

## CHANGES OF NAMES, CONTENTS AND ATTITUDES TO MATHEMATICAL UNITS

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### ABSTRACT

Will this material be on the exam? Why do I need to know this stuff? These are the sorts of questions that have been regularly asked by our mathematics students. Pre-service mathematics teachers often suggest that they do not need to learn anything that they do not have to teach. Generally, these students appear to have very little aesthetic appreciation for mathematics and its applications.

Currently, we teach five traditional mathematical content units that are provided mainly for pre-service mathematics teachers. These units have been adapted and modified over the years from units that were designed primarily for science students. They contained a heavy focus on calculus with a limited breadth of mathematical experience. After consulting widely on the best mathematical practices throughout Australia and internationally, it was decided to reform all of the mathematics units to make them more attractive to a wider audience.

The units that are currently being developed are: Profit, Loss and Gambling; Upon the Shoulders of Giants; Logic and Imagination; Modelling and Change; Algorithms, Bits and Bytes; Space, Shape, and Design; and Modelling Reality. The overall goal of this redevelopment is to improve student attitudes and motivation by exposing them to a wide range of topics in mathematics that are usable and relevant. All of these units will incorporate current technology, contain realistic problems, and include visiting speakers. Student assessment in these units will consist of portfolios, projects and examinations.

The introduction of these new units will result in students having a greater choice of the units they wish to study. In order to overcome potential logistical problems of a small mathematics department, innovative changes to the structure of the units will also be examined. This paper will provide the details of the establishment and content of these units.

# 1. Introduction

There have been many articles written on the advantages and disadvantages of using thematic approaches, modelling and applications to provoke the need for mathematical methods and techniques. The interested reader is referred to Kilpatrick (1997), Wu (1997) and Black and Atkin (1996) for discussions on reforms in mathematics education. While this paper will discuss the major issues briefly, the main focus will be on the rationale for restructuring the mathematics units, details of the units, and the actual implementation process conducted at the University of Ballarat.

## 2. Current Situation

In Australia, there is increasing concern about the declining numbers and calibre of students choosing to major in mathematics (Forgasz & Swedosh, 1997). The range of students' needs, backgrounds, learning approaches and mathematical abilities is very broad. Some of the Australian enrolment trends and student perceptions of mathematics are described in Forgasz & Leder (1998), Forgasz, Leder, & Brew (1998) and Forgasz (1998). Course restructures throughout the institution have resulted in most mathematics units being designated as electives rather than core units where a unit is defined as a full-time load for one semester. In the past, our main client group was science students. Since the restructure, numbers enrolled in mathematics units have dropped by more than 50% and the main interest is from education and computing students. In the last couple of years, there has been less than thirty students enrolled in first year mathematics units and less than fifteen in second and third year units.

An investigation into the reasons for falling mathematics numbers soon revealed some obvious problems. Students may choose their electives from a broad range of units. There is strong competition from other discipline areas and the mathematics units have not been successful in attracting these students. Current unit titles such as Mathematics, Pure Mathematics, and Introduction to Calculus and Computer Algebra Systems appear to deter both students and enrolment advisers. Further discussions revealed both students and advisers thought that the units would be difficult, boring, and irrelevant.

The units offered were designed principally for science students and were narrow in content. Unit evaluations as well as informal student comments over several semesters have confirmed the dissatisfaction of the remaining education and computing students with the domination of calculus in the current units. Another problem was that of assessment. Regular worksheets, which were really learning tasks, formed part of the assessments along with assignments and examinations. Examinations were worth about 50 % of the final grade, considerably more than in most other disciplines, and students often exhibited a high level of exam anxiety.

## 3. Proposed Solutions

Overall, it was clear that the mathematics department had to improve the image of mathematics and make it more relevant for the students undertaking these units. We wanted to encourage more education students to take mathematics as their special teaching area as well as entice students from around the university to take elective units in mathematics. We therefore needed to cover a wider range of mathematics, both in terms of topics and depth.

