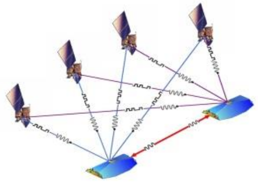

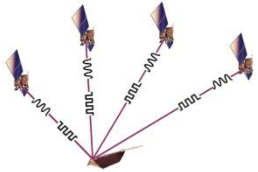

A proposal for an orbit determination procedure for short arcs of LEO with GPS SST observations

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LEO Missions (Earth Explorers)

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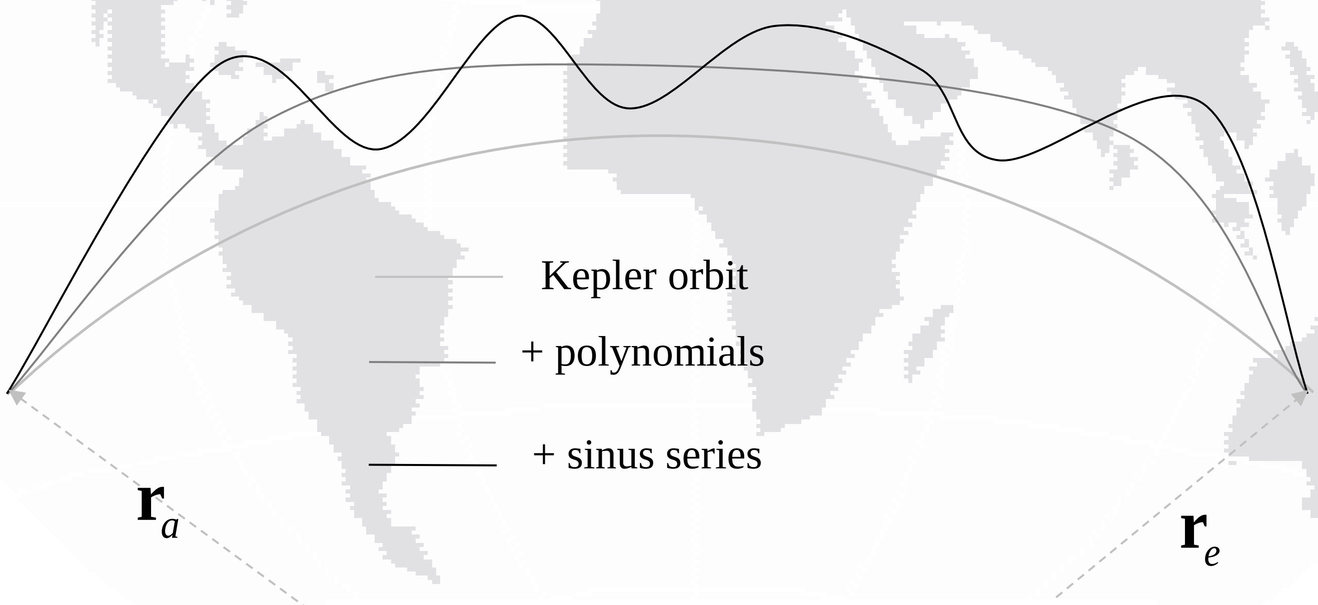
Advantages of LEO POD

- LEO orbits can be used to recover the gravity field of the Earth (SST, SGG)
- Analysis of altimetry observations require precise orbits
- Atmosphere sounding requires precise positions of the LEO satellites
- GNSS (GPS, GLONASS, GALILEO) methods play an important role in POD in addition to classical methods (e.g. SLR)

Short Arc POD

Step by step presentation of short arc LEO

$$\mathbf{r} = \underbrace{\mathbf{r}_a \cdot \frac{\sin((1-\tau)\cdot n)}{\sin(n)} + \mathbf{r}_e \cdot \frac{\sin(\tau \cdot n)}{\sin(n)}}_{\textbf{Kepler orbit}} + \underbrace{\mathbf{C}^T \mathbf{P}(\tau)}_{\textbf{Euler-Bernoulli Polynomial}} + \underbrace{\sum_{\nu=1}^{n} \bar{\mathbf{d}}_\nu \sin(\nu\pi\tau)}_{\textbf{Sinus series}}$$



representation of LEO short arc

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LEO Short Arc POD

Short arcs of LEO orbits can be represented as:

$$\mathbf{r}(\tau) = \underbrace{\mathbf{r}_a \cdot \frac{\sin((1-\tau)n)}{\sin(n)} + \mathbf{r}_e \cdot \frac{\sin(\tau \cdot n)}{\sin(n)}}_{\text{Kepler orbit}} + \underbrace{\mathbf{C}^T \mathbf{P}(\tau)}_{\text{Euler-Bernoulli Polynomial}} + \underbrace{\sum_{\nu=1}^{n} \bar{\mathbf{d}}_\nu \sin(\nu\pi\tau)}_{\text{Sinus series}}$$

