Design and Pricing of Probabilistic Quality

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Abstract

Increasingly, sellers are offering goods characterized by probabilistic quality. In such offers, buyers receive a synthetic product comprising of a lottery between two vertically differentiated goods. Given this emerging practice, I formally investigate the design and pricing of probabilistic quality. In this dissertation, I ask: How does probabilistic selling improve seller profits in vertical markets? When probabilistic quality is optimal, how is it designed; in particular, how are the associated probability, pricing, and product set determined? Further, what is the impact of transaction costs on the design of probabilistic quality? Next, what is the impact of probabilistic selling on consumer surplus? Finally, will probabilistic quality arise under demand uncertainty?

My analysis reveals that probabilistic quality can enhance seller profits via three distinct routes: profitably disposing excess capacity, better targeting of the high-quality product, and greater market coverage. In addition, transaction costs can play a critical role on the emergence and manner of emergence of probabilistic quality. Next, I find that probabilistic quality can potentially enhance consumer surplus even though its implementation necessitates a dissipative transaction cost. Finally, I find that probabilistic quality is robust to considerations of demand uncertainty.

Design and Pricing of Probabilistic Quality

1. INTRODUCTION

Consider the following observed market practices. An internet broadband service provider offers two levels of service: Gold and Palladium. The Gold service is priced at \$59.95 and offers a guaranteed download speed of 50Mbs. The Palladium service, on the other hand, is priced lower at \$49.95 but the download speed varies between 20Mbs and 50Mbps. Next, a major theme park offers two types of tickets: a higher-priced ticket that allows patrons to join a line with substantially reduced waiting times and a lower-priced offering that is restricted to the regular line. This regular line may sometimes have reduced waiting times and at other times suffer from long waiting times. Similarly, a racquet club offers two levels of pricing for court time: a higher rate for guaranteed court availability and a lower rate which provides court time only as capacity becomes available. Again, the lower rate may sometimes yield a court immediately but at other times may involve substantial waiting. Finally, a hotel offers three distinct products: a premium suite at a high price with guaranteed availability, a standard offering at a low price, and the premium suite at intermediate price but subject to availability failing which the standard offering is provided.

Notice that in all of these examples, one of the products in the product line essentially amounts to a lottery between two vertically differentiated goods. I refer to such a synthetic product as *probabilistic quality* and the associated sales practice as *probabilistic selling*, a nomenclature in the spirit of Fay and Xie (2008). Given growing use of this practice, I formally investigate the design and pricing of probabilistic quality. Specifically, my research aims to answer the following questions:

- How does probabilistic selling improve seller profits in vertical markets? What are the essential trade-offs involved in employing this selling format?
- When probabilistic quality is optimal, how should it be designed? In particular, how are the probability and price associated with probabilistic quality determined? In addition, should probabilistic quality be offered in tandem with the low-quality product, the high-quality product, or both?
- What is the impact of transaction costs on the design of probabilistic quality?
- Finally, what is the impact of probabilistic selling on consumer surplus?

I believe that my answers to these questions and attendant insights build on the burgeoning literature on the probabilistic aspects of market exchanges (see, for example, Fay 2008; Fay and Xie 2008, Fay and Xie 2010, Shugan and Xie 2000, Biyalagorsky, Gerstner, Weiss and Xie 2005). Further, the world-wide growth in services and the nature of technological developments further portends increased use of the basic segmentation scheme inherent in probabilistic selling. The reasoning behind this prediction is as follows. The implementation of probabilistic selling requires that customers be limited in their ability to arbitrage products. In the case of physical goods, customers (or entrepreneurial middleman) can always re-market the products and thereby undo the segmentation schemes of the marketer. Such arbitrage is relatively difficult in the case of services because consumption requires the presence of the customer. Similarly, from a technological perspective, the use of electronic smart cards embedded with bio-metric identification devices further constrains service delivery to the purchaser (Shugan and Xie 2004).

The rest of the study is organized in the following manner. In the next section, I briefly review the background literature and position my contributions in relation to the extant work. Then, I present my model, analysis, and findings. Finally, I conclude with a discussion of my key findings and outline directions for future research.

2. LITERATURE REVIEW AND POSITIONING

There is a growing body of research in marketing that analyses the uncertainty inherent in market exchanges. The extant literature has highlighted two main aspects of uncertainty: *uncertainty in buyer's consumption state* and *uncertainty in product offered*. Uncertainty in buyer's consumption state refers to the fact that buyers are often unsure about how much they value a future product. For example, buyers may be uncertain as to how strongly they will crave Chinese cuisine on some future occasion (Shugan and Xie 2000; Xie and Shugan 2001). In effect, this body of research introduces the notion of uncertainty in buyer consumption state to reflect the fact that the utility obtained by buyers is likely to be influenced by various personal factors such as mood, work schedule, and family situation. These researchers then demonstrate the profit-enhancing ability of advance selling. Such profit improvement arises because advance selling allows the contract to be inked when both parties are equally uncertain about future consumption utilities.

In contrast, uncertainty in product offered occurs when sellers engage in probabilistic selling. Here, the seller offers a good that essentially consists of a lottery between two distinct goods. Fay and Xie (2008) demonstrate how probabilistic selling can enhance profits via enhanced price-discrimination and market expansion. In effect, introduction of the probabilistic good allows the seller to separate consumers with strong preferences (who buy the component goods) from consumers with weak preferences (who buy the probabilistic good) thus enhancing prices for the component goods. Further, probabilistic selling facilitates market expansion by allowing the seller to promote the probabilistic good to low-valuation customers who would not have otherwise entered the market.

Finally, recent work has also *compared* the selling strategies that arise from the two sources of uncertainty, namely, uncertainty in buyer's consumption state and uncertainty in product offered. Specifically, Fay and Xie (2010) demonstrate the market characteristics that influence the profitability of advance selling relative to probabilistic selling.

Within this stream of research, my work is closest to that of Fay and Xie (2008). However, my problem context and results differ markedly from that of Fay and Xie (2008). In my context, a seller has two vertically differentiated products on hand, and, on account of demand uncertainty, faces excess capacity with respect to the high-quality product. In contrast, Fay and Xie (2008) primarily focus on a horizontal context. The key differences in results and insights are as follows. First, in Fay and Xie (2008), the probability associated with the probabilistic offer is primarily derived to be $\frac{1}{2}$; deviating from this level in either direction increases cannibalization and also reduces demand. In my work, however, the probability associated with probabilistic quality will, in general, be something other than $\frac{1}{2}$. Indeed, the derivation of this probability and related comparative statics in terms of model parameters is a key contribution of my work. In addition, I also introduce the notion of transaction costs associated with probabilistic quality. The need for such a parameter is foreshadowed in Fay and Xie (2008) and I extend their conceptualization to explicitly demonstrate its impact on the design and pricing of probabilistic quality. Next, their product line always includes the two extreme products and the intermediate probabilistic product. In contrast, in my work, the product line sometimes includes the two extreme products and the intermediate probabilistic product and at other times includes only the high-quality product and the intermediate probabilistic quality. Moreover, I am also able to demonstrate improvement in consumer surplus via two routes: lower prices enjoyed by consumers or targeting high-quality to those who value it greatly. Finally, although Fay and Xie (2008) also demonstrate the optimality of probabilistic selling for vertical markets, the vertical market that I utilize has a preference structure that is somewhat different from that employed in Fay and Xie (2008). I will clarify this difference more precisely in a later section.

In a related stream of work, Biyalagorsky, Gerstner, Weiss, and Xie (2005) analyze probabilistic quality in the context of service upgrades wherein buyers of upgradeable tickets face a lottery between two classes of service. While my work is closely related to their analysis, my problem context and results differ in the following manner. In their model, buyers of upgradeable tickets receive the higher class of service only if the high-type buyer fails to show up in the second period; moreover, this probability is exogenous to their analysis. In contrast, and as mentioned previously, I explicitly consider the probability of receiving the high-quality product as a decision variable. Further, in their model, there is no degradation in price paid by the high-type consumer for the high-quality product on account of introducing probabilistic quality. This is because the two offers are temporally separate and the high-type consumer only appears in the second period. However, in my research, as in Fay and Xie (2008), all consumers appear simultaneously. As such, the introduction of probabilistic quality leads to "cannibalization" of the margin that the seller can obtain from sale of the high-quality product. These differences distinguish my work in important ways from the research of Biyalagorsky, Gerstner, Weiss, and Xie (2005).

Finally, my study also builds on the extant research on product line design. Deneckere and McAfee (1996) and Moorthy and Png (1992) focus on a vertical market characterized by two segments that differ in their taste for quality. They then analyze issues related to optimal product line design such as purposeful degradation of quality offered to low-type consumers in order to *reduce* cannibalization. While I use many of their analysis techniques in my research, I differ primarily in that I introduce a synthetic third product which is a combination of the low-quality and high-quality product. Thus, in my research, I *increase* cannibalization on account of introduction of a product that is closer to the high-quality product than the low-quality product. Nevertheless, I demonstrate how this practice can potentially enhance seller profits. Moreover, the design and pricing of such an enhanced product that is essentially probabilistic in nature is a key contribution of my work. It is in this way that I complement the extant research on product line design.

3. MODEL AND ANALYSIS

I begin by describing the key elements of my model. Next, I present my analysis that responds to the questions that I seek to address in my research endeavor. I partition my analysis into two parts. In the main part, I derive findings pertaining to probabilistic quality in a world where capacity level for the high-quality product is given and exceeds demand. My characterization of the design and pricing of probabilistic quality is confined to this world. However, in additional analysis, I incorporate demand uncertainty and endogenize capacity choice to demonstrate that a world in which capacity exceeds demand can indeed arise in the first place.

3.1 Model

3.1.1 Basic Features

My model setting consists of a monopolist with two goods: a high-quality service and a low-quality service. In addition to these two vertically differentiated goods, the seller also has the option to include a synthetic product that essentially amounts to a lottery between the two goods. When offering probabilistic quality in this fashion, the seller offers the high-quality good with some pre-announced level of probability, $\emptyset$. Correspondingly, this implies that the seller offers the low-quality good with probability $1 - \emptyset$.

An important issue that arises here is that related to the implementation of such a synthetic product. In particular, since buyers only observe binary outcomes (high- or low-quality and not the entire distribution of probabilistic sales), there is a need for verification of the pre-announced level of probability. Absent such verification, the seller could very easily announce a probabilistic product that is priced at a premium relative to the low-quality product but pack it completely with the low-quality product. Of course, rational consumers anticipate this and probabilistic selling thus unravels. Indeed, this very issue has been foreshadowed and detailed by Fay and Xie (2008, p. 685). In response, they suggest that seller reputation, independent reviews or a channel intermediary could perform this verification function in the face of such seller opportunism. Interestingly, this issue has also been raised in Moorthy and Png (1992). In particular, they highlight the role of credible commitment by the seller with respect to the decision variables of quality, price, and the order in which the products are introduced. In like fashion, they suggest that seller reputation, industry practice, or cost of reneging all influence the credibility of commitment.

While all these mechanisms are feasible, I assume that utilizing an intermediary is the most efficient route to effect verification of the specific probability announced by the seller – building a reputation or relying on independent reviews are indirect mechanisms and will likely require more investment. Further, I posit that such verification is obtained at a transaction cost, c, per unit of the probabilistic offering that is sold. This is consistent with what is observed in the online gaming industry where online casinos post “odds.” In particular, the software platforms employed by online casinos rely on random number generators that are licensed and audited by third-party vendors (see, for example: http://www.casinocashjourney.com/merge-gaming-network-software.htm). I anticipate that sellers intent on realizing the promise inherent in probabilistic selling will likely have to follow a similar route. This is because the essence of implementing probabilistic quality revolves around the "odds" of receiving high-quality.

3.1.2 Capacity, Demand, and Valuations

The seller has capacity M for the high-quality service and N for the low-quality service with $M < N$. This last assumption is consistent with anecdotal evidence that sellers of two quality tiers generally offer more low-quality capacity. The associated variable costs for offering the high-quality and low-quality services are c_H and c_L respectively, with $c_H > c_L$ and c_L normalized to zero.

I next assume that there are two types of consumers in the market, high-type and low-type, with market sizes n_H and n_L, respectively. In my research, I assume that low-type consumers constitute the mass market and they are numerous; consequently, they exceed the entire capacity of the seller (i.e., $n_L > M + N$, and I will provide an explanation for why I invoke this assumption at a later point). However, the number of high-type customers is strictly less than the seller's capacity of high-quality (i.e., $n_H < M$). Then, an important question that arises is why the seller will ever choose to build high-quality capacity that exceeds demand. I posit the following intuitive reason: demand uncertainty. Accordingly, in additional analysis, I incorporate demand uncertainty for the high-quality product and explicitly endogenize the associated capacity choice. I then show that the optimal capacity choice for the high-quality product, M^*, exceeds the lower bound for the support of n_H. In this way, I demonstrate that the world in which my analysis is conducted (namely, $n_H < M$) can indeed arise in the first place.

