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A Model and Methodologies for The Location Problem with Logistical Components

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Abstract

This paper significantly extends traditional facility location models by introducing several logistical cost components such as holding, ordering, and transportation costs in a multi-commodity, multi-location framework. Since location and logistical costs are highly inter-related, the paper provides an integrated model, and seeks to minimize total physical distribution costs by simultaneously determining optimal locations, flows, shipment compositions, and shipment cycle times. Two sophisticated heuristic methodologies, based on Lagrangian relaxation and simulated annealing, respectively, are provided and compared in an extensive computational experiment.

Scope and purpose

Logistics has recently acquired great significance in industry, in part due to the rapidly growing interest in Supply Chain Management. One of the important open issues in logistics is the effective integration of logistical cost

components such as transportation cost with facility location models, since the two are highly inter-related in practice. In particular, locations, flows, shipment compositions, and shipment cycle times are highly inter-dependent. The determination of optimal values of these variables is crucial for minimizing physical distribution costs. This paper proposes an integrated location–consolidation model and provides two sophisticated methodologies to solve the problem. The relative performance of the two methodologies is investigated in an extensive computational experiment.

Keywords

Facility location, Logistics, Lagrangian relaxation, Simulated annealing

1. Introduction

The current interest in supply chain management has highlighted the importance of logistics, the physical distribution component of which alone averages about 7.5% of sales in the United States [1]. This paper is concerned with an important open issue in logistics, namely the integration of facility location models with logistical functionality. According to Ballou [2], (i) location models do not incorporate nonlinearities and discontinuities found in logistics, particularly in transportation and (ii) location models deal with location, transportation, and inventory decisions in a fragmented rather than integrated manner.

The primary components of logistics costs are inventory holding costs, transportation costs, and ordering/setup costs. These costs are significantly impacted by the timing and grouping (consolidation) of products into shipments that flow through the distribution network. The timing aspect, for a single link between a supplier and a destination, has been developed as the classical economic order quantity (EOQ) problem. Timing and grouping considerations in an EOQ setting, have been investigated, for a single link and multiple products in the inbound consolidation literature. However, due to the extreme difficulty, if not impossibility, of developing analytical formulae in the case of multiple locations and/or warehouses, alternate methodologies, such as mathematical programming offer practical solution approaches. As Bowersox and Closs [3] have noted, the mixed-integer programming approach 'offers considerable flexibility which enables us to incorporate many of the complexities and idiosyncrasies found in logistical applications'.

To put the current research into perspective, compare two real-world applications. The first application, involving the impact of shipment consolidation on product flows and routing, is described in Blumenfeld et al. [4]. The electronic division of General Motors (Delco) in 1981 produced parts in Milwaukee (Wisconsin), Matamoros (Mexico), and Kokomo (Indiana). The central warehouse was located in Kokomo and products were shipped from Milwaukee and Matamoros to Kokomo by truck. At Kokomo, the parts were consolidated before being sent to about 30 GM plants located in various parts of the country. The objective was to minimize the sum of transportation and holding costs. This objective was determined by decisions regarding the composition of shipments and whether shipments should be sent direct to the plants or after consolidation at Kokomo. Routings, shipment compositions and cycle times were found to be highly interrelated. The problem was eventually solved heuristically by a methodology involving the decomposition of the network into subnetworks.

The second, more recent application involves the proposed acquisition of a mid-sized lawn and garden fencing company by a larger rival. Comparing this problem to the Delco problem, it is seen that both applications seek to optimize a deterministic freight network in which the parameters of the business environment, such as cost structures and product demands, are assumed to be fixed in the short run. However, there are at least three important differences between the applications. First, the Delco problem involves known and fixed locations for plants and warehouses. In contrast, in common with most mergers/acquisitions, the proposed acquisition implies numerous system redundancies in the form of overlapping territories covered by adjacent plants, branches, distribution centers, etc., necessitating network re-design in the interest of cost effectiveness. This implies that facility locations are outcomes, rather than fixed parameters of the model.

The second important difference between the two approaches pertains to the solution methodology. The Delco application involves a methodology which does not provide a bound on the proximity of the solution provided to the optimal solution, requiring qualitative assessments of the need for further system improvements. In contrast, this paper develops a sophisticated Lagrangian methodology that provides a bound on the optimal solution, which, in turn, makes it easier to judge the benefit of any proposed modifications to the system (route changes, branch closings, etc.). Finally, this paper integrates a realistic stepwise transportation cost function into the model that mimics the fare structures of common carriers, while the Delco application uses a simpler cost structure.

A simple prototype of the 'location/consolidation' problem investigated in this paper is provided in Fig. 1. The diagram shows a two-level freight network which comprised three facilities, three warehouses, and two destinations. Currently, facilities 1 and 3, and warehouses 1 and 2 are open. The flows in the network are implied by the arrowheads. Typical questions that are crucial to the analysis are shown in the figure — for instance, (i) should warehouse 3 be open? (ii) should group 1 on the link between warehouse 1 and destination 1 comprise products 1 and 5? (iii) what are the cycle times of shipments between facility 1 and warehouse 1 (the sawtooth pattern represents the inventory level as a function of cycle time)? (iv) what are the optimal flows between warehouse 2 and destination 2? It should also be noted that every question is potentially applicable to every arc (arc and link are used inter-changeably here) on the network, complicating the analysis immensely, even on a simple prototype network.

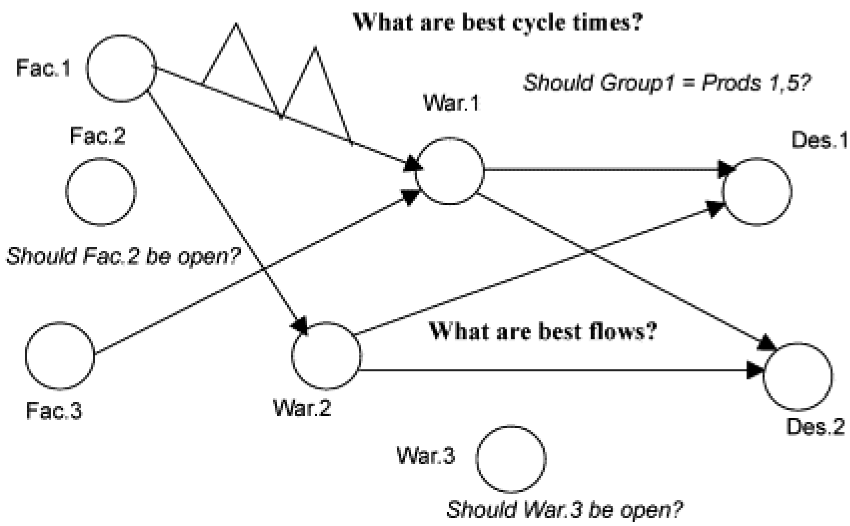

Fig. 1. The concurrent location/consolidation problem.