$\mathbf{r}(\tau) = \begin{bmatrix} x & y & z \end{bmatrix}^T$ Position vector of the LEO at time t

$\tau = \frac{t - t_a}{t_e - t_a}$ Normalized time at t

$\mathbf{r}_a, t_a, \mathbf{r}_e, t_e$ Boundary positions & times

$n = \sqrt{\frac{k^2 M}{a^3}} T$ Mean motion of the LEO satellite,

Short Arc POD

$$\mathbf{C}_{3\times4}=\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{pmatrix}$$

Euler-Bernoulli polynomial coefficients

$$\mathbf{P}(\tau)=\begin{pmatrix} -\tau+\tau^2 \\ \frac{1}{2}\tau-\frac{3}{2}\tau^2+\tau^3 \\ \tau-2\tau^3+\tau^4 \\ -\frac{1}{6}\tau+\frac{5}{3}\tau^3-\frac{5}{2}\tau^4+\tau^5 \end{pmatrix}=\begin{pmatrix} P_1(\tau) \\ P_2(\tau) \\ P_3(\tau) \\ P_4(\tau) \end{pmatrix}$$

Euler-Bernoulli polynomial

$\bar{\mathbf{d}}_f$ **amplitude vector of Fourier series**

n **degree of the Fourier series expansion.**

Short Arc POD

- This method is based on a solution of Newton's equation of motion, formulated as a boundary value problem proposed by Schneider (1968) and modified by Ilk (1976)

- Short arcs of LEO can be formulated in the:
  - kinematical mode, in the
  - dynamical mode, and everything in-between, in the
  - reduced-dynamical mode.

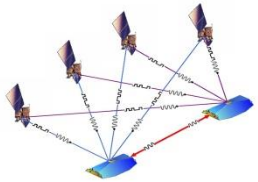
Short Arc POD-Simulation

- To test the short arc POD procedure, the method has been tested based on a GRACE twin-satellite simulation scenario.

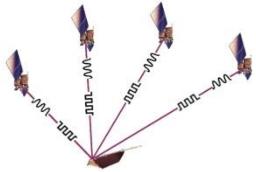
- Simulated GRACE Data

  - short arc length (~ ca. 20 min)
  - GPS high-low observations (hl-SST)
  - terrestrial ground observations to GRACE satellites (ranges (SLR), horizontal angles, vertical angles)
  - range and range-rate observations.

- The method has been simulated in the dynamical, kinematical, reduced dynamical modes, but here only the kinematical mode will be presented.



Kinematical POD-Simulation



$$\mathbf{r}(t) = \underbrace{\mathbf{r}_a \cdot \frac{\sin((1-\tau)\cdot n)}{\sin(n)} + \mathbf{r}_e \cdot \frac{\sin(\tau \cdot n)}{\sin(n)}}_{\text{Kepler orbit}} + \overbrace{\mathbf{C}^T \mathbf{P}(\tau)}^{\text{Euler-Bernoulli Polynomial}} + \underbrace{\sum_{\nu=1}^{n} \bar{\mathbf{d}}_\nu \sin(\nu\pi\tau)}_{\text{Sinus series}}$$

all coefficients have been estimated by a redundant Gauss-Markov model

Advantage:

the kinematical orbits can be used directly to recover the Earth gravity field.

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Kinematical POD-Simulation

Disadvantages:

✓the accuracy of the approximation coefficients is essential.

✓many parameters must be estimated through the adjustment procedure.

✓estimated coefficients of the dynamical method must be used as initial values for the subsequent kinematical orbit determination (in the simulation case).

