

# Essence and Necessity, and the Aristotelian Modal Syllogistic: A Historical and Analytical Study

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ESSENCE AND NECESSITY, AND THE ARISTOTELIAN MODAL SYLLOGISTIC:  
A HISTORICAL AND ANALYTICAL STUDY

by

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ABSTRACT  
ESSENCE AND NECESSITY, AND THE ARISTOTELIAN MODAL SYLLOGISTIC:  
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The following is a critical and historical account of Aristotelian Essentialism informed by recent work on Aristotle's modal syllogistic. The semantics of the modal syllogistic are interpreted in a way that is motivated by Aristotle, and also make his validity claims in the *Prior Analytics* consistent to a higher degree than previously developed interpretative models. In Chapter One, ancient and contemporary objections to the Aristotelian modal syllogistic are discussed. A resolution to apparent inconsistencies in Aristotle's modal syllogistic is proposed and developed out of recent work by Patterson, Rini, and Malink. In particular, I argue that the semantics of negation is distinct in modal context from those of assertoric negative claims. Given my interpretive model of Aristotle's semantics, in Chapter Two, I provide proofs for each of the mixed apodictic syllogisms, and propose a method of using Venn Diagrams to visualize the validity claims Aristotle makes in the *Prior Analytics*. Chapter Three explores how Aristotle's syllogistic fits within Aristotle's philosophy of science and demonstration, particularly within the context of the *Posterior Analytics*. Consideration is given to the Aristotelian understanding of the relationship among necessity, explanation, definition, and essence. Chapter Four applies Aristotelian modal logic in contemporary contexts. I contrast Aristotelian modality and essentialism with contemporary modalism based upon the semantics of possible worlds, e.g. Kripke and Putnam. I also develop an account of how Aristotelian modal logic can ground a sortal dependent theory of identity, as discussed by Wiggins.

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## ABBREVIATIONS OF THE WORKS OF ARISTOTLE

<i>APo.</i>	<i>Posterior Analytics</i>
<i>APr.</i>	<i>Prior Analytics</i>
<i>Cat.</i>	<i>Categories</i>
<i>De An.</i>	<i>de Anima</i>
<i>De In.</i>	<i>de Interpretatione</i>
<i>Meta.</i>	<i>Metaphysics</i>
<i>PA</i>	<i>de Partibus Animalium</i>
<i>Phys.</i>	<i>Physics</i>
<i>Top.</i>	<i>Topics</i>

## CHAPTER ONE

### 1.0 Introduction

Aristotle's modal logic is considered by many commentators to be a murky mess. The oft repeated phrase on this subject, coined by Günther Patzig, is that the modal syllogistic is "a realm of darkness" (1968, 86 fn.21). Patzig was specifically referencing the attempt by Albrecht Becker to interpret Aristotle's use of "modal operators" in the "traditional way". This is, by no means, a recent condemnation. Alexander of Aphrodisias reports that Aristotle's earliest commentators disagreed with many of his supposed valid forms of argument. Theophrastus and Eudemus thought that no valid mixed apodictic syllogism, a syllogism with one apodictic premise and one non-apodictic premise, could have an apodictic conclusion. Rather, they insisted that the weaker modality of the premises should rule over the conclusion.

Contemporary commentators condemn Aristotle for confusing *de re* and *de dicto* modal contexts. The difference between the contexts is a matter of where one places the "modal operator" relative to the terms and quantifiers in a logical expression. For instance, a *de re* modal claim modifies the predicate term as in "all men are necessarily mortal," whereas in *de dicto* modal claims, the operator modifies the entire proposition, as in "necessarily all men are mortal."

Many commentators, like Hintikka, follow the work of Becker in supposing that Aristotle's failure to distinguish these contexts led him to falsely identify forms

of the syllogism as valid. The most often discussed forms discussed are Barbara-LXL and Barbara-XLL. The name “Barbara” is one of several names developed by medieval commentators as mnemonic devices to identify valid syllogisms. Each vowel in the names represents the sort of categorical proposition used in the argument. Barbara is a “perfect syllogism” of the first figure, with two universal affirmative premises that lead to a universal conclusion. In Barbara-LXL, the major premise is apodictic and the minor premise is assertoric, while in Barbara-XLL the major premise is assertoric and the minor premise is apodictic. Aristotle argues that Barbara-LXL is valid while Barbara-XLL is invalid. In fact, according to Aristotle, all syllogisms of the first figure with apodictic major premises and assertoric minor premises validly conclude to an apodictic proposition, which is to say that premises lead to the conclusion that the major term belongs to the minor term by necessity. Likewise, all first figure syllogisms with assertoric major premises and apodictic minor premises do not validly lead to an apodictic conclusion. Aristotle makes use of valid first figure modal syllogism to prove the validity of second and third figure syllogisms. Hence, Lagerlund (2000, 12) refers to the “Two Barbaras” as the test for all interpretations of Aristotle, as it leads to a deeper understanding of Aristotelian modality generally. The literature dedicated to uncovering why Aristotle thinks this is so has come to be called the “Problem of the Two Barbaras” (see, for instance, Thom 1991; Patterson 1995, 75-80, 87-123; McCall 1963, 10-13). One should note that a coherent explication of the “two Barbaras” does not unravel all of the supposed inconsistencies found within the *Prior Analytics*. Rather, it is an indicator as to whether one has taken the first steps in the right direction of understanding

the Aristotelian conception of modality. So, I endeavor to take this step, and a few more with the aid of recent work by Adriane Rini (2011) and Marko Malink (2013) who, building on earlier work of Paul Thom (1996) and Richard Patterson (1995) have devised interpretive models for the syllogistic that resolve many of the apparent problems that lurk among the various modal forms.

I argue that an interpretative model of the modal syllogistic can be devised that is both faithful to Aristotle's claims, and philosophically fruitful in understanding Aristotelian metaphysics and philosophy science. Consequently, working out such an interpretative model provides us with both historical and contemporary Aristotelianism. Also, I have devised a method of diagramming modal syllogisms, which shall aid us in visually grasping the validity and invalidity of various forms of argument. In what I consider to be my most significant contribution to the literature on Aristotle's modal syllogistic, I advance a method of proof for the modal syllogistic informed by Rini's attempt to translate the modal proofs into predicate logic. I concur with Malink's heterodox interpretation of proof for modal propositions, which requires a modification to Rini's translations. Based on Malink's heterodox interpretation, and rules of conversion set forth by Aristotle, I devise definitions for the four categorical propositions and their apodictic counterparts. These definitions are largely based upon Malink's 2013 work, *Aristotle's Modal Syllogistic*, though I offer some modifications that allow for straightforward metalogical proofs for the canonical listing of valid pure and mixed apodictic syllogisms. The proofs are metalogical in the sense that they are proofs about the validity of proofs that use classical rules of induction.

My primary innovation has to do with the treatment of negation in apodictic and assertoric propositions. The negation connective, the tilde, as opposed to the complementary class, represented by a term letter with an over-bar, features in unraveling the apparent inconsistencies other commentators have struggled to make consistent. Rini, for instance, believes that Aristotle has made a subtle mistake in denying the validity to Baroco-LXL and Baroco-XLL, two syllogistic moods that involve negative premises (Rini 2011, 88). Like Malink, I propose a way to resolve the apparent inconsistencies for the mixed apodictic argument forms. Malink's solution comes from working out the meaning of  $o_L$ -predication, of which he writes that it is, "...somewhat complex and technical; it provides more a technical ad hoc solution than an independently motivated notion of  $[o_L]$ -predication (2013, 186).<sup>1</sup> To properly articulate his notion of  $o_L$ -predication, Malink must delve into defining contingency and possibility. However, I believe that my own interpretation is easily understood, and grounded in an Aristotelian discussion of negation, opposition, privation, and complementarity. Having worked this out in the First Chapter, I work through a series of proofs and a method of Venn Diagramming the arguments in the Second Chapter to verify that the interpretation coheres with Aristotle's claims in the *Prior Analytics*.

In the Third Chapter, I discuss how the modal syllogistic applies to Aristotle's philosophy of science as found in the *Posterior Analytics*, and in particular, the ways in which Aristotle connects definition, explanation, and essence to scientific

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<sup>1</sup> Work on modal syllogistics containing possibility and contingency will be briefly discussed, but a thorough treatment of all forms and discussion of whether they are consistent is beyond the focus of this project.

knowledge. The aim is to argue that Aristotelian modality is implicit in Aristotle's discussion of scientific knowledge. Importing contemporary notions of modality back onto Aristotle is not merely anachronistic, but risks misinterpreting how he conceives of science as a project by which the mind comes to understand the essences of things. I apply this new understanding of Aristotle's philosophy of science to Aristotle's biological works. I also consider ways in which Aristotelian philosophy of science, essentialism, and modalism can have applications in contemporary philosophy of science contexts. In this way, I show that a proper understanding of Aristotle's modal syllogistic is not just philosophically fruitful in properly understanding Aristotle's contributions to various fields of the natural science, and biology in particular, but I also argue for the continued philosophical usefulness of the Aristotelian perspective on essences in contemporary contexts.

The discussion in Chapter Four is of the application of Aristotle's philosophy of science, essentialism, and modalism to contemporary philosophy science, with its essentialist tendencies, leads naturally to a discussion of the comparison of Aristotelian modality and essentialism to what Kit Fine has termed, contemporary modalism, i.e. that "ordinary modal idioms (necessarily, possibly) are primitive... Only actual objects exist" (2005, 133). David Oderberg, who utilizes Fine's critique of modalism in the development of his own neo-Aristotelian account of essences, notes that modalism relies on the semantics of possible worlds, and so essences are reduced to rigid designators, terms that pick out the same objects across possible worlds (2007, 4). So, with a coherent account of necessity in Aristotle's modal syllogistic, I argue that Aristotelian essentialism is strengthened and provides an

intuitive account of essences, necessity, definition, and explanation independent of possible worlds, and the various questions they raise.

### 1.1 Symbolization:

Philosophers and commentators have adopted various ways to symbolize Aristotle's modal syllogistic. For the sake of convenience, we shall adopt conventions largely based upon Ignacio Angelelli's isomorphism to Aristotle's Greek (1979). This reinforces the fact that one should not automatically assume syntactical isomorphism of Aristotelian modal propositions and modal propositions represented in modern predicate logic. *A*, *B*, and *C* will be used to represent the terms of the syllogism. Unless otherwise indicated, *A* shall represent the major, *B* the middle, and *C* the minor in most of the proofs in the first chapter. The four categorical sentences will be represented by: a, e, i, and o. Assertoric and modal contexts will be represented by X, L, Q, and M: assertoric, necessary, contingent and possible predication respectively. Thus, for example, "Aa<sub>L</sub>B" translates as "*A* necessarily belongs to all *B*."

### 1.2 Basic Modal Syllogisms

Syllogisms are composed of three categorical propositions. Each of those propositions can be one of four different moods: (a) universal affirmative, (e) universal negative, (i) particular affirmative, and (o) particular negative. A

sylllogism with two universal affirmative premises and a universal affirmative conclusion will have the mood *AAA*. Along with mood, syllogisms can be classified by figure. Aristotle explicitly discusses three figures, though a fourth figure may be implicit in the text, according to Rini, and later developed explicitly, perhaps first by Galen, and then by later Islamic and Latin philosophers (see Rini 2011, 99; and Rescher 1966). For the purposes of this analysis, we will focus on the three figures that Aristotle identified, as it is those figures of which Aristotle considered the validity.

Figure is determined by the position of the middle term, the term that is repeated in both premises, in the major and minor premises. So given that *A* is the major term, *B* is the middle term, and *C* is the minor term, we have the following three figures:

I	II	III
AxB	BxA	AxB
<u>BxC</u>	<u>BxC</u>	<u>CxB</u>
AxC	AxC	AxC

An assertoric syllogism with a mood of *AAA* and of the first figure is valid. The scholastics named each of the valid argument forms, and *AAA-1* is known as *Barbara*, where each vowel represents a categorical proposition in the argument. In our symbolization, assertoric *Barbara* appears as:

$$1. Aa_xB \quad 2. Ba_xC \quad \therefore \quad 3. Aa_xC$$

In natural language: *A* belongs to all *B* and *B* belongs to all *C*, therefore *A* belongs to all *C*. Saying “*A* belongs to all *B*” appears to invert the way an English speaker might construct a categorical proposition; e.g. “All *B* is *A*.” Nonetheless, this is faithful to



Aristotle's construction,<sup>2</sup> and the way in which contemporary commentators work with the Aristotelian syllogistic (e.g. Patterson 1995). Furthermore, it provides some clarity that Aristotle intends to relate universals as belonging to one another, which avoids some of the ambiguous ways the copula, when expressed by the verb "to be", can be construed.<sup>3</sup> Use of the verb "to be" in constructing statements fits well with what Bäck describes as the "copulative interpretation" of the statement. On this interpretation, the statement, "...changes its logical function dependant on its sentential context" (2000, 98). Since the Greek word 'ἔστιν' can have multiple functions, e.g. to make existence claims, identity claims, or merely to connect a subject term to a predicate term, context must guide us in understanding the sort of statement we are making. However, this introduces ambiguity into the syllogism, since context dictates the way in which terms are being linked, and the sort of metaphysical assumptions one is making about those terms, e.g. whether the copula imparts existential import<sup>4</sup> upon the terms, whether the terms reference kinds or members that fall under kinds, etc. The copulative theory, which has been dominant in interpreting Aristotle, has held that "belongs to" locution makes no existential claims (*ibid.*). Bäck's aspect theory claims that "P belongs to S" should be read as, "S is existent as a P." "So for example, 'Socrates is (a) man' is to be read as 'Socrates is existent as a man' (*ibid.* 3). Propositions are compound in that they assert both that S exists, and that S is P. Indeed, this appears to be correct, if one considers that

---

<sup>2</sup> Łukasiewicz explains, "Aristotle always puts the predicate in the first place and the subject in the second. He never says, 'All B is A', but uses instead the expression 'A is predicated of all B' or more often 'A belongs to all B'" (1957, 3).

<sup>3</sup> "A belongs to all B" is expressed in a few ways by Aristotle. For example, one might see "A κατὰ παντος τοῦ B," "τὸ A παντὶ τῷ B ὑπάρχει," or "τὸ A κατηγορεῖται κατὰ παντος τοῦ B."

<sup>4</sup> The issue of existential import is more fully explored in the fourth chapter.

categorical propositions say something about terms that have underlying natures. If a categorical proposition is in any sense true, then its terms exist. So, what I shall say is that the “belongs to” locution comports with the way Aristotle typically constructs these propositions, and they do carry existential import, a topic I address more fully in my Four Chapter. This permits immediate inferences like sub-alternation, from universal claims to particular claims. Moreover, we shall hold that Aristotle permits the “belongs to” locution to be used to predicate of subjects that are either singular, e.g. “Socrates” or “this man”, or kind terms, e.g. “Man”. However, singular terms cannot be placed in the predicate position, a point I will elaborate on elsewhere.

As Malink notes, Aristotle does not provide details into his semantics. Malink develops a *dictum de omni* semantics wherein to say  $A$  is  $a_x$ -predication of  $B$ , “...just in case every member of the plurality associated with  $B$  is a member of the plurality associated with  $A$ ” (2013, 19). For  $e_x$ -predication, there is that which is *dictum de nullo*, where  $A$  is  $e_x$ -predication of  $B$  if and only if no member of the plurality associated with  $B$  is a member of the plurality associated with  $A$  (*ibid.* 19-20). Combined, the semantics of that which is *dictum de omni et de nullo* provides the semantical framework for Aristotle’s categorical propositions. However, it is not precisely clear what these pluralities are, and how they relate to the terms. The orthodox reading of this semantics is that the plurality which falls under  $B$  is composed of individuals such that since  $A$  is  $a_x$ -predicated of  $B$ , those individuals also fall under  $A$ . Likewise, if no member of a plurality associated with  $B$  is associated with  $A$ , then no individuals that fall under  $B$  will fall under  $A$ . Malink

rejects this reading of Aristotle's semantics, as it leads to problems with existential import among other things (*ibid.* 20). He defends, instead, a heterodox interpretation of Aristotle's semantics, where the plurality associated with a term is a set or proper-part of the categorical term. Consequently, the members of the plurality, if they are associated with a kind-term will themselves be kind-terms rather than individuals who exemplify the kind-term. We shall be adopting this semantical model, with some variation in the modal *dictum de nullo*.

The addition of a fourth figure allows for 256 possible assertoric syllogisms.

The fourth figure appears as follows:

$$\begin{array}{l} \text{IV} \\ \text{BxA} \\ \underline{\text{CxB}} \\ \text{AxC} \end{array}$$

Of the 256 possible assertoric syllogisms, 24 are thought to be valid, given Aristotle's three methods of proof: conversion, *reductio ad absurdum*, and *ekthesis*.<sup>5</sup> Contemporary logic textbooks count the number of valid assertoric syllogisms differently, since universal propositions lack existential import, according to the standard predicate logic, which developed out of the Fregean framework. Hence only 15 are counted as valid. If the extra variables of modality (possibility and

---

<sup>5</sup> Since the advent of modern logic this number has been reduced to 15 valid forms. This is because universal propositions, i.e. a and e propositions, are no longer thought to carry existential import. Only particular propositions carry existential import. This is why contemporary attempts to represent Aristotelian propositions in symbolic form will often make use of the existential quantifier. Hence, arguments like Barbari are said to commit the "existential fallacy" wherein a particular conclusion is drawn from universal premises. That is, they illicitly draw existential conclusions from premises that carry no existential import. Along with Barbari, other forms now discounted would include Celaront, Camestros, Cesaro, Darapti, Felapton, Bamalip, Calemos, and Fesapo.

necessity) are added, there are 6912 possible moods (Lagerlund 2000, 9).<sup>6</sup> Aristotle limits his discussion to testing only those modal syllogisms related to valid assertoric syllogisms i.e. 648 variations derived from (and including) the original 24 valid assertoric syllogism. Not all of the 648 will be valid. Our investigation will be limited, for the most part, to those pure and mixed apodictic forms of the syllogism that Aristotle identifies as valid or invalid in the first three figures.

As conversion is essential to understanding the syllogism in general, and the modal syllogism in particular, it will be helpful to explicate these rules as they will be part of the rules by which my interpretation operates.<sup>7</sup> The first rule we shall discuss is conversion. Aristotle writes:

It is necessary then that in universal attribution the terms of the negative proposition should be convertible, e.g. if no pleasure is good, then no good is pleasure; the terms of the affirmative must be convertible, not however universally, but in part, e.g. if every pleasure is good, some good must be pleasure; the particular affirmative must convert in part (for if some pleasure is good, then some good will be pleasure; but the particular negative need not convert, for if some animal is not man, it does not follow that some man is not animal (*APr.* 25a5-13).

Formally, we shall express the rules Aristotle mentions in this passage as:

$$\begin{aligned} Aa_xB &\supset BixA \text{ (Conv } ax-ix) \\ AixB &\equiv BixA \text{ (Conv } ix-ix) \\ AexB &\equiv BexA \text{ (Conv } ex-ex) \end{aligned}^8$$

---

<sup>6</sup> Lagerlund says that the number will either be 6912 or 16384, depending on whether one admits of the different uses of “possibility” throughout Aristotle’s works e.g. *Prior Analytics* and *De interpretatione* (2000, 9). Hintikka identifies two notions of possibility in Aristotle (1973, 27-8). Possibility is treated within *De interpretatione* as a contradictory of the impossible. Thus, the possible is simply that which is not impossible. In *Prior Analytics*, Aristotle sees possibility as involving what may or may not be what later philosophers have come to refer to as contingency. If we limit ourselves to the sort of possibility that Aristotle systematically treats in *Prior Analytics*, i.e. contingency, then each of the three proposition of the syllogism has the possibility of three different modalities and three different moods multiplied by four different figures, hence  $12^3 \times 4$ . More may need to be said about the different notions of possibility in Aristotle, but this falls outside of the purview of our topic.

<sup>7</sup> All rules and definitions enumerated within parentheses constitute my overall interpretation of Aristotle’s modal syllogistic. These are the first six rules. During my discussion, I will lay out rules and definitions offered by other interpreters. They will not be enumerated within parentheses.

<sup>8</sup> See Lagerlund (2000, 7) for a similar interpretation of Aristotle’s conversion rules.

In a similar manner, the necessary predications convert in this manner, according to

Aristotle:

The universal negative converts universally; each of the affirmatives converts into a particular. If it is necessary that  $A$  belongs to no  $B$ , it is necessary also that  $B$  belongs to no  $A$ . For if it is possible that it belongs to some  $A$ , it would be possible the  $A$  belongs to some  $B$ . If  $A$  belongs to all or some  $B$  of necessity, it is necessary also that  $B$  belongs to some  $A$ ; for if there were no necessity, neither would  $A$  belong to some  $B$  of necessity. But the particular negative does not convert, for the same reason which we have already stated (*APr.* 25a26-35).

Following Aristotle, and our symbolization, we can establish the following apodictic conversion rules:

$$\begin{aligned} Aa_L B &\supset Bi_L A \text{ (Conv } a_L-i_L) \\ Ai_L B &\equiv Bi_L A \text{ (Conv } i_L-i_L) \\ Ae_L B &\equiv Be_L A \text{ (Conv } e_L-e_L) \end{aligned}$$

An example of a valid purely apodictic syllogism of the second figure is Cesare-LLL:

$$1. Be_L A \quad 2. Ba_L C \quad \therefore 3. Ae_L C$$

A natural language example of Cesare-LLL might be: 1) animals necessarily belong to no plants, 2) animals necessarily belong to all things with appetitive powers, therefore, 3) plants necessarily belong to no things with appetitive powers.

Furthermore, we may convert Cesare-LLL to its related valid first figure syllogism, Celarent, by converting the major premise, or:

$$1. Ae_L B \text{ (from Conv } e_L-e_L) \quad 2. Ba_L C \quad \therefore 3. Ae_L C$$

The first premise now informs us that plant necessarily belongs to no animals.

---

### 1.3 The Two Barbaras:

Perhaps one of the more perplexing aspects of Aristotle's modal syllogistic is his admission of which syllogisms are valid and which are invalid. According to Aristotle, when Barbara has an apodictic major premise and an assertoric minor premise, it validly reaches an apodictic conclusion. But if it is the minor premise that is apodictic, while the major is assertoric, nothing is said to validly follow. Smith (1995) reports:

Many subsequent logicians have held that this is unacceptable—among them Aristotle's lifelong associate Theophrastus, who dropped the offending rule from his own modal syllogistic in favor of the simpler rule that the modality of the conclusion is always the weakest of the modalities of any premise (45).

The issue first arises in the *Prior Analytics* at the point at which Aristotle says one sort of mixed apodictic Barbara, Barbara-LXL, is valid while the other, Barbara-XLL, is invalid. He writes,

It happens sometimes also that when *one* proposition is necessary the deduction is necessary, not however when *wither* is necessary, but only when the one is related to the major is, e.g. if *A* is taken as necessarily belonging or not belonging to *B*, but *B* is taken as simply belonging to *C*; for if the propositions are taken in this way, *A* will necessarily belong or not belong to *C*. For since *A* necessarily belongs, or does not belong, to every *B*, and since *C* is one of the *B*s, it is clear that for *C* also the positive or negative relation to *A* will hold necessarily. But if *AB* is not necessary, but *BC* is necessary, the conclusion will not be necessary. For if it were, it would result both through the first figure and through the third that *A* belongs necessarily to some *B*. But this is false; for *B* may be such that it is possible that *A* should belong to none of it (*APr.* 30a15-29).<sup>9</sup>

That is, in our symbolization:

(Aa<sub>L</sub>B / Ba<sub>x</sub>C // Aa<sub>L</sub>C) is valid. (Aa<sub>x</sub>B / Ba<sub>L</sub>C // Aa<sub>L</sub>C) is invalid.

As noted above, Theophrastus and Eudemus would have rejected both Barbaras as invalid. Instead they endorse Barbara-LXX:

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<sup>9</sup> Unless otherwise indicated, all translations of Aristotle are from the Revised Oxford Translation (1995) of the *Complete Works of Aristotle*, Ed. J. Barnes.

1. AaLB 2. Ba $\times$ C  $\therefore$  3. Aa $\times$ C

Alexander of Aphrodisias, in his commentaries, while not explicitly taking a side on the matter, draws attention to the debate between Theophrastus and Eudemus, on the one hand, and the “Peripatetic” defenders of Aristotle, on the other. Alexander explains,

[Eudemus and Theophrastus] say that in all combinations of a necessary and an unqualified premiss which are put together syllogistically, the conclusion is unqualified. They take this from the <idea> that in every <sylogistic> combination the conclusion is always like the less and weaker of the premisses assumed. For the conclusion which follows from an affirmative and a negative premise is negative, and the conclusion which follows from a universal and a particular premiss is particular. And, <they say,> it is the same way in the case of mixed premisses: in the case of combinations of a necessary and an unqualified premiss the conclusion is unqualified because the unqualified is less than the necessary (Alexander of Aphrodisias 124.9-17).

So, we might describe the first justification for the Theophrastan preference for the weaker premise, the *peiores*-rule, as an argument by analogy. But a stronger argument for this is needed, if we are to uncover the rules of validity of modal syllogisms. And Alexander helpfully advances a formal argument in addition:

...[I]f *B* holds of all *C* but not by necessity, it is contingent that *B* sometimes be disjoined from *C*. But when *B* had been disjoined from *C*, *A* will also be disjoined from it. And if this is so, *A* will not hold of *C* by necessity (*ibid.* 124.18-20).

Mueller and Gould recommend that we take “*B* holds of all *C* but not by necessity” to mean Ba $\cup$ C rather than Ba $\times$ C. Given Ba $\cup$ C, Alexander says that *B* will sometimes be disjoined from *C*, which Mueller and Gould take to mean that by an AE-transformation, we can validly infer Be $\cup$ C from Ba $\cup$ C (See *APr.* 1.17 36b35-37a4). But Mueller and Gould do not think that there is any way to reach the conclusion that when *B* had been disjoined from *C*, *A* will also be disjoined from it. “...[F]rom the fact that AaB and BeC it does not follow that AeC (or even AoC)” (Alexander of

Aphrodisias 1999a, 119).<sup>10</sup> The argument, then, takes as its starting point the assumption that  $Aa_LB$  and concludes that either  $Ae_QC$  or  $Ao_QC$ . So our options are: 1) i)  $Aa_LB$  ii)  $Be_QC$ , therefore iii)  $Ae_QC$ , or 2) i)  $Aa_LB$  ii)  $Bo_QC$ , therefore iii)  $Ao_QC$ . However, neither (1) nor (2) are valid. Mueller and Gould interpret the passage as  $\neg\text{NEC}(Ba_C)$ , which I interpret to mean  $\sim(Ba_LC)$ , and take this to be equivalent to  $Ba_QC$  and transformable into  $Be_QC$ . Of course, it is far from clear that the negation of  $Ba_LC$  would be anything like  $Ba_QC/Be_QC$ , rather than the more orthodox view that it would be  $Bo_M C$  (see Thom 1996, 13 for traditional oppositions between L and M propositions; see Malink 2013, 197-201 for reasons to suspect that Aristotle was not consistent in those oppositions). Yet, if we are insisting that Aristotelian modality be treated as a copula modifier and not as a logical operator, it may be problematic to assume that we can treat negation as an operator that is always independent of the copula. At the very least, we must be attentive to how negation is being used in the syllogism. Aristotle was keenly aware of the ways in which negation can modify the copula, the predicate, or the quantifier (see *De In.* 10). The relationship among affirmations and negations, as they modify various aspects of the proposition are not always straightforward. For instance, (a) 'every man is wise' is contrary to (b) 'every man is not wise'. It may seem that (b) is logically equivalent to (c) 'every man is not-wise' but it is likely more akin to a material equivalence depending on whether the negated predicate picks out actual things in the world. "Not-wise" isn't really a predicate in the proper sense, but a privation. Further (d) not every man is wise' is the contradictory of (a) and the sub-altern of (b). This point will be picked

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<sup>10</sup> Both AEE-1 and AEO-1 commits the fallacy of the illicit process of the major term and so are invalid.



up later, as my interpretation of the modal syllogistic depends upon when we can infer that a “negative” predicate has an underlying nature or not.

So, how we ought to interpret  $\neg\text{NEC}(\text{BaC})$  raises a rather large interpretive question right at the outset, with no clear answer as to whether it ought to be either  $\text{Be}_Q\text{C}$ ,  $\text{Bo}_Q\text{C}$ , or even  $\text{Bo}_M\text{C}$ , which is the contradictory form that seems most plausible given Paul Thom’s analysis. However, i)  $\text{Aa}_L\text{B}$  ii)  $\text{Bo}_M\text{C}$ , therefore iii)  $\text{Ao}_M\text{C}$  is simply not valid. So it is not clear that the formal argument provides the sort of proof that Alexander reports.

However one ought to interpret Alexander’s formal proof of Theophrastus’s and Eudemus’s weaker modal rule, the following can be noted. First, it is far from clear that their argument is validly formulated. Second, if the argument Alexander presents is, or can be, validly formulated, this does not prevent Aristotle’s stronger modal rules from being true. I take it, instead, that Aristotle understood the implications of his mixed modal syllogisms, and that they play a vital role in his philosophy such that one can build upon first principles that are apodictic, and observations which are not apodictic, build up new apodictic knowledge, which could then advance knowledge in new demonstrations. This will be further discussed in Chapter Three. Finally, Barbara-LXX may be valid under certain assumptions that are simply not at play in an Aristotelian syllogism. For Aristotle, the modal syllogistic involves inferences between class-terms for the purposes of the sciences. When Alexander explains Theophrastus’s and Eudemus’s acceptance of Barbara-LXX but not Barbara-LXL, their terms seem to be temporally qualified and so individuated, i.e. that some member of  $B$ , call it  $a$ , sometimes does not belong to  $C$ .

Further attempts to demonstrate the invalidity of Barbara-LXL by adding content to the form may further suggest that these proofs by counter-example are not treating class-terms but instances. Consider the following:

For animal holds of every human by necessity; let human hold of all that moves; it is not true that animal holds of all that moves by necessity. Furthermore, if having knowledge is said of everything literate by necessity, and literate is said of every human unqualifiedly, it is not true that having knowledge is said of every human by necessity. And moving by means of legs is said of all that walks by necessity; let walking hold of every human; it is not true that moving <by means of legs> holds of every human by necessity (Alexander of Aphrodisias, 1999a 124.20-30).

What are we to make of these arguments? Pamela Huby thinks that,

These examples all have one premise which involves a definitional truth, e.g., All men are animals, and for that reason is necessary, and a second which could be true on occasion but is often false, e.g., All moving things are (at the present time) men. They lead to a proposed conclusion which is clearly unacceptable, e.g., All moving things are necessarily animals (2002, 95).

Again, we should note the temporal qualification Huby inserts into the premise. This is a concern that Aristotle had regarding the validity of the syllogism:

We must understand 'that which belongs to every' with no limitations in respect of time, e.g. to the present or to a particular period, but without qualification. For it is by the help of such propositions that we make deductions, since if the proposition is understood with reference to the present moment, there cannot be a deduction (*APr.* 34b7-10).

In effect, the temporal qualification introduces a fourth term into the argument. For, we must consider 1) men, 2) animals, 3) moving and 4) presently moving things. If we reduce the syllogism back to three terms by preferring (3) to (4), the syllogism is clearly unsound, and if we prefer (4) to (3) it is no longer clear that this is a counterexample to the validity of Barbara-LXL. For, if all men are necessarily animals, and all presently moving things are men, then all presently moving things are necessarily animals. Obviously it is not in virtue of being a "presently moving thing" that such things are necessarily animals, but it is in virtue that the presently moving are humans. The modality is rooted in the essence that is the subject of the

major premise, not the property that is predicated of the essence in the minor premise. There are further problems with this argument, for as a *reductio*, it is an additional difficulty that an argument invites us to permit a false, or “often false,” premise and then contemplate whether the conclusion should not follow necessarily. Such a *reductio* depends upon pumping our modal intuitions, and it is an added challenge to suppose, counterfactually, that the assertoric premise is true, and also that the conclusion ought not to follow from it. If were to suppose a situation where only men were moving, it may very well be a valid conclusion that all moving things are necessarily animals. Likewise, having knowledge necessarily belongs to all “literate things”. But the second premise is not universally true, for not all humans are literate. So it is no wonder that it is false that “knowledge” is necessarily said of every “human.”

Were it the case that, necessarily, every human is born with the ability to read and write in at least one language, given that this is a variety of knowledge, necessarily every human has knowledge. The same could be said for the final argument, which requires that we contemplate another counterfactual, that “walking” holds of every human. It certainly does not, for we do not walk at all times, nor does it hold of humans afflicted with lameness. So at best, it seems that Theophrastus and Eudemus offer only an argument from analogy for the *peiores*-rule. Their formal and material arguments are not successful.

Interestingly enough, the contemporary view, as found in, say Smith (1995), is that Theophrastus and Eudemus were the first to discover a flaw in Aristotle’s modal syllogistic, that this was reported by Alexander, and that the subsequent

development of modal syllogistics simply embraced this supposed insight by adopting the *peiores*-rule. However, this is an historical oversimplification. One could easily argue that Alexander only reports an ongoing debate between one group of Aristotelians and Theophrastus and Eudemus. Huby ultimately believes that Alexander endorses the views of Theophrastus and Eudemus (see Huby 2002, 94). And indeed we do find Alexander saying that their views “seem to be reasonable” (Alexander 1999, 59; 124.30). But Alexander makes the effort to present the three different groups of Aristotelians who defended Aristotle and to develop their responses. Interestingly, Mueller and Gould point out that Alexander admits at 129.18-20 that Aristotle’s views are reasonable too (1999a, 119). This is not to suggest which side Alexander took on the debate over mixed premises, but rather to emphasize that the debate was far from settled in antiquity, with both sides making purportedly reasonable arguments (Huby 2002, 95). Whichever way Alexander sided, he certainly did not think that the matter was clear. As Patterson (1995, 79) notes, Theophrastus’s arguments only work against a *de dicto* interpretation of Aristotle’s syllogism. So it seems that Theophrastus is operating with the assumption that Aristotle’s modal syllogisms should be read in a *de dicto* way. Hence the *peiores* rule can be understood as merely eliminating the necessity operator to allow inferences with weaker premises. Once eliminated, it would be invalid to re-introduce a necessity operator in the conclusion.

The early half of the twentieth century saw renewed interest in the “Two Barbaras” problem and the Aristotelian modal syllogistic. Becker (1933) argued that one can make sense of the “Two Barbaras” by distinguishing between *de re* and *de*

*dicto* modality. On a *de dicto* reading of the “Two Barbaras” both arguments appear invalid, suggesting a vindication of Theophrastus’s *peiores* rule. The supposedly valid form appears on the left, while the invalid form on the right:

- |                               |                               |
|-------------------------------|-------------------------------|
| 1. Necessarily AaB            | 1. AaB                        |
| 2. BaC                        | 2. Necessarily BaC            |
| 3. Therefore, Necessarily AaC | 3. Therefore, Necessarily AaC |

A *de re* reading of the “Two Barbaras” apparently resolves the issue:

- |                                 |   |
|---------------------------------|---|
| 1. (A necessarily) aB           | 1. AaB  |
| 2. BaC                          | 2. (B necessarily) aC                         |
| 3. Therefore (A necessarily) aC | 3. Therefore (A necessarily) aC <sup>11</sup> |

Based on this resolution, many have assumed that Aristotle was operating, at least some of the time, with something like our contemporary notion of *de re* modality in mind. However, the conversion rules for apodictic categorical propositions preclude the possibility of a consistent *de re* reading. For on the *de re* interpretation: (A necessarily) aB  $\rightarrow$  (B necessarily) iA is illicit. So, while it is the case that animal necessarily belongs to all dog, one cannot infer from this that some dog necessarily belong to animal. Nonetheless, Aristotle consistently appeals to such a rule in devising his modal syllogisms, and so it seems moods which are derived via conversion will be illicit as well. Therefore, the “Two Barbaras” apparently generates a troublesome trilemma. On the one hand, if the modal syllogistic is to be understood as *de dicto*, then it seems that one ought to prefer Theophrastus’s *peiores*-rule, that the conclusion should receive the weaker modality. If, on the other hand, the modal syllogistic is to be understood *de re*, then Aristotle is inconsistent in the way he proves the validity of several argument forms beyond the

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<sup>11</sup> H. Lagerlund. 2000. 12-13.

two Barbaras. Finally, if Aristotle uses a mixture of *de dicto* and *de re* modality, then he did not take the care to indicate that he was doing so, or specify which rules should be applied in which contexts. Given this trilemma the view emerging from Becker's analysis is that we must abandon Aristotle's modal syllogistic.

Recent commentators have argued that the *de re/de dicto* distinction is not an intractable, or even relevant, problem for Aristotle (see Patterson 1995, 10; Thom 1996, 3-4). If the *de re/de dicto* distinction should not be invoked to explain Aristotelian modality, three pronged trilemma can be avoided. Arguments to that effect will be developed.

Though explicated in a formal manner, the inference offered here is far from evident from the form alone. We must consider, for instance, why the major term would belong to the middle by necessity. Furthermore, why should the mere fact that the middle term belongs to the minor term entail that the major term necessarily belongs to the minor term? These insights ultimately depend upon the metaphysical analysis of that which cannot be otherwise and that which can be otherwise than it is (Rini 2011, 39; see also *APo.* 71b9-16). Many interpreters of Aristotle's logic insist that understanding metaphysics is necessary for understanding his modal syllogistic (c.f. Ross 1957, Johnson 1989, Patterson 1995, and Thom 1991 & 1996). The difficulty is in determining just how Aristotle's metaphysics affects his modal logic.

## 1.4 The Modal Copula

One might wonder why a return to Aristotelian modality is worth developing in light of the success of contemporary modal logic. There are at least a few reasons to do so beyond mere historical curiosity. The first reason is that, despite even our best efforts every logical system carries with it, or at the very least, raises serious metaphysical questions. An alternative way of conceptualizing about possibility and necessity helps us to avoid the tendency of thinking that the assumptions that we carry with our logic are indispensable and so ontologically necessary features just as, say, the indispensability of numbers led Quine to mathematical realism. For Quine reasoned that whatever is indispensable to our best scientific theories ought to be admitted into our ontology. The more that modal reasoning is utilized in the philosophy of science, counterfactuals, and natural kinds and essences, the more possible worlds may appear indispensable to our best scientific theories of the world. So we may see modal realism, the idea that possible worlds are in some sense, real, as a compelling conclusion to reach. A second reason to develop this system is to help us preserve a still-living and fruitful tradition that continually struggles to be properly understood alongside its modern and contemporary analogs. The rise of modern philosophy occasioned a massive semantic shift in philosophical terminology. Terms like “matter,” “actuality,” and “causality” have taken on subtly different meanings from ancient to scholastic, modern and contemporary times. The contemporary philosopher, looking back at the scholastic and ancient traditions often scratches her head when confronted with the seemingly

ridiculous and apparently patently false arguments. The bread and butter of the historian of philosophy is to rediscover the way in which philosophers of the past understood the terms of their own arguments, and to set up the metaphysical contexts in which their arguments thrive. Often it is the case that within those contexts, the seemingly absurd and unsound arguments come into focus as far more challenging and philosophically insightful. “Necessity,” “contingency,” “possibility,” “substance,” “essence,” and “property” have drastically shifted in meaning. A careful study of Aristotle’s modal syllogistic requires carefully thinking through how Aristotle understood those terms. So, it can help us to understand other aspects of Aristotle’s philosophy, including his science of demonstration, and his metaphysical analysis of essences. Finally, for a time, it was unfashionable to speak of essences and kinds among philosophers. With the decline of Logical Positivism, and the return of metaphysics, such talk is considered meaningful. In some ways, the discovery of a semantics for modal logic ushered in this age. At the same time, depending on the semantics of possible worlds as the best way to understand modality has placed the metaphysics of essences in a precarious position, one in which essences are tethered to debates about the nature of possible worlds. A turn to an Aristotelian analysis of modality might offer us a way to conceive of modality without depending on possible worlds and any metaphysical puzzles they raise.

Contemporary logicians use a modal operator to modify statements. This follows the precedent of treating quantifiers, and even negation, as operators or connectives that modify whole statements where a statement is understood in terms of individual objects and their predicates. However, in Aristotle’s logic, the



quantity of a statement, i.e. whether the statement is universal, particular, or indefinite, and the quality of the statement, i.e. whether the statement is affirmative or negative, is a component of the copula by which two terms are conjoined.

Rini offers a way of translating Aristotle's categorical propositions into predicate logic. For assertoric propositions, she provides the following (2011, 15):

(A)	$A$ belongs to every $B$	$(\forall x)(Bx \supset Ax)$
(E)	$A$ belongs to no $B$	$(\forall x)(Bx \supset \sim Ax)$
(I)	$A$ belongs to some $B$	$(\exists x)(Bx \ \& \ Ax)$
(O)	$A$ does not belong to some $B$	$(\exists x)(Bx \ \& \ \sim Ax)$

Likewise for the apodictic propositions, Rini offers the following definitions (see *ibid.* 52):

LA	It is necessary for $A$ to belong to every $B$	$(\forall x)(Bx \supset LAx)$
LA	It is necessary for $A$ to belong to no $B$	$(\forall x)(Bx \supset L\sim Ax)$
LI	It is necessary for $A$ to belong to some $B$	$(\exists x)(Bx \ \& \ LAx)^{12}$
LO	It is necessary for $A$ not to belong to some $B$	$(\exists x)(Bx \ \& \ L\sim Ax)$

Utilizing Rini's definitions, we are able to prove the validity of Barbara-LXL:

1.  $AaL B$
2.  $BaL C$
3.  $(\forall x)(Bx \supset LAx)$  (1 Def LA)
4.  $(\forall x)(Cx \supset Bx)$  (2 Def A)
5.  $Bu \supset LAu$  (3 UI)
6.  $Cu \supset Bu$  (4 UI)
7.  $Cu \supset LAu$  (5,6 HS)
8.  $(\forall x)(Cx \supset LAx)$  (7 UG)
9.  $AaL C$  (8 Def LA)

One also has a sense for why Barbara-XLL is invalid:

1.  $AaL B$  (Major Premise)
2.  $BaL C$  (Minor Premise)
3.  $(\forall x)(Bx \supset Ax)$  (1 Def A)
4.  $(\forall x)(Cx \supset LBx)$  (2 Def LA)
5.  $Bu \supset Au$  (3 UI)
6.  $Cu \supset L Bu$  (4 UI)

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<sup>12</sup> Note that Rini writes  $(\exists x)(Bx \ \& \ LBx)$ , but I take this to be a typo.

Now to link  $Cu$  with  $Au$ , we must take an alternative step, whereby we eliminate necessity, NE rule, something like:  $(\forall P)(\forall x)(LPx \supset Px)$ , where  $P$  stands for some predicate term. If an object has a predicate necessarily, then it has the predicate, or:

7.  $(\forall P)(\forall x)(LPx \supset Px)$  (NE)
8.  $(\forall x)(LBx \supset Bx)$  (7 UI)
9.  $LBu \supset Bu$  (8 UI)

At this point in the deduction, we can say that  $Cu$  implies  $Bu$ , and so infer that  $Cu$  implies  $Au$ :

10.  $Cu \supset Bu$  (6,9 HS)
11.  $Cu \supset Au$  (5,10 HS)

But note that this only leads us to an assertoric conclusion. There seems to be no way to predicate  $B$  of  $u$  with necessity, hence the generalization of (11) cannot yield an apodictic conclusion.

12.  $(\forall x)(Cx \supset Ax)$  (11 UG)
13.  $AaxC$  (12 Def A)

Barbara- $XLX$  is, indeed, a valid form, and we have inadvertently proved it by attempting to prove Barbra- $XLL$  (see *APr.* 30a23-32). So it seems that Rini's attempt to translate Aristotle's modal syllogistic into lower predicate calculus passes the notorious test case by helping us to explain the two Barbaras. However, Rini's interpretation is not entirely faithful to Aristotle's list of valid and invalid forms. She often boldly diverges from the canonical listing of valid and invalid syllogism (see Rini 2011, 95-105). For instance, Disamis- $LXL$  is a third figure syllogism that Aristotle identifies as invalid. Rini's translation, if it is faithful to Aristotle's understanding of modal syllogisms, apparently vindicates Disamis- $LXL$ :

1.  $A_iL B$  (Major Premise)
2.  $C_a x B$  (Minor Premise)
3.  $(\exists x)(Bx \ \& \ LAx)$  (1 LI)
4.  $(\forall x)(Bx \supset Cx)$  (2 Def A)
5.  $Bu \ \& \ LAu$  (3 EI)
6.  $Bu \supset Cu$  (4 UI)
7.  $Bu$  (5 Simp)
8.  $Cu$  (6,7 MP)
9.  $LAu$  (5 Simp)
10.  $Cu \ \& \ LAu$  (8,9 Conj)
11.  $(\exists x)(Cx \ \& \ LAx)$  (10 EG)
12.  $A_iL C$  (11 LI)

Yet Aristotle seems to refute Disamis-LXL at *Prior Analytics* 31b31-33. He uses the terms “biped” for the minor, “animal” for the middle term, and “awake” for the major term. That is, if awake belongs necessarily to some animals and biped belongs to all animals, Aristotle rejects the notion that awake belongs necessarily to some bipeds. Aristotle’s use of examples to explain invalidity can be difficult to follow, since it seems that one must share Aristotle’s modal intuitions, which are largely dictated by his conception of modality, which is precisely what is in question when attempting to translate the modal syllogistic into something like lower predicate logic. However, the broader point is that Aristotle explicitly rejects this form. Rini’s translations permit other forms of argument that Aristotle rejects as well, including Baroco-LXL, a particular controversial argument form in the literature (see Thom 1996, 59; Malink 2013, 183-186). She argues that Aristotle is committing what she refers to as the subtle mistake. She explains the nature of the mistake in the following way:

The problem is that the validity of LE-conversions depends on the genuineness restriction on the subject term, and even in an equivalence the *subject* of the proposition on one side of the equivalence is different from the subject of the proposition on the other side of the equivalence (Rini 2011, 88).

Rini's "genuineness requirement" is that certain modal conversions, in this case of  $e_L$ -propositions, the "input" proposition must be "genuine," that is, a red term. A red term is an essential term, which Rini says can be a real logical subject (see Rini 2011, 4, 44). Rini believes that Aristotle's counter-examples for proof of invalidity depend upon conversions that run afoul of the genuineness restriction because they require treating non-essential terms as the subjects of modal predications. Rini contrasts (i) All men are necessarily-animals with (ii) All moving things are necessarily-animals. She notes that (i) validly converts to "Some animals are necessary-men" because the subject of the input proposition is red, but a conversion of (ii) yields "Some animals are necessary-movers" which she argues is illicit. It is illicit, because Rini does not think necessity can be linked to "green" terms. Of course, this assumes that necessity is being treated as an operator that modifies predicate terms, and this is not, as Malink and others argue, how Aristotle would conceive of modality.

A second concern with Rini's method is that, in using predicate logic, she quantifies over objects rather than Aristotelian class-terms. Consequently, Rini treats modality as an operator that modifies the way an object within a given domain of discourse is linked to the predicate. But Aristotle's system of logic is syntactically quite different, making such a translation not only problematic, but potentially misleading.

## 1.5 Malink on the Metaphysics of Aristotelian Modality

Malink argues that to understand Aristotle's motivations in the modal syllogistic, we must understand the theory of predicables and categories present in the *Topics*. It is in the *Topics* that Aristotle develops his division of predicables into five kinds: definition, genus, differentia, proprium, and accident (See Malink 2013, 6).<sup>13</sup>

In addition, the *Topics'* theory of categories introduces, among other things, a distinction between two kinds of terms. The first group contains substance terms like 'animal' and 'man', and non-substance terms like 'color', 'redness', and 'motion'. Call these *essence terms*. The second group contains non-substance terms like 'colored', 'red', and 'moving'. Call these *non-essence terms* (Malink 2013, 7).

Based on the *Topics*, Malink develops a series of theses:

Thesis 1: If  $B$  is an essence term, then  $B$  is  $a_L$ -predicated of everything of which it is  $a_X$ -predicated.<sup>14</sup>

This means that essence terms entail a modal feature of the copula, to which the subject is joined to the predicate. Moreover, Malink argues that "...only essence terms can serve as the subjects of [ $a_L$ -predications]" (Malink 2013, 8). Thus, his second thesis is:

Thesis 2: If  $A$  is  $a_L$ -predicated of  $B$ , then  $B$  is an essence term

Out of this, Malink devises an intriguing argument for the validity of Barbara-LXL and why such an argument would not be applicable to Barbara-XLL. Essentially, Malink takes these two theses to show that the premise pair in Barbara-LXL implies the premise pair in Barbara-LLL. Since the latter argument is relatively uncontroversial, and can be derived from Barbara-LXL and the two theses, Malink

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<sup>13</sup> The number of predicables varies between four and five if one considers definition to be a predicable in itself rather than the combination of genus and differentia.

<sup>14</sup> Malink prefers to symbolize apodictic propositions with a subscript N, for necessity, rather than L, which follows the Polish convention.

argues that the validity of one is related to the validity of the other. So in Barbara-LXL, the first premise is  $AaL B$ , and by thesis two, we know that  $B$  is an essence term, and in premise two is  $Bax C$ , where  $B$  is  $ax$ -predicated of  $C$ . Hence  $B$  is implicitly  $aL$ -predicated of  $C$  even if the premise doesn't state that explicitly. So Barbara-LXL is implicitly Barbara-LLL.

## 1.6 Malink's Heterodox Interpretation of Aristotelian Semantics

Malink, following Patterson, has given some good reason to think that Aristotle conceived of modality as just another way in which a copula could be modified. Malink's evidence comes from the *Prior Analytics*, where Aristotle writes,

Every proposition states that something either belongs or must belong or may belong; of these some are affirmative, others negative, in respect of each of the three modes; again some affirmative and negative propositions are universal, others particular, others indefinite (*APr.* 1.2 25a1-5).

Malink comments, "In the tripartite syntax of categorical propositions, negative propositions are not obtained by applying a negative constituent to an affirmative proposition. Instead, they are obtained by applying a negative copula ( $\tau\acute{o} \mu\grave{\eta} \epsilon\acute{\iota}\nu\alpha\iota$ ) instead of an affirmative one ( $\tau\acute{o} \epsilon\acute{\iota}\nu\alpha\iota$ ) to two terms" (Malink 2013, 25). Like the quality of the proposition, Malink notes that quantity is specified by the copula, or the way in which one term is said to belong to another. To say that quantity and quality ways to specify the sort of copula joining the terms is not so controversial, yet the idea that modality should also be treated as a copula modifier has somehow escaped modern interpreters. But if this is so, the contemporary claim that Aristotle failed to distinguish between *de re* and *de dicto* modal contexts falls to the wayside.

Aristotle's logic simply is not flexible enough to make *de dicto* modal claims as there are no movable operators. There is a tendency to think, then, that Aristotle's understanding of modality is entirely *de re*. The short response is to say yes. But the longer answer requires that we understand the sort of "things" about which modal statements assert. For, first-order predicate logic both relates statements to one another by various connectives, and reduces all simple statements to a relationship between predicates and individuals within a domain of discourse. The universal quantifier renders the domain of discourse over all individuals in relation to some predicate. Likewise, a modal operator that is placed before a simple statement modifies the relationship between individuals and their predicates.

While Aristotle's logic accommodates singular terms, they are not the general paradigm of his logical system. The reason for this can be grounded in Aristotle's ontological square, found in the *Categories*. Aristotle notes that:

Of things that are: (a) some are *said of* a subject but are not *in* any subject. For example, man is said of a subject, the individual man, but is not in any subject. (b) Some are in a subject but not said of any subject... For example, the individual knowledge-of-grammar is in a subject, the soul, but is not said of any subject; and the individual white is in a subject, the body (for all colour is in a body), but is not said of any subject. (c) Some are both said of a subject and in a subject. For example, knowledge is in a subject, the soul, and is also said of a subject knowledge-of-grammar. (d) Some are neither in a subject nor said of a subject, for example, the individual man or the individual horse—for nothing of this sort is either in a subject or said of a subject (*Cat.* 1a20-1b6).

Singular terms refer to primary substances. As such, they cannot be in or said of another subject, which means that, at best, they can self-predicate (*Meta.* 1018a4). Given that an Aristotelian syllogism features three terms, one of which repeats twice in the premises, singular terms will only feature in those syllogism where the term will not be predicated of any non-identical terms. Therefore, singular syllogisms are metaphysically problematic in certain figures where the singular term is used as a

predicate. For instance, it is permissible to argue, "...animal belongs to all man, man belongs to Socrates, therefore animal belongs to Socrates." In this example, the singular term is never a predicate. We could put "Socrates" in the predicate position, as in "animal belongs to Socrates, and Socrates belongs to teacher of Plato, therefore animal belongs to the teacher of Plato." In this argument, "Socrates" can be the predicate of the minor term because there is arguably an identity between "Socrates" and "teacher of Plato" at least in the sense that, in extensional contexts, they are co-referring terms. However, there cannot be a sound syllogism where a singular term is predicated of some non-identical term in one of the premises.

Understanding exactly what the relationship is between terms has been an interpretive challenge. Malink distinguishes what he calls the "orthodox dictum semantics" from his own "heterodox" interpretation. On the orthodox view the pluralities associated with terms are individuals. "The set of individuals which fall under a term is often referred to as the extension of that term" (Malink 2013, 45). Malink defines the way in which the four categorical propositions are defined on the orthodox view:

$$\begin{aligned} AaxB &\equiv (\forall z)(Bz \supset Az) \\ AexB &\equiv (\forall z)(Bz \supset \sim Az) \\ AixB &\equiv (\exists z)(Bz \& Az) \\ AoxB &\equiv (\exists z)(Bz \& \sim Az) \end{aligned}$$

This is along the lines of the interpretation offered by Rini. Malink finds this view problematic for two reasons. The first has to do with the assumption that the plurality associated with the terms is composed of individuals. Such an assumption treats a categorical term as symbolic of a set of individuals. Indeed, this does seem



to suggest or favor a nominalistic reading of Aristotle. The second criticism is that it introduces into the syntactical definition of categorical syllogisms a zero-ordered quantified individual variable. Malink explains his alternative in the following way:

The heterodox *dictum* semantics is based on the assumption that the plurality associated with a term consists of exactly those items of which the term is  $a_x$ -predicated. The relation of  $a_x$ -predication is treated as a primitive preorder, in terms of which  $e_x$ -,  $i_x$ -, and  $o_x$ -predication are defined (2013, 63).

Malink offers an alternative semantics based on categorical terms, defining the four assertoric categorical propositions in the following manner:

$$\begin{aligned} A_a x B &\equiv (\forall Z)(B_a x Z \supset A_a x Z) \\ A_e x B &\equiv (\forall Z)(B_a x Z \supset \sim A_a x Z) \\ A_i x B &\equiv (\exists Z)(B_a x Z \& A_a x Z) \\ A_o x B &\equiv (\exists Z)(B_a x Z \& \sim A_a x Z) \end{aligned}$$

In quantifying over categorical terms rather than individuals, Malink avoids existential claims about individuals, while maintaining the reflexivity and transitivity of  $a_x$ -propositions that are crucial in defining  $e_x$ -  $i_x$ - and  $o_x$ -proposition relations. One should not think that ‘ $(\forall Z)$ ’ is a universal quantification over properties, as is common in second-order predicate logic. However, Malink notes that his heterodox interpretation should be considered a first-order logic in which class-terms are zero-ordered individual variables, and “ $a_x$ ,” “ $e_x$ ,” and the like are relations between the class-variables (2013, 70). So, Malink is quantifying over a subset of Aristotelian categorical terms, leaving it somewhat open as to how one ought to understand the metaphysical nature of Aristotelian terms. I shall argue, later on, that if one were to create bridge-laws between the heterodox interpretation, and classic first-order predicate logic. For, categorical terms can also

bear the “belongs to” relation of individual subjects, i.e. primary substances, as we shall see.

Malink addresses two objections to the heterodox interpretation. The first, raised by Barnes, is that, “it is more natural to read *dictum de omni* in the orthodox than in the heterodox way” (Malink 2013, 64). That is, whatever is said of a term universally must be said of the members that fall under the term universally. As Malink points out, it may just be more natural to read this as a dictum relating to class-terms, or universals, and particular individuals because that has been the dominant way of understanding the principle. There is nothing intrinsic to the principle that requires such an interpretation. Secondly, it is objected that the heterodox interpretation is circular, since it contains an *ax*-predication in defining an *ax*-predication. The orthodox definition is more informative because it avoids such circularity (*ibid.* 65). Malink says that the heterodox interpretation treats *ax*-predications as primitive.

On this view, Aristotle’s *dictum de omni et de nullo* is not intended as a definition of what *ax*-predication is. Instead, it specifies logical properties of *ax*- and *ex*-predication that account for the validity of his perfect moods and conversion rules (*ibid.* 66).

Thus, the heterodox interpretation preserves validity and Aristotelian rules without introducing metaphysical questions that lead us to suppose that Aristotle assumed any particular relationship among categorical terms and individuals exemplifying those terms. Moreover, Malink shows that the heterodox interpretation can validate *ax*-conversion, unlike the orthodox interpretation (*ibid.* 67).

There is still some way in which Malink’s interpretations are ambiguous in how one should understand the metaphysical nature of categorical terms, but also

vague with respect to negation. It is not entirely clear what is being negated in  $Ae_xB$  and  $Ao_xB$ . In  $Ae_xB$ , negation is built into the copula. But when it is defined in terms of the negation sign, this raises some questions. Is Malink doing to negation what he charges others have done with modality? That is, is he treating negation purely as a logical connective rather than a modification to the copula? Aristotle seems to treat negation in a variety of ways in *De Interpretatione* as I mentioned in Mueller and Gould's discussion of Alexander of Aphrodisias' formal argument for the *peiores* rule. To do justice to Aristotle, we must consider how the negation sign distributes over the categorical proposition. For instance, we do not want to say that  $\sim Aa_xZ$  involves only the negation of the predicate term. For Malink to make the sorts of deductions he wants to make, the negation will be of the entire expression. A more precise representation would be  $\sim(Aa_xZ)$ , in which case we are negating the claim that  $A$  belongs to all  $Z$ , which is seemingly equivalent to  $Ao_xZ$ . Of course, this introduces a bit of a challenge when defining modal propositions. Indeed, part of Malink's case for treating modality as a modifier of the copula rather than as an operator is based on the fact that Aristotle treats quantity and quality within the copula. For, in that case, we are not merely negating the quantity or quality of the copula, but the modality as well. This is complicated by the fact that, depending on the kinds of categorical terms employed in the proposition, negation of the copula may or may not be equivalent to the affirmation of the complementary class, an equivalence that is commonly called obversion. I shall argue that, in the case of apodictic propositions, which deal with substantive, essential, or counter-predicating terms, negation of the copula is equivalent to affirmation of the

complementary class. However, if the proposition is not apodictic, negation may only be of the copula alone, and equivalence to the affirmation of the complementary class is considered illicit. Put simply, I argue that a proper understanding of negation is a crucial piece of the puzzle in trying to understand Aristotle's modal syllogistic.

Malink says that 'a' propositions are a primitive preorder in that they are reflexive and transitive. This allows Malink to define the other three categorical propositions in terms of a-predication, which plays a large role in the interpretive model Malink recommends. However, applying the heterodox interpretation in a modal context is not so straightforward. Malink struggles to define his propositions such that they fit all cases, especially with respect to the o-proposition. This is because he seeks a definition for apodictic o-predicates that permit Baroco-LLL and Bocardo-LLL to be valid, but do not permit Baroco-XLL and Bocardo-NXN to be valid. Malink's tentative definitions for apodictic propositions are as follows:

$Aa_LB$  if and only if for every  $Z$ ,  $Z$  is a member associated with the plurality associated with  $B$ , then  $A$  is said of  $Z$  by necessity (*ibid.* 108)

That is:  $Aa_LB \equiv (\forall Z)(Ba_xZ \supset Aa_LZ)^{15}$

$Ae_LB$  if and only if for every  $Z$ , if  $Ba_xZ$  then not  $Aa_xZ$ , and there are  $C$  and  $D$  such that  $Ca_LA$  and  $Da_LB$  (*ibid.* 170).

That is:  $Ae_LB \equiv (\forall Z)[Ba_xZ \supset \sim(Aa_xZ)] \ \& \ (\exists C)(\exists D)(Ca_LA \ \& \ Da_LB)$

$Ai_LB$  if and only if for some  $Z$  ( $Ba_xZ$  and  $Aa_LZ$ ) or for some  $Z$  ( $Aa_xZ$  and  $Ba_LZ$ ) (*ibid.* 179).

That is:  $Ai_LB \equiv (\exists Z)(Ba_xZ \ \& \ Aa_LZ) \ \vee \ (\exists Z)(Aa_xZ \ \& \ Ba_LZ)$

$Ao_LB$  if and only if for some  $Z$ ,  $Ba_xZ$  and  $Ae_LZ$  (*ibid.* 181).

That is:  $Ao_LB \equiv (\exists Z)(Ba_xZ \ \& \ Ae_LZ)$

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<sup>15</sup> Malink defines  $(\forall Z)(Z \text{ mpaw } B)$  as  $(\forall Z)(Ba_xZ)$  and so we see that  $(\forall Z)(Z \text{ mpaw } B \supset Aa_LZ)$  just means  $(\forall Z)(Ba_xZ \supset Aa_LZ)$  later on (see Malink 2013, 111). My interpretation adopts this convention.

This is a tentative list, since Malink ultimately thinks the apodictic o-proposition is problematic. Malink must offer a more convoluted expression of particular negative propositions in order to align with Aristotle. Malink develops another definition of o-predication later on, which he thinks satisfy the demands of the proofs, but fall short in other ways. Also, Malink notes that some interpret the apodictic i-proposition as a conjunctive expression. However, he argues that a disjunctive interpretation accords with the conclusion that Darii-LXL while still allowing for conversion, since disjunctions are symmetric (*ibid.* 179). Malink notes, “[t]he disjunctive definition implies that [i<sub>L</sub>]-propositions may be true even if the predicate term is not [a<sub>L</sub>-predicated] of anything. It suffices that the subject term is [a<sub>L</sub>-predicated] of something of which the predicate term is a<sub>X</sub>-predicated” (*ibid.*).

While Malink’s interpretative definitions can be made to comport with Aristotle’s requirements of validity and invalidity across the three figures, there are some problems to consider, especially with regard to both pure and mixed Baroco syllogisms. According to Aristotle, Baroco-LLL is valid. This is unsurprising to most, since Aristotle argues that a valid purely assertoric syllogism will have a related valid purely apodictic form. Aristotle writes:

In the case of what is necessary, things are pretty much the same as in the case of what belongs; for when the terms are put in the same way, then, whether something belongs or necessarily belongs (or does not belong), a deduction will or will not result alike in both cases, the only difference being the addition of the expression ‘necessarily’ to the terms (*APr.* 29b35-30a1).

Aristotle reasons that his general account of predication applies in modal cases, and that the negative propositions have the same conversion rules. So he says that most

figures can be proved through similar means of conversion. But in the case of Baroco-LLL and Bocardo-LLL, Aristotle recognizes a difference. He says,

But in the middle figure when the universal is affirmative and the particular negative, and again, in the third figure when the universal is affirmative and the particular is negative, the demonstration will not take the same form but it is necessary by the exposition of a part of the subject, to which in each case the predicate does not belong, to make the deduction in reference to this: with terms so chosen the conclusion will be necessary. But if the relation is necessary in respect of the part exposed, it must hold of some of that term in which this part is included; for the part exposed is just some of that. And each of the resulting deduction is in the appropriate figure (*APr.* 30a6-15).

In other words, Aristotle wants to prove Baroco-LLL by way of Camestres and Bocardo-LLL by Felapton (see Ross 1957, 317). It is not the case that every valid argument with universal premises has a related valid argument with sub-alternate premises. The first figure Celarent does not provide justification for a parallel OAO argument in the first figure, though it may be related to Celaront. In other words, one should not think that whatever holds for a universal argument will also hold for a similar argument with a sub-alternated premise. It is not clear how Aristotle hoped to prove Baroco-LLL by way of Camestres, but Malink offers one interpretation:

1. Ba<sub>L</sub>A (major premise)
2. Bo<sub>L</sub>C (minor premise)
3. (∃Z)(Ca<sub>x</sub>Z & Be<sub>L</sub>Z) (from 2)
4. Ca<sub>x</sub>U & Be<sub>L</sub>U (3 EI)
5. Ca<sub>x</sub>U & Ae<sub>L</sub>U (1,4 Camestres-LLL)
6. Ao<sub>L</sub>C (5 Felapton-LXL)

This is problematic, however, since a similar proof can be made for Baroco-XLL, which is supposed to be invalid:

1. Ba<sub>x</sub>A (major premise)
2. Bo<sub>L</sub>C (minor premise)
3. (∃Z)(Ca<sub>x</sub>Z & Be<sub>L</sub>Z) (from 2)
4. Ca<sub>x</sub>U & Be<sub>L</sub>U (3 EI)
5. Ca<sub>x</sub>U & Ae<sub>L</sub>U (1,4 Camestres-XLL)

6. Ao<sub>L</sub>C (5 Felapton-LXL)

Malink is also concerned that o<sub>L</sub>-predication should make Bocardo-LLL valid, but Bocardo-LXL invalid. Indeed, his definitions provide for the validity of Bocardo-LLL:

1. Ao<sub>L</sub>B (major premise)
2. Ca<sub>L</sub>B (minor premise)
3. (∃Z)(Ba<sub>X</sub>Z & Ae<sub>L</sub>Z) (from 1)
4. Ba<sub>X</sub>U & Ae<sub>L</sub>U (3 EI)
5. (∀Z)(Ba<sub>X</sub>Z ⊃ Ca<sub>L</sub>Z) (from 2)
6. Ba<sub>X</sub>U ⊃ Ca<sub>L</sub>U (5 UI)
7. Ba<sub>X</sub>U (4 Simp)
8. Ca<sub>L</sub>U
9. Ae<sub>L</sub>U (4 Simp)
10. Ca<sub>L</sub>U & Ae<sub>L</sub>U (8,9 Conj)
11. (∃Z)(Ca<sub>L</sub>Z & Ae<sub>L</sub>Z) (10 EG)
12. Ao<sub>L</sub>C

A small quibble one might have over the way this proof runs is that Malink allows *C* to belong to all *Z* by necessity while his general definition of o<sub>L</sub>-predication requires only that the *C* belong to some *Z* assertorically. To demonstrate validity, we can add a rule that Malink uses to justify Barbara-LXL, namely L-X-subordination (see Malink 2013, 130-131). It is also possible to derive validity without this rule by simplifying out the existential component of a<sub>L</sub>-propositions. Malink defines L-X-sub is defined as:

$$Aa<sub>L</sub>B \supset Aa<sub>X</sub>B \text{ (L-X-sub } a<sub>L</sub>)}$$

He cites *Posterior Analytics* 1.2, which states, “That which signifies substance signifies just what or just a subspecies of that which is predicated” (*APo.* 83a24-5; see Malink 2013, 131). It seems reasonable to suppose that this would hold for other categorical propositions such that:

$Ae_L B \supset Ae_X B$  (L-X-sub  $e_L$ )

$Ai_L B \supset Ai_X B$  (L-X-sub  $i_L$ )

$Ao_L B \supset Ao_X B$  (L-X-sub  $o_L$ )

This would add an extra step in our inferences, but it would also preserve the definition of  $o_L$ -propositions. The problem is that Bocardo-LXL is supposed to be invalid according to Aristotle. However, a proof, similar to the one for Bocardo-LLL can be made for Bocardo-LXL:

1.  $Ao_L B$  (major premise)
2.  $Ca_X B$  (minor premise)
3.  $(\exists Z)(Ba_X Z \ \& \ Ae_L Z)$  (from 1)
4.  $Ba_X U \ \& \ Ae_L U$  (3 EI)
5.  $(\forall Z)(Ba_X Z \supset Ca_X Z)$  (from 2)
6.  $Ba_X U \supset Ca_X U$  (5 UI)
7.  $Ba_X U$  (4 Simp)
8.  $Ca_X U$
9.  $Ae_L U$  (4 Simp)
10.  $Ca_X U \ \& \ Ae_L U$  (8,9 Conj)
11.  $(\exists Z)(Ca_X Z \ \& \ Ae_L Z)$  (10 EG)
12.  $Ao_L C$

So, as we have seen, Malink's tentative definitions, especially of  $o_L$ -propositions, prove to be somewhat problematic. His proof for Baroco-LLL can be parodied to provide proofs for Baroco-XLL, which Malink admits (Malink 2013, 181). Aristotle claimed that validity for these forms can be proved through *ekthesis*, so it seems that we should be able to prove them in this way.

*Ekthesis* is a method of proof that Aristotle employs to demonstrate the validity of syllogisms containing assertoric or apodictic premises. Robin Smith enumerates the assumptions that operate in *ekthesis* as follows:

1. If  $AiB$ , then there is some  $S$  such that  $AaS$  and  $BaS$ .
2. If  $AoB$ , then there is some  $S$  such that  $AeS$  and  $BaS$ .
3. If there is some  $S$  such that  $AaS$  and  $BaS$ , then  $AiB$ .



