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Developing Preservice Teachers' Mathematical and Pedagogical Knowledge Using an Integrated Approach

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This paper describes how an integrated mathematics content and early field-experience course provides opportunities for preservice elementary teachers to develop understanding of mathematics and mathematics teaching. Engaging preservice teachers in solving and discussing mathematical tasks and providing opportunities to implement these tasks with elementary students creates an authentic context for the future teachers to reflect on their own understanding of mathematics, mathematics teaching, and students' mathematical thinking. Essential elements of the cycle of events in the integrated model of instruction are discussed: preservice students' acquisition of mathematical concepts in the context of selected tasks in the content course; subsequent posing of mathematical tasks in early field experiences; reflection on work with students; and response to instructors' feedback.

The 2008 National Council on Teacher Quality (Greenberg & Walsh) report included five standards intended to guide reform efforts for the preparation of elementary mathematics teachers. The overarching theme of the standards was knowledge. The main recommendation for design of coursework for prospective elementary teachers focused on their "unique needs," emphasizing the ways in which they need to know and understand elementary mathematics. This unique kind of understanding is frequently described by the mathematics education community as specialized mathematical knowledge for teaching, and comprises (broadly defined) mathematical and pedagogical content knowledge (Ball & Bass, 2003; Hill, Ball, & Schilling, 2008).

Preservice teachers typically arrive at a university with a procedural understanding of elementary mathematics and a strongly held belief that procedural understanding is the core of mathematics learning. Such understanding and belief, having developed through 12 years of procedure-oriented mathematics instruction, interferes with preservice teachers' abilities to acquire subject matter knowledge in a meaningful way (Ball, 1990). Strong subject matter knowledge is an important requisite for establishing pedagogical content knowledge (Capraro, Capraro, Parker, Klum, & Raulerson, 2005). Therefore it stands to reason that preservice teachers typically fail to acquire an understanding of the pedagogical components needed to teach mathematics. For example, Crespo (2003) and Crespo and Sinclair (2008) document that preservice teachers have limited abilities to select and pose good mathematical tasks that engage students in thinking about mathematics. Nicol (1999), and Moyer and Milewicz (2002) draw attention to the fact that preservice teachers' questioning skills are inadequate to probe students' understanding and move them beyond providing an answer to a problem.

Needed pedagogical skills develop slowly over time in mathematics coursework when preservice teachers explicitly engage in analysis, discussion, and reflection on students' mathematical thinking given the support and guidance of mathematics educators. Thus, effective teacher preparation programs need to provide future teachers with compelling opportunities to acquire and strengthen both components of mathematical knowledge for teaching: mathematics content and

Completing appropriate college mathematics and methods courses does not necessarily guarantee that preservice teachers will use what they learn to inform their work with students in field experiences, student teaching, or beginning practice. Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992) provide evidence of how a student teacher who had finished a significant number of college mathematics courses struggled to explain to a sixth grade class why and how the standard algorithm for dividing fractions works. Ebby (2000) argued that unless preservice teachers learn how to make direct connections between the mathematics they learn in their content courses with what they learn about teaching mathematics in methods courses and field experiences, teacher preparation programs will remain a weak intervention. As such, programs might fail to change the beliefs and the effect of the experiences that preservice teachers have as they begin their studies.

This paper explores how integrating mathematics and pedagogy in the context of *early* field experiences (prior to student teaching) induces preservice teachers to develop knowledge of mathematics and teaching mathematics that supports student learning. The authors demonstrate how interactions between teaching and learning and between knowledge and practice provide preservice teachers with authentic opportunities to analyze students' thinking and reflect on their own teaching actions. Ebby (2000) emphasized that such opportunities are essential to help preservice teachers internalize different aspects of mathematical knowledge for teaching.

Bridging Mathematical and Pedagogical Content Knowledge in the Early Field Experience: An Integrated Model of Instruction

The sequence of integrated mathematics courses discussed here was designed using recommendations from the Conference Board of the Mathematical Sciences (CBMS, 2001) as a framework. The purpose was to provide preservice teachers with opportunities to make direct connections between mathematics courses to direct work with students in early field experiences. The integrated model provided preservice teachers with authentic opportunities to reflect on their personal knowledge and practice.

Taught jointly by faculty from the Department of Mathematics, Statistics, and Computer Science and the College of Education, the integrated sequence consisted of two courses: (1) Number Systems and Operations, and (2) Algebra and Geometry for Teachers. An early field experience was integrated into each course. Preservice teachers were provided opportunities to strengthen mathematical knowledge for teaching by implementing selected mathematical tasks with elementary students in early field experiences.

