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Sufficient Conditions Used in Admittance Selection for Force-guided Assembly of Polygonal Parts

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Sufficient Conditions Used in Admittance Selection for Force-Guided Assembly of Polygonal Parts

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Abstract:

Admittance control approaches show significant promise in providing reliable force-guided assembly. An important issue in the development of these approaches is the specification of an appropriate admittance control law. This paper identifies procedures for selecting the appropriate admittance to achieve reliable force-guided assembly of planar polyhedral parts for single-point contact cases. A set of conditions that are imposed on the admittance matrix is presented. These conditions ensure that the motion that results from contact reduces part misalignment. We show that, for bounded misalignments, if an admittance satisfies the

misalignment-reduction conditions at a finite number of contact configurations, then the admittance will also satisfy the conditions at all intermediate configurations.

SECTION I. Introduction

Admittance control has been used in assembly tasks to provide force regulation and force guidance. In robotic assembly tasks, the admittance maps contact forces into changes in the velocity of the body held by the manipulator. To achieve reliable assembly, the manipulator admittance must be appropriate for the particular assembly task. Here we identify procedures used to select the appropriate manipulator admittance for planar assembly.

We consider a simple form of admittance, a linear admittance control law [1]. For planar applications, this admittance behavior has the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{A}\mathbf{w}$$

(1)

where $\mathbf{v}_0$ is the nominal twist (a three-vector for planar cases), $\mathbf{w}$ is the contact wrench (force and torque) measured in the body frame (a three-vector), $\mathbf{A}$ is the admittance matrix (a 3 × 3 matrix), and $\mathbf{v}$ is the motion of the body.

Many researchers have addressed the use of admittance for force guidance. Whitney [2], [3] proposed that the compliance of a manipulator be structured so that contact forces lead to decreasing errors. Peshkin [4] addressed the synthesis of an accommodation (inverse damping) matrix by specifying the desired force/motion relation at a sampled set of positional errors for a planar assembly task. An unconstrained optimization was then used to obtain an accommodation matrix that does not necessarily provide force guidance. Asada [5] used a similar unconstrained optimization procedure for the design of an accommodation neural network rather than an accommodation matrix. Others [6], [7] provided synthesis procedures based on spatial intuitive reasoning. None of these approaches, however, ensures that the admittance selected will, in fact, be reliable.

A reliable admittance selection approach is to design the control law so that, at each possible part misalignment, the contact force always leads to a motion that reduces the existing misalignment. The approach is referred to as force assembly, and has been successful for workpart into fixture insertion when errors are infinitesimal [1], [8], [9].

For force assembly, the motion resulting from contact must instantaneously reduce misalignment. Since the configuration space of a rigid body is non-Euclidian, there is no “natural” metric for finite spatial error. In [10], several body-specific metrics are established. One metric is based on the Euclidean distance between a single point on the body and its location when properly positioned. The specific point on the body corresponds to the location having the maximum distance from its properly mated position. This point on the body is configuration dependent.

In this paper, we consider a measure of error based on the Euclidean distance between an arbitrarily chosen single (fixed) point on the held body and its location when properly positioned. Because the selection of the reference location is arbitrary, one configuration-dependent location (point of maximum distance) can be selected to use the established metric or more than one location can be selected to further restrict the description of what constitutes error reduction in rigid body assembly.

The misalignment reduction condition of force assembly requires that, at each possible misalignment, the contact force yields a motion that reduces the misalignment. Using the point-based measure of misalignment discussed above, this condition can be expressed mathematically, if we let $\mathbf{d}$ (a three-vector for planar motion) be the line vector from the selected point at its properly mated position to its current position. Then, for error-reducing motion, the condition is

$$\mathbf{d}^T\mathbf{v} = \mathbf{d}^T(\mathbf{v}_0 + \mathbf{A}\mathbf{w}) < 0$$

(2)

which must be satisfied for all possible misalignments.

Because the line vector $\mathbf{d}$ depends on the rigid body configuration and because the number of configurations is infinite, it is impossible to impose the error-reduction condition for all misalignments. In application, however, the misalignments of the rigid body are bounded by the extremes within a contact state, or the accuracy of the robotic manipulator. Those misalignments on the “boundary” are of particular interest.

Here, we show that, by identifying an admittance matrix that satisfies the error-reduction conditions at a *finite* number of configurations on the boundary, the error-reduction requirements are also satisfied for *all* configurations within the bounded area.

This paper considers polygonal rigid body assembly involving planar motion constrained by frictionless, single-point contact. Polygonal planar bodies in single-point contact have two types of stable contact states. One is referred to as “edge-vertex” contact; the other is referred to as “vertex-edge” contact. In “edge-vertex” contact, one edge of the held body is in contact with one vertex of the mating fixtured part [Fig. 1(a)]. In “vertex-edge” contact, one vertex of the held body is in contact with one edge of its mating part [Fig. 1(b)].