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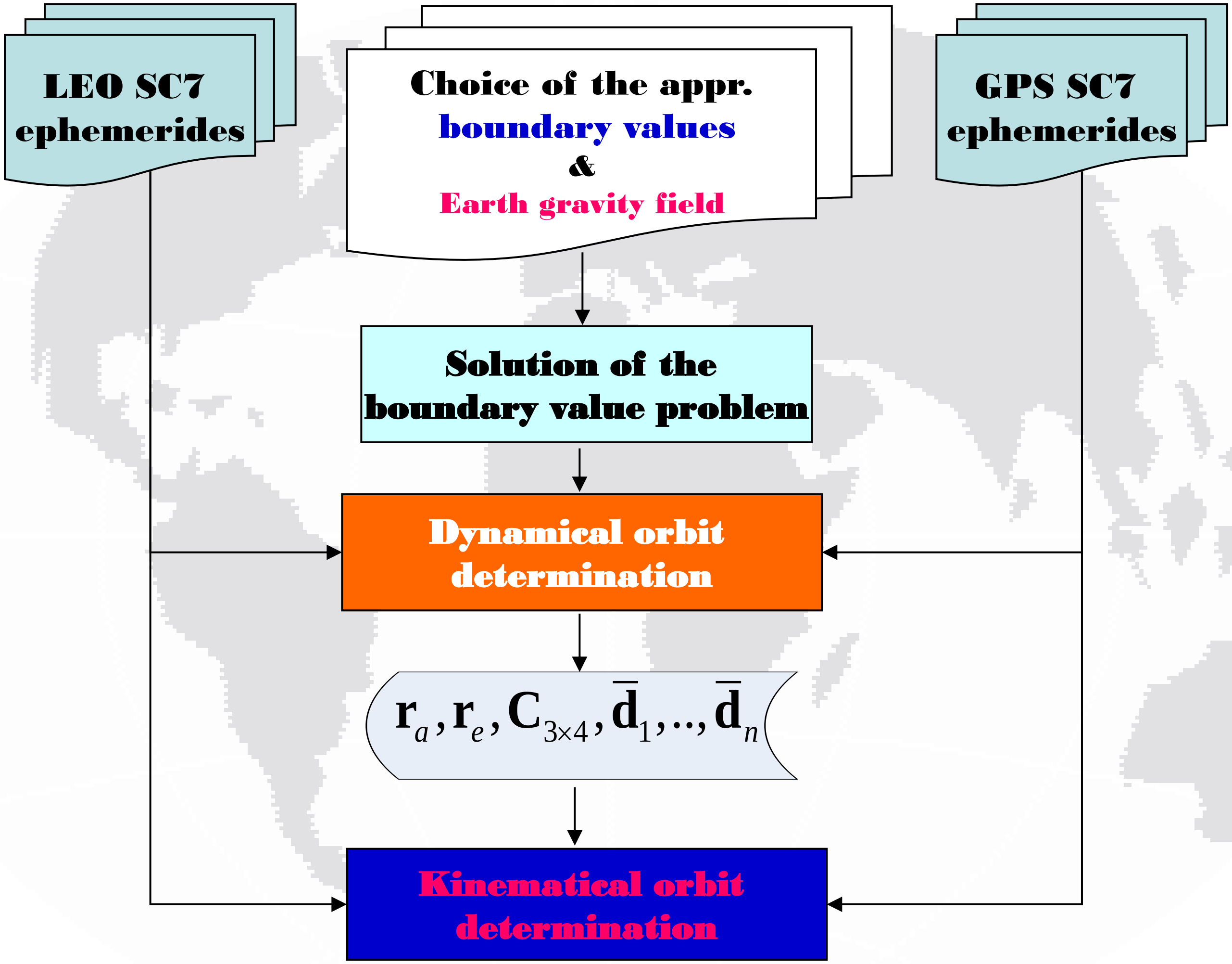
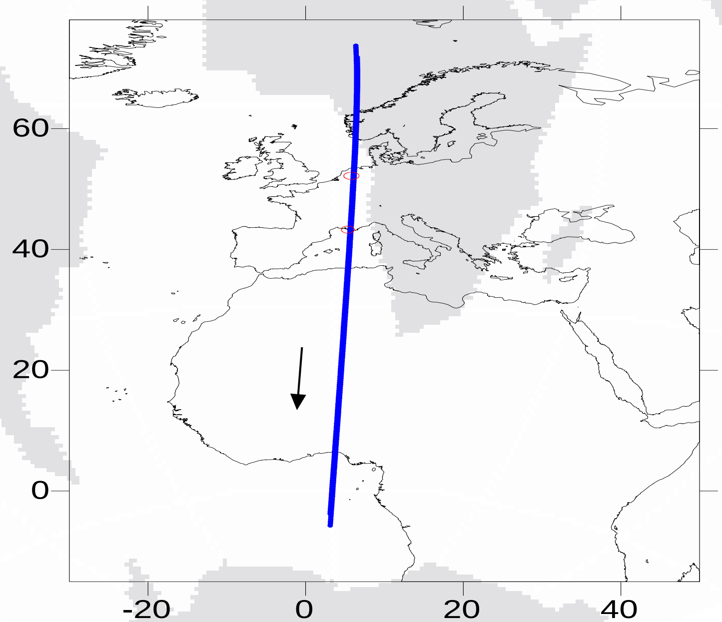
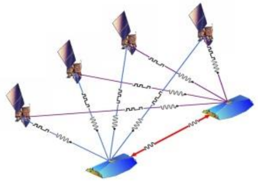
POD flowchart -Simulation



