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# Portfolio Optimization, CAPM & Factor Modeling Project Report

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# **Portfolio Optimization, CAPM & Factor Modeling**

## **Project Report**

A Directed Research Project

Submitted to the Faculty of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Professional Degree of Master of Science

in

Financial Mathematics

by

**Yijun Dong**

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December 2011

**Approved:**

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**Professor Marcel Blais, Advisor**

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**Professor Bogdan Vernescu, Head of Department**

## **Abstract**

In this Portfolio Optimization Project, we used Markowitz's modern portfolio theory for portfolio optimization. We selected fifteen stocks traded on the New York Stock Exchange and gathered these stocks' historical data from Yahoo Finance <sup>[1]</sup>. Then we used Markowitz's theory to analyze this data in order to obtain the optimal weights of our initial portfolio. To maintain our investment in a current tangency portfolio, we recalculated the optimal weights and rebalanced the positions every week.

In the CAPM project, we used the security characteristic line to calculate the stocks' daily returns. We also computed the risk of each asset, portfolio beta, and portfolio epsilons.

In the Factor Modeling project, we computed estimates of each asset's expected returns and return variances of fifteen stocks for each of our factor models. Also we computed estimates of the covariances among our asset returns. In order to find which model performs best, we compared each portfolio's actual return with its corresponding estimated portfolio return.

**Key Word:** CAPM, Modern Portfolio Theory, Factor Model, French and Fama Model

## **Acknowledgements**

This report is a joint work work done by Chenghao Xu and Yijun Dong.

Firstly, thanks to Professor Marcel Blais whose guidance and enthusiasm for this work served to make this an enjoyable exercise. Also thanks to my teammate, Chenghao Xu, this project could not have been completed without his cooperation. During this project, we learned many useful theories and methods of analysis which enhance our ability to deal with practical problems. We enjoyed this exercise.

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# **1. Portfolio Optimization Project**

## **1.1 Introduction**

Portfolio theory is an important theoretical development made by Harry Markowitz<sup>[2]</sup>. In this project, we used this theory to obtain an optimal portfolio, which contains 15 stocks trading on the New York Stock Exchange (NYSE). Based on this optimal portfolio, we formed the positions in an Interactive Brokers Paper Trading Account<sup>[3]</sup>.

We recomputed the new optimal portfolio every Friday after the market closed and rebalanced our portfolio on the following Monday. Then we calculated the actual return of our portfolio. The estimated return was determined by the tangency portfolio. By comparing the difference of these two returns, we investigated the possibility that modern portfolio theory is useful in the real world.

## **1.2 Assumptions<sup>[2]</sup>**

- 1) All investors are risk averse and only allocate their portfolios on the efficient frontier.
- 2) All investors have the same probability distribution for future rates of return.
- 3) Information is free to all investors.
- 4) All assets are properly priced with respect to the risk they bear.
- 5) Investors can borrow or invest at the risk-free rate.
- 6) There are no taxes, transaction costs, or short sale restrictions.

## **1.3 Stocks and Risk-free Rate**

### **1.3.1 Stocks**

We chose six sectors: Financial, Consumer Goods, Technology, Basic Materials, Services, and Industrial Goods. From these sectors we chose 15 stocks that had a big market cap and high liquidity. The 15 stocks we chose are shown in Table 1.

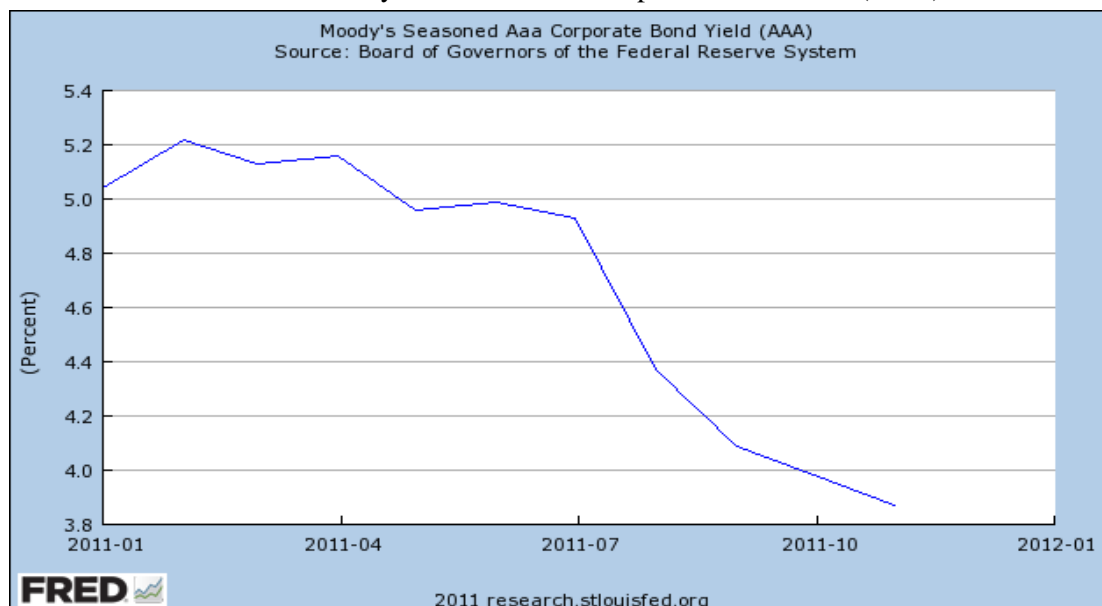
Table 1: Stocks We Chose

Symbol	Company Name	Market Cap	Sector
GS	Goldman Sachs Group, Inc	49.80B	Financial
SNE	Sony Corporation	17.74B	Consumer Goods
IBM	International Business Machines Corp.	227.40B	Technology
XOM	Exxon Mobil Corporation	387.39B	Basic Materials
F	Ford Motor Co.	41.99B	Consumer Goods
YUM	Yum! Brands, Inc.	26.40B	Services
JPM	JPMorgan Chase & Co.	126.20B	Financial
MCD	McDonald's Corp.	98.24B	Services
BA	Boeing Co.	52.67B	Industrial Goods
NKE	Nike Inc.	44.63B	Consumer Goods
EDU	New Oriental Education & Technology Group	3.88B	Services
M	Macy's, Inc.	13.86B	Services
FST	Forest Oil Corp.	1.86B	Basic Materials
CVS	CVS Caremark Corporation	49.81B	Services
AMD	Advanced Micro Devices, Inc.	3.94B	Technology

### 1.3.2 Risk-Free Rate

Assuming that an Aaa rated bond would not default in the short term, we chose Moody's Seasoned Aaa Corporate Bond yield as the risk-free rate. Chart 1 below is the Moody's Seasoned Aaa Corporate Bond Yield (AAA)<sup>[4]</sup>, and we used 4% as our annual risk-free rate. We divided the 4% annual rate by 52 to get the weekly risk-free rate.

Chart 1: Moody's Seasoned Aaa Corporate Bond Yield (AAA)<sup>[4]</sup>





## 1.4 Forming the Portfolio and Rebalancing

### 1.4.1 Parameter Estimation and Weights Calculation

In our project, we calculated the sample mean and sample covariance of the daily log returns of our portfolio. Then we inputted the sample mean and sample covariance into the function “quadprog” in MATLAB <sup>[5]</sup> to compute assets’ weights. The weights of the tangency portfolio were the data we wanted.

### 1.4.2 Initial Portfolio

Based on the tangency portfolio weights, we formed the positions in the Interactive Brokers Paper Trading Account <sup>[3]</sup>. On 12/02/2011 we formed our portfolio with an initial capital of \$500,000; however, we received the margin calls every day and were forced to close some positions. Thus, to avoid margin calls, we reset our initial capital to be \$1,000,000, and invested \$500,000 in our portfolio.

Initially we set no boundaries on our stocks’ weights so that we could short or long at a large weight. As shown in Chart 2, we have large long positions in XOM and MCD and also large short positions in JPM and NKE.

Chart 2: Initial Weights

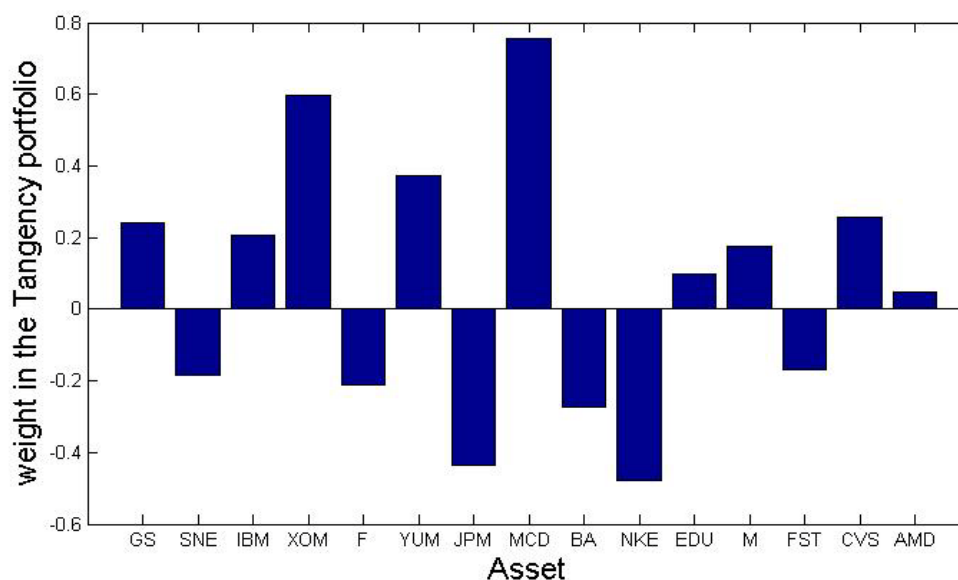
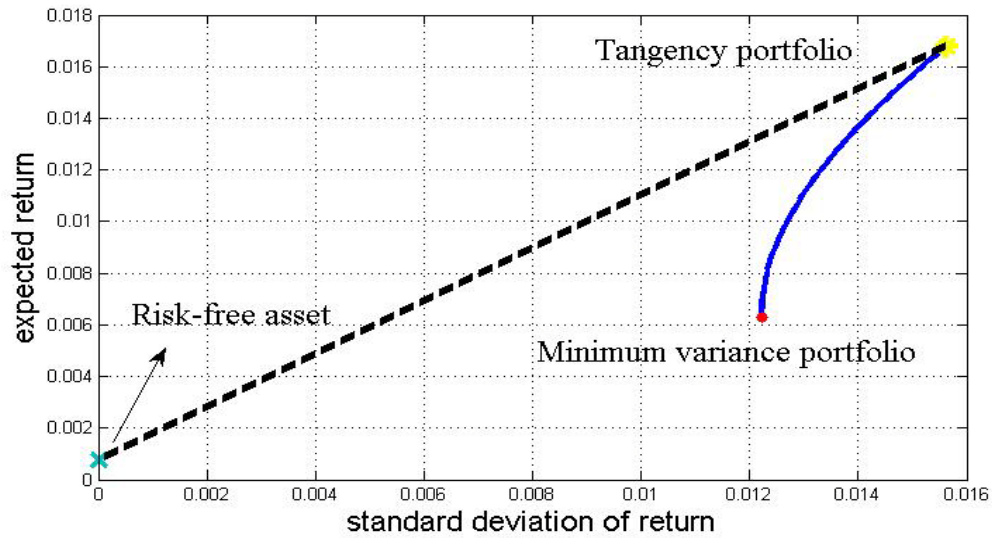


Chart 3: Plot of Tangency Portfolio



However, we still received a margin call and it automatically closed our positions in BA (Boeing) and NKE (NIKE). We repurchased these two stocks back on the second day. The theoretical positions and weights are shown in Table 2. The details about our portfolio in this period are shown in Table 3<sup>[3]</sup>.