In order to provide tools that can directly tackle the problem of inter-dependency between structural parameters of a distribution/location network, the primary goal of the current research is the development of a model and solution methodologies that will permit the simultaneous determination of optimal (i) plant and warehouse locations, (ii) flows in the resulting distribution network, and (iii) shipment compositions and frequencies in the network, taking into account relevant logistical costs such as transportation costs, warehousing costs, etc. Subsequent sections of the paper contain (a) a summary review of relevant literature in freight networks, inbound consolidation, and facility location (b) a model for concurrent location and shipment consolidation (c) alternate methodologies based on the techniques of simulated annealing and Lagrangian relaxation (d) details of a computational experiment to test the methodologies and (e) provide concluding thoughts.

2. Related literature

The primary reference fields for the location/logistics problem are freight networks, inbound consolidation, and facility location. As the literature on these topics is rather extensive, no effort is made here to provide comprehensive reviews. Freight network analysis primarily seeks to minimize the total cost of distribution by capitalizing on opportunities for consolidation in various forms. Some of these forms are inventory consolidation, vehicle consolidation, peddling (multiple deliveries to proximate customers), and terminal consolidation. Representative instances from the freight network literature include Blumenfeld et al. [4] described in the introduction, Klincewicz [5], and Benjamin [6].

Klincewicz proposes a network model consisting of multiple origins, consolidation terminals, and destinations in which the objective is to minimize the total inventory holding and transportation costs, assuming that the shipping cost functions can be adequately described by piece-wise linear functions of volume and that the quantities shipped between each origin–destination pair are known in advance. Benjamin considers a single-commodity network and separates the logistics problem into a transportation problem and an economic lot size problem. He makes the distinction between annual flow and individual shipment size and emphasizes the need for simultaneous solution of flows and shipments. However, Benjamin actually employs an iterative rather than simultaneous procedure to solve the logistics problem.

The field of inbound consolidation contains research on the problem of determining optimal shipment compositions and frequencies. Early work in consolidation [7] focused on the tradeoff between ordering and inventory holding costs recognized in the economic order quantity (EOQ) formula, which offers a simple analytical solution to the problem of minimizing the sum of ordering and holding costs. The EOQ concept has been extended to multiple products and the optimal determination of shipping packages and packaging frequencies. However, this research is confined to ordering and holding costs on a single link between two points such as a supplier and a customer. In part, this is because it is extremely difficult, if not impossible, to develop analytical formulae when considering complications such as transportation costs and/or multiple plants, destinations, and warehouses.

Mathematical programming models offer a relatively tractable alternative approach to 'closed-form' solutions for solving the location–consolidation problem. Further, as noted by Bowersox and Closs [3], the modeling flexibility provided by the mixed-integer programming approach makes it particularly suitable for incorporating the complexities that arise in logistics in general and transportation in particular. Recent instances of mathematical programming approaches to the inbound consolidation problem include Russell and Krajewski [8], and Syam and Shetty [9]. However, these studies are confined to relatively simple environments involving only a single destination rather than multiple destinations, and supplier(s) with pre-determined rather than undecided locations. Further, both of these papers were concerned with single-echelon networks, rather than bi-echelon networks including warehouses, as in the current research. This paper significantly extends previous work in inbound consolidation by using a mathematical programming approach that seeks to simultaneously and optimally determine (i) the locations of multiple plants and (ii) the composition and timing of shipments and (iii) flows between multiple plants, warehouses, and destinations.

The literature in facility location is vast, and with many variants. Of these variants, the capacitated plant location problem (CPLP) has some elements in common with the problem investigated in this paper. An extensive review of (CPLP), found in Sridharan [10], readily reveals that the problem considered here has not been investigated previously. Representative older work includes a seminal paper on multicommodity distribution by Geoffrion and Graves [11], who employed Benders' partitioning technique. Their model did not include a way to select from multiple plants and did not incorporate the fixed costs of plant operation. More recently, Pirkul and Jayaraman [12] have investigated a problem with some structural similarities to that studied in this paper. Both are capacitated and bi-echelon, and both include multiple plants, warehouses, and destinations. However, the current research includes significant complexities that go well beyond the scope of [12]. In particular, it includes many important logistical elements such as optimal shipment identification, shipment cycle times, holding costs, ordering costs, and a piecewise linear transportation cost structure that resembles those found in industry. Finally, it contains, as in the *p*-median problem, parameters regarding the specific number of plants and warehouses to open. The formulation studied in this research has neither been developed nor solved in previous research, and it is labeled, for convenience, as the location–consolidation model. This paper provides two methodologies to heuristically solve the model.

3. Mathematical model and item consolidation

The logistical framework and its mathematical representation are presented in this section. The section also contains a discussion of the manner in which the decision variables of the model facilitate inventory consolidation, an objective of the research.

3.1. Logistical framework

The logistics model encapsulates the functioning of a multi-commodity freight network, consisting of plants, intermediate warehouses for inventory consolidation, and final destinations. In this paper, a plant is sometimes referred to as a 'manufacturing facility', and a link between two points in the network is occasionally referred to as an 'arc'. In common with the freight network and inbound consolidation literature, the term 'cycle time' refers to the frequency with which a good or group of goods is shipped — once a year, twice a year, etc. The objective is to determine locations, flows, groups of commodities, and their associated cycle times so as to minimize total logistics cost, comprised of inventory holding, purchase order line-item, purchase-order header, transportation, manufacturing, material handling, and fixed costs at plants and warehouse. The constraints of the model impose various logistical requirements — for instance, that the demand for each good at each destination must be met, that freight rates are a piecewise function of volume and shipment weight, that each facility is subject to certain capacity limitations, etc.

3.2. Model structure

In the mathematical model that follows, the objective function comprises the following components: (i) inventory holding costs (ii) ordering line-item costs (iii) ordering header costs (iv) transportation costs (v) manufacturing costs (vi) material handling costs at warehouses (vii) fixed costs at plants (viii) fixed costs at warehouses. Ordering costs, which are incurred when an order is fulfilled, consist of a fixed (header) charge for each order and additional line-item charges for individual items. Inventory holding costs in the model are computed as fixed percentages of item values. Plants differ in unit manufacturing costs of items and fixed costs, and material-handling costs vary from warehouse to warehouse, as do the fixed warehouse costs. All costs are considered on an annualized basis. Transportation costs are incurred on all links and are modeled to capture freight rates that vary as a function of shipping distance and shipment weight.

In the model, constraint (1) ensures that the demand for each item at each destination is met, and constraint (2) imposes the capacity limitations at each facility by commodity (equal to zero if the facility is not open). Constraint (3) imposes material-handling capacity limits on each warehouse by commodity (equal to zero if the warehouse is not open). Constraints (4) and (5) impose freight rates on shipments according to weight. Constraint (6) ensures that at most one freight rate applies to a shipment. Constraint (7) ensures that a line-item ordering cost is incurred if an item is shipped on a link and also imposes capacity limits on links. Constraint (8) ensures that only one cycle time applies to an item on a particular arc. Constraint (9) compactly ensures that order header costs are incurred when line-item order costs and, by virtue of constraint (7), flows exist on a link.

Finally, constraints (10) and (11) impose restrictions on the numbers of open plants and warehouses, respectively.

