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Abstract

In this study, we used problem posing as a measure of the effect of middle-school curriculum on students' learning in high school. Students using a Standards-based curriculum in middle school performed equally well or better than students who used more traditional curricula in high school. The findings from this study not only show evidence of strengths one might expect of students who used the Standards-based reform curriculum, but also bolster the feasibility and validity of problem posing as a measure of curriculum effect on student learning. In addition, the findings of this study demonstrate the usefulness of employing a qualitative rubric to assess different characteristics of students' responses to the posing tasks. Instructional and methodological implications of this study, as well as future directions for research are discussed.

Key words: problem posing, curriculum, longitudinal study, assessment, problem solving

There is a long history of integrating mathematical problem solving into school mathematics (Stanic & Kilpatrick, 1988). In contrast, problem-posing research is relatively new (Cai & Hwang, 2002; Kilpatrick, 1987; Silver, 1994; Silver & Cai 1996). Nevertheless, there have been efforts around the world to incorporate problem posing into school mathematics at different educational levels (e.g., Brink, 1987; Chinese National Ministry of Education, 1986; Hashimoto, 1987; Healy, 1993; Keil, 1964/1967). There appears to be a high level of interest among many practitioners in making problem posing a more prominent feature of classroom instruction. This is understandable since problem posing as an intellectual activity has long been recognized as critically important in scientific investigations (Einstein & Infeld, 1938). Indeed, according to Einstein, posing an interesting problem is more important than solving the problem.

Despite this interest in integrating mathematical problem posing into classroom practice, little is known about the cognitive processes involved when students generate their own problems and therefore about the ways problem posing can be used as an assessment tool. Furthermore, little research has been done to identify instructional strategies that can effectively promote productive problem posing, or even to determine whether engaging students in problem-posing activities is an effective pedagogical strategy. The purpose of this study is to begin to address some of these questions by investigating the use of problem posing as a measure of the effect of curriculum on students' learning.

Background and Theoretical Bases

LieCal Project

This study is part of a large research project called *Longitudinal Investigation of the Effect of Curriculum on Algebra Learning* (LieCal Project). The LieCal Project was designed to examine the similarities and differences between a *Standards*-based curriculum called the Connected Mathematics Program (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) and more traditional curricula (hereafter called non-CMP curricula) by longitudinally comparing the effects of the two types of curricula on students' learning of algebra. We investigated not only the ways and circumstances under which the CMP and non-CMP curricula affected student achievement gains, but also the characteristics of these reform and traditional curricula that hindered or contributed to the gains. The Connected Mathematics Program was selected for investigation in the LieCal Project for several reasons, not the least of which is the fact that it has been more broadly implemented than any other so-called 'reform curriculum' at the middle-school level. It has been used in all 50 states and some foreign countries (Rivette, Grant, Ludema, & Rickard, 2003; Show-Me Center, 2002).

In previous years of the LieCal Project, we compared middle-school students' performance in classrooms that used CMP with the performance of students in classrooms that used non-CMP curricula. We found that on open-ended tasks, which assessed conceptual understanding and problem solving, the growth rate for CMP students over the three years was significantly greater than for non-CMP students (Cai, Wang, Moyer, & Nie, 2011). In particular, our analysis using Growth Curve Modeling showed that over the three middle-school years, the CMP students' scores on open-ended tasks increased significantly more than the non-CMP students' scores ($t = 2.79$, $p < .01$). CMP students had an average annual gain of 25.09 scale points whereas non-CMP students had an average annual gain of 19.39. At the same time, CMP and non-CMP students showed similar performance growth over the three middle-school years on multiple-choice tasks assessing computation and equation-solving skills. These findings suggested that the use of the CMP curriculum was associated with a significantly greater gain in conceptual understanding and problem solving than was the case for the non-CMP curricula. Moreover, those relatively greater conceptual gains did not come at the cost of basic skills, as evidenced by the comparable results attained by CMP and non-CMP students on computation and equation-solving tasks.