Table 2: Theoretical Weights and Shares on 2011.11.07

Symbol	Cash Basis	Weight	Shares to Buy
AMD	23440.70	4.69%	4112.40
BA	-137028.18	-27.41%	-2082.18
CVS	127702.08	25.54%	3342.99
EDU	49572.27	9.91%	1709.39
F	-104892.09	-20.98%	-9249.74
FST	-84586.16	-16.92%	-5711.42
GS	120876.87	24.18%	1145.75
IBM	104141.84	20.83%	557.50
JPM	-218073.69	-43.61%	-6357.83
M	88433.90	17.69%	2813.68
MCD	377164.72	75.43%	3988.63
NIKE	-239579.17	-47.92%	-2575.84
SNE	-92319.17	-18.46%	-5143.13
XOM	299329.55	59.87%	3793.78
YUM	185816.54	37.16%	3412.61
Total	500000.00	100.00%	

Table 3: Shares and Average Price from 2011.11.07 to 2011.11.11 <sup>[3]</sup>

Symbol	Quantity	Cost Price	Cost Basis	Close Price	Value	Unrealized P/L	% of NAV
AMD	4,112	5.66	23,293.48	5.68	23,356.16	62.68	1.69
BA	0	0	0	0	0	0	0
CVS	3,343	38.11	127,385.02	38.23	127,802.89	417.88	9.25
EDU	1,709	28.93	49,439.74	28.62	48,911.58	-528.16	3.54
F	-9,250	11.18	-103,423.25	11.22	-103,785.00	-361.75	16.41
FST	-5,711	16.17	-92,357.20	15.68	-89,548.48	2,808.72	14.16
GS	1,146	105.48	120,884.27	105.57	120,983.22	98.95	8.76
IBM	558	186.22	103,907.97	187.32	104,524.56	616.59	7.57
JPM	-6,358	34.02	-216,289.95	34.24	-217,697.92	-1,407.97	34.41
M	2,814	31.42	88,401.81	32.35	91,032.90	2,631.09	6.59
MCD	3,988	93.75	373,886.18	94.62	377,344.56	3,458.38	27.32
NKE	-1,376	94.44	-129,947.16	93.8	-129,068.80	878.36	20.4
SNE	-5,143	17.88	-91,978.54	17.99	-92,522.57	-544.02	14.63
XOM	3,794	78.83	299,072.05	79.35	301,053.90	1,981.85	21.79
YUM	3,412	54.28	185,216.18	54.63	186,397.56	1,181.38	13.49
Total	-2,962		737,491		748,785	11,294	

There were no dividends this week and the actual return was  $\frac{748,785}{737,491} - 1 = 1.53\%$ .

### 1.4.3 Adjusted Portfolio in Week Two

At the end of week one, we recalculated the positions according to the new historical prices and adjusted the positions on the following Monday using market orders. The details about the new portfolio from 11/14/2011 to 11/18/2011 are in Table 4<sup>[3]</sup>. Chart 4 and Chart 5 show the weights and the tangency portfolio. The expected return was 1.26%.

Chart 4: Weights in Week Two

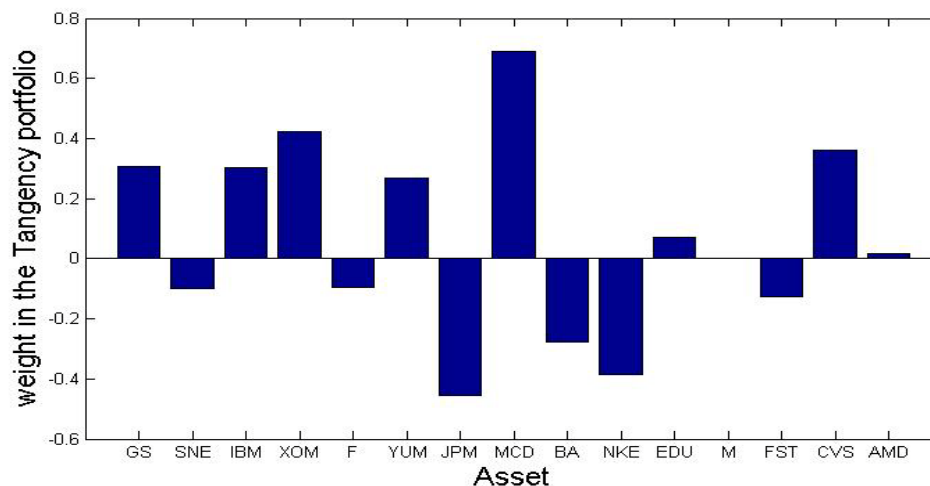


Chart 5: Plot of Tangency Portfolio in Week 2

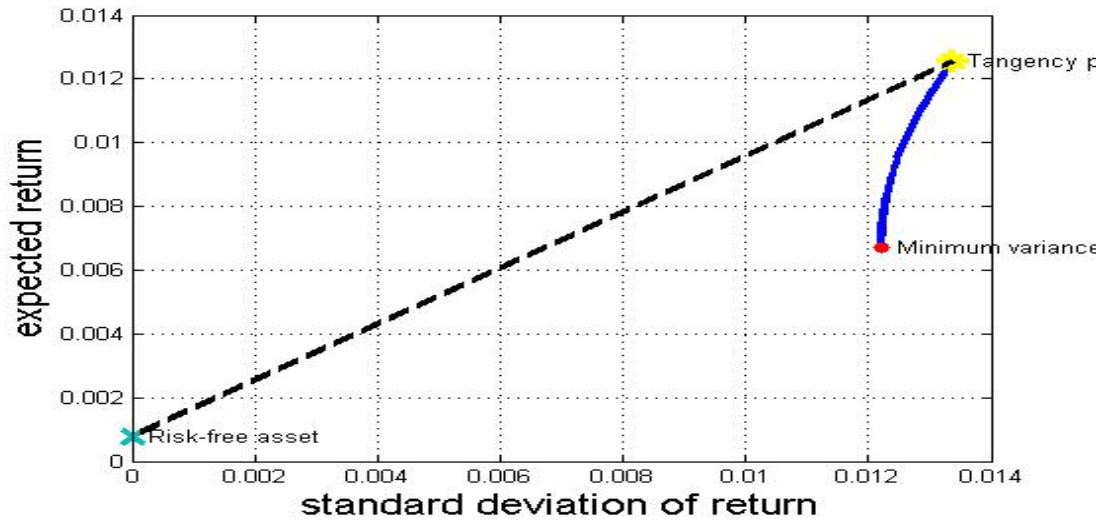


Table 4: Shares and Average Price from 2011.11.14 to 2011.11.18<sup>[3]</sup>

Symbol	Quantity	Cost Price	Cost Basis	Close Price	Value	Unrealized P/L	% of NAV
AMD	1,522	5.664343	8,621.13	5.47	8,325.34	-295.79	0.71
BA	-2078	68.501526	-142346.17	67.46	-140181.88	2164.29	19.12
CVS	4,593	38.390652	176,328.26	38.16	175,268.88	-1059.38	14.89
EDU	1,311	28.937896	37,937.58	22.78	29,864.58	-8073	2.54
F	-8,752	11.181113	-97,857.10	10.1	-88,395.20	9461.9	12.05
FST	-4,106	16.172	-66,402.23	14.7	-60,358.20	6,044.03	8.23
GS	1,504	104.22528	156,754.82	91.91	138,232.64	-18522.18	11.75
IBM	813	187.12023	152,128.74	185.24	150,600.12	-1528.62	12.8
JPM	-6,839	33.939648	-232,113.26	30.62	-209,410.18	22,703.07	28.56
M	40	31.415	1,256.60	30.36	1,214.40	-42.20	0.1
MCD	3,633	93.748157	340,587.06	92.74	336,924.42	-3,662.63	28.63
NKE	-2,011	93.973339	-188,980.39	92.75	-186,520.25	2460.14	25.44
SNE	-2,865	17.879412	-51,224.52	16.91	-48,447.15	2777.36	6.61
XOM	2,658	78.824488	209,515.49	77.9	207,058.20	-2,457.29	17.59
YUM	2,404	54.28906	130,510.90	53.84	129,431.36	-1,079.54	11
Total	-8,173		434,717		443,607	8,890	

In the second week, IBM and XOM paid total dividends at the amount of \$418.5 and \$1783.18. We adjusted the return of this week by adding the dividend. Thus the

actual return for this week was  $\frac{443,607+418.5+1783.18}{434,717} - 1 = 2.55\%$ .

### 1.4.4 Adjusted Portfolio in Week Three

At the end of week two, we recalculated the positions and rebalanced it on the following Monday when the market opened; however, in this week, for the purpose of diversification, we set the boundaries of weights to be 50%. The details of the portfolio from 11/21/2011 to 11/25/2011 are present in Table 5. Chart 6 is the plot of the weights and Chart 7 shows the tangency portfolio. The expected return we calculated was 1.31%, and it is shown in Chart 7.

Chart 6: Weights in Week 3

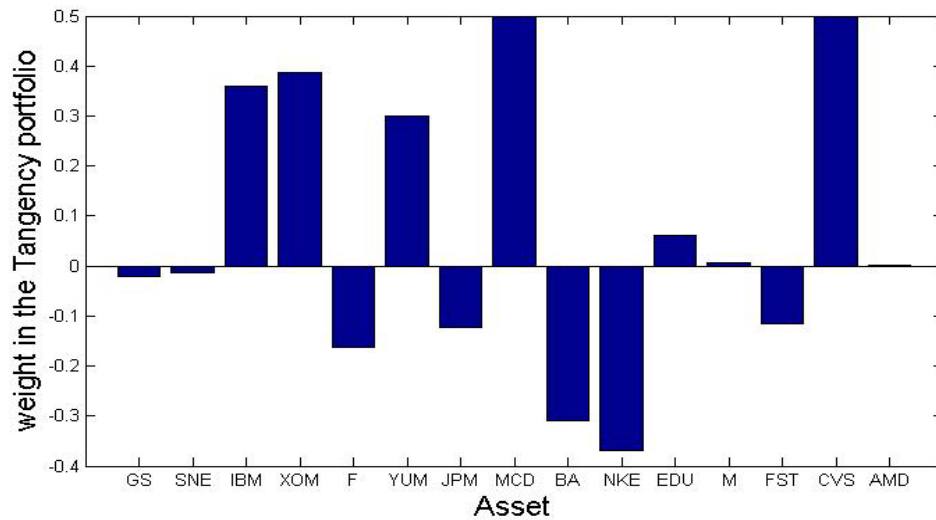


Chart 7: Plot of Tangency Portfolio in Week 3

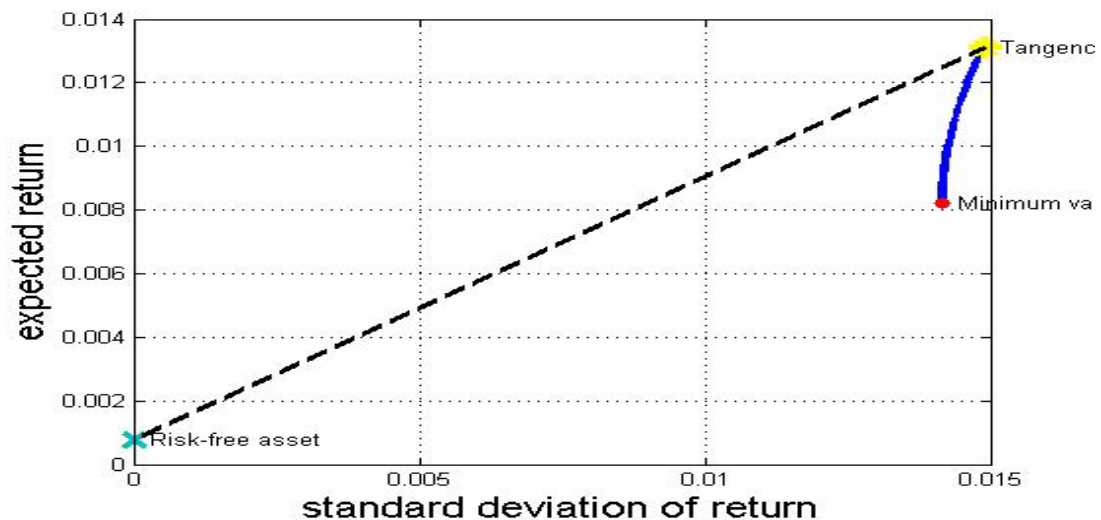


Table 5: Shares and Average Price from 11/21/2011 to 11/25/2011

Symbol	Quantity	Cost Price	Cost Basis	Close Price	Value	Unrealized P/L	% of NAV
AMD	50	5.67	283.25	4.99	249.50	-33.75	0.02
BA	-2299	68.29	-157008.42	62.78	-144331.22	12677.2	25.07
CVS	6,519	38.22	249,140.22	36.85	240,225.15	-8915.06	23.43
EDU	1,277	28.94	36,954.78	23.46	29,958.42	-6996.36	2.92
F	-12,490	10.81	-134,985.13	9.75	-121,777.50	13207.63	21.15
FST	-4,126	16.16	-66,687.23	13.3	-54,875.80	11,811.43	9.53
GS	-112	90.17	-10,098.48	88.75	-9,940.00	158.48	1.73
IBM	963	186.53	179,624.74	177.06	170,508.78	-9115.96	16.63
JPM	-2,017	33.76	-68,085.70	28.48	-57,444.16	10,641.54	9.98
M	123	30.52	3,754.24	29.45	3,622.35	-131.89	0.35
MCD	2,708	93.74	253,845.18	92.1	249,406.80	-4,438.38	24.32
NKE	-2,004	93.97	-188,319.62	90.28	-180,921.12	7398.5	31.42
SNE	-384	17.84	-6,851.83	16.96	-6,512.64	339.19	1.13
XOM	2,483	78.82	195,719.59	73.9	183,493.70	-12,225.88	17.89
YUM	2,807	54.14	151,964.60	52.72	147,985.04	-3,979.56	14.43
Total			439,250		449,647	10,397	

