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Tort Liability and Unawareness*

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Abstract

Unawareness is a form of bounded rationality where a person fails to conceive all feasible acts or consequences or to perceive as feasible all conceivable act-consequence links. We study the implications of unawareness for tort law, where relevant examples include the discovery of a new product or technology (new act), of a new disease or injury (new consequence), or that a product can cause an injury (new link). We argue that negligence has an important advantage over strict liability in a world with unawareness—negligence, through the stipulation of due care standards, spreads awareness about the updated probability of harm.

JEL codes: D83, K13.

Keywords: negligence, strict liability, tort law, unawareness.

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1 Introduction

Background. Economists traditionally model choice under uncertainty according to Savage’s theory of subjective expected utility (Savage, 1954). Savage’s theory posits a space of mutually exclusive and collectively exhaustive states of the world, representing all possible resolutions of uncertainty. It assumes that when a person chooses an act, although she is uncertain about the true state of the world and therefore about the consequences of her chosen act, she nevertheless has complete knowledge of the state space—she knows all the possible acts and all the possible consequences of each and every act.

In reality, however, a person often does not have complete knowledge of the state space. This is known as *unawareness*. A person may be unaware of some acts, some consequences, or that a known act can cause a known consequence. An extreme example of the latter is that no one was aware that supporting anti-Soviet fighters in Afghanistan in the 1980s could lead to the destruction of the World Trade Center in 2001.

Unawareness plays an important role in many legal spheres. A prime example is that people may write incomplete contracts due to unforeseen contingencies. Another example is that a person may be unaware that an act can result in criminal liability. A third example is that a person may not be aware of all the objections that she can raise to a witness’s testimony in a trial. In this paper, we study the implications of unawareness for tort law, the part of the common law that governs liability for civil wrongs.

Tort liability rules and unawareness. A central question in the field of law and economics is whether negligence or strict liability is the more efficient tort liability rule. Under negligence, a victim can recover damages for harm caused by the activity of an injurer who failed to take reasonable care when engaging in the activity. Under strict liability, by contrast, a victim can recover damages for harm caused by the activity of an injurer irrespective of whether the injurer took reasonable care. The relative efficiency of the two rules is customarily measured by the Kaldor-Hicks criterion.

A bedrock result in the economic analysis of tort law is that, in the case of unilateral accidents with fixed activity levels, negligence and strict liability are equally efficient, provided that, in the case of negligence, the court properly sets the due care standard (the legal standard for what constitutes reasonable care) (Shavell, 1987).¹ This equivalence result, however, presents something of a puzzle in light of two facts about negligence. First, negligence is the dominant rule in Anglo-American law.² Second, negligence is the more costly rule to administer, because the court must determine the due care standard and adjudicate whether the standard was met. The puzzle is that if negligence and strict liability are equally efficient but negligence is more costly to administer, why is negligence the dominant rule?

The negligence puzzle has led researchers to revisit the equivalence result by exploring departures from the standard accident model, which is based on the expected utility framework and the Bayesian paradigm. For instance, Teitelbaum (2007) and Chakravarty and Kelsey (2017) explore ambiguity (Knightian uncertainty). They assume that the relevant parties are Choquet expected utility maximizers with neo-additive beliefs about accident risk, and they find that this breaks the equivalence in favor of negligence.³

In this paper, we explore unawareness. Unawareness is the failure to conceive or perceive the entire state space. It is a form of bounded rationality in which a person fails to conceive all available acts or potential consequences or fails to perceive as feasible all conceivable act-consequence links. Unawareness creates the possibility of *growing awareness*—the expansion of the state space when a person discovers a new act, consequence, or act-consequence link.

¹In unilateral accidents, the injurer, but not the victim, can take care to reduce expected harm. In unilateral accidents with fixed activity levels, the injurer affects expected harm only through her level of care (and not through her level of activity). The equivalence result also holds in the case of bilateral accidents with fixed activity levels, provided that strict liability is coupled with the defense of contributory negligence.

²In modern Anglo-American law, strict liability applies only in a handful of accident cases, including cases involving abnormally dangerous activities or products with manufacturing defects (Dobbs et al., 2011, § 2). Indeed, certain accident cases that were traditionally governed by strict liability are now governed by negligence, including cases involving products with a design or warning defect (Dobbs et al., 2011, § 450).

³The neo-additive Choquet expected utility model was developed by Chateauneuf et al. (2007). Franzoni (2017) models ambiguity aversion according to the smooth model of Klibanoff et al. (2005). He finds that strict liability dominates negligence when the injurer has lower degrees of uncertainty aversion than the victim and can formulate more precise estimates of the probability of harm, but that negligence dominates strict liability when harm is dispersed on a very large number of victims, irrespective of the parties' respective degrees of uncertainty aversion.

Examples relevant to tort law include the discovery of a new product or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery that a known product can cause a known injury (new act-consequence link).

We study the implications of unawareness for tort law, and specifically for the negligence versus strict liability debate. To model unawareness and growing awareness, which requires a theory of how beliefs update as the state space expands, we adopt the *reverse Bayesian* approach of Karni and Vierø (2013). Karni and Vierø (2013) posit that as a person becomes aware of new acts, consequences, or act-consequence links, her beliefs update in a way that preserves the relative likelihoods of events in the original state space. More specifically, they postulate that (i) in the case of a new act or consequence, probability mass shifts proportionally away from the events in the original state space to the new events in the expanded state space, and (ii) in the case of a new act-consequence link, null events in the original state space become non-null, and probability mass shifts proportionally away from the original non-null events to the original null events that become non-null.⁴

We argue that negligence has an important advantage over strict liability in a world with unawareness and growing awareness. Under either liability rule, a tort litigation involving a new act, consequence, or act-consequence link makes the world aware of a new possibility of harm. Under negligence, however, the litigation provides the world with more information. In particular, the court's stipulation of a new due care standard serves as a knowledge transmission mechanism, providing the world with information about the updated probability of harm. This information is necessary for either rule to induce the injurers of the world to take efficient care. Negligence provides this information to injurers. Under strict liability, they would have to expend additional resources to develop this information on their own.

Related literature and our contributions. To our knowledge, this paper is the first to incorporate unawareness into the economic analysis of tort law. As such, we contribute to the tort law and economics literature and to the unawareness literature, both of which

⁴A null event is an event believed to have zero probability.

are too vast to review here. Surveys of the tort law and economics literature, which was pioneered by Brown (1973),⁵ include Shavell (2007), Schäfer and Müller-Langer (2009), and more recently Arlen (2017). Surveys of the unawareness literature, which was pioneered by Fagin and Halpern (1988),⁶ include Schipper (2014) (which offers a “gentle introduction”) and Schipper (2015) (which provides an extended review).

Karni and Vierø (2013) are among the pioneers of the choice-theoretic approach (i.e., the state-space approach) to modeling unawareness. Subsequent papers build on their approach. For instance, Grant et al. (2017) invoke their approach to model learning by experimentation in a world with unawareness; Karni and Vierø (2017) extend their model to the case where the decision maker anticipates her growing awareness; and Dominiak and Tserenjigmid (2018) generalize their model such that the decision maker perceives ambiguity in the wake of growing awareness. Karni and Vierø (2013, 2017) and Dominiak and Tserenjigmid (2018) survey the papers that take a choice-theoretic approach to modeling unawareness.

One limitation of Karni and Vierø’s (2013) model is that reverse Bayesianism alone does not fully determine the updated probability distribution over the expanded state space. This is because reverse Bayesianism implies restrictions on the updated probabilities of non-null events in the original state space, but not on the probabilities of new events in the expanded state space.⁷ We overcome this limitation by assuming that acts are statistically independent (though we show that negligence still has a partial advantage over strict liability when we relax this assumption). We show that this additional restriction is sufficient to fully determine the updated probability distribution in each case of growing awareness (new act, new consequence, and new act-consequence link). In this way, we make a contribution to the reverse Bayesian model, independent of our contributions to the economics of tort law.

⁵Other early contributions include Diamond (1974a,b), Green (1976), and Shavell (1980).

⁶Other early contributions include Modica and Rustichini (1994, 1999), Dekel et al. (1998), Halpern (2001), Heifetz et al. (2006), and Halpern and Rêgo (2008).

⁷Karni and Vierø (2013, p. 2805) highlight this feature of reverse Bayesianism in their concluding remarks: “The model presented in this article predicts that, as awareness grows and the state space expands, the relative likelihoods of events in the original state space remain unchanged. The model is silent about the absolute levels of these probabilities. In other words, our theory does not predict the probability of the new events in the expanded state space.”

A handful of papers apply unawareness models to study other legal topics. The bulk of these focus on contracts. For example, Board and Chung (2011) argue that asymmetric unawareness provides a justification for the *contra proferentem* doctrine of contract interpretation, which provides that ambiguous terms in a contract should be construed against the drafter; Zhao (2011) argues that unawareness may explain the existence of force majeure clauses in contracts; Grant et al. (2012) study aspects of differential awareness that give rise to contractual disputes; Filiz-Ozbay (2012) posits asymmetric awareness as a reason for the incompleteness of contracts; von Thadden and Zhao (2012, 2014) study the properties of optimal contracts under moral hazard when the agent may be partially unaware of her action space; and Auster (2013) introduces asymmetric unawareness into the canonical moral hazard model and analyzes the properties of the optimal contract.

Within the tort law and economics literature, the papers closest to ours include the three described above that explore the implications of ambiguity for tort law (Teitelbaum, 2007; Chakravarty and Kelsey, 2017; Franzoni, 2017).⁸ Although ambiguity and unawareness are distinct phenomena, both are types of uncertainty that the standard accident model does not admit.⁹ Hence, we share a common enterprise with the papers on tort law and ambiguity. We enrich the standard accident model to allow the parties to face not just risk but rather a more profound and realistic type of uncertainty, and we explore the implications of such uncertainty for the debate over tort liability rules.

We also share connections with Ott and Schäfer (1997), Feess and Wohlschlegel (2006), and Currie and MacLeod (2014). Currie and MacLeod (2014) develop an alternative to

⁸Also related are the papers that explore the implications of risk aversion for tort law. In the seminal paper on the topic, Shavell (1982) shows that strict liability is superior when the injurer is risk neutral and the victim is risk averse, while negligence is superior in the opposite case. Franzoni (2017, n. 10) reviews other papers on optimal tort liability rules under risk aversion and related contributions.

⁹“Under ambiguity, the agent still conceives of the space of all relevant contingencies but has difficulties to evaluate them probabilistically. Under unawareness, the agent cannot even conceive all relevant contingencies” (Schipper, 2014, p. 1). Researchers currently are exploring connections between ambiguity and unawareness. For instance, Dominiak and Tserenjigmid (2018) generalize the model of Karni and Vierø (2013) such that the decision maker perceives ambiguity in the wake of growing awareness. We draw a similar connection between growing awareness and ambiguity in Section 4.4, where we discuss circumstances under which reverse Bayesian updating results in ambiguity about the updated probability of harm.

the standard accident model that makes use of state-space representations, dubbed “Savage Tables,” to model the decision problems faced by an injurer (who is fully aware of the state space) under different tort liability rules. They apply their model to argue, *inter alia*, that negligence provides better incentives than strict liability in the case of the Good Samaritan.

Ott and Schäfer (1997) study how the due care standard in negligence develops when the court starts with no information about the efficient level of care and relies on information provided by injurers and victims through litigation. In their model, an efficient standard evolves over time as a result of a learning process based on the information acquired by the court from litigants. In a similar vein, Feess and Wohlschlegel (2006) compare negligence and strict liability when some injurers have better information than others and the court about the efficient level of care and the court does not know which injurers are informed or uninformed. They show that under certain conditions the court can learn the efficient level of care by imperfectly observing the injurer’s level of care in a large number of cases, and that under negligence (but not strict liability) the uninformed injurers can in turn learn the efficient level of care by observing the court’s due care standard.

Like Ott and Schäfer (1997) we study the evolution of the negligence due care standard in response to knowledge generated by litigants, and like Feess and Wohlschlegel (2006) we argue that negligence has an advantage over strict liability in terms of knowledge transmission. Our motivation and analysis fundamentally differ from theirs, however, as we consider a world with symmetric unawareness whereas they consider a (fully aware) world with asymmetric information. Moreover, we explicitly model the process of belief revision in the wake of growing awareness and of knowledge transmission through the due care standard, while Ott and Schäfer (1997) derive transition probabilities from one standard to another and Feess and Wohlschlegel (2006) derive steady-state beliefs in a rational expectations equilibrium.

We also contribute to the relatively nascent but rapidly growing behavioral law and economics literature. Sunstein (1997), Jolls et al. (1998), and Korobkin and Ulen (2000) were early calls for the modification of standard law and economics models to reflect advances in

behavioral economics and decision theory. Sunstein (2000) and Parisi and Smith (2005) are edited volumes that collect early papers in the literature. Zamir and Teichman (2014) and Teitelbaum and Zeiler (2018) are more recent volumes. Halbersberg and Guttel (2014) and Luppi and Parisi (2018) provide surveys of behavioral models of tort law.

