in order to satisfy the second constraint that the deviation of the adopted threshold from the optimum remains constant as the value of x changes, the value of the adopted threshold $b = \bar{b} + \delta$ also needs to be adjusted correspondingly. Hence, this constrained derivative of $h(b,x)$ can be partitioned into three parts,

$$\left.\frac{dh(b,x)}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0,\ \frac{d(b-\bar{b})}{dx}=0} = \frac{\partial h(b,x)}{\partial x} + \frac{dh(b,x)}{dp}\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0} + \frac{dh(b,x)}{db}\left.\frac{db}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0,\ \frac{d(b-\bar{b})}{dx}=0} \qquad \text{(A11)}$$

where the first term represents the direct effect due to the change in x, the second term represents the indirect effect due to the adjustment in p and the third term represents the indirect effect due to the adjustment in *b*. The direct effect represented by the first term is treated as a partial derivative because now both *p* and *b* are also functions of *x*. By (A8) and (A9), we know:

$$\frac{\partial h(b,x)}{\partial x} = -\frac{(p+c)(\sqrt{\Delta}+1)}{\lambda\sigma^2} e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1 \right)^{-1} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}} \right) \left[\frac{c}{p+c} e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} - 1 \right]$$
$$- \frac{c(\sqrt{\Delta}+1)}{\lambda\sigma^2} e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1 \right)^{-1} \left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} - e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} \right]. \quad \text{(A12)}$$

The amount of adjustment in *p* necessary to satisfy the constraint that value of $h(\bar{b},x)$ remains unchanged as the value of *x* changes is defined by the following equation,

$$\frac{\partial h(\bar{b},x)}{\partial x} + \frac{dh(\bar{b},x)}{dp}\frac{dp}{dx} = 0,$$

and is thus given by:

$$\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0} = -\frac{dh(\bar{b},x)}{dp} \Big/ \frac{\partial h(\bar{b},x)}{\partial x} > 0, \quad \text{(A13)}$$

based on (A7) and (A10). Substituting into (A13) the expressions for $\partial h(\bar{b},x)/\partial x$ and $dh(\bar{b},x)/dp$ from (A7) and (A10), respectively, we have:

$$\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0} = \frac{(p+c)(\sqrt{\Delta}+1)\left[\frac{c}{p+c} e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} - 1 \right]}{\sigma^2} > 0. \quad \text{(A14)}$$

The amount of adjustment in *b* necessary to satisfy the constraint that the deviation of *b* from the optimum $\bar{b}$ remains constant as the value of *x* changes should be exactly the same as the change in $\bar{b}$ resulting from the adjustment in *p* defined in (A14), that is,

$$\left.\frac{db}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0,\ \frac{d(b-\bar{b})}{dx}=0} = \frac{d\bar{b}}{dp}\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0}.$$

Based on the Envelope theorem, we know $\frac{d\bar{b}}{dp} = -\frac{d^2h(\bar{b},x)}{dbdp} \Big/ \frac{d^2h(\bar{b},x)}{db^2}$. The expression for $d^2h(\bar{b},x)/db^2$ has already been derived in (A4). The expression for $d^2h(\bar{b},x)/dbdp$ can be found by differentiating (A3) with respect to *p* at $b=\bar{b}$. We have:

$$\frac{d^2h(\bar{b},x)}{dbdp}=\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-1\right)^{-2}\frac{c(\sqrt{\Delta}+1)e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}}{\lambda\sigma^2}\frac{2\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}$$

$$=\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\left(e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-1\right)^{-2}\frac{2\sqrt{\Delta}e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}}{\lambda\sigma^2}.$$

Using the above result and the result given in (A4), we get:

$$\frac{d\bar{b}}{dp}=\frac{e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}}{\frac{c(\sqrt{\Delta}+1)e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}}{\sigma^2}\left[e^{\frac{2\bar{b}\sqrt{\Delta}}{\sigma^2}}-\frac{p+c}{c}e^{\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}\right]}=\frac{\sigma^2}{(p+c)(\sqrt{\Delta}+1)\left[\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}-1\right]}.$$

Interestingly, $\frac{d\bar{b}}{dp}$ turns out to be precisely the reciprocal of $\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0}$. Hence, we have:

$$\left.\frac{db}{dx}\right|_{\substack{\frac{dh(\bar{b},x)}{dx}=0,\\ \frac{d(b-\bar{b})}{dx}=0}}=\frac{d\bar{b}}{dp}\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0}=1, \tag{A15}$$

which means that a shift in x will cause an equidistant shift in $\bar{b}$ as well as in $b=\bar{b}+\delta$.