Since there was no dividend this week, thus the actual return for this week was:

$$\frac{449,647}{439,250} - 1 = 2.36\%$$

#### 1.4.5 Adjusted Portfolio in Week Four

At the end of week two, we recalculated the positions and rebalanced it on the following Monday when the market opened. In this week, we kept the boundaries at 50%. The details of the portfolio from 11/28/2011 to 12/02/2011 are in Table 6<sup>[3]</sup>. Chart 8 is the plot of the weights and Chart 9 shows the tangency portfolio. The expected return for this week was 1.19% and is shown in Chart 9.

Chart 8: Weights on 15 Stocks in Week 4

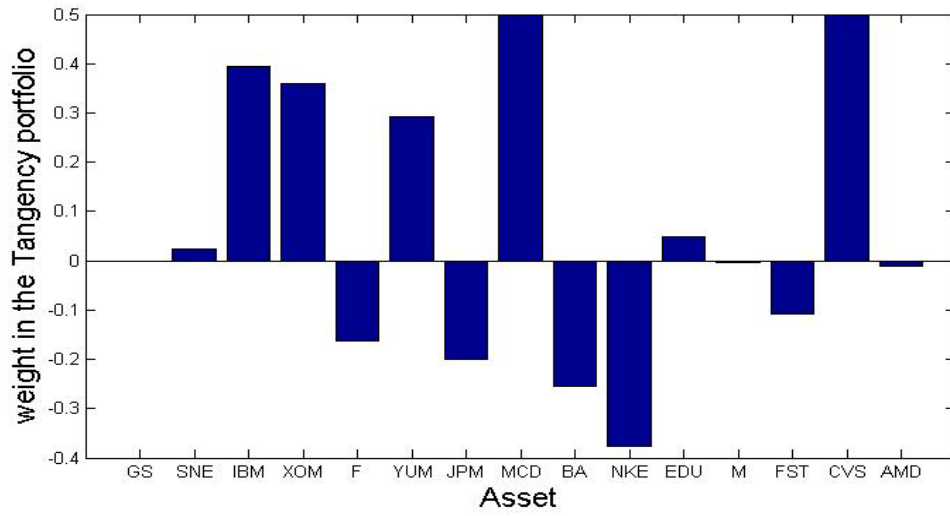


Chart 9: Plot of Tangency Portfolio in Week 3

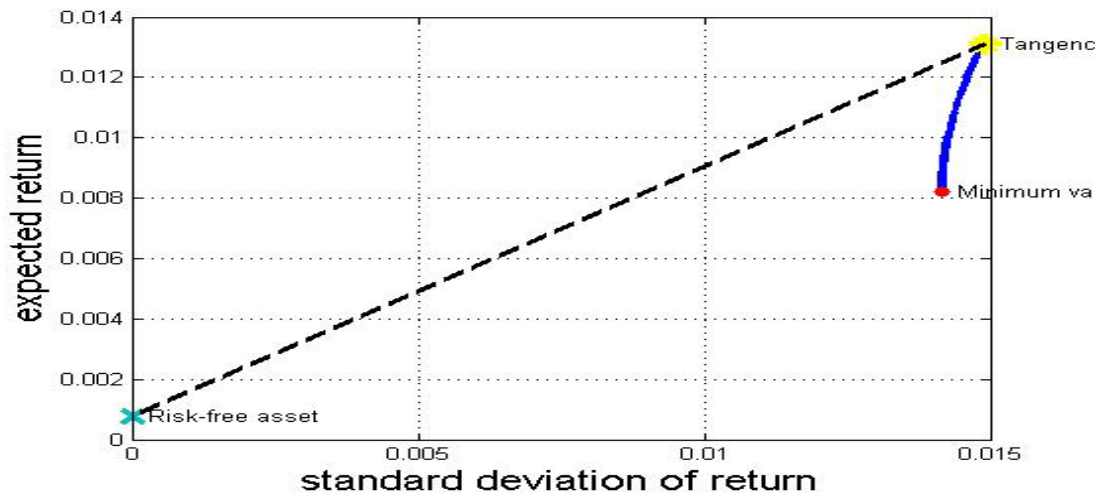


Table 6: Weight, Shares and Average Price from 11/28/2011 to 12/02/2011

Symbol	Quantity	Cost Price	Cost Basis	Close Price	Value	Unrealized P/L	% of NAV
AMD	-1081	5.33	-5758.26	5.65	-6107.65	-349.38	0.94
BA	-1970	68.23	-134418.27	71.3	-140461	-6042.73	21.51
CVS	6,764	38.20	258,360.79	38.29	258,993.56	632.77	23.62
EDU	962	28.93	27,827.47	24.94	23,992.28	-3835.19	2.19
F	-12,527	10.81	-135,354.87	10.9	-136,544.30	-1189.43	20.91
FST	-4,094	16.16	-66,168.99	16.38	-67,059.72	-890.73	10.27
GS	-4	90.17	-360.66	97.25	-389.00	-28.34	0.06
IBM	1088	186.00	202,366.50	189.66	206,350.08	3983.58	18.82
JPM	-3,425	31.82	-108,966.98	32.33	-110,730.25	-1,763.27	16.96
M	-58	30.83	-1,788.14	32.54	-1,887.32	-99.18	0.29
MCD	2,662	93.74	249,531.79	95.7	254,753.40	5,221.61	23.23
NKE	-1,971	93.96	-185,204.59	96.25	-189,708.75	-4504.16	29.06
SNE	704	17.74	12,488.16	17.77	12,510.08	21.92	1.14
XOM	2,376	78.82	187,286.38	79.79	189,581.04	2,294.66	17.29
YUM	2,674	54.13	144,744.70	56.25	150,412.50	5,667.80	13.72
Total	-7900		444585.03		443704.95	-880.07	

During this week, we received a dividend from MCD at the amount of \$1,895.60. Since we shorted GS, we had to pay dividend at the amount of \$39.20. So the actual return for this week was:

$$\frac{443,704.95 + 1,895.60 - 39.20}{444,585.03} - 1 = 0.22\%$$

## 1.5 Conclusion

We gathered the data about the change of stock weights over the four weeks. The differences between the actual returns and the expected returns are shown in Chart 11. Chart 11 illustrates that the actual returns fluctuate around the expected returns. Also the average actual return was 1.67%, and the average expected return was 1.35%, which was very close to average actual return. We concluded that the modern portfolio theory is still useful in the real market.



Chart 10: Stock Weights Change over the Week

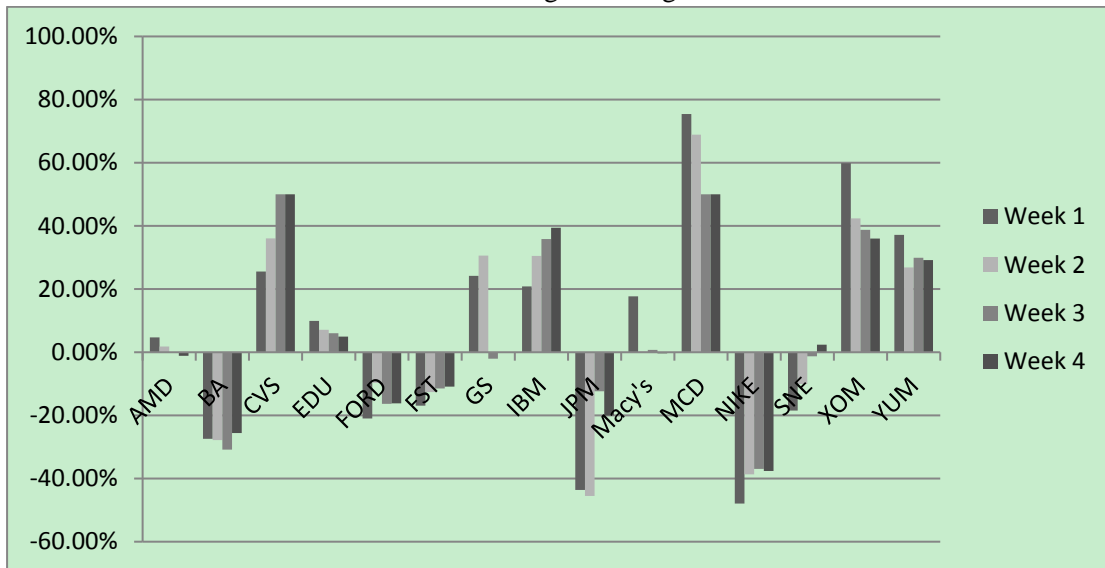
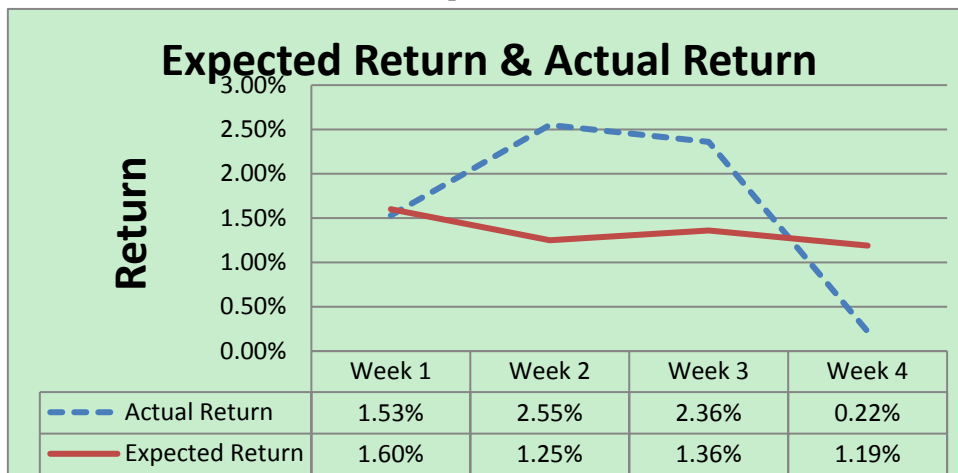


Chart 11: Differences of Expected Returns and Actual Returns



However, there were some disadvantages. We found that in the real world, if we invested all of our capital in risky assets, then it was very likely that we would receive margin calls.

Another disadvantage was the assumption that there were no transaction costs or taxes; however, in the real market, it is impossible to avoid taxes or transaction fees.

Furthermore, when we examined the weights of assets in each portfolio, we found that, before we set the bounds on the portfolio weights, we had very large long or short positions in our portfolio. This does not satisfy the goal of asset diversification.

Most importantly, in this project we only chose 15 stocks, but if we chose 200 stocks, we would need to calculate a 200\*200 covariance matrix. Thus as the number of stocks increases, the data we need to process would increase at a quadratic rate. We

concluded that for a small number of stocks, the modern portfolio theory can work, but for portfolios with a large number of stocks it may not be easy to generate useful results.