Structure of the paper. The remainder of the paper is organized as follows. Section 2 presents the accident model—a unilateral accident model featuring multiple activities with fixed levels—and derives the equivalence result. Section 3 presents the unawareness model and provides relevant examples of new acts, new consequences, and new act-consequence links. Section 4 compares and contrasts negligence and strict liability in a world with unawareness. It considers a simplified model with two acts, two consequences, quadratic care costs, and linear expected harm reduction, and separately analyzes the cases of a new act, a new consequence, and a new act-consequence link. Section 5 extends the analysis to a more general model with m acts, n consequences, convex care costs, and convex expected harm reduction. Section 6 discusses the results and suggests directions for future research. The Appendix collects the proofs of the propositions and corollaries stated in Sections 4 and 5.

2 The Accident Model

There are two agents: an injurer and a victim. Both are risk neutral subjective expected utility maximizers. The agents are strangers and not in a contractual relationship. Transaction costs are sufficiently high to preclude Coasian bargaining.

The injurer has available $m \geq 2$ activities, f_1, \dots, f_m . Each activity has the potential to cause harm to the victim, though we assume the outcomes are independent across activities. That is, we assume the activities are independent experiments, akin to m one-armed bandits. We refer to this assumption below as *act independence*.¹⁰

¹⁰While act independence is a reasonable assumption in many settings, there undoubtedly are settings in which it is not. We explore the implications of relaxing the act independence assumption in Section 4.4.

There are $n \geq 2$ potential degrees of harm, z_1, \dots, z_n , where $z_j \geq 0$ for all $j = 1, \dots, n$. Activity f_i causes harm z_j with probability π_{ij} , where $\sum_{j=1}^n \pi_{ij} = 1$ for all $i = 1, \dots, m$. Thus, activity f_i 's expected harm is $\sum_{j=1}^n \pi_{ij} z_j$. In the absence of unawareness, the agents have correct beliefs about each harm probability π_{ij} .

The injurer engages in each available activity. For each activity f_i , the injurer, but not the victim, can take care to reduce the activity's expected harm to the victim. The injurer chooses a level of care $x_i \geq 0$ having cost $c(x_i)$. Being careless is costless, $c(0) = 0$, and the marginal cost of care is positive and increasing: $c'(x_i) > 0$ and $c''(x_i) > 0$ for all x_i . Taking care reduces the activity's expected harm at a non-increasing rate: $h_i(x_i) \equiv \sum_{j=1}^n \pi_{ij} z_j \tau(x_i)$, where $\tau(x_i) \in (0, 1]$ for all x_i with $\tau(0) = 1$ and where $\tau'(x_i) < 0$ and $\tau''(x_i) \geq 0$ for all x_i . We assume that $c(\cdot)$ and $\tau(\cdot)$ are known and the same for all activities. We make the latter assumption for simplicity. It is without loss of generality given the former assumption.

If activity f_i causes harm, the victim may be entitled to damages from the injurer, depending on the applicable tort liability rule. Under negligence, the victim is entitled to damages equal to the harm if the injurer's level of care was below the due care standard for the activity, \bar{x}_i , which is stipulated by the court.¹¹ Under strict liability, the victim is entitled to damages equal to the harm irrespective of the injurer's level of care. We assume the injurer has the ability to pay any and all damages to which the victim may be entitled.

The social goal is to minimize the total social costs of the injurer's activities (the sum of the costs of care and the expected harms):

$$\underset{x_1, \dots, x_m \geq 0}{\text{minimize}} \quad \sum_{i=1}^m c(x_i) + h_i(x_i).$$

The solution $\tilde{x} = (\tilde{x}_1, \dots, \tilde{x}_m)$ is given implicitly by the first order conditions

$$c'(\tilde{x}_i) = -h'_i(\tilde{x}_i), \quad i = 1, \dots, m,$$

¹¹Following in the tradition of the tort law and economics literature, we model the due care standard as a precise stipulation. In reality, the due care standard may be less specific. For a discussion on the specificity of the due care standard at common law, see Dobbs et al. (2011, § 145).

and is given explicitly by

$$\tilde{x}_i = \xi^{-1} \left(\sum_{j=1}^n \pi_{ij} z_j \right), \quad i = 1, \dots, m,$$

where ξ^{-1} denotes the inverse of $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$.¹² We refer to \tilde{x}_i as the efficient level of care for activity f_i . It is the level of care at which the marginal cost of care equals the marginal benefit (the marginal reduction in expected harm).

Under strict liability, the injurer's problem is identical to the social goal. This is because strict liability forces the injurer to internalize the total social costs of her activities. Hence, strict liability induces the injurer to take efficient care in each activity.

Under negligence, the injurer's problem is

$$\begin{aligned} & \underset{x_1, \dots, x_m \geq 0}{\text{minimize}} && \sum_{i=1}^m c(x_i) + h_i(x_i) \chi(x_i < \bar{x}_i), \\ & \text{where} && \chi(x_i < \bar{x}_i) \equiv \begin{cases} 1 & \text{if } x_i < \bar{x}_i \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

and where \bar{x}_i is the due care standard for activity f_i . If the court sets $\bar{x}_i = \tilde{x}_i$ for all i , then the injurer takes efficient care in each activity. The reason is twofold. First, the injurer will not take more than the efficient level of care, because she faces no liability if her level of care equals or exceeds the efficient level. Second, the injurer will not take less than the efficient level of care, because then she faces strictly liability, which induces her to take efficient care.

The equivalence result follows immediately from the foregoing.

Theorem 1 (Equivalence Result). *The injurer will take efficient care in each activity under either negligence or strict liability, provided that, in the case of negligence, the court sets the due care standard for each activity equal to the efficient level of care for that activity.*

¹²Note that $\xi'(x_i) = \frac{c'(x_i)\tau''(x_i) - \tau'(x_i)c''(x_i)}{[\tau'(x_i)]^2} > 0$ for all x_i ; hence ξ is invertible.

3 The Unawareness Model

In this section we describe the model of unawareness and provide some examples of recent tort cases that illustrate the three cases of growing unawareness.

We model unawareness and growing awareness à la Karni and Vierø (2013). The primitives of the model are a finite, non-empty set F of feasible *acts* and a finite, non-empty set Z of feasible *consequences*. In our setting, the feasible acts are the injurer’s available activities and the feasible consequences are the potential harms to the victim.

States are functions from the set of acts to the set of consequences. A state assigns a consequence to each act. The set of all possible states, Z^F , defines the *conceivable state space*. With m acts and n consequences, there are n^m conceivable states.

The agents and the court (collectively, the *parties*) originally conceive the sets of acts and consequences to be $F = \{f_1, \dots, f_m\}$ and $Z = \{z_1, \dots, z_n\}$. The conceivable state space is $Z^F = \{s_1, \dots, s_{n^m}\}$, where each state $s \in Z^F$ is a vector of length m , the i th element of which, s^i , is the consequence $z_j \in Z$ produced by act $f_i \in F$ in that state of the world.

An act-consequence link, or *link*, is a causal relationship between an act and a consequence. The conceivable state space admits all conceivable links. However, the parties may perceive one or more links as infeasible, which brings them to nullify the states that admit such link. We refer to these as null states and denote them by $N \subset Z^F$. Taking only the non-null states defines the *feasible state space*, $S \equiv Z^F \setminus N$. When $N \neq \emptyset$, there are $\prod_{i=1}^m (n - \nu_i)$ feasible states, where ν_i denotes the number of nullified links involving act f_i .

The parties have common beliefs represented by a probability measure p on the conceivable state space, Z^F . The support set of p is the feasible state space, S . That is, the parties assign non-zero probability to each non-null state and zero probability to each null state.

The parties may initially fail to conceive one or more acts or consequences or to perceive as feasible one or more conceivable links. We refer to such failures of conception or perception as *unawareness*. However, the parties may later discover a new act or consequence, which expands both the feasible state space and the conceivable state space, or a new link, which

expands the feasible state space but not the conceivable state space.¹³ We refer to such discoveries and expansions as *growing awareness*.

To illustrate, suppose $S = Z^F$ and the parties discover a new consequence, z_{n+1} . Then the set of potential harms becomes $\widehat{Z} = Z \cup \{z_{n+1}\}$ and the feasible and conceivable state spaces both expand to $\widehat{S} = \widehat{Z}^F = \{s_1, \dots, s_{(n+1)^m}\}$, where each state remains a vector of length m . Alternatively, suppose the parties discover a new act, f_{m+1} . Then the set of available activities becomes $\widehat{F} = F \cup \{f_{m+1}\}$ and the feasible and conceivable state spaces both expand to $\widehat{S} = \widehat{Z}^{\widehat{F}} = \{s_1, \dots, s_{n(m+1)}\}$, where each state now is a vector of length $m + 1$. Lastly, suppose $S \subset Z^F$ because the parties initially perceive as infeasible the link from f_1 to z_n . Discovery of the link from f_1 to z_n does not alter the conceivable state space, but the feasible state space expands to coincide with the conceivable state space: $\widehat{S} = Z^F$. Illustrative depictions of conceivable and feasible state spaces and their expansion due to the discovery of new acts, consequences, and links can be found in Section 4.

In the wake of growing awareness, the parties' beliefs update in a way that preserves the relative likelihoods of the events in the original feasible state space (which are the non-null events in the original conceivable state space). In each case of growing awareness, probability mass shifts proportionally away from the events in the original feasible state space to the new events in the expanded feasible state space. In the case of a new act or consequence, the new events in the expanded feasible state space are also new events in the expanded conceivable state space. In the case of a new link, the new events in the expanded feasible state space are the null events in the original conceivable state space that become non-null.

Karni and Vierø refer to this updating as *reverse Bayesianism*. Let \widehat{p} denote the parties' updated beliefs. Formally, reverse Bayesianism implies: (i) in the case of a new consequence or link, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$; and (ii) in the case of a new act, $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$ for all $s, t \in S$, where $E(s)$ denotes the event in \widehat{S} that corresponds to state s in S ; that is, $E(s) \equiv \{t \in \widehat{S} : t^i = s^i \text{ for all } i \neq m + 1\}$ (assuming the new act is f_{m+1}).

¹³To be clear, by “new” we mean “not previously conceived” in the case of acts and consequences, and “previously conceived but perceived as infeasible” in the case of links.

The act independence assumption implies additional restrictions on \hat{p} . Let $A_i(z_j) \subset \hat{S}$ denote the event that f_i yields z_j ; that is, $A_i(z_j) \equiv \{t \in \hat{S} : t^i = z_j\}$. We refer to events of this type as *act events*. Act independence implies $A_i(z_j) \perp A_{i'}(z_{j'})$ for all i and i' where $i \neq i'$ and all j and j' .¹⁴ Take any event $E \subseteq \hat{S}$. We can express each state $s = (s^1, \dots, s^m) \in E$ as the intersection of a unique collection of independent act events: $s = \bigcap_i A_i(s^i)$. It follows that $\hat{p}(s) = \prod_i \hat{p}(A_i(s^i))$ for all $s \in E$. Observe that growing awareness, whether it entails a new act, consequence, or link, gives rise to a new event $\Delta = \hat{S} \setminus S$. Thus, in each case of growing awareness, act independence implies $\hat{p}(s) = \prod_i \hat{p}(A_i(s^i))$ for all $s \in \Delta$.

3.1 Examples of Growing Awareness

Instances of growing awareness that are relevant to tort law include the discovery of a new and potentially harmful product or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery that a known product can cause a known injury in a previously unknown way (new link). We list below a few illustrative examples.

3.1.1 New Acts

Fracking. Modern day hydraulic fracturing, or “fracking,” was developed by George Mitchell in the late 1990s, though its origins can be traced to the 1860s (Gold, 2014). In *Ely v. Cabot Oil and Gas Corporation*, a Pennsylvania jury found in favor of nine plaintiffs on claims that the defendant’s fracking activity at two natural gas wells in Susquehanna County in the mid-2000s was negligent and caused the plaintiffs’ compensable nuisance injuries by interfering with and damaging the plaintiffs’ access to water and their enjoyment of their property. The gravamen of the plaintiffs’ complaint was that the defendant’s fracking activity negligently permitted methane to flow into underground aquifers that wound up polluting the plaintiffs’ water wells.¹⁵ Less than two years after the jury verdict in 2016, and less than three months

¹⁴The symbol \perp denotes statistical independence. Acts are assumed to be independent with respect to both the ex-ante and ex-post probabilities.

¹⁵See 2017 WL 1196510 (M.D. Pa. 2017) (denying defendant’s motion for judgment as a matter of law but granting its motion for a new trial). The case settled before the second trial.

after the case was finally settled in 2017, the defendant and 25 other major oil and gas companies, under the auspices of the American Petroleum Institute, announced that they were launching a new program, called the Environmental Partnership, focused on reducing emissions of methane and other pollutants from the natural gas sector (Henry, 2017).