I further posit that the two types of consumers have different valuations for the two levels of services. The high-type consumers' value high-quality at V_{HH} and low-quality at V_{HL}, with $V_{HH} > V_{HL}$. The low-type consumers value high-quality at V_{LH} and low-quality at V_{LL}, with $V_{LH} > V_{LL}$. I also expect $V_{HH} > V_{LH}$ and $V_{HL} > V_{LL}$. Finally, I denote $[V_{HH} - V_{HL}] - [V_{LH} - V_{LL}] = [V_{HH} + V_{LL}] - [V_{HL} + V_{LH}]$ as Δ and expect $\Delta > 0$. That is, in addition to their stronger preference for a given level of quality, high-type consumers value successive levels of quality even more than low-type consumers. This assumption is consistent with the basic vertical model presented in Tirole (1988, p. 296) and is frequently employed in the marketing literature (see, for example, Moorthy and Png 1992).

At this point, I compare my preference structure to that of Fay and Xie (2008) where they extend their analysis of horizontal differentiation to the case of vertical differentiation (p. 681). I highlight that there are subtle, but important, differences in the preference structure assumed between their work and mine. While both sets of researchers assume $V_{HH} > V_{HL}$ and $V_{LH} > V_{LL}$ and $\Delta > 0$, I assume $V_{HL} > V_{LL}$ whereas Fay and Xie (2008) assume $V_{LL} > V_{HL}$. In effect, I conceptualize that high-quality users are simply more "intense" users in that they value any quality level (e.g., 50 Mbps or 20Mbs) more than the low-quality user. In contrast, Fay and Xie (2008) conceptualize that the high-type consumer demonstrates "threshold" effects with respect to quality. For instance, if the high-type watches streaming movies and the low-type browses, the high-type gets little or no value from the 20Mbs product whereas the low-type does. In my work, I consider the "intensity" interpretation but fully recognize that the "threshold" interpretation is also likely to be an equally important characterization of real-world vertical markets. Interestingly, I note that these alternate preference structures may even occur within the same product line but across different product sets. For example, in the automobile market, Lexus / Camry may have the Fay and Xie (2008) preference structure because of snob effects whereas Camry / Corolla may have my proposed preference structure. Again, this discussion suggests that vertical markets may differ in preference structure and it is important to recognize these distinctions across different research endeavors. Substantively, considering $V_{HL} > V_{LL}$, as opposed to $V_{LL} > V_{HL}$ heightens cannibalization - one can thus rightfully doubt the emergence of probabilistic quality in my setting, further justifying my formal inquiry.

Within my model, consumers and sellers behave as follows. Consumers take their valuations as given and choose a service, with its associated price, so as to maximize utility. This utility comprises of valuation for the service less the price charged by the seller. Sellers, on the other hand, take segments and valuations as given and offer products and set prices to maximize their profits. [1] I reiterate that in my model, as in Fay and Xie (2008), all consumers appear simultaneously and the resolution of which product the consumer receives upon purchasing probabilistic quality is immediate. Finally, if there are more buyers for a product

[1] An additional assumption that I invoke pertaining to valuations is: $V_{HH} - V_{HL} > C_H$. That is, it is profitable to introduce high-quality.

than offered by the seller then allocation is purely random. Before I discuss my analysis, it is useful to discuss three benchmarks that the seller has at his (or her) disposal.

3.1.3 Benchmarks

There are three strategies that the seller can adopt. In the first, the seller offers high-quality service to the high-type consumer and low-quality service to low-type consumers yielding the traditional differentiated product line strategy. I label this strategy as Benchmark 1. Following Moorthy and Png (1992), I posit that the high-quality product is targeted to the high-type segment and the low-quality product is targeted to the low-type segment. Moreover, the seller charges V_{LL} for the low-quality product and $V_{HH} - V_{HL} + V_{LL}$ for the high-quality product. The price for the high-quality product, p_H, is obtained from the incentive-compatibility constraint such that the high-type consumer is just indifferent between consuming the high- and low-quality products: $V_{HH} - p_H = V_{HL} - V_{LL}$. I call this as a "strong" differentiation strategy because the difference in price between low-quality and high-quality is the difference in the *high-type's* valuations of these qualities subject to incentive-compatibility. Prices and profits associated with Benchmark 1 can be formally stated as follows:

$$p_H = V_{HH} - V_{HL} + V_{LL} \; and \; p_L = V_{LL} \Rightarrow \pi_{B1} = n_H(V_{HH} + V_{LL} - V_{HL} - c_H) + NV_{LL}$$
$$= n_H(V_{LH} + \Delta - c_H) + NV_{LL}$$

(1)

In the second strategy, the entire capacity of the seller, $M + N$, is exhausted by setting a price of V_{LH} for the high-quality product and a price of V_{LL} for the low-quality product. In effect, the seller offers high-quality service at the low-type consumers' reservation price V_{LH} and the low-quality service at low-type consumers' reservation price V_{LL}. This also yields the traditional differentiated product line strategy but I refer to this as a "weak" differentiation strategy. This is because the difference in price between low-quality and high-quality is the difference in the *low-type's* valuations of these qualities. Here, the low-type consumers are indifferent between high-quality and low-quality goods. However, the high-type consumers will choose to buy the high-quality service because they obtain greater utility from consuming the high-quality service in this instance. Given more buyers than capacity, product assignment is random. Since seller profits are independent of the composition of buyers, prices and profits associated with Benchmark 2 are given as:

$$p_H = V_{LH} \text{ } and \text{ } p_L = V_{LL} \Rightarrow \pi_{B2} = M(V_{LH} - c_H) + NV_{LL} \qquad (2)$$

The "strong" differentiation in Benchmark 1 improves on the "weak" differentiation in Benchmark 2 by yielding a higher price for the high-quality product which follows from the fact that $\Delta > 0$. However, it does not dominate Benchmark 2 because it cannot exhaust all the high-quality capacity as in Benchmark 2. In particular, capacity of $M - n_H$ remains unfilled since $n_H < M$. As such, both benchmarks need to be retained for subsequent analysis.

A third strategy is to focus only on serving the high-type customers with high-quality product. This is an "up-market" strategy where the seller only offers the high-quality service at price V_{HH} to high-type consumers. By adopting this strategy, the seller actually gives up profits from the low-type consumers in order to realize the benefit of charging the high price of V_{HH} to high-type consumers. If the seller wants to include the low-type consumers in the market by offering the low-quality service, the price of the high-quality service has to be lowered in order to prevent the high-type consumer from choosing the service targeted to the low-type consumer. To prevent such degradation of profits, the seller chooses to focus exclusively on the high-type consumers. I label this as Benchmark 3 and the profit arising from this strategy is:

$$p_H = V_{HH} \Rightarrow \pi_{B3} = n_H(V_{HH} - c_H) \tag{3}$$

I next set a restriction to focus on the most interesting region of my parameter space In particular, I posit than an "up-market" strategy wherein the seller only sells the high-quality product to the high-types as in Benchmark 3 is dominated by the full-coverage strategies embodied in either Benchmark 1 or 2. Intuitively, I expect ignoring the low-type segment to be too costly. Mathematically, I conduct my analysis here assuming that:

$$\pi_{B3} \leq Max\,\{\pi_{B1}, \pi_{B2}\} \tag{4}$$

Of course, later, I will also analyze the case where Benchmark 3 dominates both Benchmarks 1 and 2 and explicitly demonstrate the emergence of probabilistic quality in that scenario as well. For ease in exposition, this is detailed in the Appendix.

3.1.4 Analysis

Benchmarks 1 and 2 illustrate the seller's outcomes without adopting probabilistic quality in the product line. I next examine what the seller can do by employing probabilistic quality. Before I present this analysis, I preview how probabilistic quality can enhance improve on the two benchmarks. Broadly, probabilistic quality can improve on Benchmark 1 by reducing fallow capacity associated with this benchmark. In contrast, probabilistic quality can improve on the weak differentiation associated with Benchmark 2. Specifically, by including probabilistic quality, the seller can now target the high-quality product to the high-type segment and obtain a better price for high-quality.

With some abuse of notation, I shall variously employ the symbol $\emptyset$ to reflect the probability of receiving high-quality, a subscript to denote the level of a decision variable under probabilistic quality, or even the strategy of employing probabilistic quality in the product line. Note that when probabilistic quality is targeted to a particular segment, the value that buyers place on it is a linear combination of the valuations for high- and low-quality. That is, $V_{H\phi} = \phi V_{HH} + (1-\phi)V_{HL}$ and $V_{L\phi} = \phi V_{LH} + (1-\phi)V_{LL}$. Further, the seller has to set prices with incentive-compatibility in mind. That is, buyers will compute the utility of selecting every product offered by the seller and choose the product which maximizes their utility; consequently, prices have to be set with this consideration in mind.

At this point, it is instructive to discuss when and how probabilistic quality can arise. First, although I assume $n_H < M$, I note that probabilistic quality will never arise when $n_H \geq M$. Formally, I have:

Result 1: Probabilistic quality will never arise when the number of high-type customers exceeds the available high-quality capacity, i.e., when $n_H \geq M$.

Result 1 demonstrates that probabilistic quality is never optimal when $n_H \geq M$. To prove this, first note that in this instance Benchmark 1 dominates Benchmark 2 because it has higher prices and does not now suffer from unused capacity. Including probabilistic quality to improve on Benchmark 1 implies that the seller has to decide which segment probabilistic quality should be targeted towards. Two cases are pertinent. In the first case, the seller targets probabilistic quality to the high-type segment. In its most general form, this case involves selling both high-quality and probabilistic quality to the high-type segment so that is what I will analyze. Accordingly, the seller announces some capacity of high-quality and some capacity of probabilistic quality targeted to the high-type segment with actual assignments being random. I need the following incentive-compatibility constraints:

$$V_{H\phi} - p_{\phi} \geq V_{HL} - p_L$$

$$\Rightarrow p_{\phi} = \phi(V_{HH} - V_{HL}) + V_{LL} = \phi(V_{HH} - V_{HL} + V_{LL}) + (1-\phi)V_{LL} \tag{5a}$$

$$V_{HH} - p_H \geq V_{HL} - p_L$$

$$\Rightarrow p_H = V_{HH} - V_{HL} + V_{LL} \tag{5b}$$

Equation (5a) reveals that the prices for the two components in probabilistic quality are identical to the prices for these components in Benchmark 1 (please see equation (1)). Equation (5b) reveals that the price obtained for the high-quality product is identical to Benchmark 1. As such, probabilistic quality targeted to the high-type segment cannot improve on Benchmark 1 and given transaction costs, c, actually does worse.

Next, consider the case when probabilistic quality is targeted to the low-type consumer. In its most general form, this case involves selling both probabilistic quality and low-quality to the low-type segment so that is what I will analyze. Accordingly, the seller announces some capacity of probabilistic quality and the remaining capacity of low-quality targeted to the low-type segment with actual assignments being random. I have

$$p_{\phi} = V_{L\phi} = \phi V_{LH} + (1-\phi)V_{LL} \tag{6a}$$

In addition, to guarantee that the high-type consumers buy the high-quality product I need:

$$\begin{cases} V_{HH} - p_H \geq V_{H\phi} - p_{\phi} \\ V_{HH} - p_H \geq V_{HL} - p_L \end{cases}$$

$$\Rightarrow \begin{cases} p_H \leq V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \\ p_H \leq V_{HH} - (V_{HL} - V_{LL}) \end{cases}$$

Since $\Delta = (V_{HH} - V_{LH}) - (V_{HL} - V_{LL}) \geq 0$;

$$V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \leq V_{HH} - (V_{HL} - V_{LL})$$

$$\Rightarrow p_H = V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \tag{6b}$$

In this event, the price obtained for the high-quality component in probabilistic quality is lower than that obtained by selling the high-quality to the high-type customer in Benchmark 1 ($V_{LH} < V_{HH} - V_{HL} + V_{LL}$ since $\Delta > 0$) whereas the price of the low-quality component remains unchanged. Moreover, offering probabilistic quality to the low-type also degrades the price that the seller can charge to the high-type consumer because of "cannibalization." Specifically, the price of the high-quality product drops from $V_{HH} - V_{HL} + V_{LL}$ to $V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) = V_{HH} - V_{HL} + V_{LL} - \phi\,\Delta$. Interestingly, the degradation in price is more pronounced as probabilistic quality becomes a closer substitute to the high-quality product (increasing ϕ). Finally, the transaction costs associated with probabilistic quality further degrades the profits from targeting probabilistic quality to the low-type segment. For all of these reasons, probabilistic quality targeted to the low-type segment is also unable to improve on the profits obtained via Benchmark 1.

Having shown that the seller will not target probabilistic quality to either the low- or high-type whenever $n_H \geq M$, I now focus on the case where $n_H < M$. My goal is to describe the manner in which probabilistic quality can emerge and thereby facilitate the subsequent exposition. I have:

Result 2: When optimal, the strategy of probabilistic quality, ϕ, can only take the following forms:

a. Probabilistic quality with *two* quality tiers **[H, ϕ]** where the seller offers n_H high-quality products to the high-type segment and cobbles the remaining $M - n_H$

high-quality products with N low-quality products to offer probabilistic quality to the low-type segment.

b. Probabilistic quality with *three* quality tiers **[H, ф, L]** where the seller offers n_H high-quality products to the high-type segment and creates both probabilistic quality and low-quality targeted to the low-type segment. In this event, all of the remaining $M - n_H$ high-quality products and some of the N low-quality products are used to create probabilistic quality. The remaining low-quality units are retained for separate sale to the low-type segment.