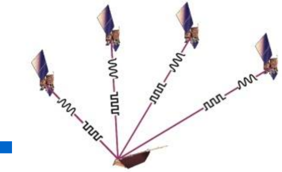
Short Arc POD-Example

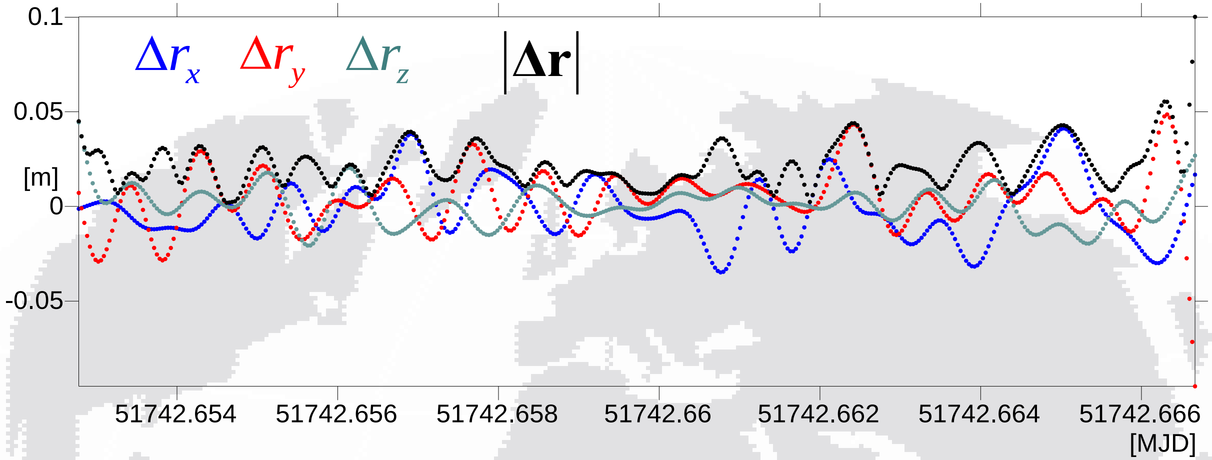
- short arcs of GRACE twin-satellites (A,B) above Europe and Africa have been selected.
- Observations :
  - GPS high-low SST pseudo-range observations.
  - range and range-rates between GRACE twin satellite
  - ground station observations (ranges, horizontal & vertical angels)



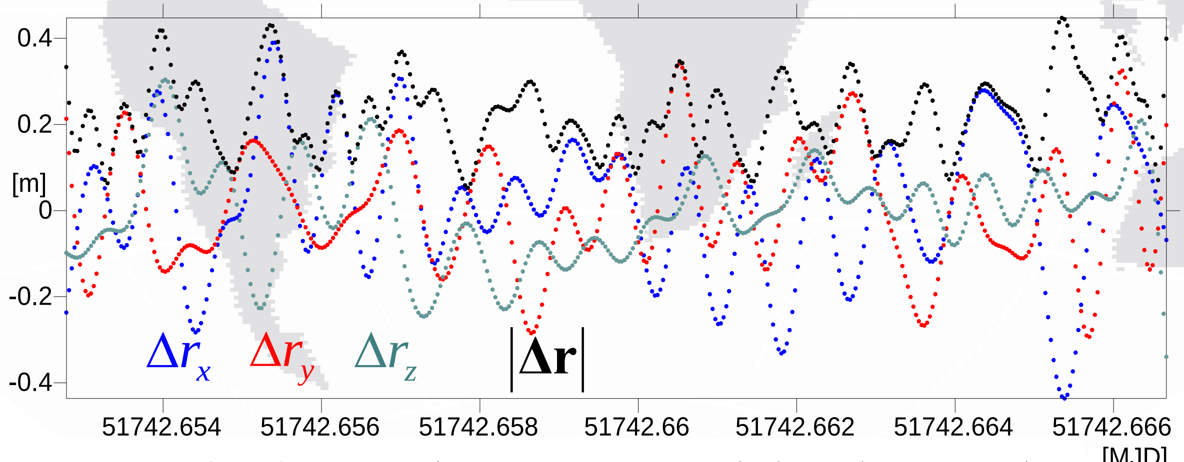


Kinematical POD-Simulation





GPS pseudo-range precision(±0,05 m)



GPS pseudo-range precision(±0,5 m)