## 2. CAPM Project

### 2.1 Introduction

In the Capital Asset Pricing Model (CAPM), the capital market line (CML) relates the excess expected return on an efficient portfolio to its risk <sup>[2]</sup>. The Security Market Line (SML) relates the excess return on an individual asset to the slope of its regression on the market portfolio <sup>[2]</sup>. However, the SML and the CML only consider the expected returns but not the risks. As such, the Security Characteristic Line has been used <sup>[2]</sup>.

In this project, we used the Capital Asset Pricing Model (CAPM) to analyze our portfolio that had been created in our “Portfolio Optimization Project”. Then, we used the Security Characteristic Line and the corresponding return-generating process to model the returns of the 15 stocks we chose. Here, the stocks’ daily adjusted closing prices are from 08/26/2011 to 11/18/2011, and we used the S&P 500 index as the market portfolio. After estimating beta ( $\beta$ ), the variance of market portfolio  $\sigma_M^2$  and the variance of epsilon  $\sigma_\epsilon^2$ , we calculated the risk ( $\sigma_j$ ) of assets and analyzed the systematic and unique component of risk.

### 2.2 Assumptions <sup>[2]</sup>

- 1) The market prices are at equilibrium.
- 2) All market participants have the same forecasts of expected returns and risks.
- 3) The market rewards people for bearing unavoidable risk, but there is no reward for unnecessary risk.
- 4) White noise terms  $\epsilon_{1,t}, \dots, \epsilon_{p,t}$  are uncorrelated
- 5)  $\epsilon_{j,t}$  and the market portfolio return  $R_{M,t}$  are uncorrelated.

## 2.3 Parameter Estimation

### 2.3.1 Model Description

Let  $R_{j,t}$  be the  $j_{th}$  asset return at time  $t$  (in days),  $R_{M,t}$  be the market portfolio return at time  $t$ , and  $\mu_{f,t}$  be the risk-free rate at time  $t$ . According to this notation, we form the Security Characteristic Line. Its equation is shown below <sup>[2]</sup>:

$$R_{j,t} = \mu_{f,t} + \beta_j \times (R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

where  $\epsilon_{j,t} \sim N(0, \sigma_{\epsilon,j}^2)$  and  $\epsilon_{1,t}, \dots, \epsilon_{p,t}$  are uncorrelated with each other. In our project  $R_{j,t}$  represented daily net return for  $j_{th}$  asset at time  $t$ . Since our sample period was from 08/26/2011 to 11/18/2011,  $t$  was ranged from 1 to 60.

### 2.3.2 Estimating the Risk-free Rate

In this project the yield of a 6-Month U.S. Treasury Bill <sup>[4]</sup> is used as the risk-free rate of return. Its annualized rate of return is 0.05%.

Secondly, we calculated the daily risk-free rates using the formula:

$$\mu_f^d = \sqrt[252]{(1 + \mu_f^a)} - 1$$

where  $\mu_f^d$  was the daily risk-free rate and  $\mu_f^a$  was the annual risk-free rate. We calculate  $\mu_f^d = 0.000002$ .

### 2.3.3 Beta

We used the linear regression method to estimate  $\beta_j$  of each asset by this formula <sup>[2]</sup>:

$$\hat{\beta}_j = \frac{\sum_{t=1}^n (R_{j,t} - \bar{R}_j)(R_{M,t} - \bar{R}_M)}{\sum_{t=1}^n (R_{M,t} - \bar{R}_M)^2}$$

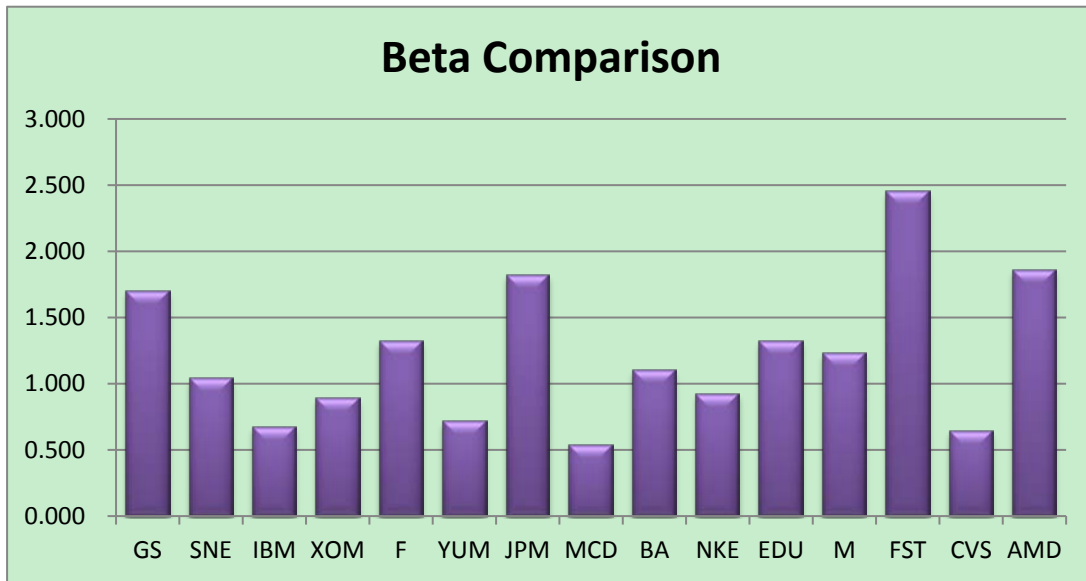
where  $n$  was the sample size.

In our project,  $n=60$ , since we had 60 trading days.  $\bar{R}_j$  was the sample mean of each asset's daily returns, and  $\bar{R}_M$  was the sample mean of daily returns of the S&P 500. Through MATLAB <sup>[5]</sup> programming (check code in appendix), then we estimated each stock's  $\beta$ . Table 7 shows each asset's  $\beta$ , and Chart 12 is the comparison of the 15 stocks'  $\beta$ .

Table 7: Estimated Stocks'  $\beta$

Stock	$\beta$
GS	1.696
SNE	1.036
IBM	0.669
XOM	0.886
F	1.319
YUM	0.712
JPM	1.820
MCD	0.535
BA	1.101
NKE	0.920
EDU	1.321
M	1.226
FST	2.447
CVS	0.639
AMD	1.856

Chart 12: Comparison of Assets'  $\beta$



### 2.3.4 Risk of Market and Assets

We computed the sample variance of the S&P 500's daily returns. Then we used this as the market component of risk, which is  $\hat{\sigma}_M^2 = 0.000329$ . We used the residual sum of squared errors to estimate the  $\sigma_{\varepsilon,j}^2$  by formula [2]:

$$\sigma_{\varepsilon}^2 \text{Residual error } Ms = \frac{SSE(p)}{n - p - 1}$$

Then we computed the risk of each asset by this formula [2]:

$$\sigma_j^2 = \beta \times \sigma_M^2 + \sigma_{\varepsilon,j}^2$$

Table 8 shows the risk of each asset.

Table 8: Each Asset's Risk

Stock	Risk of Each Asset
GS	0.00121
SNE	0.00079
IBM	0.00025
XOM	0.00032
F	0.00079
YUM	0.00036
JPM	0.00131
MCD	0.00016
BA	0.00052
NKE	0.00041
EDU	0.00219
M	0.00071
FST	0.00497
CVS	0.00022
AMD	0.00203

### 2.3.5 Portfolio Beta

We used the weights on 11/18/2011 from our Portfolio Optimization Project to calculate the portfolio  $\beta$  by formula <sup>[2]</sup>:

$$\beta_p = \sum_{j=1}^N \omega_j \beta_j$$

We calculate that  $\beta_p = 0.0234$ .

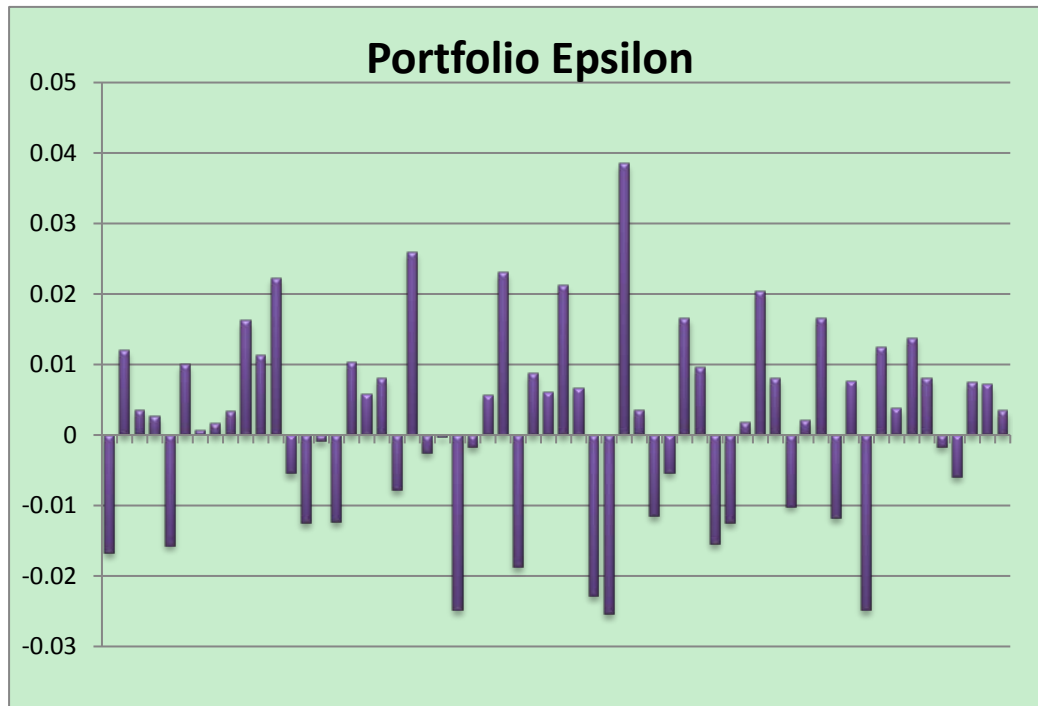
### 2.3.6 Portfolio Epsilon

For each time index, there was a portfolio epsilon. Thus, we had 60 portfolio epsilons. The formula <sup>[2]</sup> is:

$$\varepsilon_{p,t} = \sum_{j=1}^N \omega_j \varepsilon_j$$

Then, we compared the portfolio epsilons from time 1 to time 60, which are shown in Chart 13.

Chart 13: Comparison of Portfolio Epsilons



## 2.4 Conclusion

First, our portfolio  $\beta$  was  $0.0234$  which was much smaller than 1, so our portfolio was not aggressive.

Second, from Chart 13, we see that some portfolio epsilons were rather large (time point 35, time point 21). The main reason was that security characteristic line is a one factor model, which did not predict the portfolio return very well. Thus the method we used for improvement was to add more factors into our model such as the French & Fama factors.

Third, in order to reduce unsystematic risk, we should have chosen stocks in different industries and sectors. Therefore if we had the opportunity to choose our stocks again, we would have chosen stocks that had smaller correlations.

Finally, we used the S&P 500 as our market portfolio, but in the real world the S&P 500 is not sufficient to represent the whole market. In order to estimate the portfolio return and portfolio risk more accurately, we would need to use an index which contains more information about the market.

### 3. Factor Model Project

#### 3.1 Introduction

In this project we generated 7 factor models using different combinations of four important factors: CAPM, French & Fama, BofA Merrill Lynch US Corporate AAA Effective Yield (BAML Yield) <sup>[4]</sup>, and the Federal Fund Rate. First we estimated the value of  $\beta$  through the least squares estimation method. Then we estimated each asset's expected return and return variance, as well as the covariance among these assets' returns. With these estimations we computed optimal weights. At the end of the project, we used a back-testing strategy to compare the performance of the 7 factor models.

#### 3.2 Stocks and Factor Choice

For this project, we used the stocks' adjusted daily closing prices from March 1st, 2010 to Dec 31st, 2010, and then we calculated the sample return of the assets during that period.

There are two basic Factor models that we examine. One is the CAPM excess market return factor model and the other is the French & Fama factor model <sup>[2]</sup>. We then extend CAPM and French & Fama with two other factors. This generates 7 different factor models.

The first factor we chose was the Federal Funds Rate <sup>[5]</sup> (effective). The reasons for choosing this factor included <sup>[5]</sup>:

- It is a benchmark interest rate.
- It has strong relationship with prices of financial assets.
- It reflects the market supply-demand relationship and influences market expectation.
- It influences Treasury Bills, Treasury Notes, and Treasury Bonds of the United States.