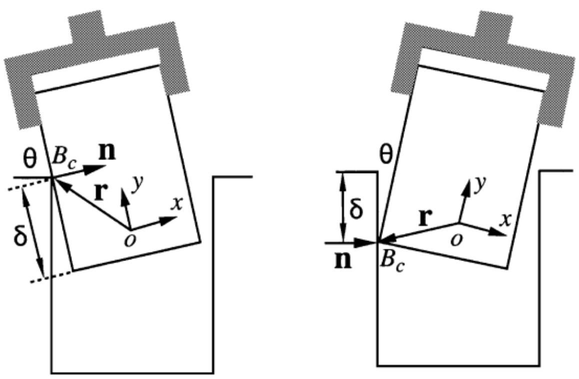

Fig. 1. Planar single-point contact. (a) Edge-vertex contact state. (b) Vertexedge contact state.

In this paper, means of calculating the motion of a constrained body and an error-reduction function are derived in Section II. Sufficient conditions for error reduction for edge-vertex and vertex-edge contact states are derived in Section III and Section IV, respectively. These conditions show that an admittance matrix satisfying the error-reduction conditions at the boundaries of a set of contact configurations, also satisfies the error-reduction conditions at all intermediate configurations. A brief discussion and a summary are presented in Section V.

SECTION II. Error-Reducing Motion of a Constrained Rigid Body

Consider a planar rigid body interacting with a surface as shown in Fig. 1. Let n (unit two-vector) be the surface normal (pointing toward the held body) at the contact point. The unit wrench associated with the normal force has the form

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{n} \\ (\mathbf{r}\times\mathbf{n})\cdot\mathbf{k} \end{bmatrix}$$

(3)

where $\mathbf{r}$ is the position vector from the origin of the coordinate frame to the point of contact, B_c, and $\mathbf{k}$ is the unit vector orthogonal to the plane.

Let ϕ be the magnitude of the normal contact force. The contact wrench is

$$\mathbf{w} = \mathbf{w}_n\phi.$$

(4)

By the control law (1), the motion of the body is

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{A}\mathbf{w}_n\phi.$$

(5)

Because the motion of the rigid body cannot penetrate the surface, the reciprocal condition [11]must be satisfied

$$\mathbf{w}_n^T\mathbf{v} = \mathbf{w}_n^T\mathbf{v}_0 + \mathbf{w}_n^T\mathbf{A}\mathbf{w}_n\phi = 0.$$

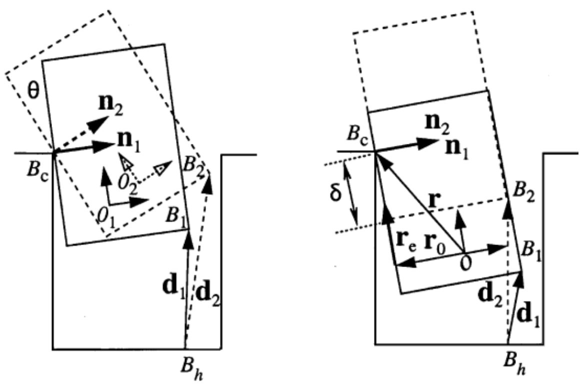

Fig. 2. Edge-vertex contact state. (a) Orientational variation. (b) Translational variation.

The magnitude ϕ is determined from

$$\phi = \frac{-\mathbf{v}_0^T\mathbf{w}_n}{\mathbf{w}_n^T\mathbf{A}\mathbf{w}_n}.$$

(6)

Substituting (6) into (5) yields

$$\mathbf{v} = \frac{(\mathbf{v}_0\mathbf{w}_n^T - \mathbf{v}_0^T\mathbf{w}_n\mathbf{I})\mathbf{A}\mathbf{w}_n}{\mathbf{w}_n^T\mathbf{A}\mathbf{w}_n}.$$

(7)

If the compliant motion is error reducing, condition (2) must be satisfied for a given point. Thus

$$E=\frac{\mathbf{d}^T(\mathbf{v}_0\mathbf{w}_n^T-\mathbf{v}_0^T\mathbf{w}_n\mathbf{I})\mathbf{A}\mathbf{w}_n}{\mathbf{w}_n^T\mathbf{A}\mathbf{w}_n}<0$$

(8)

where $\mathbf{A}$, $\mathbf{d}$, and $\mathbf{w}$ are expressed in a body frame.

Since $\mathbf{A}$ is positive definite, $\mathbf{w}_n^T\mathbf{A}\mathbf{w}_n>0$, the denominator is positive. Therefore, the error-reduction function can be expressed as

$$F_{\rm er}=\mathbf{d}^T(\mathbf{v}_0\mathbf{w}_n^T-\mathbf{v}_0^T\mathbf{w}_n\mathbf{I})\mathbf{A}\mathbf{w}_n.$$

(9)

For error-reducing motion, $F_{\rm er}$ must be negative for all contact configurations considered.

Since the contact wrench $\mathbf{w}_n$ depends on the configuration of the body, the error-reduction function in (9) is a function of configuration. As shown in Fig. 1, for both contact states, the configuration of the body can be described by two variables (δ,θ). Thus, the function $F_{\rm er}$ can be expressed as a function of (δ,θ).

In each contact case, the range for each of the variables can be transformed to be centered about zero, e.g., $[\delta_{min},\delta_{max}]\Rightarrow[-\delta_M,\delta_M]$ and $[\theta_{min},\theta_{max}]\Rightarrow[-\theta_M,\theta_M]$ to facilitate subsequent analysis.

