Time-Optimal Control of Discrete-Time Systems with Known Waveform Disturbances

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TIME-OPTIMAL CONTROL OF DISCRETE-TIME SYSTEMS WITH KNOWN WAVEFORM DISTURBANCES

by

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ABSTRACT

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In this thesis, a discrete-time observer based disturbance accommodation controller is designed that is capable of minimizing the effect of disturbances with known waveform in both the system state and measurement as fast as possible while also driving the state to zero. This control is achieved by designing a single control input to accommodate disturbances in both the system state and measurement. For controller design, the state and measurement equations are augmented, and a least squares minimization technique is used to find a control input that drives the system state and measurement to zero, guaranteeing deadbeat response. During the design it is assumed that all system and disturbance state variables are available for feedback. When this is not, an observer is needed.

When using a deadbeat controller, the only option for the observer is to also be deadbeat. Two types of deadbeat observers are used in this work: full-order and reduced-order. The full-order observer generates estimates for both the system and disturbance state variables (measureable or not) and driving the estimation error to zero. For a faster time response, a reduced-order deadbeat observer was then designed. Reduced-order observers have a faster response because a reduced-order observer only constructs estimates for the un-measureable system and disturbance state variables.

As an extension, a new model for the control input was introduced for the case when the feed-forward term in the measurement was not present. This involved using a

so-called ‘pseudo-output’ that allows the controller to indirectly minimize the effect of the disturbance in the measurement.

Simulations show that when this control scheme is used, the system state and measurement are driven to zero when no disturbance. When disturbances are present, their effects are minimized. In all cases control action is achieved in the appropriate number of time steps for the given system.

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1 INTRODUCTION

Control theory has been around for many years and, as one can expect, many researchers have expanded on it, some taking different paths. The paths have created two main types of control theory: classical and modern. One specific researcher, C. D. Johnson, used the modeling technique in modern control theory to develop a control method that greatly diminishes the effect of external disturbances with known waveform structures in real-time. The technique he developed is referred to as disturbance accommodating control theory [1]. The development of the controller proposed in this thesis utilizes this method. To begin, it is important to understand the general concepts of the tools that make this controller possible.

1.1 Control Theory

As mentioned, there are two main types of control theory: classical and modern. Classical control theory is the foundation of many controllers in industry and involves developing such controllers as PI, PD, and PID controllers. These types of control use knowledge of the transfer function of a system to design the controller. Classical control is a very useful and effective method of control but, due to the input-output nature of the models used, there is no knowledge of what is happening in the internal stages of the system. A type of control that contains information about the internal state variables is modern control theory. Modern control theory uses the system model's differential equation and defines new variables that allow the model to be described by a set of coupled first order differential equations. These new variables are the intermediate stages of the system, defined as ‘state variables’ and these variables are formed into a vector creating the state space description of a system. There are various types of controllers that have been designed based on this theory including ‘state-feedback control’ that, as the name implies, utilizes feedback of the state variables.

There are many objectives in controlling a system, some of which include stabilizing, driving a system to an operating point, optimizing a performance criterion, or driving a system to its equilibrium point. Whatever the reason might be, there may be some external forces that may disrupt the system. These disruptions are known as disturbances and are another reason to apply control to a system. One type of controller that has been proven to work effectively for disturbances that have known waveform-types has been briefly mentioned already in this introduction and is one developed by C. D. Johnson. The controller to handle these disturbances is known as a disturbance accommodation controller (DAC).

DAC is a technique that makes use of modern control theory. A state space description for the system is created along with a state space description of the waveform-type of the disturbance. When developing the control input, it is assumed that all state variables and disturbances are available (measurable) which may not be the case. In the case where not all of the state variables and disturbances are measured, the actual variables are replaced by the estimated values that are generated from an observer. In the end, a DAC has the ability to meet performance specifications for systems with disturbances of unknown magnitude and arrival time.

1.2 Observers

As previously mentioned, this DAC needs a tool that will generate estimates of the un-measurable state variables and disturbances. Over the years, many tools that can achieve this have been developed. In this thesis, the Luenberger observer is used [16]. This type of an observer creates an estimate of the state based on the given system and a term proportional to a defined output error.

1.3 Deadbeat Performance

In the design of a state feedback controller, different performance criteria can be met by designing the controller gains such that the gains set the eigenvalues of the controlled system to desired values. The controller proposed in this thesis will make use of deadbeat performance that requires the eigenvalues are set to zero. Deadbeat performance in discrete-time systems means the system state or output will reach the desired value in n-steps for an n-dimensional system. Due to the rapid response by the controller, large overshoot in the state variables and in the measurement can be expected with deadbeat controllers because of large control inputs.

When an observer is used in combination with a controller, the observer needs to have a faster response than the controller so the system has accurate estimates before the control action is finished. However, for a deadbeat controller, the only option is to use a deadbeat observer because this is the fastest response.

1.4 Previous Work involving the use of DAC theory

In 1971, C. D. Johnson introduced accommodation of external disturbances in linear regulator and servomechanism problems [2] by making use of DAC theory. He discussed the modeling method for disturbances with known waveform structures and then discussed the different modes of accommodation: absorption, minimization and maximum utilization. In an overview paper on DAC theory [1], he stated, "Disturbance-Accommodating Control Theory is a relatively new technique of modern control which enables one to design feedback controllers which can maintain performance specifications in the face of uncertain, persistent acting external disturbances." He then gave a more detailed explanation of how to develop the disturbance model. He also went through the details of the different modes of accommodation where the absorption mode of accommodation uses a control input that completely cancels out the effect of the disturbance, the minimization mode of accommodation minimizes the effect of the disturbance in some specified sense, and the utilization mode of accommodation makes use of the disturbance to assist the controller in achieving a desired control task. He discussed different applications such as controlling a low-power laser designator device mounted in a helicopter, chemical process control, and control of machine-tool chatter. He also discussed different extensions, one of which is accommodation of modeling errors. He mentioned the disturbances experienced in aircraft maneuvering and navigation are suitable for DAC theory because they have "a high degree of waveform structure."

In 1986, K. D. Reinig and A. A. Desrochers applied Johnson's DAC theory to rotating mechanical systems that experience vibrations when operating at a constant speed near resonance [3]. They used the concepts of DAC theory in the continuous time domain and applied them similarly to the frequency domain to successfully reduce the effect of the vibrations on the systems.

Also in 1986, E. Yaz extended Johnson's work to disturbances that have waveforms with nonlinear models applied to systems that also have nonlinear system models [4]. He made use of the minimization mode of accommodation when deriving the desired control input.

In 1989, T. W. Martin and E. Yaz generalized the discrete-time version of the work in [5]. They also compared the adaptive method of DAC with the nonlinear method of DAC and found that the adaptive method had better results if all of the design requirements could be met [6]. In 1990, Martin and Yaz gave conditions under which disturbance models with unknown parameters could be handled indicating the robustness property of DAC [7]. In 1992, they introduced a disturbance accommodation controller for continuous-time systems with various forms of nonlinearities [8].

In 1992, A. Azemi and E. Yaz extended DAC theory to discrete-time nonlinear stochastic systems. In their work, they introduced a 'pseudo-output' that consisted of the current measurement and the past input [9]. By adding this pseudo-output to their control input equation, they accommodated disturbances not only in the system but also in the output. By accommodating disturbances in the output, a better estimate of the state can be achieved for better overall performance compared to control without the pseudo-output included.

In 2000, H. Kim and Y. Kim developed a discrete-time controller for a system with low frequency disturbances and unknowns in the system matrices [10]. They applied their controller to a satellite altitude control problem and showed desirable results.

In 2001, I. Tshiofwe, et al. developed an LMI based disturbance reduction controller for systems with multiple delays in the state and in the input [11]. They reconstructed the state through a multiple time-delay observer that was designed using an LMI. Then they used part of the control signal to actively minimize the disturbance.

In 2003, I. Tshiofwe, et al. introduced the use of a reduced order observer in the development of a DAC [12]. They also used a linear matrix inequality (LMI) technique to design their observer. For the control input, they used the minimization mode of accommodation. In their paper, they showed successful results of accommodation of the disturbance in the state variables.

In 2003, K. Stol and M. Balas applied DAC theory to blade load mitigation in wind turbines with a periodic disturbance [13]. They derived the nonlinear model, linearized it, and developed three different types of controllers to accommodate the disturbances in their system. These controllers were a time-periodic DAC that used optimal periodic control techniques, time-invariant DAC that used a time-invariant version of the plant in the design of the controller, and a PID controller. Their results showed the periodic DAC was superior to the others.

In 2007, Z. Gao, T. Breikin, and H. Wang introduced a model that included a feed-forward term in the measurement equation that allowed for direct correction of the measurement when there were disturbances present [14]. As mentioned previously, if there are disturbances in the measurement, it is desirable to accommodate these to achieve better estimates from the observer. They made use of a proportional and integral observer for their estimates and used a control input that satisfied a given cost function.

DAC theory has come a long way over the years. There is still plenty of room for more extensions and new applications are always arising. DAC is an effective way to minimize or utilize the disturbances present in many systems as long as the disturbance has a waveform structure.

1.5 Scope of This Work and Main Contributions

This thesis proposes to expand on DAC theory applying it to linear time-invariant discrete-time systems that have disturbances in both the system state and in the measurement. The DAC that is proposed is time-optimal, reaching a minimal value as fast as possible. To achieve this, a deadbeat controller is first designed with the assumption that all variables are known. This controller will have two parts, one that will control the system state and one that will minimize the effect of the disturbance. The controller design is then followed by the design of both a full-order and reduced-order deadbeat observer where the estimates from the observer will be used in the controller. Furthermore, system conditions for the developed control technique are derived to allow a user to test the given system and decide if the technique will produce desirable results. Lastly, an extension is also proposed for systems with no control term in the measurement. The extension introduces a pseudo-output that, when used in the controller, allows for control and accommodation of disturbances in the measurement.

1.6 Thesis Organization

This thesis is comprised of four chapters. Chapter 2 consists of a derivation of the deadbeat controller, full-order deadbeat observer, and reduced-order deadbeat observer that have been proposed. Chapter 3 contains six case studies of two single-input, single-output (SISO) time-invariant systems with a variety of disturbances that have known waveform structures and both full-order and reduced-order observers are analyzed. Chapter 4 discusses the conditions for systems for the proposed DAC to work. It also includes an extension on the proposed DAC when there is no input in the measurement equation and conditions for this controller to work. Chapter 5 is a summary of the previous chapters and suggestions for future work.

2 PROPOSED CONTROL TECHNIQUE

DACs are an effective way to minimize or eliminate the effects of disturbances on a system and have been in use for many years. There are different design methods that are covered in [1] as mentioned in the previous chapter. In this work, the "disturbance-minimization mode of accommodation" [1] method is used. In previous work, this method has been used to design DACs for more restricted systems, i.e. no feed-forward (feed-through) term or no disturbance in the measurement (output). By introducing a more general system model, however, this technique can be applied to a wider range of systems.

In this chapter, an observer based deadbeat DAC design technique for discrete-time systems with known waveform-type unknown disturbances will be developed. By deriving this technique for discrete-time systems using the deadbeat concept, a desirable response will be achieved in minimum time. Two different types of closed-loop deadbeat observers, full-order and reduced-order, will be designed. Both observers give similar results that will be compared and analyzed in chapter 3. The design technique involves solving for the control and observer gains by using the canonical forms of the system matrices and then transforming the gains back to the original system's form. By doing this, it can be guaranteed that the DAC will be deadbeat.

2.1 Model

Consider the following discrete-time linear time invariant system:

$$x_{k+1} = Ax_k + Bu_k + Fw_k \tag{2.1a}$$

$$y_k = C_1 x_k + Du_k + G_1 w_k \tag{2.1b}$$

$$w_{k+1} = Ew_k + \sigma_k \tag{2.1c}$$

where $x_k \in \Re^{nx}$ is the state, $u_k \in \Re$ is the applied control input, $w_k \in \Re^{nw}$ is the state of the disturbance, $y_k \in \Re$ is the measured output, σ_k is an unknown impulse sequence which accounts for the system and measurement disturbances and occurs in a "sparsely populated" manner [1], and A, B, C_1, D, E, F, G_1 are real matrices of appropriate dimensions. The waveform of the disturbance is known (i.e. step, ramp, sinusoidal, etc.); however, the magnitude, arrival time, and duration of the disturbances are unknown.

As discussed in chapter 1, DACs are able to accommodate for these known waveform-type unknown disturbances. A deadbeat discrete-time controller will be designed to accommodate the disturbances as quickly as possible.

2.2 Controller

Designing controllers in discrete-time has many benefits. One of these benefits is the ability to drive a system to zero in a finite amount of time. Another benefit is the ability for the state to reach zero in n-steps for an n-dimensional system. The type of controller that can achieve optimal time response is called a deadbeat controller and is the type of controller that will be used in this DAC.

First, the state (2.1a) and output (2.1b) equations, are augmented to create a new system,

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} F \\ G_1 \end{bmatrix} w_k + \begin{bmatrix} B \\ D \end{bmatrix} u_k . \tag{2.2}$$

By augmenting (2.1a) and (2.1b), a single control input can be designed to provide desired control to the system while accommodating the disturbance. By designing this controller to be deadbeat, it will accommodate the disturbances as quickly as possible. The control input is considered to have two parts,

$$u_k = u_k^c + u_k^d . \tag{2.3}$$

One of these parts is a state control input, $u_k^c = L\begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix}$, which will drive the state to zero, and the other part is a disturbance accommodation input, $u_k^d = L_d w_k$, which will minimize the effect of the disturbance. This defined control input is then substituted for u_k in (2.2),

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L \right) \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \left(\begin{bmatrix} F \\ G_1 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L_d \right) w_k \tag{2.4a}$$

which is of the form

$$\chi_{k+1} = \left(A_c + B_c L \right) \chi_k + \left(\Phi + B_c L_d \right) w_k \quad \text{such that } \chi_k \in \Re^{n_x+1} . \tag{2.4b}$$

where A_c, B_c, Φ, and χ_k are obtained by matching the two equations.

To analyze the evolution of this system, the convolution summation for this system is considered,

$$\chi_k = \left(A_c + B_c L\right)^k \chi_0 + \sum_{i=0}^{k-1} \left(\Phi + B_c L_d\right)^{k-i-1} w_i \ . \tag{2.5}$$

By making use of the Cayley-Hamilton theorem which states all matrices must satisfy their characteristic equation, it is seen that the evolution of the state, χ_k, depends on the eigenvalues of $(A_c + B_c L)$ and the summation term. By choosing the eigenvalues of $(A_c + B_c L)$ to be zero (for this to be a deadbeat controller) and minimizing the norm of $\left(\Phi + B_c L_d\right)$, when designing the gains L and L_d, the state variables will reach zero and the effect of the disturbances will be minimized in $\left(n_x + 1\right)$-steps.

The system pair (A_c, B_c) must be controllable for there to exist a controller gain L that will allow the eigenvalues to be placed anywhere inside the unit circle. When the augmented system is controllable, a least squares minimization technique is performed. The system dynamic equation is minimized with respect to the controller gains,

$$\min_{L \text{ and } L_d} \left\| \left(A_c + B_c L\right) \chi_k + \left(\Phi + B_c L_d\right) w_k \right\| . \tag{2.6}$$

A completion of the squares method was used to solve for the control gains:

$$\begin{aligned}
\left(A_c + B_c L\right)^T \left(A_c + B_c L\right) &= A_c^T A_c + A_c^T B_c L + L^T B_c^T A_c + L^T B_c^T B_c L \\
&= A_c^T A_c + \left(L + L^*\right)^T B_c^T B_c \left(L + L^*\right) - L^{*T} B_c^T B_c L^* \\
&\rightarrow L^T B_c^T B_c L^* = L^T B_c^T A_c \\
&\rightarrow L^* = \left(B_c^T B_c\right)^{-1} B_c^T A_c \\
&\rightarrow L = -L^* = -\left(B_c^T B_c\right)^{-1} B_c^T A_c \\
L_d &= -B_c^{\dagger} A_c
\end{aligned}$$

and

$$\begin{aligned}
\left(\Phi + B_c L_d\right)^T \left(\Phi + B_c L_d\right) &= \Phi^T \Phi + \Phi^T B_c L_d + L_d^T B_c^T \Phi + L_d^T B_c^T B_c L_d \\
&= \Phi^T \Phi + \left(L_d + L_d^*\right)^T B_c^T B_c \left(L_d + L_d^*\right) - L_d^{*T} B_c^T B_c L_d^* \\
&\rightarrow L_d^T B_c^T B_c L_d^* = L_d^T B_c^T \Phi \\
&\rightarrow L_d^* = \left(B_c^T B_c\right)^{-1} B_c^T \Phi \\
&\rightarrow L_d = -L_d^* = -\left(B_c^T B_c\right)^{-1} B_c^T \Phi \\
L_d &= -B_c^{\dagger} \Phi
\end{aligned}$$

where $A^{\dagger}$ denotes the Moore-Penrose pseudo inverse of A [16]. It can be seen than the following control gains will satisfy the minimization condition,

$$L = -B_c^{\dagger} A_c \equiv -\begin{bmatrix} B \\ D \end{bmatrix}^{\dagger} \begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} \tag{2.7a}$$

and

$$L_d = -B_c^{\dagger} \Phi \equiv -\begin{bmatrix} B \\ D \end{bmatrix}^{\dagger} \begin{bmatrix} F \\ G_1 \end{bmatrix}. \tag{2.7b}$$

For this controller to be deadbeat, the eigenvalues of $(A_c + B_c L)$ must be equal to zero not just a minimum value. This is condition guaranteed by designing a gain $\bar{L}$ using the controllable canonical forms [15] of A_c and B_c where the eigenvalues of $(\bar{A}_c + \bar{B}_c \bar{L})$ are always zero. ($\bar{A}$ denotes the canonical form of A.) This result is demonstrated for a single input, n-dimensional system as follows:

$$
\begin{aligned}
&\bar{A}_c + \bar{B}_c \bar{L} \\
&= \bar{A}_c - \bar{B}_c (\bar{B}_c^T \bar{B}_c)^{-1} \bar{B}_c^T \bar{A}_c \\
&= (I - \bar{B}_c (\bar{B}_c^T \bar{B}_c)^{-1} \bar{B}_c^T) \bar{A}_c \\
&= \left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \right) \begin{bmatrix} -a_1 \cdots\cdots\cdots -a_n \\ 1 \quad 0 \cdots\cdots \quad 0 \\ 0 \quad \ddots \ddots \ddots \quad \vdots \\ \vdots \quad \ddots \ddots \ddots \quad \vdots \\ 0 \quad \cdots 0 \quad 1 \quad 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 1 & 0 \end{bmatrix} .
\end{aligned}
$$

Therefore, $\lambda_i(\bar{A}_c + \bar{B}_c \bar{L}) = 0$ for $i = 1, 2, \cdots, n$. The gain calculated from the canonical forms of A_c and B_c needs to be transformed to the original form of the system using the transformation technique [15]:

$$\bar{L} = -\left(\bar{B}_c\right)^{\dagger} \bar{A}_c \tag{2.8a}$$

$$L = \bar{L}\bar{W}_c W_c^{-1} \tag{2.8b}$$

where

$$W_c = \begin{bmatrix} B_c & A_c B_c & \cdots & A_c^{\,n-2} B_c & A_c^{\,n-1} B_c \end{bmatrix} \tag{2.9}$$

and

$$\bar{W}_c = \begin{bmatrix} \bar{B}_c & \bar{A}_c \bar{B}_c & \cdots & \bar{A}_c^{\,n-2} \bar{B}_c & \bar{A}_c^{\,n-1} \bar{B}_c \end{bmatrix}. \tag{2.10}$$

This controller is designed assuming x_k and w_k are known; however in most cases, not all of the state variables are known (or measurable) and the disturbances are unknown. To solve this problem a closed-loop deadbeat observer is designed that will estimate these unknowns in a minimal amount of time.

2.3 Closed-Loop Observers

2.3.1 Full-Order Observer

Observers are used to take information from the control input and the measurement to construct an estimate of the internal state variables. A commonly used observer is one proposed by D. G. Luenberger [16].

To begin the discussion of the Luenberger observer, assume an augmented system model of the unknown variables:

$$\begin{bmatrix} x_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & E \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \tag{2.11a}$$

$$y_k = \begin{bmatrix} C_1 & G_1 \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + Du_k \tag{2.11b}$$

which is can be rewritten as

$$X_{k+1} = A_{fo} X_k + B_{fo} u_k \quad \text{such that } X_k \in \Re^{n_x + n_w} \tag{2.11c}$$

$$y_k = C_{fo} X_k + Du_k \; . \tag{2.11d}$$

where A_{fo}, B_{fo}, C_{fo}, and X_k are obtained by matching (2.11a) and (2.11b) with (2.11c) and (2,11d), respectively.

Luenberger begins by defining an error signal between the actual output and the estimate of the output,

$$\varepsilon_k = y_k - \hat{y}_k \tag{2.12}$$

where

$$\hat{y}_k = C_{fo} \hat{X}_k + Du_k \; . \tag{2.13}$$

Noting that the input, u_k, is known in both the actual and estimated output, this term will cancel itself, and the output error equation can be rewritten as

$$\varepsilon_k = C_{fo} X_k - C_{fo} \hat{X}_k = C_{fo} (X_k - \hat{X}_k) \tag{2.14}$$

where the state error is

$$e_k = X_k - \hat{X}_k \; . \tag{2.15}$$

Now that the state error has been defined, it is desirable to analyze the evolution of this error,

$$e_{k+1} = \mathrm{X}_{k+1} - \hat{\mathrm{X}}_{k+1}. \tag{2.16}$$

The dynamic equation of the state estimate is obtained by using the estimate of the state plus a term proportional to the output error,

$$\begin{aligned} \hat{\mathrm{X}}_{k+1} &= A_{fo}\hat{\mathrm{X}}_k + B_{fo}u_k + K\varepsilon_k \\ &= A_{fo}\hat{\mathrm{X}}_k + B_{fo}u_k + KC_{fo}(\mathrm{X}_k - \hat{\mathrm{X}}_k) \end{aligned}. \tag{2.17}$$

Now that the dynamic equations for X_{k+1} and $\hat{\mathrm{X}}_{k+1}$ are available, the dynamic equation for the state error is written as

$$\begin{aligned} e_{k+1} &= A_{fo}\mathrm{X}_k + B_{fo}u_k - A_{fo}\hat{\mathrm{X}}_k - B_{fo}u_k - KC_{fo}(\mathrm{X}_k - \hat{\mathrm{X}}_k) \\ &= A_{fo}(\mathrm{X}_k - \hat{\mathrm{X}}_k) - KC_{fo}(\mathrm{X}_k - \hat{\mathrm{X}}_k) \\ &= (A_{fo} - KC_{fo})e_k \end{aligned}. \tag{2.18}$$

Consider the convolution summation solution for the state estimation error,

$$e_k = (A_{fo} - KC_{fo})^k e_0. \tag{2.19}$$

Again, by making use of the Cayley-Hamilton theorem, it is seen that the evolution of the error depends on the eigenvalues of $(A_{fo} - KC_{fo})$. The system pair (A_{fo}, C_{fo}) must be observable for there to exist an observer gain, K, which will allow the eigenvalues to be placed anywhere inside the unit circle. By choosing these eigenvalues to be zero, the error will reach zero in $(n_x + n_w)$ steps. A least squares minimization of the error system matrix over K is performed by completion of the square:

$$
\begin{aligned}
&\min_K \left\{(A_{fo} - KC_{fo})(A_{fo} - KC_{fo})^T\right\} = A_{fo}A_{fo}^T - A_{fo}C_{fo}^T K^T - KC_{fo}A_{fo}^T + KC_{fo}C_{fo}^T K^T \\
&= A_{fo}A_{fo}^T + (K - K^*)C_{fo}C_{fo}^T (K - K^*)^T - \bar{K}^* C_{fo}C_{fo}^T K^{*T} + K^* C_{fo}C_{fo}^T K^T = A_{fo}C_{fo}^T K^T \\
&\rightarrow K^* C_{fo}C_{fo}^T = A_{fo}C_{fo}^T \\
&\rightarrow K^* = A_{fo}C_{fo}^T (C_{fo}C_{fo}^T)^\dagger
\end{aligned}
$$

$$
K = K^* = A_{fo}C_{fo}{}^\dagger . \tag{2.20}
$$

Since (A_{fo}, C_{fo}) is observable, the observable canonical forms [15] of A_{fo} and C_{fo} can be used. Similar to the controller, by finding the gain $\bar{K}$ (which is K solved for using $\bar{A}_{fo}$ and $\bar{C}_{fo}$, the eigenvalues of $(\bar{A}_{fo} - \bar{K}\bar{C}_{fo})$ are guaranteed to be zero. This is shown on the following page for a single output and n-dimensional system:

$$\bar{A}_{fo} - \bar{K}\bar{C}_{fo} = \bar{A}_{fo} - \bar{A}_{fo}\bar{C}_{fo}{}^{\dagger}\bar{C}_{fo} = \bar{A}_{fo}(I - \bar{C}_{fo}{}^{\dagger}\bar{C}_{fo})$$

$$= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^{\dagger} \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} .$$

$$= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

Therefore, $\lambda_i(\bar{A}_{fo} - \bar{K}\bar{C}_{fo}) = 0$ for $i = 1, 2, \cdots, n$. The observer gain will then need to be transformed back into the original system's form by use of a transformation technique [15],

$$\bar{K} = \bar{A}_{fo}\bar{C}_{fo}{}^{\dagger} \tag{2.21a}$$

$$K = W_o^{-1}\bar{W}_o\bar{K} \tag{2.21b}$$

where

$$W_o = \begin{bmatrix} C_{fo} \\ C_{fo} A_{fo} \\ \vdots \\ C_{fo} A_{fo}{}^{n-2} \\ C_{fo} A_{fo}{}^{n-1} \end{bmatrix} \tag{2.22}$$

and

$$\bar{W}_o = \begin{bmatrix} \bar{C}_{fo} \\ \bar{C}_{fo} \bar{A}_{fo} \\ \vdots \\ \bar{C}_{fo} \bar{A}_{fo}{}^{n-2} \\ \bar{C}_{fo} \bar{A}_{fo}{}^{n-1} \end{bmatrix}. \tag{2.23}$$

If the system has state information in the measurement, the full-order observer is redundant because it reproduces all state variables not just the ones that are unknown. Also, the calculations of a larger order system have a longer calculation time in a digital controller than a smaller order system because there is more work to be done in the processor with the larger order system. One method of decreasing the order of the observer is to use a reduced-order observer that is designed to reconstruct only the unknown state variables, lowering the order of the augmented system created for the observer design and therefore decreasing the calculation time. A reduced-order observer will now be designed for this system using similar concepts as the full-order observer.

2.3.2 Reduced-Order Observer

In general, the order of a reduced-order observer is equal to the order of the full-order observer minus the order of the measurement. The technique in this work is derived for single input single output (SISO) systems; therefore, by using a reduced-order deadbeat observer opposed to the full-order deadbeat observer, the response time will be reduced by one time sample. Using the design technique for a Luenberger reduced-order observer in [15] as a basis, a composite vector containing the system and disturbance state is defined as

$$z_k = C_2 x_k + G_2 w_k \tag{2.24}$$

and is used to augment the measurement (2.1b) as

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} C_1 & G_1 \\ C_2 & G_2 \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + \begin{bmatrix} D \\ 0 \end{bmatrix} u_k \,. \tag{2.25}$$

Similar to the technique in [7], the matrix $\begin{bmatrix} C_1 & G_1 \end{bmatrix}$ must be of full rank and the matrices C_2 and G_2 are chosen such that $\begin{bmatrix} C_1 & G_1 \\ C_2 & G_2 \end{bmatrix}$ is invertible and $\begin{bmatrix} C_1 & G_1 \\ C_2 & G_2 \end{bmatrix}^{-1} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$, where $\boldsymbol{\Omega}_{11}$ has the dimension n_x x n_y and $\boldsymbol{\Omega}_{22}$ has the dimension n_w x (n_x+n_w-1).

Keeping in mind that the goal of the observer is to reconstruct all unmeasureable state variables and disturbances, (2.25) is rearranged to solve for the unknown variables in terms of y_k and z_k,

$$\begin{bmatrix} x_k \\ w_k \end{bmatrix} = \begin{bmatrix} C_1 & G_1 \\ C_2 & G_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ z_k \end{bmatrix} - \begin{bmatrix} D \\ 0 \end{bmatrix} u_k \right), \tag{2.26a}$$

where y_k is available but z_k is not. The vector z_k consists of the estimated state and disturbance variables given by the observer and (2.26a) is rewritten as

$$\begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} = \begin{bmatrix} C_1 & G_1 \\ C_2 & G_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} - \begin{bmatrix} D \\ 0 \end{bmatrix} u_k \right), \tag{2.26b}$$

where $\hat{x}_k$ denotes the estimate of x_k and $\hat{w}_k$ denotes the estimate of w_k.