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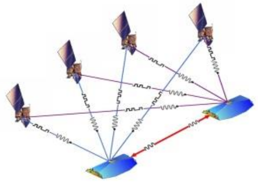
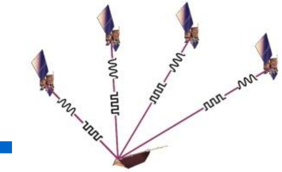
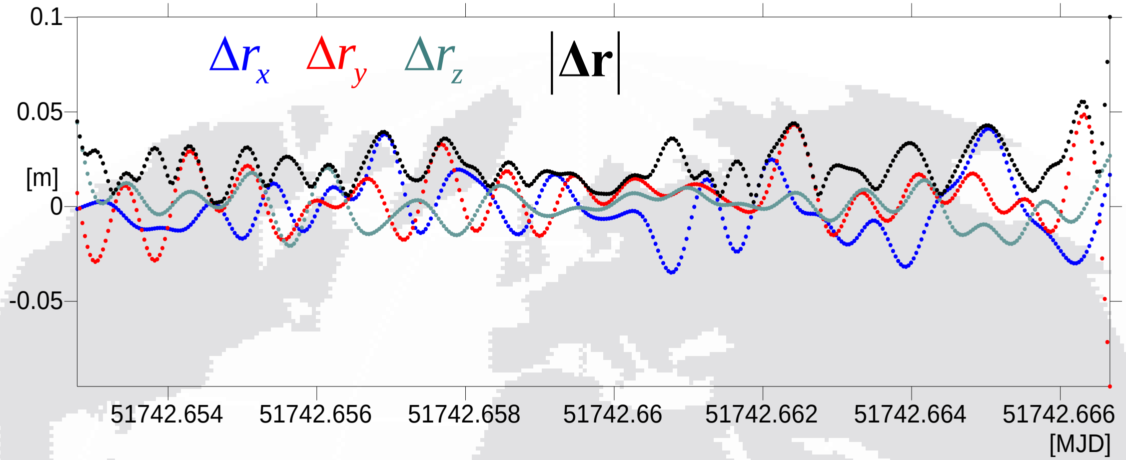
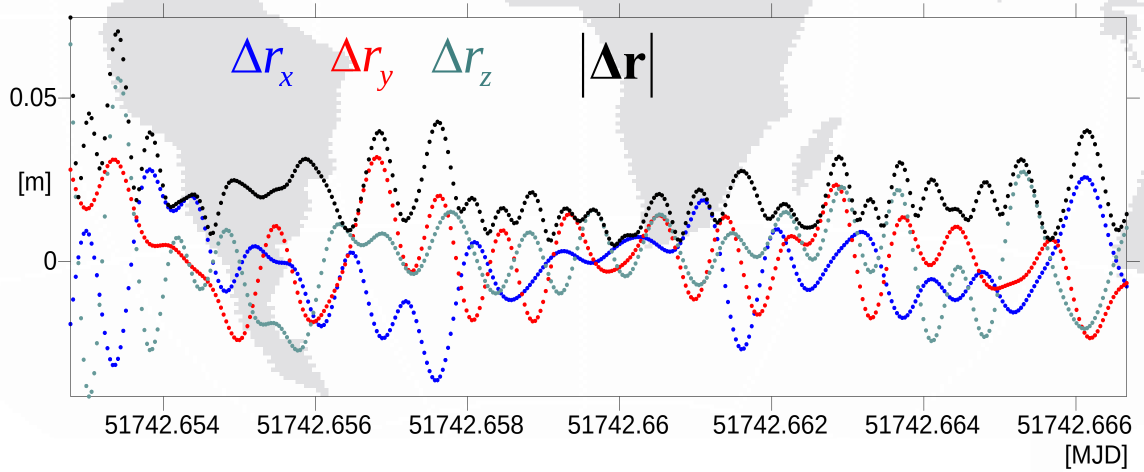


Kinematical POD-Simulation





with GRACE range & range-rate measurements



without GRACE range & range-rate measurements

Kinematical POD-Real Data

- Kinematical LEO orbit has been estimated in zero difference processing mode of GPS observations.

- IGS GPS final orbits with accuracy of ~cm, Earth rotation parameters (ERP) from IERS centre have been used in the procedure.

- because of ZD procedure, many corrections must be applied to GPS satellite positions , code and sequential time difference of carrier phase observations (e.g. GPS antenna mass centre offset, relativistic effect,…)

- no Earth gravity filed and no force models have been used in the kinematical mode (advantage of the method)

Kinematical LEO POD-Procedure

- at first, initial positions & clock offsets have been estimaed with Bancroft model.

- LEO approximation positions & clock offsets have been improved with the code pseudorange observations in accuracy of code observations. (~ meter)

- LEO position & clock offset differences have been estimated in accuracy of carrier phase sequential time difference observations.(~ cm)

- LEO abosulte positions & clock offsets from the code, position & clock offset differences from the carrier phase observations have been combined to estimate final LEO positions & clock offsets at every epoch.

