Efficiency of Underdamped dc SQUIDs as Readout Devices for Flux Qubits

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***Abstract*—The flux state quantum bit (qubit) is promising for a solid state implementation of scalable quantum computing. The simplest flux state qubit consists of an rf SQUID with two fluxoid states, which can be readout with a dc SQUID—the most sensitive magnetic flux detector. Efficient readout with less back-action is desirable for quantum computing. In this work, we report measurements of the switching flux and switching current distributions of under damped dc SQUIDs. The data show that single shot readout of flux qubit with very high efficiency ($>99\%$) can be realized using underdamped hysteretic dc SQUIDs.**

***Index Terms*—dc SQUID, qubit, readout.**

QUANTUM computing has drawn significant interest because of its massive intrinsic parallelism. In principle, any system that is able to store and coherently process information in a Hilbert space can be used to implement quantum computing. Recently, quantum logic operations have been demonstrated in several physical systems such as trapped ions [1], NMR [2], quantum electrodynamics cavities [3], and Josephson devices [4]. Because the solid state Josephson device is scalable and its parameters are readily adjustable, Josephson qubits are recognized as a very promising approach to quantum computing.

Based on the two quantum conjugate variables—charge and phase (flux)—Josephson qubits can be divided into two main types: the charge qubit and the flux qubit. Comparing with the charge qubit, the flux qubit has the advantage of being insensitive to background charge fluctuation which is a major source of decoherence. From a practical point of view, a strong single-shot readout measurement of qubit is needed for not only getting the final result of the quantum computation but also the purpose of error correction in the course of the computation. Therefore, a detailed study on the sensitivity and efficiency of dc SQUIDs as the readout devices of the flux qubits is instructive [5]. In this work, the flux modulation of switching current, the switching flux and switching current distributions, and the efficiency of underdamped dc SQUIDs as flux qubit state readout devices are presented and discussed. In addition, back-action introduced by the readout devices is also considered.

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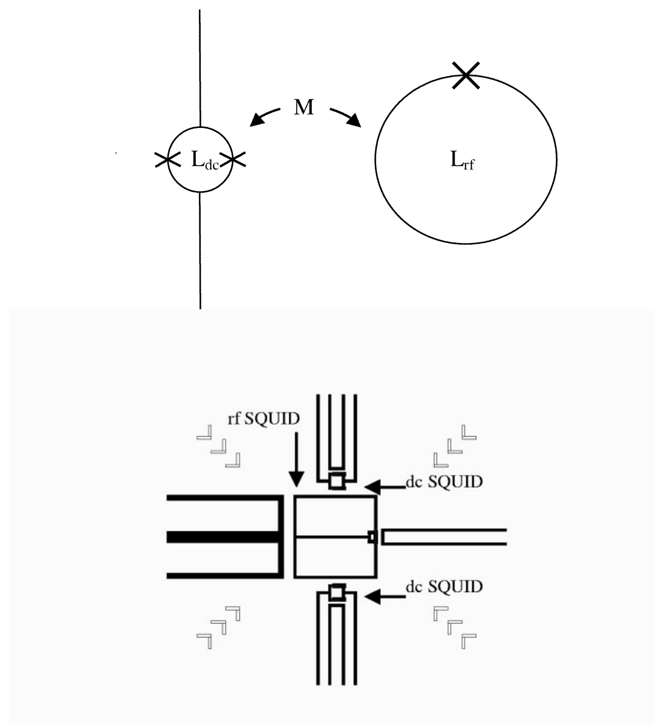

Fig. 1. Top: the equivalent circuit of a flux qubit coupled to a dc SQUID readout device through mutual inductance M. Bottom: the layout of a gradiometer flux qubit inductively coupled to two unshunted dc SQUID detectors.

