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Abstract:

An important issue in the development of force guidance assembly strategies is the specification of an appropriate admittance control law. This paper identifies conditions to be satisfied when selecting the appropriate admittance to achieve force-guided assembly of polygonal parts for multipoint contact with friction. These conditions restrict the admittance behavior for each of the various one-point and two-point contact cases and ensure that the motion that results from contact reduces part misalignment for each case. We show that, for bounded friction and part misalignments, if the identified conditions are satisfied for a finite number of contact configurations and friction coefficients, the conditions ensure that force guidance is achieved for all configurations and values of friction within the specified bounds.

SECTION I. Introduction

Admittance control is useful in assembly tasks in providing both force regulation and force guidance. In these tasks, the admittance maps contact forces into changes in the velocity of the held body. To achieve reliable assembly through force guidance, it is important that an appropriate admittance is selected. For planar motion, the linear admittance control law has the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{A}\mathbf{w}$$

(1)

where $\mathbf{v}_0$ is the nominal velocity (a 3-vector), $\mathbf{w}$ is the contact wrench measured in the body frame (a 3-vector), $\mathbf{A}$ is the admittance matrix (a 3×3 matrix), and $\mathbf{v}$ is the resulting motion (a 3-vector).

Others have addressed the use of an admittance for force guidance. Whitney [1], [2] proposed that the compliance of a manipulator be structured so that contact forces lead to decreasing errors.
Peshkin [3] addressed the synthesis of an accommodation (inverse damping) matrix by specifying the desired force/motion relation at a sampled set of positional errors for a planar assembly task. An unconstrained optimization was then used to obtain an accommodation matrix. Asada [4] used a similar optimization procedure for the design of an accommodation neural network rather than an accommodation matrix. More recently, Fasse and Broenink [5] and Marcelo et al. [6] provided synthesis procedures based on spatial intuitive reasoning. None of these approaches, however, ensures that the admittance selected will, in fact, be reliable for a specified range of friction coefficients and part misalignments.

A reliable admittance selection approach is to design the control law so that, at each possible part misalignment, the contact force always leads to a motion that instantaneously reduces the existing misalignment. The approach is referred to as *force assembly* [7]–[8] [9].

The description of rigid body motions that reduce misalignment is neither obvious nor unique. The configuration space of a rigid body is non-Euclidian; therefore, there is no natural metric for finite error. In [10], several body-specific rigid body metrics were identified. These metrics are based on the Euclidean distance between one (or more) point(s) on the body and its (their) corresponding location(s) when properly positioned.

Previously, we have considered sufficient conditions on the admittance to ensure planar force assembly in *frictionless* single-point contact [11]. In the study, we considered a *measure* of error based on the Euclidean distance between a fixed point on the held body and its location when properly positioned. The misalignment reduction condition of force assembly requires that, at each possible misalignment, the contact force yields a motion that reduces the misalignment. Using the point-based measure of misalignment discussed above, this condition can be expressed mathematically if we let $\mathbf{d}$ (a 3-vector for planar motion) be the line vector from the selected point at its proper mated position to its current position. Then, for error reducing motion, the condition is

$$\mathbf{d}^T\mathbf{v} = \mathbf{d}^T(\mathbf{v}_0 + \mathbf{A}\mathbf{w}) < 0$$

(2)

which must be satisfied for all possible misalignments.

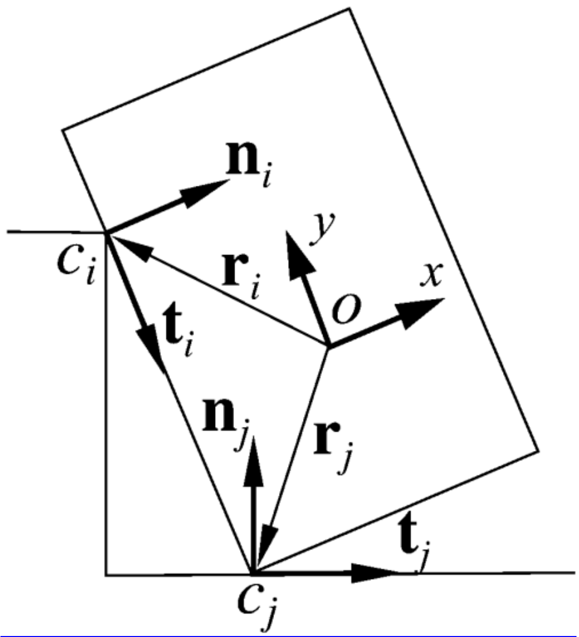

Fig. 1. Rigid body constrained by multipoint contact. The contact force at each contact point has normal and frictional components.

Here, we investigate single-and two-point *frictional* planar contact using the same error measure. We show that, by identifying an admittance matrix that satisfies the error-reduction conditions at a *finite* number of extremal contact configurations and at two specified coefficients of friction, the error-reduction requirements are also satisfied for *all* intermediate configurations and for all coefficients of friction within the range specified. The friction model considered is "hard" point contact satisfying Coulomb's law [12].

In this paper, a description of error-reducing motion for a rigid body in frictional contact is derived in Section II. The strategy used to obtain sufficient conditions to impose on the admittance is first presented in Section III. Sufficient conditions for error reduction for single-point contact and two-point contact are then obtained in Sections IV and V, respectively. These conditions show that an admittance matrix satisfying the error-reduction conditions at the boundaries of a set of contact configurations and friction coefficients also satisfies the error-reduction conditions at all intermediate configurations and all intermediate friction coefficients. A numerical example of admittance selection for a planar peg-in-hole problem is presented in Section VI, and a brief summary is provided in Section VII.

SECTION II. Error-Reducing Motion

In this section, the misalignment-reducing motions of a constrained rigid body in single-point and two-point frictional contact are mathematically described. The equation describing the constrained motion of a rigid body is first identified. Functions describing error-reducing motions for single-point and two-point contact are then obtained.

A. Constrained Rigid Body Motion

Consider planar motion of a rigid body in m-point contact with another part as shown in Fig. 1. Let $\mathbf{n}_i$ be the surface normal (unit 2-vector pointing toward the held body) and let $\mathbf{t}_{bi}$ be a unit 2-vector tangent to the surface at the contact point i. Then, the direction of friction $\mathbf{t}_i$ must be along $\mathbf{t}_{bi}$, i.e., $\mathbf{t}_i = \pm\mathbf{t}_{bi}$. Let ϕ_i be the magnitude of the contact force at i and μ_i be the coefficient of friction. Then the wrench obtained from contact at m locations is given by

$$\mathbf{w}_i = \sum_{i=1}^{m} (\mathbf{w}_{ni} + \mu_i \mathbf{w}_{ti})\phi_i$$

(3)

where

$$\begin{aligned} \mathbf{w}_{ni} &= \begin{bmatrix} \mathbf{n}_i \\ (\mathbf{r}_i \times \mathbf{n}_i) \cdot \mathbf{k} \end{bmatrix} \\ \mathbf{w}_{ti} &= \begin{bmatrix} \mathbf{t}_i \\ (\mathbf{r}_i \times \mathbf{t}_i) \cdot \mathbf{k} \end{bmatrix} \end{aligned}$$

and where $\mathbf{r}_i$ is the position vector from the origin of the coordinate frame to the point of contact i and $\mathbf{k}$ is the unit vector orthogonal to the plane.

If we denote

$$\begin{aligned} \mathbf{W}_n &= [\mathbf{w}_{n1}, \dots, \mathbf{w}_{nm}] \in \mathbb{R}^{3\times m} \\ \mathbf{W}_t &= [\mathbf{w}_{t1}, \dots, \mathbf{w}_{tm}] \in \mathbb{R}^{3\times m} \\ \boldsymbol{\mu} &= \operatorname{diag}(\mu_1, \dots, \mu_m) \in \mathbb{R}^{m\times m} \\ \boldsymbol{\phi} &= [\phi_1, \dots, \phi_m]^T \in \mathbb{R}^m \\ \mathbf{W} &= \mathbf{W}_n + \mathbf{W}_t \boldsymbol{\mu} \end{aligned}$$

then the total contact wrench is

$$\mathbf{w} = \mathbf{W}\boldsymbol{\phi}.$$

(4)

By the control law (1), the motion of the rigid body is given by

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{A}\mathbf{W}\boldsymbol{\phi}.$$

(5)

To maintain contact [13], the reciprocal condition requires

$$\mathbf{w}_{ni}^T \mathbf{v} = 0 \Longrightarrow \mathbf{W}_n^T \mathbf{v} = \mathbf{0}.$$

Substituting (5) into this and solving for ϕ, we have

$$\boldsymbol{\phi} = -[\mathbf{W}_n^T \mathbf{A}\mathbf{W}]^{-1} \mathbf{W}_n^T \mathbf{v}_0.$$

(6)

Substituting this result back into (5) yields the following equation for constrained motion:

$$\mathbf{v} = \mathbf{v}_0 - \mathbf{A}\mathbf{W}[\mathbf{W}_n^T \mathbf{A}\mathbf{W}]^{-1} \mathbf{W}_n^T \mathbf{v}_0.$$

(7)

For a given relative configuration of planar bodies, the normal $\mathbf{n}_i$ and tangent space base vector $\mathbf{t}_{bi}$ at the contact point are known. The direction of the friction force ($\mathbf{t}_i = \mathbf{t}_{bi}$ or $\mathbf{t}_i = -\mathbf{t}_{bi}$) is uniquely determined by satisfying the following conditions: 1) each component of ϕ in (6) is positive and 2) $\mathbf{v}^T\mathbf{w}_{ti} < 0$. Thus, $\mathbf{t}$ is known for each contact point and the compliant motion can be determined by (7).

To calculate the motion, the matrix $[\mathbf{W}_n^T\mathbf{AW}]$ must be full rank. Since $\mathbf{A}$ is positive definite [7], $\det[\mathbf{W}_n^T\mathbf{AW}] > 0$ for $\mu = 0$. In the following sections, we assume that, for $\mu \in [0, \mu_M]$, the inequality

$$\det[\mathbf{W}_n^T\mathbf{AW}] = \det[\mathbf{W}_n^T\mathbf{AW}_n + \mathbf{W}_n^T\mathbf{AW}_t\boldsymbol{\mu}] > 0$$

(8)

is satisfied.