Currently, we are following the same cohort of middle-school students through their high school years in the same urban school district to investigate how the use of different types of middle-school mathematics curricula affects their learning of high school mathematics. Specifically, we are examining how students' curricular experiences in the middle grades affect their learning in high school. We do this by gathering empirical evidence about the relationship between the development of conceptual understanding, symbol manipulation skills, and problem-solving skills in middle school and the learning of mathematics in high school. One way that we have assessed this relationship is by asking students to pose problems of their own.

Students' Problem Posing and Problem Solving

If problem posing is really an important intellectual activity in school mathematics, we should be able to demonstrate that teachers and students alike are capable of posing meaningful mathematics problems. So, one important line of research in problem posing has been to explore what problems teachers and students are able to pose (Cai, 1998; Cai & Hwang, 2002; English, 1997a, 1997b; Silver & Cai, 1996; Silver, Mamona-Downs, Leung, & Kenney, 1996; Stoyanova & Ellerton, 1996). In this line of inquiry, researchers usually design a problem situation and ask subjects to pose problems that can be solved using the information given in the situation. School students, pre-service teachers, and in-service teachers have been subjects in this type of mathematical problem-posing research. In general, the findings of this research have suggested that both students and teachers are capable of posing interesting and important mathematical problems.

A second important direction for investigation has been probing the links between problem posing and problem solving (e.g., Cai, 1998; Cai & Hwang, 2002; Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996). Kilpatrick (1987) provided a theoretical argument that the quality of the problems subjects pose might serve as an index of how well they can solve problems. Several researchers have also conducted empirical studies examining potential connections between problem posing and problem solving. Ellerton (1986) compared the mathematical problems generated by eight high-ability young children with those generated by eight low-ability young children, asking each to pose a mathematical problem that would be quite difficult for her or his friends to solve. Ellerton reported that the more-able students posed problems that were more complex than those posed by less-able students.

Likewise, Silver and Cai (1996) analyzed the responses of more than 500 middle-school students to a task that asked them to pose three questions based on a driving situation. The student-posed problems were analyzed according to their type, solvability, and complexity. Additionally, Silver and Cai used eight open-ended tasks to measure the students' mathematical problem-solving performance. They found that students' problem-solving performances were highly correlated with their problem-posing performances. Compared to less successful problem solvers, good problem solvers generated more, and more complex, mathematical problems.

Although Silver and Cai (1996) measured students' problem-solving performance with tasks that were mostly unrelated to the problem-posing tasks they used in their study, others have used tasks in which the posing and solving components were related to the same mathematical and contextual structure. For example, Cai and his associates (Cai, 1998; Cai & Hwang, 2002) used problem-posing and problem-solving tasks that were closely related to one another in order to examine Chinese and US students' solving and posing performances. Cai and Hwang (2002) found differential relationships between posing and solving for US and Chinese students. That is, there was a stronger link between problem solving and problem posing for the Chinese sample, whereas the link was much weaker for the US sample. Cai and Hwang construed that the weakness of the US students' links between problem solving and problem posing did not imply a lack of generality in either their problem solving or their problem posing. Instead, the weaker link between the variety of the US students' posed problems and their problem-solving success might have been attributable to the fact that the US students almost never used abstract strategies. As a result, Cai and Hwang hypothesized that the ability to pose a variety of problem types was strongly associated with the relative abstractness of the strategy used in the Chinese sample.

Problem Posing in the Curriculum

The ultimate goal of educational research is to improve students' learning. Research on problem posing is no exception. Like problem solving, problem posing can be viewed as a classroom activity, and there has been increased interest in integrating mathematical problem posing into classrooms. Researchers have suggested that student-posed problems foster student creativity (Silver, 1997; Yuan & Sriraman, 2011) and are more likely to connect mathematics to students' own interests, something that is often not the case with traditional textbook problems. Looking across disciplines, research in reading has shown that engaging students in problem posing can lead to significant gains in reading comprehension. A meta-analysis showed effect sizes of 0.36 using standardized tests and 0.86 using researcher-developed tests (Rosenshine, Meister, & Chapman, 1996).