The error between the actual and estimated state variables in terms of z_k is

$$\begin{aligned} e_k^r &= z_k - \hat{z}_k \\ &= C_2 x_k + G_2 w_k - \hat{z}_k \end{aligned} \tag{2.27}$$

and the error dynamic equation is

$$e_{k+1}^r = C_2 x_{k+1} + G_2 w_{k+1} - \hat{z}_{k+1}. \tag{2.28}$$

The update equation for the vector $\hat{z}_k$ is given by an extension on the general reduced order observer dynamic equation [15],

$$\hat{z}_{k+1} = K_1 \hat{z}_k + K_2 y_k + K_3 y_{k+1} + K_4 u_k + K_5 u_{k+1} \tag{2.29}$$

where K_1 through K_5 are the reduced-order observer gains. After substitution of $\hat{z}_k$, y_k, and y_{k+1} into (2.29) and (2.29) is substituted into (2.28), the error dynamic equation can be written as

$$\begin{aligned} e_{k+1}^r &= K_1 e_k^r \\ &+ (C_2 A - K_1 C_2 - K_2 C_1 - K_3 C_1 A) x_k \\ &+ (C_2 B - K_2 D - K_3 C_1 B - K_4) u_k \\ &+ (C_2 F + G_2 E - K_1 G_2 - K_2 G_1 - K_3 (C_1 F + G_1 E)) w_k \\ &+ (-K_3 D - K_5) u_{k+1} \end{aligned} . \tag{2.30}$$

If (2.30) can be reduced to $e_k^r = K_1 e_k^r$, the evolution of this error can be analyzed and the eigenvalues of K_1 can be chosen to be zero. By setting the last four matrix coefficients equal to zero, the e_k^r term will be the only one remaining. After setting these coefficients equal to zero, four of the observer gains can be rewritten in terms of K_3 as

$$K_1 = C_2\left(A\Omega_{12} + F\Omega_{22}\right) + G_2E\Omega_{22} - K_3\left(C_1A\Omega_{12} + (C_1F + G_1E)\Omega_{22}\right) \tag{2.31a}$$

$$K_2 = C_2\left(A\Omega_{11} + F\Omega_{21}\right) + G_2E\Omega_{21} - K_3\left(C_1A\Omega_{11} + (C_1F + G_1E)\Omega_{21}\right) \tag{2.31b}$$

$$\begin{aligned} K_4 &= C_2B - (C_2\left(A\Omega_{11} + F\Omega_{21}\right) + G_2E\Omega_{21})D \\ &\quad - K_3\left(C_1B - \left(C_1A\Omega_{11} + (C_1F + G_1E)\Omega_{21}\right)D\right) \end{aligned} \tag{2.31c}$$

$$K_5 = -K_3D\,. \tag{2.31d}$$

Now (2.31) becomes

$$\begin{aligned} e_{k+1}^r &= K_1e_k^r \\ &= \left(C_2\left(A\Omega_{12} + F\Omega_{22}\right) + G_2E\Omega_{22} - K_3\left(C_1A\Omega_{12} + (C_1F + G_1E)\Omega_{22}\right)\right)e_k^r \end{aligned} \tag{2.32}$$

In (2.33), the only unknown variable is K_3 and can be written in the form

$$e_{k+1}^r = (A_o - K_3C_o)e_k^r \tag{2.33}$$

where

$$A_o = C_2A\Omega_{12} + (C_2F + G_2E)\Omega_{22} \tag{2.34a}$$

and

$$C_o = C_1 A\Omega_{12} + (C_1 F + G_1 E)\Omega_{22}\,. \tag{2.34b}$$

Once again, the convolution summation solution is analyzed,

$$e_k^r = (A_o - K_3 C_o)^k e_0^r \tag{2.35}$$

and it is seen that the evolution of the error depends on the eigenvalues of $(A_o - K_3 C_o)$. The same minimization technique that was used in the full-order observer case is used for the reduced-order observer when the system pair $\left(A_o, C_o\right)$ is observable. The observable canonical forms of $\left(A_o, C_o\right)$ [15] are used to calculate the initial gain that will guarantee the eigenvalues of $(A_o - K_3 C_o)$ will equal zero. The same least squares minimization of the error system matrix over $\bar{K}_3$ is performed resulting in

$$\bar{K}_3 = \bar{A}_o \bar{C}_o^{\dagger}\,. \tag{2.36}$$

On the following page for a single output and n-dimensional system, it is displayed that by calculating $\bar{K}_3$ in canonical form, deadbeat response is guaranteed:

$$
\begin{aligned}
&\bar{A}_o - \bar{K}_3\bar{C}_o = \bar{A}_o - \bar{A}_o\bar{C}_o^{\dagger}\bar{C}_o = \bar{A}_o(I - \bar{C}_o^{\dagger}\bar{C}_o) \\
&= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^{\dagger} \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_{n-1} & \vdots & \ddots & \ddots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 0 \end{bmatrix} . \\
&\therefore \lambda_i(\bar{A}_o - \bar{K}_3\bar{C}_o) = 0 \text{ for i=1,2,}\cdots\text{,n}
\end{aligned}
$$

Therefore, $\lambda_i(\bar{A}_o - \bar{K}_3\bar{C}_o) = 0$ for $i = 1, 2, \cdots, n$. The observer gain is transformed back into the original system's form by use of a transformation technique [15],

$$K_3 = W_o^{-1}\bar{W}_o\bar{K}_3 \tag{2.37}$$

where W_o and $\bar{W}_o$ are the observability matrices for the system and its canonical form where

$$W_o = \begin{bmatrix} C_o \\ C_o A_o \\ \vdots \\ C_o A_o^{\,n-2} \\ C_o A_o^{\,n-1} \end{bmatrix} \tag{2.38}$$

and

$$\bar{W}_o = \begin{bmatrix} \bar{C}_o \\ \bar{C}_o \bar{A}_o \\ \vdots \\ \bar{C}_o \bar{A}_o^{\,n-2} \\ \bar{C}_o \bar{A}_o^{\,n-1} \end{bmatrix}. \tag{2.39}$$

After designing the full-order or reduced-order deadbeat observer, the estimates of x_k and w_k will be available for the controller completing the DAC design. The overall response time for the DAC will be the time it takes for the estimates to be available with zero error ($(n_x + n_w)$ steps for the full-order and $(n_x + n_w - 1)$ steps for the reduced-order) plus the time it takes the controller to drive the system state to zero while also minimizing the effect of the disturbance ($(n_x + 1)$ steps).

2.4 Conclusion

In this chapter, an observer based deadbeat DAC design technique for discrete-time systems with known waveform-type unknown disturbances was developed. Two different types of deadbeat observers, full-order and reduced-order, were introduced. It was shown that the only obvious difference between the two observers was the reduced-order observer should have a response time 1-step faster than the response time of the full-order observer. This technique involved solving for the control and observer gains by first using the canonical forms of the system matrices and then transforming the gains back to the original system's form. By doing this, it was shown that this DAC design technique is guaranteed to be deadbeat. In the following chapter, this technique will be applied to different examples and the performance will be analyzed.

3 CASE STUDIES

In this chapter, the deadbeat DAC design will be applied to two case studies: (1) a pendulum system and (2) a magnetic levitation system. There will also be three different types of disturbances considered: (1) step-type disturbances, (2) ramp-type disturbances, and (3) sinusoidal-type disturbances. The pendulum system controller is simulated for each of these disturbances while the magnetic levitation controller is simulated for the step-type disturbance to display that the technique also works for higher order systems. Each of the systems acted on by step-type disturbances will have two DACs designed for them, one using a full-order observer and one using a reduced-order observer in order to compare a reduced-order observer versus the full-order observer. Only one DAC will be designed for the pendulum system with the ramp-type and sinusoidal-type disturbances that uses the reduced-order observer.

First, models for each system are developed, linearized if necessary, put into state space form, and discretized. Then, a deadbeat DAC to minimize the effect of the disturbance in the minimal amount of time is designed for each disturbance to be considered. The last part needed for the DAC is the estimates from the observer; therefore, a deadbeat observer will then be designed following either the technique for the full-order observer or the technique for the reduced-order observer. Once the design of the observer based DAC is completed, the system response will show the effect of the disturbance being minimized as quickly as possible.

3.1 Pendulum System

3.1.1 System and Step-Type Disturbance Model

A pendulum, which is shown in figure 3.1, is a second order system that has many real world applications (i.e. an arm of a robot).

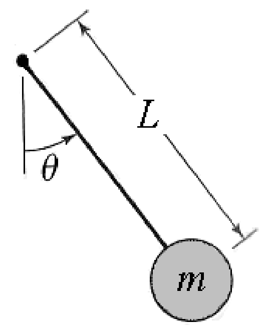

Figure 3.1 Pendulum system [17]

If the mass of the rod of the pendulum is insignificant compared to the concentrated mass at the end, m, the following dynamic equation is derived [17]

$$L\ddot{\theta} + g\sin\theta = 0\,. \tag{3.1}$$

For small angles

$$\sin\theta \approx \theta$$

which reduces (3.1) to

$$L\ddot{\theta} + g\theta = 0\,. \tag{3.2}$$

From (3.2), it is seen there is no input or disturbance included in the dynamic equation. An input and disturbance are added to allow the system to be in a form that has all of the system matrices defined in (2.1) present,

$$L\ddot{\theta} + g\theta = au + bw. \tag{3.3}$$

where *a* and *b* are coefficients that determine the strength of the input and disturbance signals, respectively. Now, for $L = 0.5m$ with $g = 9.8m/s^2$ and with the state variables defined as

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \end{aligned} \tag{3.4}$$

resulting in the matrix vector formulation,

$$\dot{x}(t) = A^c x(t) + B^c u(t) + F^c w(t) \tag{3.5}$$

$$y(t) = C^c x(t) + D^c u(t) + G^c w(t), \tag{3.6}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -19.6 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + F^c w \tag{3.7}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + 0.1u + 0.2w, \tag{3.8}$$

where F^c is left as a variable because it will change depending on the disturbance being applied. The c-superscript is used to denote the continuous time case.

The controller technique developed in this thesis utilizes discrete-time systems, so this system is discretized [15] using the sampling time $T_s = 0.1s$ and written in the matrix vector form

$$x_{k+1} = Ax_k + Bu_k + Fw_k$$

$$y_k = C_1 x_k + Du_k + G_1 w_k$$

resulting in

$$x_{k+1} = \begin{bmatrix} 0.9036 & 0.0968 \\ -1.8966 & 0.9036 \end{bmatrix} x_k + \begin{bmatrix} 0.0049 \\ 0.0968 \end{bmatrix} u_k + F w_k \tag{3.9}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + 0.1 u_k + 0.2 w_k \tag{3.10}$$

where F represents the discrete-time version of F^c. The disturbance waveform structure must now be defined.

The first disturbance to be considered is the step-type disturbance. A step input is a constant value starting at a given time, the step-type disturbance is developed using this concept. This disturbance is modeled as a constant with an unknown impulse sequence added to it,

$$w_{k+1} = w_k + \sigma_k \tag{3.11}$$

where in the pendulum with step-type disturbances example, the impulse sequence is

$$\sigma_k = \begin{cases} -0.59, & k = 5 \quad (t = 0.5s) \\ 0.59\ , & k = 15 \quad (t = 1.5s) \\ 0.8\ , & k = 25 \quad (t = 2.5s) \\ -0.8\ , & k = 35 \quad (t = 3.5s) \\ 0\ , & otherwise \end{cases}.$$

Using this impulse sequence, the disturbance will take the form of a step of magnitude -0.59 turning on at 0.5 s and turning off at 1.5s following by a step of magnitude 0.8 that turns on at 2.5s and turns off one second later. This impulse sequence is not actually known, it is solely created for simulation purposes.

For this first order disturbance,

$$F^c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

implying that the step-type disturbance will be applied to the angular velocity (x_2), which when is converted into discrete-time becomes,

$$F = \begin{bmatrix} 0.0049 \\ 0.0968 \end{bmatrix}.$$

3.1.2 Controller for Pendulum with Step-Type Disturbance

As described in section 2.2, an augmented system is created in order to find one control input for both the state and the measurement. The augmented system is

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} 0.9036 & 0.0968 & 0 \\ -1.8966 & 0.9036 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} 0.0049 \\ 0.0968 \\ 0.2000 \end{bmatrix} w_k + \begin{bmatrix} 0.0049 \\ 0.0968 \\ 0.1000 \end{bmatrix} u_k$$

which has an open loop response shown in figures 3.2 through 3.4. In these figures, the solid line is the state variable or measurement and the dotted line is the disturbance. The moment the disturbance is present in the system oscillations begin.

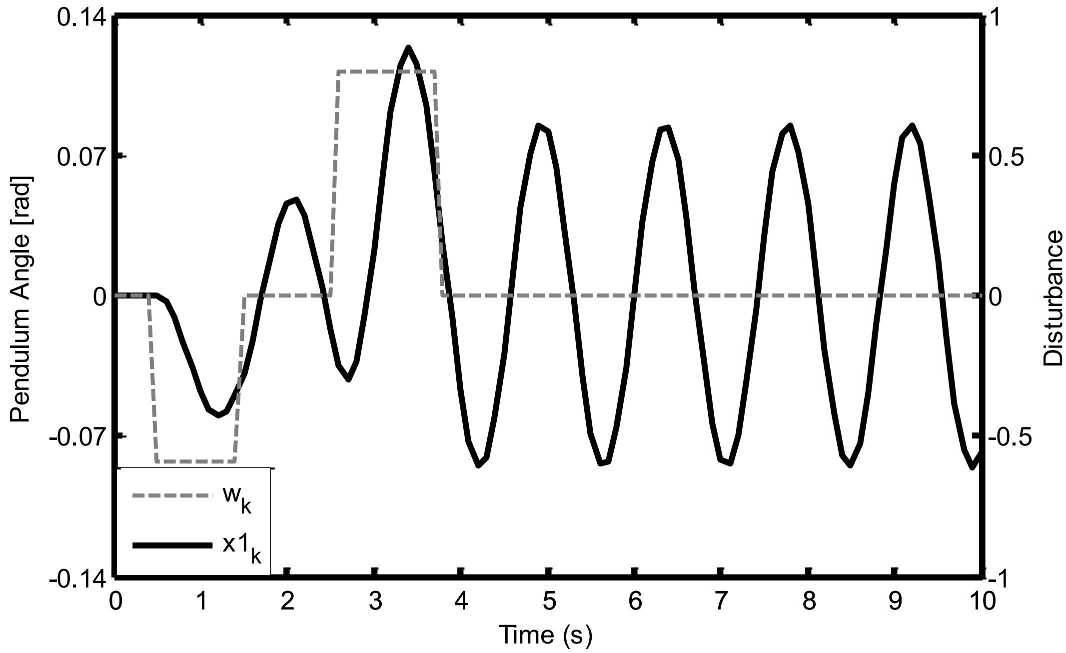

Figure 3.2 Open loop response of pendulum angle with step-type disturbance

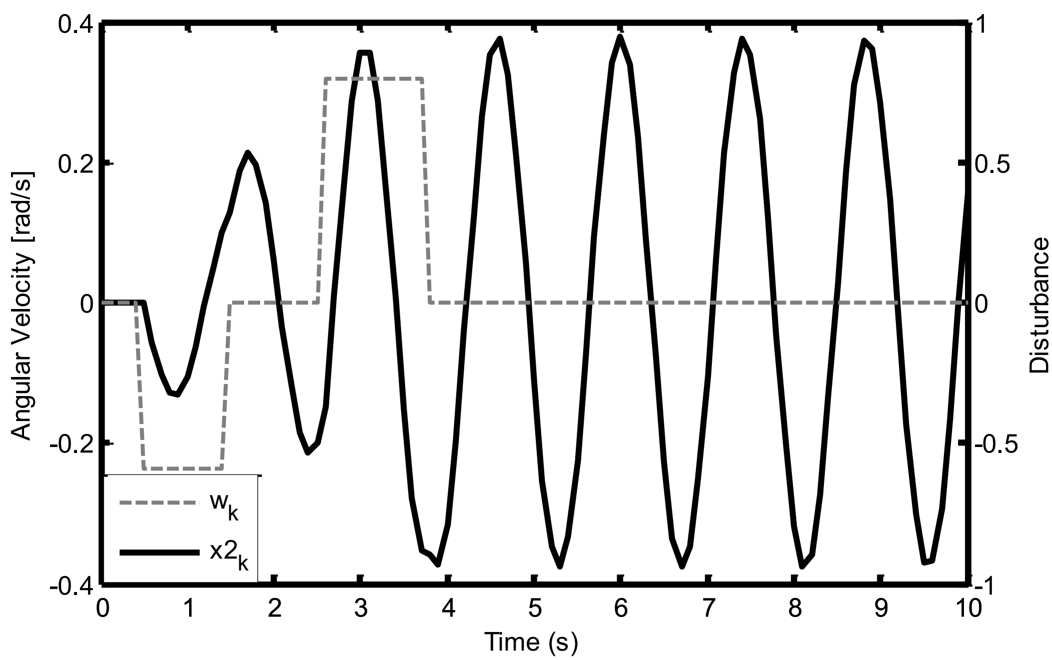

Figure 3.3 Open loop response of angular velocity of pendulum with step-type disturbance

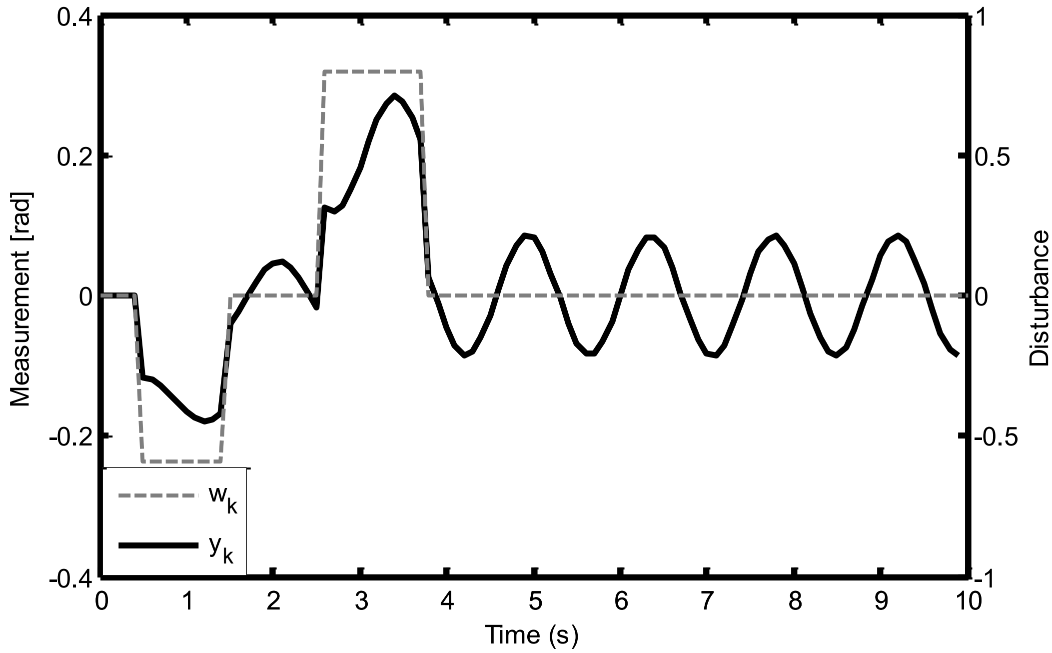

Figure 3.4 Open loop response of measurement with step-type disturbance

The controllability of the system pair $\left(A_c, B_c\right)$ is determined by finding the controllability matrix, multiplying it by its transpose and determining if the resulting matrix has non-zero eigenvalues. If there are zero eigenvalues, the augmented system is not controllable.

$$W_c = \begin{bmatrix} B_c & A_c B_c & A_c^2 B_c \end{bmatrix}$$

$$\lambda\left(W_c W_c^T\right) = \begin{bmatrix} 0.0001 \\ 0.0028 \\ 0.0253 \end{bmatrix}$$

One of the eigenvalues is close to zero and the other two are small which implies the system is close to being unstable; therefore, the control gains will be expected to be large.

The augmented system is transformed into controllable canonical form [15],

$$\bar{A}_c = \begin{bmatrix} 1.8072 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } \bar{B}_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The state control gains are formed by calculating the controllable canonical form gains (2.8a) and then using the transformation technique (2.8b); the disturbance accommodation control gains are also calculated using (2.7b),

$$\bar{L} = \begin{bmatrix} -1.8072 & 1 & 0 \end{bmatrix} \;\therefore\; L = \begin{bmatrix} -82.0495 & -14.5051 & 0 \end{bmatrix}$$

$$L_d = \begin{bmatrix} -1.5158 \end{bmatrix}.$$

As discussed in chapter 2, by following this technique, the controller will always be deadbeat. This is checked by looking at the eigenvalues of $A_c + B_c L$,

$$\lambda\left(A_c + B_c L\right) = \begin{bmatrix} 0 \\ -1.66\times10^{-8} \\ 1.66\times10^{-8} \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The eigenvalues are not exactly zero due to rounding errors in MATLAB when transforming the control gains from the canonical form to the original system's form, but they are very close to zero. Now that the control input, $u_k = L\hat{x}_k + L_d\hat{w}_k$, is found, the estimates of x_k and w_k are needed to complete the design. In section 3.1.3, a full-order observer will be developed and in section 3.1.4, a reduced-order observer will be developed.

3.1.3 Full-Order Observer for Pendulum System with Step-Type Disturbance

Recall from section 2.3.1, similar to the controller, an augmented system is created which consists of the state variables to be estimated, $\hat{x}_k$ and $\hat{w}_k$,

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{w}_{k+1} \end{bmatrix} = \begin{bmatrix} 0.9036 & 0.0968 & 0.0049 \\ -1.8966 & 0.9036 & 0.0968 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} 0.0049 \\ 0.0968 \\ 0 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + 0.1 u_k$$

The observability of the system pair $\left(A_{fo}, C_{fo}\right)$ is checked by finding the observability matrix and then finding the eigenvalues of the Gram matrix of the observability matrix, $\lambda\left(W_{fo} W_{fo}^T\right)$. The system will be observable if all these eigenvalues are non-zero.

$$W_{fo} = \begin{bmatrix} C_{fo} \\ C_{fo} A_{fo} \\ C_{fo} A_{fo}^2 \end{bmatrix}$$

$$\lambda\left(W_{fo} W_{fo}^T\right) = \begin{bmatrix} 0.0003 \\ 0.0271 \\ 2.3594 \end{bmatrix}$$

Again, one of the eigenvalues is close to zero and another one is small which means two of the observer gains will be expected to be large.

The system is converted to observable canonical form [15],

$$\bar{A}_{fo} = \begin{bmatrix} 2.8072 & 1 & 0 \\ -2.8072 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } \bar{\mathrm{C}}_{fo} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

and the full-order observer gains result are obtained following the procedure outlined in section 2.3.1, equations (2.21a) and (2.21b),

$$\bar{K} = \begin{bmatrix} 2.8072 \\ -2.8072 \\ 1 \end{bmatrix} \quad \therefore \; K = \begin{bmatrix} -1.3249 \\ 21.0462 \\ 20.6605 \end{bmatrix}.$$

The eigenvalues of $A_{fo} - KC_{fo}$ are

$$\lambda\left(A_{fo} - KC_{fo}\right) = \begin{bmatrix} -2.164\times10^{-5} \\ (1.082 + 1.874\mathrm{i})\times10^{-5} \\ (1.082 - 1.874\mathrm{i})\times10^{-5} \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The eigenvalues for the canonical form are all zero, however, due to rounding by MATLAB when going through the transformation to the original system's form, the eigenvalues are not exactly zero anymore, but they are close. A third order deadbeat controller and third order deadbeat observer have now been designed to accommodate step-type disturbances on a second order pendulum system, therefore the maximum amount of time for the DAC to minimize the disturbance is the sum of the orders of the augmented systems times the sampling time, 0.6s (see section 2.2).

3.1.4 Simulation of Deadbeat Observer Based DAC for Pendulum with Step-Type Disturbance

The full-order deadbeat observer based DAC is applied to the pendulum example upon which step-type disturbances are acting. The simulations should show a minimization of the disturbance in 0.6s while the disturbance is present, where 0.6s is the sum of the orders of the two augmented systems times the time step used for discretization. Once the disturbance is no longer acting on the system, the system is expected to reach zero (the desired value) in 0.6s. The results from the simulation are shown in figures 3.5 through 3.8. Figure 3.5 shows for deadbeat control, the control input is large with a maximum magnitude of 59.8, but it is only active for a short period of time. In the figures 3.6 through 3.8, a minimization of the disturbance is achieved in 0.6s and once the disturbance is gone, the system is controlled to zero in 0.6s. Therefore, the full-order deadbeat observer based DAC is functioning just as expected.

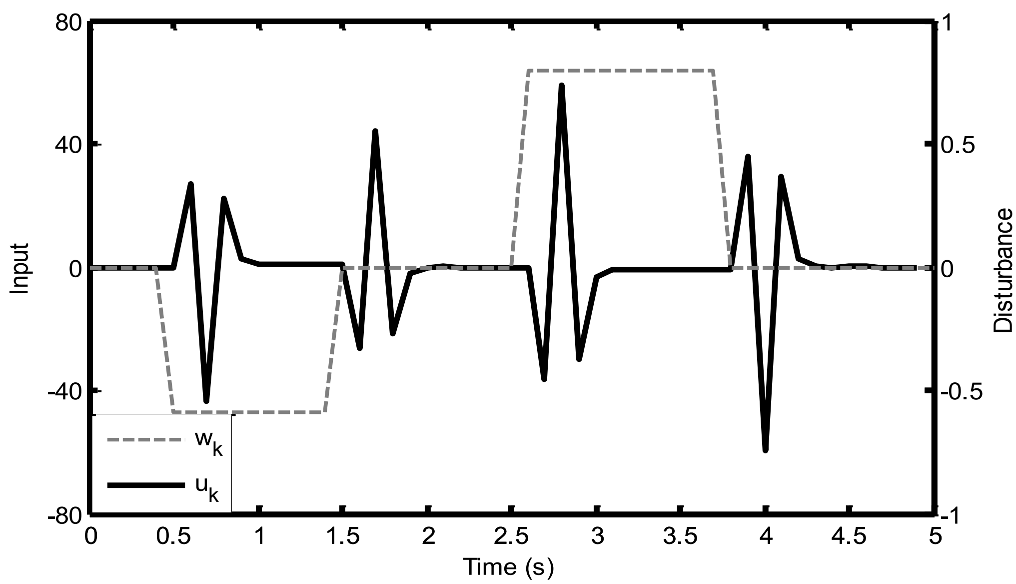

Figure 3.5 Control input for full-order observer based DAC with step-type disturbance on pendulum system

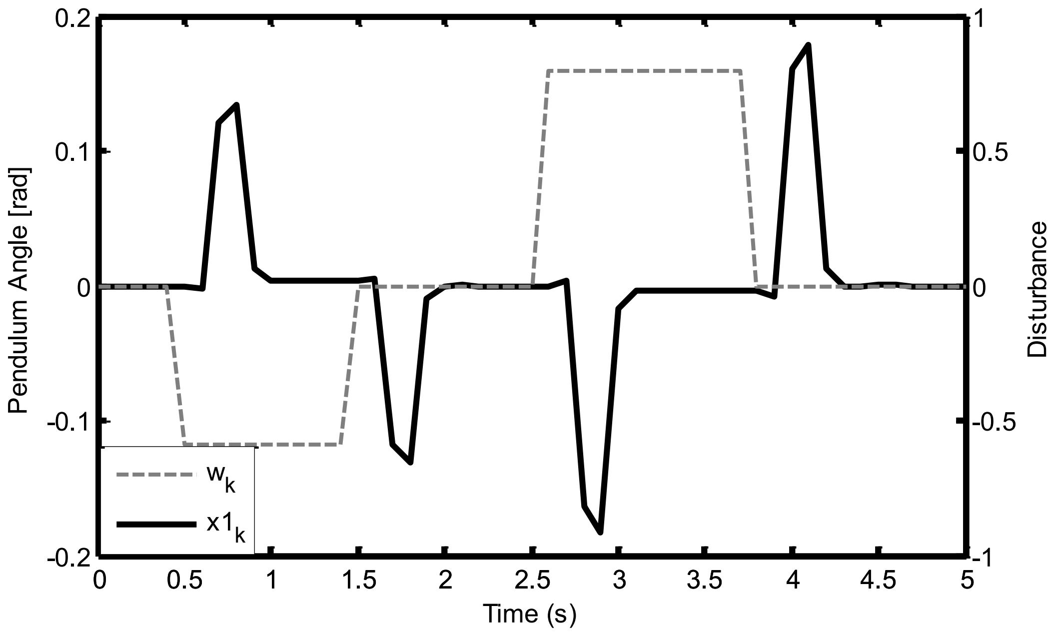

Figure 3.6 Closed loop response of pendulum angle for full-order observer based DAC with step-type disturbance

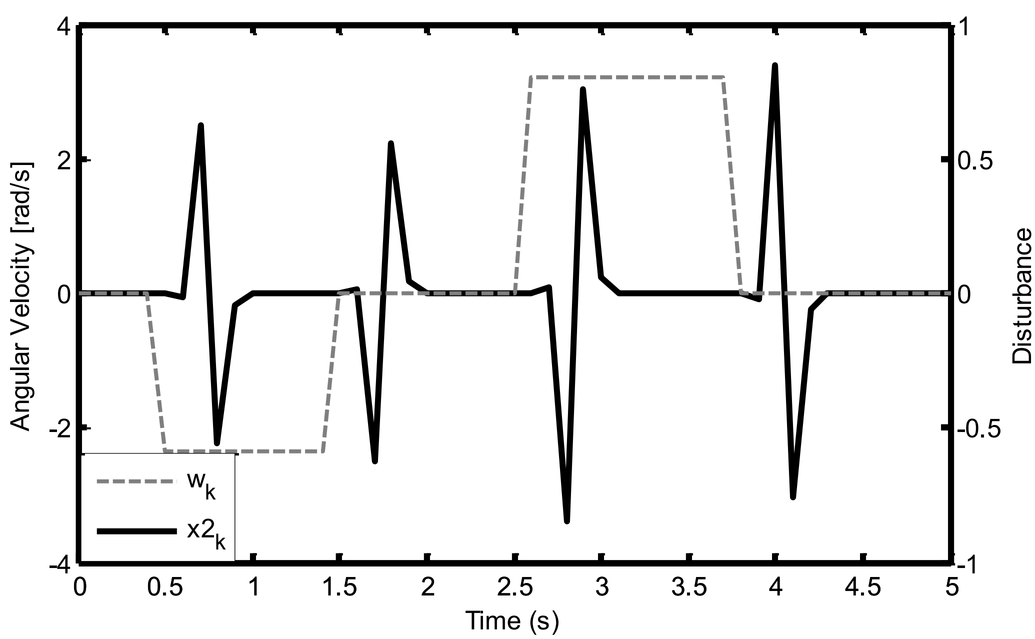

Figure 3.7 Closed loop response of angular velocity for full-order observer based DAC with step-type disturbance

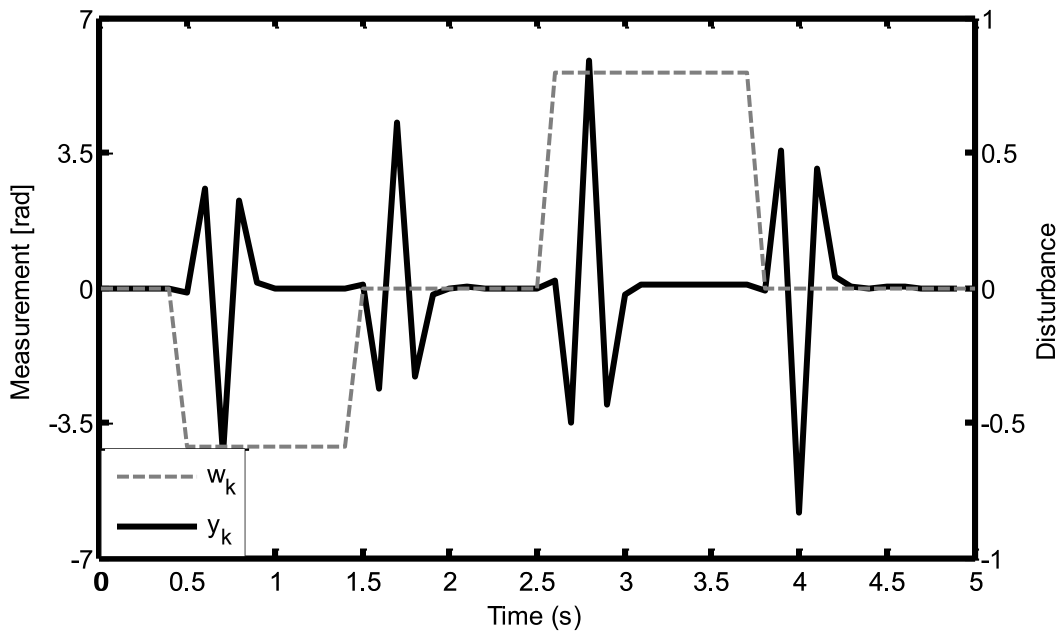

Figure 3.8 Closed loop response of measurement for full-order observer based DAC with step-type disturbance

Since one objective of this work is to design the DAC to be time-optimal, a reduced-order observer is now designed for this system for a faster response time.

3.1.5 Reduced-Order Observer for Pendulum with Step-Type Disturbance

Following the steps detailed in section 2.3.2, a reduced-order observer is designed for the pendulum system with step-type disturbances. First, the composite vector is created consisting of the variables to be estimated is made,

$$z_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k .$$

Since x_k and w_k are not available, these variables are replaced by their estimates and the composite vector is augmented with the output equation resulting in

$$\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} u_k .$$

This equation can now be solved for $\hat{x}_k$ and $\hat{w}_k$, since $\begin{bmatrix} 1 & 0 & 0.1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is invertible, as

$$\begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} - \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} u_k \right),$$

allowing the estimates, $\hat{x}_k$ and $\hat{w}_k$, to be determined when $\hat{z}_k$ is known.