The mathematics and pedagogy embedded in each task were first discussed in content courses. Then, the preservice teachers, under the supervision of the course instructor, implemented the selected tasks with students in the early field experiences. Working directly with students provided preservice teachers opportunities to analyze elementary students' mathematical thinking, reflect on their own understanding of these same concepts, and reflect on their own teaching actions. In addition, the integrated course sequence created opportunities for course instructors to continuously assess preservice teachers' mathematical and pedagogical knowledge, allowing for individualized support and intervention. To illustrate how the integrated model supports preservice teachers' learning of mathematics and pedagogy the authors use examples from the Number Systems and Operations course, specifically the fractions unit.

The Study of Fractions

For preservice teachers and elementary students, understanding fractions is one of the more difficult topics in the elementary mathematics curriculum (e.g., Lamon, 2007; Ma, 1999; Newton, 2008). Lamon (2007) argued that difficulties with understanding and teaching the concept of fractions relate to the complexity of fraction representations. Kieren (1976) originally emphasized the complex nature of fractions by identifying four different subconstructs for interpreting the meaning of fractions: ratio, measure, operator, and quotient. Each interpretation builds on the part-to-whole relationship (Behr, Harel, Post, & Lesh, 1992). In fractions literature, the part-to-whole subconstruct is defined as a comparison of one or more equal parts of a unit to the total number of equal parts into which a unit is divided. The ratio subconstruct expresses a part-to-part comparison of two quantities where the number of units in the first quantity relates to the number of units in the second quantity. The measure subconstruct represents the notion of density on the number line, emphasizing the role of unit fractions and fostering knowledge of fractions as additive quantities. The operator subconstruct supports acquisition of multiplicative reasoning. The quotient subconstruct employs two different interpretations of fraction division: partitive (how many in each group) and quotative (how many groups). The unit on fractions included in the Number Systems and Operations course utilized the different subconstructs to assist preservice teachers in developing an understanding of fractions.

Selecting Mathematical Tasks

To provide preservice teachers opportunities to develop a complex and deep understanding of fractions and to examine their own mathematical and pedagogical knowledge, the authors selected tasks to be used by the preservice teachers and their field students. The Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2000) and descriptions of worthwhile mathematical tasks (National Council of Teachers of Mathematics, NCTM, 1991) guided task selection. These frameworks provided a filter for selecting tasks with potential to move preservice teachers and their elementary students from a procedural understanding to a conceptual understanding of fractions. Selected tasks had the potential to elicit problem solving, reasoning, communication, and making connections in order to help preservice teachers build an understanding of the subconstruct models; learn the pedagogical content knowledge related to student misconceptions about fractions; explore different materials available for teaching and learning about fractions; and practice a variety of teaching strategies. In addition, selected tasks transferred to the field experience as viable problems for elementary students to solve. Example tasks are shown in Figure 1.

Subconstruct	Task Number	Mathematical Task
Part-to-Whole	1	[1]Kayla says that the shaded part of the picture can't represent $\frac{1}{4}$ because there are 3 shaded circles and 3 is more than 1, but $\frac{1}{4}$ is supposed to be less than 1. What can you tell Kayla about fractions that might help her? 
Ratio	2	Andy and his sister Amy are making lemonade for their lemonade stand. Which of the following two mixtures will make the lemoniest lemonade? Mixing three tablespoons of lemon juice with four cups of water or mixing four tablespoons of lemon juice with five cups of water? Use as many ways as you can think of to solve this problem. Each time,

Tasks adapted from Beckmann, S. (2008). *Mathematics for elementary teachers with activities manual* (2^{nd} ed.). Boston, MA: Pearson.

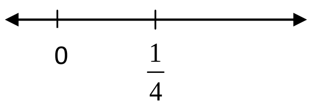
		clearly explain your thinking.
Operator	3	Demarco used $\frac{3}{4}$ cup of cheese in the pan of lasagna he made. His younger brother Anthony ate $\frac{5}{16}$ of the pan of lasagna. What fraction of a cup of cheese did Anthony consume when he ate the lasagna? Use area drawings to show how you solved the problem? Explain how your drawings helped you to solve it.
Quotient	4	Mary has $3\frac{1}{4}$ yards of fabric to make dresses for her dolls. Each dress requires $\frac{2}{3}$ of a yard of fabric. How many dresses can she make? Will she have any fabric left? How much? Use a drawing to solve the problem.
Measure	5	Locate $\frac{15}{24}$ on the number line 

Figure 1. Examples of mathematical tasks used in the fraction unit.

Data presented in the next section comes from transcriptions of videotaped preservice teachers' interactions in the content class and audiotapes of preservice teachers' interactions with field students, as well as reflective journals. These data were part of a larger project that followed 27 preservice teachers from their content class to their early field experience.