Proof: In Appendix.

Result 2 limits the options available to the seller. In particular, the seller will never sell probabilistic quality to the high-type segment. The seller always prefers to exhaust the high-quality capacity on the high-type segment in order to obtain an attractive price. The remaining unsold high-quality capacity is then sold via probabilistic quality to the low-type segment. Sometimes, all of the low-quality product is used to create probabilistic quality and targeted to the low-type segment. At other times, some low-quality capacity is reserved for separate sale to the low-type segment. This discussion reveals that offering probabilistic quality and low-quality to the low-type segment via a $[\boldsymbol{\phi}, \boldsymbol{L}]$ offering is never optimal. Similarly, since $[\boldsymbol{\phi}]$ is a special case of $[\boldsymbol{\phi}, \boldsymbol{L}]$**,** Result 2 also reveals that adopting a $[\boldsymbol{\phi}]$ strategy targeted to the low-type segment is never optimal.

Following Results 1 and 2, offering probabilistic quality implies that the seller always targets high-quality to the high-types **a**nd then cobbles together the unsold high-quality capacity $M - n_H$ and some low-quality capacity X to create the synthetic product. When consumers buy this kind of service, they are not guaranteed to receive either high-quality or low-quality; rather, consumers obtain the high-quality service with probability $\emptyset = \frac{M-n_H}{M-n_H+X}$ and low-quality service with probability $1 - \emptyset = \frac{X}{M-n_H+X}$. In addition, the reason why I invoke $n_L > M + N$ can now be made clear: this assumption ensures that there is enough demand for low-quality and probabilistic quality. This, in turn, allows us to demonstrate the essential costs and benefits of introducing probabilistic quality without being fettered by constraints on n_L.

In my analysis, a key decision variable is X, the amount of low-quality product that must be added to the excess high-quality capacity $M - n_H$ to create the synthetic product. Thus, one managerially relevant output of my model is the amount of low-quality capacity that should be set apart in order to offer probabilistic quality. A second managerially relevant output of my model is the price and probability associated with probabilistic quality, and it turns out that deriving X also defines these variables. Since the price for the low-quality product is V_{LL} and probabilistic quality is always targeted to the low-type segment, the price for the probabilistic quality is $p_\emptyset = V_{L\emptyset} = \emptyset V_{LH} + (1-\emptyset)V_{LL}$. From the incentive-compatibility constraint of the high-type customers, the price of the high-quality product is:

$$p_H = V_{HH} - \emptyset(V_{HH} - V_{LH}) - (1-\emptyset)(V_{HL} - V_{LL}) \tag{7}$$

Also, the cost for probabilistic quality is $\emptyset c_H + c$, which includes both the product cost as well as the aforementioned transaction costs. Under these considerations, the seller chooses X to maximize the following profit function:

$$\pi = n_H(p_H - c_H) + (M - n_H + X)(p_\emptyset - \emptyset c_H - c) + (N - X)p_L$$

$$= n_H(\Delta + c) + M(V_{LH} - c_H - c) + NV_{LL} - cX - \left[\frac{(M - n_H)n_H}{M - n_H + X}\right]\Delta$$

$$\text{ST. } 0 \leq X \leq N \tag{8}$$

I find it convenient to define four levels with respect to the variable that represents the transaction cost associated with offering probabilistic quality, *c*. I also find it convenient to define one specific level for the variable that represents the capacity associated with the high-quality product, *M*. In particular, I denote:

$$c_1 = \frac{(M-n_H)n_H\Delta}{[M-n_H+N]^2} \tag{9a}$$

$$c_2 = \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} \tag{9b}$$

$$c_3 = \frac{n_H\Delta}{4(M-n_H)} \tag{9c}$$

$$c_4 = \frac{(M-n_H)(V_{LH}-c_H)}{M-n_H+N} - \frac{(M-n_H)n_H\Delta}{[M-n_H+N]^2} \tag{9d}$$

$$M_0 = \frac{n_H(\Delta+V_{LH}-c_H)}{V_{LH}-c_H} \tag{9e}$$

3.1.5 Findings

I am now in a position to outline the propositions that flow from analysis. I note that my analytical findings unfold over two cases that differ with respect to the relative magnitude of n_H. More specifically, these two cases are described by $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ and $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$. Broadly, these two cases reflect how much excess high-quality capacity exists. It is straightforward to see that the amount of excess capacity is greater under the former inequality. Considering both these cases allows my work to be general and complete. In addition, I note that the critical value for high-quality capacity, $M_0 = \frac{n_H(\Delta+V_{LH}-c_H)}{V_{LH}-c_H}$, demarcates the boundary of the two Benchmarks, 1 and 2. If $M < M_0$, Benchmark 1 is relevant. Conversely, if $M > M_0$, then Benchmark 2 is germane. Intuitively, when M is relatively small, the loss stemming from the unsold capacity in Benchmark 1 is that not large; consequently, this is the region where Benchmark 1 dominates Benchmark 2. In contrast, when M is relatively large, the loss stemming from the unsold capacity in Benchmark 1 is fairly large; consequently, this is the region where Benchmark 2 dominates Benchmark 1.

My first proposition pertains to the optimality and design of probabilistic quality. Formally, I have:

Proposition 1: Regardless of the relative magnitude of n_H, the seller finds it profit-enhancing to offer probabilistic quality in two distinct regions of the parameter space. These two regions differ in the number of segments targeted. In Region A (two-segment), the seller augments probabilistic quality only with the high-quality product. In Region B (three-segment), the seller augments probabilistic quality with both the high-quality product as well as the low-quality product. Further, the products offered, probability associated with probabilistic quality, and prices are as in Tables 1a and 1b[2]. The relevant regions are depicted in Figures 1a and 1b.

Proof: In Appendix

[2] Across the two cases, I see that the difference is mainly regarding the optimality of offering probabilistic quality. In particular, probabilistic quality in the form of the **[H, ϕ, L]** solution is optimal over a greater region when $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$. Recall that the profitability of Benchmark 1 is influenced by the magnitude of n_H since it positively influences the number of high-quality sales. Thus, when $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$, Benchmark 1 is not that profitable; moreover, the amount of fallow capacity is high; consequently, probabilistic quality improves on Benchmark 1 over a greater region as shown in Figure 1a. Conversely, when $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$, Benchmark 1 is relatively profitable; moreover, the amount of fallow capacity is low; consequently, probabilistic quality is unable to improve on Benchmark 1. These considerations are not salient with respect to Benchmark 2 because n_H does not influence its profitability; consequently, the region of optimality of **[H, ϕ, L]** is not impacted. Finally, while not obvious from the Figures, the upper boundary of Region A when $M < M_0$ is also altered. Specifically, in Figure 1a, it is defined by c_1 whereas in Figure 1b it is defined by c_4.

In the figures, the horizontal axis is the capacity of the high-quality product and the vertical axis is the seller's transaction costs. The horizontal axis starts at n_H since M has to exceed n_H in order to provide excess capacity for probabilistic quality. The starting point of the vertical axis is $c = 0$.

I begin by providing some intuition for the underlying drivers behind the choice of a product and pricing strategy (**[H, ɸ]**, **[H, ɸ, L]**, Benchmark 1, or Benchmark 2) First, I compare probabilistic quality with Benchmark 1. Note that with the introduction of probabilistic quality, the price obtained for the high-quality product goes down in relation to the price obtained for the high-quality product in Benchmark 1 (cannibalization). Specifically, in the **[H, ɸ]** offering, $p_H = \Delta + V_{LH} - \frac{(M-n_H)\Delta}{M+N-n_H}$ whereas in Benchmark 1, $p_H = \Delta + V_{LH}$. Similarly, in the **[H, ɸ, L]** offering, $p_H = \Delta + V_{LH} - \sqrt{\frac{c(M-n_H)\Delta}{n_H}}$ whereas in Benchmark 1,$p_H = \Delta + V_{LH}$. This cannibalization notwithstanding, the benefit of probabilistic quality is that it can gainfully utilize fallow capacity.

At this point, it is important to note the probability associated with probabilistic quality implicitly controls the extent of the cannibalization effect. When $\emptyset$ is high, there is a greater proportion of high-quality product in the probabilistic offer; consequently, cannibalization is more severe because the products are close substitutes. On the other hand, when $\emptyset$ is low (obtained by choosing a high *X*), it leads to a large number of units that are sold via probabilistic quality. The question then arises: why does the seller not minimize the cannibalization effect by lowering the choice of $\emptyset$? The answer is that selling a large number of units via probabilistic quality degrades profits because the transaction cost, c, applies to each probabilistic sale. It is these countervailing forces that leads to an interior solution for $\emptyset$.

Next, I compare probabilistic quality to Benchmark 2. The advantage of probabilistic quality is that the seller is able to obtain a better price for the high-quality capacity by targeting the high-type customer. Specifically, in the **[H, ϕ]** offering, $p_H = \Delta + V_{LH} - \frac{(M-n_H)\Delta}{M+N-n_H}$ whereas in Benchmark 2, $p_H = V_{LH}$. It is straightforward to see that $\boldsymbol{p_H}$ in **[H, ϕ]** is always greater than $\boldsymbol{p_H}$ in Benchmark 2 since $\Delta > 0$. Similarly, in the **[H, ϕ, L]** offering, $p_H = \Delta + V_{LH} - \sqrt{\frac{c(M-n_H)\Delta}{n_H}} = \Delta + V_{LH} - \emptyset^{*}\Delta$ whereas in Benchmark 2, $p_H = V_{LH}$ (refer Table 1a and 1b) **.** Because $\emptyset^{*}$is less than 1, it is straightforward to see that p_H in **[H, ϕ, L]** is also greater than p_H in Benchmark 2.

This discussion implies that probabilistic quality can potentially improve on the weak differentiation associated with Benchmark 2. Specifically, by including probabilistic quality, the seller can now target the high-quality product to the high-type segment and obtain a better price for high-quality.

Depending on how these drivers play out, the seller employs one of four strategies: **[H, ϕ]**, **[H, ϕ, L]**, Benchmark 1, or Benchmark 2). I now discuss the specific regions that are highlighted in Figures (1a) and (1b).

Region A: **[*H, Φ*].**

Across this region, the transaction cost of using probabilistic quality is relatively low. As such, the concern about profit erosion from transaction costs is not that salient. Thus, it is optimal to minimize the cannibalization effect by exhausting all the low-quality product to create probabilistic quality. I thus obtain the **[*H, Φ*]** strategy.

Region B: **[*H, Φ, L*].**

In region B, the transaction cost of using probabilistic quality is higher than in region A for any given M. Here, the concern about profit erosion stemming from transaction costs is relevant. Consequently, the seller cannot minimize the cannibalization effect by indiscriminately using a large number of low-quality products to create probabilistic quality. As such, the seller does not exhaust the low-quality capacity while creating the probabilistic offer but rather reserves some low-quality product for separate sale via **[*H, Φ, L*].**

Benchmarks

In the Figures, I also show that as transaction cost increases, the optimal solution reverts to the benchmarks. The seller's efforts to mitigate cannibalization inherent in probabilistic quality are hampered by the profit erosion stemming from increased transaction costs; consequently, the benchmarks emerge as the optimal solution.

Boundaries of Regions

In the case of $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ (Figure 1a), the frontier between B and C can be understood as follows. For a fixed transaction cost c, as the capacity of the high-quality product *M* increases, the attractiveness of **[*H, Φ, L*]** relative to Benchmark 1 increases. This is because the profitability of Benchmark 1 is independent of *M* whereas **[*H, Φ, L*]** increases in profitability as *M* increases. Simply put, probabilistic quality is more attractive with greater excess high-quality capacity. Therefore, for a given level of transaction costs, the payoff of adopting Strategy **[*H, Φ, L*]** exceeds that of Benchmark 1 if *M* is large enough.

For both $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ and $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ (Figures 2a and 2b), the frontier between regions B and D can be understood as follows. For a fixed transaction cost c, as *M* increases, the seller switches from **[*H, Φ, L*]** to Benchmark 2. The reason is as follows. As *M* increases, the payoffs of strategies **[*H, Φ, L*]** and Benchmark 2 increase because both of them exhaust the high-quality product and increases in its capacity allow greater use of an attractive product. Thus, the comparison of **[*H, Φ, L*]** and Benchmark 2 depends on how fast profits increase in *M*. Differentiating both profits with respect to *M*, I get:

$$\frac{\partial \pi_{Benchmark\ 2}}{\partial M} = V_{LH} - c_H \tag{10a}$$

$$\frac{\partial \pi_{H,\emptyset,L}}{\partial M} = V_{LH} - c_H - \sqrt{\frac{c n_H \Delta}{M - n_H}} \tag{10b}$$

I can show that $\frac{\partial \pi_{H,\emptyset,L}}{\partial M} < \frac{\partial \pi_{Benchmark\ 2}}{\partial M}$ indicating a slower speed of increase with M for **[*H, Φ, L*]**. Thus, beyond a critical M, Benchmark 2 is better than **[*H, Φ, L*].**

Having discussed my results pertaining to Benchmarks 1 and 2, I highlight that probabilistic quality can also emerge under Benchmark 3. As mentioned previously, this analysis is detailed in the Appendix. In effect, despite cannibalization, probabilistic quality benefits the seller via sales of unsold capacity as well as the inclusion of a previously neglected segment.