Kinematical LEO POD-Code

Code pseudo-range GPS SST observations:

$$P_{r,i}^{s}\left(t\right)=\left|\mathbf{R}_{z}(\omega_{e}.\varepsilon_{r}^{s})\mathbf{r}^{s}(t-\varepsilon_{r}^{s})-\mathbf{r}_{r}(t)\right|+c\left[dt^{s}(t-\varepsilon_{r}^{s})-dt_{r}(t)\right]+I_{i}^{r}(t)+$$

$$d_{O}^{s}(t)+d_{R}^{r}(t)-d_{R}^{s}(t)+d_{C,i}^{r}(t)+d_{V,i}^{r}(t)+d_{M,P_{i}}(t)+e_{P_{i}}$$

- **GPS satellite positions must be corrected for GPS antenna-mass centre offset (SP3 files from ACC centres).**

- **special relativistic effects must be applied to GPS and LEO satellites, because of the eccentricity of the orbits.**

- **to eliminate ionosphere effect in the observation equations, ionosphere-free code observations have been used in the data processing.**

Kinematical LEO POD-Code

- LEO antenna phase centre & phase centre variation must be applied to observation equation. In GPS satellites, this effect can be neglected.

- from the code SST observations, LEO positions and clock offsets can be estimated at every epoch with enough number of GPS satellites (>4) and enough GPS satellites geometry (enough DOP).

- The accuracy of meter have been expected for the GPS code SST observations, and it depends on the GPS satellites geometry (DOP).

Kinematical LEO POD-Carrier Phase

Carrier phase ionosphere-free observation at epochs (1,2)

$$\Phi_{r,3}^{s}\left(t_1\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|+\lambda_3 N_{r,3}^{s}+c\left[dt^s(t_1-\varepsilon_1)-dt_r(t_1)\right]+$$
$$d_O^s(t_1)+d_R^r(t_1)-d_R^s(t_1)+d_{C,3}^r(t_1)+d_{V,3}^r(t_1)+d_{M,\Phi_3}(t_1)+e_{\Phi_3}$$

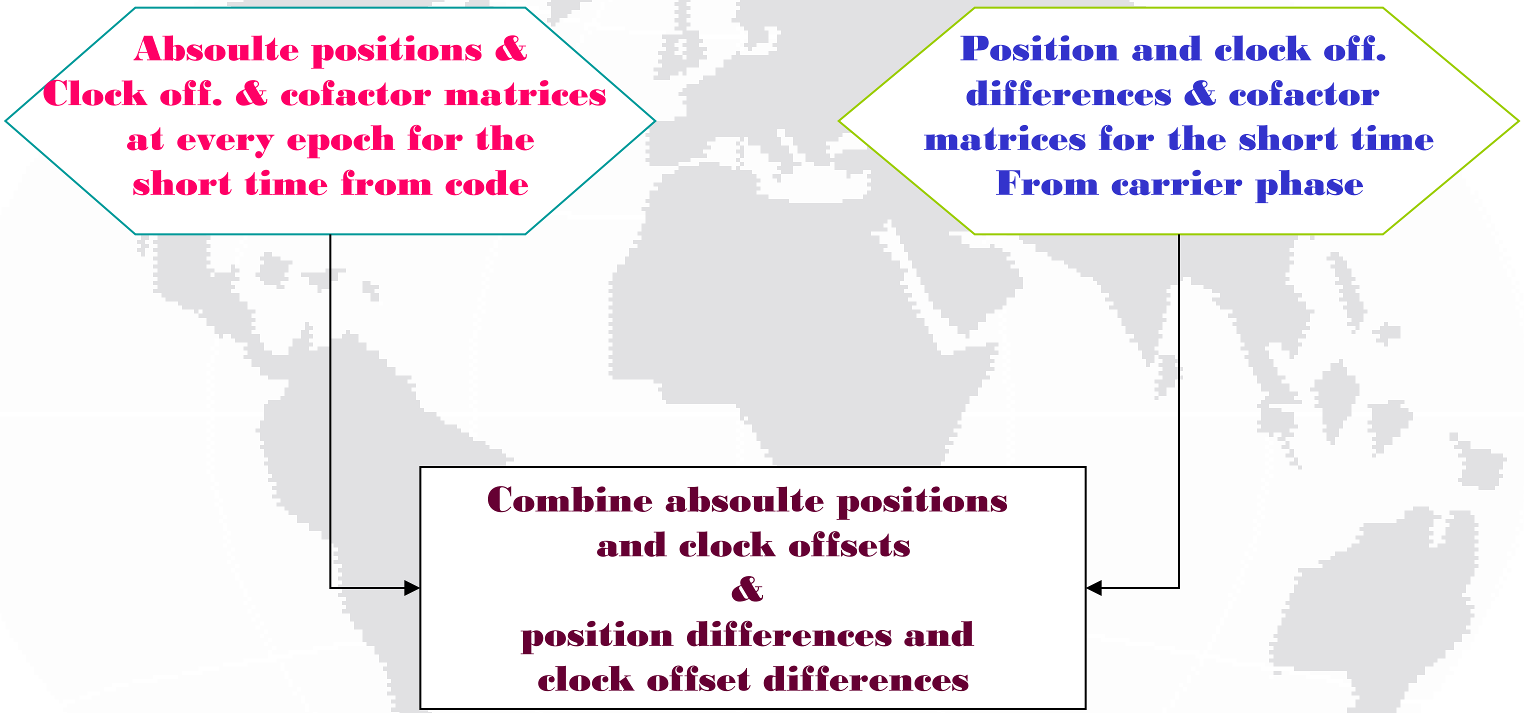
$$\Phi_{r,3}^{s}\left(t_2\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|+\lambda_3 N_{r,3}^{s}+c\left[dt^s(t_2-\varepsilon_2)-dt_r(t_2)\right]+$$
$$d_O^s(t_2)+d_R^r(t_2)-d_R^s(t_2)+d_{C,3}^r(t_2)+d_{V,3}^r(t_2)+d_{M,\Phi_3}(t_2)+e_{\Phi_3}$$