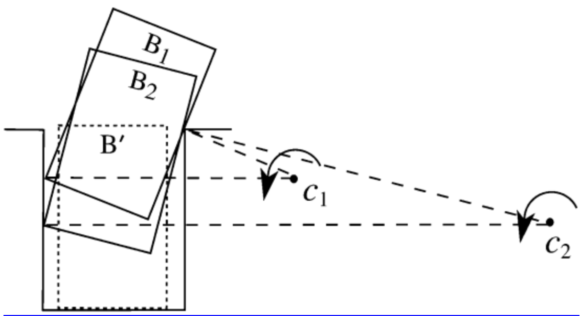

Fig. 2. Error-reducing motion for two-point contact. The angular motion of the rigid body is in the same direction for all configurations of a given two-point contact state.

B. Error-Reduction Function for Single-Point Contact

For single-point contact, $\mathbf{W}_n = [\mathbf{w}_n]$ and $\mathbf{W}_t = [\mathbf{w}_t]$. If the compliant motion is error reducing, condition (2) must be satisfied for one (or more) specified point(s) on the body. Thus

$$E = \frac{\mathbf{d}^T(\mathbf{v}_0\mathbf{w}_n^T - \mathbf{v}_0^T\mathbf{w}_n\mathbf{I})\mathbf{A}(\mathbf{w}_n + \mu\mathbf{w}_t)}{\mathbf{w}_n^T\mathbf{Aw}_n + \mu\mathbf{w}_n^T\mathbf{Aw}_t} < 0.$$

(9)

Due to (8), the denominator in (9) is positive. Therefore, the error-reduction function can be expressed as

$$F_{1p} = \mathbf{d}^T(\mathbf{v}_0\mathbf{w}_n^T - \mathbf{v}_0^T\mathbf{w}_n\mathbf{I})\mathbf{A}(\mathbf{w}_n + \mu\mathbf{w}_t).$$

(10)

Note that error-reducing motion is indicated by the sign of F_{1p} in (10).

C. Error-Reduction Function for Two-Point Contact

For planar motion of a rigid body with two-point contact, if the contact is maintained, the body has only one degree of freedom (DOF). The instantaneous motion of the body is a rotation about the body's instantaneous center.

If the instantaneous center is at infinity and contact is maintained, the motion of the body is pure translation. This is the simplest admittance design case: error reduction at extremal configurations within the contact state ensures error reduction for all configurations within that contact state.

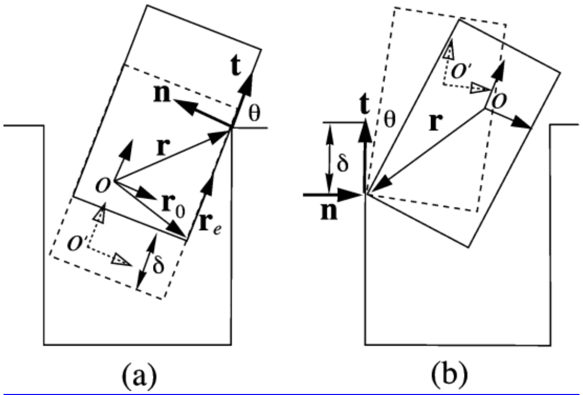

Fig. 3. Configuration variation for the same single-point contact state. Contact state configuration variation is illustrated for (a) $\{e-v\}$ (edge–vertex) contact and (b) $\{v-e\}$ (vertex–edge) contact.

If the instantaneous center is not at infinity (generic case), it is uniquely determined by the geometry of the contact for each configuration within the contact state. Because admittance design of the instantaneous center at the infinity case is trivial, the admittance of only this more general (and more difficult) type of two-point contact state is addressed.

As stated previously, the force-assembly error-reduction condition requires that, at any instant, the motion of the body must be toward its properly mated position. Consider the two-point contact state shown in Fig. 2. For error reduction, the direction of rotation of the body about the instantaneous center c_i must cause the body to move toward the properly mated configuration B'. Since error reduction must hold for any configuration, the angular motion of the body must be in the same direction for all configurations within the same contact state. Thus, the error-reducing motion for two-point contact is solely indicated by the angular velocity of the constrained body.

Now consider the angular motion in (7). Let $\mathbf{e}_3=[0,0,1]^T$ and $\mathbf{v}_0=[v_{x0},v_{y0},\omega_0]^T$, then the orientational component in (7) is

$$\omega=\mathbf{e}_3^T\mathbf{v}=\omega_0-\mathbf{e}_3^T\mathbf{AW}[\mathbf{W}_n^T\mathbf{AW}]^{-1}\mathbf{W}_n^T\mathbf{v}_0.$$

(11)

Let $[\mathbf{W}_n^T\mathbf{AW}]^*$ be the adjugate of $[\mathbf{W}_n^T\mathbf{AW}]$ as follows:

$$[\mathbf{W}_n^T\mathbf{AW}]^*=\begin{bmatrix}\mathbf{w}_{n2}^T\mathbf{Aw}_2 & -\mathbf{w}_{n2}^T\mathbf{Aw}_2\\ -\mathbf{w}_{n1}^T\mathbf{Aw}_2 & \mathbf{w}_{n1}^T\mathbf{Aw}_1\end{bmatrix}.$$

Then

$$[\mathbf{W}_n^T\mathbf{AW}]^{-1}=\frac{1}{\det(\mathbf{W}_n^T\mathbf{AW})}[\mathbf{W}_n^T\mathbf{AW}]^*.$$

Substituting this into (11) yields

$$\omega\quad=\frac{[\det(\mathbf{W}_n^T\mathbf{AW})\,\omega_0-\mathbf{e}_3^T\mathbf{AW}[\mathbf{W}_n^T\mathbf{AW}]^*\mathbf{W}_n^T\mathbf{v}_0]}{\det(\mathbf{W}_n^T\mathbf{AW})}.$$

(12)

Because in (8) we assume that $\det(\mathbf{W}_n^T\mathbf{AW}) > 0$, we now only need to consider the following function:

$$F_{2p} = \det(\mathbf{W}_n^T\mathbf{AW})\,\omega_0 - \mathbf{e}_3^T\mathbf{AW}[\mathbf{W}_n^T\mathbf{AW}]^*\mathbf{W}_n^T\mathbf{v}_0$$

which can be expressed as

$$\begin{aligned} F_{2p} \quad &= [(\mathbf{w}_{n1}^T\mathbf{Aw}_1)(\mathbf{w}_{n2}^T\mathbf{Aw}_2) - (\mathbf{w}_{n1}^T\mathbf{Aw}_2)(\mathbf{w}_{n2}^T\mathbf{Aw}_1)]\omega_0 \\ &-(\mathbf{w}_{n1}^T\mathbf{v}_0)(\mathbf{a}_3^T\mathbf{w}_1)(\mathbf{w}_{n2}^T\mathbf{Aw}_2) \\ &+(\mathbf{w}_{n1}^T\mathbf{v}_0)(\mathbf{a}_3^T\mathbf{w}_2)(\mathbf{w}_{n1}^T\mathbf{Aw}_2) \\ &+(\mathbf{w}_{n2}^T\mathbf{v}_0)(\mathbf{a}_3^T\mathbf{w}_1)(\mathbf{w}_{n2}^T\mathbf{Aw}_1) \\ &-(\mathbf{w}_{n2}^T\mathbf{v}_0)(\mathbf{a}_3^T\mathbf{w}_2)(\mathbf{w}_{n1}^T\mathbf{Aw}_1) \end{aligned}$$

(13)

where $\mathbf{a}_3$ is the third column of the admittance matrix $\mathbf{A}$ and $\mathbf{w}_i$ is the ith column of the matrix $\mathbf{W}$.

Since the function F_{2p} in (13) indicates the sign of the orientational motion for the body, it is used as the error-reduction function for the two-point contact case.

SECTION III. Solution Strategy

In general, the error-reduction functions F_{1p} in (10) and F_{2p} in (13) depend on the geometries of the parts in contact. In this section, a strategy to obtain sufficient conditions for error reduction is presented. With this strategy, a set of sufficient conditions can be obtained for bounded configurations without explicitly describing the specific variation in part configuration within a given contact state.

A. Contact States

Polygonal planar bodies in single-point contact have two basic types of contact. One is referred to as "edge–vertex" contact ($\{e - v\}$); the other is referred to as "vertex–edge" contact ($\{v - e\}$). In "edge–vertex" contact, one edge of the held body is in contact with one vertex of the mating fixtured part [Fig. 3(a)]. In "vertex–edge" contact, one vertex of the held body is in contact with one edge of its mating part [Fig. 3(b)].

Configurations within the same contact state vary. As shown in Fig. 3, δ represents a relative position along an edge of a body and θ represents a relative orientation between the two parts. The ranges for δ and θ can be determined by robot accuracy or by bounds imposed by the contact state.

The basic types of two-point contact are the various combinations of two single-point contacts. There are three types of two-point contact: 1) one $\{e - v\}$ and one $\{v - e\}$ contact ($\{e - v, v - e\}$); 2) two $\{e - v\}$ contact ($\{e - v, e - v\}$); and 3) two $\{v - e\}$ contact ($\{v - e, v - e\}$).

B. Single-Point Contact

In the following, the contact wrenches for the two basic types of single-point contact are presented. We show that, although the body configuration is determined by two variables (δ, θ), the unit contact wrench for each type of contact depends on only one of them.

1) Edge–Vertex Contact Wrenches

Consider the case for which one edge of the held body is in contact with one vertex of its mating part ($\{e - v\}$ contact state). As shown in Fig. 3(a), the direction of the contact wrench is constant in the body frame but the location of the contact varies, thus the unit contact wrench can be expressed as

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{n} \\ (\mathbf{r} \times \mathbf{n}) \cdot \mathbf{k} \end{bmatrix}$$
$$\mathbf{w}_t = \begin{bmatrix} \mathbf{t} \\ (\mathbf{r} \times \mathbf{t}) \cdot \mathbf{k} \end{bmatrix}$$

(14)

where $\mathbf{r} = \mathbf{r}_0 + \mathbf{r}_e\delta$, and where $\mathbf{r}_0$ identifies a location on the edge and $\mathbf{r}_e$ is the unit vector along the edge (both are constant in the body frame). Thus, $\mathbf{w}$ is a single-variable function in δ for a given $\{v - e\}$ contact state.