The results shown in (A14) and (A15), together with results obtained earlier in (A3) and (A7), enable us to find the exact expressions for the second and third terms in (A13). Using (A7) and (A14), we get:

$$\frac{dh(b,x)}{dp}\left.\frac{dp}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0}=\frac{(p+c)(\sqrt{\Delta}+1)}{\lambda\sigma^2}e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-1\right)^{-1}\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right)\left[\frac{c}{p+c}e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}}-1\right]. \tag{A16}$$

Using (A3) and (A15), we get:

$$\frac{dh(b,x)}{db}\left.\frac{db}{dx}\right|_{\substack{\frac{dh(\bar{b},x)}{dx}=0,\\ \frac{d(b-\bar{b})}{dx}=0}}=\left[e^{\frac{x(\sqrt{\Delta}+1)}{\sigma^2}}-e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}}\right]\frac{c(\sqrt{\Delta}+1)e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}}{\lambda\sigma^2\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-1\right)^2}\left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)}e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}}-e^{\frac{2b\sqrt{\Delta}}{\sigma^2}}-\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\right]. \tag{A17}$$

Note that the sum of (A16) and the first term in (A12) can be written as:

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$$\nu = -\frac{c(\sqrt{\Delta}+1)}{\lambda\sigma^2} e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right)^{-2} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right) e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right) \left[1 - e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}\right]$$

$$= -\frac{c(\sqrt{\Delta}+1) e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right) e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} \left[e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} + e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} - 1\right]}{\lambda\sigma^2 \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right)^2}. \quad \text{(A18)}$$

And the sum of (A17) and the second term in (A12) can be written as:

$$\rho = -\frac{c(\sqrt{\Delta}+1) e^{-\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right) e^{\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} \left[\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} - e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\right]}{\lambda\sigma^2 \left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right)^2}. \quad \text{(A19)}$$

The only difference between (A18) and (A19) lies in bracketed elements in their numerators. The sum of the bracketed elements from (A18) and (A19) is:

$$\xi = \frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} - e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} + e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} - 1. \quad \text{(A20)}$$

By the first-order condition from (A2), we have $\frac{2(p+c)\sqrt{\Delta}}{c(\sqrt{\Delta}+1)} = e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} + \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} e^{-\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}}$. So

$$\xi = e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} + \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} e^{-\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} - e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} + e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} - 1$$

$$= \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} e^{\frac{b(\sqrt{\Delta}+1)}{\sigma^2}} e^{-\frac{\bar{b}(\sqrt{\Delta}+1)}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1} + e^{\frac{\bar{b}(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}} - 1$$

$$= \frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\left[e^{\frac{(b-\bar{b})(\sqrt{\Delta}+1)}{\sigma^2}} - 1\right] + \left[e^{-\frac{(b-\bar{b})(\sqrt{\Delta}-1)}{\sigma^2}} - 1\right].$$

It can be seen that the two terms are both zero for $b = \bar{b}$ but have opposite signs for $b \neq \bar{b}$: when $b < \bar{b}$, the first term is negative and the second term is positive; and when $b > \bar{b}$, the first term is positive and the second term is negative. We can also see that the first term is an increasing function of *b* and the second term is a decreasing function of *b*. So the sign of ξ at $b \neq \bar{b}$ depends on which term dominates. We can ascertain this by differentiating ξ with respect to *b*.

$$\frac{d\xi}{db} = \frac{\sqrt{\Delta}-1}{\sigma^2} e^{\frac{(b-\bar{b})(\sqrt{\Delta}+1)}{\sigma^2}} - \frac{\sqrt{\Delta}-1}{\sigma^2} e^{-\frac{(b-\bar{b})(\sqrt{\Delta}-1)}{\sigma^2}} \begin{cases} < 0 \text{ for } b < \bar{b} \\ > 0 \text{ for } b > \bar{b} \end{cases} \tag{A21}$$

The first inequality in (A21) shows that ξ is dominated by the second term (which is positive for $b < \bar{b}$) when $b < \bar{b}$. The second inequality in (A21) shows that ξ is dominated by the first term (which is positive for $b > \bar{b}$) when $b > \bar{b}$. Thus, based on (A11) through (A21), we can conclude that, for $b \in (x, \bar{b})$ or $b \in (\bar{b}, \infty)$,

$$\left.\frac{dh(b,x)}{dx}\right|_{\frac{dh(\bar{b},x)}{dx}=0,\ \frac{d(b-\bar{b})}{dx}=0} = -\frac{c(\sqrt{\Delta}+1)\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - e^{\frac{2x\sqrt{\Delta}}{\sigma^2}}\right)\left\{\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}+1}\left[e^{\frac{(b-\bar{b})(\sqrt{\Delta}+1)}{\sigma^2}} - 1\right] + \left[e^{-\frac{(b-\bar{b})(\sqrt{\Delta}-1)}{\sigma^2}} - 1\right]\right\}}{\lambda\sigma^2 e^{\frac{x(\sqrt{\Delta}-1)}{\sigma^2}} e^{-\frac{b(\sqrt{\Delta}-1)}{\sigma^2}}\left(e^{\frac{2b\sqrt{\Delta}}{\sigma^2}} - 1\right)^2} < 0 .$$

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Table 1: Effects of changes in σ and λ on the optimal value of b.