Before the observer gains are calculated, the matrices A_o and C_o defined in section 2.3.2 are checked for observability using the same method discussed for the full-order observer system,

$$W_o = \begin{bmatrix} C_o \\ C_o A_o \end{bmatrix}$$
$$\lambda\left(W_o W_o^T\right) = \begin{bmatrix} 0.0010 \\ 0.0215 \end{bmatrix}$$

Again, one of the eigenvalues is close to zero and the other one is small which means the observer gains will be expected to be large.

The system is converted to observable canonical form [15],

$$\bar{A}_o = \begin{bmatrix} 1.9036 & 1 \\ \text{-}0.9036 & 0 \end{bmatrix} \text{ and } \bar{C}_o = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The gain, $\bar{K}_3$, is calculated and transformed into the original system via $K_3 = W_o^{-1}\bar{W}_o\bar{K}_3$, following the procedure outlined in section 2.3.2. The resulting reduced-order observer gains are

$$K_1 = \begin{bmatrix} \text{-}0.5000 & 0.1250 \\ \text{-}1.9992 & 0.5000 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} \text{-}15.0033 \\ \text{-}18.6686 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} 14.5051 \\ 20.6605 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} 1.5257 \\ 1.7652 \end{bmatrix}$$
$$K_5 = \begin{bmatrix} \text{-}1.4505 \\ \text{-}2.0660 \end{bmatrix}$$

where the eigenvalues of $A_o - K_3C_o$ are

$$\lambda\left(A_o - K_3C_o\right) = \begin{bmatrix} 1.253\text{i}\times10^{-8} \\ \text{-}1.253\text{i}\times10^{-8} \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A reduced-order observer based DAC consisting of a third order deadbeat controller and second order deadbeat observer has now been designed to accommodate step-type disturbances on a pendulum system, the maximum amount of time for the DAC to minimize the disturbance should be the sum of the orders of the augmented systems times the sampling time, 0.5s.

3.1.6 Simulation of Reduced-Order Deadbeat Observer Based DAC for Pendulum with Step-Type Disturbance

The reduced-order deadbeat observer based DAC is applied to the pendulum example that has step-type disturbances acting on it. It is expected that the simulations will show a minimization of the disturbance in 0.5s while the disturbance is present. Once the disturbance is no longer acting on the system, the system is expected to reach zero in 0.5s. The results from the simulation are shown in figures 3.9 through 3.12. Figure 3.9 shows that the reduced-order observer based DAC has a larger input than the full-order observer based DAC. In figures 3.10 through 3.12, the simulations show a minimization of the disturbance in 0.5s and once the disturbance is gone, the system is controlled to zero in 0.5s just as expected.

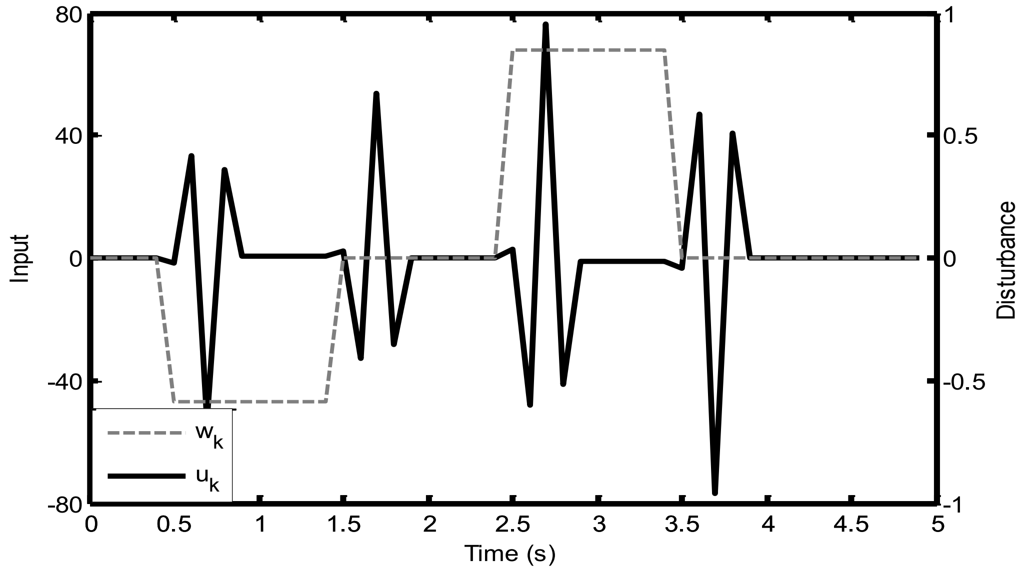

Figure 3.9 Control input for reduced-order observer based DAC with step-type disturbance on pendulum system

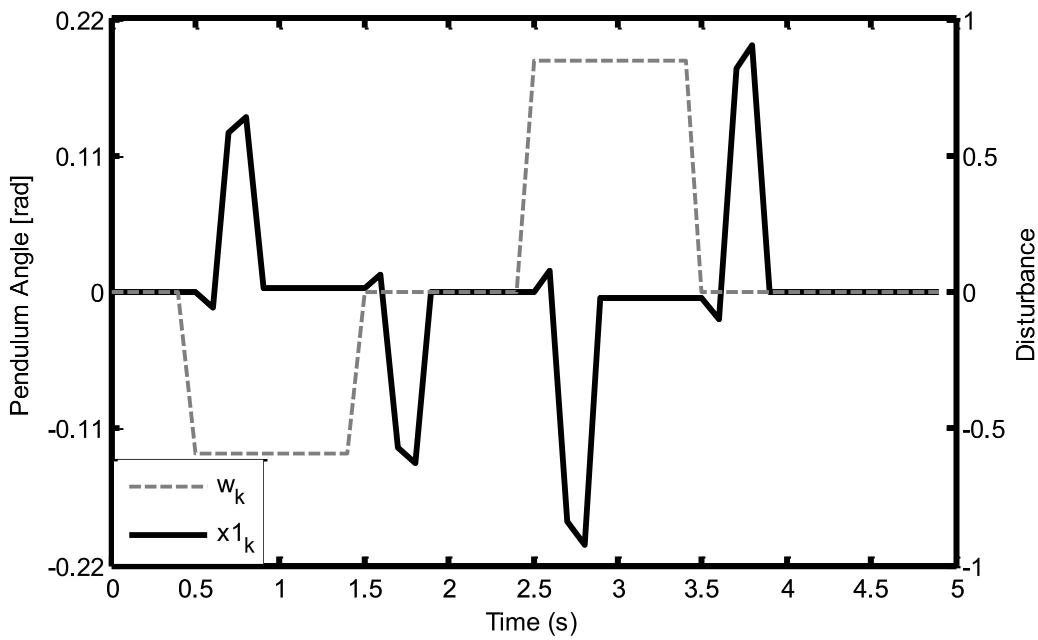

Figure 3.10 Closed loop response of pendulum angle for reduced-order observer based DAC with step-type disturbance

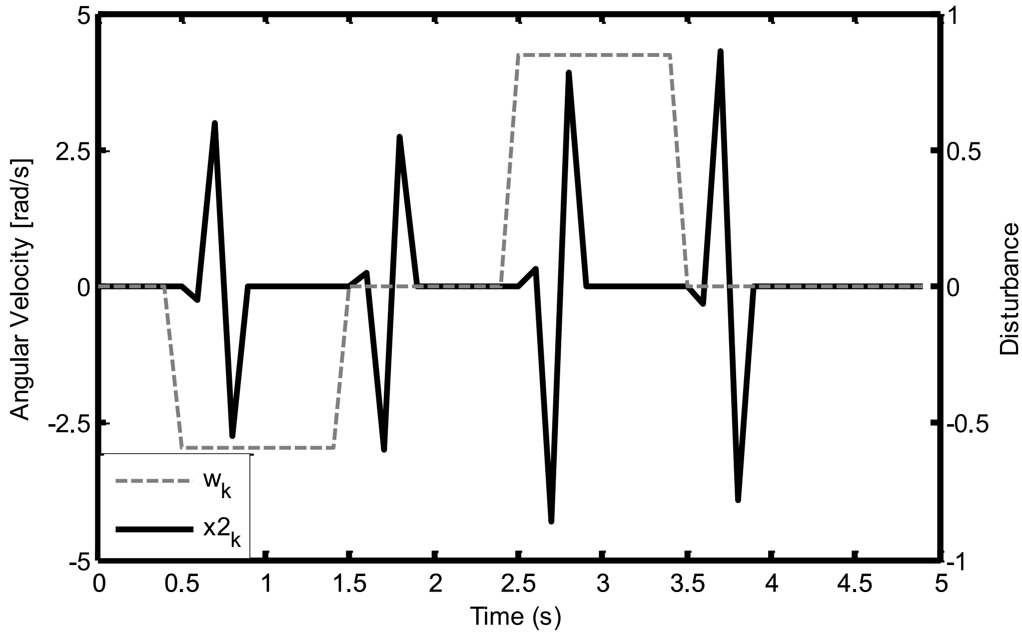

Figure 3.11 Closed loop response of angular velocity for reduced-order observer based DAC with step-type disturbance

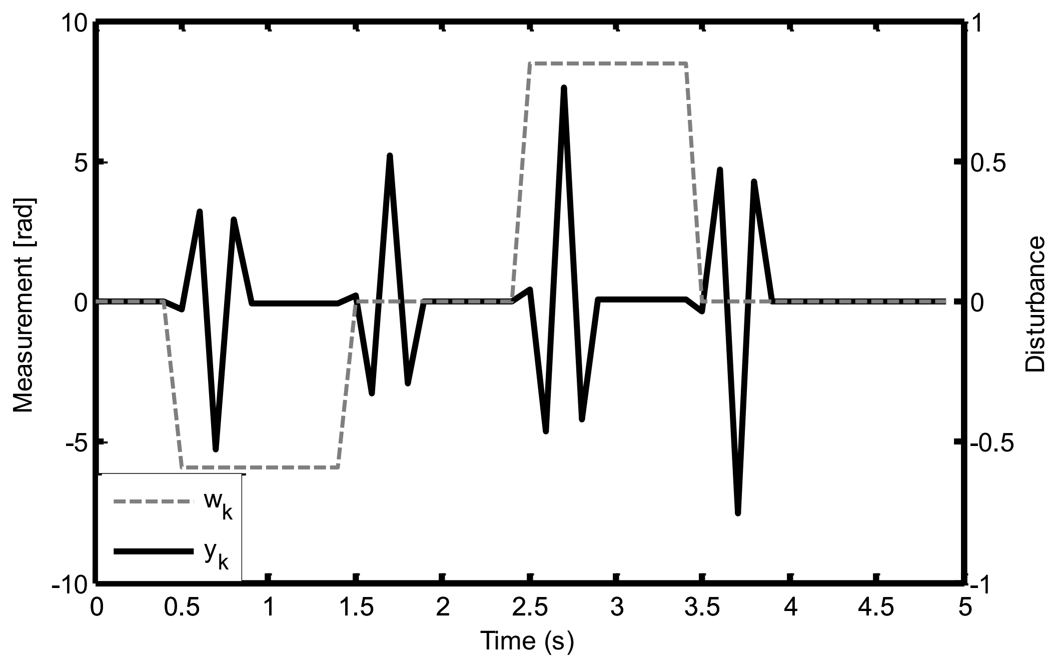

Figure 3.12 Closed loop response of measurement for reduced-order observer based DAC with step-type disturbance

While it is clear that the reduced-order observer based DAC gives a faster response than the full-order based DAC, does it have any other benefits? This question is examined by looking at a co-plot of the full-order and reduced-order based DAC responses seen in figures 3.13 through 3.16. The figures on the following pages show that for this example, the only benefit is the one already known, the reduced-order observer based DAC is able to minimize the effects of the disturbances quicker than the full-order observer based DAC.

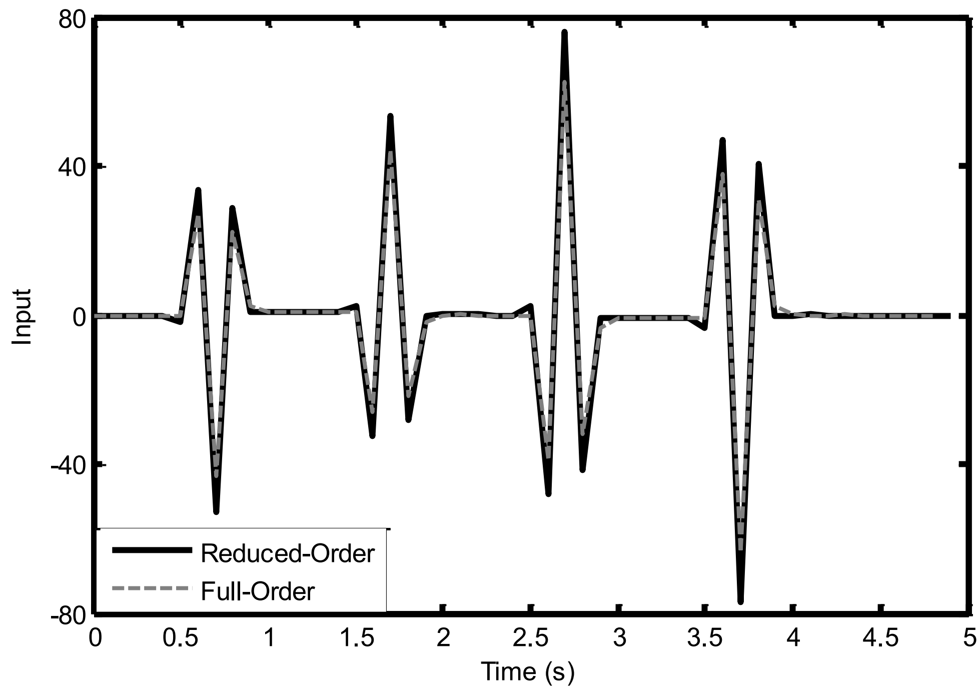

Figure 3.13 Control input comparison between reduced-order and full-order observer based DACs with step-type disturbance

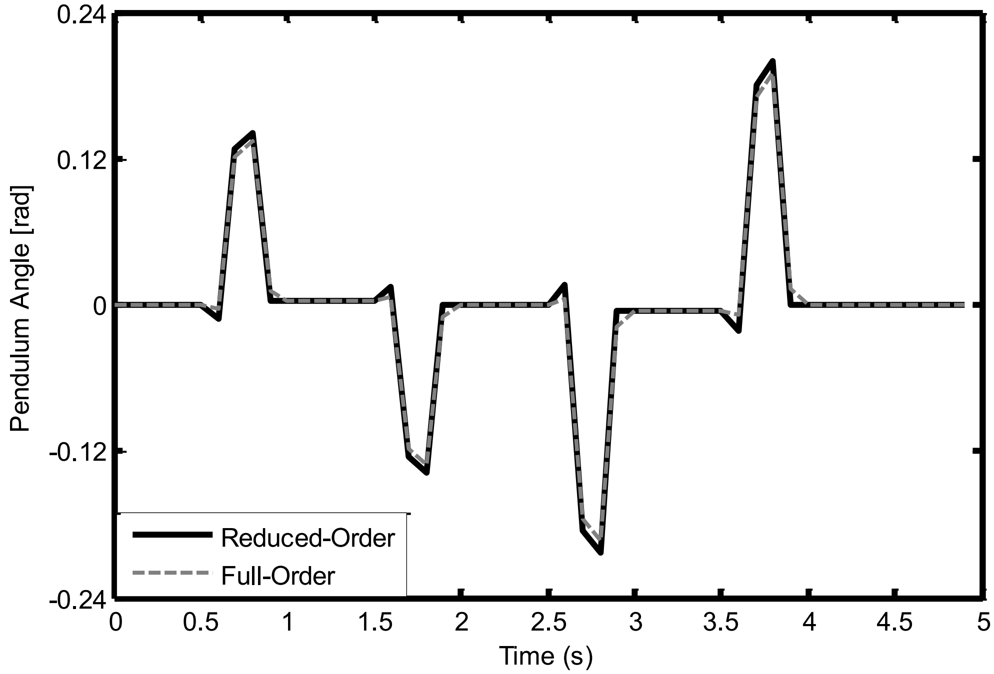

Figure 3.14 Closed loop response of pendulum angle comparison between reduced-order and full-order observer based DACs with step-type disturbance

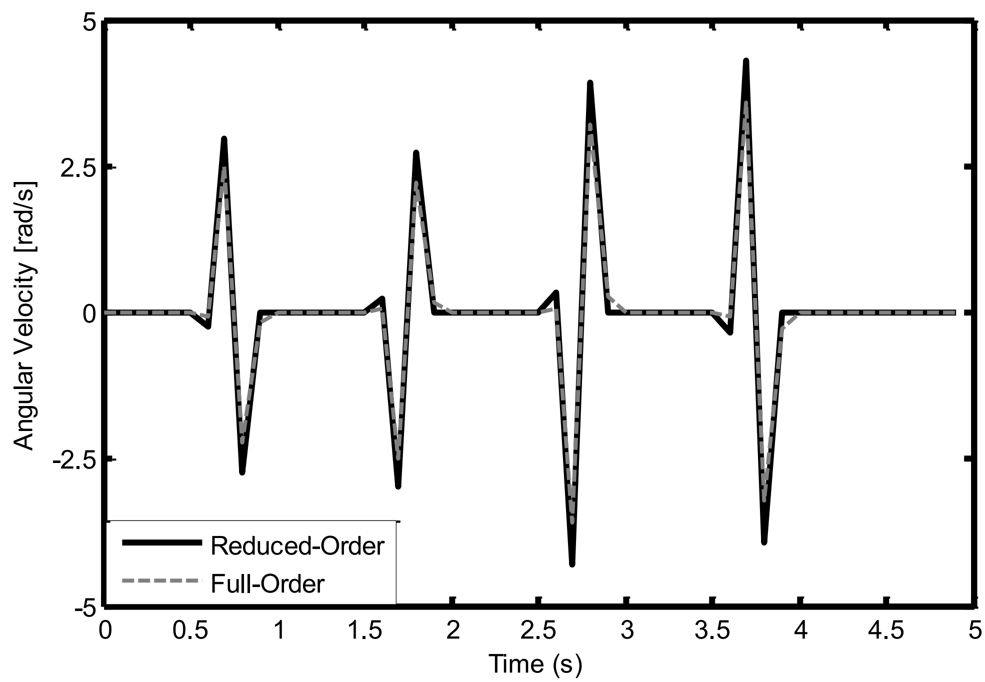

Figure 3.15 Closed loop response of angular velocity comparison between reduced-order and full-order observer based DACs with step-type disturbance

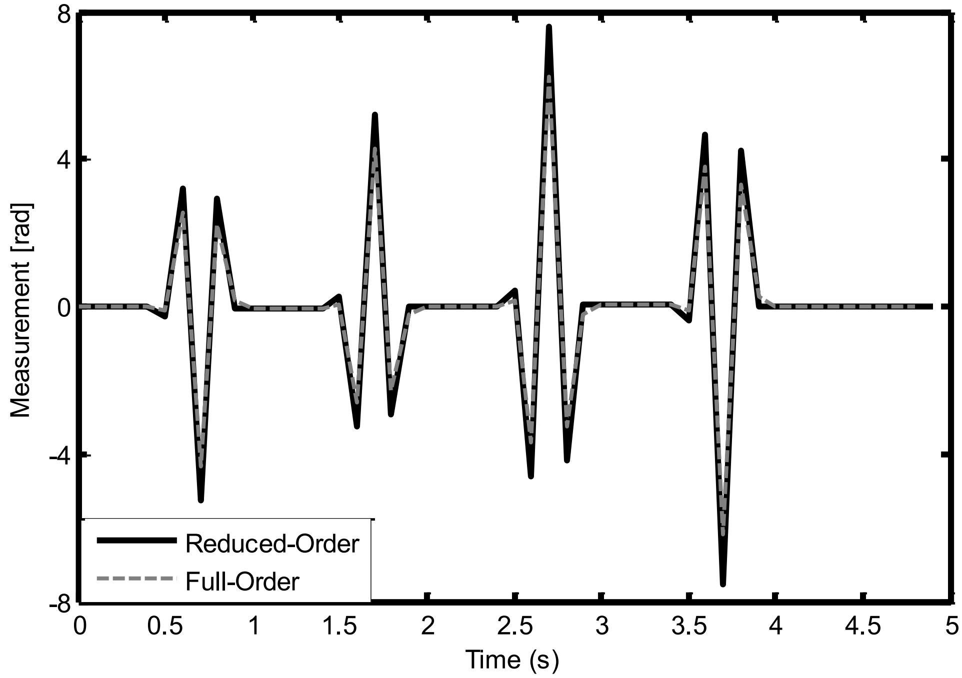

Figure 3.16 Closed loop response of measurement comparison between reduced-order and full-order observer based DACs with step-type disturbance

For convenience, in the following examples, only the reduced-order observer based DAC will be used because it has a faster response.

3.1.7 Ramp-Type Disturbance Model

A ramp-type disturbance is now being applied to the pendulum system described in section 3.1.1. A ramp is just a line and the equation for a line is $y = mx + b$; simply stated, it is a term changing at a constant rate plus a constant value. In discrete-time, its model is developed using this concept. The model of this waveform structure is

$$\begin{bmatrix} w_{k+1}^1 \\ w_{k+1}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix} + \sigma_k \tag{3.12}$$

where w_k^1 is the constant term and is the same as a step-type disturbance and w_k^2 is this constant term plus w_{k-1}^2 causing w_k^2 to change at a constant rate, making it a ramp-type disturbance. This is shown by the first four iterations of a basic example,

$$\begin{aligned} w_0^1 &= 1 \\ w_0^2 &= 0 \end{aligned}$$

$$\begin{aligned} w_1^1 &= 1 \\ w_1^2 &= 1+0=1 \end{aligned}$$

$$\begin{aligned} w_2^1 &= 1 \\ w_2^2 &= 1+1=2 \end{aligned}$$

$$\begin{aligned} w_3^1 &= 1 \\ w_3^2 &= 1+2=3 \end{aligned}.$$

In this example, there is a step-type disturbance of magnitude 1.4 occurring at 1s and lasting until 1.7s and a ramp-type disturbance occurring at 2.3s with a slope of -2.486 and lasting until 3s when it is completely gone (this involves a ramp with slope +2.486 and a step of magnitude 1.7).

For this second order disturbance, in continuous time

$$F_c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

implying the step-type disturbance (w_k^1) will be applied to the angular velocity (x_2). This matrix is transformed into discrete-time,

$$F = \begin{bmatrix} 0.0049 & 0 \\ 0.0968 & 0 \end{bmatrix}$$

and the ramp-type disturbance is acting on the measurement,

$$G_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

3.1.8 Controller for Pendulum with Ramp-Type and Step-Type Disturbances

As before, an augmented system must be created in order to find one control input for both the state and the measurement. The augmented system is

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} 0.9036 & 0.0968 & 0 \\ -1.8966 & 0.9036 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} 0.0049 & 0 \\ 0.0968 & 0 \\ 0 & 0.2 \end{bmatrix} w_k + \begin{bmatrix} 0.0049 \\ 0.0968 \\ 0.1000 \end{bmatrix} u_k$$

which has an open loop response shown in figures 3.17 through 3.19. The figures show the state or measurement as the solid signal, the dashed line is the step-type disturbance and the dotted line is the ramp-type disturbance. When the step-type disturbance is present in the state, the figures show that oscillations begin. The measurement plot shows that the measurement follows the pendulum angle until the ramp disturbance is applied, then the measurement is a ramped version of the pendulum angle. Once the ramp disturbance is no longer applied to the measurement, the measurement follows the pendulum angle again.

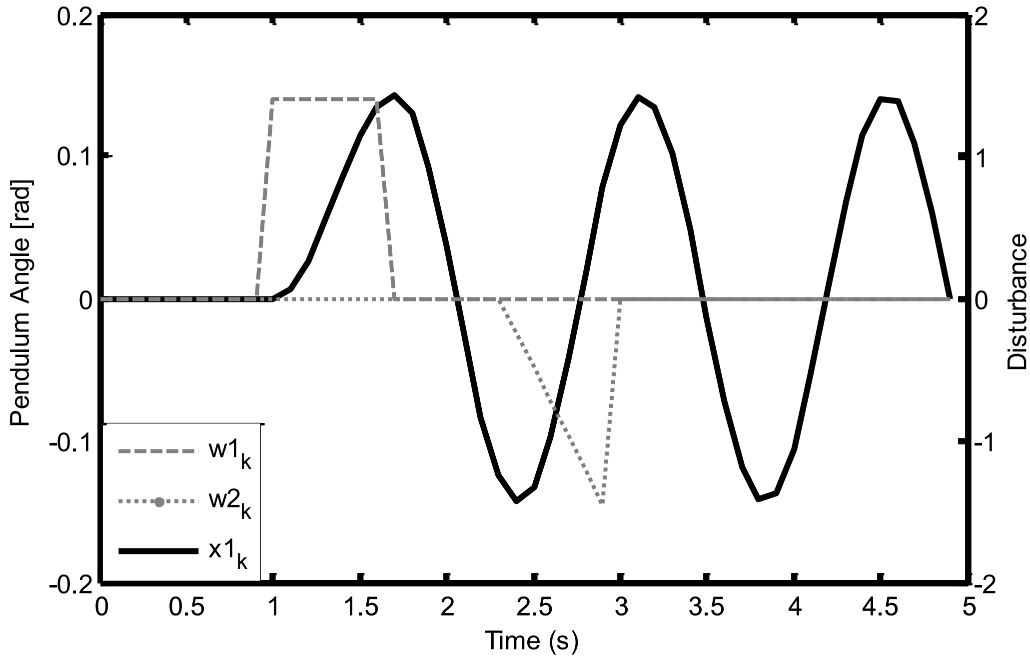

Figure 3.17 Open loop response of pendulum angle for step-type and ramp-type disturbances

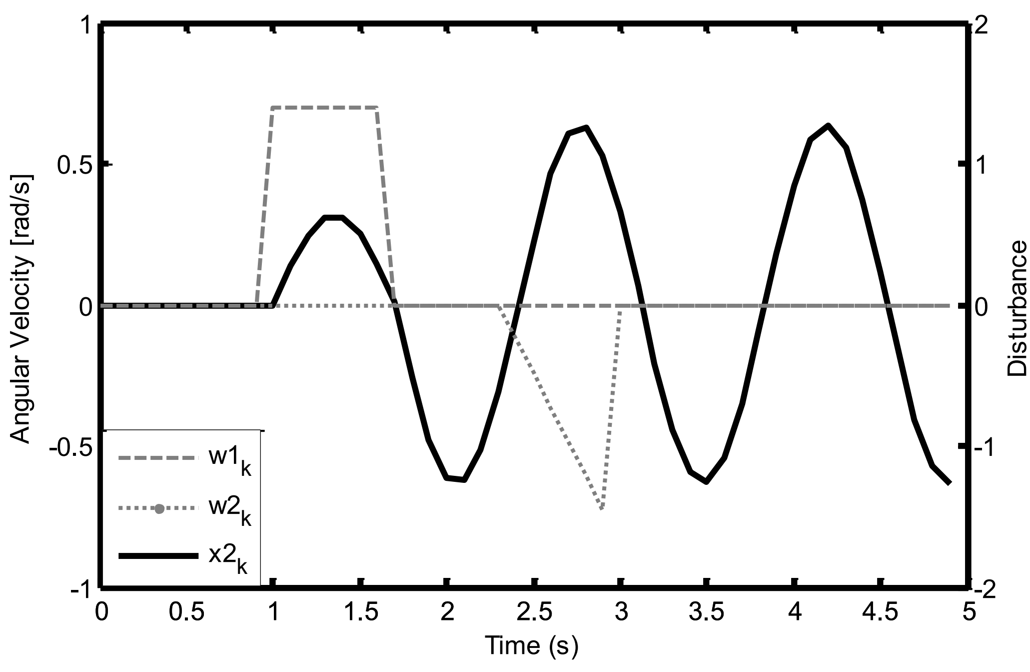

Figure 3.18 Open loop response of angular velocity for step-type and ramp-type disturbances

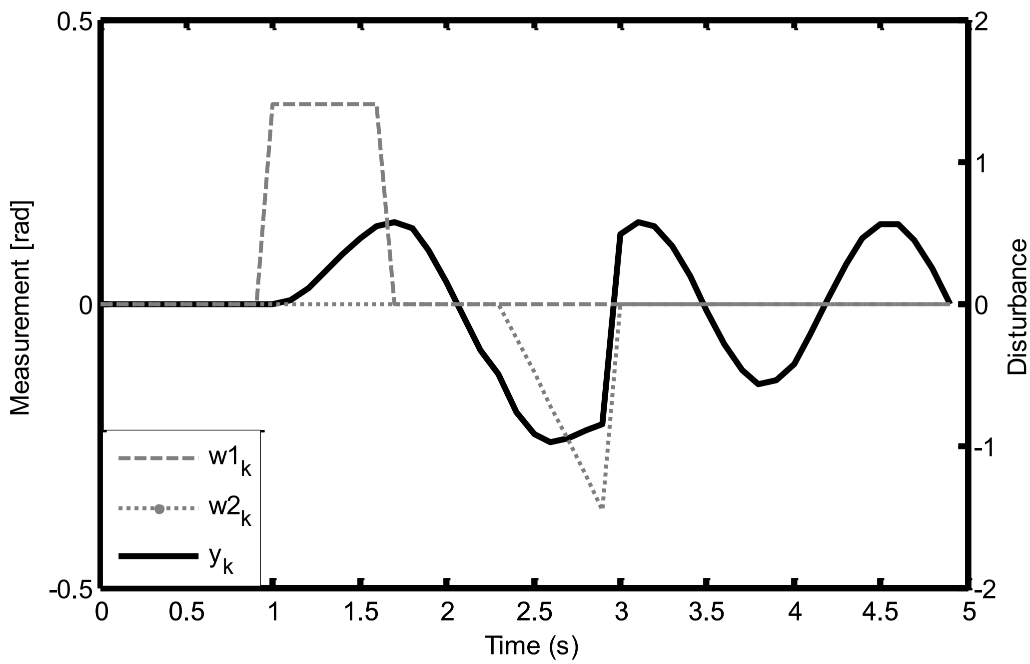

Figure 3.19 Open loop response of measurement for step-type and ramp-type disturbances

The state control portion of the controller is the same as when there was only a step-type disturbance since it does not rely on the matrices that were altered, E, F, and G_1. Therefore, the control gain L remains the same as the previous case, while the disturbance accommodation control gain will be changed using (2.7b) given in section 2.2.

$$L = \begin{bmatrix} -82.0495 & -14.5051 & 0 \end{bmatrix}$$

$$L_d = \begin{bmatrix} -0.4842 & -1.0316 \end{bmatrix}$$

Again, this technique will always place the eigenvalues at zero making this a deadbeat controller.

3.1.9 Reduced-Order Observer for Pendulum with Ramp-Type and Step-Type Disturbances

Following the procedure to design a reduced-order observer for the pendulum system with ramp-type disturbances, a fictitious vector is created consisting of the variables needing to be estimated,

$$z_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} w_k$$

where x_k and w_k are not available. These variables are replaced by their estimates and the composite vector is augmented with the output equation resulting in

$$\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k$$

which, when rearranged, gives

$$\begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} - \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k \right)$$

allowing the estimates to be determined.

The observability for this observer system is checked and found to be satisfied.