The best device to measure the fluxoid state of an rf SQUID-flux qubit is probably the dc SQUID, which is well known as the most sensitive magnetometer. A dc SQUID consists of a superconducting loop of inductance L_{dc}, interrupted by two Josephson junctions with a total critical current I_c. Because underdamped dc SQUIDs introduce much less additional dissipation and noise to the flux qubit it is more desirable to use them as readout devices than the conventional overdamped dc SQUID magnetometers. A simple readout circuit that uses unshunted dc SQUID detectors for a flux qubit (rf SQUID) is shown in Fig. 1. In the following, we will focus our discussions on two readout methods: the sweeping-current and the sweeping-flux detection modes.

For the sweeping-current mode, one ramps the bias current of the dc SQUID up while keeping the flux bias constant until it switches to the finite voltage state. Since the detector's switching current is sensitive to the amount of externally applied flux

$$\Phi_{xdc} = \Phi_{xdcs} + MI_{\mathrm{cir}}(f) \tag{1}$$

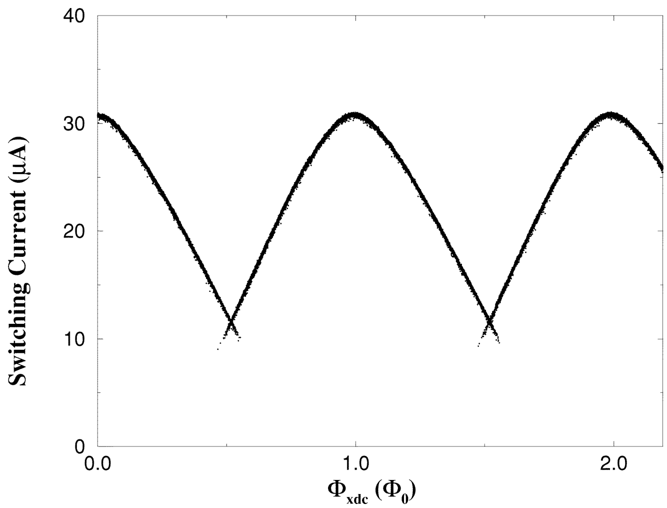

Fig. 2. The flux modulation of the switching current of the dc SQUID at $T = 1.55$ K.

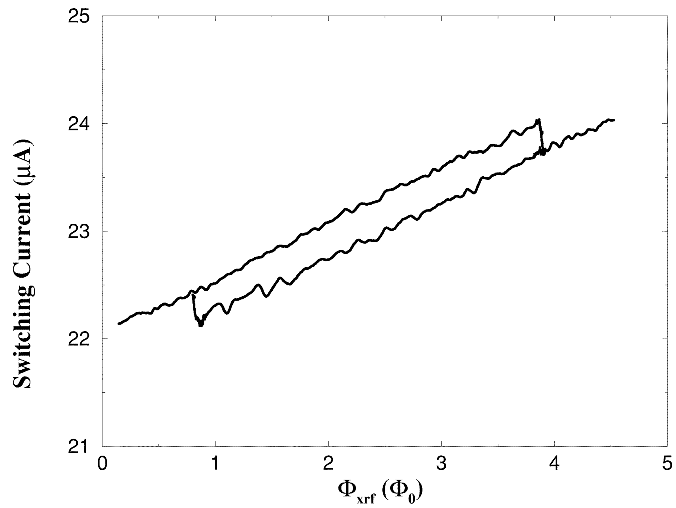

Fig. 3. The rf SQUID's hysteretic loop detected by the dc SQUID switch at 4.2 K. The higher (lower) branch of the loop corresponds to the $|0\rangle(|1\rangle)$ qubit state.

in the dc SQUID loop, where $I_{\rm cir}(f)$ is the circulation current of the qubit which depends on the fluxoid state ($f = 0$, 1) of the qubit, the flux state of the qubit can be inferred from the measured value of switching current I_S. Here, Φ_{xdcs} is the quasistatic flux bias of the dc SQUID detector and M is the mutual inductance between the qubit and the detector. For instance, for $\Phi_{xdcs}/\Phi_0 \approx 0.25$ the detector's switching current is higher when the qubit is in the $|0\rangle(f = 0)$ state, i.e., $I_S(f = 0) > I_S(f = 1)$, assuming that the detector is flux biased on one of the positive-slope sides of the $I_S(\Phi_e)$ transfer function and that the qubit and detector are weakly coupled $(M|I_{\rm cir}|/\Phi_0 \ll 1)$.

