NONLINEAR ANALYSIS AND DYNAMIC STRUCTURE IN THE ENERGY MARKET

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Nonlinear Analysis and Dynamic Structure in the Energy Market

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Abstract

This research assesses the dynamic structure of the energy sector of the aggregate economy in the context of nonlinear mechanisms. Earlier studies have focused mainly on the price of the energy products when detecting nonlinearities in time series data of the energy market, and there is little mention of the production side of the market. Moreover, there is a lack of exploration about the implication of high dimensionality and time aggregation when analyzing the market's fundamentals. This research will address these gaps by including the quantity side of the market in addition to the price and by systematically incorporating various frequencies for sample sizes in three essays. The goal of this research is to provide an inclusive and exhaustive examination of the dynamics in the energy markets.

The first essay begins with the application of statistical techniques, and it incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. It utilizes the daily spot price observations on five major products in the energy market. The results suggest that the time series daily spot prices of the energy products are highly nonlinear in their nature. They demonstrate apparent evidence of general nonlinear serial dependence in each individual

series, as well as nonlinearity in the first, second, and third moments of the series.

The second essay examines the underlying mechanism of crude oil production and identifies the nonlinear structure of the production market by utilizing various monthly time series observations of crude oil production: the U.S. field, Organization of the Petroleum Exporting Countries (OPEC), non-OPEC, and the world production of crude oil. The finding implies that the time series data of the U.S. field, OPEC, and the world production of crude oil exhibit deep nonlinearity in their structure and are generated by nonlinear mechanisms. However, the dynamics of the non-OPEC production time series data does not reveal signs of nonlinearity.

The third essay explores nonlinear structure in the case of high dimensionality of the observations, different frequencies of sample sizes, and division of the samples into sub-samples. It systematically examines the robustness of the inference methods at various levels of time aggregation by employing daily spot prices on crude oil for 26 years as well as monthly spot price index on crude oil for 41 years. The daily and monthly samples are divided into sub-samples as well. All the tests detect strong evidence of nonlinear structure in the daily spot price of crude oil; whereas in monthly observations the evidence of nonlinear dependence is less dramatic, indicating that the nonlinear serial dependence will not be as intense when the time aggregation increase in time series observations.

Dedication

This dissertation is dedicated to my beloved Ebrahim and Fereshteh

for their unconditional love and support and for being on my side throughout the years.

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Chapter 1

Introduction

Background Study. The energy sector has always had a substantial role in the aggregate economy. The energy market, particularly the petroleum price and production have significantly been influenced by exogenous shocks, such as geopolitical events, and their fluctuations have frequently impacted the global economy. Petroleum prices have been remarkably unstable over the last few years, escalating to a high of $145 in July 2008 and a declining to $30 in December 2008 as a result of global recession, reported by the Energy Information Administration (EIA) in the International Energy Outlook 2011. Moreover, during the recent political unrest in the Middle East and the majority of oil-supplying countries, the price of petroleum accelerated to nearly $113 per barrel per day in May 2011 after being relatively stable at around $80 per barrel per day since the 2008 global recession. Due to the important role of the energy market in the aggregate economy fluctuations, the correlation between oil shocks and important economic variables, such as aggregate output and employment, has been a long debated subject in economic literature. Hamilton (1983, 2003); Rotemberg and Woodford (1996) among many other seminal studies have discovered negative correlation between energy disruption and aggregate measure of main the economic variables.

However, to better examine the correlation between the energy sector and the economy, it is crucial to find an appropriate specification that fits the data generating mechanism more closely. As explained by Brockett, Hinich, and Patterson (1988), due to the complication of linear and quadratic coefficients in the estimation of time series models, it is necessary to check the nonlinear structure in the observed time series and to determine which time series are not compliant to linear time series modeling.

Research Objectives. My research assesses the dynamic structure of the energy sector of the aggregate economy in the context of nonlinear mechanisms. Previous studies in the literature mainly focused on the price of the energy market and neglected the production of crude oil, which is the variable that responds to the price. Furthermore, the existing literature has mainly focused on the daily prices of the energy products when detecting nonlinear mechanism in the market's fundamentals. Moreover, there is a lack of examination about utilizing different time aggregations and implication of high dimensionality in analyzing the data generating mechanism in the energy market. My research will address these gaps in three essays by considering the quantity side of the energy market in addition to the price, systematically incorporating various sample sizes with different frequencies, and by using high dimensional observations.

Essay One: Nonlinear Structure in Energy Products. The first essay begins with application of statistical techniques and incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. The first essay is to examine the data generation mechanisms of the five main energy products. It utilizes the daily spot price observations between January 1995 to August 2011 on crude oils, West Texas Intermediate (WTI–Cushing) and Europe Brent, New Harpor heating oil, and New York conventional gasoline regular. Moreover, daily spot prices on Henry Hub Gulf Coast natural gas from January 1997 is used for the analysis. None of the utilized univariate tests have exactly the same null hypothesis and they focus on different aspects of nonlinearity. The findings of the inference methods reveal that there is strong evidence of nonlinear structure in the time series data, indicating that each individual series exhibits general nonlinear serial dependence as well as nonlinearity in the mean, variance, and skewness functions.

Essay Two: Nonlinear Dynamics in Crude Oil Production. The second essay focuses on the dynamic properties of the crude oil production and is motivated by the neglected quantity side of the energy market. There is extensive literature on modeling the crude oil production prediction. The production of crude oil is one of the main variables that impacts the aggregate output fluctuations and has significant influence on different sectors. The aggregate economy often performs weekly after a major disruption in crude oil supply that corresponds to the increase in the price of oil. Hence, it is essential to study the dynamics of the production of crude oil to better explore the production market's fundamentals. The study will allow us to attain more plausible empirical and forecasting results by a model specification that is more close to the data generating mechanisms. The essay also examines the crude oil production time series data of the major oil producing parties by employing statistical methods and econometrics techniques. The employed techniques focus on different aspects of nonlinearity and reveal different forms of nonlinear structure in the data generating mechanism. The essay utilizes monthly observations on the U.S. field production of crude oil from January 1973 to June 2011 as well as Organization of the Petroleum Exporting Countries (OPEC), non-OPEC, and the world production of crude oil. The sample period of the last three time series observations is from January 1973 to January 2012. The tests reveal significant signs of nonlinearity in all the time series observations, excluding non-OPEC production of crude oil. The results of the underlying mechanism for non-OPEC production can be attributed to the steady growth rate of the petroleum production for those countries, which indicates that the supply of crude oil has not been significantly disrupted by exogenous shocks such as geopolitical events. The OPEC production of crude oil, however, has frequently experienced disruptions of crude oil production and clear indications of nonlinear structure is reflected in the OPEC production time series observations. The method to assess the nonlinear dynamics of crude oil production is a new approach in uncovering the supply side of the energy market.

Essay Three: Time Aggregation in Nonlinear Analysis. The third essay examines the dynamic structure of the daily prices of crude oil with different frequencies, higher dimensional cases, and by dividing the entire sample size into various sub-periods. The chapter addresses the gap in the literature for a thorough investigation on various time aggregation levels in the petroleum price market and will identify at which time frequencies the nonlinear dependence cannot be detected. To this end, the essay utilizes daily spot price of crude oil, West Texas Intermediate (WTI), from January 2, 1986 to April 30, 2012 consisting of 6642 observations. The period of time analyzed is divided into three sub-periods: January 2, 1986 to December 30, 1993 consisting of 2039 observations, January 3, 1994 to December 31, 2003 consisting of 2511 observations, and January 5, 2004 to April 30, 2012 consisting of 2092 observations. Moreover, the monthly time series observations on the real price value of crude oil (WTI) is utilized. The sample period of study is from January 1970 to March 2011 for a total of 494 observations and is divided into two sub-samples: January 1970 to December 1991 for a total of 263 observations and January 1992 to March 2011 for a total of 231 observations. Incorporating monthly observations to assess the existence of nonlinear structures in the time series data generating mechanism of crude oil, when the time between observations increases, distinguishes the approach of this chapter from the existing studies in the literature. To carry out the analysis, the most widely univariate tests to detect nonlinearity are employed. These test will explore different attributes of nonlinear serial dependence and focus on distinct aspects of nonlinearity. Hence, using the tests jointly can provide a better understanding of the nature of the nonlinearity that may exist in the data generating mechanism. The findings of the daily spot price of crude oil indicate strong evidence of nonlinear structure in the data generating mechanism, whereas the signs of nonlinear dependence in monthly observations is less significant. The chapter concludes that the volume of nonlinear dependence differs by using various levels of time aggregation on daily spot price of crude oil, and it declines by increasing the time between occurrences. The findings are consistent with the results of the study by Patterson and Ashley (2000b).

Dissertation Organization. This dissertation is organized into five chapters, including the introduction. Chapter two assesses the nonlinear dynamic in time series of various energy products. Chapter three analyzes the nonlinear dynamic structure in the crude oil production market. Chapter four examines the role of time aggregation and high dimensionality in dynamic structure of crude oil. The final chapter provides a summary of findings and discusses their economic applications.

Chapter 2

Nonlinear Structure in Time Series of the Energy Products

2.1 Introduction

The energy sector, in particular the petroleum market, has played a key role in the aggregate economy. Historically, this sector has been influenced by political disturbances. Over the last four decades, the price of petroleum has dramatically increased in response to a series of major events. For instance, during the recent political unrest in the Middle East, the price of petroleum accelerated to nearly $113 per barrel per day in May 2011 after being relatively stable at around $80 per barrel per day since the 2008 credit crisis. As a result of these various shocks, a large number of studies have centered their attention on the correlation between the energy sector disruption and the aggregate economic activity, such as Hamilton (1983, 2003); Rotemberg and Woodford (1996) among many others.

However, to attain a more precise relation between the energy sector and the economy, it is crucial to employ appropriate specifications, which are reasonably close to the data generating mechanism, and to examine whether the time series observations in the market are generated by a linear process or a nonlinear dynamic mechanism. As illustrated by Brockett et al. (1988), given the nature of confounding linear and quadratic coefficients in the estimation of time series models, it is important to detect nonlinear structure in the observed time series and to determine which time series are not compliant to linear time series modeling. If the nonlinearity is present in the data, choosing a nonlinear time series can provide more plausible post sample forecasting ability (Ashley and Patterson (2006)). Furthermore, investigating the sector's data generating process helps to resolve whether or not the market's fluctuations are exogenous, as noted by Kyrtsou, Malliaris, and Serletis (2009).

Therefore, in view of the importance of the energy sector in the aggregate economy, it is vital to reveal the nature of the time series data generating mechanism of the prices in the energy market and to assess the dynamic structure of the energy sector, which is the goal of this chapter.

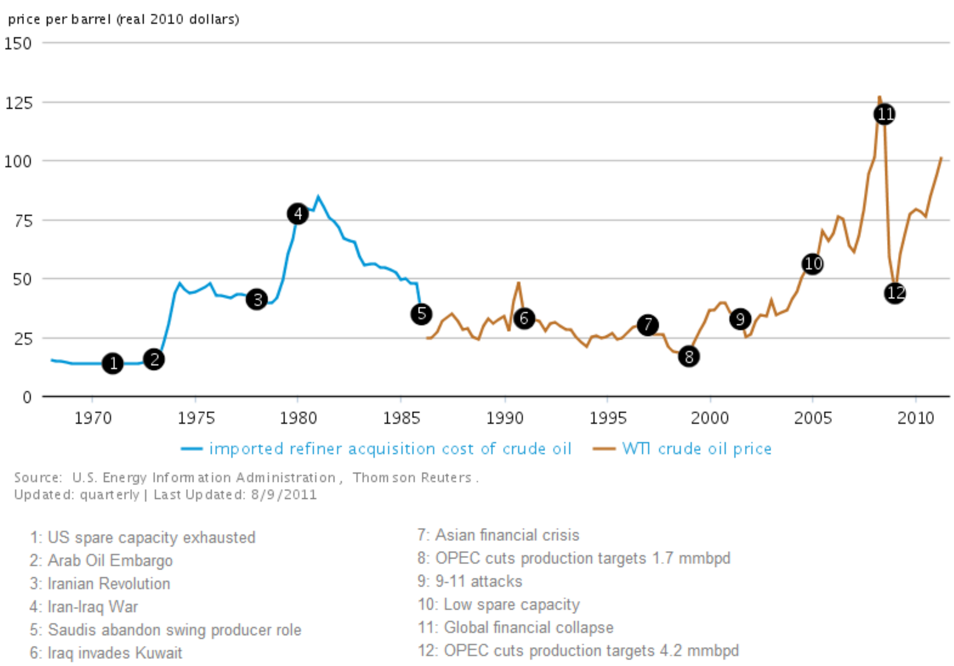
This chapter will uncover the daily data generating mechanism of observed time series of the five main energy products and assess the existence of a nonlinear structure in the market's fundamentals by employing statistical methods and econometrics techniques. This study incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. It utilizes daily spot price observations between January 1995 to August 2011 on five major products in the energy market – crude oil (West Texas Intermediate (WTI) and Europe Brent), heating oil, gasoline and natural gas. This chapter is organized as follows: the next two sections discuss the role of the energy market in the global economy; Section Three reviews the related literature; Section Four describes the data and related different unit root analysis; Section Five discusses the inference methods as well as the results of performing the nonlinearity tests to examine the markets' data generating mechanism; a brief summary and conclusion for this chapter are offered in Section Six.

2.2 Energy Products: Price and Consumption

Energy Products Price

In the early 1970s to early 1980s, the price of oil increased considerably in response to the major conflicts in the Middle East, which reduced the world supply of oil dramatically. The first fall in supply in that decade was experienced in late 1973 as a result of tightening the oil embargo by the Organization of the Petroleum Exporting Countries (OPEC). Oil production was cut by five million barrels per day and the price of oil increased 400 percent in six months (Sill (2007)). Crude oil price reaction to a variety of global geopolitical events is shown in Figure 2.1. The next dramatic increase in oil price occurred as a result of the Iranian Revolution, which began in late 1978 and resulted in a drop of 3.9 million barrels per day of Iran's crude oil production until 1981. In 1980, the Iran-Iraq war began and by 1981 OPEC production declined by seven million barrels per day from its level in 1978. The world oil price jumped from $14 per barrel in 1979 to more than $35 in 1981. The subsequent event was the Persian Gulf Crisis in 1990, when Iraq invaded Kuwait and resulted in another sudden increase in crude oil price. The price of crude oil, which was relatively stable, escalated from $16 per barrels per day in July to more than $36 per barrel per day in September 1990.

Figure 2.1: Crude Oil Prices React to Variety of Geopolitical and Economic Events



Source: U.S. Energy Information Administration (2012), Thomson Reuters. Crude Oil Prices React to a Variety of Geopolitical and Economic Events. *What Drives Crude Oil Prices.* Retrieved from: `http://www.eia.gov/finance/markets/spot-prices.cfm`

After 1990, world oil demand had a dramatic increase during the global recovery period of 2003–2007 until the global financial collapse in 2008, when the oil price escalated to $134 per barrel per day in July 2008. Once again, the energy market encountered another dramatic increase in oil prices as a result of unrest in the Middle East in 2011.

The WTI spot price accelerated to nearly $120 per barrel per day in April 2011. Those rises and falls in the energy market and the oil price shocks have influenced U.S. economy through different channels. As Hamilton (1983) has noted in his paper, seven out of eight postwar U.S. recessions were proceeded by a significant increase in price of petroleum. In another paper, Hamilton (2011) states that the count as of 2011 stands at ten out of eleven. High oil prices and energy supply disruptions may lead to economic downturns due to the variations in the business cycle because of the supply shocks. Moreover, oil price shocks may also influence the aggregate economic activity through monetary policies. If a rise in oil price is related to general price inflation, monetary authorities may adopt restrictive monetary policies, which could slow the economy's growth.[1] Bernanke, Gertler, and Watson (1997) argue that the effect of oil price shocks on the economy results in changes in monetary policies, i.e., increase in the interest rates, which causes the downturn in economy.

Therefore, historically the energy market has always had a crucial role in the economy and has a substantial impact on different sectors. Hence, it is critical to understand the nature of the energy market and discover the structure of time series of energy products' prices, which is essentially the aim of this section.

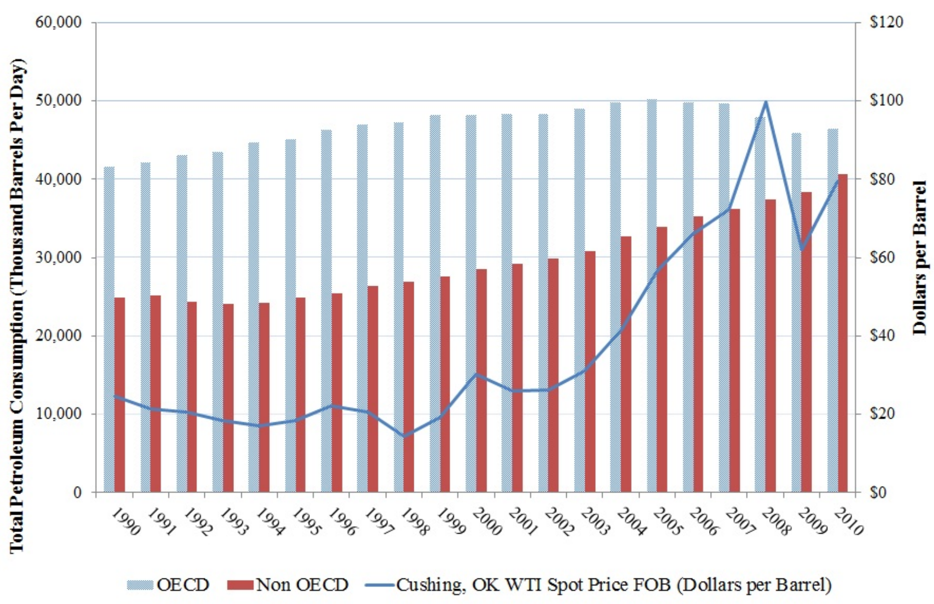
The Petroleum Consumption

Energy components' demand, particularly petroleum, has increased over time. Figure 2.2 describes the changes in total petroleum consumption from 1990 to 2010 of the Organization for Economic Cooperation and Development (OECD) countries, non-OECD

[1] Robert Pirog (2005), CRS report for congress.

countries and also the WTI price levels. It is noticeable that rising oil prices held down the growth of oil consumption growth in OECD countries in 2008 and 2009, in contrast with non-OECD countries. This is partially because of a relatively slower economic growth rate and more efficient transportation sectors, so the impact of the higher prices has been more apparent in OECD countries[2]. However, in 2010, the OECD organization consist of 34 countries, accounts for 53 percent of worldwide oil demand, and 41 percent of this number belongs to the United States. The United States stands as the first ranked consumer of the petroleum and almost all other energy components in the world by reaching nearly 19180 thousands barrels per day for petroleum consumption in 2010.

Figure 2.2: OECD and Non OECD Petroluem Consumption, WTI Crude Oil Price



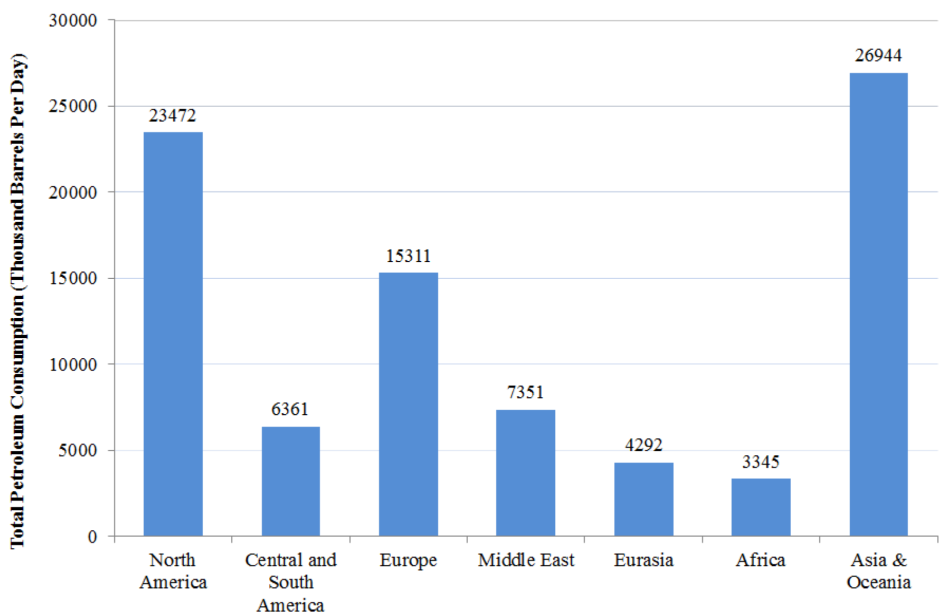
Source: Energy Information Administration (EIA)

Therefore, the industrialized countries consumption of crude oil is significantly more

[2]Oil Market Basics, Energy Information Administration (EIA).

than developing countries, and North America, dominated by the United States, is the second largest consuming area in the world, as it is demonstrated in Figure 2.3. For instance, oil consumption in North America (the United States and Canada) is nearly three gallons per day per capita[3] while in the rest of the OECD countries is equal to 1.4 gallons per day per capita, and outside the OECD the is almost 0.2 gallons per day per capita. [4]

Figure 2.3: Global Petroleum Consumption in 2010 (Thousand Barrels Per Day)

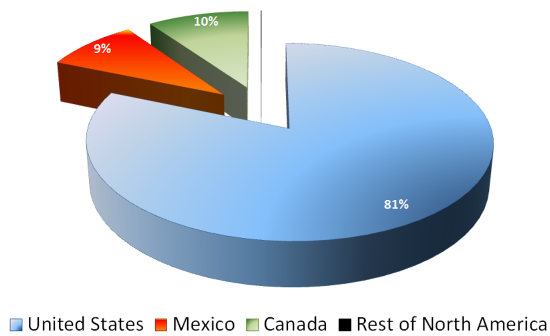


Data Source: Energy Information Administration (EIA)

The United States economy may not be as energy dependent as in the previous decades, but the market for petroleum has been known by strong demand growth over time, in particular in from 2003 to 2007 as a result of global recovery. As also demon-

[3]Oil Market Basics, Energy Information Administration (EIA).
[4]Oil Market Basics, Energy Information Administration (EIA).

Figure 2.4: Percentage of Petroleum Consumption in North America in 2011 (Thousand Barrels Per Day)



Data Source: Energy Information Administration (EIA)

strated by Figure 2.4, the United States has the first spot in consuming the petroleum product among the North American countries.

The escalation of global oil consumption, particularly by China and India, and also the declining the output from oil-producing countries (such as Libya) can potentially result in high prices for petroleum yet again and suppress the global economy.

2.3 Literature Review

There is an extensive body of literature about the energy market and its impacts on economic activities. Also, there are studies that focus on the structure of the energy market, the interaction of the energy market with other markets, and the energy related policies. This section will review the most related literature to the subject of the paper in three sections and will explain how the study contributes to the existing literature.

2.3.1 Energy Market and its Impact on the Aggregate Economy

Hamilton (1983) uses Sim's (1980) six-variable quarterly vector autoregressive (VAR) model and shows that all but one of the U.S. recessions since World War II have been preceded by a dramatic increase in the price of crude petroleum. He discusses the evidence that is presented even over the period of 1948–72, which shows the significant and nonspurious correlation, supporting the fact that oil shocks were a contributing factor in at least some of the U.S. recessions prior to 1972. The same VAR model is employed by Mork (1989) to investigate whether Hamilton's results hold when the sample is extended to include the oil market collapse. The asymmetric response to oil price increase and decrease is under particular investigation in Mork's paper. The results confirm the negative correlation with the oil price increase and the behavior of GNP growth.

In an extensive review, Hamilton (2003) analyzes the existing literature that relates the oil price shocks to economic activity and states that oil price increases are much more important than oil price decreases. Also, increases have significantly less predictive content if they correct earlier decreases. According to Hamilton's findings, the recent increase in the oil price is because of an increase in demand, which differs from past observations.

Oil price fluctuations have also affected the monetary policies. In a seminal study Bernanke et al. (1997) investigate the responses of monetary policy to economic disturbances by focusing on oil price shocks and using VAR approach. They argue that the effect of oil price shocks on the economy results from the monetary policies i.e., increase in the interest rates that responds to oil price shocks and causes the downturn in the economy. Their view has been challenged by Hamilton and Herrera (2004). They state that the monetary policies designed to offset the tightening consequences of oil price shocks are not as influential as stated by Bernanke et al. (1997). Since the oil shocks have more impact on the economy than Bernanke et al. (1997) argue, the feasibility of the monetary policy to offset even a small shock is unpersuaded. Hamilton (2011) reviews some of the literature on the macroeconomic effect of the oil shocks with a particular focus on possible nonlinearities in the relation. He includes both supply and demand shocks and concludes that the relation between GDP growth and oil prices is nonlinear.

2.3.2 Empirical Time Series Analysis of the Energy Market

Serletis (1992) examines the evidence for random walk behavior in the energy future prices by employing the daily observations for crude oil, heating oil, and unleaded gasoline. The findings indicate that the unit root hypothesis can be rejected if the possibilities of a one-time break in the intercept and the slope of the trend function at an unknown point of time are allowed. An extension to Serletis (1992) is another study by Elder and Serletis (2007) that re-examines the empirical evidence for random walk behavior in energy future prices. The paper employs a newly developed semi-parametric estimator called the Walvelet OLS Estimator. Their finding with this new estimator suggests that each energy return series displays unambiguous evidence of long memory, with no evidence of infinite unconditional variance. As they state:

> “The particular form of long memory is anti-persistence, characterized by the variance of each series being dominated by high frequency (low wavelet scale) components.”(*Elder and Serletis (2007)*)

Serletis and Herbert (1999) explore the degree of shared trends across the North America energy market. They test for unit root in univariate time series representations of six natural gas prices as well as power and fuel prices. Based on the augmented Dicky-Fuller (ADF) unit root testing procedure, one of the paper's findings shows that the random-walk hypothesis cannot be rejected for the natural gas and fuel oil prices. The power price series, however, appears to be stationary. Also, Serletis and Rangel-Ruiz (2004) discuss the strength of shared trends and shared cycles between North American natural gas and crude oil markets. Their results show that there has been "decoupling" of the prices of these two sources of energy as a result of oil and gas deregulation in the United States. In other work for analyzing the energy price behavior, Serletis and Kemp (1998) investigate the basic stylized fact of energy price movements. The results are robust compare to alternative measures of the cycle and indicate that crude oil and heating oil prices are synchronous and procyclical whereas unleaded gasoline and natural gas prices are lagging procyclically. Moreover, they find that energy prices are positively and contemporaneously correlated with consumer prices and their cycles lead the cycle of consumer prices, suggesting a possible role for energy prices in the conduct of monetary policy.

2.3.3 Nonlinearities and Chaos in Economic Data

Identifying nonlinearities and chaos in economic data has attracted considerable attention in the literature. Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1995) apply nonlinear tests to detect nonlinear behavior or chaos in various monetary aggregate data series, and discuss the controversy that has arisen about the available results. They use five inference methods to test for nonlinearity and chaos: the Hinich bispectrum test, the BDS test, the Lyapunov exponent estimator of Nychka, the White's test, and the Kaplan test. The findings provide a possible explanation for the controversies that exist regarding empirical evidence of chaos in economic data. They also state that the source of controversies can be found in the lack of robustness of the inference. In another influential study, Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1997) explore the reasons for empirical difficulties with the interpretations of nonlinear and chaos tests' results that have increased over time. They design and run a single-blind controlled competition among the aforementioned five highly regarded tests for nonlinearity or chaos with 10 simulated data series. The results shows that although there are some clear differences among the power functions of the tests, there exists some consistency in their inferences across the method of inference. They also discuss different issues that need to be taken into consideration in interpreting the results. As they state

> "One consideration is the difference in the power functions over alternative, for fixed null. The other consideration is the differences in null hypotheses of each test. The latter consideration produces a degree of noncomparability of the tests and the possibility that some of the tests could be used jointly." (*Barnett et al. (1997)*)

Barnett, Jones, and Nesmith (2004) test the existence of nonlinearity in the cointegration relations of a system containing money demand variables, by applying the Hinich bispectrum test. The findings have some evidence of nonlinearity, and therefore they find that the issue is empirically relevant.The detection of chaos in economic data is also examined by Barnett and Hinich (1993) using Divisa monetary aggregate and applying the Hinich bispectrum test. They produce a strong rejection of linearity with the Divisia M_1 data and state that these data are deeply nonlinear. Kyrtsou and Serletis (2006) discuss univariate tests for independence and hidden nonlinear deterministic structure in economic and financial time series. They apply the tests to Canadian exchange rate, using daily data over a 30-year period and they identify an interesting relationship between high-dimensional nonlinearity and shocks.

Furthermore, interest in studying the behavior of the energy market and applying the existing tests to detect the nonlinearities and chaos in this market has been growing over time. Kyrtsou et al. (2009) discuss number of widely used univariate test from dynamical system theory and apply them to the energy market. They apply these tests to daily observations of the energy market for nearly 15 years. They find indications consistent with nonlinear dependence in each of the markets. They also suggest that an effective nonlinear model of energy prices would produce a deeper perception of the energy market fluctuations than existing linear models. Sertletis and Gogas (1999) test for deterministic chaos in the North American Natural Gas Liquids Market. They use the Lyapunov exponent estimator and they find that there is evidence consistent with a chaotic nonlinear generation process in natural gas liquid markets. Serletis and Andreadis (2004) use daily observations on West Texas Intermediate crude oil prices, Henry Hub natural gas prices, and various tests from dynamical theory to support a random fractal structure for North American energy markets. The result is consistent with the reported result by Serletis and Gogas (1999) as they find evidence of nonlinear chaotic dynamics in North American natural gas liquids markets but not in crude oil and natural gas markets.

As discussed above, there are studies in the literature that investigated the energy market's fundamentals by applying univariate nonlinearity tests to detect the nonlinear structure in the energy market. This chapter will incorporate the Kaplan test, which detects the general nonlinearity, to examine the nonlinear dependence in the energy market more comprehensively. The kaplan test has not been applied to the energy products so far, hence the study will provide more insights about the structure of the market. Moreover, the study uses different sample data series for the prices of the major products in the energy market and adds one main product of the energy makrket, which has not been considered in the past studies.

2.4 Data Description and Unit Root Analysis

This essay uses daily prices data on five energy products obtained from Energy Information Administration (EIA). The descriptions of the employed daily data are as follows:

- Daily spot price on crude oil: West Texas Intermediate (WTI-Cushing)[5] and Europe Brent[6]. The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations for each series.
- Daily spot price on the New York Harbor heating oil[7]. The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations.
- Daily spot price on New York conventional gasoline regular[8]. The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations.

[5]WTI-Cushing: A crude stream produced in Texas and southern Oklahoma, which serves as a reference or "marker" for pricing a number of other crude streams, is traded in the domestic spot market at Cushing, Oklahoma. (Energy Information Administration (EIA))

[6]Brent: A blended crude stream produced in the North Sea region, which serves as a reference or "marker" for pricing a number of other crude streams. (Energy Information Administration (EIA))

[7]The location specified in either spot or futures contracts for delivery of a product in New York Harbor. (Energy Information Administration (EIA))

[8]Finished motor gasoline not included in the oxygenated or reformulated gasoline categories. Excludes

Table 2.1: Summary Statistics of Differenced Log Series

Series	Sample Mean	Sample Median	Standard Deviation	Skewness	Kurtosis	Jarque-Bera (p-value)
WTI (Crude Oil)	0.0001	0.0004	0.0110	-0.1924	7.6120	0.0000
Europe Brent	0.0001	0.0003	0.0103	-0.1111	7.8295	0.0000
Heating Oil	0.0001	0.0000	0.0116	-1.4674	39.0782	0.0000
Gasoline	0.0001	0.0006	0.0122	0.0218	6.6622	0.0000
Natural Gas	0.0000	0.0000	0.0201	0.4861	22.6005	0.0000

- Daily spot price on Henry Hub Gulf Coast natural gas[9]. The sample period of 01/07/1997 to 08/16/2011 consists of 3654 observations for each series.

The descriptive statistics of the first differenced of the log levels for the prices are reported in Table 2.1. Figures 2.5 to Figure 2.9 depict the prices of the variables during the sample periods, and Figure 2.10 to Figure 2.14 demonstrate the log levels and the first differenced log levels for each series in the Appendix A of this chapter.

Before conducting nonlinear and chaos analysis, the first step is to test for stochastic trend (unit root) in each individual series and avoid any possible spurious regression. The study employs three alternative tests for unit root to discover whether or not the series' behavior follow the random walk.

2.4.1 Unit Root Analysis

In order to carry out the nonlinear analysis, the first step is to test whether or not the log price of each individual series follows a random walk or has unit root. This chapter

reformulated gasoline blendstock for oxygenate blending (RBOB) as well as other blendstock. (Energy Information Administration (EIA))

[9]A gaseous mixture of hydrocarbon compounds, the primary one being methane delivered at the Henry Hub in Louisiana. (Energy Information Administration (EIA))

employs three alternative conventional test procedures to deal with the behavior of the data, the Augmented Dickey-Fuller test (ADF), the Philips and Perron test (PP), and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. The first employed test is the augmented Dickey-Fuller (ADF) test to check the existence of a unit root in an $AR(p)$ process, the unit root test is carried out under the null hypothesis $H_o : \beta = 0$ versus the alternative hypothesis $H_a : \beta < 0$ using the regression

$$\Delta y_t = c_t + \beta y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + e_t \tag{2.1}$$

where c_t is a deterministic function of the time index t and $\Delta y_j = y_j - y_{j-1}$ is the differenced series of y_t. The t-ratio of the statistic is computed by

$$ADF - test = \frac{\hat{\beta}}{std(\hat{\beta})} \tag{2.2}$$

where $\hat{\beta}$ denotes the least squares estimates of β, and the t-ratio is known as the *augmented Dickey-Fuller*(ADF) unit root test – see Dickey and Fuller (1981) for details. The error term is assumed to be homoscedastic and the value of p is set such that the error is serially uncorrelated as well.

Furthermore, the Philips and Perron (1988) known as (PP) unit root test is employed to test whether the log level of the series exhibit a random walk behavior. The PP test differs from the ADF test in handling the serial correlation and heteroscedasticity in the errors, and it allows for errors not to be independently and identically distributed (*iid*). The PP unit root test is essentially based on Equation 2.1, but without the

Table 2.2: Augmented Dickey-Fuller Unit Root Tests
Null Hypothesis: The log levels and the differenced log of the series have unit root
Lag length: Automatic Selection Based on SIC.

Log Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-3.183	-2.875	-2.953	-3.502	-2.894
p-value*	0.087	0.170	0.145	0.039	0.164
DLog Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-48.015	-63.855	-35.131	-61.932	-50.560
p-value*	0.000	0.000	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
Notes: The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is from January 07, 1997 to August 16, 2011.

lag differences. While the ADF test correct for the higher-order serial correlation by adding lagged difference terms to the right-hand side, the PP unit root test makes a non-parametric correction to account for residual serial correlation Maslyuk and Smyth (2008). Therefore, the PP test statistic is robust to a variety of serial correlation and time-dependent heteroscedasticity. The test regression for PP test is

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \tag{2.3}$$

where u_t is $I(0)$ and can be heteroscedastic. The PP test corrects for any serial correlations and heteroscedasticity in the error u_t of the test regression by modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$. Under the null hypothesis that $\pi = 0$, the PP statistic has the same asymptotic distribution as the ADF t-statistic and normalized bias statistic – see Philips and Perron (1988) for more details.

The t-statistics for the ADF and PP tests ($t_{(\hat{\beta})} and Z_{t_{(\hat{\pi})}}$) as well as the p-values for

Table 2.3: Philips-Perron Unit Root Test
Null Hypothesis: The log levels and the differenced log of the series have unit root
Bandwidth: (Newey-West automatic) using Bartlett Kernel

Log Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-2.970	-2.905	-3.036	-3.565	-3.094
p-value*	0.140	0.160	0.122	0.032	0.107
DLog Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-65.323	-63.855	-64.445	-61.879	-59.182
p-value*	0.000	0.000	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
Notes: The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is from January 07, 1997 to August 16, 2011.

the log levels of the series are reported in Table 2.2 and Table 2.3. In the specifications of the unit root regressions for the ADF and the PP test in log level of the individual series, the constant term as well as the time trend are included. As the results show in Table 2.2 and Table 2.3, I fail to reject the null hypotheses of a unit root for the ADF and PP tests for each of the variables in log levels at the 1% significant level.

Another test to verify the results of the ADF and the PP tests and to identify the random walk behavior in the series is employed in this chapter as well. The test is introduced by Kwiatkowski, Phillips, Schmidt, and Shin (1992) known as (KPSS) test. The ADF and the PP unit root tests are carried out under the null hypothesis of whether a time series is *I(1)*. The KPSS test, on the other hand, is known as a stationary test and will test the null hypothesis that the series is $I(0)$, that is to say $H_o : y_t \sim I(0)$. The test is conducted under the null hypothesis of either level stationary or trend stationary to investigate whether a series is $I(0)$, $I(1)$ or are not in fact informative about whether they are stationary or follow random walk behavior. Table 2.4 shows the results for the

Table 2.4: Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) Unit Root Test
Null Hypothesis: The log levels and the differenced log of the series are stationary
Bandwidth: (Newey-West automatic) using Bartlett kernel

Log Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
KPSS test statistic (LS)	7.123	7.081	7.024	7.050	3.801
KPSS test statistic (TS)	0.441	0.452	0.453	0.426	1.016
DLog Level	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
KPSS test statistic (LS)	0.038	0.042	0.044	0.026	0.039
KPSS test statistic (TS)	0.036	0.029	0.030	0.020	0.034

Notes: The 1%, 5% and 10% critical values for KPSS test statistics (LS) [given in Kwiatkowski, Phillips, Schmidt, and Shin (1992)] are 0.739, 0.463 and 0.347, respectively.
The 1%, 5% and 10% critical values for KPSS test statistics (TS) [given in Kwiatkowski et al. (1992)] are 0.216, 0.146 and 0.119, respectively.
The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 7, 1997 to August 16, 2011.

KPSS tests in which the first test statistic (LS) has the null hypothesis of level stationary and the second test statistic (TS) tests the null hypothesis of trend stationary. Both t-statistics exceed the 1%, 5% and 10% critical values given in Kwiatkowski et al. (1992). Therefore, I can reject the null hypotheses of the stationarity of the log levels at the 1%, 5% and 10% significant levels. It is to be noted that in all the regressions for unit root tests in log levels, the trend terms have been included to distinguish whether or not the series are "trend stationary" (TS) model, where a stationary component is added to a deterministic trend term.

The decision to deal with the random walk behavior is to transform the log levels into the first differenced of the logs. The ADF and PP unit root test results, after performing them on the first differenced log, indicate that I can reject the null hypotheses of unit root in first differenced levels. Moreover, the null hypotheses of the KPSS test, level and trend stationary, cannot be rejected in the first differenced log levels. Hence, I use the first differenced of the log levels for each individual series throughout the rest of the paper unless otherwise noted.

2.5 The Inference Methods

In this section, the inference methods for detecting nonlinearities in this study will be introduced. The BDS test, the Hinich bicovariance test, the Hinich bispectrum test, the Engle LM test, the McLeod-Li test, and the Tsay test. All the above tests, except Hinich bispectrum test, require to remove any serial dependence from the data via a prewhitening model. Any other serial dependence is the result of a nonlinear data generating mechanism. The Hinich bispectrum test directly tests the data generating mechanism and it is invariant to filtering of the data (Patterson and Ashley (2000a)). Moreover, the Kaplan test is included as one of the inference methods to better capture the dynamic structure of the energy market.