I next focus on how the design of probabilistic quality is impacted when the number of high-quality demanders, n_H, and transaction costs, c, change. I focus on these two variables because many firms are likely to experience changes in these variables on account of their other marketing activities and advances in technology, respectively. As such, it is managerially relevant to examine how the design of probabilistic quality is impacted by changes in these variables. Formally, I have:

Proposition 2a: Regardless of the relative magnitude of n_H, whenever the seller adopts **[*H, Φ, L*]**:

(i) The probability associated with probabilistic quality, ϕ, is decreasing in the number of high-quality demanders, n_H, and increasing in transaction costs, c.

(ii) The price of probabilistic quality, p_ϕ, is decreasing in the number of high-quality demanders, n_H, and increasing in transaction costs, c.

(iii) The quantity of product sold probabilistically, $M - n_H + X$, varies non-monotonically in the number of high-quality demanders, n_H, and is decreasing in transaction costs, c.

Mathematically:

$$\frac{\partial\phi}{\partial n_H} < 0,\ \frac{\partial\phi}{\partial c} > 0,$$

$$\frac{\partial p_\phi}{\partial n_H} < 0,\ \frac{\partial p_\phi}{\partial c} > 0,$$

$$\frac{\partial(M-n_H+X)}{\partial n_H} \geq 0 \ when\ n_H \leq \frac{M}{2} \text{ and } \frac{\partial(M-n_H+X)}{\partial n_H} < 0 \ when\ n_H > \frac{M}{2}\text{ , and}$$

$$\frac{\partial(M-n_H+X)}{\partial c} < 0$$

Proof: In Appendix

First, since p_ϕ is a linear function of ϕ, they share the same comparative statics. Thus, in the discussions below, the explanations for p_ϕ and ϕ are similar.

To facilitate understanding, I discuss the intuition behind the comparative statics associated with p_ϕ and $(M - n_H + X)$ under two cases: when n_H is either relatively small or when n_H is relatively large. Specifically, my discussion unfolds separately for the cases when $n_H \leq \frac{M}{2}$ and $n_H > \frac{M}{2}$. When the amount of high-quality demanders is relatively small, the idle high-quality capacity that can be used in creating probabilistic quality,$(M - n_H)$, is large. Since $\phi = \frac{(M-n_H)}{(M-n_H+X)}$, increases in X lower ϕ; here however, ϕ is not very sensitive to changes in X because $(M - n_H)$ is large. Given that p_ϕ increases linearly in ϕ, I can alternatively say that the price of the probabilistic product is not very sensitive to changes in X. In this light, the best way to respond to an increase in n_H is to *increase* the total amount of probabilistic product offered by *increasing* $M - n_H + X$. Such a strategy proves profit-maximizing because the drop in price for probabilistic quality is more than compensated by the increased volume associated with probabilistic quality. This is the reason why $\frac{\partial p_\phi}{\partial n_H} < 0$, $\frac{\partial \phi}{\partial n_H} < 0$ and $\frac{\partial (M-n_H+X)}{\partial n_H} > 0$ when $n_H \leq \frac{M}{2}$

Next, when the amount of high-quality demanders is relatively large, the idle high-quality capacity that can be used in creating probabilistic quality, $(M - n_H)$, is small. Since $\phi = \frac{(M-n_H)}{(M-n_H+X)}$, decreases in X improve ϕ; moreover, ϕ is very sensitive to changes in X because $(M - n_H)$ is small. Given that p_ϕ increases linearly in ϕ, I can alternatively say that the price of the probabilistic product is very sensitive to changes in X. Thus, here it is prudent for the seller to decrease the capacity utilized for probabilistic selling, $M - n_H + X$, thereby leading to an upward pressure on price. Nevertheless, since $M - n_H$ also decreases as n_H increases, this leads to a decrease in quality with an attendant downward pressure on price.

The overall impact of both these pressures on price remains negative; consequently, $\frac{\partial p_\phi}{\partial n_H} < 0$, $\frac{\partial \phi}{\partial n_H} < 0$ and $\frac{\partial (M - n_H + X)}{\partial n_H} < 0$ when $n_H > \frac{M}{2}$.

Finally, the comparative statics with respect to transaction costs, *c*, are straightforward. As costs increase, the seller's best response is to increase price as in a standard monopoly offering. This is achieved by increasing the quality of the probabilistic product, ϕ, with a corresponding increase in p_ϕ. This implies a decrease in the amount of probabilistic quality offered, $M - n_H + X$.

Proposition 2b: Regardless of the relative magnitude of n_H, whenever the seller adopts ***[H, Φ]***:

(i) The probability associated with probabilistic quality, ϕ, is decreasing in the number of high-quality demanders, n_H, but independent of the transaction cost, *c*.

(ii) The price of probabilistic quality, p_ϕ, is decreasing in the number of high-quality demanders, $n_{H,}$ but independent of the transaction cost, *c*.

(iii) The quantity of product sold probabilistically, $M - n_H + X$, decreases in the number of high-quality demanders, n_H, but is independent of the transaction costs, *c*.

Mathematically:

$\frac{\partial \phi}{\partial n_H} < 0, \frac{\partial \phi}{\partial c} = 0,$

$\frac{\partial p_\phi}{\partial n_H} < 0, \frac{\partial p_\phi}{\partial c} = 0,$

$\frac{\partial (M-n_H+N)}{\partial n_H} < 0$ and $\frac{\partial (M-n_H+N)}{\partial c} = 0$

Proof: In Appendix

As in Proposition 2a, p_ϕ and ϕ, share the same comparative statics. Thus, in the discussions below, the explanations for p_ϕ and ϕ are similar.

To understand Proposition 2b, note that the amount of product sold probabilistically is $(M - n_H + N)$ as $X = N$ in the two-segment solution. Here, the total amount of low-quality capacity involved in the probabilistic quality package is constant and equal to N; consequently, $\phi = \frac{(M-n_H)}{(M-n_H+N)}$. In this event, an increase in the number of high-type customers, n_H, means that both $M - n_H$ and $M - n_H + N$ decrease; however, the decrease in $M - n_H$ has a stronger impact on ϕ than the decrease in $M - n_H + N$, leading to a decrease in ϕ with an attendant decrease in p_ϕ. Moreover, since $X = N$, transaction costs have no impact on $M - n_H + X$, ϕ and p_ϕ.

Finally, I examine the impact of probabilistic quality on consumer surplus. I have:

Proposition 3: Regardless of the relative size of n_H:

(i) Offering probabilistic quality *improves* consumer surplus when the high-quality capacity is relatively low ($M < M_0$),

(ii) Offering probabilistic quality may either *improve* or *degrade* consumer surplus when high-quality capacity is relatively high ($M \geq M_0$) – it depends on the proportion of high-type consumers assumed to avail themselves of the high-quality product in Benchmark 2.

Proof: In Appendix.

To understand Proposition 3, first note that any surplus that arises comes only from the high-type segment through their consumption of the high-quality product since they never consume probabilistic quality - the low-type segment is always kept at their valuation. Now consider the case when the high-quality capacity is relatively low. In this event, the relevant benchmark (1) involves leaving some high-quality capacity idle. The introduction of probabilistic quality forces the seller to further lower the price of the high-quality product; consequently, high-type consumers now enjoy even greater levels of surplus. This is the source of improved consumer surplus.

Now consider the case when high-quality capacity is relatively high. In this event, the relevant benchmark (2) involves using the entire capacity. Here, the high-type consumer obtains greater surplus from consuming the high-quality product at price V_{LH} in Benchmark 2 relative to consuming the high-quality product under the strategy affiliated with probabilistic quality ($V_{LH} < V_{HH} - \emptyset(V_{HH} - V_{LH}) - (1 - \emptyset)(V_{HL} - V_{LL})$, $(please\ see\ equation\ (7))$) although both provide positive surplus. However, there is no way to fix the number of high-type consumers that are able to avail themselves of the attractive price of V_{LH} for the high-quality product under Benchmark 2 since allocation is random. Consequently, if this number is not too large, probabilistic quality provides greater surplus. In contrast, if this number is relatively large, the surplus provided by probabilistic quality is exceeded by the surplus enjoyed in Benchmark 2.

3.1.6 Additional Analysis

I now explicitly address the issue of how the seller arrives in a world where capacity level for the high-quality product exceeds demand. Consider a seller with initial capacity N of low-quality products. Now, as the market matures, there is an opportunity to target an emergent high-type segment (premium suites, 50Mps internet, etc.). Unfortunately, however, this emergent high-type segment is characterized by demand uncertainty. Nevertheless, to take advantage of this opportunity, the seller embarks to build capacity M^* for the high-quality product with some convex cost function. This capacity choice is undertaken before demand is realized. Next, demand is realized and the seller has to select a strategy given M^*, N, and the realized value of n_H. If the realization of n_H is such that $n_H < M^*$, I obtain the world that I analyze and in which probabilistic quality can emerge. In contrast, if the realization of n_H is such that $n_H \geq M^*$, the seller will never offer probabilistic quality.

To incorporate demand uncertainty, I employ the uniform distribution for high-quality demand: $n_H \sim U\ [0, n_{\bar{H}}]$. The goal of my analysis here is to demonstrate that even when the seller chooses capacity, the optimal level of high-quality capacity is greater than the lower bound for the support of n_H (i.e., $M^* > 0$). If this is true, there is a non-zero probability that M^* will be greater than the realized value of n_H. In other words, the world that I confine my analysis to, namely, one in which capacity for the high-quality product exceeds demand, can indeed arise in the first place. Formally, I obtain:

> **Proposition 4**: When the seller chooses capacity for the high-quality product in the face of demand uncertainty, the optimal capacity choice M^* is greater than the lower bound for the support of n_H . This implies that a world in which capacity for the high-quality exceeds demand can indeed arise in the first place.
>
> **Proof:** In Appendix.

Finally, for completeness, I show via numerical examples that when the seller chooses capacity for high-quality, both **[*H, Φ*]** as well as **[*H, Φ, L*]** do indeed emerge as the profit-maximizing offerings (please see Table 2). Moreover, the former arises for a relatively low value of transaction cost whereas the latter arises for a relatively high value of transaction cost. In short, endogenizing capacity choice does not eliminate the emergence of probabilistic quality.

4. DISCUSSION

My research adds to the growing literature on the probabilistic aspects of market exchanges. Indeed, the numerous marketplace examples of probabilistic quality motivate my formal study on this topic. Broadly, I contribute to this literature by focusing on the design and pricing of probabilistic quality.

In my analysis, I find that there are three distinct routes by which probabilistic quality can enhance seller profits. First, sellers employing a "strong" differentiation strategy with a view to benefit from better price realization while tolerating some fallow capacity can utilize probabilistic quality to improve capacity utilization. Here, the introduction of a product that is in between the high and low-quality offerings actually leads to increased cannibalization of the price obtained for the high-quality product; however, this is mitigated by pushing the quality inherent in probabilistic quality towards the low-quality offering by packing in more low-quality products. This mitigation, in turn, is constrained by the fact that selling large numbers of probabilistic quality erodes profits on account of the transaction costs associated with probabilistic quality. These opposing forces lead to an interior solution for the probability associated with probabilistic quality.

Second, sellers utilizing their entire capacity but sacrificing margins by following a "weak" differentiation strategy can use probabilistic quality to enhance profits by now targeting the high-quality product to high-type consumers with correspondingly higher margins. Again, to minimize cannibalization, the seller would prefer to push the quality inherent in probabilistic quality towards the low-quality offering by packing in more low-quality products. However, selling large numbers of probabilistic quality erodes profits on account of the transaction costs associated with probabilistic quality. As before, I thus obtain an interior solution.

Third, sellers following an "upmarket" strategy can also benefit from employing probabilistic quality. Here, the seller ignores the low-type segment completely despite excess high-quality capacity as well as the availability of the full amount of low-quality capacity. Although probabilistic quality induces cannibalization, the twin benefits of selling the fallow high-quality and the full amount of low-quality capacity to the low-type consumers can prove to be profit enhancing.

My research also reveals an important role for the magnitude of transaction costs in deciding which products should be included in the product line. I find that increases in transaction cost move the seller away from **[H, ɸ]** to **[H, ɸ, L].** This is because increased transaction costs degrade the profits from utilizing probabilistic quality; consequently, the seller now reserves some low-type capacity for separate sale to the low-type segment even though its inclusion in probabilistic quality could mitigate the cannibalization effect. Overall, my results here speak to the impact of transaction costs on the design of the product line when offering probabilistic quality.