Similarly, for the sweeping-flux mode of readout, one ramps up the flux bias of the dc SQUID, while keeping the bias current constant, until the detector switches to the $V \neq 0$ state. The value of the flux bias at which the detector switches depends on the qubit state.

These readout procedures can be improved by implementing simple changes. For example, when operating in the sweeping-current readout mode, back-action can be significantly reduced by setting the bias current to a level that is about halfway between $I_S(f = 1)$ and $I_S(f = 0)$ so that the detector will switch only if the qubit was in the $|1\rangle$ state.

The circuit studied here was a variable barrier rf SQUID flux qubit inductively coupled to an unshunted dc SQUID detector. The qubit has a $L_{rf} \approx 200$ pH superconducting loop that is interrupted by a small dc SQUID consisting of two 1.5×1.5 μm^2 NbN/AlN/NbN Josephson tunnel junctions. Two dc SQUID detectors are coupled to the qubit. Each detector has two nominally identical 2×2 μm^2 NbN/AIN/NbN Josephson tunnel junctions. The maximum critical current of each dc SQUID is $I_c = 2I_0 \simeq 33$ μA, the capacitance is $C \approx 0.45$ pF, and the loop inductance is $L_{dc} \approx 30$ pH. The mutual inductance between the qubit and the dc SQUIDs is $M \approx 2.9$ pH, which is determined from the measured flux dependence of switching current and the size of the flux jump produced by the qubit switching from one to the other fluxoid state.

Fig. 2 shows the periodic modulation of the dc SQUID's switching current $I_S(\Phi_{xdc})$ taken at $T = 1.55$ K. During the measurement, the qubit state was kept constant. The flux dependence of I_S is obtained by repeatedly ramping up and down the bias current of the dc SQUID while slowly (quasistatically) increasing the magnetic flux Φ_{es} at the same time. It is well known that switching from the zero voltage state to the finite voltage state is a random process and that the switching currents at a constant flux bias obey certain statistical distribution [6]. It is obvious that distributions with narrower width (σ_1) are desirable for high efficiency readout. From Fig. 2 one can see that the distribution is slightly wider for larger switching currents and narrower for smaller switching currents, as expected from theory. The double peak distribution in the vicinity of the minimum switching current is resulted from the emergence of a second set of metastable potential wells in the two-dimensional potential landscape of the dc SQUIDs that do not satisfy $\beta_T = 2\pi L_{dc} L_0/\Phi_0 \ll 1$.

To quantitatively study performance of dc SQUIDs as readout devices for flux qubits we measured the rf SQUID qubit's circulation current I_S as a function of flux Φ_{xrf} applied to the qubit. For the qubit used in this work, $I_{\rm cir}(\Phi_{xrf})$ is hysteretic because the qubit's potential shape parameter $\beta_L \equiv 2\pi L_{rf} I_{crf}/\Phi_0$ is much greater than unity. The result obtained at 4.2 K is shown in Fig. 3, where the horizontal axis is the flux applied to the rf SQUID and the vertical axis is the average switching current $\bar{I}_S$ of the dc SQUID detector. The sudden jumps in $\bar{I}_S$ correspond to the qubit changing state from $|0\rangle$ to $|1\rangle$ and vice versa. Note that in this work the fluxoid state with a clockwise (counterclockwise) circulation current is denoted as the qubit state $|0\rangle(|1\rangle)$. One can see that a transition between these two fluxoid states induces a signal of $\Delta\bar{I}_S \approx 0.3$ μA.