sequential time difference ionosphere-free carrier phase observation between epochs (1,2)

$$\Delta\tilde{\Phi}_{r,3}^{s}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$
$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

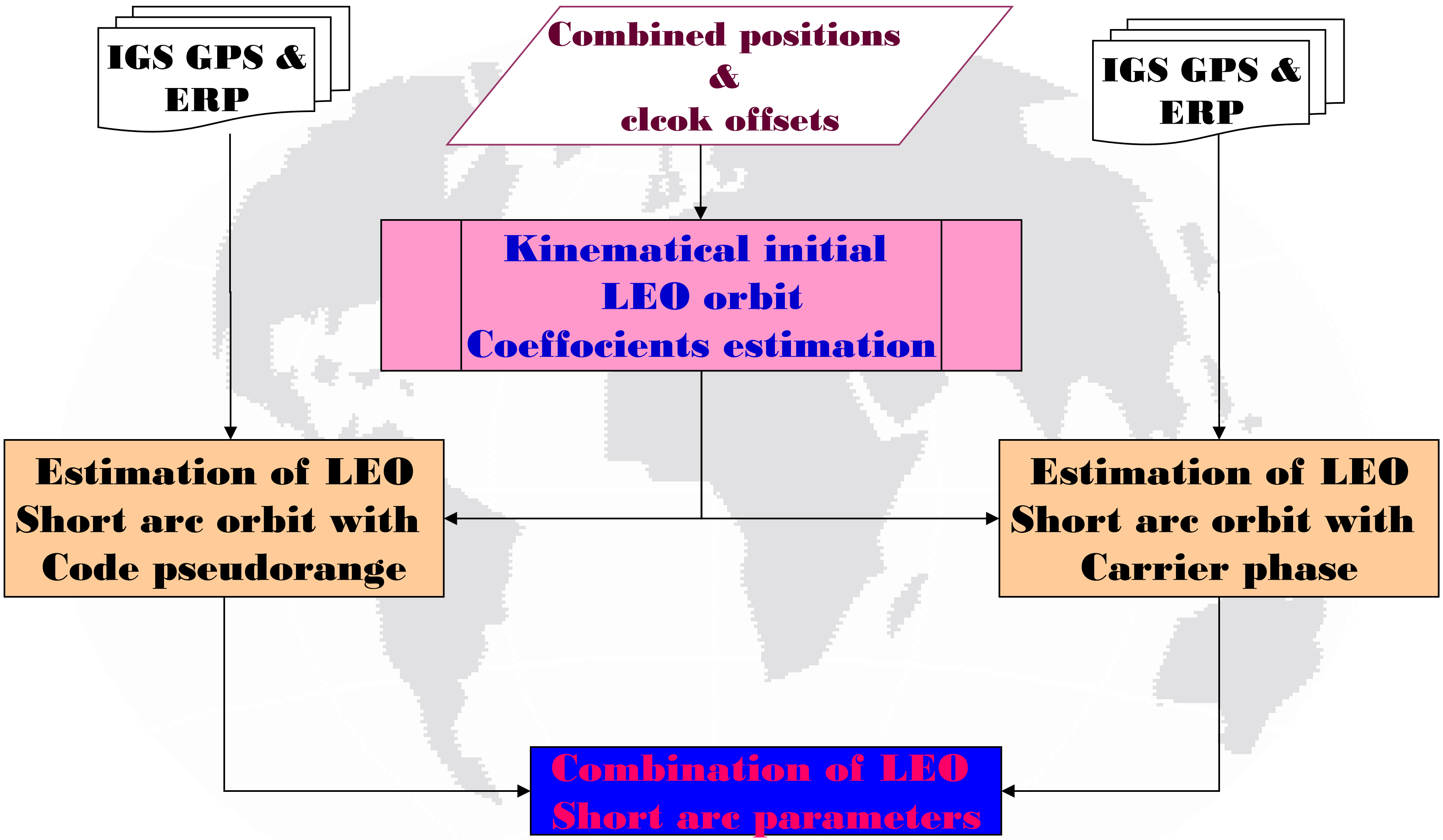
Kinematical LEO POD-Combination

- LEO position & clock offset differences can be estimated from the sequential time differenced carrier phase observations (observations without cycle slips) at every epoch.