2) Vertex–Edge Contact Wrenches

Consider the case for which one vertex of the body is in contact with one edge of its mating part ($\{v - e\}$ contact state). As shown in Fig. 3(b), the direction of the contact wrench $\mathbf{w}$ is constant in the global coordinate frame and the relative position of the contact location is constant in the body frame. Suppose that the relative body orientation changes in angle θ; then the direction of the contact force also changes in θ in the body frame. Thus, the unit contact wrench in the body frame can be expressed as

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{Rn} \\ (\mathbf{r} \times \mathbf{Rn}) \cdot \mathbf{k} \end{bmatrix}$$
$$\mathbf{w}_t = \begin{bmatrix} \mathbf{Rt} \\ (\mathbf{r} \times \mathbf{Rt}) \cdot \mathbf{k} \end{bmatrix}$$

(15)

where $\mathbf{R}$ is the rotation matrix associated with θ having the form

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

(16)

Therefore, in the body frame, w is a single-variable function in θ for a specified $\{v - e\}$ contact state.

3) Mathematical Requirement

If the parts remain in single-point contact, the planar motion of the rigid body has 2 DOF.

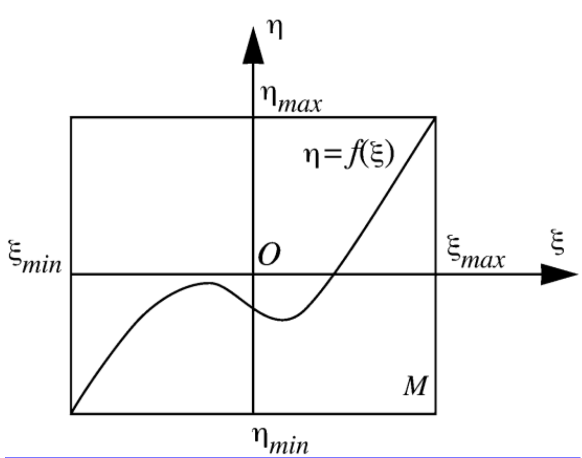

Fig. 4. Two-point contact state. In the bounded area, ξ and η are treated as two independent variables regardless of their relation $\eta = f(\xi)$.

Suppose that the range of δ and θ are $[\delta_{min}, \delta_{max}]$ and $[\theta_{min}, \theta_{max}]$, respectively, and the bounded area is $\mathbf{M}$. If in $\mathbf{M}$ the error-reduction condition is satisfied, then for any configuration considered, the admittance ensures that the error-reduction requirement is satisfied. Mathematically, this requirement can be imposed on the admittance using the function F_{1p} with the following two conditions.

1. For one point $(\delta_0, \theta_0) \in M$, the error-reduction condition is satisfied, i.e.,

$$F_{1p} = F_{1p}(\delta_0, \theta_0) < 0.$$

(17)

2. For all points in M, F_{1p} does not change sign.

C. Two-Point Contact

Since two-point contact is a combination of the two single-point contact cases, the contact wrench for two-point contact is a combination of the two corresponding single-point contact wrenches.

The error-reduction function for two-point contact calculated using (13) involves two contact wrenches $\mathbf{w}_1$ and $\mathbf{w}_2$. Since each unit wrench in (14) or (15) is, in general, a function of δ or θ, the error-reduction function can always be expressed as a function of two variables, i.e.,

$$F_{2p} = F_{2p}(\xi, \eta)$$

(18)

where ξ and η are θ or δ, depending on the contact state. For example, for the $\{e - v, e - v\}$ contact, ξ and η are the two displacement variables along their corresponding edges, δ_1 and δ_2.

If the parts remain in two-point contact, the planar motion of the rigid body has only 1 DOF. Therefore, the two parameters ξ and η in (18) must be related by the geometry of the parts, i.e., are related by a function $\eta = f(\xi)$. The error-reduction condition requires that, for all configurations on the curve $\eta = f(\xi)$, the orientational error of the body is instantaneously reduced by contact. This means that the error-reduction function F_{2p} has the appropriate sign along the curve $\eta = f(\xi)$.

Because this function is geometry-specific and difficult to determine, we consider a set of more conservative conditions based on the range of the two variables. Recall that δ represents a relative position along an edge of a body and θ represents a relative orientation between the two parts. The ranges for δ and θ can be determined from bounds on relative misalignment or by bounds determined by the contact state. Therefore, the ranges of the two parameters ξ and η are readily determined.

Suppose that the range of ξ and η are $[\xi_{\min}, \xi_{\max}]$ and $[\eta_{\min}, \eta_{\max}]$, respectively. Consider the rectangular area M bounded by $[\xi_{\min}, \xi_{\max}]$ and $[\eta_{\min}, \eta_{\max}]$, as shown in Fig. 4. If in the bounded area the error-reduction condition is satisfied, then for any configuration considered, the admittance ensures that the error-reduction requirement is satisfied. Mathematically, this requirement can be imposed on the admittance using function F_{2p} with the following two conditions.

1. For one point $(\xi_0, \eta_0) \in M$, the error-reduction condition is satisfied, i.e.,

$$F_{2p} = F_{2p}(\xi_0, \eta_0) < 0.$$

(19)

2. For all points in M, F_{2p} does not change sign.

As such, sufficient conditions for error-reducing motion are established. This conservative approach enables us to treat the parameters (ξ, η) as two independent variables regardless of the geometrical relationship between them.

D. Bounds on Misalignment

For both single-point and two-point contact cases, the misalignments considered are finite and bounded. The extremes of ξ and η are determined by the robot accuracy and the geometrical constraints of the parts limiting part misalignment. In each contact case, the range for each of the variables can be transformed to be centered about a local origin, e.g., $[\xi_{\min}, \xi_{\max}] \Rightarrow [-\xi_M, \xi_M]$ and $[\eta_{\min}, \eta_{\max}] \Rightarrow [-\eta_M, \eta_M]$ to facilitate subsequent analysis. Thus, the two variables (ξ, η) present a variation from a "central" configuration within the contact state.

In order to obtain sufficient conditions for all configurations, the bounds of ξ and η should be chosen so that they cover all possible misalignments within the contact state. In this paper, the conditions obtained are sufficient conditions as long as the local orientational variation from the "central" configuration is within $[-\theta_M, \theta_M]$ which is in the range $\left[-\left(\frac{\pi}{2}\right), \left(\frac{\pi}{2}\right)\right]$, i.e., $\theta_M \leq \left(\frac{\pi}{2}\right)$. This greatly exceeds the orientational uncertainty for any conventional robot.

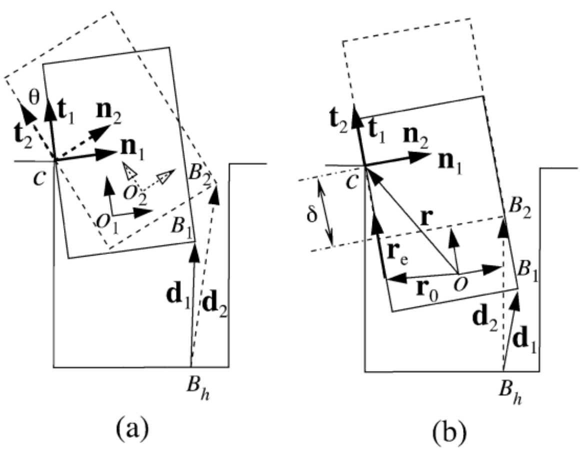

Fig. 5. Edge–vertex contact state. (a) Orientational variation: the contact wrench $\mathbf{w}$ is constant in the body frame while the error-measure vector $\mathbf{d}$ is a nonlinear function of θ. (b) Translational variation: both the contact wrench $\mathbf{w}$ and the error-measure vector $\mathbf{d}$ are functions of δ.

SECTION IV. Conditions for Single-Point Contact

In this section, single-point contact with friction is considered. For planar polygonal parts, two contact states, $\{e - v\}$ and $\{v - e\}$, are considered respectively. As shown in Fig. 3, for both contact states, the configuration of the body can be described by two variables (δ, θ). The range of μ considered is $[0, \mu_M]$.

For each contact case, we develop a set of sufficient conditions on a finite number of configurations that ensures the mathematical requirement for error-reducing motion presented in Section III-B3. In doing so, more "conservative" functions are constructed based on the error-reduction function F_{1p} in (10) and the extreme values of θ_M and δ_M. Then, by evaluating the roots of these functions at the extremal configurations, the sufficient conditions are obtained.

Note that the size of the space of admittance behaviors deemed to be acceptable depends on how conservative the sufficient conditions are. If overly conservative, the conditions would eliminate portions of the $\mathbf{A}$ space that could actually provide misalignment reduction. We seek a set of conditions that ensures force assembly and yet is not overly conservative.

A. Edge–Vertex Contact State

As shown in Fig. 3(a), the contact configuration of the body can be determined by two parameters (δ, θ). We prove that, if an admittance matrix $\mathbf{A}$ satisfies a set of conditions at the configuration "boundary" points for $\mu = 0$ and μ_M, then the $\mathbf{A}$ matrix ensures error-reducing motion for all configurations $\theta \in [-\theta_M, \theta_M], \delta \in [-\delta_M, \delta_M]$, and $\mu \in [0, \mu_M]$.

1) Error-Reduction Function

In order to obtain the error-reduction function, we first express the contact wrench and the error-measure vector $\mathbf{d}$ in terms of δ and θ.

For an edge–vertex contact state, as shown in Fig. 5(a), when the held body rotates relative to the fixtured body about the contact point, the description of the contact wrench does not change in a body-based coordinate frame. When the held body translates relative to the fixtured body, as shown in Fig. 5(b), the description of the contact wrench changes in a body-based coordinate frame because the contact point changes (although its direction is constant). Thus, the contact wrench depends only on the translational variable δ.

