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NATURE OF MATHEMATICAL MODELING TASKS FOR SECONDARY MATHEMATICS PRESERVICE TEACHERS

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This study investigated the nature of written modeling tasks reported by instructors of required courses in five secondary mathematics teacher education programs. These tasks were analyzed based on a framework addressing potential cognitive orientation (simple procedures, complex procedures, and rich tasks) and purpose (epistemological, educational, contextual, and socio-critical modeling) of the tasks. Our analysis suggests that most tasks included questions of more than one cognitive orientation and more than half of the tasks were coded as contextual modeling. We also found that tasks that were coded as contextual modeling offered opportunities for future teachers to engage with questions at all levels of cognitive orientation. The nature of several modeling tasks, along with the ideas for refining the current frameworks, are presented for future implications of analyzing and developing modeling tasks.

Keywords: Modeling, Teacher Education-Preservice, Algebra and Algebraic Thinking

Connecting educational theory to practice is critical in supporting future secondary mathematics teachers to develop the skills and understanding necessary to enact effective mathematical modeling tasks. The *Common Core State Standards for Mathematics* included modeling as a mathematical practice and content standard; modeling is described as a full iterative process used to solve rich mathematical tasks (NGO & CCSSO, 2010). However, the meaning and purpose of mathematical modeling has been found to vary widely in both theory and practice (e.g., Anhalt & Cortez, 2015; Kaiser & Sriraman, 2006). To connect current theories to practice, we applied two frameworks with distinct perspectives to analyze the nature of modeling tasks. We collected tasks as part of a larger research project Preparing to Teach Algebra (PTA), which investigated opportunities secondary teacher education programs provide for future secondary teachers to learn about mathematical modeling. As we focus on the nature of these tasks, we answer the question, "*What is the nature (i.e., potential modeling purpose and cognitive orientation) of modeling tasks reported by secondary teacher preparation programs?"*

Theoretical Framework

In this secondary analysis of the larger *PTA* study, we use two frameworks proposed to characterize intended modeling purposes (Kaiser & Sriraman, 2006) and cognitive orientations (White & Mesa, 2014) of mathematical tasks. Kaiser and Sriraman (2006) conducted a historical analysis of research papers focused on mathematical modeling and, based on their findings, proposed five categories of purposes of mathematical modeling. Of their five categories, we focus on four we found relevant to our analysis: *epistemological, educational*, *contextual*, and *socio-critical* modeling. Kaiser and Sriraman described *epistemological modeling* as mathematical modeling with the purpose of developing mathematical theory. *Educational modeling* occurs when "real-world examples and their interrelations with mathematics become a central element for the structuring of teaching and learning mathematics" (p. 306). In this type, modeling is used explicitly as a tool for teaching and learning other mathematical content. *Contextual modeling* occurs when the purpose of modeling is to develop further understanding of modeling itself by engaging in the modeling process to solve a task embedded in a real-world context. *Socio-critical modeling* supports "critical thinking about the role

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of mathematics in society" (p. 306); with this purpose, mathematical modeling is used as a tool to critically investigate and potentially change real-world situations that are relevant to students.

White and Mesa (2014) proposed a framework for differentiating potential cognitive orientations of mathematical tasks. The framework includes three main categories: *simple procedures, complex procedures,* and *rich tasks*. *Simple procedures* are defined as those tasks requiring students to draw on factual or procedural knowledge; they must "remember factual information" or "recall and apply procedures" (p. 14). Students are told which fact or procedures to use, and they must remember them and apply them in the task. *Complex procedures* include tasks requiring students to draw on procedural and conceptual knowledge: to "recognize and apply procedures" (p. 14). In these tasks, students are not told explicitly which procedures to use, but instead are expected to draw on their understanding to choose an appropriate procedure and apply it in the task. Finally, *rich tasks* include any tasks that prompt students to write explanations of procedures, to interpret, compare, or make inferences, or to analyze, evaluate, or create situations or structures. Rich tasks involve high-level mathematical thinking and offer students more opportunities to make their own decisions when solving tasks.

Method

As a part of the larger *PTA* study, we focused on the potential cognitive orientations and purposes of modeling for nine written tasks involving mathematical modeling in instructional materials collected from five universities. Because almost all tasks included multiple subquestions in which each of them varied in terms of richness, we defined our coding unit as a subquestion rather than a task. We coded subquestions in terms of potential cognitive orientation (i.e., simple, complex, rich) and purpose of modeling (i.e., epistemological, educational, contextual, socio-critical). Two researchers each coded the questions independently, and resolved all discrepancies. When we coded a question, we also considered questions prior to the one we were coding. For instance, a question was coded as rich the first time it appeared because it prompted students to analyze. But if similar questions follow in later sections, those later questions might not present new challenges to students. Answering similar questions lowers the cognitive orientation by becoming a routine procedure, thus, we coded such a question as either simple or complex rather than rich.

Results

Cognitive Orientation of Mathematical Tasks

Table 1 presents an analysis of task purposes (first column) and cognitive orientations (remaining columns). If a task is designated in one of the seven categories (e.g., R, SC) in Table 1, it means that its subquestions were coded by the cognitive orientation(s) in that category. For example, the Traffic Flow task is under the category of "Simple & Rich (SR)". This means that all questions in this task were coded as either simple or rich, with at least one in each category.