2.5.1 The BDS Test: A Test for Serial Independence

The well known Brock, Dechert, Scheinkman and LeBaron(1996) test, also known as the BDS test, is one form of portmanteau tests for independence. Portmanteau tests are residual-based tests in which the null hypothesis is well stated, but they do not have a specific alternative hypothesis. The BDS test is a popular test to detect the serial independence in time series data. The BDS test introduces a test of independence that can be applied to the estimated residuals of any time series model, if the model can be transformed into a form with independent and identically distributed errors. The test employs the correlation function (correlation integral) to calculate the test statistics. The correlation function was introduced as a method of measuring the fractal dimension of deterministic data. The correlation function (integral) measures of the sequential pattern's frequency that exist in the data – see Brock, Dechert, and Scheinkman (1986) for more details. It is to be mentioned that the correlation function is different than the correlation dimension, which is the method used in testing for chaos introduced by Grassberger and Procaccia (1983). Barnett et al. (1995) state that correlation dimension is potentially helpful in testing for chaos, however modeling for high-dimensional chaos needs a large number of variables. Moreover, the sampling properties as well as the derived distribution of the correlation dimension are unknown. Therefore, the BDS test uses the correlation function as a test statistic Barnett et al. (1995). As they explain:

> "Since the derived distribution of the correlation dimension is unknown, the BDS test uses the correlation function as the test statistic. The asymptotic distribution of the correlation function is known under the null hypothesis of whiteness (independent and identically distributed observations). As a result, the BDS test can be used to produce a formal statistical test of whiteness against general dependence. However, the sampling distribution of the BDS test statistic is not known under the null of chaos. When testing for chaos by this means, we are left with the uncomfortable choice between the correlation dimension, which produces a direct test for chaos, but only when no substantial stochastic shocks exist within the model, or the correlation function, which does have known sampling properties when there are stochastic shocks within the model, but only under a different null hypothesis (i.e. whiteness)." *Barnett et al. (1995)*

The BDS test is used to test the null of linearity against a variety of possible deviation from independence in the series including nonlinearity and chaos. The test is applied to a series of estimated residual after removing any linear structure. Under the null hypothesis of independent and identically distributed ($i.i.d$) or whiteness, the BDS statistic is

$$\sqrt{n}\frac{C_{m,n}(\epsilon) - C_1(\epsilon)^m}{\sigma_m(\epsilon)} \tag{2.4}$$

where $C_{m,n}(\epsilon)$ is the correlation integral, $\sigma_m(\epsilon)$ is the asymptotic standard deviation of the numerator and m is the embedding dimension. The test converges to $N(0,1)$ under the null hypothesis of whiteness – see Brock et al. (1986) for more details.

The BDS test statistic is a transformation of the correlation function, which asymptotically becomes a standard normal Z statistic under the null hypothesis of whiteness (Barnett et al. (1995)). I apply the BDS test to the differenced log of the individual time series of the energy data. The choice of the values of ϵ and m can be challenging in using the BDS test. The results with BDS are reported in Table 2.5 for dimension 2–8 and the chosen ϵ equals to one and two standard deviation of the data[10].

Results with the BDS Test

I produce the BDS test statistic for all the embedding dimension from two to eight, and the inferences are always the same and robust at each embedding dimension. As can be observed in the Table 2.5, the results indicate the significance at the 1%, 5% and 10% significance levels based on the asymptotic distribution. Therefore, the BDS test rejects the null hypothesis of independent and identically distributed observations and detect the nonlinearity in each energy product. The BDS test has high power against a numerous nonlinear alternatives. Therefore, accepting the null hypothesis in BDS test indicates that there are strong evidence for the null. Thus, it is suggested that the BDS test

[10] ϵ is calculated as a multiple of the standard deviation of the series.

should be the first test to run. In the case of this study in which the linearity is rejected with the BDS test, the results reflect little information to distinguish the existing forms of nonlinearity in the data. Hence, I utilize the more focused tests to identify the other possible forms of nonlinearity in the data – see Barnett et al. (1997) for more details.

2.5.2 Kaplan Test: A Test for Continuity and Determinism

There has been a wide range of methods in which reconstruction dynamics of the employed time series have been developed in order to characterize the dynamics in terms of predictability or dynamical invariant Kaplan (1994). These classifications are often employed to characterize whether the time series data are consistent with a deterministic mechanism, or a stochastic mechanism. As Kaplan (1994) mentions, it is common to test the predictability near every point in the time series in the nonlinear prediction method. Even though it might not be possible to predict future values of time series at every point, it may be likely to make accurate predictions at a few points. This may suffice for detecting the underlying determinism. Moreover, when deducing dynamics from a time series, continuity is often the only safe assumption one can make about a possible deterministic mechanism for a time series. Kaplan (1994) proposed a test for determinism in a time series based on consistency with a continuous dynamical mapping. The test answers a question like, "If two points x_i and x_j are very close together, are their images x_{i+1} and x_{j+1} also close together?" (Kaplan (1994))[11]. In other words, deterministic solution paths, unlike stochastic processes, have the property that points that are close together

[11]A test based on continuity in phase space proposed by Daniel Kaplan, Centre for Nonlinear Dynamics, Department of Physiology, McGill University.

Table 2.5: BDS Test Z-Statistic (Dimension 2–8)

Difference Log of Crude Oil- WTI

	ϵ			
m	1σ	p-values	2σ	p-values
2	8.6502	0.000	13.1413	0.000
3	11.2398	0.000	16.9129	0.000
4	12.7804	0.000	18.5466	0.000
5	13.7043	0.000	19.4291	0.000
6	14.9684	0.000	19.9851	0.000
7	16.6185	0.000	20.5845	0.000
8	18.2287	0.000	20.6910	0.000

Difference Log of Crude Oil- Europe Brent

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.2676	0.000	7.5338	0.000
3	8.4860	0.000	10.14615	0.000
4	10.4088	0.000	12.0745	0.000
5	12.2642	0.000	13.2835	0.000
6	14.2073	0.000	14.2686	0.000
7	16.1597	0.000	15.0526	0.000
8	17.9577	0.000	15.6694	0.000

Difference Log of Heating Oil

	ϵ			
m	1σ	p-values	2σ	p-values
2	8.96026	0.000	11.6395	0.000
3	11.7476	0.000	14.6171	0.000
4	13.2784	0.000	15.9821	0.000
5	14.7733	0.000	16.8227	0.000
6	16.6146	0.000	17.7109	0.000
7	16.2660	0.000	18.3812	0.000
8	19.9599	0.000	18.8056	0.000

Differenced Log of Gasoline

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.6834	0.000	9.9913	0.000
3	8.2688	0.000	11.1637	0.000
4	9.7411	0.000	12.4350	0.000
5	10.9942	0.000	13.0850	0.000
6	12.3876	0.000	13.8142	0.000
7	13.7737	0.000	14.4790	0.000
8	15.2485	0.000	15.0990	0.000

Differenced Log of Natural Gas

	ϵ			
m	1σ	p-values	2σ	p-values
2	14.8284	0.000	19.98.61	0.000
3	18.3402	0.000	22.2270	0.000
4	20.6047	0.000	23.3393	0.000
5	22.7769	0.000	24.2032	0.000
6	25.5779	0.000	24.9886	0.000
7	28.3910	0.000	25.5891	0.000
8	31.3578	0.000	26.1315	0.000

are close under their image in phase space. Therefore, when the underlying function linking image and pre-image together is continuous, if the points x_i and x_j are close their images x_{i+1} and x_{j+1} are close together as well. In the case of chaos, the output plot of the system is hardly distinguishable from a stochastic process. Therefore, detecting the continuity of the system can be a difficult procedure, even when the data is entirely deterministic. However, it is easier to detect deterministic structure when plotting the solution path in phase space (x_{t+1} plotted against x_t and lagged values of x_t) than in plotting x_t versus t (Barnett et al. (1995)). Based on the above facts, the Kaplan test has strictly positive lower bound for a stochastic process, but not for a deterministic solution path. The statistic tests the null hypothesis that the data is deterministic against the alternative, which is that the data comes from a particular stochastic process. If the test statistic is smaller for the data than for the stochastic process by a statistically significant amount, then the stochastic process is rejected as an alternative to other forms of nonwhite structure (Barnett et al. (1995)). The test is computed by an adequately large number of linear processes that plausibly might have produced the data. The test procedure involves producing a linear stochastic process surrogate data[12] for the observed data. The next stage is to determine a noisy continuous nonlinear dynamical solution path to better describe the observed data. If the value of the test statistic from the surrogate is not small enough compared to the computed value of the test statistic from the observed data, a noisy continuous dynamical solution is concluded. As described by Barnett et al. (1995), the test procedure is formally stated as follows: If the time series data arise from

[12]Surrogate data is random data generated with the same mean, variance, and autocorrelation function as the original data.

a deterministically chaotic dynamical system, the value of x_{t+1} is a single-valued function of the state of the system at time t. Let the vector $x_t = (x_t, x_{t-1}, ..., x_{t-m-1})$ embedded in m-dimensional "phase space" and obtained from a m-dimensional vector $x_i{}_{i=1}^{T}$ in state space. Then there exists a function $f(x_t)$ such that $f(x_t) = x_{t+1}$, where x_{t+1} is called the "image" of the point x_t in phase space. If the system is perfectly deterministic with a continues f, close points in m-dimensional phase space have close image, whereas in a stochastic system close points in phase space may produce different images. The Kaplan test investigates if the function f is continuous based on the evidence provided the observed time series data. In the similar delta-epsilon proofs of continuity, δ is the distance in phase space and ϵ is the distance of the images. For a given choice of embedding dimension m, the distance in the phase space is calculated as $\delta_{ij} = |x_i - x_j|$ and the distance between their image is calculated as $\epsilon_{ij} = |x_{i+1} - x_{j+1}|$ for all i and j. It is useful to construct the average of the values of ϵ_{ij} conditional on the corresponding values of δ_{ij} satisfying $\delta_{ij} < r$ and define the average as $E(r)$. It is expected to have $E(r) \to 0$ as $r \to 0$ for a perfectly deterministic system with continuous f, whereas if the underlying system is stochastic the convergence may not happen as a point x_i may have different images. The statistic for the Kaplan test is defined as $K \equiv \lim_{r\to 0} E(r)$. The non-zero value of K can be interpreted as "goodness of fit" measure from fitting a continuous model of some fixed order to an infinite amount of data. If this measure is smaller for the observed data than for surrogate data generated by a model that satisfies a stated null hypothesis, then the null hypothesis should be rejected (Barnett et al. (1995)). As stated by Garcia (2007), another way of interpreting the non-zero value of K is as the level of nondeterminism or the amount of noise in the data. If the system is stochastic the amount of K is expected to be higher for nearly deterministic ones. Therefore, we should reject the null hypothesis when K on the observed data is smaller than K on the surrogate data. In other words, the hypothesis of linearity is rejected in order to test if the value of the statistic from the surrogates is never small enough compared to the value of the statistic obtained from the original data. Since the distribution of the statistic table is not laid out, Kaplan proposes two different methods to compute the minimum value of K obtained from the surrogates. The first approach is to estimate the minimum value of K from a finite sample of surrogates, and impute that to the population of the surrogates. Another approach involves the computation of the mean and standard error of the values of K from the finite sample and the subtraction of a multiple of (2 or 3) to obtain the an estimate of population minimum Alharbi (2009). This chapter uses twenty surrogate time series using the same approach suggested by Kaplan. The surrogate data is a random realization from time series data of the energy markets generated with the same mean, variance, and autocorrelation functions as the original data. Moreover, the lag embedded time series is also generated using 2, 3, 4, and 5 dimensional spaces.

The result of the Kaplan test for daily spot prices of five energy products are reported in Table 2.6. Also, the results of the Kaplan test are graphically summarized in the Appendix B of this chapter. The plot of *delta* versus *epsilon* shows the sign of discontinuity is all cases, when *delta* goes to zero, *epsilon* does not.

Results of the Kaplan Test

The null hypothesis of the Kaplan test is stochastic linearity of the process. As mentioned by Barnett et al. (1995), the Kaplan test involves a strong power against chaos and is expected not to accept the null facing with chaotic series although current form of test can either accept or reject linearity. It is worth mentioning that the Kaplan test is designed where the dynamical functional form underlying the time series data is unknown, and the main purpose is to determine if there is evidence of deterministic mechanism or not. The results of the Kaplan test are displayed in Table 2.6 for embedding dimension(m) 2, 3, 4 and 5[13].

The mean, minimum, and standard deviations are computed over twenty surrogates for each time series. Moreover, K statistic is calculated for each series. The null of stochastic linearity is rejected when the computed K for each daily spot price of energy product is less than the minimum of K statistic from surrogates or KS_{min} that is $K < KS_{min}$. As suggested by Kaplan, the t-statistic is calculated as a tool to find the results significance as: $t = \frac{K - KS_{mean}}{KS_{sd}}$, where KS_{mean} and KS_{sd} are the mean and standard deviation for KS values for surrogates.

As displayed in Table 2.6, the test rejects the null of linearity of the daily spot price on crude oil, West Texas Intermediate, and Gasoline in all dimensions at the 1% significance level excluding dimension=2 of WTI. Moreover, the null of linearity is rejected for Heating Oil and Natural Gas in all the embedding dimensions at the 1% significance level

[13]The Kaplan test was carried out using the original MATLAB codes provided with gratitude by Professor Daniel Kaplan and modified based on the analysis in this study: The MATLAB source code for the Kaplan test can be also retrieved from `http://www.macalester.edu/ kaplan/software/` Kaplan, Daniel. (1996). Delta-Epsilon [Computer MATLAB Software]. Retrieved from: http://www.macalester.edu/ kaplan/software/.

Table 2.6: Kaplan Test Statistic: Results from Daily Spot Prices on Five Energy Products

Log Level	**Embedding Dimension**	**Mean K on surrogates**	**Std. dev. of K on surrogates**	**Min K on surrogates**	**K statistic on energy data**	***t*-Statistic**
Crude Oil WTI	2	0.0126	0.0012	0.0102	0.0102	-2
	3	0.0126	0.002	0.0086	0.0092	-1.7
	4	0.0127	0.0011	0.0105	0.0084	-3.90
	5	0.013	0.0022	0.0086	0.0077	-2.40
Brent Europe	2	0.0117	0.0009	0.0098	0.0103	-1.54
	3	0.0114	0.0011	0.0092	0.0094	-1.81
	4	0.0112	0.0016	0.008	0.0086	-1.62
	5	0.0114	0.0024	0.0066	0.008	-1.41
Heating Oil	2	0.0131	0.0015	0.0101	0.01	-2.06
	3	0.0128	0.002	0.0088	0.0089	-1.95
	4	0.013	0.0028	0.0074	0.0086	-1.57
	5	0.0128	0.0019	0.009	0.008	-2.52
Gasoline	2	0.0244	0.0024	0.0196	0.0163	-3.37
	3	0.0234	0.0037	0.016	0.0141	-2.51
	4	0.0219	0.0036	0.0147	0.0121	-2.72
	5	0.0214	0.0051	0.0112	0.0106	-2.11
Natural Gas	2	0.0138	0.0008	0.0121	0.012	-2.21
	3	0.0139	0.0008	0.0121	0.0108	-3.52
	4	0.0135	0.0017	0.0101	0.0102	-1.94
	5	0.0141	0.0018	0.0105	0.0094	-2.61

Notes: K is the Kaplan test statistic. Twenty surrogates were used to compute the mean and standard deviation. The sample period for the daily spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

excluding embedding dimension=4 . It is concluded that the null of linearity is rejected in favor of nonlinearity for the majority of embedding dimensions in all individual series. These results are consistent with the results by the BDS test. However, unexpectedly the null of linearity of the daily sport price on crude oil, Europe Brent cannot be rejected

The Kaplan test detects the evidence of general nonlinearity in observed time series. The chapter proceeds with more focused tests to investigate other possible forms of nonlinearity in the observed time series such as third order nonlinearity.

2.5.3 Tests for Nonlinearity

The Hinich Bicovariance Test

As noted by Patterson and Ashley (2000a) the Hinich Bicovariance test assumes x_t is a realization from a third-order stationary stochastic process and tests for serial independence. It uses the sample bicovariances of the data. The (r, s) sample bicovariance is defined as

$$C_3(r,s) = (N-s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s} \quad 0 \leq r \leq s. \tag{2.5}$$

The sample bicovariances, Equations 2.5, are a generalization of a skewness parameter. The $C_3(r, s)$ are all zero for zero mean, serially $i.i.d$ data.

Non-zero values for the $C_3(r, s)$ are projected from observations in which x_t depends on lagged cross-products, such as $x_{t-i}x_{t-j}$ and higher order terms.

Let $G(r, s) = (N - s)^{0.5} C_3(r, s)$ and define X_3 as

$$X_3 = \sum_{s=2}^{\phi} \sum_{r=1}^{s-1} [G(r,s)]^2 \tag{2.6}$$

Under the null hypothesis that x_t is a serially $i.i.d$ process, Hinich and Patterson (1995) show that X_3 is asymptotically distributed as $\chi^2[\phi(\phi - 1)/2]$ for $\phi < N^{0.5}$. They recommend using $\phi = N^{0.4}$ based on their simulations. Under the assumption that $E((x_t)^{0.5})$ exists, the X_3 statistic will discover nonzero third-order correlations. It can be considered as generalization of the Box-Pierce portmanteau statistics – see Hinich and Patterson (1985) for more discussion.

The Hinich Bispectrum Test

A process is said to be third-order nonlinear dependence if the skewness function in the frequency domain is not flat as a function of frequency pairs. The definition of the square of the skewness function is shown in Equation 2.8. This form of the nonlinearity is called third order, since the skewness function is a normalization of the Fourier transform of the third-order autocovariances. That Fourier transform is called the bispectrum (Barnett et al. (1997)).

The Hinich bispectrum test is a nonparametric test that examines the third-order moments (bicovariance) of the data in the frequency domain to obtain a direct test for a nonlinear generation mechanism, regardless of any linear independence that might be present in the data. Therefore, when the tests rejects the null (the skewness function is flat), there is no need to check the possibility that the linear prewhitening model has failed to remove all linear serial dependence in the data (Ashley and Patterson (2006)).

Hinich (1982) develops this test for flatness of bispectrum. He argues that the bispectrum in the frequency domain is easier to interpret than multiplicity of the third-order moments $c_{xxx}(r, s) : s \leq r, r = 0, 1, 2 \cdots$ in the domain. Barnett and Hinich (1993) explain the computation of the test statistic. For frequencies f_1 and f_2 in the principle domain

$$\Omega = (f_1, f_2) : 0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1$$

is the Hinich bispectrum of the series at frequency pair (f_1, f_2), and its double Fourier transformation of the third-moments function is:

$$B_{xxx}(f_1, f_2) = \sum_{r=-\infty}^{r=\infty} \sum_{s=-\infty}^{s=\infty} c_{xxx}(r, s) exp[-2\pi(f_1 r + f_2 s)]. \tag{2.7}$$

The square of the skewness function $\Gamma^2(f_1, f_2)$ is defined in terms of the bispectrum as:

$$\Gamma^2(f_1, f_2) = \frac{|B_{xxx}(f_1, f_2)|^2}{S_{xx}(f_1) S_{xx}(f_2) S_{xx}(f_1 + f_2)} \tag{2.8}$$

where $S_{xx}(f)$ is the (ordinary power) spectrum of x_t at frequency f. If the time series x_t is linear then the squared of skewness function $\Gamma^2(f_1, f_2)$ is constant over all frequency pairs (f_1, f_2) in Ω, and the skewness function $\Gamma^2(f_1, f_2)$ is zero over all frequencies if x_t is Gaussian. Linearity and Gaussianity can be tested using a sample estimator of the skewness function $\Gamma^2(f_1, f_2)$ – see Barnett and Hinich (1993) for more details on computation of the test and Hinich (1982) for more details on the test.

Engle LM Test

The test was proposed by Engle (1982) to examine nonlinearity in the second moment, particularly for ARCH disturbances. The test employs the Lagrangian multiplier procedure and runs the OLS regression and saves the residuals. Then, the next step is to regress the squared residuals on a constant and p lagged values of the squared residuals and test NR^2 as a χ^2_p.

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \hat{\varepsilon}_{t-j}^2 + u_t \tag{2.9}$$

The test statistic is based on the R^2 of the regression similar to the common methods in most Lagrange Multiplier tests. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the above regression is asymptotically distributed as χ^2_p.

The McLeod-Li Test

McLeod and Li (1983) developed a portmanteau test for nonlinear statistical dependence in the squared-residual autocorrelations of fitted ARMA models. The tests looks at the autocorrelation function of the squares of the prewhitened data and tests whether $corr(x_t^2, x_{t-j}^2)$ is nonzero for some j. The autocorrelation at the lag j for the squared residuals x_t^2 is estimated by

$$\hat{r}(j) = \frac{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)(x_{t-j}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)}, \quad where \hat{\sigma}^2 = \sum_{t=1}^{N} \frac{x_t^2}{N} \tag{2.10}$$

Under the null hypothesis that x_t is an $i.i.d$ process, McLeod and Li (1983) showed that, for sufficiently large and fixed L,

$$Q = N(N+2)\sum_{j=1}^{L} \frac{\hat{r}^2(j)}{N-j} \tag{2.11}$$

is asymptotically χ^2_L under the null hypothesis of a linear generating mechanism for the data. They have set $L = 20$ for their small-sample simulation in their examination.

The Tsay Test

The Tsay (1986) test explicitly look for quadratic serial dependence in the data, using quadratic terms lagged up to K periods. Let the $K = k(k+1)/2$ column vectors $V_1, ..., V_k$

contains all the unique cross-products of the form $x_{t-i}x_{t-j}$, where $i \in [i, k]$ and $j \in [j, k]$. Let $\hat{v}_{t,i}$ denote the projection of $v_{t,i}$ on the subspace orthogonal to $x_{t-1}, ..., x_{t-k}$, which is the residuals from a regression of $v_{t,i}$ on $x_{t-1}, ..., x_{t-k}$. The parameters $\gamma_i, ..., \gamma_k$ are estimated by applying OLS to the regression equation:

$$x_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i \hat{v} i, t + \eta_t \tag{2.12}$$

Then, the Tsay test statistic is the usual F statistic for testing the null hypothesis that $\gamma_1, ..., \gamma_k$ are all zero.

The Results for Nonlinearity Tests

The results for the Hinich bicovariance, the Hinich bispectrum, the McLeod-Li, the Engle, and the Tsay test are reported in Table 2.7 and Table 2.8, for both bootstrapping the significance levels and asymptotical distributions[14]. As stated by Patterson and Ashley (2000a) the described tests are only asymptotically justified similar to the most econometrics procedure. Therefore, the significance levels of all the tests are consistently bootstrapped. Also, the significance levels based on the asymptotic distributions are computed – see Patterson and Ashley (2000a) for further details on the bootstrap simulation.

Based on the Hinich and Patterson (1985)'s simulation, where N is the sample size for each individual series, $\phi = N^{0.4}$ is used in the Hinich bicovariance test. Moreover, the test

[14]The nonlinear software was thankfully provided by Professor Douglas M. Patterson. The source, instruction on running the toolkit program, and analysis can be found in Patterson and Ashley (2000a): "A Nonlinear Time Series Workshop: A Toolkit for Detecting and Identifying Nonlinear Serial Dependence", Kluwer Academic Publishers: Norwell. Available at: http://www.wkap.nl/.

is calculated using up to 15 lags and also with the number of bootstrap iterations equal to 1000. As displayed by the results, based on the bootstrapped as well as asymptotic distributions, this test rejects the null hypothesis that x_t is a serially $i.i.d$ process in every case at the 1%, 5% and 10% significance levels.

The Hinich bispectrum test, on the other hand, examines the third order moments (bicovariance) of the data in frequency domain to obtain a direct test for a nonlinear generating mechanism. More importantly, this test focuses on nonlinear serial dependence, and it substantially differs from Hinich bicovariance test in using the sample bicovariance of the data. The Hinich bispectrum test accepts the linearity if it cannot reject the flatness of bispectrum, and accepts the Gaussianity if the bispectrum is flat and also equals to zero. As can be observed in the Table 2.8, the results of Gaussianity indicate extremely small p-values for each energy components market in the case of asymptotic distribution. As a result the null hypothesis of the Gaussianity is rejected at the 10% significance level. Moreover, the null of linearity for each individual series exhibits a very significant results by very small p-values for the 80 percent fractile bispectrum linearity test for every series. Hence, in the case of asymptotic distribution, the null hypothesis of the linearity is also rejected at the 1%, 5% and 10% significance levels for each individual series. In other words, the rejection of linearity provides strong evidences for the presence of the third order nonlinearity in the data generating process as also noted by Barnett et al. (1997). Ashley and Patterson (2006) show that the bispectrum $B_{xxx}(f_1, f_2)$ is consistently estimated using an average of appropriate triple products of the Fourier representation of the observed time series. The average is taken over a square containing

M adjacent frequency pairs. Hinich (1982) showed that M must be above the $N^{0.5}$ to consistently estimate $B_{xxx}(f_1, f_2)$. The results are calculated for M to equals to $N^{0.6}$.

The Engle LM test (1982) examines nonlinearity in the second moments. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the regression Equation 2.9 is asymptotically distributed as χ^2_p. The results are reported for p (lagged values) equals to 5, and they exhibit substantially small p-values at the 10% significance level in both bootstrapped and asymptotic distributions. Therefore, the null hypothesis of nonlinearity in the second moments is rejected in all cases. Following the literature, the results are quoted for p=5.

The null hypothesis of x_t is an $i.i.d$ process in McLeod and Li (1983) test is also rejected for up to 24 lags in bootstrapped and asymptotic distributions. As shown in the tables, the results yield very small p-values at the 10% significance level. Here the results are reported for $L = 24$.

Table 2.7: Significance Level for Nonlinearity Tests
Bootstrap Simulation

Series	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.000	0.000	0.000
Engle($p = 5$)	0.000	0.000	0.000	0.000	0.000
McLeod-Li($L = 24$)	0.000	0.000	0.000	0.000	0.000
Tsay ($k = 5$)	0.000	0.000	0.000	0.000	0.000

Notes: Number of bootstrap iterations =100.
The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot price, which is January 07, 1997 to August 16, 2011.

The results for Tsay test is reported using k=5. Following the existing literature in the subject the value of $k = 5$ is used here. The reported results based on the bootstrapped as well as asymptotic distributions, are indicating that the null hypothesis is rejected in

Table 2.8: Significance Level for Nonlinearity Tests
Asymptotic Distribution

Series	**Crude Oil WTI**	**Brent Europe**	**Heating Oil**	**Gasoline**	**Natural Gas**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.000	0.000	0.000
Bispectral (Gaussianity) ($M = N^{0.6}$)	0.000	0.000	0.000	0.000	0.000
Bispectral (Linearity) ($M = N^{0.6}$)	0.000	0.000	0.000	0.000	0.000
Engle($p = 5$)	0.000	0.000	0.000	0.000	0.000
McLeod-Li($L = 24$)	0.000	0.000	0.000	0.000	0.000
Tsay ($k = 5$)	0.000	0.000	0.000	0.000	0.000

Notes: The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

10% significance level.

Therefore, based on the bootstrapped and asymptotic distributions, the results for the nonlinear tests reveal that the employed data have clear evidence of nonlinearity in their structure. The time series prices data of energy products exhibit nonlinearity in the mean, variance and skewness functions. These results are consistent with other reported findings in the literature, such as Kyrtsou et al. (2009). The evidence for significant nonlinearity in data generating mechanism in the energy market offers to model the time series data with an accurate specifications that reflects dynamics in the data, which helps to obtain valid parameter estimations.

2.6 Summary and Conclusion

This chapter employed statistical and econometrics techniques to investigate the non-linear dependence in the energy market. The techniques involve the most widely used univariate tests to detect the nonlinearity in the observed time series data. To examine whether the time series data in the energy market exhibit nonlinearity in their generating mechanism, the study utilized the daily spot prices of five major products in the energy market over 16 years. The results indicate that the daily spot prices of crude oil (West Texas Intermediate (WTI) and Europe Brent), heating oil, gasoline and natural gas exhibit deep nonlinearity in their structure.

None of the tests have exactly the same null hypothesis and they differ in the power against the alternative hypothesis. The tests focus on different aspects of nonlinearity and detect distinct features of nonlinear serial dependence in the data. Additionally, using the tests jointly can produce deeper perception into the nature of the nonlinearity that may exist in the data. The BDS test is a test of general nonlinearity in the process against all other possible alternative null hypothesis of linearity and has a high power against numerous classes of alternative hypotheses. The results of the BDS test indicate that the linearity is rejected; hence it is a compelling indication to employ more particular tests that consider the more detailed features of nonlinearity. The attributes of the Kaplan test seem to be comparable to the BDS test. However, Barnett et al. (1997) state that in their experiments the Kaplan test, unlike the BDS test, acquired the right answer with both large and small samples. The results for the Kaplan tests detect evidence of nonlinearity in all the time series data excluding the daily spot price on crude oil (Europe Brent). The Hinich bicovariance test focuses on the third-order moments (time domain) of the data and detected nonlinearity in each series. The Hinich bispectrum test examines the lack of third-order nonlinear dependence (frequency domain), and the associated Gaussianity test, is a test of a necessary and not sufficient condition for Gaussianity[15]. The results of

[15]See Barnett et al. (1997) for more details.

the Hinich bispectrum suggest that the observed time series data in the energy market are generated by a nonlinear, non-Gaussian process. The Engle Lagrangian multiplier (LM) test focuses on the nonlinearity in the second moment. The null hypothesis of no ARCH-type disturbances is rejected by the Engle-LM test. The McLeod-Li test rejects the null hypothesis of linearity in the variance as well. Finally, the Tsay test rejects the null hypothesis of linearity in each individual series. Therefore, all the tests detect strong evidence of nonlinear structure in the time series data, indicating that the employed time series in various markets of the energy products are generated by a nonlinear mechanism.

As noted by Ashley and Patterson (2006), the evaluation of the time series models is based on the goodness of fit and the post sample forecasting ability. Prediction can be improved by nonlinear models when there is evidence of nonlinearity in the data generating process (Maravall (1983); Tong (1983); Ashley and Patterson (2006)). The main implication of nonlinearity in the observed data is that the series cannot be properly forecasted with linear models when there are signs of nonlinearity in the data. Therefore, in view of the importance of the energy sector in aggregate economic activity, it is essential to check the existence of nonlinearity in the time series data of the energy market. The investigation will allow us to attain more plausible empirical results by employing an efficient time series modeling that coincides with the data generating process dynamics.

Appendix A: Data Description, Key Terms, and Definitions

The definition of the energy market products in the Appendix A of this chapter are adopted from Energy Information Administration (EIA).

Figure 2.5 represents the Cushing, OK WTI spot price FOB (Dollars per Barrel). The variable West Texas Intermediate (WTI- Cushing) is defined as follows:

- *West Texas Intermediate (WTI - Cushing:)* A crude stream produced in Texas and southern Oklahoma, which serves as a reference or "marker" for pricing a number of other crude streams and which is traded in the domestic spot market at Cushing, Oklahoma.[16]

- *Crude Oil:* A mixture of hydrocarbons that exists in liquid phase in natural underground reservoirs and remains liquid at atmospheric pressure after passing through surface separating facilities. Depending upon the characteristics of the crude stream, it may also include:

  - Small amounts of hydrocarbons that exist in a gaseous phase in natural underground reservoirs but are liquid at atmospheric pressure after being recovered from oil well (casinghead) gas in lease separators and are subsequently commingled with the crude stream without being separately measured. Lease condensate recovered as a liquid from natural gas wells in lease or field separation facilities and later mixed into the crude stream is also included;

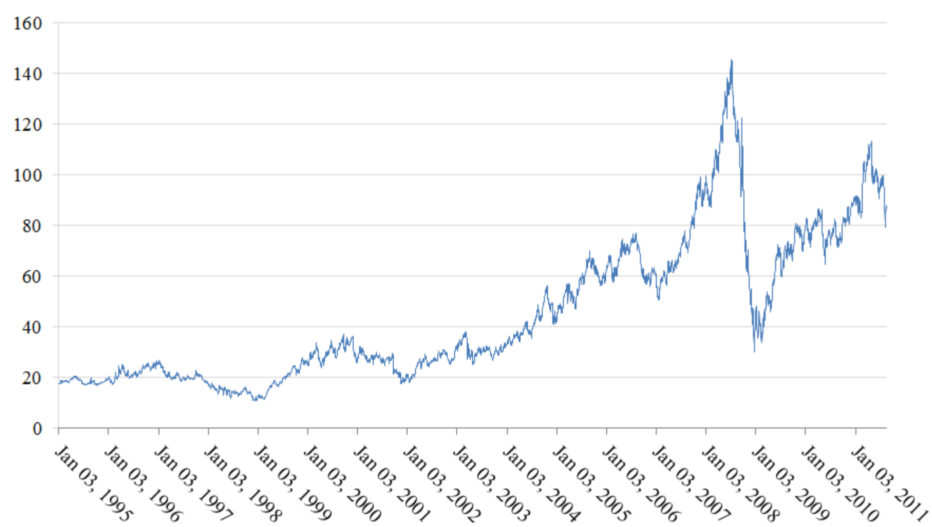
[16] Energy Information Administration (EIA).

– Small amounts of nonhydrocarbons produced with the oil, such as sulfur and various metals;

– Drip gases, and liquid hydrocarbons produced from tar sands, oil sands, gilsonite, and oil shale.

Liquids produced at natural gas processing plants are excluded. Crude oil is refined to produce a wide array of petroleum products, including heating oils; gasoline, diesel and jet fuels; lubricants; asphalt; ethane, propane, and butane; and many other products used for their energy or chemical content.[17]

Figure 2.5: Daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel)



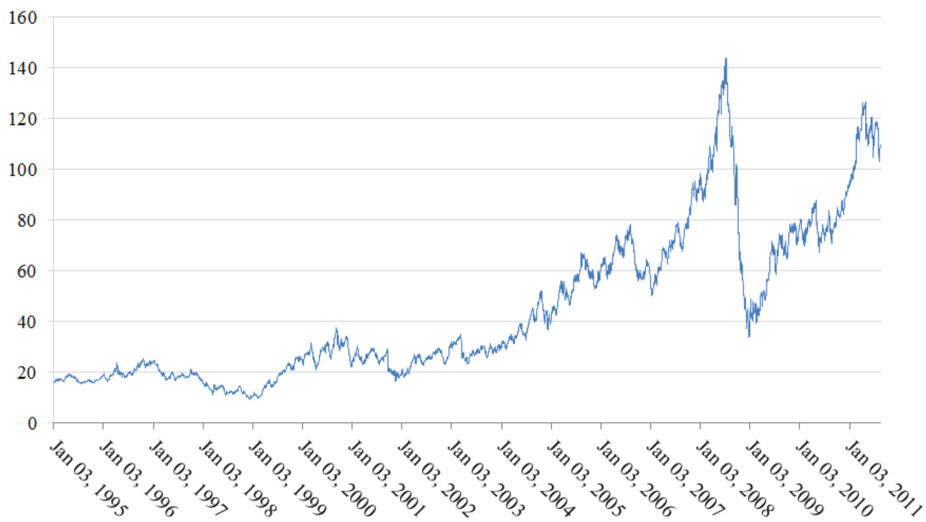
Data Source: Energy Information Administration (EIA)

Figure 2.6 represents Europe Brent spot price FOB (Dollars per Barrel). The variable Europe Brent is defined as follows:

[17]Energy Information Administration (EIA).

Brent: A blended crude stream produced in the North Sea region which serves as a reference or "marker" for pricing a number of other crude streams.[18]

Figure 2.6: Daily Europe Brent Spot Price FOB (Dollars per Barrel)



Data Source: Energy Information Administration (EIA)

Figure 2.7 represents New York Harbor No. 2 Heating Oil Spot Price FOB (Dollars per Gallon). The variable New York Harbor is defined as follows:

New York Harbor: The location specified in either spot or futures contracts for delivery of a product in New York Harbor.[19]

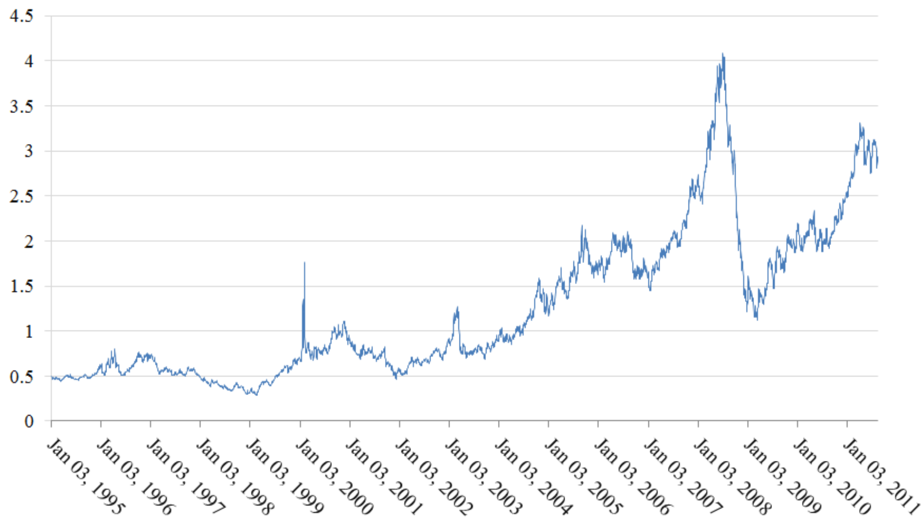
Figure 2.8 represents New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon). The variables Conventional Gasoline and the New York Harbor are defined as follows:

Conventional Gasoline: Finished motor gasoline not included in the oxygenated or reformulated gasoline categories. Excludes reformulated gasoline blendstock for oxygenate

[18] Energy Information Administration (EIA).

[19] Energy Information Administration (EIA).

Figure 2.7: Daily New York Harbor No. 2 Heating Oil Spot Price FOB (Dollars per Gallon)



Data Source: Energy Information Administration (EIA)

blending (RBOB) as well as other blendstock.[20]

New York Harbor: The location specified in either spot or futures contracts for delivery of a product in New York Harbor.[21]

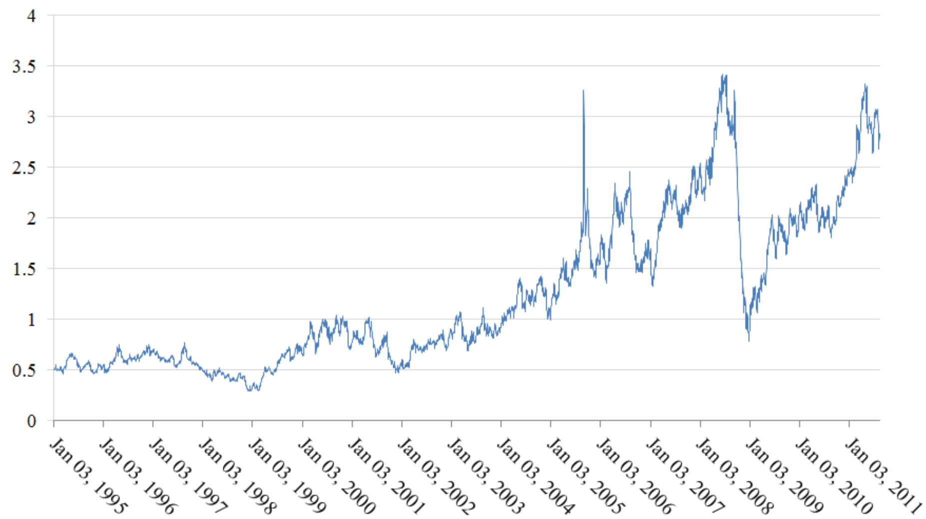
Figure 2.9 represents Henry Hub Gulf Coast Natural Gas Spot Price ($/ MMBTU). The variable U.S. Gulf Coast is defined as follows:

U.S. Gulf Coast: The location specified in either spot or futures contracts for delivery of a product in any port city along the coastline of Texas and Louisiana.[22]

Figures 2.10, 2.11, 2.12, 2.13 and 2.14 show the log and the differenced log of the individual series.