4. If there is some S such that AeS and BaS, then AoB.

Along with these rules, the following procedures are assumed:

- (5)  $AiB \vdash AaS, BaS$  (where S does not occur previously)
- (6)  $AoB \vdash AeS, BaS$  (where S does not occur previously)
- (7)  $AaS, BaS \vdash AiB$
- (8)  $AeS, BaS \vdash AoB$

Malink's heterodox interpretation could be considered a use of proof by *ekthesis*, since a mereological part of a class term is used to define the relationship between two class terms such that, for instance,  $AaxB$  is defined as  $(\forall Z)(AaxZ \supset BaxZ)$ . The variable Z picks out those mereological parts that function in the same way S functions in Smith's explication of proof by ekthesis. Unfortunately, under the heterodox interpretation, the set of valid and invalid syllogisms, particularly with respect to Baroco and Bocardo, are not consistent with Aristotle's claims. Our challenge will to introduce some modifications to Malink's definitions so as to strive towards an interpretation that follows Aristotle on each of his claims in the mixed apodictic syllogism.

### 1.7 An Adjustment to Malink's Heterodox Interpretation

Now, Malink's definitions would permit the following proof for Baroco-XXX:

- 1.  $BaxA$  (major premise)
- 2.  $BoxC$  (minor premise)
- 3.  $(\forall Z)(AaxZ \supset BaxZ)$  (1 Def ax)
- 4.  $(\exists Z)[CaxZ \ \& \ \sim(BaxZ)]$  (2 Def ox)
- 5.  $CaxU \ \& \ \sim(BaxU)$  (4 EI)
- 6.  $AaxU \supset BaxU$  (3 UI)
- 7.  $\sim(BaxU)$  (5 Simp)
- 8.  $\sim(AaxU)$  (6,7 MT)
- 9.  $CaxU$  (5 Simp)

10.  $CaxU \& \sim(AaxU)$  (8,9 Conj)
11.  $(\exists Z)[(CaxZ \& \sim(AaxZ))]$  (10 EG)
12.  $AoxC$  (11 Def  $ox$ )

And this appears to be a solid proof by ekthesis, as far as it goes. We supply some pseudonyms for various categorical parts that compose the class terms, and arrive at the same conclusion that Aristotle reached. The only adjustment thus far is to negate the whole expression in 4, i.e.  $\sim(BaxZ)$ . Thus, I adhere to Malink's definitions of assertoric propositions with only minor adjustment:

- (1)  $AaxB \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset AaxZ)$  (Def  $ax$ )
- (2)  $AexB \stackrel{\text{def}}{=} (\forall Z)[BaxZ \supset \sim(AaxZ)]$  (Def  $ex$ )
- (3)  $AixB \stackrel{\text{def}}{=} (\exists Z)(BaxZ \& AaxZ)$  (Def  $ix$ )
- (4)  $AoxB \stackrel{\text{def}}{=} (\exists Z)[BaxZ \& \sim(AaxZ)]$  (Def  $ox$ )

The apodictic propositions will differ more substantially from Malink's interpretation. It is in this respect that I hope to devise an interpretation that will allow proofs that hold to the Aristotelian canon. I offer the following:

- (5)  $AaLB \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset AaLZ)$  (Def  $aL$ )
- (6)  $AeLB \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset \bar{A}aLZ)$  (Def  $eL$ )
- (7)  $AiLB \stackrel{\text{def}}{=} (\exists Z)[(BaxZ \& AaLZ) \vee (AaxZ \& BaLZ)]$  (Def  $iL$ )
- (8)  $AoLB \stackrel{\text{def}}{=} (\exists Z)[(BaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& BaLZ)]$  (Def  $oL$ )

Class complements are used instead of negations. This is the primary modification in Malink's heterodox interpretation of Aristotle's semantics. In effect, I am modifying the semantics of *dictum de nullo* in the case of apodictic propositions such that they are treated as *dictum de omni* and plurality associated with the class-complement of the predicate is said of the subject. However, I must motivate the use of class complements, especially since it is somewhat controversial to claim that a class complement would exist for any term that is  $eL$ - or  $oL$ -predicated. Simply put, I

am arguing that a negative apodictic proposition implies that the predicate does not belong to the subject. However, since the subject is a substance or essence term, there must be a class of substance or essence terms that fall under it, which, by the negation, can be categorized by the complementary or privative term. I will provide further support for this later on.

I would also like to offer the following definitions of possible categorical propositions:

- (9)  $A_{aM}B \stackrel{\text{def}}{=} (\forall Z)(B_{aM}Z \supset A_{aM}Z)$  (Def  $a_M$ )
- (10)  $A_{eM}B \stackrel{\text{def}}{=} (\forall Z)(B_{aM}Z \supset \bar{A}_{aM}Z)$  (Def  $e_M$ )
- (11)  $A_{iM}B \stackrel{\text{def}}{=} (\exists Z)(B_{aM}Z \ \& \ A_{iM}Z)$  (Def  $i_M$ )
- (12)  $A_{oM}B \stackrel{\text{def}}{=} (\exists Z)(B_{aM}Z \ \& \ \bar{A}_{iM}Z)$  (Def  $o_M$ )

while contingent propositions could be defined as follows:

- (13)  $A_{aQ}B \stackrel{\text{def}}{=} (\forall Z)[(B_{aM}Z \supset A_{aM}Z) \ \& \ (B_{aM}Z \supset \bar{A}_{aM}Z)]$  (Def  $a_Q$ )
- (14)  $A_{eQ}B \stackrel{\text{def}}{=} (\forall Z)[(B_{aM}Z \supset A_{aM}Z) \ \& \ (B_{aM}Z \supset \bar{A}_{aM}Z)]$  (Def  $e_Q$ )
- (15)  $A_{iQ}B \stackrel{\text{def}}{=} (\exists Z)[B_{aM}Z \ \& \ (A_{iM}Z \ \& \ \bar{A}_{iM}Z)]$  (Def  $i_Q$ )
- (16)  $A_{oQ}B \stackrel{\text{def}}{=} (\exists Z)[B_{aM}Z \ \& \ (A_{iM}Z \ \& \ \bar{A}_{iM}Z)]$  (Def  $o_Q$ )

There are a few things to note with respect to these definitions. The first is that I have opted to define possibility as amplified. This is for the sake of validity across all of the syllogisms Aristotle lists in the *Prior Analytics*. What I should say is that these are operationally consistent definitions. In truth  $A_{aM}B$  could be defined as  $(\forall Z)(B_{aX}Z \supset A_{aM}Z)$ , in other words, were  $B$  to belong to all  $Z$  assertorically, it would follow that  $B$  possibly belongs to  $Z$ , which is to say that  $B_{aM}Z$  could replace any line where there is  $B_{aX}Z$ , *mutatis mutandis* for other categorical terms. However, I have found that  $(\forall Z)(B_{aX}Z \supset A_{aM}Z)$  is less useful as a definition for  $A_{aM}B$ . Nonetheless, one might note that if  $A_{oM}B$  is defined as  $(\exists Z)(B_{aM}Z \ \& \ \bar{A}_{aM}Z)$ , then its contradictory

is, strictly speaking  $(\forall Z)[B_{aM}Z \supset \sim(\bar{A}_{iM}Z)]$ , which is equivalent to  $(\forall Z)[B_{aX}Z \supset \sim(A_{oM}Z)]$  or  $(\forall Z)(B_{aX}Z \supset A_{aL}Z)$ .

The following conversion rules hold provided that the categorical terms are constants and not bounded variables or pseudonyms:

Assertoric Conversion:

- (17)  $A_{aX}B \supset B_{iX}A$  (Conv ax-ix)
- (18)  $A_{iX}B \equiv B_{iX}A$  (Conv ix-ix)
- (19)  $A_{eX}B \equiv B_{eX}A$  (Conv ex-ex)
- (20)  $A_{oX}B$  does not convert

Apodictic Conversion:

- (21)  $A_{aL}B \supset B_{iL}A$  (Conv aL-iL)
- (22)  $A_{iL}B \equiv B_{iL}A$  (Conv iL-iL)
- (23)  $A_{eL}B \equiv B_{eL}A$  (Conv eL-eL)
- (24)  $A_{oL}B$  does not convert

Now, one issue may be the conversion of  $e_L$  propositions, that is:  $(\forall Z)(B_{aX}Z \supset \bar{A}_{aL}Z) \supset (\forall Z)(A_{aX}Z \supset \bar{B}_{aL}Z)$ . Aristotle argues this by assuming the contradictory, that  $B_{iM}A$ , which he sees as a straightforward conversion to  $A_{iM}B$ , in which case A would possibly belong to some B, and non-A would necessarily belong to all B. What we must understand is that some B cannot possibly be an A while necessarily being non-A at the same time. Consequently,  $e_L$ -propositions convert simply.

Possible Conversion:

- (25)  $A_{aM}B \supset B_{iM}A$  (Conv aM-iM)
- (26)  $A_{iM}B \equiv B_{iM}A$  (Conv iM-iM)
- (27)  $A_{eM}B \equiv B_{eM}A$  (Conv eM-eM)
- (28)  $A_{oM}B$  does not convert

Contingent Conversion:

Since universal propositions distribute over the subject, something is said of the nature of the subject, and so it is assumed that it has an underlying nature. As such, the complementary term of the subject can be posited:

- (29)  $Aa_QB \supset Bi_QA$  (Conv  $a_Q-i_Q$ )  
 (30)  $Ae_QB \supset Bo_QA$  (Conv  $e_Q-o_Q$ )

In particular contingent propositions, there must be an underlying nature said of the subject, which is to say that the subject converts just in case the subject is a substance or essence term, which can be established if contingency is amplified:

- (31)  $(\exists Z)[Ba_QZ \& (Aa_MZ \& \bar{A}a_MZ)] \equiv (\exists Z)[Aa_QZ \& (Ba_MZ \& \bar{B}a_MZ)]$  (Conv  $i_Q-i_Q$ )  
 (32)  $(\exists Z)[Ba_QZ \& (Aa_MZ \& \bar{A}a_MZ)] \equiv (\exists Z)[Aa_QZ \& (Ba_MZ \& \bar{B}a_MZ)]$  (Conv  $o_Q-o_Q$ )

Here I am using the operational definitions to illustrate how a particular proposition would convert were it to have contingency amplified to both subject and predicate terms. This, again, would be based on the fact that Aristotle treats certain possibilities as “natural” and in such cases the negative is treated like the affirmative, which is precisely what my interpretation does. But, given that, we must be sensitive to whether the subject of a particular proposition has a complementary term, which cannot be assumed. In fact, were conversions permitted without ampliation, or without establishing that the term in question has a complementary term within the context of the proposition, certain illicit inferences could be made.

It is significant that Aristotle makes the following remark with respect to possibility conversions:

In respect of possible propositions, since possibility is used in several ways (for we say that what is necessary and what is not necessary and what is potential is possible), affirmative statements will all convert in a similar manner. For if it is possible that  $A$  belongs to all or some  $B$ , it will be possible that  $B$  belongs to some  $A$ . For if it could belong to none, then  $A$

could belong to no  $B$ . This has been already proved. But in negative statements the case is different. Whatever is said to be possible, either because it necessarily belongs or because it does not necessarily not belong, admits of conversion like other negative statements... The particular negative is similar. But if anything is said to be possible because it is the general rule and natural (and it is in this way we define the possible), the negative propositions can no longer be converted in the same way: the universal negative does not convert, and the particular does. This will be plain when we speak about the possible. At present we may take this much as clear in addition to what has been said: the statements that it is possible that  $A$  belongs to no  $B$  or does not belong to some  $B$  are affirmative in form; for the expression 'is possible' ranks along with 'is', and 'is' makes an affirmative always and in every case, whatever the terms to which it is added in predication, e.g. 'it is not-good' or 'it is not-white' or in a word 'it is not-this'. But this also will be proved in the sequel. In conversion these will behave like the other affirmative propositions (*APr*: 25a38-25b6, 25b13-25).

This passage tells us that negative possibility premises can be treated as though they were affirmative by use of privative class terms. Aristotle's point here just is the solution we propose to our interpretation, which is extremely significant to the case for our modification to Malink's analysis of Aristotelian semantics. Modal categorical propositions are such that the terms used are assumed to have an underlying nature that admits negation being translated as affirmatives with complementary terms. This permits mixed apodictic syllogisms with negative premises, like Baroco and Bocardo to be consistent with the rest of the syllogisms Aristotle identifies as valid.

Given my use of complementary class terms, it will also be important to specify some obversion rules.

Assertoric Obversion:

$$(33) \bar{A}e_xB \supset Aa_xB \text{ (Obv } e_x\text{-}a_x\text{)}$$

$$(34) \bar{A}a_xB \supset Ae_xB \text{ (Obv } a_x\text{-}e_x\text{)}$$

$$(35) \bar{A}o_xB \supset Ai_xB \text{ (Obv } o_x\text{-}i_x\text{)}$$

$$(36) \bar{A}i_xB \supset Ao_xB \text{ (Obv } i_x\text{-}o_x\text{)}$$

Apodictic Obversion:

$$(37) \bar{A}e_LB \equiv Aa_LB \text{ (Obv } e_L\text{-}a_L\text{)}$$

$$(38) \bar{A}a_LB \equiv Ae_LB \text{ (Obv } a_L\text{-}e_L\text{)}$$

$$(39) \bar{A}o_LB \equiv Ai_LB \text{ (Obv } o_L\text{-}i_L\text{)}$$

$$(40) \bar{A}i_L B \equiv A o_L B \text{ (Obv } i_L\text{-}o_L\text{)}$$

Possible Obversion:

$$(41) \bar{A}e_M B \equiv A a_M B \text{ (Obv } e_M\text{-}a_M\text{)}$$

$$(42) \bar{A}a_M B \equiv A e_M B \text{ (Obv } a_M\text{-}e_M\text{)}$$

$$(43) \bar{A}o_M B \equiv A i_M B \text{ (Obv } o_M\text{-}i_M\text{)}$$

$$(44) \bar{A}i_M B \equiv A o_M B \text{ (Obv } i_M\text{-}o_M\text{)}$$

Contingent Obversion:

$$(45) \bar{A}e_Q B \equiv A a_Q B \text{ (Obv } e_M\text{-}a_M\text{)}$$

$$(46) \bar{A}a_Q B \equiv A e_Q B \text{ (Obv } a_M\text{-}e_M\text{)}$$

$$(47) \bar{A}o_Q B \equiv A i_Q B \text{ (Obv } o_M\text{-}i_M\text{)}$$

$$(48) \bar{A}i_Q B \equiv A o_Q B \text{ (Obv } i_M\text{-}o_M\text{)}$$

Apodictic to Assertoric Subordination:

$$(49) A a_L B \supset (\forall Z)(B a_X Z \supset A a_X Z) \text{ (L-X-sub } a_L\text{)}$$

$$(50) A e_L B \supset (\forall Z)\{[B a_X Z \supset \bar{A} a_X Z] \& [B a_X Z \supset \sim(A a_X Z)]\} \text{ (L-X-sub } e_L\text{)}$$

$$(51) A i_L B \supset (\exists Z)(B a_X Z \& A a_X Z) \text{ (L-X-sub } i_L\text{)}$$

$$(52) A o_L B \supset (\exists Z)\{B a_X Z \& [\sim(A a_X Z) \& \bar{A} a_X Z]\} \text{ (L-X-sub } o_L\text{)}$$

Assertoric to Possible Subordination:

$$(53) A a_X B \supset (\forall Z)(B a_M Z \supset A a_M Z) \text{ (X-M-sub } a_X\text{)}$$

$$(54) A e_X B \supset (\forall Z)[B a_M Z \supset \bar{A} a_M Z] \text{ (X-M-sub } e_X\text{)}$$

$$(55) A i_X B \supset (\exists Z)(B a_M Z \& A a_M Z) \text{ (X-M-sub } i_X\text{)}$$

$$(56) A i_X B \supset (\exists Z)(B a_M Z \& \bar{A} a_M Z) \text{ (X-M-sub } i_X\text{)}$$

Apodictic to Possible Subordination:

$$(57) A a_L B \supset (\forall Z)(B a_M Z \supset A a_M Z) \text{ (L-M-sub } a_L\text{)}$$

$$(58) A e_L B \supset (\forall Z)[(B a_M Z \supset \bar{A} a_M Z) \& (B a_X Z \supset \sim(A a_M Z))] \text{ (L-M-sub } e_L\text{)}$$

$$(59) A i_L B \supset (\exists Z)(B a_M Z \& A a_M Z) \text{ (L-M-sub } i_L\text{)}$$

$$(60) A o_L B \supset (\exists Z)\{B a_M Z \& [\bar{A} a_M Z \& \sim(A a_M Z)] \text{ (L-M-sub } o_L\text{)}$$

Contradiction rules: Contradiction substitution is permitted when the terms in the proposition are constant, and not bound variables or pseudonyms.

Contradictory Assertoric Propositions:

$$(61) A a_X B \equiv \sim(A o_X B) \text{ (} a_X|o_X\text{)}$$

$$(62) A o_X B \equiv \sim(A a_X B) \text{ (} o_X|a_X\text{)}$$

$$(63) A e_X B \equiv \sim(A i_X B) \text{ (} e_X|i_X\text{)}$$

$$(64) A i_X B \equiv \sim(A e_X B) \text{ (} i_X|e_X\text{)}$$

Contradictory Apodictic and Possible Propositions:

- (65)  $Aa_L B \equiv \sim(Ao_M B) (a_L | o_M)$   
 (66)  $Ao_M B \equiv \sim(Aa_L B) (o_M | a_L)$   
 (67)  $Ae_M B \equiv \sim(Ai_L B) (e_M | i_L)$   
 (68)  $Ai_L B \equiv \sim(Ae_M B) (i_L | e_M)$   
 (69)  $Aa_M B \equiv \sim(Ao_L B) (a_M | o_L)$   
 (70)  $Ao_L B \equiv \sim(Aa_M B) (o_L | a_M)$   
 (71)  $Ae_L B \equiv \sim(Ai_M B) (e_L | i_M)$   
 (72)  $Ai_M B \equiv \sim(Ae_L B) (i_M | e_L)$

The following rules are, then, advised in the interpretative model that I propose for

Aristotle's modal syllogistic as pertaining to L-X subordination:

- (73)  $Aa_L B \supset (\forall Z)(Ba_X Z \supset Aa_X Z) (L-X\text{-sub } a_L)$   
 (74)  $Ae_L B \supset (\forall Z)\{[Ba_X Z \supset \bar{A}a_X Z] \& [Ba_X Z \supset \sim(Aa_X Z)]\} (L-X\text{-sub } e_L)$   
 (75)  $Ai_L B \supset (\exists Z)(Ba_X Z \& Aa_X Z) (L-X\text{-sub } i_L)$   
 (76)  $Ao_L B \supset (\exists Z)\{Ba_X Z \& [\sim(Aa_X Z) \& \bar{A}a_X Z]\} (L-X\text{-sub } o_L)$

Notice, in particular allows for the preservation of information from  $e_L$ - to  $e_X$ -propositions and from  $o_L$ - to  $o_X$ -propositions, namely that the complement of an essence term has been posited. It also allows deductions on the level of assertoric propositions. The preservation of the complementary class information will prove vital in the various proofs of the pure and mixed apodictic syllogisms.

My interpretation entirely depends upon the distinctive ways in which negation is used between assertoric and apodictic propositions. By making this distinction, one can generate proofs that cohere precisely with the canonical list of valid and invalid syllogisms provided by Aristotle. Further work will be needed to show that the canonical lists of problematic and contingent syllogisms are also coherent. Additional rules are likely needed, e.g. Barbara-XQM requires a realization or actualization principle that I believe is best motivated by my approach. Also, a



solid understanding of opposition rules between necessity, possibility, and contingency is needed. This, however, is outside the purview of the current work.<sup>16</sup>

Perhaps the most controversial aspect of my assertoric and apodictic propositions is that they rely upon complementary classes and obversion rules. Recall that this interpretative model is based upon quantifying over some categorical term  $Z$ . To deny that  $A$  belongs to any  $B$  is a function of the copula, no doubt, but the effect, within the scope of essence terms, is to  $a_L$ -predicate  $A$  to the complementary of the essence term. This means that negation is not univocal from apodictic to assertoric propositions. To negate an essential predication implies that one may negate the related nonessential predicate, but the two negations are not equivalent.

One might object that by obversion a class-term is negated into its complementary class in an indefinite way, for instance “some man does not belong to white” can become “some non-man belongs to white.” Malink says of this,

In the *de Interpretatione*, Aristotle states that terms such as ‘not-man’ are not names in the proper sense, but merely indefinite names. Aristotle does not use such terms in the *Prior Analytics* 1.1-22. Of course this does not mean they cannot be used in the syllogistic. But in any case, it should not be presupposed that every term possesses a complement in Aristotle’s language of categorical propositions (2013, 99).

Indeed, Aristotle makes use of obversion in *de Interpretation* 10, 19b19-21a1 where he relates four cases:

- |   |  |
|---|--|
| (a) ‘a man is just’   | (b) ‘a man is not just’<br>This is the negation of (a) |
| (d) ‘a man is not not-just’<br>This is the negation of (c). | (c) ‘a man is not-just’                                |

Aristotle says of this:

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<sup>16</sup> The appendix offers a further exploration of possibility and contingency in the modal syllogistic.

Names and verbs that are indefinite (and thereby opposite), such as ‘not-man’ and ‘not-just’, might be thought to be negations without a name and verb. But they are not, For a negation must always be true or false; but one who says not-man—without adding anything else—has no more said something true or false (indeed rather less so) than one who says man (*De Int* 20a31-36).

So the objection that because such terms are indefinite, they are meaningless or illicit for use in obversion, does not work. Aristotle thinks that they are meaningful when set within a predication. The reason that my interpretation does not utilize complementary classes when explicating assertoric propositions is because it cannot be assumed that the predication is substantive or essential, though it might be. Consequently, there cannot be an assumption that the opposition described in  $Ae_xB$  is such that  $Z$  belongs to  $B$  and  $\bar{A}$ . We cannot be sure that since everything  $B$  is not  $A$  that everything  $B$  is  $\bar{A}$ .

To motivate the use of complementary class terms in the specific context of apodictic propositions, we can note that there is a precedent in Aristotle for using obversion as a method to switch between statements like, “feathers belong to no man” and “featherless belongs to all man”. However, Aristotle specifies rules for negations and privative terms, which are related in the same way. He writes:

Let  $A$  stand for to be good,  $B$  for not to be good, let  $C$  stand for to be not-good and be placed under  $B$ , and let  $D$  stand for not to be not-good and be placed under  $A$ . Then either  $A$  or  $B$  will belong to everything, but they will never belong to the same thing; and either  $C$  or  $D$  will belong to everything, but they will never belong to the same thing. And  $B$  must belong to everything to which  $C$  belongs. For if it is true to say it is not-white, it is true also to say it is not white; for it is impossible that a thing should simultaneously be white and be not-white, or be a not-white log and be a white log; consequently if the affirmation does not belong, the denial must belong. But  $C$  does not always belong to  $B$ ; for what is not a log at all, cannot be a not-white log either. On the other hand  $D$  belongs to everything to which  $A$  belongs. For either  $C$  or  $D$  belongs to everything to which  $A$  belongs. But since a thing cannot be simultaneously not-white and white,  $D$  must belong to everything to which  $A$  belongs. For of that which is white it is true to say that it is not not-white. But  $A$  is not true of every  $D$ . For of that which is not a log at all it is not true to say  $A$ , viz. that it is a white log. Consequently  $D$  is true, but  $A$  is not true, i.e. that it is a white log. It is clear also that  $A$  and  $C$  cannot together belong to the same thing, and that  $B$  and  $D$  may belong to the same thing (A.Pr. 51b37-52a12).

This somewhat cryptic passage contain what I believe is a crucial insight in interpreting the modal syllogistic. Aristotle sets up certain implication rules, namely:

$$A \supset D \text{ and } C \supset B$$

But one cannot assume that  $A$  and  $D$  or  $C$  and  $B$  are equivalent. That is if something is good, then it is not non-good, but you cannot say that whatever is not non-good is good. Likewise, if something is non-good then it is not to be good. But whatever is not to be good is not necessarily non-good. Hence, the assertoric obversion rules are implications that move from the privative or complementary terms to negations. The inference from a negation to a complementary term or privative is not considered valid, at least when the proposition is assertoric. Perhaps the more perplexing part of the passage is that Aristotle has to do with the white log. So  $A$  is “white log”,  $B$ : “not to be a white log”,  $C$ : “to be a not-white log”, and  $D$ : “not to be a not-white log.” The issue seems to be an ambiguity in the range of the negation vis-à-vis the privative quality and substance. We cannot obvert  $D$  to  $A$  because it is unclear that there is a white-log at all, but  $A$  can obvert to  $D$ , since it is clear that there is a white log and the privative ranges over the quality only. So, Aristotle’s concern seems to be about mistaking nonessential and essential predications, as “good” and “white” are only qualities of substances and not treated essentially in and of themselves. The claim, then, is that propositions that deal strictly with essence or substance terms will avoid this ambiguity and can be obverted either way.

Aristotle argues in *Prior Analytics* A.46 that “not to be this” is not identical to “to be not-this” and so places a limit on how we might use obversion. Aristotle musters two sorts of arguments for this. The first is based on a few examples. He considers ‘he can walk’ and ‘he can not-walk’ to be analogous to ‘to be white’ and ‘to be not-white.’ Aristotle then notes that ‘he cannot walk’ is not the same as ‘he can not-walk.’ For a man who can not-walk may also be able to walk, but a man who cannot walk is not able to walk. As Ross points out, Aristotle’s argument is fallacious (Ross 1957, 422). A proper obversion of ‘he is not able to walk’ would be ‘he is unable to walk’ or that of ‘he is not that which can walk’ would be ‘he is that which cannot walk.’ Aristotle gives a similar example with ‘he does not know the good’ and ‘he knows the not-good.’ Again, Aristotle argues that someone who knows the not-good could also know the good. But Aristotle has made the same mistake. For, a proper or traditional obversion would be from ‘he is not cognizant of the good’ to ‘he is not-cognizant of the good.’ Ross believes, however, that Aristotle goes on to make a successful argument against obversion. “He points out that being not-equal presupposes a definite nature, that of the unequal, i.e. presupposes as its subject a quantitative thing, while not being equal has not such presupposition” (*ibid.*). Ross says, “Whatever may be said of the form ‘*A* is not-*B*’, which is really an invention of logicians, it is the case that such predications as ‘is unequal’, ‘is immoral’ ... do imply a certain kind of underlying nature in the subject, while ‘is not equal’, ‘is not moral’ do not” (*ibid.*). Again, indefiniteness is only an issue in certain contexts, e.g. outside of the contexts of propositions.

Connected to the notion of indefiniteness is the idea of there being paronymous pairs of terms. Aristotle does not think that nonsubstance paronyms can be the subjects of essential predication. However, the corresponding nouns of paronyms can be the subjects of essential predication and, indeed, have genera. Malink resolves an apparent contradiction in Aristotle by distinguishing between two kinds of nonsubstance terms.

The apparent contradiction can be resolved by means of a distinction that Aristotle draws between two kinds of nonsubstance terms. In the *Categories*, Aristotle distinguishes between nouns such as 'justice', 'blindness', and 'whiteness' on the one hand and corresponding terms such as 'just', 'blind' and 'white' on the other. In Aristotle's terminology, terms of the latter kind are called paronymous or paronyms (Malink 2013, 136).

Substance terms, like "man" are essence terms. However, terms falling under nonsubstance categories may be paronyms with related essential term correlates. For instance, in quantity "equal" relates to "equality" and in quality "white" relates to "whiteness". So, consider again "moving" being  $o_x$ -predicated of "man". In this case, the paronym has a related essence term, "movement" which can be essentially predicated of subjects, though not "man". Nonetheless, if "movement" did not belong to some "men", we might consider it sensible to obvert to  $i_x$ -predication "nonmovement" belongs to some "men". Or one might consider the following: movement necessarily does not belong to some movers, therefore nonmovement, i.e. immutability, necessarily belongs to some movers. The apophatic theologian might be inclined to accept such a proposition, and a large set of other apodictically predicated privatives.

In defense of my use of complementary classes, I argue that while they may be functioning as indeterminate names, those names function as terms within a domain of discourse limited to demonstration within the sciences. In other words,

within that domain, there is a presumption that terms refer to distinct natures, and that the predication of complementary terms is restricted to those natures. So, for instance, in a proposition like “non-amphibian belongs to all man”, “non-amphibian” may be indefinite by itself. However, when it is combined with the subject “man” it takes on a meaning and truth value. Whether that truth-value is equivalent to the truth-value of “amphibian belongs to no man” will depend on another factor, namely whether “amphibian” is an essence or substance term. This can be revealed by the context of the argument in terms of the modality of the premises in which “amphibian” appears. We know that “amphibian” is a natural kind, and so saying that man is non-amphibian is equivalent to saying man is not amphibian. Man is an animal. So to say that man is a non-amphibian, in the context of a demonstration of the sort of animal man is, is really just to say that man is a “non-amphibian animal”. Outside of that context, it may make little sense to talk about non-amphibians, since in and of itself, the complementary term is indefinite.

Aristotle is also not wary of using privative terms in demonstration. For instance, “illness” is  $e_L$ -predicated of “health” (*APr.* 48a8-13). Other privative terms that Aristotle employs includes “ignorance” (ἀμαθία), and “inanimate” (ἄψυχον) (see Malink 2013, 326-333). Indeed, Aristotle utilizes “illness”, “ignorance”, and “inanimate” in demonstrative syllogisms throughout the *Prior Analytics*. This suggests that he would have permitted  $a_L$ -predications of these privative terms, even though they are not proper substances. Instead, they are the complementary classes of substance or essence terms. Though they are not substance terms, or essence terms in the proper sense, they capture substances or essences which do

not fall under the complementary positive terms to which they are paired. This confers the features of substance or essence terms upon them, i.e. they can be the subjects of  $a_L$ -predication, and they can counterpredicate insofar as they are the complementary class terms of substantive or essential terms. So, for instance, while “not-white” cannot counterpredicate, “non-whiteness” can.

Deslauriers argues that *Posterior Analytics* 2.13 where, “...Aristotle describes a process like division, the procedure whereby differentiae are assigned to a genus in order to differentiate species, i.e. divide the genus: ‘of the attributes which belong to each thing there are some which are wider in extent than it but not wider than its genus...’ (2007, 23-24). This sounds strikingly similar to the Stranger’s method of *dieresis* in Plato’s *Sophist*. However, Deslauriers argues that Aristotle prohibits, or is at least critical of privative divisions. “Privative divisions are, then, problematic in two ways: they cannot be further divided because they cannot be further specified, and they make necessary the identification of the genus with one of its species (by appeal to the law of excluded middle)” (*ibid.* 29). Nonetheless, Aristotle uses privatives, which suggests that there are some contexts where their use is legitimate. To complicate the matter further, Aristotle’s views on privatives developed in his biological works. In *Parts of Animals* Aristotle writes:

...[P]rivative terms inevitably form one branch of dichotomous division, as we see in the proposed dichotomies. But privative terms in their character of privatives admit of no subdivision. For there can be no specific forms of a negation, of Featherless for instance or of Footless, as there are of Feathered and of Footed. Yet a generic differentia must be subdivisible; for otherwise what is there that makes it generic rather than specific? (*PA* 642b22-26).

This seems to suggest that by the time he was focusing on his scientific investigations, Aristotle wanted to exclude privatives generated from dichotomous

divisions from being treated as essence terms. They cannot be a genus or species to anything else. In fact, Aristotle notes that some animals, like the ant and glowworm, can be divided within the species into those with wings and the wingless. So, privative terms cannot be used in essential definitions. Nonetheless, even if a privative is accidental, it is still *per se* accidental. For Aristotle says,

...straight belongs to line and so does curved, and odd and even to number, and prime and composite, and equilateral and oblong; and for all these there belongs in the account which says what they are in the one case line, and in the others number. And similarly in other cases too it is such things that I say belong to something in itself (*APo.* 73b1-5).

Indeed, of these sorts of *per se* predications, Aristotle claims that they,

...hold both because of themselves and from necessity. For it is not possible for them not to belong, either *simpliciter* or as regards their opposites—e.g. straight or crooked to line, and odd and even to number. For the contrary is either a privation or a contradiction in the same genus—e.g. even is what is not odd among numbers, in so far as it follows. Hence if it is necessary to affirm or deny, it is necessary too for what belongs in itself to belong (*APo.* 73b18-24).

So in dichotomous divisions of *per se* accidents, it seems that we can say that it is necessary that privatives belong to the subject in some sense. Ferejohn refers to the passage as mystifying, but translating the 73b24 as “since it is necessary that everything be affirmed or denied” he believes the conclusion depends, “...ultimately on what seems to be some modalized version of the *Law of the Excluded Middle* (LEM)...” (1991, 101).<sup>17</sup> If so, the modalized *per se* predications will have opposites that fall under the same genus, which Ferejohn dubs A-pairs. To explain this, Ferejohn proposes the Principle of Opposites (PO), which states: “If  $(\Phi, \Psi)$  form an A-pair appropriate to genus G, then application of “ $\Psi$ ” and “not  $\Phi$ ” within G are intersubstitutable” (*ibid.* 102). This is to say that A-pairs are complements. This is

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<sup>17</sup> Ferejohn defines two versions of the modalized-LEM. 1) Weak MLEM: Necessarily, for every member  $x$  of G, and for every attribute F applicable within F,  $x$  either has F or lacks F. 2) Strong MLEM: for every member  $x$  of G, and for every opposite F appropriate to G,  $x$  either necessarily has G or  $x$  necessarily lacks F. Aristotle opts for the stronger *de re* version (1991, 102).



precisely the sense in which I hold that apodictic negative propositions can be obverted and contain privative predications that, being necessary to the subject, are *per se* yet accidental to the subject.

Still there is concern in using privatives. “The concern with dichotomous and privative divisions is then a concern about completeness, and so, ultimately, a concern about arbitrary divisions. This is because Aristotle, like Plato, believes that only a complete division can ensure natural or non-arbitrary divisions and hence correct definitions...” (Deslauriers 2007, 29). Nonetheless, demonstration does not always have to be of the definition. It would be a mistake for Aristotle to reject the use of privatives in scientific demonstration on the grounds that they are incomplete.

In the case of apodictic predications, my interpretation affirms that we can assume more information. This is because we are dealing with the a special kind of predication—what Rini would see as linking together two red term, or the predication of substance or essence terms, as Malink sometimes puts it (see Malink 2013, 7). Malink distinguishes between essence terms, which can be the subjects in some  $a_L$ -predications, and substance terms, which can be the subjects in some strong  $a_L$ -predications (Malink 2013, 14). This means that an  $a_L$ -predication implies that the predicate is at least an essence term and possibly a substance term. My contention is that any subject term that is predicated of by necessity will have class complements, whereas one cannot make this assumption with respect to nonessential terms. My argument for the complementarity of essence and substance terms is as follows:

- I: For any substance or essence term that is predicated of a subject, there exists a genus to which that substance or essence term belongs.
- II: For any genus to which a substance or essence term belongs, there is a genus to which the substance or essence term does not belong.
- III: If there is a genus to which a substance or essence term<sub>1</sub> does not belong, then there exists an essence or substance term<sub>2</sub> that is contrary to essence or substance term<sub>1</sub>.
- IV: If there exists an essence or substance term<sub>2</sub> that is contrary to essence or substance term<sub>1</sub>, there exists an essence or substance term that is the member of the complementary class to term<sub>1</sub>.

The basic intuition behind this argument is that no essence or substance term will be co-extensional with all essence or substance terms. So, there will always be an essence or substance that is non-identical to any term one specifies. Thus, there will always be a condition that satisfies the complementarity of any substance or essence term. From this, we can conclude that for any substance or essence term that is predicated of a subject, there exists an essence or substance term that is a member of the complementary class of that term. In the *Metaphysics* Aristotle makes an important distinction between what he calls “bare negation” and privative terms with respect to scientific knowledge. In this case, Aristotle is considering a scientific knowledge of unity and its opposites. Can there be a scientific knowledge of that which is non-unity? Aristotle answers as follows:

Now since it is the work of one science to investigate opposites, and plurality is opposite to unity, and it belongs to one science to investigate the negation and the privation because in both cases we are really investigating unity, to which the negation or the privation refers (for we either say simply that unity is not present, or that it is not present in some particular class; in the latter case the characteristic difference of the class modifies the meaning of 'unity', as compared with the meaning conveyed in the bare negation; for the negation means just the absence of unity, while in privation there is also implied an underlying nature of which the privation is predicated),—in view of all these facts, the contraries of the concepts we named above, the other and the dissimilar and the unequal, and everything else which is derived either from these or from plurality and unity, must fall within the province of the science above-named (*Meta.* 1004a10-20).<sup>18</sup>

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<sup>18</sup><< ἐπεὶ δὲ μιᾶς τάντικείμενα θεωρῆσαι, τῷ δὲ ἐνὶ ἀντίκειται πλῆθος—ἀπόφασιν δὲ καὶ στέρησιν μιᾶς ἐστὶ θεωρῆσαι διὰ τὸ ἀμφοτέρως θεωρεῖσθαι τὸ ἐν οὐ ἢ ἀπόφασιν ἢ ἢ στέρησιν (ἢ γὰρ ἀπλῶς λέγομεν ὅτι οὐχ ὑπάρχει ἐκεῖνο, ἢ τινι γένει· **ἐνθα μὲν οὖν τῷ ἐνὶ ἢ διαφορὰ πρόσεστι παρὰ τὸ ἐν τῆ**

This passage, along with *Prior Analytics* A.3 and A.46, quoted above, are the strongest evidence in support of the modification to Malink’s *Dicto de omni et nullo* semantics. In the interpretation that I offer, negative assertoric propositions are “bare negations” that do not imply an underlying nature. Indeed, scientific knowledge follows upon demonstration, as we will discuss further in the second chapter. Thus, privative predications that follow upon scientific demonstrations would be apodictic. Negative apodictic propositions assert a necessary relation to the predicate being negated. I believe an Aristotelian understanding of modality is founded upon the notion that modal properties are founded upon more primitive metaphysical notions of natures and essences that inform us of what a substance is fundamentally. Insofar as necessity is grounded in the nature of the subject, there is an underlying nature implied even by the necessity of the negation. This does not mean that the nature is itself negative, but that there is some nature that, in so far as it is affirmed, the negative predication is equivalent to a privative predication with respect to the subject and implies an underlying nature.

Still, one might worry that since obversion implies complementarity and so strong supplementation, any insistence upon strong supplementation in the case of apodictic predication would be an *ad hoc* move on my part. Malink writes, “In the

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**ἀποφάσει**, ἀπουσία γὰρ ἡ ἀπόφασις ἐκείνου ἐστίν, ἐν δὲ τῇ στερήσει καὶ ὑποκει ἡ ἀπόφασις ἐκείνου ἐστίν, ἐν δὲ τῇ στερήσει καὶ ὑποκειμένη τις φύσις γίγνεται καθ’ ἧς λέγεται ἡ στέρησις) τῷ δ’ ἐνὶ πλῆθος ἀντίκειται—ὥστε καὶ τάντικείμενα τοῖς εἰρημένοις, τό τε ἕτερον καὶ ἀνόμοιον καὶ ἄνισον καὶ ὅσα ἄλλα λέγεται ἢ κατὰ ταῦτα ἢ κατὰ πλῆθος καὶ τὸ ἔν, τῆς εἰρημένης γνωρίζειν ἐπιστήμης>> (*Meta.* 1004a10-20, emphasis mine).

preorder semantics, the existence of such complements for every term implies the mereological principle of (strong) supplementation:  $\sim(BaxA) \supset (\exists Z)\{(AaxZ) \& (\forall Y)[(ZaxY) \supset \sim(BaxY)]\}$ . This is equivalent to the claim that  $BoxA \supset (\exists Z)(AaxZ \& BexZ)$ , which Malink calls the “strong principle of  $o_X$ -ecthesis.” According to Malink, Aristotle rejects this principle in *Prior Analytics* B.22. Thus he is committed to denying the universal existence of complements satisfying the traditional laws of obversion: some terms may have such complements, but not all (Malink 2013, 99). Malink cites Brenner (2000, 342) for further evidence that the traditional laws of obversion simply will not work for X- and L-propositions. Brenner argues that, were one to admit traditional obversion rules and conversion rules, then Barbara- $XLX$  could be transformed into Celarent- $LXL$  and one would be able to conclude to Barbara- $LXL$ , i.e. that  $AaLc$  follows from the conclusion. He argues as follows:

1.  $AaxB$  (major premise)
2.  $BaLc$  (minor premise)
3.  $\sim Bax\sim A$  (1 by contraposition)<sup>19</sup>
4.  $\sim BeLc$  (2 by obversion)
5.  $CeL\sim B$  (4 by conversion)
6.  $CeL\sim A$  (3,5 Celarent- $LXL$ )
7.  $\sim AeLc$  (6 by conversion)
8.  $AaLc$  (7 by obversion)

In appealing to traditional obversion rules, Brenner allows contraposition of assertoric propositions. Since contraposition depends on the validity of obversion, the question is whether this sort of inference should be permitted. Indeed, our rules block this inference, treating obversion of assertoric propositions as an entailment rather than equivalence. If one is provided the complementary class of a term, one

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<sup>19</sup> This follows Brenner’s symbolic conventions.

can obvert it to the negation of the claim  $AaB$ , that is, if one has  $\bar{A}axB \supset AexB$ , but the inference does not work the other way. So, our interpretation does not founder on Brenner's argument. Moreover, I agree with Malink that Aristotle rejects traditional obversion. One cannot assume that every predicate term has a class-complement with, as Aristotle would say, an underlying nature.

Given that I accept Malink's interpretation of Aristotle on obversion, I do not endorse a strong principle of  $o_X$ -ekthesis. One can say that my interpretation implies a strong principle of  $o_L$ -ekthesis. I hold that  $Bo_LA \supset (\exists Z)\{(AaxZ) \& (\forall Y)[(ZaxY \supset \bar{B}axY)]\}$ , which is just to say that  $Bo_LA$  implies  $(\exists Z)(AaxZ \& \bar{B}axZ)$ , which can be shown to be a theorem given my interpretative rules:

1.  $Bo_LA$  (CP)
2.  $\sim(\exists Z)(AaxZ \& \bar{B}axZ)$  (IP)
3.  $Bo_LA \supset (\exists Z)\{AaxZ \& [\sim(BaxZ) \& \bar{B}axZ]\}$  (L-X-sub  $o_L$ )
4.  $(\exists Z)\{AaxZ \& [\sim(BaxZ) \& \bar{B}axZ]\}$  (1,3 MP)
5.  $AaxU \& [\sim(BaxU) \& \bar{B}axU]$  (4 EI)
6.  $(\forall Z)\sim(AaxZ \& \bar{B}axZ)$  (2 QN)
7.  $\sim(AaxU \& \bar{B}axU)$  (6 UI)
8.  $\sim(AaxU) \vee \sim(\bar{B}axU)$  (7 DeM)
9.  $\sim(BaxU) \& \bar{B}axU$  (5 Simp)
10.  $\bar{B}axU$  (9 Simp)
11.  $\sim\sim(\bar{B}axU)$  (10 DN)
12.  $\sim(AaxU)$  (8,11 DS)
13.  $AaxU$  (5 Simp)
14.  $AaxU \& \sim(AaxU)$  (12,13 Conj)
15.  $\sim\sim(\exists Z)(AaxZ \& \bar{B}axZ)$  (2-14 IP)
16.  $(\exists Z)(AaxZ \& \bar{B}axZ)$  (15 DN)
17.  $Bo_LA \supset (\exists Z)(AaxZ \& \bar{B}axZ)$  (1-16 CP)

While this does not alone prove that Aristotle endorsed there being a complementary class for every term that is apodictically predicated of a subject, it does demonstrate consistency on the part of my interpretation, which does not require any *ad hoc* claim of strong supplementation in apodictic contexts merely

because of my reliance on complementarity. Strong  $o_L$ -ekthesis is a theorem of my interpretation as a consequence of devising rules that follow Aristotle's canonical listing.

Malink considers the following example, where "moving" is  $o_X$ -predicated of "man." On the view that there are complements for every term, the set that is the semantic value of "non-moving man" would have to be nonempty (Malink 2013, 98).

However, Malink retorts,

...the question whether the domain of possible semantic values contains a semantic value for the term 'non-moving man' comes down to the question whether the language under consideration contains the term. Even if the language under consideration contains two terms 'man' and 'moving', there is no guarantee that it also contains the term 'not-moving man' (Malink 2013, 98-99).

While I agree with this example, the question becomes more difficult when we consider the distinction Malink makes between essence terms and nonessence terms. My argument is that apodictic predication involves an essence, substance, or what has been termed an "epistemic substance" by Goldin, wherein the set that is the semantic value of complementary terms is nonempty, or assumed as existing for the sake of the first principles of a given science.<sup>20</sup>

Finally, we might consider the objection that the use of privatives is merely to place negation in the predicate term rather than the copula. If, like modality and quantity, the negative quality of a proposition is to be treated by the copula, using complementary classes in articulating  $e_L$ - and  $o_L$ -propositions may be misleading at best, and contrary to Aristotle's tripartite structure of the proposition. It is important to say that apodictic propositions imply correlated assertoric

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<sup>20</sup> Goldin defines epistemic substances as, "...those entities the existence of which must be assumed by the science studying them, through what Aristotle calls 'hypotheses,' and that the definition of such entities and no other, will have the status of scientific first principles" (1996, 12).

propositions. So  $Ae_L B \supset Ae_x B$ . With respect to both apodictic and assertoric propositions, we should understand that the negation is in the copula, even if the heterodox interpretation captures  $Ae_x B$  as  $(\forall Z)[BaxZ \supset \sim(AaxZ)]$ . This should not be understood as definitive of  $ex$ -predication as much as it is an ekthetical explication by considering how the parts to which  $B$  belongs are not parts to which  $A$  belong. Now in my apodictic explication, this much is implied, but I also articulate the fact that the parts to which  $B$  belongs are parts to which  $\bar{A}$  belong.

### 1.8 Conclusion:

To summarize my case for using complementary classes in apodictic propositions, I have argued that my interpretation is consistent with the restrictions on negation and obversion that Aristotle makes in *De Interpretatione* 10 and *Prior Analytics* A.46. I address the concerns of indefiniteness by discussing Aristotle's use of paronyms, and also his use of privative terms is consistent with Aristotle's use of such terms throughout the *Prior Analytics*. I have constructed a philosophical argument for the complementarity of essence and substance terms, and found motivation in *Metaphysics*  $\Gamma$  for there to be an underlying nature in the case of privative terms as used in scientific demonstrations. Finally, my interpretation adheres to strong  $o_L$ -ekthesis, which is implied by the assumption that every apodictically predicated term has a complementary term. So I think the case for using complementary classes in apodictic negative predications is well-founded.

Along with my alternative definitions, I propose a graphic way of representing the modal syllogisms through modalized Venn diagrams. These Venn diagrams intend to track precisely which terms Aristotle claims are modalized substantive terms, and which are not modalized or nonessential. There is an historic precedent of using Venn Diagrams as an aid to visualizing validity and invalidity in assertoric categorical syllogisms. Various standards are set forth for symbolizing quantitative and qualitative copulation between terms. Those standards are preserved here, and a few more standards are adopted for adding modality. In constructing modalized Venn diagrams, it is as if we are adding a dimension by which we can track the modal information contained in the syllogism. Venn diagrams allow us to “...represent the relations of membership and inclusion and the operations of union, intersection, and complementation” (Baron 1969, 113).

Baron explains,

The scope and content of ancient formal logic was determined by Aristotle's *Organon* and, in particular, the *Doctrine of the Syllogism* which, as it has come down to us, contains no diagrams. Nonetheless, so suggestive is the language and manner of presentation of the syllogistic schema, that many logicians have speculated as to the possibility that Aristotle made use of spatial concepts in his actual lectures (Baron 1969, 114-115).

It is in this spirit that I have attempted to spatially conceive of the modal syllogistic, in the hopes that it may reveal what Aristotle had in mind when he proposed his perplexing canonical listing of valid and invalid argument forms. In fact, it was through playing around with modality and Venn diagrams that I came upon the idea that complementary classes and negation needed to be addressed.

The diagrams are intended to illustrate the relationship between class terms. One difference is that, while standard Venn Diagrams indicate particular



propositions by using asterisks, as if to draw our attention to particular individuals that dwell within a region, my use of asterisks is to indicate that there exists a subset within a region of a class-term that can be the subject of predication for the terms intersecting that region. What we are indicating by the asterisks is not some individual that exemplifies the class term, but a sub-class term that falls within the region. This is in keeping with Malink's heterodox interpretation. In fact, there is something more intuitive in thinking that an asterisk picks out terms that are parts that compose a region of intersecting class-terms. The alternative view, which it picks out individuals, raises problems of existential import for arguments like Darapti and Camestro. More will be said of how the diagram is intended to track modal relationships, in the next chapter.

## CHAPTER TWO

### 2.0 Explication of Argument Forms along with Venn Diagrams

It is not possible to provide a deductive proof of invalidity. At best one can point to the failure to provide a deductive proof for these arguments as *prima facie* evidence that they are invalid. So the lack of any obviously valid deduction would accord with Aristotle's findings. Aristotle sought to prove invalidity through examples, however given that his proofs were challenged nonetheless by subsequent logicians, proof by example is hardly seen as decisive. Graphic representations, such as Venn diagrams, are a useful way to indicate invalidity, though it may be somewhat circular, since one is identifying a certain visual pattern with invalidity, and so must assume certain forms are invalid in order to establish the patterns as a guide. That is, regrettably, a limit to this method. However, once those visual patterns are established, predictions can be made, as to which forms should be considered valid or invalid. The following is my explication of the modal syllogistic given my interpretation, definitions, and rules along with related Venn diagrams.

The four categorical propositions A, E, I, and O are typically diagrammed in the following manner:

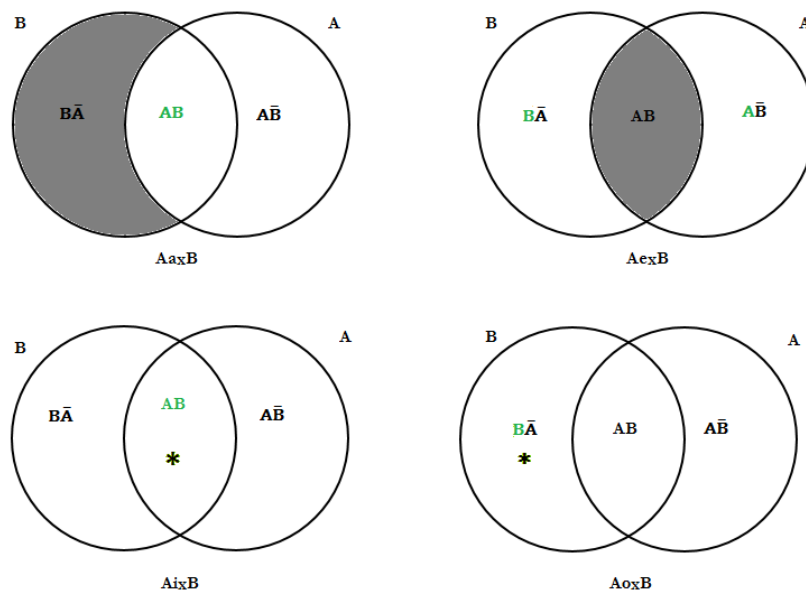


Fig. 1: Assertoric Categorical Propositions

Note that green lettering occurs in those regions where  $ax$ -prediction can be assumed for a class term. In  $AaxB$ , both terms are green because, by conversion *per accidens*, we know that  $BixA$ , in which case there would exist some  $Z$  that of which  $B$  is  $ax$ -predicated. In  $AexB$ , the class complementary terms are not green, since there is no assumption that there is an underlying nature to that which is not  $B$ , or not  $A$ . In the particular propositions, an asterisk is used to represent the existence of some  $Z$ . In the case of  $AixB$ , that  $Z$  is  $ax$ -predicated of  $A$  and  $B$ , hence both terms are colored green. In  $AoxB$ , only  $B$  is colored green, as we do not know that there is an underlying nature to that which is not  $A$ , nor do we assume that  $A$  is an essence or substance term.

We can modalize the Venn Diagrams by noting which regions or particulars are said to belong, or not belong to other regions by necessity. Here is an example of how I propose to modalize various regions.

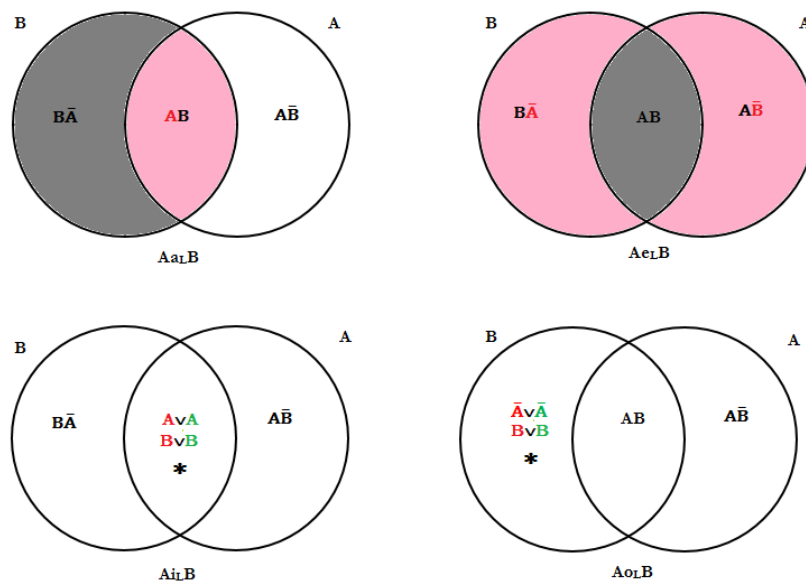


Fig. 2: Apodictic Categorical Propositions

The first thing to note is that various terms are colored red. In  $Aa_L B$ ,  $A$  is  $a_L$ -predicated of  $Z$ , hence it is red. Now one might anticipate the  $B$  in  $Aa_L B$  should be colored green, because it is  $a_X$ -predicated of  $Z$ . However, this would indicate, according to the conventions of the diagram that I am setting forth, that an assertoric premise has coincided with that region and “colored” the term. So for the sake of our method, apodictic  $a_L$ -propositions do not color the subject term green. Likewise, in  $Ae_L B$ , we see that it is the complementary terms that are highlighted red and their regions are shaded pink. This is in keeping with our interpretation that negative apodictic propositions can always be obverted. Given that  $e_L$ -propositions convert simply, both complementary terms are red, and the regions which they occupy are shaded pink. The subject terms of  $e_L$ -propositions are not colored red or green, it will be the task of other premises to link into the subject terms to help us determine validity. In the particular propositions, given the disjunctive

interpretation of Malink, it is not asserted whether a given term in a region is red or green. In  $Ai_L B$ ,  $Z$  is either the subject of  $a_L$ - or  $a_X$ -predication for  $A$  or  $B$ , so  $A$  or  $B$  could be red or green. It will take more information to determine whether the entire region should be shaded, since nothing can be assumed of the entire region.

Likewise for  $Ao_L B$ , we do not know if  $B$  or the complement of  $A$  are red or green, though it is one or the other.

Venn diagrams of categorical syllogisms involve three interlocking circles, each representing a term of the argument. The grey regions indicate regions precluded by the proposition. A red asterisks indicate that there is a modalized subset. The subset is red to indicate that it is  $a_L$ -predicated of at least one other term in the region. Pink regions indicate that two relevant terms are linked together by necessity. By relevant, I mean relevant to the conclusion.  $Aa_L B$  is equivalent to  $(\forall Z)(BaxZ \supset Aa_L Z)$ . Hence the pink  $A_L$  region indicates that all of the  $Z$ s that are  $B$  are necessarily  $A$ .

## 2.1 The First Figure

First figure syllogisms have the middle term as the subject of the major premise, and the predicate of the minor premise. Aristotle says that in the mixed apodictic syllogisms of the first figure, all of the  $LXL$  forms are valid while the  $XLL$  forms are invalid. The following uses Venn diagrams to illustrate validity and invalidity and also provides proofs based on the definitions and rules devised in the previous sections.

Below, in Figure 3, we see a comparison between Barbara-LXL and Barbara-XLL:

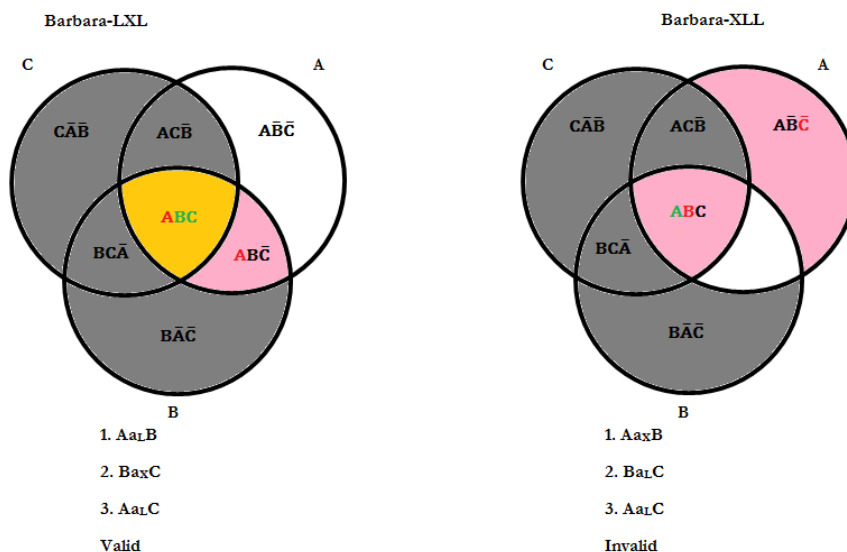


Fig. 3: Comparison of Barbara-LXL to Barbara-XLL

As we can see on the left, any term that belongs to the  $ABC$  region is necessarily  $B$ . Since all of the  $A$ s are in that region, all  $A$ s are necessarily  $B$ . To aid with perceiving the validity of apodictic syllogism, I have adopted the convention of shading a region gold under the following condition: it must be known that a term is red, and the other two terms must be colored red or green. When all three letters of the region are known to have a particular color, the region is highlighted in gold to indicate that an apodictic conclusion can be drawn. On the right, we see that the first premise allows us to shade  $A$  and  $B$  green in region  $ABC$ . The second premise allows us to color  $B$  red in  $ABC$ . Given that  $Ba\bar{L}C$  can be contraposition to  $\bar{C}a\bar{L}\bar{B}$ , we must also shade region  $A\bar{B}\bar{C}$  pink, and color  $\bar{C}$  red. Such information should not be left out, as

it will be significant for certain figures. As we can see, Barbara-XLL does not color all of the relevant terms, so an apodictic conclusion cannot validly be reached.

The validity of Barbara-LXL can also be demonstrated by our modified definitions. A proof of the invalidity of Barbara-XLL, of course, cannot be provided. Indeed, despite the best efforts of the author, no proof is available for any of the invalid mixed apodictic syllogism forms. The validity of Barbara-LXL is as follows:

Barbara-LXL (Valid):

1.  $Aa_L B$  (major premise)
2.  $Ba_X C$  (minor premise) //  $Aa_L C$  ( $\forall Z$ )( $Ca_X Z \supset Aa_L Z$ )
3. ( $\forall Z$ )( $Ba_X Z \supset Aa_L Z$ ) (1 Def  $a_L$ )
4. ( $\forall Z$ )( $Ca_X Z \supset Ba_X Z$ ) (2 Def  $a_X$ )
5.  $Ba_X U \supset Aa_L U$  (3 UI)
6.  $Ca_X U \supset Ba_X U$  (4 UI)
7.  $Ca_X U \supset Aa_L U$  (5,6 HS)
8. ( $\forall Z$ )( $Ca_X Z \supset Aa_L Z$ ) (7 UG)
9.  $Aa_L C$  (8 Def  $a_L$ )

Further examples help to show how this visualization makes sense of other moods.

As Aristotle says,

For since  $A$  necessarily belongs, or does not belong, to every  $B$ , and since  $C$  is one of the  $B$ s, it is clear that for  $C$  also the positive or negative relation to  $A$  will hold necessarily. But if  $AB$  is not necessary, but  $BC$  is necessary, the conclusion will not be necessary (*APr.* A.9 30a20-24).<sup>21</sup>

This text confirms that Aristotle thought Celarent-LXL is valid, but Celarent-XLL is invalid. Aristotle provides an example to motivate the invalidity of Barbara-XLL. The major term is movement, the middle term is animal, and the minor term is man. So, Barbara-XLL tells us:

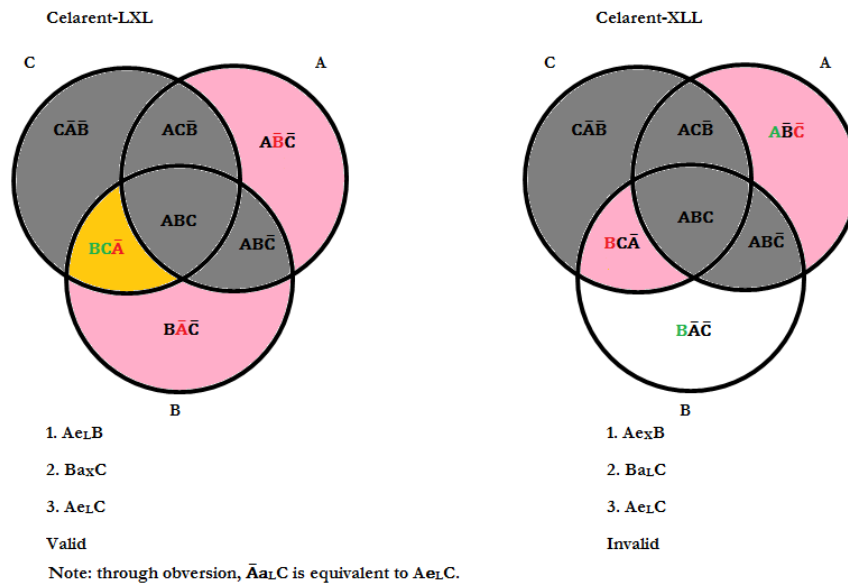
1. Movement belongs to every Animal.
2. Animal necessarily belongs to every Man.
3. Therefore, Movement necessarily belongs to every Man.

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<sup>21</sup> In this quote,  $A$  is the major term,  $B$  is the middle, and  $C$  is the minor term.

Aristotle notes that the premises are both true. But the conclusion does not follow, because movement does not necessarily belong to animals or man. This is because an animal has the capacity for movement, but isn't necessarily in motion.

Our next set of mixed apodictic syllogisms to consider is Celarent-LXL and Celarent-XLL. As is the case for all mixed apodictic syllogisms in the first figure, the LXL form is valid while the XLL form is invalid. Our Venn concurs with Aristotle's findings:



In Celarent-LXL, we see an instance of an  $e_L$ -predication. Hence, we are able to color the complementary class terms red. On the left, in region  $BC\bar{A}$  we see that the region is shaded golden because the assertoric minor premise allowed us to color  $B$  and  $C$  green. On the right, we see that the assertoric premise allows us to color  $A$  in region  $A\bar{B}\bar{C}$  green. In fact, due to contraposition of the major apodictic premise, we must



color  $\bar{C}$  red. The region is not shaded gold because  $\bar{B}$  is not colored red or green.

Celarent-LXL can be proved valid by way of the following deduction:

Celarent-LXL (Valid):

1.  $Ae_L B$  (major premise)
2.  $BaxC$  (minor premise) //  $Ae_L C (\forall Z)(CaxZ \supset \bar{Aa}_L Z)$
3.  $(\forall Z)(BaxZ \supset \bar{Aa}_L Z)$  (1 Def  $e_L$ )
4.  $(\forall Z)(CaxZ \supset BaxZ)$  (2 Def  $ax$ )
5.  $BaxU \supset \bar{Aa}_L U$  (3 UI)
6.  $CaxU \supset BaxU$  (4 UI)
7.  $CaxU \supset \bar{Aa}_L U$  (5,6 HS)
8.  $(\forall Z)(CaxZ \supset \bar{Aa}_L Z)$  (7 UG)
9.  $Ae_L C$  (8 Def  $e_L$ )

A proof is unavailable for Celarent-XLL, as we would need to relate  $CaxZ$  to  $\bar{Aa}_L Z$ . In terms of necessity, there are no regions that relate  $C$  to  $A$ . Thus, there is no reason to suppose that  $A$  would necessarily belong to no  $C$ .

The next diagram is significant in that it is the first to include particular premises.

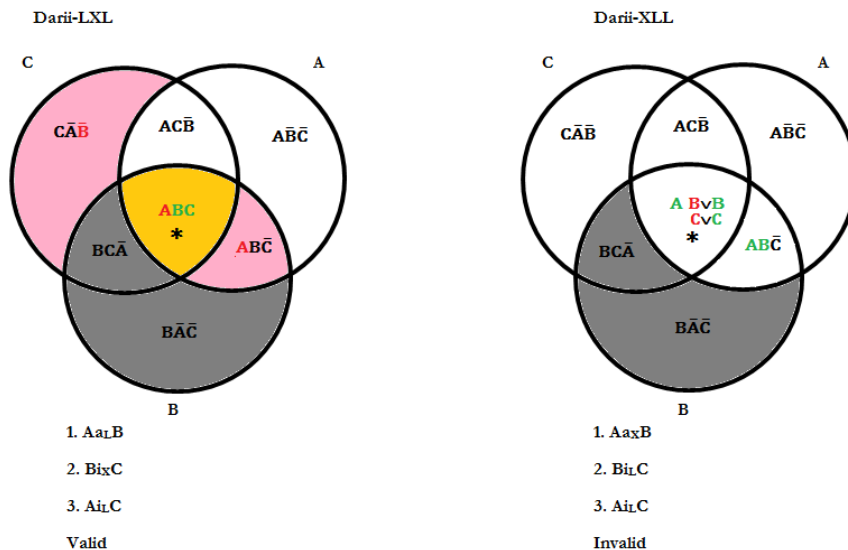


Fig. 5: Comparison of Darii-LXL to Darii-XLL

On the left we see that the assertoric minor premise guaranteed that  $B$  and  $C$  were green.  $ABC$  is shaded gold because it contains a red  $A$  and so an apodictic conclusion can be reached. Recall that  $\bar{B}$  is colored red because of the possibility of contraposition on the major premise. On the right, we see that region  $ABC$  should have a green  $A$  and  $B$ . However, the disjunctive nature of the minor premise does not allow us to infer the conclusion. We cannot know whether  $B$  or  $C$  should be colored red. As we must know whether a specific term is to be colored red in order to shade a region pink or gold, an apodictic conclusion cannot be reached. Of course, this is just to accord with Aristotle's findings regarding Darii and Ferio:

In particular deductions, if the universal is necessary, then the conclusion will be necessary; but if the particular, the conclusion will not be necessary, whether the universal proposition is negative or affirmative (*APr*: A9. 30a34-36).

Aristotle invites us to consider an argument against the validity of Darii-XLL with the terms 'movement,' 'animal,' and 'white.' So:

1. Motion belongs to all Animal [ $AaxB$ ]
2. Animal necessarily belongs to some White [ $BiLC$ ]

The question is whether White necessarily belongs to some motion. Now it is somewhat difficult to see what this example shows. It seems Aristotle wants to say that even if some of that which is white is necessarily an animal, it does not follow that some of that which is white is necessarily in motion. This is because motion is not said to belong to animal by necessity, so while animal may be said of some white by necessity, we cannot assume that all of the properties of animal are held necessarily by white in virtue of being animal, even if that white thing is animal by necessity.

On the other hand, Darii-LXL shows that the major term belongs to the minor term by necessity precisely because it is predicated of the middle term by necessity.

The proof for Darii-LXL is as follows:

Darii-LXL (Valid):

1.  $AaLB$  (major premise)
2.  $BiXC$  (minor premise)// $AiLC$   $(\exists Z)[(CaXZ \ \& \ AaLZ) \ \vee \ (AaxZ \ \& \ CaLZ)]$
3.  $(\exists Z)(CaXZ \ \& \ BaXZ)$  (2 Def ix)
4.  $(\forall Z)(BaXZ \ \supset \ AaLZ)$  (1 Def aL)
5.  $BaXU \ \& \ CaXU$  (4 EI)
6.  $BaXU$  (5 Simp)
7.  $BaXU \ \supset \ AaLU$  (4 UI)
8.  $AaLU$  (6,7 MP)
9.  $CaXU$  (5 Simp)
10.  $CaXU \ \& \ AaLU$  (8,9 Conj)
11.  $(CaXU \ \& \ AaLU) \ \vee \ (AaxU \ \& \ CaLU)$  (10 Add)
12.  $(AaxU \ \& \ CaLU) \ \vee \ (CaXU \ \& \ AaLU)$  (11 Comm)
13.  $(\exists Z)[(AaxZ \ \& \ CaLZ) \ \vee \ (CaXZ \ \& \ AaLZ)]$  (12 EG)
14.  $CaLA$  (13 Def iL)
15.  $AiLC$  (14 Conv iL-iL)

Because of the disjunctive definition of  $i_L$ -propositions, it does not seem that a parallel proof for Darii-XLL is possible. Malink defends the disjunctive definition of  $i_L$ -propositions insofar as according to the definition they are symmetric and so can convert, and they satisfy the restrictions Aristotle lays out (Malink 2013, 179).

In considering Ferio, we complete the perfect syllogisms of the first figure. The mixed modal perfect syllogisms are all valid in the LXL form, and invalid in the XLL form.