Mathematical notation and model

The following parameters and variables are used to describe the mathematical location/consolidation model. Here, '*n*' denotes arc or link, '*i*' denotes commodity, '*h*' denotes cycle-time, '*f*' denotes freight rate, '*s*' denotes manufacturing facility, '*e*' denotes warehouse, and '*d*' denotes destination.

N	set of arcs in the network
S	set of open manufacturing plants
$S-$	set of arcs outgoing from open manufacturing plants
$s-$	set of arcs outgoing from open manufacturing facility s
E	set of open warehouses
$E-$	set of arcs outgoing from open warehouses
$e-$	set of arcs outgoing from open warehouse e
$e+$	set of arcs incoming at open warehouse e
D	set of destinations
$D+$	set of incoming arcs at destinations
$d+$	set of incoming arcs at destination d
I	set of commodities (items)
H	set of cycle-times
F	set of freight rates
$\mathrm{p_s}$	parameter representing the number of open manufacturing plants
$\mathrm{p_e}$	parameter representing the number of open warehouses
$\mathrm{V_{in}}$	value of item i on arc n
$\mathrm{t_h}$	cycle-time h
$\mathrm{k_i}$	annual inventory holding cost percentage of item i
$\mathrm{w_i}$	weight per unit in pounds of item i
$\mathrm{c_{fn}}$	freight cost rate f on arc n
$\mathrm{G_n}$	fixed header cost of a purchase order on arc n
$\mathrm{a_{in}}$	line-item ordering cost of item i on arc n
$\mathrm{m_e}$	unit material handling cost at warehouse e
$\mathrm{K_s}$	fixed annual cost associated with facility s
$\mathrm{R_e}$	fixed annual cost associated with warehouse e
$\mathrm{b_{di}}$	demand for item i at destination d
$\mathrm{u_{is}}$	manufacturing capacity limit for item i at facility s
$\mathrm{u_{in}}$	capacity limit for item i on arc n
$\mathrm{C_{ei}}$	material handling capacity limit for item i at warehouse e
$\mathrm{\beta_{fn}}$	weight breakpoint f on arc n.

The decision variables of the model are:

$\mathrm{X_{fhi}^{n}}$	integer variable representing flow of item *i* on arc *n* with cycle-time *h*, freight category *f*
$\mathrm{\alpha_{hfn}}$	binary variable, equal to 1 if there is flow on arc *n* with cycle-time *h* and freight category *f*, 0 otherwise
π_{hin}	binary variable, equal to 1 if item *i* is ordered on cycle-time *h* on arc *n*, 0 otherwise
$\chi\mathrm{hn}$	binary variable, equal to 1 if an order is placed on cycle-time *h* on arc *n*, 0, otherwise
ψ_s	binary variable, equal to 1 if manufacturing facility *s* is open, 0 otherwise
φ_e	binary variable, equal to 1 if warehouse *e* is open, 0 otherwise

Using this notation the logistics model is as follows:

$$\text{Minimize} \sum_{f\in F}\sum_{h\in H}\sum_{i\in I}\sum_{n\in N}\frac{1}{2} t_h k_i V_{in} X_{fhi}^{n} + \sum_{h\in H}\sum_{i\in I}\sum_{n\in N} a_{in} t_h \pi_{hin} + \sum_{h=1}^{H}\sum_{n=1}^{N} G_n t_h \chi_{hn}$$

$$+\sum_{h=1}^{H}\sum_{n=1}^{N}\sum_{f=1}^{F} c_{fn}\sum_{i=1}^{I} X_{fhi}^{n} w_i + \sum_{f\in F}\sum_{h\in H}\sum_{i\in I}\sum_{n\in S^-} V_{in} X_{fhi}^{n}$$

$$+\sum_{e\in E}\sum_{f\in F}\sum_{h\in H}\sum_{i\in I}\sum_{n\in E^-} m_{ei} X_{fhi}^{n} + \sum_{s\in S} K_s \psi_s + \sum_{e\in E} R_e \phi_e,$$

(1)

$$\text{subject to} \sum_{f\in F}\sum_{h\in H}\sum_{n\in d^+} X_{fhi}^{n} = b_{di}, \forall d,i,$$

(2)

$$\sum_{f\in F}\sum_{h\in H}\sum_{n\in s^-} X_{fhi}^{n} \leqslant u_{is}\psi_s, \forall i,s,$$

(3)

$$\sum_{f\in F}\sum_{h\in H}\sum_{n\in e^-} X_{fhi}^{n} \leqslant C_{ei}\phi_e, \forall e,i,$$

(4)

$$t_h\sum_{i\in I} X_{fhi}^{n} w_i \leqslant \beta_{(f+1)n}\alpha_{fhn}, \forall f,h,n,$$

(5)

$$t_h\sum_{i\in I} X_{fhi}^{n} w_i \geqslant \beta_{fn}\alpha_{fhn}, \forall f,h,n,$$

(6)

$$\sum_{f\in F}\alpha_{fhn} \leqslant 1, \forall h,n,$$

(7)

$$\sum_{f\in F}\sum_{h\in H} X_{fhi}^{n} \leqslant \sum_{h\in H}\pi_{hin} u_{in}, \forall i,n,$$

(8)

$$\sum_{h\in H}\pi_{hin} \leqslant 1, \forall i,n,$$

(9)

$$\sum_{i\in I}\pi_{hin} \leqslant |I|\chi_{hn}, \forall h,n,$$

(10)

$$\sum_{s\in S}\psi_s = p_s,$$

(11)

$$\sum_{e\in E}\phi_e = p_e,$$

$$\alpha_{fhn}\in\{0,1\}\forall f,h,n, \pi_{hin}\in\{0,1\}\forall h,i,n, \chi_{hn}\in\{0,1\}\forall h,n,$$

$$\psi_s\in\{0,1\}\forall s, \phi_e\in\{0,1\}\forall e, X_{fhi}^{n}\ \text{integer}\ \forall f,h,i,n.$$

3.4. Shipment consolidation

Shipment consolidation is concerned with determining the composition and shipping frequencies of shipments (also called groups). Optimal grouping leads to the lowest possible logistical cost. The key to the concurrent determination of flows and shipments is a transformation found in network flow analysis, namely, artificially dividing (or splitting) the total flow of product on each link into smaller groups. Each group includes only the products that have the same cycle time (shipping frequency), and the same transportation freight rate. All the products in a group are shipped together, thus comprising a shipment. The model is solved to determine the optimal shipments on each link in the network.

Using the notation of the model, assume, as an instance, that the optimal (or best) solution specifies optimal flows in which variables X_{321}^2 and X_{325}^2 are positive-valued. This means that commodities 1 and 5 are, first, both shipped on arc 2, and, second, both part of the same shipment on arc 2. This shipment incurs the third freight rate and is shipped with the second shipping frequency applicable on arc 2. Therefore, one of the optimal groups on arc 2 will include both items 1 and 5. It may be noted that the constraints of the model ensure that an item cannot belong to more than one group (i.e., shipment) on any particular link. This illustrates how the form of the decision variables of the model makes it possible to concurrently determine both optimal flows as well as optimal groups (shipments). In this context, permissible cycle times (shipping frequencies) belong to a discrete set that corresponds to the specifications or needs of logistics managers.