### **3.1 New content and teaching**

In response to this situation, and following much discussion, the following goals were established for our teaching of mathematics:

- Students who do our units should be able to reason mathematically, communicate and solve problems, as well as master algorithms and remember facts.
- Students should understand and appreciate the role of mathematics and its applications in the real world.
- Education students should form a positive view of their potential careers as mathematics teachers.
- Each unit should incorporate up-to-date teaching technology and utilise methods that enhance student learning.

With these goals in mind, we set about the task of developing units that would expose students to a wide range of mathematical topics that are useable and relevant. The new focus of every unit would be in the use of themes, applications and some problem based learning to provoke the need for mathematical methods and techniques. Burrill (1993) summarises the nature of our new approach when he states that 'rather than memorising algorithms and manipulating symbols following explicit directions from a teacher, students must explore, investigate and interact with each other, and the teacher, as they develop strategies to resolve the problem. There is a strong emphasis on communication, and the ability to explain and justify a reasoning process'.

Generally, most teaching lessons examine a problem or application. The aim is to grab the attention and interest of students as well as 'the initial exploration leading to the development, discovery or invention of mathematical concepts' (de Lange, 1993). It is unreasonable to expect that every mathematics problem will have an engaging application, so there will also be times when a more traditional approach is needed. Once the problem is understood, the teacher and students attempt to identify possible strategies and any mathematical aspects of the problem. The actual process of discussing strategies can be considerable (de Lange, 1993). Students should investigate the advantages and disadvantages of various strategies and decide on the best solution. Once a strategy and solution is reached, students should evaluate their results and the mathematical methods and techniques used. These skills and insights gained can then be applied to a set of other problems that have been designed by the teacher to practice the material and methods learnt.

It should also be noted that teaching will be made more complicated by using real-world problems as the role of the teacher will be strongly geared around organiser and facilitator rather than deliverer of information. Some problems will have more than one answer or several strategies. Some teachers may also feel threatened with the apparent loss of authority, but in this case, all staff are willing to face the challenges of change.

### **3.2 New assessment**

The proposed changes in content and teaching will result in students demonstrating new ways of learning and doing maths in the classroom. Therefore, it is important to develop new ways of assessing their understanding and progress. Students demonstrate their mathematical understanding through a variety of methods such as asking important questions; making abstract connections; and applying learnt concepts to new problems. Swan (1993) identifies the following aspects of learning that can be assessed: facts, strategies, skills, concepts, appreciation and awareness, and personal attitudes and qualities. The new units will contain assessment that is varied enough to test these mathematical abilities.

The broad range of our students' mathematical abilities as well as learning styles should also be addressed in assessment tasks. Cretchley (1999) accounts for the diversity of backgrounds and abilities by presenting an attempt at allowing students to select their own items of assessment. Students were invited to submit one 'good' question from the textbook exercises that best demonstrated the concepts covered in that section and justify their selection. We have, also, successfully trialled tasks of this type and plan to extend this form of assessment. Student reports, which summarise a task or result and student-created tests are other valid assessment tasks that reveal students' knowledge and understanding of mathematics.

As much of the in-class discussion will utilise group work, it seems obvious that there should be a greater use of group work in assessment. A large amount of literature discusses the benefits and achievements of students working in groups. The interested reader is referred to Duncan & Dick (2000) for references to recent literature. The assessment should also reflect the strong emphasis on communication during in-class discussions. Gretton and Challis (1999) discuss assignments that emphasise communicating a solution and using technology. For example, students had to perform a simple regression analysis for a business. They were then asked to write a letter to the owners (non-mathematicians) that explains the mathematics used in their solution. Alongside these, student directed and group assessment tasks, examinations will still be used to test basic skills and verification of student knowledge as displayed in open assessment tasks.