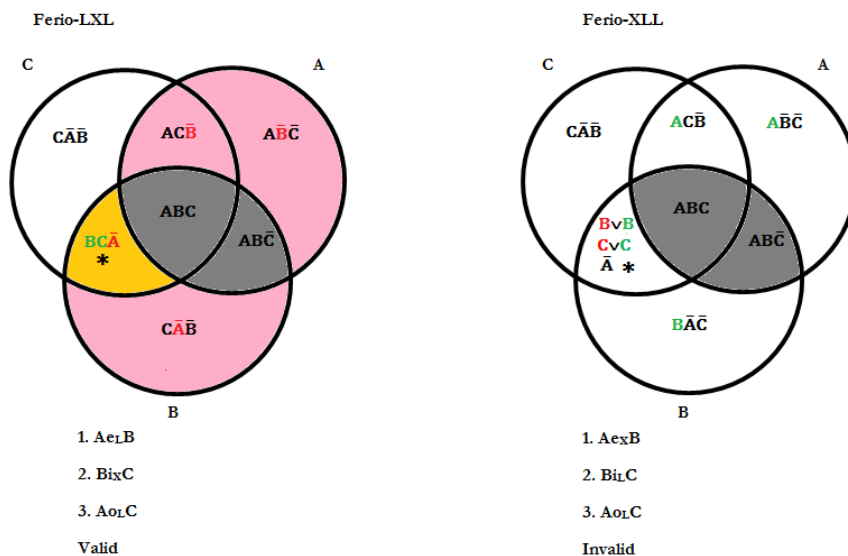


Fig. 6: Comparison of Ferio-LXL to Ferio-XLL

With our analysis of Ferio, the first figure is complete. Indeed, given our method of diagramming, we would predict that the argument on the left is valid, while the argument on the right does not permit us to reach an apodictic conclusion. Again, note that the subset indicated by the asterisk on the left side is necessarily non- $A$ , hence it is colored red. On the right, we see that there is a subset that is either  $a_L$ -

predicated of  $B$  or  $C$ , but this is insufficient to lead to the conclusion. Again, one can see how vital it is to distinguish between privative terms with underlying natures, and bare negations. In  $Ae_xB$  we have a bare negation of the predicate  $A$ , but in  $Ae_LB$ , we posit that there is, indeed, a non- $A$  and that whatever is  $B$  or  $C$  in region  $BC\bar{A}$  is necessarily a non- $A$ . We can also proceed with a deduction for Ferio-LXL, though not for Ferio-XLL.

Ferio-LXL (Valid):

1.  $Ae_LB$  (major premise)
2.  $Bi_xC$  (minor premise) //  $Ao_LC$   $(\exists Z)[(Ca_xZ \ \& \ \bar{A}a_LZ) \ \vee \ (\bar{A}a_xZ \ \& \ Ca_LZ)]$
3.  $(\exists Z)(Ca_xZ \ \& \ Ba_xZ)$  (2 Def ix)
4.  $(\forall Z)(Ba_xZ \ \supset \ \bar{A}a_LZ)$  (1 Def eL)
5.  $Ca_xU \ \& \ Ba_xU$  (3 EI)
6.  $Ba_xU \ \supset \ \bar{A}a_LU$  (4 EI)
7.  $Ba_xU$  (5 Simp)
8.  $\bar{A}a_LU$  (6,7 MP)
9.  $Ca_xU$  (5 Simp)
10.  $Ca_xU \ \& \ \bar{A}a_LU$  (8,9 Conj)
11.  $(Ca_xU \ \& \ \bar{A}a_LU) \ \vee \ (\bar{A}a_xU \ \& \ Ca_LU)$  (10 Add)
12.  $(\exists Z)[(Ca_xZ \ \& \ \bar{A}a_LZ) \ \vee \ (\bar{A}a_xZ \ \& \ Ca_LZ)]$  (11 EG)
13.  $Ao_LC$  (12 Def oL)

## 2.2 The Second Figure

Aristotle's second figure is distinguished by the appearance of the middle term in the predicate place for both the major and middle premise. In this section I will be discussing Cesare, Camestres, Festino, and Baroco. Of these moods, Baroco is one highlighted by several commentators as problematic, including Malink. He offers an analysis of o-predication to accommodate the validity of this mood, but concedes that it may be in conflict with other sections.

The first of the second figure arguments to consider is Cesare, with a universal negative major and a universal affirmative minor.

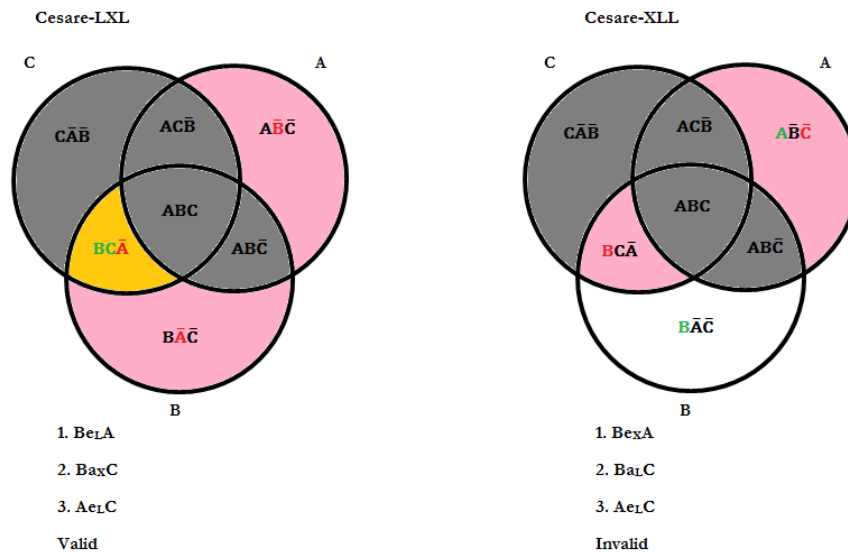


Fig. 7: Comparison of Cesare-LXL to Cesare-XLL

Cesare-LXL is considered valid by Aristotle, while Cesare-XLL is invalid. Indeed, our diagram correctly anticipates Aristotle's position. He writes, "In the second figure, if the negative proposition is necessary, then the conclusion will be necessary, but if the affirmative, not necessary" (*APr.* 30b7-9). On the left, we see that region  $BC\bar{A}$  is golden because  $\bar{A}$  is red, and both  $B$  and  $C$  are green, due to the assertoric minor premise. On the right, the same region cannot be shaded golden since  $\bar{A}$  cannot be colored red, and  $B$  and  $C$  cannot be colored by our convention. Indeed, no logical proof for Cesare-XLL is apparently possible given our definitions, but we can construct the argument for Cesare-LXL in the following manner:

Cesare-LXL (Valid):

1.  $Be_LA$  (major premise)
2.  $Ba_xC$  (minor premise) //  $Ae_LC (\forall Z)(Ca_xZ \supset \bar{A}a_LZ)$
3.  $(\forall Z)(Ca_xZ \supset Ba_xZ)$  (2 Def ax)
4.  $Ae_LB$  (1 Conv  $e_L-e_L$ )
5.  $(\forall Z)(Ba_xZ \supset \bar{A}a_LZ)$  (4 Def  $e_L$ )
6.  $Ca_xU \supset Ba_xU$  (3 UI)
7.  $Ba_xU \supset \bar{A}a_LU$  (5 UI)
8.  $Ca_xU \supset \bar{A}a_LU$  (6,7 HS)
9.  $(\forall Z)(Ca_xZ \supset \bar{A}a_LZ)$  (8 UG)
10.  $Ae_LC$  (9 Def  $e_L$ )

A related argument to Cesare is Camestres. If one were to convert the conclusion in Cesare, the result would be this argument form. Hence, while Cesare-LXL is valid, Camestres-XLL is really the corresponding argument, since the conversion of the conclusion of Cesare flips the major and minor premises. The Venn diagram is a mirror image of the previous argument:

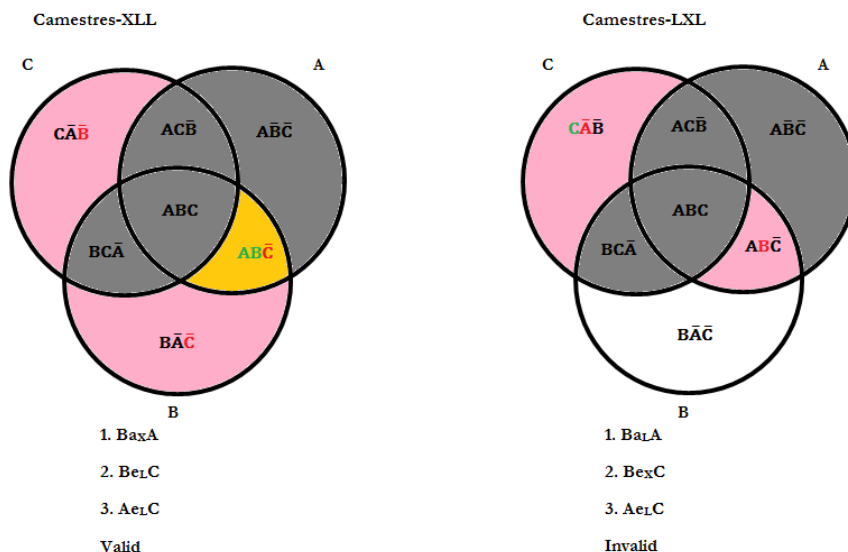


Fig. 8: Comparison of Camestres-XLL to Camestres-LXL

As the previous quote from Aristotle indicates, Camestres-XLL is valid while Camestres-LXL is invalid. Indeed, on the left side of Figure 8, we see that the minor premise plays a role in relating non- $C$  and  $A$  to one another by necessity. That is, the open region for  $A$  is necessarily non- $C$ . Given that we can obvert  $a_L$ -propositions, and convert  $e_L$  propositions simply, we can infer that  $\bar{C}a_LA$  is equivalent to  $Ce_LA$ , which is equivalent to  $Ae_LC$ , our conclusion. On the right, we do not have the right green terms to make Camestres-LXL valid. Another way to look at it is to say that Camestres-LXL is logically equivalent to Cesare-XLL, which we have already proved to be invalid. While no logical proof for Camestres-LXL can be made from our definitions, a proof for the validity of Camestres-XLL is as follows:

Camestres-XLL (Valid):

1.  $BaxA$  (major premise)
2.  $Be_LC$  (minor premise) //  $Ae_LC (\forall Z)(CaxZ \supset \bar{A}a_LZ)$
3.  $(\forall Z)(AaxZ \supset BaxZ)$  (1 Def ax)
4.  $Ce_LB$  (2 Conv  $e_L$ - $e_L$ )
5.  $(\forall Z)(BaxZ \supset \bar{C}a_LZ)$  (4 Def  $e_L$ )
6.  $AaxU \supset BaxU$  (3 UI)
7.  $BaxU \supset \bar{C}a_LU$  (5 UI)
8.  $AaxU \supset \bar{C}a_LU$  (6,7 HS)
9.  $(\forall Z)(AaxZ \supset \bar{C}a_LZ)$  (8 UG)
10.  $Ce_LA$  (9 Def  $e_L$ )
11.  $Ae_LC$  (10 Conv  $e_L$ - $e_L$ )

No proof for Camestres-LXL is forthcoming, given my definitions. And since the diagram is consistent with other invalid diagrams, we can start to see that the diagrams are a consistent predictor of validity and invalidity.

Festino is related to the first figure Ferio, which is evident from the similarities between fig. 6 and fig. 9 below. If one were to convert the major premise in Ferio, one would arrive at Festino. Since Ferio-LXL is valid, so will Festino-LXL,



and given the invalidity of Festino-XLL, we should expect Festino-LXL to be invalid as well:

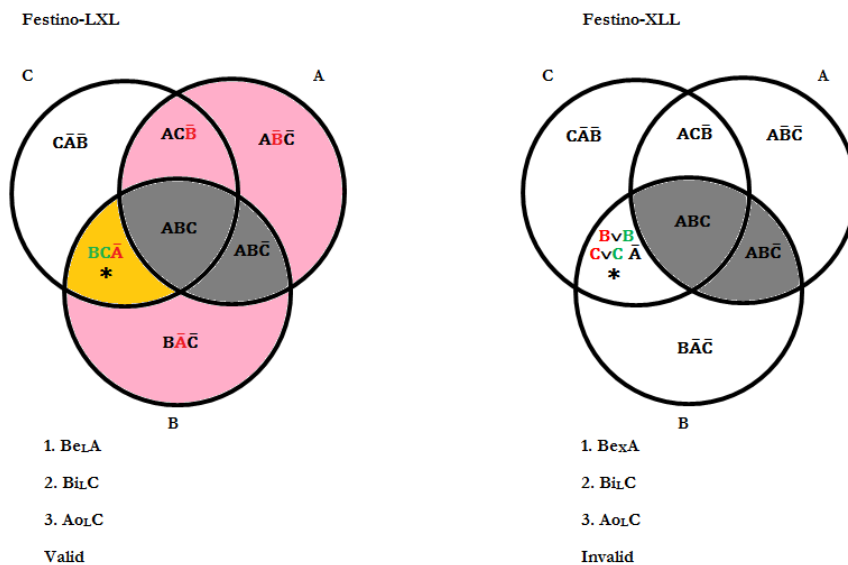


Fig.9: Comparison of Festino-LXL to Festino-XLL

According to the Venn diagram, Festino-LXL is valid while Festino-XLL is invalid. Again, note that for the left side, where there is asterisk indicating a subset, the region is  $a_L$ -predicated non- $A$ . On the right side, there is a subset that is apodictically predicated either of  $B$  or  $C$ , but we do not have sufficient modal information to determine the conclusion. A proof for Festino-LXL can be provided as follows:

Festino-LXL (Valid):

1.  $Be_L A$  (major premise)
2.  $Bi_x C$  (minor premise) //  $Ao_L C$   $(\exists Z)[(CaxZ \& \bar{A}a_L Z) \vee (\bar{A}axZ \& Ca_L Z)]$
3.  $Ae_L B$  (1 Conv  $e_L$ - $e_L$ )
4.  $(\exists Z)(CaxZ \& BaxZ)$  (2 Def  $i_x$ )
5.  $(\forall Z)(BaxZ \supset \bar{A}a_L Z)$  (3 Def  $e_L$ )
6.  $CaxU \& BaxU$  (4 EI)

7.  $BaxU \supset \bar{A}aLU$  (5 UI)
8.  $BaxU$  (6 Simp)
9.  $\bar{A}aLU$  (7,8 MP)
10.  $CaxU$  (6 Simp)
11.  $CaxU \ \& \ \bar{A}aLU$  (9,10 Conj)
12.  $(CaxU \ \& \ \bar{A}aLU) \vee (\bar{A}axU \ \& \ CaLU)$  (11 Add)
13.  $(\exists Z)[(CaxZ \ \& \ \bar{A}aLZ) \vee (\bar{A}axZ \ \& \ CaLZ)]$  (12 EG)
14.  $oLc$  (13 Def  $oL$ )

A proof for Festino-XLL does not appear possible, primarily because of the apodictic minor premise, and the disjunctive interpretation that we have adopted. Aristotle explains:

...[W]henever the negative proposition is both negative and necessary, then the conclusion will be necessary; but whenever the affirmative is universal and the negative particular, the conclusion will not be necessary (*APr.* 31a2-4).

So, our findings once again concur with Aristotle's canonical listings.

In the second figure, perhaps the most perplexing argument form is Baroco. There seems to be no consensus among commentators as to whether Aristotle was correct in his determination of whether the various forms of Baroco are valid. What's worse, it seems as though, if one adopts rules and interpretations that accord with Aristotle's claims regarding the validity and invalidity of the various forms of Baroco and Bocardo, those rules and interpretations do not appear to cohere with other argument forms.

It has been said that the two Barbaras is a litmus test for one's interpretative model of Aristotle's syllogistic. It can be seen as a first stage litmus test. A major hurdle is finding an interpretation that coheres with the canonical claims regarding Baroco and Bocardo,  $oL$ -predications being particularly difficult to handle. But if one can adhere to Aristotle's canonical listings with respect to Baroco without

invalidating other canonical arguments, then one is on the right interpretative path.

I offer the following, then, as evidence that my interpretation is on the right path.

My earlier discussion of negation and complementarity figures in how I can provide an interpretation that agrees with Aristotle with respect to Baroco without invalidating other forms. In fact, it will also feature in my discussion of Bocardo, which is a troublesome third-figure syllogism. Aristotle used ekthesis to prove the validity of modal forms of both Baroco and Bocardo. This is because, as Patterson notes, any reduction to the first figure would involve a conversion *per accidens* of a universal affirmative premise, which result in two particular premises, from which nothing follows (Patterson 1995, 70). We are essentially using a method of ekthesis in our metalogical proofs, as stated above. Attempting a second-order ekthesis would be illicit, and would fail to test the validity of Baroco. So the question of whether Aristotle was correct will rest on whether a plausible interpretation of modality is on offer. First, Baroco-XXX, the assertoric syllogism, can be proved valid by the following deduction:

Baroco-XXX (Valid):

1.  $BaxA$  (major premise)
2.  $BoxC$  (minor premise) //  $AoxC (\exists Z)[(CaxZ \ \& \ \sim(AaxZ))$
3.  $(\forall Z)(AaxZ \supset BaxZ)$  (1 Def ax)
4.  $(\exists Z)[CaxZ \ \& \ \sim(BaxZ)]$  (2 Def ox)
5.  $CaxU \ \& \ \sim(BaxU)$  (4 EI)
6.  $AaxU \supset BaxU$  (3 UI)
7.  $\sim(BaxU)$  (5 Simp)
8.  $\sim(AaxU)$  (6,7 MT)
9.  $CaxU$  (5 Simp)
10.  $CaxU \ \& \ \sim(AaxU)$  (8,9 Conj)
11.  $(\exists Z)[(CaxZ \ \& \ \sim(AaxZ))$  (10 EG)
12.  $AoxC$  (11 Def ox)

It seems rather straightforward to say that if the assertoric syllogism is valid, then the pure apodictic version will be valid too. And this is an assumption that Aristotle makes for all his pure apodictic syllogisms. “In the case of what is necessary, things are pretty much the same as in the case of what belongs (or does not belong), a deduction will or will not result alike in both cases, the only difference being the addition of the expression ‘necessarily’ to the terms” (*APr.* 29b36-30a1). However, actually providing a demonstration of that fact is not so straightforward, and the modal Venn diagram is rather unique and requires some interpretation as well.

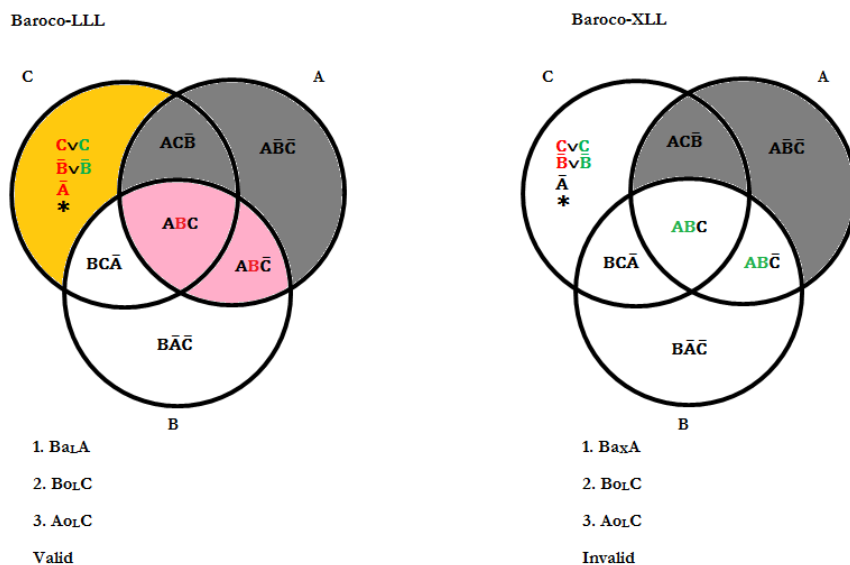


Fig.10: Comparison of Baroco-LLL to Baroco-XLL

On the left, we see that Baroco-LLL is considered valid. Indeed, we know that  $\bar{A}$  is  $a_L$ -predicated of the region, so the region can be shaded. We know that  $\bar{A}$  is  $a_L$ -predicated in  $C\bar{A}\bar{B}$  because the major premise can be contraposed. So, in that region,  $\bar{A}$  is  $a_L$ -predicated of some  $C$  and  $\bar{B}$ . Because of the disjunctive nature of the minor

premise,  $C$  and  $\bar{B}$  could be colored red or green, but either way, they are both colored either, which is sufficient to link the relevant terms together in a proof. An example of the proof will appear below. The region is shaded golden, and the proof is determined to be valid. On the right, we see that there is insufficient information to shade  $C\bar{A}\bar{B}$ , since we do not know whether the relevant subset is necessarily  $C$  or  $\bar{B}$ . Also, we have no data on  $\bar{A}$ , since it is not the case that we can contrapose the major, given that it is assertoric and we have denied that traditional obversion rules govern in those circumstances. The proof of Baroco-LLL is as follows:

Baroco-LLL (Valid):

1.  $Ba_LA$  (major premise)
2.  $Bo_LC$  (minor premise) //  $Ao_LC (\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xU \& Ca_LZ)]$
3.  $\bar{B}e_LA$  (1 Obv  $e_L-a_L$ )
4.  $Ae_L\bar{B}$  (3 Conv  $e_L-e_L$ )
5.  $Bo_LC \supset (\exists Z)\{Ca_xZ \& [\bar{B}a_xZ \& \sim(Ba_xZ)]\}$  (L-X-sub  $o_L$ )
6.  $(\exists Z)\{Ca_xZ \& [\bar{B}a_xZ \& \sim(Ba_xZ)]\}$  (2,5 MP)
7.  $(\forall Z)(\bar{B}a_xZ \supset \bar{A}a_LZ)$  (4 Def  $e_L$ )
8.  $Ca_xU \& [\bar{B}a_xU \& \sim(Ba_xU)]$  (6 EI)
9.  $\bar{B}a_xU \& \sim(Ba_xU)$  (8 Simp)
10.  $\bar{B}a_xU \supset \bar{A}a_LU$  (7 UI)
11.  $\bar{B}a_xU$  (9 Simp)
12.  $\bar{A}a_LU$  (10,11 MP)
13.  $Ca_xU$  (8 Simp)
14.  $Ca_xU \& \bar{A}a_LU$  (12,13 Conj)
15.  $(Ca_xU \& \bar{A}a_LU) \vee (\bar{A}a_xU \& Ca_LU)$  (14 Add)
16.  $(\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xU \& Ca_LZ)]$  (15 EG)
17.  $Ao_LC$  (16 Def  $o_L$ )

Indeed, it is also an important result that a parallel proof for Baroco-XLL is not forthcoming. One concern might be that a proof that uses L-X subordination on the minor premise,  $Bo_LC$ , would mean that Baroco-LXL is valid, contrary to Aristotle's canonical listing. My initial inclination was to define a rule whereby  $Bo_LC \supset (\exists Z)(Ca_xZ \& \bar{B}a_xZ)$ , which is essentially to say that  $Bo_LC \supset \bar{B}i_xC$  and then use

obversion to argue that  $\bar{B}axZ$  implies  $BexZ$ , i.e.  $\sim(BaxZ)$ . This would have added several steps to the proof, so I've defined L-X subordination to contain all of this information, so that it can be readily utilized in a proof if necessary. This means that we are able to derive  $a_L$ -predication from the fact that there is a privative complementary,  $\bar{B}$ , that has an underlying nature, as Aristotle might put it. This means that the proof for Baroco-LLL cannot be readily adapted to prove Baroco-LXL, an issue that has concerned other commentators. Paul Thom, for instance, identifies a problem with the Baroco-LLL proof by *ekthesis* and writes,

It is clear that, if Aristotle's system is taken to include the counter-examples as well as the ecthetic proofs, then it is inconsistent, being committed to the validity as well as to the invalidity of Baroco XLL and Bocardo LXL. This fact obligates the interpreter to revise Aristotle's system, either by dropping the ecthetic procedure for apodictic forms, or by allowing Baroco XLL and Bocardo LXL as valid (Thom 1993, 195).

Thom opts to preserve *ekthesis* and jettison the proofs by counter-example that Aristotle marshals against the validity of Baroco-XLL and Bocardo-LXL. Aristotle also provides an argument by examples where  $A$  is animal,  $B$  is man, and  $C$  is white.

Aristotle argues that the following is clearly invalid:

1. Animal necessarily belongs to every man
2. Animal does not belong to some white things

Therefore,

3. Man necessarily does not belong to some white things

or

4. Animal belongs to every man
5. Animal necessarily does not belong to some white

Therefore,

6. Man necessarily does not belong to some white things

The example is not exactly clear, however. Striker (Aristotle 2009, 122) remarks:

The claims that [Baroco-LXL] and [Baroco-XLL] can be shown to be invalid using the same term... creates some difficulties. We can hardly assume that the terms would be used in the same order, for then the same premisses would have to be necessary in one example, [and] assertoric in the other. Since the difference between necessary and assertoric propositions is crucial here, the counterexamples would not be acceptable. Alexander even claims (143.18ff.) that, given these terms, both the premisses and the conclusions would be necessary

Striker recommends a rearrangement of the terms from those suggested by Aristotle to *A* man, *B* white, and *C* animal. However, it is hard to see why Aristotle would think that the premises are true with this arrangement of terms. Proving invalidity by counterexamples requires that we take premises that are true, and arrive at a false conclusion.

We can think that Aristotle's counter-examples are confused or confusing, but we cannot deny that he thought Baroco-LLL was valid and Baroco-LXL and Baroco-XLL were invalid. The following diagram, following our conventions, fits with Aristotle's claims:

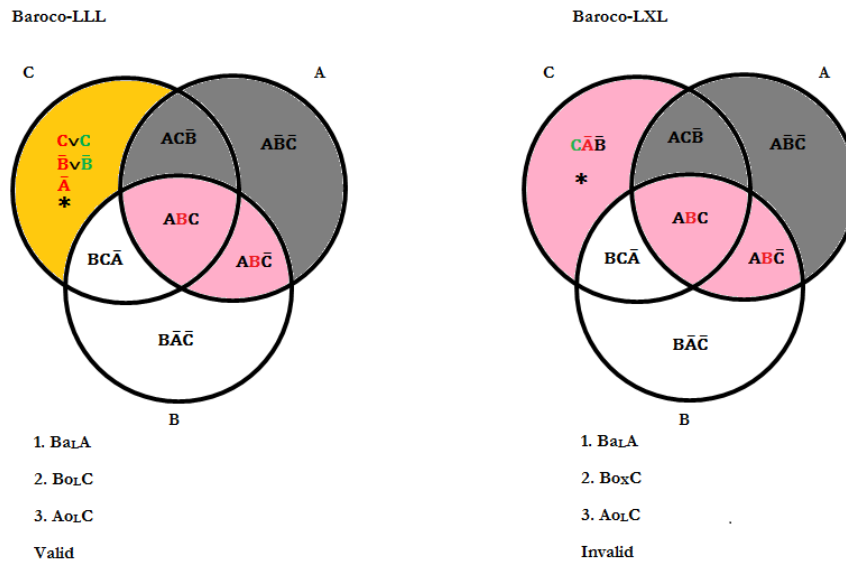


Fig. 11: Comparison of Baroco-LLL to Baroco-LXL

The diagram on the left is identical to the left side in Fig. 10, but is duplicated for the purpose of a direct visual comparison. As we can see, on the right,  $C$  is colored green due to the minor premise, but  $\bar{B}$  remains uncolored. So even though the region is shaded pink by virtue of the fact that the major premise can be contraposed, we cannot directly link  $\bar{A}$  to  $C$ . For any proof to succeed there would have to be an ax-predication of  $\bar{B}$  in the argument. Now one might argue that in contraposing the major, we arrive at  $(\forall Z)(\bar{B}axZ \supset \bar{A}aLZ)$ , but our convention has not been to color the subject terms of apodictic  $aL$ -predications. Is this an *ad hoc* move? The reason this is not done is because we cannot assume that some subset, call it  $Z_1$ , can be ax-predicated of  $C$  and  $\bar{B}$ . That would be the implication of coloring the subject terms green in  $aL$ -predication, that they are always ax-predicated of the same subgroup as all other terms in the region. This in a proof, making such an assumption would be equivalent to a violation of existential instantiation.



Indeed, Aristotle agrees that the conclusion will not be necessary when the affirmative premise is universal and necessary, or when the negative premise is particular and necessary

Again let the affirmative be both universal and necessary, and let the affirmative refer to  $B$ . If then  $A$  necessarily belongs to every  $B$ , but does not belong to some  $C$ , it is clear that  $B$  will not belong to some  $C$ , but not necessarily... Nor again, if the negative is necessary but particular, will the conclusion be necessary (*APr*: 31a10-17).

Consequently, Baroco-LXX and Baroco-XXL are valid arguments, which means that we can arrive at an assertoric conclusion even if one premise is apodictic.

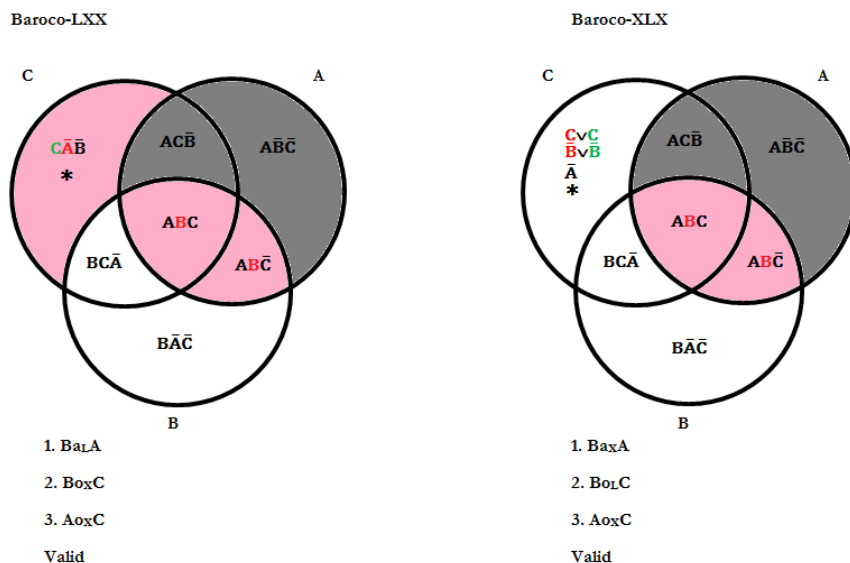


Fig. 12: Comparison of Baroco-LXX to Baroco-XXL

Proofs can also be provided for both argument forms:

Baroco-LXX (Valid):

1.  $Ba_L A$  (major premise)
2.  $Bo_X C$  (minor premise) //  $Ao_X C$  ( $\exists Z$ ) [ $(Ca_X Z \ \& \ \sim(Aa_X Z))$ ]
3.  $Ba_L A \supset (\forall Z)(Aa_X Z \supset Ba_X Z)$  (L-X-sub aL)
4.  $(\forall Z)(Aa_X Z \supset Ba_X Z)$  (1,3 MP)

5.  $(\exists Z)[CaxZ \ \& \ \sim(BaxZ)]$  (2 Def ox)
6.  $CaxU \ \& \ \sim(BaxU)$  (5 EI)
7.  $AaxU \ \supset \ BaxU$  (4 UI)
8.  $\sim(BaxU)$  (6 Simp)
9.  $\sim(AaxU)$  (7,8 MT)
10.  $CaxU$  (6 Simp)
11.  $CaxU \ \& \ \sim(AaxU)$  (9,10 Conj)
13.  $(\exists Z)[CaxZ \ \& \ \sim(AaxZ)]$  (11 EG)
14.  $AoxC$  (11 Def ox)

Baroco-XXL (Valid):

1.  $BaxA$  (major premise)
2.  $BoLc$  (minor premise) //  $AoxC \ (\exists Z)[(CaxZ \ \& \ \sim(AaxZ))]$
3.  $BoLc \ \supset \ (\exists Z)\{CaxZ \ \& \ [\bar{B}axZ \ \& \ \sim(BaxZ)]\}$  (L-X-sub oL)
4.  $(\exists Z)\{CaxZ \ \& \ [\bar{B}axZ \ \& \ \sim(BaxZ)]\}$  (2,3 MP)
5.  $(\forall Z)(AaxZ \ \supset \ BaxZ)$  (1 Def)
6.  $CaxU \ \& \ [\bar{B}axU \ \& \ \sim(BaxU)]$  (4 EI)
7.  $AaxU \ \supset \ BaxU$  (5 UI)
8.  $\bar{B}axU \ \& \ \sim(BaxU)$  (6 Simp)
9.  $\sim(BaxU)$  (8 Simp)
10.  $\sim(BaxU) \ \supset \ \sim(AaxU)$  (7 Contra)
11.  $\sim(AaxU)$  (9,10 MP)
12.  $CaxU$  (6 Simp)
13.  $CaxU \ \& \ \sim(AaxU)$  (11,12 Conj)
14.  $(\exists Z)[CaxZ \ \& \ \sim(AaxZ)]$  (13 EG)
15.  $AoxC$  (14 Def ox)

So our interpretation passes another major test by allowing us to prove the validity of Baroco-LLL, Baroco-LXX, and Baroco-XXL, while no proofs for Baroco-LXL or Baroco-XLL are available.

### 2.3 The Third Figure

In Aristotle's third figure, the middle term appears as the subject of the major and minor premises. The following diagrams and proofs conform to the canonical listing provided by Aristotle. The first for our consideration is Darapti, which would

not be considered valid under contemporary conventions where universal propositions lack existential import. Of course, under the heterodox interpretation, a particular proposition does not claim that there are individuals exemplifying various predicates, but that there is a sub-class that exists in various regions. In effect, one is not really deriving the existence of a primary substance with various properties from the class terms. Rather, for Aristotle, the intelligibility of these terms bears witness to their existence as class-terms. And the claim that there exist parts within a class-term is not such a radical claim. It is really nothing more than to admit that any class-term can be divided into proper-parts that have or lack some predicate. Consequently, in following Aristotle's convention of sub-alternation, it is licit to infer the existence of some subset within a region exists. One may draw an asterisk in a region that has two terms colored to represent that there exists a subset that can be assigned those terms:

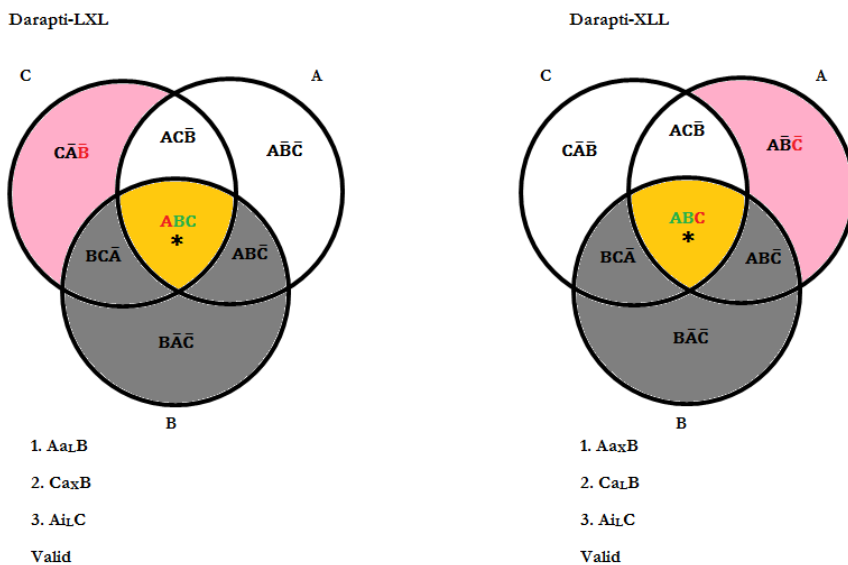


Fig. 13: Comparison of Darapti-LXL to Darapti- XLL

Both arguments are valid and appear as mirror images of one another. We must keep in mind that the major premise on the left can be contraposed and the minor premise on the right as well. On the left, we see that  $B$  and  $C$  are colored green, so some of them can be linked to  $A$  by  $a_L$  predication. Likewise, on the right,  $A$  and  $B$  can be colored green, and so we can say that there is a subset of  $B$ s and  $C$ s that are necessarily  $A$ .

Darapti-LXL (Valid):

1.  $Aa_LB$  (major premise)
2.  $Ca_xB$  (minor premise) //  $Ai_LC (\exists Z)[(Ca_xZ \& Aa_LZ) \vee (Aa_xZ \& Ca_LZ)]$
3.  $(\forall Z)(Ba_xZ \supset Aa_LZ)$  (1 Def  $a_L$ )
4.  $Ca_xB \supset Bi_xC$  (2 Conv  $a_x$ - $i_x$ )
5.  $Bi_xC$  (2,4 MP)
6.  $(\exists Z)(Ca_xZ \& Ba_xZ)$  (5 Def  $i_x$ )
7.  $Ca_xU \& Ba_xU$  (6 EI)
8.  $Ba_xU \supset Aa_LU$  (3 UI)
9.  $Ba_xU$  (7 Simp)
10.  $Aa_LU$  (8,9 MP)
11.  $Ca_xU$  (7 Simp)
12.  $Ca_xU \& Aa_LU$  (10,11 Conj)
13.  $(Ca_xU \& Aa_LU) \vee (Aa_xU \& Ca_LU)$  (12 Add)
14.  $(\exists Z)[(Ca_xZ \& Aa_LZ) \vee (Aa_xZ \& Ca_LZ)]$  (13 EG)
15.  $Ai_LC$  (14 Def  $i_L$ )

Darapti-XLL (Valid):

1.  $Aa_xB$  (major premise)
2.  $Ca_LB$  (minor premise) //  $Ai_LC (\exists Z)[(Ca_xZ \& Aa_LZ) \vee (Aa_xZ \& Ca_LZ)]$
3.  $(\forall Z)(Ba_xZ \supset Ca_LZ)$  (2 Def  $a_L$ )
4.  $Aa_xB \supset Bi_xA$  (1 Conv  $a_x$ - $i_x$ )
5.  $Bi_xA$  (2,4 MP)
6.  $(\exists Z)(Aa_xZ \& Ba_xZ)$  (5 Def  $i_x$ )
7.  $Aa_xU \& Ba_xU$  (6 EI)
8.  $Ba_xU \supset Ca_LU$  (3 UI)
9.  $Ba_xU$  (7 Simp)
10.  $Ca_LU$  (8,9 MP)
11.  $Aa_xU$  (7 Simp)
12.  $Aa_xU \& Ca_LU$  (10,11 Conj)
13.  $(Aa_xU \& Ca_LU) \vee (Ca_xU \& Aa_LU)$  (12 Add)

- 14.  $(CaxU \ \& \ AaL U) \vee (AaxU \ \& \ CaL U)$  (13 Comm)
- 15.  $(\exists Z)[(CaxZ \ \& \ AaL Z) \vee (AaxZ \ \& \ CaL Z)]$  (14 EG)
- 16.  $AiL C$  (15 Def  $iL$ )

Next, we must consider Felapton, which like Darapti arrives at a particular conclusion from universal premises. We will require that at least two terms be colored before we can legitimately assign a region an asterisk.

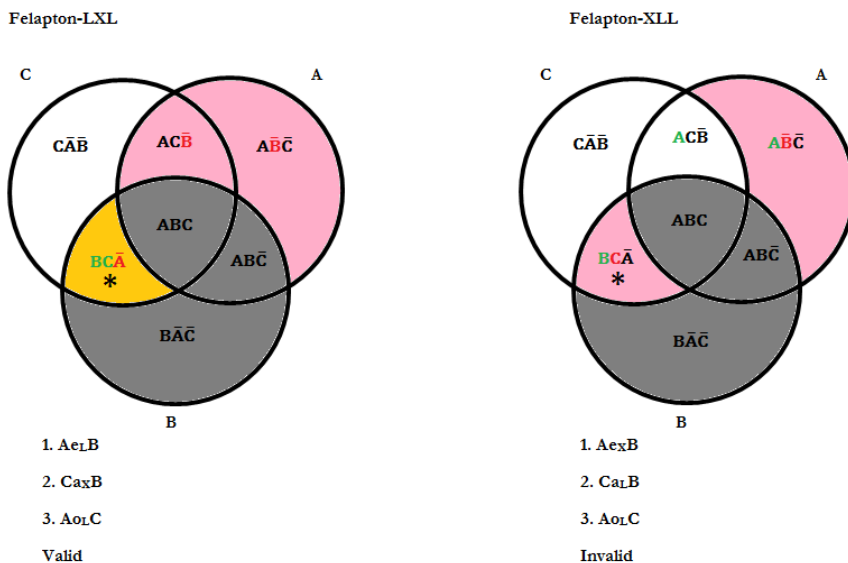


Fig. 14: Comparison of Felapton-LXL to Felapton-XLL

The comparison between Felapton-LXL and Felapton-XLL is interesting in that we can see that it is licit to draw an asterisk in region  $BC\bar{A}$ , but only one should be shaded gold to indicate a valid apodictic conclusion can be reached between the major and minor term through the middle term of that region. The proof for Felapton-LXL confirms our findings.

Felapton-LXL (Valid):

- 1.  $AeL B$  (major premise)

2.  $CaxB$  (minor premise) //  $AoLC (\exists Z)(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)$
3.  $(\forall Z)(BaxZ \supset \bar{A}aLZ)$  (1 Def eL)
4.  $CaxB \supset BixC$  (2 Conv ax-ix)
5.  $BixC$  (2,4 MP)
6.  $(\exists Z)(CaxZ \& BaxZ)$  (5 Def ix)
7.  $CaxU \& BaxU$  (6 EI)
8.  $BaxU \supset \bar{A}aLU$  (3 UI)
9.  $BaxU$  (7 Simp)
10.  $\bar{A}aLU$  (8,9 MP)
11.  $CaxU$  (7 Simp)
12.  $CaxU \& \bar{A}aLU$  (10,11 Conj)
13.  $(CaxU \& \bar{A}aLU) \vee (\bar{A}axU \& CaLU)$  (12 Add)
14.  $(\exists Z)(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)$  (13 EG)
15.  $AoLC$  (14 Def oL)

No proof for Felapton-XLL is possible. From my own attempts, this is primarily due to the apparent need to convert the minor premise, which leaves you with a disjunction and no root to a conclusion.

Disamis is related to Darii by converting the conclusion and the particular premise simply. As Darii-LXL is valid, Disamis-XLL is valid. And since Darii-XLL is invalid, one can imagine that the related conversions lead us to conclude that Darii-LXL is invalid. Our Venn diagrams confirm this insight:

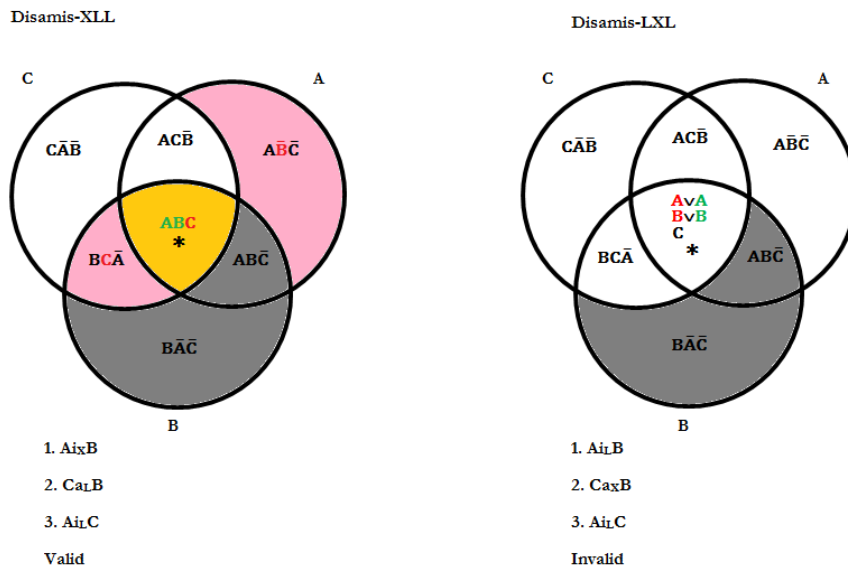


Fig. 15: Comparison of Disamis-XLL to Disamis-LXL

Disamis-XLL (Valid):

1.  $Ai_xB$  (major premise)
2.  $Ca_LB$  (minor premise) //  $Ai_LC$  ( $\exists Z$ ) [ $(Ca_xZ \& Aa_LZ) \vee (Aa_xZ \& Ca_LZ)$ ]
3.  $(\forall Z)(Ba_xZ \supset Ca_LZ)$  (2 Def al)
4.  $(\exists Z)(Ba_xZ \& Aa_xZ)$  (1 Def ix)
5.  $Ba_xU \& Aa_xU$  (4 UI)
6.  $Ba_xU$  (5 Simp)
7.  $Ba_xU \supset Ca_LU$  (3 UI)
8.  $Ca_LU$  (6,7 MP)
9.  $Aa_xU$  (5 Simp)
10.  $Aa_xU \& Ca_LU$  (8,9 Conj)
11.  $(Aa_xU \& Ca_LU) \vee (Ca_xU \& Aa_LU)$  (10 Add)
12.  $(Ca_xU \& Aa_LU) \vee (Aa_xU \& Ca_LU)$  (11 Comm)
13.  $(\exists Z)[(Ca_xZ \& Aa_LZ) \vee (Aa_xZ \& Ca_LZ)]$  (12 EG)
14.  $Ai_LC$  (13 Def il)

Based on my definitions, no proof for Disamis-LXL appears possible, which accords with Aristotle's claims.

Datisi is also related to Darii by way of converting the minor premise in Darii simply. Of course, this does not change which premise is major or minor, as happens

when the conclusion is converted. So as we might anticipate, Datisi-LXL is valid, just like Darii-LXL. And Datisi-XLL is invalid for much the same reasons Darii-XLL is invalid.

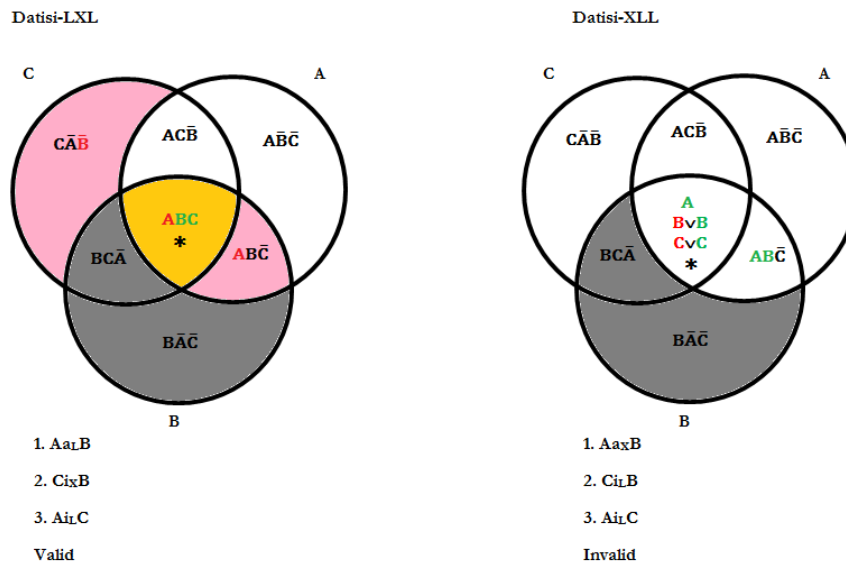


Fig. 16: Comparison of Datisi-LXL to Datisi-XLL

Based on our exposition of the other Venn diagrams, we should begin to understand why the diagram on the left tells us that the argument is valid. The argument on the right may be a little perplexing because it seems that all three terms are colored in region  $ABC$ . The reason why we cannot draw an apodictic conclusion is because we simply cannot tell which subset in the region should receive  $a_L$ -predication. Consequently, we can draw an asterisk, but we cannot say for certain that the conclusion would follow. The argument for Datisi-XLL is not forthcoming, as in the case of other canonically listed invalid syllogism, but Datisi-LXL is as follows:



Datisi-LXL (Valid):

1.  $Aa_L B$  (major premise)
2.  $Cix B$  (minor premise) //  $Ai_L C (\exists Z)[(Cax Z \& Aa_L Z) \vee (Aax Z \& Ca_L Z)]$
3.  $(\forall Z)(Bax Z \supset Aa_L Z)$  (1 Def  $a_L$ )
4.  $(\exists Z)(Bax Z \& Cax Z)$  (2 Def  $ix$ )
5.  $Bax U \& Cax U$  (4 EI)
6.  $Bax U$  (5 Simp)
7.  $Bax U \supset Aa_L U$  (3 UI)
8.  $Aa_L U$  (6,7 MP)
9.  $Cax U$  (5 Simp)
10.  $Cax U \& Aa_L U$  (8,9 Conj)
11.  $(Cax U \& Aa_L U) \vee (Aax U \& Ca_L U)$  (10 Add)
12.  $(\exists Z)[(Cax Z \& Aa_L Z) \vee (Aax Z \& Ca_L Z)]$  (11 EG)
13.  $Ai_L C$  (12 Def  $i_L$ )

After Baroco, the next major hurdle to overcome in the apodictic syllogism is Bocardo. Interestingly, if we were to apply our modified obversion rules, we could determine that Bocardo-LXL is related to Disamis-LXL, which we've already determined is invalid. On the other hand, Bocardo-XLL cannot obvert, because our obversion rule does not apply to  $ox$ -propositions. The inability to obvert the major premise does not tell us that Bocardo-XLL is invalid, but it does mean that we cannot assume it should be valid because it is somehow related to Disamis-XLL, which is valid. We will start, then with a comparison of Bocardo-LLL, considered valid by Aristotle, to Bocardo-LXL, which we anticipate should be invalid.

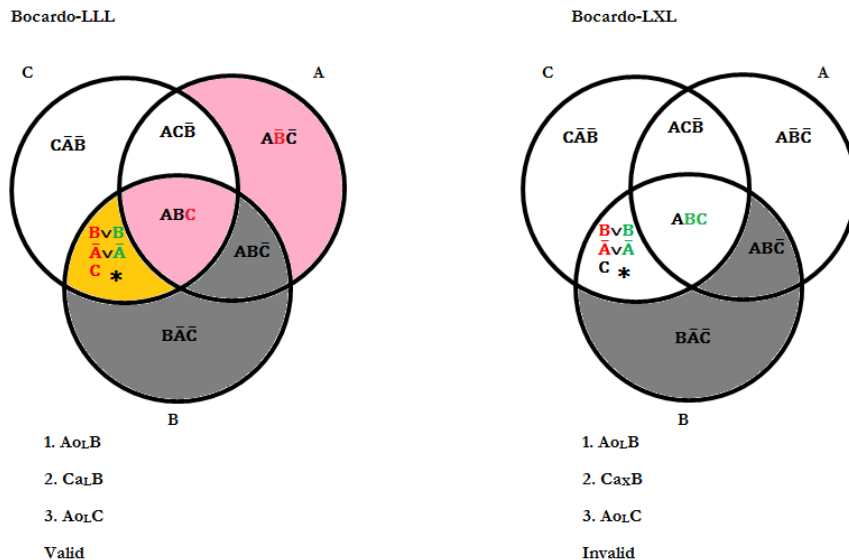


Fig. 17: Comparison of Bocardo-LLL to Bocardo-LXL

Indeed, we see on the right that, not only are all the terms colored, but that we know that the region is apodictic because of the minor premise. Hence we can assume that  $\bar{A}$  or  $C$  can be  $a_L$ -predicated of some subset,  $Z$ , in that region. On the right, no such inference can be made, other than that there is some subset in  $C\bar{B}\bar{A}$ . The proof for Bocardo-LLL is as follows:

Bocardo-LLL (Valid):

1.  $Ao_L B$  (major premise)
2.  $Ca_L B$  (minor premise) //  $Ao_L C$   $(\exists Z)[(Ca_X Z \ \& \ \bar{A}a_L Z) \vee (\bar{A}a_X Z \ \& \ Ca_L Z)]$
3.  $Ao_L B \supset (\exists Z)\{Ba_X Z \ \& \ [\bar{A}a_X Z \ \& \ \sim(Aa_X Z)]\}$  (L-X-sub  $o_L$ )
4.  $(\exists Z)\{Ba_X Z \ \& \ [\bar{A}a_X Z \ \& \ \sim(Aa_X Z)]\}$  (1,3 MP)
5.  $(\forall Z)(Ba_X Z \supset Ca_L Z)$  (2 Def  $a_L$ )
6.  $Ba_X U \ \& \ [\bar{A}a_X U \ \& \ \sim(Aa_X U)]$  (4 EI)
7.  $Ba_X U \supset Ca_L U$  (5 UI)
8.  $Ba_X U$  (6 Simp)
9.  $Ca_L U$  (7,8 MP)
10.  $\bar{A}a_X U \ \& \ \sim(Aa_X U)$  (6 Simp)
11.  $\bar{A}a_X U$  (10 Simp)
12.  $\bar{A}a_X U \ \& \ Ca_L U$  (9,11 Conj)

- 13.  $(\bar{A}xU \ \& \ CaLU) \vee (CaxU \ \& \ \bar{A}aLU)$  (12 Add)
- 14.  $(CaxU \ \& \ \bar{A}aLU) \vee (\bar{A}axU \ \& \ CaLU)$  (13 Comm)
- 15.  $(\exists Z)[(CaxZ \ \& \ \bar{A}aLZ) \vee (\bar{A}axZ \ \& \ CaLZ)]$  (14 EG)
- 16.  $AoLC$  (15 Def  $oL$ )

Note, again, the use of L-X subordination of the major premise in (3-4) might lend appearance to a parallel proof for Bocardo-XLL. However, there is no assumption in Bocardo-XLL that the assertoric major, “Some B is not A” implies that there is an essence or underlying nature to the bare negation of A. So this means that Bocardo-XLL cannot be shown to be valid by obverting it to a related Disamis-XLL, nor can it be proved through a parallel proof of Bocardo-LLL. These have been the Scylla and Charybdis that any interpretation must avoid. The Venn diagram appears as follows:

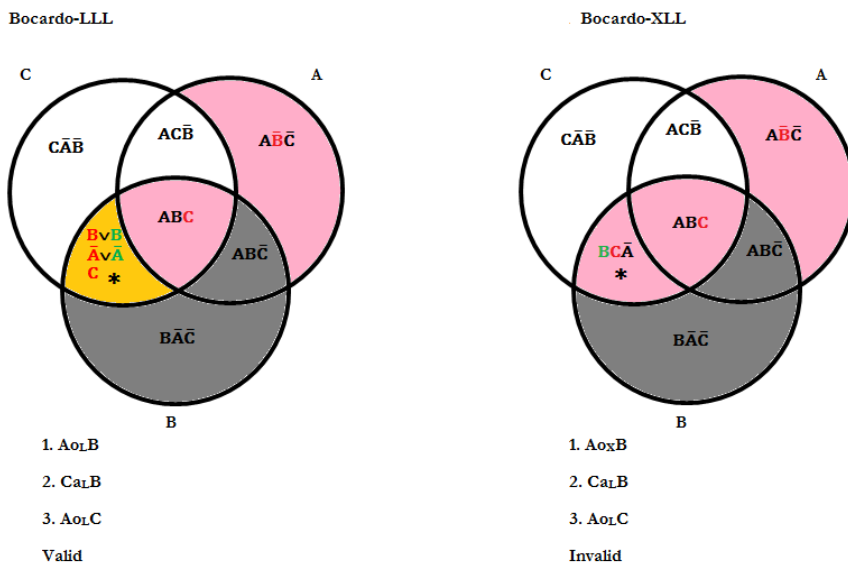


Fig. 18: Comparison of Bocardo-LLL to Bocardo-XLL

The diagram on the right of Fig. 18 is identical to Bocardo-LLL in Fig. 17. It is duplicated merely to contrast with the canonically listed invalid form. Here,

Bocardo-XLL seems very close to being valid. However, the major premise only permits us to color  $B$  green in region  $CB\bar{A}$ . While a subset can be identified as  $a_L$ -predicated of  $B$  or  $C$ , we do not have data to confirm that there is any  $\bar{A}$ s in that region, and so we cannot apodictically predicate that some  $C$  is necessarily non- $A$  if, as our interpretation implies, apodictic negative propositions imply privations with underlying natures or essences. Our diagram combined with a philosophical argument about privation and apodictic propositions suggest that Bocardo-XLL is invalid. No proof was forthcoming given my efforts, which as I will explain below, included attempts at indirect proof. If such a proof is possible, given my definitions and rules, it is not currently apparent to me.

The final argument form for our consideration is Ferison-LXL which is, of course, related to Ferio, where the minor term of Ferio is converted simply. So we might anticipate that Ferison-LXL is valid and Ferison-XLL is invalid, since the major and minor do not switch position, as when the conclusion is converted. In this case, of course, no conversion of the conclusion is possible.

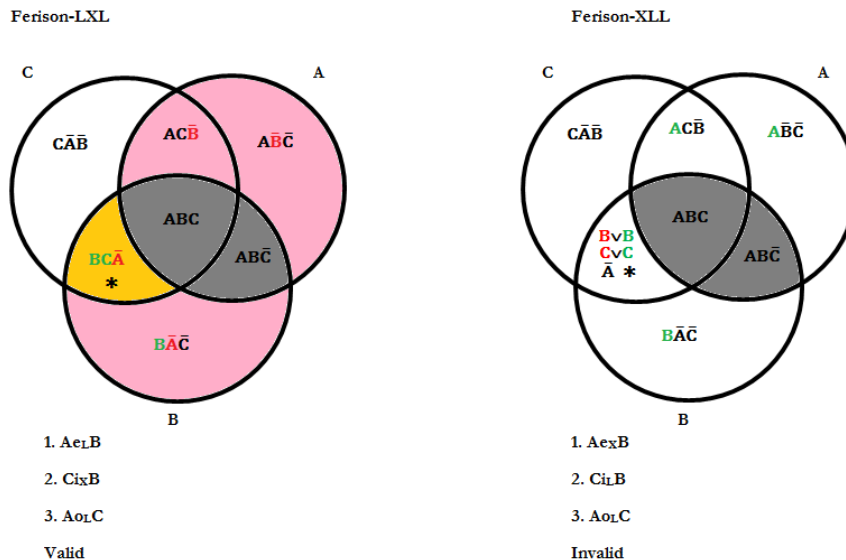


Fig. 19: Comparison of Ferison-LXL to Ferison-XLL

On the left side, we see that  $B$  and  $C$  are colored green, as per the assertoric minor premise. The major premise, being apodictic, permits us to color  $\bar{A}$  red. Accordingly, the region  $CB\bar{A}$  is shaded gold. As for the argument on the right side, we note a similar problem that we have seen with other arguments where the only apodictic premise is particular. We simply cannot determine whether the  $a_L$ -predication belongs to  $B$  or  $C$ . What's more, we have no reason to think that there is an underlying nature to  $\bar{A}$ . And, of course, for whatever it is worth, no proof for Ferison-XLL was apparent.

Ferison-LXL (Valid):

1.  $Ae_L B$  (major premise)
2.  $Ca_L B$  (minor premise) //  $Ao_L C$   $(\exists Z)[(Ca_x Z \& \bar{A}a_L Z) \vee (\bar{A}a_x Z \& Ca_L Z)]$
3.  $(\forall Z)(Ba_x Z \supset \bar{A}a_L Z)$  (1 Def  $e_L$ )
4.  $(\exists Z)(Ba_x Z \& Ca_x Z)$  (2 Def  $ix$ )
5.  $Ba_x U \& Ca_x U$  (4 EI)
6.  $Ba_x U$  (5 Simp)

7.  $BaxU \supset \bar{A}aLU$  (3 UI)
8.  $\bar{A}aLU$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \ \& \ \bar{A}aLU$  (8,9 Conj)
11.  $(CaxU \ \& \ \bar{A}aLU) \vee (\bar{A}axU \ \& \ CaLU)$  (10 Add)
12.  $(\exists Z)[(CaxZ \ \& \ \bar{A}aLZ) \vee (\bar{A}axZ \ \& \ CaLZ)]$  (11 EG)
13.  $AoLC$  (12 Def  $o_L$ )

There are, of course, other syllogisms to consider, including those in the fourth figure, which Aristotle apparently did not include in his own discussion of valid categorical forms. Further work must be done to show which other apodictic argument forms like Barbari, Celaront, Cesaro, and Camestros are valid.

## 2.4 Conclusion

The preceding exposition gives us cause to think that some model of the Aristotelian syllogistic can be made consistent—i.e. the canonically listed pure and mixed apodictic syllogisms. This is not to say that every mood that Aristotle claimed to be valid is in fact valid as our discussion has left out a discussion of one and two-way possibility as it relates overall. Malink's heterodox interpretation, along with his definitions of problematic and contingent propositions makes great strides towards providing an interpretation that coheres with each claim Aristotle has made. Unfortunately, as these models become more complex, it becomes more difficult to defend the idea that Aristotle had such technically convoluted notions of modal opposition and inference. Taking Malink's lead, I have embraced various aspects of the heterodox interpretation, while adding modifications to various articulations of propositions, especially with respect to the negative propositions.

One concern that might be raised about my interpretation, and that of Malink or Rini, is the use of first-order, or lower predicate, calculus on Aristotelian logic. Rini remarks, "Some of the scholars who eschew [lower predicate calculus] notion do so because they feel that predicates of individuals distorts Aristotle's text too much" (2011, 236). However, so long as we build upon well formed formulae in an attempt to distort Aristotle as little as possible, we can have a way of understanding Aristotle's modal syllogistic. In other words, so long as we can reasonably translate between Aristotle's syntax and the syntax devised by our use of lower predicate calculus, we have a functional apparatus for manipulating Aristotelian propositions in truth-functional ways. To that end, I think it is preferable to adopt Malink's mereological interpretation over Rini's use of individuals set within a domain of discourse. Related to this is a second concern as to whether Aristotelian notions of modality can be utilized in contexts outside of Aristotle's particular syntax. Part of the reason that Aristotle's modal syllogistic avoids the *de re/de dicto* distinction is because of the tripartite structure of categorical propositions wherein propositional modifiers affect the copula. It is hard to imagine, then, a way in which the modal logic could leave behind Aristotelian syntax altogether. Moreover, there will always be something deeply metaphysical about the way in which Aristotle conceives of his logic. It is the logic of categorical terms, not of individuals and predicates, or sentences and logical connectives. This is not to say that Aristotle's logic is alien to sentential or predicate logic, but that it pertains to logical relations on a different level, one that can be translated into LPC relationships, but is not fundamental to them. This is not to say that our modern logic is somehow superior or more

metaphysically neutral. For, indeed, we could just as well say that sentential and predicate logic is not fundamental to Aristotle's logic. He didn't need these modern tools to understand the logical relationships among categorical terms and copulae. But for us, these modern tools are like a cipher, which helps us to decode Aristotle's metaphysical-logical intuitions of these relationships.

What we have, then, is an interpretation that makes sense of the canonical listing provided by Aristotle. This interpretation accords with the view that modality should be understood as a copula modifier, and so Aristotle's propositions cannot be neatly bifurcated into those that are *de re* and *de dicto*. Secondly, we must recognize that Aristotle's modal logic tends to focus on substance or essence terms or modal claims about particulars *qua* essential predications. This is significant, especially when considering negative apodictic propositions, and is central to our interpretative model. Thirdly, we can see that Aristotle intended his logic to be used in demonstration, which we explore more in the third chapter. Thus, Aristotle envisions his modal logic to be the logic of scientific inferences. In apprehending the universal nature of things, we are going to be relying upon the only mood that provides universal affirmations, namely Barbara. While Theophrastus, Eudemus, Alexander, and later commentators had their concerns about the "two Barbaras" I think it has been demonstrated that there are several interpretive models upon which it can be shown that one needs only one apodictic premise to reach an apodictic conclusion.

Finally, in devising a method of diagramming apodictic syllogisms, it is my hope that future work will be done to devise a list of valid and invalid apodictic



moods not explicitly mentioned by Aristotle, e.g. in the fourth figure. It is my belief that this method of proof will aid in the accessibility of Aristotelian modal logic to students, and those who are interested in a form of modal logic independent of possible worlds, but instead rooted in the relationship between terms, and the natures those terms reference. The appendix of this work works through the proofs for assertoric, and problematic syllogism not mentioned in this chapter. What one will find there is that the interpretation offered does not significantly disrupt other forms of the syllogism. There are, however a few arguments determined to be valid that are not commonly considered part of the canonical list. They are not listed as invalid, however, and some commentators have included them among the valid syllogisms. Those arguments include: Disamis-QXQ, Bocardo-QXQ, Bocardo-XQM, Bocardo-QLQ. More work will be needed to devise Venn diagrams for the problematic syllogisms, which I think would help to improve our understanding of the way in which Aristotle reasons about possibility and contingency.

## CHAPTER THREE:

### 3.0 Introduction

In the preceding chapter, we saw a plausible interpretation of the modal syllogistic that relies on an innovative way of understanding of negation and privation in Aristotle. This account resolves apparent inconsistencies, particularly in the mixed apodictic syllogisms, without sacrificing fidelity to what Aristotle might have had in mind when devising his rules. Here, our task will be to argue for the applicability of the modal syllogistic, as we have interpreted it, in Aristotle's scientific enterprise. So, we will be examining Aristotle's scientific methodology, as found in the *Posterior Analytics*, and in some of the biological works, like *Parts of Animals*. We will also consider whether there is a unity in Aristotle's approach to modality by which various kinds of necessity and possibility can be employed by the modal syllogistic.

In examining the relationship of modality to Aristotelian science, we will be examining the relationship among nominal definitions, essential definitions, explanations, causes, and demonstrations. We will also consider the various sorts of *per se* predications a term might have. Through examining these aspects of Aristotle's philosophy of science, we will come to understand the role Aristotle's modal metaphysics and logic play within Aristotle's account of demonstrative knowledge.

### 3.1 Transcending the *De Re/De Dicto* Distinction:

Our modifications to Marko Malink's interpretive model offers a plausible alternative to the *de re* interpretation of Aristotelian modal claims, as seen in Becker. While previous interpreters have taken Aristotle to be predicating of the predicate term or whole proposition by necessity, Malink's mereological interpretation suggests that the necessity modifies the copula. Two class-terms are related to one another through this modified copula and Malink then considers how a sub-group, which belongs to the subject, might belong to the predicate class. According to Malink, the *de re* interpretation quantifies over individuals, where properties are *necessarily said* of the individuals that exemplify them. While many commentators tend to think that Aristotle's apodictic propositions are *de re*, and that a *de re* reading is needed to make sense of the modal syllogistic, adopting this interpretation of modality will not allow Aristotle to perform the conversions that he requires throughout the *Prior Analytics* (Malink 2013, 88). For many commentators, this just means that Aristotle did not understand the distinction between *de re* and *de dicto*, for his conversion rules seemingly work under a *de dicto* context. The heterodox interpretation, which Malink advances need not make any distinctions between the two modal contexts and, among other advantages, it preserves conversion rules.

When the contemporary modalist mentions the significance of the *de re/de dicto* distinction, she has something like the following in mind. There is the *de dicto* proposition:

(1) Necessarily, 2 is an even prime number.

A parallel, though not semantically identical *de re* modal claim, predicates not of the whole proposition but of a property within the proposition, as in:

(2) 2 is necessarily an even prime number.

One might wonder, then, if working out a consistent account of Aristotle's logic should just be a matter of clarifying where the modal operator should lie in a given proposition. However, if we accept that, in different contexts, Aristotle intended both *de re* and *de dicto* claims, another problem arises. For, it is prevalent among many contemporary philosophers and logicians to be skeptical about *de re* modality, what Alvin Plantinga refers to as a kind of "uneasiness":

The feeling persists that there must be something incoherent or unintelligible about *de re* modality—modality *de dicto* may be at any rate marginally respectable, but modality *de re* makes sense only if explicable only in terms of the former. One source of this feeling... is the notion, endorsed by Quine, that necessity resides in the way we speak of things, not in the things we speak of (1974, 27).

Quinean concerns are not merely rooted in emotional reactions or apprehensiveness to *de re* modality. In fact, Quine's own reason for thinking that necessity resides in the way we speak rather than in things themselves is due to the referential opacity of *de re* contexts, which do not preserve the substitution of co-referential terms *salve veritate*. This problem most clearly manifests itself when one translates a *de re* proposition into quantified modal logic. So, (1) could properly be translated as:

(3)  $\Box(\forall x)[(x = 2) \rightarrow (x \text{ is even} \ \& \ x \text{ is prime})]$

and (2) above as:

(4)  $(\forall x)\{(x = 2) \rightarrow [(\Box x \text{ is even}) \ \& \ (\Box x \text{ is prime})]\}$

The problem with (4) is that  $x$  could be something like “the number of pages I have left to read”. We can see that substituting “the number of pages I have left to read” generates no problem, but (4) seems to suggest that if the number of pages one has left to read happens to be identical to 2, then it is necessarily the case that the number of pages one has left to read is even and prime. However, it would be absurd to imply that the number of pages I have left to read is necessarily even and prime, i.e. two. Hence Quine concludes,

...Necessity as semantical predicate reflects a non-Aristotelian view of necessity: necessity resides in the way which we say things, and not in the things we talk about. Necessity as statement operator is capable, we saw, of being reconstrued in terms of necessity as a semantical predicate, but has, nevertheless, its special dangers; it makes for an excessive and idle elaboration of laws of iterated modality. This last complicates the logic of singular terms; worse, it leads us back into the metaphysical jungle of Aristotelian essentialism (Quine 1966, 174).