Indeed, there are at least two reasons why one might believe that engaging in problem-posing activities should have a positive impact on students' learning. First, problem-posing activities are usually cognitively demanding tasks (Cai & Hwang, 2002). Whether it involves generating new problems based on a given situation or reformulating an existing problem, problem posing often requires the poser to go beyond problem-solving procedures to reflect on the larger structure and goal of the task. As tasks with different cognitive demands are likely to induce different kinds of learning (Doyle, 1983), the high cognitive demand of problem-posing activities can provide intellectual contexts for students' rich mathematical development. Such activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interest and curiosity (NCTM, 1991). Second, problem-solving processes often involve the generation and solution of subsidiary problems (Polya, 1957). Thus, the ability to pose complex problems should allow for more robust problem-solving abilities (e.g., Cai & Hwang, 2002). Encouraging students to generate problems is therefore not only likely to foster student understanding of problem situations, but also to nurture the development of more advanced problem-solving strategies.

Although it is theoretically sound to engage students in problem-posing activities in an attempt to understand and improve their learning of mathematics, more empirical studies are needed to demonstrate any actual effects. The research that has been conducted on reading can serve as a model for systematically investigating the effect of mathematical exploration in general, and problem-posing activities in particular, on students' learning of mathematics. Building on an international comparative study (Cai & Hwang, 2002), Lu and Wang and their associates (Lu & Wang, 2006; Wang & Lu, 2000) launched a project on mathematical situations and problem posing. The project had two interrelated key components. The first was the systematic development of teaching materials that fostered the ability to develop problem-posing tasks based upon given mathematical situations. These teaching materials—including mathematical situations and problem-posing tasks—were not intended to replace textbooks; instead, they were used to supplement regular textbook problems. The second component of the project was the systematic implementation of the teaching materials. By 2006, more than 300 schools in 10 provinces in China had participated in the project. Teachers received training to use mathematical situations and problem-posing tasks along with their regular curriculum. Most importantly, teachers received training on how to develop mathematical situations and pose problems (Lu & Wang, 2006). Student performance was improved by engaging in the problem-posing activities (Cai & Nie, 2007; Lu & Wang, 2006).

Singer and her associates (Singer, 2009; Singer & Moscovici, 2008) have also documented the effect of systematic training that focused on problem-posing strategies.

These problem-posing strategies, including problem transposition using various representations, problem extension by adding new operations or conditions, comparison of various problems by assessing similarities and differences, and analysis of incomplete or redundant problems, improved students' ability to pose meaningful problems. It is interesting to note, though, that the few studies involving problem posing have mainly used problem posing as a measure of the effect of engaging students in problem-posing activities. There has been little work that has used problem posing to measure the broader effect of a curriculum or program on mathematics learning.

Problem Posing as a Measure of Curriculum Effect

There are many unanswered research questions in problem-posing research (Cai, 2011). One such question concerns the use of problem posing in assessment. Advocates for problem posing have typically addressed its potential benefits for student learning—namely, that experiences with mathematical problem posing can promote students' creativity and engagement in authentic mathematical activity by providing encounters with many problems, methods, and solutions rather than only one of each. However, some researchers have also found that problem-posing tasks can reveal interesting and important aspects of students' mathematical thinking (Cai, 1998; Cai & Hwang, 2002). For example, Cai and Hwang examined the problems posed by Chinese students in the context of a sequence of dot patterns. They found that the typical progression of posed problems illuminated a corresponding progression in the students' thinking when solving pattern problems: gathering data, analyzing the data for trends, and finally making predictions.

In addition, if problem-posing activities were to form a regular part of instruction in a curriculum, it would become appropriate to ask whether problem posing itself should be assessed as an object of instruction. If so, how might this be done? Even if problem posing were not the object of instruction, when problem posing was used in the classroom as a means to engage students in learning important concepts and skills and to enhance their problem-solving competence, it would seem reasonable to consider also using generative, problem-posing activities as part of assessments related to those same concepts, skills, and competencies. What would one expect to find when using problem posing as a part of such assessments?