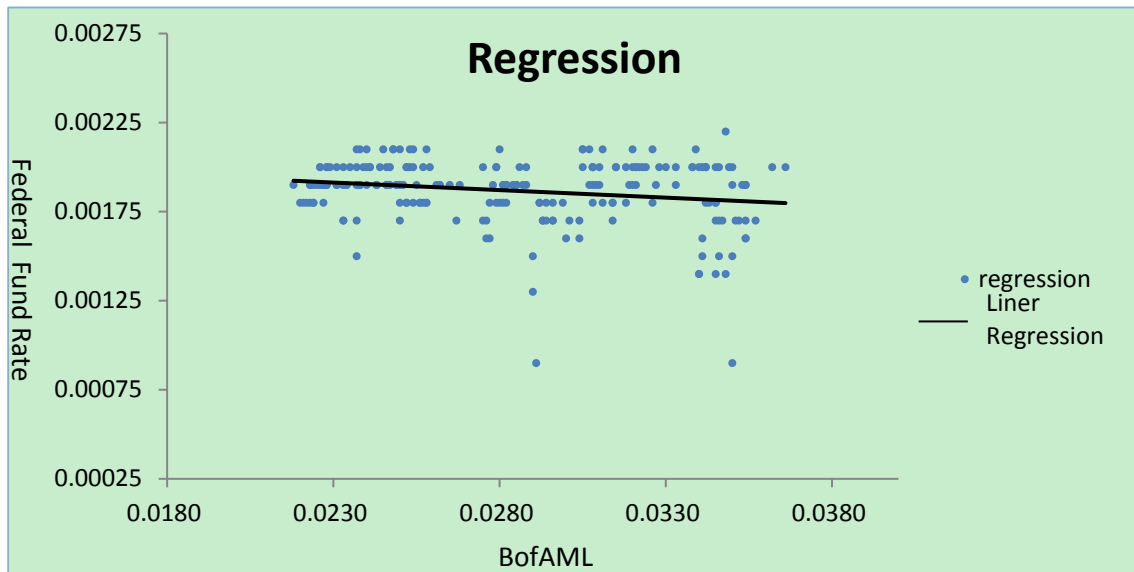
The second factor we chose was the BAML Yield <sup>[4]</sup>. The reasons for choosing this factor included:

- It has a higher return compared to U.S. Treasury Bonds and its yield will influence the stock market.
- It has little relationship with the Federal Funds Rates through regression analysis. Table 9 tells us that the Adjusted  $R^2$  was only 0.0346. It is therefore a good supplement to the first factor. More importantly, a smaller Adjusted  $R^2$  would make the assets' risk



(variances of the assets) smaller. This is very useful in risk reduction.

Chart 14: Linear Relationship between Federal Fund Rate and BofAML Yield<sup>[7]</sup>



Regression analysis output:

Table 9: Regression Analysis for Federal Fund Rate and BAML Yield

Regration	
Multiple R	0.19782
R Square	0.03913
Adjusted R	0.03460
Std Dev	0.00018
Observed	214

### 3.3 Method and Process

#### 3.3.1 Risk-free Rate

In this project, the yield of a 6-Month U.S. Treasury Bill<sup>[4]</sup> from 03/01/2010 to 12/31/2010 is used as the risk-free rate of return.

Secondly, we calculated the daily risk-free rates using the formula:

$$\mu_f^d = \sqrt[252]{1 + \mu_f^a} - 1$$

where  $\mu_f^d$  was the daily risk-free rate and  $\mu_f^a$  was the annual risk-free rate. We calculated that  $\mu_f^d = 0.0000079$ .

### 3.3.2 Estimation of Beta

The generalized factor model is <sup>[2]</sup>

$$R_{j,t} = \mu_{f,t} + \beta_{0,j} + \beta_{1,j}F_{1,t} + \dots + \beta_{p,j}F_{p,t} + \varepsilon_{j,t}$$

where  $F_{1,t} \dots F_{p,t}$  are the values of  $p$  factors at time  $t$ . In this project the sample period was from 03/01/2010 to 12/31/2012.

We used least squares estimation to find the  $\hat{\beta}_0, \dots, \hat{\beta}_p$  that minimize

$$\sum_{i=1}^N [Y_i - (\hat{\beta}_0 + \hat{\beta}_1 F_{i,1} + \hat{\beta}_2 F_{i,2} + \dots + \hat{\beta}_p F_{i,p})]^2$$

where  $N=214$  was the sample size.

Then we generated the beta Matrix <sup>[2]</sup>:

$$\beta = \begin{pmatrix} \beta_{1,1} & \dots & \beta_{1,15} \\ \vdots & \ddots & \vdots \\ \beta_{p,1} & \dots & \beta_{p,15} \end{pmatrix}$$

### 3.3.3 Estimate $\Sigma_F$

We estimated  $\Sigma_F$  by calculating the sample covariance matrix of factors, where

$\Sigma_F$  is the  $p \times p$  covariance matrix of the  $p$  factors <sup>[2]</sup>

$$\Sigma_F = \begin{pmatrix} \hat{\sigma}_{F_1}^2 & \hat{\sigma}_{F_1 F_2} & \dots & \dots & \hat{\sigma}_{F_1 F_{p-1}} & \hat{\sigma}_{F_1 F_P} \\ \hat{\sigma}_{F_2 F_1} & \hat{\sigma}_{F_2}^2 & \dots & \dots & \vdots & \hat{\sigma}_{F_2 F_P} \\ \vdots & \vdots & \ddots & \dots & \vdots & \vdots \\ \vdots & \vdots & \dots & \ddots & \vdots & \vdots \\ \hat{\sigma}_{F_{p-1} F_1} & \vdots & \dots & \dots & \hat{\sigma}_{F_{p-1}}^2 & \hat{\sigma}_{F_{p-1} F_P} \\ \hat{\sigma}_{F_P F_1} & \hat{\sigma}_{F_P F_2} & \dots & \dots & \hat{\sigma}_{F_P F_{p-1}} & \hat{\sigma}_{F_P}^2 \end{pmatrix}$$

### 3.3.4 Estimate $\Sigma_\varepsilon$

For  $\Sigma_\varepsilon$ , we used the residual sum of squared errors to estimate the variance of each asset <sup>[2]</sup>:

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{n-p-1} \sum_1^N \hat{\varepsilon}_i^2$$

where the estimation was unbiased, and  $N=214$ . Since  $\varepsilon_{1,t} \dots \varepsilon_{p,t}$  were uncorrelated, the covariance between assets was 0. Therefore  $\Sigma_\varepsilon$  should be a  $15 \times 15$  diagonal matrix <sup>[2]</sup>

$$\Sigma_{\varepsilon} = \begin{pmatrix} \widehat{\sigma}_{\varepsilon,1}^2 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \widehat{\sigma}_{\varepsilon,j}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \widehat{\sigma}_{\varepsilon,15}^2 \end{pmatrix}.$$

### 3.3.5 Estimating Expectations, Variances and Covariance of Asset Returns

We had already estimated  $\beta$ ,  $\sum_{\varepsilon}$  and  $\sum_F$ . Thus we could obtain <sup>[2]</sup>

$$Cov(R_t) = \beta^T \sum_F \beta + \sum_{\varepsilon}$$

and

$$Var(R_j) = \beta_j^T \sum_F \beta_j + \sigma_{\varepsilon}^2$$

At the same time, we computed  $E(F_{j,t})$  for each asset and estimated each asset's expected return  $E(R_{j,t})$  <sup>[2]</sup>

$$E(R_{j,t}) = \mu_f + \widehat{\beta}_{0,j} + \widehat{\beta}_{1,j}E(F_{1,t}) + \widehat{\beta}_{2,j}E(F_{2,t}) + \cdots + \widehat{\beta}_{p,j}E(F_{p,t}) \quad (3.3.5)$$

### 3.3.6 Computing Optimization Weights

The last step was to input  $\sum R$ ,  $E(R)$ , and the risk-free rate to MATLAB function “optimalPorfolio,” <sup>[6]</sup> which we designed for computing the tangency portfolio. Since a portfolio with a large weight on a single asset would not satisfy the goal of asset diversification, we set a bound of 0.25 for each asset in the function “optimalPorfolio” <sup>[6]</sup>. Thus we were able to avoid unreasonable results.

Then we obtained an optimization weight for each asset and estimated the portfolio's return by <sup>[2]</sup>:

$$E(R_p) = \sum_{i=1}^N \omega_j E(R_j)$$

where  $N=15$  (number of stocks).

### 3.3.7 Tracking Stocks from 01/03/2011 to 10/31/2011

The profit rate for the  $t^{th}$  holding period is

$$R_t = \frac{P_t}{P_1} - 1$$

where  $t=10$ , and  $P_1$  and  $P_t$  were the stock's prices on 01/03/2011 and 10/31/2011, respectively.

Table 10: Prices of Each Asset on 01/03/2011 and 10/31/2011 and Profit Rate

Stock (symbol)	Prices(01/03/2011)	Prices (10/31/2011)	Profit Rate
GS	171.06	109.13	-36.20%
SNE	36.38	20.97	-42.36%
IBM	145.03	183.89	26.79%
XOM	72.82	77.63	6.61%
Ford	17.25	11.68	-32.29%
YUM	48.11	53.57	11.35%
JPM	42.66	34.76	-18.52%
MCD	74.35	92.16	23.95%
BOEING	64.81	65.37	0.86%
NIKE	84.84	95.99	13.14%
EDU	26.37	29.64	12.40%
Macy's	25.19	30.53	21.20%
FST	38.43	11.66	-69.66%
CVS	34.56	36.33	5.12%
AMD	8.47	5.83	-31.17%

We set our original capital to be \$500,000. Based on the weights and profit rates we were able to calculate the total profits.

### 3.4 Factors Models

The details of the 7 factor models are shown below.

#### 3.4.1 French & Fama (FF)

Chart 15: Optimal Weights of FF Model



Table 11: Weights, Capital and Profit of FF Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.095285	47642.44	-36.20%	-17248.31
SNE	-0.015936	-7967.83	-42.36%	3375.05
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
FORD	0.134993	67496.51	-32.29%	-21794.52
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.250000	-125000.00	-18.52%	23148.15
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.168811	84405.56	12.40%	10466.67
MACY	0.054401	27200.62	21.20%	5766.23
FST	0.058305	29152.42	-69.66%	-20307.32
CVS	-0.083150	-41574.82	5.12%	-2129.27
AMD	-0.162710	-81354.89	-31.17%	25357.37
Total	1	500000	N/A	107860.76

Table 12: Estimated Returns and Return Variance of FF Model

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000381	0.000054
SNE	0.000394	0.000367	-0.000006
IBM	0.000799	0.000124	0.000200
XOM	0.000709	0.000131	0.000177
FORD	0.001947	0.000555	0.000263
YUM	0.001925	0.000203	0.000481
JPM	0.000246	0.000361	-0.000062
MCD	0.001018	0.000094	0.000254
BOEING	0.000403	0.000336	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000520	0.000278
MACY	0.001636	0.000585	0.000089
FST	0.001891	0.000635	0.000110
CVS	0.000293	0.000232	-0.000024
AMD	0.000523	0.000740	-0.000085
Total	N/A	N/A	0.001947