With that, Quine rejects quantified modal logic altogether. “Its referential opacity has been shown by a breakdown in the operation of putting one constant singular term for another which names the same object” (Quine 1966, 171). Quine also believes there is a related implication to quantified modal logic, namely Aristotelian essentialism, which he describes as “...the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to the thing, and others accidental” (Quine 1966, 173). For Quine, Aristotelian essentialism amounts to the position that properties are attached to things by necessity, and this could only be explicated in quantified modal logic. So, Plantinga’s project, in the *Nature of Necessity* is, among other things, to rescue essences from Quine’s critique—to find a way to translate *de re* modal claims to equivalent *de dicto* statements that are more palatable and referentially transparent. Notice that in *de dicto* modal statements, because the operator ranges

over the entire expression, what is said to be necessary is the way in which a property is predicated of a subject or by which two simple statements relate to one another, as opposed to attributes relating to things. Aristotle's modal context compares terms to one another—terms that, were they instantiated, would be said of in simple propositional forms. Malink says, "For Aristotle, the predicative relation holding true between a singular term and a general term ('man' belongs to 'Socrates') is of the same kind as the predicative relation holding true between two general terms ('animal' belongs to 'man')" (Malink 2006, 2). We must, as interpreters, be attentive to when the subject of a statement is intended to be a singular term, i.e. an individual, or a class term.

The question, then, is whether Aristotle has an account of modality that is coherent, metaphysically robust in helping us to understand the nature of essence, and pragmatically useful in advancing scientific knowledge despite the fact that it makes necessity claims about things in reality. The key is to understand that the things in reality of which Aristotle is making necessity claims, is class-terms, which can be exemplified in particular things. So, with this understanding in mind, we will explore the relationship among necessity, definition, explanation, and essence, especially with respect to Aristotle's *Posterior Analytics*.

### 3.2 The Modality of Definition, Explanation, and Essence

Recall, as I noted in the first chapter, one of the primary critiques against Aristotle's modal syllogistic advanced in the 20<sup>th</sup>-century by Becker, Łukasiewicz,

and Hintikka is that Aristotle unwittingly switches between *de dicto* and *de re* modal contexts. However, this view hinges on the idea that Aristotle treated modality as an operator that could range over entire propositions or predications of individual entities. Marko Malink, David Charles, and other contemporary commentators, like Richard Patterson, are emphatic that Aristotelian modality involves modifications directly to the copula by which terms are joined. Indeed, we have found that by adopting the position that modality is a copula modifier, we can reconcile far more of Aristotle's claims regarding the validity of modal syllogisms, which was our task in Chapter One. Moreover, such an interpretation is well motivated by the *Prior Analytics* itself. As David Charles explains,

Aristotle treats parallel 'belonging', 'belonging from necessity', and being capable of belonging (*Pr. An.* 29<sup>b</sup>29<sup>ff</sup>). This strategy makes sense on the assumption that modal notions are treated as modifying 'belonging' in different ways. For, 'belonging' is that feature of a verb which indicates a mode of combination of (e.g. man with Theaetetus) (2000, 381).

Malink selects the same passage in the *Prior Analytics* when devising his tripartite model of modal categorical statements. So, construing necessity as a sentential operator is not merely anachronistic, but fails to cohere with the sort of modal claims Aristotle makes. Indeed, Aristotle has offered a robust account of necessity that can account for essences, but the semantics and logical grammar undergirding this account went largely ignored due to historical happenstance and subsequent misinterpretations, as we have argued. Dispelling such misunderstandings will bring us closer to grasping the interconnection and unity among Aristotle's theory of essences, modality, and his overarching scientific methodology.

Charles, in his seminal work on Aristotle and definition, *Aristotle on Meaning and Essence*, believes that Aristotle faces two challenges if he is to explain the relationship between essences and necessity,

He must show how the distinction between necessary and essential features can be maintained. It cannot be enough to point to the fact that we have intuitions about essences distinct from (and more demanding than) those concerning merely necessary properties. He needs to establish that these intuitions are metaphysically well grounded. Further, he must indicate how we are to make sense of the logical grammar of claims involving necessity and essential features without referring to possible worlds (2000, 18).

Charles makes the case that Aristotle is able to distinguish between necessary and essential features, and indeed, we shall examine a 2013 article where he and Steven Williams explore the relationship in detail. It is less clear, however, how Charles thinks that Aristotle can avoid the semantics of possible worlds. It is admittedly an odd way of putting it when Charles insists that Aristotle avoids the semantics of possible worlds more than a millennia before such semantics were ever devised, but the advice is well-taken for contemporary commentators and interpreters. Indeed, the previous chapter provides just such an account.

According to Charles, Aristotle believes signification develops in stages. By these stages, the understanding leverages a greater insight into the nature of things themselves. These stages of definition transition from the nominal to the essential.

Charles summarizes the three stages as follows (2000, 24):

Stage 1: This stage is achieved when one knows an account of what a name or another name-like expression signifies.

Stage 2: This stage is achieved when one knows that what is signified by a name or name-like expression exists

Stage 3: This stage is achieved when one knows the essence of the object/kind signified by a name or name-like expression.

Charles provides the example of a triangle at Stage 1, which one might know has three angles before knowing that such figures exist. All that is related in the



definition is what the word signifies. In setting forth the nominal definition of a term, one is able to progress to the next stage. For if one knows that “triangle” signifies “three-sided plane figure” and one discovers that there are some three-sided plane figures, one learns that there actually are some triangles in reality. With the existence of triangles established, a search for the cause of three-sidedness can be undertaken. That is, there can be a search for the reason why, τὸ διότι, for which a triangle should have three-sides. In what follows, we will consider how the varieties of definition act as a springboard for explanatory and eventually essential definitions of terms.

To understand this, we must consider more closely the role modality plays in Aristotle’s definitions. A definition, ὀρισμὸς, is a proposition that, as Aristotle says, “...seems to be of what a thing is, and what a thing is in every case universal and affirmative” (*APo.* 90b4-5).<sup>22</sup> As such, it can be broken down into the tripartite structure of any other categorical proposition. So any definition would include the quantity, quality, and modality within the copula as well. Oddly enough, when Steven Williams and David Charles analyze definition in their exposition of the Master Craftsman argument, they treat necessity in the contemporary way, as a sentential operator. In doing so, they end up raising metaphysical conundrums foreign to Aristotle’s conception of natures and essences.

Williams and Charles want to answer three questions: (A) how does explanatory role figure in the proper understanding of essence, (B) why should essence have both definitional and explanatory roles, and (C) how can essences

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<sup>22</sup> ὁ μὲν γὰρ ὀρισμὸς τοῦ τί ἐστὶν εἶναι δοκεῖ, τὸ δὲ τί ἐστὶν ἅπαν καθόλου καὶ κατηγορικόν.

explain why kind members must have certain properties (2013 121). Williams and Charles argue that Aristotle's,

...notion of essence has two interrelated roles: one definitional, the other explanatory. On the one hand, it specifies what it is for something to belong to a particular kind of thing, the specification being, of necessity, both necessary and sufficient for membership of the kind; while, on the other hand, it explains not just why members of such kinds have certain further properties, but ultimately why they *must* have such properties (*ibid*).

To unpack precisely how Aristotle understands the definitional and explanatory roles of essences, Williams and Charles turns to the *Posterior Analytics* and Aristotle's discussion of the essential definition of thunder. Aristotle develops his definition of "thunder" in three stages. He first answers the question "what is thunder" and concludes that it is a noise in the clouds. As to the second question, why the noise occurs in the clouds, he identifies the cause, which he believes is that a fire is being extinguished. So, the full definition of thunder becomes a "...noise of fire being extinguished in the clouds" (*APo.* 90a14). The definiens contains both the explanandum and explanans of "thunder". In the case of thunder, the explanandum, a noise [in the clouds], is explained by a fire that is being extinguished [in the clouds], and both taken together provide a definition.

Such a picture is powerful and beautifully economical; and it is one which can evidently be extended consistently to the other examples Aristotle considers here, using the same threefold specification of the phenomenon: *being an F, a G, and an F brought on by [its being] a G* (Williams & Charles 2013, 124).

However, Williams and Charles doubt that Aristotle can successfully extend this method of definition to cover all cases. The method is suited for defining phenomena that are efficiently caused to be, and they believe that Aristotle was attempting to extend this method of definition to teleology so as to be able to define other substances.

However, while teleology is ideal for the elucidation of artifact kinds, and a sort of functional explanation that is not simply *faute de mieux* might even now be appropriate in some areas of biology, it would be dogmatic to insist that all natural kinds, even biological kinds, be definable this way (Williams & Charles 2013, 124-5).

So, Aristotle's aim may not have been to provide a singular solution to connect essential definitions to causal explanations. Rather, it may be that Aristotle saw the usefulness of doing so, sometimes by appealing to efficient causality, and sometimes by appealing to other sorts of causality, for example, teleology. Whether the combination of utilizing both sorts of causality would provide essential definitions for all natural kinds is doubtful. Williams and Charles point out, chemical elements are defined by their atomic structure rather than teleology or efficient causality. Aristotle might have been happy to admit atomic structure as, itself, some elucidation of the formal and material causes, so perhaps the explanation need not be restricted to one or two types of causality. For example, the chemist might define "gold" as that element with the atomic number of 79. Also, the chemist might explain why gold never tarnishes by appealing to the implications of having 79 electrons and protons, along with the requisite 118 neutrons. So the form can have explanatory power. Indeed, Aristotle gestures towards four types of explanation, that:

...there are four types of explanation (one, what it is to be a thing; one, that if certain things hold necessarily that this does; another, what initiated the change, and fourth, the aim), all these are proved through the middle term (*APo.* 94a20-24).

So, there is no need to restrict ourselves to teleological explanations when deriving definitions. Indeed, if a relationship between definition and explanation is to be established, it is natural to turn to causality, and so all four causes could be invoked, which is precisely what Aristotle does in the above passage. It should be noted,

however, that Aristotle never claims that he can prove that there are four causes, though he does seem to think there are just four, given that no thinker has uncovered a cause beyond them (Williams & Charles 2013, 125; see *Meta.* 988b16-19). Regardless of whether there are explanations beyond the four causes, the point seems to be that certain definitions depend upon explanation, and that efficient causality and teleology are two common kinds of explanation, especially in biology and physics. So it seems that Williams and Charles can provide a plausible account of Aristotle's project for connecting essences with explanations and definitions. The question that connects definition, explanation, and essence to Aristotelian science is how Aristotle could argue that certain kinds must have precisely the properties they have. Williams and Charles lay out the argument is as follows:

Let G be essence, K be kind, and F be property

(5)  $\Box(\forall x)(Kx \leftrightarrow Gx)$  (Def)<sup>23</sup>

(6)  $\Box(\forall x)(Gx \rightarrow Fx)$  (Exp)<sup>24</sup>

(7)  $\Box(\forall x)(Kx \rightarrow Fx)$  (from 6,7)<sup>25</sup>

(8)  $(\forall x)(Kx \rightarrow \Box Fx)$ <sup>26</sup>

Therefore,

(9)  $\Box(\forall x)(Kx \rightarrow \Box Fx)$  (from 3,4)

Furthermore, Williams and Charles point out that (9) can be derived with Def, Exp, and Ess or Ess\*:

(10)  $\Box(\forall x)(Kx \rightarrow \Box Kx)$  (Ess)

(11)  $\Box(\forall x)(Gx \rightarrow \Box Gx)$  (Ess\*)

That is, there are multiple paths in this argument by which one can arrive at (9), that necessarily thing K must have property F. That is, (9) can be explained by Def,

<sup>23</sup> Necessarily, a thing is kind K if and only if it has essence G.

<sup>24</sup> Necessarily, a thing is kind G only if it has property F.

<sup>25</sup> Necessarily, if a thing is kind K, then it has property F.

<sup>26</sup> If a thing is kind K, then necessarily it has property F.

Exp, and (8), or by combining Def and Exp with either Ess or Ess\*. Williams and Charles provide justification for (7), but (8) appears to be an independent premise for which Williams and Charles provide little textual or philosophical justification. Indeed, it seems to be inserted so that can arrive at their intended conclusions, which appears *ad hoc*. Perhaps (8) is intended to be based on repeated observations of the thing defined, so it would be based upon induction (ἐπαγωγή) where several observations of the particular give rise to knowledge, or perhaps it is by νοῦς, a kind of intuitive grasp (see Lesher 1973, 45). More on Aristotelian induction will be said later on. Williams and Charles say that the definition explains why members of K must have F, but they do not provide any way to derive that from Def or Exp.

Williams and Charles give us the example of the lemon:

...it is a superficial, but universal, property of lemons that they are tart, and that this is a property of lemons *as such*, i.e. a property that lemons have in virtue of their being lemons or in virtue of having the essence of lemons (2013, 132).

So Exp will represent the explanation for why lemons are tart. This might be, as Williams and Charles suggest, having a certain kind of genetic code, G, such that any x that has G will be tart. The question, though, still remains as to how Williams and Charles justify moving from the *de dicto* claim connecting tartness to having a certain kind of genetics, to the *de re* claim that things which have a certain genetics are necessarily tart.

Williams and Charles clarify that, “It should be emphasized here that while we will occasionally make use of resources not available to Aristotle himself, none is (in our view) materially out of sympathy with his overall metaphysics” (*ibid.* 127-8). Unfortunately, in explicating the premises of their argument, certain

anachronisms emerge that are troublingly. For instance, in explaining definitional necessity, the duo appeals to Leibniz's indiscernibility of identicals, that "...objects drawn from any ontological category are identical only if they have all of their properties in common..." (*ibid.* 129). From this, Williams and Charles deduce:

(12)  $\Box(\text{being such as to have } G = \text{being such as to have } G) \text{ iff } \Box(\text{being } K = \text{being such as to have } G)$  (*ibid.*).

But Williams and Charles also realize that explaining definition in terms of the necessity operator and identity relation can lead certain objections anticipated by Kripke in *Naming and Necessity*, that is

(13) The first Postmaster General of the US = the inventor of bifocals

and

(14)  $\Box(\text{The inventor of bifocals} = \text{the inventor of bifocals})$

So it would seem that by the transitivity of identity:

(15)  $\Box(\text{The first Postmaster General of the US} = \text{the inventor of bifocals})$   
(Williams & Charles 2013, 130; see also Kripke 1980, 98).

But (15) is false, "What makes it only contingently true that the first Postmaster General of the US = the inventor of bifocals is that at least one of the specifications of the individual Ben Franklin applies to him only contingently and the specifications are not such as to co-vary necessarily" (Williams & Charles 2013, 131). Of course, the question should be as to whether this should be problematic for Aristotle. Williams and Charles argue that Aristotle is concerned with a different metaphysical question, that "...the property of being a K = the property of being such as to have G, since, according to the Aristotelian picture, being a K and being such as to have G are different specifications of what it is to be a K, one tautological,

the other elucidatory" (*ibid.*). Individuals, like Benjamin Franklin are only contingently related to properties like "being the inventor of bifocals", but Aristotle is concerned with kinds and essences that relate one to another necessarily rather than contingently. To then raise the question of identity between individual things being in certain ways is not part of Aristotle's project.<sup>27</sup> Williams and Charles' claim that "what it is to be a K" is a tautological specification of "being K". So it is not "being Socrates" that is the same as "being a human", for "being Socrates" is not "what it is to be a human". Otherwise one might think that a snubness of the nose is somehow essential to what it is to be human. Rather, "being a rational animal" is "being human", and this is tautological with "what it is to be a human". The fact that there is a concern with Kripkean puzzles surrounding singular terms like "Benjamin Franklin" betrays a fundamental difference in the way Williams and Charles are constructing definitional necessity. Aristotle says, "For definition seems to be of what a thing is" (*APo.* 90b4). What a thing is will relate not to some sort of *haecceity*, or individual essence, but those secondary substance terms and essential properties that follow upon them.

While one can agree that Aristotle has different metaphysical concerns with respect to definition, I am reticent to assent to Williams's and Charles's account that this can be explained because the semantic description of being kind K is "the property of being a K". The metaphysics implicit in describing kinds as sorts of properties unduly suggests that the Aristotelian notion of definition is primarily about relating a thing to its property by necessity. This seems to reverse the order of

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<sup>27</sup> In the fourth chapter I shall argue for bridge laws, which will allow one to transition between Aristotelian modal contexts and quantified modal logic.

explanation. That is, it is the fact that one is being kind K that explains why it is necessarily the case that it is predicated with K, and that the substance, i.e. the subject of the predication, in question would simply not exist were property K ever fail to be exemplified. Aristotle says,

The terms 'being' and 'non-being' are employed firstly with reference to the categories, and secondly with reference to the potentiality or actuality of these or their potentiality or actuality of these or their opposites, while being and non-being are in the strictest sense are truth and falsity. The condition of this in the objects is their being combined or separated, so that he who thinks the separated to be separated and the combined to be combined has the truth, while he whose thought is in a state contrary to that of the objects is in error. This being so, when is what is called truth or falsity present, and when is it not? We must consider what is meant by these terms. It is not because we think that you are white, that you *are* white, but because you are white we who say this have the truth (*Meta.* 1051a35-1051b8).

In the case of having whiteness, what makes it true is that there exists a person who has the quality of whiteness. But for other categories of being, such as substantial form, the truth-maker will not be an individual possessing this or that essential property, but the existence of the primary substance insofar as it is a τὸ τί ἦν εἶναι. Moreover, in the case of essential definition, as opposed to mere nominal definitions of terms like "goat-stag", the existence of the primary substance seems to be a requirement, for Aristotle tells us, "Since we must know the existence of the thing and it must be given, clearly the question is *why* the matter is some individual thing" (*Meta.* 1041b4-5). A definition, then, does not define an individual in terms of a property that it necessarily has. Rather, a definition should explain why an individual is what it is insofar as it exists *qua* its essence, or more properly what an essence is in terms of its genus and difference. So, suppose we say x exists as a triangle and triangle is necessarily a plane figure with three angles. We can explain what it is, i.e. triangle, not because it, the individual x, is necessarily a plan figure with three angles, but we explain that it exists as a triangle because that which it is,



triangle, is necessarily a plan figure with three angles. Williams and Charles want to say:

- (16)  $\Box$  (being triangle = being such as to have three-angles in a closed plane figure)
- (17)  $\Box$   $(\forall x)(x \text{ is a Triangle} \leftrightarrow x \text{ is a three-angled closed plane figure})$  (16 Def)
- (18)  $\Box$   $(\forall x)(x \text{ is a three-angled closed plane figure} \rightarrow x \text{ has three sides})$  (Exp)
- (19)  $(\forall x)(x \text{ is a three-angled closed plane figure} \rightarrow \Box x \text{ has three sides})$  (premise)
- (20)  $\Box$   $(\forall x)(x \text{ is a three-angled closed plane figure} \rightarrow \Box x \text{ has three sides})$  (from 17,18, and 19)

Now, at (19) the claim is made that anything that is three-angled is necessarily three-sided. The question Williams and Charles raise about actualism is tantamount to asking the question of whether some individual  $x$  might be, say, possibly not three-angled, and so possibly not three-sided. Does this mean that Aristotle is concerned with defining individuals in terms of properties in the manner that Williams and Charles do? I think an important distinction can be made between defining an individual thing *qua* individual and defining the individual thing *qua* essence. The Kripkean examples trade on taking there being accidental features of an individual and using those features to define the individual *qua* individual rather than *qua* essence. We intuitively know that these accidental features are contingent, and so are inappropriate in specifying the essential definition of a thing. For a definition is, "...a phrase signifying a thing's essence" (*Top.* 101b35). To put the issue a bit differently, if the subject of a definition is the thing, *qua* individual, rather than its substantial form, and a definition creates a necessary identity relation between a thing and its essence, it would follow that every individual is identical to

its own essence and there would be no members that fall under the same species. Williams and Charles are misled by adopting quantified modal logic in order to describe Aristotelian definitions. Thus they are forced to treat necessity as an operator that functions independently from the definition's copula, permitting identity substitutions that can be problematic without specifying *ad hoc* metaphysical restrictions to kind-terms. Even still, it is not even clear that the *de re* step at (19) is true. One might adopt the metaphysical position that an individual can subsist even when it does not exemplify its kind term. So much is explicit in the metaphysics of Thomas Aquinas, who believes that an individual intellect can subsist while ceasing to be a human substance (cf. SCG II.79). If so, it would be the case that "human" does not belong to some individual x, even though that x still subsists as a disembodied intellect.

Along with a use of the necessity operator, I have noted that Williams and Charles appeal to Leibniz's laws of identity in explaining the relationship between definition and necessity. This is anachronistic not only by appealing to Leibnizian metaphysical principles, but also because, as Allan Bäck writes, "...Aristotle does not make much of identity statements" (2000, 182). Bäck points to the *Topics* where Aristotle describes numerical identity as the use of, "...two appellations of the same thing" (*ibid.* 183; see *Top.* 103a32-3). In explicating Aristotle's theory of predication, Bäck goes so far as to say, "...The claim of the sameness of a term and its definition must not be understood as modern identity..." (2000, 196). Yet this is precisely what Williams and Charles have done in order to explain the sort of definitional

predication on finds in Aristotle. So it is problematic, from an interpretative standpoint, to reduce definitions to identity claims about individuals.

We must return to the question of what is being defined in the definition. In predicate logic, one is defining individuals within a universe of discourse. That is, for all  $x$ , if  $x$  is a  $K$ , then  $x$  is a  $G$ . So  $x$ 's that have the property  $K$  also have the property  $G$ . This tells us something about the way  $x$ 's are to be defined. Aristotle, on the other hand, wants to be able to utilize definitions within his categorical demonstrations. This problem is similar to one that Owen Goldin (1996, 57) locates in Gomez-Lobo's account of τὸ εἶναι τι and εἰ ἔστι. Goldin explains:

Gomez-Lobo follows the lead of Kahn in detecting a predicative use of εἶναι in many passages in which an existential use had been taken for granted. He interprets Aristotelian hypotheses as statements asserting that a given substrate is a certain kind of thing, where this identification of a thing makes possible a demonstration that certain attributes inhere in the thing (*ibid.* 52).

Crucial to Gomez-Lobo's interpretation is his exposition of Aristotle's universal predication, which he thinks has the general form of  $(x)(Tx \rightarrow Rx)$  (see Gomez-Lobo 1980 77-78; Goldin 1996 56-57). Goldin responds:

For Aristotle the proposition "All T's are R's" is a predication; at issue is its subject. If *every triangle* (πᾶν τρίγωνον [71a19-20]), then must we not know that every kind of triangle exists in order to know the premise?... Aristotle resolves the problem, which he identifies as a version of the *Meno* problem, by denying that a premise is grasped without restriction (ἄπλῶς) (71a27-28). The subject term of a universal proposition regarding triangles is not *every triangle* but *triangle* itself. The proposition is grasped *as* holding of every case and thus allows us to know that a certain predicate inheres anything we know to be a triangle (Goldin 1996, 57).

Notice that this also alleviates the question of actualism, as we are not left wondering whether this triangle could be a banana, as we are talking about triangle itself. Gomez-Lobo's interpretation places emphasis and existential import of things. It is the structure of predicate calculus to color one's interpretation of predication in Aristotle with respect to existential import. What is the subject of predication

$(\forall x)(Tx \rightarrow Rx)$ , e.g. in  $Ta$ ? There is an underlying supposition in predicate logic that the subject and predicate of are not of the same ontological category. The subject is assumed to be some individual, while the predicate is most often construed as some sort of property. Intentionally or not, this leads us to suppose that individual things are conceptually separable from their properties. One might suppose that this is unproblematic for some properties, but on the Aristotelian picture, the substance cannot be separated from the individual in an intelligible way. Take, for instance, the claim “this man is a rational animal”. Predicate calculus would break this down to the expression  $(\exists x)[Mx \ \& \ (Rx \ \& \ Ax)]$  where  $x$  is some individual and “man”, “rational” and “animal” are predicated of it. As far as logical translations into predicate calculus, this is useful, but it tends to suggest that what we have defined is a “this” as a “man” & “rational” & “animal” rather than elucidate that this is a man, and men are rational animals.

Aristotle’s categorical syllogisms allow one to predicate both across ontological categories and within the same ontological category. For Aristotle, an essential definition defines a substance in terms of a more generic essence and the *differentia*, which is an essential property or necessary quality. Of course, in defining secondary substances, we come to understand primary substances. According to Aristotle, “...animal is predicated of man and therefore also of the individual man; for were it predicated of none of the individual men it would not be predicated of man at all” (*Cat.* 2a36-38). Primary substances are neither said of nor in a subject. The identification of primary substances does not answer the τί ἐστὶ question. If we were to interpret the variable in  $(\exists x)Mx$  as, say, a primary substance, then we start

to see how it is that Aristotelian propositions do not quite mesh with contemporary syntax. This is, in my estimation, the source of much confusion. Consider, for instance, the a-propositions are commonly translated into predicate logic. Suppose, for instance, rational belongs to all men. In predicate logic, we universally quantify over individuals that are predicated of “man” and “rational”, so the result is the conditional:  $(\forall x)(Mx \supset Rx)$ . Logicians are taught to translate universal propositions into material conditionals, in which the connection between antecedent and consequent is defined by a truth-table. This is antithetical to Aristotle’s conception of logic wherein terms are related to one another through predication. The Aristotelian statement Aristotle’s logic slices up the logical space differently by directly relating predicating qualities of substances, differentia of genera, etc. For example, material implication is defined in a truth-functional manner, as one might see in a truth table, as in:

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

The truth-functional account of conditionals, which traces back to Philo’s interpretation of conditionals, “...does not require any connection or relevance between the if-clause and the main clause for a conditional to be true” (Sanford 1989, 24). Consequently, on this account it is true that “if fish are warm-blooded animals, fish lay eggs” and it is true that “if fish are warm-blooded animals, fish are cold-blooded animals,” and it is true that “if fish are warm-blooded animals, humans are cold-blooded animals”. So long as the antecedent is false, the statement is true.

For scientific propositions to be true, within an Aristotelian framework, there must be a connection between predicates, and this connection is underscored by modally conditioning the copula. That these predicates belong to individuals or “primary substances” is incidental to the logic itself. Thus, if Aristotle were to tell us that rational belongs necessarily to man, he is not making a claim about what a primary substance must be predicated of an individual. Many of the issues of *de re* modality arise because of this, i.e. because in the *de re* context it is being said that some individual thing necessarily is predicated in some way or another. Aristotle joins class terms together and so is operating, in a sense, on a higher order of logic.

Returning to Williams’s and Charles’s exposition on essences and modality, we see that explanation is understood as, “...the claim that necessarily anything with the defining essential properties of K, namely G, is F” (Williams and Charles 2013, 131). Essences must do some explanatory work, and so a good essential definition will provide a *differentia* that not only qualitatively distinguishes it from other kinds within a genus, but also causally explains what it is in some way. Williams and Charles consider the tartness of a lemon and so structure explanation this way:

$$(20) (\forall x)(x\text{'s being such as to have } G \text{ causes } x\text{'s being tart)}$$

Since this claim will be necessary, where G is some genus and T is tartness:

$$(21) \Box(\forall x)(Gx \rightarrow Tx)$$

But this is to explain that some instance of x’s being G implies that it will also be T. It does not explain that Lemon is G and this implies that Lemon is T. Williams and Charles make an important concession at this point, “Of course, although Aristotle did concern himself with claims about particular individuals such as that this K is F,

he is, through explanatory syllogisms, principally concerned in the cases we have been considering with general claims such as that all Ks are F" (2013, 133). They use this as a response to a Hempelian model that it could be, as Williams and Charles say, a massive coincidence that everything that has G is F, which threatens our ability to apprehend essences (*ibid.*). Hempel, like Aristotle, offers a deductive model for explanation. This "deductive nomological" account of explanation relies on empirical generalizations,

...connecting different observable aspects of the phenomena under scrutiny... But science raises the question 'why?' also with respect to the uniformities expressed by such laws, and often answers it in basically the same manner, namely, by subsuming the uniformities under more inclusive laws, and eventually under comprehensive theories (Hempel 1962, 11).

Uniformity is not to be explained in terms of essences and natures, but natural laws, which subsume one to another. Ultimately, Hempel's approach to explanation is Humean in that these empirical generalizations are inferred from constant conjunction. Williams's and Charles's problem is related to a form of actualism where  $\diamond(\exists x)(x \text{ is a purple lemon})$  implies, by the Barcan formula,  $(\exists x)\diamond(x \text{ is a purple lemon})$ . While it may be useful to be able to move the possibility operator inside an existential quantifier, this kind of actualism raises a peculiar metaphysical question, namely, how it could be that there is an individual that is possibly a purple lemon. Does this mean that the possibility is inherent within the individual *qua* individual to be a purple lemon? What sorts of individuals have that possibility, or do individuals *qua* individuals have certain essences by virtue of which they have the potential to have certain combinations of essential and accidental properties? The Barcan Formula, combined with Actualism, makes the individual something akin to

prime matter in that the individual is pure potency. What is it about the individual that permits of this possibility? Likewise, it could be accidental that all individuals that have G are F, since there is the potential that they could have been anything and no necessity in a Gx being an Fx. But then it is accidental to their having G, and merely coincidental that the possibility of having F is exemplified. Again, is it *qua* individual that the possibility of F is always exemplified? Being an individual does not do any explanatory work though, which is probably why Williams and Charles seek to escape a Hempelian critique by actually returning to Aristotle's project of explaining substances in terms of essences. Williams and Charles explain that G need not be an essence; it just has to be law-like, presumably in a nonHumean manner. Also, we might agree that G would not be the essence, but the question arises whether 'G & (G→F)' is the essence, or at least an attempt at deriving the essence in some way. These questions still need to be addressed by them.

Williams and Charles argue that Aristotle, "...embraces an idea of objects as 'this suches', individuated as the objects they are by the kinds under which they fall. Further, they hold that Aristotle thinks that the cause of the object being a 'such' and being one object is the same: the relevant cause accounts for the object being one persisting object and to its being a K" (2013, 134). Their reason for saying this is found in the *Metaphysics*:

... [T]he proximate matter and the form are one and the same thing, the one potentially, the other actually. Therefore to ask the cause of their being one potentially, the other actually is like asking the cause of unity in general; for each thing is a unity, and the potential and the actual are somehow one. Therefore there is no other cause here unless there is something which caused the movement from potentiality into actuality (*Meta.* 1045b17-27).



Indeed, this text points to a causal unity in the proximate matter and form in that which is being a 'such', however this does not explain the unity between substance and the definition of a thing, which is the move Williams and Charles want to make in understanding essences and their relationship to the existence of things. "In effect, therefore, there is some feature sufficient for being a K whose loss entails both that the object (which suffers the loss) ceases to be a K and is no longer one persisting object" (Williams and Charles 2013, 134). In other words, substantial changes destroy what a thing is, and so other things result from the substantial changes. For instance, a human undergoes a substantial change at death and the body becomes a corpse. Some modern companies can cremate the corpse, extract the carbon, and create a laboratory diamond from the matter so that a loved one can be cherished and worn as an adornment. But, there is no longer a human there, or even a corpse, just a diamond. Does this mean that  $(\exists x)[x \text{ is a human} \ \& \ \diamond(x \text{ is a diamond})]$ ? Aristotle's answer is that human is not a possible diamond, but it is the proximate matter of the primary substance which remains and undergoes change. In this sense, Aristotle is an actualist who can affirm something like the Barcan Formula.

William and Charles raise this question again, "Why can't anything be anything? (*ibid.* 136). To answer the question, they turn to a modified version of Leibniz's Law, "...the *default* position is surely always that if a thing changes any of its properties, ordinarily conceived, it must be thought of as a different thing, *unless there is a good reason for thinking otherwise*" (*ibid.*). However, it is unclear that this law, so stated, will help address the metaphysical question of why anything cannot

be anything else. For they construe the law in terms of the way a thing must be thought rather than how it in fact is. Perhaps the reason that one must think it is a different thing is based on essential and accidental properties, but that is not apparent. Furthermore, Leibniz's law defaults to the position that all properties are essential, unless it can be demonstrated otherwise. Why should this be the default position? Such a position is metaphysically and epistemologically risky, as one may be unable to disprove some different attribute, say skin color or eye color, is not indicative of an entirely distinct substance. In fact, we do not see Aristotle's science develop in this way, i.e. taking as default the position that a change in properties is indicative of a change of what a thing is. It seem more reasonable to say that the default position is agnosticism when any change in properties occurs, unless there are good reasons to say whether those changes were with respect to accidental or essential properties. Otherwise one will find that updating our understanding of essences insurmountably difficult, even with inductive inferences. One's concept of the essence will be solidified upon first inspection rather than repeated experience. Once those features are incorporated into the essence, stronger defeaters will be needed to change the definition. In other words, this principle biases essential definitions that default towards narrowness in definitions established upon first observation. Aristotle is far too concerned with distinguishing between kinds of change (κίνησις), e.g. the generation (γένεσις), and destruction (φθορά) common to substantial change as opposed to alteration (ἀλλοίωσις) (Cohen 1978, 389). "The general idea is that an alteration of a thing leaves it in existence, but with (or

without) an attribute (πάθος) it previously lacked (or had). Substantial change, on the other hand, involves a thing's coming or going out of existence" (*ibid.*).

Williams and Charles use the master craftsman as a way to illuminate the notion of essences. They say that an artifact's essence is its function, suitably defined. Of course, they admit that Aristotle would preclude artifacts as instances of essences on the grounds that they lack a natural teleology qua artifact, though the natural materials out of which they are composed will be suitable for the various ends the craftsman intends for the object. So a particular kind of wood that is supple and yet sufficiently hard is good for a cricket bat insofar as one intends the bat to hit the ball. The master craftsman is one who knows how to utilize materials in such a way as to integrate them into a functioning unity. But, the master craftsman needs only to know those properties of their medium that is relevant to the design. As Williams and Charles note, igloo-builders needs to know that ice and snow remain solid at certain temperatures, and melt at other temperatures, and other relevant aspects of their medium. They do not require an exhaustive knowledge of their materials. Still, they need to be able to correctly identify the materials they need. To do this, they must be able to answer both the explanatory question "Why do these things behave as they do" and the definitional question "What is it to be such an object" (Williams and Charles 2013, 138). Williams and Charles contrast the craftsman with the scientific experimentalists. While the former is concerned with "what works" and so is only concerned with an exact understanding of nature insofar as it serves this pragmatic end, the scientific experimentalist wants to properly "cut nature at its joints" (*ibid.*). The craftsman is primarily concerned with

individual things insofar as he wants to create this chair or that table. Knowledge of the essences of those individuals is of secondary importance, so long as the individual materials function appropriately. This could be true even if the materials undergo substantial changes, so long as the appropriate accidental properties remain or change in the appropriate ways. Williams and Charles turn to the example of a wooden chair that still functions as a seat even after the wood petrifies.

Likewise, a craftsman working with cement uses it precisely because the substance and accidents change in predictable and useful ways. It can be poured into a mould, and as the components chemically react, the liquid hardens. So cement is useful as an individual thing because it is initially easy to shape and the later hard to deform. The artisan does not care whether the object remains what it is before and after it undergoes the change from liquid to solid. Substantial change can also work against the craftsman. For example, he might use an iron nail because it is strong, but it will oxidize to the point where the rusty material can no longer hold two pieces of wood together. In contrast, the experimentalist is not concerned with individual things as such, but only insofar as they exemplify essences. "A tree that has petrified has nothing at the level of biological, chemical or physical theory to warrant the supposition that it continues to exist. It may look like a tree, but it will only continue to exist as such homonymously" (*ibid.* 138). The experimentalist might know that there is not anything like a tree there now, but the craftsman might note that the former tree and present petrified tree both are suited for the same project. Nonetheless, the craftsman does need to apprehend something of the essence of a thing to know how to use a specific instance of a thing well.

Williams and Charles raise the question as to which the master craftsman's knowledge being superseded by the experimentalist. In *Metaphysics A*, Aristotle suggests as much when he writes,

...all men suppose what is called wisdom to deal with the first causes and the principles of things. This is why, as has been said before, the man of experience is thought to be wiser than the possessors of any perception whatever, the artist wiser than the men of experience, the master-worker than the mechanic, and the theoretical kinds of knowledge to be more of the nature of wisdom than the productive. Clearly then wisdom is knowledge about certain causes and principles (*Meta.* 981b26-a3).

Or is the craftsman's view inferior to that of the experimentalist? They speculate "How far is this from Aristotle's own picture, particularly in regard to the master craftsman's being superseded by the experimentalist? Maybe it is not clear what Aristotle would say at this point" (Williams and Charles 2013, 139). In the end, they want to say that the experimentalist knowledge is better, and yet, as we have seen, it is difficult to rid oneself of the notion that "individual" is more primitive than "essence" and that it survives even substantial change. An anti-essentialist might suggest, then, that there is a sense in which the craftsman, while knowing something practical, namely that this nail will become rust, or this piece of leather will soften with time and wear, know something greater. For the craftsman knows what ends individual objects can fulfill even after such objects have ceased to be what they are. It is a notion that seems to operate in the very predicate logic Williams and Charles use to explain Aristotelian explanatory definitions. So,  $(\exists x)(Wx \text{ at } t_1 \ \& \ Px \text{ at } t_2)$  tells us that there is some  $x$  that is wooden at one time, and petrified at another. How is this not a more exact description of reality, cut at the joints, than the one that the experimentalist wants to provide for the relationship between wood or petrified-wood and hardness? I do not want to suggest that Williams and Charles question

whether Aristotle, or Aristotle's essentialism, leaves open the question of whether the experimentalist supersedes the master-craftsman. Rather, I want to suggest that the question only arises for Williams and Charles in a genuinely perplexing way because of the very logic they use to explicate Aristotle's essentialism.

### 3.3: Towards an Aristotelian Demonstrative Science

Williams and Charles want to explain "...what it is about essences that ensures that they have the twin definitional explanatory roles?" (2013, 140). So Williams and Charles argue that essences play a definitional role insofar as they are explanatorily basic and informative rather than redundant. This provides a helpful distinction between necessary properties and essential predication. For, all essential properties will be necessary, but many necessary properties are non-informative or derivative of more basic properties. The alternative, which Oderberg refers to as "modalism", is the idea that essences just are necessary properties (2007, 7). But, as we have seen, this leads to puzzles regarding individuals that lead some philosophers, like Quine, to suspect that modality is referentially opaque, and essences ontologically superfluous. For other philosophers, like Kripke, the solution is rigid designation, which individual essences. Returning to Aristotelian modality, then, is to navigate between the anti-essentialism of Quine and the *haecceitism* of Kripke, where essences are reduced to necessary identity relations across possible worlds.

Indeed, I am sympathetic to William and Charles in regarding essences as having definitional roles that are basic and informative. Furthermore, insofar as essences define, they define by necessity. However, in understanding the relationship between essences and necessity, I think it is more helpful to turn to Aristotle's modal syllogistic rather than predicate logic. This is because predicate logic raises the question of the subsistence of individuals over substantial change. And this question can be raised with force when necessity is split into *de re* and *de dicto* contexts. Williams and Charles attempted to circumvent those contexts by combining them within their definitions of definition, explanation, and essence. Indeed, we find that since  $\Box(\forall x)(Kx \leftrightarrow Gx)$ , according to definition, and  $\Box(\forall x)(Gx \rightarrow Fx)$ , according to explanation,  $(\forall x)(Kx \rightarrow \Box Kx)$  and  $\Box(\forall x)(Gx \rightarrow \Box Gx)$ , if something is a K, we can conclude that x is  $\Box K$ ,  $\Box G$ , and  $\Box F$ . And this raised some the difficult question of why it cannot be the case that anything could be anything. That is, what justifies Williams and Charles's liberal use of the necessity operator aside from an *ad hoc* attempt to rescue their interpretation? However, this leads to a certain untoward form of actualism where a thing could exist such that it is possibly something else—a seeming consequence of Barcan's formula. That is, they want to avoid the odd situation where  $\Box(\forall x)(Kx \leftrightarrow Gx)$ ,  $\Box(\forall x)(Gx \rightarrow Fx)$ , and  $(\forall x)(Kx \rightarrow \Diamond \sim Kx)$ , i.e. that an individual could subsist and has the potential to not be of the kind it is if we specify that if x is kind K, then it necessarily is kind K. However, this raises the metaphysically ugly question of what explains this *de re* necessity: the individual, possible worlds, or the property itself? As Williams and Charles allude, contemporary essentialists struggle to distinguish the predication of an

essence from the predication of essential and logically necessary properties.

However, it is not clear that a liberal use of the necessity operator is the solution to this problem. Perhaps Williams and Charles cannot be blamed in the sense that there does not seem to be a plausible alternative to discuss essences and modality other than quantified modal logic. With an alternative metaphysics of modality in place, one that is Aristotelian, we can now relate essence, definition, and explanation to necessity without *ad hoc* rescues, the threat of this untoward form of actualism, referential opacity, speculative modal realism about possible worlds, or any other fears.

Allan Bäck writes, "Aristotle's modal theory often ends up being central to his philosophical endeavors. After all, Aristotle's science is concerned with what is *necessarily*." (1995, 86). Bäck has in mind the *Posterior Analytics*, where Aristotle says:

We think we understand a thing *simpliciter* (and not in the sophistic fashion accidentally) whenever we think we are aware both that the explanation because of which the object is its explanation, and it is not possible for this to be otherwise (*APo.* 71b9-12).

To understand the explanation of something is to understand something necessary about it. Aristotle then discusses the necessity of demonstrative understanding to, "...depend on things which are true and primitive and immediate and more familiar than and prior to and explanatory of the conclusion" (*APo* 71b21-22). Bäck argues that Aristotle's modal logic should be used to understand Aristotle's general investigation into the essences of things. This is controversial for a couple of reasons since, a) Aristotle does not seem to take up the modal logic in other works after dealing with it in Book I of the *Prior Analytics*, b) the necessity of demonstration of



which Aristotle refers in the *Posterior Analytics* may only refer to the necessity of valid inferences rather than the necessity of predicating something of a subject, and

c) Aristotle's modal logic has been viewed as inconsistent throughout most of the history of Western Philosophy, so if that is the logic upon which Aristotle's philosophy of science is to be understood, so much for Aristotle's philosophy of science. Jonathan Barnes advocates the view that Aristotle simply did not use his modal syllogistic in scientific investigations:

First, Book A does not contain a theory of scientific methodology. Aristotle does not pretend to be offering guidance to the scientist—or, for that matter, to the historian or the philosopher—on how to best pursue his researches or how most efficiently to uncover new truth; nor, of course, did Aristotle attempt to carry out his own scientific researches in accordance with the canons of the *Analytics* (xii).

In fact, one does not find that Aristotle employs the modal syllogistic outside of the *Prior Analytics*. However, Aristotle does state that the subject of the *Prior Analytics* is demonstration, connecting it specifically with science:

Science and demonstrative deductions are not concerned with things which are indefinite, because the middle term is uncertain; but they are concerned with things that are natural, and as a rule arguments and inquiries are made about things which are possible in this sense. Deductions indeed can be made about the former, but it is unusual at any rate to inquire about them (*APr.* 32b18-22).

Despite this, Barnes argues that Aristotle has no intention of using the modal syllogistic in his scientific program:

Aristotle is indeed clear that all the propositions involved in a paradigm demonstration will be necessary; and it is also true... that some of his remarks might lead us to think that *modal* syllogistic is the logic of demonstrative reasoning, so that the propositions appearing in a paradigm demonstration will have the form '*Necessarily P*'. But Aristotle nowhere says that this is his view, nor do any of his illustrative examples contain an explicit modal operator (xxi).

For Aristotle, "necessarily" is not a modal operator, but a way to modify the copula. One reason why Aristotle may not have stated his examples in explicitly modal terms is that it is sufficient to refer to them as "demonstrations" to indicate that the

conclusion is established with apodictic force. So, in what sense is “necessarily” being used throughout Aristotle’s *Analytics*? Perhaps “necessarily” refers to the force of the valid inference itself rather than the modality of the premises. According to Aristotle,

...a deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by this last phrase that it follows because of them, and by this, that no further term is required from without in order to make the consequence necessary (*APr.* 24b19-23).

If “necessarily” only refers to deduction in the *Posterior Analytic*, then the distinction between demonstration and deduction breaks down and all deductions are demonstrations. But Aristotle is quite explicit that, “...deduction is more general: a demonstration is a sort of deduction, but not every deduction is a demonstration” (*APr.* 25b26-30).

Aristotle’s discussion of demonstration, or ἀπόδειξις, can be confusing since he sometimes claims to have provided an ἀπόδειξις of an assertoric deduction (see *APr.* 29b11). Thus, it seems as though an ἀπόδειξις need not have apodictic premises, but need only conclude with necessity. Rather, Aristotle views his deduction of various valid assertoric syllogisms as demonstrative, even though he does not take the time to explicitly modalize the premises. Aristotle is clear in the *Prior Analytics* that demonstrative premises are assumed on the basis of the first principles of a given science (*APr.* 24a29). Further, he affirms in the *Posterior Analytics*, “Demonstration, therefore, is deduction from what is necessary. We must, therefore grasp on what things and what sort of things demonstrations depends” (*APo.* 73a23-24). Ferejohn takes this to mean that the premises of scientific demonstrations must be necessary as well (1991, 68). In the *Posterior Analytics*,

Aristotle describes demonstration as a sort of understanding based on what is necessary. Aristotle writes,

That the deduction must depend on necessities is evident from this too: if, when there is a demonstration, a man who has not got an account of the reason why does not have understanding, and if it might be that *A* belongs to *C* from necessity but that *B*, the middle term through which it was demonstrated, does not hold from necessity, then he does not know the reason why. For this is not so because of the middle term; for it is possible for that not to be the case, whereas the conclusion is necessary (*APo.* 74b27-33).

In other words, Aristotle requires that if *A* belongs to *C* by necessity, then there is at least one apodictic premise whereby *A* or *C* is connected to *B* by necessity. There is little doubt that Aristotle intends us to use modal reasoning when using demonstration. And through demonstration, we should come to know the reason why.

Malink points out that Aristotle suggests an implicit modal feature of scientific knowledge when he says:

It is also evident from this that it is not possible to opine and to know the same thing at the same time. For one would at the same time hold the belief that the same thing can be otherwise and cannot be otherwise, which is not possible (*APo.* 89a38-b1).

However, Malink notes that Aristotle believes that demonstration must come through a middle term “that is necessary too” (*APo.* 87b23).

That the deduction must depend on necessities is evident from this too; if, when there is a demonstration, a man who has not got an account of the reason why does not have understanding, and if it might be that *A* belongs to *C* from necessity but that *B*, the middle term through which it was demonstrated, does not hold from necessity, then he does not know the reason why. For this is not so because of the middle term; for it is possible for that not to be the case, whereas the conclusion is necessary” (*APo.* 74b27-32).

So it seems that both premises must be necessary for a deduction to be a scientific demonstration. However, in the same chapter, it seems as though Aristotle directly affirms Barbara-LXL as a valid demonstration:

...nothing prevents the middle term through which it was proved from being non-necessary’ for one can deduce a necessity from a non-necessity, just as one can deduce a truth from non-truths. But when the middle term is from necessity, the conclusion too is

necessary, just as from truths it is always true; for let  $A$  be said of  $B$  from necessity, and this of  $C$ —then that  $A$  belongs to  $C$  is also necessary (*APo.* 75a1-8).

If Aristotle is saying “and this of  $C$ ” assertorically, then that just is Barbara-LXL. But then why say that if man is to have demonstrative understanding, the demonstration must be through a middle term that is necessary too (*APo.* 75a11-13)?

Malink’s solution to this is to suggest the following, namely that if  $Aa_LB$  is true, then  $B$  is what has come to be known as a *per se<sub>3</sub>* term, that is, they are terms that are not said of any other underlying subject but are “in themselves,” that is a “this” or a τὸδε τι (see Malink 2014, 7; *APo.* 73b5-10).<sup>28</sup> Thus, given that  $B$  is a *per se<sub>3</sub>* term, if it is assertorically predicated of some other subject, that implies that it is also apodictically predicated of it. Effectively, if  $Aa_LB$ , then  $B$  is *per se<sub>3</sub>*. If  $B$  is *per se<sub>3</sub>* and  $Ba_XC$ , then  $Ba_LC$ . This means that Barbara-LXL effectively entails Barbara-LLL. Malink turns to Posterior Analytics 1.22 to argue that “[t]hat which signifies substance signifies just what or just a subspecies of what it of which it is predicated” (Malink 2014, 8). Of course, if a substance term is predicated synonymously or of sub-species, it can belong apodictically. In other words, the middle term is predicated necessarily, whether one explicitly realizes it or not. The mixed modal syllogisms merely show that one need not be aware of this to arrive at valid apodictic conclusions. Malink argues that if i)  $Aa_LB$  is true, then  $B$  is a *per se<sub>3</sub>* term, and ii) given that  $B$  a *per se<sub>3</sub>* term, if  $Ba_XC$  is true,  $Ba_LC$  is true.

Another possible interpretation is that the mixed-modal syllogisms are intrinsic to scientific inquiry, but they are not scientific demonstrations.

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<sup>28</sup> For a full account of the various kinds of *per se* terms, i.e. *per se<sub>1</sub>*- *per se<sub>4</sub>* see Ferejohn (1991).

Nonetheless, their use is in deriving apodictic conclusions that can be used as premises in other syllogisms that are scientific demonstrations. This means that one can bootstrap oneself into scientific knowledge even if one is starting from a position that is less than scientific.

If Malink is correct, we might be able to offer support to Williams and Charles's analysis of definition by way of Aristotle's modal syllogistic. The modal syllogistic is connected to demonstration rather than definition. But Aristotle has a good deal to say about how the two are related to one another. As noted earlier, Aristotle specifies three sorts of definitions in the *Posterior Analytics* (Ross 1957, 634).<sup>29</sup> The first type is a nominal definition, which is "...an account of what the name, or a different name-like account, signifies—e.g. what triangle signifies" (*APo.* 93b29-31). Aristotle specifies that the other sort of definition,

...is an account which makes clear why a thing is. Hence the former type of definition signifies but does not prove, whereas the latter evidently will be a sort of demonstration of what a thing is, differing in position from the demonstration (*APo.* 93b37-94a1).

What is clear is that not all definitions are demonstrative, but at least some are. And those that are demonstrative are the ones that do explanatory work as well. In *Posterior Analytics* 11 Aristotle discusses four kinds of explanation and specifies that these are "proved through the middle term" (*APo.* 94a24). Ross identifies the four sorts of explanation as "...the essence, the conditions that necessitate consequent, the efficient cause, the final cause" (1957, 637). So, if we are to discuss explanation and definition in terms of demonstration, essences, necessary causes and consequences, we ought to use the modal syllogisms. Aristotle is quite clear that

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<sup>29</sup> Ross notes that it initially seems like the four parts of the discussion correspond to four types of definitions, but says that Aristotle's first and third kinds of definition are nominal, the second is causal, and fourth is really a substantial definition.

this will be done through the middle term, which is yet another reason to consider housing our exposition of essential definitions within the modal syllogistic given its use of class terms for the middle terms.

In Aristotle's definition of "thunder," we find that the definition follows as the conclusion of a demonstration where it is proved that thunder is a noise in the clouds. To capture the argument in a modal syllogism, we might say that  $A$  is 'thunder',  $B$  is 'noise in the clouds', and  $C$  is 'extinction of fire in the clouds'. So  $A$  is linked to  $B$  through  $C$ , the middle term, which is an efficient cause of  $B$  belonging to all  $A$ . For the demonstration to obtain, we know that, at the very least,  $B$  must necessarily belong to all  $C$ . So long as  $C$  belongs to all  $A$ , we have a demonstrative and necessary conclusion. Thus, it turns out that Barbara-LXL is very special, indeed. It is both the only syllogism to conclude to a universal affirmative, and also does so through a non-apodictic minor premise. We can begin with a non-essential definition of "thunder" and, through explanation, arrive at an essential definition:

- (i) the extinguishing of fire in the clouds necessarily belongs to all noises in the clouds,
- (ii) A noise in the clouds belongs to all thunder,

hence,

- (iii) the extinguishing of fire in the clouds necessarily belongs to all thunder.

So, whether or not thunder is necessarily a noise in the clouds, we can derive that it is necessarily an extinguishing of fire in the clouds. If this is the correct analysis, what we see is the ability to transition from nominal definitions to essential

definitions by way of explanation, or more specifically, through a scientific understanding of causality.<sup>30</sup>

Necessity emerges in the account of essences insofar as necessity is used to explain the causal connection, in this case, between the quenching of fire and a noise, and the definition. We begin by defining what the word “thunder” means, and then when we identify an instance of that nominal definition, we can analyze the cause and eventually reveal the essence of a thing. Barbara-LXL, therefore, is helpful for moving from something that may be universally affirmed, but not essential, in the minor premise to an essential conclusion so long as the major premise is apodictic. Hence Aristotle says,

...[I]n cases in which we know accidentally that a thing is, necessarily we have to hold on what it is; for we do not even know that it is, and to seek what it is without grasping that it is, is to seek nothing. But in cases where we grasp something, it is easier. Hence in so far as we grasp that it is, to that extent we also have some hold on what it is (*APo.* 93a25-29).

In the case of thunder, we may have some accidental knowledge that it is some sort of noise in the clouds, an essential demonstration occurs when it is known that ‘noise in the cloud’ is necessarily caused by ‘extinguishing of fire in the cloud’. According to Goldin (1996, 106) the examples found in *Posterior Analytics* 2.8, including the preceding syllogism on thunder, are commonly thought to be demonstration not of essences, but of material definitions. This interpretation has its roots in Philoponus, but remains the dominant view today. In contrast, Goldin argues that Aristotle is concerned with demonstration of essences. The idea that

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<sup>30</sup> It is admittedly controversial to identify this passage as a demonstration given that there does not appear to be anything substantial to thunder in the proper sense. Nonetheless, some commentators, e.g. Ross (1957, 535) says that definitions can be recast as demonstrations, and specifically cites the “causal definition” of thunder as a recasting of a demonstration ‘Where fire is quenched there is noise, Fire is quenched in clouds. Therefore there is noise in the clouds’.

essences could be demonstrated in this way seems at odds with the fact that the demonstration proceeds through some non-essential feature of the natural kind in question. That is, what thunder is, essentially, is a noise in the clouds. The quenching of fire explains why there is a noise in the clouds. But for the demonstration to move from explanation to essential definition, we must know an essential property of the explanans, even if the explanans is καθ' αὐτό συμβεβηκότα relative to the definiendum. According to Ross,

...[s]ince a definition is either a premiss (i.e. a minor premiss defining one of the subjects of the science in question), it a demonstration recast, or a conclusion of demonstration, it must be a universal proposition defining not an individual thing but a species (1957, 535).

So when Aristotle references thunder in the *Posterior Analytics*, he is referring to a species and not an individual or even a collection of individual observed. To the contemporary mind unfamiliar with Aristotle's modal syllogistic, it seems bizarre that an essential definition could be formed from accidental features one has observed. But a proper understanding of the modal syllogistic not only properly frames Aristotle's endeavors in forging explanatory definitions, but reveals an underlying method that transitions from empirical perceptions of non-essential καθ' αὐτό properties to an intellectual grasp of the natural kind itself. What is required is explanatory knowledge of the καθ' αὐτό property that is apodictically predicated of the middle term in a demonstration. The conclusion just is an explanatory definition of the second variety identified by Aristotle in the *Posterior Analytics*.

The Master Craftsman argument that Charles and Williams present can be reconstituted as an Aristotelian syllogism with the following form:



- (22) F necessarily belongs to all G (Causal Exp)
- (23) G belongs to all K (Nominal Def)
- (24) F necessarily belongs to all K (22, 23 Barbara-LXL)

In this argument (24) is analogous Williams' and Charles' (9). In my exposition of their argument, I found (8) to be unmotivated. To recall, (8) states  $(\forall x)(Kx \rightarrow \Box Fx)$ , so it is the *de re* claim that if x is kind K, then necessarily x has property F. If there task was to show us how explanation and definition can give rise to genuine knowledge about the sorts of properties various kinds have, much of the heavy lifting is done here. The explanatory and definitional premises remain *de dicto*, so it is (8) that does most of the explanatory work by providing the bridge needed to reach a *de re* conclusion. By using Aristotle's modal syllogistic, we can avoid the *de re/de dicto* distinction, a distinction which is anachronistic to Aristotle's work. So we are able to arrive at how even a non-modal definitional claim can be combined with a modal explanatory claim to derive essential properties. One could use mixed modal syllogisms to lay the groundwork for necessary premises that can be taken up into new scientific, i.e. demonstrative, arguments.

One can parallel Williams and Charles another way by strengthening (23) into an essential definition. Nothing prevents me from deriving essential properties in (24). Through this can arrive at the conclusion via Barbara-LXL, combining a nominal definition with a modal explanation, or by Barbara-LLL, which combines an essential definition and explanation, depending on how confident I am in my scientific findings. Both modal syllogisms are valid according to Aristotle. So we have:

- (25) G belongs necessarily to all K (Ess).

We should note, however, that Williams and Charles offered two definitions of essence, (10)  $\Box(\forall x)(Kx \rightarrow \Box Kx)$  (Ess), and (11)  $\Box(\forall x)(Gx \rightarrow \Box Gx)$  (Ess\*). It is as if, in recognizing the implicit bifurcation between individuals and properties, Williams and Charles have doubled down on the necessity operator as a way to really glue an essence to its instantiation. They ask, “Indeed, outside of the realm of pure logic, why does any object *have to be* anything at all? What is to prevent anything from being anything?” (Williams and Charles 2013, 134). But this is precisely the problem that is raised by the semantics of possible worlds, of which Charles has warned us. Why is it that something that is a K is necessarily K? The answer is to say that the proposition is necessary itself. Consider the common example used to highlight the *de re/de dicto* distinction, bachelor. We know that it is false to say “all bachelors are necessarily unmarried males” because we suppose that a bachelor could become married. For the Aristotelian, this is because “bachelor” is accidental to the substance in question, and the essence of that substance is “human” or “rational animal”. An Aristotelian would have less of a problem with *de re* statements like “all humans are necessarily rational animals”. Williams and Charles are offering a claim that is both *de re* and *de dicto*; i.e. “necessarily if x is a human, x is necessarily a human”. Now suppose you want to derive an essential property from this claim. For instance, we might say “necessarily, if x is a human, x is rational”. Williams and Charles claim to conclude “necessarily, if x is a human, x is necessarily rational” on a fairly modest system of modal logic, namely K4. Hence, they can show how Ess and Exp can explain why certain kinds have the properties that they have.

However, given that and quantifiers are involved, it will be necessary to invoke the Barcan Formula and the Converse of the Barcan Formula, which is precisely why actualism emerges as an assumption in Williams and Charles' argument. Though they do not present the validity of the argument explicitly, I would reconstruct it as follows, where H is human, R is rational:

- (25)  $\Box(\forall x)(Hx \rightarrow \Box Hx)$  (premise)
- (26)  $\Box(\forall x)(Hx \rightarrow Rx)$  (premise)
- (27)  $(\forall x)\Box(Hx \rightarrow \Box Hx)$  (25 CBF)
- (28)  $\Box(Hu \rightarrow \Box Hu)$  (27 UI)
- (29)  $\Box Hu \rightarrow \Box\Box Hu$  (28  $\Box$ Dist)
- (30)  $(\forall x)\Box(Hx \rightarrow Rx)$  (26 CBF)
- (31)  $\Box(Hu \rightarrow Ru)$  (30 UI)
- (32)  $\Box\Box(Hu \rightarrow Ru)$  (31 Axiom 4)
- (33)  $\Box(\Box Hu \rightarrow \Box Ru)$  (32  $\Box$ Dist)
- (34)  $\Box\Box Hu \rightarrow \Box\Box Ru$  (33  $\Box$ Dist)
- (35)  $\Box Hu \rightarrow \Box\Box Ru$  (29, 34 HS)
- (36)  $\Box(Hu \rightarrow \Box Ru)$  (35  $\Box$ Dist)
- (37)  $(\forall x)\Box(Hx \rightarrow \Box Rx)$  (36 UG)
- (38)  $\Box(\forall x)(Hx \rightarrow \Box Rx)$  (37 BF)

So, indeed, it is possible to derive (8) from (Ess) and (Exp). Nonetheless, there is still the question of whether we must invoke the semantics of worlds here. If the above argument is what Williams and Charles have in mind, then they must rely on BF and CBF, which is controversial. Indeed, BF and CBF are proved invalid on Kripke's formal semantics of first-order modal logic (Williamson Forthcoming, 4). Furthermore, if actualism is implied by the argument, we need to assess whether this actualism is consistent with Aristotle's metaphysics. Some forms of actualism have been enthusiastically dubbed "Aristotelian actualism" insofar as the possible is ontologically dependent upon actual individuals (see Fitch 1996, 68). G.W. Fitch describes Aristotelian actualism as one which, "...takes very seriously the idea that

the only things that exist are basic actual objects and things composed of actual objects" (*ibid.* 57). We should consider, then, how Aristotelian modal metaphysics might inform our understanding of Barcan's formula and whether the sort of actualism it entails is untoward or controversial (see, for instance, Menzel 2015).

Williams and Charles are not the first to rely on Barcan's Formula to unpack key Aristotelian insights. Ebert and Nortmann (2007) also double up on modal operators so as to make Aristotelian apodictic propositions simultaneously *de dicto* and *de re*. Rini (2011, 56) worries that the Ebert and Nortmann's use of Barcan's Formula in proving EL-conversions involves making use of tools to which Aristotle did not have access. It is interesting, though, that the closer one is able to approximate Aristotle's modal syllogistic in terms of contemporary quantified modal logic, the need for Barcan's formula arises, and the question of its implications for actualism are raised. Indeed, Aristotle is an actualist.

I would go so far as his metaphysics is implicitly in line with the sort of thesis that falls out of the Barcan Formula, so long as we do not allow the syntactical differences between Aristotle and contemporary quantified modal logic to be a stumbling block. Suppose, for instance, in  $\Box(\forall x)(Hx \rightarrow \Box Rx)$ ,  $x$  is a variable that ranges over Aristotle's primary substance. While a value for  $x$  can be named, it is neither said of nor in anything else, but things can be said of it. If so, we might be able to make sense of the implications of the Barcan Formula, namely that the possibility that there exists an  $x$  that is  $F$  implies that there actually exists an  $x$  that is possibly, or has the potential to be  $F$ . This is the Aristotelian insight that potentials are housed, so to speak, in that which actually exists. Now, some actualists take this

to mean that the possibility that I have a biological brother, which I actually do not, means that there actually exists some  $x$  that has the property of possibly being my brother. What  $x$  could satisfy this demand? Certainly no presently existing person could have the parents they have and be my biological brother. Still, the Aristotelian would insist that the possibility of Human actually exists in presently existing humans. That is to say that the possibility of the instantiation of the kind-term "Human" exists in humans.

Whether this comports with the sort of actualism implied by the Barcan Formula is far from clear. That is, Aristotle might be an actualist insofar as he would reject the possibilism of Lewis. After all, it would be difficult to understand what potencies and contingencies were if all possibilities are equally real and only called actual insofar as certain possibilities obtain in a world designated as actual. However, it is doubtful that Aristotle would explain the actuality of modal truths in terms of maximal descriptions of worlds, sets, etc. His actualism is grounded in the act/potency distinction itself. For Aristotle, actuality is prior to potentiality, in time, in substance, and in the sense that the perishable proceeds from the imperishable, necessary, and eternal (see *Meta.*  $\Theta.8$ ). As Aristotle puts it, "For from the potential the actual is always produced by an actual thing, e.g. man by man, musician by musician; there is always a first mover, and the mover already exists actually" (*Meta.* 1049b24-29). Specifically, with respect to priority in substance, Aristotle holds that the substance or form is actuality, which is always prior, or as he says, "...matter exists in a potential state, just because it may attain to its form; and when it exists *actually*, then it is in its form" (*Meta.* 1050a15-15). So, whatever potentials

exist, do so insofar as their existence is grounded in what is actually existing, be it prior existing individuals of the species, the formal aspect of the substance, or the divine. Such things would account for the actual existence of these potentials. The potential, then, for Horse, is found among horses, the natural kind, Horse, and the eternal first causes by which all else is possible. But note that such this is still to talk of an actualism with respect to kind-terms rather than individual entities. So, the sorts of puzzles that arise in contemporary metaphysics of modality, e.g. the possible son of Wittgenstein, does not have to have its existence in some particular thing, would not arise for Aristotle.

Insofar as these concerns can be circumvented by adopting the Aristotelian modal syllogistic where, for the most part, terms represent kinds, the question arises as to how one can transition from a pre-theoretical nominal definition to an essential definition about the natural kinds that actually exist. Suppose one has a working definition that captures the relevant kinds in question. To devise an essential definition, one need not merely identify some property that necessarily belongs to some kind-term, but that the property must be the cause of the existence of the kind-term. Moreover, it is not enough to simply induce from constant conjunction, through some Humean analysis, that kind K of essence G always has property F and place a modal operator on the claim. The cause must explain must bring about, or actualize, the substance in question. For Williams and Charles, Aristotle must make essential claims that are simultaneously *de dicto* and *de re*, but for Aristotle, such a distinction makes little sense. Moreover, to require both

contexts is *ad hoc* in the sense that, while it allows the inference to go through, it does not seem well motivated by Aristotle's metaphysics itself.

While Williams and Charles seek to express Aristotle's modal claims as simultaneously *de re* and *de dicto*, Malink's interpretation is simultaneously *de dicto*-like and *de re*-like without being either (2013, 139). While *de re* claims tell us that some individual has a property necessarily, Aristotle's modal propositions tell us that a predicate term belongs to a subject term necessarily. So those individuals that exemplify the subject term will relate to the predicate term by necessity as well. In this sense, it is a modal statement that relates to *res* but not insofar as an individual exemplifies its properties, but insofar as an individual exemplifies a given term and so must, or must not, exemplify other terms. According to Malink,

Instead of the scope of sentential necessity operators, the scale between *de dicto*-likeness and *de re*-likeness is determined by the degree to which non-atomic terms are relevant for the truth-conditions of propositions. If only atomic terms (the extensions of terms) are taken into account, we obtain a *de re*-like reading without analytic *de dicto* components. If all proper parts of the subject term - not the subject term itself - are taken into account, we obtain an intermediate reading with some more *de dicto* components. Finally, if all improper parts of the subject term - including the subject term itself - are taken into account, we obtain the full strength of combined *de re*-likeness and *de dicto*-likeness. In a similar way, the reverse direction yields a scale whose starting point is, again, a combined *de re* and *de dicto*-like reading (Malink 2006b, 6).

It is worth noting that this full-strength is precisely what Williams and Charles can appeal to in their argument through (10) or (11) above. That is,  $\Box(\forall x)(Kx \rightarrow \Box Fx)$  can be derived from  $\Box(\forall x)(Kx \leftrightarrow Gx)$  (Def) and  $\Box(\forall x)(Gx \rightarrow Fx)$  (Exp), along with either  $\Box(\forall x)(Kx \rightarrow \Box Kx)$  (Ess) or (11)  $\Box(\forall x)(Gx \rightarrow \Box Gx)$  (Ess\*). If we were to establish parallel principles of Ess or Ess\* according to Malink's heterodox interpretation, the resulting proposition would have the full strength of being both *de re*-like and *de dicto*-like insofar as the subject-term would have to

included as an improper part of the expression . So, Williams and Charles may have the right intuition, but in translating Ess and Ess\* into first-order logic, they raise the actualism concerns we have just mentioned.

### 3.4 Knowledge of Essences

The question remains as to how we can arrive at *de re-like* knowledge of essences and explanations when building up a science. If Ess or Ess\* are going to play a crucial role in deriving further apodictic knowledge, and advancing a science, we will need some account for how knowledge of essences is gained. It is by induction, *voũç*, or some combination thereof? Aristotle discusses *voũç* in the *Posterior Analytics* within the context of explaining how indemonstrable principles can be known. James Lesher identifies the role that *voũç* plays in the following passage,

...neither is all understanding demonstrative, but in the case of the immediate it is non-demonstrable—and that this is necessary is evident; for if it is necessary to understand the things which are prior and on which the demonstration depends, and it comes to a stop at some time, it is necessary for these immediate to be non-demonstrable. So as to that we argue thus; and we also say that there is not only understanding but also some principle of understanding by which we become familiar with definitions (*APo.* 72b19-24).

Though *voũç* is not directly named in the passage, Lesher makes the connection by pointing out that it is later named as the source of scientific knowledge in *Posterior Analytics* 88b36 (Lesher 1973, 52). If Lesher's connection is correct, *voũç* would be relevant to the question of definitions, explanation, and ultimately to the foundation of demonstrative knowledge through the modal syllogistic. Yet, what *voũç* is exactly and how it contributes to our knowledge is a notoriously difficult question.



To answer that question, Leshner examines passages like *Posterior Analytics* 100a3-9 where Aristotle explains the process by which scientific understanding comes about. We proceed from sense-perception to memory and from memory to experience. Out of this experience arises the skill of the craftsman and the knowledge of the scientist. It is this passage which Williams and Charles allude to when they discuss the master craftsmen in contrast to the experimentalists. This suggests that, for Aristotle, νοῦς is connected to a process of induction. However, one might object to Aristotle's reliance on induction, if it is to be the source of demonstrative ἐπιστήμη. The modern reader of Aristotle may be concerned that would seem to have the effect of weakening the strength of demonstration, which relies on premises that are necessarily true. This sort of epistemological problem runs to the core of science itself, in that no empirical premise can be known with absolute certainty. The necessity, then, which these scientific demonstrations enjoy, should not be understood as logically necessary and epistemologically certain, but necessary given what one has reason to believe is true. Indeed Aristotle thought that, "...it is necessary for us to become familiar with the primitives by induction; for perception too instills the universal in this way" (*APo.* 100b4-5). A crucial aspect in all of this appears to be whether there is a middle term.