The system is converted to observable canonical form [15],

$$\bar{A}_o = \begin{bmatrix} 2.9036 & 1 & 0 \\ -2.8072 & 0 & 1 \\ 0.9036 & 0 & 0 \end{bmatrix} \text{ and } \bar{C}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The gain, $\bar{K}_3$, is calculated and transformed into the original system via $K_3 = W_o^{-1}\bar{W}_o\bar{K}_3$, the resulting reduced-order observer gains are

$$K_1 = \begin{bmatrix} 4.1862 & 7.0483 & 1.0334 \\ -2.5092 & -4.3137 & -0.5000 \\ -4.3783 & -8.2719 & 0.1276 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 28.7563 \\ -23.4310 \\ -40.8847 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -33.9234 \\ 25.9310 \\ 45.2469 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} -2.6120 \\ 2.2155 \\ 3.8659 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} 3.3923 \\ -2.5931 \\ -4.5247 \end{bmatrix}$$

where the eigenvalues of $A_o - K_3C_o$ are approximately equal to zero.

The reduced-order observer based DAC consists of a third order deadbeat controller and third order deadbeat observer which have now been designed to accommodate step-type disturbances on a pendulum system, the maximum amount of time for the DAC to minimize the disturbance is six time samples, or 0.6s.

3.1.10 Simulation of Reduced-Order Deadbeat Observer Based DAC for Pendulum with Ramp-Type and Step-Type Disturbances

The reduced-order deadbeat observer based DAC is applied to the pendulum example that has a ramp-type disturbance on the measurement and a step-type disturbance on the angular velocity. It is expected that the simulations will show a minimization of the disturbance in 0.6s while the disturbance is present. Once the disturbance is no longer acting on the system, the system is expected to reach zero in 0.6s. The results from the simulation are shown in figures 3.20 through 3.23. Figure 3.20 is the control input where the very large magnitude between 3s and 3.5s is because when the ramp disturbance abruptly goes to zero, that is actually the same as a new ramp disturbance plus a step disturbance at the same time that causes the control to work harder. In figures 3.21 through 3.23, the simulations show a minimization of the disturbance in 0.6s and once the disturbance is gone, the system is controlled to zero in 0.6s just as expected. The magnitude of the response when the ramp-type disturbance is gone is very large due to a ramp-type disturbance actually being a ramp-type disturbance plus a step-type disturbance at the same time. This causes the DAC to work harder (causing a larger overshoot in the response).

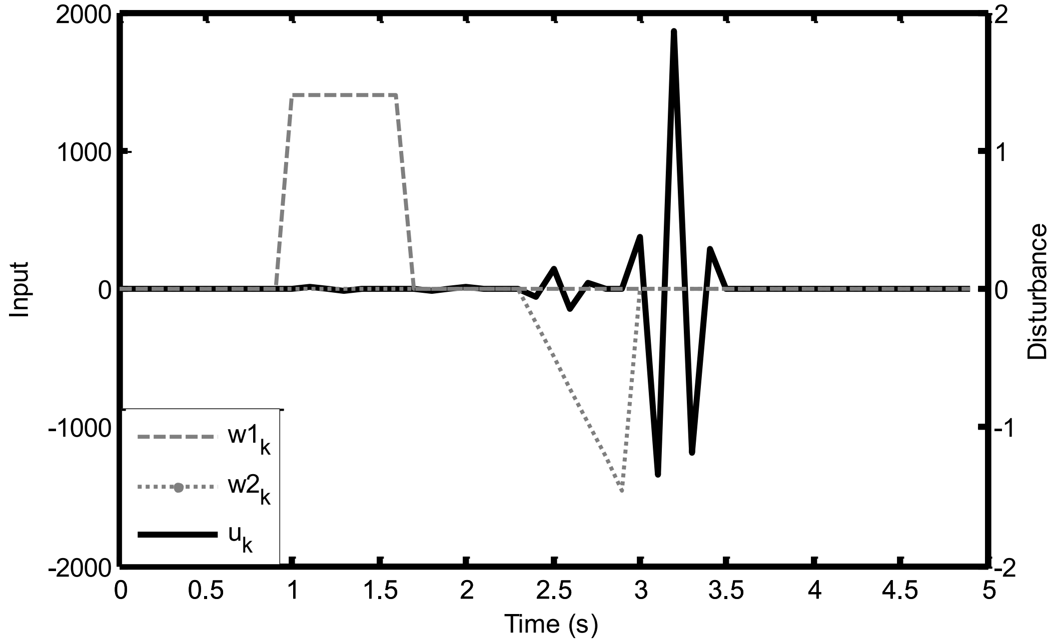

Figure 3.20 Control input for step-type and ramp-type disturbances

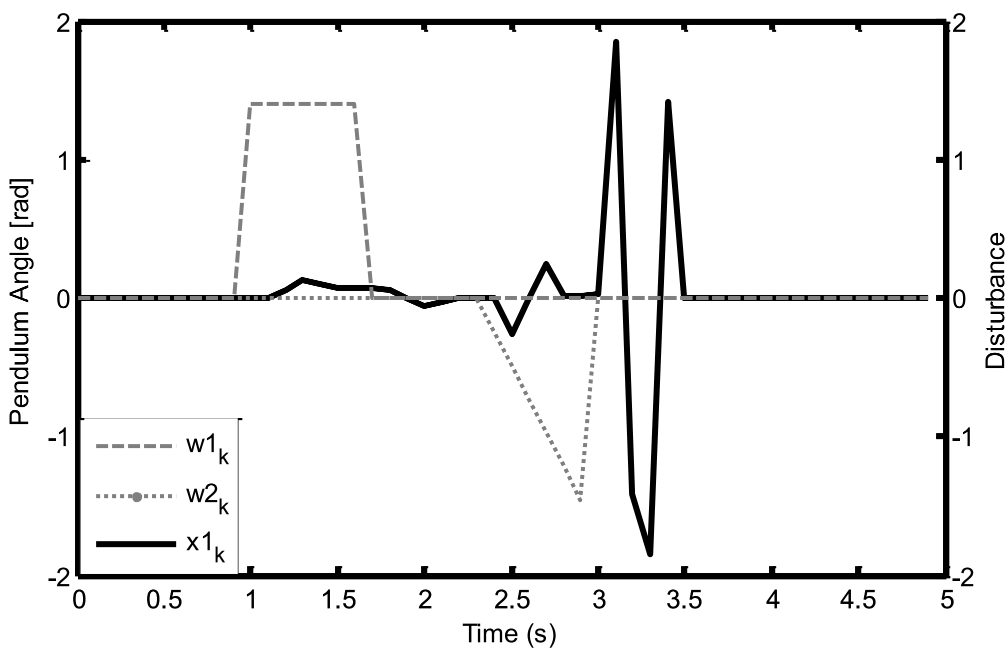

Figure 3.21 Closed loop response of pendulum angle for step-type and ramp-type disturbances

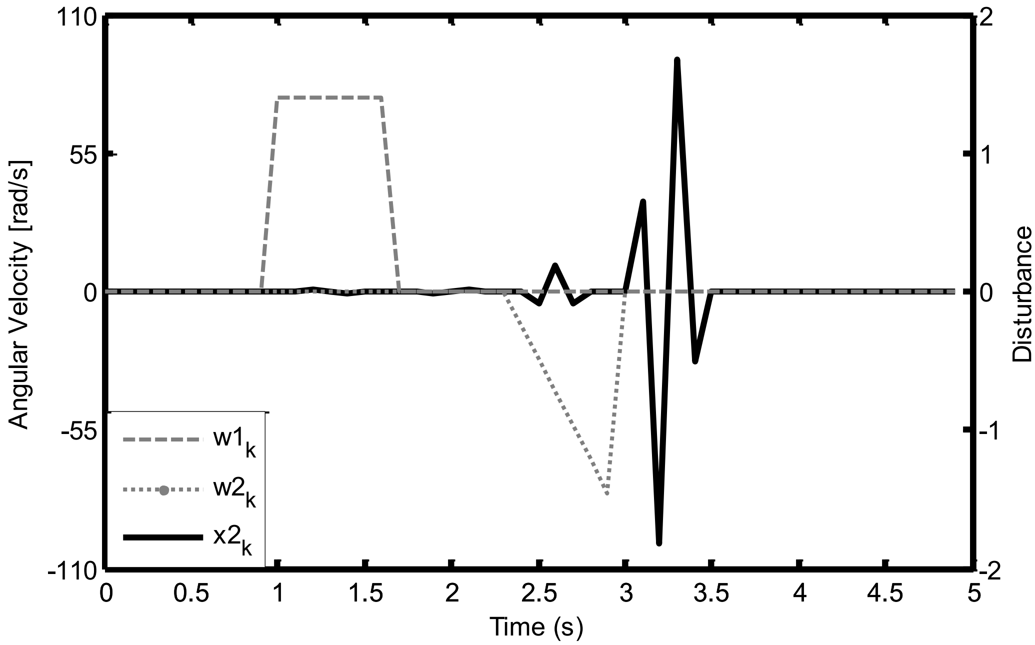

Figure 3.22 Closed loop response of angular velocity for step-type and ramp-type disturbances

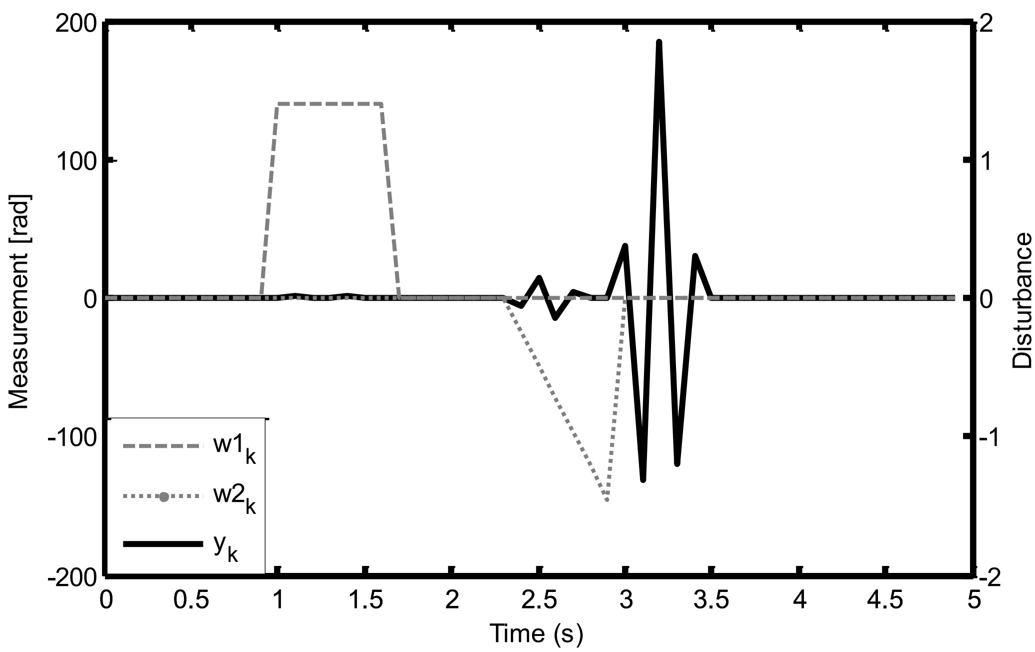

Figure 3.23 Closed loop response of measurement for step-type and ramp-type disturbances

3.1.11 Sinusoidal-Type Disturbance Model

The development of the sinusoidal-type disturbance requires a little more detail. Let

$$w_k^1 = A\sin(\omega kT + \theta) \tag{3.13}$$

$$w_k^2 = A\cos(\omega kT + \theta) \tag{3.14}$$

and consider the disturbance one step later

$$w_{k+1}^1 = A\sin(\omega(k+1)T + \theta) \tag{3.15}$$

$$w_{k+1}^2 = A\cos(\omega(k+1)T + \theta)\,. \tag{3.16}$$

Using trigonometric identities, the two components of the disturbance can be written as

$$\begin{aligned} w_{k+1}^1 &= \underbrace{A\sin(\omega kT + \theta)}_{w_k^1}\cos(\omega T) + \underbrace{A\cos(\omega kT + \theta)}_{w_k^2}\sin(\omega T) = \cos(\omega T)w_k^1 + \sin(\omega T)w_k^2 \\ w_{k+1}^2 &= -\underbrace{A\sin(\omega kT + \theta)}_{w_k^1}\sin(\omega T) + \underbrace{A\cos(\omega kT + \theta)}_{w_k^2}\cos(\omega T) = -\sin(\omega T)w_k^1 + \cos(\omega T)w_k^2 \end{aligned}$$

which results in the following equation where σ_k, the random impulse sequence, has also been added,

$$\begin{bmatrix} w_{k+1}^1 \\ w_{k+1}^2 \end{bmatrix} = \begin{bmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{bmatrix} \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix} + \sigma_k\,. \tag{3.17}$$

In (3.17), the sinusoidal functions are written in terms of the disturbance's radian frequency and the sample time, so it is noted that for sinusoidal disturbances, it is not sufficient to only know the waveform is sinusoidal, the radian frequency must also be known. For this example, the radian frequency is chosen to be 1rad/s and as stated earlier the sampling time, T, is 0.1s resulting in the model,

$$\begin{bmatrix} w_{k+1}^1 \\ w_{k+1}^2 \end{bmatrix} = \begin{bmatrix} \cos(0.1) & \sin(0.1) \\ -\sin(0.1) & \cos(0.1) \end{bmatrix} \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix} + \sigma_k$$

where the impulse sequence causes a sine disturbance with magnitude 3 to occur at 0.5s and last until 1.5s and a cosine disturbance with magnitude 3 to occur at 2.5s and last until 3.5s.

For this example, the disturbance variables are chosen to act on the same state and measurement as in the previous example resulting in the same values in F where the state has the first disturbance state (the sine) applied and the measurement has the second disturbance state (the cosine) applied.

3.1.12 Controller for Pendulum with Sinusoidal-Type Disturbances

As before, an augmented system must be created in order to find one control input for both the state and the measurement. The augmented system is

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} 0.9036 & 0.0968 & 0 \\ -1.8966 & 0.9036 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} 0.0049 & 0 \\ 0.0968 & 0 \\ 0 & 0.2 \end{bmatrix} w_k + \begin{bmatrix} 0.0049 \\ 0.0968 \\ 0.1000 \end{bmatrix} u_k$$

which has an open loop response shown in figures 3.24 through 3.26. The figures show the sine disturbance as the dashed line, the cosine disturbance as the dotted line, and the state or measurement as the solid line. Once the disturbance is applied to the system, the figured show that the system goes into oscillations. The measurement, again, follows the pendulum angle until the disturbance is applied to it, and then it is a distorted version of the pendulum angle until the disturbance is no longer applied.

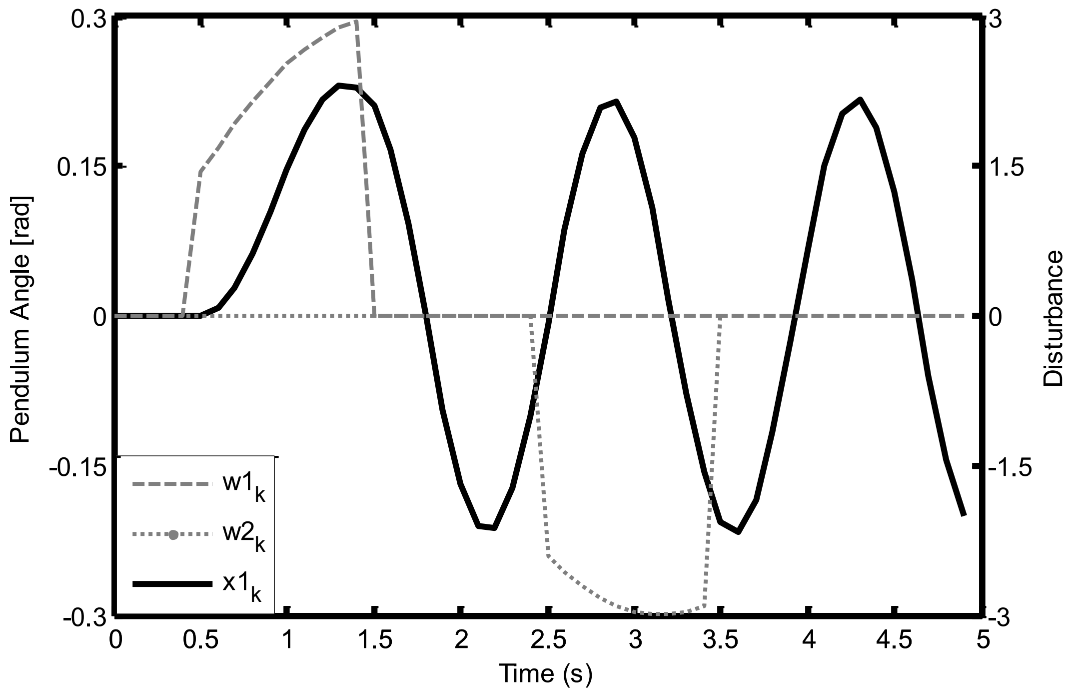

Figure 3.24 Open loop response of pendulum angle for sinusoidal-type disturbances

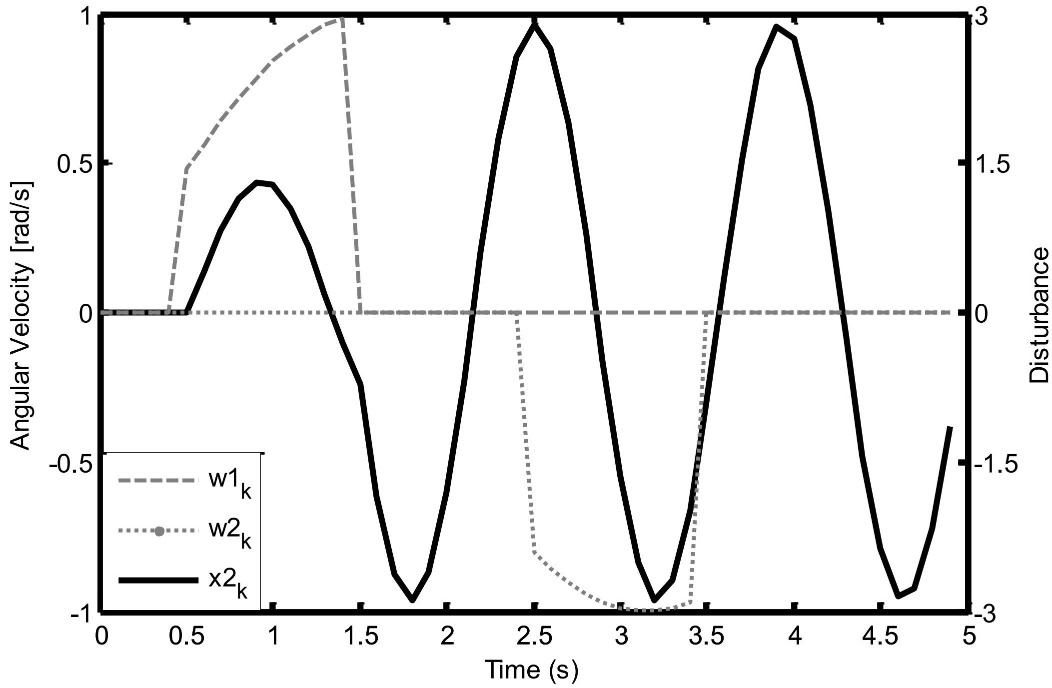

Figure 3.25 Open loop response of angular velocity for sinusoidal-type disturbances

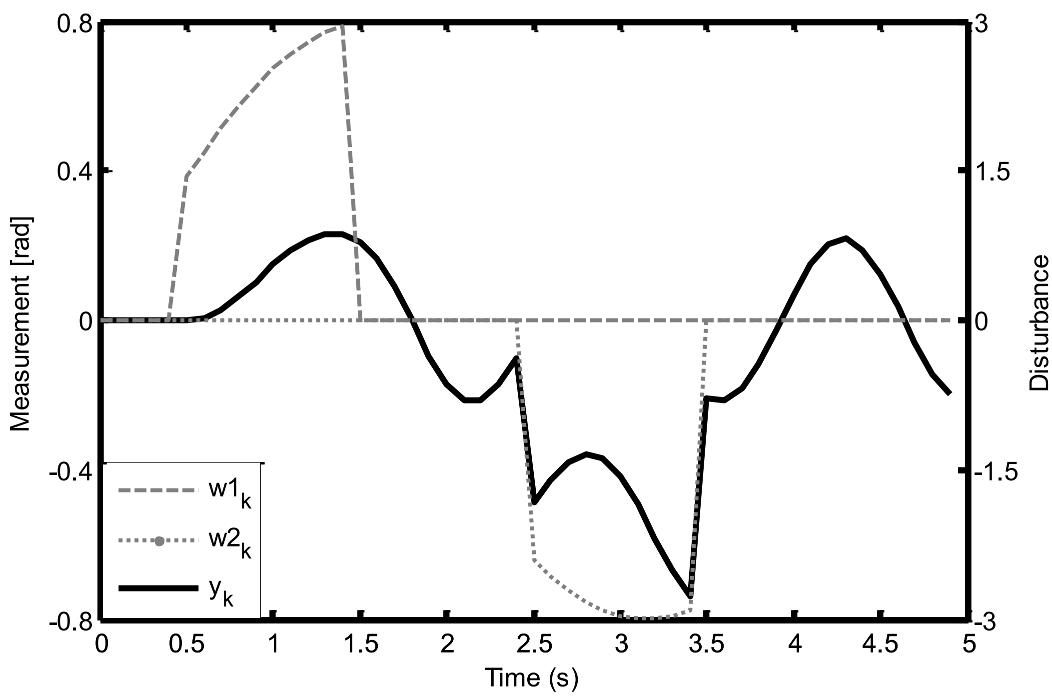

Figure 3.26 Open loop response of measurement for sinusoidal-type disturbances

The state control portion of the controller is the same as the two previous examples, and the only matrix in the system that has changed is the disturbance dynamic matrix, E; therefore the disturbance accommodation control gain is the same as when there was a ramp-type disturbance.

$$L = \begin{bmatrix} -82.0495 & -14.5051 & 0 \end{bmatrix}$$

$$L_d = \begin{bmatrix} -0.4842 & -1.0316 \end{bmatrix}$$

It is important to keep in mind, the F matrix could have changed for this example from the previous one resulting in different controller gains. Also, remember this technique will always place the eigenvalues at zero making this a deadbeat controller.

3.1.13 Reduced-Order Observer for Pendulum with Sinusoidal-Type Disturbances

Following the procedure to design a reduced-order observer for the pendulum system with sinusoidal-type disturbances, a composite vector is created consisting of the variables needing to be estimated,

$$z_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} w_k$$

where x_k and w_k are not available. These variables are replaced by their estimates and the composite vector is augmented with the output equation resulting in

$$\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k$$

which, when rearranged, gives

$$\begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} - \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k \right)$$

allowing the estimates to be determined.

The observability for this augmented system is checked and found to be satisfied. The system is converted to observable canonical form [15],

$$\bar{A}_o = \begin{bmatrix} 0.9036 & 1 & 0 \\ 0.9801 & 0 & 1 \\ -0.8856 & 0 & 0 \end{bmatrix} \text{ and } \bar{C}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

After calculating the canonical form of the observer gain, $\bar{K}_3$, this gain is transformed to the original system's form and the reduced-order observer gains are

$$K_1 = \begin{bmatrix} 0.0059 & 0.2364 & 3.9020 \\ -6.0241 & 1.9318 & 23.7390 \\ 0.2402 & -0.1372 & -1.9377 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -10.2792 \\ -56.2524 \\ 2.2433 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 9.2769 \\ 62.2544 \\ -2.4826 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 1.0790 \\ 5.3190 \\ -0.2121 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} -0.9277 \\ -6.2254 \\ 0.2483 \end{bmatrix}$$

where the eigenvalues of $A_o - K_3 C_o$ are approximately equal to zero.

The reduced-order observer based DAC consists of a third order deadbeat controller and third order deadbeat observer have now been designed to accommodate step-type disturbances on a pendulum system, the maximum amount of time for the DAC to minimize the disturbance is six time samples, or 0.6s.

3.1.14 Simulations of Reduced-Order Deadbeat Observer Based DAC for Pendulum with Sinusoidal-Type Disturbances

The reduced-order deadbeat observer based DAC is applied to the pendulum example that has sinusoidal-type disturbances acting on it. It is expected that the simulations will show a minimization of the disturbance in 0.6s while the disturbance is present. Once the disturbance is no longer acting on the system, the system is expected to reach zero in 0.6s. The results from the simulation are shown in figures 3.27 through 3.30. The control input is shown in figure 3.27. In figures3.28 through 3.30, the simulations show a minimization of the disturbance in 0.6s and once the disturbance is gone, the system is controlled to zero in 0.6s just as expected.

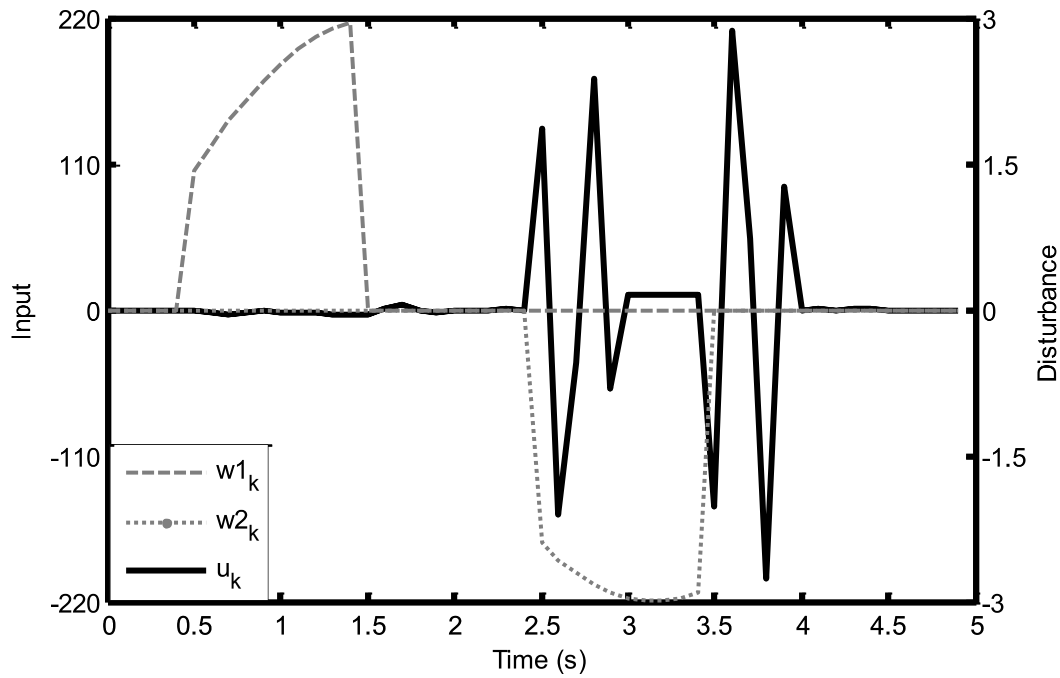

Figure 3.27 Control input for sinusoidal-type disturbances

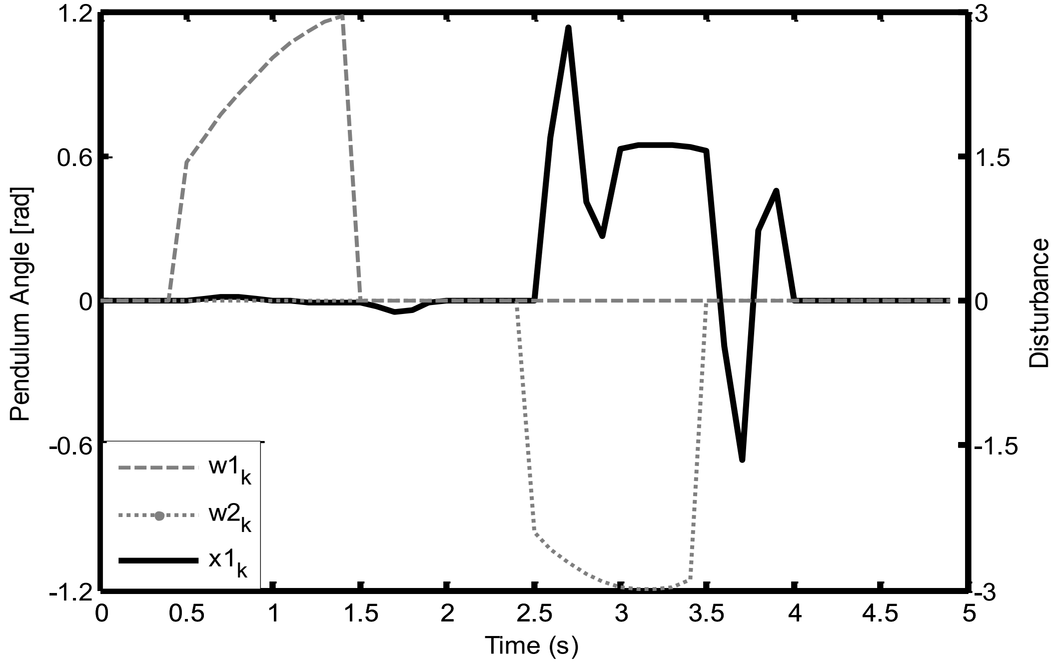

Figure 3.28 Closed loop response of pendulum angle for sinusoidal-type disturbances

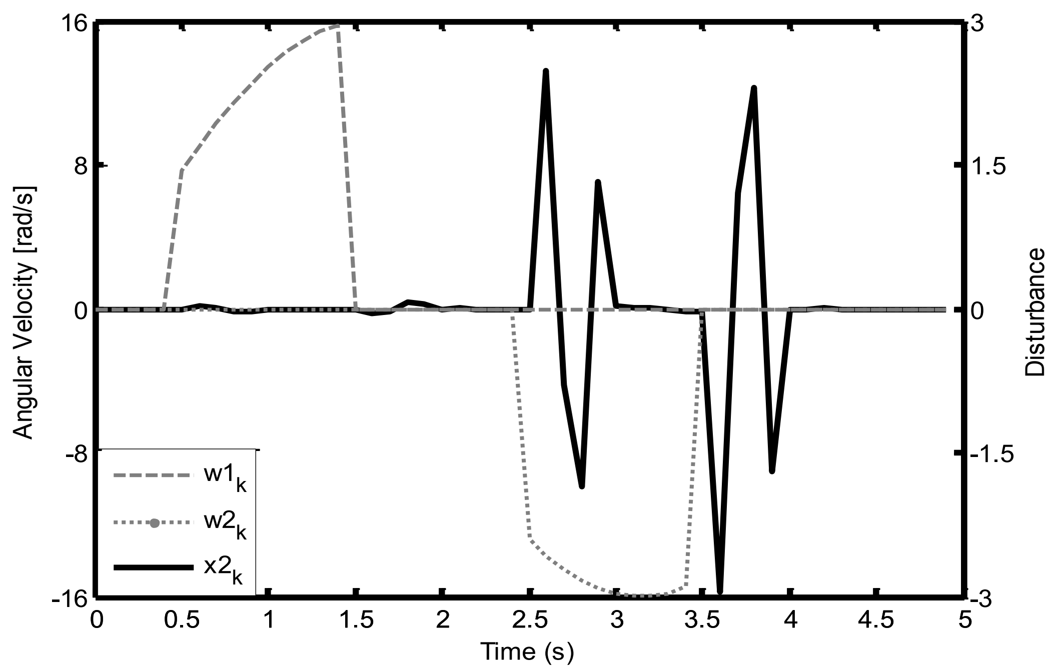

Figure 3.29 Closed loop response of angular velocity for sinusoidal-type disturbances

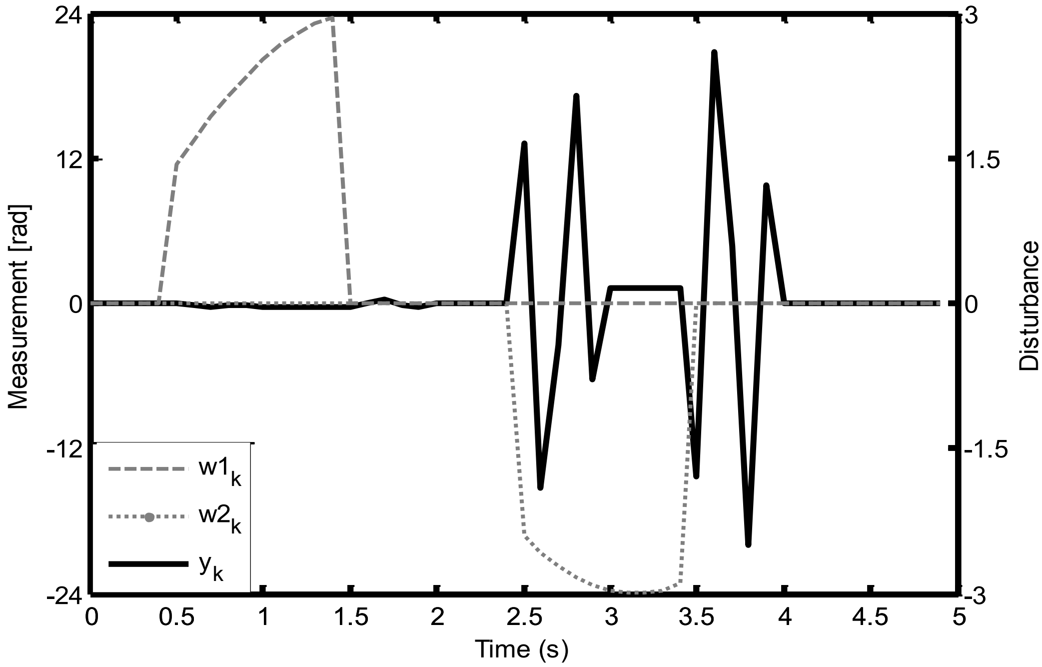

Figure 3.30 Closed loop response of measurement for sinusoidal-type disturbances

3.2 Magnetic Levitation System

3.2.1 System and Step-Type Disturbance Model

The next example is based off a basic magnetic levitation system, which is of third order, seen in figure 3.31. One major application of magnetic levitation systems is the maglev train.