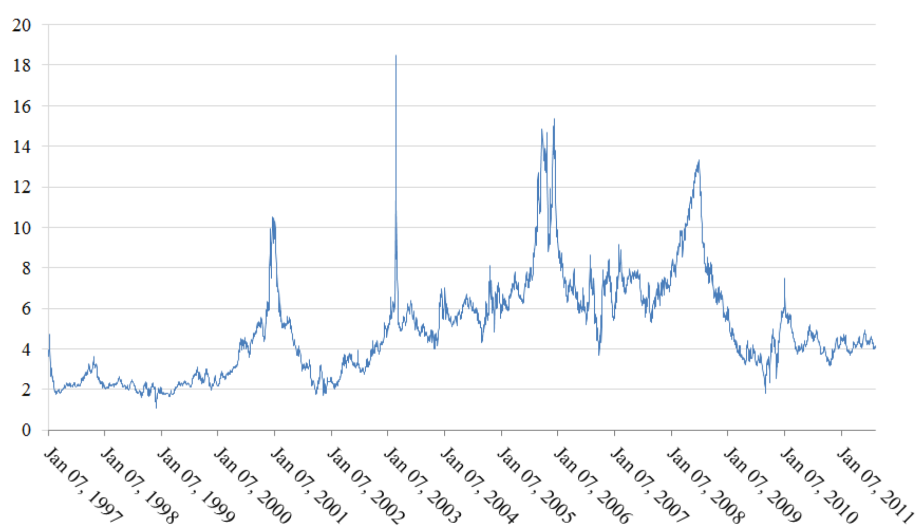
[20]Energy Information Administration (EIA).
[21]Energy Information Administration (EIA).
[22]Energy Information Administration (EIA).

Figure 2.8: Daily New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon)



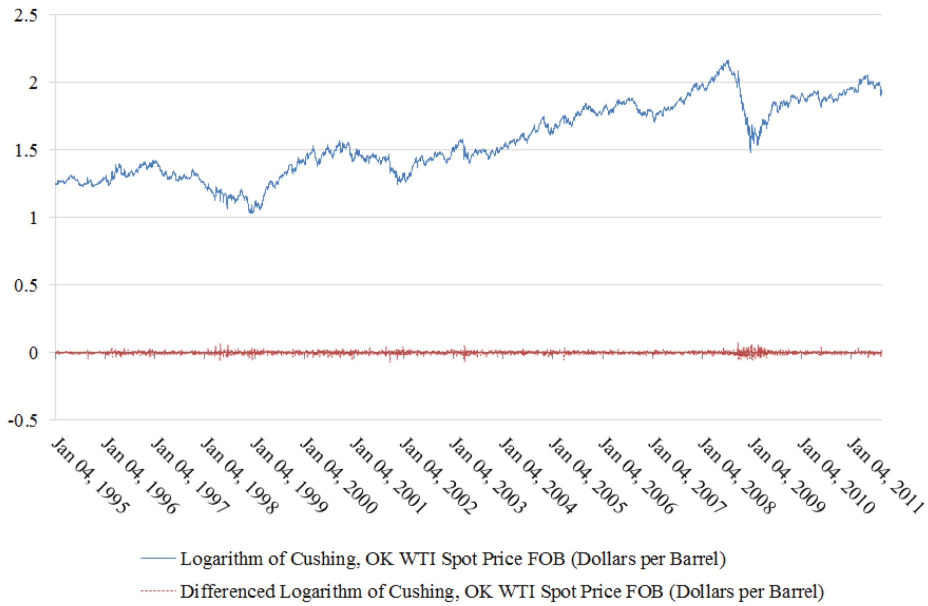
Data Source: Energy Information Administration (EIA)

Figure 2.9: Daily Henry Hub Gulf Coast Natural Gas Spot Price ($/MMBTU)



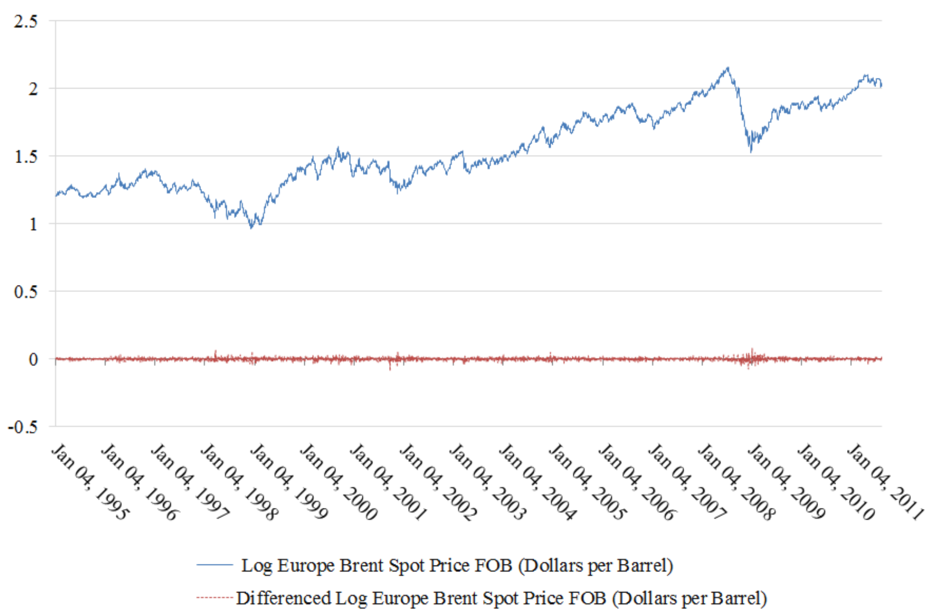
Data Source: Energy Information Administration (EIA)

Figure 2.10: Log and Differenced Log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel)



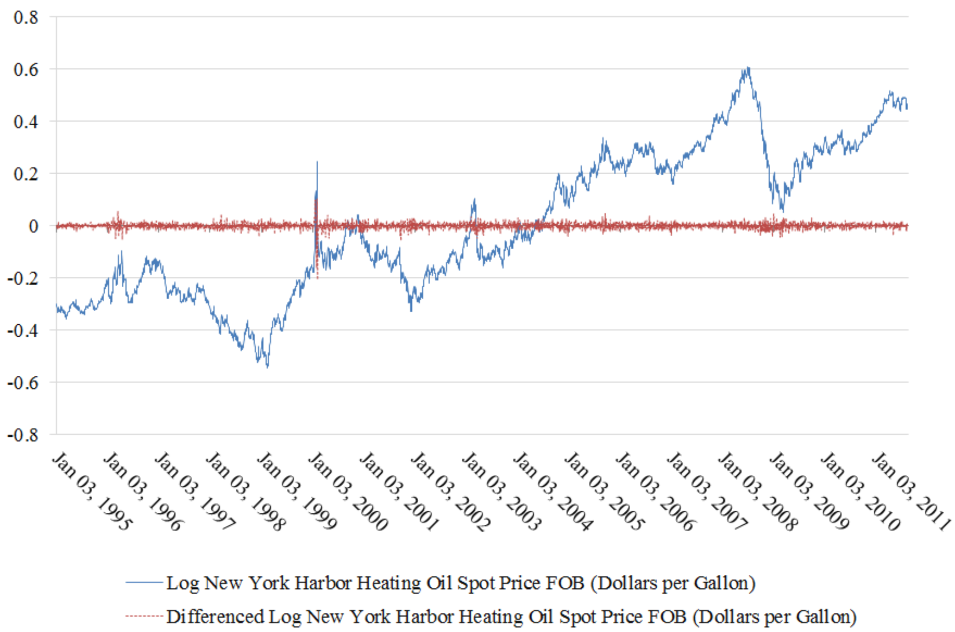
Data Source: Energy Information Administration(EIA)

Figure 2.11: Log and Differenced Log of Europe Brent Spot Price (Dollars/Barrel)



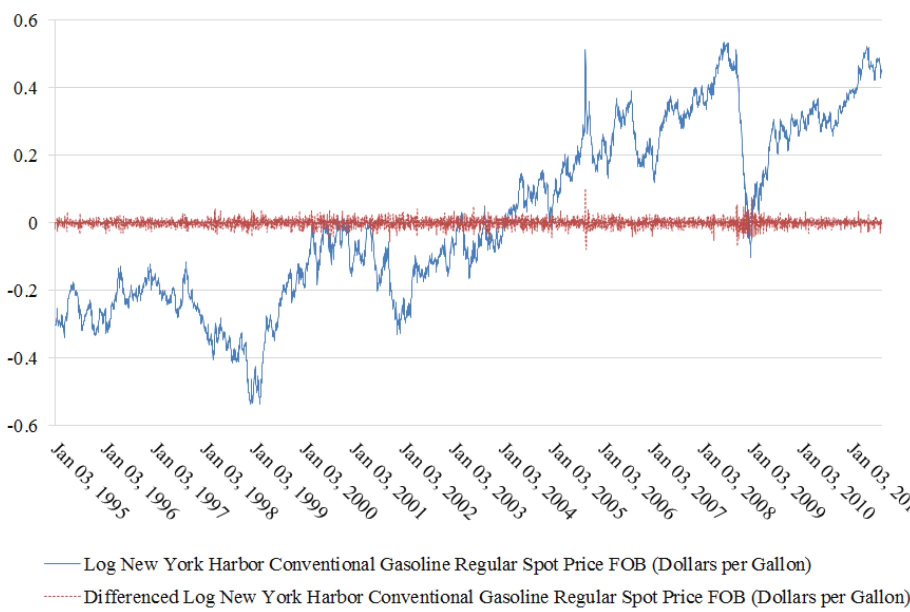
Data Source: Energy Information Administration(EIA)

Figure 2.12: Log and Differenced Log of New York Harbor Heating Oil Spot Price FOB (Dollars per Gallon)



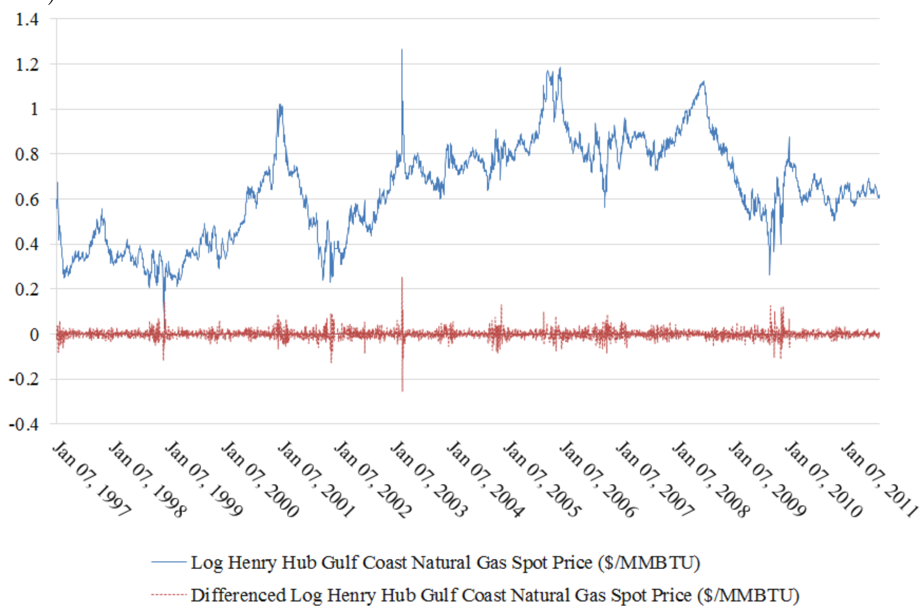
Data Source: Energy Information Administration(EIA)

Figure 2.13: Log and Differenced Log of New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon)



Data Source: Energy Information Administration(EIA)

Figure 2.14: Log and Differenced Log of Henry Hub Gulf Coast Natural Gas Spot Price ($/MMBTU)



Data Source: Energy Information Administration(EIA)

Appendix B: Figures of the Kaplan Results for Embedding Dimension 2 − 5

Figures 2.15 to 2.18 display the Kaplan test results. In other words, the plots of δ versus ϵ are shown in Figures 2.15 to 2.15. The signs of discontinuity are revealed in the plots: as δ goes to zero, ϵ does not in each daily spot prices of energy products.

Figure 2.15: *Delta* vs. *Epsilon*, The Kaplan Test Results from Daily Spot Prices on Five Energy Products, Lag Embedded=2

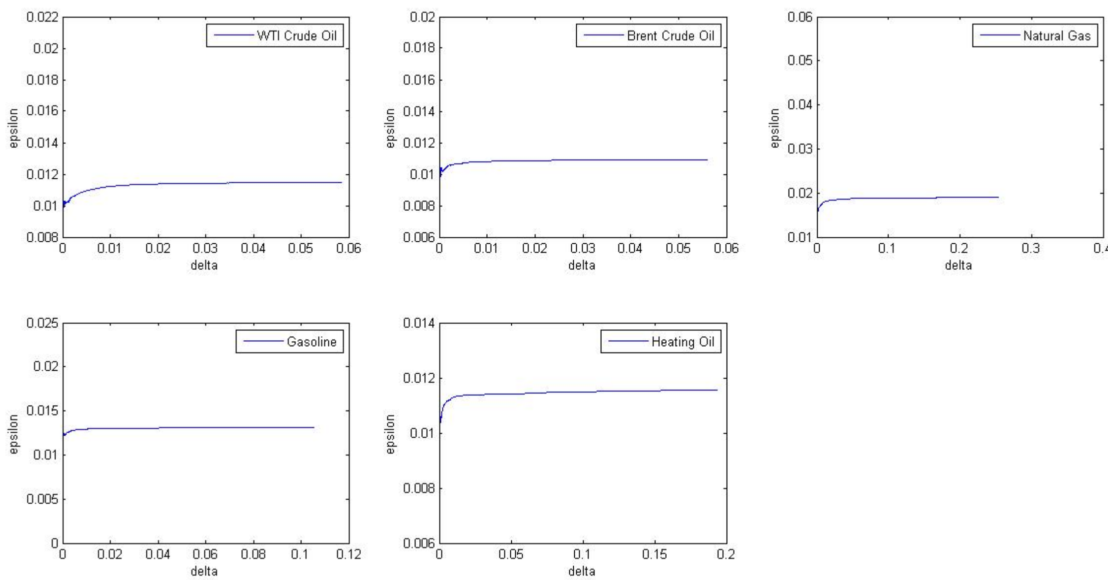


Figure 2.16: *Delta* vs. *Epsilon*, The Kaplan Test Results from Daily Spot Prices on Five Energy Products, Lag Embedded=3

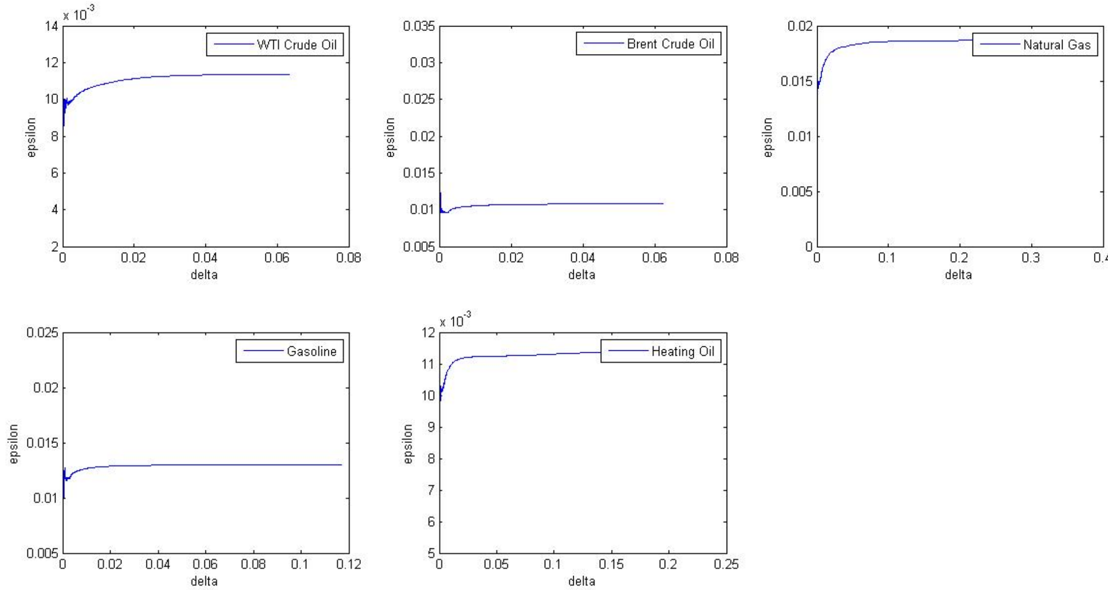


Figure 2.17: *Delta* vs. *Epsilon*, The Kaplan Test Results from Daily Spot Prices on Five Energy Products, Lag Embedded=4

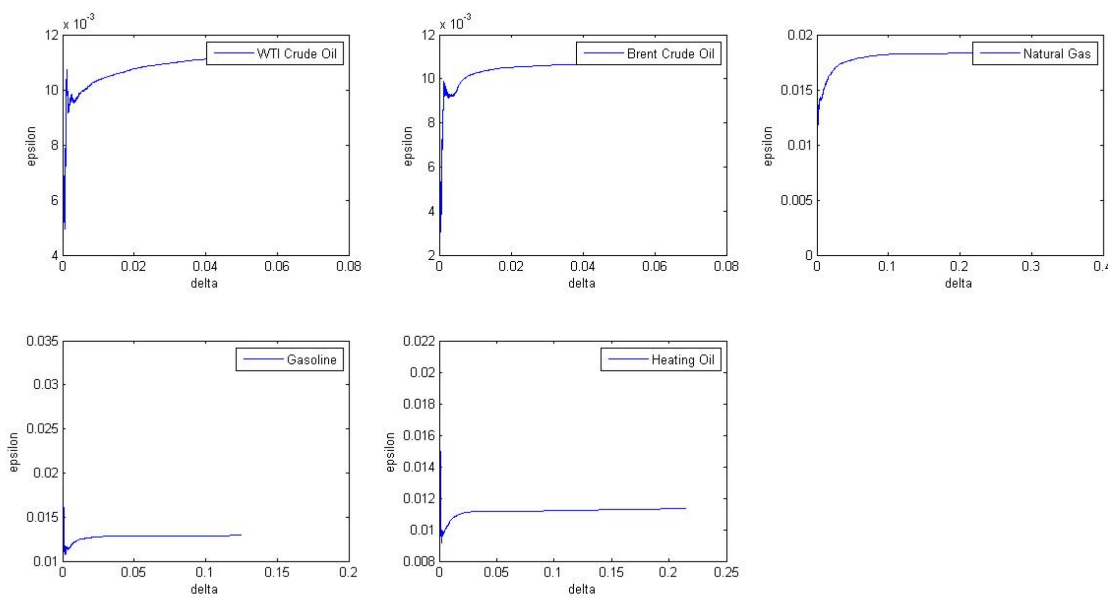
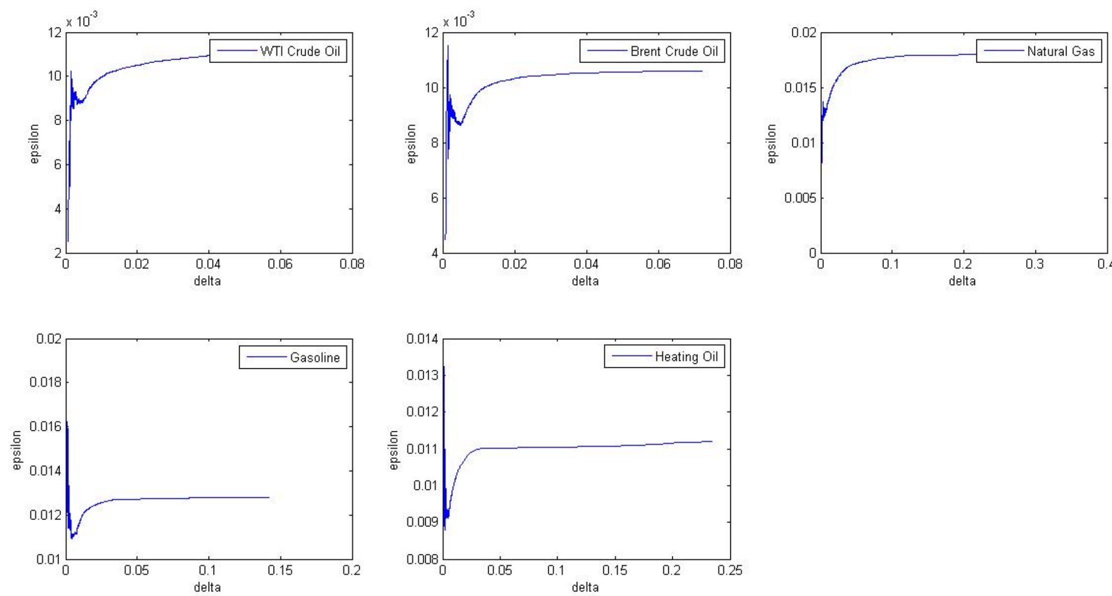


Figure 2.18: *Delta* vs. *Epsilon*, The Kaplan Test Results from Daily Spot Prices on Five Energy Products, Lag Embedded=5



Chapter 3

Nonlinear Dynamic Structure in Crude Oil Production

3.1 Introduction

There has been a great interest in various literature on crude oil supply and projection of oil production. Among many approaches of modeling and forecasting the future of crude oil production, the "Hubbert Curve"[1], which was presented in 1956 by M. King Hubbert, has been widely used as a basic tool in forecasting the market. Based on Hubbert's model, the cumulative production can be characterized by a logistic function, and the first derivation of the logistic function would define the yearly production by a bell-shaped curve. The Hubbert Model correctly predicted that the United States oil production would peak in the early 1970s. Since then the model has been widely used

[1] The Hubbert Theory was named after the petroleum geologist with Shell Oil, M. King Hubbert

in forecasting the peak of the world's oil production. Campbell and Laherrere (1998) predicted the global oil production by enhancing the Hubbert model. Campbell and his co-author's models have been the most widely published bottom-up models of crude oil production for global prediction, as noted by Brandt (2010). Therefore, the question of when "the maximum production, a peak, would occur" is a magnitude of those studies that predict crude oil production.

Furthermore, since the first oil shock of the 1970s and political unrest in Organization of the Petroleum Exporting Countries (OPEC), crude oil production has been a crucial variable in defining real GDP growth rate as well as monetary policies that might lead to CPI inflation (Bernanke et al. (1997); Hamilton (1983) among many others). Hence, it is important to consider production of crude oil, which is the variable that responds to price, when analyzing the energy market because the disruption in the production variable has a major impact in aggregate economic activity as well. As a results, the production of crude oil is a central variable to be properly forecasted. However, to define a more accurate projection of crude oil production, it is crucial to employ appropriate specifications, which are close to the data generating mechanism and to examine whether the time series observations in the market are generated by a linear process or a nonlinear dynamic mechanism. If the nonlinearity is present in the data, choosing a nonlinear time series can provide more plausible post-sample forecasting ability (Ashley and Patterson (2006)).

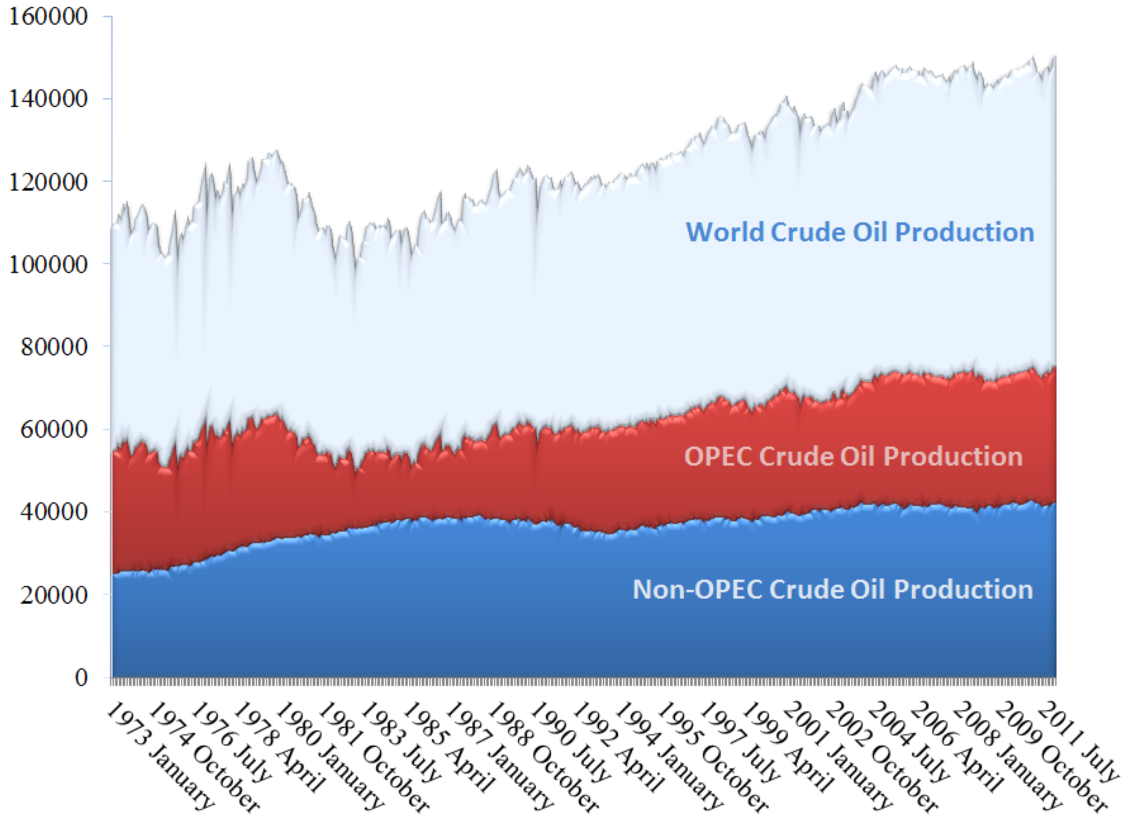
This essay, along with the other chapters in this dissertation will provide a comprehensive analysis on the structure of the energy market. This chapter is motivated by the largely neglected quantity side of the energy market and the characteristics of the dynamics properties in the time series of crude oil production. The dynamic structure of the time series data in context of the nonlinear mechanism in production of crude oil has never been assessed. This study incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. It utilizes monthly observations on the U.S. field production of crude oil from January 1920 to June 2011 as well as OPEC production of crude oil, non-OPEC countries production of crude oil, and the world production of crude oil. The sample period for last three time series observations is from January 1973 to January 2012. This chapter begins with a review of the disruption of crude oil and its impact on the aggregate economy. Section Three reviews the related literature. Section Four describes the data and various unit root analyses. Section Five discusses the inference methods as well as the results of performing the nonlinearity tests to examine the markets' data generating mechanism. Lastly, a brief summary and conclusion for this chapter are discussed in Section Six.

3.2 Exogenous Shocks in Production of Petroleum

Crude oil is the primary energy source of the world's total energy demand. The production of crude oil has involved a dramatic growth rate from 1973 to 2012 by nearly 39 percent. However, this rate has been affected several times by major events. The first fall in supply was experienced in late 1973 as a result of tightening the oil embargo by OPEC. Oil production was cut by five million barrels per day and the price of oil increased by 400 percent in six months (Sill (2007)), attributed to the “first oil shocks”. The next dramatic drop in production occurred as a result of the Iranian Revolution, which began in late 1978 and resulted in a drop of 3.9 million barrels per day of Iran’s crude oil production until 1981. In 1980, the Iran-Iraq war began and by 1981 the OPEC production declined by seven million barrels per day from its level in 1978. In 1973 the average of OPEC oil production was nearly 29.6 million barrels per day and by April 1982, when the next drop in production occurred, the OPEC supply averaged nearly 15 million barrels per day. The share of the production of crude oil by OPEC dropped from 53 percent in 1973 to 28 percent in 1982. OPEC increased the production of crude oil in subsequent years, but oil producers operating outside OPEC contributed to major percentage of the world’s oil in recent years. In 2009, OPEC participated almost 40 percent in the world’s total production while non-OPEC supply of crude oil represents nearly 60 percent of total world supply in the last ten years. Only ten countries produce about three-quarters of total non-OPEC oil supply, with the largest producer Russia as noted by Cline (2010). Figure 3.1 displays the area of crude oil production by each individual series.

It is worth mentioning that despite escalations in oil prices and efficient energy policies to reduce the energy intensity in recent years, the demand for energy components, and as a result production, has increased over time. In 2010 the Organization for Economic Cooperation and Development (OECD) countries, accounts for 53 percent of worldwide oil demand and 41 percent of this number belongs to the United States. Therefore, the growth in consumption resulted in rising in production such that by January 2012 world

Figure 3.1: Crude Oil Production, OPEC, Non-OPEC Countries, and the World (Thousand Barrels per Day)

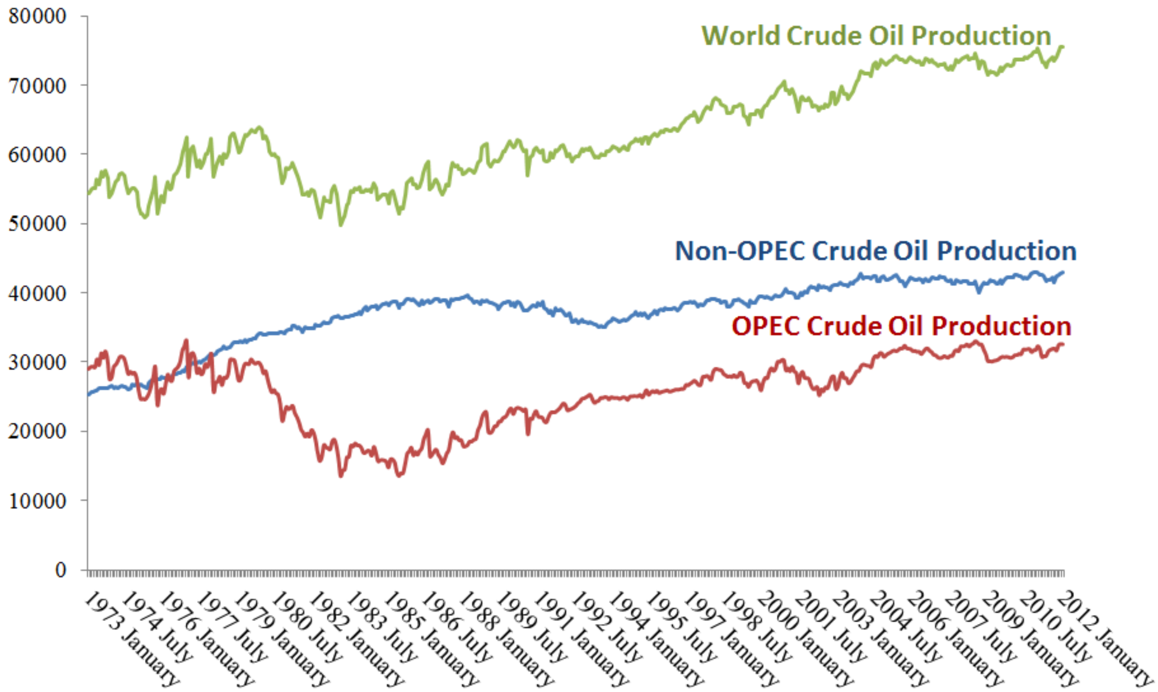


Data Source: Energy Information Administration(EIA)

oil production has reached nearly 76 million barrels per day in which 43 percent is share of OPEC countries and 57 percent is non-OPEC countries crude oil production.

As can be seen in Figure 3.2, the rise in the non-OPEC production of crude oil exhibits slow and steady growth rate and does not display major fluctuations. Considering the nature of the production market and the constraint of the petroleum availability, there are major debates around forecasting the world's peak oil production for OPEC and non-OPEC. The related literature is to be discussed in the next section.

Figure 3.2: Crude Oil Production Overview (Thousand Barrels per Day)



Data Source: Energy Information Administration(EIA)

3.3 Literature Review

3.3.1 Nonlinear Approach in Crude Oil production

There are studies in different literatures that consider crude oil production to be nonlinear. A major study in the petroleum industry is by Hubbert (1959), who introduced a bell-shaped function to capture nonlinear dynamics in the oil production. In the Hubbert model, the cumulative production is assumed to be a logistic function, and the annual rate of production would be characterized by a bell-shaped curve over time (Kaufmann and Cleveland (2001)). The Hubbert Model correctly predicted that the United States oil production would peak in early 1970s. Since then the model has been widely used in forecasting the world's peak oil production. Campbell and Laherrere (1998) predicted the global oil production by enhancing the Hubbert model. Campbell and his co-author's models have been most widely published bottom-up models of crude oil production for global prediction, as noted by Brandt (2010). Haubrich and Meyer (2007) state that there are some economic reasons for considering a nonlinear (logistic curve) pattern for the production data. Using two different approaches, linear and nonlinear, they estimate the peak in oil production. In analyzing the nonlinear unit root properties, Maslyuk and Smyth (2009) test for nonlinearities and unit root in crude oil production. Their findings indicate that for eleven countries, out of seventeen OPEC and non-OPEC members, a unit root was present in both regimes that they consider in their study. Brandt (2010) assesses existing oil supply models and their accuracy in predicting the future of oil production. He classifies the main present models into five categories along with four dimensions of variability. Brandt (2010) states that the existing models have proceeded not very successfully in describing the future of global oil production.

To acquire a more accurate prediction of the world's peak oil production, it is crucial to employ appropriate specifications, which are reasonably close to the data generating mechanism, and to examine whether the time series observations in the market are generated by a linear process or a nonlinear dynamic mechanism. As explained by Brockett et al. (1988), given the nature of confounding linear and quadratic coefficients in the estimation of time series models, it is important to test for significant nonlinearity in the observed time series and to determine which time series are not amenable to linear time series modeling. If the nonlinearity is present in the data, choosing a nonlinear time series can provide a more plausible post-sample forecasting ability (Ashley and Patterson (2006)). Therefore, the conventional linear time series modeling may not yield the most plausible results when there is significant nonlinearity in the data generating mechanism. Hence, it is essential to check the existence of the nonlinearity in the data before applying the empirical analysis. Numerous studies have addressed the issue in other literature, such as monetary economics, by applying univariate nonlinearity tests to detect the nonlinear structure in the economic data as well as the energy market.

Barnett et al. (1995) apply nonlinear tests to detect nonlinear behavior or chaos in various monetary aggregate data series, and discuss the controversy that has arisen about the available results. They use five inference methods to test for nonlinearity and chaos: the Hinich bispectrum test, the BDS test, the Lyapunov exponent estimator of Nychka, the White's test, and the Kaplan test. The findings provide a possible explanation for the controversies that exist regarding empirical evidence of chaos in economic data. They also state that the source of controversies can be found in the lack of robustness of the inference. In another influential study, Barnett et al. (1997) explore the reasons for empirical difficulties with the interpretations of nonlinear and chaos tests' results that have increased over time. They design and run a single-blind controlled competition among the aforementioned five highly regarded tests for nonlinearity or chaos with 10 simulated data series. The results shows that although there are some clear differences among the power functions of the tests, there exists some consistency in their inferences across the method of inference. Barnett et al. (2004) test the existence of nonlinearity in the cointegration relations of a system containing money demand variables, by applying the Hinich bispectrum test. The findings have some evidence of nonlinearity, and therefore they find that the issue is empirically relevant. Kyrtsou and Serletis (2006) discuss univariate tests for independence and hidden nonlinear deterministic structure in economic and financial time series. They apply the tests to Canadian exchange rate, using daily data over a 30-year period and they identify an interesting relationship between high-dimensional nonlinearity and shocks.

Furthermore, interest in studying the behavior of the energy market and applying the existing tests to detect the nonlinearities and chaos in this market has been growing over time. Kyrtsou et al. (2009) discuss number of widely used univariate test from dynamical system theory and apply them to the energy market. They apply these tests to daily observations of the energy market for nearly 15 years. They find indications consistent with nonlinear dependence in each of the markets. They also suggest that an effective nonlinear model of energy prices would produce a deeper perception of the energy market fluctuations than existing linear models. Sertletis and Gogas (1999) test for deterministic chaos in the North American Natural Gas Liquids Market. They use the Lyapunov exponent estimator and they find that there is evidence consistent with a chaotic nonlinear generation process in natural gas liquid markets. Serletis and Andreadis (2004) use daily observations on West Texas Intermediate crude oil prices, and Henry Hub natural gas prices and various tests from dynamical theory to support a random fractal structure for North American energy markets. The result is consistent with the reported result by Serletis and Gogas (1999) as they find evidence of nonlinear chaotic dynamics in North American natural gas liquids markets but not in the crude oil and natural gas markets.

As discussed above, some studies in the literature have investigated the energy market's fundamentals by applying univariate nonlinearity tests to detect the nonlinear structure in the energy market. However, existing literature focuses mainly on the price of the energy markets, and there is little mention of the production of the petroleum, which is the variable that responds to the price. Therefore, in order to provide a more inclusive study of the energy market structure, it is essential to study the quantity side of the market. This chapter will address the gap and assess the existence of any possible nonlinear structure in the data generating mechanism of crude oil production in the United States as well as OPEC, non-OPEC, and the world production of crude oil.

3.4 Data Description and Unit Root Analysis

This essay employs monthly observations on four different crude oil production time series obtained from Energy Information Administration (EIA).

- Monthly observations of the U.S. field production of crude oil. The sample period is from January 1920 to June 2011.
- Monthly observations of OPEC production of crude oil. The sample period is from January 1973 to January 2012.
- Monthly observations of the Non-OPEC production of crude oil. The sample period is from January 1973 to January 2012.
- Monthly observations of the world production of crude oil. The sample period is from January 1973 to January 2012.

Table 3.1: Summary Statistics of Differenced Log Series—Production of Crude Oil

Crude Oil Production	**Sample Mean**	**Sample Median**	**Standard Deviation**	**Skewness**	**Kurtosis**
US Field	0.0006	0.0004	0.0239	0.0500	5.9722
OPEC Members	0.0001	0.0010	0.0160	-1.6405	8.5220
Non-OPEC Members	0.0003	0.0005	0.0039	-0.1376	0.5552
World	0.0003	0.0007	0.0071	-1.5682	8.9481

Notes: The sample period for the U.S. field production of crude oil is from 1920:01 to 2002:12. The sample period for OPEC, Non-OPEC, and the World production of crude oil is from January 1973 to January 2012.

The descriptive statistics of the first difference of the log levels production is reported in Table 3.1. Figures 3.3 to 3.6 in the Appendix A depict monthly observations on the U.S., OPEC members, non-OPEC members, and the world production of crude oil, respectively.

Figures 3.7, 3.8, 3.9, and 3.10 show the log and the differenced log of the individual series.

3.4.1 Unit Root Analysis

In order to conduct the nonlinear analysis, the first step is to test whether or not the log level of the time series of crude oil production follows a random walk or has unit root. The two most well-known test procedures are employed to deal with the random walk behavior of the data, the Augmented Dickey-Fuller test (ADF) and the Philips and Perron test (PP).

The augmented Dickey-Fuller (ADF) test checks the existence of a unit root in an $AR(p)$

Table 3.2: Augmented Dickey-Fuller Unit Root Tests
Null Hypothesis: The log levels and the differenced log of the series have unit root
Lag length: Automatic Selection Based on SIC.

Log Level	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-1.833	-2.3092	-2.4640	-3.5446
p-value*	0.687	0.4267	0.346	0.0359
DLog Level	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-7.693	-22.0506	-27.3432	-23.5878
p-value*	0.000	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
Notes: The sample period for the U.S. field production of crude oil is from 1920:01 to 2002:12. The sample period for OPEC, Non-OPEC, and the World production of crude oil is from January 1973 to January 2012.

process, the unit root test is carried out under the null hypothesis $H_o : \beta = 0$ versus the alternative hypothesis $H_a : \beta < 0$ using the regression

$$\Delta y_t = c_t + \beta y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + e_t \tag{3.1}$$

where c_t is a deterministic function of the time index t and $\Delta y_j = y_j - y_{j-1}$ is the differenced series of y_t. The t-ratio of the statistic is computed by

$$ADF - test = \frac{\hat{\beta}}{std(\hat{\beta})} \tag{3.2}$$

where $\hat{\beta}$ denotes the least squares estimates of β, and the t-ratio is known as the *augmented Dickey-Fuller*(ADF) unit root test – see Dickey and Fuller (1981) for details. The error term is assumed to be homoscedastic and also the value of p is set such that the error is serially uncorrelated.

Furthermore, the Philips and Perron (1988) known as (PP) unit root test is employed to test whether or not the log level of the series exhibit a random walk behavior.