### **3.3 New Structure**

Currently, there is no choice of subjects for our mathematics students. These students indicated that they would find their degree more enjoyable if they were offered some choice of the units they could take. With small class sizes and low staff numbers, offering more choices seemed to be unrealistic. To address this issue, a new structure has been proposed in which each unit may be taken at one of two levels. That is, some units will be offered at introductory/intermediate level and others at intermediate/advanced level. Students will then have some choice of which units they would like to take at a lower level and those they would prefer to take at a higher level. One teacher will take a unit at both levels. The first four hours in a week will involve all students. An extra hour will be spent extending the topic for those students who take the unit at the higher level. Laboratory classes and tutorials will be shared by students working at different levels. Some questions and learning tasks will be common to both levels while other tasks will be level specific. Since, in our current classes, students work on a range of problems, largely at their own pace, these changes are not expected to cause major logistical problems.

### **3.4 New Unit Names**

Since the current titles of mathematics units seem to have a negative impact on students when making their choices for elective subjects, it was decided to market our units with names that might attract students' interest. A full list of the new unit titles along with the levels at which they will be taught is provided in table 1.

### **3.5 Details of Units**

In this paper, space precludes a full discussion of the specific content, technology and problems investigated. Some examples of these issues will be discussed for the units being developed for the upcoming year.

Unit Title	Level
Upon the Shoulders of Giants	1/2
Modelling and Change	1/2
Bits, Bytes, and Algorithms	1/2
Profit, Loss, and Gambling	1/2
Shape, Space, and Design	1/2
Logic and Imagination	2/3
Modelling Reality	2/3

**Table 1: List of Units and Corresponding Levels**

### **Profit, Loss and Gambling**

This unit lends itself nicely to introducing real life problems and demonstrating mathematical concepts to solve these problems. Enthusiasm can be generated by spending the first week both discussing the history of gambling and investigating probability through exploration with cards and dice. The next three weeks are spent introducing basic statistical concepts that will be required to understand gambling games. These concepts include probability rules, independence, mutually exclusive, odds, house margin, expected values, and probability intervals. The tutorials of these weeks involve illustrating the concepts with simple games like a player receiving \$10 for an even number and \$25 for an odd number on the outcome of a roll of a die. Students can then calculate expected values, standard deviations, probability intervals, house margins, and discuss fair games. Excel can be used to simulate dice throwing and empirically explore concepts such as expected values.

There is an assignment where students work in groups and invent their own simple dice or card gambling games and demonstrate their understanding of the statistical concepts by calculating house margins of their own devised games. A tutorial class is spent playing these games and investigating the empirical results. Higher-level students are introduced to Bayes' theorem, Chebyshev's inequality and gambler's ruin in the corresponding weeks.

Each of the next five weeks of the unit focus on one gambling game with discussion of the mathematical concepts necessary to determine probabilities, odds and house margins. The games covered are lotteries, Keno, Roulette, Two-Up, Craps, and gaming machines. Other applications are then used to reinforce the techniques learnt. For example, Lotto games demonstrate the use of combinations that will be reinforced by calculating probabilities of poker hands and drawing balls out of urns. Most of the lectures begin with a gambling game and a discussion of how to calculate various probabilities involved in the game. Once students realise the complexity of calculating these probabilities manually, simpler techniques can be introduced or reinforced. For example, the number of ways that 6 balls can be selected from 45 in a lotto game would be laborious if students wanted to write down the sample space. The discussion will turn into breaking the problem into simpler manageable tasks and investigating the patterns. Higher-level students are introduced to a variety of probability distributions, calculating means and variances of these distributions, and calculating moment generating functions.

This unit also addresses basic ideas in financial mathematics such as the time value of money, annuities, superannuation and investment strategies. Again, problems such as calculating compound interest are introduced with a simple real problem. Student exploration of the problem

by calculating the yearly amount accrued, should lead to the need of a simple formula. Assignments are provided that contain real life applications such as comparing several loan providers or how to save for superannuation. The advanced level contains more content on stocks and bonds as well as linear programming.

Visiting speakers discussing the social/psychological issues of gambling and casino operations will strengthen students' appreciation of the context of this mathematics. The internet is used to play various games and empirically test different gambling strategies (a great assignment task). Packages such as Excel are also used to simulate events such as throwing dice and simulating data from various statistical distributions.