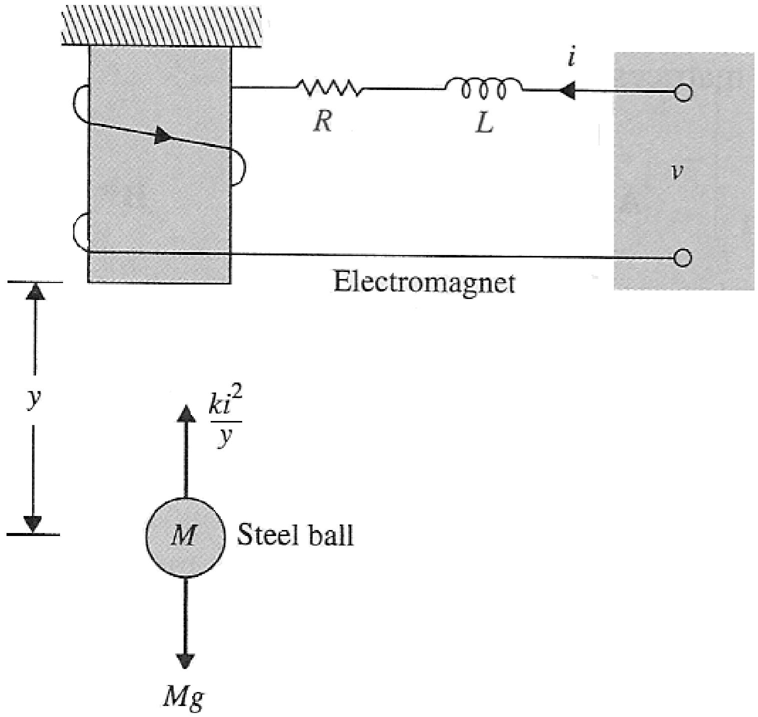

Figure 3.31 Magnetic levitation system [18]

A basic magnetic levitation system is described by the following dynamic equations [18]

$$M\ddot{y} = Mg - \frac{ki^2}{y}$$

$$v = Ri + L\dot{i}\,.$$

These dynamic equations are put into state space form by defining the following state variables

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ x_3 &= i \\ u &= v \end{aligned}.$$

The nonlinear differential equations for these state variables are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - \frac{kx_3^2}{Mx_1}$$
$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{1}{L}u$$

where $R = 1\Omega$, $L = 0.01H$, $k = 1$, and $M = 1kg$. This system is linearized about $x_1 = 0.5m$ and a step-type disturbance is added to the position of the ball and the measurement. This step-type disturbance has the same model that is described in section 3.1.1. The state space equations result in

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 64.4 & 0 & -16 \\ 0 & 0 & -100 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x + 0.1u + 0.2w .$$

This continuous time state space description is then discretized [15] using the sample time $T_s = 0.1s$,

$$x_{k+1} = \begin{bmatrix} 1.3397 & 0.1111 & -0.0157 \\ 7.1538 & 1.3397 & -0.2042 \\ 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} -0.0687 \\ -1.5731 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 0.1111 \\ 0.3397 \\ 0 \end{bmatrix} w_k$$

$$y_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_k + 0.1u_k + 0.2w_k .$$

3.2.2 Controller for Magnetic Levitation with Step-Type Disturbance

An augmented system is created to find the control input for both the state and the measurement. The augmented system is

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} 1.3397 & 0.1111 & -0.0157 & 0 \\ 7.1538 & 1.3397 & -0.2042 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} 0.1111 \\ 0.3397 \\ 0 \\ 0.2 \end{bmatrix} w_k + \begin{bmatrix} -0.0687 \\ -1.5731 \\ 1 \\ 0.1 \end{bmatrix} u_k$$

which has an open loop response shown in figures 3.32 through 3.35. The figures show the moment the disturbance is applied to the position of the ball with no control, the ball goes unstable along with the velocity of the ball. The current does not change because without feedback, it does not realize the ball is no longer at equilibrium.

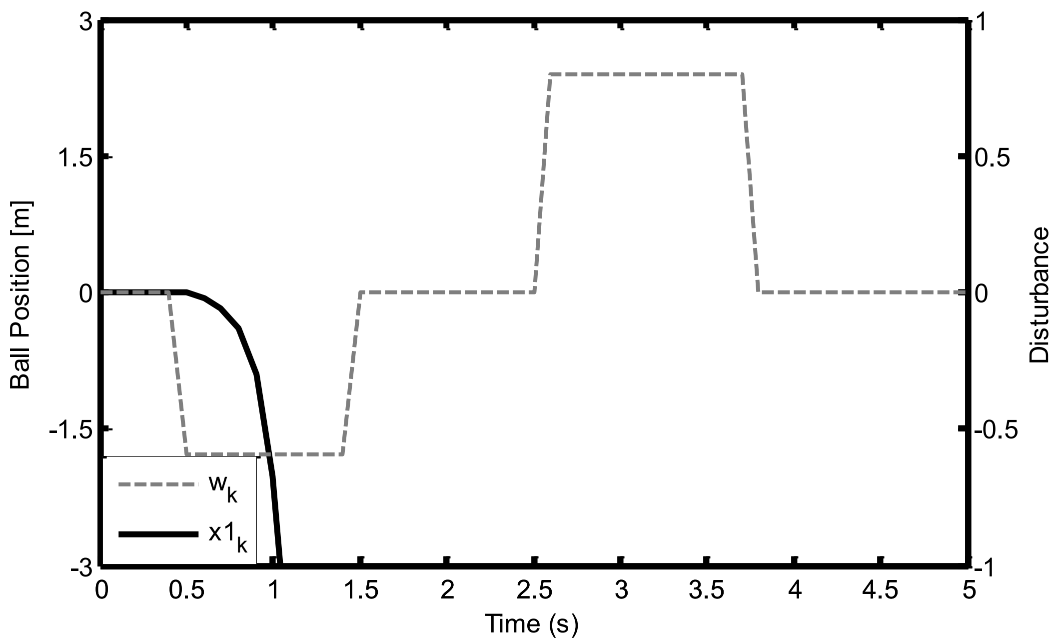

Figure 3.32 Open loop response of position of the ball for step-type disturbances

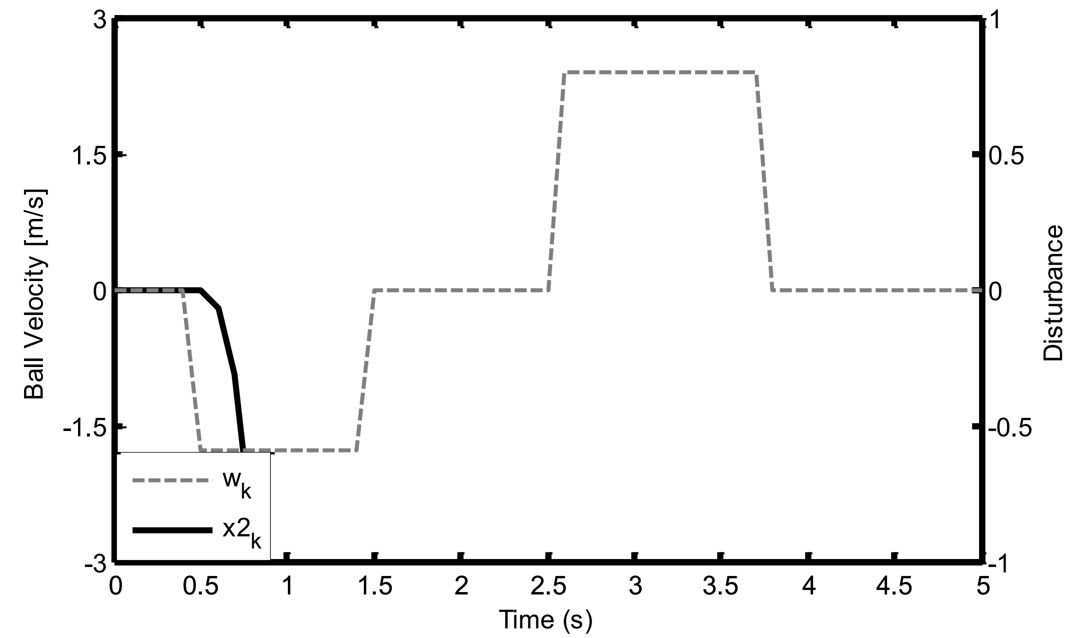

Figure 3.33 Open loop response of velocity of the ball for step-type disturbances

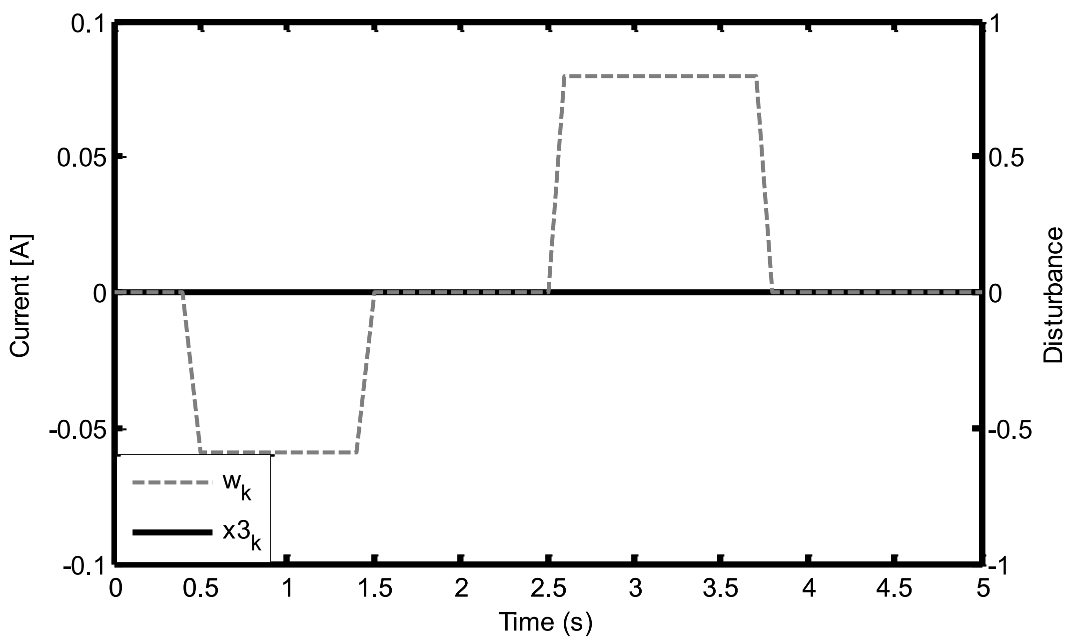

Figure 3.34 Open loop response of current for step-type disturbances

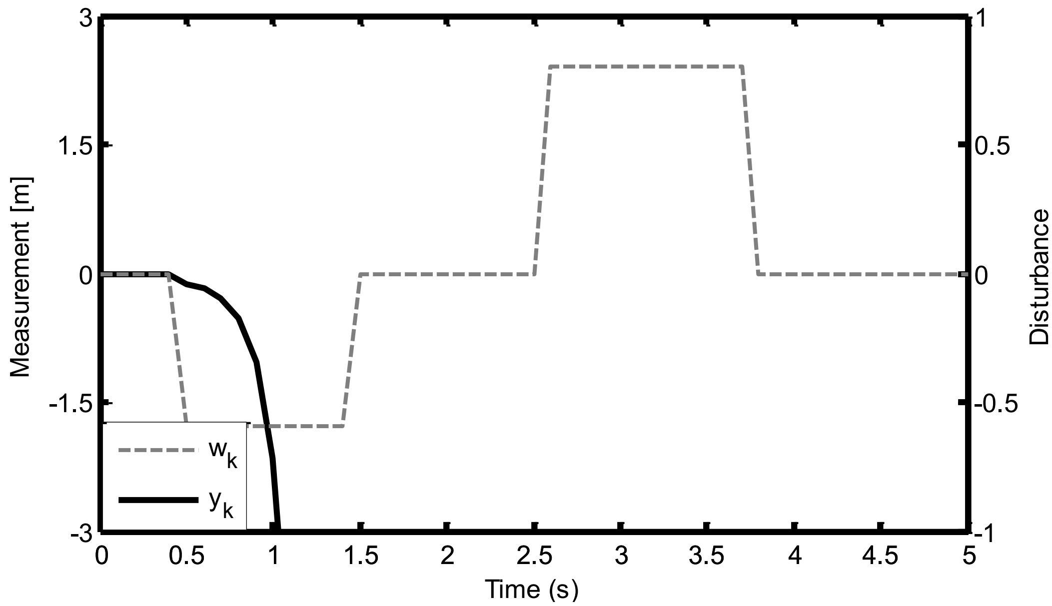

Figure 3.35 Open loop response of measurement for step-type disturbances

This system is controllable and is converted into controllable canonical form [15],

$$\bar{A}_c = \begin{bmatrix} 2.6794 & -1.001 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } \bar{B}_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The state control gains are calculated using the transformation technique [15] along with the disturbance accommodation control gains.

$$\bar{L} = \begin{bmatrix} -2.6794 & 1.0001 & 0 & 0 \end{bmatrix} \quad \therefore \; L = \begin{bmatrix} 10.6170 & 1.1346 & -0.1656 & 0 \end{bmatrix}$$

$$L_d = \begin{bmatrix} 0.1496 \end{bmatrix}$$

The eigenvalues are of $A_c + B_c L$ are approximately zero.

3.2.3 Full-Order Observer for Magnetic Levitation with Step-Type Disturbance

An augmented system is created which consists of the variables to be estimated,

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{w}_{k+1} \end{bmatrix} = \begin{bmatrix} 1.3397 & 0.1111 & -0.0157 & 0.1111 \\ 7.1538 & 1.3397 & -0.2042 & 0.3397 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} -0.0687 \\ -1.5731 \\ 1 \\ 0 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + 0.1 u_k$$

This system is observable and converted into observable canonical form [15].

$$\bar{A}_{fo} = \begin{bmatrix} 3.6794 & 1 & 0 & 0 \\ -3.6795 & 0 & 1 & 0 \\ 1.002 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \bar{\mathrm{C}}_{fo} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

The full-order observer gains result in

$$K = \begin{bmatrix} 5.1515 \\ 47.2322 \\ 0 \\ -7.3607 \end{bmatrix}$$

where the eigenvalues of $A_{fo} - KC_{fo}$ are approximately zero.

A fourth order deadbeat controller and fourth order deadbeat observer have been designed to accommodate step-type disturbances on a magnetic levitation system, the maximum amount of time for the DAC to minimize the disturbance is eight time samples, or 0.8s.

3.2.4 Simulation of Full-Order Deadbeat Observer Based DAC for Magnetic Levitation with Step-Type Disturbance

The full-order deadbeat observer based DAC is applied to the magnetic levitation example that has step-type disturbances acting on it. It is expected that the simulations will show a minimization of the disturbance in 0.8s while the disturbance is present. Once the disturbance is no longer acting on the system, the system is expected to reach zero in 0.8s. The results from the simulation are shown in figures 3.36 through 3.40. In the figures, the simulation shows a minimization of the disturbance in 0.8s and once the disturbance is gone, the system is controlled to zero in 0.8s just as expected. While the disturbance is present, the designed gains are only able to minimize the effect of the disturbance but not drive it all the way to zero. In the previous pendulum example, the input and disturbance were both present in the angular velocity and could be directly accommodated. However, in the magnetic levitation system, the control is on the current and the disturbance is on the position of the ball so the control input is indirectly accommodating the disturbance and causes error to exist while the disturbance is present. This error is most notably seen in figure 3.37.

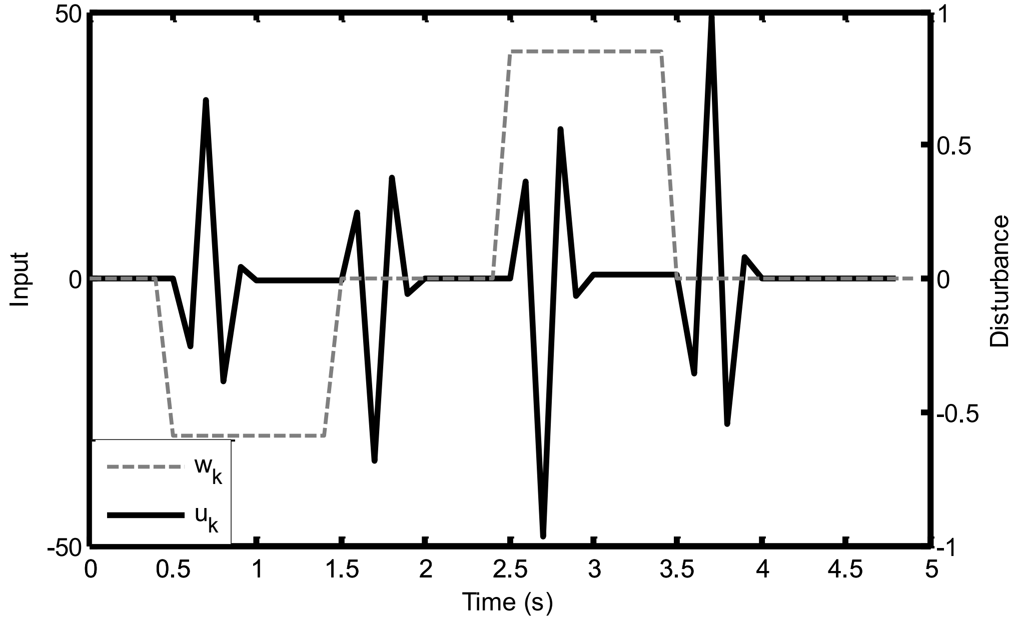

Figure 3.36 Control input for full-order observer based DAC with step-type disturbance on magnetic levitation system

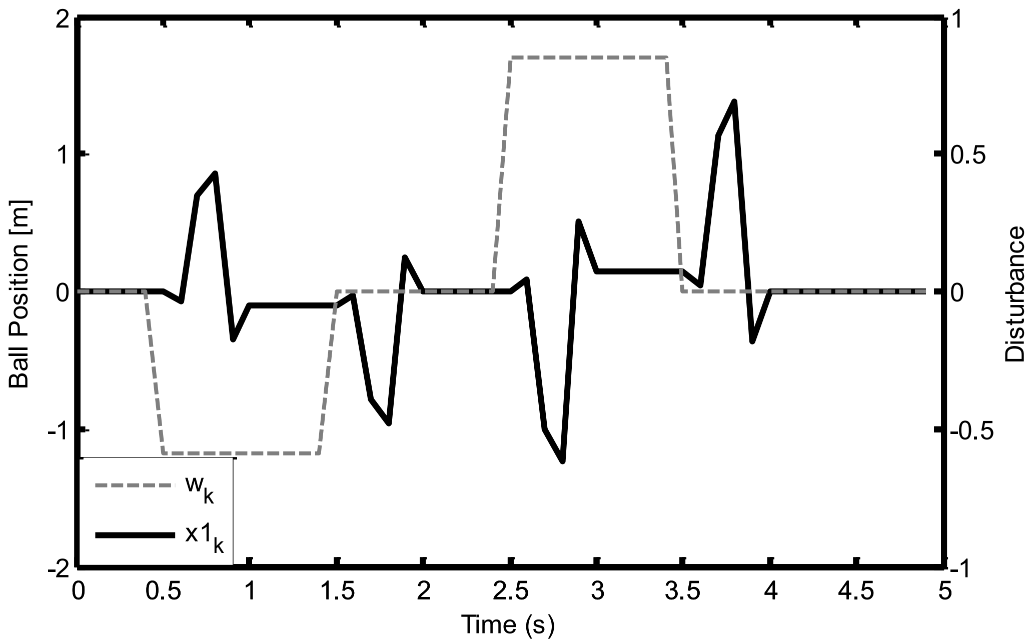

Figure 3.37 Closed loop response of position of the ball for full-order observer based DAC with step-type disturbance

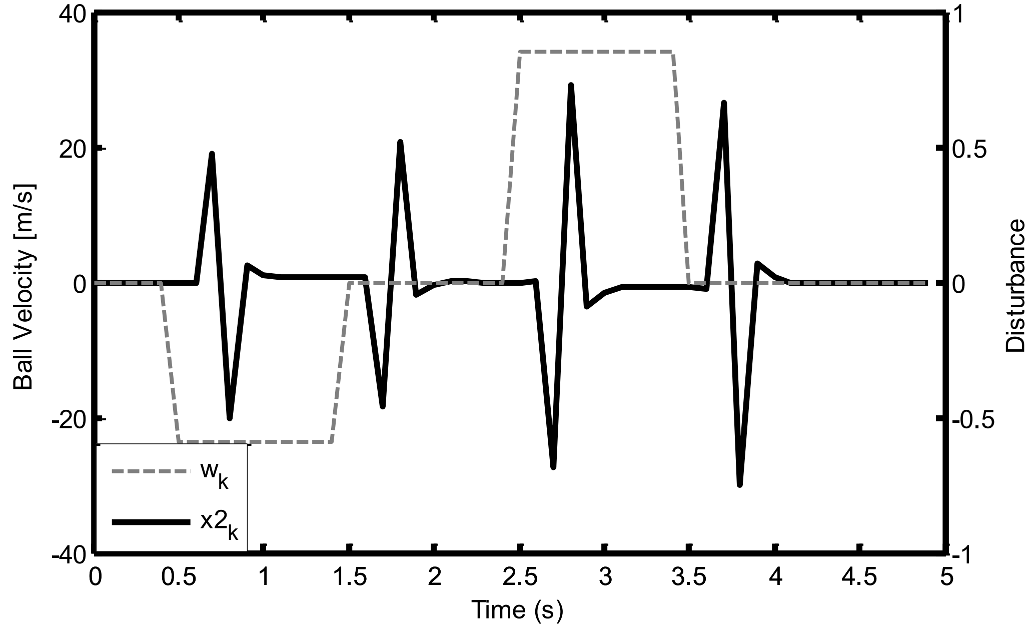

Figure 3.38 Closed loop response of velocity of the ball for full-order observer based DAC with step-type disturbance

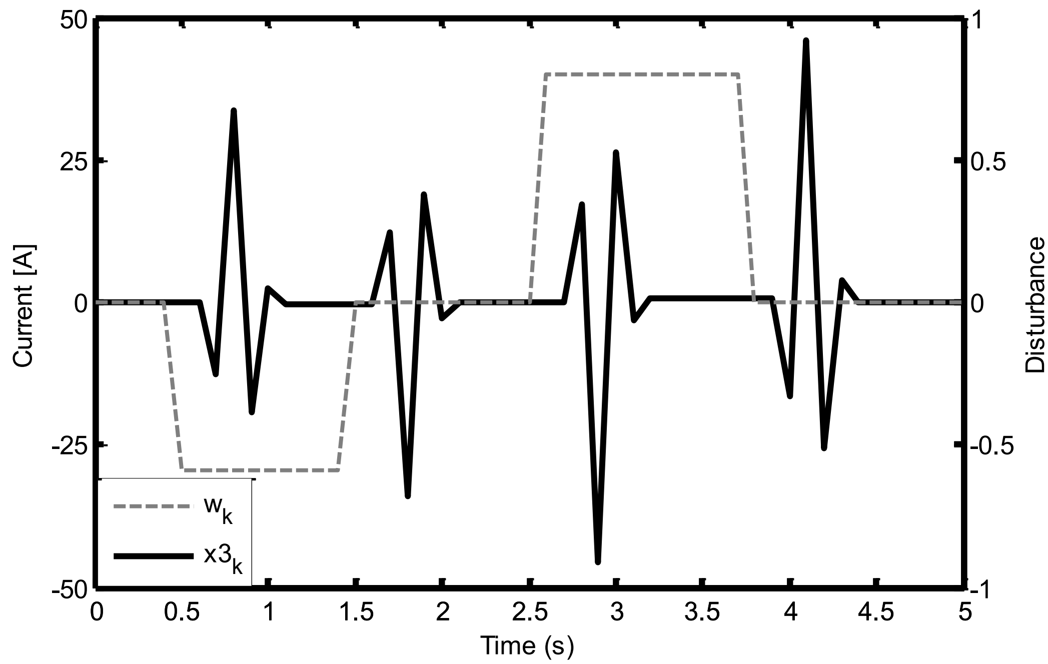

Figure 3.39 Closed loop response of current for full-order observer based DAC with step-type disturbance

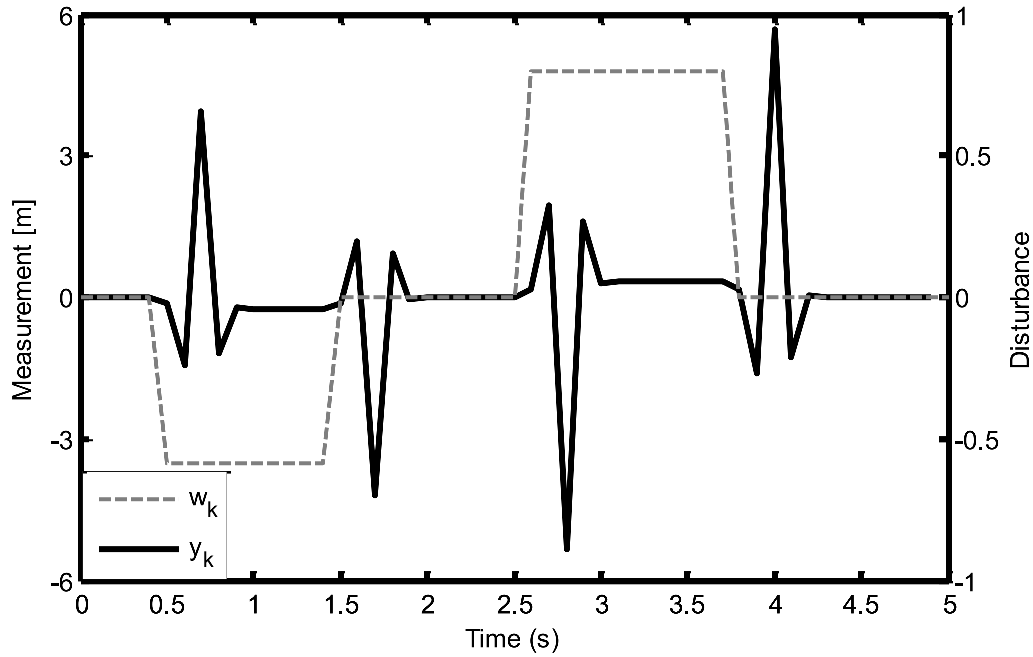

Figure 3.40 Closed loop response of measurement for full-order observer based DAC with step-type disturbance

The reduced-order observer will now be designed and applied to this example.

3.2.5 Reduced-Order Observer for Magnetic Levitation with Step-Type Disturbance

A composite vector is created consisting of the variables needing to be estimated,

$$z_k = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w_k$$

where x_k and w_k are not available. These variables are replaced by their estimates and the composite vector is augmented with the output equation resulting in

$$\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k$$

which, when rearranged, gives

$$\begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} y_k \\ \hat{z}_k \end{bmatrix} - \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_k \right)$$

allowing the estimates to be determined.

This system pair (A_o, C_o) is observable and is converted to observable canonical form [15].

$$\overline{A}_o = \begin{bmatrix} 2.3397 & 1 & 0 \\ -1.3398 & 0 & 1 \\ 0.0001 & 0 & 0 \end{bmatrix} \text{ and } \overline{\mathrm{C}}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The canonical form of the observer gain, $\overline{K}_3$, is calculated and transformed into the original system's form. The reduced-order observer gains are then calculated resulting in

$$K_1 = \begin{bmatrix} -1.3177 & 0.1721 & -2.1234 \\ 0 & 0 & 0 \\ 0.8177 & -0.1158 & 1.3176 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -24.8932 \\ 0 \\ 9.8608 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 23.9218 \\ 0 \\ -7.3607 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 2.5585 \\ 1 \\ -1.4914 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} -2.3922 \\ 0 \\ 0.7361 \end{bmatrix}$$

where the eigenvalues of $A_o - K_3 C_o$ are approximately zero.

A reduced-order observer based DAC consisting of a fourth order deadbeat controller and third order deadbeat observer has been designed to accommodate step-type disturbances on a magnetic levitation system, the maximum amount of time for the DAC to minimize the disturbance is seven time samples, or 0.7s.

3.2.6 Simulation of Reduced-Order Deadbeat Observer Based DAC for Magnetic Levitation with Step-Type Disturbance

The reduced-order deadbeat observer based DAC is applied to the magnetic levitation example that has step-type disturbances acting on it. It is expected that the simulations will show a minimization of the disturbance in 0.7s while the disturbance is present. Once the disturbance is no longer acting on the system, the system is expected to reach zero in 0.7s. The results from the simulation are shown in figures 3.41 through 3.45. Figure 3.41 is the control input. In figures 3.42 through 3.45, the simulations show a minimization of the disturbance in 0.7s and once the disturbance is gone, the system is controlled to zero in 0.7s just as expected.