In the following two sections, the variables θ and δ considered are within the ranges of $[-\theta_M,\theta_M]$ and $[-\delta_M,\delta_M]$, respectively. Error-reduction conditions are obtained for the two single-point contact states illustrated in Fig. 1.

SECTION III. Edge-Vertex Contact State

Consider edge-vertex contact. We prove that, if an admittance matrix $\mathbf{A}$ satisfies a set of conditions at the "boundary" points, then the A matrix ensures error-reducing motion for all intermediate configurations $\theta\in[-\theta_M,\theta_M]$.

A. Error-Reduction Function

In order to obtain the error-reduction function, we first express the contact wrench and the error-measure vector $\mathbf{d}$ in terms of δ and θ.

For an edge-vertex contact state as shown in Fig. 2(a), when the held body rotates relative to the fixtured body about the contact point, the description of the contact wrench does not change in a body-based coordinate frame. When the held body translates relative to the fixtured body as shown in Fig. 2(b), the description of the contact wrench changes in a body-based coordinate frame as the contact point changes (although its direction is constant). Thus, the contact wrench is a function involving only the translational variable δ.

For all edge-vertex cases, the direction of the surface normal is constant in the body frame while the position vector of the contact point, $\mathbf{r}$, varies. For an arbitrary δ, $\mathbf{r}$ can be expressed as

$$\mathbf{r}_\delta=\mathbf{r}_0+\mathbf{r}_e\delta$$

(10)

where $\mathbf{r}_0$ is a vector from the body frame to a center point of the edge (constant) and $\mathbf{r}_e$ is the unit vector along the edge. By (3), the unit wrench corresponding to the surface normal is

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{n} \\ (\mathbf{r}_\delta \times \mathbf{n}) \cdot \mathbf{k} \end{bmatrix}.$$

(11)

It can be seen that in the body frame, the direction of $\mathbf{w}_n$ is constant, while the last component (the moment term) is a linear function of δ.

Let $\mathbf{d}_0'$ be the error-measure two-vector at $(\theta, \delta) = (0{,}0)$, then for an arbitrary δ with θ=0, the error-measure vector $\mathbf{d}'$ is

$$\mathbf{d}_\delta' = \mathbf{d}_0' + \mathbf{r}_e \delta, \delta \in [-\delta_M, \delta_M]$$

(12)

where $\mathbf{r}_e$ is a unit vector along the contacting edge. Note that $\mathbf{d}_0'$ is constant in the global coordinate frame, while $\mathbf{r}_e$ is constant in a body frame. Thus, for an arbitrary orientation $\theta \in [-\theta_M, \theta_M]$ and $\delta \in [-\delta_M, \delta_M]$, the error-measure two-vector $\mathbf{d}'$ is a function of δ and θ having the form

$$\mathbf{d}'(\delta, \theta) = \mathbf{R}\mathbf{d}_0' + \mathbf{r}_e \delta$$

(13)

where $\mathbf{R}$ is the rotation matrix having the form

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

(14)

The line vector associated with $\mathbf{d}'(\delta, \theta)$ can be calculated

$$\mathbf{d}(\delta, \theta) = \begin{bmatrix} \mathbf{R}\mathbf{d}_0' \\ (\mathbf{r}_B \times \mathbf{R}\mathbf{d}_0') \cdot \mathbf{k} \end{bmatrix} + \delta \begin{bmatrix} \mathbf{r}_e \\ (\mathbf{r}_B \times \mathbf{r}_e) \cdot \mathbf{k} \end{bmatrix}$$

(15)

where $\mathbf{r}_B$ is the position vector from the body frame origin O to the error measure point B (constant in body frame).

Thus, for any intermediate configuration (δ, θ), because $\mathbf{w}_n$ in (11) only contains first-order terms in δ and $\mathbf{d}(\delta, \theta)$ in (15) only contains first-order terms in $\sin\theta$, $\cos\theta$, and δ, the error-reduction function (9) can be expressed as a third-order polynomial in δ in the form

$$F_{\mathrm{er}}(\delta, \theta) = f_3\delta^3 + f_2\delta^2 + f_1\delta + f_0$$

(16)

where the coefficients f_i's have the form

$$f_i = a_i \cos\theta + b_i \sin\theta$$

(17)

where a_i and b_i are functions of the admittance $\mathbf{A}$.

B. Sufficient Conditions for Error Reduction

The error-reduction condition requires that the error-reduction function in (16) must be negative in the range of configurations considered. In order to obtain sufficient conditions, we construct two functions, F_0 and F_M, by replacing the $\cos\theta$ terms in (16) with 1 and $\cos\theta_M$, respectively

$$\begin{aligned} F_0(\delta,\theta) = \quad & (a_3\delta^3 + a_2\delta^2 + a_1\delta + a_0) \\ & +(b_3\delta^3 + b_2\delta^2 + b_1\delta + b_0)\sin\theta \\ F_M(\delta,\theta) = \quad & (a_3\delta^3 + a_2\delta^2 + a_1\delta + a_0)\cos\theta_M \\ & +(b_3\delta^3 + b_2\delta^2 + b_1\delta + b_0)\sin\theta\,. \end{aligned}$$

(18)(19)

For small θ (e.g., $\theta \le (\pi/8)$), F_0 and F_M are close approximations of $F_{\rm er}$, and for any (δ,θ) in the range considered

$$\min\{F_0, F_M\} \le F_{\rm er} \le \max\{F_0, F_M\}\,.$$

(20)

Thus, if both F_0 and F_M are negative over the range $\delta \in [-\delta_M, \delta_M]$ and $\theta \in [-\theta_M, \theta_M]$, error-reducing motion is ensured over this range.