...for where there is a middle term the deduction proceeds through the middle term; when there is no middle term, through induction. And in a way induction is opposed to deduction; for the latter proves the extreme to belong to the third term by means of the middle, the former proves the extreme to belong to the middle by means of the third (*APr.* 68b31-35).

Likewise, νοῦς is a grasp of an unmiddled premise, which provides a starting point or first principle for the scientific demonstration. Leshner believes νοῦς and induction are "...complementary aspects of the same activity" (Leshner 1973, 62). If

this is so, then what differentiates them? Lesher raises a couple of possibilities: 1) that νοῦς is a final act of insight by which a general principle is grasped after and induction, or 2) νοῦς validates or sanctions inductive inferences. Lesher does not think the answer is in any way clear (*ibid.* 59). The key may be that νοῦς is connected in a special way to definitions, explanation, and first principles.

What I am suggesting is that νόησις, the mental act of contemplative thought, or grasping, is not merely the act of sanctioning a separate inductive inference. Rather, our understanding of essences will ultimately be grounded not in an inductive leap from particulars, but will be grasped through a combination of self-evident first principles, postulates, and definitions lying at the foundation of a given science.

Utilizing modal syllogisms in coming to new knowledge is a return to the science of essences, which, for Tuomas Tahko, is nothing less than a return to first philosophy, as Aristotle envisioned it. Tahko argues for a neo-Aristotelian position that he believes is faithful to Aristotle, but inspired by contemporary accounts of essences developed by Fine and Lowe. His central thesis is that *essence precedes existence* (2013, 56). For Tahko this is both an ontological and epistemic claim. He states that the ontological priority of essence to existence is not to claim that essences are the existential ground of an entity, for an entity might be ontological dependent on many other things. Rather, he thinks of essence as “...a statement of what the being of the entity consists in; its existence, identity, and persistence conditions” (*ibid.* 57). Agreeing with Kit Fine, Tahko takes an essential property to be a sufficient, but not a necessary, condition for being a necessary property. Some

necessary properties do not directly answer the “what is it” question, and so are less fundamental than the essential *propria* are. So, truths about essences have ontological priority over non-essential modal truths. Tahko also makes the terse remark that it is “...important to note that on this view, we should *not* reduce essence to *de re* modal properties” (2013, 58). This warning fits well with our thesis, as Aristotle’s modalism does not make such a reduction.

Tahko’s view is an important springboard in another regard. It suggests how a first philosophy, in which essences have primacy, can integrate with and aid the natural sciences. It is in this regard that we can connect back to our discussion of *voũç* as a capacity to grasp first principles and necessary truths and the implications those principles will have for the kinds of entities the science anticipates to discover. Tahko considers the example of the recent discovery of the Higgs boson. In searching for a particle that fulfills the explanatory role needed, a range of essential natures were already available for consideration by theoretical physicists. The question that two independent teams of researchers were trying to answer was not “what kind of essences could fulfill this explanatory role,” for the theoretical physicists had already done this. The question was which possible essence actually exists. Thus Tahko writes:

The genuine, actual essences must be determined with the help of empirical evidence. Notice that I do not say *discovered*, because the role of the empirical work is merely to confirm which of the candidate essences are genuine. Hence, the problem of *propria* will ultimately be addressed by empirical science, but not without prior study of candidate essences (2013, 63).

If Tahko is correct, the theoretical physicist can utilize a metaphysical mode of contemplation when considering what sort of thing could do the explanatory work

needed., With a range of explanatory hypotheses available, i.e. we are looking for a sort of entity that has the following features, the experimental scientist can confirm which, among the range, happen to exist in actuality. Finally, the experimentalist can then fill in which *propria* are entailed by the essence. The range of essences is that which fulfills a certain explanatory role, and each has their own definition.

Observation and induction helps us to know which particle actually exists, but it is a grasp of the first principles of particle physics and mathematics that lead to a limited range of options available. In that sense, one could argue that contemporary science still engages *voũç* by theorizing about the sorts of beings that are possible and would do explanatory work given the principles of a science.

Moreover, we can see how this sort of process aids the scientist in overcoming Meno's debater's paradox. To recall, Plato presents the paradox as follows:

MENO: How will you look for it, Socrates, when you do not know at all what it is? How will you aim to search for something you do not know at all? If you should meet with it, how will you know that this is the thing you did not know?  
 SOCRATES: I know what you want to say, Meno. Do you realize what a debater's argument you are bringing up, that a man cannot search either for what he knows or for what he does not know? He cannot search for what he knows—since he knows it, there is no need to search - nor for what he does not know, for he does not know what to look for (Meno, 80d-e)

Indeed, Aristotle directly references this paradox at the outset of the *Posterior Analytics*, using the example of knowing that a triangle has angles equal to two right angles:

Before the induction or before getting a deduction, you should perhaps be said to understand in a way—but in another way not. For if you did not know if it is *simpliciter*, how did you know it has two right angles *simpliciter*? But it is clear that you understand it in *this* sense—that you understand it universally—but you do not understand it *simpliciter*. (Otherwise the puzzle in the Meno will result; for you will learn either nothing or what you know.) (*APo.* 71a25-30).

What the theoretical physicist understands is a range of universal essences, but she does not know which one exists in reality.

What we have seen, then, in this chapter, is that Aristotle's modal syllogistic can be adapted to scientific demonstrations, which was his intention. The modal syllogistic avoids the problems of the *de re/de dicto* dichotomy by offering features of both while avoiding the problems of referential opacity, and perplexing questions raised by actualism. Crucially, Aristotle's modality is dealing primarily with class terms rather than individuals. When it deals with individuals, it cannot treat them as the middle term in some demonstration, which limits their appearance in the syllogism.

Aristotle's modality can help us to understand the way in which definition, explanation, and essence are interconnected. Indeed, it is because Aristotle allows for mixed modal syllogisms, where an apodictic conclusion can be reached when only one premise is apodictic, that the transition from mere nominal definitions to demonstrable knowledge can occur. We can then see how  $\nu\omicron\upsilon\varsigma$  interplays with demonstration to arrive at definitions, explanations and new scientific knowledge. The grasp of essences and explanations play a crucial role along the way. In the coming chapter we will consider how Aristotle's essentialism avoids some of the pitfalls that contemporary modalism is susceptible. But already, we have a keen idea of how an Aristotelian science can utilize the modal syllogistic in fruitful ways.

For Aristotle's modal syllogistic to be applicable both to interpretations of Aristotelian science as it was described in biological works like *Parts of Animals* and to contemporary scientific methodology, more must be said about the nature of

necessity and its connection to essences. We must also explore the various senses of necessity used by Aristotle and whether it can be applied to the nature of essences and the modal syllogistic. There are three notions of necessity found in *Parts of Animals* salient to our discussion of modality. Also, we will build upon Katherine Koslicki's case studies on Aristotle's biological works and the question of essence and necessity. Finally, we will return to the Tuomas Tahko's claim that something like Aristotelian essences can play a role in contemporary science.

There appears, in *Parts of Animals*, three distinct notions of necessity. Allan Bäck identifies them as: 1) absolute necessity, 2) material necessity, and 3) teleological, or hypothetical necessity.<sup>31</sup> In support of the first mode, absolute necessity, Bäck has in mind the sort of necessity that follows from Aristotelian definitions.

The conclusions of demonstration are necessary. They derive from universal truths, which hold always, and not from those particular truths that hold only sometimes, and so they are not limited by the factual circumstances of particular individuals (Bäck 1995, 96).

In support of this, Bäck cites *Categories* 13b15-18, in which certain propositions about particular individuals cannot be necessarily true, since the individual can cease to exist. Bäck's example is "Socrates has a sense of humor," which is not absolutely necessary, since it is not necessary that there is a Socrates. However, "man has a sense of humor" is true insofar as it follows from the essential definition

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<sup>31</sup> Bäck cites the following passage for support (1995, 90): "For many things are produced, simply as the results of necessity. It may, however, be asked, of what modes of necessity are we speaking when we say this. For it can be neither of the two modes which are set forth in the philosophical treatises. There is, however, the third mode, in such things at any rate are generated. For instance, we say that food is necessary in neither of the two modes, but because an animal cannot possibly do without it. This third mode is what may be called hypothetical necessity. For if a piece of wood is to be split with an axe, the axe must of necessity be made of bronze or iron (*PA* 642a1-11).

of “man” (*ibid.*). So this necessity, we might say, is tied to the *secondary substance* or essential nature and what is said of it with respect to various predicates.

The second sort of necessity, material necessity, which Bäck sees as a kind of conditional necessity, though not in the same senses as hypothetical necessity is said to be conditional. “In general, the second mode concerns those necessities based upon present, contingent material circumstances and their causal implications” (Bäck 1995, 95). This necessity is sometimes referred to as Democritean necessity in that future states of affairs are inferred to be necessary given previous materials states. Aristotle is critical of focusing on Democritean necessity to the exclusion of other types of necessity, but he does accept it (Cooper 1987, 259). Cooper cites the *Generation of Animals* where Aristotle discusses and distinguishes between types of necessities that pertain to the shedding and replacement of front teeth in some animals.

Once [the front teeth] are formed, they fall out on the one hand for the sake of the better, because what is sharp quickly gets blunted, so that [the animal] must get other new ones to do the work [tearing food off];... on the other hand they fall out from necessity, because of the roots of the front teeth are in a thin part [of the jaw], so that they are weak and easily work loose... Democritus, however, neglecting to mention that for the sake of which [things happen in the course of nature], refers to necessity all the things that nature uses – things that are necessitated in that way, but that does not mean that they are not for the sake of something, and for the sake of what is better in each case. So nothing prevents [the front teeth] from... falling out in the way he says, but it is not on account of those factors (*dia tauta*) that they do, but on account of the end (*dia to telos*): they are causes as sources of motion and the instruments and matter (Cooper 1987, 258; *GA* 5.8 789a8-789b8)

So we see that there is a necessity that the teeth will fall out because of the material weakness of the jaw and the thinness of the front teeth. Those represent Democritean necessity.

The third necessity, hypothetical necessity, moves in the opposite direction as compared to Democritean necessity. That is, it reasons from some future aim that

will come about to the necessity of certain present conditions obtaining. In the previous example of sharp front teeth, it is noted that the need to tear food requires sharpness of teeth, and the sharpness of teeth is maintained by the replenishing of the front teeth. The Democritean necessity is complementary to the hypothetical necessity found in the telos. Bäck specifies that, "...to be hypothetical in this sense, a necessity 1) must follow from a hypothesis of a future goal of something already in existence and 2) must concern things needed as external means, or necessary conditions for realizing that goal (Bäck 1995, 93).

Hypothetical necessity, interestingly enough, gives ontological meat to the notion of potency. As Bäck explains, infant Socrates has the full substantial form of human, but the infant lacks certain abilities that adult humans have.

Aristotle seems to believe that potentialities have an inherent drive to become actual. [*Ph.* 192b13-5; 193b12-8] So the present existence of an individual thing at a particular time, with certain potentialities, serves as a ground for a certain state of affairs in the future. The postulation of a future state of affairs in a teleologically necessary claim is not a psychological product of wishful thinking, but rather something objectively grounded on potentialities that actually exist (Bäck 1994, 94).

The complementarity of hypothetical and Democritean necessities is found in this idea that present actualities, that is, whatever actual substance there is, determines the sort of potentials that will be realized, and those future actualities are not merely what we would like to happen, or what we think could happen. They are, in a sense, determined to be given what actually is the case now, and given that there will be no interruption to the normal course of events.

So we see that these modalities are rooted in substances, and more fundamentally in the act/potency distinction that underlies Aristotelian metaphysics. Bäck concludes that these are the three sorts of necessity operating in



Aristotle. The question arises, however, whether these sorts of notions of necessity can be accommodated by a singular modal syllogistic, as developed in the *Prior Analytics*. The initial concern, that Aristotle's syllogistic is too inconsistent to worry about adapting to scientific questions, has been mitigated by our analysis in Chapter One. Moreover, we have provided reason to think that the modal syllogistic is the proper the logic of scientific demonstration in the *Posterior Analytics*, giving reason to think that Aristotle had this intention in mind. Broader questions remain, however. For instance, are the modal syllogistic and the *Posterior Analytics* consistent with the method Aristotle actually employs in his biological works? Also, can this method carry over into contemporary contexts, especially with the renewed interest in neo-Aristotelianism and essentialism in the philosophy of science?

Contemporary modal logic often requires specifications and various axiomatic commitments contextualized by the sorts of necessity in question. For instance, one might adopt system S5 when talking about broadly logical or metaphysical questions, as some natural theologians do when they discuss modal ontological arguments for God's existence. However, when talking about physical necessity, it would be inappropriate to treat what is necessary in one possible world as transitive with another possible world, as one does in the S5 system. One might say that it is physically necessary that, for instance, given Newton's first law of motion  $\sum F = 0 \leftrightarrow \frac{dv}{dt} = 0$  a certain object will not spontaneously begin to move without some other object acting upon it, but it would be a rather strong claim to apply any particularly strong modal system to physical necessity claims. There may be a possible world where objects spontaneously start and stop moving. In fact, one

should be cautious to use even S4 to say that the physical necessity of some motion obeying Newton's first law is itself necessary, rather than contingent on a law that might very well be contingent in any broader grades of modality. But what of Aristotle's modal syllogistic? Can it accommodate different sorts of necessity? For even Aristotle admits that "necessary" can have many meanings.

Turning to *Metaphysics V*, one finds a seemingly disorganized list of senses that "necessary" can take. Aristotle provides five senses: 1) that, without which, as a condition, a thing cannot live, 2) the condition without which a good cannot be or come to be, 3) the compulsory and compulsion, 4) that which cannot be otherwise, and 5) demonstrations. Aristotle says that (4) is the primary sense of necessity, and, "...from this sense of necessity all the others are somehow derived" (*Meta.* 1015a34-35). Bäck reduces the five senses to his three categories of necessity by noting that the coming to be of something necessary fits with hypothetical necessity, the necessity that results from something is or is compelled comports to Democritean necessity. Thus, (1)-(3) are hybrid ways of expressing the forward-looking and backward-looking material and teleological necessity. Then there is (4) and (5). Are they to be collapsed into absolute necessity? One way to think of this might be that (4) is the absolute necessity of substances, and (5) is the absolute necessity of reason or logic when considering essences that, for Aristotle, is isomorphic with, and makes intelligible, the absolute necessity of substances. In collapsing the accounts of necessity to three, and unifying them, Bäck's account has certain advantages. As he notes,

My account does explain how the conclusions of demonstrations are necessary y, and how the primary assumptions of demonstrative science gain their necessity. My account also has

the merit of explaining why Aristotle never explicitly equates the necessary (in the first mode) with the essential (*kath'auto*) or with the always true (*kata pantos*), even though he seems to assume their coextension often enough (Bäck 1995, 104).

Bäck concludes that all three modes of necessity are relative, and can be seen as a sort of entailment relationship (see Bäck 1995, 104; fn. 33).

It seems odd, though, that (4) and (5) should be related in some way. On this point, Bäck seems to agree that it is somewhat odd. He writes,

...from a modern perspective, he runs together the logically necessary, like the principle of non-contradiction, and the physically necessary ('every horse is an animal') into his first mode of necessity. Indeed, he has little interest in the logically necessary anyways: his interest lies with the necessary features of those essences that manifest themselves in individual substances. Still necessary statements are necessary because the predicates follow from the essences of the subjects... (Bäck 1995, 106).

But logical necessity seems to be quite distinct. In fact, the conclusions of assertoric syllogisms are logically necessary, and this seems to threaten the distinction between modal and non-modal syllogisms. Here, I think we have to be careful to distinguish what it is that corresponds, in reality, to the logical necessity of valid non-modal syllogisms. It is not the material content that is necessary itself, but the form of the argument itself. Given Aristotle's isomorphism, there is some sense in which the form of the argument corresponds to a necessary reality. This does not, of course, have to take on a Platonic element, since we could very well say that Aristotle is describing, in the *Prior Analytics*, necessary forms of rational thought that exist in rational souls. "Actual knowledge," according to Aristotle, "is identical with its object..." (*De An.* 430a20, 431a1). So, given that logical reasoning is an object of knowledge, it is identical with our knowing it. So there is an absolute necessity in the logical form of the syllogism, which is identical to a real necessity in the mode of thought that exists in the intellect.

An argument from analogy offers tantalizing support for Bäck's position. We find in the *Metaphysics* the claim that Being is said in many ways. "There are many senses in which a thing may be said to 'be', but they are related to once central point, one definite kind of thing, and are not homonymous" (*Meta.* 1003a33-34). Of course, the thrust of the *Metaphysics* is that although Being is said in many ways, it is πρὸς ἓν, and so there is a primary sense of Being, which unifies all senses, and permits a science of being, metaphysics, and all other science. Necessity is also said in many ways, there is a sense that is πρὸς ἓν for all other senses. So, by analogy, if the unity of Being permits scientific inquiry into beings, the unity of necessity permits scientific knowledge of essences, accidents, necessity in causation, and necessity in reasoning. Aristotle argues:

... [T]here are many senses in which a thing is said to be, but all refer to one starting-point; some things are said to be because they are substances, others because they are affections of substance, other because they are a process towards substance, or of thing which are relative to substance, or negations of some of these things or of substance itself... It is clear then that it is the work of one science also to study the thing that are, *qua* being.—But everywhere science deals chiefly with that which is primary, and on which other things depend, and in virtue of which they get their names (*Meta.* 1003b5-18).

For Aristotle, the unity of Being allows not only for a science of Being, necessitates deductive logic:

...[S]ince there is one kind of thinker who is even above the natural philosopher (for nature is only one particular genus of being), the discussion of these truths also will belong to him whose inquiry is universal and deals with primary substance. Natural science also is a kind of wisdom, but it is not the first kind.—And the attempts of some who discuss the terms on which truth should be accepted, are due to a want of training in logic; for they should know these things already when they come to a special study, and not be inquiring into them while they are pursuing it.—Evidently then the philosopher, who is studying the nature of all substance, must inquire also into the principles of deduction (*Meta.* 1004b34-1005a7).

If there can generally be a logic that underpins the science of Being, one that maps onto what really is the case, and what cannot be otherwise, then the scientist can reach conclusions such that they are both materially and formally necessary. Thus,

one can develop sciences that are based on first principles, which are both necessarily true and whose justification also has the mark of necessity.

Unlike contemporary modal logic, which sees narrow logical necessity and broad metaphysical necessity as distinct, Aristotle holds that they are really unified in much the way reality is unified with reason through being fundamentally intelligible. That we can have a logic of Being that permits valid inferences, despite the plurality of senses of Being, is far more surprising than that there could be a logic of Necessity, and so possibility, that is grounded in the one sense of necessity to which all other senses point. Thus, it is not the case that metaphysics is more fundamental to Aristotle than logic when compared to contemporary systems. Rather, Aristotle doesn't separate the metaphysical sense of necessity from the logical sense in a way that is utterly equivocal. They mutually support one another in a logic that permits inferences about what actually is and *must be* the case in reality. So, metaphysical claims about essences, and necessary predications, can be more than the flighty speculations of the arm-chair metaphysician. They can be the inferences of a scientifically and empirically minded metaphysician, who is able to combine universal claims with the first principles, and generate new scientific knowledge.

Aristotle says, "Now spoken sounds are symbols of affections in the soul, and written marks the symbols of spoken sound... But what these are in the first place signs of—affections of the soul—are the same for all; and what these affections are likenesses of—actual things—are also the same" (*De Int.* 16a4-9). This point echoes Aristotle's claim in *De Anima* III, 5 and 7, that knowledge is identical with its object.

As [Aristotle] says at the beginning of *On Interpretation*, names are primarily in the mental language, which seems to have little redundancy and which seems, if we but think clearly, to have a structure reflecting reality. Indeed, the representation is supposed to be isomorphic with the structure of being. I have argued that this isomorphism does not reflect a naïve projection of the structure of the Greek language onto the world, but rather requires a sophisticated regimentation of the structure of the Greek language according to Aristotle's ontology (Bäck 2000, 177).

Indeed, among the “sound” or “names” treated in *De Interpretatione* are modal names like “possible” “impossible” and “necessary.” Nonetheless, modality is clearly a part of Aristotle's ideal protocol language given that he treats it in great detail within the *Prior Analytics* and *De Interpretatione*.

That there is an isomorphism between language, and in particular, the technical language Aristotle is developing, and reality, tells us that modal properties are features of actual things, and are not mere logical fictions useful in understanding whether a property can or cannot fail to hold of an object. Hence, the *de re*-like aspect of Aristotle's thought. So we have a unified science of necessity that corresponds with reality and can be configured within the syllogistic to provide demonstration. The modal syllogistic, then, is not merely an inconsequential logical exercise. It is intended to cleave reality at the joints, and aid us in making real scientific inferences about the world that are backed by deductive necessity.

Given that there can be a unity of necessity in Aristotle, and out of unity, a science of modality grounded in real essences, it is apparent that Aristotle's modal syllogistic was intended to be a system that provides genuine knowledge about what is necessarily the case for substances of various sciences.

### 3.5 Aristotelian Essentialism and Aristotelian Science

To see how this might work in practice, we can consider Katherine Koslicki's case study of the multiple stomachs of the camel to better understand how necessary features of a thing, "...can be traced back to facts about the essence and hence explained by appeal to definitions..." (Koslicki 2012, 20).

Koslicki denies that Aristotle has a modal conception of essence, but by this, she means the sort of contemporary modalism in the style of those like Kripke and Putnam. Koslicki takes her cue from Kit Fine in not reducing essence to modality (Koslicki 2012, 188). Fine, Koslicki, and others like David Oderberg, are critical of the type of essentialism where essential properties just are properties that necessarily hold for a given kind of thing. Fine notes that under contemporary essentialism essences are reduced to properties that an object has just in case it has those properties necessarily. In other words, the necessity of the property is sufficient to establish it as an essential property (Fine 1994, 3-4). Fine's position on essences is described as non-modal by Koslicki (see Koslicki 2012, 189). This is somewhat misleading since Fine does grant that essential properties are necessarily had (Fine 1994, 4)

Likewise, Koslicki identifies Aristotle as one who rejects the reduction of essences to modality.

Both Aristotle and Fine, in their conception of the relation between essence and modality, rely on a distinction between what belongs to the essence of an object and what merely follows from the essence of an object. On both Fine's and Aristotle's conception, the essential truths characterize the essence of an object and state what features are essential to it, while the necessary truths characterize what merely follows from the essence of an object and state what features are necessary (but non-essential) to it (Koslicki 2012, 188).

It is correct that Aristotle does not reduce essence to mere modality (there is something distinctive about essences) it is not quite right to conceptualize Aristotle's essences as "non-modal". While Aristotelian essences ought not to be characterized as properties that obtain across possible worlds, they are necessary, at least in the sense that is relevant to the modal syllogistic. However, Koslicki is quite right that Aristotle views essences as explanatorily and causally prior to other necessary properties.

Aristotle's central idea, to trace the explanatory power of definitions to the causal power of essences, has the potential to open the door to a philosophically satisfying response to the question of how the necessary features of an object are related to its essential features (Koslicki 2012, 189).

Koslicki references those important distinctions made in the *Posterior Analytics* between understanding the facts and the reason why. Aristotle's example of the planets serves to highlight this distinction:

...let *C* be the planets, *B* not twinkling, *A* being near. Thus it is true to say *B* of *C*; for the planets do not twinkle. But also to say *A* of *B*; for what does not twinkle is near (let this be got through induction or through perception). So it is necessary that *A* belongs to *C*; so that it has been demonstrated that the planets are near. Now this deduction is not of the reason why but of the fact; for it is not because they do not twinkle that they are near, but because they are near they do not twinkle.

But it is also possible for the latter to be proved through the former, and the demonstration will be of the reason why—e.g. let *C* be the planets, *B* being near, *A* not twinkling. Thus *B* belongs to *C* and *A* to *B*; so that *A* belongs to *C*. And the deduction is of the reason why; for the primitive explanation has been assumed (*APo.* 78a30-78b2).

Koslicki characterizes the arguments as follows:

- (1)<sup>32</sup>
  - c) Heavenly bodies which are near do not twinkle.
  - a) Planets are heavenly bodies that are near.
  - b) Therefore, planets are heavenly bodies which do not twinkle.
- (2)
  - d) Heavenly bodies which do not twinkle are near.
  - b) Planets are heavenly bodies that do not twinkle.
  - a) Therefore, planets are heavenly bodies that are near.

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<sup>32</sup> Koslicki labels both arguments as (2), which I believe is a typo.



(1) is a demonstration of why and (2) is a demonstration of the fact. Koslicki says, “Aristotle would characterize both arguments as deductively valid; but only one of them; viz., the first, succeeds in meeting the additional criteria imposed on deductively valid arguments which are also demonstrative” (Koslicki 2012, 199). Still, Aristotle seems to be comfortable distinguishing between demonstrations of facts and demonstrations of why:

Ἔτι ἐφ’ ὧν τὸ μέσον ἔξω τίθεται. καὶ γὰρ ἐν τούτοις τοῦ ὅτι καὶ οὐ τοῦ διότι ἡ ἀπόδειξις· οὐ καὶ γὰρ ἐν τούτοις τοῦ ὅτι καὶ οὐ τοῦ διότι ἡ ἀπόδειξις· οὐ γὰρ λέγεται τὸ αἴτιον (APo. 78b13-15).

Again, in cases in which the middle is positioned outside—for in these too the demonstration is of the fact and not of the reason why; for the explanation is not mentioned.

Koslicki insists, “In a proper demonstrative argument, the middle term must be *explanatory* of the conclusion, in a very specific sense: the middle term must state what properly belongs to the *definition* of the kind of phenomenon in question (viz., in this case, planets)” (Koslicki 2012, 199). There is a sense of priority in the demonstrations τὸ διότι in that it provides some sort of causal or explanatory account. At the same time, Aristotle makes clear in *Posterior Analytics* A.4 that demonstration is a deduction from what is necessary. In subtle contrast, Goldin explains that a demonstration of the reason why,

...is the mainstay of the sciences: such demonstrations are grounded entirely on atomic predications that make clear *why* the conclusion holds. A demonstration of [the fact], like the former kind, has as its premises objects of scientific knowledge. But because these are not necessarily atomic predications, it is possible that the conclusion of such a demonstration states the cause of one of the premises. Such a demonstration serves as a proof that the conclusion is the case but does not explain why it is the case. Every explanatory demonstration can also function as a proof that the conclusion of the demonstration is true,

but not every proof that a conclusion is true serves to explain that conclusion (Goldin 1996, 120).

So explanatory demonstrations serve a dual function of proving both τοῦ ὅτι and τοῦ διότι. Goldin explores how Aristotle seeks to explain why there are eclipses of the moon. Initial attempts at such a syllogism, though not explanatory, can still lead to apodictic conclusions. I contend that such syllogisms are crucial for building up a body of knowledge within a science from which explanatory deductions can be derived, i.e. what Ross calls a “demonstration recast” (1957, 535).

Consider, for instance, Koslicki’s (2), which demonstrably concludes that planets are heavenly bodies that are near. Suppose we accept all that has been said up to this point that demonstrations are apodictic in nature. Let us imagine a progression in science such that one begins only with the principle that heavenly bodies which do not twinkle are near. Suppose this principle is taken to be apodictically certain. One can derive that planets are necessarily heavenly bodies that are near merely from the non-apodictic observation that planets are heavenly bodies that do not twinkle. This knowledge can then be utilized in an explanatory demonstration effectively supporting the necessity of the minor premise. So, once the explanation is known, and given that it too will be apodictic, one will have a pure apodictic explanatory demonstration through the middle term. Thus, a case could be made for the use of the demonstration of facts where the middle term is not predicated of necessity in the minor premise. This leads to useful information for later scientific discovery. The key, I think, is an insight Goldin elucidates in his treatment of Posterior Analytics 2.16, namely that, “Throughout Aristotle’s positive argument for the convertibility of a subject and a demonstrated predicate rests on

the convertibility of the *premises* of the demonstration” (Goldin 1996, 143). So we see in Koslicki’s examination of the demonstration regarding planets that (1c) is converted to (2d). This conversion, or recasting, combined with the apodictic conclusion from (2a) can provide an apodictic minor, and so allows us to generate a genuinely scientific explanatory demonstration. Goldin explains,

If a demonstration has more than one middle term, each middle term will convert with that before it, until we reach a middle term that is definitional of its subject. If the demonstration is well formed, that middle term will not be the genus of the subject. (Otherwise, it will be that genus, and not a subject that falls under that genus, that is the true cause of the inherence of the attribute in question.) It must therefore be the differentia of the subject. Throughout his career Aristotle maintained that a differentia converts with its subject (cf. *Cat.* 3.1b16-24; *Metaph.* 7.12). Accordingly, every demonstrated attribute, as scientifically understood, converts with its subject (Goldin 1996, 147).

This is to say that such attributes will be counterpredicable, and so be essential or substantive in the relevant senses needed for apodictic premises of a modal syllogism. In effect, a science is built up through such a process of bootstrapping from first principles and universals that are definitional, towards those that are explanatory, and through combining mixed apodictic demonstrations of the fact (a “demonstration that” as opposed to a “demonstration why”) with explanations, come to new scientific knowledge.

### 3.6: Conclusion

In this chapter, I have argued for a central place of the modal syllogistic in the sort of demonstrations that Aristotle discusses in the *Posterior Analytics* where definition and explanation enrich the soil of a science to the point where some things are understood, in a certain respect, prior to knowing what things are the

case. Mixed apodictic syllogisms, in particular, play a crucial role in this process, as they permit apodictic inferences even when one of the extreme terms is not joined to the middle by apodictic force. The result is an increase in scientific knowledge. This, I believe, is Aristotle's ultimate response to Meno's paradox. In effect, the paradox prompted Aristotle to devise a system of deduction and demonstration, whereby definitions can be recast into explanations, which in turn are built upon the principles of a science and expand the breadth of what is known within that science. As Ferejohn puts it, "...the distinction Aristotle brings to bear on the *Meno* paradox at 71a29-21 is indeed one that is at the very heart of his own theory of *apodeixis*..." (1988, 100). Ferejohn notes that Aristotle's response is a two stage process of diaresis and demonstration,

My proposal is that the pre-syllogistic "framing" stage of *apodeixis* is seen by [Aristotle] as a method of transforming "merely universal" and "qualified" knowledge into genuine, *de re* knowledge *haplōs*. It is a method wherein some set of *horoi* (definitional *archai*) that have been apprehended previously (as in *An. Post.* B 19) are "placed," or "set out" upon a scientifically interesting genus of entities whose existence and place in the broad scheme of things is already recognized or assumed (*ibid.* 109).

Ferejohn argues that Aristotle understood that Plato's response to Meno's paradox in the *Sophist* and *Statesman*, i.e. the use of diaresis to arrive at definitions, was insufficient. As David Charles points out, "Aristotle wishes to allow that one can come to know of the existence as well as the nature of the kind" (2000, 76). Like Ferejohn, Charles view Aristotle's response to Meno as a process that begins in definition and ends with a demonstration of the existence of the kind in question. This is only part of the pre-scientific task, seeding a science with some definitions. Other definitions will be arrived at through demonstration. These can then be transitioned and recast into genuine scientific demonstrations. Charles laments,

however, “It is difficult to see how the modern essentialist can account for one’s coming to know of the kind’s existence. For, that is already presupposed in his understanding of the term” (*ibid.*) Indeed, in this chapter, it was my aim to show that we are no longer shackled to the modal equipment contemporary modalists and essentialists use. In the next chapter we shall explore other virtues that an Aristotelian understanding of modality and modal logic can offer.

## CHAPTER FOUR

### 4.0 Introduction

Aristotelian modal logic should not be seen as a competitor to the variety of modal logics we have today. Nor should it be seen as an outmoded way of conceiving of modality. Rather, I will argue here that Aristotle's modal syllogistic is a complementary way of reasoning about the relationships among terms, be they accidental or essential. There is, however, a certain prejudice against the old Aristotelian system, e.g. given the orthodox interpretation of Aristotelian *dictum de omni et nullo* semantics, it is commonly thought that there is an issue with existential import. So, we have adopted Malink's heterodox interpretation of this semantics, which treats the pluralities said or denied of categorical terms as terms in their own right. So, if one is to critique or endorse Aristotelian logic, one must first understand what Aristotle means by "term" (ὄρος). Aristotle writes, "I call a term that into which the proposition is resolved, i.e. both the predicate and that of which it is predicated, 'is' or 'is not' being added" (*APr.* 24b16-17). It is the basic element of a categorical proposition. Striker explains that ὄρος, meaning limit, is appropriated by Aristotle from mathematics, where ὄροι are the terms of the ratio. "His subsequent explanations of the phrases 'to be in something as in a whole; and 'to be predicated of all/none' suggest that he takes terms to refer to classes of individual objects of which the respective expressions (man, runs, etc.) are true" (Striker 2009, 78). What we mean by "classes of individual objects" is of particular

significance. The term is not merely some word or phrase applied to a collection or assemblage of things, but constitutes the formal element said of a thing by which that thing is intelligible to a mind. It is what is said of a thing. In the *Categories*, Aristotle devises a two-tiered ontology of the primary (what is not said of a subject) and the secondary (what is said of a subject). Categorical propositions predicate a class-term of another class-term. Such propositions say something about a subject that can, itself be said of a subject. Hence, categorical propositions convert, and subjects can be turned into predicates. Yet, it is also possible to predicate of a subject which, itself, cannot be said of a subject, i.e. of that which cannot be a predicate of a proposition. Such things are the primary, the individual, and for Aristotle, it is not merely an ill-formed expression to predicate an individual of a subject, but it is ontologically incorrect to do so. Aristotle writes, “Neither can individuals be predicated of other things, though other things can be predicated of them” (*APr*: 43a40). So, it is clearly possible to treat individuals as the subjects of propositions and to predicate of them. Aristotle explains,

Whenever one thing is predicated of another as of a subject, all things said of what is predicated will be said of the subject also. For example, man is predicated of the individual man, and animal of man; so animal will be predicated of the individual man also—for the individual man is both a man and an animal (*Cat* 1b10-15).

We must take note that it is the primary substance that is the subject in Aristotle’s example. So, when “man” is the “subject” and “animal” is predicated of “man”, they are both said of the subject, which is the individual man.

Fundamental to Aristotle’s logic, the very elements out of which the logic is composed, is a two-tiered system where terms are the formal elements that inhere in individuals. The logic of the syllogistic is nothing other than the method by which

these formal elements can be combined validly in propositions—propositions about the ways in which things exist, which we are capable of comprehending. Striker goes on to say, “One should note, however, that classes like those of white things, running things, etc., as opposed to genera and species, on the one hand, substances, and attributes, on the other, do not seem to have a place in Aristotle’s official ontology” (Striker 2009, 78). Striker concludes from this that Aristotle’s syllogistic, as developed in the *Prior Analytics* is independent of his metaphysics. Indeed, terms are not parts of his ontology. However, I think Striker is fundamentally mistaken to say that Aristotle’s syllogistic is independent of his metaphysics or that terms bear no relation to ontology. Rather, class terms belongs to one half of Aristotle’s ontological square, i.e. that which can be said of a subject. And it is this half to which terms belong that make intelligible the other half, which constitutes individuals, which are always the subjects in any proposition. So while terms are not included along with genera and species in his ontology, it is rather the case that terms are a broad section of Aristotle’s ontology, indeed the formal elements, which include genera, species, and other sorts of predicates. One might, then, object that in the logic of opposition among universal and particular statements; that it appears to be overly burdened by metaphysical suppositions. It is supposed that logic, being a tool for reasoning about various metaphysical positions, should be neutral towards all. We could, one might suggest, treat classical predicate logic as the metaphysically neutral position, but even that goes too far. For, predicate logic treats existence as a second-order property, so treats all existence claims with the existential quantifier,



assumes the principle of bivalence, and assumes a non-empty domain of discourse. These are not, properly speaking, metaphysically neutral positions.

The prejudice of thinking that Aristotle's modal syllogistic is somehow defective has led many contemporary commentators to suggest various ways of translating Aristotle into first-order predicate calculus, and the modal syllogistic into a kind of quantified modal logic that is somewhat arbitrary and inconsistent in where it places the "box" or "diamond," hence inconsistently treating *de dicto* and *de re* contexts as equivalent. But such interpretations tend to be confused in regard to the universe of discourse with which the Aristotelian syllogistic dealt.

So, in this chapter, where we shall discuss the various virtues of having an Aristotelian modal logic, it will be important to deal with the common objection that Aristotle's logic founders on the existential fallacy.

First, it is worth noting that predicate logic is thought to avoid problems with existential import. Universally quantified propositions carry no existential import, we are told. This could not be further from the truth. In fact, one might say that all universal propositions carry existential import insofar as the domain of discourse in classical predicate logic is never empty. For example, consider the following argument, where  $Hx$  is "x is a human" and  $Rx$  is "x is rational" :

1.  $(\forall x)(Hx \supset Rx)$  (premise)
2.  $Hu \supset Ru$  (1 UI)
3.  $(\exists x)(Hx \supset Rx)$  (2 EG)

This is literally to say that there exists some  $x$  such that if  $x$  is human, then  $x$  is rational. One might be tempted to say that this is just a hypothetical claim. The

material conditional, though an odd rendition of our natural language conditional, is truth-functional and so logically useful. It is equivalent to:

$$4. (\exists x)(\sim Hx \vee Rx) \text{ (3 Impl)}$$

In other words, we can conclude that there exists some  $x$  that is either not human, or rational. So, these implications are nothing more than disjunctive statements, and it is at least plausible that one can have existential claims that are disjunctive. Indeed, the idea of disjunctive properties is a topic that is often debated in contemporary metaphysics (see Clapp 2001). So, if we are willing to grant that something could have one property, or another, as Nelson Goodman's "grue"<sup>33</sup> is green or red, depending on the time and circumstance, then it is at least plausible to claim that all disjunctive statements that are existentially quantified carry existential import. If the "∃" is to tell us something about what exists in the world, (4) tells us there is at least something, though it is not decided what predicate or property this something has. We might say that  $x$  has the property of either not being a human or of being rational. Moreover, we could conclude that this  $x$  cannot be human and not rational at the same time and in the same way. This is to say something very definite about this  $x$ .

To make this point another way, suppose for a moment, if we can, that there is not anything—nothing in the true Leibnizian sense of the question "why is there

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<sup>33</sup> Goodman provides a new riddle of induction through his example of an object that has the predicate "grue". An object is grue if it is green up until some time, after which it is blue (Goodman 1983, 74). Paul Audi writes, "The question of whether there are disjunctive properties concerns the structure of properties. It is obvious that some predicates have a disjunctive structure. And an important lesson of Nelson Goodman's work is that, with a little ingenuity, any property can be expressed by a disjunctive predicate, just as greenness can be expressed by 'is grue and observed before 2500 or else bleen'" (2013, 748).

something rather than nothing at all.” If there were not anything, then (4) above would be false. It would be false because there would not be any  $x$  for which the disjunct could be true. Someone might be tempted to say that  $(\exists x)\sim Hx$ , but that would mean that  $(\exists x)\sim Hx$  is equivalent to  $\sim(\exists x)Hx$ . Could it really be the case that the claim “there exists an  $x$  such that  $x$  is not human” is equivalent to “it is not the case that there exists an  $x$  such that  $x$  is human” ? The latter is compatible with there being nothing at all whereas the former clearly asserts that there is some  $x$ . Of course one might respond that there cannot be “nothing” in the Leibnizian sense. Indeed, we cannot even formulate this fundamental question in first-order predicate logic. We cannot form the proposition, “There is not anything.” However, the most general statements about existence, e.g. self-identity can lead directly to existential claims.

A common way to translate “something exists” in first-order predicate logic is to use the identity relation. So  $x$  exists if and only if  $(\exists x)(\exists y)(x = y)$ . William Vallicella criticizes the assumption that the existential quantifier does the job of making claims with existential import. He considers the following argument (2002, 58).

1.  $(\forall x)(x = x)$  (principle of identity)
2. God = God (1 UI)
3.  $(\exists x)(x = \text{God})$  (2 EG)
4. Therefore, God exists (3 by translation into ordinary idiom)

Before theists rejoice at this proof, there is obviously something wrong with it. The argument proves too much, i.e. it could prove the existence of anything. Vallicella

argues that there is an ambiguity in the way we understand “ $\exists$ ”. The inference from (2) to (3), “...trades on a confusion between the particular with the existential quantifier. It trades on a confusion of 'something' with 'something that exists'” (*ibid.*). Likewise, one could prove the existence of something through first-order predicate logic merely with the principle of identity, as in:

5.  $(\forall x)(x = x)$  (principle of identity)
6.  $u = u$  (5 UI)
7.  $(\exists x)(x = x)$  (6 EG)

Now, one might object that this does not actually prove the existence of something, but that, given a domain of discourse in which (5) is true, i.e. in a non-empty set, the conclusion follows trivially. However, it is at least a puzzling question as to whether (5) is false in a domain of discourse that is empty. If so, it seems that the very act of quantifying over individuals would result in a proposition that is false. We could not even claim  $\sim(\forall x)(x = x)$  as that would be equivalent to  $(\exists x)\sim(x = x)$ , which if we take the existential quantifier to assert that there exists an  $x$ , then it is just to say that something non-self-identical exists. This surely does not follow from domain of discourse being empty. Indeed, no existentially quantified theorem is valid in an empty domain, which was one of the motivations for developing free logic (see Nolt 2014).

However, one should not assume that free logic is metaphysically neutral. The very act of parsing reality up into individuals which can exemplify properties or predicates, whether any such individuals exist, is itself a metaphysical claim at least insofar as one considers the content of logical propositions to say something true

about reality. If not, one is left with a skeptical position in which logic is the mere manipulation of symbols according to arbitrary rules, for what it would mean to be truth-preserving if these symbols do not even refer to any metaphysical realities whatsoever! Moreover, if classical first-order predicate logic is not metaphysically neutral in presuming non-empty domains in its theorems, free logic is not metaphysically neutral in treating existence as a predicate, a notion explicitly rejected by Kantians and Fregeans alike.

All this is well and good, you might say. Contemporary logic is at least less metaphysical than Aristotelian logic, and while there might be some questionable existential import given the assumed domains in first-order predicate logic, Aristotelian logic is even stronger in its commitments. It doesn't just commit us to some  $x$  that either exhibits one property or another, but by sub-alternation, one can seemingly say that if "Mammal belongs to all Unicorn" then "Mammal belongs to some Unicorn." So, an abundance of mythical creatures seemingly must be added to our ontology. The concern over Aristotle and existential import relates back to criticisms of the square of opposition, which has been used as a pedagogical tool in logic for millennia.

As Mario Mignucci points out, in the early twentieth century many logicians considered there to be counter-examples to contrariety, sub-contrariety, and sub-alternation (see Mignucci 2007, 123-124). However, those counter-examples depend upon certain assumptions about how one ought to translate Aristotle's categorical propositions into first-order logic, such assumptions that underpin criticisms of Aristotelian modal logic as well. Mignucci points out that the key to

translating between contemporary logic and categorical propositions seems to be sourced in the *Prior Analytics*, where Aristotle says of universals,

...we say that one term is predicated of all of another, whenever nothing can be found of which the other term cannot be asserted; 'to be predicated of none' must be understood in the same way.<sup>34</sup>

This has been translated, according to Mignucci, into first-order predicate logic in the following manner:  $AaB \stackrel{\text{def}}{=} \sim(\exists x)(Bx \ \& \ \sim Ax)$  (*ibid.* 125). Since  $\sim(\exists x)(Bx \ \& \ \sim Ax) \equiv (\forall x)(Bx \supset Ax)$ , universal affirmative categorical propositions have been commonly translated as a universally quantified conditional statement. Hence, Aristotle is committed, in his universal propositions, to statements that seemingly carry no existential import. Yet, many of his inferences seem to commit the existential fallacy by reaching an existential conclusion from universal premises, as the “unicorn” example shows above. Like Mignucci, I ultimately reject the notion that  $AaB$  should be translated as or defined as  $(\forall x)(Bx \supset Ax)$  (*ibid.* 134). My solution has been to follow Malink in interpreting the relationship among categorical terms as a mereological relationship among higher-ordered class-terms. In defining  $AaxB \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset AaxZ)$ , we are not relating individuals to predicates. So the inference from  $AaxB$  to  $AixB$ , or  $(\forall Z)(BaxZ \supset AaxZ)$  to  $(\exists Z)(BaxZ \ \& \ AaxZ)$  does not explicitly commit us to individual subject of which the class-terms belong. Indeed, Aristotle's logic allows one to devise a nominal definition prior to understanding whether there are any instances of the thing in the world. The traditional oppositions of contrariety, sub-contrariety, and sub-alternation, are not

<sup>34</sup> *APr.* 24b26-30: λέγομεν δὲ ἐν τῷ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ἢ λαβεῖν τῶν τοῦ ὑποκειμένου καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως. Mignucci notes that Ross omits τοῦ ὑποκειμένου, though explains that it would be tacitly understood even if omitted (Mignucci 2007, 124).

falsified by there not being any instances that fall under the terms. Rather, the failure of there being any individuals under the term would mean that the term could not be understood—in principle. The very act of predication, or of using a copula to link such terms, depends upon there being a nature grounding each term. For example, if we attempt to predicate “animal” of “goat-stag”, the result would, for Aristotle, not have a truth value. It would be akin to what the logical positivists say of unverifiable propositions. This is not because “goat-stag” has not been verified, but so long as that name refers to no nature, there is nothing true or false that could be definitively said of it. As Aristotle puts it, it is impossible to know what a goatstag is (*APo.* 92b7). This is not merely an epistemological point, though. The impossibility is rooted in an ontological point. There simply is no primary substance, so there is nothing for the senses to be acquainted with and there is not a true secondary substance for  $\nu\ \omicron\ \tilde{\upsilon}\zeta$  to grasp. If it is impossible to know what a goatstag is, there simply is no truth or falsity to any affirmative or privative claim made about goat-stags. There simply is no fact of the matter. What we have, then, with the goat-stag, is the Aristotelian response to the charge that his categorical propositions run afoul of the existential fallacy.

Just as the contemporary objection to Aristotelian logic is based on the incorrect assumption that Aristotle failed to understand existential import, the contemporary objection to Aristotelian modal logic has been that Aristotle failed to distinguish *de re* from *de dicto*. The previous chapters have shown that this objection is based on a misunderstanding of how Aristotle’s logic dissects the

world. Every logical system carries with it basic assumptions, and Aristotelian logic is no different. While there is great virtue in devising a theory or system that is neutral to a wide variety of logical positions, there is little hope for a system of logic that is entirely neutral. Rather, the best recourse is to understand in which respects a theory or system is neutral, and in which respects it presupposes some metaphysical outlook. So, in the latter half of the twentieth century, we have seen the development of a wide array of logical systems, including modal logic, which are explicit in the presuppositions, or axioms, of the system. For example, the dialetheist champions paraconsistent logics not merely out of curiosity, but because they believe that reality is such that there are true contradictions. Relevance logics were developed to better capture our meaning when we speak of implication, even though this may include metaphysically laden notions of causality and explanation. Finally, in modal logic, there is a multitude of systems that make various assumptions about the relationship among possible worlds, and whether one thinks that S4 or S5 are appropriate systems to employ will depend upon the metaphysical context, and in part, on what metaphysical weight one gives to possible worlds themselves. Aristotelian modal logic can join the constellation of logical systems that do helpful theoretical and deductive work. Rather than shun metaphysically loaded logical systems, or strive towards those systems which are the “least metaphysical” whatever that might mean, one should be aware of the metaphysics that underpins the systems one uses, and acknowledge that those metaphysical commitments are part of the inference one is making.



In the following sections, we shall consider a few metaphysical puzzles which are somewhat intractable for contemporary modal logicians, to which Aristotelian modal logic may offer insights, especially pertaining to identity. I will argue for the virtue of having a conceptually distinct way of discussing modal properties as a guard against what may be called the indispensability argument for possible worlds. Lastly, I shall make some concluding remarks about Aristotelian modal logic, and his metaphysics of modality—grounded in his essentialism and theory of substances.

#### 4.1 The Circularity Problem

David Oderberg raises a fundamental problem with contemporary modalism. The criterion for what counts as a possible world is necessarily circular, in that it will require the notion of modality to identify possible worlds, while the relationship among possible worlds grounds the meaning of “possibility” and “necessity.”

Oderberg writes,

...even if the realist could get around the circularity problem, say by postulating possible worlds as primitive existents, as a modal given –rather than as entities for which we have to have modality-involving criteria – he would end up merely relocating the analysandum. Instead of having to understand the modal properties of objects within a world, we will have to come to terms with the modal properties of the worlds themselves. What is it for the worlds to have the modal properties they do (at the very least being possible, and perhaps also necessary)? We are still faced with unanalysed modality, only it has moved somewhere else. Now there may, as realists believe, be net theoretical benefits to be gained from explaining the modality of individuals within worlds in terms of the worlds themselves, but unless one is wedded, implausibly, to a cost–benefit approach to metaphysics, this will not be satisfactory. We want to know why objects have the modal properties they do. To answer that this is (at least in part) because worlds have the modal properties they do is only to push the problem from one place in the rug to another (2007, 2).

In other words, if we define modal properties in terms of worlds, then we must also define worlds in terms of modal properties. If we try to escape this by treating modal properties as basic to worlds, then we cease to explain modal properties as they seemingly exist in the objects of our actual experience.

In a similar manner, Boris Kment's project has been to develop a metaphysics of modality where modal facts are just facts about essences, grounding relationships, and explanation. He seeks a solution to the circularity problem in terms of closeness among metaphysically possible worlds and the actual world (2014, 27). While I find in Kment an ally who takes essences seriously when it comes to modality, his project is still ultimately wedded to a metaphysical geography of worlds.

The advantage of Aristotelian modal logic is that modality is merely a copula modifier. While there are defined relationships among "necessity", "contingency", and "possibility", those relationships do not define "essence" and "accident" or "actuality" and "potentiality" for Aristotle. So, there is no circularity in saying that an essential predication of a term is apodictic. Of course it will be, but given that *propria*, and other *per se* predications will be apodictic, it is clearly not circular.

Raising the circularity critique is not merely to suggest that there are problems with contemporary modalism that are not present or operating in Aristotelian modality. Rather, I would like to suggest that the two notions of modality could operate side by side and ultimately be informative of one another. With certain bridge-laws in place, we could understand the modal properties of

worlds, essences, and individuals as different ways of conceptualizing of modality. In the previous section, I suggested that Aristotelian modality should be considered analogous to higher-ordered logic, and that what is said to be necessary of a subject term, may not be predicated apodictically of an individual, who is an instance of the kind. This is at least true, unless we make certain assumptions about the relationship an individual bears to its essence, e.g. that an individual cannot survive the loss of an essential property, or in any way exist at a time, or in a world without exemplifying that property. A fully worked out system bridging the two-logics is beyond my purview here, but I will say that with such rules in place, and with clear distinctions in the meanings of modal properties as they bear upon essences, worlds, and individuals, much of the concern over circularity and primitiveness will be relegated to specific contexts, e.g. in the realm of broader metaphysical modality beyond nomological necessity and possibility, and outside of what Aristotle's empiricism would advise us to speculate upon. If "possibility" is said to be a primitive feature of worlds, it is not the same feature that a human has when it is said that a human possibly laughs. Whether and how the notions are related will depend upon whether is a modal realist or not.

## 4.2 Actualism

In the previous chapter, we touched upon the topic of actualism with respect to Williams' s and Charles' s argument regarding explanation. In order to reach their conclusions, within the context of first-order quantified modal logic, they required

the use of the Barcan Formula and the Converse of the Barcan Formula. I argued that it was a reliance on those modal axioms that raised the question for Williams and Charles about whether an individual could be anything other than what it is essentially. I connected this to the question of actualism. Here, I am to address the question of actualism more directly, and to provide what I take to be a distinctly authentic Aristotelian sort of actualism, one that I shall argue avoids the sorts of problems that arise in first-order quantified modal logic. Moreover, I shall apply insights from this section to my discussion of identity to provide an account of identity that avoids some classical paradoxes. Both actualism and identity suffer problems that I believe are ultimately rooted in how first-order predicate logic carves up reality.

Timothy Williamson provides an informal argument against the Barcan Formulas and the actualism it seems to imply, when applied to first-order quantified modal logic. He explains that the Barcan Formula (BF) says that there is an object that “could have been a child of Wittgenstein” (Williamson forthcoming, 5). Now given that Wittgenstein never had a child, it is highly unintuitive that there is now something that possibly is the child of Wittgenstein. However, this is a direct implication of BF combined with the less controversial *de dicto* claim that possibly, there is something that is the child of Wittgenstein. That is, where  $Cxy$  means “x is the child of y”, and  $w$  is Wittgenstein, according to BF:  $\diamond(\exists x)C_xw \supset (\exists x)\diamond C_xw$ . Given the plausible claim that  $\diamond(\exists x)C_xw$ , we are forced to conclude  $(\exists x)\diamond C_xw$ , which seems unacceptable. What could this thing be? Would it be a human, or is it the

matter in the world, which presently exists, that could have been articulated and formed into a person who was the genetic offspring of Wittgenstein?

The Barcan Formula and its converse suffer from another problem, namely that on Kripkean semantics, it is invalid (See Kripke 1963, Williamson forthcoming, 5-7). Those who want to be able to apply the Barcan Formula must adopt a different form of semantics from Kripke's model theory. Now, this objection is true given the model Kripke proposes, but Williamson takes the informal arguments as ultimately the larger problem.

Interestingly, Williamson argues that these problems dissolve when we transition to second-order quantified modal logic. The difference between first- and second-order quantified modal logic is that in first-order, the domain of the quantifiers is allowed to vary among worlds (Williamson forthcoming, 14).

Since the intensions over which the second-order quantifiers range are restricted to those that for each world deliver a subset of its first-order domain as the extension, they are sensitive to the variability of the first-order domains. For instance, the intension corresponding to self-identity delivers at each world the first-order domain of that world as the extension, so those extensions vary exactly as much as the first-order domains. However, that cross-world variation in extension within an intension induces no cross-world variation in the domain of the second-order quantifiers (the set of 1-place intensions) (*ibid.*).

As Williamson succinctly puts it, the second-order domain is fixed while the first-order domain is not. Consequently, the second-order versions of BF and CBF are valid even while their first-order correlates are invalid (*ibid.*).

This is significant for our discussion insofar as Aristotle's modal logic, following Malink's interpretation, quantifies over kind-terms, and the mereological parts of those terms. Even though Aristotelian modality is based upon a modification to the copula, and so there is no operator that can "jump" the

quantifier, it may instead be fair to say *a fortiori* the domain is fixed and does not vary, since we are talking about predicates and kind terms apart from the semantics of possible worlds. The fear, then, that an Aristotelian form of actualism would succumb to the sorts of problems that plague first-order quantified modal logic is doubly put to rest. For the domain is fixed, given Aristotelian semantics, and even if one were to introduce Kripkean semantics and somehow merge it with Aristotelian modality, creating a hybrid of modality based on worlds and essences, the domain would still be fixed, given Williamson's argument. So, for example, suppose one were to raise the question of the essence of bats, and that all bats are winged, according to their essence. Thus, "winged" necessarily belongs to all "bat".

Moreover, one might say that there are many worlds in which bats exist, and in such worlds, bats are winged by their very nature. It would still be the case that any Aristotelian claim about the essence of a bat would be about class-terms across possible worlds, rather than individuals. In other words, according to BF, under quantified modal logic, where  $Bx$  is "x is a bat" and  $Wx$  is "x is winged," the claim that  $(\forall x)\Box(Bx \supset Wx) \supset \Box(\forall x)(Bx \supset Wx)$ . The corollary, of course, is:  $\Diamond(\exists x)(Bx \ \& \ \sim Wx) \supset (\exists x)\Diamond(Bx \ \& \ \sim Wx)$ , and this is what many metaphysicians and philosophers of logic find unacceptable. Now, suppose we were to hybridize our interpretation of Aristotelian of modality with a possible-worlds semantics. For instance, suppose we were to discuss the Aristotelian terms of "bat" and "winged" :  $\Diamond\bar{W}axB$ , we could very well draw the implication  $\bar{W}aMB$ . This would not lead to concerns over there currently existing possible wingless bats. It would

merely imply that the nature of “Bat” is such that possibly being “Wingless” could be said of it. Those who take “winged” to be essential to “bat” would object that the example is false, but would not be forced to speculate over what presently existing actual things are possible wingless bats. At best, it would be to claim that an individual bat, insofar as it exemplifies “batness,” has the potential to be wingless. Another example might be that trunked animals are possibly hairy, like the extinct mastodon and mammoth. We could express this as:  $\diamond HaxT$  and one might infer from this that  $H_{aM}T$  and  $\bar{H}_{aM}T$ , which is to say that “hairy” and “hairless” are possibly said of “trunked animal”.<sup>35</sup>

The question, I think, is what this Aristotelian actualism looks like. So, I should like to propose the following analogue to the Barcan Formulas, namely an Aristotelian Barcan Formula, or ABF:

$$1. \diamond BaxA \supset B_{aM}A \text{ (ABF)}$$

Now, it is an interesting question, as to whether one can go from an Aristotelian modal claim to a claim about individuals. I would offer the following:

2.  $B_{aM}A \supset (\exists x)Ax$  (premise)
3.  $(B_{aM}A \ \& \ (\exists x)Ax) \supset \diamond(\exists x)Bx$  (premise)<sup>36</sup>
4.  $[(B_{aM}A \ \& \ (\exists x)Ax) \supset \diamond(\exists x)Bx] \supset (\exists x)\diamond Bx$  (premise)

So from,

$$5. B_{aM}A \text{ (Assumption)}$$

We can infer,

$$6. (\exists x)Ax \text{ (From 2,5)}$$

<sup>35</sup> Thanks to Owen Goldin for this example.

<sup>36</sup> This line adds a step that is superfluous in a sense, but it is meant to illustrate a parallel to the Barcan Formula.

Also, the *de dicto* claim

$$6. \diamond(\exists x)Bx \text{ (From 2,3,5)}$$

And the *de re* claim,

$$7. (\exists x)\diamond Bx \text{ (From 2,4,6)}$$

So, if we have the assertoric proposition  $\diamond B_{xA}$ , we can infer  $B_{xA}$ , and given that possibility, for Aristotle, will imply that there is an underlying nature and existence of individuals, we can make *de re* and *de dicto* claims of individuals in first-order predicate logic. For if a subject did not have an underlying nature, there would be instances of the kind “goat-stag”. So, (1) just is the claim that if  $B$  possibly belongs to all  $A$ , it is only insofar as there are  $A$ s in existence. The *de dicto* and *de re* possibility claims then follow insofar as  $B_{xA}$  implies both.<sup>37</sup>

One can make inferences about Socrates and his essential properties, but we must understand that Aristotelian modal logic is a higher-ordered logic when we are also quantifying over individuals, and so the modal relationships held among class-terms are not identical to the modal relationships held between individuals, i.e. primary substances, and what is said of them. Aristotelian essentialism requires a fixed domain, and so inferences about individuals, which possess natures in the actual world. So, if  $B_{xA}$  is true, we can infer, at the very least,  $(\forall x)(Ax \supset Bx)$ , and arguably  $(\forall x)\Box(Ax \supset Bx)$ . This is not to say, of course, that  $B_{xA}$  is logically or semantically equivalent to  $(\forall x)\Box(Ax \supset Bx)$ . It isn't. Rather, it is to express a bridge between

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<sup>37</sup> As for apodictic rules, we might establish the following:

$$1. B_{xA} \supset \Box B_{xA}$$

$$2. \Box B_{xA} \supset (\forall x)\Box(Ax \supset Bx) \text{ (premise)}$$

So from,

$$3. B_{xA} \text{ (Assumption)}$$

We can infer,

$$4. (\exists x)\Box(Ax \ \& \ Bx) \text{ (From 8,9,10)}$$



the two logics in much the same way Aristotle suggests one can infer from the fact that “Man” belongs to some individual man, that “Animal” belongs to him as well. Indeed, one could be permitted to infer  $(\exists x)\Box(Ax \& Bx)$  from  $Ba \wedge Aa$ —though not directly from  $(\forall x)\Box(Ax \supset Bx)$ . In terms of Aristotelian modality and the individual  $x$ , we could say that the  $B$ , which the  $x$  expresses, necessarily belongs to all  $A$ , of which  $x$  is an instance. This suggests, again, that there is a problem of existential import in Aristotle, but as I have argued above, that would be a mistake. Rather, it is to say if we may use the parlance of second-order modal speech, that is, if we were to bridge between the Aristotelian modal syllogistic, and quantify both over kind terms and individuals, we could make inferences to the effect that modal categorical propositions imply the existence of things that have those properties. If they did not exist, it would not be intelligible to say that  $B$  belongs to  $A$  by necessity, as that just is a claim about nature. To insist otherwise is merely to insist on a form of nominalism, but then modality is no longer being grounded on essences, and the only alternative is to ground such claims on the basis of worlds.

### 4.3 Possible Worlds and Essences

Kripke and Putnam have been credited with heralding the return of essentialism in the 20<sup>th</sup> century (Oderberg 2007, 1). However, their understanding of essences made necessity a more primitive concept. The result has been, in contemporary analytic philosophy, to equate essential properties with just those properties that are possessed by necessity. Such a view has been challenged by Kit Fine, with his

singleton set argument against what he has called “modalism”, a term that I introduced in the first chapter. Fine’s point is that the necessity of an essential property is a consequence of the property being an essence. Thus Fine believes that all essential properties are necessary, but not all necessary properties are essential. There are necessary properties that are uninformative or trivial with respect to an object. Yet the modalist must include such features as part of the essence of a thing. For example, that Socrates occupies worlds in which there is an infinity of even numbers is a rather trivial modal truth, yet it could be considered a necessary property of Socrates by the contemporary modalist. This suggests to Fine that the modal criterion for essences is incorrect.

Similarly, consider the singleton set that contains Socrates. Fine grants that “containing Socrates” is part of the essence of the singleton set, but it seems unfitting to think that, by some sort of symmetrical relation, we should also think that “being contained by the singleton set ‘Socrates’” is part of the essence of Socrates, even if such containment is necessarily true of Socrates (Fine 1994, 5). It is as if the modalist has settled for necessary properties and nothing more, rather than the source or explanation for why certain properties should be necessary. While Fine does not see a live alternative to modality and possible world semantics, he does argue that “...the traditional assimilation of essence to definition is better suited to the task of explaining what essence is” (Fine 1994, 3). And, of course, this tradition extends back to Plato and Aristotle. Neo-Aristotelians like Oderberg and

Tuomas Tahko want to return to the source of these modalities, to the things themselves, so to speak (see Tahko 2013, 58; Oderberg 2007, 12).

Aristotelian essentialism is intended to relate essential terms to one another, rather than to define essences in terms of properties necessarily predicated of individuals. So while Socrates has an essence, apodictic predication will occur among those essence terms, the relevant *propria*, and various other *per se* predications.

A second issue pertaining to the contemporary understanding of essences is that *de re* modal statements can be reduced to *de dicto* statements about properties holding across possible worlds. Creating such a bridge was one of Plantinga's projects in the *Nature of Necessity* (1974) and as I mentioned in the third chapter, his motivation for devising this reduction was the uneasiness some philosophers have with *de re* properties. The tendency now, though, as with any reduction, is to say that *de re* properties are nothing but short-handed ways of expressing *de dicto* truths. So Kenneth Konyndyk writes, "...the statement that Socrates is necessarily or essentially rational could be read as saying that the statement that Socrates is rational is true in every possible world" (1986, 88). There is, however, a certain cost one must pay in adopting this approach, according to Konyndyk.

One of the first is that contingent entities cannot be said to have essential properties. Presumably rationality is one of Socrates' essential properties. However, it is not true in every possible world that Socrates is rational... it seems that the reason why there are worlds in which it is not true that Socrates is rational is that Socrates does not exist in these worlds (*ibid.* 89).

One could opt for an understanding of essences such that, *x*, has a property *P* *de re* essentially if and only if in every world where the *x* exists, *x* had *P* (*ibid.* 106).

This, of course, has led to further conundrums. For one thing, if existence is a property, then everything has existence essentially, including contingent individuals. Another concern, raised by Konyndyk, is that if contingent individuals exist in only one possible world, then all properties it has, it has essentially. David Lewis offers the counter-part theory wherein an individual could be said to have a property essentially if and only if in every world where there is a counterpart, the counterpart has that property. However, this threatens to be circular if the property of being a counterpart entails having some set of properties in common with an individual in another world. Then counter-part theory doesn't explain *de re* essential properties, but instead it requires those properties to explain in what sense two individuals are counter-parts.

#### 4.4 Transworld Individuals and Counterpart Theory

One interesting debate that has arisen, as a result of what I consider to be the threat of the “indispensibility” of possible world semantics is between transworld individualists and counterpart theorists. A transworld individualist is one who holds that an individual in some possible world,  $W$ , can hold an identity relation with another individual in an accessible world, call it  $W_1$ . The counterpart theorist says that identity relations cannot hold across possible worlds. So, when we say that it is possible that Barak Obama was a marine, we mean that there is a possible world where a counterpart who lived a similar life to Barak Obama was a

marine in that world. There is something of an antinomy between the transworld theorist and the counter-part theorist, however.

There is intuitive plausibility in the transworld identity position, for when we speak of possibilities that might have occurred to an actual individual, we imagine that such a person exemplifies that possibility, not just someone like the individual. We think of possibilities as proper to us, which is to say that I have a property of possibly being bald. However, transworld identity is not without its philosophical problems. Michael Loux references the indiscernibility of identicals as problematic for transworld identity. For, suppose there is some person,  $p$ , that has a set of actual properties,  $F$ , and a set of possible properties,  $G$ , in world  $W_1$ . Now, there is a person,  $q$ , in  $W_2$ , and let us suppose that  $q$  in  $W_2$  is identical to  $p$  in  $W_1$ . If the indiscernibility of identity holds, then  $p$  and  $q$  should have all of their properties in common. However,  $p$ 's properties, which are contained in sets  $F$  and  $G$ , have different members than the sets of actual and possible properties  $q$  has. So it seems that they cannot be identical (See Loux 1979, 37).

Another problem Loux highlights for transworld identity is the problem of transitivity. The crux of the problem is to suppose that we have two non-identical objects,  $x$  and  $y$ , that actually exist in world  $W_1$ . We then suppose that the properties between  $x$  and  $y$  are exchanged such that  $x$  and  $y$  have traded one property in  $W_2$ , that first property and another property in  $W_3$ , and so on until we reach world  $W_n$  wherein all of the properties that  $x$  had in  $W_1$   $y$  has in  $W_n$  and vice versa. It seems that, at some point,  $x$  and  $y$  became identical with one another, even though they lack identity in  $W_1$ .

Lastly, Loux notes that there is, “...to use Alvin Plantinga’s useful expression, no ‘empirically manifest property’ we can appeal to...” when we seek to determine which object or individual in another possible world is identical to some object or individual in our world. For it may be that the very set of properties that we set out to look for in identifying an object or individual are just those properties that happen to be different, or that some other object or individual has those very same properties in the world in question.

These objections are not without rejoinders. For instance, it has long been noted that the indiscernibility of identity is not definitional of identity, and often admits of counter-examples. Those who hold to identity across time must contend with the fact that identity can hold between two temporally distinct individuals who have different sets of properties. The identity is held some other way-- say by a set of properties essential to making that individual self-identical. Likewise, if there is an essential set of properties, then the transitivity problem can be well explained. There cannot be self-sameness between two worlds while slowly interchanging away essential properties.

Konyndyk points out that the “interchange argument” operates on the assumption that “...*all* of a thing’s properties are essential to it—a very implausible position” (1986, 116). Konyndyk thinks that the argument trades on the idea that we cannot identify an essential set of properties, and so we willingly give them all up piecemeal.

The alternative to transworld identity is counterpart theory, championed by David Lewis. This is the idea that there cannot be identity across worlds. Rather, there is a sort of similarity between objects in possible worlds.

The strategy is to hold that while each individual is worldbound, an object existing in one possible world can have a counterpart in some other world. As Lewis puts it, an individual's counterparts are things that resemble it 'closely in both content and context in important respects' (Lewis 1979, 40).

There is no problem of the indiscernibility of identicals because the counterpart theorist doesn't hold that a counterpart is identical. We need not worry as to how to empirically identify the same object across worlds. Rather, content and context sufficiently pick out which individual is functioning as a counterpart.

Counterpart theory is, of course, not without its detractors. For it seems that under counterpart theory, our ordinary modal speech is vastly incorrect. For when I say that Barack Obama is possibly a Republican, I am speaking about a possibility that he has in virtue of himself, and not the idea that someone else in a possible world, who sufficiently resembles Obama, who also happens to be a member of the Republican party. It really doesn't tell us anything about Obama's own capabilities, or potentials. Likewise, if someone were to say that Socrates is necessarily human, that person seems to suggest that in all worlds where Socrates exists, he exists as a human. There is not, for instance, a world where Socrates, the individual who is identical to the teacher of Plato in the actual world, exists as a goldfish in some other world.