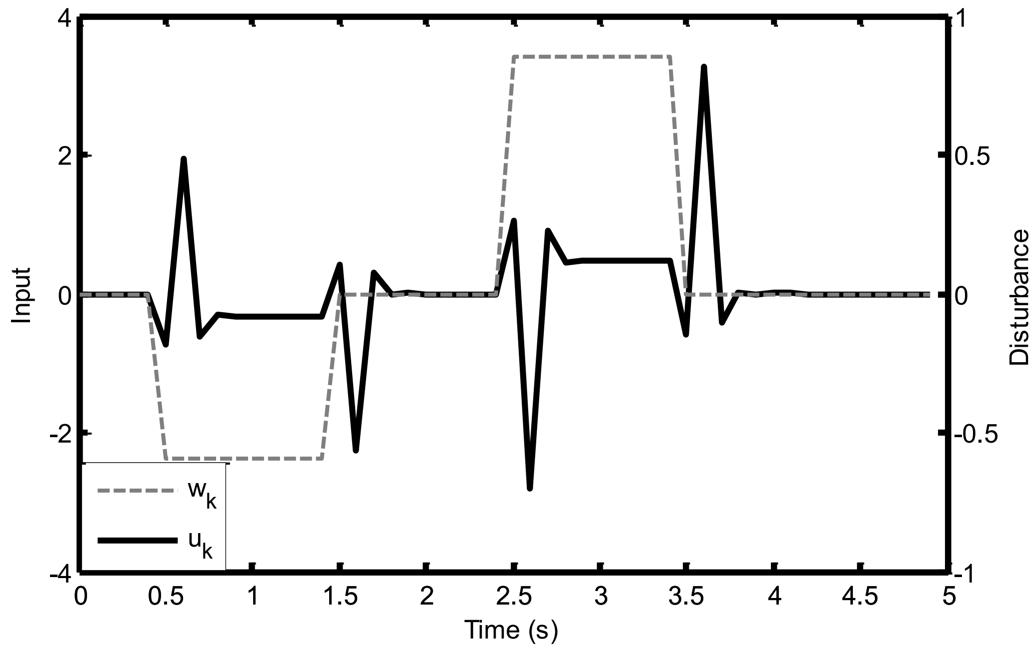

Figure 3.41 Control input for reduced-order observer based DAC with step-type disturbance on magnetic levitation system

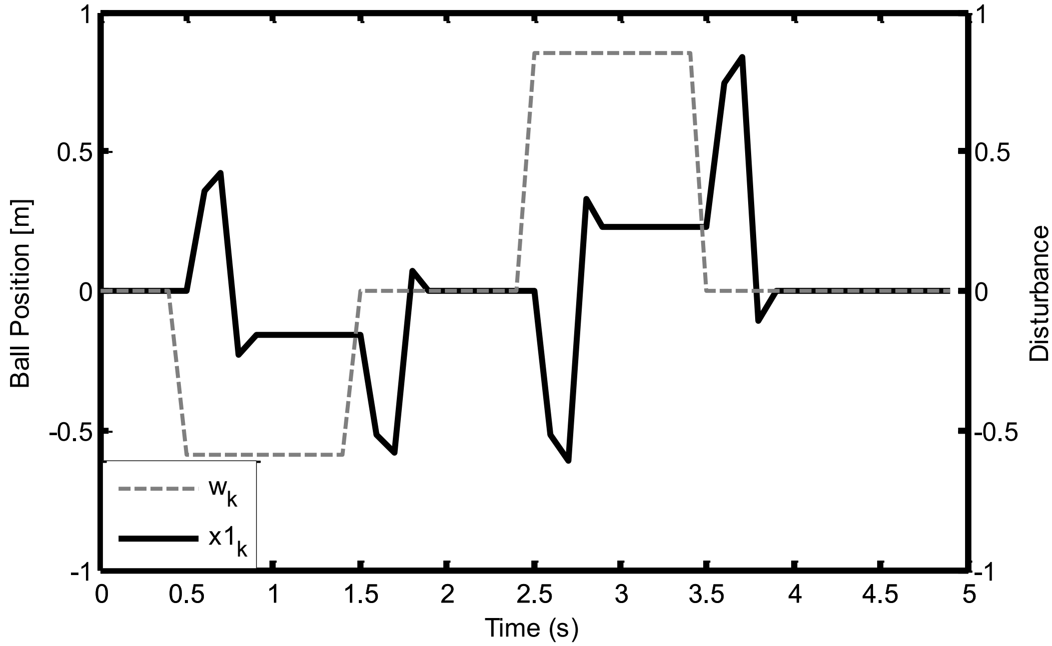

Figure 3.42 Closed loop response of position of the ball for reduced-order observer based DAC with step-type disturbance

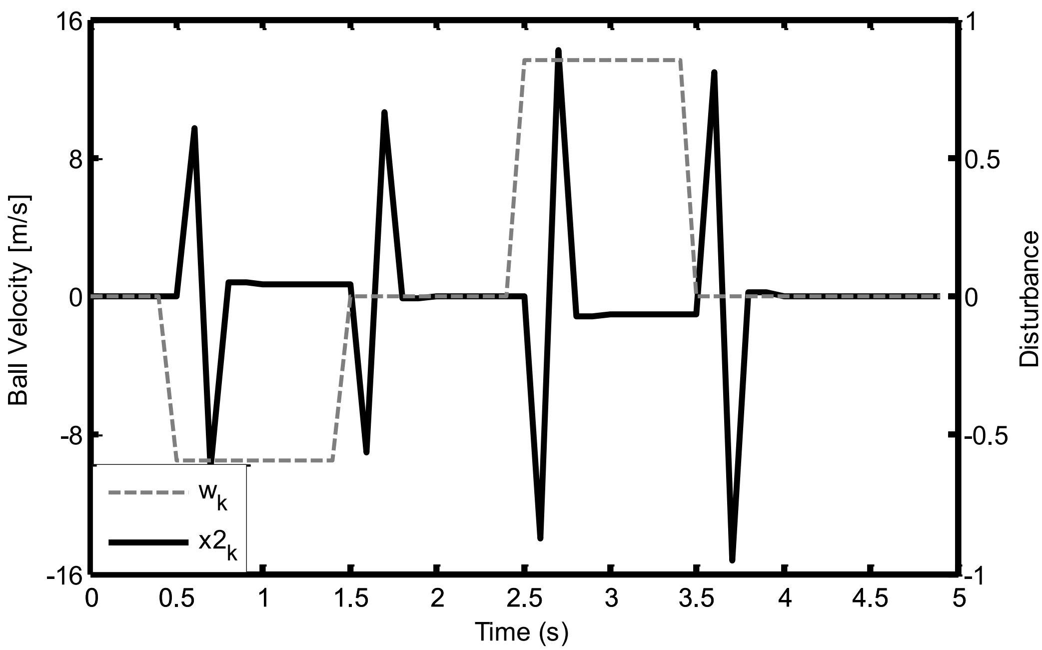

Figure 3.43 Closed loop response of velocity of the ball for reduced-order observer based DAC with step-type disturbance

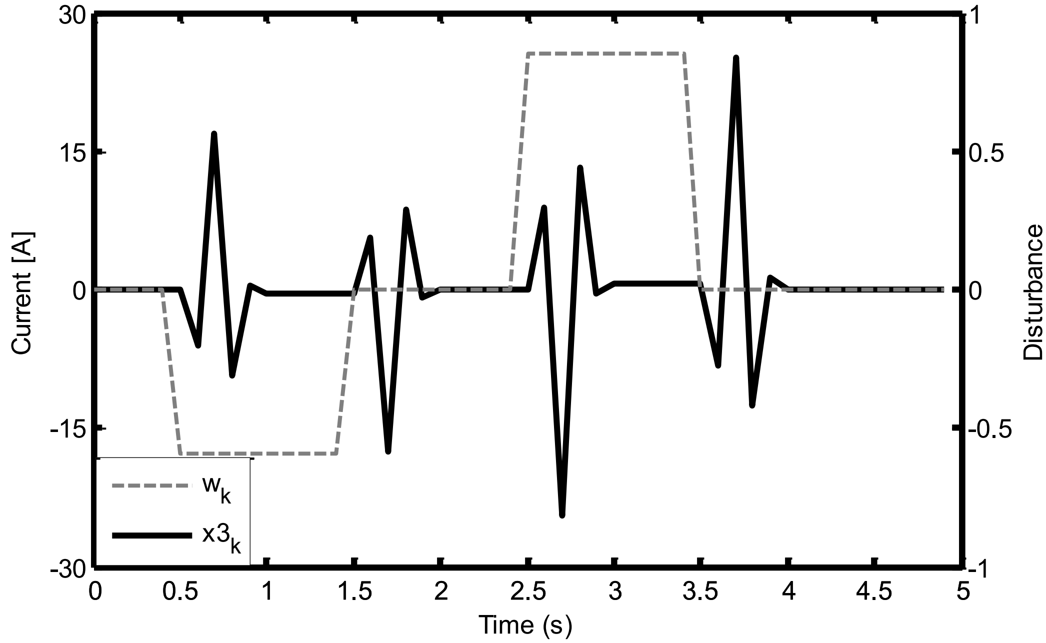

Figure 3.44 Closed loop response of current for reduced-order observer based DAC with step-type disturbance

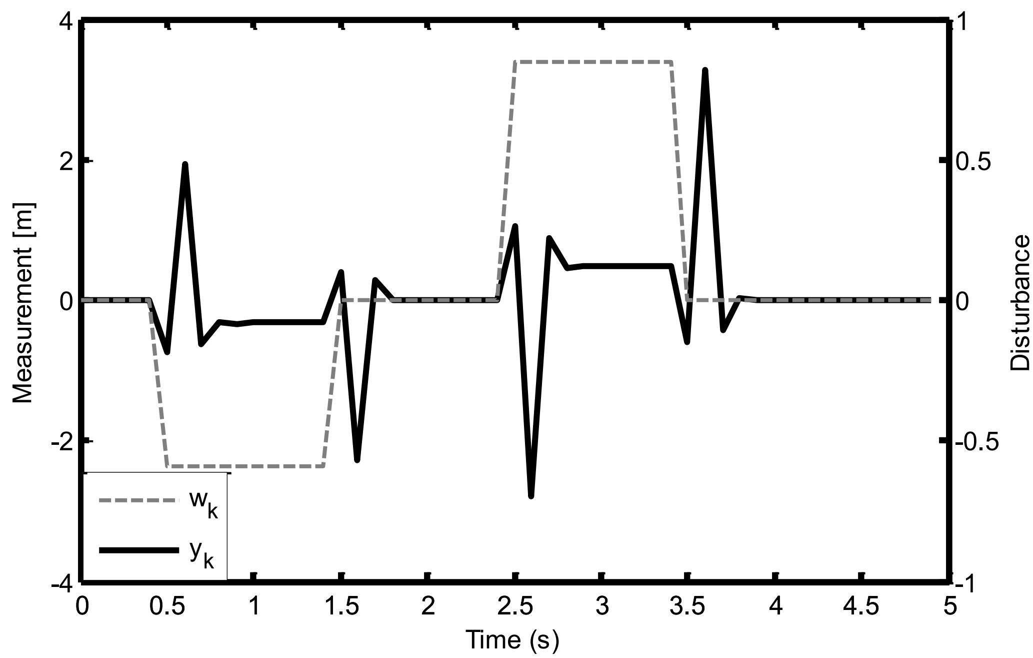

Figure 3.45 Closed loop response of measurement for reduced-order observer based DAC with step-type disturbance

3.3 Conclusion

In this chapter, several observer based deadbeat DACs have been designed. Four were designed for a pendulum system and two for a magnetic levitation system. Three different types of disturbances were considered: (1) step-type disturbances, (2) ramp-type disturbances, and (3) sinusoidal-type disturbances. First, each of the models were found, linearized, put into state space form, and discretized. Then, an observer based deadbeat DAC that minimized the effect of the disturbance in the minimal amount of time was designed for the given case. Once the design of the observer based DAC was completed and applied to the system, the system response showed the effect of the disturbance being minimized as quickly as possible. The full-order based DAC minimized the disturbance in $\left(2n_x + n_w + 1\right)$ steps while the reduced-order based DAC minimized the disturbance in $\left(2n_x + n_w\right)$ steps. In the next chapter, system conditions that must be met for this technique to work are developed and an extension is created for when one of the system conditions is not met.

4 CONDITIONS AND EXTENSION

In this chapter, system conditions for the developed control technique are derived to allow for a user to test the given system and decide if the technique will produce desirable results. When one of the system conditions, namely the feed-forward term, is not met, an extension is derived that minimizes the effect of the disturbance on the measurement. The extension is then compared to the results of the original technique and conclusions are made.

4.1 System Conditions for Proposed Control Scheme

In chapter two, a DAC was derived which made use of the following control input which consists of two parts,

$$u_k = L\begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + L_d w_k \,, \tag{4.1}$$

where L is the controller gain to drive the state variables to zero and L_d is the controller gain to minimize the disturbance.

This control input is substituted into the system equation (2.4a),

$$\begin{bmatrix} x_{k+1} \\ y_k \end{bmatrix} = \left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L \right) \begin{bmatrix} x_k \\ y_{k-1} \end{bmatrix} + \left(\begin{bmatrix} F \\ G_1 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L_d \right) w_k \,, \tag{4.2}$$

where the effect of this control input should drive the state variables to zero. In general, the control gains can be derived by considering the minimum norm solution for

$\left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L\right)$ and $\left(\begin{bmatrix} F \\ G_1 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L_d\right)$, which are given in (2.7a) and (2.7b) in chapter two. In order to be able to have a gain L that will place the eigenvalues of $\left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} L\right)$ at the desired values, the system pair $\left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix}, \begin{bmatrix} B \\ D \end{bmatrix}\right)$ needs to be controllable.

The controllability of the system pair $\left(\begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix}, \begin{bmatrix} B \\ D \end{bmatrix}\right)$ is checked by analyzing the controllability matrix of the system. If the determinant of the controllability matrix of the system is zero, the system pair is not controllable; therefore, to set conditions on the SISO system matrices, the determinant of the controllability matrix is analyzed. The controllability matrix for this augmented system is

$$W_c = \begin{bmatrix} B_c & A_c B_c & \cdots & A_c^{\,n-2} B_c & A_c^{\,n-1} B_c \end{bmatrix} \quad \text{for } A_c \in \Re^{n \text{ x } n} \tag{4.3}$$

where

$$A_c = \begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} \in \Re^{(n_x+1) \text{ x } (n_x+1)} \tag{4.4}$$

$$B_c = \begin{bmatrix} B \\ D \end{bmatrix} \tag{4.5}$$

which results in the following controllability matrix,

$$W_c = \begin{bmatrix} B & AB & A^2 B & \cdots & A^{n_x} B \\ D & C_1 B & C_1 AB & & C_1 A^{n_x - 1} B \end{bmatrix}. \tag{4.6}$$

Since the controllability matrix is being analyzed for conditions that will ensure it does not equal zero, the columns are rearranged into a form that is easily simplified. The column order can be rearranged without changing the overall result because the determinant of a matrix does not change from a nonzero value to zero by changing the column order in the matrix. The controllability matrix simplifies to

$$\begin{bmatrix} A\mathbb{C} & B \\ C_1\mathbb{C} & D \end{bmatrix}$$

where

$$\mathbb{C} = \begin{bmatrix} B & AB & \cdots & A^{n_x-2}B & A^{n_x-1}B \end{bmatrix} . \tag{4.7}$$

Now, the determinant of this matrix can be analyzed,

$$\begin{aligned} \left| \begin{bmatrix} A\mathbb{C} & B \\ C_1\mathbb{C} & D \end{bmatrix} \right| &= D \left| A\mathbb{C} - \frac{BC_1\mathbb{C}}{D} \right| \\ &= D \left| A - \frac{BC_1}{D} \right| \left| \mathbb{C} \right| \end{aligned}$$

and if and only if the following conditions hold

$$\begin{aligned} &(1) \;\; D \neq 0 \\ &(2) \;\; |\mathbb{C}| \neq 0 \Rightarrow (A, B) \text{ controllable} \\ &(3) \;\; \left| A - \frac{BC_1}{D} \right| \neq 0 \end{aligned}$$

will $W_c \neq 0$.

The third statement is analyzed further by considering a third order example where controllable canonical forms [15] of matrices A and B are used,

$$\left|\bar{A}-\frac{\bar{B}C_1}{D}\right| = \left| \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} c_{11} & c_{12} & c_{13} \end{bmatrix}}{D} \right|$$

$$= \left| \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} - \left(\frac{1}{D}\right)\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} -a_1-\frac{c_{11}}{D} & -a_2-\frac{c_{12}}{D} & -a_3-\frac{c_{13}}{D} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right|$$

$$= -a_3 - \frac{c_{13}}{D}$$

This resulting term should be non-zero which implies $a_3 \neq -\frac{c_{13}}{D}$ and in general,

$$a_n \neq -\frac{c_{1n}}{D}.$$

Thus, it can be stated that this technique will work for systems that satisfy the following three conditions:

1. $D \neq 0$
2. $|\mathbb{C}| \neq 0 \Rightarrow (A, B)$ controllable

3. $\left|A-\frac{BC_1}{D}\right| \neq 0 \Rightarrow a_n \neq \frac{c_{1n}}{D}$.

However, for many systems there is no feed-forward term ($D=0$) , which would imply this technique will not accommodate the effect of the disturbance in the measurement. This technique is extended in a way that allows it to work for systems that do not have a feed-forward term.

4.2 Control Scheme for Systems with No Feed-forward Term

Consider the following system with known waveform-type disturbances,

$$x_{k+1} = Ax_k + Bu_k + Fw_k \tag{4.8}$$

$$y_k = Cx_k + Gw_k \tag{4.9}$$

$$w_{k+1} = Ew_k + \sigma_k \, . \tag{4.10}$$

Having no feed-forward term is an issue because when a disturbance is present in the measurement, there is no control term to minimize the disturbance's effect. This issue is taken care of by introducing a term similar to the pseudo-output [9] introduced by A. Azemi and E. Yaz consisting of the current measurement and the previous control input,

$$z_k = \phi y_k + \gamma u_{k-1} \tag{4.11}$$

where the dynamic equation of this term is

$$z_{k+1} = \phi CAx_k + \left(\phi CB + \gamma\right)u_k + \left(\phi\left(CF + GE\right)\right)w_k \, . \tag{4.12}$$

This dynamic equation is augmented with the system equation allowing for one control input to be designed which can control both the system state and the measurement indirectly through z_k,

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ \phi CA & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix} u_k + \begin{bmatrix} F \\ \phi\left(CF + GE\right) \end{bmatrix} w_k \,. \tag{4.13}$$

Let the control input include terms proportional to the system state, the disturbance, and the pseudo-output:

$$u_k = L_{c1} x_k + L_{c2} z_k + L_d w_k \tag{4.14}$$

and substitute u_k into (4.13),

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \left(\begin{bmatrix} A & 0 \\ \phi CA & 0 \end{bmatrix} + \begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix} \begin{bmatrix} L_{c1} & L_{c2} \end{bmatrix} \right) \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \left(\begin{bmatrix} F \\ \phi\left(CF + GE\right) \end{bmatrix} + \begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix} L_d \right) w_k \,. \tag{4.15}$$

The same technique in chapter two can now be used to solve for the controller gains. The minimum norm solution for the state and disturbance control inputs are obtained as

$$\begin{bmatrix} L_{c1} & L_{c2} \end{bmatrix} = -\begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix}^{\dagger} \begin{bmatrix} A & 0 \\ \phi CA & 0 \end{bmatrix} \tag{4.16}$$

$$L_d = -\begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix}^{\dagger} \begin{bmatrix} F \\ \phi\left(CF + GE\right) \end{bmatrix}. \tag{4.17}$$

Next, as in the previous section, the controllability of the state control system is analyzed. The controllability matrix of the system pair $\left(\begin{bmatrix} A & 0 \\ \phi CA & 0 \end{bmatrix}, \begin{bmatrix} B \\ \phi CB + \gamma \end{bmatrix} \right)$ is

$$W_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n_xB} \\ \phi CB+\gamma & \phi CAB & \phi CA^2B & & \phi CA^{n_x}B \end{bmatrix} \tag{4.18}$$

where

$$\left|\begin{bmatrix} B & AB & A^2B & \cdots & A^{n_xB} \\ \phi CB+\gamma & \phi CAB & \phi CA^2B & & \phi CA^{n_x}B \end{bmatrix}\right| \neq 0$$

for this system pair to be controllable. Again, the columns of this controllability matrix can be rearranged to allow for simplification without changing the result,

$$\left|\begin{bmatrix} A\mathbb{C} & B \\ \phi CA\mathbb{C} & \phi CB+\gamma \end{bmatrix}\right| \neq 0$$

for $\mathbb{C}$ defined in (4.7). The determinant is analyzed further to show what conditions must be met for this system to be controllable:

$$\begin{aligned} \left|\begin{bmatrix} A\mathbb{C} & B \\ \phi CA\mathbb{C} & \phi CB+\gamma \end{bmatrix}\right| &= |\phi CB+\gamma|\left|A\mathbb{C} - \frac{B\phi CA\mathbb{C}}{\gamma+\phi CB}\right| \\ &= |\phi CB+\gamma|\left|I_{n_x} - \frac{BC}{\frac{\gamma}{\phi}+CB}\right||A||\mathbb{C}| \end{aligned}.$$

From here, the second term is analyzed more by using a third order example with the controllable canonical form of B,

$$\left| I_3 - \frac{\bar{B}C}{\frac{\gamma}{\phi} + C\bar{B}} \right| = \left| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \end{bmatrix}}{\frac{\gamma}{\phi} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \right|$$

$$= \left| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left(\frac{1}{\frac{\gamma}{\phi} + c_{11}} \right) \right|$$

$$= \left| \begin{bmatrix} 1 - \frac{c_{11}}{\frac{\gamma}{\phi} + c_{11}} & -\frac{c_{12}}{\frac{\gamma}{\phi} + c_{11}} & -\frac{c_{13}}{\frac{\gamma}{\phi} + c_{11}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right|$$

$$= 1 - \frac{1}{\frac{\gamma}{c_{11}\phi} + 1}$$

This result implies $\frac{\gamma}{c_{11}\phi} \neq 0$. From the given statement, it is determined that γ should not equal zero and $c_{11}\phi$ should not be much greater than γ for this condition to hold.

Thus it can be stated that for this formulation to work for this system the following conditions must be met:

1. $\gamma + \phi CB \neq 0 \Rightarrow \gamma \neq -\phi CB$

2. $\frac{\gamma}{c_{11}\phi} \neq 0$

3. $|A| \neq 0$

4. $|\mathbb{C}| \neq 0 \Rightarrow (A, B)$ controllable.

The first two conditions help the user choose appropriate phi and gamma values and the last two conditions are on the system. If these conditions are met, the developed extension can be used to give more desirable results than the original technique in the absence of a feed-forward term.

4.3 Simulations and Analysis

A system is created with a variable feed-forward term, D, so comparisons can be made when this term is present versus not present,

$$x_{k+1} = \begin{bmatrix} 0.9 & 0.7 \\ -1.8 & 0.9 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} w_k$$

$$y_{k+1} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + Du_k + \begin{bmatrix} 0 & 1 \end{bmatrix} w_k$$

$$w_{k+1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} w_k + \sigma_k.$$

4.3.1 Original Technique with a Feed-Forward Term Present

The conditions mentioned in section 4.1 are checked for this system:

1. $D \neq 0$? Yes, $D = 1$.

2. System pair (A, B) controllable? This is checked by looking at the eigenvalues of the matrix resulting from the product of the controllability matrix and its transpose,

$$\lambda_i\left(W_c W_c^T\right) = \begin{bmatrix} 0.2135 \\ 1.9749 \\ 4.9793 \end{bmatrix}.$$

   None of the eigenvalues are close to zero that implies this system pair is controllable.

3. $a_n \neq -\frac{c_{1n}}{D}$? This is determined by looking at the characteristic equation's coefficients to get $a_2 = 2.0700$ and $-\frac{c_{12}}{D} = 0$. Therefore no matter what value D is, this condition is met.

Now that the system has proven to meet the conditions for the given system, the original proposed technique in this thesis is used to develop a controller where the control input is

$$u_k = Lx_k + L_d w_k \ .$$

The simulation shown in figure 4.1 (and a zoomed in view in figure 4.2) displays that when the feed-forward term is present, the measurement is close to the expected value (zero) while the disturbance is present. The simulation shows that this controller is able to minimize the effect of the disturbances in the measurement with a maximum overshoot magnitude of 106.1 rad and a maximum error of 0.0734 rad while the disturbance is present.

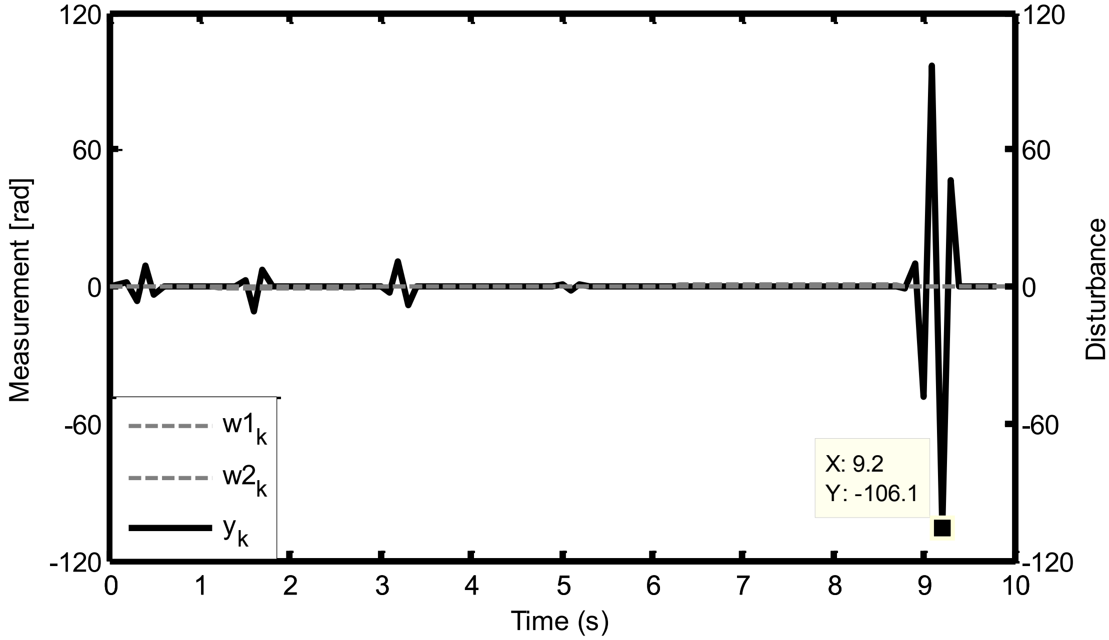

Figure 4.1 (Original technique) Controller response of the measurement (solid) co-plotted with the disturbances when $D=1$

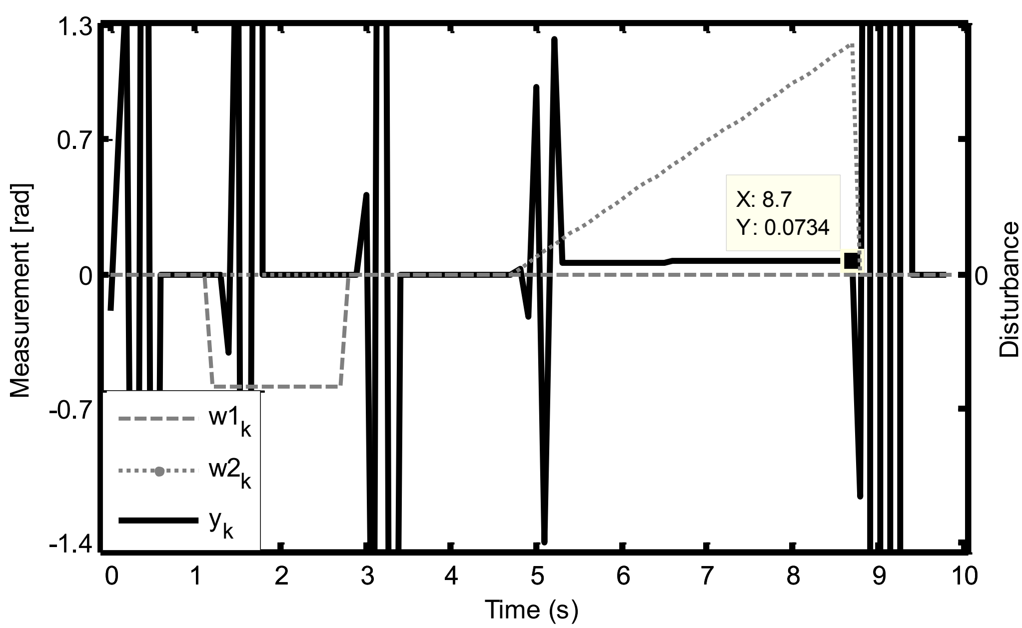

Figure 4.2 (Original technique) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D=1$

4.3.2 Original Technique with a Feed-Forward Term Absent

One of the conditions for this technique is $D \neq 0$ so it is expected that when the feed-forward term is gone, the simulation will show a worse result in the measurement. This result can be seen in figure 4.3 (with a zoomed in view in figure 4.4). When the feed-forward term is gone, the result of the measurement shows that this controller technique is unable to minimize the disturbance present in the measurement (keeping in mind the step disturbance that is minimized is present in the state and the ramp disturbance which is not minimized is present in the measurement).