For a given θ, both F_0 and F_M are third-order polynomials in δ. To obtain conditions on F_0 and F_M, we first evaluate the bounds on the coefficients of these two polynomials.

By (18) and (19), the coefficients of δ^i in F_0 and F_M are

$$\begin{aligned} f_i^0(\theta) = \quad & a_i + b_i\sin\theta \\ f_i^M(\theta) = \quad & a_i\cos\theta_M + b_i\sin\theta\,. \end{aligned}$$

(21)(22)

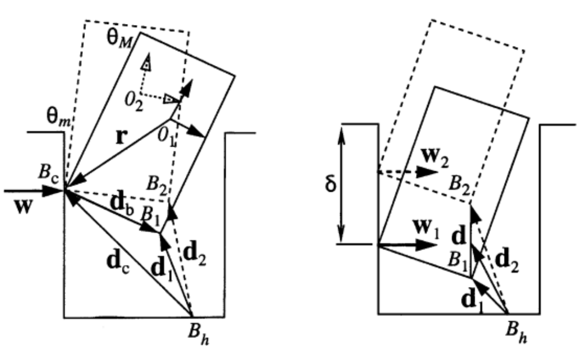

Fig. 3. Vertex-edge contact state. (a) Orientational variation. (b) Translational variation.

In the range of $|\theta| \leq (\pi/8)$, both $f_i^0(\theta)$ and $f_i^M(\theta)$ are monotonic. Thus, the maximum (minimum) values of f_i^0 and $f_i^M(\theta)$ are determined from their values at the two boundary points: $\theta = \pm\theta_M$. Denote

$$\begin{aligned} s_M = &\quad \max\{|f_i^0(\pm\theta_M)|, |f_i^M(\pm\theta_M)|, i = 1,2,3\} \\ s_0 = &\quad \min\{|f_0^0(\pm\theta_M)|, |f_0^M(\pm\theta_M)|\}. \end{aligned}$$

(23)(24)

We prove that if

$$\frac{s_0}{s_M + s_0} > \delta_M$$

(25)

then both F_0 and F_M have no root for all $\delta \in [-\delta_M, \delta_M]$, $\theta \in [-\theta_M, \theta_M]$.

To prove this, consider the function F_0 in (18). For an arbitrary $\theta_0 \in [-\theta_M, \theta_M]$, F_0 is a third-order polynomial in a single-variable δ

$$\begin{aligned} F_0(\delta, \theta_0) = &\quad c_3\delta^3 + c_2\delta^2 + c_1\delta + c_0 \\ \text{where} & \\ c_i = &\quad a_i + b_i \sin\theta_0 . \\ \text{Let} & \\ c_M = &\quad \max\{|c_1|, |c_2|, |c_3|\} \end{aligned}$$

(26)(27)(28)

then, as shown in the Appendix, each root of F_0, ξ, must satisfy

$$|\xi| \geq \frac{|c_0|}{c_M + |c_0|}.$$

(29)

Since $\theta_0 \in [-\theta_M, \theta_M]$, by (23) and (24), we have

$$\begin{aligned} c_M \leq &\quad s_M, |c_0| \geq s_0. \\ \text{Therefore} & \\ \frac{s_M}{s_0} \geq &\quad \frac{c_M}{c_0} \end{aligned}$$

(30)(31)

which leads to

$$|\xi| \geq \frac{|c_0|}{c_M + |c_0|} \geq \frac{s_0}{s_M + s_0} > \delta_M.$$

(32)

Thus, F_0 has no root in $[-\delta_M, \delta_M]$ for all $\theta \in [-\theta_M, \theta_M]$. The same reasoning applies to F_M. Therefore, the functions F_0 and F_M do not change sign if inequality (25) is satisfied. By (20), $F_{\rm er}$ has no root in the same bounded area. Since s_M in (23) and s_0 in (24) are functions of the admittance $\mathbf{A}$, (25)imposes a constraint on $\mathbf{A}$. In summary, we have the following proposition.

Proposition 1

For an edge-vertex contact state, if at the configuration $(\delta, \theta) = (0,0)$, the admittance satisfies the error-reduction condition (2), and (25) is satisfied for the configuration boundary points $[\pm\delta_M, \pm\theta_M]$, then the admittance will satisfy the error-reduction conditions for all configurations bounded by these four configurations.

Thus, to ensure that contact yields error-reducing motion for the body for an edge-vertex contact state, only two conditions [(2) and (25)] need be satisfied.