The PP test, unlike the ADF test, allows for errors not to be independently and identically distributed (*iid*), and it is essentially based on Equation 3.1, but without the lag differences. While the ADF test correct for the higher-order serial correlation by adding lagged difference terms to the right-hand side, the PP unit root test makes a non-parametric correction to account for residual serial correlation Maslyuk and Smyth (2008). Therefore, the PP test statistic is robust to a variety of serial correlation and time-dependent heteroscedasticity. The test regression for PP test is

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \tag{3.3}$$

where u_t is $I(0)$ and can be heteroscedastic. The PP test corrects for any serial correlations and heteroscedasticity in the error u_t of the test regression by modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$.

Under the null hypothesis that $\pi = 0$, the PP statistic has the same asymptotic distribution as the ADF t-statistic and normalized bias statistic – see Philips and Perron (1988) for more details.

The t-statistics for the ADF test ($t_{(\hat{\beta})} and Z_{t_{(\hat{\pi})}}$) as well as the p-values for the log levels and differenced log of the production time series are reported in Table 3.2.

A constant term as well as the time trend are included in the specifications of the unit root regressions for the ADF and the PP tests in log level for each individual time series. As displayed by the results in Tables 3.2 and 3.3, the null hypotheses of a unit root for the ADF and PP tests in log levels are rejected at the 1% significance level in

Table 3.3: Philips-Perron Unit Root Test
Null Hypothesis: The log levels and the differenced log of the series have unit root
Bandwidth: (Newey-West automatic) using Bartlett Kernel

Log Level	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-3.232	-2.060	-2.5768	-3.332
p-value*	0.018	0.5662	0.2913	0.0624
DLog Level	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-64.110	-22.781	-27.952	-26.3351
p-value*	0.000	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
Notes: The sample period for the U.S. field production of crude oil is from 1920:01 to 2002:12. The sample period for OPEC, Non-OPEC, and the World production of crude oil is from January 1973 to January 2012.

each case.

The decision to deal with the random walk behavior is to transform the log levels into the first differenced of the logs. The ADF and PP unit root test results in 3.2 and 3.3 indicate that the null hypotheses of unit root in first differenced levels can be rejected. Hence, the first differenced of the log for each individual production times series throughout the rest of the paper will be used unless otherwise noted.

3.5 The Inference Methods

The following inference methods for detecting nonlinearities are employed in this section: The BDS test, the Hinich bicovariance test, the Hinich bispectrum test, the Engle LM test, the McLeod-Li test, and the Tsay test. Each of the aforementioned tests excluding the Hinich bispectrum test, require to remove any serial dependence from the data via a prewhitening model. Any other serial dependence is the result of a nonlinear data generating mechanism. The Hinich bispectrum test directly tests the data generating mechanism and it is invariant to filtering of the data (Patterson and Ashley (2000a)). Moreover, the Kaplan test is considered in the inference methods to assess the dynamic structure of the market more precisely.

3.5.1 The BDS Test: A Test for Serial Independence

The widely held Brock, Dechert, Scheinkman and LeBaron(1996) test, also known as the BDS test, is one form of portmanteau tests for independence. Portmanteau tests are residual-based tests in which the null hypothesis is well stated, but they do not have a specific alternative hypothesis. The BDS test Brock et al. (1986) is a popular test to detect the serial independence in time series data. The BDS test introduces a test of independence that can be applied to the estimated residuals of any time series model, if the model can be transformed into a form with independent and identically distributed errors. The test employs the correlation function (correlation integral) to calculate the test statistics. The correlation function was introduced as a method of measuring the fractal dimension of deterministic data. The correlation function (integral) measures of the sequential pattern's frequency that exist in the data – see Brock et al. (1986) for more details.

The BDS test is used to test the null of linearity against a variety of possible deviation from independence in the series including nonlinearity and chaos. The test is applied to a series of estimated residual after removing any linear structure. Under the null hypothesis of independent and identically distributed ($i.i.d$) or whiteness, the BDS statistic is

$$\sqrt{n}\frac{C_{m,n}(\epsilon) - C_1(\epsilon)^m}{\sigma_m(\epsilon)} \tag{3.4}$$

where $C_{m,n}(\epsilon)$ is the correlation integral, $\sigma_m(\epsilon)$ is the asymptotic standard deviation of the numerator and m is the embedding dimension. The test converges to $N(0,1)$ under the null hypothesis of whiteness [The details for the test statistic and the formula can be found in Brock et al. (1986)].

The BDS test is applied to the differenced log of the time series of the U.S., OPEC, non-OPEC, and the the world production of crude oil. The choice of the values of ϵ and m may be a challenge in using the BDS test. The results with BDS are reported in Table 3.4 for dimension 2–8 and the chosen ϵ equals to one and two standard deviation of the data[2].

Results with the BDS Test

The BDS test statistic is produced for all the embedding dimension from two to eight. As can be observed in the Table 3.4, the results indicate the significance at the 10% level based on the asymptotic distribution for the U.S. and OPEC production of crude oil. The null hypothesis of the BDS test is rejected at the 1% significance level for the non-OPEC and the world production of crude oil indicating that the nonlinear structure is significant in the U.S. and OPEC production of crude oil.

The BDS test represent a high power against numerous nonlinear alternatives. Therefore, accepting the null hypothesis in the BDS test indicates that there are strong evidence

[2] ϵ is calculated as a multiple of the standard deviation of the series.

Table 3.4: BDS Test Z-Statistic (Dimension 2-8)

U.S. Production of Crude Oil (1920:01–2011:06)

	ϵ			
m	1σ	p-values	2σ	p-values
2	23.3336	0.000	16.8553	0.000
3	25.7004	0.000	14.2563	0.000
4	27.6393	0.000	11.4692	0.000
5	31.3215	0.000	9.0335	0.000
6	37.8373	0.000	7.5150	0.000
7	49.5211	0.000	7.0203	0.000
8	65.3457	0.000	6.2050	0.000

OPEC Members Production of Crude Oil (1973:01–2012:01)

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.8381	0.000	5.3932	0.000
3	8.2316	0.000	5.6121	0.000
4	9.7325	0.000	6.1709	0.000
5	11.5644	0.000	6.5206	0.000
6	13.6929	0.000	6.6346	0.000
7	16.2910	0.000	6.6855	0.000
8	19.5460	0.000	6.7479	0.000

Non-OPEC Members Production of Crude Oil (1973:01–2012:01)

	ϵ			
m	1σ	p-values	2σ	p-values
2	3.47	0.0005	4.1717	0.0000
3	3.5567	0.0004	4.1202	0.0000
4	3.4460	0.0006	4.0617	0.0000
5	2.9808	0.0029	3.1990	0.0014
6	2.7839	0.0054	2.7547	0.0059
7	2.5461	0.0109	2.5563	0.0106
8	2.1983	0.0279	2.3173	0.0205

World Production of Crude Oil (1973:01–2012:01)

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.3063	0.0112	6.3063	0.0000
3	8.0802	0.0116	8.0802	0.0000
4	9.5669	0.0092	9.5669	0.0000
5	11.1495	0.0226	11.1495	0.0000
6	12.9776	0.0101	12.9776	0.0000
7	15.2389	0.0089	15.2389	0.0000
8	17.8683	0.0104	17.8683	0.0000

for the null. Hence, it is recommended that the BDS test should be the first test to perform. The current results reflect little information to distinguish the existing forms of nonlinearity in the time series data of oil production. The Kaplan test is employed to verify the BDS tests results. Then the chapter will proceed to utilize more focused tests to identify the other possible forms of nonlinearity in the data – see Barnett et al. (1997) for more details.

3.5.2 Kaplan Test: A Test for Continuity and Determinism

There has been a wide range of methods in which reconstruction dynamics of the employed time series have been developed in order to characterize the dynamics in terms of predictability or dynamical invariant Kaplan (1994). These classifications are often employed to characterize whether the time series data are consistent with a deterministic mechanism, or a stochastic mechanism. As Kaplan (1994) mentions, it is common to test the predictability near every point in the time series in the nonlinear prediction method. Even though it might not be possible to predict future values of time series at every point, it may be likely to make accurate predictions at a few points. This may suffice for detecting the underlying determinism. Moreover, when deducing dynamics from a time series, continuity is often the only safe assumption one can make about a possible deterministic mechanism for a time series. Kaplan (1994) proposed a test for determinism in a time series based on consistency with a continuous dynamical mapping. The test answers a question like, "If two points x_i and x_j are very close together, are their images x_{i+1}

and x_{j+1} also close together?" (Kaplan (1994))[3]. In other words, deterministic solution paths, unlike stochastic processes, have the property that points that are close together are close under their image in phase space. Therefore, when the underlying function linking image and pre-image together is continuous, if the points x_i and x_j are close their images x_{i+1} and x_{j+1} are close together as well. In the case of chaos, the output plot of the system is hardly distinguishable from a stochastic process. Therefore, detecting the continuity of the system can be a difficult procedure, even when the data is entirely deterministic. However, it is easier to detect deterministic structure when plotting the solution path in phase space (x_{t+1} plotted against x_t and lagged values of x_t) than in plotting x_t versus t (Barnett et al. (1995)). Based on the above facts, the Kaplan test has strictly positive lower bound for a stochastic process, but not for a deterministic solution path. The statistic tests the null hypothesis that the data is deterministic against the alternative, which is that the data comes from a particular stochastic process. If the test statistic is smaller for the data than for the stochastic process by a statistically significant amount, then the stochastic process is rejected as an alternative to other forms of nonwhite structure (Barnett et al. (1995)).

The test is computed by an adequately large number of linear processes that plausibly might have produced the data. The test procedure involves producing a linear stochastic process surrogate data[4] for the observed data. The next stage is to determine a noisy continuous nonlinear dynamical solution path that describes the observed data more ac-

[3]A test based on continuity in phase space proposed by Daniel Kaplan, Centre for Nonlinear Dynamics, Department of Physiology, McGill University.

[4]Surrogate data is random data generated with the same mean, variance, and autocorrelation function as the original data.

curately. If the value of the test statistic from the surrogate is not small enough compared to the computed value of the test statistic from the observed data, a noisy continuous dynamical solution is concluded. As described by Barnett et al. (1995), the test procedure is formally stated as follows: If the time series data arise from a deterministically chaotic dynamical system, the value of x_{t+1} is a single-valued function of the state of the system at time t. Let the vector $x_t = (x_t, x_{t-1}, ..., x_{t-m-1})$ embedded in m-dimensional "phase space" and obtained from a m-dimensional vector $x_{i}{}_{i=1}^{T}$ in state space. Then there exists a function $f(x_t)$ such that $f(x_t) = x_{t+1}$, where x_{t+1} is called the "image" of the point x_t in phase space. If the system is perfectly deterministic with a continues f, close points in m-dimensional phase space have close image, whereas in a stochastic system close points in phase space may produce different images. The Kaplan test investigates if the function f is continuous based on the evidence provided the observed time series data.

In the equivalence delta-epsilon proofs of continuity, δ is the distance in phase space and ϵ is the distance of the images. For a given choice of embedding dimension m, the distance in the phase space is calculated as $\delta_{ij} = |x_i - x_j|$ and the distance between their image is calculated as $\epsilon_{ij} = |x_{i+1} - x_{j+1}|$ for all i and j. It is useful to construct the average of the values of ϵ_{ij} conditional on the corresponding values of δ_{ij} satisfying $\delta_{ij} < r$ and define the average as $E(r)$. It is expected to have $E(r) \to 0$ as $r \to 0$ for a perfectly deterministic system with continuous f, whereas if the underlying system is stochastic the convergence may not happen as a point x_i may have different images. The statistic for the Kaplan test is defined as $K \equiv \lim_{r\to 0} E(r)$. The non-zero value of K can be interpreted as "goodness of fit" measure from fitting a continuous model of some fixed order to an infinite amount of data. If this measure is smaller for the observed data than for surrogate data generated by a model that satisfies a stated null hypothesis, then the null hypothesis should be rejected (Barnett et al. (1995)). As stated by Garcia (2007), another way of interpreting the non-zero value of K is as the level of nondeterminism or the amount of noise in the data. If the system is stochastic the amount of K is expected to be higher for nearly deterministic ones. Therefore, we should reject the null hypothesis when K on the observed data is smaller than K on the surrogate data. Since the distribution of the statistic table is not laid out, Kaplan proposes two different methods to compute the minimum value of K obtained from the surrogates. The first approach is to estimate the minimum value of K from a finite sample of surrogates, and impute that to the population of the surrogates. Another approach involves the computation of the mean and standard error of the values of K from the finite sample and the subtraction of a multiple of (2 or 3) to obtain the an estimate of population minimum (Alharbi (2009)).

This chapter uses twenty surrogate time series using the same approach suggested by Kaplan. The Surrogate data is a random realization from time series data of the energy markets generated with the same mean, variance, and autocorrelation function as the original data. Moreover, the lag embedded time series is also generated using 2, 3, 4, and 5 dimensional spaces.

The result of the Kaplan test for the production of crude oil for the four series are reported in Table 3.5[5]. Also, the results of the Kaplan test are graphically summa-

[5]The Kaplan test was carried out using the original MATLAB codes provided with gratitude by

rized in Appendix B of this chapter. The plot of *delta* versus *epsilon* shows the sign of discontinuity is all cases, when *delta* goes to zero, *epsilon* does not.

Results of the Kaplan Test

The null hypothesis of the Kaplan test is stochastic linearity of the process. As mentioned by Barnett et al. (1995), the Kaplan test involves a strong power against chaos and is expected not to accept the null facing with chaotic series although current form of test can either accept or reject linearity. The Kaplan test is designed where the dynamical functional form underlying the time series data is unknown, and the main purpose is to study if there is evidence of deterministic mechanism or not.

The results with the Kaplan test are displayed in Table 3.5 for embedding dimension(m) 2, 3, 4 and 5. The mean, minimum, and standard deviations are computed over twenty surrogates for each time series. Moreover, K statistic is calculated for each series. The null of stochastic linearity is rejected when the computed K for each daily spot price of energy product is less than the minimum of K statistic from surrogates or KS_{min} that is $K < KS_{min}$. As recommended by Kaplan, the t-statistic is calculated on the results significance as: $t = \frac{K - KS_{mean}}{KS_{sd}}$, where KS_{mean} and KS_{sd} are the mean and standard deviation for KS values for surrogates.

As can be observed from Table 3.5, the test rejects the null of linearity in the U.S. production of crude oil, OPEC members production of crude oil, and the world production of crude oil in all dimensions. The interesting fact is the null of linearity cannot be rejected

Professor Daniel Kaplan and modified based on the analysis in this study:
Kaplan, Daniel. (1996). Delta-Epsilon [Computer MATLAB Software]. Retrieved from: http://www.macalester.edu/ kaplan/software/.

Table 3.5: Kaplan Test Statistic: Production of Crude Oil

Log Level	Embedding Dimension	Mean K on surrogates	Std. dev. of K on surrogates	Min K on surrogates	K statistic on energy data	t-statistic
U.S. Production	2	0.0233	0.0021	0.0191	0.0152	-3.857
	3	0.0227	0.003	0.0167	0.0121	-3.533
	4	0.0233	0.0037	0.0159	0.0091	-3.837
	5	0.0214	0.0035	0.0144	0.0072	-4.057
OPEC Production	2	0.0181	0.0018	0.0145	0.0122	-3.277
	3	0.0186	0.0033	0.012	0.01	-2.606
	4	0.018	0.0031	0.0118	0.0082	-3.161
	5	0.0184	0.0036	0.0112	0.0074	-3.055
Non-OPEC Production	2	0.0043	0.0004	0.0034	0.0039	-0.962
	3	0.0042	0.0006	0.0029	0.0038	-0.664
	4	0.0042	0.0005	0.0030	0.0038	-0.677
	5	0.0041	0.001	0.0021	0.0037	-0.4
World Production	2	0.0078	0.0006	0.0065	0.006	-2.883
	3	0.008	0.0009	0.0060	0.0052	-2.821
	4	0.008	0.0011	0.0058	0.0045	-3.181
	5	0.0082	0.0013	0.0056	0.0041	-3.153

Notes: K is the Kaplan test statistic. Twenty surrogates were used to compute the mean and standard deviation. The sample period for the U.S. field production of crude oil is from 1920:01 to 2002:12. The sample periods for OPEC, Non-OPEC, and the World production of crude oil is from January 1973 to January 2012.

for non-OPEC supply of crude oil. The phenomenon can be attributed to slow growth rate in oil supply in those countries. As can be observed in the plot of time series of non-OPEC crude oil production members, the growth rate of the series is a slow trend, indicating that the non-OPEC members production of crude oil has not been significantly influenced by the various fluctuations in the market.

3.5.3 Tests for Nonlinearity

In this section, other forms of nonlinearity will be examined by employing more focused tests such as Hinich bispectrum test, which explore third order nonlinearity.

The Hinich Bicovariance Test

As noted by Patterson and Ashley (2000a), the Hinich Bicovariance test assumes x_t is a realization from a third-order stationary stochastic process and tests for serial independence. It uses the sample bicovariances of the data. The (r, s) sample bicovariance is defined as

$$C_3(r, s) = (N - s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s} \quad 0 \leq r \leq s. \tag{3.5}$$

The sample bicovariances, Equations 3.5, are a generalization of a skewness parameter. The $C_3(r, s)$ are all zero for zero mean, serially $i.i.d$ data. Non-zero values for the $C_3(r, s)$ are projected from observations in which x_t depends on lagged cross-products, such as $x_{t-i}x_{t-j}$ and higher order terms.

Let $G(r, s) = (N - s)^{0.5} C_3(r, s)$ and define X_3 as

$$X_3 = \sum_{s=2}^{\phi} \sum_{r=1}^{s-1} [G(r, s)]^2 \tag{3.6}$$

Under the null hypothesis that x_t is a serially $i.i.d$ process, Hinich and Patterson (1995) show that X_3 is asymptotically distributed as $\chi^2[\phi(\phi - 1)/2]$ for $\phi < N^{0.5}$. They recommend using $\phi = N^{0.4}$ based on their simulations. Under the assumption that $E((x_t)^{0.5})$ exists, the X_3 statistic will discover nonzero third-order correlations. It can be considered as generalization of the Box-Pierce portmanteau statistics – see Hinich and Patterson (1985) for more discussion.

The Hinich Bispectrum Test

A process is said to be third-order nonlinear dependence if the skewness function in the frequency domain is not flat as a function of frequency pairs. The definition of the square of the skewness function is shown in Equation 3.8. This form of the nonlinearity is called third order, since the skewness function is a normalization of the Fourier transform of the third-order autocovariances. That Fourier transform is called the bispectrum (Barnett et al. (1997)).

The Hinich bispectrum test is a nonparametric test that examines the third-order moments (bicovariance) of the data in the frequency domain to obtain a direct test for a nonlinear generation mechanism, regardless of any linear independence that might be present in the data. Therefore, when the tests rejects the null (the skewness function is flat), there is no need to check the possibility that the linear prewhitening model has failed to remove all linear serial dependence in the data (Ashley and Patterson (2006)).

Hinich (1982) develops this test for flatness of bispectrum. He argues that the bispectrum in the frequency domain is easier to interpret than multiplicity of the third-order moments $c_{xxx}(r, s) : s \leq r, r = 0, 1, 2 \cdots$ in the domain. Barnett and Hinich (1993) explain the computation of the test statistic. For frequencies f_1 and f_2 in the principle domain

$$\Omega = (f_1, f_2) : 0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1$$

is the Hinich bispectrum of the series at frequency pair (f_1, f_2), and its double Fourier transformation of the third-moments function is:

$$B_{xxx}(f_1, f_2) = \sum_{r=-\infty}^{r=\infty} \sum_{s=-\infty}^{s=\infty} c_{xxx}(r, s) exp[-2\pi(f_1 r + f_2 s)]. \tag{3.7}$$

The square of the skewness function $\Gamma(f_1, f_2)$ is defined in terms of the bispectrum as:

$$\Gamma^2(f_1, f_2) = \frac{|B_{xxx}(f_1, f_2)|^2}{S_{xx}(f_1) S_{xx}(f_2) S_{xx}(f_1 + f_2)} \tag{3.8}$$

where $S_{xx}(f)$ is the (ordinary power) spectrum of x_t at frequency f. If the time series x_t is linear then the squared of skewness function $\Gamma^2(f_1, f_2)$ is constant over all frequency pairs (f_1, f_2) in Ω, and the skewness function $\Gamma^2(f_1, f_2)$ is zero over all frequencies if x_t is Gaussian. Linearity and Gaussianity can be tested using a sample estimator of the skewness function $\Gamma^2(f_1, f_2)$ – see Barnett and Hinich (1993) for more details on computation of the test and Hinich (1982) for more details on the test.

Engle LM Test

The test was proposed by Engle (1982) to examine nonlinearity in the second moment, particularly for ARCH disturbances. The test employs the Lagrangian multiplier procedure and runs the OLS regression and saves the residuals. Then the next procedures is to regress the squared residuals on a constant and p lagged values of the squared residuals and test NR^2 as a χ^2_p.

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \hat{\varepsilon}_{t-j}^2 + u_t \tag{3.9}$$

As most Lagrange multiplier tests, the test statistic is based on the R^2 of the regression. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the above regression is asymptotically distributed as χ^2_p.

The McLeod-Li Test

McLeod and Li (1983) developed a portmanteau test for nonlinear statistical dependence in the squared-residual autocorrelations of fitted ARMA models. The tests looks at the autocorrelation function of the squares of the prewhitened data and tests whether $corr(x_t^2, x_{t-j}^2)$ is nonzero for some j. The autocorrelation at the lag j for the squared residuals x_t^2 is estimated by

$$\hat{r}(j) = \frac{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)(x_{t-j}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)}, \quad where \hat{\sigma}^2 = \sum_{t=1}^{N} \frac{x_t^2}{N} \tag{3.10}$$

Under the null hypothesis that x_t is an $i.i.d$ process, McLeod and Li (1983) showed that, for sufficiently large and fixed L,

$$Q = N(N+2)\sum_{j=1}^{L} \frac{\hat{r}^2(j)}{N-j} \tag{3.11}$$

is asymptotically χ^2_L under the null hypothesis of a linear generating mechanism for the data. They have set $L = 20$ for their small-sample simulation in their examination.

The Tsay Test

The Tsay (1986) test explicitly looks for quadratic serial dependence in the data, using quadratic terms lagged up to K periods. Let the $K = k(k+1)/2$ column vectors $V_1, ..., V_k$

contains all the unique cross-products of the form $x_{t-i}x_{t-j}$, where $i \in [i,k]$ and $j \in [j,k]$. Let $\hat{v}_{t,i}$ denote the projection of $v_{t,i}$ on the subspace orthogonal to $x_{t-1},...,x_{t-k}$, which is the residuals from a regression of $v_{t,i}$ on $x_{t-1},...,x_{t-k}$. The parameters $\gamma_i,...,\gamma_k$ are estimated by applying OLS to the regression equation:

$$x_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i \hat{v} i, t + \eta_t \tag{3.12}$$

Then, the Tsay test statistic is the usual F statistic for testing the null hypothesis that $\gamma_1,...,\gamma_k$ are all zero.

The Results for Nonlinearity Tests

The results (significance levels) for the Hinich bicovariance, the Hinich bispectrum, the McLeod-Li, the Engle, and the Tsay test are reported in Table 3.6 and 3.7, for both asymptotical distribution and bootstrapping simulation[6].

As stated by Patterson and Ashley (2000a), the described tests are only asymptotically justified similar to most econometrics procedures. Therefore, the significance levels of all the tests are routinely bootstrapped. Also, the significance levels based on the asymptotic distributions are computed – see Patterson and Ashley (2000a) for further details on the bootstrap simulation.

In the Hinich bicovariance test, I use $\phi = N^{0.4}$ based on the Hinich and Patterson (1985)'s simulation, where N is the sample size for each individual series. The test is

[6] The source of the nonlinear software was thankfully provided by Professor Douglas M. Patterson. The source, instruction on running the toolkit program, and analysis can be found in Patterson and Ashley (2000a): "A Nonlinear Time Series Workshop: A Toolkit for Detecting and Identifying Nonlinear Serial Dependence", Kluwer Academic Publishers: Norwell. Available at: http://www.wkap.nl/.

calculated using up to 15 lags and also with the number of bootstrap iterations equal to 100. As displayed by the results, based on the bootstrapped as well as asymptotic distributions, the test rejects the null hypothesis that the time series of oil production is a serially *i.i.d* process at the 10% significance level for the U.S., OPEC, and the world production of crude oil. The null hypothesis cannot be rejected in non-OPEC production of crude oil at the 5% significance level.

The Hinich bispectrum test examines the third order moments (bicovariance) of the data in frequency domain to obtain a direct test for a nonlinear generating mechanism. More importantly, this test focuses on nonlinear serial dependence, and it is different than the procedure of the sample bicovariance data than the Hinich bicovariance test described earlier. The Hinich bispectrum test accepts the linearity if it cannot reject the flatness of bispectrum, and accepts the Gaussianity if the bispectrum is flat and is also equal to zero. As can be observed in the Table 3.6, the results of Gaussianity indicate extremely small p-values for oil production variables in the case of asymptotic distribution. As a result the null hypothesis of the Gaussianity is rejected at the 10% significance level.

Moreover, the null of linearity for time series of oil production exhibits significant results by very small p-values for the 80 percent fractile bispectrum linearity test for time series data of crude oil production. Hence, the null hypothesis of the linearity is also rejected at the 10% significance level. In other words, the rejection of linearity provides strong evidence for the presence of the third order nonlinearity in the data generating process of crude oil production as also noted by Barnett et al. (1997).

Ashley and Patterson (2006) show that the bispectrum $B_{xxx}(f_1, f_2)$ is consistently estimated using an average of appropriate triple products of the Fourier representation of the observed time series. The average is taken over a square containing M adjacent frequency pairs. Hinich (1982) showed that M must be above the $N^{0.5}$ to consistently estimate $B_{xxx}(f_1, f_2)$. The results here are caculated for M equals to $N^{0.6}$.

Table 3.6: Significance Level for Nonlinearity Tests
Asymptotic Distribution

Series	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.085	0.000
Bispectral (Gaussianity) ($M = N^{0.6}$)	0.000	0.000	0.000	0.000
Bispectral (Linearity) ($M = N^{0.6}$)	0.000	0.005	0.000	0.000
Engle($p = 5$)	0.000	0.014	0.066	0.000
McLeod-Li($L = 24$)	0.000	0.000	0.004	0.001
Tsay ($k = 5$)	0.000	0.000	0.203	0.000

Notes: The sample period for the U.S. field production of crude oil is from 1920:01 to 2002:12. The sample period for OPEC, non-OPEC, and the world production of crude oil is from January 1973 to January 2012.

Table 3.7: Significance Level for Nonlinearity Tests
Bootstrap Simulation

Series	**U.S. Production**	**OPEC Members Production**	**Non-OPEC Members Production**	**World Production**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.120	0.000
Engle($p = 5$)	0.003	0.030	0.090	0.000
McLeod-Li($L = 24$)	0.001	0.000	0.000	0.000
Tsay ($k = 5$)	0.000	0.000	0.220	0.000

Notes: Number of bootstrap iterations =100
The sample period for the U.S. field production of crude oil is 1920:01 to 2002:12. The sample period for OPEC, non-OPEC, the world production of crude oil is from January 1973 to January 2012.

The Engle LM test (1982) examines nonlinearity in the second moment. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the regression Equation 3.9 is asymptotically distributed as χ^2_p. The results are reported for p (lagged values) equals to 5, and they exhibit substantially small p-values for the U.S. and the world production of crude oil at the 10% significance level in both asymptotic and bootstrapped distributions. The null hypothesis of the Engle-LM test is rejected at the 5% significance level for the OPEC crude oil production. The Engle-LM test rejects the null hypothesis for non-OPEC production of crude oil at the 1% significance level. Following the literature, the results are reported for p=5 for the Engle-LM test.

The null hypothesis of the McLeod and Li (1983) test is rejected for up to 24 lags in bootstrapped and asymptotic distributions. As shown in Tables 3.6 and 3.7, the results yield very small p-values at the 10% significance level. The results are calculated for $L = 24$ for the McLeod and Li test.

The result of the Tsay test is reported for the value of $k = 5$. The reported results based on the bootstrapped as well as asymptotic distributions indicate that the null hypothesis is rejected at the 10% significance level of all the series excluding the non-OPEC production of crude oil. The null hypothesis of the Tsay test cannot be rejected in non-OPEC crude oil production at the 10% significance level.

Therefore, based on the bootstrapped and asymptotic distributions, the results for the nonlinear tests reflect that the employed time series of the U.S., OPEC, and the world production of crude oil have clear evidence of nonlinearity in their structure. The time series data of non-OPEC production of crude oil does not reveal signs of nonlinearity in the data generating mechanism.

3.6 Summary and Conclusion

There exists an extensive literature on modeling the future of crude oil production. In order to attain an accurate projection about the future of oil supply, it is essential to employ a model specification that is supported by the data and reflects the underlying mechanism of the market's dynamics. This chapter presents a new approach to assess the dynamic structure in the data generating mechanism of crude oil production in the context of a nonlinear mechanism. The study employs statistical and econometrics techniques, which involves the most widely used univariate tests, to investigate the nonlinear dependence in the supply side of the energy market.

To study the time series data of crude oil production in context of the nonlinear mechanism, the study utilized monthly observations of the U.S. field production of crude oil from January 1920 to June 2011, OPEC production, non-OPEC, and the world production of crude oil from January 1973 to January 2012. The results indicate that the observed time series data on production of crude oil ,excluding non-OPEC countries production, exhibit deep nonlinearity in their structure.

The BDS test is a test of general nonlinearity in the process, against all other possible alternative nulls of linearity and has a high power against the numerous classes of alternative hypotheses. The results of the BDS test indicate that the linearity is rejected in all the time series data at the 5% level of significance, excluding non-OPEC production of crude oil. However, the null of nonlinearity is rejected at the 1% level of significance in the non-OPEC time series data. The Kaplan's test features seem to be comparable to the BDS test. However, Barnett et al. (1997) state that in their experiments the Kaplan test, unlike the BDS test, acquired the right answer with both large and small samples. The results for Kaplan tests detect evidence of nonlinearity in all the time series data apart from the non-OPEC production of crude oil. Given the results from the BDS test and the Kaplan test, there are convincing evidence of the importance of employing more particular tests that explore the more detailed features of nonlinearity.

The Hinich bicovariance test focuses on the third-order moments (time domain) of the data and detected nonlinearity in each series excluding non-OPEC at the 5% significance level. The Hinich bispectrum test examines the lack of third-order nonlinear dependence (frequency domain), and the associated Gaussianity test, is a test of a necessary and not sufficient condition for Gaussianity[7]. The results of the Hinich bispectrum suggest that the observed time series data in the supply side of the energy market are generated by a nonlinear and a non-Gaussian process. The Engle Lagrangian multiplier (LM) test focuses on the nonlinearity in the second moment. The null hypothesis of no ARCH-type disturbances is rejected by the Engle-LM test in the U.S. field and the world production of crude oil. The null hypothesis of non-ARCH disturbances is rejected in OPEC and non-OPEC production of crude oil at the 1% and 5% levels of significance, respectively. The McLeod-Li test also rejects the null hypothesis of linearity in the variance for each individual series. Finally, the Tsay test rejects the null hypothesis of linearity in each individual series. However, the null hypothesis cannot be rejected in the case of the non-OPEC countries production of crude oil in Tsay test. Therefore, all the tests detect strong evidence of nonlinear structure in the time series data of the U.S. field, OPEC, and the world production of crude oil, indicating that the employed series are generated

[7] See Barnett et al. (1997) for more details.

by a nonlinear mechanism. However, non-OPEC production of crude oil reveals different and notable results. The dynamics of the non-OPEC production time series data is not nonlinear in its structure.

The results of the underlying mechanism for non-OPEC production can be attributed to the nature of the market for those countries. As displayed in Figure 3.2, the growth rate of the production market for non-OPEC production exhibits a steady rate and has not been significantly influenced by exogenous shocks, and non-OPEC time series production data does not reflect evidence of nonlinear structure in its data generating mechanism. OPEC production of crude oil, however, has been frequently disrupted as a result of geopolitical events, and clear indications of nonlinearity are reflected in the OPEC production time series observations.

To enhance the projection of the production of the crude oil market, one needs to consider the nature of the energy market in order to examine nonlinear dynamics in its data generating mechanism. As explained by Ashley and Patterson (2006), if the nonlinearity is present in the data, choosing a nonlinear time series can provide more reasonable post-sample forecasting ability. Therefore, in consideration of the significance of the production of crude oil in the aggregate economy, detecting nonlinear dynamics in the market's fundamentals will allow researchers to utilize a more accurate time series modeling, which is reasonably close to the data generating mechanism. A compliant model that is supported by the data will provide an accurate empirical results for projection of the crude oil production market.

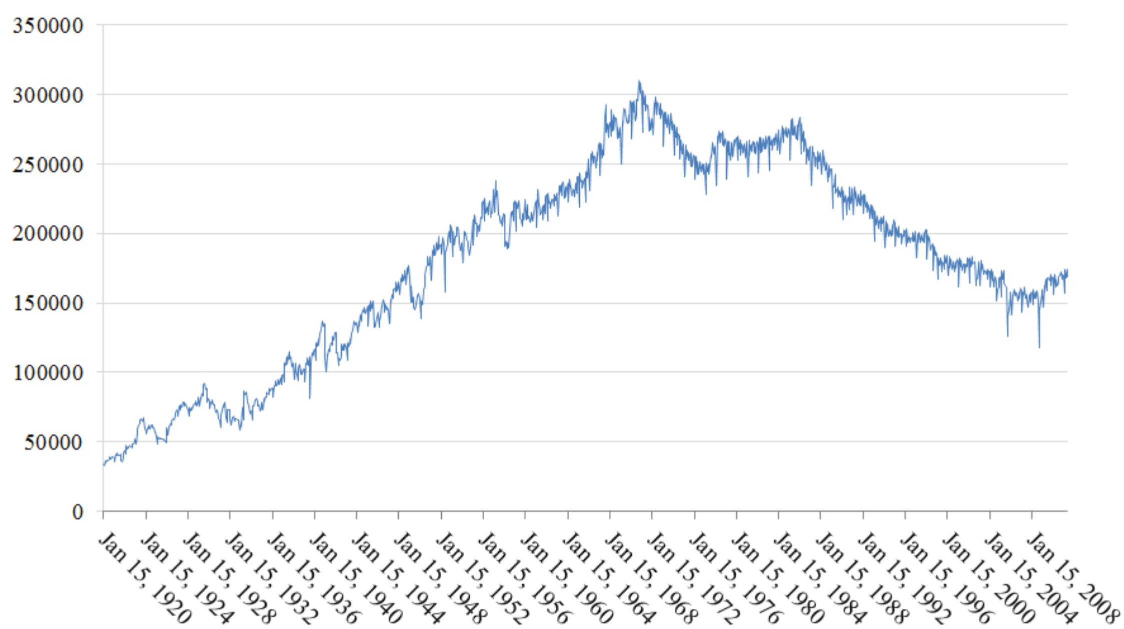
Appendix A: Data Description, Key Terms and Definitions

The variable Crude Production is defined as follows:

The volume of crude oil produced from oil reservoirs during given periods of time. The amount of such production for a given period is measured as volumes delivered from lease storage tanks (i.e., the point of custody transfer) to pipelines, trucks, or other media for transport to refineries or terminals with adjustments for (1) net differences between opening and closing lease inventories, and (2) basic sediment and water (BS&W).[8]

Figure 3.3 represents the U.S. field production of crude oil.

Figure 3.3: Monthly U.S. Field Production of Crude Oil (Thousand Barrels/Day)

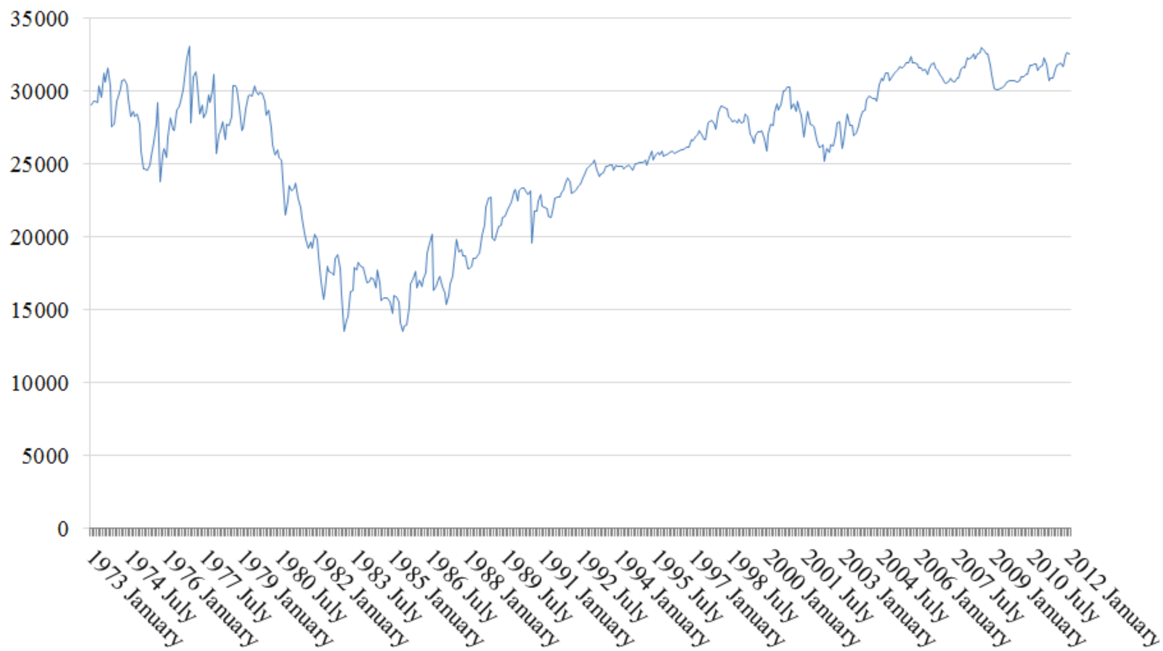


Data Source: Energy Information Administration (EIA)

[8]Energy Information Administration (EIA).

Figure 3.4 represents the Organization of the Petroleum Exporting Countries (OPEC) production of crude oil. The growth rate of the production of OPEC countries from January 1973 to January 2012 is nearly 12 percent.

Figure 3.4: Monthly OPEC Production of Crude Oil (Thousand Barrels/Day)



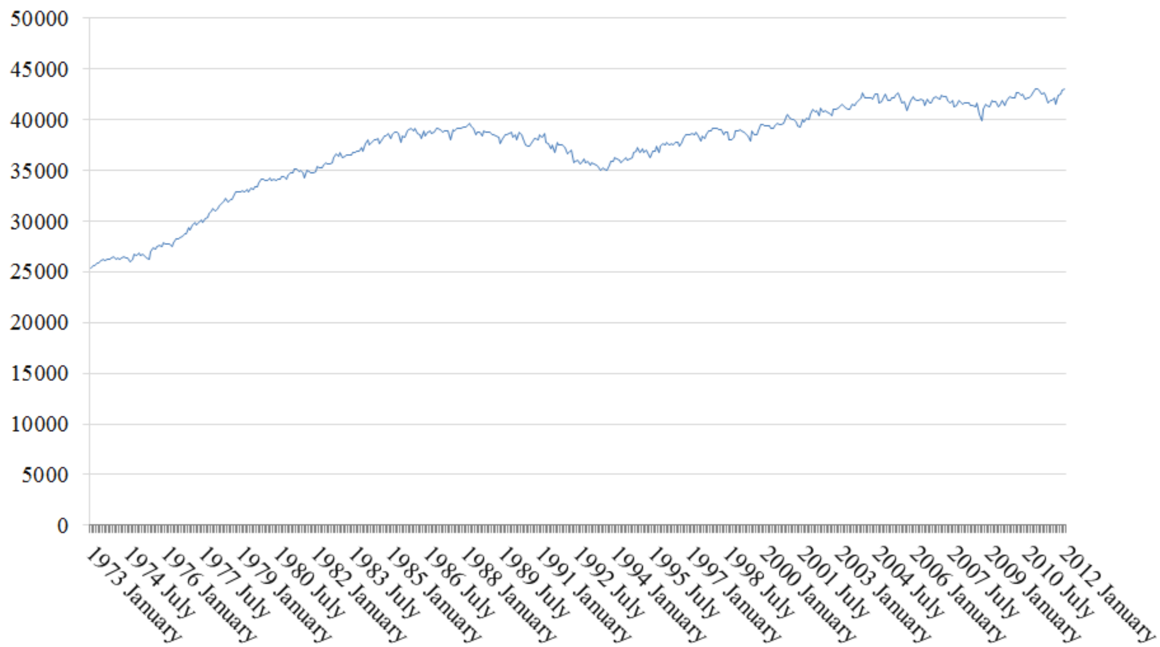
Data Source: Energy Information Administration (EIA)

Figure 3.5 represents the non-OPEC Countries production of crude oil. The growth rate of the production of non-OPEC countries from January 1973 to January 2012 is nearly 70 percent.