### **Upon the Shoulders of Giants**

It is important to have an introductory mathematics unit that provides students with an overview of the fundamental skills in number, function, algebra, and geometry required in the other mathematics units. Rather than repeating the same material covered in the secondary school curriculum, the students will be guided through the development of these ideas from an historical perspective. By looking at the origin of fundamental concepts, it is envisaged that students will improve their understanding of these concepts. This unit will show students that mathematics was not discovered in the polished form of our textbooks, but often developed in intuitive and experimental fashion out of a need to solve problems (Katz, 1998).

Number theory, Euclidean and Cartesian geometry, astronomy, trigonometry, number systems, algebra, functions and probability are covered in this course at introductory levels. This unit has a one hour lecture and three hours of tutorials per week. The lecture takes the students through the historical and social context of each topic covered and presents appropriate anecdotes and biographies. The tutorials are used to present and apply the underlying mathematical skills inherent in the concepts discussed in the preceding lecture. The skills covered include basic trigonometry, algebraic manipulation, scientific notation and evaluation of functions. At the advanced level, students attend one extra tutorial hour in which they are challenged with problems which combine concepts in an applied situation. For example, one advanced tutorial problem asks students to estimate astronomical distances using scientific notation, trigonometry and algebra.

The assessment for this unit involves a group presentation on topics such as  $\pi$ , mental calculation or the golden ratio. There is one assignment which covers elementary algebra, geometry, trigonometry and functions and includes an essay style question on mathematics history. Students are also expected to hand in a portfolio of problems as discussed in section 3.2. Current technology is used to replicate some of the early explorations, unlike Rheticus (1524-1576), a mathematical astronomer who spent 12 years with hired human computers to produce two trigonometric tables!!

### **Logic and Imagination**

All mathematics courses need to find an appropriate place to introduce students to fundamental mathematical reasoning and proof. This unit aims to do this in a variety of topics which are often found in Discrete Mathematics courses. By starting with logical puzzles and informal discussion of paradoxes, we hope to gain the students' interest in more abstract reasoning. Elementary Number Theory is a good source of some of the simplest theorems, and will be taken far enough to enable a discussion of Public Key Cryptography. (One of our aims is to provide a course that will be useful and interesting for more mathematically inclined computing students to take as an elective.) The imagination of mathematicians over the centuries will be highlighted by a discussion of number systems leading up to complex numbers, and for the more advanced students a brief look at the

quaternions. We will also include other topics of interest to computing students, such as hierarchies of algorithmic complexity.

In the initial implementation of this unit, it has not been necessary to run it at two levels, however the students form two relatively different groups. These groups have two classes in common and each have two other tutorial classes separately. One group consists of future teachers (many primary, but some aiming to be secondary mathematics teachers). These students find the abstractions in the course difficult, and so their tutorials are used mainly for practice on easier problems. The other group is mainly computer science students, and it has proved possible to go a bit further and introduce extra material in their tutorial classes.

The following paragraphs summarise the topics covered in the units that will be developed over the next year.

### **Modelling and Change**

The major focus of this unit will be on learning and applying standard calculus techniques to model motion, growth, and change. Mathematical modelling by its very nature will be based on practical work and examples. The first couple of weeks will be spent on developing basic modelling skills. The rest of the unit will consider problems such as the analysis of velocity and acceleration for vehicles and athletes, growth and decline of populations under different environmental constraints, and marginal costs for business. Generally, each week will be focussed on a particular problem that utilises a mathematical modelling concept. Further exercises and examples will be used each week to reinforce skills learnt. This unit is one of the easiest to teach at two levels. Students that take the unit at the lower level will use algebra, differentiation, and integration to solve modelling problems. The higher-level students will also use these techniques as well as differential equations, optimisation techniques, and calculations of area and volume. These students will often work on similar problems each week to the others, but will be required to handle more demanding differentiations and integrations.

### **Bits, Bytes, and Algorithms**

This unit is compulsory for computing students but is also valuable to prospective teachers. Students will explore the representation and manipulation of numbers and symbols, the mathematical structures that underlie the storage of information, the algorithms that underlie computer software programs, introductory number theory, matrix operations, and solving linear equations using matrices. Higher-level students will apply algorithms for traversal and optimisation of networks and graphs as well as developing recursive algorithms.