Cyberbullying. The World Wide Web was invented by Tim Berners-Lee in 1989 (Gillies and Cailliau, 2000). In *D.C. v. R.R.*, a Los Angeles high school student brought an action against several of his fellow students—who had posted messages at his Web site making derogatory comments about his perceived sexual orientation and threatening him with bodily harm—claiming, *inter alia*, defamation and intentional infliction of emotional distress.¹⁶ According to the Cyberbullying Research Center (2016), the rate of cyberbullying offending among U.S. middle and high school students decreased by 40 percent in the six years after the key decision in the case by the California Court of Appeal in 2010.

3.1.2 New Consequences

HIV/AIDS. The first HIV/AIDS cases were reported in the United States in 1981 (U.S. Centers for Disease Control and Prevention, 2011). In *Quintana v. United Blood Services*, a Denver jury held the defendant liable to the plaintiff for negligently supplying her with HIV contaminated blood in 1983 (Talavera, 1993).¹⁷ In the three decades since the case was filed, U.S. blood banks, through “the use of donor educational material, specific deferral questions, and advances in HIV donor testing . . . have reduced the risk of HIV transmission from blood transfusion from about 1 in 2500 units . . . to a current estimated residual risk of about 1 in 1.47 million transfusions” (U.S. Food and Drug Administration, 2015, p. 2).

¹⁶See 182 Cal. App. 4th 1190 (2010) (affirming the trial court’s denial of defendants’ motion to strike under California’s strategic lawsuit against public participation (SLAPP) statute).

¹⁷The jury also held the defendant liable to the plaintiff’s husband for negligent infliction of emotional distress and loss of consortium. The case was initially filed in the late 1980s and the final verdict was rendered in 1992 (Talavera, 1993).

Mad cow disease. Mad cow disease, also known as bovine spongiform encephalopathy (BSE), was first discovered in the United Kingdom in 1986 (Collee and Bradley, 1997). In 2008, Ridley Inc., a Winnipeg cattle feed supplier, settled a class action by Canadian cattle farmers claiming that the defendant negligently supplied them with BSE contaminated feed in the early 1990s (Dowd, 2008). Still pending is a related class action against the Canadian government for negligently allowing BSE infected cattle to be imported into Canada in the late 1980s and used for feed ingredients in the early 1990s (Kienlen, 2017). In 2007, two years after the class action was filed, the Canadian government implemented an enhanced feed ban aimed at preventing the spread of BSE (Stephenson, 2015).

3.1.3 New Links

Agent Orange and cancer. Agent Orange, a chemical herbicide, was developed in the 1940s and used by the U.S. military as part of its herbicidal warfare program, Operation Ranch Hand, during the Vietnam War (Schuck, 1987). The first recorded case of cancer hails from ancient Egypt, and the origin of the word cancer is credited to the ancient Greek physician Hippocrates (Sudhakar, 2009). In *In re Agent Orange Product Liability Litigation*, Vietnam veterans brought a class action against the manufacturers of Agent Orange alleging, *inter alia*, that their exposure to Agent Orange in Vietnam resulted in a variety of cancers and other diseases in the veterans and birth defects in their children.¹⁸ The class action was filed in 1979. In 1983, seven months before the parties reached a settlement, Dow Chemical Company, the lead defendant in the class action, announced that it was permanently discontinuing production of 2,4,5-trichlorophenoxyacetic acid, the component of Agent Orange that was responsible for its toxicity (Holusha, 1983; American Chemical Society, 1985).

American football and CTE. The National Football League (NFL) was founded in 1920 (Crepeau, 2014). Chronic traumatic encephalopathy (CTE), a neurodegenerative brain disease found in people with a history of repetitive head injuries, was first reported in boxers

¹⁸See 597 F. Supp. 740 (E.D.N.Y. 1984). The case was settled for \$180 million.

in 1928 and in NFL players in 2005 (Lindsley, 2017). In *In re National Football League Players Concussion Injury Litigation*, retired NFL players brought a class action against the NFL alleging that the league had failed to take reasonable actions to protect the players from CTE and other chronic risks of head injuries in football.¹⁹ The action was filed in 2011. Seven months before the parties reached a settlement in 2013, the NFL announced a new concussion protocol, which includes guidelines for sideline evaluation and rules on preseason education, baseline testing, and the establishment of personnel to conduct evaluations (Flynn, 2016).

In each of the foregoing examples the initial lawsuit increased awareness and precaution undertaken by injurers.

4 Illustrative Results

In this section and the next, we compare and contrast negligence and strict liability in a world with unawareness. In both sections, we assume the parties are fully rational apart from unawareness. We further assume that when the parties are unaware of an act, consequence, or link, their beliefs, although incorrect with respect to the absolute likelihoods of events, are nevertheless correct with respect to the relative likelihoods of non-null events. Without this assumption, the parties could not have correct beliefs when they become fully aware, which would be inconsistent with the standard accident model.

In this section we consider a simple model with two acts, $F = \{f_1, f_2\}$; two consequences, $Z = \{z_1, z_2\}$, where $z_1 = 0$ and $z_2 > 0$; quadratic care costs, $c(x_i) = (x_i)^2$; and linear expected harm reduction, $\tau(x_i) = (1 - x_i)$.²⁰ Our analysis of this simple model illustrates all of the main ideas of the paper. In the next section we show that the results extend to a more general model. In both sections we maintain the assumption that $c(\cdot)$ and $\tau(\cdot)$ are known and the same for all activities. As previously stated, we make the latter assumption for simplicity; it is without loss of generality given the former assumption.

¹⁹See 821 F.3d 410 (3d Cir. 2016). The case was settled for approximately \$1 billion.

²⁰To preserve the condition $\tau(x_i) > 0$ for all x_i , we assume $x_i \in [0, 1)$ in this section.

With two acts (activities), f_1 and f_2 , and two consequences (harms), $z_1 = 0$ and $z_2 > 0$, the conceivable state space, Z^F , comprises four states: $s_1 = (0, 0)$, $s_2 = (0, z_2)$, $s_3 = (z_2, 0)$, and $s_4 = (z_2, z_2)$. Let $p_k \equiv p(s_k)$, $k = 1, \dots, 4$, denote the parties' common beliefs on Z^F . We can depict the original conceivable state space and the parties' beliefs as follows:

p	p_1	p_2	p_3	p_4
$F \setminus Z^F$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

4.1 New Link

We start with the case of a new link. We assume the parties initially perceive activity f_1 as safe and activity f_2 as risky (i.e.. capable of resulting in harm). That is, we assume they initially perceive the event $\{s_3, s_4\}$ as infeasible (null). This implies $p_3 = p_4 = 0$. We can depict the original feasible state space, $S \subset Z^F$, as follows:

p	p_1	p_2
$F \setminus S$	s_1	s_2
f_1	0	0
f_2	0	z_2

Given S and p , the efficient levels of care are

$$\tilde{x}_1 = 0 \quad \text{and} \quad \tilde{x}_2 = \frac{p_2 z_2}{2}.$$

Under negligence, the court stipulates $\bar{x}_1 = \tilde{x}_1$ and $\bar{x}_2 = \tilde{x}_2$ as the due care standards for f_1 and f_2 , respectively.

Suppose the parties discover that activity f_1 is risky. In particular, suppose that the injurer engages in f_1 , that it results in harm z_2 , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands to coincide with the conceivable state space, $\widehat{S} = Z^F$, and the parties update their beliefs from p to \widehat{p} :

\widehat{p}	\widehat{p}_1	\widehat{p}_2	\widehat{p}_3	\widehat{p}_4
$F \setminus \widehat{S}$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

We assume that, by virtue of the suit, the parties learn that activity f_1 yields harm z_2 with probability $\delta > 0$. The fact is the parties have the incentive to expend resources to develop this knowledge. As the Hand formula makes plain,²¹ the probability of harm is an essential component of a negligence case. Even in a strict liability case, the probability of harm is relevant to the issues of foreseeability and proximate cause.²²

Note that δ is the total probability of the new states in the expanded feasible state space. It is a measure of the likelihood of the event of which the parties were previously unaware. Thus, we interpret δ as the *degree of unawareness*.

By reverse Bayesianism,

$$\frac{p_1}{p_2} = \frac{\widehat{p}_1}{\widehat{p}_2}.$$

In addition, $\delta = \widehat{p}_3 + \widehat{p}_4$ (by definition) and $\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_3 + \widehat{p}_4 = 1$. Moreover, by act independence,

$$\widehat{p}_3 = (\widehat{p}_3 + \widehat{p}_4)(\widehat{p}_1 + \widehat{p}_3) \quad \text{and} \quad \widehat{p}_4 = (\widehat{p}_3 + \widehat{p}_4)(\widehat{p}_2 + \widehat{p}_4).$$

It follows that:

Proposition 1. $\widehat{p}_1 = (1 - \delta)p_1$, $\widehat{p}_2 = (1 - \delta)p_2$, $\widehat{p}_3 = \delta p_1$, and $\widehat{p}_4 = \delta p_2$.

²¹See *United States v. Carroll Towing Co.*, 159 F.2d 169 (2d Cir. 1947); Posner (1998).

²²Alternatively, we could assume the court learns δ by virtue of a sequence of suits (cf. Ott and Schäfer, 1997; Feess and Wohlschlegel, 2006).

Remark 1. Note that p is the Bayesian update of \hat{p} conditional on the event $\{s_1, s_2\}$; hence the term reverse Bayesianism.

Given \hat{S} and \hat{p} , the efficient levels of care are

$$\hat{x}_1 = \frac{(\hat{p}_3 + \hat{p}_4)z_2}{2} = \frac{\delta z_2}{2} \quad \text{and} \quad \hat{x}_2 = \frac{(\hat{p}_2 + \hat{p}_4)z_2}{2} = \frac{p_2 z_2}{2}.$$

Note that $\hat{x}_1 > \tilde{x}_1$ but $\hat{x}_2 = \tilde{x}_2$. Thus, the discovery that f_1 is risky necessitates the stipulation of a new due care standard for f_1 but not for f_2 .

Under negligence, the court stipulates $\bar{x}_1 = \hat{x}_1$ as the new due care standard for f_1 and holds the injurer liable to pay damages of z_2 to the victim.²³ This makes the injurers and victims of the world aware that f_1 is risky. Moreover, they can deduce δ from \hat{x}_1 ; specifically, $\delta = 2\hat{x}_1/z_2$. As a result, they can learn \hat{p} and $\hat{h}_1(x_1) = \delta z_2 \tau(x_1)$, without expending additional resources to learn about δ . Knowledge of $\hat{h}_1(x_1)$ is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_2 to the victim. This makes the injurers and victims of the world aware that f_1 is risky. However, they cannot deduce δ or learn \hat{p} or $\hat{h}_1(x_1)$. Without knowledge of $\hat{h}_1(x_1)$, strict liability cannot induce injurers to take efficient care.

Remark 2. If $c(\cdot)$ and $\tau(\cdot)$ are the same for all injurers, then knowledge of $\hat{h}_1(x_1)$ is not strictly necessary to induce them to take efficient care under negligence, because they can just blindly adopt \bar{x}_1 as their level of care, without bothering to deduce δ from \bar{x}_1 and learn $\hat{h}_1(x_1)$. If, however, either $c(\cdot)$ or $\tau(\cdot)$ varies across injurers, then they need to deduce δ from \bar{x}_1 in order to learn their own $\hat{h}_1(x_1)$, which is necessary for them to take efficient care.

²³Recall that $\bar{x}_1 = \tilde{x}_1 = 0$ before the parties discover that f_1 is potentially harmful. Under negligence, therefore, the injurer will have exercised no care in conjunction with f_1 . However, even if the court does not hold the injurer liable and award damages (perhaps by recognizing a civil ex post facto doctrine which prohibits retroactive application of a due care standard in a negligence suit), our results below would not change, because the world already knows the set of potential harms.

Moreover, injurers always need to know $\widehat{h}_1(x_1)$ in order to take efficient care under strict liability, because the court does not stipulate a due care standard under strict liability.

4.2 New Act

We next consider the case of a new act. We assume the original feasible state space coincides with the original conceivable state space (i.e., $S = Z^F$):

p	p_1	p_2	p_3	p_4
$F \setminus S$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

Given S and p , the efficient levels of care are

$$\tilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \tilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates $\bar{x}_1 = \tilde{x}_1$ and $\bar{x}_2 = \tilde{x}_2$ as the due care standards for f_1 and f_2 , respectively.