Figure 3.6 represents the World's production of crude oil. The growth rate of the world production from January 1973 to January 2012 is nearly 38.96 percent.

Figures 3.7, 3.8, 3.9, and 3.10 show the log and the differenced log of the individual

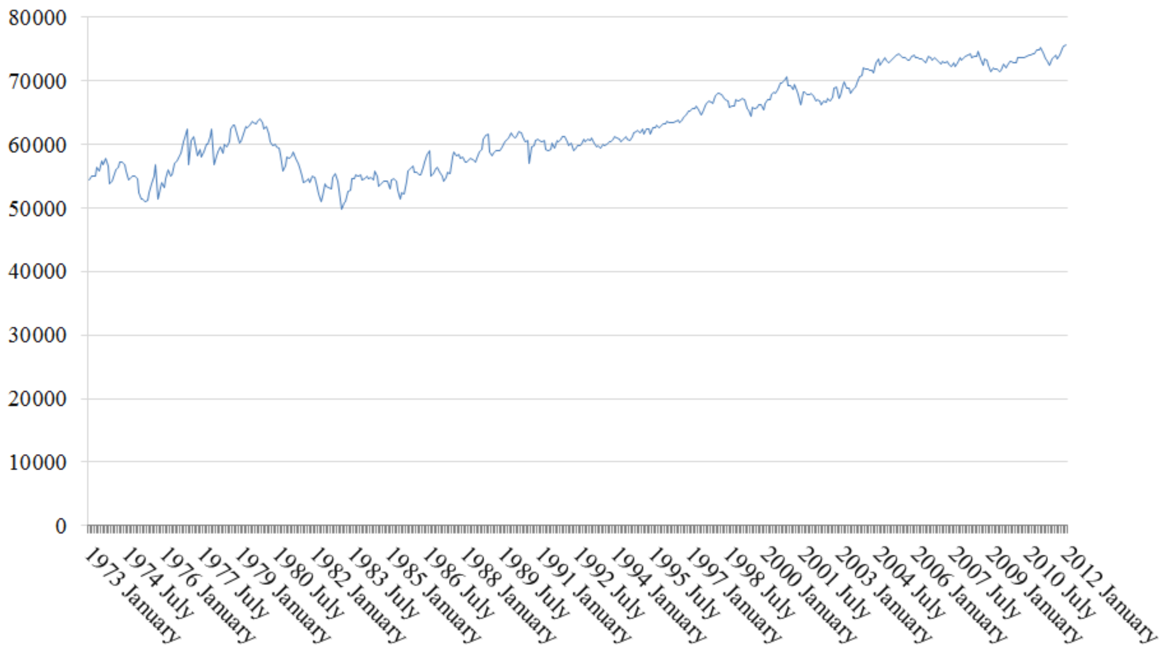
Figure 3.5: Monthly non-OPEC Production of Crude Oil (Thousand Barrels/Day)



Data Source: Energy Information Administration (EIA)

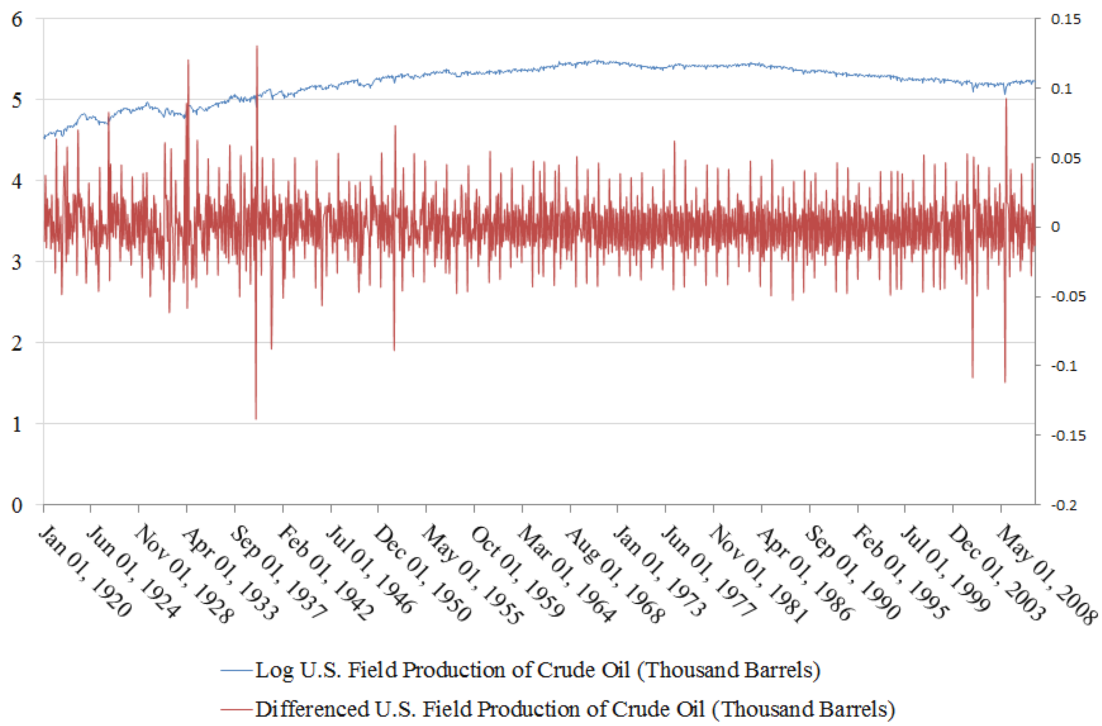
series.

Figure 3.6: Monthly World Production of Crude Oil (Thousand Barrels/Day)



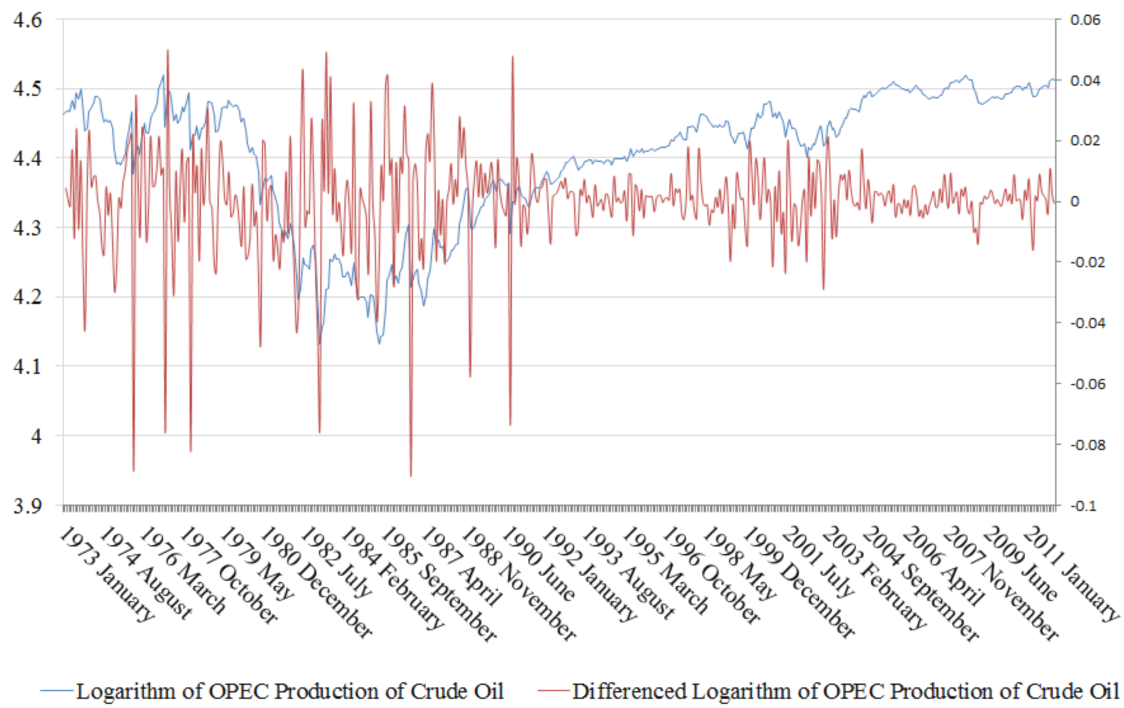
Data Source: Energy Information Administration (EIA)

Figure 3.7: Log and Differenced log of U.S. Field Production of Crude Oil (Thousand Barrels)



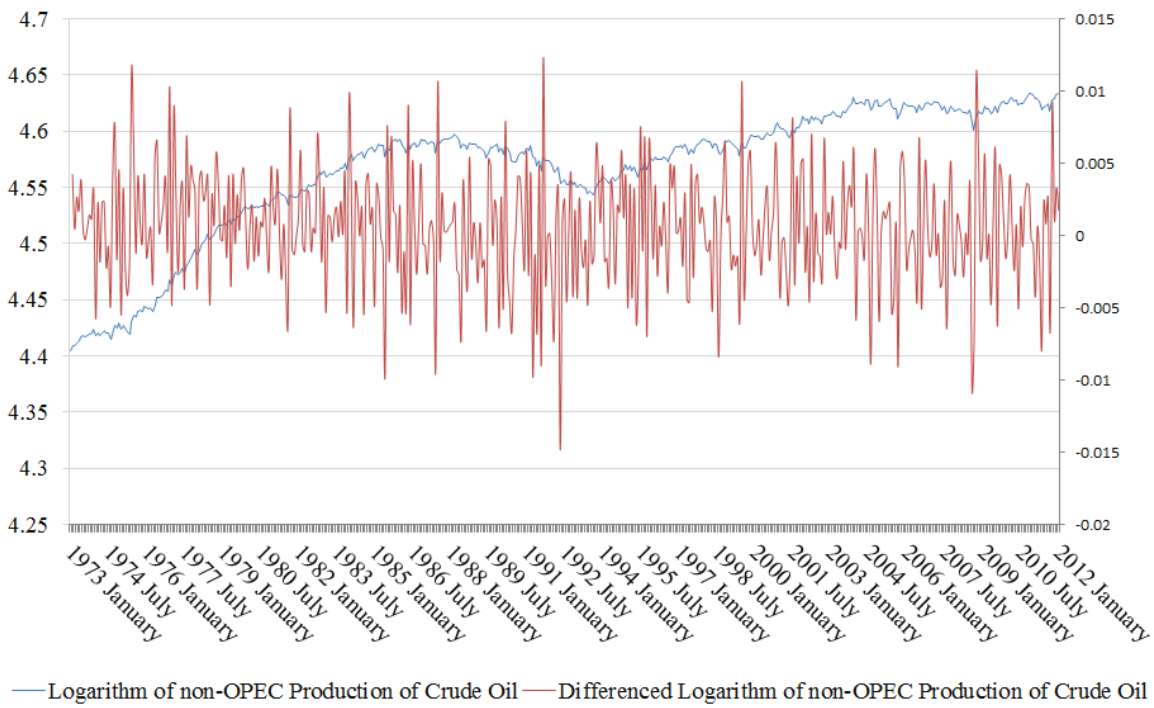
Data Source: Energy Information Administration(EIA)

Figure 3.8: Log and Differenced log of OPEC Production of Crude Oil (Thousand Barrels/Day)



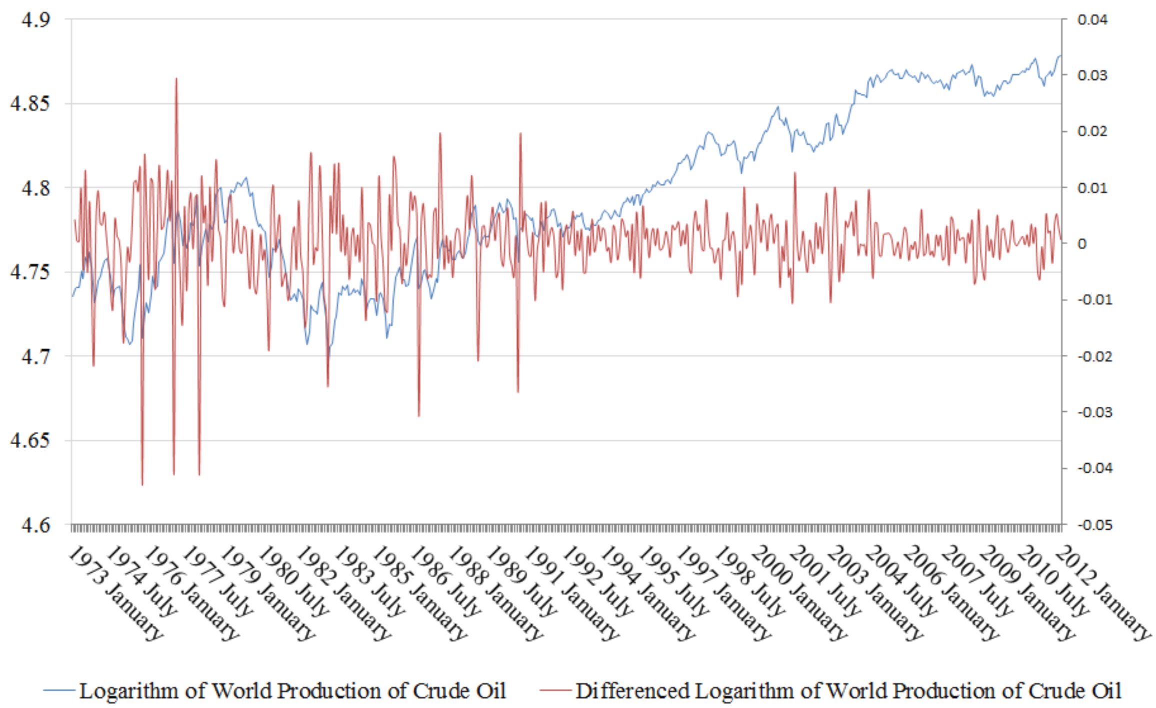
Data Source: Energy Information Administration(EIA)

Figure 3.9: Log and Differenced log of non-OPEC Production of Crude Oil (Thousand Barrels/Day)



Data Source: Energy Information Administration(EIA)

Figure 3.10: Log and Differenced log of World Production of Crude Oil (Thousand Barrels/Day)



Data Source: Energy Information Administration(EIA)

Appendix B: Figures of the Kaplan Results for Embedding Dimension $2 - 5$

Figures 3.11 to 3.14 display the Kaplan tests results. In other words, the plots of δ versus ϵ are shown in Figures 3.11 to 3.14. The signs of continuity are revealed when δ goes to zero, so ϵ does. The legend of each graph is explained as:

- U.: U.S. Production of Crude Oil
- OP: OPEC Members Production of Crude Oil
- No: Non-OPEC Members Production of Crude Oil
- Wo: World Production of Crude Oil

Figure 3.11: *Delta* vs. *Epsilon*, The Kaplan Test Results for Production of Crude Oil, Lag Embedded=2

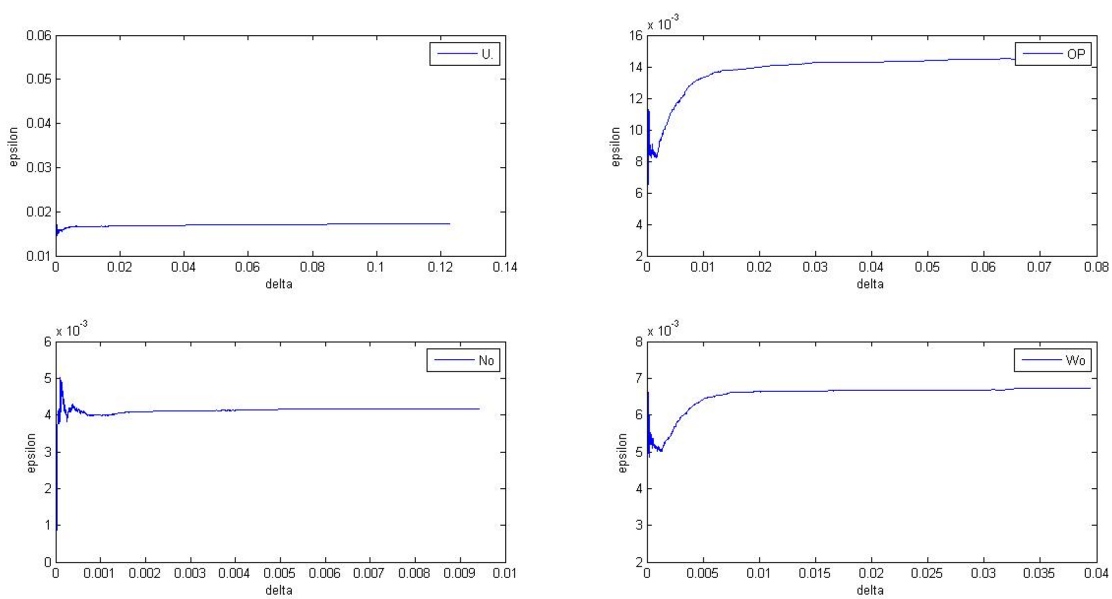


Figure 3.12: *Delta* vs. *Epsilon*, The Kaplan Test Results of Production for Crude Oil, Lag Embedded=3

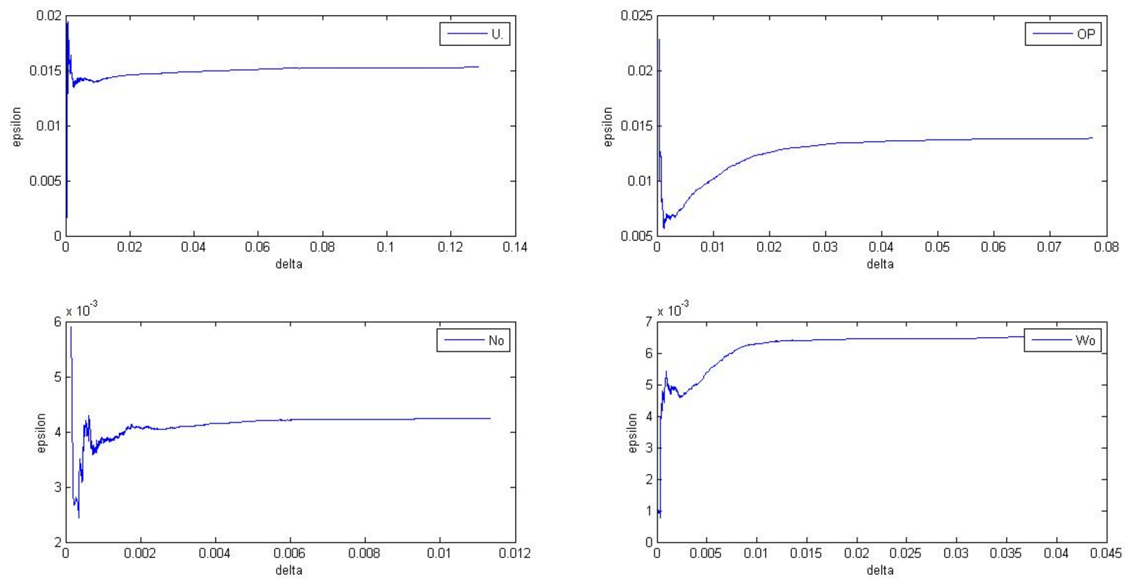


Figure 3.13: *Delta* vs. *Epsilon*, The Kaplan Test Results of Production for Crude Oil, Lag Embedded=4

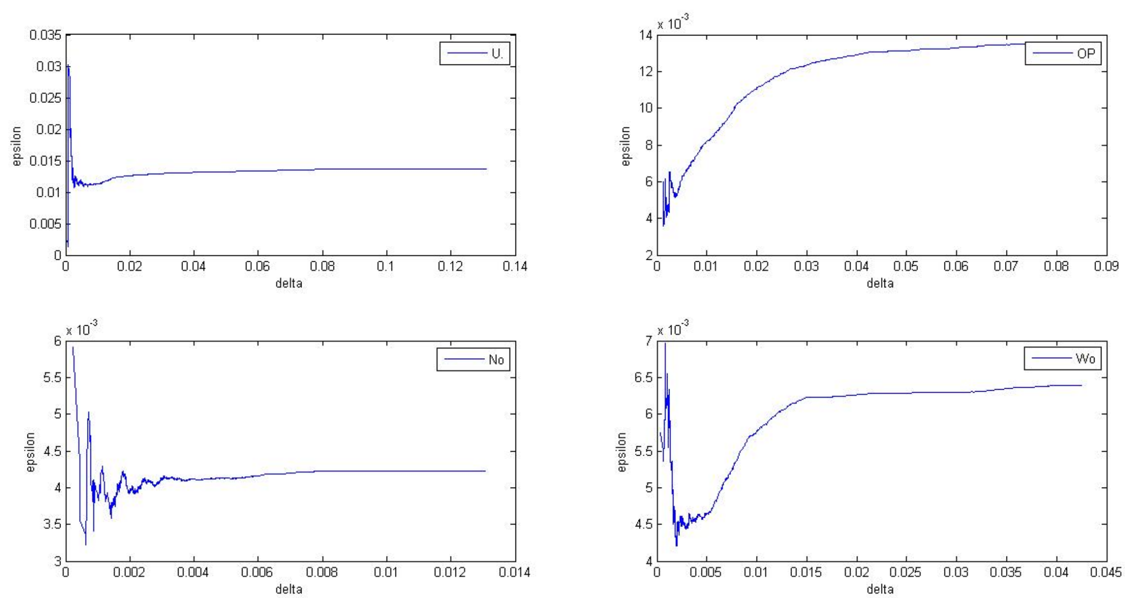
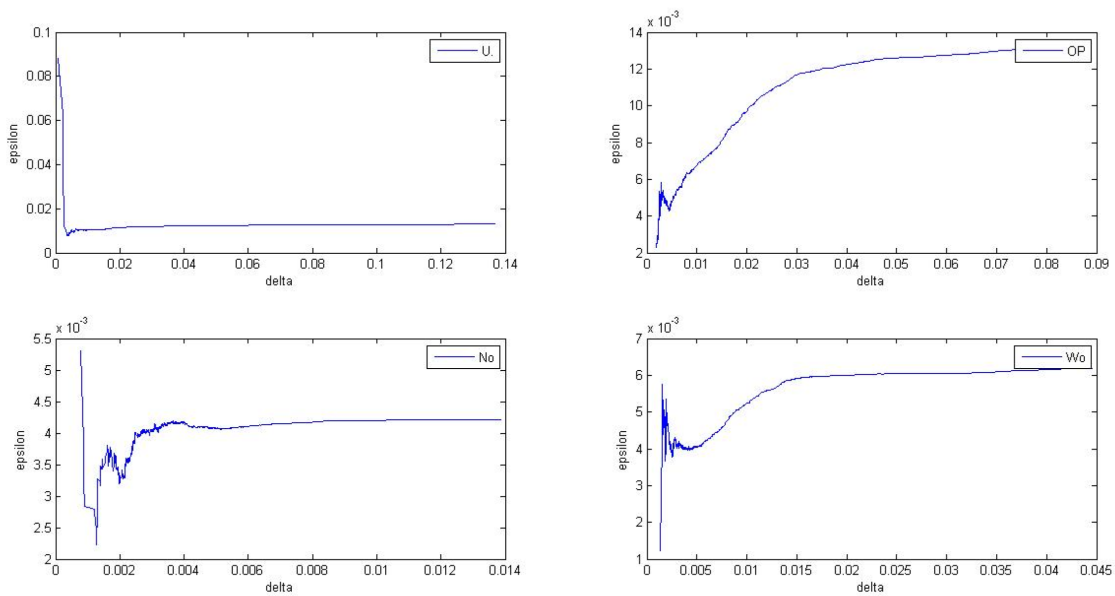


Figure 3.14: *Delta* vs. *Epsilon*, The Kaplan Test Results of Production for Crude Oil, Lag Embedded=5



Chapter 4

Dynamic Structure of the Spot Price of Crude Oil: Does Time Aggregation Matter?

4.1 Introduction

The majority of existing studies on dynamic structure of crude oil price have focused on daily prices of the market, and there is little mention of existence of nonlinearity in the other time frequencies such as monthly prices. Main studies that utilized daily observations of the energy market, such as Kyrtsou et al. (2009) among many others, have found evidence of nonlinear dependencies in the energy market. The main goal of this chapter is to employ various levels of time aggregation of the energy market including higher dimensional cases, different sample sizes and frequencies, and dividing the daily observations into sub-periods to assess the dynamic structure of the energy sector in context of nonlinear mechanism. The chapter addresses the gap in the literature for an inclusive investigation at different level of time aggregation of the energy market and is following the approach of Patterson and Ashley (2000b) on the analysis of stock market return. As they state, nonlinearity is considered as a possible procedure of stochastic dependence. The volume of the dependence decreases with the increase in the time between observations and nonlinear stochastic cannot be captured if the time within observations is adequately large.

Nonlinearity in energy market was also examined by Kyrtsou and Serletis (2006), where they discuss a number of widely used univariate tests from dynamical system theory. They apply the tests to daily observations of the energy market for nearly 15 years and find indications consistent with nonlinear dependencies in each of the markets. Identifying nonlinearity in the price of crude oil is a vital key to plausibly and accurately forecast this major variable, which is one the most influential factors in the aggregate economy. This chapter, motivated by uncovering the energy market fundamentals, will discover that at which time aggregation level the stochastic dependence or nonlinearity cannot be detected in the price of crude oil. To this end, this study incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. It employs daily spot prices on crude oil, West Texas Intermediate (WTI-Cushing) from January 2, 1986 to April 30, 2012 consists of 6642 observations obtained from Energy Information Administration (EIA). The period of time analyzed is divided into three sub-periods: January 2, 1986 to December 30, 1993 consisting of 2039 observations, January 3, 1994 to December 31, 2003 consists of 2511 observations, and January 5, 2004 to April 30, 2012 consists of 2092 observations. Moreover, the monthly time series observations on the price of crude oil is utilized that is real values on the price of crude oil, West Texas Intermediate. The sample period of study is from January 1970 to March 2011 for a total of 494 observations as well as two sub-samples: January 1970 to December 1991 for a total of 263 observations and January 1992 to March 2011 for a total of 231 observations. Incorporating monthly observations to assess the existence of nonlinear structures in the time series data generating mechanism of crude oil, when the time between observations increases, distinguishes the approach of this chapter from existing literature. This chapter is organized as follows. The next section reviews the related literature. Section Three describes the various employed datasets and related unit root analysis. Section Four discusses the inference methods as well as the results of performing the nonlinearity tests to examine the market data generating mechanism. A brief summary and conclusion for this chapter are offered in Section Five.

4.2 Literature Review

A large body of literature in analyzing the behavior of the energy market assesses the dynamic structure of daily observations. Kyrtsou et al. (2009) discuss a number of widely used univariate tests from dynamical system theory and apply them to the energy market. They apply these tests to daily observations of the energy market for nearly 15 years. They find indications consistent with nonlinear dependencies in each of the markets. They also suggest that an effective nonlinear model of energy prices would produce a deeper perception of the energy market fluctuations than existing linear models. Sertletis and Gogas (1999) test for deterministic chaos in the North American Natural Gas Liquids Market. They use the Lyapunov exponent estimator and they find that there is evidence consistent with a chaotic nonlinear generation process in natural gas liquid markets. Serletis and Andreadis (2004) use daily observations on West Texas Intermediate crude oil prices and Henry Hub natural gas prices and various tests from dynamical theory to support a random fractal structure for North American energy markets. The result is consistent with the reported result by Serletis and Gogas (1999) as they find evidence of nonlinear chaotic dynamics in North American natural gas liquids markets but not in crude oil and natural gas markets. Identifying nonlinearities and chaos in economic and financial data has attracted considerable attention as well.

Patterson and Ashley (2000b) analyze the behavior of the stock market return by examining daily, weekly, and monthly returns. Their results indicate that strong nonlinear dependence exists in daily and weekly sample intervals, however the nonlinear dependence is considerably reduced in monthly observations. Kyrtsou and Serletis (2006) discuss univariate tests for independence and hidden nonlinear deterministic structure in economic and financial time series. They apply the tests to Canadian exchange rate, using daily data over a 30-year period and they identify an interesting relationship between high-dimensional nonlinearity and shocks. Barnett et al. (1995) apply nonlinear tests to detect nonlinear behavior or chaos in various monetary aggregate data series, and discuss the controversy that has arisen about the available results. They use five inference methods to test for nonlinearity and chaos: the Hinich bispectrum test, the BDS test, the Lyapunov exponent estimator of Nychka, the White test, and the Kaplan test. The findings provide a possible explanation for the controversies that exist regarding empirical evidence of chaos in economic data. They also state that the source of controversies can be found in the lack of robustness of the inference. In another influential study, Barnett et al. (1997) explore the reasons for empirical difficulties with the interpretations of nonlinear and chaos tests' results that have increased over time. They design and run a single-blind controlled competition among the aforementioned five highly regarded tests for nonlinearity or chaos with 10 simulated data series. The results shows that although there are some clear differences among the power functions of the tests, there exists some consistency in their inferences across the method of inference. They also discuss different issues that need to be taken into consideration in interpreting the results.

As mentioned earlier, there are studies in the literature that focus on the daily time series of the energy market to examine the market's fundamentals. However, existing literature mainly focuses on the daily time series data when analyzing the market. In order to attain an inclusive perception of the data structure in the energy market, this chapter will incorporate monthly observations as well as carrying out the analysis by dividing daily observations into sub-periods. The approach will address the gap in the literature by exhausting all possible cases in time series of crude oil price.

4.3 Data Description and Unit Root Analysis

4.3.1 Daily Data

This chapter assesses the dynamic structure of the energy market by employing daily spot price on crude oil, West Texas Intermediate, from January 2, 1986 to April 30, 2012 consisting of 6642 observations obtained from Energy Information Administration (EIA). To perform the analysis on daily data, the data is divided into three sub-periods as follows:

- The first daily spot price sub-period is from January 2, 1986 to December 30, 1993 consisting of 2039 observations.
- The second daily spot price sub-period is from January 3, 1994 to December 31, 2003 consisting of 2511 observations.
- The third daily spot price sub-period is from January 5, 2004 to April 30, 2012 consisting of 2092 observations.

The sub-periods are divided such that at least one oil price shock or counter shock, when oil price experiences a sudden decline due to oversupply or recession, are in the period under investigation.

Unit Root Analysis

In order to conduct the nonlinear analysis, the first step is to test whether or not the log price of each individual series follows a random walk or has unit root. I employ two alternative conventional test procedures to deal with the behavior of the data, the augmented Dickey-Fuller test (ADF) and the Philips and Perron test (PP). The augmented

Table 4.1: Augmented Dickey-Fuller Unit Root Tests - Daily Spot Prices on WTI
Null Hypothesis: The log levels and the differenced log of the series have unit root
Lag length: Automatic Selection Based on SIC.

Log Level	**Daily Price WTI 01/02/1986 - 12/30/1993**	**Daily Price WTI 01/03/1994 - 12/31/2012**	**Daily Price WTI 01/05/2004 - 04/30/2012**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-3.134	-2.353	-1.852
p-value*	0.0984	0.4048	0.6790
DLog Level	**Daily Price WTI 01/02/1986 - 12/30/1993**	**Daily Price WTI 01/03/1994 - 12/31/2003**	**Daily Price WTI 01/05/2004 - 04/30/2012**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-18.606	-29.934	-23.759
p-value*	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.

Dickey-Fuller (ADF) test checks the existence of a unit root in an $AR(p)$ process. The unit root test is carried out under the null hypothesis $H_o : \beta = 0$ versus the alternative hypothesis $H_a : \beta < 0$ using the regression

$$\Delta y_t = c_t + \beta y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + e_t \tag{4.1}$$

where c_t is a deterministic function of the time index t and $\Delta y_j = y_j - y_{j-1}$ is the differenced series of y_t. The t-ratio of the statistic is computed by

$$ADF - test = \frac{\hat{\beta}}{std(\hat{\beta})} \tag{4.2}$$

where $\hat{\beta}$ denotes the least squares estimates of β, and the t-ratio is known as the *augmented Dickey-Fuller*(ADF) unit root test – see Dickey and Fuller (1981) for details. The error term is assumed to be homoscedastic and also the value of p is set such that the error is serially uncorrelated.

Furthermore, the Philips and Perron(1988) known as (PP) unit root test is employed to test whether or not the log level of the series exhibits a random walk behavior. The PP test differs from the ADF test in handling the serial correlation and heteroscedasticity in the errors, and it allows for errors not to be independently and identically distributed (*iid*). The PP unit root test is essentially based on Equation 4.1, but without the lag differences. While the ADF test corrects for the higher-order serial correlation by adding lagged difference terms to the right-hand side, the PP unit root test makes a non-parametric correction to account for residual serial correlation Maslyuk and Smyth (2008). Therefore, the PP test statistic is robust to a variety of serial correlation and time-dependent heteroscedasticity. The test regression for PP test is

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \tag{4.3}$$

where u_t is $I(0)$ and can be heteroscedastic. The PP test corrects for any serial correlations and heteroscedasticity in the error u_t of the test regression by modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$. Under the null hypothesis that $\pi = 0$, the PP statistic have the same asymptotic distribution as the ADF t-statistic and normalized bias statistic – see Philips and Perron (1988) for more details.

The t-statistics for the ADF and PP tests ($t_{(\hat{\beta})} and Z_{t_{(\hat{\pi})}}$) as well as the p-values for the log levels of the series are reported in Table 4.1 and Table 4.2.

In the specification of the unit root regressions for the ADF and the PP test in log level of the individual series, I included the constant term as well as the time trend to distinguish whether or not the series are "trend stationary" (TS) model, where a stationary component is added to a deterministic trend term. As the results show in

Table 4.2: Philips-Perron Unit Root Test - Daily Spot Prices on WTI
Null Hypothesis: The log levels and the differenced log of the series have unit root
Bandwidth: (Newey-West automatic) using Bartlett Kernel

Log Level	**Daily Price WTI 01/02/1986 - 12/30/1993**	**Daily Price WTI 01/03/1994 - 12/31/2012**	**Daily Price WTI 01/05/2004 - 04/30/2012**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-3.379	-2.723	-2.457
p-value*	0.0984	0.2265	0.3497
DLog Level	**Daily Price WTI 01/02/1986 - 12/30/1993**	**Daily Price WTI 01/03/1994 - 12/31/2003**	**Daily Price WTI 01/05/2004 - 04/30/2012**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-37.459	-49.174	-43.924
p-value*	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.

Tables 4.1 and 4.2, I fail to reject the null hypotheses of a unit root for the ADF and PP tests for each of the variables in log levels at the 1% significant level.

The decision to deal with the random walk behavior is to transform the log levels into the first differenced of the logs. The ADF and PP unit root test results indicate that the null hypotheses of unit root in first differenced levels at the 10% significance level can be rejected.

The descriptive statistics of the first difference of the log levels for the daily price of crude oil are reported in Table 4.3. All the three sub-periods reveals sample kurtosis larger than three, which is the kurtosis value for normal distribution, and imply "leptokurtic distributions". Figures 4.1, 4.2, and 4.3 display the differenced log levels for the previously mentioned sub-periods. As it is noticeable in those plots there are major variations during different times such as towards the last months of 1991 as a result of the Persian Gulf War or in summer 2008 as a consequence of the Global Financial Crisis. These kurtosis values underline the image of unstable crude oil market and its price fluctuations in response to different geopolitical and economics events in the sub-periods.

Table 4.3: Summary Statistics of Differenced Log Series - WTI Daily Spot Price

WTI Daily Spot Price	**Sample Mean**	**Sample Median**	**Standard Deviation**	**Skewness**	**Kurtosis**
01/02/1986–12/30/1993	0.0006	0.0004	0.0239	0.0500	5.9722
01/03/1994–12/31/2003	0.0001	0.0004	0.0107	-0.3715	4.8853
01/05/2004–04/30/2012	0.0002	0.0004	0.0109	-0.0010	4.4603

4.3.2 Monthly Data

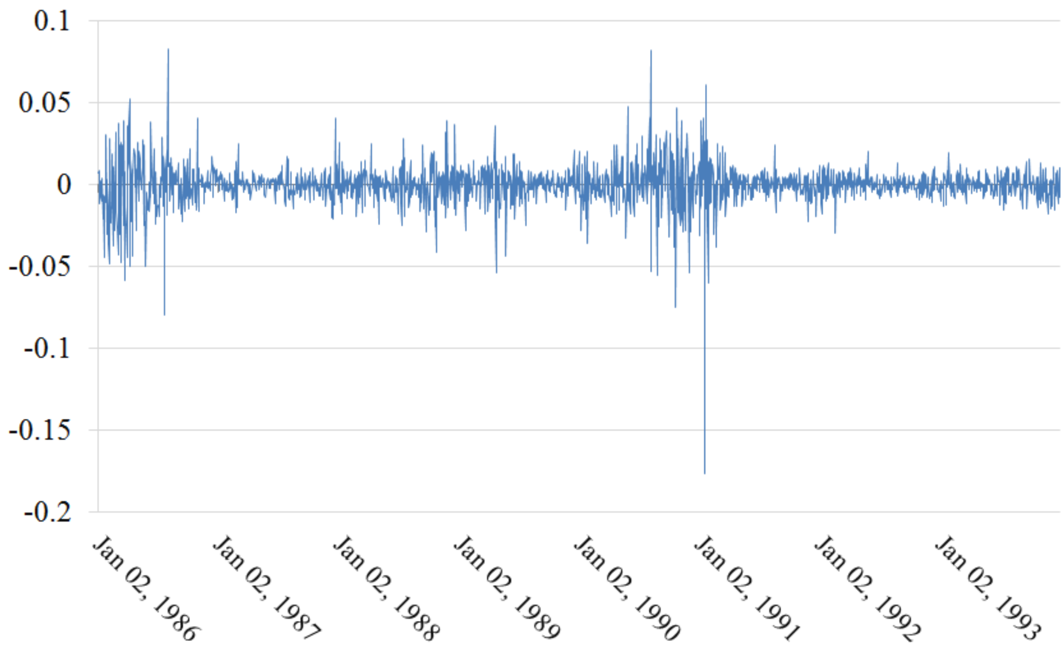
The monthly data includes real values on the spot price of crude oil, West Texas Intermediate. The sample period of January 1970 to March 2011 consists of 494 observations obtained from International Financial Statistics (IFS). To carry out the analysis, the monthly data is divided into two sub-samples: January 1970 to December 1991 for a total of 263 observations and January 1992 to March 2011 for a total of 213 observations. The monthly sample and sub-sample are defined as following:

- Monthly data is real values on the price of crude oil, West Texas Intermediate, from January 1970 to March 2011 for a total of 494 observations.
- First sub-sample monthly data on the spot price index is from January 1970 to December 1991 for a total of 263 observations.
- Second sub-sample monthly data on the spot price index is from January 1992 to March 2011 for a total of 231 observations.