It is clear that the ratio between the switching current jump and the widths of the corresponding switching current distributions, $\Delta\bar{I}_S/\sigma_1$ determines the single-shot readout efficiency (SSRE) of the sweeping-current readout mode. Here, SSRE is defined as the probability of correctly discriminating the qubit state $|0\rangle$ from $|1\rangle$ via a single measurement. It is obvious that smaller σ_1 and larger ΔI_S result in higher SSRE. In fact, it can

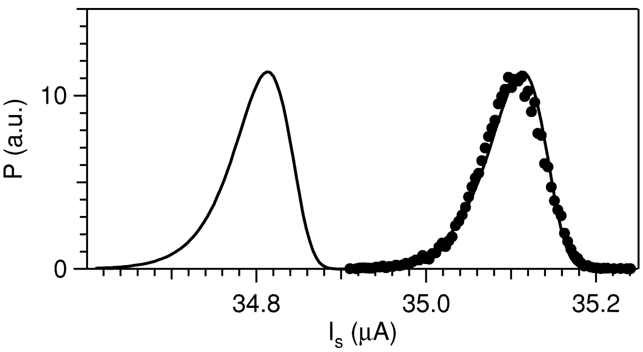

Fig. 4. Switching current distribution of a dc SQUID detector measured at $T = 10$ mK (solid dots). The solid lines are calculated from MQT theory. The distribution on the left is separated by 0.3 μA from the one on the right. The overlapping part of the two distributions, when normalized, is less than 10^{-4}.

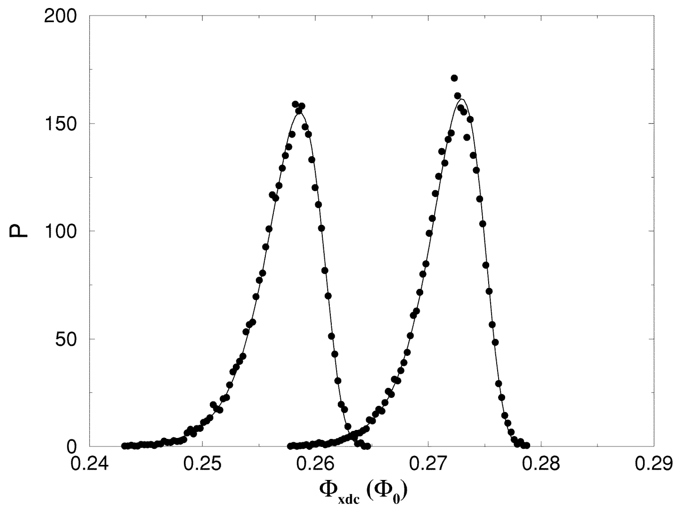

Fig. 5. The flux distributions of the dc SQUID detector at $T = 1.4$ K. Since the detector was flux biased on the negative-slope side of $I_S(\Phi_{xdc})$ the left (right) one corresponds to the qubit in state $|0\rangle (|1\rangle)$. These distributions result in a single-shot readout efficiency of about 98.8%.

be shown that in the sweeping-current detection mode, the maximum SSRE is given by

$$\mathrm{SSRE}_{\max} = 1 - \int_{-\infty}^{\infty} P_0(I_S) P_1(I_S) dI_S, \tag{2}$$

where, $P_0(I_S)$ and $P_1(I_S)$ are the switching current distribution of the detector for the qubit in state $|0\rangle$ and $|1\rangle$, respectively. Similarly, in the sweeping-flux detection mode the maximum SSRE is obtained by replacing switching current I_S in (2) with switching flux Φ_{xdcs} [5]. Recently, it has been shown that in both the classical (thermal) and quantum regimes the switching current distributions of dc SQUIDs agree very well with the theoretical predictions of thermal activation and macroscopic quantum tunneling (MQT) [7]. For the detector used in this experiment the minimum width of $P_f(I_S)$, where $f = 0$, 1, is expected to be about 40 nA in the quantum regime. The calculated overlap integral in (2) for two such switching current distributions, with their mean separated by about 0.3 μA, is less than 0.01% which corresponds to a $\mathrm{SSRE}_{\max} > 99.99\%$ at temperatures below the classical-quantum crossover temperature of $T_{co} = 0.26$ K (see Fig. 4).