Suppose the parties discover a new activity, f_3 , which has the potential to cause harm. In particular, suppose that the injurer discovers and engages in f_3 , that it results in harm z_2 , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to eight states:

\widehat{p}	\widehat{p}_1	\widehat{p}_2	\widehat{p}_3	\widehat{p}_4	\widehat{p}_5	\widehat{p}_6	\widehat{p}_7	\widehat{p}_8
$F \setminus \widehat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
f_1	0	0	z_2	z_2	0	0	z_2	z_2
f_2	0	z_2	0	z_2	0	z_2	0	z_2
f_3	0	0	0	0	z_2	z_2	z_2	z_2

The expanded feasible state space contains two copies of the original feasible state space, one in which f_3 results in no harm and one in which f_3 results in harm z_2 . Stated differently, the expanded space splits each of the original states into two depending on whether f_3 yields no harm or harm z_2 . For each state in the original feasible state space there is a corresponding event in the expanded feasible state space. For instance, the event $\{s_1, s_5\} \in \widehat{S}$ corresponds to state $s_1 \in S$, the event $\{s_2, s_6\} \in \widehat{S}$ corresponds to state $s_2 \in S$, etc.²⁴

As before, we assume that, by virtue of the suit, the parties learn that activity f_3 yields harm z_2 with probability $\delta > 0$. By reverse Bayesianism,

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_2 + \widehat{p}_6}, & \frac{p_1}{p_3} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_3 + \widehat{p}_7}, & \frac{p_1}{p_4} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_4 + \widehat{p}_8}, \\ \frac{p_2}{p_3} &= \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_3 + \widehat{p}_7}, & \frac{p_2}{p_4} &= \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_4 + \widehat{p}_8}, & \text{and} & \frac{p_3}{p_4} &= \frac{\widehat{p}_3 + \widehat{p}_7}{\widehat{p}_4 + \widehat{p}_8}. \end{aligned}$$

In addition, $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8$ (by definition) and $\widehat{p}_1 + \dots + \widehat{p}_8 = 1$. Moreover, by act independence,

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \text{and } \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8). \end{aligned}$$

It follows that:

Proposition 2. $\widehat{p}_1 = (1 - \delta)p_1, \quad \widehat{p}_2 = (1 - \delta)p_2, \quad \widehat{p}_3 = (1 - \delta)p_3, \quad \widehat{p}_4 = (1 - \delta)p_4,$
 $\widehat{p}_5 = \delta p_1, \quad \widehat{p}_6 = \delta p_2, \quad \widehat{p}_7 = \delta p_3, \quad \text{and} \quad \widehat{p}_8 = \delta p_4.$

²⁴Note that the conceivable state space also expands from four to eight states, so $\widehat{S} = Z^{\widehat{F}}$.

Given \widehat{S} and \widehat{p} , the efficient levels of care are

$$\begin{aligned}\widehat{x}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8) z_2}{2} = \frac{(p_3 + p_4) z_2}{2}, \\ \widehat{x}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8) z_2}{2} = \frac{(p_2 + p_4) z_2}{2}, \\ \text{and } \widehat{x}_3 &= \frac{(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) z_2}{2} = \frac{\delta z_2}{2}.\end{aligned}$$

Thus, the discovery of f_3 necessitates the stipulation of a new due care standard, $\widehat{\bar{x}}_3$, but it does not necessitate the stipulation of a new due care standard for f_1 or f_2 .

Under negligence, the court stipulates $\widehat{\bar{x}}_3 = \widehat{x}_3$ as the due care standard for the new activity f_3 and holds the injurer liable to pay damages of z_2 to the victim.²⁵ This makes the injurers and victims of the world aware of f_3 (and that it is risky). Moreover, they can deduce δ from $\widehat{\bar{x}}_3$; specifically, $\delta = 2\widehat{\bar{x}}_3/z_2$. As a result, they can learn \widehat{p} and $\widehat{h}_3(x_3) = \delta z_2 \tau(x_3)$, without expending additional resources to learn about δ . Knowledge of $\widehat{h}_3(x_3)$ is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_2 to the victim. This makes injurers and victims aware of f_3 (and that it is risky). However, they cannot deduce δ or learn \widehat{p} or $\widehat{h}_3(x_3)$. Without knowledge of $\widehat{h}_3(x_3)$, strict liability cannot induce injurers to take efficient care.

4.3 New Consequence

We last consider the case of a new consequence. As with the case of a new act, we assume $S = Z^F$:

p	p_1	p_2	p_3	p_4
$F \setminus S$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

²⁵Again, because the world already knows the set of potential harms, our results below would not change if the court does not hold the injurer liable and award damages.

Given S and p , the efficient levels of care are

$$\tilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \tilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates $\bar{x}_1 = \tilde{x}_1$ and $\bar{x}_2 = \tilde{x}_2$ as the due care standards for f_1 and f_2 , respectively.

Suppose the parties discover a new consequence, $z_3 > z_2$, which they link to f_1 and f_2 . In particular, suppose that the injurer engages in f_1 and f_2 , that each results in harm z_3 , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to nine states:

\hat{p}	\hat{p}_1	\hat{p}_2	\hat{p}_3	\hat{p}_4	\hat{p}_5	\hat{p}_6	\hat{p}_7	\hat{p}_8	\hat{p}_9
$F \setminus \hat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
f_1	0	0	z_2	z_2	z_3	z_3	0	z_2	z_3
f_2	0	z_2	0	z_2	0	z_2	z_3	z_3	z_3

The expanded feasible state space is characterized by three events, one in which f_1 results in no harm, one in which f_1 results in harm z_2 , and one in which f_1 results in harm z_3 . Each event contains three states, one in which f_2 results in no harm, one in which f_2 results in z_2 , and one in which f_2 results in z_3 .²⁶

We assume that, by virtue of the suit, the parties learn that activity f_1 yields z_3 with probability $\alpha > 0$ and activity f_2 yields z_3 with probability $\beta > 0$. (This is analogous to assuming the parties learn δ in the case of a new link or act.) By reverse Bayesianism,

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{\hat{p}_1}{\hat{p}_2}, & \frac{p_1}{p_3} &= \frac{\hat{p}_1}{\hat{p}_3}, & \frac{p_1}{p_4} &= \frac{\hat{p}_1}{\hat{p}_4}, \\ \frac{p_2}{p_3} &= \frac{\hat{p}_2}{\hat{p}_3}, & \frac{p_2}{p_4} &= \frac{\hat{p}_2}{\hat{p}_4}, & \text{and} & \frac{p_3}{p_4} &= \frac{\hat{p}_3}{\hat{p}_4}. \end{aligned}$$

²⁶Note that the conceivable state space also expands from four to nine states, so $\hat{S} = \hat{Z}^F$.

In addition, $\alpha = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9$ and $\beta = \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$ (both by definition) and $\widehat{p}_1 + \cdots + \widehat{p}_9 = 1$.
By act independence,

$$\begin{aligned}\widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), & \widehat{p}_6 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\ \widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), & \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), \\ \text{and } \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9).\end{aligned}$$

It follows that:

Proposition 3. $\widehat{p}_1 = (1 - \alpha)(1 - \beta)p_1$, $\widehat{p}_2 = (1 - \alpha)(1 - \beta)p_2$, $\widehat{p}_3 = (1 - \alpha)(1 - \beta)p_3$,
 $\widehat{p}_4 = (1 - \alpha)(1 - \beta)p_4$, $\widehat{p}_5 = \alpha(1 - \beta)(p_1 + p_3)$, $\widehat{p}_6 = \alpha(1 - \beta)(p_2 + p_4)$,
 $\widehat{p}_7 = \beta(1 - \alpha)(p_1 + p_2)$, $\widehat{p}_8 = \beta(1 - \alpha)(p_3 + p_4)$, and $\widehat{p}_9 = \alpha\beta$.

Note that the degree of unawareness is $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \alpha + \beta - \alpha\beta$ and that $1 - \delta = (1 - \alpha)(1 - \beta)$. We can rewrite \widehat{p} in terms of δ as follows:

Corollary 1. $\widehat{p}_1 = (1 - \delta)p_1$, $\widehat{p}_2 = (1 - \delta)p_2$, $\widehat{p}_3 = (1 - \delta)p_3$, $\widehat{p}_4 = (1 - \delta)p_4$,
 $\widehat{p}_5 = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_1 + p_3) = (\delta - \beta)(p_1 + p_3)$, $\widehat{p}_6 = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_2 + p_4) = (\delta - \beta)(p_2 + p_4)$,
 $\widehat{p}_7 = \frac{\beta}{1 - \beta}(1 - \delta)(p_1 + p_2) = (\delta - \alpha)(p_1 + p_2)$, $\widehat{p}_8 = \frac{\beta}{1 - \beta}(1 - \delta)(p_3 + p_4) = (\delta - \alpha)(p_3 + p_4)$,
and $\widehat{p}_9 = \alpha + \beta - \delta$.

Given \widehat{S} and \widehat{p} , the efficient levels of care are,

$$\begin{aligned}\widehat{\widehat{x}}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)z_2 + (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)z_3}{2} = \frac{(1 - \alpha)(p_3 + p_4)z_2 + \alpha z_3}{2} \\ \text{and } \widehat{\widehat{x}}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6)z_2 + (\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9)z_3}{2} = \frac{(1 - \beta)(p_2 + p_4)z_2 + \beta z_3}{2}.\end{aligned}$$

Note that $\widehat{\widehat{x}}_1 > \widetilde{x}_1$ and $\widehat{\widehat{x}}_2 > \widetilde{x}_2$. Thus, the discovery of z_3 necessitates the stipulation of new due care standards for both f_1 and f_2 .

Under negligence, the court stipulates $\widehat{\widehat{x}}_1 = \widehat{\widehat{x}}_1$ and $\widehat{\widehat{x}}_2 = \widehat{\widehat{x}}_2$ as the new due care standards for f_1 and f_2 , respectively. The court holds the injurer liable to pay damages of z_3 to the

victim with respect to each of f_1 and f_2 . This makes the injurers and victims of the world aware of z_3 (and that it is linked to f_1 and f_2).²⁷ Moreover, they can deduce α and β (and, therefore, δ) from \widehat{x}_1 and \widehat{x}_2 ; specifically,

$$\alpha = \frac{p_3 z_2 - 2\widehat{x}_1 + p_4 z_2}{p_3 z_2 - z_3 + p_4 z_2} \quad \text{and} \quad \beta = \frac{p_2 z_2 - 2\widehat{x}_2 + p_4 z_2}{p_2 z_2 - z_3 + p_4 z_2}.$$

As a result, they can learn \widehat{p} and

$$\begin{aligned} \widehat{h}_1(x_1) &= [(1 - \alpha)(p_3 + p_4)z_2 + \alpha z_3]\tau(x_1) \\ \text{and } \widehat{h}_2(x_2) &= [(1 - \beta)(p_2 + p_4)z_2 + \beta z_3]\tau(x_2), \end{aligned}$$

without expending additional resources to learn about α and β . Knowledge of $\widehat{h}_1(x_1)$ and $\widehat{h}_2(x_2)$ is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_3 for each instance of harm. This makes the injurers and victims of the world aware of z_3 (and that it is linked to f_1 and f_2). However, they cannot deduce α or β or learn \widehat{p} , $\widehat{h}_1(x_1)$, or $\widehat{h}_2(x_2)$. Without knowledge of $\widehat{h}_1(x_1)$ and $\widehat{h}_2(x_2)$, strict liability cannot induce efficient care.

4.4 Act Independence

Before turning to the general results, we conclude this section with a few remarks about the act independence assumption. In short, we argue that it is a useful simplifying assumption, but that it is not crucial. Even without act independence, negligence would reveal useful information to injurers regarding the updated probability of harm.

As revealed by the proofs of Propositions 1 through 3, reverse Bayesianism alone is not sufficient to fully determine the updated probability distribution \widehat{p} . To borrow a term from the econometrics literature, reverse Bayesianism only partially identifies \widehat{p} . The reason is

²⁷Even if the court does not hold the injurer liable and award damages, the victim's claims make the world aware of z_3 (and that it is linked to f_1 and f_2).

that reverse Bayesianism implies restrictions on the updated probabilities of non-null states in the original state space (or, in the case of a new act, their corresponding events in the expanded state space), but not on the probabilities of new states in the expanded state space. In other words, reverse Bayesianism prescribes how probability mass shifts away from non-null states in the original state space to the corresponding states or events in the expanded state space, but it does not dictate how the shifted probability mass is distributed among the new states in the expanded state space. This is where act independence comes in. It determines how the shifted probability mass is apportioned among the new states. Together, reverse Bayesianism and act independence fully identify \hat{p} .

How realistic is act independence? The answer depends on the nature of the specific activities in question. For instance, the risk that fracking for natural gas results in groundwater contamination is likely to be independent of the risk that importing liquefied natural gas results in a fire or explosion. By contrast, the risk of contracting HIV from sharing drug injection needles is likely to be correlated with the risk of contracting HIV from having unprotected sex, since both depend on the prevalence of HIV in the population.

Because there exist activities whose outcomes are not independent, it is useful to investigate the importance of the act independence assumption for our results.