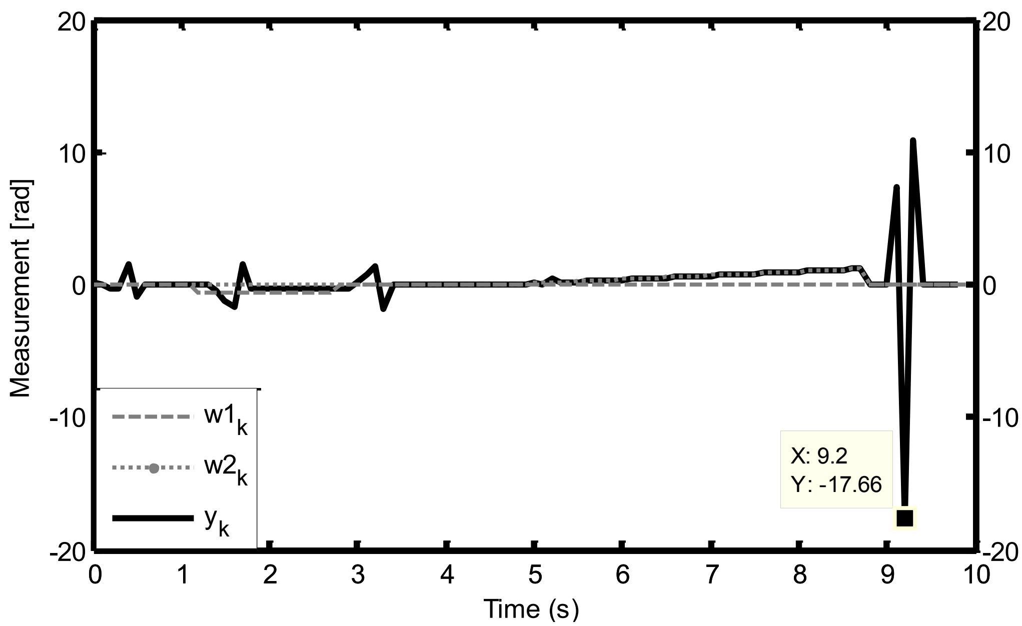

Figure 4.3 (Original technique) Controller response of x1(solid) and x2 (dash-dotted) co-plotted with the disturbances when $D = 0$

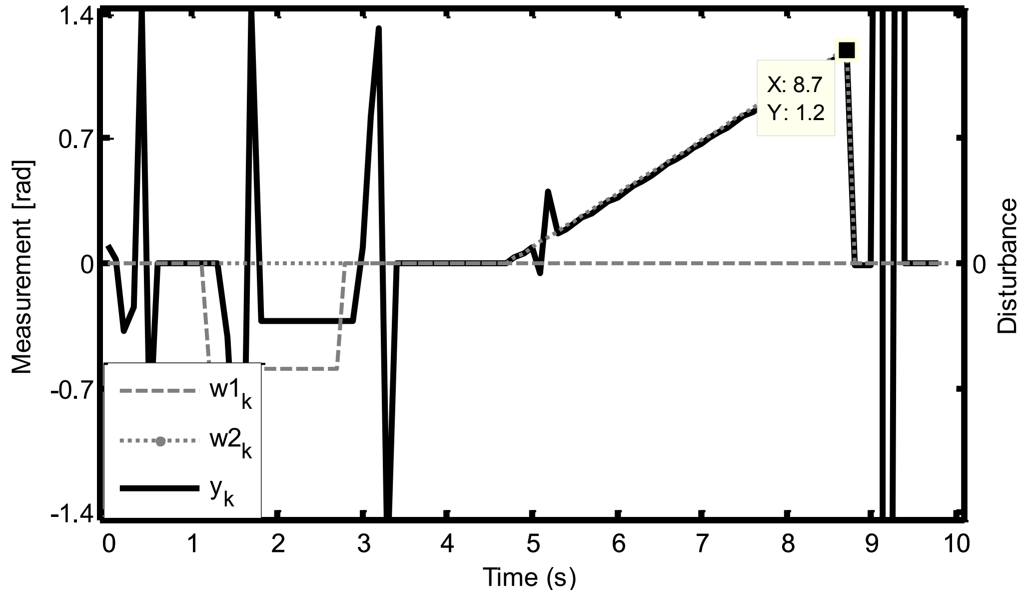

Figure 4.4 (Original technique) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D = 0$

To minimize the effect of the disturbance present in the measurement in the absence of a feed-forward term, a controller is developed using the proposed extension to the original controller technique.

4.3.3 Extension of Original Technique with Feed-Forward Term Absent

First, the system conditions must be checked:

1. $\gamma \neq \phi CB$? $CB = 0$ so if $\gamma \neq 0$ is chosen, this condition is met

2. $\frac{\gamma}{c_{11}\phi} \neq 0$? For this system $c_{11} = 1$, so if γ and ϕ are chosen such that $\frac{\gamma}{\phi} \neq 0$, this condition is met

3. $|A| \neq 0$? The determinant of A is 2.0700

4. System pair (A, B) controllable? This was shown to be met in section 4.3.1.

Now, let

$$\gamma = 1$$
$$\phi = 1$$

in the technique described in section 4.2. This controller is then substituted into the given system equation and the simulation is displayed in figure 4.5 (figure 4.6 is a zoomed in view). In figure 4.6, it is seen that the maximum error while the disturbance is present has been reduced from 1.2 rad to 0.7903 rad.

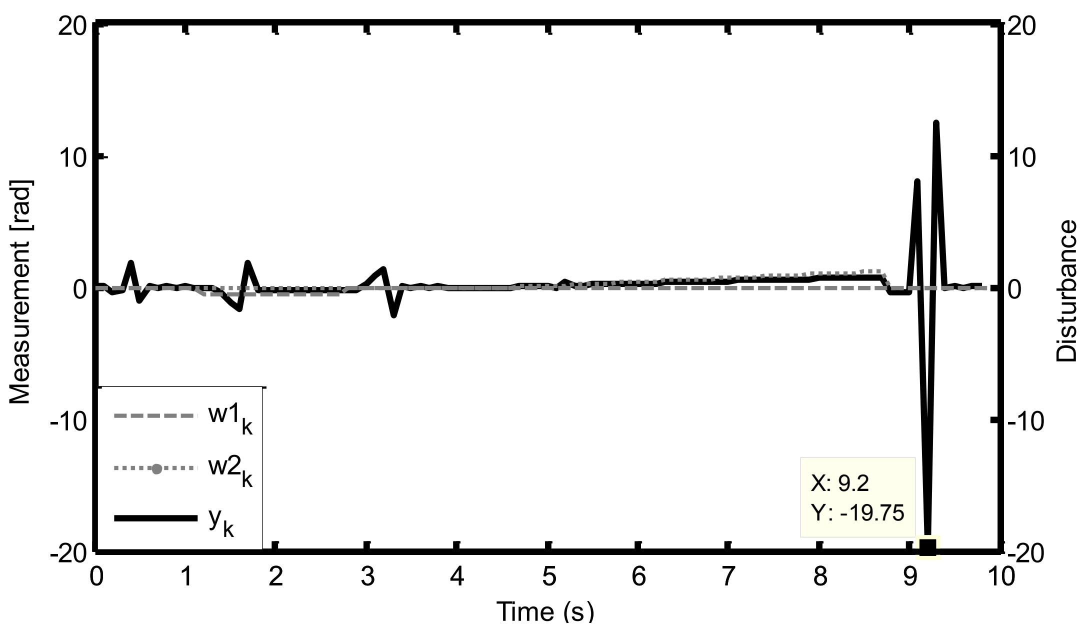

Figure 4.5 (Extension) Controller response of the measurement (solid) co-plotted with the disturbances when $D = 0$, $\phi = 1$, and $\gamma = 1$

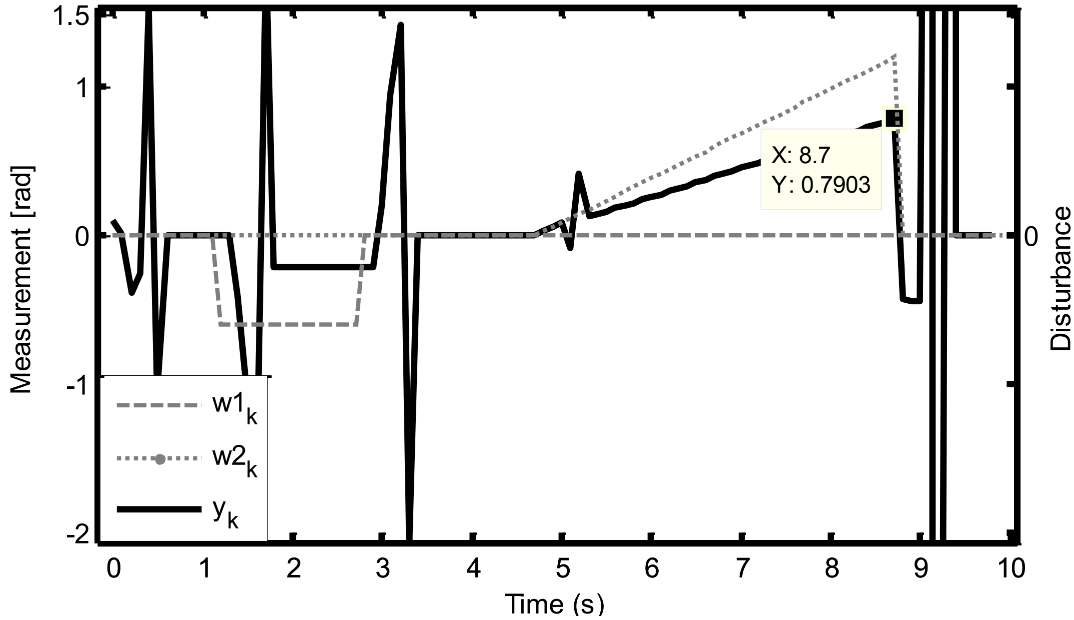

Figure 4.6 (Extension) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D=0$, $\phi=1$, and $\gamma=1$

Now, larger values for γ and ϕ are used. Let

$$\phi=10$$
$$\gamma=10$$

and apply the controller to the given system. The simulation of the controller response using these new values for ϕ and γ is shown in figure 4.7 with a zoomed in view in figure 4.8. In figure 4.7, it is seen that the maximum overshoot value increased slightly from 19.75 rad to 21.8 rad. Figure 4.8 shows that larger values for γ and ϕ decrease the maximum error while the disturbance is present.

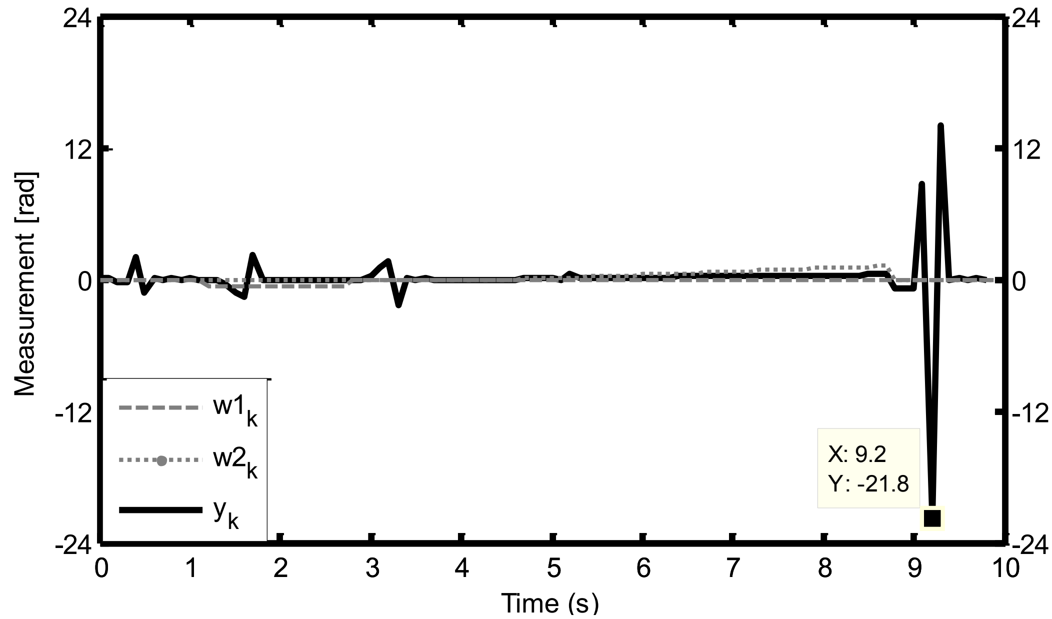

Figure 4.7 (Extension) Controller response of the measurement (solid) co-plotted with the disturbances when $D = 0$, $\phi = 10$, and $\gamma = 10$

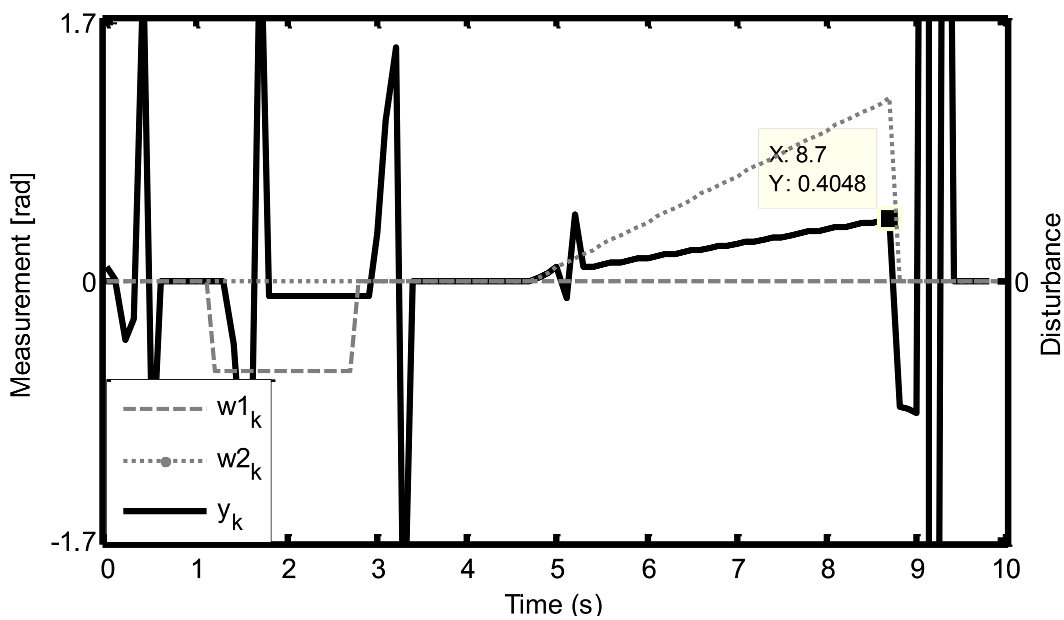

Figure 4.8 (Extension) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D = 0$, $\phi = 10$, and $\gamma = 10$

Since increasing the values of γ and ϕ showed improvement, their values are raised once more. Let

$$\phi = 50$$
$$\gamma = 50$$

in the extension technique and apply the controller to the given system. The simulation of the controller response using these new values for ϕ and γ is shown in figure 4.9 with a zoomed in view in figure 4.10. The figures show improvement again; figure 4.9 shows a slight increase in the maximum overshoot from 21.8 rad to 21.84 rad; and figure 4.10 shows a decrease from 0.4048 rad to 0.3974 rad in the maximum error while the disturbance is present.

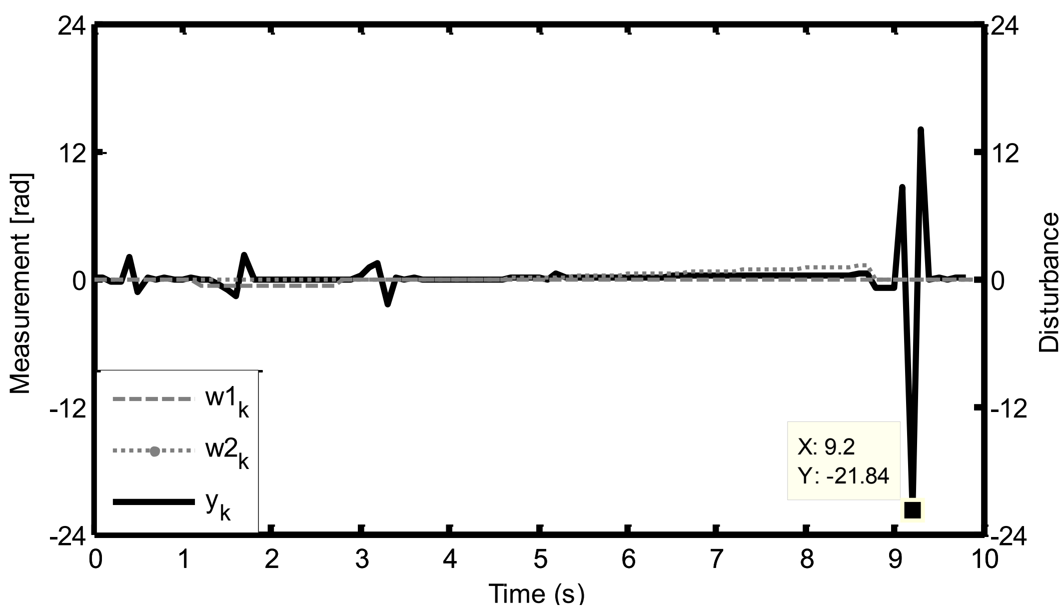

Figure 4.9 (Extension) Controller response of the measurement (solid) co-plotted with the disturbances when $D = 0$, $\phi = 50$, **and** $\gamma = 50$

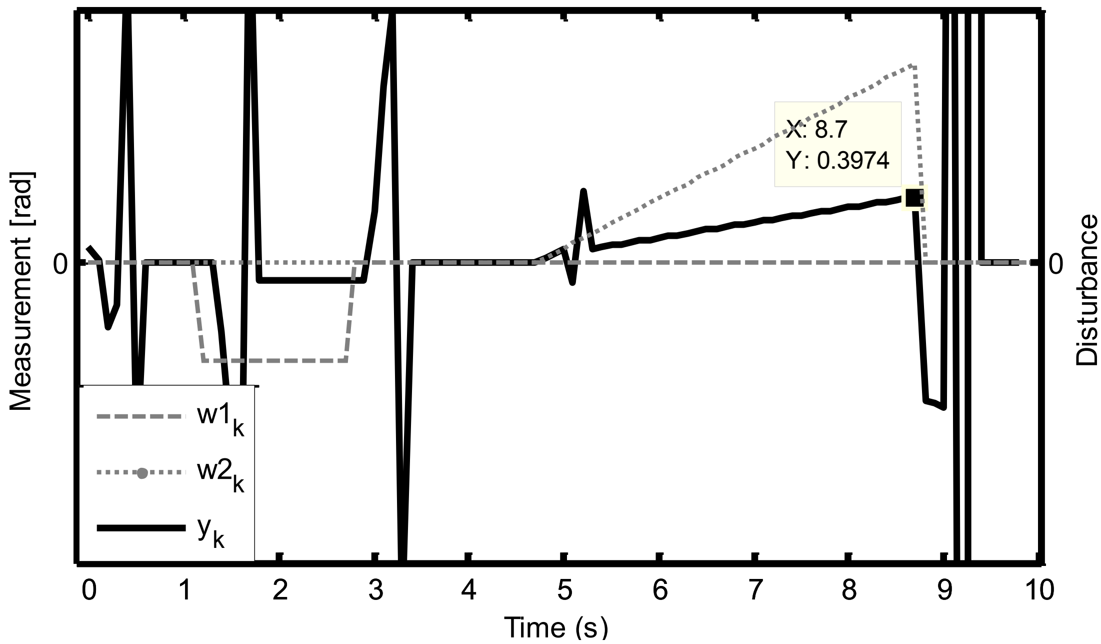

Figure 4.10 (Extension) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D = 0$, $\phi = 50$, and $\gamma = 50$

As both γ and ϕ are increased higher than 50, no further improvement is seen.

Different ratios for γ and ϕ were then considered where the sums of the absolute error in the measurement are compared and are shown in a table in Figure 4.11. In this table, the best results are seen with a phi to gamma ratio of 1.4 resulting in a sum of the absolute error in the measurement of around 67.7.

phi	gamma	Absolute Error Summation
1	1	70.5048
1	5	69.5534
1	10	69.6061
1	50	69.6958
5	5	68.3968
5	10	69.0107
5	100	69.6498
10	10	68.2663
10	50	69.4253
10	100	69.5746
50	50	68.2229
50	100	68.9733
100	100	68.2216

phi	gamma	Absolute Error Summation
1	1	70.5048
2	1	69.7531
5	1	104.2981
5	5	68.3968
7	5	67.8185
10	5	81.7388
10	10	68.2663
15	10	67.8311
20	10	83.6286
50	50	68.2229
70	50	67.6217
80	50	70.4477
100	100	68.2216
140	100	67.6202
160	100	70.4617

Figure 4.11 Sum of the absolute value of the error in the measurement with varied phi and gamma values

A simulation using the phi to gamma ratio of 1.4 is shown in Figure 4.12. This figure shows at the initial presence of the ramp, the controller is transient and once this transience is finished the maximum magnitude of the error while the disturbance is present is 0.08505.

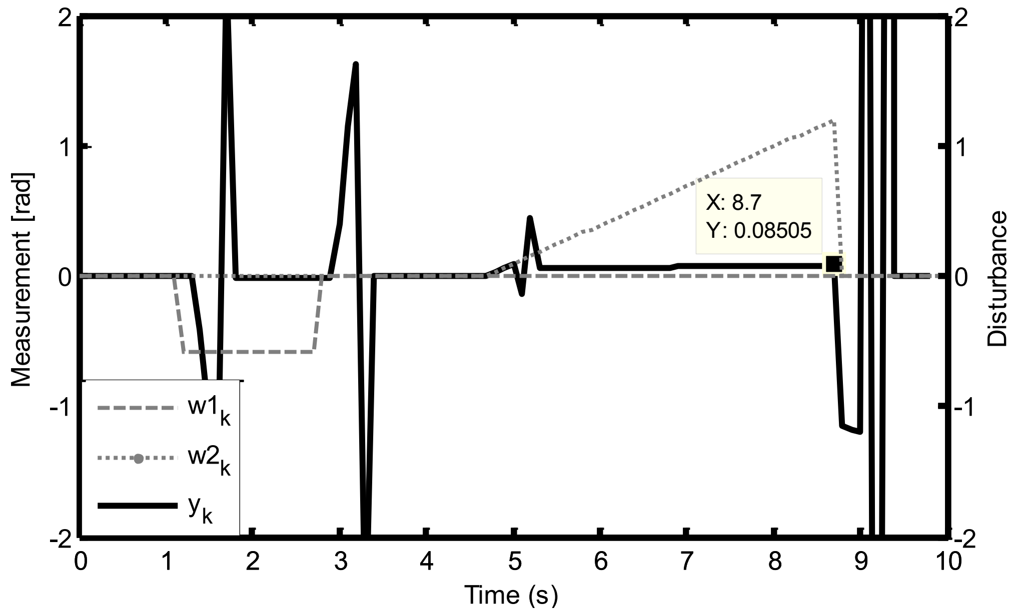

Figure 4.12 (Extension) Controller response zoomed in on the measurement (solid) co-plotted with the disturbances when $D=0$, $\phi=14$, **and** $\gamma=10$

4.4 Conclusion

In this chapter, system conditions for the control technique developed in chapter two were derived by analyzing the controllability matrix of the augmented system. When one of the conditions, namely the feed-forward term being non-zero, is not met, an extension was developed that improved the results given by the original technique. This extension made use of a 'pseudo-output' that consisted of the current measurement and the past input. The pseudo-output made it possible for the controller to indirectly minimize the effect of the disturbance in the measurement.

5 CONCLUSION AND FUTURE WORK

5.1 Summary

In this thesis, a discrete-time observer based deadbeat disturbance accommodating controller was designed that is capable of minimizing the effect of disturbances with known waveform structures in both the system state and the measurement as fast as possible. This was achieved by designing a single control input to accommodate disturbances in both the state and the measurement. To do this, it was necessary to augment the state and measurement equations. Once this augmented system was created, a least squares minimization technique was used along with completion of the squares to find a control input which would drive the system and measurement to zero (the desired value). The method that was used to design the control input also guarantees a deadbeat response when the controllable canonical forms [15] of the system matrices are used. Throughout the design of the controller, the assumption was made that all state variables and disturbances were known and directly available for feedback to the controller. This is never the case, however, which leads to the necessity of an observer.

When using a combination of an observer and a controller, the observer's response should be faster than the controller's response so accurate estimated values are being used in the controller. When using a deadbeat controller, the only option for the observer is to also be deadbeat. Two types of deadbeat observers were used in this work: full-order and reduced-order. The full-order deadbeat observer that was designed drives the estimation error to zero in minimal time. Minimum response time is achieved because of the deadbeat characteristic added to the design. The time response expected for deadbeat action is the number of time samples equal to the order of the system. Knowing this, a reduced-order deadbeat observer was then designed which has a response time faster than the full-order deadbeat observer because a reduced-order observer only generates estimates for the un-measureable state variables and disturbances, reducing the overall order of the observer.

In an extension, a new model for the control input was introduced for the case when the feed-forward control term in the measurement was not present. This extension of the original technique involved using a so-called ‘pseudo-output’ that allows the controller to indirectly minimize the effect of the disturbance in the measurement.

5.2 Conclusion

The combination of the observer and control input create an observer based disturbance accommodating controller. This DAC was applied to two different systems, a pendulum system and a magnetic levitation system, with a variety of disturbances being applied. Each system was put into state space form in continuous-time and was then converted into discrete-time. A model for the specific waveform structure was developed.

First, the pendulum system had a step-type disturbance applied to it and both a full-order and reduced-order deadbeat observer based DAC were designed and compared. The simulations showed the reduced-order observer based DAC had a faster response time as expected. This result lead to focusing on the reduced-order observer based DAC for the remaining cases involving the pendulum system. After the step-type disturbance was applied, step-type and ramp-type disturbances were applied followed by a sinusoidal-type disturbance. The simulations showed the reduced-order deadbeat observer based deadbeat DAC was able to minimize the effect of disturbances in both the state and the measurement in a minimal amount of time.

Next, the magnetic levitation system had the same step-type disturbance applied to it and, again, both a full-order and reduced-order deadbeat observer based DAC were designed. The simulations showed similar results to those seen with the pendulum system. The reduced-order observer based DAC showed a faster response than the full-order observer based DAC; however, the reduced-order observer based DAC had a larger steady state error while the disturbance was present than the full-order observer based DAC.

Lastly, an extension was developed which improved the minimization of disturbances in the measurement when there is no feed-forward term present. To show this improvement, a full-order observer based DAC was designed which used the control input proposed in the extension. The simulations showed this extension improved the minimization of disturbances in the measurement compared to the original technique when the feed-forward term is not present. Furthermore, a small study was done on the effect of the ratio between γ and ϕ where it was found that the most desirable results were seen with a phi to gamma ratio of 1.4.

5.3 Future Work

This work brings out new ideas in the area of discrete-time DAC theory. One of the main objectives of this work was to achieve a minimal response time, but a trade off was large input magnitudes. One way to extend this work would be to set a hard limit on the magnitude of the control input while also striving to achieve a minimum response time. Another way to reduce the control input would be to minimize the energy, which would place a soft constraint on the input.

The technique developed in this work was only applied to second and third order systems but can be applied to systems of any order. In addition, other disturbances with a waveform structure could be applied to the systems that fit the conditions for this technique, such as an exponential disturbance.

Furthermore, this technique was developed for linear, time-invariant, single input single output, deterministic systems. A new technique which uses this work as a basis can be developed for certain classes of nonlinear, multiple input, multiple output, time-invariant, or stochastic systems.

Also, it was seen that γ and ϕ in the extension have values that give a better result than others, so γ and ϕ can be analyzed further to be able to choose their optimal values for the best performance in both the system state and the measurement.

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APPENDIX A: MATLAB Code and Simulink Diagrams

A1. MATLAB code for Full-Order Observer Based DAC for Pendulum System with Step Disturbance

First, using simulink, generate the disturbance:

Then simulate the code:

```
clear w
%Generates step-type disturbance
%    (Note the final time may need to be changed)
for i=1:51
    w(:,i)=[yout(i,1)];      %w=step
end

%Clearing variables (note w is not included)
clear t x y u Xs_hat Xs e k error
%Closing all figures
close all

%Continuous-time system matrices
A=[0 1;-9.8/.5 0];B=[0;1];C1=[1 0];D=0.1;F=[0;1];G1=[0.2];e=[1];
%Dimensions of x, w, and y, respectively
nx=2;nw=1;ny=1;
```

```
%M-Matrix to discretize A and B (When the dimension of
%     A changes, more zeros will need to be added)
M1=[A B;0 0 0];
%Intermediate variable for discretization
N1=expm(M1*0.1);
%M-Matrix to discretize F (When the dimension of
%     A and F change, more zeros will need to be added)
M2=[A F;[0 0 0]];
%Intermediate variable for discretization
N2=expm(M2*0.1);

%Choosing appropriate sections of N1 and N2 for
%   the discretized matrices
a=N1(1:nx,1:nx);b=N1(1:nx,(nx+1));f=N2(1:nx,(nx+1):(nx+nw));
%Discrete measurement matrices remain the same
c1=C1;d=D;g1=G1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%****************OBSERVER***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Augmented system for observer design-Ao matrix (depending on
%  the dimension of a and e more zero may be needed)
Ao=[a f;0 0 e];
%Augmented system for observer design-Co matrix
Co=[c1 g1];
%Obtaining transfer function
[numo,deno]=ss2tf(Ao,[b;0],Co,1);
%Generating observable canonical form of Ao (depending on the
%     dimension of Ao, Ao_bar's rows of 1's and 0's will need
%     to be modified)
Ao_bar=[-[deno(2:(nx+nw+1))'],[1;0;0],[0;1;0]];
%Observable canonical form of Co
%     (zeros may need to be added when dimensions change)
Co_bar=[1 0 0];
%Observability matrix for (Ao,Co) pair
Wo=obsv(Ao,Co);
%Observability matrix for (Ao_bar,Co_bar) pair
Wo_bar= obsv(Ao_bar,Co_bar);
%Observer gain in canonical form
K_bar=Ao_bar*Co_bar'*pinv(Co_bar*Co_bar');
%Tranformation to original system's form
K=inv(Wo)*Wo_bar*K_bar;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************CONTROLLER****************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Augmented system for controller design-Ac matrix (depending on
%  the dimension of a more zero may be needed)
Ac=[a [0;0];c1 0];
%Augmented system for observer design-Bc matrix
Bc=[b;d];
%Obtaining transfer function
[numc,denc]=ss2tf(Ac,Bc,[1 0 0],1);
```

```
%Generating controllable canonical form of Ac (depending on the
%     dimension of Ac, Ac_bar's rows of 1's and 0's will need
%     to be modified)
Ac_bar=[-denc(2:4);1 0 0;0 1 0];
%Controllable canonical form of Bc
%     (zeros may need to be added when dimensions change)
Bc_bar=[1;0;0];
%Controllablilty matrix for (Ac,Bc) pair
Wc=ctrb(Ac,Bc);

%Controllablilty matrix for (Ac_bar,Bc_bar) pair
Wc_bar=ctrb(Ac_bar,Bc_bar);
%State control gain in canonical form
L_bar=-pinv(Bc_bar)*Ac_bar;
%Transformation to original system's form
L=L_bar*Wc_bar*inv(Wc);
%Disturbance minimization gain
Ld=-pinv([b;d])*[f;g1];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%****************Simulation***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Final simulation time (seconds)
ftime=5.1;
%Sampling time
T=0.1;
%Number of sampling points
kf=ceil(ftime/T);
%Initial conditions in the state (x) and the augmented state
%   estimate (Xs_hat=[x_hat;w_hat])
x(:,1)=[0;0];
Xs_hat(1:(nx+nw),1)=[0;0;0];

for k=1:kf-1;
    %Control input (for open loop simulation let equal to 0)
    u(k)=[L(1:nx),Ld]*Xs_hat(:,k);
    %State equation
    x(:,k+1)=a*x(:,k)+b*u(:,k)+f*w(:,k);
    %Measurement equation
    y(:,k)=c1*x(:,k)+d*u(:,k)+g1*w(:,k);
    %Observer estimate update equation
    Xs_hat(:,k+1)=([a f;0 0 e]-K*[c1
g1])*Xs_hat(:,k)+[b;0]*u(:,k)+K*y(:,k)-K*[d]*u(:,k);
end

t=T*[0:kf-1];
ty=T*[0:kf-2];
%Sets all signals as black for default
set(0,'DefaultAxesColorOrder',[0 0 0]);
```

```
figure(1)
%Plots the state and disturbance on the same plot with
%   different y-axes
[AX,H1,H2] = plotyy(t,x(1,:),t,w,'plot');xlabel('Time (s)');
%Y-axis labels
set(get(AX(1),'Ylabel'),'String','Pendulum Angle [rad]');
set(get(AX(2),'Ylabel'),'String','Disturbance');
%Setting the axes line thickness
set(AX(1),'LineWidth',2.5);
set(AX(2),'LineWidth',2.5);
%Defining the y-scale for the disturbance
set(AX(2),'Ylim',[-1 1],'Ytick',[-1 -.5 0 .5 1]);
%Setting the state to be a solid line and 2.5 points thick
set(H1,'LineStyle','-','LineWidth',2.5);
%Setting the disturbance to be a dashed line, 1.5 points thick
%   and to be a dark gray
set(H2,'LineStyle','--','LineWidth',1.5,'Color',[.5 .5 .5]);
%Legend for disturbance and state
legend('w_k','x1_k');

%The rest of the plots are generated similarly to what is shown

%NOTE: when a larger dimension system is used, plots will
%   need to be added so all state variables are plotted
```