As shown in Fig. 5(b), in the body frame, the direction of the surface normal is constant while the position vector of the contact point, $\mathbf{r}$, varies. For an arbitrary δ,$\mathbf{r}$ can be expressed as

$$\mathbf{r}_\delta = \mathbf{r}_0 + \mathbf{r}_e\delta$$

where $\mathbf{r}_0$ is a vector from the body frame to a center point of the edge (constant) and re is the unit vector along the edge.

By (14), the unit wrench corresponding to the surface normal and friction are

$$\begin{aligned} \mathbf{w}_n &= \begin{bmatrix} \mathbf{n} \\ (\mathbf{r}_\delta \times \mathbf{n}) \cdot \mathbf{k} \end{bmatrix} \\ \mathbf{w}_t &= \begin{bmatrix} \mathbf{t} \\ (\mathbf{r}_\delta \times \mathbf{t}) \cdot \mathbf{k} \end{bmatrix}. \end{aligned}$$

(20)

It can be seen that, in the body frame, the directions of $\mathbf{w}_n$ and $\mathbf{w}_t$ are constant while the last components (the moment terms) are linear functions of δ.

Let $\mathbf{d}'_0$ be the error-measure 2-vector at $(\theta, \delta) = (0{,}0)$, then for an arbitrary δ with $\theta = 0$, the error-measure vector $\mathbf{d}'$ is

$$\mathbf{d}'_\delta = \mathbf{d}'_0 + \mathbf{r}_e\delta, \delta \in [-\delta_M, \delta_M]$$

where $\mathbf{r}_e$ is a unit vector along the contacting edge. Note that $\mathbf{d}'_0$ is constant in the global coordinate frame, while re is constant in the body coordinate frame. Thus, for an arbitrary orientation $\theta \in [-\theta_M, \theta_M]$ and $\delta \in [-\delta_M, \delta_M]$, the error-measure 2-vector $\mathbf{d}'$ is a function of δ and θ having the form

$$\mathbf{d}'(\delta, \theta) = \mathbf{R}\mathbf{d}'_0 + \mathbf{r}_e\delta$$

where $\mathbf{R}$ is the rotation matrix having the form of (16).

The line vector associated with $\mathbf{d}'(\delta, \theta)$ can be calculated as

$$\mathbf{d}(\delta,\theta) = \begin{bmatrix} \mathbf{R}\mathbf{d}_0{}' \\ (\mathbf{r}_B \times \mathbf{R}\mathbf{d}_0') \cdot \mathbf{k} \end{bmatrix} + \delta \begin{bmatrix} \mathbf{r}_e \\ (\mathbf{r}_B \times \mathbf{r}_e) \cdot \mathbf{k} \end{bmatrix}$$

(21)

where $\mathbf{r}_B$ is the position vector from the body frame origin to point B.

Thus, for any intermediate configuration (δ,θ), because $\mathbf{w}_n$ and $\mathbf{w}_t$ in (20) each only contain first-order terms in δ, and $\mathbf{d}(\delta,\theta)$ in (21) only contains first-order terms in $\sin\theta$, $\cos\theta$ and δ, the error-reduction function (10) can be expressed as a third-order polynomial in δ in the form

$$F_{1p}(\delta,\theta) = f_3\delta^3 + f_2\delta^2 + f_1\delta + f_0$$

(22)

where the coefficients f_i have the form

$$f_i = a_i \cos\theta + b_i \sin\theta + c_i.$$

(23)

Also note that μ appears in the coefficients of $\mathbf{w}_t$. Therefore, the coefficients a_i, b_i, and c_i have the form $(h_i + \mu h'_i)$, where h_i and h_i' are functions of the admittance $\mathbf{A}$.

2) Sufficient Conditions for Error Reduction

The error-reduction condition requires that the error-reduction function in (22) must be negative in the range of configurations considered. In order to obtain sufficient conditions, we construct two functions F_0 and F_M by replacing the $\cos\theta$ terms in (23) with 1 and $\cos\theta_M$, respectively, as follows:

$$\begin{aligned} F_0(\delta,\theta) = \quad & (a_3\delta^3 + a_2\delta^2 + a_1\delta + a_0) \\ & +(b_3\delta^3 + b_2\delta^2 + b_1\delta + b_0)\sin\theta \\ & +(c_3\delta^3 + c_2\delta^2 + c_1\delta + c_0) \\ F_M(\delta,\theta) = \quad & (a_3\delta^3 + a_2\delta^2 + a_1\delta + a_0)\cos\theta_M \\ & +(b_3\delta^3 + b_2\delta^2 + b_1\delta + b_0)\sin\theta \\ & +(c_3\delta^3 + c_2\delta^2 + c_1\delta + c_0). \end{aligned}$$

(24)(25)

For any (δ,θ) in the range considered, we have

$$\min\{F_0, F_M\} \le F_{1p} \le \max\{F_0, F_M\}.$$

(26)

Thus, if both F_0 and F_M are negative over the range $\delta \in [-\delta_M, \delta_M]$ and $\theta \in [-\theta_M, \theta_M]$, error-reducing motion is ensured.

For a given θ, both F_0 and F_M are third-order polynomials in δ. To obtain conditions on F_0 and F_M, we first evaluate the bounds on the coefficients of these two polynomials.

By (24) and (25), the coefficients of δ^i in F_0 and F_M have the form

$$
\begin{aligned}
f_i^0(\mu,\theta) &= (p_i + p_i'\mu) + (q_i + q_i'\mu)\sin\theta \\
f_i^M(\mu,\theta) &= (p_i + p_i'\mu)\cos\theta_M + (q_i + q_i'\mu)\sin\theta
\end{aligned}
$$

where p_i, p'_1, q_i, and q'_i are functions of the admittance **A**.

If the range of μ is $[0,\mu_M]$, it can be proved that f_i^0 and f_i^M achieve their maximum and minimum values only at the boundary points $(0,\pm\theta_M)$ and $(\mu_M,\pm\theta_M)$. This can be verified by evaluating the Hessian matrices of f_i^0 and f_i^M. In fact, the Hessian matrix of f_i^0 with respect to (μ,θ) is

$$
\mathrm{Hess}(f_i^0) = \begin{bmatrix} 0 & q_i'\cos\theta \\ q_i'\cos\theta & -(q_i + q_i'\mu)\sin\theta \end{bmatrix}.
$$

Since $\det(\mathrm{Hess}) = -{q_i}'^2\cos^2\theta < 0$, the Hessian is indefinite, and the function f_i^0 cannot have a maximum or minimum in the interior of the area $[0,\mu_M]\times[-\theta_M,\theta_M]$ [14]. Thus, the maximum (minimum) values of f_i^0 can be chosen from its four values at the four boundary points: $(0,\pm\theta_M)$ and $(\mu_M,\pm\theta_M)$. The same property holds true for f_i^M.

Denote

$$
\begin{aligned}
s_M &= \max\{|f_i^0|, |f_i^M|, i = 1,2,3\} \\
s_0 &= \min\{|f_0^0|, |f_0^M|\}.
\end{aligned}
$$

(27)(28)

We prove that if

$$
\frac{s_0}{s_M + s_0} > \delta_M
$$

(29)

then both F_0 and F_M have no root for all $\delta \in [-\delta_M,\delta_M]$, $\theta \in [-\theta_M,\theta_M]$ and $\mu \in [0,\mu_M]$.

Consider the function F_0 in (24). For an arbitrary $\theta_0 \in [-\theta_M,\theta_M]$ and an arbitrary $\mu_0 \in [0,\mu_M]$, F_0, F_0 is a third-order polynomial in a single-variable δ

$$
F_0(\delta,\theta_0) = g_3\delta^3 + g_2\delta^2 + g_1\delta + g_0
$$

where

$$
g_i = (p_i + \mu_0 p_i') + (q_i + \mu_0 q_i')\sin\theta_0.
$$

Let

$$
g_M = \max\{|g_1|, |g_2|, |g_3|\}.
$$

Then, as shown in [11], a root of F_0, ξ, must satisfy

$$
|\xi| \geq \frac{|g_0|}{g_M + |g_0|}.
$$

Since $\theta_0 \in [-\theta_M,\theta_M]$ and $\mu_0 \in [0,\mu_M]$, by (27) and (28), we have

$$g_M \leq s_M, |g_0| \geq s_0.$$

Therefore

$$\frac{s_M}{s_0} \geq \frac{g_M}{g_0}$$

which leads to

$$|\xi| \geq \frac{|g_0|}{g_M + |g_0|} \geq \frac{s_0}{s_M + s_0} > \delta_M.$$

Thus, F_0 has no root in $[-\delta_M, \delta_M]$ for all $\theta \in [-\theta_M, \theta_M]$ and $\mu \in [0, \mu_M]$. The same reasoning applies to F_M. Therefore, the functions F_0 and F_M do not change sign if inequality (29) is satisfied. By (26), F_{1p} has no root in the same bounded area. Since the sM in (27) and s0 in (28) are functions of the admittance $\mathbf{A}$, (29) imposes a constraint on $\mathbf{A}$. In summary, we have the following.

Proposition 1

For an edge–vertex contact state, if at the configuration $(\delta, \theta) = (0,0)$, the admittance satisfies the error-reduction condition (2) and condition (29) is satisfied for the configuration boundary points $[\pm\delta_M, \pm\theta_M]$ and the minimum and maximum values of friction coefficient $\mu = 0, \mu_M$, then the admittance will satisfy the error-reduction conditions for all configurations bounded by these four configurations and friction coefficient $\mu \leq \mu_M$.

Thus, for an edge–vertex contact state, to ensure that contact yields error-reducing motion for the body, only four configuration extremals at two extremal coefficients of friction need be tested.

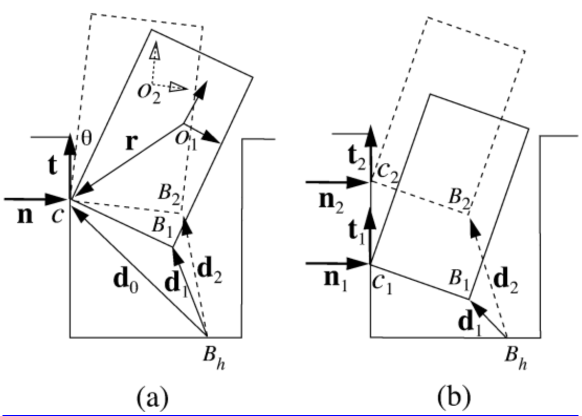

Fig. 6. Vertex–edge contact state. (a) Orientational variation. (b) Translational variation.