Fig. 5 shows the two switching flux distributions of the dc SQUID detector at $T = 1.14$ K. The left distribution was taken with the qubit in the $|0\rangle$ state while the right one was taken with the qubit in the $|1\rangle$ state. The data (solid circles) agree with the theory (solid lines) quite well. The width of the left distribution in Fig. 5 is $\sigma_{|1\rangle} \simeq 7.2$ mΦ_0 and the right is $\sigma_{|0\rangle} \simeq 6.9$ mΦ_0, respectively. The separation between the peaks of the two switching flux distributions is about 14.4 mΦ_0. The single-shot readout efficiency of sweeping-flux readout mode was then evaluated by integrating the overlapping part of the two distributions. The value of the overlap obtained was about 1.2%, corresponding to a single-shot readout efficiency of 98.8%. Notice that for the sweeping-flux mode, SSRE can also be improved by cooling the detector to $T \ll T_{co}$. Therefore, although using an unshunted dc SQUID to detect the state of a flux qubit is intrinsically a statistical measurement, our data show that for flux qubit high efficiency single-shot readout can be realized if the parameters of the dc SQUID are properly chosen.

Because $\Delta I_S = M[I_{\mathrm{cir}}(f=1) - I_{\mathrm{cir}}(f=0)](\partial I_S / \partial \Phi_{xdc})$ and the terms in the square (curved) brackets depend only on the parameters of the qubit (detector), stronger coupling between the qubit and detector leads to larger signal, and thus higher readout efficiency. Unfortunately, increasing coupling will also result in stronger back-action that causes gate errors and decoherence in the qubit. Therefore, SSRE should be improved without increasing the coupling between the detector and qubit. Since the width of switching current distribution at $T \ll T_{co}$ scales approximately with $C^{-1/3} I_c^{2/3}$ and $\partial I_S / \partial \Phi_{xdc} \approx 2I_c/\Phi_0$ for detectors with $\beta_T \ll 1$ we have

$$\mathrm{SSRE} \propto \frac{\Delta I_S}{\sigma_l} \propto \frac{M I_c}{I_c^{2/3} C^{-1/3}} = M I_c^{1/3} C^{1/3}. \tag{3}$$

Hence, SSRE can be raised by increasing the critical current and/or shunt capacitance of the Josephson junctions. However, increasing I_c results in other problems such as heating, larger β_T (thus smaller $\partial I_S / \partial \Phi_{xdc}$), or smaller M (in order to keep $\beta_T \ll 1$). Therefore, the simplest way of improve SSRE is to use junctions with large capacitive shunt.

A problem that has not been studied very much before is back-action from dc SQUID detectors onto the flux qubits. From the point of view of reducing back-action from the detector to the qubit one must keep the coupling as weak as possible. In the following discussions we estimate the amount of back-action generated from ramping the dc SQUID's bias current. For the qubit-detector circuit tested in this experiment the back-action flux is $MI_{dc} \approx 1.4 \times 10^{-2}\, \Phi_0$, where $I_{dc} \approx 10\, \mu$A is the typical value of a dc SQUID's bias current. Since a change of less than 1 mΦ_0 in flux bias is large enough to significantly alter the energy level spectrum of a typical rf SQUID flux qubit this amount of back-action must be reduced by at least a factor of 10^2. This can be achieved by the use of detectors with much smaller critical current and clever qubit-detector coupling schemes. One of the methods is to decouple the external (symmetric) mode of the dc SQUID's bias current from the qubit while maintaining sufficient coupling between the internal mode (i.e., circulating current) of the detector and the qubit. Another way of having sufficient readout efficiency while keeping back-action to minimum is to use the variable flux transformers so that coupling between the qubit and detector can be switched on/off *in situ* [8].