4. Solution methodologies

In its entirety, the model is a 0–1 integer representation of a *p*-median problem with an embedded multicommodity distribution sub-problem. The *p*-median problem and the multicommodity distribution problem are known to be NP-complete [13] making the model very difficult to solve. Hence, the solution methodologies involve the development of heuristic procedures. Two methodologies that have performed well on difficult combinatorial problems are simulated annealing and Lagrangian relaxation. Both these methodologies are heuristic in nature but often provide good solutions to NP-complete and other combinatorial problems. In this paper, the two methodologies are applied to identical instances of the location–consolidation problem and their relative performance is assessed.

Simulated annealing and Lagrangian relaxation have to be highly tailored to a specific problem in order to perform well. In this respect using either of the methodologies is an art as well as a science, and a certain degree of experimentation is inevitable. The following sub-sections will describe the implementation of these methodologies in this paper.

4.1. Simulated annealing procedure

Simulated annealing, as the name may suggest, is the mathematical analog of the metallurgical process of annealing (slow cooling), the purpose of which is to impart certain desirable properties to the metal being treated. The cooling schedule, which is specific to a particular metal, is critical to the success of the process — schedules that are too fast or too slow usually impart certain undesirable properties to the metal. The cooling schedule is very important in the mathematical analog as well. The first application of simulated annealing to combinatorial optimization problems was reported by Kirkpatrick et al. [14]. Many successful applications have been reported since then, and no effort is made here to provide a complete survey. Representative (and relevant) applications include the application of annealing to the quadratic assignment problem [15], and a double-annealing procedure for the vertex-constrained multi-site location problem reported in Righini [16].

The methodology involves the iterative interaction of a primary step (P) and a secondary step (Q). In the primary step, the simulated annealing procedure determines which plants and warehouses should be open. In the secondary step, a Lagrangian procedure accepts the assignments of the master step and solves the flow and consolidation problem in the resulting network. The heuristic methodology alternates between the master and secondary steps. It is terminated when an acceptable degree (as compared to a pre-determined tolerance) of convergence between successive solutions to the master problem is achieved or if the annealing procedure has traversed a pre-determined number of 'epochs' (discussed below).

In this study, we use annealing to solve the ‘outer’ problem of determining the optimal sets of open plants and warehouses. The procedure starts with a random solution (assignment). A number of ‘epochs’ follow, in which the solution is perturbed by the systematic interchanging of a number of open and closed plants. The ‘temperature’, which determines the probability of a solution being accepted, is reduced in successive epochs. A new solution is evaluated by solving the ‘inner’ consolidation problem. The objective function of the location–consolidation model is neither concave nor convex in the decision (flow) variables. As a result, gradient-based solution methods may terminate in relatively inferior local optima. An important characteristic of simulated annealing is that it ameliorates this problem by occasionally accepting non-improving solutions with a gradually decreasing probability. The annealing procedure is terminated when either (i) all epochs have been traversed or (ii) convergence, as compared to a pre-determined tolerance, is achieved between successive solutions. A general overview and diagram of the simulated annealing procedure are provided below.

4.1.1. Simulated annealing procedure

(1)	*Initialization*		
	(a)		Select an initial solution, s∈S — this implies randomly opening ps plants and pe warehouses (with sufficient capacity to meet demand). Initially *s* is both the incumbent (best) as well as current solution (s^{n}). Solve the inner Lagrangian problem to obtain an objective function value $f(s)$.
	(b)		Fix the number of epochs, *E*, an initial high temperature, $T > 0$, the initial number of iterations per epoch, *I*, and a convergence parameter, ε.
	(c)		Set the epoch number, $e = 0$ and fix a temperature modification factor, *t*, and iteration modification factor, *k*.
(2)	*Epoch processing*		
	(a)	$\mathrm{e} = \mathrm{e} + 1.$	
			If $e > E$, stop and accept the incumbent solution as best If e>1, modify the temperature i.e., $T = t * T$, and modify the number of iterations/epoch i.e., $I = k * I$. Set the iteration number, $i = 0$.
	(b)	*Iteration*	
		(i)	$i = i + 1$.
			If $i > I$, start new epoch, i.e., go to step 2a.
		(ii)	Generate a new solution, s′, by warehouse interchange and facility interchange with the current solution (s^{n}). Solve the inner problem (Q) to obtain the objective function value f(s′) using Lagrangian relaxation of constraints (2), (3), (6), (8), and (10) [details in Section 4.3].
		(iii)	Calculate $\Delta = \mathrm{f}(\mathrm{s}') - \mathrm{f}(\mathrm{s})$.
			If $\mathrm{Abs}(\Delta) < \varepsilon$, stop and accept the incumbent solution as best.
		(iv)	If $\Delta < 0$, then sn=s′, else if Random(0,1)<Exp(−Δ/T) then s^{n}=s′s=s^{n} if f(s^{n})<f(s).
		(v)	Go to new iteration i.e., step 2b(i).
		End Iteration	
	End Epoch		
End Procedure			

In practice, simulated annealing procedures have to be tailored to the problem at hand (Fig 2). In particular, their behavior has to be closely monitored because they usually ‘plateau’ after a while. To ‘plateau’ in this context is to execute at length without noticeable improvement in the objective value. The point at which a particular annealing procedure begins to plateau depends on the structure of the problem and also on the values of the annealing parameters. As a result of this, simulated annealing has elements of an art as well as a science. In the case of the location/consolidation problem, the critical annealing parameters are the annealing schedule and the choice of neighborhood. Details regarding the effects of these parameters are provided in the computational results below.