B. Vertex–Edge Contact State

As shown in Fig. 3(b), the configuration of the body can be determined by the orientation of the body θ and the location of the contact point δ. We prove that, if an admittance matrix A satisfies a set of conditions at a finite number of configurations for $\mu = 0, \mu_M$, then the $\mathbf{A}$ matrix ensures error-reducing motion for all configurations $\theta \in [-\theta_M, \theta_M]$, $\delta \in [-\delta_M, \delta_M]$, and all coefficients of friction $\mu \in [0, \mu_M]$.

In this case, however, the error-reduction function linearly depends on a single configuration parameter when considering translational variation separately. As a consequence, a somewhat simpler evaluation is used. To use this simpler approach (similar to that used for the frictionless case [11]), we first consider orientational and translational variation separately. Then, by combining the two separate variation cases, sufficient conditions for all configurations within the contact state are obtained.

1) Orientational Variation

Consider only orientation variation as illustrated in Fig. 6(a). In this case, both the direction of the error-reduction vector $\mathbf{d}$ and the direction of the contact wrench $\mathbf{w}$ (in the body frame) are changed by changing the orientation. We prove that, for variation $\theta_M \leq \left(\frac{\pi}{2}\right)$, if $\mathbf{A}$ satisfies a set of conditions at orientation $\theta = 0$, then an error-reducing motion is ensured for all configurations $\theta \in [-\theta_M, \theta_M]$.

Consider a rotational variation of the configuration given by an angle change θ. Let $\mathbf{n}_0$ and $\mathbf{t}_0$ be the unit vectors in the directions of the normal force and friction force, respectively, when $\theta = 0$. Then the unit contact normal and friction wrenches calculated using (15) are

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{Rn}_0 \\ (\mathbf{r} \times \mathbf{Rn}_0) \cdot \mathbf{k} \end{bmatrix}, \mathbf{w}_t = \begin{bmatrix} \mathbf{Rt}_0 \\ (\mathbf{r} \times \mathbf{Rt}_0) \cdot \mathbf{k} \end{bmatrix}$$

(30)

where $\mathbf{r}$ is the position vector from the origin of the body frame to the contact point (constant).

Since all configurations considered correspond to pure rotation about the contact point, the position vector of B relative to its properly mated position for an intermediate configuration can be expressed in the body frame as

$$\mathbf{d}'_\theta = \mathbf{Rd}'_0 + \mathbf{d}'$$

(31)

where $\mathbf{d}'_0$ is the position vector from B_h to the contact point c and $\mathbf{d}'$ is the position vector from c to point B_1. Note that $\mathbf{d}'_0$ is a constant in the global frame and $\mathbf{d}'$ is constant in the body frame. Then, the line vector identifying the position of B relative to its properly mated position B_h (expressed in the body frame) is

$$\mathbf{d}_\theta = \begin{bmatrix} \mathbf{d}_\theta{}' \\ (\mathbf{r}_B \times \mathbf{d}'_\theta) \cdot \mathbf{k} \end{bmatrix}$$

(32)

where $\mathbf{r}_B$ is the vector from the body frame origin to point B.

Since $\mathbf{d}_\theta$, $\mathbf{w}_n$,, and $\mathbf{w}_t$ each involve first-order terms in $\sin\theta$ and $\cos\theta$, the error-reduction function (10) can be expressed as a third-order polynomial in $\sin\theta$ and $\cos\theta$. Further, by the relation $\sin^2\theta = 1 - \cos^2\theta$, the function can be written in the form

$$F_{1p}(\theta) = c_1 \cos^3\theta + c_2 \sin\theta\cos^2\theta + c_3\cos^2\theta \\ + c_4 \sin\theta\cos\theta + c_5\sin\theta + c_6\cos\theta + c_7$$

(33)

where the c_i 's are functions of the admittance matrix $\mathbf{A}$ and the friction coefficient μ having the form

$$c_i = a_i + \mu b_i, i = 1, \dots, 7.$$

(34)

2) Error-Reduction Conditions

To achieve error reduction at all other configurations and for any value of friction less than μ_M considered, $F_{1p}(\theta)$ must be negative for all $\theta \in [-\theta_M, \theta_M]$ and $\mu \in [0, \mu_M]$. Now consider F_{1p} as a function of (θ, μ), then $F_{1p}(\theta, \mu)$ only contains a first-order term in μ. In the following, we first obtain error-reduction conditions for $\theta \in [-\theta_M, \theta_M]$ for both $\mu = 0$ and $\mu = \mu_M$. Then, we prove that the conditions for the extremal friction coefficients ensure error-reducing motion for any intermediate $\mu \in [0, \mu_M]$.

By an appropriate rearrangement, (33) can be written as

$$F_{1p}(\theta,\mu) = (c_1 \cos^3\theta + c_3 \cos^2\theta + c_6 \cos\theta + c_7) \\ +(c_2 \cos^2\theta + c_4 \cos\theta + c_5)\sin\theta\,.$$

(35)

For $\mu = 0, c_i = a_i$. A conservative "more positive" function $F_0^+(\theta)$ for $\theta > 0$ is constructed based on (35) by the following:

- if $a_i > 0$, replace the corresponding $\cos\theta$ with 1 (by setting $\theta = 0$);
- if $a_i < 0$, replace the corresponding $\cos\theta$ with $\cos\theta_M$.

As such, $F_0^+(\theta)$ has the form $F_0^+(\theta) = a + a^+ \sin\theta$.. It can be seen that, for any $0 \le \theta \le \theta_M$, $F(\theta)|_{\mu=0} \le F_0^+(\theta)$..

For $\theta < 0$, a conservative "more positive" function $F_0^-(\theta)$ is constructed based on (35) by the following.

- For the terms involving $\sin\theta$, if $a_i > 0$, replace the corresponding $\cos\theta$ with $\cos\theta_M$; if $a_i < 0$, replace the corresponding $\cos\theta$ with 1.
- For the terms involving only $\cos\theta$, if $a_i > 0$, replace the corresponding $\cos\theta$ with 1; if $a_i < 0$, replace the corresponding $\cos\theta$ with $\cos\theta_M$.

As such, F_0^- has the form $F_0^-(\theta) = a + a^- \sin\theta$.. It can be seen that, for any $-\theta_M \le \theta \le 0$,, we have $F(\theta)|_{\mu=0} \le F_0^-(\theta)$..

Because $\sin\theta$ is a monotonic function over $[-(\pi/2), (\pi/2)]$, $F_0^+(0) < 0$ and $F_0^+(\theta_M) < 0$ ensure that $F_0^+(\theta) < 0$ for all $\theta \in [0, \theta_M]$; $F_0^-(0) < 0$ and $F_0^-(-\theta_M) < 0$ ensure that $F_0^-(\theta) < 0$ for all $\theta \in [-\theta_M, 0]$. Since $F_0^-(0) = F_0^+(0)$, the following set of three inequalities:

$$\begin{aligned} a &< 0 \\ a + a^+ \sin\theta_M &< 0 \\ a - a^- \sin\theta_M &< 0 \end{aligned}$$

(36)(37)(38)

ensures that $F(\theta)|_{\mu=0} < 0$ for all $\theta \in [-\theta_M, \theta_M]$.

Using the same procedure for $F(\theta)|_{\mu=\mu_M}$, two conservative "more positive" functions $F_{\mu_M}^-(\theta)$ and $F_{\mu_M}^+(\theta)$ are constructed as follows:

$$\begin{aligned} F_{\mu_M}^+(\theta) &= e + e^+ \sin\theta \\ F_{\mu_M}^-(\theta) &= e + e^- \sin\theta\,. \end{aligned}$$

Thus, the following set of three inequalities:

$$\begin{aligned} e &< 0 \\ e + e^{+}\sin\theta_M &< 0 \\ e - e^{-}\sin\theta_M &< 0 \end{aligned}$$

(39)(40)(41)

ensures that $F(\theta)|_{\mu=\mu_M} < 0$ for all $\theta \in [-\theta_M, \theta_M]$.

Although inequalities (36)–(38) and (39)–(41) are constructed for two friction coefficients $\mu = 0, \mu_M$, they are sufficient error-reduction conditions for all $\mu \in [0, \mu_M]$. In fact, since the error-reduction function F_{1p} contains only a first-order term in μ, then, for any $\theta \in [-\theta_M, \theta_M]$ and $\mu \in [0, \mu_M]$, we have

$$\begin{aligned} \min\{F_{1p}(\theta,0), F_{1p}(\theta,\mu_M)\} &\le F_{1p}(\theta,\mu) \\ &\le \max\{F_{1p}(\theta,0), F_{1p}(\theta,\mu_M)\}. \end{aligned}$$

(42)

Since the sets of inequalities (36)–(38) and (39)–(41) ensure $F_{1p}(\theta,0) < 0$ and $F_{1p}(\theta,\mu_M) < 0$, thus, from (42), the desired result $F_{1p}(\theta,\mu) < 0$ for $\theta \in [-\theta_M, \theta_M]$, $\mu \in [0, \mu_M]$ is ensured by these inequalities.

3) Translational Variation

Now consider the translational variation of the contact configuration illustrated in Fig. 6(b). In this case, only translation along the edge is allowed, and the contact force does not change in the body frame. The configuration of the body can be determined by a vector d [Fig. 6(b)].

Suppose that, at the two extremal configurations characterized by $\mathbf{d}_1$ and $\mathbf{d}_2$, the error-reduction conditions are satisfied

$$\begin{aligned} \mathbf{d}_1{}^T\mathbf{v}_0 + \mathbf{d}_1{}^T\mathbf{A}\mathbf{w}_1 &< 0 \\ \mathbf{d}_2{}^T\mathbf{v}_0 + \mathbf{d}_2{}^T\mathbf{A}\mathbf{w}_2 &< 0 \end{aligned}$$

(43)(44)

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are total contact wrenches at the two locations c_1 and c_2.