New link. In the case of a new link, reverse Bayesianism alone implies $\hat{p}_1 = (1 - \delta)p_1$, $\hat{p}_2 = (1 - \delta)p_2$, and $\hat{p}_3 + \hat{p}_4 = \delta$. Importantly, reverse Bayesianism alone is not sufficient to separately identify \hat{p}_3 and \hat{p}_4 . As it turns out, this does not create an issue with respect to activity f_1 . Recall that, by assumption, the parties learn δ (the probability that f_1 yields z_2). Because the efficient level of care for f_1 is a function of the sum $\hat{p}_3 + \hat{p}_4$, the court can stipulate a new due care standard for f_1 in terms of δ . This is sufficient to make the injurers and victims of the world aware that f_1 is potentially harmful. Moreover, they can deduce δ from the new due care standard for f_1 and, in turn, learn $\hat{h}_1(x_1) = \delta z_2 \tau(x_1)$.

Relaxing act independence, however, creates ambiguity with respect to the updated risk of activity f_2 . Because the efficient level of care for f_2 is a function of the sum $\widehat{p}_2 + \widehat{p}_4$, without act independence (or another assumption that separately identifies \widehat{p}_3 and \widehat{p}_4), the court cannot stipulate a precise new due care standard for f_2 . The best the court can do is specify lower and upper bounds, using the knowledge that $\widehat{p}_4 \in (0, \delta)$. Given these bounds, the best the injurers and victims of the world can do is infer bounds on $\widehat{h}_2(x_2)$.

Of course, the ambiguity can be resolved if, by virtue of the suit, the parties learn more about \widehat{p} . For instance, if the parties learn not only δ but also either $\widehat{p}_2 + \widehat{p}_4$ (the updated probability that f_2 yields z_2) or \widehat{p}_4 (the joint probability that f_1 and f_2 yield z_2), this is sufficient to separately identify \widehat{p}_3 and \widehat{p}_4 . With this, the court can stipulate a precise new due care standard for f_2 , from which the injurers and victims of the world can deduce $\widehat{p}_2 + \widehat{p}_4$ and, in turn, learn $\widehat{h}_2(x_2) = (\widehat{p}_2 + \widehat{p}_4)z_2\tau(x_2)$.

New act. In the case of a new act, reverse Bayesianism alone implies $\widehat{p}_1 + \widehat{p}_5 = p_1$, $\widehat{p}_2 + \widehat{p}_6 = p_2$, $\widehat{p}_3 + \widehat{p}_7 = p_3$, $\widehat{p}_4 + \widehat{p}_8 = p_4$, and $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 = \delta$. Recall that the efficient level of care for f_1 is a function of the sum $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8$; the efficient level of care for f_2 is a function of the sum $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8$; and the efficient level of care for f_3 is a function of the sum $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8$. Hence, even without act independence, the court's information is sufficiently precise (i) to know that it need not stipulate new due care standards for activities f_1 and f_2 and (ii) to stipulate a due care standard for the new activity f_3 . This makes the injurers and victims of the world aware of f_3 (and that it is risky). Moreover, they can deduce δ from the due care standard for f_3 and, in turn, learn $\widehat{h}_3(x_3) = \delta z_3 \tau(x_3)$.

New consequence. In the case of a new consequence, reverse Bayesianism alone implies $\widehat{p}_1 = (1 - \delta)p_1$, $\widehat{p}_2 = (1 - \delta)p_2$, $\widehat{p}_3 = (1 - \delta)p_3$, $\widehat{p}_4 = (1 - \delta)p_4$, and $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \delta$. By assumption, the parties learn $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9 = \alpha$ (the probability that f_1 yields z_3) and $\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \beta$ (the probability that f_2 yields z_3). Assume the parties also learn \widehat{p}_9 (the joint probability that f_1 and f_2 yield z_3), and let $\widehat{p}_9 = \gamma$. Note that $\delta = \alpha + \beta - \gamma$.

Recall that the efficient level of care for activity f_1 is a function of α and the sum $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$ (the updated probability that f_1 yields z_2), and the efficient level of care for activity f_2 is a function of β and the sum $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$ (the updated probability that f_2 yields z_2). Without act independence, the sums $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$ and $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$ are only partially identified (because \widehat{p}_6 and \widehat{p}_8 are not separately identified), creating ambiguity with respect to the updated risks of both activities. As a result, the court cannot stipulate precise new due care standards for activities f_1 and f_2 . The best the court can do is specify lower and upper bounds:

$$\widehat{x}_1 \in \left(\frac{(1-\delta)(p_3+p_4)z_2 + \alpha z_3}{2}, \frac{((1-\delta)(p_3+p_4) + \delta)z_2 + \alpha z_3}{2} \right)$$

and $\widehat{x}_2 \in \left(\frac{(1-\delta)(p_2+p_4)z_2 + \beta z_3}{2}, \frac{((1-\delta)(p_2+p_4) + \delta)z_2 + \beta z_3}{2} \right).$

Given these bounds, and given that the victim's claims make the world aware of z_3 (and that it is linked to f_1 and f_2), the injurers and victims of the world can deduce α , β , and δ ; however, the best they can do is infer bounds on $\widehat{h}_1(x_1)$ and $\widehat{h}_2(x_2)$.

As before, the ambiguity can be resolved if, by virtue of the suit, the parties learn more about \widehat{p} . For instance, if the parties learn not only δ and γ but also either $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$ or $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$, this is sufficient to separately identify \widehat{p}_5 , \widehat{p}_6 , \widehat{p}_7 , and \widehat{p}_8 . With this, the court can stipulate precise new due care standards for f_1 and f_2 , from which injurers and victims of the world can learn $\widehat{h}_1(x_2)$ and $\widehat{h}_2(x_2)$.

In summary, without act independence, reverse Bayesianism only partially identifies \widehat{p} . This does not create an issue in the case of a new act—the court's information is sufficiently precise to stipulate a due care standard with respect to each activity. In the case of a new link or consequence, however, the partial identification of \widehat{p} creates ambiguity with respect to the updated risk of one or both activities, leading to imprecise due care standards. In short, we might say that, without act independence, negligence achieves only “boundedly optimal” deterrence. That said, negligence still has a partial advantage over strict liability.

What's more, the ambiguity in any case can be resolved if the parties learn more about \hat{p} . In other words, the more the parties learn about the updated probability of harm, the less important is the act independence assumption for our results.²⁸

5 General Results

In this section, we show that our results extend to a more general model with m acts and n consequences. We also relax the shape restrictions on the care cost and expected harm reduction functions and assume only that each is convex.

Let $F = \{f_1, \dots, f_m\}$ be the set of activities and $Z = \{z_1, \dots, z_n\}$ be the set of harms, where $0 \leq z_1 < z_2 < \dots < z_n$. For each activity f_i , the cost of taking care $x_i \geq 0$ is $c(x_i)$, where $c(0) = 0$, $c'(x_i) > 0$, and $c''(x_i) > 0$ for all $x_i \geq 0$. Activity f_i 's expected harm is $h_i(x_i) \equiv \sum_{j=1}^n \pi_{ij} z_j \tau(x_i)$, where (i) π_{ij} is the probability that f_i causes z_j and (ii) $\tau(x_i) \in (0, 1]$, $\tau(0) = 1$, $\tau'(x_i) < 0$, and $\tau''(x_i) \geq 0$ for all $x_i \geq 0$.²⁹

Given F and Z , the conceivable state space is Z^F , where each state $s \in Z^F$ is a vector of length m , the i th element of which, s^i , is the harm $z_j \in Z$ caused by activity $f_i \in F$ in that state. The feasible state space is $S \equiv Z^F \setminus N$, where $N \subset Z^F$ is the set of null states. Each state in N is induced by a nullified link between an activity f_i and a harm z_j .

Let p represent the parties' common beliefs on Z^F . The support set of p is S . That is, $p(s) > 0$ for all $s \in S$ and $p(s) = 0$ for all $s \in N$.

Given S and p , the efficient levels of care are $\tilde{x}_i = \xi^{-1} \left(\sum_{j=1}^n \pi_{ij} z_j \right)$, $i = 1, \dots, m$, where (i) ξ^{-1} denotes the inverse of $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$ and (ii) $\pi_{ij} = \sum_{s \in S: s^i = z_j} p(s)$. Under negligence, the court stipulates $\bar{x}_i = \tilde{x}_i$ as the due care standard for each activity f_i .

²⁸Recall that the "parties" are the injurer, the victim, and the court.

²⁹For example, we could have $\tau(x_i) = e^{-x_i}$.

5.1 New Link

Assume $N \neq \emptyset$, so $S \subset Z^F$. Suppose the parties discover a new link from f_l to z_k for some $l \in \{1, \dots, m\}$ and $k \in \{1, \dots, n\}$. Let \widehat{S} denote the expanded feasible state space and \widehat{p} denote the parties' updated beliefs. In addition, let $\Delta = \widehat{S} \setminus S$. We assume that, by virtue of a tort litigation, the parties learn that f_l yields z_k with probability $\delta > 0$.

By reverse Bayesianism, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$. In addition, $\delta = \widehat{p}(\Delta)$ by definition and $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$. Moreover, by act independence, $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$ for all $s = (s^1, \dots, s^m) \in \Delta$, where $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$ is the event that activity f_i yields harm z_j .

Given any $s \in \Delta$, let $L(s) \equiv \{t \in S : t^i = s^i, \forall i \neq l\}$ denote the event in S that corresponds to $s \in \Delta$. It follows that:

Proposition 4. *In the case of a new link involving f_l :*

- (i) $\widehat{p}(s) = (1 - \delta)p(s)$ for all $s \in S$; and
- (ii) $\widehat{p}(s) = \delta p(L(s))$ for all $s \in \Delta$.

Given \widehat{S} and \widehat{p} , the efficient levels of care are $\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^n \widehat{\pi}_{ij} z_j \right)$, $i = 1, \dots, m$, where $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$. Specifically:

Proposition 5. *In the case of a new link from f_l to z_k :*

- (i) $\widehat{x}_l = \xi^{-1} \left(\sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k \right)$; and
- (ii) $\widehat{x}_i = \widetilde{x}_i$ for all $i \neq l$.

Corollary 2. $\widehat{x}_l = \widetilde{x}_l$ if and only if $z_k = \sum_{j=1}^n \pi_{lj} z_j$.

Thus, the discovery that f_l can yield z_k necessitates the stipulation of a new due care standard for f_l (unless $z_k = \sum_{j=1}^n \pi_{lj} z_j$) but not for the other activities.

Under negligence, the court stipulates $\widehat{x}_l = \widehat{\widetilde{x}}_l$ as the new due care standard for f_l (or restipulates $\widehat{x}_l = \widetilde{x}_l$ if $z_k = \sum_{j=1}^n \pi_{lj} z_j$) and holds the injurer liable to pay damages of z_k to the victim if $\widehat{\widetilde{x}}_l > \widetilde{x}_l$. This, along with the victim's claim, makes the injurers and victims of the world aware that f_l can yield z_k . Moreover, they can deduce δ from $\widehat{\widetilde{x}}_l$.

Proposition 6. *In the case of a new link from f_l to z_k , $\delta = \frac{c'(\hat{x}_l) + \sum_{j=1}^n \pi_{lj} z_j \tau'(\hat{x}_l)}{\sum_{j=1}^n \pi_{lj} z_j \tau'(\hat{x}_l) - z_k \tau'(\hat{x}_l)}$.*

As a result, they can learn \hat{p} and $\hat{h}_l(x_l) = \sum_{j=1}^n [(1 - \delta)\pi_{lj} z_j + \delta z_k] \tau(x_l)$, without expending additional resources to learn about δ . Knowledge of $\hat{h}_l(x_l)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_k to the victim. This makes the injurers and victims of the world aware that f_l can yield z_k . However, they cannot deduce δ or learn \hat{p} or $\hat{h}_l(x_l)$. Without knowledge of $\hat{h}_l(x_l)$, strict liability cannot induce injurers to take efficient care.

5.2 New Act

Assume $S \subseteq Z^F$. Suppose the parties discover a new act, f_{m+1} . Let \hat{S} denote the expanded feasible state space and \hat{p} denote the parties' updated beliefs. We assume that, by virtue of a tort litigation, the parties learn that f_{m+1} yields z_j with probability $\delta_j > 0$ for all $j = 1, \dots, n$.³⁰ Note that $\sum_{j=1}^n \delta_j = 1$.

By reverse Bayesianism, $p(s)/p(t) = \hat{p}(E(s))/\hat{p}(E(t))$ for all $s, t \in S$, where $E(s) \equiv \{t \in \hat{S} : t^i = s^i, \forall i \neq m+1\}$ denotes the event in \hat{S} that corresponds to $s \in S$. Note that $\{E(s) : s \in S\}$ forms a partition of \hat{S} and that $|E(s)| = n$ for all $s \in S$. With a slight abuse of notation, index the states in each $E(s)$ by $j = 1, \dots, n$.