### 3.4.2 CAPM and Federal Fund Rate (C&FFR)

Chart 16: Optimal Weights of C&FFR

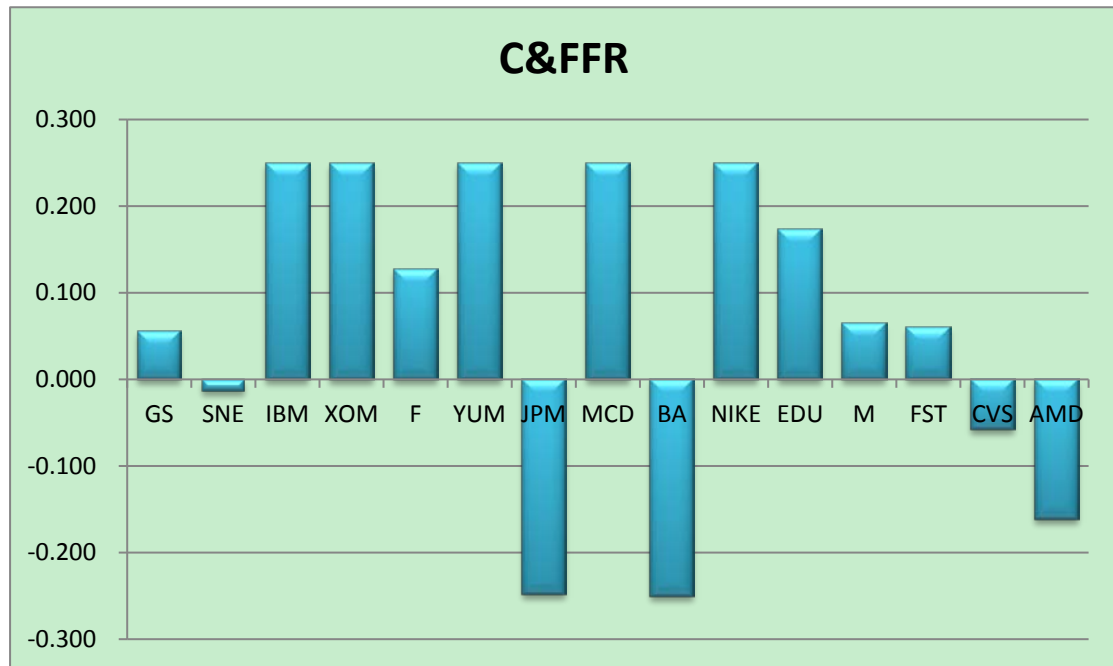


Table 13: Weights, Capital and Profit of C&FFR Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.055125	27562.67	-36.20%	-9978.70
SNE	-0.013357	-6678.69	-42.36%	2828.99
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.127261	63630.43	-32.29%	-20546.17
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.248129	-124064.50	-18.52%	22974.91
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.173049	86524.73	12.40%	10729.46
M	0.064623	32311.63	21.20%	6849.71
FST	0.060652	30325.92	-69.66%	-21124.77
CVS	-0.058113	-29056.57	5.12%	-1488.14
AMD	-0.161111	-80555.61	-31.17%	25108.24
Total	1	500000	N/A	116580.24

Table 14: Estimated Returns and Return Variance of C&FFR

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000380	0.000031
SNE	0.000394	0.000366	-0.000005
IBM	0.000799	0.000124	0.000200
XOM	0.000709	0.000131	0.000177
F	0.001947	0.000554	0.000248
YUM	0.001925	0.000203	0.000481
JPM	0.000246	0.000361	-0.000061
MCD	0.001018	0.000094	0.000254
BOEING	0.000403	0.000336	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000518	0.000285
M	0.001636	0.000584	0.000106
FST	0.001891	0.000634	0.000115
CVS	0.000293	0.000231	-0.000017
AMD	0.000523	0.000738	-0.000084
Total	N/A	N/A	0.001947

### 3.4.3 French & Fama and Federal Fund Rate (FF&FFR)

Chart 17: Optimal Weights of FF&FFR

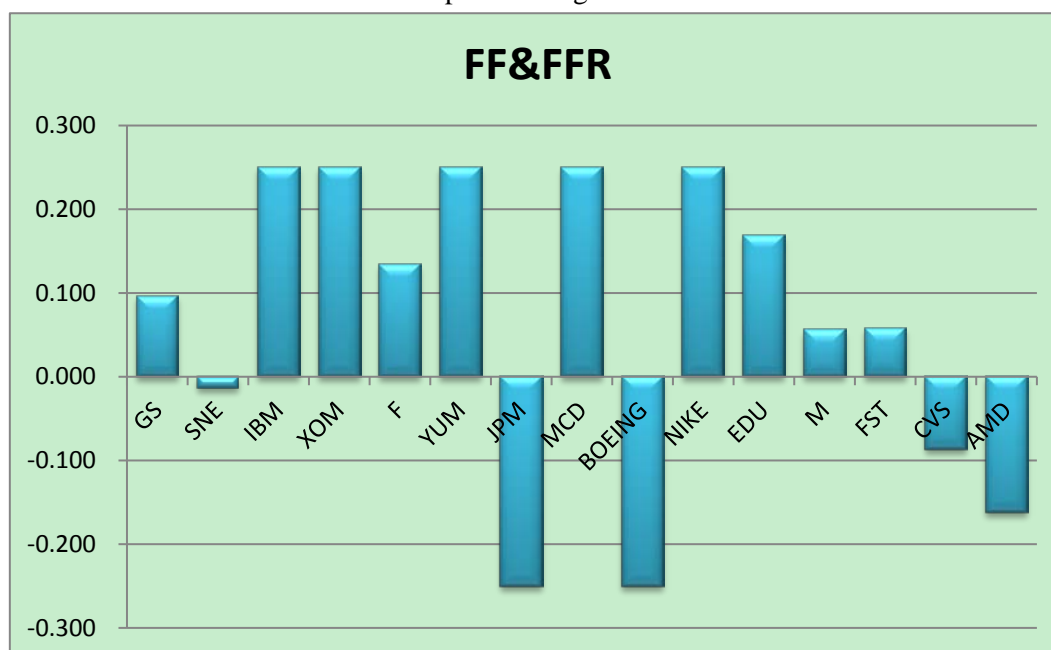


Table 15: Weights, Capital and Profit of FF&FFR Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.096118	48058.90	-36.20%	-17399.09
SNE	-0.013871	-6935.64	-42.36%	2937.83
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.133686	66842.85	-32.29%	-21583.46
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.250000	-125000.00	-18.52%	23148.15
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.169049	84524.42	12.40%	10481.41
M	0.055910	27955.10	21.20%	5926.17
FST	0.057849	28924.72	-69.66%	-20148.70
CVS	-0.086490	-43244.76	5.12%	-2214.79
AMD	-0.162251	-81125.59	-31.17%	25285.90
Total	1	500000	N/A	107660.13

Table 16: Estimated Returns and Return Variance of FF&FFR

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000382	0.000054
SNE	0.000394	0.000368	-0.000005
IBM	0.000799	0.000125	0.000200
XOM	0.000709	0.000131	0.000177
F	0.001947	0.000556	0.000260
YUM	0.001925	0.000204	0.000481
JPM	0.000246	0.000361	-0.000062
MCD	0.001018	0.000094	0.000254
BOEING	0.000403	0.000337	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000522	0.000278
M	0.001636	0.000587	0.000091
FST	0.001891	0.000637	0.000109
CVS	0.000293	0.000232	-0.000025
AMD	0.000523	0.000741	-0.000085
Total	N/A	N/A	0.001947



### 3.4.4 CAPM and BofAML Yield (C&BY)

Chart 18: Optimal Weights of C&BY

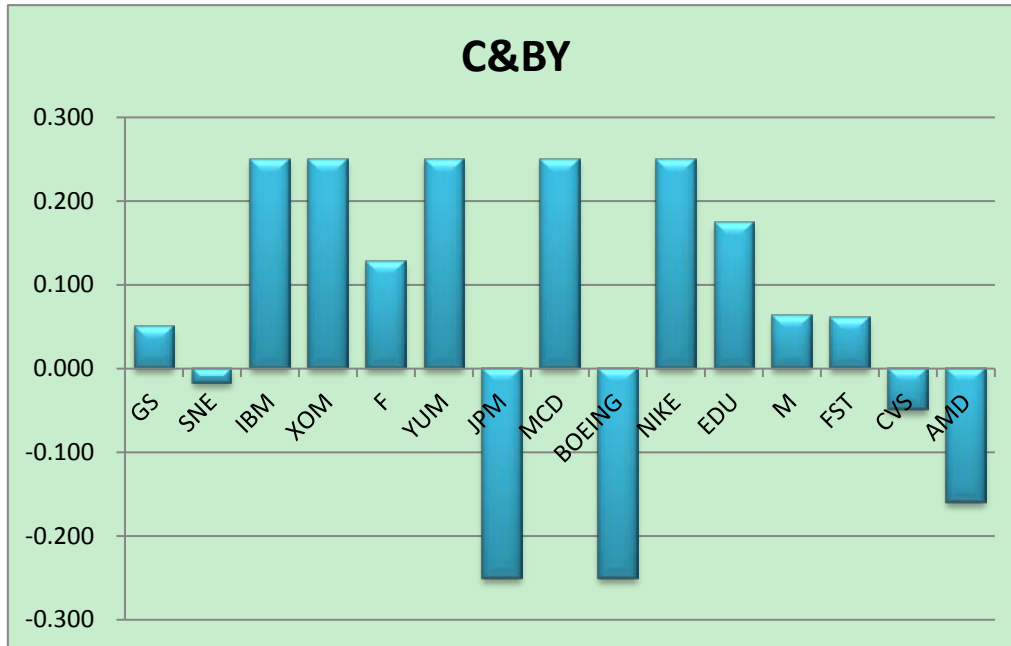


Table 17: Weights, Capital and Profit of C&BY Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.050540	25270.17	-36.20%	-9148.73
SNE	-0.017747	-8873.58	-42.36%	3758.71
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.127848	63924.20	-32.29%	-20641.03
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.250000	-125000.00	-18.52%	23148.15
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.174182	87090.82	12.40%	10799.66
M	0.063090	31545.12	21.20%	6687.21
FST	0.061201	30600.28	-69.66%	-21315.88
CVS	-0.049249	-24624.50	5.12%	-1261.15
AMD	-0.159865	-79932.51	-31.17%	24914.03
Total	1	500000	N/A	118167.67

Table 18: Estimated Returns and Return Variance of C&BY

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000380	0.000029
SNE	0.000394	0.000366	-0.000007
IBM	0.000799	0.000124	0.000200
XOM	0.000709	0.000131	0.000177
F	0.001947	0.000554	0.000249
YUM	0.001925	0.000203	0.000481
JPM	0.000246	0.000361	-0.000062
MCD	0.001018	0.000094	0.000254
BOEING	0.000403	0.000336	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000518	0.000287
M	0.001636	0.000584	0.000103
FST	0.001891	0.000634	0.000116
CVS	0.000293	0.000231	-0.000014
AMD	0.000523	0.000738	-0.000084
Total	N/A	N/A	0.001947

### 3.4.5 French & Fama and BAML Yield (FF&BY)

Chart 19: Optimal Weights of FF&BY

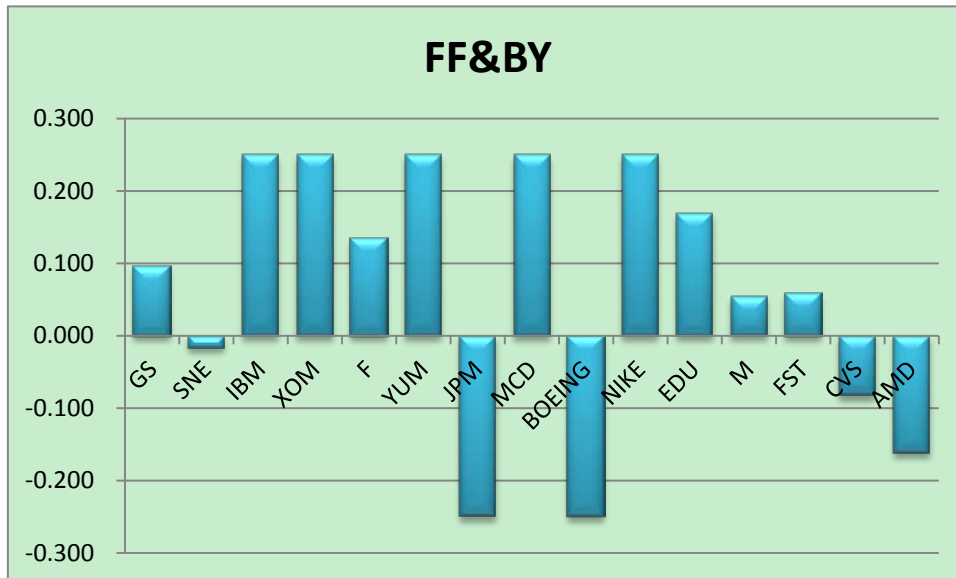


Table 19: Weights, Capital and Profit of FF&amp;BY Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.095104	47551.98	-36.20%	-17215.56
SNE	-0.016741	-8370.48	-42.36%	3545.61
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.134863	67431.52	-32.29%	-21773.54
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.250000	-125000.00	-18.52%	23148.15
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.169305	84652.70	12.40%	10497.32
M	0.054208	27103.88	21.20%	5745.72
FST	0.058167	29083.48	-69.66%	-20259.30
CVS	-0.082878	-41439.05	5.12%	-2122.31
AMD	-0.162028	-81014.02	-31.17%	25251.12
Total	1	500000	N/A	108043.92