For any $\alpha, \beta \ge 0$, we have

$$(\alpha\mathbf{d}_1 + \beta\mathbf{d}_2)^T\mathbf{v}_0 + (\alpha\mathbf{d}_1 + \beta\mathbf{d}_2)^T\mathbf{A}\mathbf{w} < 0.$$

(45)

At any intermediate configuration, the d vector is expressed as a convex combination of the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$, i.e.,

$$\mathbf{d} = \alpha\mathbf{d}_1 + \beta\mathbf{d}_2$$

(46)

where $\alpha, \beta \ge 0$ and $\alpha + \beta = 1$.

Since the contact wrench $\mathbf{w}$ is the same in the body frame for all contact configurations, $\mathbf{w} = \mathbf{w}_1 = \mathbf{w}_2$. Substituting (46) into (45) yields

$$F_{1p} = \mathbf{d}^T\mathbf{v}_0 + \mathbf{d}^T\mathbf{A}\mathbf{w} < 0.$$

Thus, for translational variation, if at two configurations the error-reduction condition is satisfied, then the error-reduction condition must be satisfied for all intermediate configurations bounded by these two configurations.

Note that the contact wrench $\mathbf{w}_i$'s in (43) and (44) include friction. Because the coefficient of friction μ is linear in F_{1p}, satisfying the error-reduction conditions at $\mu = 0, \mu_M$ ensures that the same conditions are satisfied for all $\mu \in [0, \mu_M]$.

4) General Case

Similar to the frictionless case presented in [11], because of the linear dependence of the error-reduction function on the boundary configurations for the translational-only variation, the results presented in Sections IV-B2 and IV-B3 can be generalized to all configurations within the vertex–edge contact state (i.e., those involving both translational and orientational variation). Thus, we have the following.

Proposition 2

For a vertex–edge contact state with variation of orientation $[-\theta_M, \theta_M]$ and variation of translation $[-\delta_M, \delta_M]$, if at the two configurations with different contact boundary locations $[-\delta_M, \delta_M]$ the admittance satisfies inequalities (36)–(38) and (39)–(41) for $\mu = 0, \mu_M$, then the admittance will satisfy the error-reduction condition for all configurations bounded by the four configurations, $(-\delta_M, -\theta_M), (-\delta_M, \theta_M), (\delta_M, -\theta_M), (\delta_M, \theta_M)$, for all $\mu \in [0, \mu_M]$.

Therefore, for an edge–vertex contact state, to ensure that the motion response due to contact is error reducing for all configurations considered, function values at only two configuration extremals and two coefficients of friction need be tested.

SECTION V. Conditions for Two-Point Contact

In this section, sufficient conditions are obtained for each type of two-point contact state. Below, for each type of contact: 1) the error-reduction function F_{2p} is specified; 2) bounds of the coefficients in F_{2p} are identified; and 3) specific conditions for satisfying error reduction are presented.

Using the notation in Section III-C, we denote the area of bounded configurations as M. Since the contact force $\mathbf{w}_i$ contains only a linear term in the friction coefficient μ_i, the error-reduction function F_{2p} in (13) contains only linear and quadratic terms in μ_i.

A. Conditions for $\{e - v, v - e\}$ Contact

In this case, the two-point contact wrench is a combination of the two corresponding single-point contact wrenches.

Using the notation developed in Section III-B, the contact wrenches for the $\{v - e\}$ and $\{e - v\}$ contact are obtained by (15) and (14), respectively, as follows:

$$
\begin{aligned}
\mathbf{w}_{n1} &= \begin{bmatrix} \mathbf{Rn}_1 \\ (\mathbf{r}_1 \times \mathbf{Rn}_1) \cdot \mathbf{k} \end{bmatrix} \\
\mathbf{w}_{t1} &= \begin{bmatrix} \mathbf{Rt}_1 \\ (\mathbf{r}_1 \times \mathbf{Rt}_1) \cdot \mathbf{k} \end{bmatrix} \\
\mathbf{w}_{n2} &= \begin{bmatrix} \mathbf{n}_2 \\ (\mathbf{r}_2 \times \mathbf{n}_2) \cdot \mathbf{k} \end{bmatrix} \\
\mathbf{w}_{t2} &= \begin{bmatrix} \mathbf{t}_2 \\ (\mathbf{r}_2 \times \mathbf{t}_2) \cdot \mathbf{k} \end{bmatrix}
\end{aligned}
$$

where $\mathbf{R}$ is the rotation matrix defined in (16) and $\mathbf{r}_2$ has the form

$$\mathbf{r}_2 = \mathbf{r}_{02} + \mathbf{r}_{e2}\delta$$

where $\mathbf{r}_{02}$ and $\mathbf{r}_{e2}$ are constant vectors associated with contact point 2 described in (14).

Since $\mathbf{w}_{n1}$ and $\mathbf{w}_{t1}$ contain only first-order terms in $\cos\theta$ and $\sin\theta$, and $\mathbf{w}_{n2}$ and $\mathbf{w}_{t2}$ contain only a linear term in δ, for a given μ_1 and μ_2, the error-reduction function (13) can be expressed as a function of (θ, δ) in the form

$$F_{2p}(\theta, \delta) = q_2\delta^2 + q_1\delta + q_0$$

(47)

where q_i's are functions of θ having the form

$$q_i = a_i \cos^2\theta + b_i \cos\theta \sin\theta + c_i.$$

(48)

For a given μ_1 and μ_2, if at a given configuration within the range considered error reduction is satisfied: $F_{2p} < 0$, then, in order for all θ and δ within the specified range to satisfy the condition, we need to obtain conditions such that F_{2p} has no root in this range.

1) Bounds on the Coefficients

In order to analyze the root of the function F_{2p} for a given μ, we evaluate the bounds on the coefficients q_i's in (48).

First, consider the term involving $\cos\theta$ alone ($a_i \cos^2\theta$) in (48). If we denote

$$
\begin{aligned}
p_{i1}^{+} &= \max\{a_i \cos^2\theta_M, a_i\} \\
p_{i1}^{-} &= \min\{a_i \cos^2\theta_M, a_i\}
\end{aligned}
$$

then for $\forall\theta \in [-\theta_M, \theta_M]$ and $i = 0,1,2$, we have

$$p_{i1}^{-} \le a_i \cos^2\theta \le p_{i1}^{+}.$$

Consider the term involving $\sin\theta$ ($b_i \cos\theta \sin\theta$) in (48). Denote

$$
\begin{aligned}
p_{i2}^{+} &= \max\{0, b_i \sin\theta_M, b_i \cos\theta_M \sin\theta_M\} \\
p_{i2}^{-} &= \min\{0, b_i \sin\theta_M, b_i \cos\theta_M \sin\theta_M\}.
\end{aligned}
$$

Then, for $\forall\theta \in [-\theta_M, \theta_M]$ and $i = 0,1,2$, we have

$$p_{i2}^{-} \le b_i \cos\theta \sin\theta \le p_{i2}^{+}.$$

Thus

$$(p_{i1}^{-} + p_{i2}^{-} + c_i) \le q_i \le (p_{i1}^{+} + p_{i2}^{+} + c_i).$$

If we denote

$$\begin{aligned} q_{Mi} &= p_{i1}^{+} + p_{i2}^{+} + c_i \\ q_{mi} &= p_{i1}^{-} + p_{i2}^{-} + c_i \end{aligned}$$

(49)(50)

then the bounds for q_i's are determined as follows:

$$q_{mi} \le q_i \le q_{Mi}, i = 0,1,2$$

(51)

where all q_{mi}'s and q_{Mi}'s are functions of the admittance matrix (independent of the configuration).

2) Sufficient Conditions for Given Friction Coefficients

Since the bounds of q_i's are determined, a single-variable polynomial is constructed for which the method used for single-point contact case (Section IV-A2) is applied.

First, the error-reduction condition must be satisfied at one configuration in the range considered [say, at $(\theta, \delta) = (0,0)$].

To consider all configurations, we construct a polynomial given by

$$P_{2p}(\delta) = q_{M2}\delta^2 + q_{M1}\delta + q_{m0}$$

(52)

where the coefficients are constants defined in (49) and (50). Denote $q_M = \max\{|q_{M2}|, |q_{M1}|\}$.. It is proved [11] that if

$$\frac{|q_{m0}|}{q_M + |q_{m0}|} > \delta_M$$

(53)

then $P_{2p}(\delta)$ has no root in $[-\delta_M, \delta_M]$. Since the coefficients of $P_{2p}(\delta)$ are extremal values of q_i's in the range considered, condition (53) ensures that the function $F_{2p}(\theta, \delta)$ in (47) has no root in $[-\delta_M, \delta_M]$ for any given $\theta \in [-\theta_M, \theta_M]$. In fact, for a given $\theta \in [-\theta_M, \theta_M]$, $F_{2p}(\theta, \delta)$ is a polynomial in δ. As shown in [11], a root of $F_{2p}(\theta, \delta)$, δ_θ, must satisfy

$$\delta_\theta \ge \max\left\{\frac{|q_0|}{|q_1| + |q_0|}, \frac{|q_0|}{|q_2| + |q_0|}\right\} \ge \frac{|q_{m0}|}{q_M + |q_{m0}|} > \delta_M$$

which ensures that $F_{2p}(\theta, \delta)$ has no root in $[-\delta_M, \delta_M]$. Thus, if for a given set of μ_i 's (53) is satisfied, then F_{2p} does not change sign for all configurations (θ, δ) within the range $\theta \in [-\theta_M, \theta_M]$ and $\delta \in [-\delta_M, \delta_M]$.

3) Sufficient Conditions for Arbitrary μ_i 's

Now consider arbitrary $\mu_1 \leq \mu_{M1}$ and $\mu_2 \leq \mu_{M2}$. In order for all μ_i's to satisfy the error-reduction condition, additional conditions must be considered. As stated previously, F_{2p} contains only linear and quadratic terms in μ_i. Thus, F_{2p} can be expressed as

$$F_{2p} = h_1\mu_1^2 + h_2\mu_1\mu_2 + h_3\mu_2^2 + h_4\mu_1 + h_5\mu_2 + h_6$$

(54)

where h_i's are functions of (θ, δ) and have the form

$$h_i = q_{2i}\delta^2 + q_{1i}\delta + q_{0i}$$

where q_{ji}'s have the same form as q_i's in (48).