Yet another important contribution of my work is the explicit delineation of the probability and pricing associated with probabilistic quality. I demonstrate the impact of two important parameters, namely, the number of high-quality demanders, n_H, and transaction costs, c, on the pricing and probability associated with probabilistic quality. My focus on these variables is particularly germane since many firms are likely to experience changes in these variables on account of their other marketing activities and advances in technology, respectively.

A very interesting finding of my study is that probabilistic quality can improve consumer surplus *even in the presence of transaction costs*. This accrues because of two reasons: lower prices for the high-type consumer (relative to Benchmark 1) and potentially steering more high-quality product to the high-type consumer (relative to Benchmark 2). Here, it is interesting to contrast my results with Leffler, Rucker and Munn (2000) where the authors find that although the initiative to measure quality lies with the seller (this is also the case in Milgrom and Weber 1982), such measurements are at the expense of consumer surplus. However, in my study, the seller chooses to *introduce* uncertainty in the actual product that the customer receives. While this introduces a dissipative transaction cost, lower prices or additional quantity enjoyed by high-type consumers improves surplus.

Finally, I explicitly demonstrate that a world in which the high-quality capacity exceeds demand can indeed arise even when capacity choice is a decision variable. Specifically, given uncertainty in the demand associated with the high-quality product, I find that the seller will choose a level of high-quality capacity that is strictly greater than the lower bound of the demand distribution. This finding provides the proper logical motivation for my model context.

Of course, my work is not without limitations. While this study is sharply focused on probabilistic quality in a vertical context to demonstrate its viability, I have omitted certain aspects that merit further attention. First, given uncertainty in product, it is natural that customer risk-aversion will play a major role. However, generalizing this to the real marketplace would be an empirical question where customer valuations and risk-preferences need to be examined carefully. Second, given the viability of probabilistic quality, it is possible that the monopolist can offer multiple tiers of probabilistic quality, with varying probabilities of obtaining the high-quality product. Given recent research on customized pricing in high-technology environment (see, for example, Syam, Ruan and Hess 2005, Chen and Iyer 2002, Ansari and Mela 2003), this is an important consideration where the seller can, in principle, offer as many tiers as the number of customers. However, deciding on the number of tiers and the associated probabilities is not a simple task; thus, it is beyond the scope of the current work. Finally, including temporal variation in willingness to pay (see, for example, Desiraju and Shugan 1999) and examining the impact of competition are also worthy of future research consideration.

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Table1a: Product Offerings across the Parameter Space - case where $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

<table>
<tr><th>$0 \le M \le M_0$</th><th>$M > M_0$</th></tr>
<tr><td colspan="2">$0 \le c < c_1$

Strategy [*H*, *Φ*]: Region A

$p_H = \Delta + V_{LH} - \frac{(M-n_H)\Delta}{M+N-n_H}$, $p_\emptyset = V_{LL} + \frac{(M-n_H)(V_{LH}-V_{LL})}{M+N-n_H}$,

$\emptyset^* = \frac{M-n_H}{M-n_H+N}$

$\pi_{H,\emptyset} = M(V_{LH}-c_H) + NV_{LL} + \frac{n_H N\Delta}{M+N-n_H} - c(M-n_H+N)$</td></tr>
<tr><td>$c_1 \le c \le c_2$

Strategy [*H*, *Φ* and *L*]: Region B

$p_H = \Delta + V_{LH} - \sqrt{\frac{c(M-n_H)\Delta}{n_H}}$

$p_\emptyset = V_{LL} + (V_{LH}-V_{LL})\sqrt{\frac{c(M-n_H)\Delta}{n_H}}$

$p_L = V_{LL}$

$\emptyset^* = \sqrt{\frac{c(M-n_H)}{n_H\Delta}}$

$\pi_{H,\emptyset,L} = n_H\Delta + M(V_{LH}-c_H) + NV_{LL} - 2\sqrt{c(M-n_H)n_H\Delta}$</td><td>$c_1 \le c \le c_3$

Strategy [*H*, *Φ* and *L*]: Region B
Same as the case involving $c_1 \le c \le c_2$</td></tr>
<tr><td>$c > c_2$

Benchmark 1 : Region C

$p_H = V_{HH} + V_{LL} - V_{HL} = \Delta + V_{LH}$

$p_L = V_{LL}$

$\pi_1 = n_H[\Delta + V_{LH} - c_H] + NV_{LL}$</td><td>$c > c_3$

Benchmark 2: Region D

$p_H = V_{LH}$ *and* $p_L = V_{LL}$

$\pi_2 = M(V_{LH}-c_H) + NV_{LL}$</td></tr>
</table>

Table1b: Product Offerings across the Parameter Space - case where $n_H \ge \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

<table>
<tr><th>$0 \le M \le M_0$</th><th>$M > M_0$</th></tr>
<tr><td>

$0 \le c < c_4$

Strategy [*H*, *Φ*] : Region A

$\boldsymbol{p_H = \Delta + V_{LH} - \frac{(M-n_H)\Delta}{M+N-n_H}},$

$$\boldsymbol{p_\emptyset = V_{LL} + \frac{(M-n_H)(V_{LH}-V_{LL})}{M+N-n_H}}$$

$$\emptyset^* = \boldsymbol{\frac{M-n_H}{M-n_H+N}}$$

$$\boldsymbol{\pi_{H,\emptyset} = M(V_{LH}-c_H) + NV_{LL} + \frac{n_H N\Delta}{M+N-n_H} - c(M-n_H+N)}$$

</td><td>

$0 \le c \le c_1$

Prices and profits are the same as the case with $\boldsymbol{0 \le c < c_4}$

</td></tr>
<tr><td rowspan="2">

$c > c_4$

Benchmark 1 : Region C

$\boldsymbol{p_H = V_{HH} + V_{LL} - V_{HL} = \Delta + V_{LH}}$

$\boldsymbol{p_L = V_{LL}}$

$\boldsymbol{\pi_1 = n_H[\Delta + V_{LH} - c_H] + NV_{LL}}$

</td><td>

$c_1 \le c \le c_3$

Strategy [*H*, *Φ* and *L*] : Region B

$$\boldsymbol{p_H = \Delta + V_{LH} - \sqrt{\frac{c(M-n_H)\Delta}{n_H}}}$$

$$\boldsymbol{p_\emptyset = V_{LL} + (V_{LH}-V_{LL})\sqrt{\frac{c(M-n_H)\Delta}{n_H}}}$$

$\boldsymbol{p_L = V_{LL}}$

$$\emptyset^* = \boldsymbol{\sqrt{\frac{c(M-n_H)}{n_H\Delta}}}$$

$\boldsymbol{\pi_{H,\emptyset,L} = n_H\Delta + M(V_{LH}-c_H) + NV_{LL} - 2\sqrt{c(M-n_H)n_H\Delta}}$

</td></tr>
<tr><td>

$c > c_3$

Benchmark 2 : Region D

$\boldsymbol{p_H = V_{LH}\ and\ p_L = V_{LL}}$

$\boldsymbol{\pi_2 = M(V_{LH}-c_H) + NV_{LL}}$

</td></tr>
</table>

Table 2: Numerical Example to Demonstrate Emergence of Probabilistic Quality Under Capacity Choice

Example 1: Low contracting costs leading to [H, $\emptyset$] solution for some realizations of $n_{\bar{H}}$ $(c = 0)$

$V_{HH} = 4.4, V_{HL} = 2.5,\ V_{LH} = 2.3,\ V_{LL} = 1.3,\ c_H = 0.3,\ c = 0,\ l = 0.04,\ N = 40$ and $n_{\bar{H}} = 9$.

$M^* = 7.22$ and $\pi^* = 6.94543$

n_H	0	1	2	3	4	5	6	7	8	9
Strategy	B2	$[H, \emptyset]$	$[H, \emptyset]$	$[H, \emptyset]$	$[H, \emptyset]$	$[H, \emptyset]$	$[H, \emptyset]$	$[H, \emptyset]$	B1	B1
X	0	40	40	40	40	40	40	40	0	0

Example 2: High contracting costs leading to [H, $\emptyset, L$] solution for some realizations of $n_{\bar{H}}$ $(c = 0.1)$

$V_{HH} = 4.4, V_{HL} = 2.5,\ V_{LH} = 2.3,\ V_{LL} = 1.3,\ c_H = 0.3,\ c = 0.1,\ l = 0.04,\ N = 40$ and $n_{\bar{H}} = 9$.

$M^* = 7.18$ and $\pi^* = 6.94489$

n_H	0	1	2	3	4	5	6	7	8	9
Strategy	B2	B2	B2	$[H, \emptyset, L]$	$[H, \emptyset, L]$	$[H, \emptyset, L]$	$[H, \emptyset, L]$	B1	B1	B1
X	0	0	0	6	8	8	7	0	0	0

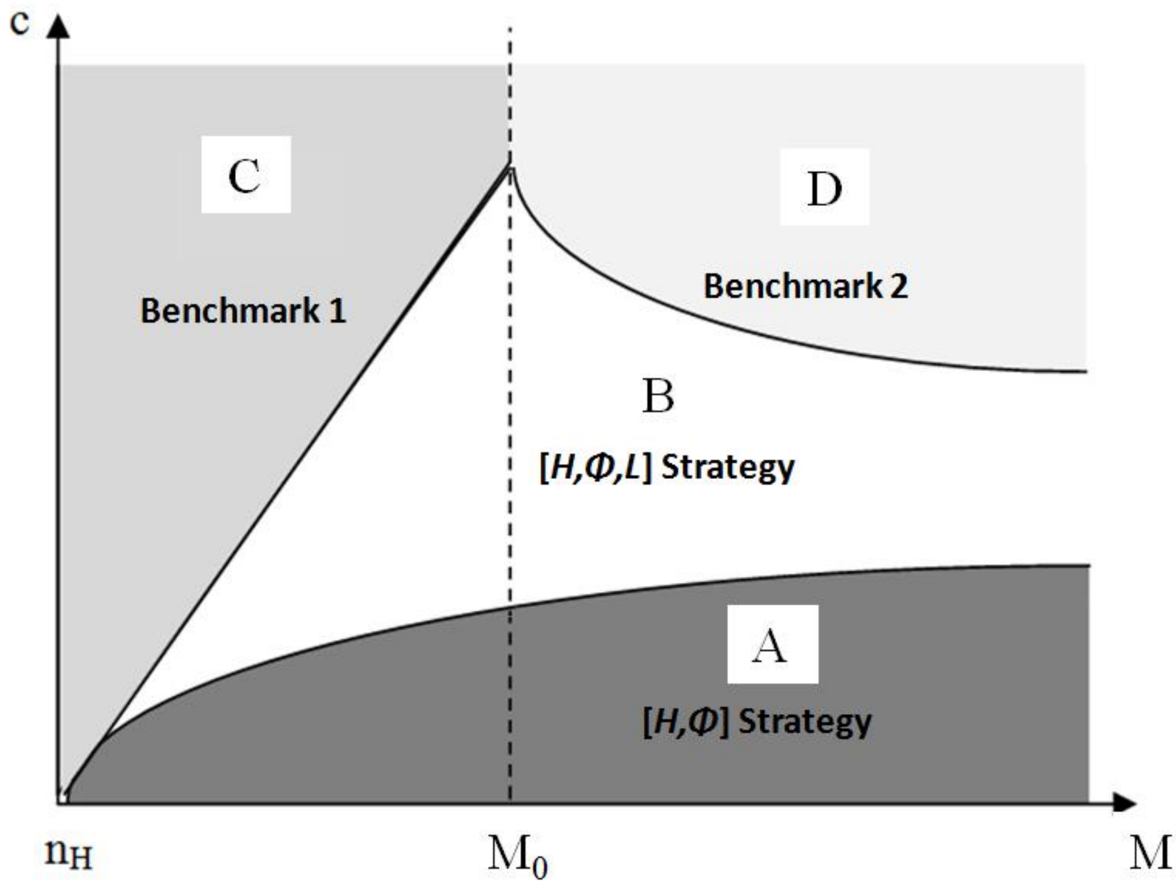


Figure 1a: Selling Strategies, $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