### **Space, Shape, and Design**

This unit will investigate the patterns in the shapes of nature, art, architecture, and industry. It will provide students with some experience in the thinking and techniques necessary to establish evidence of general patterns and calculations related to spatial measurement and design. Topics that will be covered include two-dimensional and three-dimensional shapes, geometric properties, tessellations, symmetry, topology, graph theory, fractals, kaleidoscopes, and trigonometry. Activities will include constructing 3D shapes, working out fencing lines for land subdivisions, finding paths to fit constraints, and analysing optimum shapes for industrial design. Higher-level students will have further experience in the formal use of mathematics to solve spatial problems.

### **Modelling Reality**

This is the second unit of modelling that students can undertake if they have completed the unit Modelling and Change. The topics in this unit include an introduction to multivariate calculus, numerical methods, interpolation, linear algebra, and consolidating topics previously encountered

in Modelling and Change. Higher-level students will work on the same topics but will have more challenging problems.

## **4. Steps to Implement the Solutions**

This section contains the different stages that the mathematics department actually undertook in getting the new units established. The process began in the middle of last year (2001) and the units will be running in first semester 2002.

### **Stage 1: Discussion and Research**

There was lots of dialogue with current students, staff across the University, at conferences (for example: Delta Symposium on Teaching Undergraduate Mathematics and MERGA), and at other institutions. Staff read current mathematics education books and journals.

### **Stage 2: A 'brain storm' and response**

A loosely structured, imaginative list of 'possibilities' was designed to elicit clear responses from the mathematics staff. Following a discussion at a meeting of all staff, there was unanimous 'in principle' agreement. Consensus was then reached on unit names and broad areas of content. Tasks were assigned to facilitate the preparing of a formal, detailed proposal for change.

### **Stage 3: Development of formal proposals for change**

This stage involved more detailed discussions with colleagues at other institutions, current mathematics students, and students not currently undertaking any mathematics electives. The staff then had to gather resources and ideas and prepare a list of mathematical content required by our various client groups.

A planning day was arranged when all of the mathematics department could attend. A large matrix containing the unit names as rows and mathematical topics as columns was drawn on a whiteboard. A number (1,2 or 3) indicating the level at which the content would be taught was placed in the appropriate grids. This method quickly highlighted any areas of mathematical content would be omitted or repeated. It also helped to define whether a unit would be offered at introductory/intermediate or intermediate/advanced levels. Based on the agreed matrix, pairs of staff prepared formal unit outlines and a plan of the possible sequences of the unit was detailed.

There was agreement on an assessment policy that was general enough to be common to all units. For each unit, students will be required to submit a portfolio consisting of an annotated selection of their work to demonstrate their achievement of specific learning objectives. All students will be required to participate in projects or presentations, which in most units will involve group work. Tests or an examination will be directed at assessing basic skills and verification of knowledge and concepts demonstrated in non-supervised work.

## **5. Current Feedback**

Even before the units have been taught, there has been feedback and interest shown from a variety of sources. Student responses to the names and synopses for units has been overwhelmingly positive with comments like: 'Oh that sounds interesting', 'I'd like to do that' or 'why didn't I have the chance to do those units?'. Non-mathematics staff have been more circumspect, especially about the name 'Upon the shoulders of giants' for the basic unit. However, in contrast to the past, we have not been met with a neutral response. The titles have



generated interest and discussion that have allowed us the opportunity to share our enthusiasm for our discipline!

## 6. Conclusion

The new units will provide a broad range of mathematical concepts that should be suitable for students from a variety of disciplines. Students will be provided with a greater choice of units and an increase in the breadth of mathematical experiences. There will be more emphasis on mathematics that is useable and relevant without reducing the content that pre-service teachers are expected to cover. The goal of these changes is to improve students' motivation, perception and attitude towards mathematics. At the time of writing, we have still to face the big test of implementation of this new program. All issues of the first semester in teaching these units will be shared at the conference. Responses from both the teachers and students about the successes and problems found in structure, content and assessment of the units will be presented and discussed.

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