SECTION IV. Vertex-Edge Contact State

In this section, vertex-edge contact is considered. As shown in Fig. 3, the configuration of the body can be determined by the orientation of the body θ and the location of the contact point δ.

Suppose that θ varies within the range of $[-\theta_M, \theta_M]$, and δ varies within the range of $[-\delta_M, \delta_M]$. We prove that, if an admittance matrix $\mathbf{A}$ satisfies a set of conditions determined at the “boundary” configurations, then the same admittance will ensure that the motion is error reducing for any intermediate configuration $\theta \in [-\theta_M, \theta_M]$, $\delta \in [-\delta_M, \delta_M]$.

To prove the results, we first consider configuration variations in orientation and translation separately. Then, by combining the two cases, general results are obtained.

A. Configuration Variation in Orientation

Consider only orientational variation of the contact configuration as illustrated in Fig. 3(a). In this case, both the direction of the error-reduction vector $\mathbf{d}$ and the direction of the contact force are changed by changing the orientation. We prove that, for $\theta_M \le (\pi/4)$, if A satisfies a set of conditions at $\theta = 0$, then an error-reducing motion is ensured for all configurations $\theta \in [-\theta_M, \theta_M]$.

1. Error-Reduction Function

Let $\mathbf{w}_{n0}$ be the wrench, and $\mathbf{d}_0$ be the position vector associated with $\theta = 0$. Suppose at $\theta = 0$, an error-reducing motion is obtained, i.e.,

$$\mathbf{d}_0{}^T \mathbf{v}_0 + \mathbf{d}_0{}^T \mathbf{A}\mathbf{w}_{n0} < 0.$$

(33)

Consider a rotation given by an angle change $\theta \in [-\theta_M, \theta_M]$. If we denote $\mathbf{n}_0$ as the surface normal associated with $\theta = 0$, then in the body coordination frame, the surface normal associated with θ is

$$\mathbf{n}_\theta = \mathbf{R}(\theta)\mathbf{n}_0$$

(34)

where $\mathbf{R}$ is the rotation matrix having the form of (14).

Since contact is frictionless, the contact force is along the surface normal at the contact point. Thus, the unit contact wrench is

$$\mathbf{w}_n(\theta) = \begin{bmatrix} \mathbf{n}_\theta \\ (\mathbf{r} \times \mathbf{n}_\theta) \cdot \mathbf{k} \end{bmatrix} = \begin{bmatrix} \mathbf{R}\mathbf{n}_0 \\ (\mathbf{r} \times \mathbf{R}\mathbf{n}_0) \cdot \mathbf{k} \end{bmatrix}$$

(35)

where $\mathbf{r}$ is the position vector from the origin of the body frame to the contact point (constant), and $\mathbf{k}$ is the unit vector in the direction of the z axis.

Since the two configurations correspond to pure rotation about the contact point, the error-measure two-vector $\mathbf{d}'$ for an intermediate configuration can be expressed in the body frame as

$$\mathbf{d}'_\theta = \mathbf{R}\mathbf{d}'_c + \mathbf{d}'_b$$

(36)

where $\mathbf{d}'_c$ is the position two-vector from B_h to the contact point B_c, and $\mathbf{d}'_b$ is the position two-vector from B_c to point B_1. Note that $\mathbf{d}'_c$ is a constant in the global frame, and $\mathbf{d}'_b$ is constant in the body frame. Then, in the body frame, the line vector associated with $\mathbf{d}'$ is obtained

$$\mathbf{d}_\theta = \begin{bmatrix} \mathbf{d}'_\theta \\ \left(\mathbf{r}_B \times \mathbf{d}'_\theta\right) \cdot \mathbf{k} \end{bmatrix}$$

(37)

where $\mathbf{r}_B$ is the position vector from the body frame origin to point B.

By (9), the error-reduction function can be written as

$$F_{\text{er}}(\theta) = \mathbf{d}_\theta^T \mathbf{v}_0 (\mathbf{w}_n^T \mathbf{A} \mathbf{w}_n) - \mathbf{d}_\theta^T \mathbf{A} \mathbf{w}_n (\mathbf{w}_n^T \mathbf{v}_0).$$

(38)

From (35) and (37), it can be seen that $\mathbf{d}_\theta$ and $\mathbf{w}_n$ involve first-order terms in $\sin\theta$ and $\cos\theta$. Thus, $F_{\text{er}}(\theta)$ can be expressed in the form

$$\begin{aligned} F_{\text{er}}(\theta) = \quad & g_1 \sin^3\theta + g_2 \cos^3\theta + g_3 \sin^2\theta \cos\theta \\ & + g_4 \sin\theta \cos^2\theta + g_5 \sin^2\theta + g_6 \cos^2\theta \\ & + g_7 \sin\theta \cos\theta + g_8 \sin\theta + g_9 \cos\theta + g_{10}. \end{aligned}$$

(39)

Using the relation

$$\sin^2\theta = 1 - \cos^2\theta$$

to eliminate all $\sin^2\theta$ terms in (39), $F_{\text{er}}(\theta)$ can be written in the form

$$\begin{aligned} F_{\text{er}}(\theta) = \quad & c_1 \cos^3\theta + c_2 \sin\theta \cos^2\theta + c_3 \cos^2\theta \\ & + c_4 \sin\theta \cos\theta + c_5 \sin\theta + c_6 \cos\theta + c_7 \end{aligned}$$

(40)

where the c_i's are functions of the admittance matrix $\mathbf{A}$.