		λ			
	σ^2	4%	15%	25%	50%
$\frac{p+c}{c}=5$	100.0	100.6857	46.7095	35.2793	24.3272
	10.0	51.0954	18.9695	13.5198	8.7951
	5.0	46.1210	15.5941	10.7614	6.7599
	1.0	41.5109	11.9342	7.5894	4.2678
	.1	40.3661	10.8589	6.5663	3.3457
$\frac{p+c}{c}=2$	100.0	53.9586	25.8981	19.7208	13.7093
	10.0	24.2687	9.7408	7.1139	4.7484
	5.0	21.1152	7.6959	5.4720	3.5569
	1.0	18.1543	5.3974	3.5123	2.05445
	.1	17.4131	4.7048	2.8559	1.4684
$\frac{p+c}{c}=1.5$	100.0	38.0616	18.5947	14.2127	9.9240
	10.0	15.7929	6.7060	4.9678	3.3627
	5.0	13.2601	5.1619	3.7444	2.4839
	1.0	10.8212	3.3449	2.2304	1.3521
	.1	10.2067	2.7727	1.6910	.8790
$\frac{p+c}{c}=1.01$	100.0	5.0803	2.6022	2.0117	1.4197
	10.0	1.6675	.8386	.6455	.4536
	5.0	1.2067	.5999	.4605	.3228
	1.0	.5984	.2822	.2141	.1483
	.1	.2990	.1111	.0798	.0524

Figure 1: Project value as a function of control threshold.