Unit Root Analysis

The two most widely used conventional tests, the ADF and the PP tests, are employed to check the existence of unit root in monthly data. The methods of the test are explained in the previous section. In the specification of the unit root regressions for the ADF

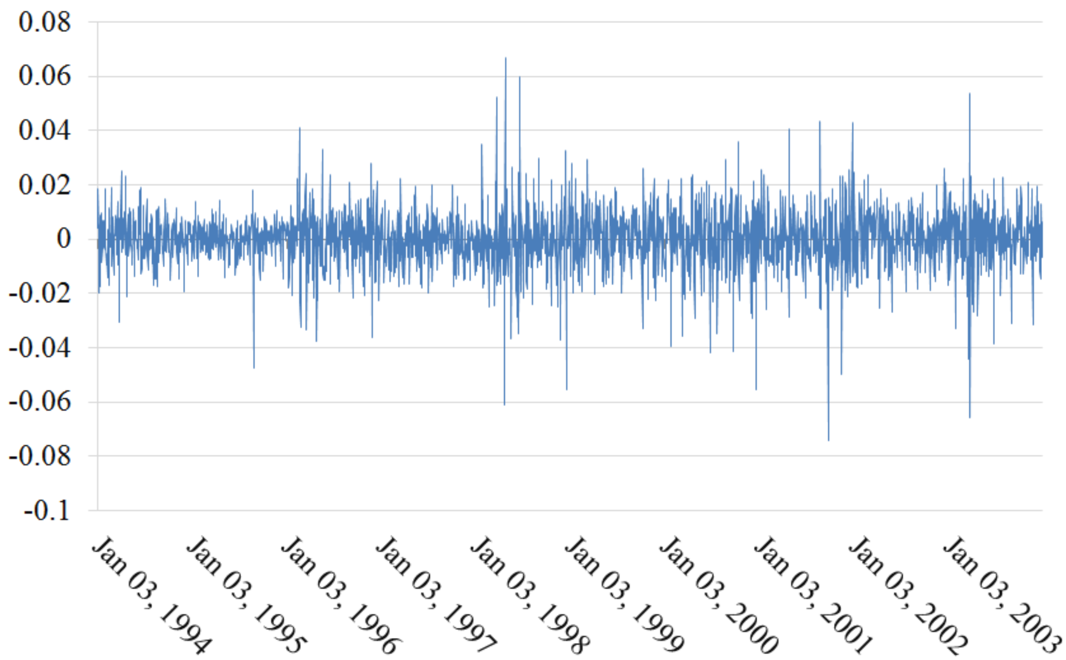
Figure 4.1: Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) - 01/02/1986 – 12/30/1993



Data Source: Energy Information Administration(EIA)

and the PP test in log level of the individual series, I included the constant term as well as the time trend to distinguish whether or not the series are "trend stationary" (TS) model, where a stationary component is added to a deterministic trend term. As the results show in Tables 4.4 and 4.5, I fail to reject the null hypotheses of a unit root for the ADF and PP tests for each of the variables in log levels at the 10% significant level. The decision to deal with the random walk behavior is to transform the log levels into the first differenced of the logs. The ADF and PP unit root test results indicate that I can reject the null hypotheses of unit root in first differenced levels at the 10% significance level. Hence, I use the first differenced of the log levels for each daily individual series

Figure 4.2: Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) - 01/03/1994 – 12/31/2003



Data Source: Energy Information Administration(EIA)

throughout the rest of the paper unless otherwise noted.

The descriptive statistics as well as the plot of differenced log levels of the monthly observations on prices are displayed in Table 4.6, Figures 4.4, 4.5, and 4.6, respectively. The kurtosis statistic is particulary large and implies a leptokurtic distribution for the sample periods of January 1970 to March 2011 and January 1970 to December 1991. As shown by Figure 4.4, the extreme fluctuations are indications of the volatile market and as a result a heavy tail distribution. The first significant deviation occurs around January 1974, when the first oil shock happened in late 1973 and early 1974. Other extreme fluctuations took place as a result of OPEC oversupply about February 1986, the Persian Gulf War around August 1990, and the Global Financial Crisis in 2008. In addition to

Table 4.4: Augmented Dickey-Fuller Unit Root Tests - Monthly Spot Price Indices on WTI

Null Hypothesis: The log levels and the differenced log of the series have unit root
Lag length: Automatic Selection Based on SIC.

Log Level	**Monthly Price WTI 1970:01 - 2011:04**	**Monthly Price WTI 1970:01 - 1991:12**	**Monthly Price WTI 1992:01 - 2011:04**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-2.409	-1.468	-2.944
p-value*	0.374	0.8381	0.150
DLog Level	**Monthly Price WTI 1970:01 - 2011:04**	**Monthly Price WTI 1970:01 - 1991:12**	**Monthly Price WTI 1992:01 - 2011:04**
ADF Test Statistic ($t_{(\hat{\beta})}$)	-17.336	-12.816	-11.640
p-value*	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
The sample period for monthly price of crude oil, West Texas Intermediate (WTI), is from 1970:01 to 2011:04 for a total 495 observations. The sample sub-periods for the monthly spot prices: January 1970 - December 1991 and January 1992 - April 2011, a total of 264 and 231 observations, respectively.

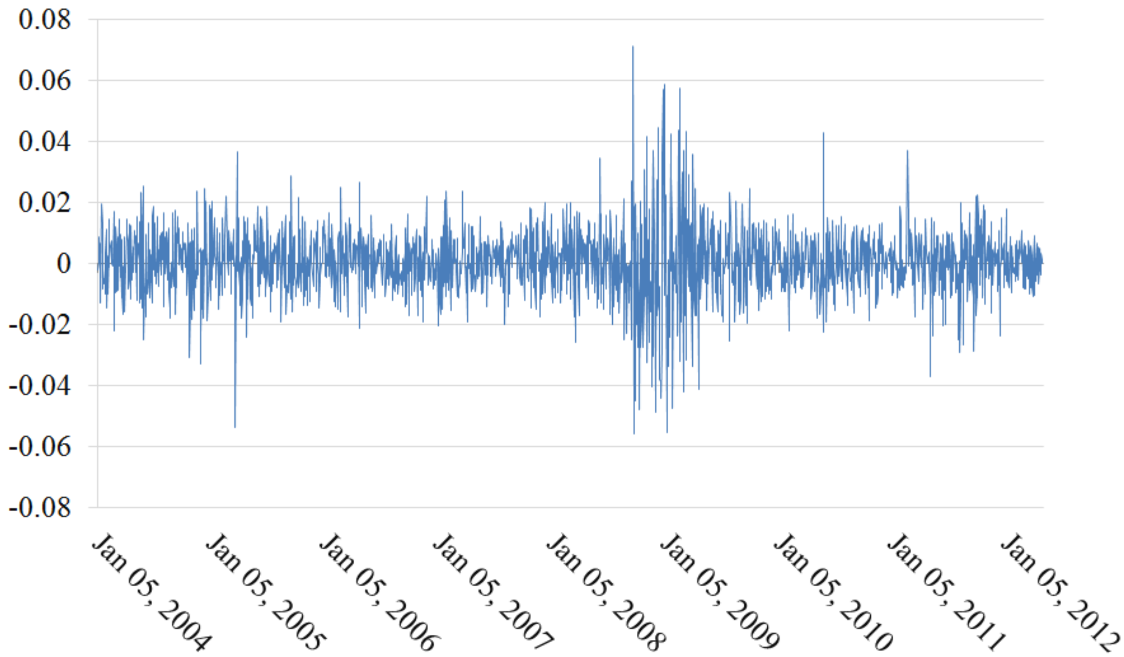
Table 4.5: Philips-Perron Unit Root Test - Monthly Spot Price Indices on WTI

Null Hypothesis: The log levels and the differenced log of the series have unit root
Bandwidth: (Newey-West automatic) using Bartlett Kernel

Log Level	**Monthly Price WTI 1970:01 - 2011:04**	**Monthly Price WTI 1970:01 - 1991:12**	**Monthly Price WTI 1992:01 - 2011:04**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-2.290	-1.288	-2.88
p-value*	0.4378	0.888	0.169
DLog Level	**Monthly Price WTI 1970:01 - 2011:04**	**Monthly Price WTI 1970:01 - 1991:12**	**Monthly Price WTI 1992:01 - 2011:04**
PP Test Statistic ($Z_{t_{(\hat{\pi})}}$)	-17.336	-12.666	-11.640
p-value*	0.000	0.000	0.000

* MacKinnon (1996) one-sided p-values.
The sample period for monthly price of crude oil, West Texas Intermediate (WTI), is from 1970:01 to 2011:04 for a total 495 observations. The sample sub-periods for the monthly spot prices: January 1970 - December 1991 and January 1992 - April 2011, a total of 264 and 231 observations, respectively.

Figure 4.3: Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) - 01/05/2004 – 04/30/2012



Data Source: Energy Information Administration(EIA)

the aforementioned events, there are yet other occasions that influenced the crude oil market price to be more unstable. The extreme value of the kurtosis statistic is the reflection of the crude oil market's nature throughout the years. The second sub-period (1992:01 – 2011:03), however, reveals a smaller value than three for kurtosis statistic, which implied a smaller tail-frequency and a flatter top than the normal distribution (Platykurtic distribution).

Table 4.6: Summary Statistics of Differenced Log Series - West Texas Intermediate (WTI) Monthly Spot Price Index

Monthly Spot Price Index	**Sample Periods Mean**	**Sample Median**	**Standard Deviation**	**Skewness**	**Kurtosis**
WTI (1970:01–2011:03)	0.0030	0.000	0.0358	1.8815	24.7710
WTI (1970:01–1991:12)	0.00291	0.000	0.0366	3.9287	41.8937
WTI (1992:01–2011:03)	0.0031	0.0063	0.0356	-0.7759	1.9473

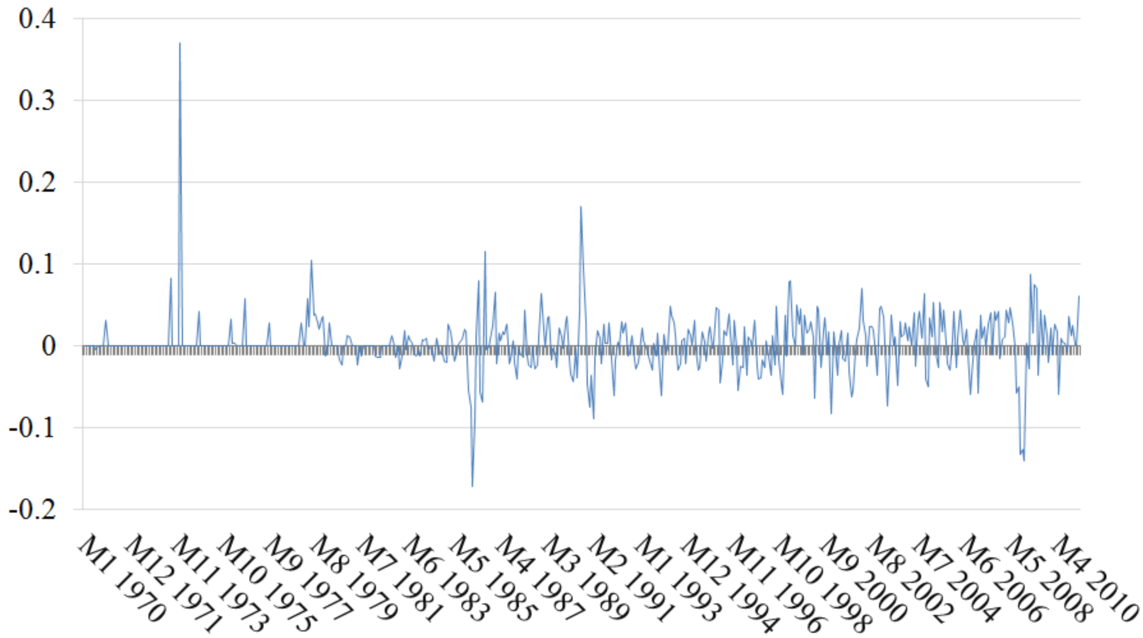
4.4 The Inference Methods

This section introduces the inference methods for statistically detecting nonlinearities in data generating mechanism of the employed time series observations: The BDS test, the Kaplan test, the Hinich bicovariance test, the Hinich bispectrum test, the Engle LM test, the McLeod-Li test, and the Tsay test. It is to be noted that all the above tests, except Hinich bispectrum test, require to remove any serial dependence from the data via a prewhitening model. Any other serial dependence is the result of a nonlinear data generating mechanism. The Hinich bispectrum test directly tests the data generating mechanism and it is invariant to filtering of the data (Patterson and Ashley (2000a)).

4.4.1 The BDS Test: A Test for Serial Independence

The well known Brock, Dechert, Scheinkman and LeBaron(1996) test, also known as the BDS test, is one form of portmanteau tests for independence. Portmanteau tests are residual-based tests in which the null hypothesis is well stated, but they do not have a specific alternative hypothesis. The BDS test Brock et al. (1986) is a popular test to

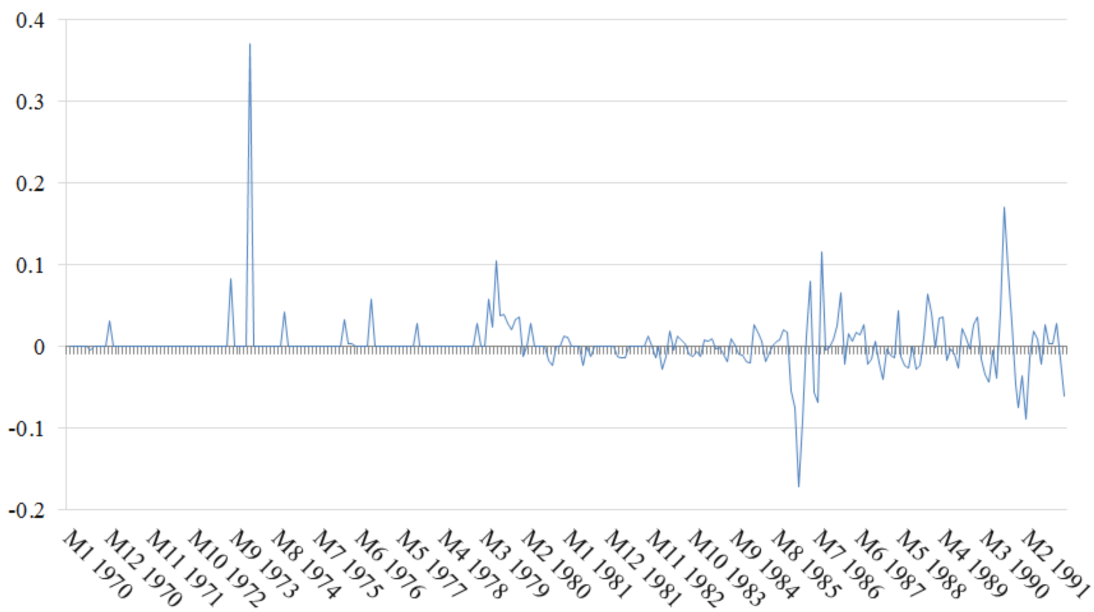
Figure 4.4: Differenced log of West Texas Intermediate (WTI) Monthly Spot Price Index (Dollars/Barrel)



Data Source: International Financial Statistics (IFS)

detect the serial independence in time series data. The BDS test introduces a test of independence that can be applied to the estimated residuals of any time series model, if the model can be transformed into a form with independent and identically distributed errors. The test employs the correlation function (correlation integral) to calculate the test statistics. The correlation function was introduced as a method of measuring the fractal dimension of deterministic data. The correlation function (integral) measures of the sequential pattern's frequency that exist in the data – see Brock et al. (1986) for more details. It is to be mentioned here that the correlation function is different than the correlation dimension, which is the method used in testing for chaos introduced by Grassberger and Procaccia (1983). Barnett et al. (1995) state that correlation dimension

Figure 4.5: Differenced log of West Texas Intermediate (WTI) Monthly Spot Price Index (Dollars/Barrel) January 1970 - March 2011



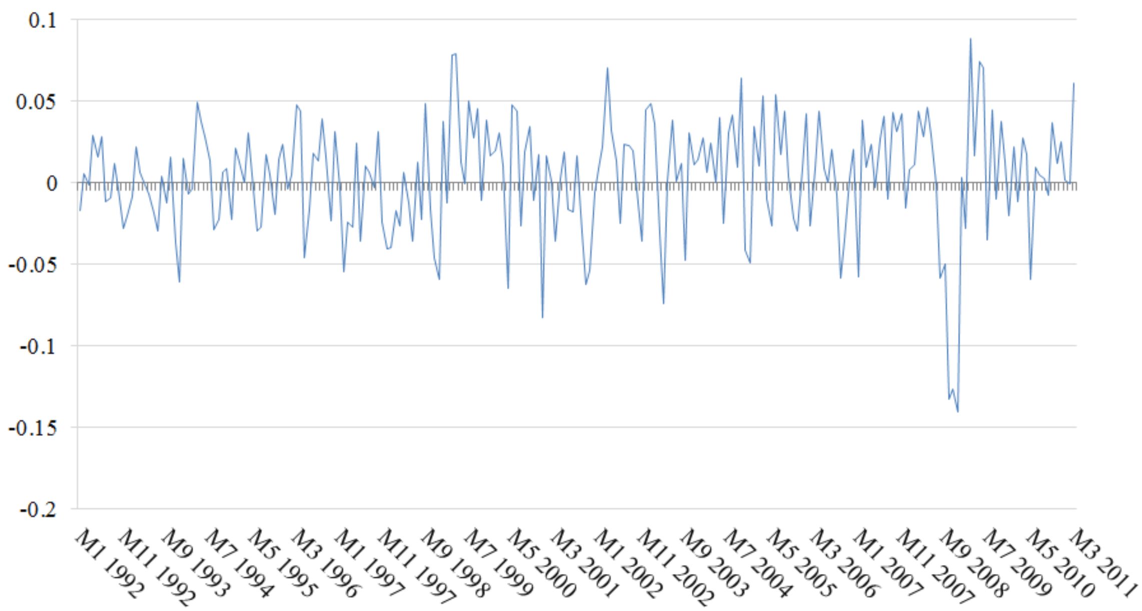
Data Source: International Financial Statistics (IFS)

is potentially helpful in testing for chaos, however modeling for high-dimensional chaos needs a large number of variables. Moreover, the sampling properties as well as the derived distribution of the correlation dimension are unknown, therefore the BDS test uses the correlation function as a test statistic Barnett et al. (1995).

The BDS test is used to test the null of linearity against a variety of possible deviation from independence in the series including nonlinearity and chaos. The test is applied to a series of estimated residual after removing any linear structure. Under the null hypothesis of independent and identically distributed ($i.i.d$) or whiteness, the BDS statistic is

$$\sqrt{n}\frac{C_{m,n}(\epsilon) - C_1(\epsilon)^m}{\sigma_m(\epsilon)} \tag{4.4}$$

Figure 4.6: Differenced log of West Texas Intermediate (WTI) Monthly Spot Price Index (Dollars/Barrel)



Data Source: International Financial Statistics (IFS)

where $C_{m,n}(\epsilon)$ is the correlation integral, $\sigma_m(\epsilon)$ is the asymptotic standard deviation of the numerator and m is the embedding dimension. The BDS test statistic is a transformation of the correlation function, which asymptotically becomes a standard normal Z statistic under the null hypothesis of whiteness Barnett et al. (1995). I apply the BDS test to the differenced log of the daily and monthly price of crude oil. To carry out the BDS test, the data is prefiltered by fitting the linear ARMA model, and the BDS test is applied against the remaining nonlinear structure in residuals. The choice of the values of ϵ and m can be challenging in using the BDS test. The results with BDS are reported in Tables 4.7 and 4.8 for dimension 2-8 and the value of ϵ equals to 1 and 2 standard deviation of the data[1].

[1] ϵ is calculated as a multiple of the standard deviation of the series.

Results with the BDS Test

I produce the BDS test statistic for the embedding dimension from two to eight, and the inferences are always the same and robust at each embedding dimension. The BDS method tests the null hypothesis of linearity of the process, which has high power against numerous nonlinear alternatives. The test is run for the three sub-periods of daily spot prices of crude oil as well as monthly spot prices in the entire sample size from January 1970 to March 2011 and the two sub-periods of the monthly prices as described in the data description section.

Daily Data

The BDS test results for the three sub-periods of daily spot prices of crude oil are displayed in Table 4.7. As can be observed, the results indicate the significance at the 1%, 5% and 10% levels based on the asymptotic distribution. Therefore, the BDS test rejects the null hypothesis of independent and identically distributed observations and detects the nonlinearity in all the daily sub-period time series observations. Therefore, when the time between observations is not large, the BDS test detect nonlinearity in all cases and shows an underlying nonlinear system.

Monthly Data

The results of monthly data for the entire sample and the two sub-samples are displayed in Table 4.8. The results reveal interesting facts about monthly data. Strong nonlinear dependence is shown in the whole sample and the first sub-sample at all dimensions.

Table 4.7: BDS Test Z-Statistic (Dimension 2-8)

Daily Spot Price of Crude Oil (WTI) (01/02/1986–12/30/1993)

	ϵ			
m	1σ	p-values	2σ	p-values
2	15.5429	0.000	11.5245	0.000
3	19.3594	0.000	14.1430	0.000
4	22.7812	0.000	16.2379	0.000
5	26.0529	0.000	17.7021	0.000
6	29.7307	0.000	18.9850	0.000
7	33.7293	0.000	19.8804	0.000
8	38.6443	0.000	20.6548	0.000

Daily Spot Price of Crude Oil (WTI) (01/03/1994–12/31/2003)

	ϵ			
m	1σ	p-values	2σ	p-values
2	4.3339	0.000	6.9100	0.000
3	5.9073	0.000	9.1127	0.000
4	6.9195	0.000	9.7959	0.000
5	7.7156	0.000	10.2777	0.000
6	8.7967	0.000	10.7110	0.000
7	10.0576	0.000	11.0819	0.000
8	11.3930	0.000	11.2764	0.000

Daily Spot Price of Crude Oil (WTI) (01/05/2004–04/30/2012)

	ϵ			
m	1σ	p-values	2σ	p-values
2	7.8861	0.000	11.9767	0.000
3	10.2783	0.000	15.3180	0.000
4	11.8994	0.000	17.2187	0.000
5	12.8264	0.000	18.1921	0.000
6	13.8714	0.000	18.8699	0.000
7	15.0360	0.000	19.4220	0.000
8	16.2019	0.000	19.7599	0.000

However, the nonlinear dependence is not as strong in the second sub-sample, particulary where ϵ is calculated as a first multiple of the standard deviation of the series.

The BDS test has high power against numerous nonlinear alternatives. Therefore, accepting the null hypothesis in BDS test indicates that there is strong evidence for the null. In that sense, it is suggested that the BDS test should be the first test to run. In the case of this study in which the linearity is rejected with the BDS test in most cases, the results reflect little information to distinguish the existing forms of nonlinearity in the data. To verify the BDS test, the Kaplan test is employed, which is a similar model to the BDS test. Furthermore, I utilize the more focused tests to identify the other possible forms of nonlinearity in the data – see Barnett et al. (1997) for more details.

4.4.2 Kaplan Test: A Test for Continuity and Determinism

There has been a wide range of methods in which reconstruction dynamics of the employed time series have been developed in order to characterize the dynamics in terms of predictability or dynamical invariant Kaplan (1994). These classifications are often employed to characterize whether the time series data are consistent with a deterministic mechanism, or a stochastic mechanism. As Kaplan (1994) mentions, it is common to test the predictability near every point in the time series in the nonlinear prediction method. Even though it might not be possible to predict future values of time series at every point, it may be likely to make accurate predictions at a few points. This may suffice for detecting the underlying determinism. Moreover, when deducing dynamics from a time series, continuity is often the only safe assumption one can make about a

Table 4.8: BDS Test Z-Statistic (Dimension 2-8)

Monthly Spot Price of Crude Oil (WTI) (1970:01–2011:03)

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.6402	0.000	7.6051	0.000
3	8.2962	0.000	8.4290	0.000
4	9.5298	0.000	8.9864	0.000
5	10.5471	0.000	8.9948	0.000
6	12.6705	0.000	9.2969	0.000
7	14.6477	0.000	9.3050	0.000
8	17.0018	0.000	9.2616	0.000

Monthly Spot Price of Crude Oil (WTI) (1970:01–1991:12)

	ϵ			
m	1σ	p-values	2σ	p-values
2	6.5397	0.000	4.2893	0.000
3	8.0014	0.000	5.3004	0.000
4	8.1749	0.000	5.8527	0.000
5	8.3150	0.000	5.9218	0.000
6	8.8078	0.000	6.2110	0.000
7	9.1136	0.000	6.2970	0.000
8	9.3985	0.000	6.2975	0.000

Monthly Spot Price of Crude Oil (WTI) (1992:01–2011:03)

	ϵ			
m	1σ	p-values	2σ	p-values
2	2.5360	0.0112	4.0298	0.0001
3	2.5245	0.0116	3.9660	0.0001
4	2.6035	0.0092	4.2307	0.000
5	2.2795	0.0226	4.0933	0.000
6	2.5717	0.0101	4.1728	0.000
7	2.6174	0.0089	3.9522	0.0001
8	2.5629	0.0104	3.7664	0.0002

possible deterministic mechanism for a time series. Kaplan (1994) proposed a test for determinism in a time series based on consistency with a continuous dynamical mapping. The test answers a question like, "If two points x_i and x_j are very close together, are their images x_{i+1} and x_{j+1} also close together?" (Kaplan (1994))[2]. In other words, deterministic solution paths, unlike stochastic processes, have the property that points that are close together are close under their image in phase space. Therefore, when the underlying function linking image and pre-image together is continuous, if the points x_i and x_j are close their images x_{i+1} and x_{j+1} are close together as well. In the case of chaos, the output plot of the system is hardly distinguishable from a stochastic process. Therefore, detecting the continuity of the system can be a difficult procedure, even when the data is entirely deterministic. However, it is easier to detect deterministic structure when plotting the solution path in phase space (x_{t+1} plotted against x_t and lagged values of x_t) than in plotting x_t versus t (Barnett et al. (1995)). Based on the above facts, the Kaplan test has strictly positive lower bound for a stochastic process, but not for a deterministic solution path. The statistic tests the null hypothesis that the data is deterministic against the alternative, which is that the data comes from a particular stochastic process. If the test statistic is smaller for the data than for the stochastic process by a statistically significant amount, then the stochastic process is rejected as an alternative to other forms of nonwhite structure (Barnett et al. (1995)). The test is computed by an adequately large number of linear processes that plausibly might have produced the data. The test procedure involves producing a linear stochastic process

[2]A test based on continuity in phase space proposed by Daniel Kaplan, Centre for Nonlinear Dynamics, Department of Physiology, McGill University.

surrogate data[3] for the observed data. The next stage is to determine a noisy continuous nonlinear dynamical solution path better describes the observed data. If the value of the test statistic from the surrogate is not small enough compared to the computed value of the test statistic from the observed data, a noisy continuous dynamical solution is concluded. As described by Barnett et al. (1995), the test procedure is formally stated as follows: If the time series data arise from a deterministically chaotic dynamical system, the value of x_{t+1} is a single-valued function of the state of the system at time t. Let the vector $x_t = (x_t, x_{t-1}, ..., x_{t-m-1})$ embedded in m-dimensional "phase space" and obtained from a m-dimensional vector $x_{i}{}_{i=1}^{T}$ in state space. Then there exists a function $f(x_t)$ such that $f(x_t) = x_{t+1}$, where x_{t+1} is called the "image" of the point x_t in phase space. If the system is perfectly deterministic with a continues f, close points in m-dimensional phase space have close image, whereas in a stochastic system close points in phase space may produce different images. The Kaplan test investigates if the function f is continuous based on the evidence provided the observed time series data. In the equivalence delta-epsilon proofs of continuity, δ is the distance in phase space and ϵ is the distance of the images. For a given choice of embedding dimension m, the distance in the phase space is calculated as $\delta_{ij} = |x_i - x_j|$ and the distance between their image is calculated as $\epsilon_{ij} = |x_{i+1} - x_{j+1}|$ for all i and j. It is useful to construct the average of the values of ϵ_{ij} conditional on the corresponding values of δ_{ij} satisfying $\delta_{ij} < r$ and define the average as $E(r)$. It is expected to have $E(r) \to 0$ as $r \to 0$ for a perfectly deterministic system with continuous f, whereas if the underlying system is stochastic the convergence may

[3]Surrogate data is random data generated with the same mean, variance, and autocorrelation function as the original data.

not happen as a point x_i may have different images. The statistic for the Kaplan test is defined as $K \equiv \lim_{r \to 0} E(r)$. The non-zero value of K can be interpreted as "goodness of fit" measure from fitting a continuous model of some fixed order to an infinite amount of data. If this measure is smaller for the observed data than for surrogate data generated by a model that satisfies a stated null hypothesis, then the null hypothesis should be rejected (Barnett et al. (1995)). As stated by Garcia (2007), another way of interpreting the non-zero value of K is as the level of nondeterminism or the amount of noise in the data. If the system is stochastic the amount of K is expected to be higher for nearly deterministic ones. Therefore, we should reject the null hypothesis when K on the observed data is smaller than K on the surrogate data. In other words, the hypothesis of linearity is rejected in order to test if the value of the statistic from the surrogates is never small enough compared to the value of the statistic obtained from the original data. Since the distribution of the statistic table is not laid out, Kaplan proposes two different methods to compute the minimum value of K obtained from the surrogates. The first approach is to estimate the minimum value of K from a finite sample of surrogates, and impute that to the population of the surrogates. Another approach involves the computation of the mean and standard error of the values of K from the finite sample and the subtraction of a multiple of (2 or 3) to obtain the an estimate of population minimum (Alharbi (2009)). This chapter uses twenty surrogate time series using the same approach suggested by Kaplan. The Surrogate data is a random realization from time series data of the energy markets generated with the same mean, variance, and autocorrelation function as the original data. Moreover, the lag embedded time series is also generated using 2, 3, 4, and

5 dimensional spaces.

Results with the Kaplan Test

The null hypothesis for the Kaplan test is stochastic linearity of the process. As mentioned by Barnett et al. (1995), the Kaplan test involves a strong power against chaos and is expected not to accept the null with chaotic series although the current form of test can either accept or reject linearity. It is worth mentioning that the Kaplan test is designed where the dynamical functional form underlying the time series data is unknown, and the main purpose is to decide if there is evidence of a deterministic mechanism in the observed data.

Daily Data

The results with the Kaplan test for the daily spot price of the sub-periods are displayed in Table 4.9 for embedding dimensions (m) 2, 3, 4 and 5. The mean, minimum, and standard deviations are computed over twenty surrogates for each time series. Moreover, the K statistic is calculated for each series. The results of the Kaplan test are graphically summarized in Appendix B of this chapter. The null hypothesis of stochastic linearity is rejected when the computed K for each daily spot price of crude oil is less than the minimum of the K statistic from surrogates or KS_{min} that is $K < KS_{min}$. As suggested by Kaplan, the t-statistic is calculated as a tool to identify the results' significance as: $t = \frac{K - KS_{mean}}{KS_{sd}}$, where KS_{mean} and KS_{sd} are the mean and standard deviation for KS values for surrogates.

As it can be observed in Table 4.9, the test rejects the null of linearity in the first

Table 4.9: Kaplan Test Statistic: Results from Daily Spot Price on Crude Oil

Daily Price	**Embedding Dimension**	**Mean K on surrogates**	**Std. dev. of K on surrogates**	**Min K on surrogates**	**K statistic on energy data**	**t-statistic**
Crude Oil WTI	2	0.0135	0.0018	0.0099	0.0075	-3.31
(01/02/1986 - 12/30/1993)	3	0.0132	0.0024	0.0084	0.0063	-2.83
	4	0.0132	0.0038	0.0056	0.0056	-1.99
	5	0.0144	0.0041	0.0062	0.0061	-2.01
Crude Oil WTI	2	0.012	0.0006	0.0106	0.0099	-3.042
(01/03/1994 - 12/31/2003)	3	0.0124	0.0013	0.0098	0.0105	-1.46
	4	0.0123	0.0017	0.0089	0.0089	-1.99
	5	0.0119	0.0024	0.0071	0.0072	-1.93
Crude Oil WTI	2	0.0123	0.0014	0.0095	0.0065	-4.08
(01/05/2004 - 04/30/2012)	3	0.0123	0.0013	0.0097	0.0023	-7.64
	4	0.0121	0.0018	0.0085	0.0129	0.48
	5	0.0125	0.0019	0.0087	0.0063	-3.25

Notes: K is the Kaplan test statistic. Twenty surrogate were used to compute the mean, standard deviation, and minimum over the 20 surrogate. The sample sub-periods for the daily spot prices: January 2, 1986 - December 30, 1993, January 3, 1994 - December 31, 2003, and January 5 2004 - April 30, 2012 consists of 2039, 2511, and 2092 observations, respectively.

sub-period of the daily spot price of crude oil in all dimensions, excluding embedding dimension=4. The null hypothesis cannot be rejected in the second sub-period of crude oil daily spot price in embedding dimensions equal to 3, 4, and 5. The test rejects the null of linearity of the embedding dimension=2. The null of linearity is rejected in the third sub-periods of crude oil daily spot price in dimensions equal to 2, 3, and 5. However, in embedding dimension=4, the test cannot reject the null hypothesis of linearity.

Monthly Data

The Kaplan test detects the evidence of general nonlinearity in observed time series. The Kaplan test rejects the null in the first sample of monthly data, which includes the entire observations. However, in the monthly sub-samples the null hypothesis is rejected only in embedding dimension=2. This chapter proceeds with more focused tests to investigate other possible forms of nonlinearity in the observed time series, such as

Table 4.10: Kaplan Test Statistic: Results from Monthly Spot Price Index on Crude Oil

Monthly Price	Embedding Dimension	Mean K on surrogates	Std. dev. of K on surrogates	Min K on surrogates	K statistic on energy data	t-statistic
Crude Oil WTI	2	0.0517	0.0054	0.0409	0.0230	-5.3
(01:1970 - 03:2011)	3	0.0496	0.0064	0.0368	0.0293	-3.17
	4	0.0498	0.0094	0.031	0.0226	-2.89
	5	0.0539	0.0086	0.0367	0.0231	-3.58
Crude Oil WTI	2	0.0389	0.0021	0.0347	0.0270	-5.66
(01:1970 - 12:1991)	3	0.0398	0.004	0.0318	0.0218	-4.49
	4	0.0372	0.0069	0.0234	0.0345	-0.39
	5	0.0375	0.007	0.0235	0.0491	-1.66
Crude Oil WTI	2	0.0447	0.0042	0.363	0.0296	-3.59
(01:1992 - 03:2011)	3	0.0418	0.0083	0.0252	0.0293	-1.5
	4	0.0395	0.0088	0.0219	0.0290	-1.19
	5	0.048	0.0126	0.0228	0.0193	-2.27

Notes: K is the Kaplan test statistic. Twenty surrogates were used to compute the mean and standard deviation. The sample period for monthly price of crude oil, West Texas Intermediate (WTI), is from 1970:01 to 2011:04 for a total 495 observations. The sample sub-periods for the monthly spot prices: January 1970 - December 1991 and January 1992 - April 2011, a total of 264 and 231 observations, respectively.

third order nonlinearity.

4.4.3 Tests for Nonlinearity

The Hinich Bicovariance Test

As noted by Patterson and Ashley (2000a), this test assumes x_t is a realization from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The (r, s) sample bicovariance is defined as

$$C_3(r,s) = (N-s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s} \quad 0 \leq r \leq s. \tag{4.5}$$

Therefore, the sample bicovariances are a generalization of a skewness parameter. The $C_3(r, s)$ are all zero for zero mean, serially $i.i.d$ data. One would expect non-zero values for the $C_3(r, s)$ from data in which x_t depends on lagged cross-products, such as $x_{t-i}x_{t-j}$ and higher order terms. Let $G(r, s) = (N - s)^{0.5}C_3(r, s)$ and define X_3 as

$$X_3 = \sum_{s=2}^{\phi}\sum_{r=1}^{s-1}[G(r, s)]^2 \tag{4.6}$$

Under the null hypothesis that x_t is a serially $i.i.d$ process, Hinich and Patterson (1995) show that X_3 is asymptotically distributed as $\chi^2[\phi(\phi - 1)/2]$ for $\phi < N^{0.5}$. Based on their simulation, they recommend using $\phi = N^{0.4}$. Under the assumption that $E((x_t)^{0.5})$ exists, the X_3 statistic detects nonzero third-order correlations. It can be considered as generalization of the Box-Pierce portmanteau statistics – see Hinich and Patterson (1985) for more discussion.

The Hinich Bispectrum Test

A process is said to be third-order nonlinear dependence if the skewness function in the frequency domain is not flat as a function of frequency pairs. The definition of the square of the skewness function is shown in Equation 4.8. This form of the nonlinearity is called third order, since the skewness function is a normalization of the Fourier transform of the third-order autocovariances. That Fourier transform is called the bispectrum (Barnett et al. (1997)).

The Hinich bispectrum test is a nonparametric test that examines the third-order moments (bicovariance) of the data in the frequency domain to obtain a direct test for a nonlinear generation mechanism, regardless of any linear independence that might be present in the data. Therefore, when the tests rejects the null (the skewness function is flat), there is no need to check the possibility that the linear prewhitening model has failed to remove all linear serial dependence in the data. Ashley and Patterson (2006)

Hinich (1982) develops this test for flatness of bispectrum. He argues that the bispectrum in the frequency domain is easier to interpret than multiplicity of the third-order moments $c_{xxx}(r,s) : s \leq r, r = 0, 1, 2\cdots$ in the domain. Barnett and Hinich (1993) explain the computation of the test statistic. For frequencies f_1 and f_2 in the principle domain

$\Omega = (f_1, f_2) : 0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1$

is the Hinich bispectrum of the series at frequency pair (f_1, f_2), and its double Fourier transformation of the third-moments function is:

$$B_{xxx}(f_1, f_2) = \sum_{r=-\infty}^{r=\infty} \sum_{s=-\infty}^{s=\infty} c_{xxx}(r,s) exp[-2\pi(f_1 r + f_2 s)]. \tag{4.7}$$

The square of the skewness function $\Gamma^2(f_1, f_2)$ is defined in terms of the bispectrum as:

$$\Gamma^2(f_1, f_2) = \frac{|B_{xxx}(f_1, f_2)|^2}{S_{xx}(f_1) S_{xx}(f_2) S_{xx}(f_1 + f_2)} \tag{4.8}$$

where $S_{xx}(f)$ is the (ordinary power) spectrum of x_t at frequency f. If the time series x_t is linear then the squared of skewness function $\Gamma^2(f_1, f_2)$ is constant over all frequency pairs (f_1, f_2) in Ω, and the skewness function $\Gamma^2(f_1, f_2)$ is zero over all frequencies if x_t is Gaussian. Linearity and Gaussianity can be tested using a sample estimator of the skewness function $\Gamma^2(f_1, f_2)$ – see Barnett and Hinich (1993) for more details on computation of the test and Hinich (1982) for more details on the test.

Engle LM Test

The test was proposed by Engle (1982) to examine nonlinearity in the second moment, particularly for ARCH disturbances. The test employs the Lagrangian multiplier procedure and runs the OLS regression and saves the residuals. Then the next procedures is to regress the squared residuals on a constant and p lagged values of the squared residuals and test NR^2 as a χ^2_p.

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \hat{\varepsilon}_{t-j}^2 + u_t \tag{4.9}$$

As most Lagrange multiplier tests, the test statistic is based on the R^2 of the regression. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the above regression is asymptotically distributed as χ^2_p.

The McLeod-Li Test

McLeod and Li (1983) developed a portmanteau test for nonlinear statistical dependence in the squared-residual autocorrelations of fitted ARMA models. The tests looks at the autocorrelation function of the squares of the prewhitened data and tests whether $corr(x_t^2, x_{t-j}^2)$ is nonzero for some j. The autocorrelation at the lag j for the squared residuals x_t^2 is estimated by

$$\hat{r}(j) = \frac{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)(x_{t-j}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)}, \quad where \hat{\sigma}^2 = \sum_{t=1}^{N} \frac{x_t^2}{N} \tag{4.10}$$

Under the null hypothesis that x_t is an $i.i.d$ process, McLeod and Li (1983) showed that, for sufficiently large and fixed L,

$$Q = N(N+2)\sum_{j=1}^{L}\frac{\hat{r}^2(j)}{N-j} \tag{4.11}$$

is asymptotically χ^2_L under the null hypothesis of a linear generating mechanism for the data. They have set $L = 20$ for their small-sample simulation in their examination.