Overall, three out of nine tasks included subquestions that fell into only one cognitive orientation. For example, all questions from the task Bezier's Curve were coded as simple. One question asks to find $x(0)$ in terms of the constants a_0, a_1 and a_2 if $x(t) = a_0 + a_1t + a_2t^2$. Even though the overall task presented an interesting mathematical problem, each smaller question required using simple procedures (e.g., substituting 0 for t). Only one task (i.e., Traffic Flow) included questions that fell into exactly two cognitive orientations. Most tasks included a variety of questions, with at least one in each of the three cognitive orientations. For instance, the Wooody's Film Frame task focused on using computer animators to tell stories about changes in Wooody's position using linear transformations. One question that was coded as *simple* asked to compute the product of two matrices. Another question stated, "Wooody discovered a pensieve at (1, -2). What is the linear transformation here?" Though this question required students to analyze, students were asked similar questions immediately before, but because the question does not specify a particular procedure, we coded it as complex. At the end of the task, a question asked students to write an ending to the story of Wooody's moving and to describe how to illustrate it. This question involved creating and analyzing a new situation and was coded as rich. We found that most tasks included questions from more than one cognitive orientation and thus provided students with multiple cognitive orientations in modeling tasks.

Table 1: Task Richness According to Questions: Simple (S), Rich (R), Complex (C)

	S	C	R	SC	SR	CR	SCR
Educational Modeling							
Traffic Flow						X	
Heat Transfer		X					
Bezier's Curve	X						
Contextual Modeling							
Wooody's Film Frames							X
Google Page Rank Algorithm							X
Movie Money Making							X
Ferris Wheel Problem							X
Egg Launch Problem							X
Epistemological Modeling							
Quadratic and Its Secondary Difference			X				
Socio-critical Modeling							
(none)							

Modeling Purpose of Mathematical Tasks

Overall, we coded three tasks as educational modeling, five as contextual modeling, and one as epistemological modeling. We saw no socio-critical modeling tasks. Although we were open to coding individual questions within a task with multiple purposes (similar to our cognitive orientation analysis), we did not find any with multiple purposes. Subquestions presented in the Traffic Flow task, which we coded as educational modeling, provided detailed guideline for preservice teachers (PSTs) to solve the problem that embedded specific concepts in linear algebra (e.g., write the augmented matrix). The instructor's main purpose seemed to be for PSTs to practice linear algebra skills rather than building their own models. The Google Page Rank Algorithm task, on the other hand, involved a realistic context and provided PSTs an opportunity to explore different structures of web networks; thus we coded it as contextual modeling. Specifically, this task provided opportunities for PSTs to investigate how Google uses a stochastic matrix along with a Markov chain and its steady-state vector to determine the PageRank of each page on the website. The Quadratic and Its Secondary Difference task was coded as epistemological modeling because of the potential for PSTs to generate relationships between quadratics and second differences through modeling. We found that more than half of the tasks addressed realistic problems and sometimes instructors used modeling mainly to practice the newly learned concepts. Another intriguing point was that tasks coded as Contextual Modeling included subquestions of all types of cognitive orientations.

Discussion and Conclusion

In this study, we described the use of two theoretical frameworks addressing purposes and cognitive orientations of tasks to support the analysis and potential construction of effective modeling tasks. We noticed that tasks' subquestions coded as "rich" can entail varying degrees of richness. We saw three ways to structure a potential rich task in our data. One way was to give an open question (e.g., in the Egg Launch task, one question asks students which of the three teams will win the contest and to explain why) without providing subquestions for scaffolding. The second way is to give a series of questions with each question building on the previous ones. The open question appeared at the end of the task. We argue that even though both tasks included open questions, the first type was richer than the second because the first type required PSTs to create their own process to reach the final goal. The third way is a combination of the first two cases: starting with an open question, providing a series of questions, and restating the same open question at the end of the task. The benefit of the last type is to provide students with differentiated instruction because the instructor can provide the opportunity for students to either explore the task or follow the scaffolding questions depending on students' academic needs.

White and Mesa (2014) argued that the "rich task" category included the subcategories: Understanding, Applying Understanding, Analyzing, Evaluating, and Creating. They mapped these five categories across four types of knowledge: factual, procedural, conceptual, and meta-cognitive. In our analysis, we found that tasks in which students were asked to *create* a mathematical object or process were much richer than tasks where students were asked to simply *explain* a result or process. As we see different levels of cognitive demand within the "rich task" category, we suggest that instructors pay attention to these differences and present various opportunities for students to experience different types of rich tasks.

In terms of our analysis on purposes of modeling, we found no tasks to be socio-critical in nature and would recommend that PSTs are given opportunities to design and modify such tasks. PSTs need to encounter thinking processes, such as those described by Cirillo, Bartell, and Wager (2016) when they converted a modeling task "Dairy Queen" into a task involving social justice. PSTs can also be benefited by posing modeling problems as they lean about characteristics of modeling problems that address social justice issues (I, Jung, & Son, 2017). Opportunities for PSTs to learn about modeling can be enhanced when instructors consider different modeling purposes and cognitive orientation of tasks described in our study.

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