If the counterpart theory is correct, what exactly are we saying? It seems that we are saying that in order to be a counterpart of Socrates, one must be a human. *De*

*re* necessity, then, says something about what it takes to resemble Socrates, but that is not what most people think they are saying when they say that some *x* is necessarily *F*. That is, they don't have in mind that they have set forth some sort of resemblance condition for being a counterpart. As Loux explains:

...counterpart theory fails to accommodate our ordinary intuitions modal intuitions. Kripke argues this point in an informal way, focusing on the case of counterfactual discourse. He asks us to suppose that

(17) If Nixon had bribed a certain senator, he would have gotten Carswell through is true, and he points out that the truth of (17) gives Nixon grounds for regret. "If only I had offered the bribe!" Nixon might say. But Kripke contends that the counterpart theorist can make no sense of Nixon's regret here, for on his account, (17) isn't a claim about Nixon or Carswell at all, but a claim about their counterparts..." (Loux 1979, 40-41).

Aristotelian modality can offer a third alternative. While singular terms can belong to predicates with necessity, contingency, or possibility, those modal terms are not themselves understood within the semantics of possible worlds, but within the semantics of essences and accidents. Consequently, when one argues that "Mortal" necessarily belongs to "Human" and "Human" belongs to Socrates, I can conclude that mortal belongs to Socrates, not in virtue of an individual essence, or haecceity, but because a primary substance can be the subject of essential predications. Indeed, the "belongs" locution is used by Aristotle for both predication among class-terms and predication of class-terms to singularly named individuals (*APr.* 43a40-41). Moreover, if we assume that Socrates rigidly designates a particular human, then it may be permissible to say that Socrates is necessarily mortal in virtue of the fact that he is necessarily human and humans are necessarily mortal. Now this may seem as though I am taking back Aristotle's mixed apodictic syllogisms, which held that we could reach apodictic conclusions if



the right premise was apodictic, e.g. Barbara-LXL. So, why can we not construct the following syllogism:

1. Mortal necessarily belongs to all Human.
2. Human belongs to Socrates.
3. Therefore, mortal necessarily belongs to Socrates.

There is a difference in the way “Human” belongs to “Socrates” and “Mortal” belongs to “Human”. Consequently, if we take the model I am seeking to establish to bridge between terms and individuals, we might propose the following, where “Mortal” is  $B$ , “Human” is  $A$ .

1.  $Ba_LA \supset (\forall x)\Box(Ax \supset Bx)$  (premise)

In other words, given the necessity of  $B$  belonging to  $A$ , we can infer that it is necessary that should any individual  $x$  be  $A$ , it would have  $B$  as well. Indeed, if we suppose, where  $s$  stands for Socrates.

2.  $As$  (premise)

Then, we could only infer,  $Bs$ . That inference is only possible, if we assume the following:

3.  $Ba_LA$  (premise)
4.  $(\forall x)\Box(Ax \supset Bx)$  (1,3 MP)
5.  $\Box(As \supset Bs)$  (4 UI)
6.  $As \supset Bs$  (5 NE)
7.  $Bs$  (2,6 MP)

To reach the claim that Socrates is necessarily mortal, we would need a premise that states,

8.  $\square$ As (premise)

This is to say that Socrates is necessarily a human, which occasions some of the metaphysical questions we have raised about actualism, and whether it is not possible for Socrates to have ever existed without his essence. Ultimately, we may remain agnostic on the point, but allow that, should anyone make such an assertion, one could conclude:

9.  $\square$ Bs (5,8 MMP)

That is, if Socrates is necessarily a human, and since necessarily, if Socrates is a human, then Socrates is mortal, then by Modal Modus Ponens, we can reach that modal claim in the conclusion. Now this goes beyond Aristotle's specific project in the Modal Syllogistic, but it is not outside of the realm of Aristotelian metaphysics. In Aristotle's account of change, he distinguishes between two kinds, what has come to be called accidental change and substantial change. The example of accidental change that he provides is that of the unmusical man becoming musical (*Phys.* 190a13-31). According to this account of change, while the unmusical does not remain when the man becomes musical, the man survives the change. In other words, it is possible for a man, like Socrates, to become musical and still be essentially the same man. However, in substantial change, the underlying matter of a substance is utilized to make a new substance, which is not essentially the same individual as the one before. The matter may remain the same, but the substance does not survive. If so, we might reasonably infer that Socrates, the individual, can have *de re* properties, which obtain in him so long as he subsists as a human being.

That mortal necessarily belongs to all animal is to say something about the substance-term “animal” and the essential properties it has, not with respect to every possible world where individual animals exist, but a necessity founded on the what-it-is question that is so fundamental to Aristotle. The question of whether individuals exist across possible worlds, whether through some mereological composite or by some other means of identity, can be left an open question for the Aristotelian. The question of what is possible, actual, and necessary, are questions that revolve around a different ontological level, on the level of the secondary substances, as I have argued, or what he analyzes as substance in Book Zeta of the *Metaphysics*. In effect, Aristotle’s modal syllogistic completely avoids the Quinean objection against so-called “Aristotelian Essentialism” because it deals with what is essential to kind-terms rather than individuals. While Quine worries that *de re* modality will make it necessarily true that there are nine planets, Aristotle’s modality is concerned with the nature of “Planet” as a kind. Only after that is settled can we navigate the scientific question of how many planets can properly be counted as instances of that kind, and which ones are more properly classified as planetoids, dwarf planets, or moons. The Aristotelian, too, may object to Quine’s “Aristotelian Essentialism”.<sup>38</sup>

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<sup>38</sup> George Goodwin points out that process philosophers also objected to Quine, precisely due to the issue of Transworld identity. He writes, “The problem of transworld identity, then, requires (1) a clear resolution of the logical paradox about identity and difference and (2) a clear formulation of the criterion used to distinguish essential from accidental properties. Quine, of course, supposes that the problem of transworld identity cannot be resolved, because proponents of Aristotelian essentialism cannot provide a clear criterion for identifying essential properties... And, in general, he assumes that Aristotelian essentialism is fraught with problems because metaphysics itself is fraught with

So, Aristotelian modality and essentialism ground predications that individuals have in their natures. Such predications do not raise the question of the identity of individuals across worlds. Moreover, essential and accidental predications are said of individuals in virtue of their natures and not in virtue of resemblances to other individuals in other worlds. Nonetheless, even Aristotelian essentialism, as we understand it, raise some interesting questions about identity, since it will be the case that the individual, Socrates, can survive a certain degree of change but cannot survive the loss of certain essential properties predicated of him. Aristotle's metaphysics of change runs up against the Leibnizian laws of identity insofar as an individual substance can change and remain, in a certain sense, the

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problems (2003, 184). Goodwin argues that any form of substance metaphysics, including Aristotelian essentialism, has difficulty specifying a criterion for individual identity (2003, 191). He contrasts strict identity, which Goodwin defines as changeless and concrete, with the "neo-classical" genetic identity of A.N. Whitehead and Charles Hartshorne, which is changeless and abstract. According to Goodwin, the Aristotelian view of substance metaphysics depends upon a notion of identity that is strict, and this leads to the sort of referential opacity problems that Quine exploits in his critique of quantified modal logic, and essentialist metaphysics generally. Goodwin believes the neo-classical alternative escapes Quine's critique: "Consider now the Whiteheadian-Hartshornean interpretation of the statement, 'Mary was sick yesterday, but she is well today.' Each momentary subject (concrete state, event) is strictly identical with itself; but the identity which binds them together as instances of the same substantial subject is genetic, rather than strict. From the different momentary subjects, Mary-sick-at-time-x and Mary-well-at-time-y, we abstract the common features that constitute Mary-as-identical. The identity of the concrete events is strict, and the identity of abstract substances is genetic. And the events contain the substance. We have, then, two proposals for explaining individual identity: "Either there is one subject sick-now and well-later—that is, with sickness and health coexistent in diverse temporal parts of itself (which spatializes time and contradicts the very meaning of temporal)—or there is a sick subject actual now, and later another nonsick subject may be actualized belonging to the same genetic series." My argument has been that the former option, "Aristotelian essentialism," interprets substantial identity in a way that should present problems to Quine and to the rest of us as well; but that the neoclassical interpretation of substantial identity provides a coherent alternative that avoids these problems" (2003, 192-193). What I should like to suggest is that there is nothing particularly "neo-classical" in this view. Indeed, the view ascribed to Aristotle is a modern invention that results from attempting to interpret Aristotelian essentialism within the rubric of first-order logic and definite descriptions. Genetic identity, insofar as it seeks to abstract common features, is more akin to the logic of categories, in which it is terms that are predicated of one another.

same substance. So we shall examine the question of identity and Aristotelian essences more closely in the next few sections.

#### 4.5 Identity and Essence

Our contemporary use of identity as a primitive logical connective, or as a binary relation, has a perceived history that is rooted in the Leibnizian laws<sup>39</sup> and perpetuated by Gottlob Frege so that it is accepted as common currency among those in the analytic tradition. The Leibnizian principles, i.e. *the principle of the indiscernibility of identicals*, *the principle of the identity of indiscernibles*, and *the principle of substitutivity*, are said to formally regulate our usage of identity as a connective within deductive systems. For Frege, and others, identity is primitive in the sense that it cannot be defined. This is expressed by Frege while in correspondence with Husserl, where he wrote:

Since any definition is an equation, identity itself cannot be defined. Leibniz's explanation could be called a principle that brings out the nature of the relation of identity, and as such is of fundamental importance" (Frege 1997, 226).

I take Frege's point to be that any definition of identity will have to assume identity within the definition, and so will be circular. This puts any metaphysical discussion of identity in peril, since we are not even able to set out a definition as a starting point. Moreover, we are inexorably led to paradoxes. Given my defense of

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<sup>39</sup> Leibniz's Laws could be said to be related to Aristotle's own discussion of sameness: "Whether two things are the same or different, in the most strict of the meanings ascribed to 'sameness' (and we said that the same applies in the most strict sense to what is numerically one), may be examined in the light of their inflections and coordinates and opposites. For if justice is the same as courage, then too the just man is the same as the courageous man, and justly is the same as courageously" (*Top.* 151b25-33).

the coherence of Aristotelian modality, as rooted in the substantive nature of things, in the following sections the reader will find an alternative perspective on these regulative principles in light of the debate between David Wiggins, with his sortal dependency theory, and relative identity theorists, like Peter Geach. My ambition is to synthesize the insights of both views, and use Aristotelian modal logic as an analog to devise a modification to the Leibnizian principles informed by a semantics of identity where Aristotelian categorical terms, properly construed, supplants sortals as the terms upon which identity depends.

#### 4.5.1 Leibniz's Laws and Regulating Identity Relations

The rules surrounding identity have taken on the aura of logical principles attached to “=”, designating specific inferences governing this binary relation.

Many logic texts treat rules that they associate with “=” as formal and to be used in logical deduction alongside *modus ponens* and the disjunctive syllogism.

Formally, we can express these rules as follows:

- (I)  $(\forall x)(\forall y)[(x = y) \supset (\forall \Phi)(\Phi x \equiv \Phi y)]$  [indiscernibility of identicals]
- (II)  $(\forall x)(\forall y)[(\forall \Phi)(\Phi x \equiv \Phi y) \supset (x = y)]$  [identity of indiscernibles]

These can be combined to express one law:

- (III)  $(\forall x)(\forall y)[(x = y) \equiv (\forall \Phi)(\Phi x \equiv \Phi y)]$  [Leibniz's law]

Pascal Engles notes that, from (I) and (II), or from (III) above, “...all of the properties characteristic of the relation of identity can be deduced”, namely:

- (IV)  $(\forall x)(x = x)$  [reflexivity]

- (V)  $(\forall x)(\forall y)((x = y) \supset (y = x))$  [symmetry]  
 (VI)  $(\forall x)(\forall y)(\forall z)((x = y) \& (y = z) \supset (x = z))$  [transitivity]

The *substitutivity of identical* is formulated by Leibniz in the following manner: *eadem sunt quorum unum potest substitute alteri salve veritate*, or “...terms of which one can be substituted for another without affecting the truth [of the proposition in which they figure] are identical” (Engel 1991, 184). These are the regulative principles that Frege believes brings out the nature of identity, even if they do not define identity.

Engel notes that each of these principles has *prima facie* plausibility, and in cases like the reflexivity of identity, appear to be axiomatic, or intuitively obvious. However, one of Frege’s greatest contributions to philosophy grows out of his recognition of the limits of something as fundamental as the reflexivity of identity. For, on the one hand, identity is a binary relation that appears to relate two entities. If there are two distinct entities, then clearly they are not numerically identical. On the other hand, it seems trivial or vacuous to say that one entity is identical with itself. Frege’s solution to this problem is to say that identity is not a binary relationship between objects, but names. Hence, the question of whether the “Morning Star” is the same as the “Evening Star” is resolved by uncovering whether the reference is the same for two different senses to which we give a name. Once we uncover that the Morning Star is identical to the Evening star, we discover that we are referencing one object with two different senses. While sense determines the reference, there can be more than one path from various senses to the same referent. So, despite the difference in sense, if the Morning Star and the Evening Star

refer to an object that is numerically the same, then the Morning Star and the Evening Star should share all properties in common.

This occasions further difficulties for Leibniz's laws, for as Engel points out, "If we acknowledge the Fregean principle according to which the real reference and the apparent reference of the expression diverges in [non-extensional] contexts, we have no reason to expect that this principle will be valid" (Engel 1991, 186). Indeed, we have seen that Quine's rejection of Aristotelian essentialism has hinged on a similar point. This point will become crucial latter on, so it is important that I explain why this is so. For instance, suppose that our friend Viola is masquerading as Cessario. We might say that Viola is identical to the one who is masquerading as Cessario. But, Olivia might believe that the one masquerading as Cessario *is* Cessario, while she clearly does not think that Viola *is* Cessario. This seems to invite contradiction, since the transitivity of identity would suggest that Olivia both thinks and does not think that Viola is Cessario. Comedy ensues, but we do not think that the metaphysics of identity has broken down, even if our formal principles have failed us. So those who are committed to sustaining these regulative principles as expressing the nature of identity in an absolute sense will restrict the inference of the identity relations to extensional contexts. Nonetheless, this is the least of all problems for the absolutist view of identity that follows from a straightforward Fregean embrace of Leibniz. There are many known paradoxes pertaining to identity. Harry Deutsch (2007) provides a helpful overview of the more common paradoxes of identity including Chrysippus' paradox, which will serve as our



paradigm example of a change paradox in this section. So in Chrysippus' paradox we imagine a dog, call him Oscar. Deutsche explains:

Suppose that at some point  $t'$  in the future poor Oscar loses his tail. Consider the proper part of Oscar, as he is now (at  $t$ ), consisting of the whole of Oscar minus his tail. Call this object 'Oscar-minus'. Chrysippus wished to know which of these objects — Oscar or Oscar-minus — survives at  $t'$ . According to the standard account of identity, Oscar and Oscar-minus are distinct at  $t$  and hence, by [the necessity of distinctness], they are distinct at  $t'$ . (Intuitively, Oscar and Oscar-minus are distinct at  $t'$  since Oscar has a property at  $t'$  that Oscar-minus lacks, namely, the property of having had a tail at  $t$ . Notice that this argument involves a tacit appeal to [the necessity of distinctness] — or [the necessity of identity], depending on how you look at it).<sup>40</sup> Hence, if both survive, we have a case of two distinct physical objects occupying exactly the same space at the same time. Assuming that is impossible, and assuming, as commonsense demands, that Oscar survives the loss of his tail, it follows that Oscar-minus does not survive. This conclusion is paradoxical because it appears that *nothing happens* to Oscar-minus in the interval between  $t$  and  $t'$  that would cause it to perish (2007, n.p.).

Since Oscar is distinct from Oscar-minus, they are not the same, yet both seem to occupy the same spatio-temporal locations at certain times, and have the same causal history. This paradox troubles those who hold to a strict and absolute view of numerical identity and to non-discernability as central to identity. In a sense, the absolutist view of identity is a hangover from a Parmenidean view of being. But while Aristotle has long ago teased out “becoming” from being through the act/potency distinction, the Leibnizian laws have held the fort for the absolutist even despite these paradoxes. And, in fact, there are deep connections between the metaphysics of being and identity, so perhaps it is appropriate that, like Being, Aristotle notes,

...sameness is a term used in many ways, see whether things that are the same in one way are the same also in a different way. For there is either no necessity or even no possibility that things are the same specifically or generically should be numerically the same—and we face the question whether they are or are not the same in that sense” (*Top.* 152b30-34).

For Aristotle views numerical sameness as a strict identity relation, but modally qualifies the relationship between what we might call type and token identity

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<sup>40</sup> NI states:  $(a = b) \rightarrow \Box (a = b)$ ; ND states:  $(a \neq b) \rightarrow \Box(a \neq b)$

depending on the type concept in question. This provides the backdrop for my own speculations about identity vis-à-vis the contemporary projects of Geach and Wiggins who both seek, in somewhat similar ways, a return to the Aristotelian insight enunciated in the *Topics*. With a clearer conception of the metaphysics and logic of Aristotelian modality in place, it is my aim to suggest ways in which a sortal relative or dependent theory of identity can be advanced within an Aristotelian framework. So, we shall see that such a view might not just resolve issues surrounding transworld identity, but identity paradoxes tied to change in the actual world.

Engel notes two sorts of reactions from contemporary philosophers in light of these change paradoxes (1991, 188). Some have attempted to ground identity on some principle of individuation; viz. through physical or spatio-temporal continuity, or by continuity of memories as a ground for personal identity. But even with a principle of individuation in hand, it is not clear that all of the regulative rules of identity would apply. Other philosophers recognize identity to be defined by a relative criterion for individuation. We shall consider the latter philosophers first.

#### **4.5.2 Relative Theory of Identity**

Those who advance the relative theory of identity point out that our ordinary usage of identity does not match up with the absolute notion of identity, regulated by Leibniz's law. According to the relative identity theorists, like Peter Geach, "It is possible for object  $x$  and  $y$  to be the same  $F$  and yet not be the same  $G$ , (where  $F$  and

G are predicates representing kinds of things (apples, ships, passengers) rather than mere properties of things (colors, shapes)” (Deutsche 2007, n.p.). In each case, ‘same’ cannot mean absolute identity.” Though some relative identity theorists deny absolute identity, the modest form discussed here simply states that at least some identity claims are not absolute. Geach holds that the expression “ $x$  is identical to  $y$ ” is an incomplete expression, “...it is short for ‘ $x$  is the same  $A$  as  $y$ ’, where  $A$  represents some count noun, understood from the context of utterance – or else, it is just a vague expression of a half formed thought” (Geach 1972, 238). Many metaphysical puzzles and paradoxes trade on such half-formed expressions, according to Geach.

To understand Geach, we must situate his theory within his dialogue with Quine, who advances an absolutist view of identity.

Absolute identity seems at first sight to be presupposed in the branch of formal logic called identity theory. Classical identity theory may be obtained by adjoining a single scheme to ordinary quantification theory (for bound name-variables)...Quine in his *Set Theory and its Logic* attributes to Hao Wang the recognition that [the theorem:  $Fa \equiv (\forall x)(x = a)$ ] will serve as a single axiom schema for identity theory (*ibid.* 238).

Put simply, Quine wants to treat identity strictly as a binary relation, set within some theory  $T$ . Under Quine’s conception of identity, this binary relation will maintain truth-conditions across any theory. Geach explains:

“...if we find an I-predicable in a theory  $T$ , we should construe the range of quantifiers in  $T$  as a class of objects for which the I-predicable expresses absolute identity, and construe the other predicable of  $T$  correspondingly. In a wider system  $T'$  the range of the quantifiers may on this principle turn out to be different; and then, although each complete sentence of  $T$  will survive unchanged in  $T'$  with the same truth-conditions as before, the *parts* of each sentence will need reconstructing in definite ways” (*ibid.* 242).

Quine is concerned with preserving truth-conditions across theories, but Geach is disturbed by the need to “reconstruct” parts of each sentence. And while Geach

agrees with the insight that identity must be set within a theory, his contention is that identity will have to be relative to the theory in which it is set. "If... we consider a predicable that is an I-predicable *relative to a theory T*, then this need only express indiscernibility relative to the ideology of T, and need not express strict unqualified identity" (*ibid.* 241).

Geach does not think that Quine is such a novice as to commit himself to a logical error in his absolutist treatment of identity. So he grants that truth-conditions can be preserved as one moves from wider or alternate theoretical systems. For, while truth-conditions can be maintained between the systems despite changes in ideology, Geach argues that the import of the sentence appears to change (*ibid.* 243). Geach provides a concrete example in considering a theory T that treats two tokens of a word in a particular volume as instances of the same type such that *Exy* could be interpreted in two different ways: 1) *x* is a token equiform with *y*, and 2) *x* is identical with *y*. He says that Quine would have us stipulate that, under T, we apply (2) as our interpretation of *Exy*. By broadening his theory to T<sup>1</sup> by introducing a predicable, Geach allows one to distinguish between the instances of tokens of the same word (*ibid.* 242). The change in theory means that the quantifiers direct us to (1) as the interpretation of *Exy*. While Quine is correct to say that for the truth-conditions of *Exy*, the token relation clearly has a different sense than the type relation, it seems as though Quine's pragmatism leads him to think that truth-functional equivalence between the two is sufficient to preserve his absolutist view. But Geach reprimands Quine by his own principles, saying:

...this way of interpreting quantifiers so as to get an absolute identity involves a sin against a highly intuitive methodological programme, clearly enunciated by Quine himself—and

obviously Quine himself ought to be worried about this objection. The programme is that as our knowledge expands we should unhesitatingly expand out ideology, our stock of predicables, but should be much more wary about altering our ontology, the interpretation of our bound-name variable (*ibid.* 243).

In other words, in order for Quine to achieve his aim of preserving identity, he must alter what the predicable *Exy* means between systems. As odd as it is to say, Quine's bloated ontology appears to be a convolution dedicated to making elliptical orbits appear circular. Quine is, in effect, altering the ontological structure between T and T<sup>1</sup> by changing what the predicable names and then declaring that because this change is a substitution *salve veritate*, absolute identity is preserved. Geach turns the tables on Quine by showing that his preference for absolute identity leads to a less than parsimonious, i.e. un-Quinean, ontology.

Geach's solution to identity could be said to have the advantage of being ontologically parsimonious, since it does not lead to new senses for every predicate between theories. But perhaps the more important feature is that it allows us to make sense of some of the paradoxes of identity previously mentioned. Consider, again, Oscar and Oscar-minus. Oscar is said to be non-identical to Oscar-minus, since Oscar is discernibly different, being larger than the part we individuated as Oscar-minus. Geach's relative identity theory allows us to say that Oscar is the same *dog* as Oscar-minus, but that Oscar does not have the same *proper parts* as Oscar-minus, since Oscar quite clearly has a tail. So, in one relative sense, viz. with respect to proper parts, Oscar and Oscar-minus are non-identical.

For Geach, identity is relative to the theory, but he also maintains in *Logic Matters* that, "...any equivalence relation, any relation that is non-empty, reflexive in its field, transitive, and symmetrical, can be used to specify a criterion of relative

identity” (*ibid.* 249). A couple of decades later, Geach seemed to recognize that his view stands at odds with the Leibnizian principles, but that did not seem to be his initial aim when he devised this theory of identity (see Noonan 2009, n.p. ).

### 4.5.3 Sortal Dependency Theory of Identity

David Wiggins rejects the relativity of identity as he argues that it is in violation of Leibniz’s laws. His argument is an indirect proof which aims to show that Geach’s relative identity theory cannot be consistent and maintain Leibniz’s laws. The argument is as follows, let  $a =_F b$  be “a is the same F as b” and  $a \neq_G b$  be “a is not the same G as B”:

1.  $[(a =_F b) \& (a \neq_G b)] \& Ga$  (Assumption IP)
2.  $(\forall \phi)(\forall x)(\forall y)[(x =_F y) \rightarrow (\phi x \equiv \phi y)]$  (Indiscernibility of Identicals)
3.  $(a =_F b) \& (a \neq_G b)$  (Simp 1)
4.  $(a =_F b)$  (Simp 3)
5.  $(\forall x)(\forall y)[(x =_F y) \rightarrow [(x =_G x) \equiv (x =_G y)]]$  (UI 2)
6.  $(\forall y)[(a =_F y) \rightarrow [(a =_G a) \equiv (a =_G y)]]$  (UI 5)
7.  $(a =_F b) \rightarrow [(a =_G a) \equiv (a =_G b)]$  (UI 6)
8.  $(a =_G a) \equiv (a =_G b)$  (4,7 MP)
9.  $Ga$  (1 Simp)
10.  $Ga \rightarrow (a =_G a)$  (9 Reflexivity of Identity)
11.  $a =_G a$  (9,10 MP)
12.  $[(a =_G a) \rightarrow (a =_G b)] \& [(a =_G b) \rightarrow (a =_G a)]$  (8 Equiv)
13.  $(a =_G a) \rightarrow (a =_G b)$  (12 Simp)
14.  $(a =_G b)$  (11,13 MP)
15.  $(a \neq_G b)$  (3 Simp)
16.  $(a =_G b) \& (a \neq_G b)$  (14,15 Conj.)
17.  $\sim\{[(a =_F b) \& (a \neq_G b)] \& Ga\}$  (1-16 IP)<sup>41</sup>

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<sup>41</sup> This argument is adapted from Engel (1991, 190-191) with my own modifications.

The above proof appears to be a decisive refutation of the compatibility of the relativity of identity with Leibniz's laws. Hence, Deutsche (2007) reports, "[Relativity of Identity] and [Leibniz's Law] are incompatible in the sense that within the framework of a single fixed language for which [Leibniz's Law]..." Engel considers whether we can abandon Leibniz's Law, "What would be the objects which would be individuated according to a relative principle of identity of indiscernibles? *Relative* objects? But can there be such an ontology of *relative* objects?" (Engles 1991, 191). As mentioned in the previous section, Geach comes to recognize this, but the question will be whether this is a decisive blow to the relative theory of identity and one's commitment to Leibniz's laws.

Wiggins' program is not merely to be destructive of Geach's theory. He attempts to rescue some of the insights of the relative theory while preserving Leibniz's law. Wiggins' theory of identity, which he calls the sortal dependence of individuation, or **D**, maintains the idea that an identity will relate to some unspecified sortal concept, but it is not said to be a relative theory of identity, since it attempts to reconcile sortal dependence with Leibniz's laws.<sup>42</sup> Wiggins explains **D** in the following manner:

**D:**  $a = b$  if and only if there exists a sortal concept  $f$  such that

1.  $a$  and  $b$  fall under  $f$ ;
2. to that  $x$  falls under  $f$  or that  $x$  is in  $f$  is to say what  $x$  is (in the sense Aristotle isolated)
3.  $a$  is the same  $f$  as  $b$ , that is coincides with  $b$  under  $f$  in the manner of coincidence required for members of  $f$ ... (Wiggins 2001, 56).

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<sup>42</sup> See Engel (1991, 191) for a succinct explanation of Wiggins' project in developing a sortal dependency theory of identity.

Wiggins writes, “**D** unfolds and articulates the collateral and coeval ideas by the possession of which we deploy the ideas of entity, identity, and substance upon that which confronts us in experience” (*ibid.*). Yet, if **D** unfolds such metaphysical concepts, it also does so, according to Wiggins, according to an Aristotelian framework in which an entity belongs to a substance term. I shall argue that if  $x$  is to be in  $f$  in the way Aristotle isolated, then we must recognize the two-tiered distinction of primary and secondary substances, be attentive to how Aristotle distinguishes between modal claims of categorical terms, and what can be said of singular terms. I will recommend, then, that Wiggins adopt the modality of Aristotle to understand identity statements.

Engel notes that this avoids the previous problem, demonstrated in the indirect proof against the relative theory, because “... $a$  can be the same  $F$  as  $b$  without being the same  $G$  as  $b$ . This is because here we are making the hypothesis that if  $a = b$ ,  $a$  is the same  $F$  as  $b$ , but cannot be the same  $G$  as  $b$  if the predicates  $F$  and  $G$  state distinct conditions of continuity” (Engel 1991, 192). This means that when we sortal-predicate  $F$  of  $x$  at some time  $t$ , it is not possible for  $x$  to be some other sortal predicate  $G$  at  $t$ . Wiggins, quite explicitly, is pushing Geach’s insight in an even more Aristotelian direction, solving what we have characterized as a Parmenidean problem of identity by returning to an idea of essences as mutually excluding sortal-types (Engel 1991, 195). As Wiggins puts it,

Where  $f$ s make up a natural kind, the only possible worlds we need consider, for these are the only possible worlds having within them any entity that is an  $f$ , are worlds sufficiently similar in nomological respect to the actual world to exhibit specimens relevantly similar to actual specimens. This means that, for the very special case of natural kind, we find that ordinary necessity virtually coincides with physical necessity, and ordinary possibility with physical possibility (Wiggins 2001, 85).



So, Wiggins is concerned to discuss natural kinds and the essential and accidental properties they exhibit given worlds very much like the actual world.

To return to the example of Oscar, Oscar cannot be both a *dog* and a *collection of the proper parts* that make Oscar at the same time. Oscar is, in the Aristotelian sense, a dog, not, as Quine might suppose as a possible interpretation, undetached-dog-parts (see Wiggins 2001, 128). Engel aptly explains,

In this sense, for Wiggins, the paradigmatic sortal concepts are comparable with 'natural kinds' terms like 'tiger', 'aluminium' or 'apricot.' As Leibniz and Locke had already noted, and as Putnam and Kripke have pointed out again, when we use these terms, we are supposing that there is an underlying common essence, without our use resting on their association with a list of necessary and sufficient characteristic that define them (Engel 1991, 195).

However, Engel raises a worry associated with Wiggins' sortal dependency theory. For, in returning to what Wiggins believes is something like Aristotelian essences, identity will seemingly depend upon or implicitly entail *de re* modal quantification (Engel 1991, 193). Engel writes, "Since all quantification supposes identity, it is therefore to acknowledge that Leibniz's laws can be valid in these contexts and, consequently, the *substitutivity* of co-referential expressions in these contexts that it implies" (Engel 1991, 193). If Wiggins is not careful in how he develops his notion of identity, he risks infecting all aspects of quantified logic with referentially opaque contexts not unlike the one's Shakespeare's comedies so often trade upon. Engel's solution to this problem is to distinguish between weak and strong essential properties. Strong essential properties, "...are supposed to be necessary and sufficient individuation conditions of an object; and *weak* or *trivial* essential properties... provide no individuation conditions of an object" (Engles 1991, 157). Engel argues that Wiggins's theory of identity "...only presupposes a weak

essentialism" (Engel 1991, 193). Whether Engel is correct about Wiggins's is difficult to adjudicate. For, on the one hand, Wiggins's primary concern is to outline precisely how, and to what extent his sortal dependence theory can simultaneously support a principle of identity and individuation. So if Wiggins's sortals provide the necessary and sufficient conditions for at least some sorts of essences, then he advances a strong essentialism. However, Wiggins seems to anticipate this problem in some ways, or is generally wary of *de re* modal contexts, and so says,

We have resisted the idea that a theory of individuation must be a set of judgments about all possible worlds, or occupy itself with problems that are special to the making of statements of explicit necessity *de dicto* or *de re*..." (Wiggins 2001, 107).

To weaken his essentialism, Wiggins goes on to stipulate a series of rules which happen to help him avoid an explicitly *de re* essentialism. And these rules amount to placing the principle of individuation not in the sortal concept, as previous chapters seemed to suggest, but in conceivability (see Wiggins 2001 110-111). Wiggins is concerned to avoid the semantics of possible worlds, and other pitfalls of the contemporary modalist account of essences. So, Wiggins elucidates *de re* predication in terms of whether it is possible to conceive or not conceive that  $x$  is  $\phi$ . However, Wiggins also explains that "The position of the boundary between what one can conceive of  $x$  and what one cannot conceive of  $x$  depends on  $x$ , i.e. depends on which thing the thing  $x$  *actually is*" (2001, 111). This is, however, somewhat confusing, for is it  $x$  that specifies how we conceive the limits of whether  $x$  could be  $\phi$ , or is it *what  $x$  is* which determines those limits? Why should  $x$  itself determine anything about what properties it has *de re*? Finally, does this not reduce the conceptual aspect of Wiggins theory to the role of understanding or grasping the

ontological facts regarding  $x$ ? This latter question makes the conceptual aspect of Wiggins's theory appear to be only an epistemological component of **D**, i.e. how can we know what sort of accidental and essential properties  $x$  has. The former questions appear circular, for we would already have to know what sort  $x$  is to determine whether  $x$  could be  $\phi$ . While Wiggins attempts a synthesis between realism and anti-realism, referring to his position as *conceptualist realism*, he seems to recognize that this move is unpalatable, as he remarks that it will invite open hostility among his critics, both from anti-conceptualist realists, and anti-realist conceptualists (Wiggins 2001, 140). Stephen Yablo shares my concern that, "...[t]he challenge for Wiggins is to explain what else but conceptualism could legitimate the move from 'we have a certain style of tracing' to 'there is a corresponding sort of object' (2003, 4). Aristotle, with his two-tiered ontology, would not seek to understand whether  $x$  could be  $\phi$  by turning to some conceptual analysis of  $x$ . Rather, we must turn to an analysis of the kind term, determine its nature, and what can be said apodictically or contingently about it. Once we have grasped this, we can say whether some  $x$  is likewise predicated with such features. I would suggest that this process would involve Aristotelian science as it is conceived in the previous chapter, where definitions and explanations are built up from first principles and demonstrations, into a full knowledge of the essence of things.

It seems that Wiggins wants to ground identity in an explicitly Aristotelian essentialism, but then retreat to conceptualism when explaining how his sortal concepts individuate. Wiggins addresses this issue of circularity, saying,

In order to trace  $a$ , one must find out what  $a$  is, or so I have said. But what counts as knowing what  $a$  is in the Aristotelian sense? What counts as a sortal concept for a continuant?

Suppose the answer depends in part on the idea of a principle of continuity for *a*. What then of the determinable *continuity* (or *coincidence*) of such and such sort-specified kind? Has the nature of this determinable been explained?

Here we are caught in a circle which would be vicious if we thought we were bringing the concept of identity into being by means of other ideas prior and better understood. The circle would be vicious if we could not appeal to some extant *a priori* understanding of the identity relation or we could not invoke a going practice which will effect [sic] the sortal articulation of individual continuants as *this* *f* or *that* *g* or whatever. Happily, though, we lack none of these things, unless we want to offer up Leibniz's Law to some numen of confusion (*ibid.* 59).

My concern, however, is whether Leibniz's Laws sufficiently map onto the sortal concept. Wiggins go so far as to say that *f* is what *x* is in the Aristotelian sense, where *x* coincides with *y* under sortal concept *f* if and only if, "...the way in which *x* is *f*-related to *y* suffices for whatever is true of *x* to be true of *y* and whatever is true of *y* to be true of *x*" (*ibid.* 60). It seems, then, that *f* is a sortal term if two objects that coincide under *f* can be substituted for one another *salve veritate*. But knowing if this is so will, itself, require an understanding of the *de re* possible and necessary predicates said of *f* and cannot be settled by merely considering whether *x* and *y* can be substituted for one another under *f*. In other words, to determine what *x* is (or *y* is) we need to know what *f* is, and our knowledge of *f* is not going to be determined by considering whether *x* and *y* are substitutable under *f*. As Socrates might say to Meno, this is to give a swarm of bees when we need to know the nature of Bee (Meno 72a-b). Indeed, how could we know whether *x* and *y* can be substituted one for the other under *f* unless we knew what *f* is. Only then could we know that *x* and *y* are at all *f*-related. In other words, we must overcome Meno's paradox in order to discuss whether any individual thing bears an identity relation under a sortal. Aristotle's solution is found in the *Posterior Analytics*, and insofar as I have argued that the *Posterior Analytics* depends upon the modal syllogistic for demonstration, if

Wiggins wants an Aristotelian solution to this problem of circularity, he may also want to turn to Aristotle's demonstrative science and his modal syllogistic.

For he knows that quantifying over *de re* modal contexts are non-extensional. And if his sortal-dependency leads to a strong form of essentialism, then his theory of identity, and all quantifications supposed on that theory, are referentially opaque, and invalid. So, Engles writes,

Essentialism, according to Wiggins, can be limited to the assertion: for every object of reference, there is an essential sortal feature by virtue of which, if we distinguish this object as *this F*, it cannot fail to be *this F*.. Wiggins' s essentialism is realism in so far as it supposes that there are objects in the world independent of our mind that cannot fail to have the essential features they have. This is a conceptualism in so far as the essential properties that we are capable of articulating depend on *our* classifications" (Engel 1991, 195).

To some, Wiggins' solution comes across as a marriage of convenience—an *ad hoc* attempt to merge so may divergent positions together: identity that is relative to and dependent upon a sortal-concept and regulated by absolute Leibnizian laws, sortal dependency understood as an Aristotelian τόδε τι but articulated and determined by our conceptions. For if identity depends upon sortals, and sortals depend upon *our* classifications, then would it seem that Wiggins's theory ultimately can be understood as a conceptualist-dependent theory?

Setting aside, for the moment, Wiggins's *conceptual* realism, another concern is that an identity that is relative to a conceptual "general term" would also involve an intensional context. For under a purely conceptualist-dependency model the full expression would be something like: *x* is identical to *y* if and only if there is some concept *F* such that *x* and *y* belong to *F*. But surely under a pure conceptualism, concept *F* is itself a particular that exists in a mind thinking about identity at different moments. So we might end up with a case where *x* and *y* are annexed to

concept F at time  $t$  and  $x$  and  $y$  are annexed to concept G at time  $t^1$ . If  $x$  and  $y$  are to preserve identity conditions relative to successive concepts, upon which they depend, then we must posit that F is identical to G, but this is only if there is a concept, say F', to which F and G belong. A conceptualist-dependent identity theory cannot avoid regress while maintaining spatio-temporal identity of the very concept upon which identity is grounded.<sup>43</sup> So much the worse for a pure conceptualism, you might say, but in terminating the regress at any point in the conceptualist account, it becomes apparent that there is going to be a particular concept set within a particular mind. So saying that  $x$  and  $y$  are identical if and only if there is some concept F to which they belong, is, to borrow from Geach, an incomplete expression. It is really to say that  $x$  and  $y$  are identical if and only if there is a mind thinking about some concept F to which  $x$  and  $y$  both belong (in the relevant sense of belonging). And this is to set conceptualist dependent identity within an intensional, i.e. referentially opaque, context, e.g.  $x$  and  $y$  are identical when I think of some concept F to which they belong. Again, Wiggins might respond that his *conceptual realism* avoids the particularism of a pure conceptualism. And perhaps it does! But can it really navigate between the non-extensional extremes of a *de re* Charybdis and the Scylla of conceptualism? Or, to put the question differently, how does an

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<sup>43</sup> My critique of conceptualism is loosely based on Husserl's critique in the *Logical Investigations*. In writing against Locke, Berkeley and Hume, he writes, "We cannot predicate exact likeness of two things, without stating the respect in which they are thus alike. Each exact likeness relates to a Species, under which the objects compared are subsumed: this Species is not, and cannot be, merely 'alike' in the two cases, if the worst of infinite regresses is not to become inevitable (Husserl 2005, 242; LU I.2.1.2, 3). This regress has some similarity to the third man argument, in which an infinite regress of forms threatens against extreme realism.

extensional identity arise when it is dependent on the union of referentially opaque contexts? To his credit, Wiggins humbly admits to his critics, "I shall be content for the most part to clarify the claims of the particular conceptualism defended here to be one form of realism" (2001, 142). Wiggins assures us that conceptual realism, properly understood, is a mundane theory. I should like to venture an alternative solution through an even bolder return to Aristotelian metaphysics. In so doing, he might posit that the kind terms to which  $x$  and  $y$  belong, are real sortals of which the mind grasps and conceives. The question, then, would be the extent to which that sort of realism would be "conceptualist". If it is conceptualist in the sense that the formal aspects of the term inform the individual and the concept, then this sort of realism may just be called an Aristotelian realism, or moderate realism, where the mind is capable of conceiving of what objects actually are because the intellect is capable of grasping the formal aspects of objects.

#### **4.5.4 Identity Grounded on Aristotelian Essentialism and Modality**

Consider some possible directions one might take the arguments of Geach and Wiggins. First, I think it would be healthy for us to admit that our language and intuitions about identity do not seem to match up with the rules and principles philosophers have developed around identity. In a certain respect, both Geach and Wiggins are correct. To say that  $a$  and  $b$  are the same, is an incomplete expression, so we do need to contextualize it. And it seems that sortals or essences are a sound

way to contextualize a sameness that is not strict numerical sameness—a point that I take Aristotle to have expressed first.

If Wiggins seriously wants his sortal concepts to say what a thing is “in the sense Aristotle isolated,” and if Aristotle’s logic of “what a thing is” is given in his modal syllogistic, then that is the modality to be preferred. An advantage to adopting the Aristotelian modal syllogistic is that it makes no use of possible worlds semantics, or the *de dicto/de re* distinction as I have explained earlier..

It has only been in the couple of decades that work on Aristotle’s modal syllogistic has revealed a system that is arguably consistent, and which this work has ventured an interpretation to resolve certain apparent inconsistencies. So it is no fault of Wiggins that he did not consider Aristotelian modality as a viable option. Prior to that, the received history was that the modal syllogistic was, as I have said, a murky and inconsistent mess. Still, the adoption of a modal syllogistic may not solve all of Wiggins’ woes. New interpretive models do seem to suggest that Aristotle’s modal notions are not susceptible to Quine’s charge against *de re* modality as referentially opaque (Rini 2011, 34-35). Frank Lewis (2013, 141) likewise argues that there is not much reason to think Aristotle’s *de re* essentialism is subject to referential opacity. Lewis argues that Aristotle’s essences are not individual essences, and that once one disambiguates essential predications, issues of referential opacity will not arise. The question of substance in *Metaphysics* Zeta is the question of what it is for X to be F, or as Lewis puts it: what it is for-X [the owner] to be  $\phi$  [the content] (*ibid.*). The distinction between owner and content



seems to be a difference in emphasis, whether it is on the “thisness” of substance as opposed to the “whatness” of substance:

There are now two positions inside a designation of essence at which to look for referential opacity. What, first, of the position devoted to giving the content of the essence? How does substitution fare in these cases? For example, we may imagine that all and only men are capable of laughter (to borrow Aristotle’s example). Still, we would not think that what it is to be a man (the essence as of man) is the same as what it is to be a laugher (the essence as of a laugher). Kinds are not sets, and while the set of men and the set of laughers are one and the same, man and the laugher are not the same kind. (Indeed, man is a kind but the laugher is not: it is a proprium, and unlike a kind it does not indicate the essence of a thing, *Topics* A5.) So even if man and the laugher are exemplified by exactly the same things, they are not identical. And if they are not identical, we cannot intersubstitute terms for the two, and require that the result name the same essence. This is a failure of identity, however, and in no way goes to show that the expression “what it is to be (a) man” is referentially opaque at the position indicated (2013, 142).

Neither the content nor the owner of an essence seems to prevent substitution, however, one must be aware not to confuse essences with propria in making identity substitutions. This sounds similar to a sortal relative or dependent theory of identity.

Given this, a case could be made that Aristotelian modality is extensional in that apodictic claims are claims about essences and substance terms. Identity relations among individuals whose identities are relative to, or depend upon, the kinds to which they belong will substitute *salve veritate*. Aristotle’s discussion of identity is sparse, as I have noted earlier. However, a concept of identity rooted in Aristotelian essentialism may go far to resolving many of the paradoxes of identity metaphysicians or constantly discussing.

Another problem is that Wiggins’s theory depends upon a theory of individuation. But the modal syllogistic relates natural kind terms to one another, and it is rare for Aristotle to treat singular propositions within the modal syllogistic

(Patterson 1995, 72). Then again, Wiggins may be able to utilize an Aristotelian principle of individuation, independently of his logic of sortal types. Aristotle offers a discussion of individuation in *Metaphysics* Δ where he says:

Again, some things are one in number, others in species, others in genus, and others by analogy: in number, things of which the material is one, in species things of which the articulation is one, in genus, things to which the same manner of predication applies, and by analogy, as many things as are in the condition that something else is, in relation to something else (Meta. 1016b31-2).

Interestingly, this passage seems to be closer to Geach's perspective, since the principle of individuation will be relative to far more than a sortal concept, but can be adapted to numerical unities, and even to that which is an individual by analogy. Perhaps Aristotle, like Wiggins, will give priority to individuation of things that are one in number. If so, one alternative to a conceptualist principle of individuation would be a material principle of individuation, where matter is dependent on form for intelligibility as an identity, and an "en-formed" individual is dependent upon matter for its unity in being numerically the self-same.

However, we see in Wiggins a turn towards the question of the identity of artifacts and so a concern to have a principle of individuation for artifacts. He writes,

...It should go almost without saying that the distinction of natural thing from artifact is presented here in a fashion conformable at every point and in every particular with the plausible scientific belief that, however we arrived at the individuation of a given thing, the thing will be subject to the fundamental laws of physics and chemistry. For the purposes of that belief, there is no more difficulty in our distinction between natural things and other things than there is in making a further distinction *within* the class of natural things, between (a) those things which, being alive, are not in chemical and energy equilibrium with their surroundings but such from their environment the energy that they need for their typical activity or their molecular self-renewal and replacement, and (b) those natural things that maintain a typical mode of activity without being alive and cannot help but be in equilibrium with their surroundings (2001, 90).

An artifact is not, for Aristotle, a sub-set of the natural kinds. Rather, it is an assembly of natural substances, whose materials and related teleology are brought

together for some purpose that the artificer intends. Consequently, if we are truly to use an Aristotelian notion of kind-terms, i.e. natural kind-terms, then the identity of artifacts would be nothing more than a sort of fiction in the way that the object created by the artificer has its own kind-term, e.g. “table,” “chair,” or “ship”.

Wiggins turns to Book Two of Aristotle’s *Physics*, where the distinction between natural and artificial is drawn out. The passage is as follows:

animals and their organs and the elementary stuffs . . . differ from what is not naturally constituted in that each of these things has within it a principle of change and of staying unchanged, whether in respect of place or in respect of quantitative change, as in growth and decay, or in respect of qualitative change. But a bedstead or a cloak or whatever, *qua* receiving the designation ‘bed’ or ‘cloak’ . . . , i.e. in so far as it is the product of craft, has within itself no inherent tendency to any particular sort of change. Though in so far as an artifact happens to be composed . . . of whatever mixture of natural elements, it does incidentally, as so considered, have within itself the principle of change which inheres in its matter. So nature is a *source or principle of change and staying unchanged* in that to which it belongs primarily i.e. in virtue of the thing itself and not in virtue of an incidental attribute of the thing. I say ‘not in virtue of an incidental attribute’ because, for instance, a man who is a doctor might cure himself. Nevertheless it is not in so far as he is a patient that he possessed the art of medicine. It was incidental that he satisfied both descriptions. And something similar holds of everything that is an artificial product (192b8-28) (Wiggins 2001, 89).

Wiggins is critical of the “*qua*” locution along with “insofar as,” and “considered as,” in obfuscating precisely to what existent natural and artificial objects differ. So it seems that Wiggins wants an Aristotelian notion of “kind” but then extends that notion to artifacts in a way that is foreign to Aristotle. Nonetheless, Wiggins admits that, “Artefact identity does... present some difficult problems...” (*ibid.* 91). He notes that the artifact-word “clock” in “*x* coincides with *y* under the concept *clock*” seems problematic since a clock may not continuously function, which seems to defy at least one of his several principles of **D**, namely **D(v)** (*ibid.*).<sup>44</sup> As Wiggins goes on to

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<sup>44</sup> Wiggins lists 11 **D** principles. Our point here is not to revisit the variety of Wiggins’s sortal dependent principles in detail, but it is worth listing a few to understand the nature of his project,

analyze the problem of identity for artifacts, he considers the famous ship of Theseus, and engages in a lengthy dialectical discussion of which principles apply, in which do not to such an artifact as a ship, whose planks are slowly replaced over a period of time. Wiggins's quest can appear as though he is devising "just so" **D** principles to fit with his intuitions about the identity of artifacts. For Aristotle, the question would not really arise, at least with respect to artifacts. A deeper diagnosis of the problem with Wiggins's analysis, at least from the perspective an Aristotelian essentialist, is that Wiggins wants sortal dependent principles that individuate objects, seeking generalized rules about when a thing falls under a substance-concept, and the extent to which a thing can change while remaining under a substance-concept, but what is required is an understanding of the substance term itself, and what belongs to it necessarily and contingently. Once that is understood, it will also be said of the individuals that "fall under" that term. Such individuals will

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which is to specify various ways in which individuals depend on their sortal terms to exist and to relate to other objects by identity. The following are the first five principles (2001, 64, 70, 72).

**D(i):**  $(x)(t)[(x \text{ exists at } t) \rightarrow (\exists g)(g(x) \text{ at } t)]$ . That is, for all individuals,  $x$ , and times  $t$ , if  $x$  exists at  $t$ , then there exists some sortal  $g$  under which  $x$  falls at  $t$ . Wiggins does not think this principle will be strong enough for identity statements he wants for **D**. So he posits:

**D(ii):**  $(x)(\exists g)(t)[(x \text{ exists at } t) \rightarrow (g(x) \text{ at } t)]$ . Note that **D(i)** allows for  $x$  to fall under different sortals at different times, whereas in **D(ii)** specifies that a thing falls under a certain sortal throughout its existence.

**D (iii):**  $a$  is identical to  $b$  if and only if there is some concept  $f$  such that (1)  $f$  is a substance-concept under which an object that belongs to  $f$  can be singled out, traced and distinguished from other  $f$  entities and distinguished from other entities; (2)  $a$  coincides under  $f$  with  $b$ ; (3) ' $\text{coincides with } f$ ' stands for a congruence relation; i.e. all pairs  $\langle x, y \rangle$  that are members of the relation satisfy the Leibnizian schema  $\phi y$ .

**D(iv):**  $f$  is a substance-concept only if the grasp of  $f$  determines (with or without help of further empirical information about the class of  $f$ s) what can and cannot befall any  $x$  in the extension of  $f$ , and what changes  $x$  tolerates without their ceasing to exist such a thing as  $x$ ; moreover  $f$  is only a substance-concept if the grasp of  $f$  determines (with or without the help of further empirical information about the class of  $f$ s) the relative importance or unimportance to the survival of  $x$  of various classes of changes befalling specimens of  $f$  (e.g. how close they may bring  $x$  to actual extinction).

**D(v):**  $f$  is a substance-concept only if  $f$  determines either a principle of *activity*, a principle of *functioning*, or a principle of *operation* for members of its extension.

admit of the same potentials for accidental change, and have the same set of essential propria. So there is less of a need for an elaboration of individuating principles independent of a genuine grasp of the natural kinds. This will be problematic for artifacts since they are fabricated, as is the term designating whatever apparent unity they express. There is no principle of individuation to be grasped or understood in artifacts, but only principles that are fabricated according to the needs of the artificer. For we are not trying to understand nature when we investigate artifacts, we are trying to understand only the intentions and desires of the designer. The Ship of Theseus, then, may be paradoxical because there is no singular principle of individuation to be grasped. Instead, there is just a continuum of changes of an assemblage of material parts that, so long as it functions as a ship, remains for all intents and purposes, a ship. So, many metaphysicians want to extend the Ship of Theseus paradox, by analogy, to humans, wondering if our parts, replaced over time, result in a similar sorites paradox. For Aristotle, the question would not be resolved by a consideration of how many atoms are retained, but by an examination of “human”, a natural kind which has the potential to grow, take in nutrients, and replace some of its tissue. Such a natural kind can admit of a certain degree of change without ceasing to be “what it is” fundamentally.

Keeping in mind Aristotle’s point from the *Topics* that sameness is said in many ways, and that “there is either no necessity or even no possibility that things that are the same specifically or generically should be numerically the same” (*Top.* 152b30-34), it is at least an open question whether something numerically the same

with respect to its matter might be individuated analogically or by some other principle and turn out not to have identity vis-à-vis the species-form.

We might envision a way in which Aristotle's modal logic applies to a dependent theory of identity in the following way: Suppose there is an essence  $A$  such that two individuals,  $x$  and  $y$  are identical instances of the essence. From this, we might infer that whatever is apodictically predicated of the essence, say  $B$ , both  $x$  and  $y$  are instances of  $B$ . However, insofar as a predicate is contingently or possibly predicated of an essence, the predicate may or may not obtain with respect to the individual instances, even if they are numerically identical.

Let,

$x =_A y$ ...  $x$  is the same instance of essence  $A$  as  $y$

1.  $Ba_{LA}$
2.  $(\exists x)(\exists y)(\exists t) (x =_A y \text{ at } t)$
3.  $(\forall x)(\forall y)(\forall t)\{[(x =_A y \text{ at } t) \ \& \ Ba_{LA}] \supset (Bx \ \& \ By \text{ at } t)\}$

So,

4.  $(\exists x)(\exists y) (\exists t) (Bx \ \& \ By \text{ at } t)$

Note that on this Aristotelian schema, apodictic predication of kind terms permits an inference with respect to individuals, but that inference does not include *de re* necessity of individuals. This is because *de re* necessity of the modal syllogistic remains on the ontological level of the essence rather than the instance. It is on that level that there is no referential opacity. So, on this schema, given that  $B$  is apodictically predicated of  $A$ , and  $x$  and  $y$  are numerically identical under the same

natural kind term, it would follow that they do not differ with respect to having *B*. So there is no time where *x* would have *B* and *y* would lack *B*.

1.  $B_{aM}A$
  2.  $(\exists x)(\exists y)(\exists t)(x =_A y \text{ at } t)$
  3.  $(\forall x)(\forall y) (\forall t)\{[(x =_A y \text{ at } t) \ \& \ B_{aM}A] \supset \diamond(Bx \ \& \ By \text{ at } t)\}$
- So,
4.  $(\exists x)(\exists y)(\exists t)\diamond(Bx \ \& \ By \text{ at } t)$

Note that on this schema, at a certain time it is only possible that *x* and *y* will exhibit *B*. Nonetheless, it cannot be inferred from this that *x* could exhibit *B* while *y* does not, at the same time. Also, this might worry some philosophers who are wary of actualism, i.e. that there exists individuals with possible properties. However, under the Aristotelian schema, this makes sense insofar as the inference only runs insofar as we already know that there exists some *x* and some *y* that are the same instance of essence *A*.

Lastly, we should consider contingency:

1.  $B_{aQ}A$
  2.  $(\exists x)(\exists y)(\exists t) (x =_A y \text{ at } t)$
  3.  $(\forall x)(\forall y) (\forall t)\{[(x =_A y \text{ at } t) \ \& \ B_{aQ}A] \supset [[\diamond(Bx \ \& \ By \text{ at } t) \vee \diamond\sim(Bx \ \& \ By \text{ at } t)]]\}$
- So,
4.  $(\exists x)(\exists y)(\exists t)[\diamond(Bx \ \& \ By \text{ at } t) \ \& \ \diamond\sim(Bx \ \& \ By \text{ at } t)]$

That is, if it is contingent that *B* belongs to *A*, and if *x* and *y* are numerically identical *As* at a given time, then possibly they are both *Bs* at that time, or possibly they are not *Bs* at that time. What does not follow is that *Bx* and  $\sim By$  could occur at the same time. From this, we could conclude that, even though *x* and *y* are the same instance

of  $A$ , that identity does not prevent that  $B$  may or may not be predicated of  $x$  and  $y$  so long as it is not at the same time. For then it would run afoul of the principle of non-contradiction. It does permit us to comprehend change, and so discernible differences, without violating identity. For we could say that there is some  $Bx$  that is the same instance of essence  $A$  as  $y$ , and  $\diamond \sim By$  precisely because  $B$  contingently belongs to all  $A$ .

In sum, if Wiggins is to devise a sortal dependent theory of identity that makes use of sortals in precisely an Aristotelian way, then he may want to ground his *de re* claims in demonstrations made about natural kinds by use of the modal syllogistic. This would help him to escape the circularity problem to which I have eluded – a problem that Wiggins does not see as particularly vicious, but which I see as potentially leading to the sorts of problems raised by Meno's paradox. Moreover, grounding Wiggins  $D$  in Aristotelian kind terms avoids difficult disputes over the identity of artifacts. It is little wonder that the identity of artifacts is difficult to define, since there is no nature to define a fabricated term. Hence, there will always be a certain level of vagueness that attends to discussions of the identity of artifacts, a vagueness that is the consequence of the lack of there being a proper science of the artifact which can specify demonstrative knowledge of the sortals upon which the identity of individual artifacts would depend.



#### 4.6 Conclusion, or Counteracting Indispensability

It is somewhat surprising that such a committed naturalist as Quine would endorse a form of mathematical Platonism. He did so because he thought that every effort to dispense with mathematics and reduce it out of our best science was a failure. He thought that whatever was indispensable to our best scientific theories must be admitted into our ontology.

I propose that a similar argument is being raised within the metaphysics of modality. Already, possible worlds have become an essential part to our understanding of counterfactual conditionals. In turn, counterfactual conditionals have proved to be extremely useful, e.g. in defining central concepts to science, including causality, in a way that is independent from temporality – allowing for a coherent account of backward causation and causal loops. If the essentialist trend continues in science, and there is a place for essential and modal properties there, possible worlds could become viewed not merely as useful conceptual tools, but as an indispensable part of our best theories about causation, time, essential properties, and more. Perhaps it is controversial to say so, but our best scientific theories, from the potentials of quantum mechanics, to causality in closed time-like curves, has become intertwined with modal concepts.<sup>45</sup> Given the pervasiveness of modal logic, and as it ventures into more domains of philosophy and science, we risk

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<sup>45</sup> See, for example, Lombardi & Dieks (2016) for a survey of some of the ways in which modality has been used to understand and interpret quantum mechanics. See Lewis (1973) for his account of causation, which permits backward causation, and Handley (2004) to see Lewis's definition of causality applied to understanding causality in closed time-like loops.

becoming dependent upon possible worlds. There certainly will be and has been a push to admit possible worlds into our ontology.

Perhaps possible worlds should be admitted into our ontology. However, a fully developed Aristotelian modal logic allows us to conceive of modal properties in a different way, without reference to worlds. This may provide us with some space to continue to view the semantics of possible worlds as a framework or model rather than an indispensable feature. Ultimately, Aristotelian logic is limited in its domain, as it deals primarily with categorical terms as they relate to one another. On the other hand, a skeptic of possible worlds may not need much more than an Aristotelian conception of modality to make intelligible the claims of science and philosophy.

There may be a certain tendency in the philosophy of logic, perhaps it is a naïve tendency, but it seems to see some systems of logic less metaphysically loaded, and as such, more universally applicable. What I have argued, however, is that the aim to create a logical position that is completely neutral is impossible. What we have seen, instead, in the 20<sup>th</sup> and 21<sup>st</sup> centuries, is the development of multiple non-competing systems of logic, be they free, relevant, many-valued, paraconsistent, or modal. Within modal logic itself, we have seen the development of several systems based on their own axioms. Rather than viewing the Aristotelian modal syllogism as in competition with any other logical system, it is best to come to understand it and appreciate the place it has as a logical system that is deeply Aristotelian in its assumptions about essences and natures.

Aristotelian modal logic has a long history of being considered problematic. Theophrastus and Eudemus proposed counter-examples to Barbara-LXL, arguing for a more modest modal system, but one that I believe would be deficient in bootstrapping Aristotelian scientific claims from first principles and observations alone. Becker and many others find that the problem emerges from a confusion between *de re* and *de dicto* modal contexts. We have addressed this objection, and it seems to be ill-founded, and basically misunderstands the nature of the Aristotelian term, the very element out of which his logic is composed. Lastly, we have worked to resolve the remaining difficulties in Aristotle's modal syllogistic and have found that they tend to revolve around contexts in which a term can be said to have an underlying nature supportive of the predicate term in question. This became especially crucial in particular negative statements, and we should pause for a moment to consider why they, in particular, were problematic. It is, I believe, for two reasons: (1) being particular statements, we have a tendency to treat them as though they are statements about individuals, and so it is perhaps assumed that they are to be treated as primary substances. Secondly, there is an ambiguity of negation in modal contexts and outside of modal contexts. When the copula is negated and modified modally, we may treat that negation as equivalent to a complementary predication. This, as I have shown, is a practice that Aristotle himself employs. But, when there is no modal context, we cannot presume that there is an underlying nature to the private of a term.

Likewise, it was assumed by logicians in the 20<sup>th</sup> century that Aristotle's logic did not carefully distinguish between *de re* and *de dicto* contexts. Consequently,

Aristotelian modality was not properly conceived as a modification of a copula joining two class terms. Even contemporary Aristotelian commentators still use first-order predicate logic to understand Aristotle's claims, and run up against problems pertaining to actualism and the use of the Barcan Formula and its converse.

This dissertation, then, should be seen as a defense of Aristotelian modality as a consistent system. I believe that its insights can be communicated and understood visually, through the method of diagramming that I proposed. Moreover, I have argued that Aristotelian modality is the logic by which we should understand Aristotle's philosophy of science. It is through demonstration that we can build up knowledge of the world around us. Thus, we can move beyond nominal definitions and first principles towards a robust knowledge of the world around us. Indeed, it is a knowledge in which explanations are rooted in essences and causes, and which makes sense of the fact that, often, our theoretical knowledge proceeds from empirical verification.

I should like to see Aristotle's modal logic studied in its own right, and to see it applied both in understanding Aristotle's scientific writings, and in contemporary scientific contexts. If the topics I have explored in this chapter are any indication, then it would provide us with an understanding of the actual world, and the modal properties that kinds have. It would do this without leading to confusing questions about transworld-identity, or treating essences as rigidly-designated names and descriptions that somehow hold across worlds. It is a modality and a logic that is grounded upon what a thing is.

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## APPENDIX

Complete List of Rules Along with Proofs for Modal Syllogisms Using Heterodox Interpretation, LPC Rules, and Aristotelian Definitions and Rules Listed Below  
Operational Definitions of Assertoric, Apodictic, Possible, and Contingent  
Categorical Propositions:

Assertoric:

- (1)  $AaxB \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset AaxZ)$  (Def ax)
- (2)  $AexB \stackrel{\text{def}}{=} (\forall Z)[BaxZ \supset \sim(AaxZ)]$  (Def ex)
- (3)  $AixB \stackrel{\text{def}}{=} (\exists Z)(BaxZ \ \& \ AaxZ)$  (Def ix)
- (4)  $AoxB \stackrel{\text{def}}{=} (\exists Z)[BaxZ \ \& \ \sim(AaxZ)]$  (Def ox)

Apodictic:

- (5)  $AaLb \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset AaLZ)$  (Def aL)
- (6)  $AeLb \stackrel{\text{def}}{=} (\forall Z)(BaxZ \supset \bar{A}aLZ)$  (Def eL)
- (7)  $AiLb \stackrel{\text{def}}{=} (\exists Z)[(BaxZ \ \& \ AaLZ) \vee (AaxZ \ \& \ BaLZ)]$  (Def iL)
- (8)  $AoLb \stackrel{\text{def}}{=} (\exists Z)[(BaxZ \ \& \ \bar{A}aLZ) \vee (\bar{A}axZ \ \& \ BaLZ)]$  (Def oL)

Possible:

- (9)  $AaMb \stackrel{\text{def}}{=} (\forall Z)(BamZ \supset AamZ)$  (Def aM)
- (10)  $AeMb \stackrel{\text{def}}{=} (\forall Z)(BamZ \supset \bar{A}amZ)$  (Def eM)
- (11)  $AiMb \stackrel{\text{def}}{=} (\exists Z)(BamZ \ \& \ AamZ)$  (Def iM)
- (12)  $AoMb \stackrel{\text{def}}{=} (\exists Z)(BamZ \ \& \ \bar{A}amZ)$  (Def oM)

Contingent:

- (13)  $AaQb \stackrel{\text{def}}{=} (\forall Z)[(BamZ \supset AamZ) \ \& \ (BamZ \supset \bar{A}amZ)]$  (Def aQ)
- (14)  $AeQb \stackrel{\text{def}}{=} (\forall Z)[(BamZ \supset AamZ) \ \& \ (BamZ \supset \bar{A}amZ)]$  (Def eQ)
- (15)  $AiQb \stackrel{\text{def}}{=} (\exists Z)[BamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (Def iQ)
- (16)  $AoQb \stackrel{\text{def}}{=} (\exists Z)[BamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (Def oQ)

The following conversion rules hold provided that the categorical terms are constants and not bounded variables or pseudonyms:

Assertoric Conversion:

- (17)  $AaxB \supset BixA$  (Conv ax-ix)
- (18)  $AixB \equiv BixA$  (Conv ix-ix)
- (19)  $AexB \equiv BexA$  (Conv ex-ex)
- (20)  $AoxB$  does not convert

### Apodictic Conversion:

- (21)  $Aa_L B \supset Bi_L A$  (Conv  $a_L-i_L$ )
- (22)  $Ai_L B \equiv Bi_L A$  (Conv  $i_L-i_L$ )
- (23)  $Ae_L B \equiv Be_L A$  (Conv  $e_L-e_L$ )
- (24)  $Ao_L B$  does not convert

### Possible Conversion:

- (25)  $Aa_M B \supset Bi_M A$  (Conv  $a_M-i_M$ )
- (26)  $Ai_M B \equiv Bi_M A$  (Conv  $i_M-i_M$ )
- (27)  $A\exists e_M B \equiv Be_M A$  (Conv  $e_L-e_L$ )
- (28)  $Ao_M B$  does not convert

### Contingent Conversion:

Since universal propositions distribute over the subject, something is said of the nature of the subject, and so it is assumed that it has an underlying nature. As such, the complementary term of the subject can be posited:

- (29)  $Aa_Q B \supset Bi_Q A$  (Conv  $a_Q-i_Q$ )
- (30)  $Ae_Q B \supset Bo_Q A$  (Conv  $e_Q-o_Q$ )

In particular contingent propositions, there must be an underlying nature said of the subject, which is to say that the subject converts just in case the subject is a substance or essence term, which can be established if contingency is amplified:

- (31)  $(\exists Z)[Ba_Q Z \& (Aa_M Z \& \bar{A}a_M Z)] \equiv (\exists Z)[Aa_Q Z \& (Ba_M Z \& \bar{B}a_M Z)]$  (Conv  $i_Q-i_Q$ )
- (32)  $(\exists Z)[Ba_Q Z \& (Aa_M Z \& \bar{A}a_M Z)] \equiv (\exists Z)[Aa_Q Z \& (Ba_M Z \& \bar{B}a_M Z)]$  (Conv  $o_Q-o_Q$ )

Note that in (25) and (26) bounded categorical propositions were not converted, rather, I am using the operational definitions to illustrate how a particular proposition would convert were it to have contingency amplified to both subject and predicate terms. This, again, would be based on the fact that Aristotle treats certain possibilities as “natural” and in such cases, the negative is treated like the affirmative, which is precisely what my interpretation does. But, given that, we must be sensitive to whether the subject of a particular proposition has a complementary term, which cannot be assumed. In fact, were conversions permitted without ampliation, or without establishing that the term in question has a complementary term within the context of the proposition, certain illicit inferences could be made.

## Assertoric Obversion:

(33)  $\bar{A}e_xB \supset Aa_xB$  (Obv  $e_x$ - $a_x$ )

(34)  $\bar{A}a_xB \supset Ae_xB$  (Obv  $a_x$ - $e_x$ )

(35)  $\bar{A}o_xB \supset Ai_xB$  (Obv  $o_x$ - $i_x$ )

(36)  $\bar{A}i_xB \supset Ao_xB$  (Obv  $i_x$ - $o_x$ )

## Apodictic Obversion:

(37)  $\bar{A}e_LB \equiv Aa_LB$  (Obv  $e_L$ - $a_L$ )

(38)  $\bar{A}a_LB \equiv Ae_LB$  (Obv  $a_L$ - $e_L$ )

(39)  $\bar{A}o_LB \equiv Ai_LB$  (Obv  $o_L$ - $i_L$ )

(40)  $\bar{A}i_LB \equiv Ao_LB$  (Obv  $i_L$ - $o_L$ )

## Possible Obversion:

(41)  $\bar{A}e_MB \equiv Aa_MB$  (Obv  $e_M$ - $a_M$ )

(42)  $\bar{A}a_MB \equiv Ae_MB$  (Obv  $a_M$ - $e_M$ )

(43)  $\bar{A}o_MB \equiv Ai_MB$  (Obv  $o_M$ - $i_M$ )

(44)  $\bar{A}i_MB \equiv Ao_MB$  (Obv  $i_M$ - $o_M$ )

## Contingent Obversion:

(45)  $\bar{A}e_QB \equiv Aa_QB$  (Obv  $e_M$ - $a_M$ )

(46)  $\bar{A}a_QB \equiv Ae_QB$  (Obv  $a_M$ - $e_M$ )

(47)  $\bar{A}o_QB \equiv Ai_QB$  (Obv  $o_M$ - $i_M$ )

(48)  $\bar{A}i_QB \equiv Ao_QB$  (Obv  $i_M$ - $o_M$ )

## Apodictic to Assertoric Subordination:

(49)  $Aa_LB \supset (\forall Z)(Ba_xZ \supset Aa_xZ)$  (L-X-sub  $a_L$ )

(50)  $Ae_LB \supset (\forall Z)\{[Ba_xZ \supset \bar{A}a_xZ] \& [Ba_xZ \supset \sim(Aa_xZ)]\}$  (L-X-sub  $e_L$ )

(51)  $Ai_LB \supset (\exists Z)(Ba_xZ \& Aa_xZ)$  (L-X-sub  $i_L$ )

(52)  $Ao_LB \supset (\exists Z)\{Ba_xZ \& [\sim(Aa_xZ) \& \bar{A}a_xZ]\}$  (L-X-sub  $o_L$ )

## Assertoric to Possible Subordination:

(53)  $Aa_xB \supset (\forall Z)(Ba_MZ \supset Aa_MZ)$  (X-M-sub  $a_x$ )

(54)  $Ae_xB \supset (\forall Z)[Ba_MZ \supset \bar{A}a_MZ]$  (X-M-sub  $e_x$ )

(55)  $Ai_xB \supset (\exists Z)(Ba_MZ \& Aa_MZ)$  (X-M-sub  $i_x$ )

(56)  $Ai_xB \supset (\exists Z)(Ba_MZ \& \bar{A}a_MZ)$  (X-M-sub  $i_x$ )

## Apodictic to Possible Subordination:

(57)  $Aa_LB \supset (\forall Z)(Ba_MZ \supset Aa_MZ)$  (L-M-sub  $a_L$ )

(58)  $Ae_LB \supset (\forall Z)[(Ba_MZ \supset \bar{A}a_MZ) \& (Ba_xZ \supset \sim(Aa_MZ))]$  (L-M-sub  $e_L$ )

(59)  $Ai_LB \supset (\exists Z)(Ba_MZ \& Aa_MZ)$  (L-M-sub  $i_L$ )

(60)  $Ao_LB \supset (\exists Z)\{Ba_MZ \& [\bar{A}a_MZ \& \sim(Aa_MZ)]\}$  (L-M-sub  $o_L$ )



Contradiction rules: Contradiction substitution is permitted when the terms in the proposition are constant, and not bound variables or pseudonyms.