For a given configuration, the shape of F_{2p} is determined by its Hessian matrix $H(F_{2p})$. Suppose that, at the four point $(0,0)$, $(\mu_{M1}, 0)$, $(0, \mu_{M2})$ and (μ_{M1}, μ_{M2}), the error-reduction function $F_{2p} < 0$. Based on the property of the Hessian matrix $H(F_{2p})$, sufficient conditions are obtained for the following cases.

a) Positive Definite $H(F_{2p})$

In this case, for all $\mu_1 \in [0, \mu_{M1}]$ and $\mu_2 \in [0, \mu_{M2}]$, $F_{2p} < 0$. A positive definite $H(F_{2p})$ requires $h_1 > 0, 4h_1h_3 - h_2^2 > 0$.. Note that the h_i's are quadratic in δ having the same form of (47), and the bounds of h_1 and h_2 within the range $[\pm\theta_M, \pm\delta_M]$ can be determined. If we denote

$$h_i = q_{2i}\delta^2 + q_{1i}\delta + q_{0i}$$

where q_{ji} has the same form as q_i in (48), then, using the same process for q_i in Section V-A1, the upper and lower bounds of q_{ji}, q_{Mji}, and q_{mji} can be determined by (49) and (50). Consider the values of h_i when the coefficients q_{ji} and δ take their bound values, then we have a set S with a finite number of elements $S_i = \{q_{2i}\delta_M^2 \pm q_{1i}\delta_M + q_{0i}: q_{ji} = q_{Mji}, q_{mji}\}$.. If we denote $h_{Mi} = maxS_i, h_{mi} = minS_i$, then, for all configurations considered, $h_{mi} \leq h_i \leq h_{Mi}$. Thus, the condition

$$h_{m1} > 0, 4h_{m1}h_{m3} - h_{M2}^2 > 0$$

(55)

ensures that F_{2p} does not change sign for all $\mu_i \in [0, \mu_{Mi}]$ and for all configurations within the range considered.

b) Negative Definite $H(F_{2p})$

In this case, the stationary point of F_{2p} is determined by setting

$$\frac{\partial F_{2p}}{\partial \mu_1} = 0, \frac{\partial F_{2p}}{\partial \mu_2} = 0.$$

By solving the two linear equations, the stationary point of F_{2p}, (μ_{s1}, μ_{s2}), is obtained as follows:

$$\mu_{s1} = f_1(\theta, \delta), \mu_{s2} = f_2(\theta, \delta).$$

Since θ and δ are bounded, the lower bound for f_i in the considered configuration range can be obtained. Let μ_{mi} be lower bounds of μ_{si} for all of the configurations considered. Then, the condition

$$\mu_{mi} \geq \mu_{Mi}$$

(56)

ensures that F_{2p} does not change sign for all $\mu_i \in [0, \mu_{Mi}]$ and for all configurations within the range considered.

c) Indefinite $H(F_{2p})$

In this case, for a given configuration, the maximum value of F_{2p} must occur on the boundary determined by $[0, \mu_{Mi}]$. Thus, if along the four line segments

$$\{L_1: \mu_2 = 0, \mu_1 \in [0, \mu_{M1}]\}, \{L_2: \mu_1 = 0, \mu_2 \in [0, \mu_{M2}]\},$$
$$\{L_3: \mu_2 = \mu_{M2}, \mu_1 \in [0, \mu_{M1}]\},$$
$$\{L_4: \mu_1 = \mu_{M1}, \mu_2 \in [0, \mu_{M2}]\}, H(F_{2p})$$

has no root, then $H(F_{2p})$ does not change sign for all μ_i for the given configuration.

Now consider the boundary segment $\{L_1: \mu_2 = 0, \mu_1 \in [0, \mu_{M1}]\}$. The function F_{2p} can be expressed as $F_{2p} = h_{i1}'^{\mu_1^2} + h_{i2}'^{\mu_1} + h_{i3}'$.. The stationary point of F_{2p} is determined by

$$\mu_{s1} = -\frac{h'_{i2}}{2h'_{i1}}.$$

If F_{2p} has the same sign for $\mu_1 = 0, \mu_{M1}$ and μ_{s1} is not in the interval $[0, \mu_{M1}]$, then F_{2p} does not change sign over $[0, \mu_{M1}]$. If we denote h'_{mi} and h'_{Mi} as the lower and upper bounds of h'_{i2} and h'_{i1}, respectively, then the condition

$$\left|\frac{h'_{mi}}{2h'_{Mi}}\right| > \mu_{M1}$$

ensures that F_{2p} does not change sign for all $\mu_1 \in [0, \mu_{M1}]$ and for all configurations within the range considered. Applying the same process to all four line segments, we have the set of conditions

$$\left|\frac{h'_{mi}}{2h'_{Mi}}\right| > \mu_{Mi}, i = 1, \ldots, 4.$$

(57)

In summary, we have the following proposition.

Proposition 3

For an $\{e - v, v - e\}$ contact state, if: 1) at a configuration (δ_0, θ_0) with area M, the admittance satisfies the error-reduction condition (19) 2) condition (53) is satisfied at the extremal values of friction coefficients $(\mu_1, \mu_2) = (0,0), (\mu_{M1}, 0), (0, \mu_{M2})$, and (μ_{M1}, μ_{M2}); and 3) the appropriate conditions in either (55), (56), or (57) are satisfied, then the admittance will satisfy the error-reduction conditions for all configurations in M and friction coefficient $\mu_i \leq \mu_{Mi}$.

B. Conditions for $\{e - v, e - v\}$ Contact State

In this case, the two-point contact wrench is a combination of the two $\{e - v\}$ contact wrenches.

Using the notation developed in Section III-B, the contact wrenches for the two $\{e-v\}$ contacts are obtained by (14) as follows:

$$\begin{aligned}\mathbf{w}_{ni} &= \begin{bmatrix}\mathbf{n}_i \\ (\mathbf{r}_i \times \mathbf{n}_i)\cdot\mathbf{k}\end{bmatrix} \\ \mathbf{w}_{ti} &= \begin{bmatrix}\mathbf{t}_i \\ (\mathbf{r}_i \times \mathbf{t}_i)\cdot\mathbf{k}\end{bmatrix}\end{aligned}$$

where $\mathbf{r}_i = \mathbf{r}_{0i} + \mathbf{r}_{ei}\delta_i$ and where $\mathbf{r}_{0i}$ and $\mathbf{r}_{ei}$ are constant vectors associated with edge i.

For a given μ_1 and μ_2, the error-reduction function (13) can be expressed in terms of two variables δ_1 and δ_2 as follows

$$\begin{aligned}F(\delta_1,\delta_2) = (a_2\delta_1^2 + b_2\delta_1 + c_2)\delta_2^2 + (a_1\delta_1^2 + b_1\delta_1 + c_1)\delta_2 \\ +(a_0\delta_1^2 + b_0\delta_1 + c_0)\end{aligned} \tag{58}$$

where the a_i's, b_i's, and c_i's are all functions of the admittance matrix $\mathbf{A}$.

Denote

$$q_i(\delta_1) = a_i\delta_1^2 + b_i\delta_1 + c_i, i = 0,1,2.$$

(59)

Since the q_i's are quadratic functions, it is not difficult to determine their extreme values for $\delta_1 \in [-\delta_{1M}, \delta_{1M}]$. In fact, if we denote

$$\begin{aligned}q_{mi} &= min\{q_i(-\delta_{1M}), q_i(\delta_{1M}), q_i(-\frac{b_i}{2a_i})\} \\ q_{Mi} &= max\{q_i(-\delta_{1M}), q_i(\delta_{1M}), q_i(-\frac{b_i}{2a_i})\} \\ q_M &= max\{q_{M1}, q_{M2}\}\end{aligned}$$

then, for all $\delta_1 \in [-\delta_{1M}, \delta_{1M}]$, $q_{mi} \leq q_i \leq q_{Mi}$.. By the same reasoning used for the $\{e-v, v-e\}$ contact case addressed in Section V-A2, a similar condition is obtained as follows:

$$\frac{|q_{m0}|}{q_M + |q_{m0}|} > \delta_M.$$

(60)

This condition ensures that, for the given μ, $F_{2p}(\delta_1, \delta_2)$ has no root for all $\delta_1 \in [-\delta_{1M}, \delta_{1M}]$ and $\delta_2 \in [-\delta_{2M}, \delta_{2M}]$.

Now consider $\mu_i \leq \mu_{Mi}$. Similar to the $\{e-v, v-e\}$ contact case, F_{2p} can be expressed as a quadratic polynomial in μ_1 and μ_2 having the form of (54). Thus, the same process used for the $\{e-v, v-e\}$ contact case applies to the $\{e-v, e-v\}$ contact case. Three sets of conditions can be obtained based on the definiteness of $H(F_{2p})$. If we use the same notation as used in Section V-A, the three sets conditions have the forms of (55) – (57) for positive and negative definite and indefinite $H(F_{2p})$, respectively. Therefore, we have the following.

Proposition 4

For an $\{e-v, e-v\}$ contact state, if: 1) at a configuration $[\delta_0, \theta_0]$ with area M, the admittance satisfies the error-reduction condition (19); 2) condition (60) is satisfied at the extremal values of friction coefficient $(\mu_1, \mu_2) = (0,0), (\mu_{M1}, 0), (0, \mu_{M2})$, and (μ_{M1}, μ_{M2}); and 3) the appropriate conditions in either (55), (56), or (57) for this contact are satisfied, then the admittance will satisfy the error-reduction conditions for all configurations in M and friction coefficient $\mu_i \leq \mu_{Mi}$.

C. Conditions for the $\{v-e, v-e\}$ Contact State

In this case, the two-point contact wrench is a combination of the two $\{v-e\}$ contact wrenches.