- Final positions of LEO have been used to estimate the initial values of LEO orbit parameters (in the real case).

Kinematical Short Arc POD



Kinematical Short Arc POD-CODE

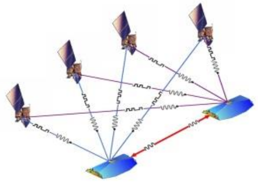
Kinematical short arc of LEO can be presented as:

$$\mathbf{r}_r(t) = \mathbf{r}_a . \frac{\sin((1-\tau).n)}{\sin(n)} + \mathbf{r}_e . \frac{\sin(\tau.n)}{\sin(n)} + \mathbf{C}.\mathbf{P}(\tau) + \sum_{f=1}^{n} \bar{\mathbf{d}}_f . \sin(\pi f \tau)$$

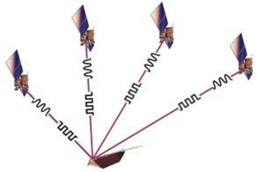
with CODE pseudorange SST observations as:

$$P_{r,i}^{s}(t) = \left| \mathbf{R}_z(\omega_e.\varepsilon_r^s)\mathbf{r}^s(t-\varepsilon_r^s) - \mathbf{r}_r(t) \right| + c\left[dt^s(t-\varepsilon_r^s) - dt_r(t) \right] + I_i^r(t) +$$

$$d_O^s(t) + d_R^r(t) - d_R^s(t) + d_{C,i}^r(t) + d_{V,i}^r(t) + d_{M,P_i}(t) + e_{P_i}$$



Kinematical POD-Real Data



CODE observation equations can be written as:

$$P_{r,i}^{s}(t) =$$

$$\left| \mathbf{R}_z(\omega_e.\varepsilon_r^s)\mathbf{r}^s(t-\varepsilon_r^s) - (\mathbf{r}_a.\frac{\sin((1-\tau).n)}{\sin(n)} + \mathbf{r}_e.\frac{\sin(\tau.n)}{\sin(n)} + \mathbf{C}_{3\times 4}\mathbf{P}(\tau) + \sum_{f=1}^{n} \bar{\mathbf{d}}_f.\sin(\pi f \tau)) \right|$$

$$+c\left[dt^s(t-\varepsilon_r^s) - dt_r(t)\right] + I_i^r(t) + d_O^s(t) + d_R^r(t) - d_R^s(t) + d_{C,i}^r(t) + d_{V,i}^r(t) + d_{M,P_i}(t) + e_{P_i}$$

unknows: LEO short arc parameters (boundary values, polynomial coefficients, amplitudes of Fourier series).

solutions: Gauss-Markov model

convergence & accuracy: after ~5 iterations, ~ meter.

Kinematical POD-Carrier Phase

sequential time differenced carrier phase observations can be written as:

$$\Delta\tilde{\Phi}^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

$$\Delta\tilde{\Phi}^{s}_{r,3}(t_1,t_2)=$$
$$\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-(\mathbf{r}_a.\frac{\sin((1-\tau_2).n)}{\sin(n)}+\mathbf{r}_e.\frac{\sin(\tau_2.n)}{\sin(n)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_2)+\sum_{f=1}^{n}\bar{\mathbf{d}}_f.\sin(\pi f\tau_2))\right|-$$
$$\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-(\mathbf{r}_a.\frac{\sin((1-\tau_1).n)}{\sin(n)}+\mathbf{r}_e.\frac{\sin(\tau_1.n)}{\sin(n)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_1)+\sum_{f=1}^{n}\bar{\mathbf{d}}_f.\sin(\pi f\tau_1))\right|-$$
$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

Kinematical POD-Carrier Phase

- unknows: boundary values, polynomial coefficients, amplitudes of Fourier series.
- solutions: Gauss-Markov model
- convergence & accuracy: after ~5 iterations, ~ cm.

- LEO short arc orbit parameters must be combined to obtain final LEO orbit coefficients.
- LEO short arc orbit can be estimated in accuracy of dm (until now).

Conclusions and remarks

- GPS SST code and carrier phase observations must be cleaned from the outliers and the cycle slips.

- because of Zero Difference (ZD) procedure, many corrections have to be considered in the GPS SST data processing.

- LEO short arcs orbit coefficients can be used directly to recover of the Earth gravity field in the global or regional form.

Thank you for your attentions