By definition, $\delta_j = \hat{p}(A_{m+1}(z_j))$, where $A_i(z_j) \equiv \{t \in \hat{S} : t^i = z_j\}$ is the event that activity f_i yields harm z_j . In addition, $\sum_{s \in \hat{S}} \hat{p}(s) = 1$. Moreover, by act independence, $\hat{p}(s) = \prod_{i=1}^{m+1} \hat{p}(A_i(s^i))$ for all $s = (s^1, \dots, s^{m+1}) \in \hat{S}$.

It follows that:

Proposition 7. *In the case of a new act f_{m+1} , for all $s \in S$ and corresponding $E(s) \subset \hat{S}$, $\hat{p}(s_j) = \delta_j p(s)$ for all $s_j \in E(s)$, $j = 1, \dots, n$.*

³⁰Assuming $\delta_j > 0$ for all $j = 1, \dots, n$ is without loss of generality. We can deal with the case where $\delta_j = 0$ for some j by assuming $\delta_j > 0$ for the first $k < n$ and changing n to k as necessary in the statements below.

Given \widehat{S} and \widehat{p} , the efficient levels of care are $\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^n \widehat{\pi}_{ij} z_j \right)$, $i = 1, \dots, m+1$, where $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$. Specifically:

Proposition 8. *In the case of a new act f_{m+1} :*

- (i) $\widehat{x}_i = \widetilde{x}_i$ for all $i \neq m+1$; and
- (ii) $\widehat{x}_{m+1} = \xi^{-1} \left(\sum_{j=1}^n \delta_j z_j \right)$.

Thus, the discovery of f_{m+1} necessitates the stipulation of a new due care standard, \widehat{x}_{m+1} , but it does not necessitate the stipulation of new due care standards for f_1, \dots, f_m .

Under negligence, the court stipulates $\widehat{x}_{m+1} = \widetilde{x}_{m+1}$ as the due care standard for the new activity f_{m+1} and holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of f_{m+1} (and that it is risky). Although they cannot separately deduce each δ_j from \widehat{x}_{m+1} , they nevertheless can infer $\widehat{h}_{m+1}(x_{m+1})$ from \widehat{x}_{m+1} , without expending additional resources to learn all δ_j .³¹

Proposition 9. *In the case of a new act f_{m+1} , $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{x}_{m+1})}{\tau'(\widehat{x}_{m+1})} \tau(x_{m+1})$.*

Knowledge of $\widehat{h}_{m+1}(x_{m+1})$ is necessary to induce the world's injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of f_{m+1} (and that it is risky). However, they do not learn $\widehat{h}_{m+1}(x_{m+1})$. Without knowledge of $\widehat{h}_{m+1}(x_{m+1})$, strict liability cannot induce injurers to take efficient care.

5.3 New Consequence

Assume $S \subseteq Z^F$. Suppose the parties discover a new consequence, z_{n+1} . Let \widehat{S} denote the expanded feasible state space and \widehat{p} denote the parties' updated beliefs. In addition, let

³¹Note, however, that if each z_j is a different type of harm that requires a different type of care, then the court would stipulate a different due care standard $\widehat{x}_{m+1,j}$ with respect to each z_j , in which case the injurers and victims of the world could separately deduce each δ_j .

$\Delta = \widehat{S} \setminus S$ and $\delta = \widehat{p}(\Delta)$. We assume that, by virtue of a tort litigation, the parties learn that f_i yields z_{n+1} with probability $\alpha_i > 0$ for all $i = 1, \dots, m$.³² Note that $1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$.

By reverse Bayesianism, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$. In addition, $\alpha_i = \widehat{p}(A_i(z_{n+1}))$ by definition and $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$. Moreover, by act independence, $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$ for all $s = (s^1, \dots, s^m) \in \Delta$.

Given any $s \in \Delta$, let $I(s) \equiv \{i \in \{1, \dots, m\} : s^i = z_{n+1}\}$ denote the indices of the acts that yield z_{n+1} in that state of the world, let $\bar{I}(s) \equiv \{i \in \{1, \dots, m\} : s^i \neq z_{n+1}\}$ denote the indices of the acts that do not yield z_{n+1} in that state of the world, and let $C(s) \equiv \{t \in S : t^i = s^i, \forall i \in \bar{I}(s)\}$ denote the event in S that corresponds to $s \in \Delta$ on $\bar{I}(s)$.

It follows that:

Proposition 10. *In the case of a new consequence z_{n+1} :*

- (i) $\widehat{p}(s) = (\prod_{i=1}^m (1 - \alpha_i)) p(s)$ for all $s \in S$;
- (ii) $\widehat{p}(s) = \left(\prod_{i \in I(s)} \alpha_i \right) \left(\prod_{i \in \bar{I}(s)} (1 - \alpha_i) \right) p(C(s))$ for all $s \in \Delta$ such that $I(s) \subsetneq \{1, \dots, m\}$;
- (iii) $\widehat{p}(s) = \prod_{i=1}^m \alpha_i$ for the $s \in \Delta$ such that $I(s) = \{1, \dots, m\}$.

Given \widehat{S} and \widehat{p} , the efficient levels of care are $\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j \right)$, $i = 1, \dots, m$, where $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$. Specifically:

Proposition 11. *In the case of a new consequence z_{n+1} , $\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1} \right)$ for all $i = 1, \dots, m$.*

Corollary 3. $\widehat{x}_i = \widetilde{x}_i$ if and only if $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$.

Thus, the discovery of z_{n+1} necessitates the stipulation of new due care standards for each activity f_i such that $z_{n+1} \neq \sum_{j=1}^n \pi_{ij} z_j$.

Under negligence, the court stipulates $\widehat{x}_i = \widetilde{x}_i$, $i = 1, \dots, m$, as the new due care standards for f_1, \dots, f_m (or restipulates $\widehat{x}_i = \widetilde{x}_i$ if $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$) and holds the injurer liable to pay damages of z_{n+1} to the victim with respect to each activity f_i such that $\widehat{x}_i > \widetilde{x}_i$. This,

³² Assuming $\alpha_i > 0$ for all i is without loss of generality. We can deal with the case where $\alpha_i > 0$ for some i by assuming $\alpha_i > 0$ for the first $l < m$ and changing m to l as necessary in the statements below.

along with the victim's claims, makes the injurers and victims of the world aware of z_{n+1} (and that it is linked to f_1, \dots, f_m). Moreover, they can deduce $\alpha_1, \dots, \alpha_m$ from $\widehat{x}_1, \dots, \widehat{x}_m$.

Proposition 12. *In the case of a new consequence z_{n+1} , $\alpha_i = \frac{c'(\widehat{x}_i) + \sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i)}{\sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i) - z_{n+1} \tau'(\widehat{x}_i)}$, for all $i = 1, \dots, m$.*

As a result, they can learn \widehat{p} and $\widehat{h}_1(x_1), \dots, \widehat{h}_m(x_m)$, without expending additional resources to learn about $\alpha_1, \dots, \alpha_m$. Knowledge of $\widehat{h}_1(x_1), \dots, \widehat{h}_m(x_m)$ is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_{n+1} to the victim with respect to each activity f_i . This makes the injurers and victims of the world aware of z_{n+1} (and that it is linked to f_1, \dots, f_m). However, they cannot deduce $\alpha_1, \dots, \alpha_m$, and hence learn \widehat{p} and $\widehat{h}_1(x_1), \dots, \widehat{h}_m(x_m)$. Again, without knowledge of $\widehat{h}_1(x_1), \dots, \widehat{h}_m(x_m)$, strict liability cannot induce efficient care.

6 Discussion

This paper extends the economic analysis of tort law to incorporate unawareness. We compare and contrast negligence and strict liability in a unilateral accident model with unawareness and growing awareness, and find that negligence has a key advantage—the due care standard serves as a knowledge transmission mechanism. Under either tort liability rule, a suit involving a newly discovered act, consequence, or act-consequence link makes the world aware of a new possibility of harm. But only negligence, through the stipulation of new due care standards, spreads awareness about the updated probability of harm. As such, the negligence due care standard is like a public good. The social benefit of spreading awareness about the updated probability of harm is that potential injurers and victims need not expend additional resources to develop this knowledge—wastefully duplicative activity that would be necessary to achieve optimal deterrence under strict liability. In a sense, negligence is akin to patents; both carry social costs (negligence is more costly to administer; patents

create monopolies and deadweight loss), yet both provide social benefits in terms of knowledge transmission. One should bear in mind, however, that we do not purport to undertake a full welfare analysis. We do not claim that negligence is superior to strict liability in all circumstances. Rather, we claim that negligence is more robust to unawareness.

To model unawareness and growing awareness, we adopt the reverse Bayesian approach of Karni and Vierø (2013). The reverse Bayesian model has (at least) two attractive features. The first is transparency. Karni and Vierø (2013) provide an axiomatic foundation for the model, and so one can judge the theory by the axioms.³³ The second attractive feature of the model is its accessibility. The model is built upon a familiar choice-theoretic framework (subjective expected utility theory), and the upshot is a belief revision theory that mirrors the process of Bayesian updating.³⁴ At the same time, the model has its shortcomings. For instance, Chambers and Hayashi (2018) criticize its empirical content from a revealed preference perspective. They show that, in the case of a new consequence, the model does not make singular predictions about observable choices over feasible acts. A second shortcoming of the model is that it assumes a naive or myopic unawareness—people are unaware that they are unaware. A sophisticated unawareness, where people are aware that they are unaware, may be more realistic. Aware of this shortcoming, Karni and Vierø (2017) extend their model to the case of sophisticated unawareness. The end result is a generalization that maintains the flavor of reverse Bayesianism and nests the naive model as a special case.

The pros and cons of the model aside, one might question the importance of our results in a world with safety regulation in addition to tort liability. In such a world, one could argue, there are regulators and other non-court actors who can spread awareness about newly discovered risks. While this may be correct, it is orthogonal to our inquiry. We are contributing to the negligence versus strict liability debate. We therefore consider a world where the tort system is the only mechanism for regulating risky activities, and we compare

³³The key axioms of the model are the “consistency” axioms, which essentially require that preferences conditional on the original state of awareness are not altered by growing awareness.

³⁴This feature prompts Dominiak and Tserenjigmid (2018, p. 3) to describe Karni and Vierø’s (2013) model as “elegant.”

and contrast the two primary tort liability rules. If one were to consider a world with safety regulation in addition to tort law, one would have to wade into the liability versus regulation debate (e.g., Shavell, 1984a,b; Posner, 2010), and conclude that regulation is the more efficient method of social control, before one could assert that the possibility of safety regulation renders moot the debate over tort liability rules.

The importance of unawareness and growing awareness—via technological progress, scientific discovery, or otherwise—plainly extends beyond the case of unilateral accidents with fixed activity levels. Natural extensions of this paper, therefore, would entail introducing unawareness into other accident settings. In addition, future research could examine the implications of unawareness for the economic analysis of other areas of law such as contract remedies and criminal law or of other legal topics such as litigation and settlement.