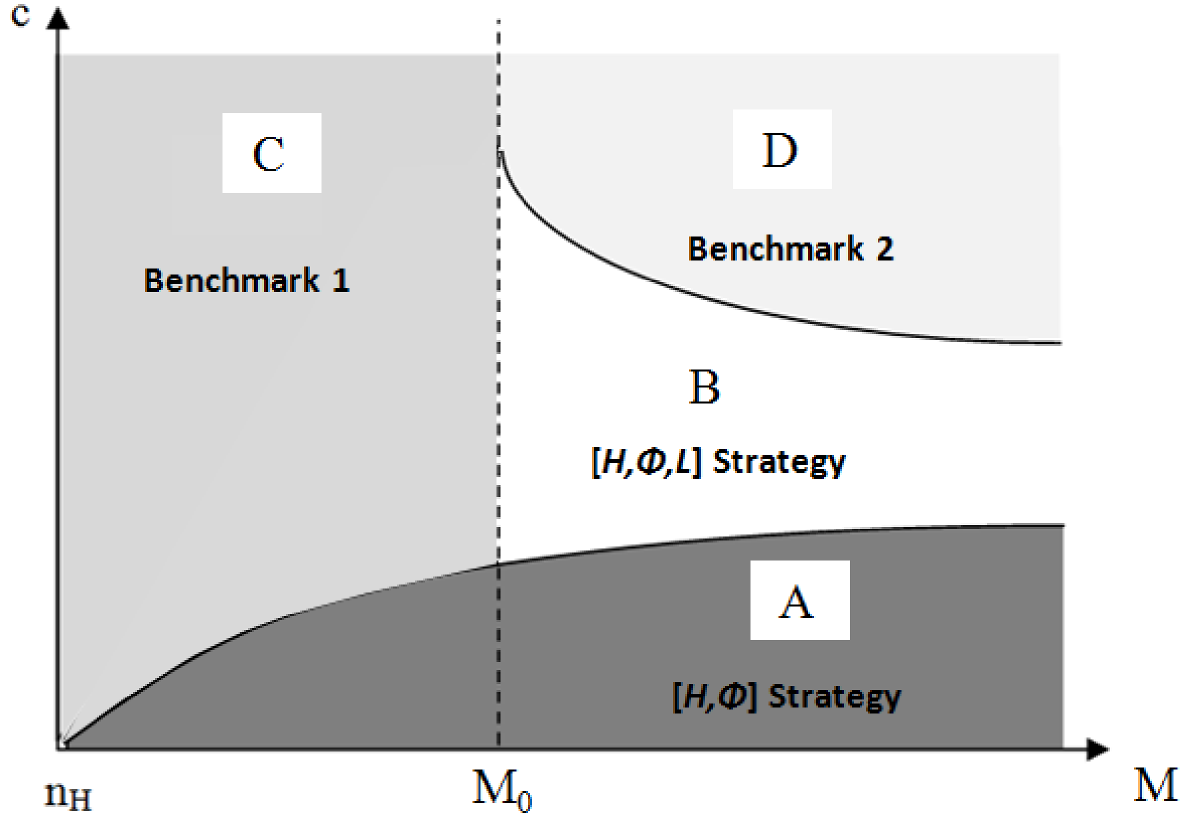


Figure 1b: Selling Strategies, $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

Appendix

Notation:

M: Capacity of high-quality product

N: Capacity of low-quality product

n_H: Market size of high-type consumers

n_L: Market size of low-type consumers

V_{HH}: High-type consumers' valuation of high-quality product

V_{HL}: High-type consumers' valuation of low-quality product

V_{LH}: Low-type consumers' valuation of high-quality product

V_{LL}: Low-type consumers' valuation of low-quality product

$V_{H\phi}$: High-type consumers' valuation of probabilistic quality product

$V_{L\phi}$: Low-type consumers' valuation of probabilistic quality product

ϕ: Probability of obtaining high-quality product under probabilistic quality

p_H: Price of high-quality product

p_L: Price of low-quality product

p_ϕ: Price of probabilistic quality product

c_H: Unit cost of offering high-quality product

c_L: Unit cost of offering low-quality product

c_ϕ: Unit cost of offering probabilistic quality product

c: Unit contracting cost of offering probabilistic quality product

X: Amount of low-quality used in probabilistic quality product

First, observe that there is no benefit in targeting probabilistic quality to the high-type segment. This is understood as follows. Consider the comparison of introducing probabilistic quality with Benchmark 1. Since $n_H < M$, if the seller wants to target the high-type with probabilistic quality then each high-type buyer of probabilistic quality is precluded from buying the high-quality offering directly. Prices are obtained via incentive-compatibility and are identical to that described in equations (5a) and (5b). Accordingly, the profit from selling a unit of probabilistic quality is $p_\phi - \phi\, c_H - c = \phi(V_{HH} - V_{HL} + V_{LL} - c_H) + (1-\phi)V_{LL} - c$ and the profit from selling a unit of high-quality is $V_{HH} - V_{HL} + V_{LL} - c_H$. Since the latter exceeds the former by the positive quality, $(1-\phi)(V_{HH} - V_{HL} - c_H) + c$, it is straightforward to conclude that the seller will not replace direct sales of high-quality by probabilistic quality.

In contrast, targeting probabilistic quality to the low-type segment has the potential to: (i) increase unit sales from low-type consumers in Benchmark 1 by utilizing fallow capacity, (ii) increase price charged for the high-quality product sold to high-type consumers in Benchmark 2, and, (iii) increase unit sales by serving the ignored low-type consumers in Benchmark 3. Thus, probabilistic quality targeted to the low-type segment has the potential to increase profits.

Now, the two segments value probabilistic quality as follows:

$$V_{H\phi} = \phi V_{HH} + (1-\phi)V_{HL} \quad \text{(A1)}$$

$$V_{L\phi} = \phi V_{LH} + (1-\phi)V_{LL} \quad \text{(A2)}$$

Price of the low-quality product:

$$p_L = V_{LL} \quad \text{(A3)}$$

The price of probabilistic quality is obtained from the following incentive-compatibility constraint:

$$V_{L\phi} - p_{\phi} \geq V_{LL} - p_L$$

$$\Longrightarrow p_{\phi} \leq V_{L\phi} - V_{LL} + p_L \text{ ; for } p_L = V_{LL}$$

$$\Longrightarrow p_{\phi} = V_{L\phi} = \phi V_{LH} + (1-\phi)V_{LL} \qquad \text{(A4)}$$

The price of the high-quality product is obtained from the following incentive-compatibility constraints:

I need $V_{HH} - p_H \geq V_{H\phi} - p_{\phi}$ and $V_{HH} - p_H \geq V_{HL} - p_L$ to guarantee that high-type consumers buy high-quality product.

$$\Longrightarrow \begin{cases} p_H \leq V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \\ p_H \leq V_{HH} - (V_{HL} - V_{LL}) \end{cases}$$

Since $(V_{HH} - V_{LH}) - (V_{HL} - V_{LL}) = \Delta \geq 0$

$$V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \leq V_{HH} - (V_{HL} - V_{LL})$$

$$\Longrightarrow p_H = V_{HH} - \phi(V_{HH} - V_{LH}) - (1-\phi)(V_{HL} - V_{LL}) \qquad \text{(A5)}$$

I now turn to the specific proofs. I begin with Result 2 and then turn to the propositions.

Manner of Emergence of Probabilistic Quality (Result 2) and Proposition 1:

First, I show that $[\boldsymbol{\phi}, \boldsymbol{L}]$ is never optimal.

Proof: With $[\boldsymbol{\phi}, \boldsymbol{L}]$ the seller does not offer high-quality product to the market, the entire high-quality capacity is used to create the probabilistic quality. Suppose the seller uses Z low-quality and M high-quality products in the probabilistic offer. Thus, $\phi = \frac{M}{M+Z}$

The seller's profit is:

$$\pi = (M+Z)\left(p_{\phi} - \phi c_H - c\right) + (N-Z)p_L$$

$$\Rightarrow \pi = (M+Z)\left[\frac{M(V_{LH} - V_{LL})}{M+Z} + V_{LL} - \frac{Mc_H}{M+Z} - c\right] + (N-Z)V_{LL}$$

$$\Rightarrow \pi = M(V_{LH} - c_H) + NV_{LL} - c(M+Z)$$

This profit is lower than $\pi_{B2} = M(V_{LH} - c_H) + NV_{LL}$. Thus, $[\boldsymbol{\phi}, \boldsymbol{L}]$ is dominated by Benchmark 2.

Next, I examine the three-segment solution: high-quality + probabilistic quality + low-quality strategy:

Here, X low-quality units are sold with $M - n_H$ high-quality units together as probabilistic quality. The decision variable in the analysis is X. The probabilistic quality product is targeted to low-type consumers, n_H high-quality products are targeted to high-type consumers while probabilistic quality and low-quality product are sold to low-type consumers.

Probability of getting high-quality product = $\phi = \frac{M-n_H}{M-n_H+X}$ (A6)

<u>Profits from Offering Probabilistic Quality</u>

The seller's profit is:

$$\pi = n_H(p_H - c_H) + (M - n_H + X)\big(p_\phi - \phi c_H - c\big) + (N - X)p_L$$

$$= n_H(\Delta + c) + M(V_{LH} - c_H - c) + NV_{LL} - cX - \left[\frac{(M-n_H)n_H}{M-n_H+X}\right]\Delta \quad \text{(A7)}$$

First order condition yields:

$$\frac{\partial\pi}{\partial X} = 0 \Longrightarrow X^* = \sqrt{\frac{(M-n_H)n_H\Delta}{c}} - (M - n_H) \quad \text{(A8)}$$

$$\Rightarrow \phi^* = \sqrt{\frac{c(M-n_H)}{n_H\Delta}} \quad \text{(A9)}$$

Thus, $p_\phi^* = V_{LL} + (V_{LH} - V_{LL})\sqrt{\frac{c(M-n_H)}{n_H\Delta}}$ (A10)

And, $\pi^* = n_H\Delta + M(V_{LH} - c_H) + NV_{LL} - 2\sqrt{c(M - n_H)n_H\Delta}$ (A11)

Second order condition yields $\frac{\partial^2\pi}{\partial X^2} = -\frac{2(M-n_H)n_H[(V_{HH}-V_{LH})-(V_{HL}-V_{LL})]}{[(M-n_H)+X]^2} \leq 0$, since $\Delta = [(V_{HH} - V_{LH}) - (V_{HL} - V_{LL})] > 0$

$\Longrightarrow X^* = \sqrt{\frac{(M-n_H)n_H\Delta}{c}} - (M - n_H)$ maximizes profit.

I now have two conditions involving X^* that shape the probabilistic quality offer:

1. $X^* > 0$

From the expression for X^*; $X^* > 0$ implies $c < \frac{n_H\Delta}{(M-n_H)}$. (Note that this inequality, given (A9), ensures that the probability associated with high-quality never exceeds 1).

When $X^* \leq 0$, probabilistic quality does not arise.

2. $X^* \leq N$

From the expression for X^*, $X^* \leq N$ implies $c \geq \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$.

When $X^* > N$, there is not enough low-quality product to put in the probabilistic quality. The maximum amount of low-quality product that can be used is N. Thus, when $X^* > N$, the seller sets $X^* = N$. I thus have the two segment solution **[*H, Φ*]** where the seller just offers the high-quality and the probabilistic quality product. The associated profit is thus obtained using the expressions for the prices, ϕ and X^* (which is now equal to N) in (A7) to get:

$$\pi^*_{H\phi} = M(V_{LH} - c_H) + NV_{LL} + \frac{n_H N\Delta}{M+N-n_H} - c(M - n_H + N) \qquad \text{(A12)}$$

When $0 < X^* \leq N$, the seller finds it optimal to offer probabilistic quality while setting some low-quality capacity aside to be offered as it is and I get the three segment solution **[*H, Φ, L*].** The condition $0 < X^* \leq N$, given the expression for X^* from (A8), translates to $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)}$ and the associated profit is obtained by substituting the expressions for the prices and X^* in (A7) to get (as expressed in equation (A11)).

$$\pi^*_{H\phi L} = n_H\Delta + M(V_{LH} - c_H) + NV_{LL} - 2\sqrt{c(M - n_H)n_H\Delta}$$

Summarizing, I obtain the following two probabilistic quality strategies **[*H, ϕ, L*]** and **[*H, Φ*]** which is ***Result 2*** for the seller based on different regions of the parameter space:

When $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)}$, I get the three segment solution **[*H, Φ, L*]**: "high quality plus probabilistic quality plus low quality product" strategy. The profit is:

$$\pi^*_{H\phi L} = n_H\Delta + M(V_{LH} - c_H) + NV_{LL} - 2\sqrt{c(M-n_H)n_H\Delta}$$

When $c < \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$, I have the two segment solution **[*H, Φ*]** where high and probabilistic quality are offered (no low-quality product). The profit is:

$$\pi^*_{H\phi} = M(V_{LH} - c_H) + NV_{LL} + \frac{n_H N\Delta}{M+N-n_H} - c(M - n_H + N)$$

I have:

$$\begin{cases} \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)} \Longrightarrow \text{I have the three segment solution } [\boldsymbol{H}, \boldsymbol{\Phi}, \boldsymbol{L}] \\ c < \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow \text{I have the two segment solution } [\boldsymbol{H}, \boldsymbol{\Phi}] \end{cases} \tag{A13}$$

where the critical value of the transaction cost $c = \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} = c_1$, from equation (9a).