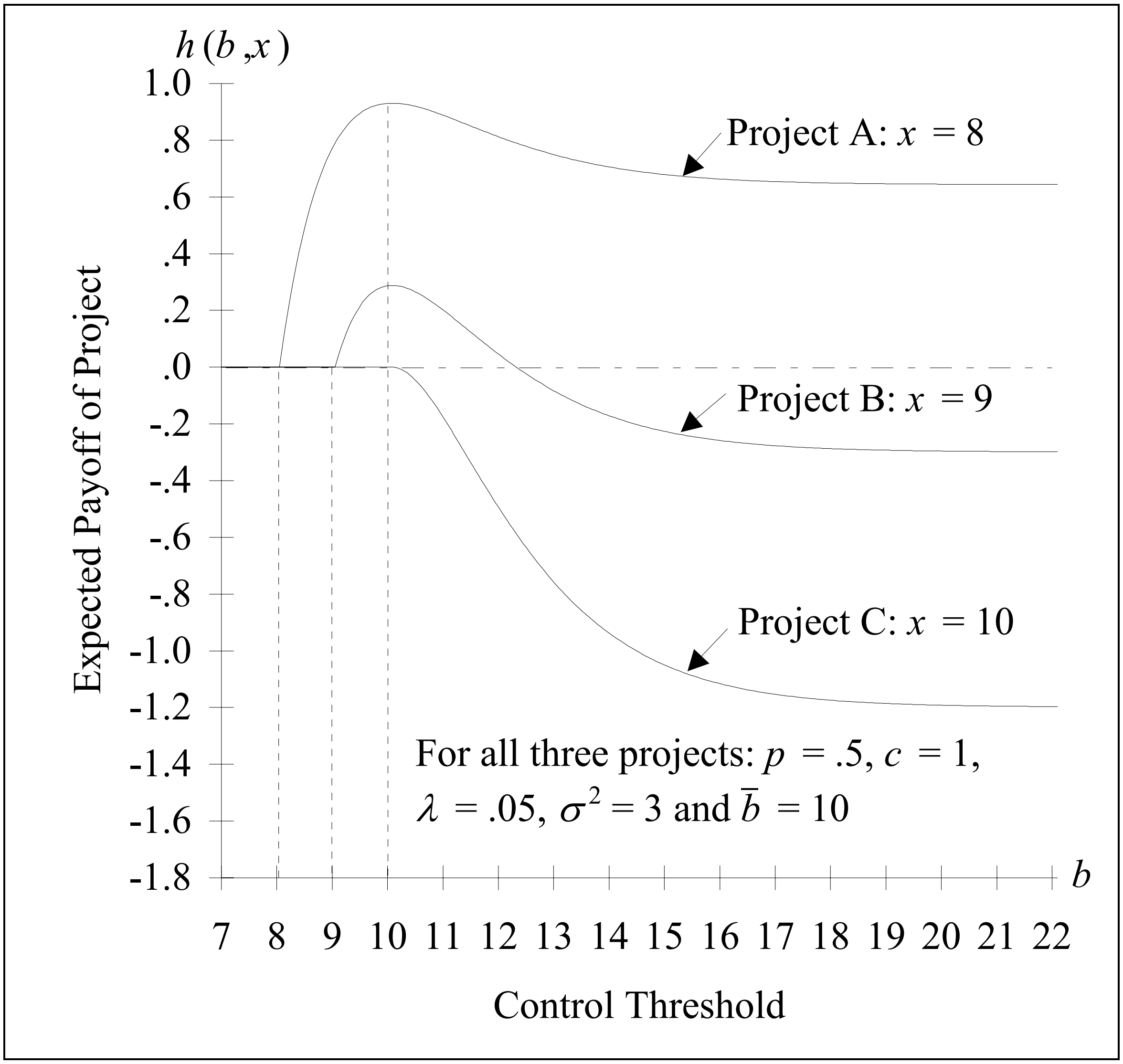
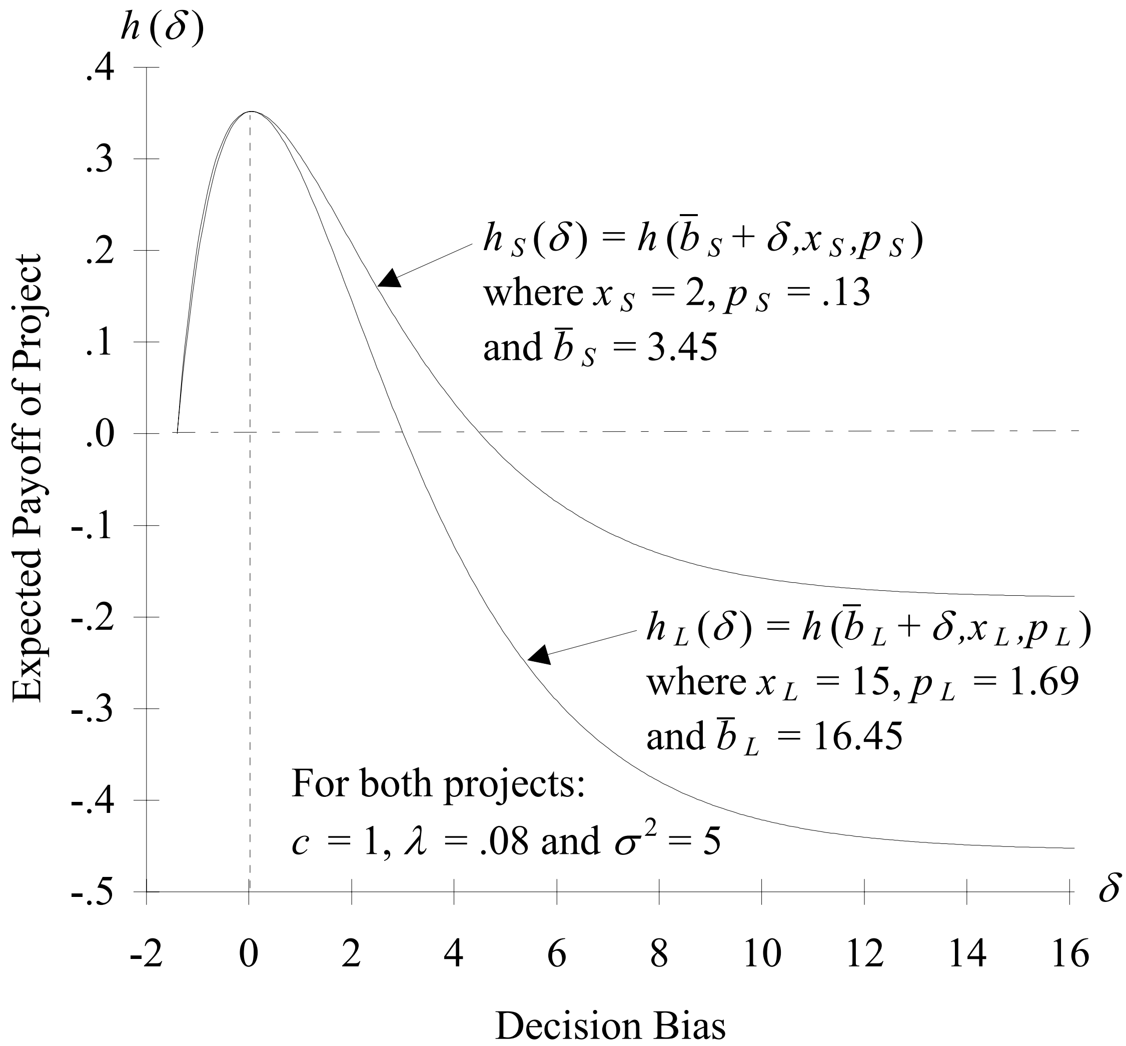


Figure 2: Impacts of a decision bias on projects of differing time to completion.



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