This study addresses this question by using problem posing as a tool to investigate the effects of the CMP curriculum on long-term student learning. Because the CMP curriculum can be characterized as a problem-based curriculum, it is instructive to compare the findings from the LieCal Middle School Project (Cai et al., 2011) to findings on the effectiveness of Problem-Based Learning (PBL) on the performance of medical students (Barrows, 2000; Hmelo-Silver, 2004; Norman & Schmidt, 1992; Vernon & Blake, 1993). Researchers have found that PBL students performed better than non-PBL (e.g., lecturing) students on clinical components in which conceptual understanding and problem-solving ability were assessed. In addition, PBL and non-PBL students performed similarly on measures of factual knowledge. When these same medical students were assessed again at a later time, the PBL students not only performed better than the non-PBL students on clinical components, but also on measures of factual knowledge (Norman & Schmidt, 1992; Vernon & Blake, 1993). This result may imply that the conceptual understanding and problem-solving abilities learned in the context of PBL facilitate the retention and acquisition of factual knowledge over longer time intervals.

Analogous to these results from research on the learning of medical students using the PBL approach, CMP students outperformed non-CMP students on measures of conceptual understanding and problem solving during middle school. In addition, CMP and non-CMP students performed similarly on measures of computation and equation solving. Therefore, it is reasonable to hypothesize that the superior conceptual understanding and problem-solving abilities gained by CMP students in middle school may result in better performance on a delayed assessment of manipulation skills, such as equation solving, in addition to better performance on tasks assessing conceptual understanding and problem solving in high school. In this paper, we test this hypothesis using problem posing as a measure.

Method

Subjects

As we indicated, this study is part of a large longitudinal research project. In this study, we examined the long-term effect on students' mathematics learning of their prior use of the CMP or non-CMP curricula in middle school, using 11^{th} graders' problem posing as the measure. A total of 390 11^{th} graders were included in this study (243 former CMP and 147 former non-CMP students). These 11^{th} graders had all taken a baseline examination at the start of the LieCal Project when they were middle-school students. They were now attending high schools with a diverse student population in the same urban school district that they attended in middle school.

Tasks and Task Administration

Figure 1 shows the two tasks used in this study. Each task had a problem-solving component and a problem-posing component. Within each task, both components were situated in the same mathematical context.

(Insert Figure 1 about here)

The first task presented students with a system of two linear equations, both of which were in the standard form $Ax + By = C$. Students were asked to solve the system and to pose a real-life problem that could be solved by using the system. The students could solve the system any way they chose, as the question did not specify a method. Students were asked to show their work. The system of equations task assessed multiple facets of students' procedural and conceptual knowledge. First, it assessed their knowledge of solving systems with respect to the methods used (procedural) and with respect to what it means to be a solution to a system (conceptual). The posing portion of the problem challenged students to make sense of the system in a realistic context. The problem assessed their understanding of the variables involved in the equations. In other words, students had to understand that the variables could represent quantities whose values varied, but that only one set of values would actually work in this system because of the functional relationships defined by the equations (conceptual). Additionally, students had to understand the relationship between the variables' units in each equation (conceptual).

The second task involved a linear graph. Students were asked to find an equation for the graph and then to pose a real-life situation that could be represented by the graph. More specifically, students were asked to write an equation for the line $y = 0.5x + 2$ based on its graph in the first quadrant (where $x \geq 0$). Again, students were not asked to use a specific solution method. This task assessed multiple facets of students' conceptual and procedural knowledge. For example, it assessed students' understanding of the relationship between linear equations and their graphs (conceptual). The posing part of the problem assessed conceptual knowledge more than procedural, evaluating, for example, the students' knowledge that a line that points up and to the right is increasing by describing an increasing relationship in their word problem (conceptual). Students' real-world understanding of the meaning of the *y*-intercept was also tested by this problem (conceptual), together with the related understanding of real life situations that are proportional versus those that are linear but not proportional versus those that are not linear. Lastly, the problem evaluated students' understanding of the functional relationship between *x* and *y* (conceptual). They had to understand that *x* and *y* were quantities related in a specific way beyond the simple use of the letters *x* and *y* as placeholders. Thus, the students' understanding of variables in functions was assessed.