Solving Mathematical Tasks in the Content Course

To stimulate their thinking about fractions as a relationship between part-to-whole and part-to-part, the preservice teachers worked in the content class in small groups on tasks similar to Task 1 (see Figure 1). The transcription below illustrates a discussion as the preservice teachers shared their thinking, anticipating various ways elementary students might reason while solving these types of tasks.

Instructor: How would you explain that this [referring to the picture in Task1] represents one fourth? Karen?

Karen: I think there would be two ways to do it. One way was if you put it into four. If you put a box around all the four different groups of three circles and then showed it as each cluster is one part.

Instructor: Do you want a big box around all of those? [referring to the picture in Task1]

Karen: Well, around each. You can make one bar and then separate each group of three [instructor draws a vertical line between each collection of three circles to separate each group]. Then you could see that as one fourth. Then I thought another way you could do it, is you counted, I don't know if this makes it more difficult, but if you counted all of the circles and you made it to three over twelve and then you could reduce it to one fourth. But I don't know if that would be too difficult.

Instructor: Okay, those are both good ideas. Somebody want to add something to that?

Carrie: You could explain to the kids that, one little circle is not the whole, in this case, the whole is, all the circles together.

Instructor: So, what's really important here is that you define the whole. So the whole is twelve circles, right? Once I know that, then I can say that this is three out of twelve. If they say that's one third [pointing to the three shaded circles], what are they thinking about [this situation] if they think that [the picture] represents one third rather than one fourth? One group of three is shaded and, how many are not shaded? Three groups of three. So, they're really thinking, this part to this part [pointing at one group of shaded circles and three groups of unshaded circles], and actually, that's a ratio. So, fractions are part to whole, you have to know what the whole is. And the whole is twelve circles.

Gina: I was thinking of it in terms of groups of shaded and unshaded circles. It's three. Some kids think that if it's three, that's thirds, so, but I don't know? But I am … Is this reciprocal thinking?

Instructor: That's interesting, I never thought of it that way. So the reciprocal of three is one third, that's true. But, I don't think that's what kids are thinking when you ask them what fraction of the circles is shaded, and they say one third. One group of three is shaded and how many are not shaded? Three groups of three. So, they're really thinking, this part to this part [pointing at shaded and unshaded groups of circles], and actually, that's a ratio. When you do part to part, all right? So, fractions as part to whole, you have to know what the whole is. And the whole is twelve

> circles. So it is very deceiving—you can see three out of twelve or one to three.

The mathematical task provided a context for preservice teachers to consider different interpretations of fractions. They engaged in a discussion about the part-to-whole subconstruct. Karen's contributions indicated two different views of the whole: a collection of four groups of three circles, and a collection of 12 circles. Karen's first approach identified one-fourth directly, as one group of three shaded circles out of four groups of three circles. Karen's second approach focused class discussion on interpreting three-twelfths as one-fourth, indicating a different view of the whole, a collection of 12 individual circles.

The preservice teachers also considered different kinds of pedagogical content knowledge needed to implement this task with elementary students. Discussion created an opportunity to examine and reflect on possible students' interpretations and misconceptions about the meaning of fractions. For example, Carrie emphasized that a teacher needed to discuss the meaning of the whole while working with students. Gina pointed out that students might focus on the relationship between groups of shaded and unshaded circles, providing an opportunity for another discussion of students' misconceptions. In addition, during class discussions preservice teachers considered various materials to support students' thinking about fractions and various questions they might pose during the early field experience.

Posing Mathematical Tasks for Students in the Early Field Experience

Each week during the early field experience preservice teachers worked with a classroom teacher, assisting the teacher in conducting a 60-minute mathematics lesson. Then each preservice teacher worked directly with two students from the classroom, conducting a 30-minute activity session. The activity sessions provided the preservice teachers with opportunities to pose selected mathematical tasks for their students. Each session each session, preservice teachers reviewed the audiotape and reflected on their teaching actions, as illustrated by the transcript excerpt, which documents Karen's interactions with a student while she implemented Task 1.

Student: [reading the problem] Kayla says that the shaded part of the picture can't represent one fourth because there are three shaded circles and three is more than one but one fourth is supposed to be less than one. What can you tell Kayla about fractions that might help her?

Karen: So, how do you think she got that? She said the shaded part of the picture can't be one fourth because there are three shaded circles and three is more than one.

Student: So it can't be one third.

Karen: One third?

Student: All three of them are colored.

Karen: If you just looked at that picture, what does it show you?

Student: Okay, I know that it is one fourth.

Karen: Four groups? Or four, just four circles?