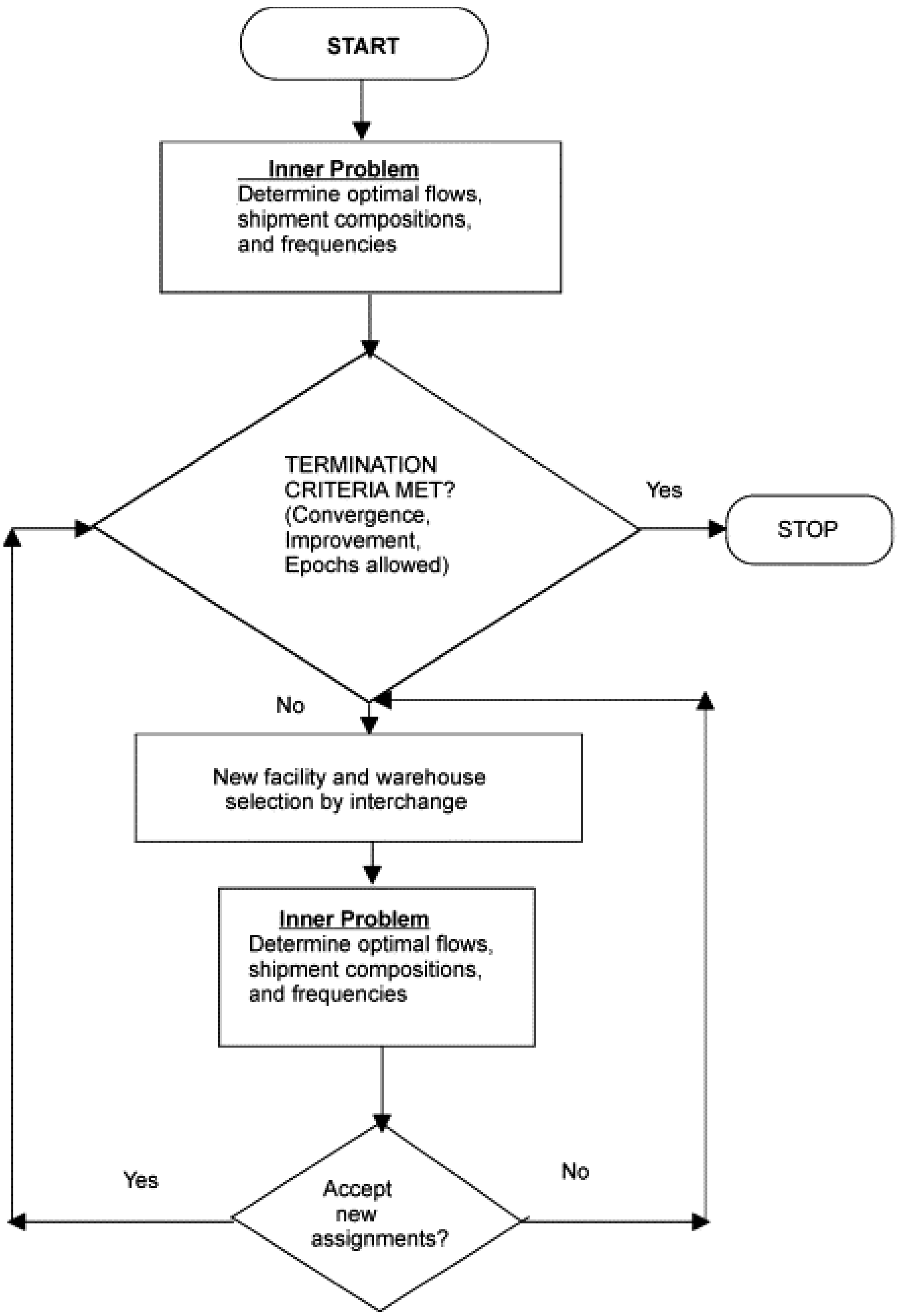

Fig. 2. Overview of simulated annealing.

4.2. Lagrangian methodology

The critical logistical issue in the current research is the determination of optimal locations for plants and warehouses. The secondary issue (which is also important) is the determination of optimal consolidation policies, given the set of open sites. The simulated annealing approach uses Lagrangian relaxation to address the secondary issue and annealing to determine optimal sites. The purely Lagrangian methodology, on the other hand, uses Lagrangian relaxation in order to determine both the optimal sites as well as optimal consolidation policies. Thus, the two methodologies differ in their approach to the critical logistical issue. This has a significant impact on their relative performance as assessed in the computational experiment described in Section 5.

Lagrangian methodology [17] involves the relaxation of constraints that complicate the model. The objective function is penalized to the extent that the relaxed model violates the omitted constraints. This is achieved by adding the relaxed constraints to the objective function multiplied by suitable penalties or multipliers. The solution to the relaxed problem with suitable fixed values of the multipliers provides a lower bound on the original problem. The Lagrangian dual problem is to find the values of the multipliers that provide the tightest possible lower bound i.e., maximizes the optimal solution to the relaxed problem.

The dual problem is usually solved by updating the multipliers in a systematic manner. Typically, the updated multipliers improve the lower bound progressively, but not necessarily monotonically. The method used in this research to update the multipliers is the subgradient method [18], convergence results for which may be found in Polyak [19]. In the absence of a duality gap, the optimal objective value of the dual problem coincides with that of the primal (original) problem. Lagrangian methods are typically terminated when there is acceptable convergence between upper bounds obtained from primal-feasible solutions and lower bounds generated by the relaxed problem. The lower bounding methodology is described next, followed by a description of the upper bounding mechanism.

4.3. Lower bounding technique

The technique is initiated by associating non-negative multipliers $\omega_{is}^{1}, \omega_{ei}^{2}, \mu_{fhn}^{1}, \mu_{fhn}^{2}, \lambda_{in}^{1}$, and λ_{hn}^{2} with the complicating constraints , , , , , and (9), respectively. These constraints are complicating in the sense that their removal greatly facilitates the solution of the transformed problem. Lagrangian relaxation is carried out by multiplying these constraints by their respective multipliers and adding them to the objective function (OF) of the original problem. The sub-problems associated with these relaxations have the integrality property [20], which implies that the resulting lower bounds cannot be tighter than those from a linear programming relaxation of the problem. Nevertheless, the Lagrangian approach is used because the resulting sub-problems can be solved very rapidly.

The relaxed problem has the following objective function:

$$OF_{lag}=OF+\sum f\in F\sum h\in H\sum n\in N\mu_{fhn}^{1}t_{h}\sum i\in IX_{fhi}^{n}w_{i}-\beta_{(f+1)n}\alpha_{fhn}+\sum i\in I$$

$$\sum s\in S\omega_{is}^{1}\sum f\in F\sum h\in H\sum n\in s-X_{fhi}^{n}-u_{is}\psi_{is}+\sum e\in E\sum i\in I\omega_{ei}^{2}\sum f\in F\sum h\in H$$

$$\sum n\in s-X_{fhi}^{n}-C_{ei}\phi_{e}\sum f\in F\sum h\in H\sum n\in N\mu_{fhn}^{2}\beta_{fn}\alpha_{fhn}-t_{h}\sum i\in IX_{fhi}w_{i}+\sum i\in I$$

$$\sum n\in N\lambda_{in}^{1}\sum f\in F\sum h\in HX_{fhi}^{n}-\sum h\in H\pi_{hin}u_{in}+\sum h\in H$$

$$\sum n\in N\lambda_{hn}^{2}\sum i\in I\pi_{hin}-|I|\chi_{hn}.$$

The Lagrangian dual problem is represented as follows:

$$\mathrm{Max}\,\varphi(\omega_{is}^{1},\omega_{ei}^{2},\mu_{fhn}^{1},\mu_{fhn}^{2},\lambda_{in}^{1},\lambda_{hn}^{2}),$$

where $\varphi(\omega_{is}^{1},\omega_{ei}^{2},\mu_{fhn}^{1},\mu_{fhn}^{2},\lambda_{in}^{1},\lambda_{hn}^{2})$ is the following model (Q1):

$\mathrm{Min}OF_{lag}$ s.t. constraints (1), (6), (8), (10), and (11).