A2. MATLAB Code for Reduced-Order Observer Based DAC for Pendulum System with Step Disturbance

First simulation the calculation for the gains:

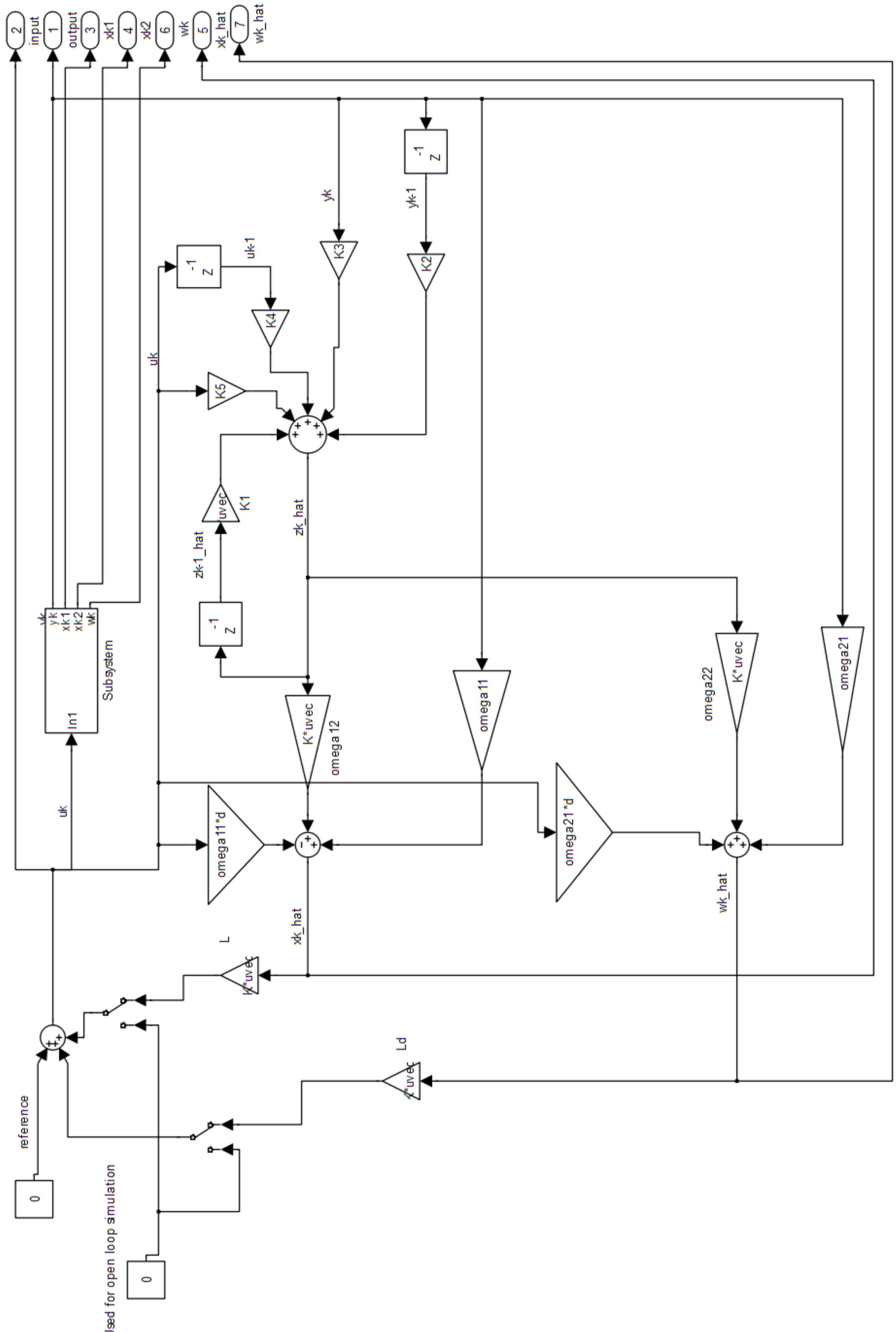
```
clear t x y u Xs_hat Xs error k err z z_hat
%Sampling time
T=0.1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************GAIN CALCULATIONS***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Dimensions of x, w, and y, respectively
nx=2;nw=1;ny=1;
%Continuous-time system matrices
A=[0 1;-9.8/.5 0];B=[0;1];C1=[1 0];D=.1;F=[0;1];G1=[.2];e=[1];
%M-Matrix to discretize A and B (When the dimension of
%     A changes, more zeros will need to be added)
M1=[A B;0 0 0];
%Intermediate variable for discretization
N1=expm(M1*0.1);
%M-Matrix to discretize F (When the dimension of
%     A and F change, more zeros will need to be added)
M2=[A F;[0 0 0]];
%Intermediate variable for discretization
N2=expm(M2*0.1);
```

```
%Choosing appropriate sections of N1 and N2 for
%   the discretized matrices
a=N1(1:nx,1:nx);b=N1(1:nx,(nx+1));f=N2(1:nx,(nx+1):(nx+nw));
%Discrete measurement matrices remain the same
c1=C1;d=D;g1=G1;
%Composite vector's coefficient matrices
c2=[0 1;0 0];g2=[0;1];
%Creating matrix that is used to solve for estimates
O=inv([c1 g1;c2 g2]);
%Partitioning O into the defined omega matrices
omega11=O(1:nx,1:ny);
omega12=O(1:nx,(ny+1):(nx+nw));
omega21=O((nx+1):(nx+nw),1:ny);
omega22=O((nx+1):(nx+nw),(ny+1):(nx+nw));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************OBSERVER***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Defined Ao matrix calculation
Ao=c2*a*omega12+(c2*f+g2*e)*omega22;
%Defined Co matrix calculation
Co=c1*a*omega12+(c1*f+g1*e)*omega22;
%Obtaining transfer function
[numo,deno]=ss2tf(Ao,[1;0],Co,1);
%Generating observable canonical form of Ao (depending on the
%     dimension of Ao, Ao_bar's rows of 1's and 0's will need
%     to be modified)
Ao_bar=[-[deno(2:3)'],[1;0]];
%Observable canonical form of Co
%     (zeros may need to be added when dimensions change)
Co_bar=[1 0];
%Observability matrix for (Ao,Co) pair
Wo=obsv(Ao,Co);
%Observability matrix for (Ao_bar,Co_bar) pair
Wo_bar=obsv(Ao_bar,Co_bar);
%Observer gain K3 in canonical form
K3_bar=Ao_bar*Co_bar'*pinv(Co_bar*Co_bar');
%Transformation of K3
K3=inv(Wo)*Wo_bar*K3_bar;
%Calculation of all other observer gains dependent on K3
K5=-K3*d;
K2=c2*a*omega11-K3*c1*a*omega11+c2*f*omega21+g2*e*omega21-
K3*(c1*f+g1*e)*omega21;
K1=c2*a*omega12-K3*c1*a*omega12+c2*f*omega22+g2*e*omega22-
K3*(c1*f+g1*e)*omega22;
K4=c2*b-K2*d-K3*c1*b;
```

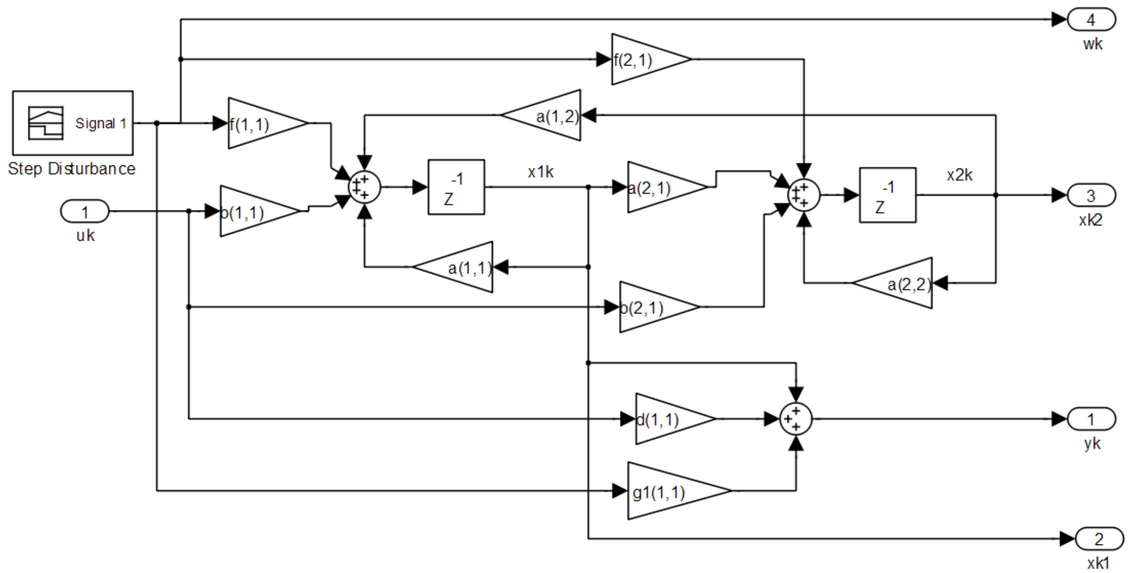
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************CONTROLLER***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Augmented system for controller design-Ac matrix (depending on
%  the dimension of a more zero may be needed)
Ac=[a [0;0];c1 0];
%Augmented system for observer design-Bc matrix
Bc=[b;d];
%Obtaining transfer function
[numc,denc]=ss2tf(Ac,Bc,[1 0 0],1);
%Generating controllable canonical form of Ac (depending on the
%     dimension of Ac, Ac_bar's rows of 1's and 0's will need
%     to be modified)
Ac_bar=[-denc(2:4);1 0 0;0 1 0];
%Controllable canonical form of Bc
%     (zeros may need to be added when dimensions change)
Bc_bar=[1;0;0];
%Controllablilty matrix for (Ac,Bc) pair
Wc=[Bc Ac*Bc Ac*Ac*Bc];
%Controllablilty matrix for (Ac_bar,Bc_bar) pair
Wc_bar=[Bc_bar Ac_bar*Bc_bar Ac_bar*Ac_bar*Bc_bar];
%State control gain in canonical form
L_bar=-pinv(Bc_bar)*Ac_bar;
%Transformation to original system's form
L=L_bar*Wc_bar*inv(Wc);
%Disturbance minimization gain
Ld=-pinv([b;d])*[f;g1];
```

Then simulate the block diagram shown on the following page:

NOTE: The gain blocks must be changed to handle matrices, this is done by simply double clicking the gain block and changing the property to Matrix(K*u) (u vector)


input
output
xk1
xk2
wk
xk_hat
wk_hat
2
1
3
4
6
5
7
yk
yk-1
uk-1
uk
K1
K2
K3
K4
K5
uvec
zk_hat
zk-1_hat
Subsystem
In1
yk
xk1
xk2
wk
omega11
omega12
omega21
omega22
K*uvec
omega11*d
omega21*d
L
Ld
reference
Used for open loop simulation
0

The subsystem block changes depending on the system. For the pendulum system with a step disturbance (the same step disturbance shown in A1), the following subsystem was used

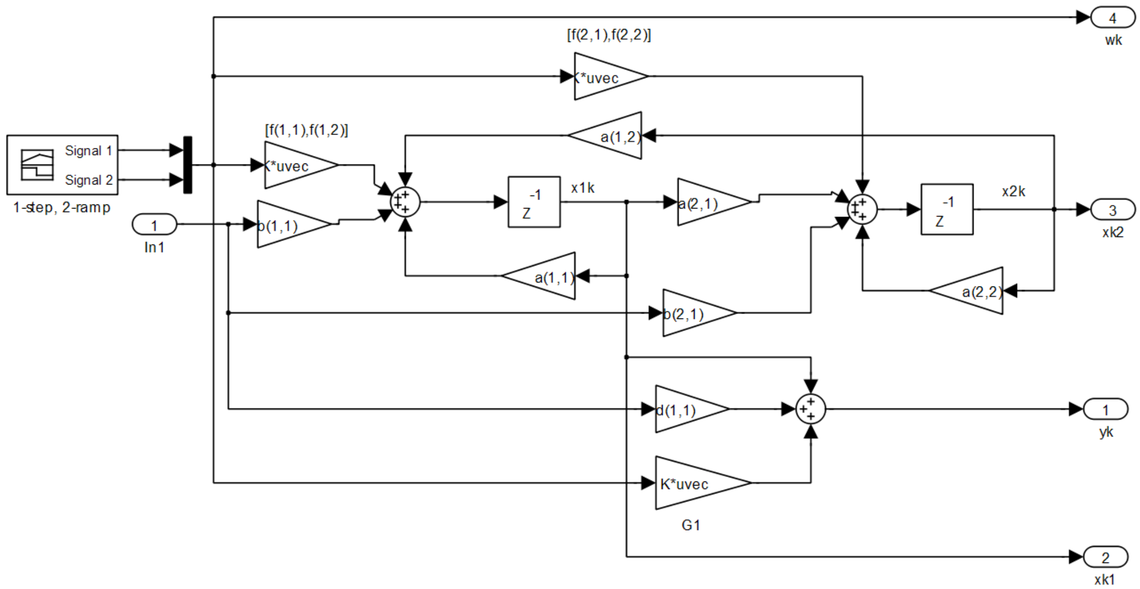


Once the system has been simulated, the following plots are created:

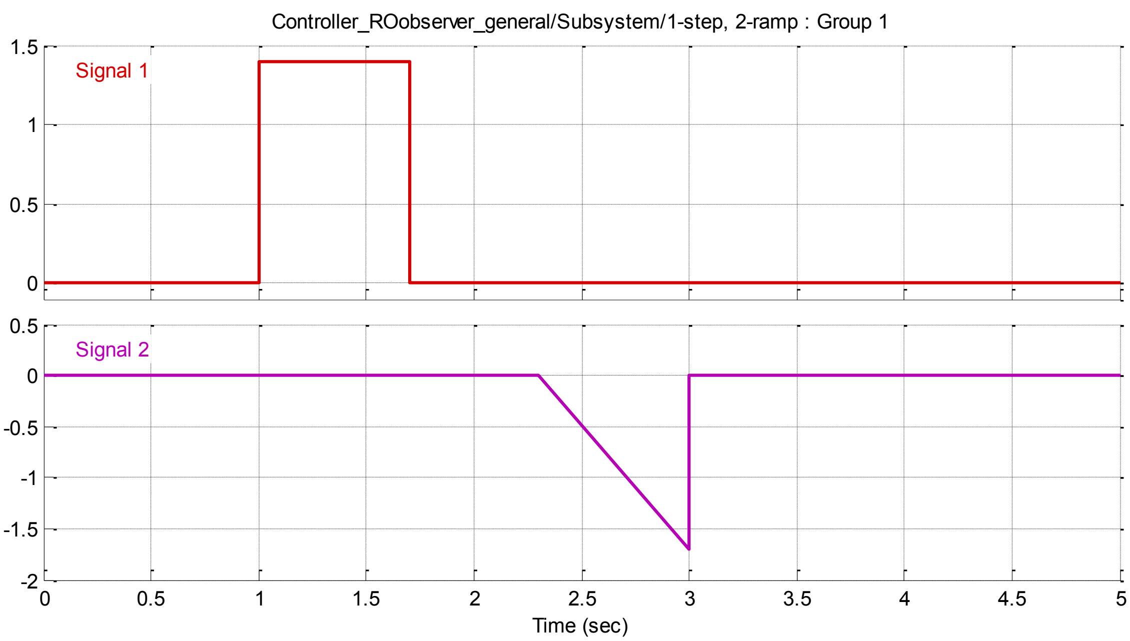
```
clc
clear y u x x_hat w w_hat
%Assigning variable names for the outputs from simulink
for i=1:50
    y(:,i)=[yout(i,1)];
    u(:,i)=[yout(i,2)];
    x(:,i)=[yout(i,3);yout(i,4)];
    x_hat(:,i)=[yout(i,5);yout(i,6)];
    w(:,i)=[yout(i,7)];
    w_hat(:,i)=[yout(i,8)];
end

%Plots are generated similarly to what is shown in A1
```

A3. Subsystem for Reduced-Order Observer Based DAC for Pendulum System with Step and Ramp Disturbances

The subsystem including the step and ramp disturbances is shown on the following page:

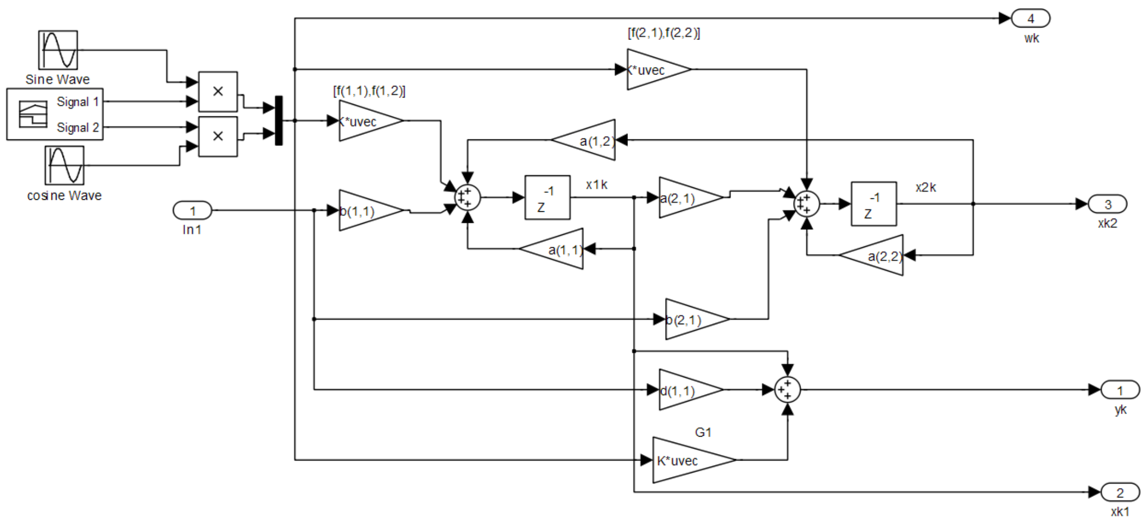


where the disturbance signals are:

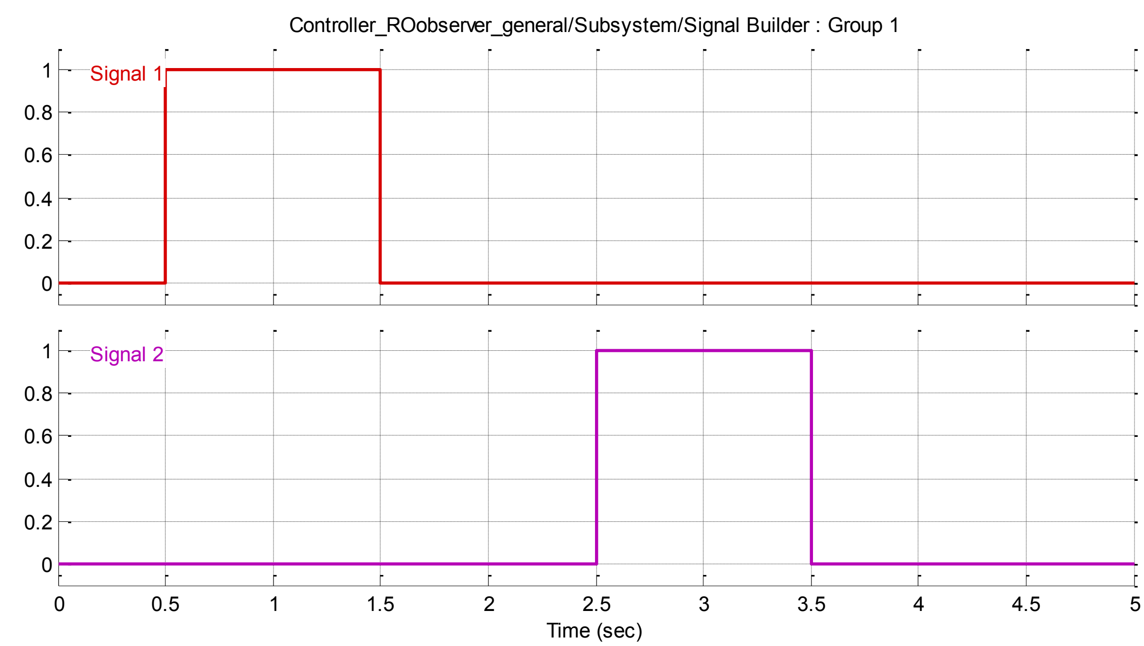


A4. Subsystem for Reduced-Order Observer Based DAC for Pendulum System with Sinusoidal Disturbances

The simulink subsystem is

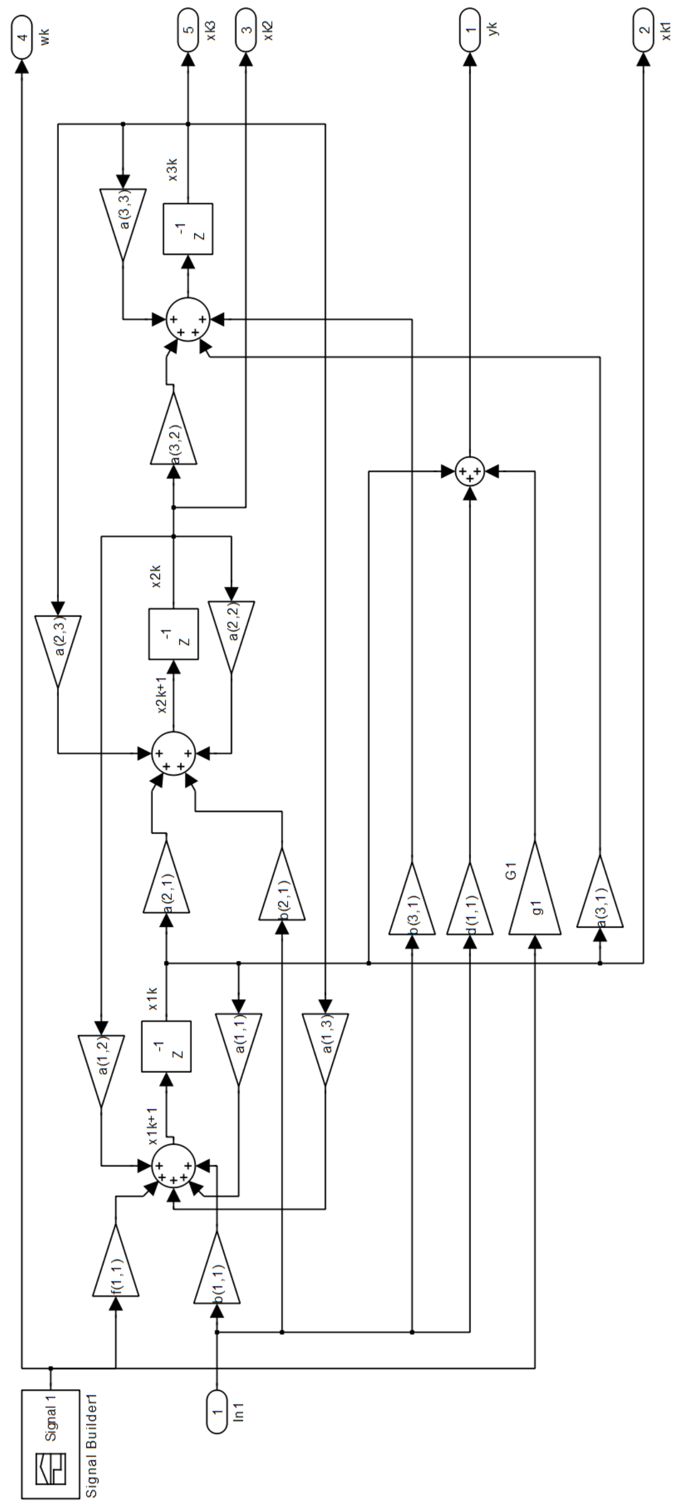


where the sine and cosine are of magnitude 3 and frequency of 1 rad/s. The signal block consists of pulses that allow the sine and cosine to 'turn on' and 'turn off' and are shown:



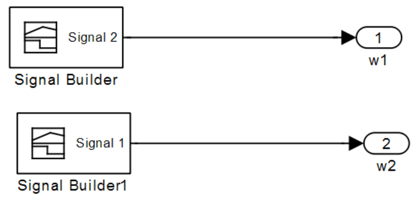
A5. Subsystem for Reduced-Order Observer Based DAC for Magnetic Levitation System with Step Disturbance

The subsystem is shown where the disturbance is the same as the disturbance in A1:

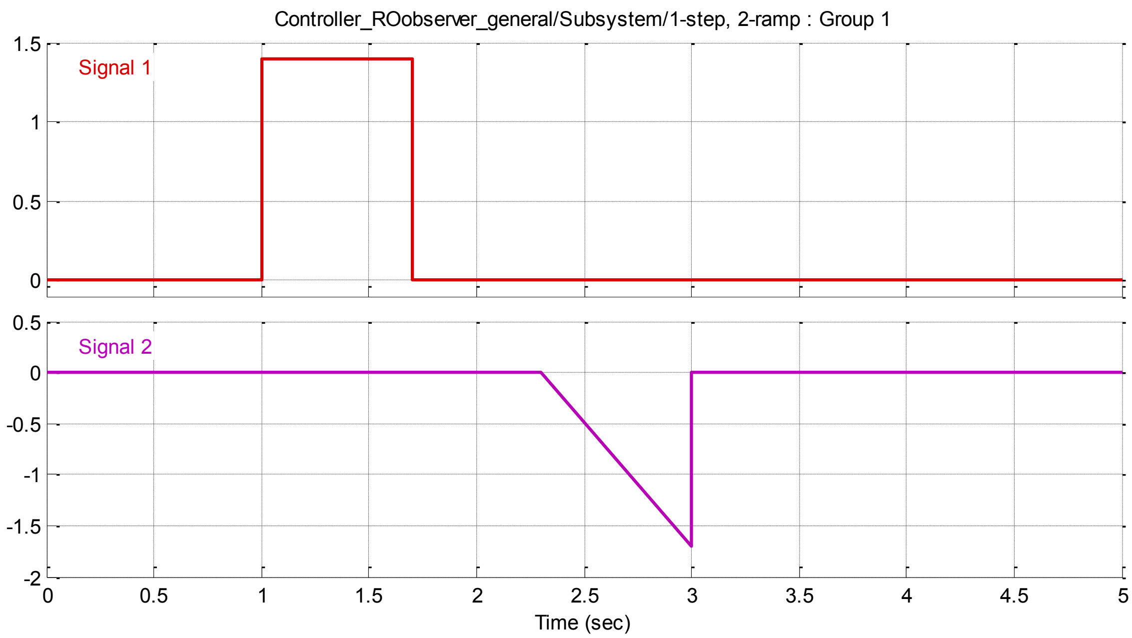


A6. MATLAB Code for Pseudo-Output Method DAC for a System with Step and Ramp Disturbances:

First simulate the disturbance:



where



Then run the following code:

```
clear w
for i=1:10
   %Generates disturbances such that w1=step, w2=ramp
    w(:,i)=[yout(i,2);yout(i,1)];
end

close all
clear t x y u Xs_hat Xs error k err L L_bar phi gamma z ty

%Sampling time
T=0.1;
%Discrete-time system
a=[.9 .7;-1.8 .9];b=[0;1];c1=[1 0];d=0;f=[0 0;1 0];g1=[0 1];e=[1 0;1
1];
%Dimensions of x, w, and y, respectively
nx=2;nw=2;ny=1;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************OBSERVER***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Augmented system for observer design-Ao matrix
Ao=[a f;[0 0;0 0] e];
%Augmented system for observer design-Co matrix
Co=[c1 g1];
%Obtaining transfer function
[numo,deno]=ss2tf(Ao,[1;0;0;0],Co,0);
%Generating observable canonical form of Ao
Ao_bar=[-[deno(2:5)'],[1;0;0;0],[0;1;0;0],[0;0;1;0]];
%Observable canonical form of Co
Co_bar=[1 0 0 0];
%Observability matrix for (Ao,Co) pair
Wo=obsv(Ao,Co);
%Observability matrix for (Ao_bar,Co_bar) pair
Wo_bar= obsv(Ao_bar,Co_bar);
%Observer gain in canonical form
K_bar=Ao_bar*Co_bar'*pinv(Co_bar*Co_bar');
%Transformation to the original system's form
K=inv(Wo)*Wo_bar*K_bar;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%***************CONTROLLER***************%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%NOTE:When the extension is used the first four
%%%%  lines are used, when the original technique
%%%%  is being used (to compare) the second
%%%%  four lines are used (comment out the set not used)

%%%%%%%%%%FIRST SET OF FOUR%%%%%%%%%%
%Define phi and gamma
phi=50;gamma=50;
%Augmented system using pseduo-output-Ac matrix
Ac=[a [0;0];phi*c1*a 0];
%Augmented system using pseduo-output-Bc matrix
Bc=[b;gamma+phi*c1*b];
%Augmented system using pseduo-output-Fc matrix
Fc=[f;phi*(c1*f+g1*e)];

%%%%%%%%%%SECOND SET OF FOUR%%%%%%%%%%
% %Define phi and gamma
% phi=0;gamma=0;
% %Augmented system using actual output-Ac matrix
% Ac=[a [0;0];c1 0];
% %Augmented system using  actual output-Bc matrix
% Bc=[b;d];
% %Augmented system using actual output-Fc matrix
% Fc=[f;g1];

%Obtaining transfer function
[numc,denc]=ss2tf(Ac,Bc,[1 0 0],0);
```

```
%Generating controllable canonical form of Ac
Ac_bar=[-denc(2:4);1 0 0;0 1 0];
%Controllable canonical form of Bc
Bc_bar=[1;0;0];
%Controllablilty matrix for (Ac,Bc) pair
Wc=ctrb(Ac,Bc);
%Controllablilty matrix for (Ac_bar,Bc_bar) pair
Wc_bar=ctrb(Ac_bar,Bc_bar);
%State control gain in canonical form
L_bar=-pinv(Bc_bar)*Ac_bar;
%Transformation to original system's form
L=L_bar*Wc_bar*inv(Wc);
%Disturbance minimization gain
Ld=-pinv(Bc)*Fc;

%Final simulation time (seconds)
ftime=10;
%Number of sampling points
kf=ceil(ftime/T);
%Initial conditions in the state (x), the augmented state
%   estimate (Xs_hat=[x_hat;w_hat]), and the pseudo-output (z)
x(:,1)=[0;0];
Xs_hat(1:4,1)=[0;0;0;0];
z(:,1)=0;
for k=1:kf-1;
    %Control input)
    u(k)=[L(1:2),Ld]*Xs_hat(:,k)+L(3:3)*z(k);
    %State equation
    x(:,k+1)=a*x(:,k)+b*u(:,k)+f*w(:,k);
    %Measurement equation
    y(:,k)=c1*x(:,k)+d*u(:,k)+g1*w(:,k);
    %Observer estimate update equation
    Xs_hat(:,k+1)=(Ao-K*Co)*Xs_hat(:,k)+[b;0;0]*u(:,k)+K*y(:,k)-
K*d*u(:,k);
    %Pseudo-output update equation

z(k+1)=[phi*c1*a,phi*(c1*f+g1*e)]*Xs_hat(:,k)+(phi*c1*b+gamma)*u(k);
end

          %Plots are generated similarly to what is shown in A1
```