Table 20: Estimated Returns and Return Variance of FF&amp;BY

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000382	0.000054
SNE	0.000394	0.000368	-0.000007
IBM	0.000799	0.000125	0.000200
XOM	0.000709	0.000131	0.000177
F	0.001947	0.000556	0.000263
YUM	0.001925	0.000204	0.000481
JPM	0.000246	0.000361	-0.000062
MCD	0.001018	0.000094	0.000254
BOEING	0.000403	0.000337	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000522	0.000279
M	0.001636	0.000587	0.000089
FST	0.001891	0.000637	0.000110
CVS	0.000293	0.000232	-0.000024
AMD	0.000523	0.000741	-0.000085
Total	N/A	N/A	0.001947

### 3.4.6 CAPM, Federal Fund Rate and BAML Yield (C&FFR&BY)

Chart 20: Optimal Weights of C&FFR&BY

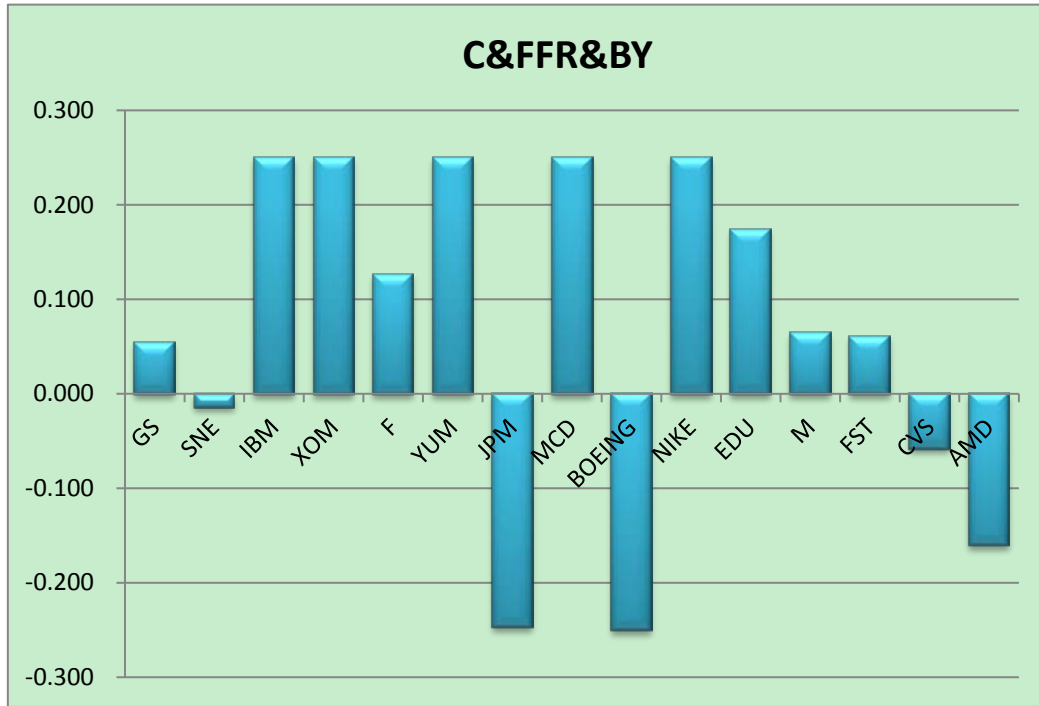


Table 21: Weights, Capital and Profit of C&FFR&BY Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.054472	27235.84	-36.20%	-9860.38
SNE	-0.014567	-7283.49	-42.36%	3085.17
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.126801	63400.26	-32.29%	-20471.85
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.247314	-123657.19	-18.52%	22899.48
MCD	0.250000	125000.00	23.95%	29942.84
BA	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.173929	86964.74	12.40%	10784.02
M	0.064407	32203.72	21.20%	6826.83
FST	0.060680	30340.19	-69.66%	-21134.71
CVS	-0.057963	-28981.59	5.12%	-1484.30
AMD	-0.160445	-80222.49	-31.17%	25004.41
Total	1	500000	N/A	116875.39

Table 22: Estimated Returns and Return Variance of C&FFR&BY

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000381	0.000031
SNE	0.000394	0.000367	-0.000006
IBM	0.000799	0.000124	0.000200
XOM	0.000709	0.000131	0.000177
F	0.001947	0.000555	0.000247
YUM	0.001925	0.000203	0.000481
JPM	0.000246	0.000361	-0.000061
MCD	0.001018	0.000094	0.000254
BA	0.000403	0.000336	-0.000101
NIKE	0.001274	0.000223	0.000318
EDU	0.001647	0.000520	0.000286
M	0.001636	0.000585	0.000105
FST	0.001891	0.000635	0.000115
CVS	0.000293	0.000232	-0.000017
AMD	0.000523	0.000740	-0.000084
Total	N/A	N/A	0.001947

### 3.4.7 French & Fama, Federal Fund Rate and BAML Yield (FF&FFR&BY)

Chart 21: Optimal Weights of FF&FFR&BY

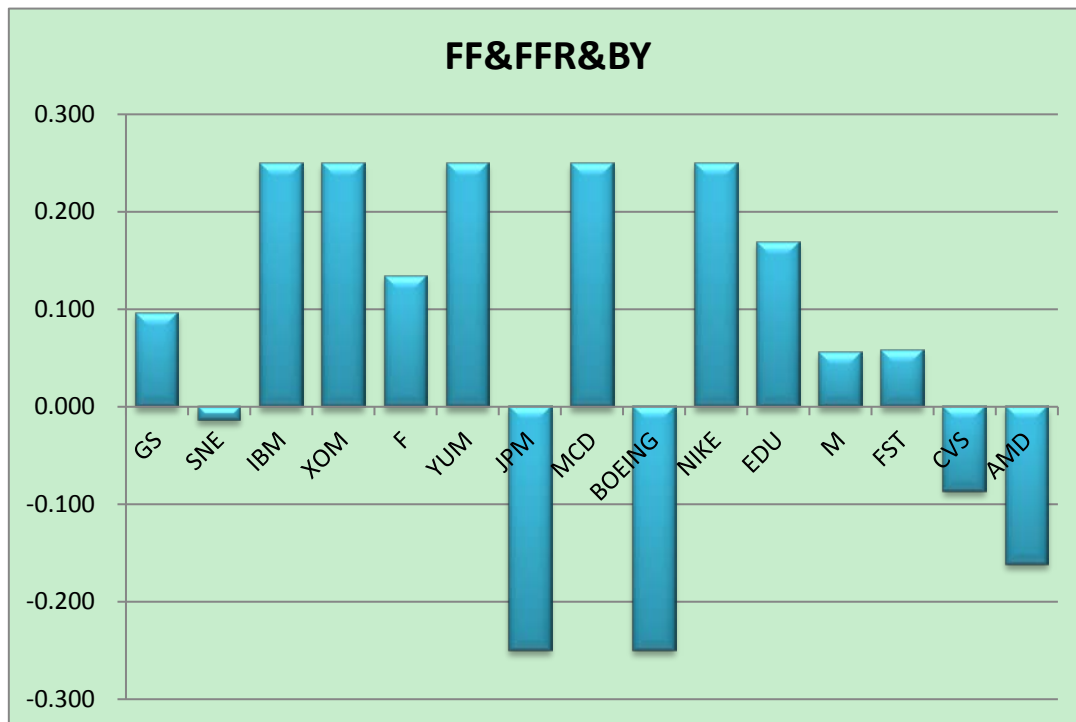


Table 23: Weights, Capital and Profit of FF&amp;FFR&amp;BY Model

Stock (symbol)	Optimal Weights	Asset's Capital	Profit rate	Profit
GS	0.096468	48233.95	-36.20%	-17462.46
SNE	-0.014189	-7094.38	-42.36%	3005.07
IBM	0.250000	125000.00	26.79%	33493.07
XOM	0.250000	125000.00	6.61%	8256.66
F	0.133820	66909.99	-32.29%	-21605.14
YUM	0.250000	125000.00	11.35%	14186.24
JPM	-0.250000	-125000.00	-18.52%	23148.15
MCD	0.250000	125000.00	23.95%	29942.84
BOEING	-0.250000	-125000.00	0.86%	-1080.08
NIKE	0.250000	125000.00	13.14%	16427.98
EDU	0.169112	84556.21	12.40%	10485.35
M	0.055730	27865.10	21.20%	5907.09
FST	0.057705	28852.30	-69.66%	-20098.26
CVS	-0.086835	-43417.71	5.12%	-2223.65
AMD	-0.161811	-80905.47	-31.17%	25217.29
Total	1	500000	N/A	107600.15

Table 24: Estimated Returns and Return Variance of FF&amp;FFR&amp;BY

Stock(symbol)	Estimated Returns	Return Variance	Weighted Returns
GS	0.000566	0.000383	0.000055
SNE	0.000394	0.000369	-0.000006
IBM	0.000799	0.000125	0.000200
XOM	0.000709	0.000132	0.000177
F	0.001947	0.000557	0.000261
YUM	0.001925	0.000204	0.000481
JPM	0.000246	0.000362	-0.000062
MCD	0.001018	0.000095	0.000254
BOEING	0.000403	0.000337	-0.000101
NIKE	0.001274	0.000224	0.000318
EDU	0.001647	0.000524	0.000278
M	0.001636	0.000588	0.000091
FST	0.001891	0.000638	0.000109
CVS	0.000293	0.000233	-0.000025
AMD	0.000523	0.000743	-0.000085
Total	N/A	N/A	0.001947

### 3.4.8 Actual Return, Return Variance and Covariance Return for Holding period from 01/01/2011 to 10/31/2011

Table 25: Actual Return, Return Variance

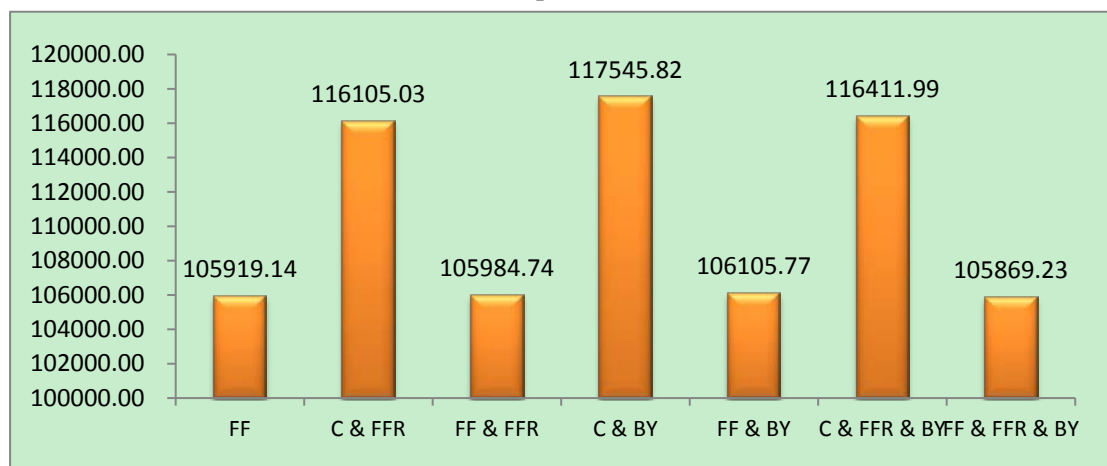
Stocks(symbol)	Actual Return	Return Variance
GS	-0.001747	0.000511
SNE	-0.002309	0.000446
ibm	0.001252	0.000195
XOM	0.000525	0.000258
ford	-0.001407	0.000634
YUM	0.000649	0.000268
JPM	-0.000556	0.000583
MCD	0.001071	0.000118
BOEING	0.000305	0.000362
NIKE	0.000821	0.000393
EDU	0.001106	0.001082
Macy's	0.001213	0.000548
FST	-0.004539	0.001881
CVS	0.000379	0.000207
AMD	-0.001014	0.001207

## 3.5 Comparison

### 3.5.1 Profit Comparison

We formed our positions on 01/03/2011 and closed our positions on 10/31/2011. We compared the total profits of 7 factor models. From Chart 22, the C&BY model was better than the others because it obtained the highest profit. FF&FFR&BY was the worst, because it obtained the lowest profit.