These two tasks were administered to students along with 10 other open-ended problem-solving tasks using a matrix sampling design. Because only a limited number of the problem-posing and problem-solving tasks could be administered within one class period, the matrix sampling design was used to have a better estimate of student ability level.

Data Coding and Inter-Rater Reliability

The problem-solving component of each task was coded as correct or incorrect. Each of the posed problems was coded along several dimensions using a qualitative rubric. Coders judged whether the student made an attempt to pose a problem and whether the posed problem was situated in a realistic context. The posed problems were also coded according to whether they correctly represented none, one, or both of the task conditions. For the system of equations task, each equation was considered one of the task conditions. For the graph task, the slope and intercept were considered the task conditions. In addition, the posed problems were coded according to whether the posed problem was valid for the given situation. A valid problem for a task refers to a posed problem that fit both conditions in the task and that could be solved using the information provided in the problem-posing situation. Finally, the situation the student created for the graph task was coded as to whether or not it reflected linearity.

Two researchers coded the data. To ensure inter-rater reliability, the two researchers independently coded a subset of the responses (12% of the responses to the system of equations task and 7% of the responses to the graph task). Inter-rater agreement was checked for each dimension of the coding. The range of agreement was between 92% and 100% for the system of equations task and between 96% and 100% for the graph task.

Results

Performance on the Solving and Posing Tasks

Based on their performance on the tasks, students appeared to find the problem-solving components challenging. Only 22% of the students were able to solve the system of equations correctly (n = 210). Similarly, only 19% of the students were able to write a correct equation describing the graph (n = 338). The problem-posing component was also challenging. Table 1 reports the problem-posing results along the different coding dimensions. It shows both the percent of students who made an attempt to pose a mathematical problem, whether valid or invalid, and the percent of students who posed valid problems. In addition, it shows the percent of students whose posed problems were situated in a context beyond the bare mathematics, whose problem situations reflected the linearity of the graph, and whose problems matched at least one of the two conditions in the task situation (the two equations in the system task and the slope and intercept in the graph task).

(Insert Table 1 about here)

In general, the graph task seemed more accessible to the students. They were more likely to pose a valid problem situation for the graph task and more likely to supply a context for their responses to the graph task. Only about one third of the students attempted to pose a problem for the system of equations task. Students were almost twice as likely to make an attempt to pose a problem for the graph task. Nevertheless, for both tasks only a small proportion of the students posed valid problems. Recall that the graph task involved both slope and *y*-intercept and the system of equations task involved two equations. As described above, a valid problem for a task thus refers to a posed problem that fit both conditions in the task and that could be solved using the information provided in the problem-posing situation. For example, one student wrote the following valid problem situation for the graph task: "The cost of renting a bike is a $2 payment plus $0.50 for each day you keep it." Another student posed the following valid problem for the system of equations task: "Jack has a number of nickels. Jenny has a number of groups of 7 pennies. Together they have 50¢ and 8 coins/coin clusters. How many does each have?"[1]

[1] In US currency, nickels and pennies are coins worth 5¢ and 1¢, respectively.

Even though about one third of the students for the system of equations task and two thirds of the students for the graph task attempted to pose problems, less than 20% of the students were able to pose problems that matched even one of the conditions. For example, the following response addressed the meaning of only one of the equations in the system of equations task: “I have *x* number of apples and *y* number of oranges. Together, they equal 8. How many apples and oranges do I have?” Similarly, the following student response reflected the meaning of the intercept of the line shown in the graph task, but only addressed the sign of the slope, not its value: “The life expectancy rate might start at 2 but over time it grows (increases).”

In addition to assessing validity and the number of conditions addressed, we were interested in students’ problem-posing skills in relation to their problem-solving skills. Therefore, we compared students’ problem-solving performance on a task with their problem-posing performance on the same task. For each of the problem-posing characteristics shown in Table 1, students who were successful problem solvers displayed, on the whole, superior problem-posing performance on the corresponding task ($p < .0001$ in all cases). For example, 24% of students who correctly solved the system of equations task also posed a valid problem corresponding to the situation. In contrast, only 1% of students who failed to solve the system of equations posed a valid problem. Similarly, 84% of students who found the correct equation for the graph task were able to embed their posed problems in a context beyond bare numbers. In contrast, only 52% of students who failed to write a correct equation could do so. These results suggest a robust relationship between problem solving and problem posing.