2. Error-Reduction Conditions

To achieve error reduction at all other configurations considered, $F_{\mathrm{er}}(\theta)$ must be negative for all $\theta \in [-\theta_M, \theta_M]$. Therefore, the function $F_{\mathrm{er}}(\theta)$ in (38) must have no root for $\theta \in [-\theta_M, \theta_M]$. In order to determine the range of roots for $F_{\mathrm{er}}(\theta)$, we construct a polynomial that limits the high and low value of the components of $F_{\mathrm{er}}(\theta)$ over $[-\theta_M, \theta_M]$.

It can be verified that, for $0 \le \theta \le (\pi/4)$, the following inequalities are valid:

$$\begin{aligned}
1-\frac{\theta^2}{2} \le \quad & \cos\theta \le 1, \theta - \frac{\theta^3}{3!} \le \sin\theta \le \theta \\
1-\theta^2 \le \quad & \cos^2\theta \le 1, \theta - \frac{4\theta^3}{3!} \le \sin\theta\cos\theta \le \theta \\
1-\frac{3\theta^2}{2} \le \quad & \cos^3\theta \le 1, \theta - \frac{7\theta^3}{3!} \le \sin\theta\cos^2\theta \le \theta.
\end{aligned}$$

(41)(42)(43)

Since $0 \le \theta \le (\pi/4)$, these inequalities are valid throughout the range of investigated θ. For $\theta \ge 0$, a "more positive" conservative polynomial approximation of $F_{\mathrm{er}}(\theta)$, $P(\theta)$ can be constructed by the following:

- if $c_i > 0$, replace the corresponding trigonometric term by the upper bound polynomial term in (41)–(43);
- if $c_i < 0$, replace the corresponding trigonometric term by the lower bound polynomial term in (41)–(43).

As such, a third-order polynomial is obtained

$$P(\theta) = p_3\theta^3 + p_2\theta^2 + p_1\theta + p_0.$$

(44)

Note that the variation between $P(\theta)$ and $F_{\mathrm{er}}(\theta)$ is quite small $[o(\theta^4)]$, and that

$$F_{\mathrm{er}}(\theta) \le P(\theta), \forall\theta \in [0, \theta_M].$$

(45)

Thus, if $P(\theta)$ has no root for $\theta \in [0, \theta_M]$, then $F_{\mathrm{er}}(\theta)$ has no root in the same range, and the error-reduction condition is satisfied.

Since $P(\theta)$ is a third-order polynomial, the roots can be expressed analytically as a function of the coefficients ai. A constraint that ensures that the error-reduction condition is satisfied throughout the range can be obtained by requiring that any positive root be greater than θ_M. Alternatively, a much simpler sufficient condition on the coefficients can be used to ensure the error-reduction condition. If we denote

$$p_M = \max\{|p_1|, |p_2|, |p_3|\} > 0$$

then, it can be proved (see the Appendix) that any root of $P(\theta)$, θ', must satisfy

$$|\theta'| \ge \frac{|p_0|}{p_M + |p_0|}.$$

Thus, the condition

$$\frac{|p_0|}{p_M + |p_0|} \geq \theta_M$$

(46)

guarantees that the function $F_{\mathrm{er}}(\theta)$ has no root over $[0, \theta_M]$ and, together with (33), that error reduction for these configurations is ensured.

Now, consider the case where – $(\pi/4) \leq \theta < 0$. In (41)–(43), the inequalities involving only $\cos\theta$ are still valid, while the inequalities involving $\sin\theta$ change directions, i.e.,

$$\begin{aligned} \theta \leq \quad & \sin\theta \leq \theta - \frac{\theta^3}{3!} \\ \theta \leq \quad & \sin\theta\cos\theta \leq \theta - \frac{4\theta^3}{3!} \\ \theta \leq \quad & \sin\theta\cos^2\theta \leq \theta - \frac{7\theta^3}{3!}. \end{aligned}$$

(47)(48)(49)

For $\theta < 0$, a “more positive” conservative polynomial approximation of $F_{\mathrm{er}}(\theta)$, $Q(\theta)$ can be constructed by the following.

- For the terms involving $\sin\theta$, if $c_i > 0$, replace the corresponding trigonometric term by the upper-bound polynomial term in (47)–(49); if $c_i < 0$, replace the corresponding trigonometric term by the lower-bound polynomial term in (47)–(49).
- For the terms involving only $\cos\theta$, if $c_i > 0$, replace the corresponding trigonometric term by the upper-bound polynomial term in (41)–(43); if $c_i < 0$, replace the corresponding trigonometric term by the lower-bound polynomial term in (41)–(43).