The Tsay Test

The Tsay test introduced by Tsay (1986) examines the nonlinearity in the mean while Engle (1982) test checks the evidence for nonlinearity in the variance. The Tsay (1986) test explicitly look for quadratic serial dependence in the data, using quadratic terms lagged up to K periods. Let the $K = k(k+1)/2$ column vectors $V_1, ..., V_k$ contains all the unique cross-products of the form $x_{t-i}x_{t-j}$, where $i \in [i,k]$ and $j \in [j,k]$. Let $\hat{v}_{t,i}$ denote the projection of $v_{t,i}$ on the subspace orthogonal to $x_{t-1}, ..., x_{t-k}$, which is the residuals from a regression of $v_{t,i}$ on $x_{t-1}, ..., x_{t-k}$. The parameters $\gamma_i, ..., \gamma_k$ are estimated by applying OLS to the regression equation:

$$x_t = \gamma_0 + \sum_{i=1}^{k}\gamma_i\hat{v}i, t + \eta_t \tag{4.12}$$

Then, the Tsay test statistic is the usual F statistic for testing the null hypothesis that $\gamma_1, ..., \gamma_k$ are all zero.

4.4.4 The Results for Nonlinearity Tests

Daily Data

Table 4.11: Significance Level for Nonlinearity Tests - Daily Spot Price of Crude Oil
Asymptotic Distribution

Sample	Daily Price WTI 01/02/1986 - 12/30/1993	Daily Price WTI 01/03/1994 - 12/31/2012	Daily Price WTI 01/05/2004 - 04/30/2012
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.000
Bispectral (Gaussianity) ($M = N^{0.6}$)	0.000	0.000	0.000
Bispectral (Linearity) ($M = N^{0.6}$)	0.000	0.001	0.000
Engle($p = 5$)	0.000	0.000	0.000
McLeod-Li($L = 24$)	0.000	0.000	0.000
Tsay ($k = 5$)	0.000	0.000	0.000

Notes: The sample sub-periods for the daily spot prices: January 2, 1986 - December 30, 1993, January 3, 1994 - December 31, 2003, and January 5 2004 - April 30, 2012 consists of 2039, 2511, and 2092 observations, respectively.

Table 4.12: Significance Level for Nonlinearity Tests - Daily Spot Price of Crude Oil
Bootstrap Simulation

Sample	Daily Price WTI 01/02/1986 - 12/30/1993	Daily Price WTI 01/03/1994 - 12/31/2012	Daily Price WTI 01/05/2004 - 04/30/2012
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.000
Engle($p = 5$)	0.000	0.000	0.000
McLeod-Li($L = 24$)	0.000	0.000	0.000
Tsay ($k = 5$)	0.000	0.000	0.000

Notes: The sample sub-periods for the daily spot prices: January 2, 1986 - December 30, 1993, January 3, 1994 - December 31, 2003, and January 5 2004 - April 30, 2012 consists of 2039, 2511, and 2092 observations, respectively.

The results of the Hinich bicovariance, the Hinich bispectrum, the McLeod-Li, the Engle, and the Tsay tests for the daily spot prices are reported in Table 4.11 and 4.12 for both asymptotical and bootstrapping distributions[4]. As stated by Patterson and Ashley (2000a), the described tests are only asymptotically justified similar to most econometrics

[4]The source of the nonlinear software was thankfully provided by Professor Douglas M. Patterson. The source, instruction on running the toolkit program, and analysis can be found in Patterson and Ashley (2000a): "A Nonlinear Time Series Workshop: A Toolkit for Detecting and Identifying Nonlinear Serial Dependence", Kluwer Academic Publishers: Norwell. Available at: http://www.wkap.nl/.

procedures. Therefore, the significance levels of all the tests are normally bootstrapped. Also, the significance levels based on the asymptotic distributions are computed – see Patterson and Ashley (2000a) for further details on the bootstrap simulation.

In the Hinich bicovariance test, I use $\phi = N^{0.4}$ based on Hinich and Patterson (1985)'s simulation, where N is the sample size for each individual series. Moreover, the test is calculated using up to 15 lags and also with the number of bootstrap iterations equal to 100. As displayed by the results, based on the bootstrapped as well as asymptotic distributions, this test rejects the null hypothesis that x_t is a serially $i.i.d$ process in every case at the 1%, 5% and 10% significance levels.

The Hinich bispectrum test, on the other hand, examines the third order moments (bicovariance) of the data in frequency domain to obtain a direct test for a nonlinear generating mechanism. More importantly, this test focuses on nonlinear serial dependence and it substantially changes the usage of the sample bicovariance data more than the Hinich bicovariance test described earlier. The Hinich bispectrum test accepts the linearity if it cannot reject the flatness of bispectrum, and accepts the Gaussianity if the bispectrum is flat and also equals to zero. As can be observed in Table 4.11, the results of Gaussianity indicate extremely small p-values for each energy components market in the case of asymptotic distribution. As a result the null hypothesis of the Gaussianity is rejected at the 1%, 5% and 10% significance levels. Moreover, the null of linearity for each individual series exhibits very significant results by very small p-values for the 80 percent fractile bispectrum linearity test for every series. Hence, in the case of asymptotic distribution, the null hypothesis of the linearity is also rejected at the 1%, 5% and 10%

significance levels for each daily sub-period time series. In other words, the rejection of linearity provides strong evidence for the presence of the third order nonlinearity in the data generating process as also noted by Barnett et al. (1997). Ashley and Patterson (2006) show that the bispectrum $B_{xxx}(f_1, f_2)$ is consistently estimated using an average of appropriate triple products of the Fourier representation of the observed time series. The average is taken over a square containing M adjacent frequency pairs. Hinich (1982) showed that M must be above the $N^{0.5}$ to consistently estimate $B_{xxx}(f_1, f_2)$. The results here are reported for M to the integer nearly equals to $N^{0.6}$.

The Engle LM test (1982) examines nonlinearity in the second moment. Under the null hypothesis of a linear generating mechanism for x_t, NR^2 for the regression Equation 4.9 is asymptotically distributed as χ^2_p. The results are reported for p (lagged values) equals to five, and they exhibit substantially small p-values at the 1%, 5% and 10% significance levels in both bootstrapped and asymptotic distributions. Therefore, the null hypothesis of nonlinearity in the second moment is rejected in all cases. Following the literature, the results are quoted for p=5.

The null hypothesis of the McLeod and Li (1983) test that is x_t is an $i.i.d$ process is also rejected for up to 24 lags in bootstrapped and asymptotic distributions. As shown in the results, the results yield very small p-values at the 1%, 5% and 10% significance levels. The results are calculated for $L = 24$.

The Tsay test (Tsay (1986)) examines the nonlinearity in the quadratic terms. Following the existing literature in the subject, the value of $k = 5$ is used here. The reported results, based on the bootstrapped as well as asymptotic distributions, indicate that the null hypothesis is rejected at the 1%, 5%, and 10% significance levels.

Therefore, based on the bootstrapped and asymptotic distributions, the results for the nonlinear tests reveal that the daily data in crude oil production for any considered sub-period have clear evidence of nonlinearity in its structure. The price of crude oil exhibits nonlinearity in mean, variance and skewness functions in all of the daily sub-periods. These results are consistent with other reported findings in the literature, such as Kyrtsou et al. (2009). The evidence for significant nonlinearity in data generating mechanism in the energy market encourage modeling the time series data into an appropriate specification in order to obtain valid parameter estimations.

Monthly Data

Table 4.13: Significance Level for Nonlinearity Tests - Monthly Spot Price of Crude Oil
Asymptotic Distribution

Sample	**Monthly Price WTI (1970:01–2011:04)**	**Monthly Price WTI (1970:01–1991:12)**	**Monthly Price WTI (1992:01–2011:04)**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.000	0.000
Bispectral (Gaussianity) ($M = N^{0.6}$)	0.000	0.000	0.000
Bispectral (Linearity) ($M = N^{0.6}$)	0.000	0.998	0.725
Engle($p = 5$)	0.645	0.950	0.000
McLeod-Li($L = 24$)	1.000	1.000	0.000
Tsay ($k = 5$)	0.002	0.003	0.001

Notes: The sample period for monthly price of crude oil, West Texas Intermediate (WTI), is from 1970:01 to 2011:04 for a total of 495 observations. The sample sub-periods for the monthly spot prices: January 1970 - December 1991 and January 1992 - April 2011, total of 264 and 231 observations, respectively.

The results of the nonlinearity tests for the monthly spot prices of crude oil are displayed in Tables 4.13 and 4.14. The parameter values for each test are set to similar values of the daily spot prices. The McLeod-Li test and the Engle test have distinctively

Table 4.14: Significance Level for Nonlinearity Tests - Monthly Spot Price of Crude Oil Bootstrap Distribution

Sample	**Monthly Price WTI (1970:01–2011:04)**	**Monthly Price WTI (1970:01–1991:12)**	**Monthly Price WTI (1992:01–2011:04)**
Bicovariance ($\phi = N^{0.4}$)	0.000	0.020	0.000
Engle($p = 5$)	0.180	0.360	0.000
McLeod-Li($L = 24$)	0.600	0.750	0.000
Tsay ($k = 5$)	0.010	0.040	0.000

Notes: The sample period for monthly price of crude oil, West Texas Intermediate (WTI), is from 1970:01 to 2011:04 for a total of 495 observations. The sample sub-periods for the monthly spot prices: January 1970 - December 1991 and January 1992 - April 2011, total of 264 and 231 observations, respectively.

high power against alternative in the first monthly sample period and the sub-sample of January 1970 to December 1991 for both asymptotic distribution and bootstrap simulation. However, the null hypotheses of the McLeod-Li test and the Engle test are strongly rejected in the monthly sample of January 1992 to April 2011. The results of Gaussianity shows extremely small p-values for each daily sample in the case of asymptotic distribution. Hence, the null hypothesis of the Gaussianity is rejected in 10% significance level. The null of linearity cannot be rejected in the sub-samples of the monthly observations. However, the null of linearity for the sample of January 1970 to April 2011 series exhibits a very significant result by very small p-values for the 80 percent fractile bispectrum linearity test for the series. Hence, in the case of asymptotic distribution, the null hypothesis of the linearity is rejected at the 10% significance level for the first sample of monthly observations and the rejection of linearity provides strong evidence for the presence of the third order nonlinearity in the data generating process as also noted by Barnett et al. (1997). The null hypotheses of the Hinich bicovariance and the Tsay tests are rejected for all the monthly samples in bootstrap simulation and asymptotic distribution.

4.5 Summary and Conclusion

The goal of this chapter is to carry out the nonlinear analysis by employing various datasets with different features. The study explores the robustness of the inference methods, including higher dimensional cases, observations with different frequencies, and division of the daily and monthly time series periods into sub-periods. As Patterson and Ashley (2000b) mentioned, nonlinearity can be considered as a type of stochastic dependence and this dependence will fade away as the time between observation increases. The chapter not only utilizes statistical techniques to investigate the nonlinear dependence in the energy market, but also examines whether the time series data of spot price of crude oil in daily and monthly frequencies exhibit any difference in terms of nonlinearity in their generating mechanism.

To perform the analysis and achieve the objectives, the study utilizes daily spot prices on crude oil (WTI) from the period of January 1, 1986 to April 30, 2012 for a total of 6642 observations and divides them into three sub-periods as described in the data description section. Moreover, the monthly spot price index on crude oil is analyzed. The sample period is from January 1970 to March 2011 consists of 494 observations, which is divided into two sub-samples as well.

To carry out the analysis the most widely used univariate tests to detect the nonlinearity in the observed time series data are employed. It is to be noted that none of the tests have exactly the same null hypothesis and they differ in the power against the alternative hypothesis and focus on different aspects of nonlinearity. They will detect distinct attributes of nonlinear serial dependence in the data. Furthermore, using the tests jointly can produce better perception of the nature of the nonlinearity that may exist in the data.

The BDS test is a test of general nonlinearity in the process, against all other possible alternative null hypothesis of linearity, and has high power against the numerous categories of alternative hypotheses. The results of the BDS tests for the daily data indicate that the linearity is rejected in all the sub-periods. In the case of monthly time series observations, the BDS test rejects the null of nonlinearity in all three monthly samples.

The Kaplan test features seem to be comparable to the BDS test. However, as Barnett et al. (1997) state in their experiments, the Kaplan test provides the right answer with both large and small samples. In the case of daily sub-periods, the results of the Kaplan tests detect evidence of nonlinearity in the first and third sub-periods, excluding the embedding dimension four. The test rejects the null of linearity in the second daily sub-period only in embedding dimension one. The Kaplan test for monthly observation displays signs of nonlinear dependence for the first sample in all embedding dimensions. The results of the Kaplan test for the second and thirds sub-samples are rather nonspecific. The null of linearity is rejected in two out of four embedding dimensions in those sub-samples. Hence, the findings of the BDS test and the Kaplan test suggest to proceed with more detailed tests that consider the specific features of nonlinearity.

The Hinich bicovariance test focuses on the third-order moments (time domain) of the data and detected nonlinearity in each series. The Hinich bispectrum test examines the lack of third-order nonlinear dependence (frequency domain), and the associated Gaussianity test, which is a test of a necessary and not sufficient condition for Gaussianity[5]. The results of the Hinich bispectrum for the sub-period of daily data suggest that the observed spot price of the crude oil are generated by a nonlinear, non-Gaussian process. The monthly data, however, reveals different results for the second and third sub-samples. The Gaussianity is rejected in all three monthly samples, but linearity is strongly accepted in the second and third sub-samples. The Engle Lagrangian multiplier (LM) test focuses on the nonlinearity in the second moment. The null hypothesis of no ARCH-type disturbances is rejected by the Engle-LM test for the three daily sub-periods. The McLeod-Li test also rejects the null hypothesis of linearity in the variance for daily observations. The Engle and the McLeod-Li tests, which are sensitive to the ARCH-type disturbances, exhibit high power for the second and the third sub-sample monthly data, whereas the Engle and the McLeod-Li tests reject the null for the entire monthly observation. The Tsay test rejects the null hypothesis of linearity for daily as well as monthly observations for each sup-period.

Therefore, all the tests detect strong evidence of nonlinear structure in the daily spot price of crude oil, whereas in monthly observations the evidence of nonlinear dependence is less dramatic. The findings suggests that nonlinear dynamic dependence is remarkable in daily spot prices of crude oil. Since prediction can be improved by nonlinear models when there is evidence of nonlinearity in the data generating process (Maravall (1983); Tong (1983); Ashley and Patterson (2006)), the series cannot be accurately forecasted with a linear model. Therefore, the variation of nonlinear dependence by utilizing different time aggregations on crude oil observations needs to be taken into consideration when predicting the price of crude oil.

[5]See Barnett et al. (1997) for more details.

In the case of forecasting the daily spot price of crude oil, a model specification that reflects the nonlinear dynamics of the market will provide more accurate empirical results (Ashley and Patterson (2006)) since nonlinear dependence can be detected in daily time series observations. However, in the case of monthly time aggregation, when signs of nonlinear data generating mechanism is less significant, utilizing the other models that comply with the market's structures will ensure proper model specification. This chapter exhausts all of the possible cases for studying the dynamics of crude oil price and provides insightful understanding of the crude oil market data generating mechanism.

Appendix A: Data Description, Key Terms and Definitions

Figures 4.7 to 4.18 in this section represent the Cushing, OK WTI spot price FOB (Dollars per Barrel) for daily and monthly frequencies. The variable West Texas Intermediate (WTI- Cushing) is defined as follows:

- *West Texas Intermediate (WTI - Cushing:)* A crude stream produced in Texas and southern Oklahoma, which serves as a reference or "marker" for pricing a number of other crude streams and which is traded in the domestic spot market at Cushing, Oklahoma.

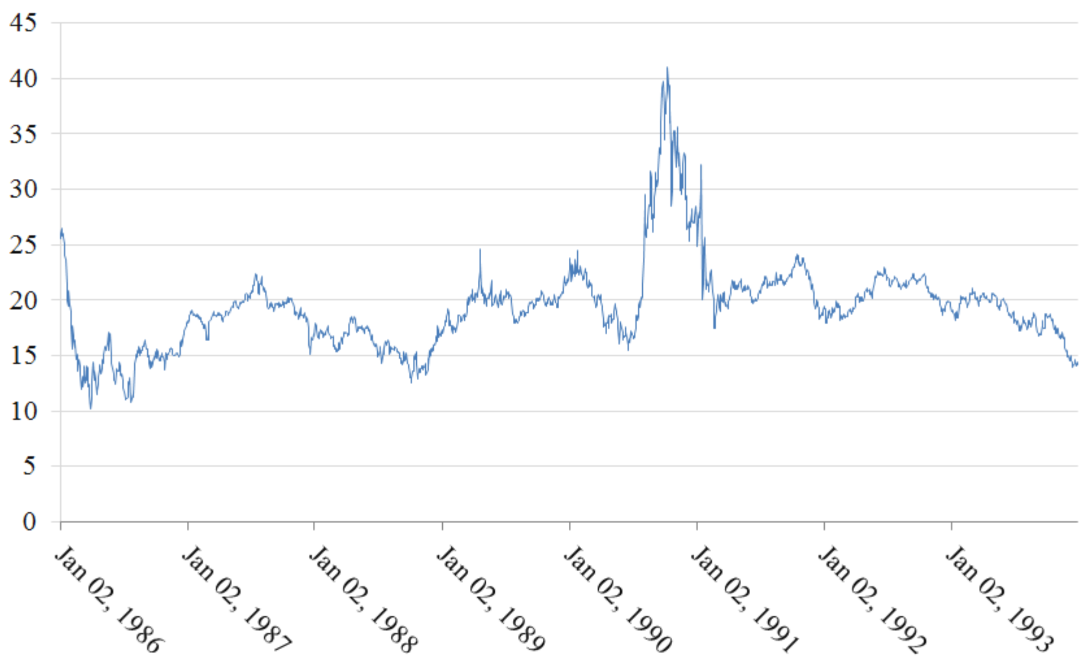
- *Crude Oil:* A mixture of hydrocarbons that exists in liquid phase in natural underground reservoirs and remains liquid at atmospheric pressure after passing through surface separating facilities. Depending upon the characteristics of the crude stream, it may also include:

  - Small amounts of hydrocarbons that exist in a gaseous phase in natural underground reservoirs but are liquid at atmospheric pressure after being recovered from oil well (casinghead) gas in lease separators and are subsequently commingled with the crude stream without being separately measured. Lease condensate recovered as a liquid from natural gas wells in lease or field separation facilities and later mixed into the crude stream is also included;
  - Small amounts of nonhydrocarbons produced with the oil, such as sulfur and various metals;

– Drip gases, and liquid hydrocarbons produced from tar sands, oil sands, gilsonite, and oil shale.

4.5.1 Daily Data

Figure 4.7: Daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel) (01/02/1986 – 12/30/1993)

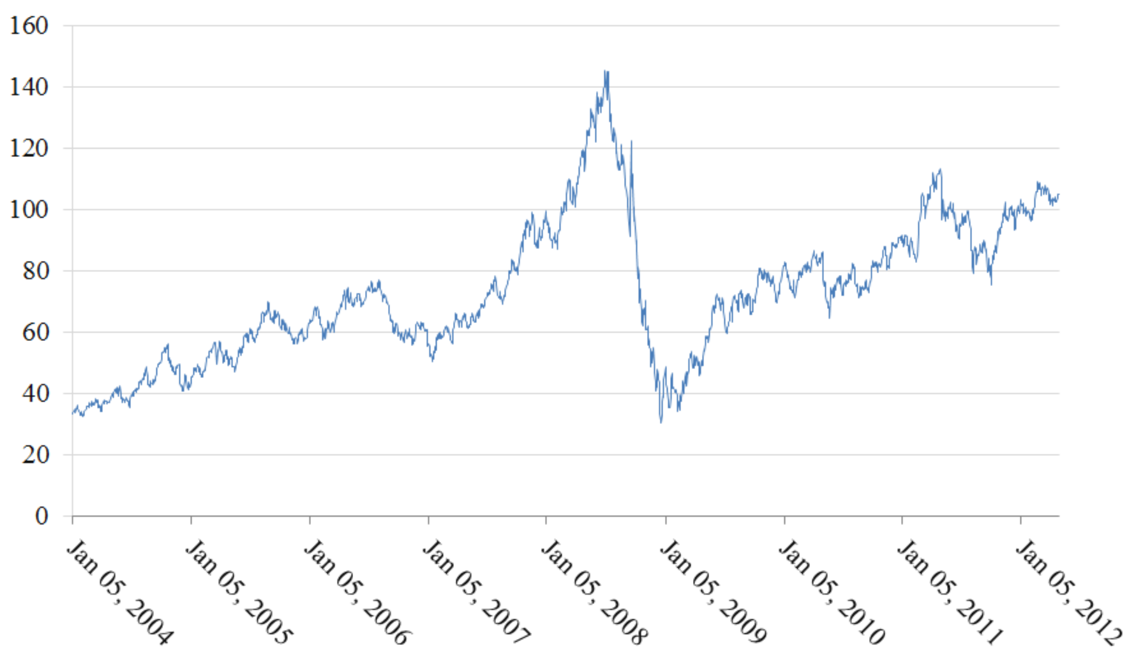


Data Source: Energy Information Administration (EIA)

Figure 4.8: Daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel) (01/03/1994 – 12/31/2003)

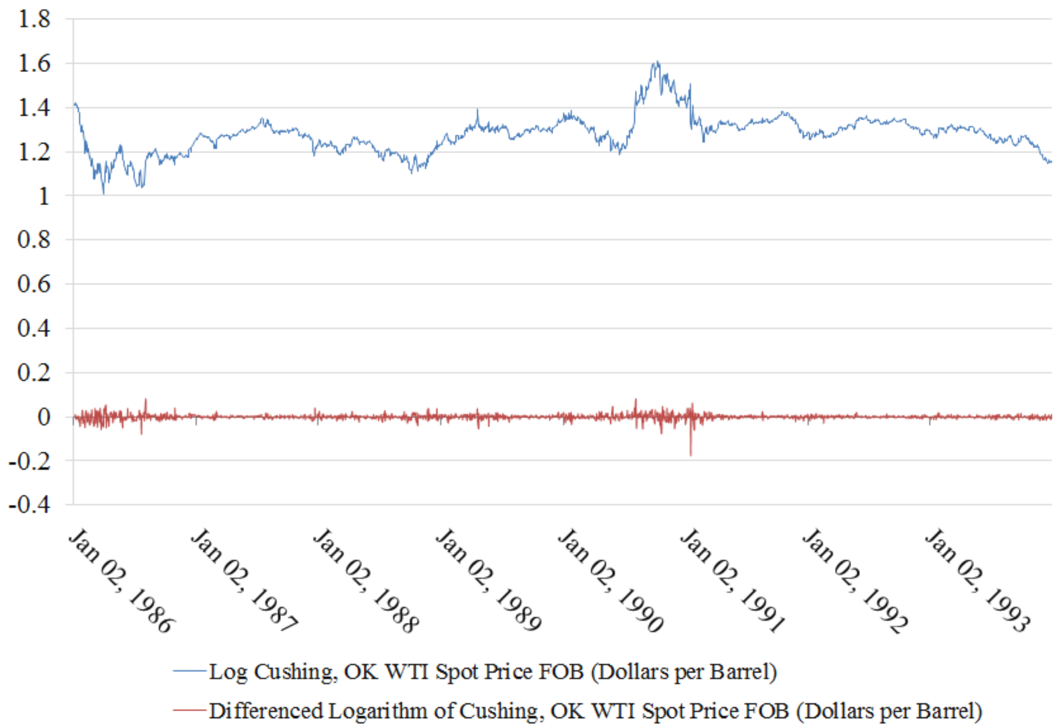
Data Source: Energy Information Administration (EIA)

Figure 4.9: Daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel) (01/05/2004 – 04/30/2012)



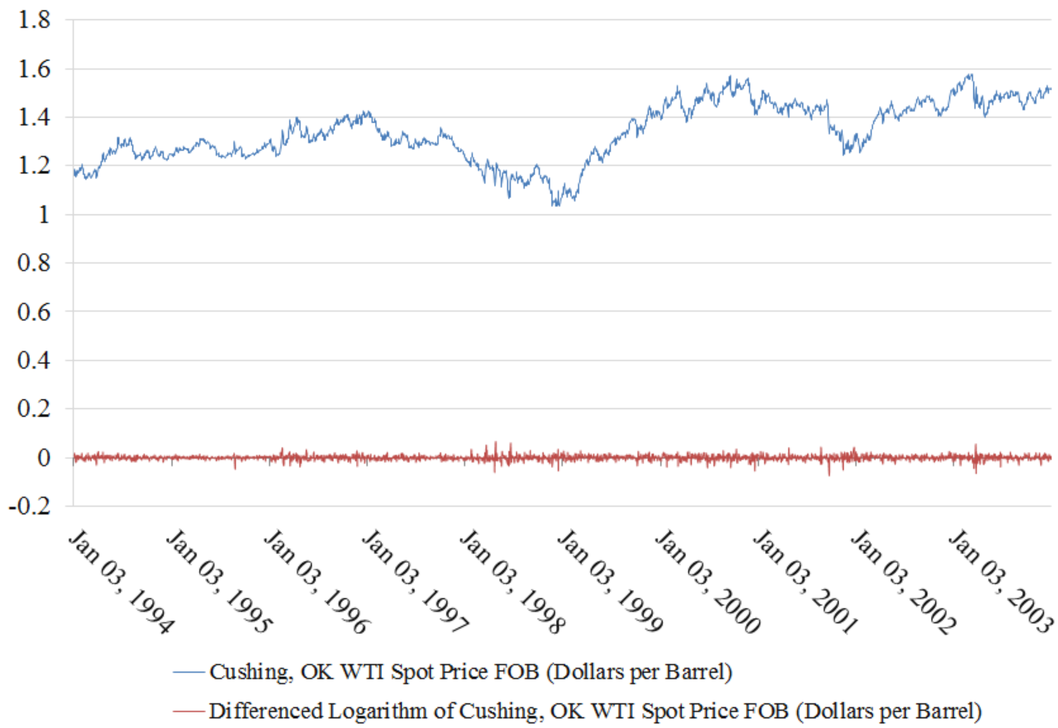
Data Source: Energy Information Administration (EIA)

Figure 4.10: Log and Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) (01/02/1986 – 12/30/1993)



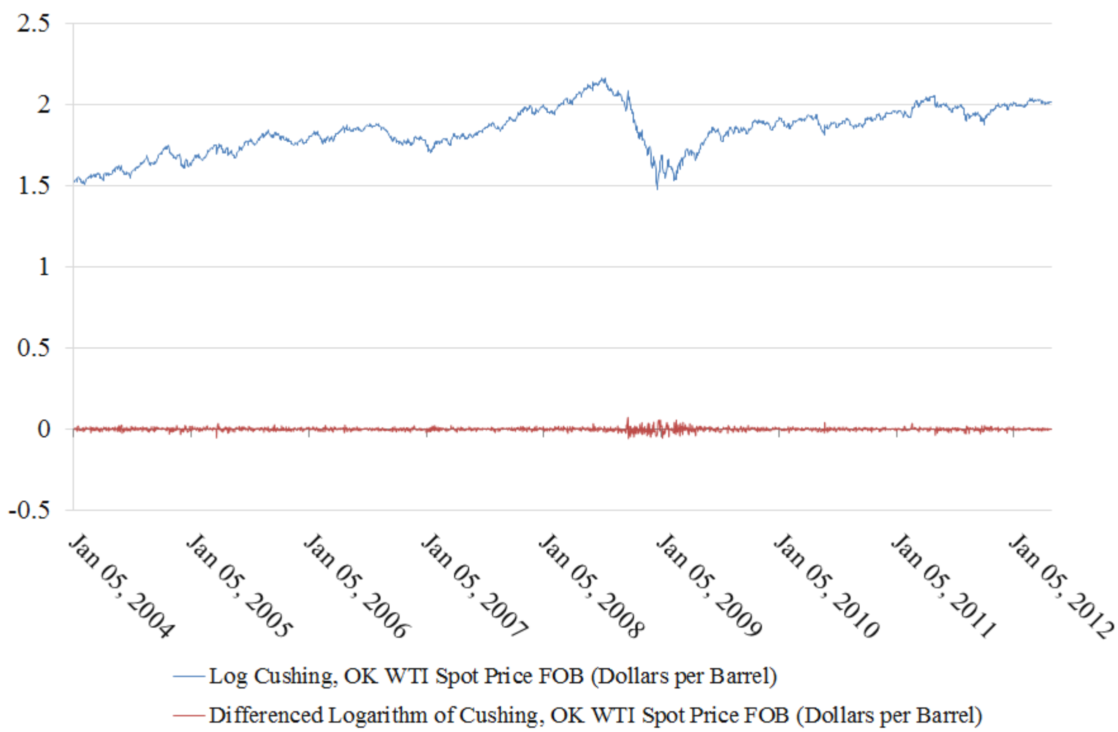
Data Source: Energy Information Administration(EIA)

Figure 4.11: Log and Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) (01/03/1994 – 12/31/2003)



Data Source: Energy Information Administration(EIA)

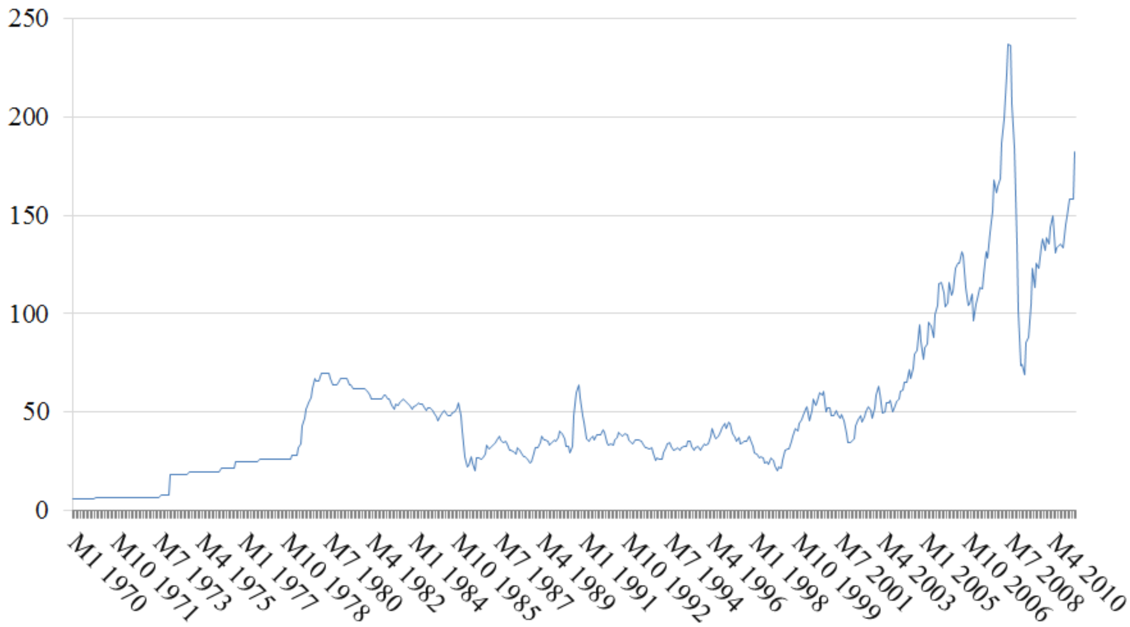
Figure 4.12: Log and Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel) (01/05/2004 – 04/30/2012)



Data Source: Energy Information Administration(EIA)

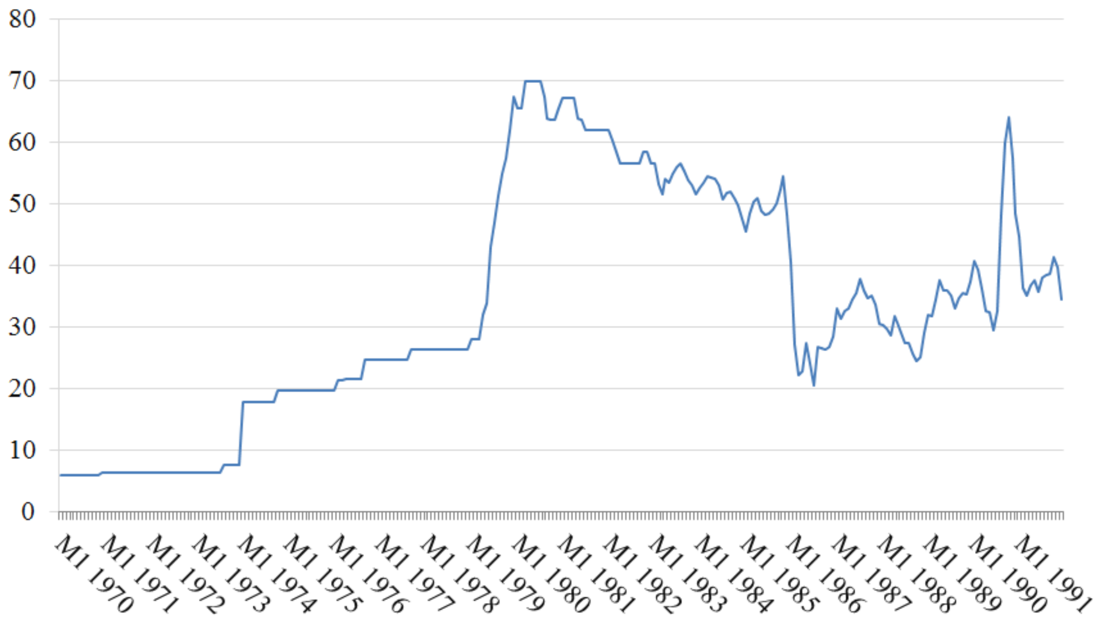
4.5.2 Monthly Data

Figure 4.13: WTI Monthly Spot Price Index (Dollars per Barrel) (1970:01 – 2011:03)



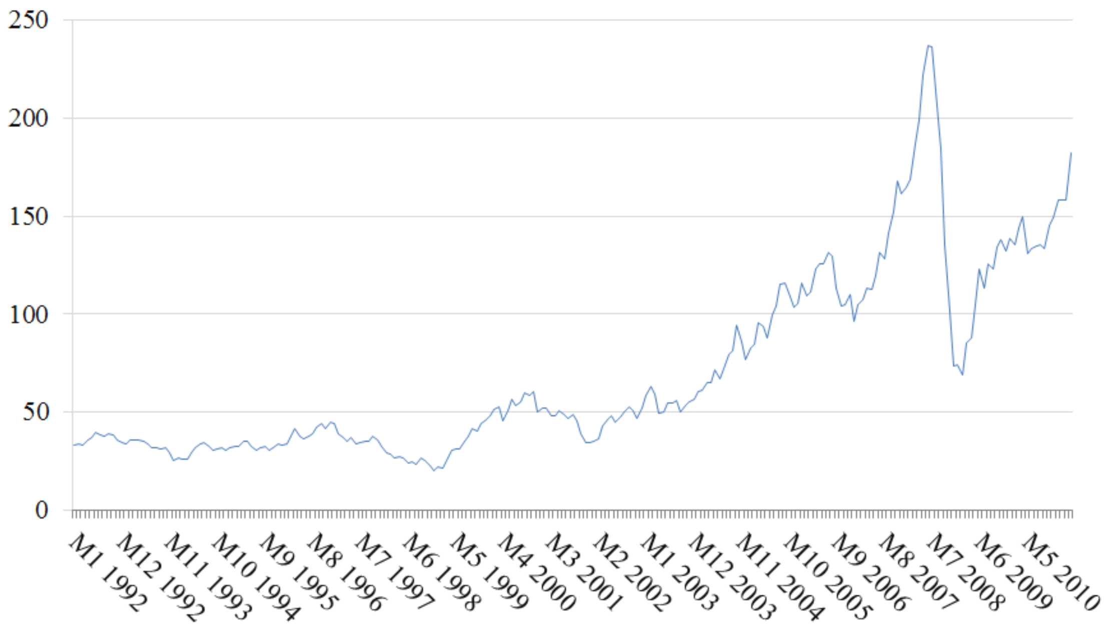
Data Source: International Financial Statistics (IFS)

Figure 4.14: WTI Monthly Spot Price Index (Dollars per Barrel) (1970:01 – 1991:12)



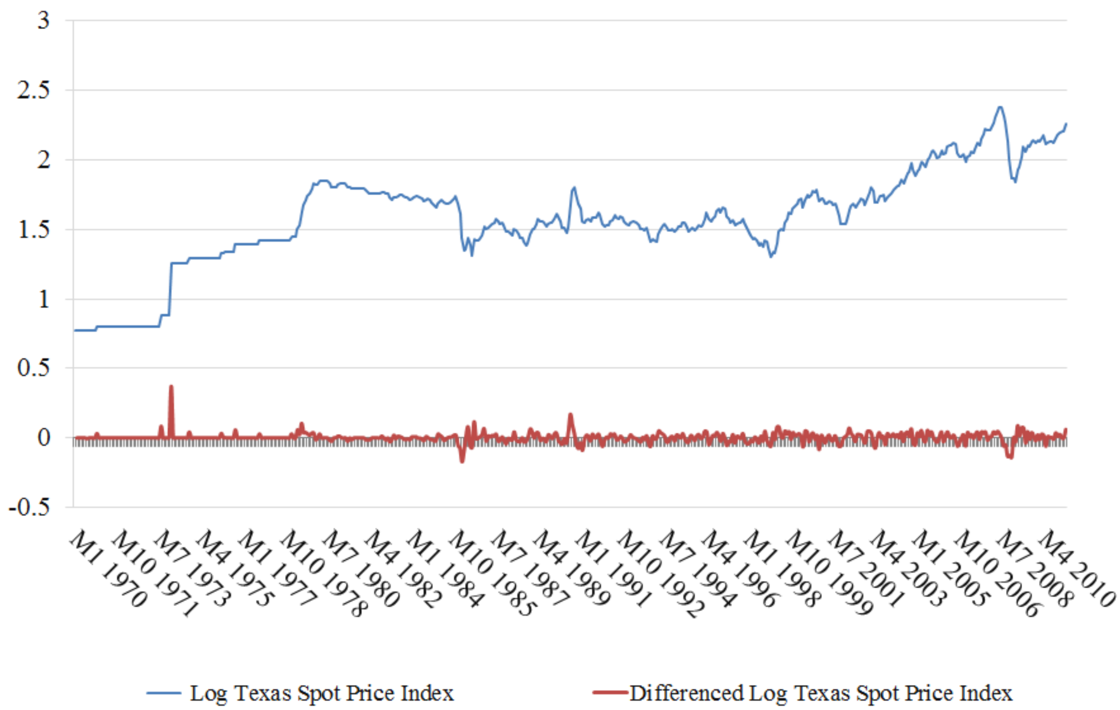
Data Source: International Financial Statistics (IFS)

Figure 4.15: WTI Monthly Spot Price Index (Dollars per Barrel) (1992:01 – 2011:03)



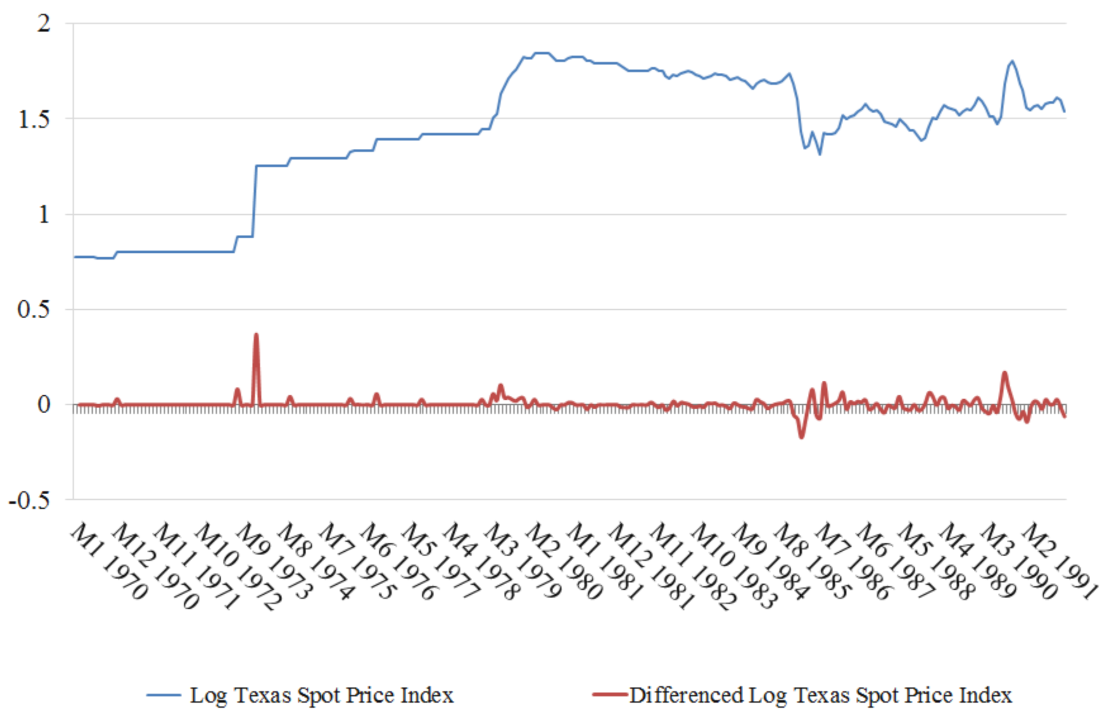
Data Source: International Financial Statistics (IFS)

Figure 4.16: Log and Differenced Log of WTI Monthly Spot Price Index (Dollars per Barrel) (1970:01 – 2011:03)



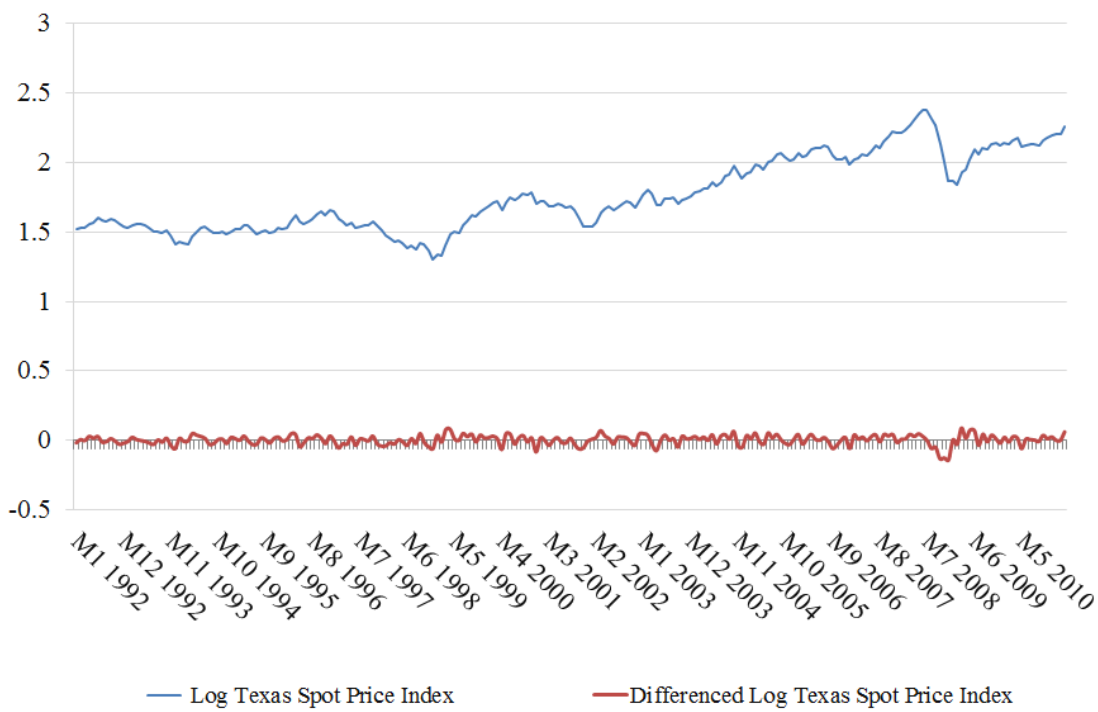
Data Source: International Financial Statistics (IFS)

Figure 4.17: Log and Differenced Log of WTI Monthly Spot Price Index (Dollars per Barrel) (1970:01 – 1991:12)



Data Source: International Financial Statistics (IFS)

Figure 4.18: Log and Differenced Log of WTI Monthly Spot Price Index (Dollars per Barrel) (1992:01 – 2011:03)



Data Source: International Financial Statistics (IFS)

Appendix B: Figures of the Kaplan Results for Embedding Dimension 2 – 5

4.5.3 Daily Spot Price of Crude Oil

Figures 4.19 to 4.22 display the Kaplan tests results for daily spot price of crude oil. In other words, the plots of δ versus ϵ are shown in Figures 4.19 to 4.22. The signs of continuity are revealed when δ goes to zero, so ϵ does.