Contradictory Assertoric Propositions:

- (61)  $AaxB \equiv \sim(AoxB) (ax|ox)$
- (62)  $AoxB \equiv \sim(AaxB) (ox|ax)$
- (63)  $AexB \equiv \sim(AixB) (ex|ix)$
- (64)  $AixB \equiv \sim(AexB) (ix|ex)$

Contradictory Apodictic and Possible Propositions:

- (65)  $AaLb \equiv \sim(AoMb) (aL|oM)$
- (66)  $AoMb \equiv \sim(AaLb) (oM|aL)$
- (67)  $AeMb \equiv \sim(AiLb) (eM|iL)$
- (68)  $AiLb \equiv \sim(AeMb) (iL|eM)$
- (69)  $AaMb \equiv \sim(AoLb) (aM|oL)$
- (70)  $AoLb \equiv \sim(AaMb) (oL|aM)$
- (71)  $AeLb \equiv \sim(AiMb) (eL|iM)$
- (72)  $AiMb \equiv \sim(AeLb) (iM|eL)$

The following list of valid syllogism concurs with the canonical list provided by Ross (1957, 285) unless another source is mentioned.

**First Figure:**

Barbara-XXX (Valid):

1.  $AaxB$  (major premise)
2.  $BaxC$  (minor premise) //  $AaxC (\forall Z)(CaxZ \supset AaxZ)$
3.  $(\forall Z)[(BaxZ \supset AaxZ)]$  (1 Def ax)
4.  $(\forall Z)(CaxZ \supset BaxZ)$  (2 Def ax)
5.  $BaxU \supset AaxU$  (3 UI)
6.  $CaxU \supset BaxU$  (4 UI)
7.  $CaxU \supset AaxU$  (5,6 HS)
8.  $(\forall Z)(CaxZ \supset AaxZ)$  (7 UG)
9.  $AaxC$  (8 Def ax)

Barbara-LLL (Valid):

1.  $AaLb$  (major premise)
2.  $BaLc$  (minor premise) //  $AaLc (\forall Z)(CaxZ \supset AaLZ)$
3.  $BaLc \supset (\forall Z)(CaxZ \supset BaxZ)$  (L-X-sub  $aL$ )
4.  $(\forall Z)(CaxZ \supset BaxZ)$  (2,3 MP)

5.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def  $a_L$ )
6.  $CaxU \supset BaxU$  (4 UI)
7.  $BaxU \supset AaLU$  (5 UI)
8.  $CaxU \supset AaLU$  (6,7 HS)
9.  $(\forall Z)(CaxZ \supset AaLZ)$  (8 UG)
10.  $AaLC$  (9 Def  $a_L$ )

Barbara-LXL (Valid):

1.  $AaLB$  (major premise)
2.  $BaxC$  (minor premise) //  $AaLC$   $(\forall Z)(CaxZ \supset AaLZ)$
3.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def  $a_L$ )
4.  $(\forall Z)(CaxZ \supset BaxZ)$  (2 Def  $ax$ )
5.  $BaxU \supset AaLU$  (3 UI)
6.  $CaxU \supset BaxU$  (4 UI)
7.  $CaxU \supset AaLU$  (5,6 HS)
8.  $(\forall Z)(CaxZ \supset AaLZ)$  (7 UG)
9.  $AaLC$  (8 Def  $a_L$ )

Barbara-QQQ (Valid):

1.  $AaQB$  (major premise)
2.  $BaQC$  (minor premise) //  $AaQC$   $(\forall Z)[(CamZ \supset AamZ) \& (CamZ \supset \bar{A}amZ)]$  (Def  $aQ$ )
3.  $(\forall Z)[(BamZ \supset AamZ) \& (BamZ \supset \bar{A}amZ)]$  (1 Def  $aQ$ )
4.  $(\forall Z)[(CamZ \supset BamZ) \& (CamZ \supset \bar{B}amZ)]$  (2 Def  $aQ$ )
5.  $(BamU \supset AamU) \& (BamU \supset \bar{A}amU)$  (3 UI)
6.  $(CamU \supset BamU) \& (CamU \supset \bar{B}amU)$  (4 UI)
7.  $BamU \supset AamU$  (5 Simp)
8.  $CamU \supset BamU$  (6 Simp)
9.  $CamU \supset AamU$  (7,8 HS)
10.  $BamU \supset \bar{A}amU$  (5 Simp)
11.  $\sim(\bar{A}amU)$  (CP)
12.  $\sim(BamU)$  (10,11 MT)
13.  $\sim(CamU)$  (8,12 MT)
14.  $\sim(\bar{A}amU) \supset \sim(CamU)$  (11-13 CP)
15.  $CamU \supset \bar{A}amU$  (14 Contra)
16.  $(CamU \supset AamU) \& (CamU \supset \bar{A}amU)$  (9,15 Conj)
17.  $(\forall Z)[(CamZ \supset AamZ) \& (CamZ \supset \bar{A}amZ)]$  (16, UG)
18.  $AaQC$  (17 Def  $aQ$ )

Barbara-QXQ (Valid):

1.  $Aa_0B$  (major premise)
2.  $Ba_xC$  (minor premise)// $Aa_0C$   $(\forall Z)[(Ca_mZ \supset Aa_mZ) \& (Ca_mZ \supset \bar{A}a_mZ)]$  (Def a<sub>0</sub>)
3.  $(\forall Z)[(Ba_mZ \supset Aa_mZ) \& (Ba_mZ \supset \bar{A}a_mZ)]$  (1 Def a<sub>0</sub>)
4.  $Ba_xC \supset (\forall Z)(Ca_mZ \supset Ba_mZ)$  (X-M-sub ax)
5.  $(\forall Z)(Ca_mZ \supset Ba_mZ)$  (2,4 MP)
6.  $(Ba_mU \supset Aa_mU) \& (Ba_mU \supset \bar{A}a_mU)$  (3 UI)
7.  $Ba_mU \supset Aa_mU$  (6 Simp)
8.  $Ca_mU \supset Ba_mU$  (5 UI)
9.  $Ca_mU \supset Aa_mU$  (7,8 HS)
10.  $Ba_mU \supset \bar{A}a_mU$  (6 Simp)
11.  $Ca_mU \supset \bar{A}a_mU$  (8,10 HS)
12.  $(Ca_mU \supset Aa_mU) \& (Ca_mU \supset \bar{A}a_mU)$  (9,11 Conj)
13.  $(\forall Z)[(Ca_mZ \supset Aa_mZ) \& (Ca_mZ \supset \bar{A}a_mZ)]$  (12 UG)
14.  $Aa_0C$  (13 Def a<sub>0</sub>)

Barbara-XQM (Valid):

1.  $Aa_xB$  (major premise)
2.  $Ba_0C$  (minor premise)// $Aa_mC$   $(\forall Z)(Ca_mZ \supset Aa_mZ)$
3.  $Aa_xB \supset (\forall Z)(Ba_mZ \supset Aa_mZ)$  (X-M-sub ax)
4.  $(\forall Z)[(Ca_mZ \supset Ba_mZ) \& (Ca_mZ \supset \bar{B}a_mZ)]$  (2 Def a<sub>0</sub>)
5.  $(\forall Z)(Ba_mZ \supset Aa_mZ)$  (1,3 MP)
6.  $(Ca_mU \supset Ba_mU) \& (Ca_xU \supset \bar{B}a_mU)$  (4 UI)
7.  $Ba_mU \supset Aa_mU$  (5 UI)
8.  $Ca_mU \supset Ba_mU$  (6 Simp)
9.  $Ca_mU \supset Aa_mU$  (7,8 HS)
10.  $(\forall Z)(Ca_mZ \supset Aa_mZ)$  (9 UG)
11.  $Aa_mC$  (10 Def a<sub>m</sub>)

Barbara-QLQ (Valid):

1.  $Aa_0B$  (major premise)
2.  $Ba_lC$  (minor premise)// $Aa_0C$   $(\forall Z)[(Ca_mZ \supset Aa_mZ) \& (Ca_mZ \supset \bar{A}a_mZ)]$  (Def a<sub>0</sub>)
3.  $(\forall Z)[(Ba_mZ \supset Aa_mZ) \& (Ba_mZ \supset \bar{A}a_mZ)]$  (1 Def a<sub>0</sub>)
4.  $Ba_lC \supset (\forall Z)(Ca_mZ \supset Ba_mZ)$  (L-M-sub a<sub>l</sub>)
5.  $(Ba_mU \supset Aa_mU) \& (Ba_mU \supset \bar{A}a_mU)$  (3 UI)
6.  $(\forall Z)(Ca_mZ \supset Ba_mZ)$  (2,4 MP)
7.  $Ca_mU \supset Ba_mU$  (6 UI)
8.  $Ba_mU \supset Aa_mU$  (5 Simp)
9.  $Ca_mU \supset Aa_mU$  (7,8 HS)
10.  $Ba_mU \supset \bar{A}a_mU$  (5 Simp)

11.  $C_{aM}U \supset \bar{A}_{aM}U$  (7,10 HS)
12.  $(C_{aM}U \supset A_{aM}U) \& (C_{aM}U \supset \bar{A}_{aM}U)$  (9,11 Conj)
13.  $(\forall Z)[(C_{aM}Z \supset A_{aM}Z) \& (C_{aM}Z \supset \bar{A}_{aM}Z)]$  (12, UG)
14.  $A_{aQ}C$  (13 Def aQ)

Barbara-LQM (Valid):

1.  $A_{aL}B$  (major premise)
2.  $B_{aQ}C$  (minor premise) //  $A_{aM}C (\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (Def aM)
3.  $A_{aL}B \supset (\forall Z)(B_{aM}Z \supset A_{aM}Z)$  (L-M sub aL)
4.  $(\forall Z)(B_{aM}Z \supset A_{aM}Z)$  (1,3 MP)
5.  $(\forall Z)[(C_{aM}Z \supset B_{aM}Z) \& (C_{aM}Z \supset \bar{B}_{aM}Z)]$  (2 Def aQ)
6.  $(C_{aM}U \supset B_{aM}U) \& (C_{aM}U \supset \bar{B}_{aM}U)$  (5 UI)
7.  $B_{aM}U \supset A_{aM}U$  (4 UI)
8.  $C_{aM}U \supset B_{aM}U$  (5 Simp)
9.  $C_{aM}U \supset A_{aM}U$  (7,8 HS)
10.  $(\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (9 UG)
11.  $A_{aM}C$  (10 Def aM)

Celarent-XXX (Valid):

1.  $A_{eX}B$  (major premise)
2.  $B_{aX}C$  (minor premise) //  $A_{eX}C (\forall Z)[C_{aX}Z \supset \sim(A_{aX}Z)]$
3.  $(\forall Z)[B_{aX}Z \supset \sim(A_{aX}Z)]$  (1 Def eX)
4.  $(\forall Z)(C_{aX}Z \supset B_{aX}Z)$  (2 Def aX)
5.  $B_{aX}U \supset \sim(A_{aX}U)$  (3 UI)
6.  $C_{aX}U \supset B_{aX}U$  (4 UI)
7.  $C_{aX}U \supset \sim(A_{aX}U)$  (5,6 HS)
8.  $(\forall Z)[C_{aX}Z \supset \sim(A_{aX}U)]$  (7 UG)
9.  $A_{eX}C$  (8 Def eX)

Celarent-LLL (Valid):

1.  $A_{eL}B$  (major premise)
2.  $B_{aL}C$  (minor premise) //  $A_{eL}C (\forall Z)(C_{aX}Z \supset \bar{A}_{aL}Z)$
3.  $B_{aL}C \supset (\forall Z)(C_{aX}Z \supset B_{aX}Z)$  (L-X-sub aL)
4.  $(\forall Z)(C_{aX}Z \supset B_{aX}Z)$  (2,3 MP)
5.  $(\forall Z)[B_{aX}Z \supset \bar{A}_{aL}Z]$  (1 Def eL)
6.  $B_{aX}U \supset \bar{A}_{aL}U$  (5 UI)
7.  $C_{aX}U \supset B_{aX}U$  (4 UI)
8.  $C_{aX}U \supset \bar{A}_{aL}U$  (6,7 HS)
9.  $(\forall Z)(C_{aX}Z \supset \bar{A}_{aL}Z)$  (8 UG)
10.  $A_{eL}C$  (9 Def eL)

Celarent-LXL (Valid):

1.  $Ae_L B$  (major premise)
2.  $Ba_x C$  (minor premise) //  $Ae_L C (\forall Z)(Ca_x Z \supset \bar{A}_a L Z)$
3.  $(\forall Z)(Ba_x Z \supset \bar{A}_a L Z)$  (1 Def  $e_L$ )
4.  $(\forall Z)(Ca_x Z \supset Ba_x Z)$  (2 Def  $a_x$ )
5.  $Ba_x U \supset \bar{A}_a L U$  (3 UI)
6.  $Ca_x U \supset Ba_x U$  (4 UI)
7.  $Ca_x U \supset \bar{A}_a L U$  (5,6 HS)
8.  $(\forall Z)(Ca_x Z \supset \bar{A}_a L Z)$  (7 UG)
9.  $Ae_L C$  (8 Def  $e_L$ )

Celarent-QQQ (Valid):

1.  $Ae_Q B$  (major premise)
2.  $Ba_Q C$  (minor premise) //  $Ae_Q C (\forall Z)[(Ca_M Z \supset Aa_M Z) \& (Ca_M Z \supset \bar{A}_a M Z)]$  (Def  $e_Q$ )
3.  $(\forall Z)[(Ba_M Z \supset Aa_M Z) \& (Ba_M Z \supset \bar{A}_a M Z)]$  (1 Def  $e_Q$ )
4.  $(\forall Z)[(Ca_x Z \supset Ba_M Z) \& (Ca_x Z \supset \bar{B}_a M Z)]$  (2 Def  $a_Q$ )
5.  $(Ba_M U \supset Aa_M U) \& (Ba_M U \supset \bar{A}_a M U)$  (3 UI)
6.  $(Ca_M U \supset Ba_M U) \& (Ca_M U \supset \bar{B}_a M U)$  (4 UI)
7.  $Ba_M U \supset Aa_M U$  (5 Simp)
8.  $Ca_M U \supset Ba_M U$  (6 Simp)
9.  $Ca_M U \supset Aa_M U$  (7,8 HS)
10.  $Ba_M U \supset \bar{A}_a M U$  (5 Simp)
11.  $\sim(\bar{A}_a M U)$  (CP)
12.  $\sim(Ba_M U)$  (10,11 MT)
13.  $\sim(Ca_M U)$  (8,12 MT)
14.  $\sim(\bar{A}_a M U) \supset \sim(Ca_M U)$  (11-13 CP)
15.  $Ca_M U \supset \bar{A}_a M U$  (14 Contra)
16.  $(Ca_M U \supset Aa_M U) \& (Ca_M U \supset \bar{A}_a M U)$  (9,15 Conj)
17.  $(\forall Z)[(Ca_M Z \supset Aa_M Z) \& (Ca_M Z \supset \bar{A}_a M Z)]$  (16, UG)
18.  $Ae_Q C$  (17 Def  $e_Q$ )

Celarent-QXQ (Valid):

1.  $Ae_Q B$  (major premise)
2.  $Ba_x C$  (minor premise) //  $Ae_Q C (\forall Z)[(Ca_M Z \supset Aa_M Z) \& (Ca_M Z \supset \bar{A}_a M Z)]$  (Def  $e_Q$ )
3.  $(\forall Z)[(Ba_M Z \supset Aa_M Z) \& (Ba_M Z \supset \bar{A}_a M Z)]$  (Def  $e_Q$ )
4.  $Ba_x C \supset (\forall Z)(Ca_M Z \supset Ba_M Z)$  (X-M-sub  $a_x$ )
5.  $(\forall Z)(Ca_M Z \supset Ba_M Z)$  (2,4 MP)
6.  $(Ba_M U \supset Aa_M U) \& (Ba_M U \supset \bar{A}_a M U)$  (3 UI)
7.  $Ba_M U \supset Aa_M U$  (6 Simp)
8.  $Ca_M U \supset Ba_M U$  (5 UI)

9.  $C_{aM}U \supset A_{aM}U$  (7,8 HS)
10.  $B_{aM}U \supset \bar{A}_{aM}U$  (6 Simp)
11.  $C_{aM}U \supset \bar{A}_{aM}U$  (8,10 HS)
12.  $(C_{aM}U \supset A_{aM}U) \& (C_{aM}U \supset \bar{A}_{aM}U)$  (9,11 Conj)
13.  $(\forall Z)[(C_{aM}Z \supset A_{aM}Z) \& (C_{aM}Z \supset \bar{A}_{aM}Z)]$  (12 UG)
14.  $A_{eQC}$  (13 Def  $eQ$ )

Celarent-XQM (Valid):

1.  $A_{eXB}$  (major premise)
2.  $B_{aQC}$  (minor premise) //  $A_{eMC} (\forall Z)(C_{aMZ} \supset \bar{A}_{aMZ})$
3.  $A_{eXB} \supset (\forall Z)(B_{aMZ} \supset \bar{A}_{aMZ})$  (X-M sub ax)
4.  $(\forall Z)[(C_{aMZ} \supset B_{aMZ}) \& (C_{aMZ} \supset \bar{B}_{aMZ})]$  (2 Def  $aQ$ )
5.  $(\forall Z)(B_{aMZ} \supset \bar{A}_{aMZ})$  (1,3 MP)
6.  $(C_{aMU} \supset B_{aMU}) \& (C_{aXU} \supset \bar{B}_{aMU})$  (4 UI)
7.  $B_{aMU} \supset \bar{A}_{aMU}$  (5 UI)
8.  $C_{aMU} \supset B_{aMU}$  (6 Simp)
9.  $C_{aMU} \supset \bar{A}_{aMU}$  (7,8 HS)
10.  $(\forall Z)(C_{aMZ} \supset \bar{A}_{aMZ})$  (9 UG)
11.  $A_{eMC}$  (10 Def  $eM$ )

Celarent-QLQ (Valid):

1.  $A_{eQB}$  (major premise)
2.  $B_{aLC}$  (minor premise) //  $A_{eQC} (\forall Z)[(C_{aMZ} \supset A_{aMZ}) \& (C_{aMZ} \supset \bar{A}_{aMZ})]$  (Def  $eQ$ )
3.  $(\forall Z)[(B_{aMZ} \supset A_{aMZ}) \& (B_{aMZ} \supset \bar{A}_{aMZ})]$  (1 Def  $eQ$ )
4.  $B_{aLC} \supset (\forall Z)(C_{aMZ} \supset B_{aMZ})$  (L-M-sub  $aL$ )
5.  $(B_{aMU} \supset A_{aMU}) \& (B_{aMU} \supset \bar{A}_{aMU})$  (3 UI)
6.  $(\forall Z)(C_{aMZ} \supset B_{aMZ})$  (2,4 MP)
7.  $C_{aMU} \supset B_{aMU}$  (6 UI)
8.  $B_{aMU} \supset A_{aMU}$  (5 Simp)
9.  $C_{aMU} \supset A_{aMU}$  (7,8 HS)
10.  $B_{aMU} \supset \bar{A}_{aMU}$  (5 Simp)
11.  $C_{aMU} \supset \bar{A}_{aMU}$  (7,10 HS)
12.  $(C_{aMU} \supset A_{aMU}) \& (C_{aMU} \supset \bar{A}_{aMU})$  (9,11 Conj)
13.  $(\forall Z)[(C_{aMZ} \supset A_{aMZ}) \& (C_{aMZ} \supset \bar{A}_{aMZ})]$  (12, UG)
14.  $A_{eQC}$  (13 Def  $eQ$ )

Celarent-LQX (Valid):

1.  $A_{eLB}$  (major premise)
2.  $B_{aQC}$  (minor premise) //  $A_{eXC} (\forall Z)[C_{aXZ} \supset \sim(A_{aXZ})]$  (Def  $eX$ )
3.  $(\forall Z)[(C_{aMZ} \supset B_{aMZ}) \& (C_{aMZ} \supset \bar{B}_{aMZ})]$  (2 Def  $aQ$ )
4.  $A_{eLB} \supset (\forall Z)\{[B_{aMZ} \supset \bar{A}_{aMZ}] \& [B_{aMZ} \supset \sim(A_{aMZ})]\}$  (L-M-sub  $eL$ )

5.  $(\forall Z)\{[B_{aM}Z \supset \bar{A}_{aM}Z] \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (1,4 MP)
6.  $A_{ix}C$  (IP)
7.  $A_{ix}C \supset (\exists Z)(C_{aM}Z \& A_{aM}Z)$  (X-M-sub ix)
8.  $(\exists Z)(C_{aM}Z \& A_{aM}Z)$  (6,7 MP)
9.  $C_{aMU} \& A_{aMU}$  (8 EI)
10.  $(C_{aMU} \supset B_{aMU}) \& (C_{aMU} \supset \bar{B}_{aMU})$  (3 UI)
11.  $[B_{aMU} \supset \bar{A}_{aMU}] \& [B_{aMU} \supset \sim(A_{aMU})]$  (5 UI)
12.  $C_{aMU} \supset B_{aMU}$  (10 Simp)
13.  $C_{aMU}$  (9 Simp)
14.  $B_{aMU}$  (12,13 MP)
15.  $A_{aMU}$  (9 Simp)
16.  $\sim\sim(A_{aMU})$  (15 DN)
17.  $B_{aMU} \supset \sim(A_{aMU})$  (11 Simp)
18.  $\sim(B_{aMU})$  (16,17 MT)
19.  $B_{aMU} \& \sim(B_{aMU})$  (14,18 Conj)
20.  $\sim(A_{ix}C)$  (6-19 IP)
21.  $A_{ex}C$  (20 ex|ix)

Celarent-LQM (Valid):

1.  $A_{eLB}$  (major premise)
2.  $B_{aQC}$  (minor premise)// $A_{em}C (\forall Z)(C_{aM}Z \supset \bar{A}_{aM}Z)$  (Def ex)
3.  $(\forall Z)[(C_{aM}Z \supset B_{aM}Z) \& (C_{aM}Z \supset \bar{B}_{aM}Z)]$  (2 Def aQ)
4.  $A_{eLB} \supset (\forall Z)\{[B_{aM}Z \supset \bar{A}_{aM}Z] \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (L-M-sub eL)
5.  $(\forall Z)\{[B_{aM}Z \supset \bar{A}_{aM}Z] \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (1,4 MP)
6.  $A_{iL}C$  (IP)
7.  $A_{iL}C \supset (\exists Z)(C_{aM}Z \& A_{aM}Z)$  (L-M-sub iL)
8.  $(\exists Z)(C_{aM}Z \& A_{aM}Z)$  (6,7 MP)
9.  $C_{aMU} \& A_{aMU}$  (8 EI)
10.  $(C_{aMU} \supset B_{aMU}) \& (C_{aMU} \supset \bar{B}_{aMU})$  (3 UI)
11.  $[B_{aMU} \supset \bar{A}_{aMU}] \& [B_{aMU} \supset \sim(A_{aMU})]$  (5 UI)
12.  $C_{aMU} \supset B_{aMU}$  (10 Simp)
13.  $C_{aMU}$  (9 Simp)
14.  $B_{aMU}$  (12,13 MP)
15.  $A_{aMU}$  (9 Simp)
16.  $\sim\sim(A_{aMU})$  (15 DN)
17.  $B_{aMU} \supset \sim(A_{aMU})$  (11 Simp)
18.  $\sim(B_{aMU})$  (16,17 MT)
19.  $B_{aMU} \& \sim(B_{aMU})$  (14,18 Conj)
20.  $\sim(A_{iL}C)$  (6-19 IP)
21.  $A_{em}C$  (20 em|iL)

Darii-XXX (Valid):

1.  $A_{ax}B$  (major premise)

2.  $BixC$  (minor premise) //  $AixC (\exists Z)(CaxZ \& AaxZ)$
3.  $CixB$  (2 Conv ix-ix)
4.  $(\exists Z)(BaxZ \& CaxZ)$  (3 Def ix)
5.  $(\forall Z)(BaxZ \supset AaxZ)$  (1 Def ax)
6.  $BaxU \& CaxU$  (4 EI)
7.  $BaxU$  (6 Simp)
8.  $BaxU \supset AaxU$  (5 UI)
9.  $AaxU$  (7,8 MP)
10.  $CaxU$  (6 Simp)
11.  $AaxU \& CaxU$  (9,10 Conj)
12.  $(\exists Z)(AaxZ \& CaxZ)$  (11 EG)
13.  $CixA$  (11 Def ix)
14.  $AixC$

Darii-LLL (Valid):

1.  $AaLb$  (major premise)
2.  $BiLc$  (minor premise) //  $AiLc (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$
3.  $ciLb$  (2 Conv  $iL-iL$ )
4.  $(\exists Z)(BaxZ \& CaLZ)$  (3 Def  $iL$ )
5.  $AaLb \supset (\forall Z)(BaxZ \supset AaxZ)$  (L-X sub  $aL-ax$ )
6.  $(\forall Z)(BaxZ \supset AaxZ)$  (1,5 MP)
7.  $BaxU \& CaLU$  (4 EI)
8.  $BaxU$  (7 Simp)
9.  $BaxU \supset AaxU$  (6 UI)
10.  $AaxU$  (8,9 MP)
11.  $CaLU$  (6 Simp)
12.  $AaxU \& CaLU$  (10,11 Conj)
13.  $(AaxU \& CaLU) \vee (CaxU \& AaLU)$  (12 EG)
14.  $(\exists Z)[(AaxZ \& CaLZ) \vee (CaxZ \& AaLZ)]$
14.  $ciLA$  (13 Def  $iL$ )
15.  $AiLc$  (14 Conv  $iL-iL$ )

Darii-LXL (Valid):

1.  $AaLb$  (major premise)
2.  $BixC$  (minor premise) //  $AiLc (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$
3.  $(\exists Z)(CaxZ \& BaxZ)$  (2 Def ix)
4.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def  $aL$ )
5.  $BaxU \& CaxU$  (4 EI)
6.  $BaxU$  (5 Simp)
7.  $BaxU \supset AaLU$  (4 UI)
8.  $AaLU$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \& AaLU$  (8,9 Conj)



11.  $(CaxU \& AaLU) \vee (AaxU \& CaLU)$  (10 Add)
12.  $(AaxU \& CaLU) \vee (CaxU \& AaLU)$  (11 Comm)
13.  $(\exists Z)[(AaxZ \& CaLZ) \vee (CaxZ \& AaLZ)]$  (12 EG)
14.  $CaLA$  (13 Def  $iL$ )
15.  $AiLC$  (14 Conv  $iL$ - $iL$ )

Darii-QQQ (Valid):

1.  $AaQB$  (major premise)
2.  $BiQC$  (minor premise)// $AiQC$   $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def  $iQ$ )
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{A}aMZ)]$  (1 Def  $aQ$ )
4.  $(\exists Z)[CaMZ \& (BaMZ \& \bar{B}aMZ)]$  (2 Def  $iQ$ )
5.  $CaMU \& (BaMU \& \bar{B}aMU)$  (4 EI)
6.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{A}aMU)$  (3 UI)
7.  $BaMU \supset AaMU$  (6 Simp)
8.  $BaMU \& \bar{B}aMU$  (5 Simp)
9.  $BaMU$  (8 Simp)
10.  $AaMU$  (7,9 MP)
11.  $BaMU \supset \bar{A}aMU$  (6 Simp)
12.  $\bar{A}aMU$  (9,11 MP)
13.  $AaMU \& \bar{A}aMU$  (10,12 Conj)
14.  $CaMU$  (5 Simp)
15.  $CaMU \& (AaMU \& \bar{A}aMU)$  (13,14 Conj)
16.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (15, EG)
17.  $AiQC$  (16 Def  $iQ$ )

Darii-QXQ (Valid):

1.  $AaQB$  (major premise)
2.  $BiXC$  (minor premise)// $AiQC$   $(\exists Z)[CaQZ \& (AaMZ \& \bar{A}aMZ)]$  (Def  $iQ$ )
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{A}aMZ)]$  (1 Def  $aQ$ )
4.  $BiXC \supset (\exists Z)(CaMZ \& BaMZ)$  (X-M-sub  $iX$ )
5.  $(\exists Z)(CaMZ \& BaMZ)$  (2,4 MP)
6.  $CaMU \& BaMU$  (5 EI)
7.  $BaMU$  (6 Simp)
8.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{A}aMU)$  (3 UI)
9.  $BaMU \supset AaMU$  (8 Simp)
10.  $AaMU$  (7,9 MP)
11.  $BaMU \supset \bar{A}aMU$  (8 Simp)
12.  $\bar{A}aMU$  (7,11 MP)
13.  $AaMU \& \bar{A}aMU$  (10,12 Conj)
14.  $CaMU$  (6 Simp)
15.  $CaMU \& (AaMU \& \bar{A}aMU)$  (13,14 Conj)
16.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (15, EG)
17.  $AiQC$  (16 Def  $iQ$ )

Darii-XQM (Valid):

1.  $AaxB$  (major premise)
2.  $BiQC$  (minor premise)// $AiMC (\exists Z)(CaMZ \& AaMZ)$  (Def im)
3.  $AaxB \supset (\forall Z)(BaMZ \supset AaMZ)$  (X-M-sub ax)
4.  $(\exists Z)[CaMZ \& (BaMZ \& \bar{BaMZ})]$  (2 Def iQ)
5.  $CaMU \& (BaMU \& \bar{BaMU})$  (4 EI)
6.  $(\forall Z)(BaMZ \supset AaMZ)$  (1,3 MP)
7.  $BaMU \supset AaMU$  (6 UI)
8.  $BaMU \& \bar{BaMU}$  (5 Simp)
9.  $BaMU$  (8 Simp)
10.  $AaMU$  (7,9 MP)
11.  $CaMU$  (5 Simp)
12.  $CaMU \& AaMU$  (10,11 Conj)
13.  $(\exists Z)(CaMZ \& AaMZ)$  (12, EG)
14.  $AiMC$  (13 Def im)

Darii-QLQ (Valid):

1.  $AaQB$  (major premise)
2.  $BiLC$  (minor premise)// $AiQC (\exists Z)[CaMZ \& (AaMZ \& \bar{AaMZ})]$  (Def iQ)
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{AaMZ})]$  (1 Def aQ)
4.  $BiLC \supset (\exists Z)(CaMZ \& BaMZ)$  (L-M-sub iL)
5.  $(\exists Z)(CaMZ \& BaMZ)$  (2,4 MP)
6.  $CaMU \& BaMU$  (5 EI)
7.  $BaMU$  (6 Simp)
8.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{AaMU})$  (3 UI)
9.  $BaMU \supset AaMU$  (8 Simp)
10.  $AaMU$  (7,9 MP)
11.  $BaMU \supset \bar{AaMU}$  (8 Simp)
12.  $\bar{AaMU}$  (7,11 MP)
13.  $AaMU \& \bar{AaMU}$  (10,12 Conj)
14.  $CaMU$  (6 Simp)
15.  $CaMU \& (AaMU \& \bar{AaMU})$  (13,14 Conj)
16.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{AaMZ})]$  (15, EG)
17.  $AiQC$  (16 Def iQ)

Darii-LQM (Valid):

1.  $AaLB$  (major premise)
2.  $BiQC$  (minor premise)// $AiMC (\exists Z)(CaMZ \& AaMZ)$  (Def iQ)
3.  $AaLB \supset (\forall Z)(BaMZ \supset AaMZ)$  (L-M-sub aL)
4.  $(\exists Z)[CaMZ \& (BaMZ \& \bar{BaMZ})]$  (2 Def iQ)
5.  $CaMU \& (BaMU \& \bar{BaMU})$  (4 EI)

6.  $(\forall Z)(B_{aM}Z \supset A_{aM}Z)$  (1,3 MP)
7.  $B_{aM}U \supset A_{aM}U$  (6 UI)
8.  $B_{aM}U \ \& \ \bar{B}_{aM}U$  (5 Simp)
9.  $B_{aM}U$  (8 Simp)
10.  $A_{aM}U$  (7,9 MP)
11.  $C_{aM}U$  (5 Simp)
12.  $C_{aM}U \ \& \ A_{aM}U$  (9,11 Conj)
13.  $(\exists Z)(C_{aM}Z \ \& \ A_{aM}Z)$  (12, EG)
14.  $A_{iM}C$  (13 Def  $i_Q$ )

Ferio-XXX (Valid):

1.  $A_{eX}B$  (major premise)
2.  $B_{iX}C$  (minor premise)// $A_{oXC} (\exists Z)[C_{aX}Z \ \& \ \sim(A_{aX}Z)]$
3.  $(\exists Z)(C_{aX}Z \ \& \ B_{aX}Z)$  (2 Def  $i_X$ )
4.  $(\forall Z)[B_{aX}Z \supset \sim(A_{aX}Z)]$  (1 Def  $e_X$ )
5.  $C_{aX}U \ \& \ B_{aX}U$  (3 EI)
6.  $B_{aX}U \supset \sim(A_{aX}U)$  (4 EI)
7.  $B_{aX}U$  (5 Simp)
8.  $\sim(A_{aX}U)$  (6,7 MP)
9.  $C_{aX}U$  (5 Simp)
10.  $C_{aX}U \ \& \ \sim(A_{aX}U)$  (8,9 Conj)
11.  $(\exists Z)[C_{aX}Z \ \& \ \sim(A_{aX}Z)]$  (10 EG)
12.  $A_{oXC}$  (11 Def  $o_X$ )

Ferio-LLL (Valid):

1.  $A_{eL}B$  (major premise)
2.  $B_{iL}C$  (minor premise)// $A_{iLC} (\exists Z)[(C_{aX}Z \ \& \ \bar{A}_{aL}Z) \vee (\bar{A}_{aX}Z \ \& \ C_{aL}Z)]$
3.  $B_{iL}C \supset B_{iX}C$  (L-X-sub  $i_L$ )
4.  $B_{iX}C$  (2,3 MP)
5.  $(\exists Z)(C_{aX}Z \ \& \ B_{aX}Z)$  (4 Def  $i_X$ )
6.  $(\forall Z)(B_{aX}Z \supset \bar{A}_{aL}Z)$  (1 Def  $e_L$ )
7.  $C_{aX}U \ \& \ B_{aX}U$  (5 EI)
8.  $B_{aX}U \supset \bar{A}_{aL}U$  (6 EI)
9.  $B_{aX}U$  (7 Simp)
10.  $\bar{A}_{aL}U$  (8,9 MP)
11.  $C_{aX}U$  (7 Simp)
12.  $C_{aX}U \ \& \ \bar{A}_{aL}U$  (10,11 Conj)
13.  $(C_{aX}U \ \& \ \bar{A}_{aL}U) \vee (\bar{A}_{aX}U \ \& \ C_{aL}U)$  (12 Add)
14.  $(\exists Z)[(C_{aX}Z \ \& \ \bar{A}_{aL}Z) \vee (\bar{A}_{aX}Z \ \& \ C_{aL}Z)]$  (13 EG)
15.  $A_{oLC}$  (14 Def  $o_L$ )

Ferio-LXL (Valid):

1.  $AeLB$  (major premise)
2.  $BixC$  (minor premise)// $AoLC (\exists Z)[(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)]$
3.  $(\exists Z)(CaxZ \& BaxZ)$  (2 Def ix)
4.  $(\forall Z)(BaxZ \supset \bar{A}aLZ)$  (1 Def eL)
5.  $CaxU \& BaxU$  (3 EI)
6.  $BaxU \supset \bar{A}aLU$  (4 EI)
7.  $BaxU$  (5 Simp)
8.  $\bar{A}aLU$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \& \bar{A}aLU$  (8,9 Conj)
11.  $(CaxU \& \bar{A}aLU) \vee (\bar{A}axU \& CaLU)$  (10 Add)
12.  $(\exists Z)[(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)]$  (11 EG)
13.  $AoLC$  (12 Def oL)

Ferio-QQQ (Valid):

1.  $AeQB$  (major premise)
2.  $BiQC$  (minor premise)// $AoQC (\exists Z)[CamZ \& (AamZ \& \bar{A}amZ)]$  (Def oQ)
3.  $(\forall Z)[(BamZ \supset AamZ) \& (BamZ \supset \bar{A}amZ)]$  (1 Def eQ)
4.  $(\exists Z)[CamZ \& (BamZ \& \bar{B}amZ)]$  (2 Def iQ)
5.  $CamU \& (BamU \& \bar{B}amU)$  (4 EI)
6.  $(BamU \supset AamU) \& (BamU \supset \bar{A}amU)$  (3 UI)
7.  $BamU \supset AamU$  (6 Simp)
8.  $BamU \& \bar{B}amU$  (5 Simp)
9.  $BamU$  (8 Simp)
10.  $AamU$  (7,9 MP)
11.  $BamU \supset \bar{A}amU$  (6 Simp)
12.  $\bar{A}amU$  (9,11 MP)
13.  $AamU \& \bar{A}amU$  (10,12 Conj)
14.  $CamU$  (5 Simp)
15.  $CamU \& (AamU \& \bar{A}amU)$  (13,14 Conj)
16.  $(\exists Z)[CamZ \& (AamZ \& \bar{A}amZ)]$  (15, EG)
17.  $AoQC$  (16 Def oQ)

Ferio-QXQ (Valid):

1.  $AeQB$  (major premise)
2.  $BiXC$  (minor premise)// $AoQC (\exists Z)[CamZ \& (AamZ \& \bar{A}amZ)]$  (Def oQ)
3.  $(\forall Z)[(BamZ \supset AamZ) \& (BamZ \supset \bar{A}amZ)]$  (1 Def eQ)
4.  $BiXC \supset (\exists Z)(CamZ \& BamZ)$  (X-M-sub ix)

5.  $(\exists Z)(C_{AM}Z \ \& \ B_{AM}Z)$  (2,4 MP)
6.  $C_{AM}U \ \& \ B_{AM}U$  (5 EI)
7.  $B_{AM}U$  (6 Simp)
8.  $(B_{AM}U \supset A_{AM}U) \ \& \ (B_{AM}U \supset \bar{A}_{AM}U)$  (3 UI)
9.  $B_{AM}U \supset A_{AM}U$  (8 Simp)
10.  $A_{AM}U$  (7,9 MP)
11.  $B_{AM}U \supset \bar{A}_{AM}U$  (8 Simp)
12.  $\bar{A}_{AM}U$  (7,11 MP)
13.  $A_{AM}U \ \& \ \bar{A}_{AM}U$  (10,12 Conj)
14.  $C_{AM}U$  (6 Simp)
15.  $C_{AM}U \ \& \ (A_{AM}U \ \& \ \bar{A}_{AM}U)$  (13,14 Conj)
16.  $(\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (15, EG)
17.  $A_{OQ}C$  (16 Def  $o_Q$ )

Ferio-XQM (Valid):

1.  $A_{eX}B$  (major premise)
2.  $B_{iQ}C$  (minor premise)// $A_{oM}C \ (\exists Z)(C_{AM}Z \ \& \ \bar{A}_{AM}Z)$  (Def  $o_M$ )
3.  $A_{eX}B \supset (\forall Z)(B_{AM}Z \supset \bar{A}_{AM}Z)$  (X-M-sub ax)
4.  $(\exists Z)[C_{AM}Z \ \& \ (B_{AM}Z \ \& \ \bar{B}_{AM}Z)]$  (2 Def  $i_Q$ )
5.  $C_{AM}U \ \& \ (B_{AM}U \ \& \ \bar{B}_{AM}U)$  (4 EI)
6.  $(\forall Z)(B_{AM}Z \supset \bar{A}_{AM}Z)$  (1,3 MP)
7.  $B_{AM}U \supset \bar{A}_{AM}U$  (6 UI)
8.  $B_{AM}U \ \& \ \bar{B}_{AM}U$  (5 Simp)
9.  $B_{AM}U$  (8 Simp)
10.  $\bar{A}_{AM}U$  (7,9 MP)
11.  $C_{AM}U$  (5 Simp)
12.  $C_{AM}U \ \& \ \bar{A}_{AM}U$  (10,11 Conj)
13.  $(\exists Z)(C_{AM}Z \ \& \ \bar{A}_{AM}Z)$  (12, EG)
14.  $A_{oM}C$  (13 Def  $o_M$ )

Ferio-QLQ (Valid):

1.  $A_{eQ}B$  (major premise)
2.  $B_{iL}C$  (minor premise)// $A_{oQ}C \ (\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (Def  $o_Q$ )
3.  $(\forall Z)[(B_{AM}Z \supset A_{AM}Z) \ \& \ (B_{AM}Z \supset \bar{A}_{AM}Z)]$  (1 Def  $e_Q$ )
4.  $B_{iL}C \supset (\exists Z)(C_{AM}Z \ \& \ B_{AM}Z)$  (L-M-sub  $i_L$ )
5.  $(\exists Z)(C_{AM}Z \ \& \ B_{AM}Z)$  (2,4 MP)
6.  $C_{AM}U \ \& \ B_{AM}U$  (5 EI)
7.  $B_{AM}U$  (6 Simp)
8.  $(B_{AM}U \supset A_{AM}U) \ \& \ (B_{AM}U \supset \bar{A}_{AM}U)$  (3 UI)
9.  $B_{AM}U \supset A_{AM}U$  (8 Simp)
10.  $A_{AM}U$  (7,9 MP)
11.  $B_{AM}U \supset \bar{A}_{AM}U$  (8 Simp)
12.  $\bar{A}_{AM}U$  (7,11 MP)

13.  $A_{aM}U$  &  $\bar{A}_{aM}U$  (10,12 Conj)
14.  $C_{aM}U$  (6 Simp)
15.  $C_{aM}U$  & ( $A_{aM}U$  &  $\bar{A}_{aM}U$ )(13,14 Conj)
16.  $(\exists Z)[C_{aM}Z$  & ( $A_{aM}Z$  &  $\bar{A}_{aM}Z$ )] (15, EG)
17.  $A_{oQ}C$  (16 Def  $oQ$ )

Ferio-LQX (Valid):

1.  $A_{eL}B$  (major premise)
2.  $B_{iQ}C$  (minor premise)// $A_{oX}C$   $(\exists Z)(C_{aX}Z$  &  $\bar{A}_{aX}Z)$  (Def  $oX$ )
3.  $A_{eL}B \supset (\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$ (L-M-sub  $e_L$ )
4.  $(\exists Z)[C_{aM}Z$  & ( $B_{aM}Z$  &  $\bar{B}_{aM}Z$ )] (2 Def  $iQ$ )
5.  $C_{aM}U$  & ( $B_{aM}U$  &  $\bar{B}_{aM}U$ ) (4 EI)
6.  $A_{aX}C$  (IP)
7.  $A_{aX}C \supset (\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (L-M-sub  $e_L$ )
8.  $(\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (6,7 MP)
9.  $C_{aM}U \supset A_{aM}U$  (8 UI)
10.  $C_{aM}U$  (5 Simp)
11.  $A_{aM}U$  (9,10 MP)
12.  $(\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (1,3 MP)
13.  $[B_{aM}U \supset \bar{A}_{aM}U]$  &  $[B_{aM}U \supset \sim(A_{aM}U)]$  (12 UI)
14.  $B_{aM}U$  &  $\bar{B}_{aM}U$  (5 Simp)
15.  $B_{aM}U$  (14 Simp)
16.  $B_{aM}U \supset \sim(A_{aM}U)$  (12 Simp)
17.  $\sim(A_{aM}U)$  (15,16 MP)
18.  $A_{aM}U$  &  $\sim(A_{aM}U)$  (11,17 Conj)
19.  $\sim(A_{aX}C)$  (6-18 IP)
20.  $A_{oX}C$  (19  $oX|aX$ )

**Second Figure:**

Cesare-XXX (Valid):

1.  $B_{eX}A$  (major premise)
2.  $B_{aX}C$  (minor premise)// $A_{eX}C$   $(\forall Z)[C_{aX}Z \supset \sim(A_{aX}Z)]$
3.  $(\forall Z)(C_{aX}Z \supset B_{aX}Z)$  (2 Def  $aX$ )
4.  $A_{eX}B$  (1 Conv  $eX-eX$ )
5.  $(\forall Z)[B_{aX}Z \supset \sim(A_{aX}Z)]$  (4 Def  $e_L$ )
6.  $C_{aX}U \supset B_{aX}U$  (3 UI)
7.  $B_{aX}U \supset \sim(A_{aX}U)$  (5 UI)
8.  $C_{aX}U \supset \sim(A_{aX}U)$  (6,7 HS)

9.  $(\forall Z)[CaxZ \supset \sim(AaxZ)]$  (8 UG)
10.  $AexC$  (9 Def ex)

Cesare-LLL (Valid):

1.  $BeLA$  (major premise)
2.  $BaLC$  (minor premise) //  $AeLC (\forall Z)(CaxZ \supset \bar{AaLZ})$
3.  $BaLC \supset (\forall Z)(CaxZ \supset BaxZ)$  (L-X-sub  $aL$ - $ax$ )
5.  $(\forall Z)(CaxZ \supset BaxZ)$  (2,3 MP)
6.  $AeLB$  (1 Conv  $eL$ - $eL$ )
7.  $(\forall Z)(BaxZ \supset \bar{AaLZ})$  (6 Def  $eL$ )
8.  $CaxU \supset BaxU$  (5 UI)
9.  $BaxU \supset \bar{AaLU}$  (7 UI)
10.  $CaxU \supset \bar{AaLU}$  (8,9 HS)
11.  $(\forall Z)(CaxZ \supset \bar{AaLZ})$  (10 UG)
12.  $AeLC$  (11 Def  $eL$ )

Cesare-LXL (Valid):

1.  $BeLA$  (major premise)
2.  $BaxC$  (minor premise) //  $AeLC (\forall Z)(CaxZ \supset \bar{AaLZ})$
3.  $(\forall Z)(CaxZ \supset BaxZ)$  (2 Def  $ax$ )
4.  $AeLB$  (1 Conv  $eL$ - $eL$ )
5.  $(\forall Z)(BaxZ \supset \bar{AaLZ})$  (4 Def  $eL$ )
6.  $CaxU \supset BaxU$  (3 UI)
7.  $BaxU \supset \bar{AaLU}$  (5 UI)
8.  $CaxU \supset \bar{AaLU}$  (6,7 HS)
9.  $(\forall Z)(CaxZ \supset \bar{AaLZ})$  (8 UG)
10.  $AeLC$  (9 Def  $eL$ )

Cesare-XQM (Valid):

1.  $BexA$  (major premise)
2.  $BaqC$  (minor premise) //  $AemC (\forall Z)(CamZ \supset \bar{AamZ})$  (Def  $em$ )
3.  $AexB$  (1 Conv  $ex$ - $ex$ )
4.  $AexB \supset (\forall Z)(BamZ \supset \bar{AamZ})$  (X-M-sub  $ax$ )
5.  $(\forall Z)[(CamZ \supset BamZ) \& (CamZ \supset \bar{BamZ})]$  (2 Def  $aQ$ )
6.  $(\forall Z)(BamZ \supset \bar{AamZ})$  (3,4 MP)
7.  $(CamU \supset BamU) \& (CaxU \supset \bar{BamU})$  (5 UI)
8.  $BamU \supset \bar{AamU}$  (6 UI)
9.  $CamU \supset BamU$  (7 Simp)
10.  $CamU \supset \bar{AamU}$  (8,9 HS)
11.  $(\forall Z)(CamZ \supset \bar{AamZ})$  (9 UG)

12.  $Ae_M C$  (11 Def  $e_M$ )

Cesare-LQX (Valid):

1.  $Be_L A$  (major premise)
2.  $Ba_Q C$  (minor premise) //  $Ae_X C (\forall Z)[Ca_X Z \supset \sim(Aa_X Z)]$  (Def  $e_X$ )
3.  $Ae_L B$  (1 Conv  $e_L$ - $e_L$ )
4.  $(\forall Z)[(Ca_M Z \supset Ba_M Z) \& (Ca_M Z \supset \bar{B}a_M Z)]$  (2 Def  $a_Q$ )
5.  $Ae_L B \supset (\forall Z)\{[Ba_M Z \supset \bar{A}a_M Z] \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (L-M-sub  $e_L$ )
6.  $(\forall Z)\{[Ba_M Z \supset \bar{A}a_M Z] \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (3,5 MP)
7.  $Ai_X C$  (IP)
8.  $Ai_X C \supset (\exists Z)(Ca_M Z \& Aa_M Z)$  (X-M-sub  $i_X$ )
9.  $(\exists Z)(Ca_M Z \& Aa_M Z)$  (7,8 MP)
10.  $Ca_M U \& Aa_M U$  (9 EI)
11.  $(Ca_M U \supset Ba_M U) \& (Ca_M U \supset \bar{B}a_M U)$  (4 UI)
12.  $[Ba_M U \supset \bar{A}a_M U] \& [Ba_M U \supset \sim(Aa_M U)]$  (6 UI)
13.  $Ca_M U \supset Ba_M U$  (11 Simp)
14.  $Ca_M U$  (10 Simp)
15.  $Ba_M U$  (13,14 MP)
16.  $Aa_M U$  (10 Simp)
17.  $\sim\sim(Aa_M U)$  (16 DN)
18.  $Ba_M U \supset \sim(Aa_M U)$  (12 Simp)
19.  $\sim(Ba_M U)$  (17,18 MT)
20.  $Ba_M U \& \sim(Ba_M U)$  (15,19 Conj)
21.  $\sim(Ai_X C)$  (7-20 IP)
22.  $Ae_X C$  (21  $e_X|i_X$ )

Cesare-LQX (Valid):

1.  $Be_L A$  (major premise)
2.  $Ba_Q C$  (minor premise) //  $Ae_X C (\forall Z)[Ca_X Z \supset \sim(Aa_X Z)]$  (Def  $e_X$ )
3.  $Ae_L B$  (1 Conv  $e_L$ - $e_L$ )
4.  $(\forall Z)[(Ca_M Z \supset Ba_M Z) \& (Ca_M Z \supset \bar{B}a_M Z)]$  (2 Def  $a_Q$ )
5.  $Ae_L B \supset (\forall Z)\{[Ba_M Z \supset \bar{A}a_M Z] \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (L-M-sub  $e_L$ )
6.  $(\forall Z)\{[Ba_M Z \supset \bar{A}a_M Z] \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (3,5 MP)
7.  $Ai_L C$  (IP)
8.  $Ai_L C \supset (\exists Z)(Ca_M Z \& Aa_M Z)$  (L-M-sub  $i_L$ )
9.  $(\exists Z)(Ca_M Z \& Aa_M Z)$  (7,8 MP)
10.  $Ca_M U \& Aa_M U$  (9 EI)
11.  $(Ca_M U \supset Ba_M U) \& (Ca_M U \supset \bar{B}a_M U)$  (4 UI)
12.  $[Ba_M U \supset \bar{A}a_M U] \& [Ba_M U \supset \sim(Aa_M U)]$  (6 UI)
13.  $Ca_M U \supset Ba_M U$  (11 Simp)
14.  $Ca_M U$  (10 Simp)
15.  $Ba_M U$  (13,14 MP)
16.  $Aa_M U$  (10 Simp)



17.  $\sim\sim(Aa_MU)$  (16 DN)
18.  $Ba_MU \supset \sim(Aa_MU)$  (12 Simp)
19.  $\sim(Ba_MU)$  (17,18 MT)
20.  $Ba_MU \& \sim(Ba_MU)$  (15,19 Conj)
21.  $\sim(Ai_LC)$  (7-20 IP)
22.  $Ae_MC$  (21  $e_M|i_L$ )

Camestres-XXX (Valid):

1.  $Ba_xA$  (major premise)
2.  $Be_xC$  (minor premise) //  $Ae_xC (\forall Z)[Ca_xZ \supset \sim(Aa_LZ)]$
3.  $(\forall Z)(Aa_xZ \supset Ba_xZ)$  (1 Def ax)
4.  $Ce_xB$  (2 Conv  $e_L-e_L$ )
5.  $(\forall Z)[Ba_xZ \supset \sim(Ca_LZ)]$  (4 Def ex)
6.  $Aa_xU \supset Ba_xU$  (3 UI)
7.  $Ba_xU \supset \sim(Ca_LU)$  (5 UI)
8.  $Aa_xU \supset \sim(Ca_LU)$  (6,7 HS)
9.  $(\forall Z)[Aa_xZ \supset \sim(Ca_LZ)]$  (8 UG)
10.  $Ce_xA$  (9 Def ex)
11.  $Ae_xC$  (10 Conv ex-ex)

Camestres-LLL (Valid):

1.  $Ba_LA$  (major premise)
2.  $Be_LC$  (minor premise) //  $Ae_LC (\forall Z)(Ca_xZ \supset \bar{A}a_LZ)$
3.  $Ba_LA \supset (\forall Z)(Aa_xZ \supset Ba_xZ)$  (L-X-sub  $a_L$ -ax)
4.  $(\forall Z)(Aa_xZ \supset Ba_xZ)$  (1,3 MP)
5.  $Ce_LB$  (2 Conv  $e_L-e_L$ )
6.  $(\forall Z)(Ba_xZ \supset \bar{C}a_LZ)$  (5 Def  $e_L$ )
7.  $Aa_xU \supset Ba_xU$  (4 UI)
8.  $Ba_xU \supset \bar{C}a_LU$  (6 UI)
9.  $Aa_xU \supset \bar{C}a_LU$  (7,8 HS)
10.  $(\forall Z)(Aa_xZ \supset \bar{C}a_LZ)$  (9 UG)
11.  $Ce_LA$  (10 Def  $e_L$ )
12.  $Ae_LC$  (11 Conv  $e_L-e_L$ )

Camestres-XLL (Valid):

1.  $Ba_xA$  (major premise)
2.  $Be_LC$  (minor premise) //  $Ae_LC (\forall Z)(Ca_xZ \supset \bar{A}a_LZ)$
3.  $(\forall Z)(Aa_xZ \supset Ba_xZ)$  (1 Def ax)
4.  $Ce_LB$  (2 Conv  $e_L-e_L$ )
5.  $(\forall Z)(Ba_xZ \supset \bar{C}a_LZ)$  (4 Def  $e_L$ )
6.  $Aa_xU \supset Ba_xU$  (3 UI)

7.  $BaxU \supset \bar{C}aLU$  (5 UI)
8.  $AaxU \supset \bar{C}aLU$  (6,7 HS)
9.  $(\forall Z)(AaxZ \supset \bar{C}aLZ)$  (8 UG)
10.  $CeLA$  (9 Def  $e_L$ )
11.  $AeLC$  (10 Conv  $e_L$ - $e_L$ )

Camestres-QXM (Valid):

1.  $BaqA$  (major premise)
2.  $BexC$  (minor premise)// $AemC$   $(\forall Z)(CamZ \supset \bar{A}amZ)$  (Def  $e_M$ )
3.  $CexB$  (1 Conv  $e_X$ - $e_X$ )
4.  $CexB \supset (\forall Z)(BamZ \supset \bar{C}amZ)$  (X-M-sub  $a_X$ )
5.  $(\forall Z)[(AamZ \supset BamZ) \& (AamZ \supset \bar{B}amZ)]$  (1 Def  $a_Q$ )
6.  $(\forall Z)(BamZ \supset \bar{C}amZ)$  (3,4 MP)
7.  $(AamU \supset BamU) \& (AaxU \supset \bar{B}amU)$  (5 UI)
8.  $BamU \supset \bar{C}amU$  (6 UI)
9.  $AamU \supset BamU$  (7 Simp)
10.  $AamU \supset \bar{C}amU$  (8,9 HS)
11.  $(\forall Z)(AamZ \supset \bar{C}amZ)$  (9 UG)
12.  $CemA$  (11 Def  $e_M$ )
13.  $AemC$  (12 Conv  $e_M$ - $e_M$ )

Camestres-QLX (Valid):

1.  $BaqA$  (major premise)
2.  $BelC$  (minor premise)// $AexC$   $(\forall Z)[CaxZ \supset \sim(AaxZ)]$  (Def  $e_X$ )
3.  $CelB$  (1 Conv  $e_L$ - $e_L$ )
4.  $(\forall Z)[(AamZ \supset BamZ) \& (AamZ \supset \bar{B}amZ)]$  (1 Def  $a_Q$ )
5.  $CelB \supset (\forall Z)\{[BamZ \supset \bar{C}amZ] \& [BamZ \supset \sim(CamZ)]\}$  (L-M-sub  $e_L$ )
6.  $(\forall Z)\{[BamZ \supset \bar{C}amZ] \& [BamZ \supset \sim(CamZ)]\}$  (3,5 MP)
7.  $AixC$  (IP)
8.  $AixC \supset (\exists Z)(CamZ \& AamZ)$  (X-M-sub  $i_X$ )
9.  $(\exists Z)(CamZ \& AamZ)$  (7,8 MP)
10.  $CamU \& AamU$  (9 EI)
11.  $(AamU \supset BamU) \& (AamU \supset \bar{B}amU)$  (4 UI)
12.  $[BamU \supset \bar{C}amU] \& [BamU \supset \sim(CamU)]$  (6 UI)
13.  $AamU \supset BamU$  (11 Simp)
14.  $AamU$  (10 Simp)
15.  $BamU$  (13,14 MP)
16.  $CamU$  (10 Simp)
17.  $\sim\sim(CamU)$  (16 DN)
18.  $BamU \supset \sim(CamU)$  (12 Simp)
19.  $\sim(BamU)$  (17,18 MT)
20.  $BamU \& \sim(BamU)$  (15,19 Conj)
21.  $\sim(AixC)$  (7-20 IP)

22.  $Ae_xC$  (21 ex|ix)

Camestres-QLM (Valid):

1.  $Ba_0A$  (major premise)
2.  $Be_LC$  (minor premise)// $Ae_xC$  ( $\forall Z$ )[ $CaxZ \supset \sim(AaxZ)$ ] (Def ex)
3.  $Ce_LB$  (1 Conv  $e_L$ - $e_L$ )
4. ( $\forall Z$ )[( $Aa_MZ \supset Ba_MZ$ ) & ( $Aa_MZ \supset \bar{B}a_MZ$ )] (1 Def  $a_0$ )
5.  $Ce_LB \supset (\forall Z)\{[Ba_MZ \supset \bar{C}a_MZ] \& [Ba_MZ \supset \sim(Ca_MZ)]\}$  (L-M-sub  $e_L$ )
6. ( $\forall Z$ ){ $[Ba_MZ \supset \bar{C}a_MZ]$  &  $[Ba_MZ \supset \sim(Ca_MZ)]$ } (3,5 MP)
7.  $Ai_LC$  (IP)
8.  $Ai_LC \supset (\exists Z)(Ca_MZ \& Aa_MZ)$  (L-M-sub  $i_L$ )
9. ( $\exists Z$ )( $Ca_MZ \& Aa_MZ$ ) (7,8 MP)
10.  $Ca_MU \& Aa_MU$  (9 EI)
11. ( $Aa_MU \supset Ba_MU$ ) & ( $Aa_MU \supset \bar{B}a_MU$ ) (4 UI)
12.  $[Ba_MU \supset \bar{C}a_MU]$  &  $[Ba_MU \supset \sim(Ca_MU)]$  (6 UI)
13.  $Aa_MU \supset Ba_MU$  (11 Simp)
14.  $Aa_MU$  (10 Simp)
15.  $Ba_MU$  (13,14 MP)
16.  $Ca_MU$  (10 Simp)
17.  $\sim\sim(Ca_MU)$  (16 DN)
18.  $Ba_MU \supset \sim(Ca_MU)$  (12 Simp)
19.  $\sim(Ba_MU)$  (17,18 MT)
20.  $Ba_MU \& \sim(Ba_MU)$  (15,19 Conj)
21.  $\sim(Ai_LC)$  (7-20 IP)
22.  $Ae_MC$  (21 ex|ix)

Festino-XXX (Valid):

1.  $BexA$  (major premise)
2.  $BixC$  (minor premise)// $AoxC$  ( $\exists Z$ )[ $CaxZ \& \sim(AaxZ)$ ]
3.  $AexB$  (1 Conv ex-ex)
4. ( $\exists Z$ )(  $CaxZ \& BaxZ$ ) (2 Def ix)
5. ( $\forall Z$ )( $BaxZ \supset \sim(AaxZ)$ ) (3 Def ex)
6.  $CaxU \& BaxU$  (4 EI)
7.  $BaxU \supset \sim(AaxU)$  (5 UI)
8.  $BaxU$  (6 Simp)
9.  $\sim(AaxU)$  (7,8 MP)
10.  $CaxU$  (6 Simp)
11.  $CaxU \& \sim(AaxU)$  (9,10 Conj)
12. ( $\exists Z$ )[ $CaxZ \& \sim(AaxZ)$ ] (11 EG)
13.  $AoxC$  (12 Def  $ox$ )

Festino-LLL (Valid):

1.  $Be_LA$  (major premise)
2.  $Bi_LC$  (minor premise) //  $Ao_LC (\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xZ \& Ca_LZ)]$
3.  $Bi_LC \supset (\exists Z)(Ca_xZ \& Ba_xZ)$  (L-X-sub  $i_L$ )
4.  $(\exists Z)(Ca_xZ \& Ba_xZ)$  (2,3 MP)
5.  $Ae_LB$  (1 Conv  $e_L-e_L$ )
6.  $(\forall Z)(Ba_xZ \supset \bar{A}a_LZ)$  (5 Def  $e_L$ )
7.  $Ca_xU \& Ba_xU$  (4 EI)
8.  $Ba_xU \supset \bar{A}a_LU$  (6 UI)
9.  $Ba_xU$  (7 Simp)
10.  $\bar{A}a_LU$  (8,9 MP)
11.  $Ca_xU$  (7 Simp)
12.  $Ca_xU \& \bar{A}a_LU$  (10,11 Conj)
13.  $(Ca_xU \& \bar{A}a_LU) \vee (\bar{A}a_xU \& Ca_LU)$  (12 Add)
14.  $(\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xZ \& Ca_LZ)]$  (13 EG)
15.  $Ao_LC$  (14 Def  $o_L$ )

Festino-LXL (Valid):

1.  $Be_LA$  (major premise)
2.  $Bi_xC$  (minor premise) //  $Ao_LC (\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xZ \& Ca_LZ)]$
3.  $Ae_LB$  (1 Conv  $e_L-e_L$ )
4.  $(\exists Z)(Ca_xZ \& Ba_xZ)$  (2 Def  $i_x$ )
5.  $(\forall Z)(Ba_xZ \supset \bar{A}a_LZ)$  (3 Def  $e_L$ )
6.  $Ca_xU \& Ba_xU$  (4 EI)
7.  $Ba_xU \supset \bar{A}a_LU$  (5 UI)
8.  $Ba_xU$  (6 Simp)
9.  $\bar{A}a_LU$  (7,8 MP)
10.  $Ca_xU$  (6 Simp)
11.  $Ca_xU \& \bar{A}a_LU$  (9,10 Conj)
12.  $(Ca_xU \& \bar{A}a_LU) \vee (\bar{A}a_xU \& Ca_LU)$  (11 Add)
13.  $(\exists Z)[(Ca_xZ \& \bar{A}a_LZ) \vee (\bar{A}a_xZ \& Ca_LZ)]$  (12 EG)
14.  $Ao_LC$  (13 Def  $o_L$ )

Festino-XQM (Valid):

1.  $Be_xA$  (major premise)
2.  $Bi_QC$  (minor premise) //  $Ao_MC (\exists Z)(Ca_MZ \& \bar{A}a_MZ)$  (Def  $o_M$ )
3.  $Ae_xB$  (1 Conv  $e_x-e_x$ )
4.  $Ae_xB \supset (\forall Z)(Ba_MZ \supset \bar{A}a_MZ)$  (X-M-sub  $a_x$ )
5.  $(\exists Z)[Ca_MZ \& (Ba_MZ \& \bar{B}a_MZ)]$  (2 Def  $i_Q$ )
6.  $Ca_MU \& (Ba_MU \& \bar{B}a_MU)$  (5 EI)
7.  $(\forall Z)(Ba_MZ \supset \bar{A}a_MZ)$  (3,4 MP)
8.  $Ba_MU \supset \bar{A}a_MU$  (7 UI)

9.  $B_{AM}U$  &  $\bar{B}_{AM}U$  (6 Simp)
10.  $B_{AM}U$  (9 Simp)
11.  $\bar{A}_{AM}U$  (8,10 MP)
12.  $C_{AM}U$  (6 Simp)
13.  $C_{AM}U$  &  $\bar{A}_{AM}U$  (11,12 Conj)
14.  $(\exists Z)(C_{AM}Z \& \bar{A}_{AM}Z)$  (13, EG)
15.  $A_{oM}C$  (14 Def  $oM$ )

Festino-LQX (Valid):

1.  $BeLA$  (major premise)
2.  $BiQC$  (minor premise)// $A_{oX}C$   $(\exists Z)[C_{aX}Z \& \sim(A_{aX}Z)]$  (Def  $oM$ )
3.  $AeLB$  (1 Conv  $eL-eL$ )
4.  $AeLB \supset (\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (L-M-sub  $eL$ )
5.  $(\exists Z)[C_{aM}Z \& (B_{aM}Z \& \bar{B}_{aM}Z)]$  (2 Def  $iQ$ )
6.  $C_{aM}U$  &  $(B_{aM}U \& \bar{B}_{aM}U)$  (5 EI)
7.  $A_{aX}C$  (IP)
8.  $A_{aX}C \supset (\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (L-M-sub  $eL$ )
9.  $(\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (7,8 MP)
10.  $C_{aM}U \supset A_{aM}U$  (9 UI)
11.  $C_{aM}U$  (6 Simp)
12.  $A_{aM}U$  (10,11 MP)
13.  $(\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (3,4 MP)
14.  $[B_{aM}U \supset \bar{A}_{aM}U] \& [B_{aM}U \supset \sim(A_{aM}U)]$  (13 UI)
15.  $B_{aM}U$  &  $\bar{B}_{aM}U$  (6 Simp)
16.  $B_{aM}U$  (15 Simp)
17.  $B_{aM}U \supset \sim(A_{aM}U)$  (13 Simp)
18.  $\sim(A_{aM}U)$  (16,17 MP)
19.  $A_{aM}U$  &  $\sim(A_{aM}U)$  (12,18 Conj)
20.  $\sim(A_{aX}C)$  (7-19 IP)
21.  $A_{oX}C$  (20  $ox|ax$ )

Baroco-XXX (Valid):

1.  $B_{aX}A$  (major premise)
2.  $B_{oX}C$  (minor premise)// $A_{oX}C$   $(\exists Z)[(C_{aX}Z \& \sim(A_{aX}Z)$
3.  $(\forall Z)(A_{aX}Z \supset B_{aX}Z)$  (1 Def  $ax$ )
4.  $(\exists Z)[C_{aX}Z \& \sim(B_{aX}Z)]$  (2 Def  $oX$ )
5.  $C_{aX}U$  &  $\sim(B_{aX}U)$  (4 EI)
6.  $A_{aX}U \supset B_{aX}U$  (3 UI)
7.  $\sim(B_{aX}U)$  (5 Simp)
8.  $\sim(A_{aX}U)$  (6,7 MT)
9.  $C_{aX}U$  (5 Simp)
10.  $C_{aX}U$  &  $\sim(A_{aX}U)$  (8,9 Conj)
11.  $(\exists Z)[(C_{aX}Z \& \sim(A_{aX}Z)$  (10 EG)

12. Ao<sub>x</sub>C (11 Def o<sub>x</sub>)

Baroco-LLL (Valid):