As such, a third-order polynomial is obtained

$$Q(\theta) = q_3\theta^3 + q_2\theta^2 + q_1\theta + q_0.$$

(50)

Again, note that the variation between $Q(\theta)$ and $F_{\mathrm{er}}(\theta)$ is small $[o(\theta^4)]$, and that

$$F_{\mathrm{er}}(\theta) \leq Q(\theta), \forall\theta \in [-\theta_M, 0].$$

(51)

If we denote

$$q_M = \max\{|q_1|, |q_2|, |q_3|\} > 0$$

then, the condition

$$\frac{|q_0|}{q_M + |q_0|} \geq \theta_M$$

(52)

guarantees that the error-reduction function $F_{\mathrm{er}}(\theta)$ has no root over $[-\theta_M, 0]$ and, together with (33), that error reduction for these configurations is ensured.

Combining (46) and (52), the error-reduction condition for any $|\theta| \leq \theta_M$ is obtained. Note that

$$p_0 = q_0 = F_{\mathrm{er}}(0).$$

If we denote

$$F_M = \max\{|p_1|, |p_2|, |p_3|, |q_1|, |q_2|, |q_3|\}$$

(53)

then, the condition $F_{\mathrm{er}}(0) < 0$, and

$$\frac{|F_{\mathrm{er}}(0)|}{F_M + |F_{\mathrm{er}}(0)|} \geq \theta_M$$

(54)

ensures that $F_{\mathrm{er}}(\theta)$ is negative for all $\theta \in [-\theta_M, \theta_M]$. Since all coefficients p_i's and q_i's are functions of the admittance matrix $\mathbf{A}$, inequality (54) imposes a constraint on $\mathbf{A}$.

Thus, for orientation variation with $\theta_M \leq (\pi/4)$, a sufficient condition for error-reducing motion is that at the center angle, the error-reduction condition (33) is satisfied, and the admittance matrix $\mathbf{A}$ satisfies inequality (54).

B. Configuration Variation in Translation

Now consider the translational variation of the contact configuration illustrated in Fig. 3(b). In this case, only translation along the edge is allowed, and the contact force does not change in the body frame. The configuration of the body can be determined by a vector d [Fig. 3(b)].

Suppose that, at the two locations $\mathbf{d}_1$ and $\mathbf{d}_2$, the error-reduction conditions are satisfied

$$\begin{aligned} \mathbf{d}_1^T \mathbf{v}_0 + \mathbf{d}_1^T \mathbf{A} \mathbf{w}_{n1} &< 0 \\ \mathbf{d}_2^T \mathbf{v}_0 + \mathbf{d}_2^T \mathbf{A} \mathbf{w}_{n2} &< 0 \end{aligned}$$

(55)(56)

where $\mathbf{w}_{n1}$ and $\mathbf{w}_{n2}$ are the contact wrenches at $\mathbf{d}_1$ and $\mathbf{d}_2$, respectively. Thus, for any $\alpha, \beta \geq 0$

$$(\alpha \mathbf{d}_1 + \beta \mathbf{d}_2)^T \mathbf{v}_0 + (\alpha \mathbf{d}_1 + \beta \mathbf{d}_2)^T \mathbf{A} \mathbf{w}_n < 0.$$

(57)

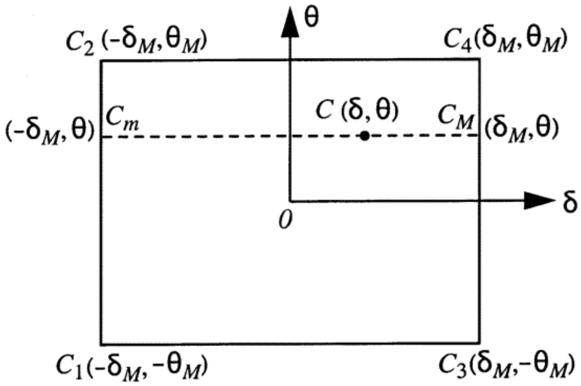

Fig. 4. General vertex-edge contact state. The conditions at four boundary configurations ensure the error-reducing motion for all intermediate configurations.

Consider an arbitrary configuration d between $\mathbf{d}_1$ and $\mathbf{d}_2$. Since the ends of these three vectors must be on a straight line, d is a convex combination of the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$, i.e.,

$$\mathbf{d} = \alpha \mathbf{d}_1 + \beta \mathbf{d}_2$$

(58)

where $\alpha, \beta \geq 0$, and $\alpha + \beta = 1$.

Since the contact wrench $\mathbf{w}_n$ is the same in the body frame for all contact configurations, $\mathbf{w}_n = \mathbf{w}_{n1} = \mathbf{w}_{n2}$. Substituting (58) into (57) yields

$$\mathbf{d}^T \mathbf{v}_0 + \mathbf{d}^T \mathbf{A} \mathbf{w}_\mathbf{n} < 0.$$

Thus, if at two configurations $(-\delta_M, \theta)$ and (δ_M, θ) the error-reduction condition is satisfied, then the error-reduction condition must be satisfied for all intermediate configurations (δ, θ) with $\delta \in -[\delta_M, \delta_M]$.

C. General Case

The results presented in Sections IV-Aand IV-B can be generalized to intermediate edge-vertex contact configurations involving both translational and orientational variations from configurations at which the conditions were imposed.