Terms may be combined in OF_{lag}, resulting in the following coefficients:

For $n\in s^{-}$, $R_{fhi}^{n}=\{\omega_{is}^{1}+1/2t_{h}k_{in}V_{in}+c_{fn}w_{i}+m_{ei}+(\mu_{fhn}^{1}-\mu_{fhn}^{2})t_{h}w_{i}+\lambda_{in}^{1}\}$.
For $n\in e^{-}$, $R_{fhi}^{n}=\{\omega_{ei}^{2}+1/2t_{h}k_{in}V_{in}+c_{fn}w_{i}+m_{ei}+(\mu_{fhn}^{1}-\mu_{fhn}^{2})t_{h}w_{i}+\lambda_{in}^{1}\}$.
$S_{hin}=\{(a_{in}/t_{h})-\lambda_{in}^{1}+\lambda_{hn}^{2}\}$, $T_{hn}=\{(G_{n}/t_{h})-|I|\lambda_{hn}^{2}\}$,
$\hat{K}_{s}=K_{s}-\sum_{i\in I}\omega_{is}^{1}u_{is}$, $\hat{R}_{e}=R_{e}-\sum_{i\in I}\omega_{ei}^{2}C_{ei}$, $A_{fhn}=\mu_{fhn}^{2}\beta_{fn}-\mu_{fhn}^{1}\beta_{(f+1)n}$.

For fixed values of the multipliers, model (Q1) separates into the following six sub-problems, the sum of whose objective functions provides a lower bound (LB) on objective value of the primal problem.

Model X
(12)
$\mathrm{Min}Z_{1}=\sum_{f\in F}\sum_{h\in H}\sum_{i\in I}\sum_{n\in N}R_{fhi}^{n}X_{fhi}^{n}$
s.t. $\sum_{f\in F}\sum_{h\in H}\sum_{n\in d^{+}}X_{fhi}^{n}=b_{di}\;\forall d,i,$
X_{fhi}^{n} integer $\forall f,h,i,n.$
Model α
(13)
$\mathrm{Min}Z_{2}=\sum_{f\in F}\sum_{h\in H}\sum_{n\in N}A_{fhn}\alpha_{fhn}$
s.t. $\sum_{f\in F}\alpha_{fhn}\leqslant 1\;\forall f,h,n,$
$\alpha_{fhn}\in\{0,1\}\;\forall f,h,n.$
Model π
(14)
$\mathrm{Min}Z_{3}=\sum_{f\in F}\sum_{h\in H}\sum_{n\in N}S_{fhn}\pi_{fhn}$
s.t. $\sum_{h\in H}\pi_{hin}\leqslant 1\;\forall i,n,$
$\pi_{hin}\in\{0,1\}\;\forall h,i,n.$
Model χ
$\mathrm{Min}Z_{4}=\sum_{h\in H}\sum_{n\in N}T_{hn}$. (unconstrained)
Model ψ
(15)
$\mathrm{Min}Z_{5}=\sum_{s\in S}\hat{K}_{s}\psi_{s}$
s.t. $\sum_{s\in S}\psi_{s}=p_{s},$
$\psi_{s}\in\{0,1\}\;\forall s.$
Model φ
(16)
$\mathrm{Min}Z_{6}=\sum_{e\in E}\hat{R}_{e}\phi_{e},$
s.t. $\sum_{e\in E}\phi_{e}=p_{e},$
$\phi_{e}\in\{0,1\}\;\forall e.$

The last two models are not necessary in the simulated annealing methodology because open sites and warehouses are determined by the annealing procedure outside of the Lagrangian framework. The six models are all linear knapsack problems that can be solved extremely rapidly using relatively simple greedy algorithms [21].

4.4. Upper bound heuristic

Upper bounds are obtained from feasible solutions. In the Lagrangian methodology, a primal feasible solution is obtained at each iteration of the subgradient optimization. Similarly, the simulated annealing methodology generates a primal feasible solution for each accepted assignment of plants. For both the methodologies, generation of a primal feasible solution is done in three steps.

(a)	The first step is the determination of open plants. In the pure Lagrangian methodology, the solutions of the ψ_s and ϕ_e subproblems in the lower bounding procedure provide readily available sets of open plants and warehouses, respectively. The simulated annealing methodology uses the annealing procedure as described previously to select open plants.
(b)	Secondly, the flow problem (model X) is solved, restricted to the sets of selected plants and warehouses. The resulting allocations are likely to violate freight rate breakpoint constraints , , since these have been relaxed.
(c)	Third, and finally, a routine is applied that (i) restores feasibility to violated freight rate breakpoint constraints and (ii) determines cycle times on each link that minimize costs for the flows found in model X. The routine is described in procedure Resfeas below. The objective value of the feasible solution at an iteration (current upper bound or CUB) is a candidate to replace the incumbent upper bound (IUB). It does so if it is smaller in value than the IUB.

Procedure Resfeas

A.	Loop until all arcs have been processed:				
		Compute total flow on arc, and total weight sent on arc			
		(1) Set mincyccost=0, mincyc=0			
			Loop until all cycles are examined:		
				Compute total cost on arc assuming all flows use current cycle	
				If total cost < mincyccost, mincyccost=totalcost,mincyc=currentcycle	
			End cycle loop		
		(2) Set appfrtrate=0, Compute shipment weight=Total weight ∗ mincyc			
			Loop until applicable freight rate is found:		
				If rate minimum weight<shipmentweight<rate maximum weight then appfrtrate=currentfreightrate	
			End freight rate loop		
		(3) Convertallflowsonarcsothatcycle=mincyc,freightrate=appfrtrate			
	End arc loop				
B.	Compute upper				

	bound as sum of costs corresponding to feasible flows found in step A and fixed costs of facility and warehouse assignments				

4.5. Algorithm for the location–consolidation problem

The combination of the upper and lower bounding schemes described above provides an algorithm for the flow-consolidation problem. An outline of this algorithm follows, and an overview is provided in Fig. 3.

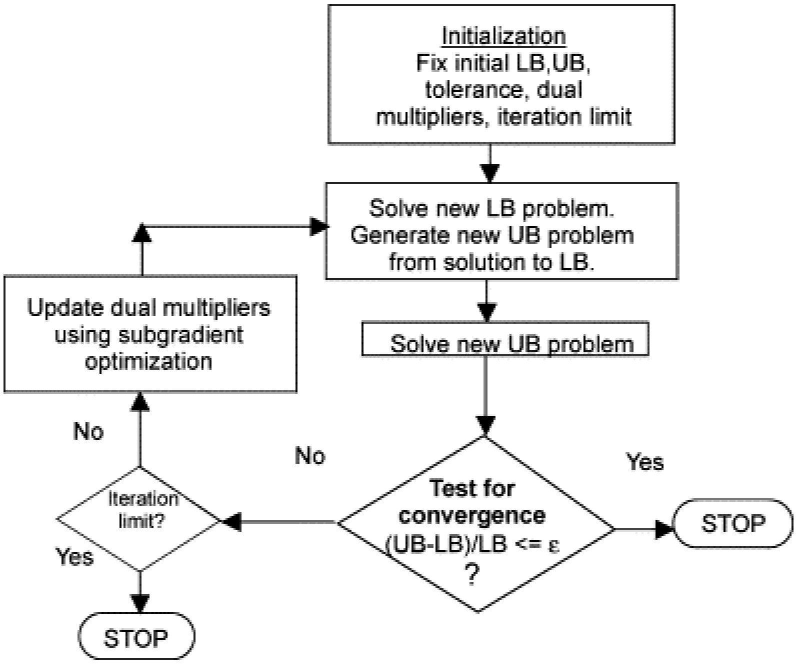

Fig. 3. Overview of lagrangian methodology.