Student: Well it's four circles, no, four groups.

Karen: Four groups. Okay. And then, so what is that? Is that our numerator or denominator?

Student: Denominator.

Karen: Which one? What do we call the pieces each?

Student: Denominator.

Karen: Denominator, right. So, if we have four groups, that makes our whole. What's our numerator?

Student: One.

Karen: Why is it one?

Student: Because, the one that she shaded in, she shaded in one group out of four.

Karen: Okay, so that's one-fourth. How did she get three? It says that the picture can't be one fourth because there's three shaded circles.

Student: And one group is three circles and she shaded the three circles out of one group so that's how she got three.

Karen: Out of one group? So that's how she got that?

Student: Hm-hm.

Karen: But, we know that's not right? Because we see that there's four groups, right?

Student: Yeah. It's four groups, but she took one group and shaded three things out of one.

Karen: Right. So, our fraction right there is one-fourth, right?

Student: Yes.

The transcript excerpt shows that Karen guided the student probing the student's understanding of a fraction, a ratio, and the difference between the two. She failed to provide a full explanation for the difference between part-to-whole and part-to-part constructs embedded in the task. She used the terms *whole, numerator,* and *denominator* explicitly throughout her work with the student, without connecting these terms to the picture in the problem or to the symbolic notation for the fraction $\frac{1}{4}$. She failed to make connections with the picture and build the meaning of one-fourth and three-twelfths based on what the student said and thought about the fraction. It is not clear what the student knew and understood about the meaning of fractions.

Karen's reflection on her interactions with the student reveals her lack of awareness of her limited mathematical and pedagogical knowledge. Instructors provided feedback and intervention through individual conferences and follow-up discussions in the content course to address Karen's deficiencies and help her strengthen and link her mathematical and pedagogical knowledge.

> I used Kayla's problem (Task 1) to help me explain to the students that a fraction is a part-to-whole relationship not a ratio or part-part relationship. This problem called for students to take a look at the group of shaded circles in comparison to the other circles. For this, they had to understand that this one group of three was one shaded group out of four groups of three circles each, and therefore, [one-fourth]. This problem, I feel, clarified the idea of part to whole relationships. It helped me find new ways of explaining these concepts to the students. It gave me new ways of looking at normal fraction problems and gave me the confidence I needed to be able to teach these to my students. Basically it provided a framework of thought that helped me look at math in the perspective of a teacher trying to get a point across, rather than a student finding answers. Now I have both perspectives (both teacher and student) to help me find ways to better tutor my students.

Feedback focused on mathematics and pedagogy and thinking about the whole, to clarify the part-to-whole meaning of fractions, and to build on student's ideas that could possibly lead to part-to-part interpretation. Instructor feedback created the opportunity for Karen to re-examine her mathematical and pedagogical content knowledge to focus on students' learning and classroom instruction prior to returning to the field. The integrated model created for Karen a sustained cycle of learning, teaching, and reflecting on her own practice.

Conclusion and Recommendations

Each element in the cycle of events described in this paper—discussing mathematical concepts in the context of selected tasks in the content course, selecting and posing mathematical tasks in the early field experience, reflecting on work with students, and responding to instructors' feedback—engages preservice teachers in a dialogue about the teaching and learning of mathematics that contributes to the development of their mathematical knowledge for teaching. As illustrated in Figure 2, the integrated model of instruction provides a way for preservice teachers to examine the connections between mathematical and pedagogical knowledge.

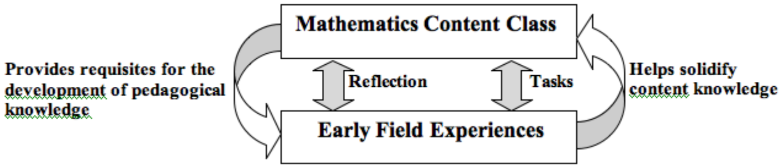


Figure 2. Developing mathematics knowledge for teaching in the context of integrated instruction.

The integrated instruction model gives preservice teachers authentic opportunities to connect their learning of mathematics with their learning about how to teach mathematics in practice.
with their learning about how to teach mathematics in practice.
The mathematical tasks serve as a bridge linking preservice

teachers' learning of mathematics and pedagogy. There are many mathematics topics that preservice teachers do not experience in this way. More work is needed to identify and to develop mathematical tasks to help preservice teachers examine mathematical concepts, elicit their thinking about how to teach these concepts, and heighten awareness of students' mathematical thinking and learning. These tasks must support the interrelated goals of strengthening preservice teachers' mathematical and pedagogical knowledge and at the same time be viable problems for elementary students to solve in the early field experience.

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