<u>Optimal Solution: Comparing Probabilistic Quality to Benchmarks 1 and 2</u>

Comparing Benchmark 1 and Benchmark 2, I have

$$\begin{cases} 0 < M \leq M_0 \Longrightarrow \pi_{B1} \geq \pi_{B2} \\ M > M_0 \Longrightarrow \pi_{B1} < \pi_{B2} \end{cases} \tag{A14}$$

(where $M_0$is as defined in (9e))

I now consider the above two cases separately:

A: *High- quality capacity is relatively low:* $0 < M \leq M_0$ (Benchmark 1 is relevant)

Comparing profits using equations 1 and 2, it can be easily seen that $\pi_{B1} \geq \pi_{B2}$ in this region and therefore the relevant benchmark in this region is $\pi_{B1} = n_H(V_{HH} + V_{LL} - V_{HL} - c_H) + NV_{LL}$.

When $c \geq \frac{n_H\Delta}{(M-n_H)}$; probabilistic quality is never optimal as this results in $X^* \leq 0$.

When $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)}$, as seen earlier, I have the three segment solution ***[H, Φ, L]***.

$$\pi^*_{H\phi L} = n_H\Delta + M(V_{LH} - c_H) + NV_{LL} - 2\sqrt{c(M-n_H)n_H\Delta}$$

Comparing this to π_{B1}, I obtain

$$\begin{cases} c < \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} \Longrightarrow \pi^*_{H\phi L} > \pi_{B1} \\ c \geq \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} \Longrightarrow \pi^*_{H\phi L} \leq \pi_{B1} \end{cases} \tag{A15}$$

where the critical value of the transaction cost $c = \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} = c_2$ from equation (9b).

Now comparing the profit of the two segment solution using (A12) with benchmark 1 yields:

$$\begin{cases} c < \frac{(M-n_H)(V_{LH}-c_H) - \frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N} \Longrightarrow \pi^*_{H\phi} > \pi_{B1} \\ c \geq \frac{(M-n_H)(V_{LH}-c_H) - \frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N} \Longrightarrow \pi^*_{H\phi} \leq \pi_{B1} \end{cases} \tag{A16}$$

where the critical value of the transaction cost $c = \frac{(M-n_H)(V_{LH}-c_H)-\frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N} = c_4$ from equation (9d).

From equations (A13) and (A15) above (and using the appropriate values of c_1 and c_2), it is evident that the three segment solution would arise only when $c_1 < c < c_2$ which is possible only when

$$\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} < \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta}.$$

This implies:

$$n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)} \quad \text{(A17)}$$

Thus, when n_H exceeds this critical limit, the three segment solution does not arise when $0 < M \leq M_0$ as depicted in Figure 1b. If n_H is below this critical value, the three segment solution arises as in Figure 1a.

Finally, looking at the optimality of the two segment solution using (A13) and (A16), I find that the two segment solution is optimal when $c < Min\{c_1, c_4\}$. Comparing these using equations (9a) and (9d), I get

$Min\{c_1, c_4\} = c_1$ when $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ and $Min\{c_1, c_4\} = c_4$ when $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$ again corresponding to figures 1a and 1b.

The above comparisons are summarized below:

When $n_H < \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

- $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} < \frac{(M-n_H)(V_{LH}-c_H)-\frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N}$
- $\frac{n_H\Delta}{(M-n_H)} \geq \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta}$
- $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} < \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta}$

$$\Longrightarrow \begin{cases} when\ 0 \leq c < \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow [H,\phi]\ is\ \ optimal \\ when\ \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} \Longrightarrow [H,\phi,L]\ is\ optimal \\ when\ c \geq \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta} \Longrightarrow benchmark\ 1\ is\ optimal \end{cases}$$

When $n_H \geq \frac{(M+N)(V_{LH}-c_H)}{2\Delta+(V_{LH}-c_H)}$

- $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \geq \frac{(M-n_H)(V_{LH}-c_H)-\frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N}$
- $\frac{n_H\Delta}{(M-n_H)} \geq \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta}$
- $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \geq \frac{(M-n_H)(V_{LH}-c_H)^2}{4n_H\Delta}$

$$\Longrightarrow \begin{cases} when\ 0 \leq c < \frac{(M-n_H)(V_{LH}-c_H)-\frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N} \Longrightarrow [H,\phi]\ is\ optimal \\ [H,\phi,L]\ is\ never\ optimal \\ when\ c \geq \frac{(M-n_H)(V_{LH}-c_H)-\frac{(M-n_H)n_H}{(M-n_H)+N}\Delta}{(M-n_H)+N} \Longrightarrow benchmark\ 1\ is\ optimal \end{cases}$$

B: *High- quality capacity is relatively high:* $M > M_0$ (Benchmark 2 is relevant)

Using equations 1 and 2, I can see that $\pi_{B2} \geq \pi_{B1}$, and thus the relevant benchmark is $\pi_{B2} = M(V_{LH} - c_H) + NV_{LL}$.

Again, when $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)}$, $\pi^*_{H\phi L} > \pi^*_{H\phi}$ and I have the three segment solution:

$$\pi^*_{H\phi L} = n_H\Delta + M(V_{LH} - c_H) + NV_{LL} - 2\sqrt{c(M-n_H)n_H\Delta}$$

Comparing this profit with π_{B2}, I have:

$$\begin{cases} c < \frac{n_H\Delta}{4(M-n_H)} \Longrightarrow \pi^*_{H\phi L} > \pi_{B2} \\ c \geq \frac{n_H\Delta}{4(M-n_H)} \Longrightarrow \pi^*_{H\phi L} \leq \pi_{B2} \end{cases} \tag{A18}$$

where the critical value of the transaction cost $c = \frac{n_H\Delta}{4(M-n_H)} = c_3$ from equation (9c)

When $c < \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$, $\pi^*_{H\phi L} < \pi^*_{H\phi}$ and I have the two segment solution.

$$\pi^*_{H\phi} = (V_{LH} - c_H) + NV_{LL} + \frac{n_H N\Delta}{M+N-n_H} - c(M - n_H + N)$$

Comparing this two segment probabilistic quality solution profit with Benchmark 2, I have

$$\begin{cases} c < \frac{Nn_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow \pi^*_{H\phi} > \pi_{B2} \\ c \geq \frac{Nn_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow \pi^*_{H\phi} \leq \pi_{B2} \end{cases} \tag{A19}$$

Similar to the logic in part A earlier, I compare the boundaries of c in the discussions above:

(a) $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$ and $\frac{Nn_H\Delta}{[(M-n_H)+N]^2}$

From $N > M$ (since I assume greater amount of low-quality capacity),

I get $\frac{Nn_H\Delta}{[(M-n_H)+N]^2} > \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$

$$\Longrightarrow when\ c \le \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow \pi^*_{H\phi} > \pi_{B2}$$

(b) $\frac{n_H\Delta}{4(M-n_H)}$ and $\frac{n_H\Delta}{(M-n_H)}$

$\frac{n_H\Delta}{4(M-n_H)} \le \frac{n_H\Delta}{(M-n_H)}$ is trivially true

(c) $\frac{n_H\Delta}{4(M-n_H)}$ and $\frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$

From $N > M$

$$\frac{n_H\Delta}{4(M-n_H)} - \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} =$$

$$n_H\Delta\left[\frac{[(M-n_H)+N]^2-4\left((M-n_H)\right)^2}{4(M-n_H)(M-n_H+N)}\right] = n_H\Delta\left[\frac{N^2+2N(M-n_H)-3(M-n_H)^2}{4(M-n_H)(M-n_H+N)}\right] >$$

$$n_H\Delta\left[\frac{N^2-(M-n_H)^2}{4(M-n_H)(M-n_H+N)}\right] > 0$$

since $N > M > M - n_H$

Thus, $\frac{n_H\Delta}{4(M-n_H)} > \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2}$ is always true.

This allows us to clearly summarize the optimal regions as follows:

$$\Rightarrow \begin{cases} when\ 0 \leq c < \dfrac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \Rightarrow [H,\phi]\ is\ optimal \\ when\ \dfrac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \dfrac{n_H\Delta}{4(M-n_H)} \Rightarrow [H,\phi,L]\ is\ optimal \\ when\ c \geq \dfrac{n_H\Delta}{4(M-n_H)} \Rightarrow benchmark\ 2\ is\ optimal \end{cases}$$

Proposition 2a: Regardless of the relative magnitude of n_H,whenever the seller adopts **[*H, Φ, L*]**:

(i) The probability associated with probabilistic quality, ϕ, is decreasing in the number of high-quality demanders, n_H, and increasing in transaction costs, c.

(ii) The price of probabilistic quality, p_ϕ, is decreasing in the number of high-quality demanders, n_H, and increasing in transaction costs, c.

(iii) The quantity of product sold probabilistically, $M - n_H + X$, varies non-monotonically in the number of high-quality demanders, n_H, and is decreasing in transaction costs, c.

Mathematically:

$\frac{\partial \phi}{\partial n_H} < 0,\ \frac{\partial \phi}{\partial c} > 0,$

$\frac{\partial p_\phi}{\partial n_H} < 0,\ \frac{\partial p_\phi}{\partial c} > 0,$

$\frac{\partial (M-n_H+X)}{\partial n_H} \geq 0\ when\ n_H \leq \frac{M}{2}$ and $\frac{\partial (M-n_H+X)}{\partial n_H} < 0\ when\ n_H > \frac{M}{2}$, and

$\frac{\partial(M-n_H+X)}{\partial c} < 0$

Proof: For the three segment solution **[*H, Φ, L*]**, I note that $\phi^* = \sqrt{\frac{c(M-n_H)}{n_H\Delta}}$ from equation (A9) and the comparative statics for ϕ immediately follow. The results for the probabilistic price also follow since from equation (A4), I see that p_ϕ is just a linear, increasing function of ϕ.

For the results involving n_H and c, in the **[*H, Φ, L*]** solution, I first note that the amount of probabilistic quality sold is $M - n_H + X^*$. Now, $X^* = \sqrt{\frac{(M-n_H)n_H\Delta}{c}} - (M - n_H)$; thus,

$$M - n_H + X^* = \sqrt{\frac{(M-n_H)n_H\Delta}{c}}$$

$$\frac{\partial(M-n_H+X^*)}{\partial c} = -\frac{1}{2}\sqrt{\frac{(M-n_H)n_H\Delta}{c^3}} < 0$$

$$and \;\; \frac{\partial(M-n_H+X^*)}{\partial n_H} = \frac{1}{2}\sqrt{\frac{\Delta}{c(M-n_H)n_H}}(M-2n_H)\begin{cases} \geq 0, & when\; n_H \leq \frac{M}{2} \\ < 0, & when\; n_H > \frac{M}{2} \end{cases}$$

proving the results relating to variation in n_H and c.

***Proposition 2b*:** Regardless of the relative magnitude of n_H, whenever the seller adopts **[*H, Φ*]**:

(i) The probability associated with probabilistic quality, $\phi,$ is decreasing in the number of high-quality demanders, n_H, but independent of the transaction cost, c.

(ii) The price of probabilistic quality, p_ϕ, is decreasing in the number of high-quality demanders, $n_{H,}$ but independent of the transaction cost, c.

(iii) The quantity of product sold probabilistically, $M - n_H + X$, decreases in the number of high-quality demanders, n_H, but is independent of the transaction costs, c.

Proof: For the two segment solution **[*H, ϕ*],** the seller exhausts the entire low-quality capacity towards building the probabilistic quality. Thus, $X^* = N$ and therefore using (A6),

$$\phi^* = \frac{M-n_H}{M-n_H+N}$$

$$\Rightarrow \frac{\partial \phi^*}{\partial c} = 0$$

$$and\ \frac{\partial \phi^*}{\partial n_H} = -\frac{Nn_H}{(M-n_H+N)^2} < 0$$

Again, as in proposition 2a, the price of the probabilistic quality $p_\phi$is simply an increasing linear function of ϕ and thus shares the same relationship with c and n_H.

Finally, since $M - n_H + X^* = M - n_H + N$, for the two segment solution,

$$\frac{\partial(M - n_H + X^*)}{\partial c} = \frac{\partial(M - n_H + N)}{\partial c} = 0$$

and

$$\frac{\partial(M - n_H + X^*)}{\partial n_H} = -1 < 0$$

Results when Benchmark 3 is relevant

Here: $\pi_{B3} > Max\,\{\pi_{B1}, \pi_{B2}\}$

Results 1 and Result 2 still hold when Benchmark 3 is the relative benchmark. The explanation follows.

In my earlier discussion of Result 1, I prove that when $n_H \geq M$, all the strategies using probabilistic quality are dominated by Benchmark 1. If $\pi_{B3} > Max\,\{\pi_{B1}, \pi_{B2}\}$, it is obvious that probabilistic quality is still dominated. As such, Result 1 still holds.

With regard to Result 2, note that, as before, the seller always prefers to exhaust the high-quality capacity on the high-type segment to obtain an attractive price. The remaining unsold high-quality capacity is then sold via probabilistic quality to the low-type segment. In my earlier discussion of Result 2, I demonstrate that offering probabilistic quality and low-quality to the low-type segment via a **[**$\boldsymbol{\phi}, \boldsymbol{L}$] offering is never optimal because **[**$\boldsymbol{\phi}, \boldsymbol{L}$] strategy is dominated by Benchmark 2. When $\pi_{B3} > Max\,\{\pi_{B1}, \pi_{B2}\}$, it is obvious that **[**$\boldsymbol{\phi}, \boldsymbol{L}$] strategy is also dominated by Benchmark 3.