We also conducted separate analyses for the CMP and non-CMP groups. The relationships between problem-solving and problem-posing performance remained ($p < .0001$ in all cases for the CMP group, $p < .05$ in all cases for the non-CMP group). That is, for both CMP and non-CMP students, those who answered the solving component correctly were more likely to attempt to pose problems and to pose valid problems than those students who answered the solving component incorrectly.

Effect of Middle-school Curriculum on High School Students' Problem Posing

In order to compare the high school performance of those students who had used the CMP curriculum in middle school to that of students who had used more traditional curricula, we divided their scores from the baseline examination taken in the 6th grade into thirds. This grouping process was done according to two baseline measures, one based on open-ended tasks assessing 6th graders' conceptual understanding and problem solving, and the other based on multiple-choice tasks assessing equation-solving skills. The main purpose of this grouping was to control for their prior achievement as measured by the baseline assessment. The results of using a two-way ANOVA (3 types of groups by 2 types of curricula) showed that CMP and non-CMP students scoring in the same third on the baseline assessment had similar mean scores on the open-ended tasks [$F(1, 130) = 0.13$, $p = 0.7164$] and on equation solving [$F(1, 129) = 3.38$, $p = 0.0681$] .

We then compared the problem-posing performance of the CMP students in each third to the non-CMP students in the same third. Generally, the CMP students performed as well or better than the non-CMP students in the same third. In the case of the system of equations task, for both the open-ended and equation-solving groupings, there were no statistically significant differences (at the .05 level) between the matched thirds on any of the problem-posing characteristics. For the graph task, when grouped into thirds using the baseline open-ended scores, there were no statistically significant differences between the CMP and non-CMP groups. When grouped into thirds using the baseline equation-solving scores, the CMP students in the top third were more likely ($z = 2.01$, $p < .05$) to generate a problem situation that matched at least one of the graph conditions (slope and intercept). Similarly, the CMP students in the top third were more likely to generate a problem situation that reflected the linearity of the graph ($z = 2.40$, $p < .05$).

Discussion

The high school students in this study clearly found both the problem-solving and problem-posing components of the tasks to be challenging. However, there was a strong connection between the students' ability to solve a problem and their ability to pose valid problems within the same mathematical context. This aligns with both the theoretical argument (Kilpatrick, 1987) and previous empirical results (e.g., Cai & Hwang, 2002; Silver & Cai, 1996) that have suggested that strong problem posers are also strong problem solvers. Moreover, students whose posed problems exhibited positive characteristics, such as linearity for the graph task, and students who embedded their problem in a real-life context for either task, were similarly more likely to be strong problem solvers. Thus, the relationship between problem posing and problem solving appears to exist along dimensions beyond simply the validity of the problems posed. This echoes the findings of Cai and Hwang (2002), who noted parallels between the sequence of problems Chinese students posed and the thought process they used to solve problems.

With respect to the impact of middle-school curriculum on students' learning in high school, CMP students performed equally well on the system of equations task, but performed better than the non-CMP students on the graph task after controlling for prior achievement levels. Given the findings from research on problem-based learning in medical education, we had hypothesized that the high school results would show that the problem-based nature of the CMP curriculum and the superior performance of CMP students on conceptual measures in middle school would result in better performance in later years. However, the differences we identified were limited to the graph task and were evident only among those students who had high baseline equation-solving scores. Nevertheless, the differences that were evident reflected strengths one might expect of CMP students, given that the CMP curriculum emphasizes making sense of mathematical relationships in real-life contexts. Thus, one might expect CMP students to remain more likely to attend to the linearity of the relationship in the graph and to the implications of the slope and intercept in a real-life context. The findings from this study do, therefore, support the hypothesis, but the evidence is not as strong as we would have expected.