Appendix

Proof of Proposition 1

By reverse Bayesianism, the definition of δ , and $\hat{p}_1 + \hat{p}_2 + \hat{p}_3 + \hat{p}_4 = 1$, we have two linearly independent equations,

$$\hat{p}_2 = \frac{p_2}{p_1} \hat{p}_1 \quad \text{and} \quad \hat{p}_1 + \hat{p}_2 = 1 - \delta,$$

and two unknowns, \hat{p}_1 and \hat{p}_2 . Substituting the first equation into the second, we have

$$\hat{p}_1 + \frac{p_2}{p_1} \hat{p}_1 = 1 - \delta,$$

which implies

$$\hat{p}_1 = \frac{(1 - \delta)p_1}{p_1 + p_2} = (1 - \delta)p_1,$$

where the last equality follows from $p_1 + p_2 = 1$. It follows that

$$\hat{p}_2 = \frac{p_2}{p_1} (1 - \delta)p_1 = (1 - \delta)p_2.$$

By act independence and the definition of δ , we have

$$\hat{p}_3 = \delta(\hat{p}_1 + \hat{p}_3) \quad \text{and} \quad \hat{p}_4 = \delta(\hat{p}_2 + \hat{p}_4),$$

which imply

$$\widehat{p}_3 = \frac{\delta}{1-\delta}\widehat{p}_1 \quad \text{and} \quad \widehat{p}_4 = \frac{\delta}{1-\delta}\widehat{p}_2.$$

It follows that

$$\widehat{p}_3 = \frac{\delta}{1-\delta}(1-\delta)p_1 = \delta p_1 \quad \text{and} \quad \widehat{p}_4 = \frac{\delta}{1-\delta}(1-\delta)p_2 = \delta p_2.$$

Proof of Proposition 2

Reverse Bayesianism implies three linearly independent conditions:³⁵

$$\begin{aligned} p_2(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_2 + \widehat{p}_6), \\ p_3(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_3 + \widehat{p}_7), \\ \text{and } p_4(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_4 + \widehat{p}_8). \end{aligned}$$

Summing the left- and right-hand sides, and adding $p_1(\widehat{p}_1 + \widehat{p}_5)$ to each side, yields

$$(p_1 + p_2 + p_3 + p_4)(\widehat{p}_1 + \widehat{p}_5) = (\widehat{p}_1 + \cdots + \widehat{p}_8)p_1.$$

Because $p_1 + p_2 + p_3 + p_4 = 1$ and $\widehat{p}_1 + \cdots + \widehat{p}_8 = 1$, we have $\widehat{p}_1 + \widehat{p}_5 = p_1$. Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 + \widehat{p}_5 = p_1, \quad \widehat{p}_2 + \widehat{p}_6 = p_2, \quad \widehat{p}_3 + \widehat{p}_7 = p_3, \quad \text{and} \quad \widehat{p}_4 + \widehat{p}_8 = p_4.$$

By act independence and the definition of δ , we have

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_1 + \widehat{p}_5)\delta, \\ \widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_2 + \widehat{p}_6)\delta, \\ \widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_3 + \widehat{p}_7)\delta, \\ \text{and } \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_4 + \widehat{p}_8)\delta, \end{aligned}$$

where the second equality follows from iterative application of act independence.³⁶ These imply

$$\widehat{p}_5 = \frac{\delta}{1-\delta}\widehat{p}_1, \quad \widehat{p}_6 = \frac{\delta}{1-\delta}\widehat{p}_2, \quad \widehat{p}_7 = \frac{\delta}{1-\delta}\widehat{p}_3, \quad \text{and} \quad \widehat{p}_8 = \frac{\delta}{1-\delta}\widehat{p}_4.$$

³⁵Note that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

³⁶For example, $(\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7) = \widehat{p}(\{s_1, s_2, s_5, s_6\})\widehat{p}(\{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_2, s_5, s_6\} \cap \{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_5\}) = \widehat{p}_1 + \widehat{p}_5$.

It follows that

$$\widehat{p}_1 + \frac{\delta}{1-\delta}\widehat{p}_1 = p_1, \quad \widehat{p}_2 + \frac{\delta}{1-\delta}\widehat{p}_2 = p_2, \quad \widehat{p}_3 + \frac{\delta}{1-\delta}\widehat{p}_3 = p_3, \quad \text{and} \quad \widehat{p}_4 + \frac{\delta}{1-\delta}\widehat{p}_4 = p_4.$$

These imply

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4,$$

which in turn imply

$$\widehat{p}_5 = \delta p_1, \quad \widehat{p}_6 = \delta p_2, \quad \widehat{p}_7 = \delta p_3, \quad \text{and} \quad \widehat{p}_8 = \delta p_4.$$

Proof of Proposition 3

Reverse Bayesianism implies three linearly independent conditions:³⁷

$$\begin{aligned} p_2\widehat{p}_1 &= p_1\widehat{p}_2, \\ p_3\widehat{p}_1 &= p_1\widehat{p}_3, \\ \text{and } p_4\widehat{p}_1 &= p_1\widehat{p}_4. \end{aligned}$$

Summing the left- and right-hand sides, and adding $p_1\widehat{p}_1$ to each side, yields

$$(p_1 + p_2 + p_3 + p_4)\widehat{p}_1 = (\widehat{p}_1 + \cdots + \widehat{p}_4)p_1.$$

By $p_1 + p_2 + p_3 + p_4 = 1$, $\widehat{p}_1 + \cdots + \widehat{p}_9 = 1$, and $\delta \equiv \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$, we have $\widehat{p}_1 = (1-\delta)p_1$. Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4.$$

By act independence and the definitions of α and β , we have

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5) = \alpha(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), \\ \widehat{p}_6 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6) = \alpha(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\ \widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7), \\ \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8), \\ \text{and } \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \alpha\beta. \end{aligned}$$

³⁷Note again that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

These imply

$$\begin{aligned}\widehat{p}_5 &= \frac{\alpha}{1-\alpha}(\widehat{p}_1 + \widehat{p}_3), & \widehat{p}_6 &= \frac{\alpha}{1-\alpha}(\widehat{p}_2 + \widehat{p}_4), \\ \widehat{p}_7 &= \frac{\beta}{1-\beta}(\widehat{p}_1 + \widehat{p}_2), & \widehat{p}_8 &= \frac{\beta}{1-\beta}(\widehat{p}_3 + \widehat{p}_4), \\ \text{and } \widehat{p}_9 &= \alpha\beta.\end{aligned}$$

From $\widehat{p}_9 = \alpha\beta$, it follows that $\delta \equiv \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \alpha + \beta - \alpha\beta$, which implies $1 - \delta = (1 - \alpha)(1 - \beta)$. (Observe that $1 - \delta = (1 - \alpha)(1 - \beta)$ also follows directly from act independence.) It follows that

$$\begin{aligned}\widehat{p}_1 &= (1 - \alpha)(1 - \beta)p_1, & \widehat{p}_2 &= (1 - \alpha)(1 - \beta)p_2, \\ \widehat{p}_3 &= (1 - \alpha)(1 - \beta)p_3, & \text{and } \widehat{p}_4 &= (1 - \alpha)(1 - \beta)p_4,\end{aligned}$$

and in turn that

$$\begin{aligned}\widehat{p}_5 &= \alpha(1 - \beta)(p_1 + p_3), & \widehat{p}_6 &= \alpha(1 - \beta)(\widehat{p}_2 + \widehat{p}_4), \\ \widehat{p}_7 &= \beta(1 - \alpha)(\widehat{p}_1 + \widehat{p}_2), & \widehat{p}_8 &= \beta(1 - \alpha)(\widehat{p}_3 + \widehat{p}_4), \\ \text{and } \widehat{p}_9 &= \alpha\beta.\end{aligned}$$

Proof of Corollary 1

We establish in the proof of Proposition 3 that

$$\begin{aligned}\widehat{p}_1 &= (1 - \delta)p_1, & \widehat{p}_2 &= (1 - \delta)p_2, \\ \widehat{p}_3 &= (1 - \delta)p_3, & \text{and } \widehat{p}_4 &= (1 - \delta)p_4.\end{aligned}$$

We also observe that $1 - \delta = (1 - \alpha)(1 - \beta)$ and $\delta = \alpha + \beta - \alpha\beta$. It follows that

$$\begin{aligned}\widehat{p}_5 &= \alpha(1 - \beta)(p_1 + p_3) = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_1 + p_3) = (\delta - \beta)(p_1 + p_3), \\ \widehat{p}_6 &= \alpha(1 - \beta)(\widehat{p}_2 + \widehat{p}_4) = \frac{\alpha}{1 - \alpha}(1 - \delta)(\widehat{p}_2 + \widehat{p}_4) = (\delta - \beta)(\widehat{p}_2 + \widehat{p}_4), \\ \widehat{p}_7 &= \beta(1 - \alpha)(\widehat{p}_1 + \widehat{p}_2) = \frac{\beta}{1 - \beta}(1 - \delta)(\widehat{p}_1 + \widehat{p}_2) = (\delta - \alpha)(\widehat{p}_1 + \widehat{p}_2), \\ \widehat{p}_8 &= \beta(1 - \alpha)(\widehat{p}_3 + \widehat{p}_4) = \frac{\beta}{1 - \beta}(1 - \delta)(\widehat{p}_3 + \widehat{p}_4) = (\delta - \alpha)(\widehat{p}_3 + \widehat{p}_4), \\ \text{and } \widehat{p}_9 &= \alpha + \beta - \delta.\end{aligned}$$

Proof of Proposition 4

(i) Take any $s \in S$. By reverse Bayesianism, we have $|S| - 1$ linearly independent equations:

$$\widehat{p}(t) = \frac{p(t)}{p(s)} \widehat{p}(s), \quad \forall t \in S, t \neq s. \quad (4.1)$$

By the definition of δ and $\sum_{t \in \widehat{S}} \widehat{p}(t) = 1$, we have

$$\sum_{t \in S} \widehat{p}(t) = 1 - \delta. \quad (4.2)$$

Substituting (4.1) into (4.2), we have

$$\widehat{p}(s) + \sum_{t \in S: t \neq s} \frac{p(t)}{p(s)} \widehat{p}(s) = 1 - \delta,$$

which implies

$$\widehat{p}(s) = \frac{(1 - \delta)p(s)}{\sum_{t \in S} p(t)} = (1 - \delta)p(s), \quad (4.3)$$

where the last equality follows from $\sum_{t \in S} p(t) = 1$.

(ii) Take any $s \in \Delta$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i)).$$

Observe that $\widehat{p}(A_l(s^l)) = \widehat{p}(A_l(z_k)) = \delta$ and $\bigcap_{i \neq l} A_i(s^i) = L(s) \cup \{s\}$. It follows that

$$\begin{aligned} \widehat{p}(s) &= \delta \prod_{i \neq l} \widehat{p}(A_i(s^i)) = \delta \widehat{p}\left(\bigcap_{i \neq l} A_i(s^i)\right) \\ &= \delta \widehat{p}(L(s) \cup \{s\}) = \delta [\widehat{p}(L(s)) + \widehat{p}(s)], \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\delta}{1 - \delta} \widehat{p}(L(s)). \quad (4.4)$$

Observe that $L(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \neq l$. It follows that

$$\widehat{p}(L(s)) = \sum_{t \in L(s)} \widehat{p}(t) = \sum_{t \in L(s)} (1 - \delta)p(t) = (1 - \delta)p(L(s)), \quad (4.5)$$

where the second equality follows from (4.3). Substituting (4.5) back into (4.4), we have $\widehat{p}(s) = \delta p(L(s))$.

Proof of Proposition 5

(i) Observe that $\sum_{j=1}^n \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \widehat{\pi}_{lj} z_j + \delta z_k$. By Proposition 4,

$$\sum_{j \neq k} \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \left[\sum_{s \in \widehat{S}: s^l = z_j} \widehat{p}(s) \right] z_j = \sum_{j \neq k} \left[\sum_{s \in S: s^l = z_j} (1 - \delta)p(s) + \sum_{s \in \Delta: s^l = z_j} \delta p(L(s)) \right] z_j.$$

Observe that $s^l = z_k$ for all $s \in \Delta$. It follows that, for all $j \neq k$,

$$\sum_{s \in \Delta: s^l = z_j} \delta p(L(s)) = 0.$$

Thus, we have

$$\sum_{j \neq k} \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \left[\sum_{s \in S: s^l = z_j} (1 - \delta)p(s) \right] z_j = \sum_{j \neq k} (1 - \delta) \pi_{lj} z_j = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j,$$

where the last equality follows from $\pi_{lk} = 0$. Hence, $\sum_{j=1}^n \widehat{\pi}_{lj} z_j = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k$.

(ii) Take any $i \neq l$ and any j . By Proposition 4,

$$\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} (1 - \delta)p(s) + \sum_{s \in \Delta: s^i = z_j} \delta p(L(s)).$$

Observe that $L(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \neq l$. Thus,

$$\sum_{s \in \Delta: s^i = z_j} p(L(s)) = \sum_{t \in S: t^i = z_j} p(t).$$

Hence,

$$\widehat{\pi}_{ij} = \sum_{s \in S: s^i = z_j} (1 - \delta)p(s) + \sum_{s \in S: s^i = z_j} \delta p(s) = \sum_{s \in S: s^i = z_j} p(s) = \pi_{ij}.$$

It follows that $\widehat{x}_i = \widetilde{x}_i$ for all $i \neq l$.

Proof of Corollary 2

By Proposition 5, $\xi(\widehat{x}_l) = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k$. Observe that $\xi(\widetilde{x}_l) = \sum_{j=1}^n \pi_{lj} z_j$. It follows that $\xi(\widehat{x}_l) = \xi(\widetilde{x}_l)$ if and only if $z_k = \sum_{j=1}^n \pi_{lj} z_j$. Because $\xi'(x_i) > 0$ for all x_i , we have $\widehat{x}_l = \widetilde{x}_l$ if and only if $z_k = \sum_{j=1}^n \pi_{lj} z_j$.