The legend of each graph is explained as:

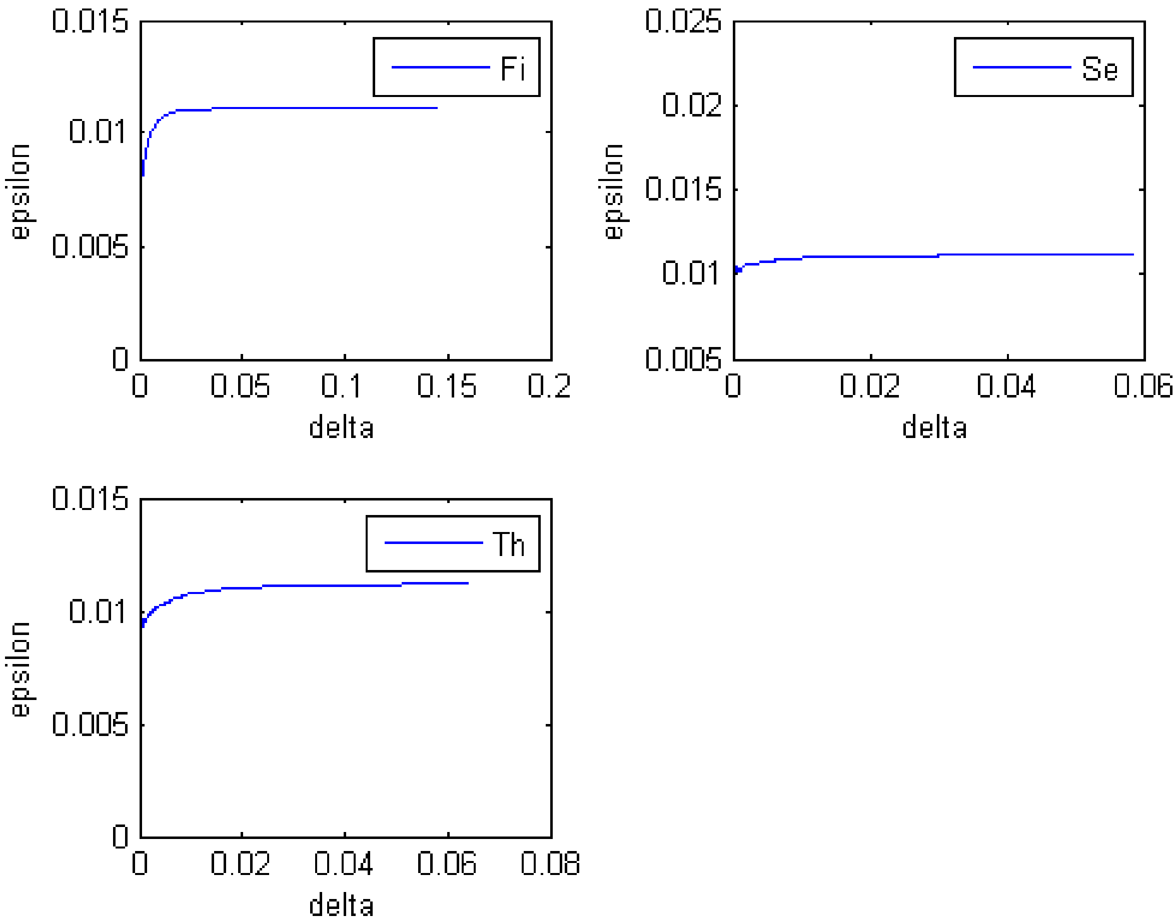
- Fi: The first daily spot price sub-period is from January 2, 1986 to December 30, 1993 consists of 2039 observations.
- Se: The second daily spot price sub-period is from January 3, 1994 to December 31, 2003 consists of 2511 observations.
- Th: The third daily spot price sub-period is from January 5, 2004 to April 30, 2012 consists of 2092 observations.

4.5.4 Monthly Spot Price Index of Crude Oil

Figures 4.19 to 4.22 display the Kaplan tests results for monthly spot price index of crude oil. In other words, the plots of δ versus ϵ are shown in Figures 4.19 to 4.22. The signs of continuity are revealed when δ goes to zero, so ϵ does.

The legend of each graph is explained as:

Figure 4.19: *Delta* vs. *Epsilon*, The Kaplan Test Results of Daily Price for Sub-Periods, Embedding Dimension=2



- 1s: Monthly spot price index of crude oil. The sample period is from January 1970 to December 1991 consists of 494 observations.
- 2n: Monthly spot price index of crude oil. The sample period is from January 1992 to March 2011 consists of 263 observations.
- 3r: Monthly spot price index of crude oil. The sample period is from January 1970 to March 2011 consists of 231 observations.

Figure 4.20: *Delta* vs. *Epsilon*, The Kaplan Test Results of Daily Price for Sub-Periods, Embedding Dimension=3

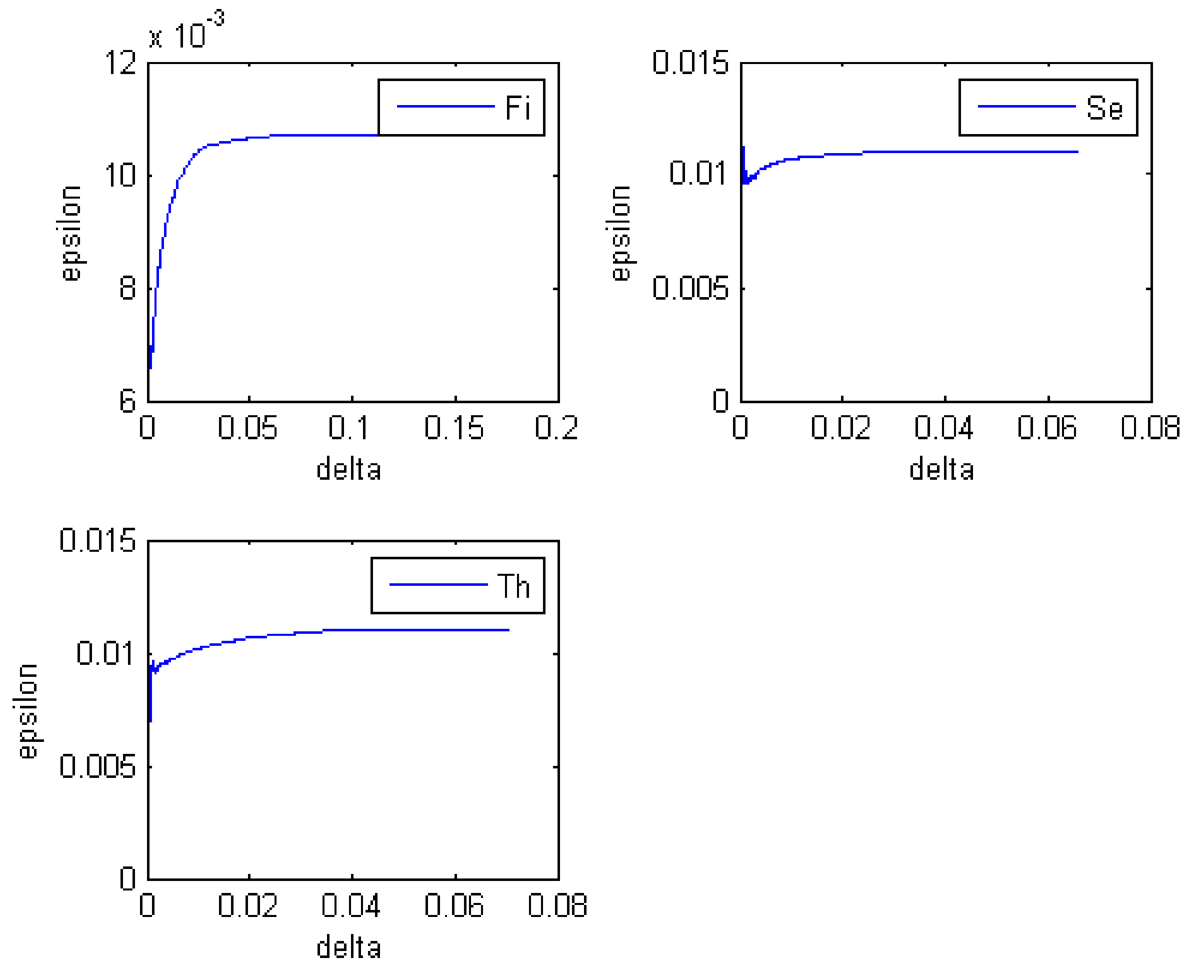


Figure 4.21: *Delta* vs. *Epsilon*, The Kaplan Test Results of Daily Price for Sub-Periods, Embedding Dimension=4

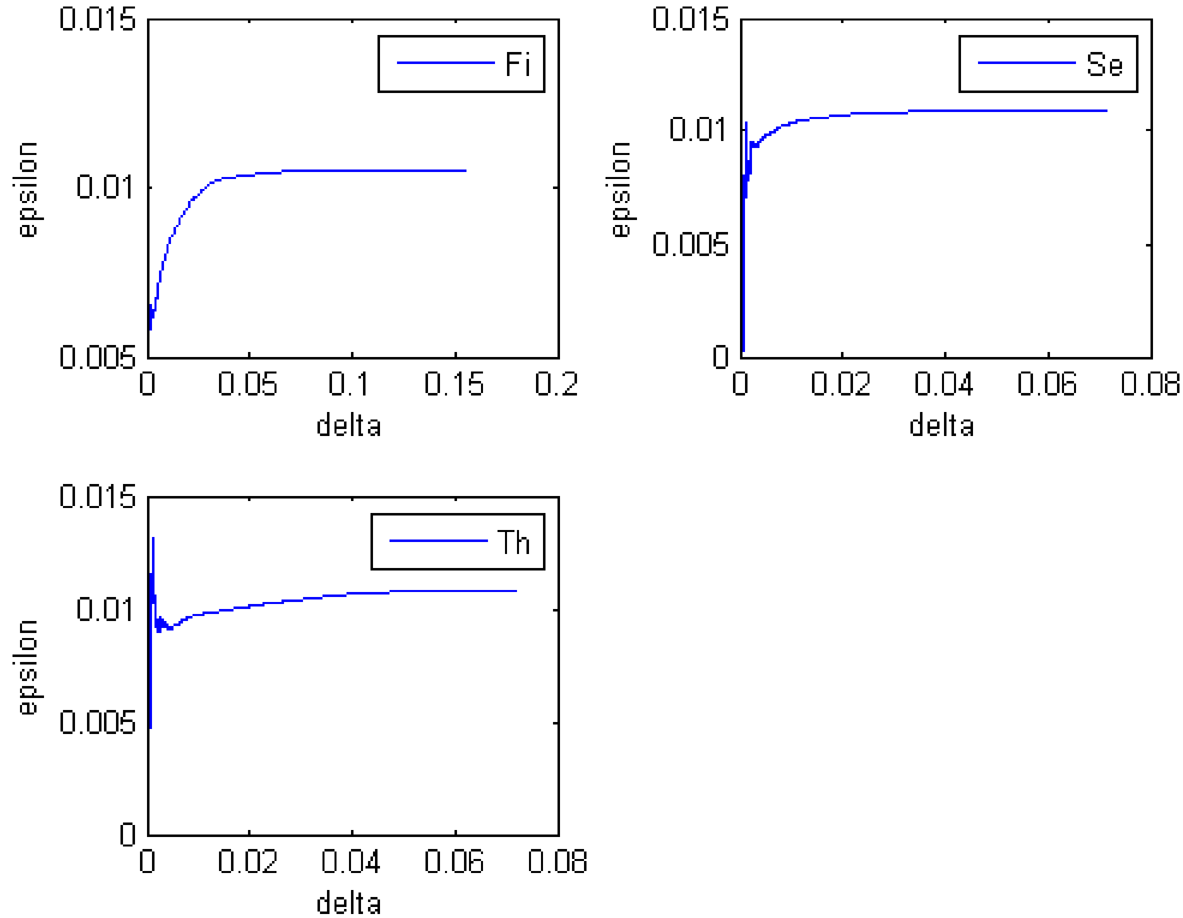


Figure 4.22: *Delta* vs. *Epsilon*, The Kaplan Test Results of Daily Price for Sub-Periods, Embedding Dimension=5

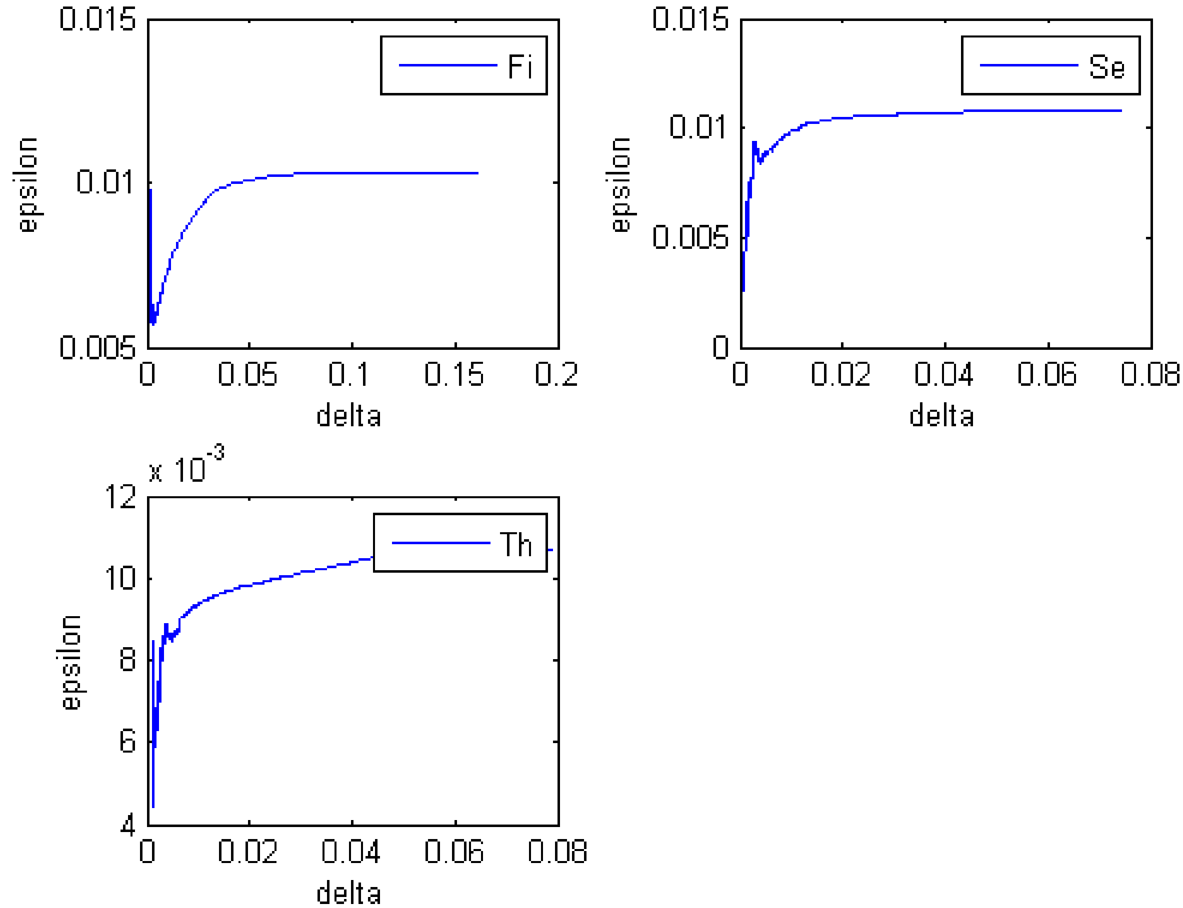


Figure 4.23: *Delta* vs. *Epsilon*, The Kaplan Test Results of Montlhy Spot Price on WTI, Embedding Dimension=2

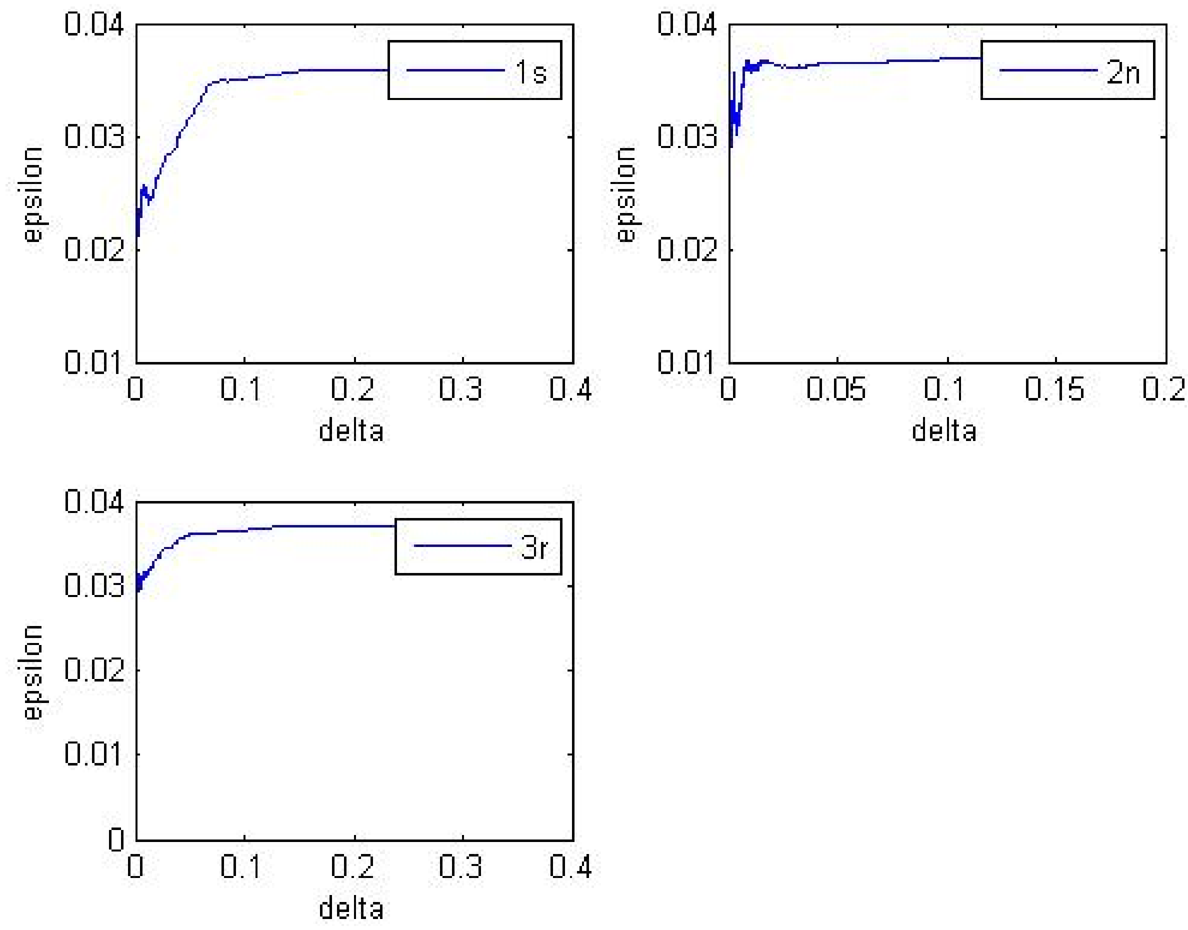


Figure 4.24: *Delta* vs. *Epsilon*, The Kaplan Test Results of Montlhy Spot Price on WTI, Embedding Dimension=3

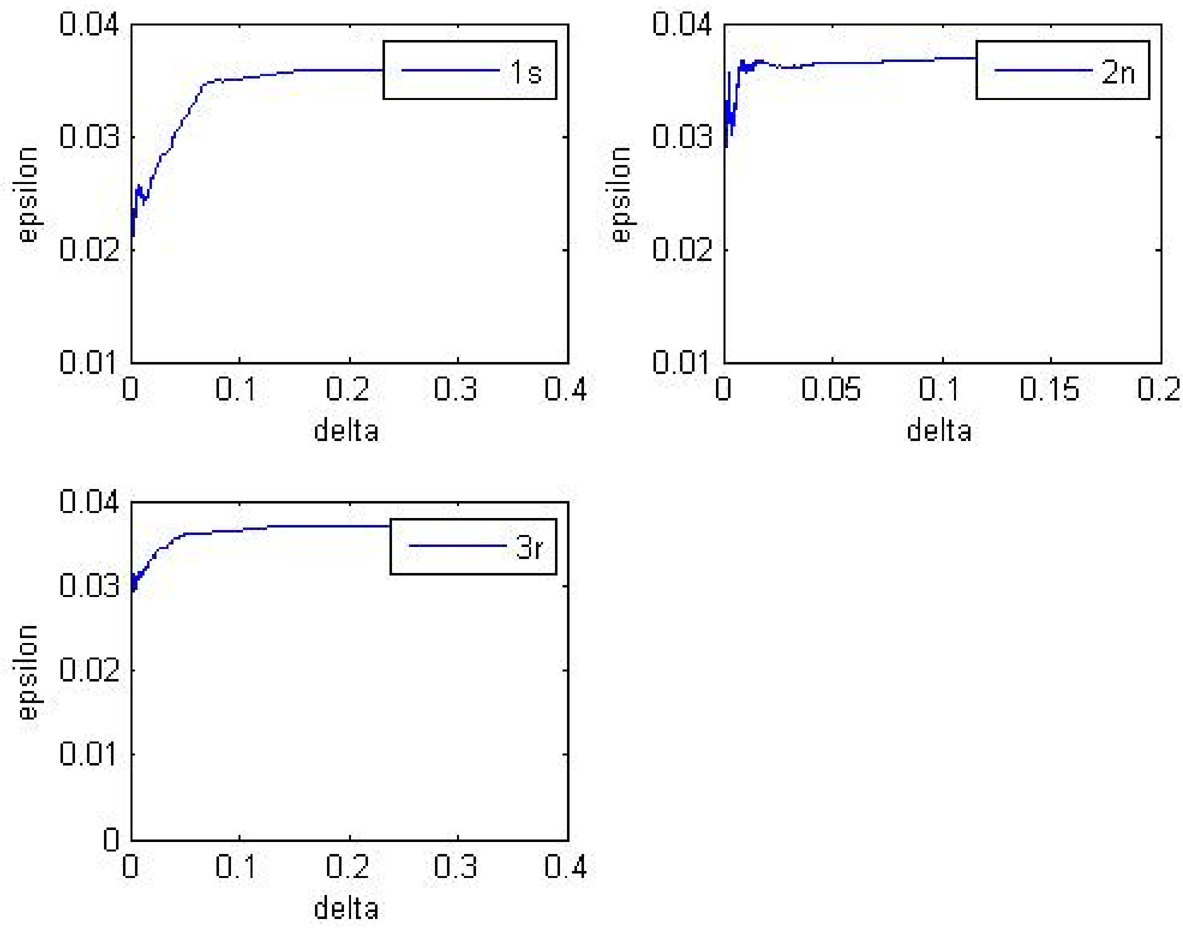


Figure 4.25: *Delta* vs. *Epsilon*, The Kaplan Test Results of Montlhy Spot Price on WTI, Embedding Dimension=4

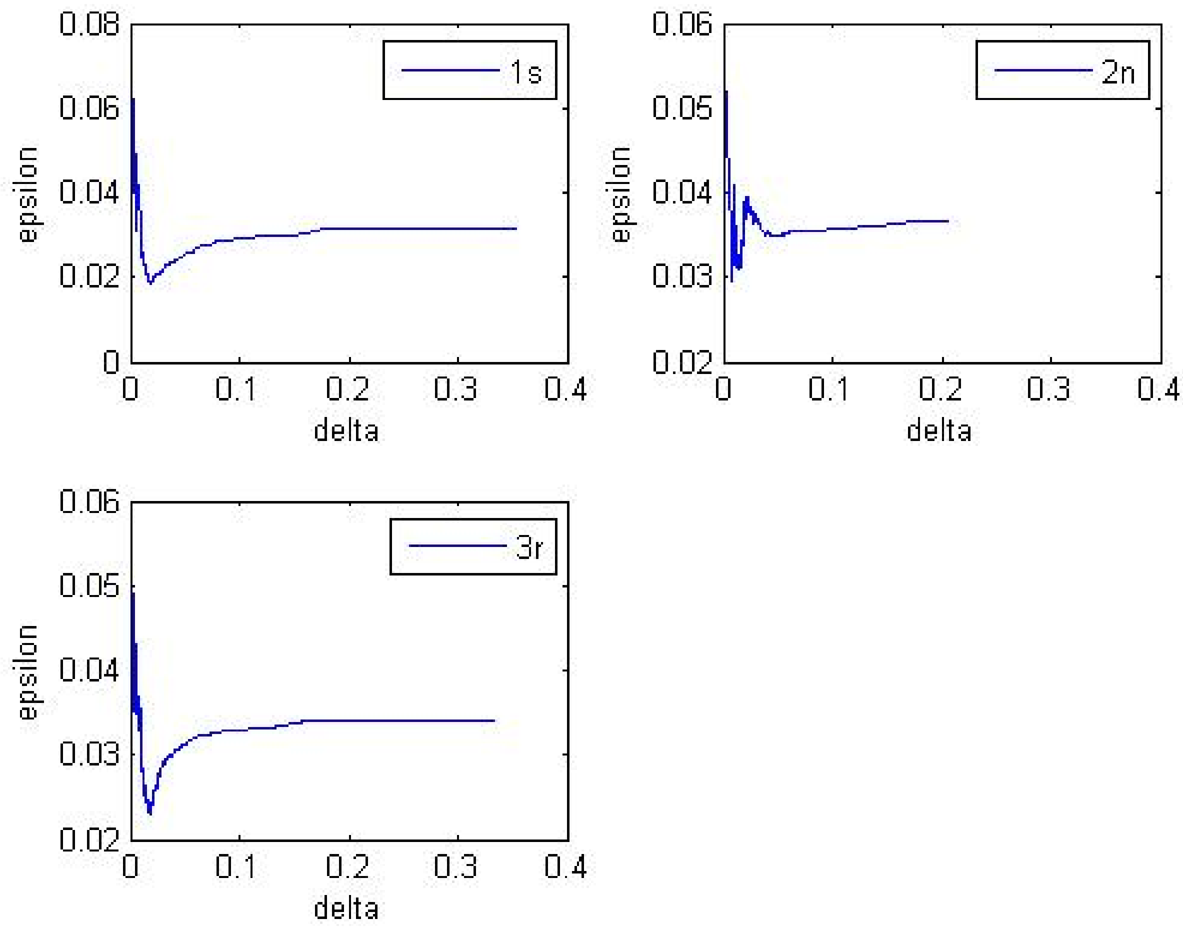
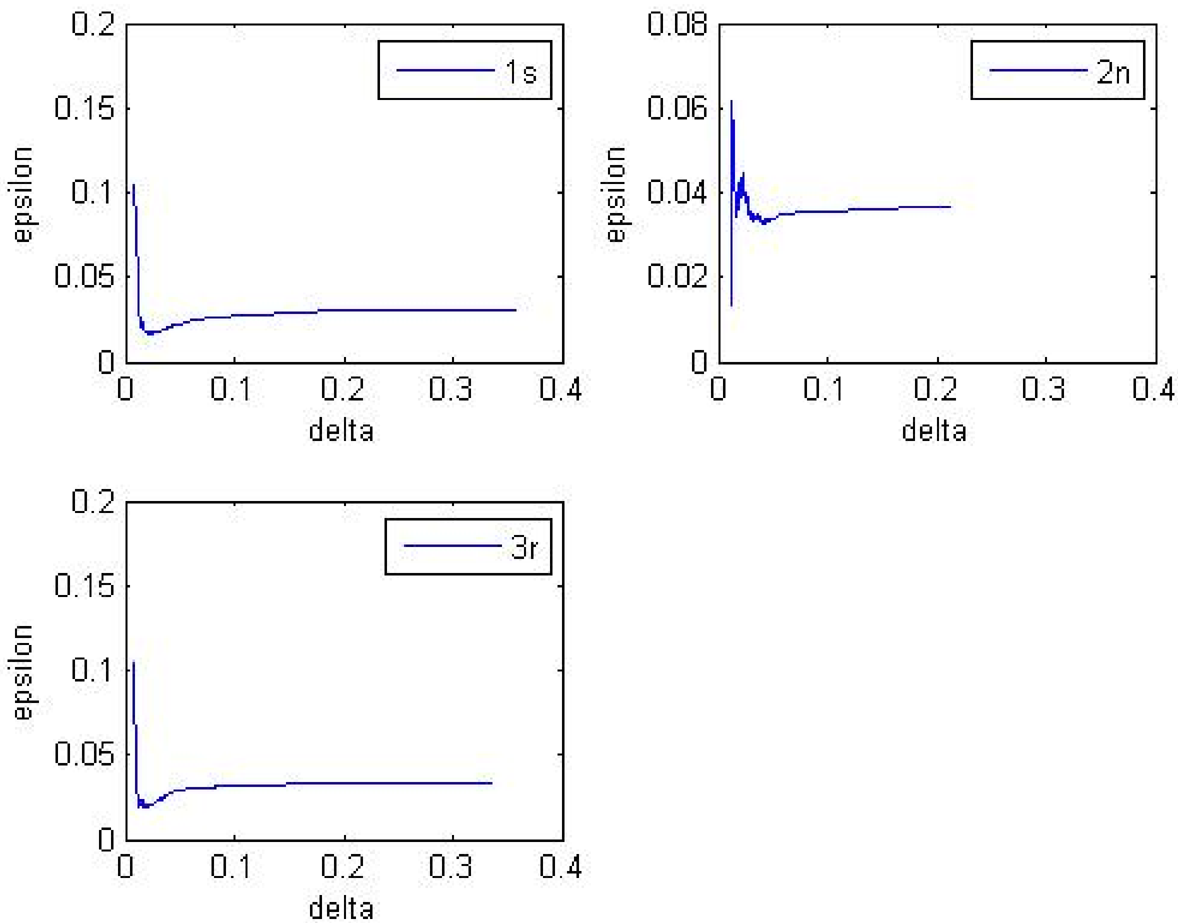


Figure 4.26: *Delta* vs. *Epsilon*, The Kaplan Test Results of Montlhy Spot Price on WTI, Embedding Dimension=5



Chapter 5

Conclusion and Economic Application

This dissertation includes three essays and assesses different features of the nonlinear dynamic structure of the energy sector in the context of nonlinear mechanisms. The research incorporates the production side of the energy market and explores the implications of high dimensionality and time aggregation when analyzing the market's fundamentals. Earlier studies, however, mainly focused on the price of the energy products and utilized daily frequency time series observations when detecting nonlinearities in the energy markets.

The first essay begins with application of statistical techniques and incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives hypothesis. The main goal of the first essay is to uncover the data generation mechanisms of the five main energy products. It utilizes the daily spot prices observations on crude oils, West Texas Intermediate (WTI-Cushing) and Europe Brent, New Harpor heating oil, and New York conventional gasoline regular, and Henry Hub Gulf Coast natural. These tests reveal different forms of nonlinearity in the data generating mechanism and detect strong evidence of nonlinear structures in the time series data. The results indicate that each individual series exhibits general nonlinear serial dependence, as well as nonlinearity in the mean, variance, and skewness functions.

The second essay explores the nonlinear dynamics of the crude oil production, and it is motivated by the largely neglected quantity side of the energy market. The production of crude oil is one of the central variables in defining the aggregate economy fluctuations and has significant impacts on various sectors. Hence, in view of the importance of the production of crude oil, understanding the dynamics of the production of crude oil is crucial. The examination of the production market's fundamentals will provide more accurate empirical and forecasting results by employing a closer specification to the data generating mechanisms. This essay employs statistical methods to examine the underlying mechanism of the time series data in the production market of crude oil. This essay uses monthly observations on the U.S. field production of crude oil, Organization of the Petroleum Exporting Countries (OPEC), non-OPEC, and the world production of crude oil. The tests detect strong evidence of nonlinearity in all the time series observations, with the exception of non-OPEC production of crude oil. The dynamics in the non-OPEC production time series data can be attributed to the nature of the market for those countries. The petroleum production for those countries exhibits a steady growth rate and has not been significantly influenced by exogenous shocks. On the other hand, OPEC production has often experienced disruptions of crude oil production and clear indications of nonlinear structure are reflected in the OPEC production time series observations.

The third essay focuses on the time aggregation and high dimensionality when assessing the nonlinear dynamics in the daily prices of crude oil. This essay utilizes daily spot price and monthly real price of crude oil, West Texas Intermediate (WTI) for over 26 years. The entire sample is divided into three sub-periods consisting of 2039 observations, 2511 observations, and 2092 observations. The sample period of study is from January 1970 to March 2011 for a total of 494 observations as well as two sub-samples: January 1970 to December 1991 for a total of 263 observations and January 1992 to March 2011 for a total of 231 observations. Incorporating monthly observations to assess the existence of nonlinear structures in the time series data generating mechanism of crude oil, when the time between observations increases, distinguishes the approach of this chapter from existing studies in the literature. To perform the analysis the most widely univariate tests to detect nonlinearity are employed to explore different aspects of nonlinear serial dependence. Daily spot price of crude oil reveals strong evidence of nonlinear structure in the data generating mechanism. In monthly observations, however, the nonlinear dependence is less significant. In summary, the power of nonlinear dependence varies by using different levels of time aggregation on daily spot price of crude oil. This chapter considered all of the possible cases for studying the dynamics of crude oil price and provides insightful understanding of the crude oil market data generating mechanism.

The nonlinearity and testing for nonlinear dynamics in the data generating mechanism have important economic applications and are becoming more common in empirical economics. The growing interests resulted from the fact that economic events are not essentially linear and a macroeconomic model may yield more plausible empirical results if the nonlinear features are considered in its nature. Employing linear time series models may cause misspecifications when the utilized time series is nonlinear. As Ashley and Patterson (1989) state "if the null hypothesis of linearity can be rejected for macroeconomic time series variables, then there would be serious misspecifications in the model by employing linear time series modeling". Also, Brockett et al. (1988) state that usual linear model coefficients can be shown to be biased in the face of a nonlinear time series structure. In the case of this research, where strong evidence of nonlinear structure in most of the utilized time series is detected, it is critical to employ an appropriate specification that reflects the dynamics of the data in the energy markets. In related studies, Hamilton (2011b) investigates nonlinearities and the macroeconomic effects of oil shocks and concludes that the relation between GDP growth and oil price is nonlinear. Also, Hamilton (2003) uses a flexible approach to identify the nonlinear relation between oil price change and GDP growth.

Moreover, interest in nonlinear forecasting models in economic literature has been growing in recent years. If nonlinearity is present in the data, choosing a nonlinear time series can provide more plausible post-sample forecasting ability (Ashley and Patterson (2006)). Various studies in the literature have concluded that nonlinear models yield better empirical results than linear models, such as Tersvirta (2005); Matias and Reboredo (2012); Suarez and Lopez (2011) among many others. As Tersvirta (2005) states, a potential effective model involves a systematic examination for finding a proper model that reflects the dynamic of the data and can be found only among the well-defined set of nonlinear models, such as: smooth transition regression (STR) models, switching regression and threshold autoregressive models, Markov-switching model, autoregressive neural network (ANN) models, time-varying autoregressive model, and nonlinear moving average models.

Another important implication of nonlinear structure in the energy market time series observations is inferences relevant to perfect markets. Under the perfect market assumption of complete contingent claims with perfect competition, perfect arbitrage, and free entry, general equilibrium time solutions are shocked martingales, having no information in past realizations that can be used for speculation about the future. But nonlinear stochastic processes contain structure that can be used for profitable speculation. The energy market is characterized not only by unpredictable exogenous shocks, but by multiple forms of market failure, such as barriers to entry (e.g., cartels on the supply side) and incomplete contingent claims. As a result, the relevant theory cannot rule out informative nonlinearity, which this study finds.

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