It may be that the overall low level of success with the system of equations task muted any potential group differences in that context. Indeed, nearly 70% of the students in both the CMP and non-CMP groups did not even attempt to pose a problem for that task, leaving little room for differentiation. It is not entirely clear why the system of equations was so daunting a context for problem posing in this study. Perhaps the need to encapsulate two linear relationships in a single problem situation was perceived as more challenging or confusing than working with the single linear relationship in the graph task. Further research into the challenges associated with posing problems of different types may help to illuminate this result.

In this study, we purposefully chose to use problem-posing tasks as part of our effort to measure the effect of the CMP curriculum. The strong relationships between posing and solving, both in the literature and in the results of this study, bolster the feasibility and validity of this choice. In addition, our use of a qualitative rubric to assess different characteristics of the students' responses to the posing tasks (such as whether the student provided an appropriate realistic context for their problem) allowed us to analyze students' thinking and the relationships between their problem solving and posing with a finer lens. In order to identify different strengths that might be associated with different curriculum approaches, it is important to look at student performance along several dimensions. The qualitative coding thus acted as a useful complement to our quantitative analysis.

It is important to note, however, that even though it may be valid and feasible to use problem-posing tasks as a measure of student learning, such tasks would likely provide more useful data when students have had more direct experiences with this type of task in the classroom. Indeed, the relatively low overall success rates in the posing tasks suggest that students are not yet well-prepared to engage with such tasks in assessments. Moreover, the limited differences we identified between the CMP and non-CMP groups may have reflected the students' limited experience with problem-posing tasks. As noted above, the large proportion of students in both groups who did not even attempt to pose a problem may have reduced the power of the problem-posing tasks to discern group differences in performance. Future research might therefore focus on ways to integrate problem posing into regular classroom activities so that students may both gain the potential learning benefits of engaging in regular problem posing and also have greater experience with posing tasks for assessment. In addition, work is needed to systematically develop problem posing as a regular tool for assessment. Our development and use of qualitative coding rubrics for the tasks in this study show that the finer-grained analyses they afford are well worth the effort.

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System of Equations Task

a. Solve the following system of equations.

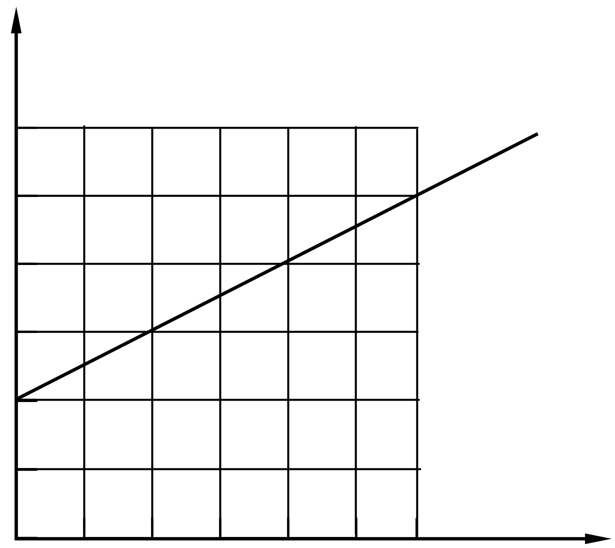
$$\begin{cases} x + y = 8 \\ 5x + 7y = 50 \end{cases}$$

Answer: x = y = Show your work.

b. Write a real life problem that could be solved using the above system of equations. Be specific.

Graph Task

Use the graph below to answer the following questions.



a. Write an equation that will produce the above graph when x is greater than or equal to zero.

b. Write a real life situation that could be represented by this graph. Be specific.

*Figure 1.*The system of equations and graph tasks. Part a of each task is the problem-solving component. Part b is the problem-posing component.

Table 1

Percent of Student Reponses to the Problem-Posing Tasks with Various Characteristics

	Attempted to pose problem	Valid Problem	Situated in a context	Reflected linearity	Matched at least one condition
System of Equations (*n* = 210)	32.4	6.2	27.6	N/A	14.3
Graph Task (*n* = 338)	63.3	16.6	58.3	26.9	18.3