By comparing π_{B3} with π_{B1} and π_{B2}, I conclude that

$$\begin{cases} When\ n_H > \frac{NV_{LL}}{V_{LH}-V_{LL}}, & \pi_{B3} > \pi_{B1} \\ When\ n_H \le \frac{NV_{LL}}{V_{LH}-V_{LL}}, & \pi_{B3} \le \pi_{B1} \end{cases}$$

and

$$\begin{cases} When\ n_H > \frac{M(V_{LH}-V_{LL})+NV_{LL}}{(V_{HH}-c_H)}, & \pi_{B3} > \pi_{B2} \\ When\ n_H \le \frac{M(V_{LH}-V_{LL})+NV_{LL}}{(V_{HH}-c_H)}, & \pi_{B3} \le \pi_{B2} \end{cases}$$

Summary:

$When\ n_H > Max\left\{\frac{NV_{LL}}{V_{LH}-V_{LL}}, \frac{M(V_{LH}-V_{LL})+NV_{LL}}{(V_{HH}-c_H)}\right\}, \quad \pi_{B3}$ is the relative benchmark

$When\ n_H \le Max\left\{\frac{NV_{LL}}{V_{LH}-V_{LL}}, \frac{M(V_{LH}-V_{LL})+NV_{LL}}{(V_{HH}-c_H)}\right\}, \quad \pi_{B1}\ and\ \pi_{B2}$ are the relative benchmarks

When Benchmark 3 is the relative benchmark, comparison of the profits of Benchmark 3 with profits of **[*H, ϕ*]** and **[*H, ϕ, L*]** yields:

(1) $\pi_{B3}^* \ge \pi_{H\emptyset L}^*$

$\Longrightarrow c \ge c_5$

$c_5 = \frac{[(M-n_H)(V_{LH}-c_H)-n_H(V_{HL}-V_{LL})+NV_{LL}]^2}{4(M-n_H)n_H\Delta}$

(2) $\pi_{B3}^* \ge \pi_{H\emptyset}^*$

$\Longrightarrow c \ge c_6$

$c_6 = \frac{(M-n_H)(V_{LH} - c_H)-n_H(V_{HL} - V_{LL})+NV_{LL}}{(M-n_H)+N}$

(3) As I discussed earlier on **[*H*, *ϕ*]** and **[*H*, *ϕ*, *L*]** strategy, I have:

$$\begin{cases} \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \leq c < \frac{n_H\Delta}{(M-n_H)} \Longrightarrow \text{I have the three segment solution } [\boldsymbol{H}, \boldsymbol{\Phi}, \boldsymbol{L}] \\ c < \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} \Longrightarrow \text{I have the two segment solution } [\boldsymbol{H}, \boldsymbol{\Phi}] \end{cases}$$

where the critical value of the transaction cost $c = \frac{(M-n_H)n_H\Delta}{[(M-n_H)+N]^2} = c_1$, from equation (9a).

Summary:

When $0 \leq c < min\{c_1, c_6\} \Longrightarrow \pi^*_{H\emptyset}$ is the optimal

When $c_1 \leq c < min\left\{c_5, \frac{n_H\Delta}{(M-n_H)}\right\} \Longrightarrow \pi^*_{H\emptyset L}$ is the optimal

Otherwise $\Longrightarrow \pi^*_{B3}$is the optimal.

I conclude that when Benchmark 3 is the relative benchmark, Results 1 and 2 still hold and probabilistic quality can still emerge as long as contracting costs are not too high.

Proposition 3: Regardless of the relative size of n_H:

(i) offering probabilistic quality *improves* consumer surplus when the high-quality capacity is relatively low ($M < M_0$)

(ii) offering probabilistic quality *degrades* consumer surplus when high-quality capacity is relatively high ($M \geq M_0$).

The consumer surplus in benchmark 1, benchmark 2, and probabilistic quality strategies are:

Surplus from Benchmark 1:

In Benchmark 1, low-type consumers don't enjoy any surplus. Surplus only comes from high-type consumers and is given as $(V_{HL} - V_{LL})$. So, the overall consumer surplus is:

$S_1 = n_H(V_{HL} - V_{LL})$.

Surplus from Benchmark 2:

In this Benchmark, low-type consumers again have zero surplus. Surplus comes from high-type consumers and is given as $(V_{HH} - V_{LH})$. However, the number of high-type consumers able to avail themselves of the high-quality product is not defined a priori (product assignment is random between the high-type and low-type). Suppose there are k $(0 \leq k \leq n_H)$ high-type consumers who end up purchasing high-quality product at V_{LH}. Then, the overall consumer surplus is:

$S_2 = k(V_{HH} - V_{LH})$.

Surplus from Probabilistic Quality Strategy:

In probabilistic quality strategy also, the low type consumers do not enjoy surplus. The high-type consumer's surplus is the same for both solutions (three- and two- segment) and is:

$$\phi(V_{HH} - V_{LH}) + (1 - \phi)(V_{HL} - V_{LL})$$

Thus, the overall consumer surplus here is:

$$S_\phi = n_H[\phi(V_{HH} - V_{LH}) + (1 - \phi)(V_{HL} - V_{LL})]$$

Comparing this to the benchmarks, I get

- $S_\phi - S_1 = n_H[\phi(V_{HH} - V_{LH}) + (1 - \phi)(V_{HL} - V_{LL})] - n_H(V_{HL} - V_{LL})$

  $= n_H\phi[(V_{HH} - V_{LH}) + (V_{HL} - V_{LL})] > 0$

  $\Longrightarrow S_\phi \geq S_1$.

- $S_\phi - S_2 = n_H[\phi(V_{HH} - V_{LH}) + (1 - \phi)(V_{HL} - V_{LL})] - k(V_{HH} - V_{LH})$

  $$\Longrightarrow \begin{cases} S_\phi \geq S_2 & , \ when\ k \leq \frac{n_H[\phi(V_{HH}-V_{LH})+(1-\phi)(V_{HL}-V_{LL})]}{(V_{HH}-V_{LH})} \\ S_\phi < S_2 & , \ when\ k > \frac{n_H[\phi(V_{HH}-V_{LH})+(1-\phi)(V_{HL}-V_{LL})]}{(V_{HH}-V_{LH})} \end{cases}$$

Thus, the aggregate consumer surplus with probabilistic quality is greater than the consumer surplus in Benchmark 1, and is conditionally better than the consumer surplus in Benchmark 2. It depends on the number of high-type consumers who are able to avail of the opportunity of buying the high-quality product, k.

Proposition 4: When the seller chooses capacity for the high-quality product in the face of demand uncertainty, the optimal capacity choice M^* is greater than the lower bound for the support of n_H. This implies that a world in which capacity for the high-quality exceeds demand can indeed arise in the first place.

Following the discussion with no demand uncertainty; there are four strategies the seller can adopt:

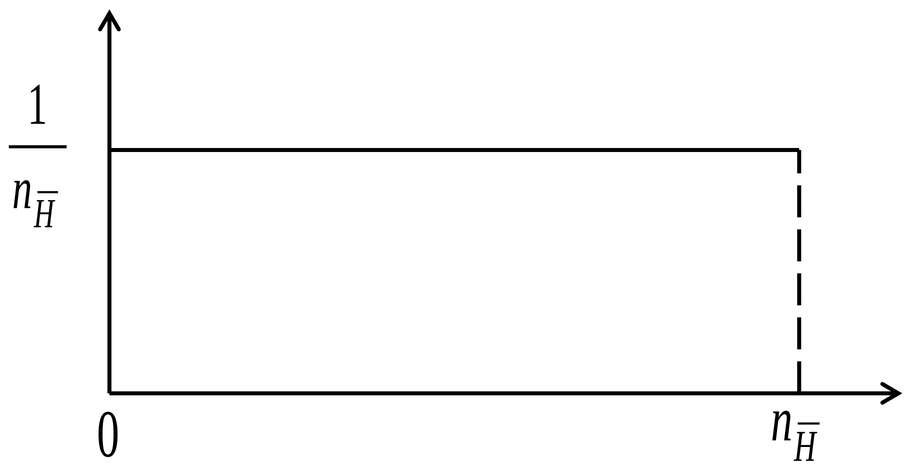
(1) Benchmark 1: Sell high-quality to the high-type consumers at $V_{HH} + V_{LL} - V_{HL}$ and sell low-quality to low-type consumers at V_{LL}.

(2) Benchmark 2: Sell high-quality to both low- and high- type consumers at V_{LH} and sell low-quality to low-type consumers at V_{LL}.

(3) Benchmark 3: Only sell high-quality to high-type consumers at V_{HH}.

(4) Probabilistic quality: Sell high-quality to high-type consumers, and sell both probabilistic and low-quality products to low-type consumers.

Assume the demand of high-type consumers follows a uniform distribution between 0 to $n_{\overline{H}}$. The probability density is $\frac{1}{n_{\overline{H}}}$.



Assuming a convex cost for capacity of the form, $\frac{lM^2}{2}$, the seller maximizes the following objective function:

$$\text{Max}_M\, \pi = \int_0^{n_{\overline{H}}} \frac{1}{n_{\overline{H}}}\, Max\,(\pi_{B1}, \pi_{B2}, \pi_{B3}, \pi_{Prob})\, dn_H - \frac{lM^2}{2}$$

Where:

$$\pi_{B1} = Min\{\, n_H, M\} * (V_{HH} + V_{LL} - V_{HL} - c_H) + NV_{LL}$$

$$\pi_{B2} = M(V_{LH} - c_H) + NV_{LL}$$

$$\pi_{B3} = Min\{\, n_H, M\} * (V_{HH} - c_H)$$

$$\pi_{Prob} = \begin{cases} \text{Domintated by B1, B2 or B3} & , \quad M \leq n_H \\ \underset{X}{\text{Max}}\, n_H(\Delta \mp c) + M(V_{LH} - c_H - c) + NV_{LL} - cX - \dfrac{(M - n_H)n_H\Delta}{(M - n_H) + X} & , \quad M > n_H \end{cases}$$

The next step is to prove that the optimal level of high-quality product offered M^* is positive:

When the seller chooses $M^* = 0$, the seller offers only low-quality products. The only strategy the seller can adopt is to sell low-quality product to low-type consumers at $p_L = V_{LL}$ and the profit is NV_{LL}. I then relax the condition on M from $M = 0$ to $M \geq 0$. If I can find *any* strategy with a positive value of M that exceed profits of NV_{LL}, I then conclude the optimal level of M^* of the profit maximization problem:

$$\underset{M}{\text{Max}}\, \pi = \int_0^{n_{\bar{H}}} \frac{1}{n_{\bar{H}}}\, Max\,\{\pi_{B1}, \pi_{B2}, \pi_{B3}, \pi_{Prob}\}\, dn_H - \frac{lM^2}{2}$$

is greater than zero.

Consider the following strategy. Suppose the seller sells M high-quality products to high type consumers at $V_{HH} + V_{LL} - V_{HL}$ and sells N low-quality product to low type consumers at V_{LL}.

$$\pi = \int_0^M \frac{1}{n_{\bar{H}}} n_H (V_{HH} + V_{LL} - V_{HL} - c_H)\, dn_H + \int_M^{n_{\bar{H}}} \frac{1}{n_{\bar{H}}} M (V_{HH} + V_{LL} - V_{HL} - c_H)\, dn_H + NV_{LL} - \frac{lM^2}{2},$$

$$= (V_{HH} + V_{LL} - V_{HL} - c_H)M - \frac{(V_{HH}+V_{LL}-V_{HL}-c_H)M^2}{2n_{\bar{H}}} + NV_{LL} - \frac{lM^2}{2}$$

F.O.C

$$\frac{\partial \pi}{\partial M} = (V_{HH} + V_{LL} - V_{HL} - c_H) - \frac{(V_{HH}+V_{LL}-V_{HL}-c_H)M}{n_{\bar{H}}} - lM = 0$$

S.O.C

$$\frac{\partial^2 \pi}{\partial M^2} = -\frac{(V_{HH}+V_{LL}-V_{HL}-c_H)}{n_{\bar{H}}} - l < 0$$

$\Longrightarrow$ When $M^* = \frac{n_{\bar{H}}[V_{HH}+V_{LL}-V_{HL}-c_H]}{(V_{HH}+V_{LL}-V_{HL}-c_H)+ln_{\bar{H}}}$, π gets the maximum $\pi^* = \frac{n_{\bar{H}}(V_{HH}+V_{LL}-V_{HL}-c_H)^2}{2[(V_{HH}+V_{LL}-V_{HL}-c_H)+ln_{\bar{H}}]} + NV_{LL} > NV_{LL}$.

Since $(V_{HH} + V_{LL} - V_{HL} - c_H) > 0$, then $M^* = \frac{n_{\bar{H}}[V_{HH}+V_{LL}-V_{HL}-c_H]}{(V_{HH}+V_{LL}-V_{HL}-c_H)+ln_{\bar{H}}} > 0$.