Let $C(\delta, \theta)$ be an arbitrary configuration with $\delta \in [-\delta_M, \delta_M]$ and $\theta \in [-\theta_M, \theta_M]$, as shown in Fig. 4. Suppose that at the four extremal configurations $C_1(-\delta_M, -\theta_M)$, $C_2(-\delta_M, \theta_M)$, $C_3(\delta_M, -\theta_M)$, and $C_4(\delta_M, \theta_M)$, the error-reduction condition is satisfied, and that at $\theta = -\theta_M, \theta_M$ (54)is satisfied.

Consider first, the two configurations C_m and C_M determined by $(-\delta_M, \theta)$ and (δ_M, θ), respectively. Since at configurations C_1 and C_2 the error-reduction condition (33) and inequality (54) are satisfied, by the results presented in Section IV-A, the error-reduction condition must be satisfied at configuration C_m. By the same reasoning, the error-reduction condition is also satisfied at the configuration C_M. Then, because the error-reduction condition is satisfied at C_m and C_M, by the results presented in Section IV-B, the error-reduction condition must also be satisfied for any $\delta \in [-\delta_M, \delta_M]$. Thus we have the following proposition.

Proposition 2

For a vertex-edge contact state with variation of orientation $[-\theta_M, \theta_M]$ and variation of translation $[-\delta_M, \delta_M]$, if at the two configurations with different contact boundary locations and the same zero angle $\left((\delta, \theta) = (-\delta_M, 0), (\delta_M, 0)\right)$ the admittance satisfies the error reduction conditions, and inequality (54) is satisfied for both $-\delta_M$ and δ_M, then the admittance will satisfy the error-reduction condition for all configurations bounded by four configurations, $-\delta_M, -\theta_M$), $-\delta_M, \theta_M$), ($\delta_M, -\theta_M$), (δ_M, θ_M).

Thus, for an edge-vertex contact state, to ensure that the motion response due to contact is error reducing for all configurations considered, only four conditions need be satisfied.

SECTION V. Discussion and Summary

In this paper, error reduction of a single point on the held body is considered. If that point corresponds to that which is maximally displaced from its proper position, an established metric is used as a measure of error reduction. Alternately, the results could be applied to a finite set of points to further restrict the description of error reduction. If, for example, n points on the body are considered, the conditions in *Propositions 1* and *2* must be satisfied for all of the n points.

Note that the relative size of the space of acceptable admittance matrices is determined by the difficulty of the assembly task. Easier tasks yield a larger space of acceptable admittance matrices.

Also note that, because the conditions imposed are for instantaneous motions and the imposed error-reduction measure does not explicitly consider rotation, it is possible to reduce the error measure while increasing the orientational misalignment of the parts. Reliability is increased when the range of orientational misalignments considered is larger than that expected for a given manipulator.

In summary, we have presented an approach for admittance selection of a planar rigid body for force-guided assembly. We have shown that, for single-point contact cases, the admittance control law can be selected based on their behavior at a *finite* number of configurations. If the error-reduction conditions are satisfied at these configurations, the error-reduction conditions will be satisfied for all intermediate configurations.

In ongoing work, we are investigating more general problems involving friction and multipoint contact.

Appendix

Consider an n th-order polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

Suppose ξ is a root of $f(x)$, then it is proved [12] that

$$|\xi| \le \max\left\{1 + \left|\frac{a_{n-1}}{a_n}\right|, 1 + \left|\frac{a_{n-2}}{a_n}\right|, \cdots, 1 + \left|\frac{a_0}{a_n}\right|\right\}.$$

Consider the transformation defined by

$$\xi = \frac{1}{\eta}$$

then

$$f\left(\frac{1}{\eta}\right) = \frac{a_n}{\eta^n} + \frac{a_{n-1}}{\eta^{n-1}} + \cdots + \frac{a_1}{\eta} + a_o = 0$$

which leads to

$$a_0 \eta^n + a_1 \eta^{n-1} + \cdots + a_{n-1}\eta + a_n = 0.$$

Thus, η is a root of the polynomial

$$h(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n.$$

Therefore

$$\left|\frac{1}{\xi}\right| = |\eta| \le \max\left\{1 + \left|\frac{a_1}{a_0}\right|, 1 + \left|\frac{a_2}{a_0}\right|, \cdots, 1 + \left|\frac{a_n}{a_0}\right|\right\}$$

which implies

$$|\xi| \ge \left(\max\left\{1 + \left|\frac{a_1}{a_0}\right|, 1 + \left|\frac{a_2}{a_0}\right|, \cdots, 1 + \left|\frac{a_n}{a_0}\right|\right\}\right)^{-1}.$$

Let

$$a_M = \max\{|a_1|, |a_2|, \cdots, |a_n|\} > 0$$

then

$$max \left\{1 + \left|\frac{a_1}{a_0}\right|, 1 + \left|\frac{a_2}{a_0}\right|, \cdots, 1 + \left|\frac{a_n}{a_0}\right|\right\} = 1 + \left|\frac{a_M}{a_0}\right|.$$

Therefore

$$|\xi| \ge \left(1 + \left|\frac{a_M}{a_0}\right|\right)^{-1} = \frac{|a_0|}{a_M + |a_0|}.$$

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