Proof of Proposition 6

By Proposition 5 and $\widehat{x}_l = \widehat{x}_l$, we have $\xi(\widehat{x}_l) = \sum_{j=1}^n (1 - \delta)\pi_{lj}z_j + \delta z_k$. It follows that

$$\delta = \frac{\xi(\widehat{x}_l) - \sum_{j=1}^n \pi_{lj}z_j}{z_k - \sum_{j=1}^n \pi_{lj}z_j}.$$

Observe that $\xi(\widehat{x}_l) = -c'(\widehat{x}_l)/\tau'(\widehat{x}_l)$. Thus,

$$\delta = \frac{c'(\widehat{x}_l) + \sum_{j=1}^n \pi_{lj}z_j\tau'(\widehat{x}_l)}{\sum_{j=1}^n \pi_{lj}z_j\tau'(\widehat{x}_l) - z_k\tau'(\widehat{x}_l)}.$$

Proof of Proposition 7

Take any $s \in S$. By reverse Bayesianism, we have $|S| - 1$ linearly independent equations:

$$p(t)\widehat{p}(E(s)) = p(s)\widehat{p}(E(t)), \quad \forall t \in S, t \neq s.$$

Summing the left- and right-hand sides, and adding $p(s)\widehat{p}(E(s))$ to each side, yields

$$\widehat{p}(E(s)) \sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(E(t)).$$

Because $\sum_{t \in S} p(t) = 1$ and $\sum_{t \in S} \widehat{p}(E(t)) = 1$, we have

$$\widehat{p}(E(s)) = p(s). \tag{7.1}$$

Take any $s_j \in E(s)$, $j \in \{1, \dots, n\}$. By act independence,

$$\widehat{p}(s_j) = \prod_{i=1}^{m+1} \widehat{p}(A_i(s_j^i)).$$

Observe that $\widehat{p}(A_{m+1}(s_j^{m+1})) = \widehat{p}(A_{m+1}(z_j)) = \delta_j$ and $\bigcap_{i=1}^m A_i(s_j^i) = E(s)$. It follows that

$$\widehat{p}(s_j) = \delta_j \prod_{i=1}^m \widehat{p}(A_i(s_j^i)) = \delta_j \widehat{p}\left(\bigcap_{i=1}^m A_i(s_j^i)\right) = \delta_j \widehat{p}(E(s)). \tag{7.2}$$

Substituting (7.1) into (7.2), we have $\widehat{p}(s_j) = \delta_j p(s)$.

Proof of Proposition 8

(i) Recall that $\{E(s) : s \in S\}$ forms a partition of \widehat{S} . Take any $i \neq m+1$ and any j . By Proposition 7,

$$\begin{aligned}\widehat{\pi}_{ij} &= \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in \widehat{S}: s^i = z_j} \left[\sum_{s_l \in E(s)} \widehat{p}(s_l) \right]. \\ &= \sum_{s \in \widehat{S}: s^i = z_j} \left[\sum_{l=1}^n \delta_l p(s) \right] = \sum_{s \in \widehat{S}: s^i = z_j} p(s) \left[\sum_{l=1}^n \delta_l \right].\end{aligned}$$

Note that $\sum_{l=1}^n \delta_l = 1$. Thus, $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} p(s) = \pi_{ij}$. It follows that $\widehat{x}_i = \widetilde{x}_i$ for all $i \neq m+1$.

(ii) By definition, $\widehat{\pi}_{m+1,j} = \delta_j$ for all $j = 1, \dots, n$. Hence, $\widehat{x}_{m+1} = \xi^{-1} \left(\sum_{j=1}^n \delta_j z_j \right)$.

Proof of Proposition 9

Observe that $\widehat{h}_{m+1}(x_{m+1}) = \sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j \tau(x_{m+1})$ and $\widehat{x}_{m+1} = \widehat{\widetilde{x}}_{m+1} = \xi^{-1} \left(\sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j \right)$. The latter implies $\xi(\widehat{\widetilde{x}}_{m+1}) = \sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j$. Thus, $\widehat{h}_{m+1}(x_{m+1}) = \xi(\widehat{\widetilde{x}}_{m+1}) \tau(x_{m+1})$. Recall that $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$. Hence, $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{\widetilde{x}}_{m+1})}{\tau'(\widehat{\widetilde{x}}_{m+1})} \tau(x_{m+1})$.

Proof of Proposition 10

(i) Take any $s \in S$. By reverse Bayesianism, we have $|S| - 1$ linearly independent equations:

$$p(t)\widehat{p}(s) = p(s)\widehat{p}(t), \quad \forall t \in S, t \neq s.$$

Summing the left- and right-hand sides, and adding $p(s)\widehat{p}(s)$ to each side, yields

$$\widehat{p}(s) \sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(t).$$

Observe that $\sum_{t \in S} p(t) = 1$ and $\sum_{t \in S} \widehat{p}(t) = 1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$. Thus,

$$\widehat{p}(s) = (1 - \delta)p(s) = \left(\prod_{i=1}^m (1 - \alpha_i) \right) p(s). \quad (10.1)$$

(ii) Take any $s \in \Delta$ such that $I(s) = \{k\}$ for any $k \in \{1, \dots, m\}$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i)).$$

Observe that $\widehat{p}(A_k(s^k)) = \widehat{p}(A_k(z_{n+1})) = \alpha_k$. Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \bar{I}(s)} \widehat{p}(A_i(s^i)).$$

Observe that $I(s) = \{k\}$ implies $\bigcap_{i \in \bar{I}(s)} A_i(s^i) = C(s) \cup \{s\}$. Hence,

$$\begin{aligned} \widehat{p}(s) &= \alpha_k \prod_{i \in \bar{I}(s)} \widehat{p}(A_i(s^i)) = \alpha_k \widehat{p}\left(\bigcap_{i \in \bar{I}(s)} A_i(s^i)\right) \\ &= \alpha_k \widehat{p}(C(s) \cup \{s\}) = \alpha_k (\widehat{p}(C(s)) + \widehat{p}(s)), \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}(C(s)). \quad (10.2)$$

Observe that $C(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \in \bar{I}(s)$. It follows that

$$\widehat{p}(C(s)) = \sum_{t \in C(s)} \widehat{p}(t) = \sum_{t \in C(s)} (1 - \delta)p(t) = (1 - \delta)p(C(s)), \quad (10.3)$$

where the second equality follows from (10.1). Substituting (10.3) back into (10.2), we have

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} (1 - \delta)p(C(s)) = \alpha_k \prod_{i \in \bar{I}(s)} (1 - \alpha_i)p(C(s)),$$

where the last equality follows from $1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$.

Next take any $s \in \Delta$ such that $I(s) = \{k, l\}$ for any $\{k, l\} \subset \{1, \dots, m\}$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i)).$$

Observe that $\widehat{p}(A_k(s^k)) = \widehat{p}(A_k(z_{n+1})) = \alpha_k$. Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \{\bar{I}(s) \cup \{l\}\}} \widehat{p}(A_i(s^i)).$$

Observe that $I(s) = \{k, l\}$ implies $\bigcap_{i \in \{\bar{I}(s) \cup \{l\}\}} A_i(s^i) = D(s) \cup \{s\}$, where $D(s) \equiv \{r \in \Delta : r^i = s^i, \forall i \in \{\bar{I}(s) \cup \{l\}\}\}$. Hence,

$$\begin{aligned} \widehat{p}(s) &= \alpha_k \prod_{i \in \{\bar{I}(s) \cup \{l\}\}} \widehat{p}(A_i(s^i)) = \alpha_k \widehat{p}\left(\bigcap_{i \in \{\bar{I}(s) \cup \{l\}\}} A_i(s^i)\right) \\ &= \alpha_k \widehat{p}(D(s) \cup \{s\}) = \alpha_k (\widehat{p}(D(s)) + \widehat{p}(s)), \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}(D(s)). \quad (10.4)$$

Observe further that $I(r) = \{l\}$ for all $r \in D(s)$. It follows that

$$\begin{aligned}
\widehat{p}(D(s)) &= \sum_{t \in D(s)} \widehat{p}(t) \\
&= \sum_{t \in D(s)} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(t)) \\
&= \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)).
\end{aligned} \tag{10.5}$$

Substituting (10.5) back into (10.4), we have

$$\begin{aligned}
\widehat{p}(s) &= \frac{\alpha_k}{1 - \alpha_k} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)). \\
&= \alpha_k \alpha_l \prod_{i \in \bar{I}(s)} (1 - \alpha_i) p(C(s)).
\end{aligned}$$

Proceeding in this fashion to consider $s \in \Delta$ such that $I(s)$ is an ι -element subset of $\{1, \dots, m\}$ for all $\iota = 3, \dots, m - 1$, we establish that

$$\widehat{p}(s) = \left(\prod_{i \in I(s)} \alpha_i \right) \left(\prod_{i \in \bar{I}(s)} (1 - \alpha_i) \right) p(C(s))$$

for all $s \in \Delta$ such that $I(s) \subset \{1, \dots, m\}$.

(iii) Take the $s \in \Delta$ such that $I(s) = \{1, \dots, m\}$. By act independence, $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$. Observe that $\widehat{p}(A_i(s^i)) = \widehat{p}(A_i(z_{n+1})) = \alpha_i$ for all $i \in I(s)$. Because $I(s) = \{1, \dots, m\}$, we have $\widehat{p}(s) = \prod_{i=1}^m \alpha_i$.

Proof of Proposition 11

Take any $i \in \{1, \dots, m\}$. Observe that

$$\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j \right) = \xi^{-1} \left(\sum_{j=1}^n \widehat{\pi}_{ij} z_j + \alpha_i z_{n+1} \right). \tag{11.1}$$

Let $\Gamma(\alpha_l, s) \equiv \left(\prod_{l \in I(s)} \alpha_l \right) \left(\prod_{l \in \bar{I}(s)} (1 - \alpha_l) \right)$ for all $s \in \Delta$. By Proposition 10,

$$\widehat{\pi}_{ij} = \sum_{s \in \widetilde{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} \prod_{l=1}^m (1 - \alpha_l) p(s) + \sum_{s \in \Delta: s^i = z_j} \Gamma(\alpha_l, s) p(C(s)),$$

for all $j \neq n + 1$. Observe that

$$\sum_{s \in S: s^i = z_j} \prod_{l=1}^m (1 - \alpha_l) p(s) = \prod_{l=1}^m (1 - \alpha_l) \sum_{s \in S: s^i = z_j} p(s) = (1 - \delta) \pi_{ij}$$

and that

$$\begin{aligned}
\sum_{s \in \Delta: s^i = z_j} \Gamma(\alpha_l, s) p(C(s)) &= \sum_{s \in \Delta: s^i = z_j} \left(\prod_{l \in I(s)} \alpha_l \right) \left(\prod_{l \in \bar{I}(s)} (1 - \alpha_l) \right) p(C(s)) \\
&= \sum_{s \in \Delta: s^i = z_j} \frac{\prod_{l \in I(s)} \alpha_l}{\prod_{l \in I(s)} (1 - \alpha_l)} (1 - \delta) p(C(s)) \\
&= \sum_{I \subset \{1, \dots, m\} \setminus \{i\}} \frac{\prod_{l \in I} \alpha_l}{\prod_{l \in I} (1 - \alpha_l)} (1 - \delta) \pi_{ij} \\
&= \frac{1 - \prod_{l \neq i} (1 - \alpha_l)}{\prod_{l \neq i} (1 - \alpha_l)} (1 - \delta) \pi_{ij}.
\end{aligned}$$

Thus,

$$\begin{aligned}
\widehat{\pi}_{ij} &= (1 - \delta) \pi_{ij} + \frac{1 - \prod_{l \neq i} (1 - \alpha_l)}{\prod_{l \neq i} (1 - \alpha_l)} (1 - \delta) \pi_{ij} \\
&= (1 - \delta) \pi_{ij} \left(\frac{1}{\prod_{l \neq i} (1 - \alpha_l)} \right) = (1 - \delta) \pi_{ij} \left(\frac{1 - \alpha_i}{1 - \delta} \right) = (1 - \alpha_i) \pi_{ij}. \quad (11.2)
\end{aligned}$$

Substituting (11.2) back into (11.1), we have $\widehat{x}_i = \xi^{-1} \left(\sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1} \right)$.

Proof of Corollary 3

By Proposition 11, $\xi(\widehat{x}_i) = \sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + (1 - \alpha_i) z_{n+1}$. Observe that $\xi(\widetilde{x}_i) = \sum_{j=1}^n \pi_{ij} z_j$. It follows that $\xi(\widehat{x}_i) = \xi(\widetilde{x}_i)$ if and only if $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$. Because $\xi'(x_i) > 0$ for all x_i , we have $\widehat{x}_i = \widetilde{x}_i$ if and only if $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$.

Proof of Proposition 12

Take any $i \in \{1, \dots, m\}$. By Proposition 11, $\widehat{x}_i = \widehat{\widetilde{x}}_i = \xi^{-1} \left(\sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1} \right)$, which implies $\xi(\widehat{x}_i) = \sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1}$. It follows that

$$\alpha_i = \frac{\xi(\widehat{x}_i) - \sum_{j=1}^n \pi_{ij} z_j}{z_{n+1} - \sum_{j=1}^n \pi_{ij} z_j}.$$

Observe that $\xi(\widehat{x}_i) = -c'(\widehat{x}_i) / \tau'(\widehat{x}_i)$. Thus,

$$\alpha_i = \frac{c'(\widehat{x}_i) + \sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i)}{\sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i) - z_{n+1} \tau'(\widehat{x}_i)}.$$

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