Algorithm Loccons

A.	Initialize the dual multipliers to zero, and the incumbent lower bound, ILB, to −∝. Set a tolerance, ε, for convergence between upper and lower bounds, and fix an iteration limit. Set the value of the incumbent upper bound, IUB to +∝.
B.	Solve the lower bound problem. $LB=Z_1+Z_2+Z_3+Z_4+Z_5+Z_6$. If LB>ILB, then ILB=LB. Generate the upper bound, CUB, from the lower bound solution. If CUB<IUB,thenIUB=CUB.
C.	Check for convergence. If IUB−ILB)/IUB⩽ε, the solution is acceptable, therefore terminate. Otherwise, use subgradient optimization to update the multipliers as described below. If the number of iterations is less than the iteration limit, return to step B, otherwise terminate.

4.6. Updating of dual multipliers

The subgradients of the function $\varphi(\omega_{is}^{1},\omega_{ei}^{2},\mu_{fhn}^{1},\mu_{fhn}^{2},\lambda_{in}^{1},\lambda_{hn}^{2})$ are the following:

$sg\mu1_{fhn}=t_h\sum_{i\in I}X_{fhi}^{n}w_i-\beta_{(f+1)n}\alpha_{fhn}, sg\mu2_{fhn}=\beta_{fn}\alpha_{fhn}-t_h\sum_{i\in I}X_{fhi}w_i,$

$sg\lambda1_{in}=\sum_{f\in F}\sum_{h\in H}X_{fhi}^{n}-\sum_{h\in H}\pi_{hin}u_{in}, sg\lambda2_{hn}=\sum_{i\in I}\pi_{hin}-|I|\chi_{hn},$

$sg\omega1_{is}=\sum_{f\in F}\sum_{h\in H}\sum_{n\in s-}X_{fhi}^{n}-u_{is}\psi_s, sg\omega2_{ei}=\sum_{f\in F}\sum_{h\in H}\sum_{n\in e-}X_{fhi}^{n}-C_{ei}\phi_e.$

The step size at iteration *i* is given by $s^i=\delta^i(IUB-ILB)/||\eta^i||^2$, where δ^i is a scalar between 0 and 2, and η^i is the vector of subgradients at iteration *i*. Using this step size, convergence results for which are provided in Polyak [19], the multipliers are systematically updated in the following manner: Letting ϑ represent, in a general way, any of the six multipliers, with $sg\vartheta$ representing the corresponding subgradient, $\vartheta^{i+1}=\vartheta^i+s^i(sg\vartheta)$. The updated multipliers are used to solve the lower bound problem in the next iteration.

5. Computational testing

A computational experiment was conducted to assess the computational effectiveness of the Lagrangian and simulated annealing algorithms. This involved solving several sets of test problems of varying size, ranging from small to very large on a Pentium (400 megahertz) personal computer. Several model parameters are kept common to the different problem sets, in order to focus on the effectiveness of the algorithms as a function of problem size. In order to provide a better sense of performance of the algorithms, a number of smaller problems were solved to optimality by a process of total enumeration i.e., considering all the possible combinations. In addition, the performance of the simulated annealing algorithm was evaluated as a function of experimental parameters such as annealing speed and neighborhood size. These factors are discussed in greater detail in the section below.

5.1. Computational experiment

The solution methodologies for the location/consolidation problem were tested by solving 10 test problems for each of a number of configurations which vary by size from small to very large. The following parameters are common to the problem sets: (i) number of freight rates=5 (ii) number of items=5 (iii) number of cycle times=5 (iv) fixed costs of plants are uniformly distributed: U($200000, $20000) (v) fixed costs of warehouses are uniformly distributed: U($100000, $10000). Other distributions of costs, such as those for procurement, holding, and ordering costs, and transportation rates are also kept common to the various problem sets.

Early 'trial runs' of the simulated annealing methodology suggested the following: (i) the rate of cooling influenced the quality of the final solution (ii) the greatest improvements in solution value are experienced in the initial epochs, rather than the later ones, and that if the algorithm stalled at a particular temperature epoch, it generally did not improve until movement to a different epoch (iii) the choice of a neighborhood is important: if a neighboring solution is defined as one that is extremely close to an incumbent solution, then the possibility of 'getting stuck' in a particular neighborhood increases; on the other hand, neighboring solutions that are relatively distant from the incumbent solution may cause the algorithm to 'bounce around' continuously without improvement of the objective value. In order to explore the practical issues discussed above, the simulated annealing procedure included the following design parameters:

(a)	Two cooling schedules, 'fast' and 'slow' were tested on each problem. In the slow schedule, the temperature at each epoch following the initial epoch was set at 95% of the temperature at the previous epoch. The fast schedule used a corresponding cooling rate of 55%, i.e., 45% reduction at each epoch.
(b)	Three different neighborhood interchange options were tested: (i) option 1 in which 50% of the incumbent locations were replaced in a new solution (ii) option 2 in which 75% were replaced and (iii) option 3 in which 100% were replaced. Option 1 moves incrementally around the solution space, option 2 explores the solution space at a medium space, and option 3 moves relatively rapidly around the solution space.
(c)	The number of iterations at an epoch was determined in the following manner: the first epoch used 10 iterations, and the number of iterations in each of the subsequent epochs was 95% of the number in the previous epoch. In addition, processing at an epoch was curtailed if no solutions were accepted by the annealing algorithm in three consecutive iterations.

5.2. Computational results

The first set of results are for relatively small problems which can be solved to optimality by evaluating all possible combinations of open plants and warehouses. The purpose of solving these problems is to provide an idea of the relative proximity of the actual optimal objective values to those provided by the Lagrangian and simulated annealing algorithms. While the Lagrangian procedure provides a bound on the optimal solution, the difference between the best obtained and optimal objective values is often smaller than the bound. The computational result tables use the following notation:

(a)	‘#CL’=the number of candidate plant locations.
(b)	‘#CW’=the number of candidate warehouse locations.
(c)	‘#PL’=the permitted number of plants.
(d)	‘#PW’=the permitted number of warehouses.
(e)	‘ANS’=annealing speed, fast or slow.
(f)	‘NS’=neighborhood size, small, medium or large.
(g)	‘LCV%’=percent convergence between upper and lower bounds in the Lagrangian methodology.
(h)	‘LOP%’=for small problems, percent difference between the best objective value from the Lagrangian feasible solutions and the optimal objective value.
(i)	‘SOP%(a,b)’=for small problems, percent difference between the best objective value from the simulated annealing feasible solutions and the optimal objective value. The first percent applies when the annealing is run until 10% improvement is achieved, and the second percent applies when the annealing is restricted to the same time limit as the Lagrangian method.
(j)	‘LSEC’=computational seconds for Lagrangian methodology.
(k)	‘SSEC’=computational seconds for simulated annealing methodology.