1. Ba<sub>L</sub>A (major premise)
2. Bo<sub>L</sub>C (minor premise) // Ao<sub>L</sub>C (∃Z)[(Ca<sub>x</sub>Z &  $\bar{A}$ <sub>aL</sub>Z) ∨ ( $\bar{A}$ <sub>aX</sub>U & Ca<sub>L</sub>Z)]
3.  $\bar{B}$ <sub>eL</sub>A (1 Obv e<sub>L</sub>-a<sub>L</sub>)
4. Ae<sub>L</sub> $\bar{B}$  (3 Conv e<sub>L</sub>-e<sub>L</sub>)
5. Bo<sub>L</sub>C ⊃ (∃Z){Ca<sub>x</sub>Z & [ $\bar{B}$ <sub>aX</sub>Z & ∼(Ba<sub>x</sub>Z)]} (L-X-sub o<sub>L</sub>)
6. (∃Z){Ca<sub>x</sub>Z & [ $\bar{B}$ <sub>aX</sub>Z & ∼(Ba<sub>x</sub>Z)]} (2,5 MP)
7. (∀Z)( $\bar{B}$ <sub>aX</sub>Z ⊃  $\bar{A}$ <sub>aL</sub>Z) (4 Def e<sub>L</sub>)
8. Ca<sub>x</sub>U & [ $\bar{B}$ <sub>aX</sub>U & ∼(Ba<sub>x</sub>U)] (6 EI)
9.  $\bar{B}$ <sub>aX</sub>U & ∼(Ba<sub>x</sub>U) (8 Simp)
10.  $\bar{B}$ <sub>aX</sub>U ⊃  $\bar{A}$ <sub>aL</sub>U (7 UI)
11.  $\bar{B}$ <sub>aX</sub>U (9 Simp)
12.  $\bar{A}$ <sub>aL</sub>U (10,11 MP)
13. Ca<sub>x</sub>U (8 Simp)
14. Ca<sub>x</sub>U &  $\bar{A}$ <sub>aL</sub>U (12,13 Conj)
15. (Ca<sub>x</sub>U &  $\bar{A}$ <sub>aL</sub>U) ∨ ( $\bar{A}$ <sub>aX</sub>U & Ca<sub>L</sub>U) (14 Add)
16. (∃Z)[(Ca<sub>x</sub>Z &  $\bar{A}$ <sub>aL</sub>Z) ∨ ( $\bar{A}$ <sub>aX</sub>U & Ca<sub>L</sub>Z)] (15 EG)
17. Ao<sub>L</sub>C (16 Def o<sub>L</sub>)

Baroco-LXX (Valid):

1. Ba<sub>L</sub>A (major premise)
2. Bo<sub>x</sub>C (minor premise) // Ao<sub>x</sub>C (∃Z)[(Ca<sub>x</sub>Z & ∼(Aa<sub>x</sub>Z)]
3. Ba<sub>L</sub>A ⊃ (∀Z)(Aa<sub>x</sub>Z ⊃ Ba<sub>x</sub>Z) (L-X-sub a<sub>L</sub>)
4. (∀Z)(Aa<sub>x</sub>Z ⊃ Ba<sub>x</sub>Z) (1,3 MP)
5. (∃Z)[Ca<sub>x</sub>Z & ∼(Ba<sub>x</sub>Z)] (2 Def o<sub>x</sub>)
6. Ca<sub>x</sub>U & ∼(Ba<sub>x</sub>U) (5 EI)
7. Aa<sub>x</sub>U ⊃ Ba<sub>x</sub>U (4 UI)
8. ∼(Ba<sub>x</sub>U) (6 Simp)
9. ∼(Aa<sub>x</sub>U) (7,8 MT)
10. Ca<sub>x</sub>U (6 Simp)
11. Ca<sub>x</sub>U & ∼(Aa<sub>x</sub>U) (9,10 Conj)
13. (∃Z)[Ca<sub>x</sub>Z & ∼(Aa<sub>x</sub>Z)] (11 EG)
14. Ao<sub>x</sub>C (11 Def o<sub>x</sub>)

Baroco-XXL (Valid):

1. Ba<sub>x</sub>A (major premise)
2. Bo<sub>L</sub>C (minor premise) // Ao<sub>x</sub>C (∃Z)[(Ca<sub>x</sub>Z & ∼(Aa<sub>x</sub>Z)]
3. Bo<sub>L</sub>C ⊃ (∃Z){Ca<sub>x</sub>Z & [ $\bar{B}$ <sub>aX</sub>Z & ∼(Ba<sub>x</sub>Z)]} (L-X-sub o<sub>L</sub>)
4. (∃Z){Ca<sub>x</sub>Z & [ $\bar{B}$ <sub>aX</sub>Z & ∼(Ba<sub>x</sub>Z)]} (2,3 MP)
5. (∀Z)(Aa<sub>x</sub>Z ⊃ Ba<sub>x</sub>Z) (1 Def)

6.  $CaxU \& [\bar{B}axU \& \sim(BaxU)]$  (4 EI)
7.  $AaxU \supset BaxU$  (5 UI)
8.  $\bar{B}axU \& \sim(BaxU)$  (6 Simp)
9.  $\sim(BaxU)$  (8 Simp)
10.  $\sim(BaxU) \supset \sim(AaxU)$  (7 Contra)
11.  $\sim(AaxU)$  (9,10 MP)
12.  $CaxU$  (6 Simp)
13.  $CaxU \& \sim(AaxU)$  (11,12 Conj)
14.  $(\exists Z)[CaxZ \& \sim(AaxZ)]$  (13 EG)
15.  $AoxC$  (14 Def ox)

### Third Figure:

#### Darapti-XXX (Valid):

1.  $AaxB$  (major premise)
2.  $CaxB$  (minor premise)// $AixC (\exists Z)(CaxZ \& AaxZ)$
3.  $(\forall Z)(BaxZ \supset AaxZ)$  (1 Def ax)
4.  $CaxB \supset BixC$  (Conv ax-ix)
5.  $BixC$  (2,4 MP)
6.  $(\exists Z)(CaxZ \& BaxZ)$  (5 Def ix)
7.  $CaxU \& BaxU$  (6 EI)
8.  $BaxU \supset AaxU$  (3 UI)
9.  $BaxU$  (7 Simp)
10.  $AaxU$  (8,9 MP)
11.  $CaxU$  (7 Simp)
12.  $CaxU \& AaxU$  (10,11 Conj)
13.  $(\exists Z)(CaxZ \& AaxZ)$  (12 EG)
14.  $AixC$  (13 Def ix)

#### Darapti-LLL (Valid):

1.  $AaL B$  (major premise)
2.  $CaL B$  (minor premise)// $AiL C (\exists Z)[(CaxZ \& AaL Z) \vee (AaxZ \& CaL Z)]$
3.  $(\forall Z)(BaxZ \supset AaL Z)$  (1 Def aL)
4.  $CaL B \supset BiL C$  (2 Conv aL-il)
5.  $BiL C \supset (\exists Z)(CaxZ \& BaxZ)$  (L-X-sub iL)
6.  $CaL B \supset (\exists Z)(CaxZ \& BaxZ)$  (4,5 HS)
7.  $(\exists Z)(CaxZ \& BaxZ)$  (2,6 MP)
8.  $CaxU \& BaxU$  (7 EI)
9.  $BaxU \supset AaL U$  (3 UI)

10.  $BaxU$  (8 Simp)
11.  $AaLU$  (9,10 MP)
12.  $CaxU$  (8 Simp)
13.  $CaxU \& AaLU$  (11,12 Conj)
14.  $(CaxU \& AaLU) \vee (AaxU \& CalU)$  (13 Add)
15.  $(\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CalZ)]$  (14 EG)
16.  $AiLC$  (15 Def  $iL$ )

Darapti-LXL (Valid):

1.  $AaLB$  (major premise)
2.  $CaxB$  (minor premise) //  $AiLC (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CalZ)]$
3.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def  $aL$ )
4.  $CaxB \supset BixC$  (2 Conv  $ax-ix$ )
5.  $BixC$  (2,4 MP)
6.  $(\exists Z)(CaxZ \& BaxZ)$  (5 Def  $ix$ )
7.  $CaxU \& BaxU$  (6 EI)
8.  $BaxU \supset AaLU$  (3 UI)
9.  $BaxU$  (7 Simp)
10.  $AaLU$  (8,9 MP)
11.  $CaxU$  (7 Simp)
12.  $CaxU \& AaLU$  (10,11 Conj)
13.  $(CaxU \& AaLU) \vee (AaxU \& CalU)$  (12 Add)
14.  $(\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CalZ)]$  (13 EG)
15.  $AiLC$  (14 Def  $iL$ )

Darapti-XLL (Valid):

1.  $AaxB$  (major premise)
2.  $CalB$  (minor premise) //  $AiLC (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CalZ)]$
3.  $(\forall Z)(BaxZ \supset CalZ)$  (2 Def  $aL$ )
4.  $AaxB \supset BixA$  (1 Conv  $ax-ix$ )
5.  $BixA$  (2,4 MP)
6.  $(\exists Z)(AaxZ \& BaxZ)$  (5 Def  $ix$ )
7.  $AaxU \& BaxU$  (6 EI)
8.  $BaxU \supset CalU$  (3 UI)
9.  $BaxU$  (7 Simp)
10.  $CalU$  (8,9 MP)
11.  $AaxU$  (7 Simp)
12.  $AaxU \& CalU$  (10,11 Conj)
13.  $(AaxU \& CalU) \vee (CaxU \& AaLU)$  (12 Add)
14.  $(CaxU \& AaLU) \vee (AaxU \& CalU)$  (13 Comm)
15.  $(\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CalZ)]$  (14 EG)
16.  $AiLC$  (15 Def  $iL$ )



Darapti-QQQ (Valid):

1.  $AaQb$  (major premise)
2.  $CaQb$  (minor premise)// $AiQc$   $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def iQ)
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{A}aMZ)]$  (1 Def aQ)
4.  $BiQc$  (2, Conv aQ-iQ)
5.  $(\exists Z)[CaMZ \& (BaMZ \& \bar{B}aMZ)]$  (4 Def iQ)
6.  $CaMU \& (BaMU \& \bar{B}aMU)$  (5 EI)
7.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{A}aMU)$  (3 UI)
8.  $BaMU \supset AaMU$  (7 Simp)
9.  $BaMU \& \bar{B}aMU$  (6 Simp)
10.  $BaMU$  (9 Simp)
11.  $AaMU$  (8,10 MP)
12.  $BaMU \supset \bar{A}aMU$  (7 Simp)
13.  $\bar{A}aMU$  (10,12 MP)
14.  $AaMU \& \bar{A}aMU$  (11,13 Conj)
15.  $CaMU$  (6 Simp)
16.  $CaMU \& (AaMU \& \bar{A}aMU)$  (14,15 Conj)
17.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (16, EG)
18.  $AiQc$  (17 Def iQ)

Darapti-QXQ (Valid):

1.  $AaQb$  (major premise)
2.  $CaXB$  (minor premise)// $AiQc$   $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def iQ)
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{A}aMZ)]$  (1 Def aQ)
4.  $BiXC$  (2, Conv aX-iX)
5.  $BiXC \supset (\exists Z)(CaMZ \& BaMZ)$  (X-M-sub iX)
6.  $(\exists Z)(CaMZ \& BaMZ)$  (4,5 MP)
7.  $CaMU \& BaMU$  (6 EI)
8.  $BaMU$  (7 Simp)
9.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{A}aMU)$  (3 UI)
10.  $BaMU \supset AaMU$  (9 Simp)
11.  $AaMU$  (8,10 MP)
12.  $BaMU \supset \bar{A}aMU$  (9 Simp)
13.  $\bar{A}aMU$  (8,12 MP)
14.  $AaMU \& \bar{A}aMU$  (11,13 Conj)
15.  $CaMU$  (7 Simp)
16.  $CaMU \& (AaMU \& \bar{A}aMU)$  (14,15 Conj)
17.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (16, EG)
18.  $AiQc$  (17 Def iQ)

Darapti-XQM (Valid):

1.  $AaxB$  (major premise)
2.  $CaQB$  (minor premise)// $AiMC (\exists Z)(CaMZ \& AaMZ)$  (Def  $iM$ )
3.  $BiQC$  (2, Conv  $aQ-iQ$ )
4.  $(\exists Z)[CaMZ \& (BaMZ \& \bar{B}aMZ)]$  (3 Def  $iQ$ )
5.  $AaXB \supset (\forall Z)(BaMZ \supset AaMZ)$  (X-M-sub  $aX$ )
6.  $CaMU \& (BaMU \& \bar{B}aMU)$  (4 EI)
7.  $(\forall Z)(BaMZ \supset AaMZ)$  (1,5 MP)
8.  $BaMU \supset AaMU$  (7 UI)
9.  $BaMU \& \bar{B}aMU$  (6 Simp)
10.  $BaMU$  (9 Simp)
11.  $AaMU$  (8,10 MP)
12.  $CaMU$  (6 Simp)
13.  $CaMU \& AaMU$  (11,12 Conj)
14.  $(\exists Z)(CaMZ \& AaMZ)$  (13, EG)
15.  $AiMC$  (14 Def  $iM$ )

Darapti-QLQ (Valid):

1.  $AaQB$  (major premise)
2.  $CaLB$  (minor premise)// $AiQC (\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def  $iQ$ )
3.  $(\forall Z)[(BaMZ \supset AaMZ) \& (BaMZ \supset \bar{A}aMZ)]$  (1 Def  $aQ$ )
4.  $BiLC$  (2 Conv  $aL-iL$ )
5.  $BiLC \supset (\exists Z)(CaMZ \& BaMZ)$  (L-M-sub  $iL$ )
6.  $(\exists Z)(CaMZ \& BaMZ)$  (4,5 MP)
7.  $CaMU \& BaMU$  (6 EI)
8.  $BaMU$  (7 Simp)
9.  $(BaMU \supset AaMU) \& (BaMU \supset \bar{A}aMU)$  (3 UI)
10.  $BaMU \supset AaMU$  (9 Simp)
11.  $AaMU$  (8,10 MP)
12.  $BaMU \supset \bar{A}aMU$  (9 Simp)
13.  $\bar{A}aMU$  (8,12 MP)
14.  $AaMU \& \bar{A}aMU$  (11,13 Conj)
15.  $CaMU$  (7 Simp)
16.  $CaMU \& (AaMU \& \bar{A}aMU)$  (14,15 Conj)
17.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (15 EG)
18.  $AiQC$  (17 Def  $iQ$ )

Darapti-LQM (Valid):

1.  $AaLB$  (major premise)
2.  $CaQB$  (minor premise)// $AiMC (\exists Z)(CaMZ \& AaMZ)$  (Def  $iM$ )
3.  $BiLA$  (1, Conv  $aL-iL$ )

4.  $Bi_L A \supset (\exists Z)(A_{aM}Z \& B_{aM}Z)$  (L-M-sub  $i_L$ )
5.  $(\exists Z)(A_{aM}Z \& B_{aM}Z)$  (3,4 MP)
6.  $A_{aM}U \& B_{aM}U$  (4 EI)
7.  $(\forall Z)[(B_{aM}Z \supset C_{aM}Z) \& (B_{aM}Z \supset \bar{C}_{aM}Z)]$  (1 Def  $a_Q$ )
8.  $(B_{aM}U \supset C_{aM}U) \& (B_{aM}U \supset \bar{C}_{aM}U)$  (7 UI)
9.  $B_{aM}U \supset C_{aM}U$  (8 Simp)
10.  $B_{aM}U$  (6 Simp)
11.  $C_{aM}U$  (9,10 MP)
12.  $A_{aM}U$  (6 Simp)
13.  $C_{aM}U \& A_{aM}U$  (11,12 Conj)
14.  $(\exists Z)(C_{aM}Z \& A_{aM}Z)$  (13, EG)
15.  $Ai_M C$  (14 Def  $i_M$ )

Felapton-XXX (Valid):

1.  $Ae_x B$  (major premise)
2.  $Ca_x B$  (minor premise) //  $Ao_x C (\exists Z)[Ca_x Z \& \sim(Aa_x Z)]$
3.  $(\forall Z)[Ba_x Z \supset \sim(Aa_x Z)]$  (1 Def  $e_x$ )
4.  $Ca_x B \supset Bi_x C$  (2 Conv  $a_x$ - $i_x$ )
5.  $Bi_x C$  (2,4 MP)
6.  $(\exists Z)(Ca_x Z \& Ba_x Z)$  (5 Def  $i_x$ )
7.  $Ca_x U \& Ba_x U$  (6 EI)
8.  $Ba_x U \supset \sim(Aa_x U)$  (3 UI)
9.  $Ba_x U$  (7 Simp)
10.  $\sim(Aa_x U)$  (8,9 MP)
11.  $Ca_x U$  (7 Simp)
12.  $Ca_x U \& \sim(Aa_x U)$  (10,11 Conj)
13.  $(\exists Z)[Ca_x Z \& \sim(Aa_x Z)]$  (12 EG)
14.  $Ao_x C$  (13 Def  $o_x$ )

Felapton-LLL (Valid):

1.  $Ae_L B$  (major premise)
2.  $Ca_L B$  (minor premise) //  $Ao_L C (\exists Z)(Ca_x Z \& \bar{A}a_L Z) \vee (\bar{A}a_x Z \& Ca_L Z)$
3.  $(\forall Z)(Ba_x Z \supset \bar{A}a_L Z)$  (1 Def  $e_L$ )
4.  $Ca_L B \supset Bi_L C$  (2 Conv  $a_x$ - $i_x$ )
5.  $Bi_L C \supset (\exists Z)(Ca_x Z \& Ba_x Z)$  (L-X-sub  $i_L$ )
6.  $Ca_L B \supset (\exists Z)(Ca_x Z \& Ba_x Z)$  (4,5 HS)
7.  $(\exists Z)(Ca_x Z \& Ba_x Z)$  (2,6 MP)
8.  $Ca_x U \& Ba_x U$  (7 EI)
9.  $Ba_x U \supset \bar{A}a_L U$  (3 UI)
10.  $Ba_x U$  (8 Simp)
11.  $\bar{A}a_L U$  (9,10 MP)
12.  $Ca_x U$  (8 Simp)
13.  $Ca_x U \& \bar{A}a_L U$  (11,12 Conj)

14.  $(C_{xU} \& \bar{A}_{aL}U) \vee (\bar{A}_{ax}U \& C_{aL}U)$  (13 Add)
15.  $(\exists Z)(C_{xZ} \& \bar{A}_{aL}Z) \vee (\bar{A}_{ax}Z \& C_{aL}Z)$  (14 EG)
16.  $A_{oL}C$  (15 Def  $o_L$ )

Felapton-LXL (Valid):

1.  $A_{eL}B$  (major premise)
2.  $C_{xB}$  (minor premise) //  $A_{oL}C$   $(\exists Z)(C_{xZ} \& \bar{A}_{aL}Z) \vee (\bar{A}_{ax}Z \& C_{aL}Z)$
3.  $(\forall Z)(B_{ax}Z \supset \bar{A}_{aL}Z)$  (1 Def  $e_L$ )
4.  $C_{xB} \supset B_{ix}C$  (2 Conv  $ax-ix$ )
5.  $B_{ix}C$  (2,4 MP)
6.  $(\exists Z)(C_{xZ} \& B_{ax}Z)$  (5 Def  $ix$ )
7.  $C_{xU} \& B_{ax}U$  (6 EI)
8.  $B_{ax}U \supset \bar{A}_{aL}U$  (3 UI)
9.  $B_{ax}U$  (7 Simp)
10.  $\bar{A}_{aL}U$  (8,9 MP)
11.  $C_{xU}$  (7 Simp)
12.  $C_{xU} \& \bar{A}_{aL}U$  (10,11 Conj)
13.  $(C_{xU} \& \bar{A}_{aL}U) \vee (\bar{A}_{ax}U \& C_{aL}U)$  (12 Add)
14.  $(\exists Z)(C_{xZ} \& \bar{A}_{aL}Z) \vee (\bar{A}_{ax}Z \& C_{aL}Z)$  (13 EG)
15.  $A_{oL}C$  (14 Def  $o_L$ )

Felapton-QQQ (Valid):

1.  $A_{eQ}B$  (major premise)
2.  $C_{aQ}B$  (minor premise) //  $A_{oQ}C$   $(\exists Z)[C_{aM}Z \& (A_{aM}Z \& \bar{A}_{aM}Z)]$  (Def  $i_Q$ )
3.  $(\forall Z)[(B_{aM}Z \supset A_{aM}Z) \& (B_{aM}Z \supset \bar{A}_{aM}Z)]$  (1 Def  $e_Q$ )
4.  $B_{iQ}C$  (2 Conv  $a_Q-i_Q$ )
5.  $(\exists Z)[C_{aM}Z \& (B_{aM}Z \& \bar{B}_{aM}Z)]$  (4 Def  $i_Q$ )
6.  $C_{aM}U \& (B_{aM}U \& \bar{B}_{aM}U)$  (5 EI)
7.  $(B_{aM}U \supset A_{aM}U) \& (B_{aM}U \supset \bar{A}_{aM}U)$  (3 UI)
8.  $B_{aM}U \supset A_{aM}U$  (7 Simp)
9.  $B_{aM}U \& \bar{B}_{aM}U$  (6 Simp)
10.  $B_{aM}U$  (9 Simp)
11.  $A_{aM}U$  (8,10 MP)
12.  $B_{aM}U \supset \bar{A}_{aM}U$  (7 Simp)
13.  $\bar{A}_{aM}U$  (10,12 MP)
14.  $A_{aM}U \& \bar{A}_{aM}U$  (11,13 Conj)
15.  $C_{aM}U$  (6 Simp)
16.  $C_{aM}U \& (A_{aM}U \& \bar{A}_{aM}U)$  (14,15 Conj)
17.  $(\exists Z)[C_{aM}Z \& (A_{aM}Z \& \bar{A}_{aM}Z)]$  (16, EG)
18.  $A_{oQ}C$  (16 Def  $o_Q$ )

Felapton-QXQ (Valid):

1.  $Ae_0B$  (major premise)
2.  $Ca_xB$  (minor premise)// $Ao_0C$   $(\exists Z)[Ca_mZ \& (Aa_mZ \& \bar{A}a_mZ)]$  (Def  $o_0$ )
3.  $Bi_xC$  (2 Conv  $ax-ix$ )
4.  $Bi_xC \supset (\exists Z)(Ca_mZ \& Ba_mZ)$  (X-M-sub  $ix$ )
5.  $(\exists Z)(Ca_mZ \& Ba_mZ)$  (3,4 MP)
6.  $Ca_mU \& Ba_mU$  (5 EI)
7.  $(\forall Z)[(Ba_mZ \supset Aa_mZ) \& (Ba_mZ \supset \bar{A}a_mZ)]$  (1 Def  $e_0$ )
8.  $(Ba_mU \supset Aa_mU) \& (Ba_mU \supset \bar{A}a_mU)$  (7 UI)
9.  $Ba_mU \supset Aa_mU$  (8 Simp)
10.  $Ba_mU$  (6 Simp)
11.  $Aa_mU$  (9,10 MP)
12.  $Ba_mU \supset \bar{A}a_mU$  (8 Simp)
13.  $\bar{A}a_mU$  (10,12 MP)
14.  $Aa_mU \& \bar{A}a_mU$  (11,13 Conj)
15.  $Ca_mU$  (6 Simp)
16.  $Ca_mU \& (Aa_mU \& \bar{A}a_mU)$  (14,15 Conj)
17.  $(\exists Z)[Ca_mZ \& (Aa_mZ \& \bar{A}a_mZ)]$  (16, EG)
18.  $Ao_0C$  (17 Def  $o_0$ )

Felapton-XQM (Valid):

1.  $Ae_xB$  (major premise)
2.  $Ca_0B$  (minor premise)// $Ao_mC$   $(\exists Z)(Ca_mZ \& \bar{A}a_mZ)$  (Def  $o_m$ )
3.  $Ae_xB \supset (\forall Z)(Ba_mZ \supset \bar{A}a_mZ)$  (X-M-sub  $ax$ )
4.  $Bi_0C$  (2 Conv  $a_0$ )
5.  $(\exists Z)[Ca_mZ \& (Ba_mZ \& \bar{B}a_mZ)]$  (4 Def  $i_0$ )
6.  $Ca_mU \& (Ba_mU \& \bar{B}a_mU)$  (5 EI)
7.  $(\forall Z)(Ba_mZ \supset \bar{A}a_mZ)$  (1,3 MP)
8.  $Ba_mU \supset \bar{A}a_mU$  (7 UI)
9.  $Ba_mU \& \bar{B}a_mU$  (6 Simp)
10.  $Ba_mU$  (9 Simp)
11.  $\bar{A}a_mU$  (8,10 MP)
12.  $Ca_mU$  (6 Simp)
13.  $Ca_mU \& \bar{A}a_mU$  (11,12 Conj)
14.  $(\exists Z)(Ca_mZ \& \bar{A}a_mZ)$  (13 EG)
15.  $Ao_mC$  (14 Def  $o_m$ )

Felapton-QLQ (Valid):

1.  $Ae_0B$  (major premise)
2.  $Ca_lB$  (minor premise)// $Ao_0C$   $(\exists Z)[Ca_mZ \& (Aa_mZ \& \bar{A}a_mZ)]$  (Def  $o_0$ )
3.  $(\forall Z)[(Ba_mZ \supset Aa_mZ) \& (Ba_mZ \supset \bar{A}a_mZ)]$  (1 Def  $e_0$ )
4.  $Bi_lC$  (2 Conv  $a_l$ )

5.  $Bi_L C \supset (\exists Z)(Ca_M Z \& Ba_M Z)$  (L-M-sub  $i_L$ )
6.  $(\exists Z)(Ca_M Z \& Ba_M Z)$  (4,5 MP)
7.  $Ca_M U \& Ba_M U$  (6 EI)
8.  $Ba_M U$  (7 Simp)
9.  $(Ba_M U \supset Aa_M U) \& (Ba_M U \supset \bar{A}a_M U)$  (3 UI)
10.  $Ba_M U \supset Aa_M U$  (9 Simp)
11.  $Aa_M U$  (8,10 MP)
12.  $Ba_M U \supset \bar{A}a_M U$  (9 Simp)
13.  $\bar{A}a_M U$  (8,12 MP)
14.  $Aa_M U \& \bar{A}a_M U$  (11,13 Conj)
15.  $Ca_M U$  (7 Simp)
16.  $Ca_M U \& (Aa_M U \& \bar{A}a_M U)$  (14,15 Conj)
17.  $(\exists Z)[Ca_M Z \& (Aa_M Z \& \bar{A}a_M Z)]$  (16, EG)
18.  $Ao_Q C$  (17 Def  $o_Q$ )

Felapton-LQX (Valid):

1.  $Ae_L B$  (major premise)
2.  $Ca_Q B$  (minor premise)// $Ao_X C (\exists Z)(Ca_X Z \& \bar{A}a_X Z)$  (Def  $o_X$ )
3.  $Ae_L B \supset (\forall Z)\{(Ba_M Z \supset \bar{A}a_M Z) \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (L-M-sub  $e_L$ )
4.  $Bi_Q C$  (2 Conv  $a_Q$ )
5.  $(\exists Z)[Ca_M Z \& (Ba_M Z \& \bar{B}a_M Z)]$  (4 Def  $i_Q$ )
6.  $Ca_M U \& (Ba_M U \& \bar{B}a_M U)$  (5 EI)
7.  $Aa_X C$  (IP)
8.  $Aa_X C \supset (\forall Z)(Ca_M Z \supset Aa_M Z)$  (L-M-sub  $e_L$ )
9.  $(\forall Z)(Ca_M Z \supset Aa_M Z)$  (7,8 MP)
10.  $Ca_M U \supset Aa_M U$  (9 UI)
11.  $Ca_M U$  (6 Simp)
12.  $Aa_M U$  (10,11 MP)
13.  $(\forall Z)\{(Ba_M Z \supset \bar{A}a_M Z) \& [Ba_M Z \supset \sim(Aa_M Z)]\}$  (1,3 MP)
14.  $[Ba_M U \supset \bar{A}a_M U] \& [Ba_M U \supset \sim(Aa_M U)]$  (13 UI)
15.  $Ba_M U \& \bar{B}a_M U$  (6 Simp)
16.  $Ba_M U$  (15 Simp)
17.  $Ba_M U \supset \sim(Aa_M U)$  (13 Simp)
18.  $\sim(Aa_M U)$  (16,17 MP)
19.  $Aa_M U \& \sim(Aa_M U)$  (12,18 Conj)
20.  $\sim(Aa_X C)$  (6-18 IP)
21.  $Ao_X C$  (19  $o_X|a_X$ )

Disamis-XXX (Valid):

1.  $Ai_X B$  (major premise)
2.  $Ca_X B$  (minor premise)// $Ai_X C (\exists Z)(Ca_X Z \& Aa_X Z)$
3.  $(\forall Z)(Ba_X Z \supset Ca_X Z)$  (2 Def  $a_X$ )
4.  $(\exists Z)(Ba_X Z \& Aa_X Z)$  (1 Def  $i_X$ )

5.  $B_{xU} \& A_{xU}$  (4 UI)
6.  $B_{xU}$  (5 Simp)
7.  $B_{xU} \supset C_{xU}$  (3 UI)
8.  $C_{xU}$  (6,7 MP)
9.  $A_{xU}$  (5 Simp)
10.  $C_{xU} \& A_{xU}$  (8,9 Conj)
11.  $(\exists Z)(C_{xZ} \& A_{xZ})$  (10 EG)
12.  $A_{ix}C$  (11 Def  $i_x$ )

Disamis-LLL (Valid):

1.  $A_{iL}B$  (major premise)
2.  $C_{aL}B$  (minor premise) //  $A_{iL}C (\exists Z)[(C_{aZ} \& A_{aZ}) \vee (A_{aZ} \& C_{aZ})]$
3.  $(\forall Z)(B_{aZ} \supset C_{aZ})$  (2 Def  $a_L$ )
4.  $A_{iL}B \supset (\exists Z)(B_{aZ} \& A_{aZ})$  (L-X-sub  $i_L$ )
5.  $(\exists Z)(B_{aZ} \& A_{aZ})$  (1,4 MP)
6.  $B_{xU} \& A_{xU}$  (5 EI)
7.  $B_{xU}$  (6 Simp)
8.  $B_{xU} \supset C_{aU}$  (3 UI)
9.  $C_{aU}$  (7,8 MP)
10.  $A_{xU}$  (6 Simp)
11.  $A_{xU} \& C_{aU}$  (9,10 Conj)
12.  $(A_{xU} \& C_{aU}) \vee (C_{xU} \& A_{aU})$  (11 Add)
13.  $(C_{xU} \& A_{aU}) \vee (A_{xU} \& C_{aU})$  (12 Comm)
14.  $(\exists Z)[(C_{aZ} \& A_{aZ}) \vee (A_{aZ} \& C_{aZ})]$  (13 EG)
15.  $A_{iL}C$  (14 Def  $i_L$ )

Disamis-XLL (Valid):

1.  $A_{ix}B$  (major premise)
2.  $C_{aL}B$  (minor premise) //  $A_{iL}C (\exists Z)[(C_{aZ} \& A_{aZ}) \vee (A_{aZ} \& C_{aZ})]$
3.  $(\forall Z)(B_{aZ} \supset C_{aZ})$  (2 Def  $a_L$ )
4.  $(\exists Z)(B_{aZ} \& A_{aZ})$  (1 Def  $i_x$ )
5.  $B_{xU} \& A_{xU}$  (4 UI)
6.  $B_{xU}$  (5 Simp)
7.  $B_{xU} \supset C_{aU}$  (3 UI)
8.  $C_{aU}$  (6,7 MP)
9.  $A_{xU}$  (5 Simp)
10.  $A_{xU} \& C_{aU}$  (8,9 Conj)
11.  $(A_{xU} \& C_{aU}) \vee (C_{xU} \& A_{aU})$  (10 Add)
12.  $(C_{xU} \& A_{aU}) \vee (A_{xU} \& C_{aU})$  (11 Comm)
13.  $(\exists Z)[(C_{aZ} \& A_{aZ}) \vee (A_{aZ} \& C_{aZ})]$  (12 EG)
14.  $A_{iL}C$  (13 Def  $i_L$ )

Disamis-QQQ (Valid):

1.  $AiQB$  (major premise)
2.  $CaQB$  (minor premise)// $AiQC (\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def iQ)
3.  $(\forall Z)[(BaMZ \supset CaMZ) \& (BaMZ \supset \bar{C}aMZ)]$  (2 Def aQ)
4.  $(\exists Z)[BaMZ \& (AaMZ \& \bar{A}aMZ)]$  (1 Def iQ)
5.  $BaMU \& (AaMU \& \bar{A}aMU)$  (4 EI)
6.  $(BaMU \supset CaMU) \& (BaMU \supset \bar{C}aMU)$  (3 UI)
7.  $BaMU \supset CaMU$  (6 Simp)
8.  $BaMU$  (5 Simp)
9.  $CaMU$  (7,8 MP)
10.  $(AaMU \& \bar{A}aMU)$  (5 Simp)
11.  $CaMU \& (AaMU \& \bar{A}aMU)$  (9,10 Conj)
12.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (11, EG)
13.  $AiQC$  (12 Def iQ)

Disamis-QXQ (Rini identifies as Valid at 40a39-b2):

1.  $AiQB$  (major premise)
2.  $CaXB$  (minor premise)// $AiQC (\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def iQ)
3.  $(\exists Z)[BaMZ \& (AaMZ \& \bar{A}aMZ)]$  (1 Def iQ)
4.  $CaXB \supset (\forall Z)(BaMZ \supset CaMZ)$  (X-M-sub ax)
5.  $(\forall Z)(BaMZ \supset CaMZ)$  (2,4 MP)
6.  $BaMU \& (AaMU \& \bar{A}aMU)$  (5 EI)
7.  $BaMU$  (6 Simp)
8.  $BaMU \supset CaMU$  (3 UI)
9.  $CaMU$  (7,8 MP)
10.  $(AaMU \& \bar{A}aMU)$  (6 Simp)
11.  $CaMU \& (AaMU \& \bar{A}aMU)$  (9,10 Conj)
12.  $(\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (11, EG)
13.  $AiQC$  (12 Def iQ)

Disamis-XQM (Valid):

1.  $AiXB$  (major premise)
2.  $CaQB$  (minor premise)// $AiMC (\exists Z)(CaMZ \& AaMZ)$  (Def iM)
3.  $AiXB \supset (\exists Z)(BaMZ \& AaMZ)$  (X-M-sub ax)
4.  $(\forall Z)[(BaMZ \supset CaMZ) \& (BaMZ \supset \bar{C}aMZ)]$  (2 Def aQ)
5.  $(\exists Z)(BaMZ \& AaMZ)$  (1,3 MP)
6.  $BaMU \& AaMU$  (1,3 MP)
7.  $(BaMU \supset CaMU) \& (BaMU \supset \bar{C}aMU)$  (6 UI)
8.  $BaMU \supset CaMU$  (7 Simp)
9.  $BaMU$  (6 Simp)
10.  $CaMU$  (8,9 MP)
11.  $AaMU$  (6 Simp)



12.  $C_{AM}U \ \& \ A_{AM}U$  (10,11 Conj)
13.  $(\exists Z)(C_{AM}Z \ \& \ A_{AM}Z)$  (12, EG)
14.  $A_{iM}C$  (13 Def  $i_M$ )

Disamis-QLQ (Valid):

1.  $A_{iQ}B$  (major premise)
2.  $C_{AL}B$  (minor premise) //  $A_{iQ}C (\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (Def  $i_Q$ )
3.  $(\exists Z)[B_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (1 Def  $i_Q$ )
4.  $C_{AL}B \supset (\forall Z)(B_{AM}Z \supset C_{AM}Z)$  (L-M-sub  $a_L$ )
5.  $(\forall Z)(B_{AM}Z \supset C_{AM}Z)$  (2,4 MP)
6.  $B_{AM}U \ \& \ (A_{AM}U \ \& \ \bar{A}_{AM}U)$  (5 EI)
7.  $B_{AM}U$  (6 Simp)
8.  $B_{AM}U \supset C_{AM}U$  (3 UI)
9.  $C_{AM}U$  (7,8 MP)
10.  $(A_{AM}U \ \& \ \bar{A}_{AM}U)$  (6 Simp)
11.  $C_{AM}U \ \& \ (A_{AM}U \ \& \ \bar{A}_{AM}U)$  (9,10 Conj)
12.  $(\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (11, EG)
13.  $A_{iQ}C$  (12 Def  $i_Q$ )

Disamis-LQM (Valid):

1.  $A_{iL}B$  (major premise)
2.  $C_{AQ}B$  (minor premise) //  $A_{iM}C (\exists Z)(C_{AM}Z \ \& \ A_{AM}Z)$  (Def  $i_Q$ )
3.  $A_{iX}B \supset (\exists Z)(B_{AM}Z \ \& \ A_{AM}Z)$  (X-M-sub  $a_X$ )
4.  $(\forall Z)[(B_{AM}Z \supset C_{AM}Z) \ \& \ (B_{AM}Z \supset \bar{C}_{AM}Z)]$  (2 Def  $a_Q$ )
5.  $(\exists Z)(B_{AM}Z \ \& \ A_{AM}Z)$  (1,3 MP)
6.  $B_{AM}U \ \& \ A_{AM}U$  (1,3 MP)
7.  $(B_{AM}U \supset C_{AM}U) \ \& \ (B_{AM}U \supset \bar{C}_{AM}U)$  (6 UI)
8.  $B_{AM}U \supset C_{AM}U$  (7 Simp)
9.  $B_{AM}U$  (6 Simp)
10.  $C_{AM}U$  (8,9 MP)
11.  $A_{AM}U$  (6 Simp)
12.  $C_{AM}U \ \& \ A_{AM}U$  (10,11 Conj)
13.  $(\exists Z)(C_{AM}Z \ \& \ A_{AM}Z)$  (12, EG)
14.  $A_{iM}C$  (13 Def  $i_M$ )

Datisi-XXX (Valid):

1.  $A_{AX}B$  (major premise)
2.  $C_{iX}B$  (minor premise) //  $A_{iX}C (\exists Z)(C_{AX}Z \ \& \ A_{AX}Z)$
3.  $(\forall Z)(B_{AX}Z \supset A_{AX}Z)$  (1 Def  $a_L$ )
4.  $(\exists Z)(B_{AX}Z \ \& \ C_{AX}Z)$  (2 Def  $i_X$ )
5.  $B_{AX}U \ \& \ C_{AX}U$  (4 EI)
6.  $B_{AX}U$  (5 Simp)

7.  $BaxU \supset AaxU$  (3 UI)
8.  $AaxU$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \& AaxU$  (8,9 Conj)
11.  $(\exists Z)(CaxZ \& AaxZ)$  (10 EG)
12.  $AixC$  (11 Def ix)

Datisi-LLL (Valid):

1.  $AaLB$  (major premise)
2.  $CaLB$  (minor premise) //  $AiLC (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$
3.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def aL)
4.  $CaLB \supset (\exists Z)(BaxZ \& CaxZ)$  (L-X-sub iL)
5.  $(\exists Z)(BaxZ \& CaxZ)$  (2,4 MP)
6.  $BaxU \& CaxU$  (5 EI)
7.  $BaxU$  (6 Simp)
8.  $BaxU \supset AaLU$  (3 UI)
9.  $AaLU$  (7,8 MP)
10.  $CaxU$  (6 Simp)
11.  $CaxU \& AaLU$  (9,10 Conj)
12.  $(CaxU \& AaLU) \vee (AaxU \& CaLU)$  (11 Add)
13.  $(\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$  (12 EG)
14.  $AiLC$  (13 Def iL)

Datisi-LXL (Valid):

1.  $AaLB$  (major premise)
2.  $CixB$  (minor premise) //  $AiLC (\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$
3.  $(\forall Z)(BaxZ \supset AaLZ)$  (1 Def aL)
4.  $(\exists Z)(BaxZ \& CaxZ)$  (2 Def ix)
5.  $BaxU \& CaxU$  (4 EI)
6.  $BaxU$  (5 Simp)
7.  $BaxU \supset AaLU$  (3 UI)
8.  $AaLU$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \& AaLU$  (8,9 Conj)
11.  $(CaxU \& AaLU) \vee (AaxU \& CaLU)$  (10 Add)
12.  $(\exists Z)[(CaxZ \& AaLZ) \vee (AaxZ \& CaLZ)]$  (11 EG)
13.  $AiLC$  (12 Def iL)

Datisi-QQQ (Valid):

1.  $AaQB$  (major premise)
2.  $CiQB$  (minor premise) //  $AiQC (\exists Z)[CaMZ \& (AaMZ \& \bar{A}aMZ)]$  (Def iQ)
3.  $(\forall Z)[(BamZ \supset AaMZ) \& (BamZ \supset \bar{A}aMZ)]$  (1 Def aQ)

4.  $(\exists Z)[B_{AM}Z \ \& \ (C_{AM}Z \ \& \ \bar{C}_{AM}Z)]$  (2 Def iq)
5.  $B_{AM}U \ \& \ (C_{AM}U \ \& \ \bar{C}_{AM}U)$  (4 EI)
6.  $(B_{AM}U \ \supset \ A_{AM}U) \ \& \ (B_{AM}U \ \supset \ \bar{A}_{AM}U)$  (3 UI)
7.  $B_{AM}U \ \supset \ A_{AM}U$  (6 Simp)
8.  $B_{AM}U$  (5 Simp)
9.  $A_{AM}U$  (7,8 MP)
10.  $B_{AM}U \ \supset \ \bar{A}_{AM}U$  (6 Simp)
11.  $\bar{A}_{AM}U$  (8,10 MP)
12.  $A_{AM}U \ \& \ \bar{A}_{AM}U$  (9,11 Conj)
13.  $C_{AM}U \ \& \ \bar{C}_{AM}U$  (5 Simp)
14.  $C_{AM}U$  (13 Simp)
15.  $C_{AM}U \ \& \ (A_{AM}U \ \& \ \bar{A}_{AM}U)$  (12,14 Conj)
16.  $(\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (15, EG)
17.  $AiQC$  (16 Def iq)

Datisi-QXQ (Valid):

1.  $AaQB$  (major premise)
2.  $CixB$  (minor premise) //  $AiQC \ (\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (Def iq)
3.  $(\forall Z)[(B_{AM}Z \ \supset \ A_{AM}Z) \ \& \ (B_{AM}Z \ \supset \ \bar{A}_{AM}Z)]$  (1 Def aQ)
4.  $CixB \ \supset \ (\exists Z)(B_{AM}Z \ \& \ C_{AM}Z)$  (X-M-sub ix)
5.  $(\exists Z)(B_{AM}Z \ \& \ C_{AM}Z)$  (2,4 MP)
6.  $B_{AM}U \ \& \ C_{AM}U$  (5 EI)
7.  $B_{AM}U$  (6 Simp)
8.  $(B_{AM}U \ \supset \ A_{AM}U) \ \& \ (B_{AM}U \ \supset \ \bar{A}_{AM}U)$  (3 UI)
9.  $B_{AM}U \ \supset \ A_{AM}U$  (8 Simp)
10.  $A_{AM}U$  (7,9 MP)
11.  $B_{AM}U \ \supset \ \bar{A}_{AM}U$  (8 Simp)
12.  $\bar{A}_{AM}U$  (7,11 MP)
13.  $A_{AM}U \ \& \ \bar{A}_{AM}U$  (10,12 Conj)
14.  $C_{AM}U$  (6 Simp)
15.  $C_{AM}U \ \& \ (A_{AM}U \ \& \ \bar{A}_{AM}U)$  (13,14 Conj)
16.  $(\exists Z)[C_{AM}Z \ \& \ (A_{AM}Z \ \& \ \bar{A}_{AM}Z)]$  (15, EG)
17.  $AiQC$  (16 Def iq)

Datisi-XQM (Valid):

1.  $AaxB$  (major premise)
2.  $CiqB$  (minor premise) //  $AimC \ (\exists Z)(C_{AM}Z \ \& \ A_{AM}Z)$  (Def im)
3.  $AaxB \ \supset \ (\forall Z)(B_{AM}Z \ \supset \ A_{AM}Z)$  (X-M-sub ax)
4.  $(\exists Z)[B_{AM}Z \ \& \ (C_{AM}Z \ \& \ \bar{C}_{AM}Z)]$  (2 Def iq)
5.  $B_{AM}U \ \& \ (C_{AM}U \ \& \ \bar{C}_{AM}U)$  (4 EI)
6.  $(\forall Z)(B_{AM}Z \ \supset \ A_{AM}Z)$  (1,3 MP)
7.  $B_{AM}U \ \supset \ A_{AM}U$  (6 UI)
8.  $B_{AM}U$  (5 Simp)

9.  $A_{aM}U$  (8 Simp)
10.  $(C_{aM}U \ \& \ \bar{C}_{aM}U)$  (7,9 MP)
11.  $C_{aM}U$  (5 Simp)
12.  $C_{aM}U \ \& \ A_{aM}U$  (10,11 Conj)
13.  $(\exists Z)(C_{aM}Z \ \& \ A_{aM}Z)$  (12, EG)
14.  $A_{iM}C$  (13 Def  $i_M$ )

Datisi-QLQ (Valid):

1.  $A_{aQ}B$  (major premise)
2.  $C_{iL}B$  (minor premise)// $A_{iQ}C$   $(\exists Z)[C_{aM}Z \ \& \ (A_{aM}Z \ \& \ \bar{A}_{aM}Z)]$  (Def  $i_Q$ )
3.  $(\forall Z)[(B_{aM}Z \ \supset \ A_{aM}Z) \ \& \ (B_{aM}Z \ \supset \ \bar{A}_{aM}Z)]$  (1 Def  $a_Q$ )
4.  $C_{iL}B \ \supset \ (\exists Z)(B_{aM}Z \ \& \ C_{aM}Z)$  (L-M-sub  $i_L$ )
5.  $(\exists Z)(B_{aM}Z \ \& \ C_{aM}Z)$  (2,4 MP)
6.  $B_{aM}U \ \& \ C_{aM}U$  (5 EI)
7.  $B_{aM}U$  (6 Simp)
8.  $(B_{aM}U \ \supset \ A_{aM}U) \ \& \ (B_{aM}U \ \supset \ \bar{A}_{aM}U)$  (3 UI)
9.  $B_{aM}U \ \supset \ A_{aM}U$  (8 Simp)
10.  $A_{aM}U$  (7,9 MP)
11.  $B_{aM}U \ \supset \ \bar{A}_{aM}U$  (8 Simp)
12.  $\bar{A}_{aM}U$  (7,11 MP)
13.  $A_{aM}U \ \& \ \bar{A}_{aM}U$  (10,12 Conj)
14.  $C_{aM}U$  (6 Simp)
15.  $C_{aM}U \ \& \ (A_{aM}U \ \& \ \bar{A}_{aM}U)$  (13,14 Conj)
16.  $(\exists Z)[C_{aM}Z \ \& \ (A_{aM}Z \ \& \ \bar{A}_{aM}Z)]$  (15, EG)
17.  $A_{iQ}C$  (16 Def  $i_Q$ )

Datisi-LQM (Valid):

1.  $A_{aL}B$  (major premise)
2.  $C_{iQ}B$  (minor premise)// $A_{iM}C$   $(\exists Z)(C_{aM}Z \ \& \ A_{aM}Z)$  (Def  $i_Q$ )
3.  $A_{aL}B \ \supset \ (\forall Z)(B_{aM}Z \ \supset \ A_{aM}Z)$  (L-M-sub  $a_L$ )
4.  $(\exists Z)[B_{aM}Z \ \& \ (C_{aM}Z \ \& \ \bar{C}_{aM}Z)]$  (2 Def  $i_Q$ )
5.  $B_{aM}U \ \& \ (C_{aM}U \ \& \ \bar{C}_{aM}U)$  (4 EI)
6.  $(\forall Z)(B_{aM}Z \ \supset \ A_{aM}Z)$  (1,3 MP)
7.  $B_{aM}U \ \supset \ A_{aM}U$  (6 UI)
8.  $B_{aM}U$  (5 Simp)
9.  $A_{aM}U$  (7,8 MP)
10.  $(C_{aM}U \ \& \ \bar{C}_{aM}U)$  (5 Simp)
11.  $C_{aM}U$  (10 Simp)
12.  $C_{aM}U \ \& \ A_{aM}U$  (9,11 Conj)
13.  $(\exists Z)(C_{aM}Z \ \& \ A_{aM}Z)$  (12, EG)
14.  $A_{iM}C$  (13 Def  $i_Q$ )

Bocardo-XXX (Valid):

1.  $AoxB$  (major premise)
2.  $CaxB$  (minor premise)// $AoxC (\exists Z)[CaxZ \& \sim(AaxZ)]$
3.  $(\exists Z)[BaxZ \& \sim(AaxZ)]$  (1 Def ox)
4.  $(\forall Z)(BaxZ \supset CaxZ)$  (2 Def ax)
5.  $BaxU \& \sim(AaxU)$  (3 EI)
6.  $BaxU \supset CaxU$  (4 UI)
7.  $BaxU$  (5 Simp)
8.  $CaxU$  (6,7 MP)
9.  $\sim(AaxU)$  (5 Simp)
10.  $CaxU \& \sim(AaxU)$  (8,9 Conj)
11.  $(\exists Z)[CaxZ \& \sim(AaxZ)]$  (10 EG)
12.  $AoxC$  (11 Def ox)

Bocardo-LLL (Valid):

1.  $AoLB$  (major premise)
2.  $CaLB$  (minor premise)// $AoLC (\exists Z)[(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)]$
3.  $AoLB \supset (\exists Z)\{BaxZ \& [\bar{A}axZ \& \sim(AaxZ)]\}$  (L-X-sub oL)
4.  $(\exists Z)\{BaxZ \& [\bar{A}axZ \& \sim(AaxZ)]\}$  (1,3 MP)
5.  $(\forall Z)(BaxZ \supset CaLZ)$  (2 Def aL)
6.  $BaxU \& [\bar{A}axU \& \sim(AaxU)]$  (4 EI)
7.  $BaxU \supset CaLU$  (5 UI)
8.  $BaxU$  (6 Simp)
9.  $CaLU$  (7,8 MP)
10.  $\bar{A}axU \& \sim(AaxU)$  (6 Simp)
11.  $\bar{A}axU$  (10 Simp)
12.  $\bar{A}axU \& CaLU$  (9,11 Conj)
13.  $(\bar{A}axU \& CaLU) \vee (CaxU \& \bar{A}aLU)$  (12 Add)
14.  $(CaxU \& \bar{A}aLU) \vee (\bar{A}axU \& CaLU)$  (13 Comm)
15.  $(\exists Z)[(CaxZ \& \bar{A}aLZ) \vee (\bar{A}axZ \& CaLZ)]$  (14 EG)
16.  $AoLC$  (15 Def oL)

Bocardo-LXX (Valid):

1.  $AoLB$  (major premise)
2.  $CaxB$  (minor premise)// $AoxC (\exists Z)[CaxZ \& \sim(AaxZ)]$
3.  $AoLB \supset (\exists Z)\{BaxZ \& [\bar{A}axZ \& \sim(AaxZ)]\}$  (L-X-sub oL)
4.  $(\exists Z)\{BaxZ \& [\bar{A}axZ \& \sim(AaxZ)]\}$  (1,3 MP)
5.  $(\forall Z)(BaxZ \supset CaxZ)$  (2 Def ax)
6.  $BaxU \& [\bar{A}axU \& \sim(AaxU)]$  (4 EI)
7.  $BaxU \supset CaxU$  (5 UI)
8.  $BaxU$  (6 Simp)
9.  $CaxU$  (7,8 MP)

10.  $\bar{A}axU \ \& \ \sim(AaxU)$  (6 Simp)
11.  $\sim(AaxU)$  (10 Simp)
12.  $CaxU \ \& \ \sim(AaxU)$  (9,11 Conj)
13.  $(\exists Z)[CaxZ \ \& \ \sim(AaxZ)]$  (12 EG)
14.  $AoxC$  (13 Def  $ox$ )

Bocardo-QQQ (Valid):

1.  $AoqB$  (major premise)
2.  $CaqB$  (minor premise)// $AoqC$   $(\exists Z)[CamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (Def iq)
3.  $(\forall Z)[(BamZ \supset CamZ) \ \& \ (BamZ \supset \bar{C}amZ)]$  (2 Def aq)
4.  $(\exists Z)[BamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (1 Def oq)
5.  $BamU \ \& \ (AamU \ \& \ \bar{A}amU)$  (4 EI)
6.  $(BamU \supset CamU) \ \& \ (BamU \supset \bar{C}amU)$  (3 UI)
7.  $BamU \supset CamU$  (6 Simp)
8.  $BamU$  (5 Simp)
9.  $CamU$  (7,8 MP)
10.  $(AamU \ \& \ \bar{A}amU)$  (5 Simp)
11.  $CamU \ \& \ (AamU \ \& \ \bar{A}amU)$  (9,10 Conj)
12.  $(\exists Z)[CamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (11, EG)
13.  $AoqC$  (12 Def oq)

Bocardo-QXQ (Rini identifies as Valid at 40a39-b2):

1.  $AoqB$  (major premise)
2.  $CaxB$  (minor premise)// $AoqC$   $(\exists Z)[CamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (Def oq)
3.  $(\exists Z)[BamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (1 Def oq)
4.  $CaxB \supset (\forall Z)(BamZ \supset CamZ)$  (X-M-sub ax)
5.  $(\forall Z)(BamZ \supset CamZ)$  (2,4 MP)
6.  $BamU \ \& \ (AamU \ \& \ \bar{A}amU)$  (5 EI)
7.  $BamU$  (6 Simp)
8.  $BamU \supset CamU$  (3 UI)
9.  $CamU$  (7,8 MP)
10.  $(AamU \ \& \ \bar{A}amU)$  (6 Simp)
11.  $CamU \ \& \ (AamU \ \& \ \bar{A}amU)$  (9,10 Conj)
12.  $(\exists Z)[CamZ \ \& \ (AamZ \ \& \ \bar{A}amZ)]$  (11, EG)
13.  $AoqC$  (12 Def oq)

Bocardo-XQM<sup>46</sup> (not validated by Aristotle's methods):

1.  $AoxB$  (major premise)
2.  $CaqB$  (minor premise)// $AomC$   $(\exists Z)(CamZ \ \& \ \bar{A}amZ)$  (Def im)
3.  $AoxB \supset (\exists Z)(BamZ \ \& \ \bar{A}amZ)$  (X-M sub ax)

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<sup>46</sup> Following Mignucci (See Thom 1996, 75)

4.  $(\forall Z)[(B_{aM}Z \supset C_{aM}Z) \& (B_{aM}Z \supset \bar{C}_{aM}Z)]$  (2 Def aQ)
5.  $(\exists Z)(B_{aM}Z \& \bar{A}_{aM}Z)$  (1,3 MP)
6.  $B_{aMU} \& \bar{A}_{aMU}$  (1,3 MP)
7.  $(B_{aMU} \supset C_{aMU}) \& (B_{aMU} \supset \bar{C}_{aMU})$  (6 UI)
8.  $B_{aMU} \supset C_{aMU}$  (7 Simp)
9.  $B_{aMU}$  (6 Simp)
10.  $C_{aMU}$  (8,9 MP)
11.  $\bar{A}_{aMU}$  (6 Simp)
12.  $C_{aMU} \& \bar{A}_{aMU}$  (10,11 Conj)
13.  $(\exists Z)(C_{aMZ} \& \bar{A}_{aMZ})$  (12, EG)
14.  $A_{oMC}$  (13 Def oM)

Bocardo-QLQ (Valid according to Rini 2011, 212):

1.  $A_{oQB}$  (major premise)
2.  $C_{aLB}$  (minor premise)// $A_{oQC} (\exists Z)[C_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (Def oQ)
3.  $(\exists Z)[B_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (1 Def oQ)
4.  $C_{aLB} \supset (\forall Z)(B_{aMZ} \supset C_{aMZ})$  (L-M-sub aL)
5.  $(\forall Z)(B_{aMZ} \supset C_{aMZ})$  (2,4 MP)
6.  $B_{aMU} \& (A_{aMU} \& \bar{A}_{aMU})$  (5 EI)
7.  $B_{aMU}$  (6 Simp)
8.  $B_{aMU} \supset C_{aMU}$  (3 UI)
9.  $C_{aMU}$  (7,8 MP)
10.  $(A_{aMU} \& \bar{A}_{aMU})$  (6 Simp)
11.  $C_{aMU} \& (A_{aMU} \& \bar{A}_{aMU})$  (9,10 Conj)
12.  $(\exists Z)[C_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (11, EG)
13.  $A_{oQC}$  (12 Def oQ)

Bocardo-LQX (Valid)

1.  $A_{oLB}$  (major premise)
2.  $C_{aQB}$  (minor premise)// $A_{oXC} (\exists Z)[C_{aMZ} \& \sim(A_{aMZ})]$  (Def iQ)
3.  $A_{oLB} \supset (\exists Z)\{B_{aMZ} \& [\bar{A}_{aMZ} \& \sim(A_{aMZ})]\}$  (L-M-sub aL)
4.  $(\forall Z)[(B_{aMZ} \supset C_{aMZ}) \& (B_{aMZ} \supset \bar{C}_{aMZ})]$  (2 Def aQ)
5.  $(\exists Z)\{B_{aMZ} \& [\bar{A}_{aMZ} \& \sim(A_{aMZ})]\}$  (1,3 MP)
6.  $B_{aMU} \& [\bar{A}_{aMU} \& \sim(A_{aMU})]$  (5 EI)
7.  $A_{aXC}$  (IP)
9.  $A_{aXC} \supset (\forall Z)(C_{aMZ} \supset A_{aMZ})$  (X-M sub aX)
8.  $(\forall Z)(C_{aMZ} \supset A_{aMZ})$  (7 Def aX)
9.  $C_{aMU} \supset A_{aMU}$  (8 UI)
10.  $\bar{A}_{aMU} \& \sim(A_{aMU})$  (6 Simp)
11.  $\sim(A_{aMU})$  (10 Simp)
12.  $\sim(C_{aMU})$  (9,11 Simp)
13.  $(B_{aMU} \supset C_{aMU}) \& (B_{aMU} \supset \bar{C}_{aMU})$  (4 UI)
14.  $B_{aMU} \supset C_{aMU}$  (13 Simp)

15.  $\sim(Ba_MU)$  (12, 14 MT)
16.  $Ba_MU$  (6 Simp)
17.  $Ba_MU \& \sim(Ba_MU)$  (15,16 Conj)
18.  $\sim(Aa_xC)$  (7-17 IP)
19.  $Ao_xC$  (18 ax|ox)

Ferison-XXX (Valid):

1.  $Ae_xB$  (major premise)
2.  $CixB$  (minor premise)// $Ao_xC (\exists Z)[CaxZ \& \sim(AaxU)]$
3.  $(\forall Z)[BaxZ \supset \sim(AaxZ)]$  (1 Def ex)
4.  $(\exists Z)(BaxZ \& CaxZ)$  (2 Def ix)
5.  $BaxU \& CaxU$  (4 EI)
6.  $BaxU$  (5 Simp)
7.  $BaxU \supset \sim(AaxU)$  (3 UI)
8.  $\sim(AaxU)$  (6,7 MP)
9.  $CaxU$  (5 Simp)
10.  $CaxU \& \sim(AaxU)$  (8,9 Conj)
11.  $(\exists Z)[CaxZ \& \sim(AaxU)]$  (10 EG)
12.  $Ao_xC$  (11 Def ox)

Ferison-LLL (Valid):

1.  $Ae_LB$  (major premise)
2.  $Ci_LB$  (minor premise)// $Ao_LC (\exists Z)[(CaxZ \& \bar{A}a_LZ) \vee (\bar{A}axZ \& Ca_LZ)]$
3.  $(\forall Z)(BaxZ \supset \bar{A}a_LZ)$  (1 Def eL)
4.  $Ci_LB \supset (\exists Z)(BaxZ \& CaxZ)$  (L-X-sub iL)
5.  $(\exists Z)(BaxZ \& CaxZ)$  (2,4 MP)
6.  $BaxU \& CaxU$  (5 EI)
7.  $BaxU$  (6 Simp)
8.  $BaxU \supset \bar{A}a_LU$  (3 UI)
9.  $\bar{A}a_LU$  (7,8 MP)
10.  $CaxU$  (6 Simp)
11.  $CaxU \& \bar{A}a_LU$  (9,10 Conj)
12.  $(CaxU \& \bar{A}a_LU) \vee (\bar{A}axU \& Ca_LU)$  (11 Add)
13.  $(\exists Z)[(CaxZ \& \bar{A}a_LZ) \vee (\bar{A}axZ \& Ca_LZ)]$  (12 EG)
14.  $Ao_LC$  (13 Def oL)

Ferison-LXL (Valid):

1.  $Ae_LB$  (major premise)
2.  $CixB$  (minor premise) // $Ao_LC (\exists Z)[(CaxZ \& \bar{A}a_LZ) \vee (\bar{A}axZ \& Ca_LZ)]$
3.  $(\forall Z)(BaxZ \supset \bar{A}a_LZ)$  (1 Def eL)
4.  $(\exists Z)(BaxZ \& CaxZ)$  (2 Def ix)
5.  $BaxU \& CaxU$  (4 EI)



6.  $B_{xU}$  (5 Simp)
7.  $B_{xU} \supset \bar{A}_{aLU}$  (3 UI)
8.  $\bar{A}_{aLU}$  (6,7 MP)
9.  $C_{xU}$  (5 Simp)
10.  $C_{xU} \& \bar{A}_{aLU}$  (8,9 Conj)
11.  $(C_{xU} \& \bar{A}_{aLU}) \vee (\bar{A}_{axU} \& C_{aLU})$  (10 Add)
12.  $(\exists Z)[(C_{axZ} \& \bar{A}_{aLZ}) \vee (\bar{A}_{axZ} \& C_{aLZ})]$  (11 EG)
13.  $A_{oLC}$  (12 Def  $oL$ )

Ferison-QQQ (Valid):

1.  $A_{eQB}$  (major premise)
2.  $C_{iQB}$  (minor premise)// $A_{oQC} (\exists Z)[C_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (Def  $oQ$ )
3.  $(\forall Z)[(B_{aMZ} \supset A_{aMZ}) \& (B_{aMZ} \supset \bar{A}_{aMZ})]$  (1 Def  $eQ$ )
4.  $(\exists Z)[B_{aMZ} \& (C_{aMZ} \& \bar{C}_{aMZ})]$  (2 Def  $iQ$ )
5.  $B_{aMU} \& (C_{aMU} \& \bar{C}_{aMU})$  (4 EI)
6.  $(B_{aMU} \supset A_{aMU}) \& (B_{aMU} \supset \bar{A}_{aMU})$  (3 UI)
7.  $B_{aMU} \supset A_{aMU}$  (6 Simp)
8.  $B_{aMU}$  (5 Simp)
9.  $A_{aMU}$  (7,8 MP)
10.  $B_{aMU} \supset \bar{A}_{aMU}$  (6 Simp)
11.  $\bar{A}_{aMU}$  (8,10 MP)
12.  $C_{aMU} \& \bar{C}_{aMU}$  (5 Simp)
13.  $C_{aMU}$  (12 Simp)
14.  $A_{aMU} \& \bar{A}_{aMU}$  (9,11 Conj)
15.  $C_{aMU} \& (A_{aMU} \& \bar{A}_{aMU})$  (13,14 Conj)
16.  $(\exists Z)[C_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (15, EG)
17.  $A_{oQC}$  (16 Def  $oQ$ )

Ferison-QXQ (Valid):

1.  $A_{eQB}$  (major premise)
2.  $C_{iXB}$  (minor premise)// $A_{oQC} (\exists Z)[C_{aMZ} \& (A_{aMZ} \& \bar{A}_{aMZ})]$  (Def  $oQ$ )
3.  $(\forall Z)[(B_{aMZ} \supset A_{aMZ}) \& (B_{aMZ} \supset \bar{A}_{aMZ})]$  (1 Def  $eQ$ )
4.  $C_{iXB} \supset (\exists Z)(B_{aMZ} \& C_{aMZ})$  (X-M-sub  $iX$ )
5.  $(\exists Z)(B_{aMZ} \& C_{aMZ})$  (2,4 MP)
6.  $B_{aMU} \& C_{aMU}$  (5 EI)
7.  $B_{aMU}$  (6 Simp)
8.  $(B_{aMU} \supset A_{aMU}) \& (B_{aMU} \supset \bar{A}_{aMU})$  (3 UI)
9.  $B_{aMU} \supset A_{aMU}$  (8 Simp)
10.  $A_{aMU}$  (7,9 MP)
11.  $B_{aMU} \supset \bar{A}_{aMU}$  (8 Simp)
12.  $\bar{A}_{aMU}$  (7,11 MP)
13.  $A_{aMU} \& \bar{A}_{aMU}$  (10,12 Conj)
14.  $C_{aMU}$  (6 Simp)

15.  $C_{AM}U \& (A_{AM}U \& \bar{A}_{AM}U)$  (13,14 Conj)
16.  $(\exists Z)[C_{AM}Z \& (A_{AM}Z \& \bar{A}_{AM}Z)]$  (15, EG)
17.  $A_{oQC}$  (16 Def  $oQ$ )

Ferison-XQM (Valid):

1.  $A_{eXB}$  (major premise)
2.  $C_{iQB}$  (minor premise)// $A_{oMC} (\exists Z)(C_{AM}Z \& \bar{A}_{AM}Z)$  (Def  $oM$ )
3.  $A_{eXB} \supset (\forall Z)(B_{AM}Z \supset \bar{A}_{AM}Z)$  (X-M-sub  $ax$ )
4.  $(\exists Z)[B_{AM}Z \& (C_{AM}Z \& \bar{C}_{AM}Z)]$  (2 Def  $iQ$ )
5.  $B_{AM}U \& (C_{AM}U \& \bar{C}_{AM}U)$  (4 EI)
6.  $(\forall Z)(B_{AM}Z \supset \bar{A}_{AM}Z)$  (1,3 MP)
7.  $B_{AM}U \supset \bar{A}_{AM}U$  (6 UI)
8.  $B_{AM}U$  (5 Simp)
9.  $\bar{A}_{AM}U$  (7,8 MP)
10.  $(C_{AM}Z \& \bar{C}_{AM}Z)$  (5 Simp)
11.  $C_{AM}U$  (10 Simp)
12.  $C_{AM}U \& \bar{A}_{AM}U$  (9,11 Conj)
13.  $(\exists Z)(C_{AM}Z \& \bar{A}_{AM}Z)$  (12, EG)
14.  $A_{oMC}$  (13 Def  $oM$ )

Ferison-QLQ (Valid):

1.  $A_{eQB}$  (major premise)
2.  $C_{iLB}$  (minor premise)// $A_{oQC} (\exists Z)[C_{AM}Z \& (A_{AM}Z \& \bar{A}_{AM}Z)]$  (Def  $oQ$ )
3.  $(\forall Z)[(B_{AM}Z \supset A_{AM}Z) \& (B_{AM}Z \supset \bar{A}_{AM}Z)]$  (1 Def  $eQ$ )
4.  $C_{iLB} \supset (\exists Z)(B_{AM}Z \& C_{AM}Z)$  (L-M-sub  $iL$ )
5.  $(\exists Z)(B_{AM}Z \& C_{AM}Z)$  (2,4 MP)
6.  $B_{AM}U \& C_{AM}U$  (5 EI)
7.  $B_{AM}U$  (6 Simp)
8.  $(B_{AM}U \supset A_{AM}U) \& (B_{AM}U \supset \bar{A}_{AM}U)$  (3 UI)
9.  $B_{AM}U \supset A_{AM}U$  (8 Simp)
10.  $A_{AM}U$  (7,9 MP)
11.  $B_{AM}U \supset \bar{A}_{AM}U$  (8 Simp)
12.  $\bar{A}_{AM}U$  (7,11 MP)
13.  $A_{AM}U \& \bar{A}_{AM}U$  (10,12 Conj)
14.  $C_{AM}U$  (6 Simp)
15.  $C_{AM}U \& (A_{AM}U \& \bar{A}_{AM}U)$  (13,14 Conj)
16.  $(\exists Z)[C_{AM}Z \& (A_{AM}Z \& \bar{A}_{AM}Z)]$  (15, EG)
17.  $A_{oQC}$  (16 Def  $oQ$ )

Ferison-LQX (Valid):

1.  $A_{eLB}$  (major premise)
2.  $B_{iQC}$  (minor premise)// $A_{oXC} (\exists Z)(C_{AX}Z \& \bar{A}_{AX}Z)$  (Def  $oX$ )

3.  $Ae_L B \supset (\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (L-M-sub e<sub>L</sub>)
4.  $(\exists Z)[B_{aM}Z \& (C_{aM}Z \& \bar{C}_{aM}Z)]$  (2 Def i<sub>Q</sub>)
5.  $B_{aM}U \& (C_{aM}U \& \bar{C}_{aM}U)$  (4 EI)
6.  $AaxC$  (IP)
7.  $AaxC \supset (\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (L-M-sub e<sub>L</sub>)
8.  $(\forall Z)(C_{aM}Z \supset A_{aM}Z)$  (6,7 MP)
9.  $C_{aM}U \supset A_{aM}U$  (8 UI)
10.  $C_{aM}U \& \bar{C}_{aM}U$  (5 Simp)
11.  $C_{aM}U$  (10 Simp)
12.  $A_{aM}U$  (9,11 MP)
13.  $(\forall Z)\{(B_{aM}Z \supset \bar{A}_{aM}Z) \& [B_{aM}Z \supset \sim(A_{aM}Z)]\}$  (1,3 MP)
14.  $[B_{aM}U \supset \bar{A}_{aM}U] \& [B_{aM}U \supset \sim(A_{aM}U)]$  (12 UI)
15.  $B_{aM}U$  (5 Simp)
16.  $B_{aM}U \supset \sim(A_{aM}U)$  (14 Simp)
17.  $\sim(A_{aM}U)$  (15,16 MP)
18.  $A_{aM}U \& \sim(A_{aM}U)$  (12,17 Conj)
19.  $\sim(AaxC)$  (6-18 IP)
20.  $AoxC$  (19 ox|ax)