Using the notation developed in Section III-B, the contact wrenches for the two $\{v-e\}$ contact are obtained by (15) as follows:

$$\begin{aligned} \mathbf{w}_{ni} &= \begin{bmatrix} \mathbf{Rn}_i \\ \mathbf{r}_i \times \mathbf{Rn}_i \end{bmatrix} \\ \mathbf{w}_{ti} &= \begin{bmatrix} \mathbf{Rt}_i \\ \mathbf{r}_i \times \mathbf{Rt}_i \end{bmatrix} \end{aligned}$$

where $\mathbf{R}$ is the rotation matrix associated with θ.

The error-reduction function (13) can be used directly. For a given μ_1 and μ_2, since the wrenches involve only one variable θ, the error-reduction function is a single-variable function in the form

$$\begin{gathered} F_{2p}(\theta) = a_1 \cos^4\theta + a_2 \cos^3\theta \sin\theta + a_3 \cos^2\theta \sin\theta \\ + a_4 \cos\theta \sin\theta + a_5 \cos\theta + a_6 \sin\theta + a_0 \end{gathered}$$

where ai is a function of the admittance matrix $\mathbf{A}$.

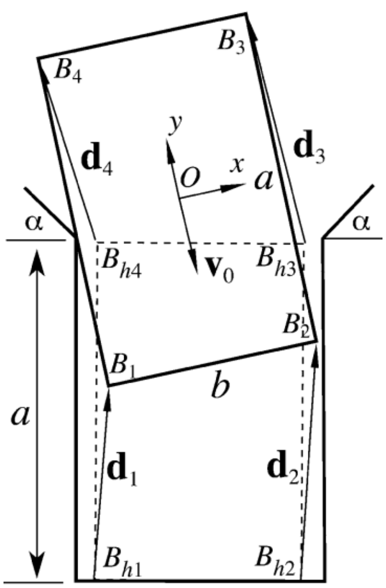

Fig. 7. Example: the four vertices of the body are chosen as the feature points for the error measure.

If we denote

$$\begin{aligned} p_1(\theta) &= a_1 \cos^4\theta + a_5 \cos\theta + a_0 \\ p_2(\theta) &= a_2 \cos^3\theta + a_3 \cos^2\theta + a_4 \cos\theta + a_6 \end{aligned}$$

then $f(\theta)$ can be expressed as $F_{2p}(\theta) = p_1(\theta) + p_2(\theta)\sin\ \theta$.. Since $|\theta| \le \theta_M \le (\pi/2)$, the bounds for the single-variable functions $p_1(\theta)$ and $p_2(\theta)$ can be obtained by the approach used for the single-point contact case (Section IV-B2).

Let p_{mi} and p_{Mi} be the bounds of p_i, i.e.,

$$\begin{aligned} p_{m1} &\le p_1 \le p_{M1} \\ p_{m2} &\le p_2 \le p_{M2} \\ p_M &= max\{|p_{2m}|, |p_{2M}|\}. \end{aligned}$$

Then, it can be proved that the conditions

$$\begin{aligned} p_{1m} - p_M \sin\ \theta_M &< 0 \\ p_{1m} + p_M \sin\ \theta_M &< 0 \end{aligned}$$

(61)(62)

ensure that, for all $\theta \in [-\theta_M, \theta_M], F_{2p}(\theta) < 0$.

Similar to the previous two-point contact cases, F_{2p} can be expressed as a quadratic polynomial in μ_1 and μ_2 in the form of (54). Using the same procedure presented in Sections V-A and V-B, three sets of conditions for intermediate $\mu_i \le \mu_{Mi}$ can be obtained. Thus, we have the following proposition.

Proposition 5

For a $\{v-e, v-e\}$ contact state, if: 1) at a configuration $[\delta_0, \theta_0]$ with area M, the admittance satisfies the error-reduction condition (19); 2) conditions (61)–(62) are satisfied at the extremal values of friction coefficients $(\mu_1, \mu_2) = (0,0), (\mu_{M1}, 0), (0, \mu_{M2})$, and (μ_{M1}, μ_{M2}); and 3) the appropriate conditions in either (55), (56), or (57) for this contact state are satisfied, then the admittance will satisfy the error-reduction conditions for all configurations in M and friction coefficient $\mu_i \le \mu_{Mi}$.

SECTION VI. Numerical Example

In this section, a numerical example is provided to demonstrate how the obtained sufficient conditions can be used in finding an admittance matrix $\mathbf{A}$ and to demonstrate the power of the force-assembly approach.

Consider the peg-in-hole problem illustrated in Fig. 7. The peg is rectangular with length $a = 3$ and width $b = 2$. The chamfer length is 0.25 and chamfer angle $\alpha = 45°$. The width of the hole is 2.2. The body frame Oxy is located at the center of the peg with x and y axes parallel to the edges of the peg. The nominal motion of the peg is along the body-frame y direction, $\mathbf{v}_0 = [0, -1, 0]^T$.

The four vertices of the peg, $B_i (i = 1, \ldots, 4)$, were chosen to be "feature points," points at which the error-reduction conditions are imposed. Let B_{hi} be the location of B_i when the peg is properly mated (as shown in Fig. 7), and $\mathbf{d}_i$ be the position vector from B_{hi} to B_i. By the error-reduction requirement for single-point contact, all $\mathbf{d}_i$'s must be monotonically reduced, i.e., condition (2) must be satisfied for all $\mathbf{d}_i$'s.

Here, the largest orientation variation considered for both single-point and two-point contact is 10° (i.e., $\theta \in [-\theta_M, \theta_M], \theta_M = 5°$). For single-point contact, the "boundary configurations" are chosen based on the extremal locations of the contact point within the contact state and the largest angular variations considered. For example, when considering the $\{v-e\}$ contact state on the chamfer, the four "boundary configurations" are the configurations where the contact points are at the two ends of the chamfer and at the two largest orientation variations $\pm\theta_M$. For two-point contact ($\{e-v, v-e\}$ contact state), the variation for δ is 1 (i.e., $\delta \in$

$[-\delta_M, \delta_M], \delta_M = 0.5)$ and the largest friction coefficients considered at the two contact locations are the same, i.e., $\mu_{M1} = \mu_{M2} = \mu_M$.

To ensure force assembly, we need to find an admittance matrix $\mathbf{A}$ that:

1. yields misalignment-reducing motion for any friction coefficient less than the (initially unknown) maximum friction coefficient μ_M;
2. yields misalignment-reducing motion for all one-point and two-point contact configurations, i.e.,

$$\begin{aligned} F_{1p}(\mathbf{A}, \mu_M) &< 0, \text{for all one-point contact} \\ F_{2p}(\mathbf{A}, \mu_M) &< 0, \text{for all two-point contact.} \end{aligned}$$

(63)(64)

Note that conditions (63) and (64) would have to be imposed on an infinite number of configurations. Using the sufficient conditions presented in *Propositions 1–5*, inequalities (63) and (64) can be replaced by a set of inequalities at a finite number of configurations. For example, for an $\{e - v\}$ contact case, the conditions in *Proposition 1* require that inequality (29) be satisfied at four configuration extremals and two extremal coefficients of friction. For this contact state, eight inequality constraints are imposed on $\mathbf{A}$ for each error-measure vector $\mathbf{d}_i$. Thus, for a single $\{e - v\}$ contact, 32 constraints are needed for the four error-measure vectors $\mathbf{d}_i$. The constraints for all other contact states can be imposed in the same way. Since all constraints are inequalities, a standard optimization can be performed. For this problem, conditions are imposed for all possible configurations, yielding a total of 296 constraints.

In the optimization, the design variables are the elements of $\mathbf{A}$ and μ_M. The objective function is defined as $F = \mu_M$..

The optimization described below was performed using MATLAB and its optimization toolbox:

1. maximize F;
2. subject to error-reduction constraints at all boundary configurations for single-point and two-point contact.

The maximum value of μ_M, obtained from the optimization, was

$$\mu_M = 0.8314$$

and the admittance matrix obtained was

$$\mathbf{A} = \begin{bmatrix} 24.2029 & -1.6472 & -2.6031 \\ -1.6472 & 35.7328 & 0.0004 \\ -2.6031 & 0.0004 & 0.5331 \end{bmatrix}.$$

Since the conditions for all one-point and two-point contact cases are imposed simultaneously, for any $\mu_1, \mu_2 \in [0, \mu_M]$, the obtained admittance $\mathbf{A}$ ensures error-reducing motion of the peg for all possible part misalignments within the range of orientational misalignment of $\pm 5°$ and translational misalignment of the chamfer width. Note that the friction coefficient obtained is higher than the static friction coefficient for clean steel on steel (0.58) [15].

SECTION VII. Discussion and Summary

In this paper, we identified procedures for selecting the appropriate admittance to achieve reliable planar force-guided assembly for multipoint contact cases with friction. Conditions imposed on the admittance matrix for each of the various types of one-point and two-point frictional contact are presented. We show that, for bounded misalignments, if the conditions are satisfied for a finite number of contact configurations and friction coefficients, the conditions ensure that force guidance is achieved for all configurations and friction coefficients within the specified bounds.

In Section II-A, it was assumed that inequality (8) is satisfied. In fact, due to the continuity of the motion and in order to avoid singularity, this inequality is required for determining the constrained motion of the rigid body. As such, in selecting an admittance, inequality (8) is used as an optimization constraint.

In this paper, we used a type of *measure* of rigid body misalignment based on the Euclidean distance between several fixed points on the held body and their locations when properly positioned. The sufficient conditions obtained are all based on this measure. If a different type of measure is used, the restrictions on $\mathbf{A}$ would be different and the results (optimal $\mathbf{A}$ and μ_M) would, most likely, also be different.

The conditions obtained are sufficient conditions as long as the misalignments are within the user-specified bounds δ_M and θ_M. Although the identified conditions have been mathematically proven to be sufficient over a large range $[-(\pi/2), (\pi/2)]$ in orientation), larger values within this range yield more restrictive conditions on $\mathbf{A}$. Overly conservative estimates of part misalignments yield overly conservative conditions on $\mathbf{A}$. Therefore, in practice, δ_M and θ_M should be determined by robot inaccuracy and/or part geometry and not be overly conservative.

In ongoing work, we are extending these results to address the design of the appropriate admittance matrix for force assembly of a polyhedral rigid body in spatial motion. However, since the kinematic description of a spatial body is significantly more difficult than that of a planar part, a different way to characterize and bound misalignments of a spatial part is needed.

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