On the basis of the results for small problems, it may be surmised that the Lagrangian methodology often provides best feasible solutions that are closer to the optimal solution than indicated by the Lagrangian bounds themselves. For the small problems, it also appears that the simulated annealing procedure generally provides solutions that are closer to the optimal solution, when required to run until at least 10% improvement in objective value is achieved. However, with a few exceptions, annealing is generally inferior when executed for the same time as the Lagrangian (Table 1).

Table 1. Small problem configurations and results

Set	**#CL**	**#PL**	**#CW**	**#PW**	**ANS**	**NS**	**LCV%**	**LOP%**	**SOP% (a,b)**	**LSEC**	**SSEC**
1.1	10	2	4	2	0.95	1.00	2.05	1.55	0.61, 1.85	0.5	1.0
1.2	10	2	4	2	0.55	1.00	1.04	0.85	0.26, 1.48	0.20	1.0
1.3	10	2	4	2	0.95	0.75	2.12	1.61	0.66, 0.66	1.0	1.0
1.4	10	2	4	2	0.55	0.75	1.03	0.88	0.24, 2.09	0.2	1.0
1.5	10	2	4	2	0.95	0.50	1.13	0.98	0.65, 1.49	0.5	0.9
1.6	10	2	4	2	0.55	0.50	1.43	1.30	0.35, 0.35	1.0	1.0
1.7	10	4	6	4	0.95	1.00	1.51	1.39	0.75, 2.13	0.8	2.0
1.8	10	4	6	4	0.55	1.00	1.82	1.66	0.94, 1.92	1.0	2.0
1.9	10	4	6	4	0.95	0.75	1.38	0.87	1.34, 2.12	1.0	2.0
1.10	10	4	6	4	0.55	0.75	0.63	0.42	0.52, 4.17	0.20	2.0
1.11	10	4	6	4	0.95	0.50	0.73	1.15	0.98, 3.14	0.5	2.0
1.12	10	4	6	4	0.55	0.50	1.50	1.23	1.67, 3.13	0.7	2.0

To facilitate analysis, further results are shown for medium-sized and large problems, which are too large to solve to optimality. These results are:

(a)	‘L<S%(a)’=percent difference between the best objective values from the Lagrangian and simulated annealing procedures for problems in which the Lagrangian solution is superior (lower). The number of problems (out of a set of ten) where the Lagrangian approach provided better solutions is shown in parentheses. Set (a) applies to normal runs where parameters require the annealing procedure to

	achieve at least 10% improvement relative to the initial solution (or stop at 1000 s). Set (b) applies where the annealing procedure is constrained to the same time as the Lagrangian method.
(b)	‘S<L%’=percent difference between the best objective values from the simulated annealing and Lagrangian procedures for problems in which the simulated annealing solution is superior (lower). The number of problems (out of a set of 10) where the annealing procedure provides better solutions is shown in parentheses.

The computational results for the second set of problems are meant to provide an idea of the comparative performance of the two methodologies with respect to objective function value and computational speed. The results suggest that, with very few exceptions, (i) the Lagrangian methodology finds superior (with lower objective function value) solutions and (ii) the Lagrangian approach is quicker than the simulated annealing procedure, with the time differential widening with problem size.

The finding above appears to apply regardless of the ‘fine-tuning’ of the annealing procedure by altering the annealing speed and neighborhood search scheme, suggesting that the Lagrangian procedure should be preferred for medium-sized and large problems (Table 2). An examination of the ‘*L*<*S*%’ and ‘*S*<*L*%’ cells further emphasizes this -the average percentage differences between the best objective function values obtained from the two methodologies are in the vicinity of 5–6% when the Lagrangian procedure is superior, and in the neighborhood of 0.5–3% in the few cases where the annealing procedure finds better solutions. It is also clear from the ‘*L*<*S*% (b)’ results that the annealing procedure is not competitive when constrained to the same time as the Lagrangian method.

Table 2. Medium and large problem configurations and results

Set	#CL	#PL	#CW	#PW	ANS	NS	LCV%	*L*<*S*% (a)	*L*<*S*% (b)	*S*<*L*%	LSEC	SSEC
2.1	25	5	5	2	0.95	1.00	2.73	4.43(10)	5.41(10)	—	3	5
2.2.	25	5	5	2	0.55	1.00	2.70	4.86(8)	5.09(10)	3.34(2)	2	5
2.3	25	5	5	2	0.95	0.75	3.49	1.85(8)	6.48(10)	0.38(2)	3	5
2.4	25	5	5	2	0.55	0.75	2.96	1.63(4)	2.73(10)	0.62(6)	2	5
2.5	25	5	5	2	0.95	0.50	3.43	2.10(6)	4.88(10)	0.73(4)	3	5
2.6	25	5	5	2	0.55	0.50	4.00	2.73(6)	6.06(10)	3.81(4)	3	5
2.7	50	10	10	4	0.95	1.00	2.75	5.29(10)	6.96(10)	—	19	45
2.8	50	10	10	4	0.55	1.00	3.48	4.86(10)	6.85(10)	—	22	45
2.9	50	10	10	4	0.95	0.75	3.02	5.63(10)	6.36(10)	—	23	44
2.10	50	10	10	4	0.55	0.75	3.71	4.53(10)	4.87(10)	—	20	44
2.11	50	10	10	4	0.95	0.50	3.18	5.87(10)	6.19(10)	—	22	43
2.12	50	10	10	4	0.55	0.50	4.41	2.84(10)	5.50(10)	—	21	43
2.13	100	20	20	8	0.95	1.00	4.32	4.24(10)	6.62(10)	—	149	324
2.14	100	20	20	8	0.55	1.00	4.01	5.65(10)	7.56(10)	—	152	294
2.15	100	20	20	8	0.95	0.75	3.67	6.66(10)	7.36(10)	—	150	321
2.16	100	20	20	8	0.55	0.75	4.81	4.15(10)	6.70(10)	—	148	329
2.17	100	20	20	8	0.95	0.50	3.68	5.88(10)	7.58(10)	—	150	328
2.18	100	20	20	8	0.55	0.50	4.04	5.18(10)	7.75(10)	—	152	324

Within the annealing procedure, performance does vary between combinations of annealing speed and neighborhood size. However, a pattern with regard to performance is hard to detect, suggesting that experimentation will be required if an annealing methodology is applied to any particular instance of the location–consolidation problem. In any event, the current results indicate that there is a high probability that a Lagrangian methodology will either out-perform or be very competitive with a simulated annealing approach to this problem.

6. Conclusion

This paper is concerned with the exposition and solution of an important problem in logistics, referred to as the location–consolidation problem. It involves simultaneously determining facility locations, flows, shipment compositions, and shipment cycle times in a multicommodity, multiple plant and multiple warehouse environment. The problem is combinatorial in nature (NP-complete) and extremely difficult to solve. Two competing methodologies, based on simulated annealing and Lagrangian relaxation, respectively, are provided and compared in an extensive computational experiment. The Lagrangian methodology provides tight bounds and out-performs the annealing procedure for medium-sized and large problems, with respect to both computational time and solution quality. The annealing procedure provides better solutions for small problems, when allowed to run for somewhat longer than the Lagrangian method.

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