Chart 22: Profit Comparison of Factor Models



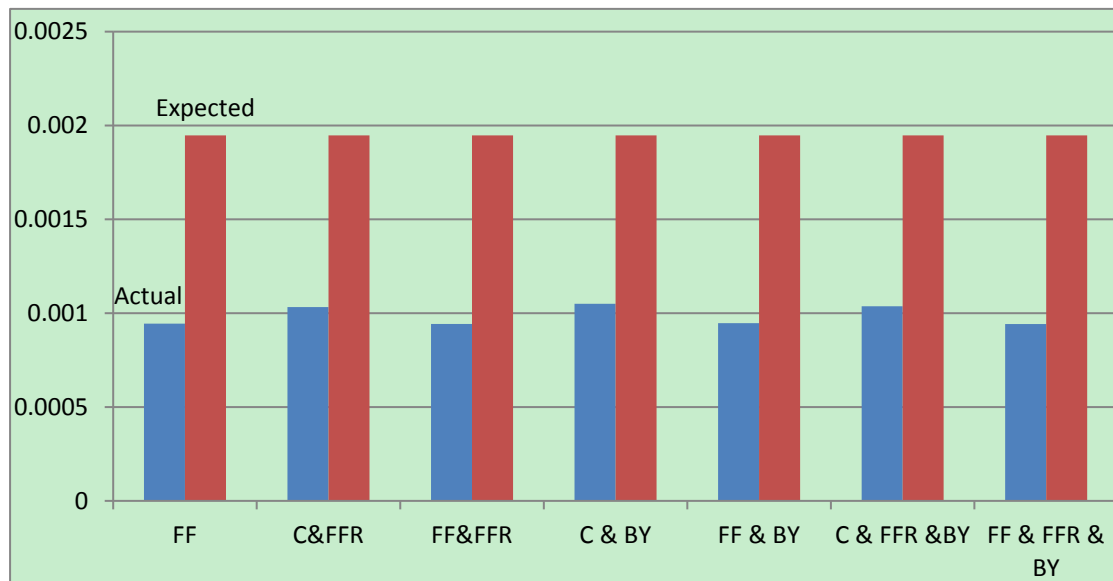
### 3.5.2 Portfolio Return Comparison

We compared the actual portfolio returns and the expected portfolio returns of the 7 models. From Table 26 and Chart 23, we see that C&BY was also the best because its expected portfolio return was closer to its actual portfolio return. The FF model was the worst because the difference between its actual portfolio return and expected portfolio return was larger than the others.

Table 26: Comparing Actual Portfolio Return with Expected Portfolio Return

Factor Model	Actual Profolio Return	Expected Portfolio Return
FF	0.00094433	0.001947
C&FFR	0.00103266	0.001947
FF&FFR	0.00094238	0.001947
C&BY	0.00105001	0.001947
FF&BY	0.00094704	0.001947
C&FFR&BY	0.00103675	0.001947
FF&FFR&BY	0.00094224	0.001947

Chart 23: Comparing Actual Portfolio Return with Expected Portfolio Return





### **3.6 Conclusion**

First, the French & Fama factor model and CAPM had the greatest influence in deciding the portfolio's returns. According to Table 26, all of models that included the French & Fama factor had lower profits than all models that included CAPM.

Second, we concluded that the Federal Fund Rate Factor had less influence on the assets' returns than the other factors. For example, the actual portfolio return of CAPM with the BAML Yield was 0.00105001, very close to that of CAPM with the BAML Yield and the Federal Fund Rate, which was 0.00103675. Adding the Federal Fund Rate Factor only made a slight difference for the actual portfolio's return.

Third, from Table 26 we see that the Fund Federal Rate factor had a negative effect, in that adding this factor led to a decrease in the portfolio return. For example, the actual portfolio return of FF&BY was 0.00094704, and the actual portfolio return of FF&FFR&BY was 0.00094224.

Fourth, we found that adding more factors did not lead to better results. For instance, the FF&FFR&BY model, which had five factors, was not the best factor model according to our analysis above.

Finally, from Chart 23 we found that there were big differences between actual portfolio returns and expected portfolio returns. The main reason was that our multiple-factor models were based on a significant assumption that returns of assets were stationary; however, in the real world such assumptions are not typically satisfied.

## 4. References

### References - Literature

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2. *Statistics and Finance An Introduction*, David Ruppert, Springer, Corrected as of the 2nd printing, 2006
3. <http://www.interactivebrokers.com/ibg/main.php> Interactive brokers LLC
4. <http://research.stlouisfed.org/fred2/> Federal Reserve Economic Data
5. <http://www.newyorkfed.org/markets/omo/dmm/fedfundsdata.cfm> Federal Funds rate.
6. Matlab R2010a, the Mathworks
7. *Statistical Inference*, George Casella, Roger L. Berger, 2nd edition, June 18, 2001
8. <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html> CAPM and French & Fama models choice

## Appendix

MATLAB code

Portfolio Optimization Project:

```
% Use function optimalWeights to plot an efficient frontier for n risky
% assets
% Read the historical adjust prices of 10 stocks,

data1=xlsread('weekly.xlsx','B2:P41');           % historical data

% Calculate the log return of the adjust closed price
LogReturn=logreturn(data1);

% Calculate the Expected Return of the histrocall data
mu=mean(LogReturn)';

% Calculate the Covariance Matrix of the histrocall data
CovMatrix=cov(LogReturn);

% Calculate the Efficient Frontier for 1 year
n=length(mu);
Aeq=[ones(1,n);mu'];
ngrid = 100 ;
muP = linspace(0,2*max(mu),ngrid) ;           % muP grid
rf = 0.0398/52;
omegaP = zeros(n,ngrid) ;
sigmaP=zeros(1,100);
LB=-0.5*ones(n,1);
UB=0.5*ones(n,1);

for ii = 1:ngrid ;
    omegaP(:,ii) = quadprog(CovMatrix,zeros(n,1),[],[],Aeq,[1;muP(ii)],LB,UB) ;
    sigmaP(ii) = sqrt(omegaP(:,ii)'*CovMatrix*omegaP(:,ii)) ;
end ;

imin=find(sigmaP==min(sigmaP)) ;
Ieff = (muP >= muP(imin)) ;
sharperatio = (muP-rf) ./ sigmaP ;
Itangency = find(sharperatio == max(sharperatio));

% Return the weight of these stocks
Position=500000*omegaP(:,Itangency)
```

```

% Creat figure 1 about efficient frontier
figure(1)
clf;
fsize = 16;
fsize2 = 10;
p =
plot(sigmaP(Ieff),muP(Ieff),sigmaP(Itangency),muP(Itangency),'*',sigmaP(imin),muP
(imin),'.',0,rf,'x') ;
l1=line([0 sigmaP(Itangency)],[rf,muP(Itangency)]);
t1= text(sigmaP(imin),muP(imin),' Minimum variance portfolio','fontsize',fsize2) ;
t2= text(sigmaP(Itangency),muP(Itangency),' Tangency potfolio','fontsize',fsize2) ;
t3=text(0,rf,' Risk-free asset','fontsize',fsize2) ;
% Change line widths, marker sizes, and colors for better apperance
set(p,'markersize',12);
set(p,'linewidth',3);
set(p(2), 'color', 'yellow');
set(p(3),'markersize',20);
set(p(3), 'color', 'red');
set(l1,'linewidth',3) ;
set(l1,'linestyle','--') ;
set(l1, 'color', 'black') ;
% Lable Axes
xlabel('standard deviation of return','fontsize',fsize) ;
ylabel('expected return','fontsize',fsize) ;
grid;

% Creat figure 3 about the bar of weight in the Tangency variance
% portfolio
figure(2)
bar(omegaP(:,Itangency));
% Lable Axes
xlabel('Asset','fontsize',fsize) ;
ylabel('weight in the Tangency portfolio','fontsize',fsize) ;

function r = logreturn(Price)
% Input m*n Price matrix
% Output of (m-1)*n logreturn Matric
%
logprice=log(Price);
[M,N]=size(Price);
r=zeros(M-1,N);
for i=1:M-1

```

```

    r(i,:)=logprice(i,:)-logprice(i+1,:);
end
end

CAPM Project:
clear all
SP500=xlsread('capmproject.xlsx','sheet1','q2:q61');
Stocks=xlsread('capmproject.xlsx','sheet1','b2:p61');
Weights=xlsread('capmproject.xlsx','sheet1','b75:p75');
Mstocks=mean(Stocks);%input the expected return vectors
Msp500=mean(SP500);
rf=0.000002;
riskMkt=var(SP500);
for jj=1:15

beta(jj)=((Stocks(:,jj)-Mstocks(jj))*(SP500-Msp500))/((SP500-Msp500)*(SP500-Ms
p500));% estimate beta
end
for ii=1:15 % estimate the epsilon

Residual(:,ii)=Stocks(:,ii)-rf*ones(length(SP500),1)-beta(ii)*(SP500-rf*ones(length(S
P500),1));
    sigmaEpsilon(:,ii)= (Residual(:,ii)* Residual(:,ii))/(length(SP500)-1-1);
end
% risk of each asset
sigmaAsset=riskMkt*(beta.^2)+sigmaEpsilon
% epsilon
Epsilon=Residual;
PorfBeta=Weights*beta';
PorfEpsilon=Weights*Epsilon';
PorfRisk=Weights*sigmaAsset';

Factor Model Project:
clear all
y=xlsread('FactorModelProject.xlsx','NetReturn','b2:p215');
X=xlsread('FactorModelProject.xlsx','CapmBofa','a2:c215');
InitialCapital=500000;
rf=0.0000079; % Risk free rate
X(:,1)=ones(length(y(:,1)),1); % first column of Factor Matrix should be 1
y=y-rf;
ExpectFactor=mean(X);
for ii=1:15

```

```

    EstimateBeta(:,ii)=regress(y(:,ii),X);% get the least square estimator beta
end
ExpectAssetReturn=ExpectFactor*EstimateBeta+rf;
CovFactor=cov(X(:,2:length(X(1,:))));
yfit=X*EstimateBeta;
Epsilon=y-yfit;
EstimateEpsilon=(diag(diag(Epsilon'*Epsilon)))/(length(X(:,1))-length(X(1,:)));
CovReturnMatrix=(EstimateBeta(2:length(X(1,:)),:))*CovFactor*EstimateBeta(2:length(X(1,:),:))+EstimateEpsilon;
[optimalWeights,optimalInvestment]=optimalPorfolio(ExpectAssetReturn,CovReturnMatrix,rf,InitialCapital);
ExpectPorfolioReturn=ExpectAssetReturn*optimalWeights;
figure(1)
bar(optimalWeights);
ExpectPortfolioReturn=ExpectAssetReturn*optimalWeights;

```

```

function[optimalWeights,optimalInvestment]=optimalPorfolio(expReturns,CovMatrix,rf,InitialCapital)
%expReturns=(mean(date))';%input the expected return vectors
%CovMatrix=cov(date); input the covariance matrix
mup=linspace(min(expReturns),max(expReturns),100);
Aeq=[ones(1,length(expReturns));expReturns];
omegap=zeros(length(expReturns),100);
sigmap=zeros(length(mup),1);
for ii=1:100
    omegap(:,ii)
=quadprog(CovMatrix,zeros(length(expReturns),1),eye(length(expReturns)),0.25*ones(length(expReturns),1), Aeq,[1;mup(ii)]);
    sigmap(ii)=(omegap(:,ii)'*CovMatrix*omegap(:,ii))^0.5;
end
imin=find(sigmap==min(sigmap));
ieff=find(mup>=mup(imin));
sharperatio=(mup-rf)./sigmap';
itangency=find(sharperatio==max(sharperatio))
optimalWeights=omegap(:,itangency);
optimalInvestment=InitialCapital*optimalWeights;
end

```