Finally, let's examine the amount of additional damping introduced onto a flux qubit from a dc SQUID detector. For this purpose the detector can be modeled as a resistor R_{dc} in parallel with an inductor L_{dc} that is coupled to the qubit through a mutual inductance M. It is straightforward to show that for frequencies $\omega \ll R_{dc}/L_{dc}$ the effect of the detector is equivalent to shunt the qubit with an effective damping resistor $R_{\text{eff}} = (L_{rf}/M)^2 R_{dc}$. Because R_{eff} is an increasing function of ω its effect becomes much weaker at high frequencies $(\omega \gg R_{dc}/L_{dc})$. Taking a typical value of $L_{rf}/M \approx 100$ and $R_{dc} \approx 100$ kΩ for unshunted dc SQUIDs at low temperature, we found $R_{\text{eff}} \approx 1$ GΩ, which will have negligible effect on the energy relaxation and dephasing of the flux qubits.

In summary, the switching current and switching flux distributions of unshunted dc SQUID detectors were measured in the thermal and quantum regimes. In the sweeping current readout mode a single-shot efficiency of better than 99.99% was demonstrated at 10 mK. In the sweeping flux readout mode a single-shot efficiency of about 98.8% was obtained at 1.4 K. These results show that when used as the readout device for rf SQUID flux qubits one can achieve very high efficiency single-shot detection so that the readout can almost be regarded as deterministic. It is also shown that dc SQUID detectors with lower crossover temperature are desirable for having higher SSRE and that the right approach is to increasing shunt capacitance rather than decreasing critical current of the junctions. In addition, our analysis shows that care must be taken to reduce back-action. Finally, it is also shown that the amount of additional damping onto the flux qubits from the dc SQUID detectors is negligible.

REFERENCES

[1] C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, "Demonstration of a fundamental quantum logic gate," *Phys. Rev. Lett.*, vol. 75, pp. 4714–4717, 1995.

[2] N. A. Gershenfeld and I. L. Chang, "Bulk spin-resonance quantum computation," *Science*, vol. 275, pp. 350–353, 1997.

[3] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, "Measurement of conditional phase shifts for quantum logic," *Phys. Rev. Lett.*, vol. 75, pp. 4710–4713, 1995.

[4] Y. Makhlin, G. Schön, and A. Shnirman, "Quantum-state engineering with Josephson-junction devices," *Rev. Mod. Phys.*, vol. 73, pp. 357–400, 2001.

[5] C. Cosmelli, F. Sciamanna, M. G. Castellano, R. Leoni, G. Torrioli, P. Carelli, and F. Chiarello, "Stroboscopic single-shot detection of flux state in a radio frequency superconducting quantum interference device," *Appl. Phys. Lett.*, vol. 80, pp. 3150–3152, 2002.

[6] J. Kurkijarvi, "Intrinsic fluctuations in a superconducting ring closed with a Josephson junction," *Phys. Rev. B*, vol. 6, pp. 832–832, 1972.

[7] Li Shao-Xiong, Y. Yu, Y. Zhang, W. Qiu, S. Han, and Z. Wang, "Quantitative study of macroscopic quantum tunneling in a dc SQUID: A system with two degrees of freedom," *Phys. Rev. Lett.*, vol. 89, pp. 098 301-1–098 301-4, 2002.

[8] T. V. Filippov, S. K. Tolpago, J. Mannik, and J. E. Lukens, "Tunable transformer for Qubits based on flux states," IEEE Trans. Appl. Supercon., 2002, submitted for publication.