Worcester Polytechnic Institute
Digital WPI

Masters Theses (All Theses, All Years)

Electronic Theses and Dissertations

2015-03-20

Refining Learning Maps with Data Fitting Techniques

Seth Akonor Adjei
Worcester Polytechnic Institute

Refining Learning Maps with Data Fitting Techniques

By

Seth Akonor Adjei

A Thesis

Submitted to the Faculty

Of

WORCESTER POLYTECHNIC INSTITUTE

In partial fulfillment of the requirements for the

Degree of Master of Science

In

Computer Science

____

March 2014

APPROVED:

____
Dr. Neil T. Heffernan
Advisor – WPI

____
Dr. Joseph Beck
Reader – WPI

____
Professor Craig Wills
Head of Computer Science Department

Abstract

Learning maps have been used to represent student knowledge for many years. These maps are usually hand made by experts in a given domain. However, these hand-made maps have not been found to be predictive of student performance. Several methods have been proposed to find better fitting learning maps. These methods include the Learning Factors Analysis (LFA) model and the Q-matrices.

In this thesis I report on the application of one of the proposed operations in the LFA method to a small section of a skill graph and develop a greedy search algorithm for finding better fitting models for this graph. Additionally an investigation of the factors that influence the search for better data fitting models using the proposed algorithm is reported. I also present an empirical study in which PLACEments, an adaptive testing system that employs a skill graph, is modified to test the strength of prerequisite skill links in a given learning map and propose a method for refining learning maps based on those findings.

It was found that the proposed greedy search algorithm performs as well as an original skill graph but with a smaller set of skills in the graph. Additionally it was found that, among other factors, the number of unnecessary skills, the number of items in the graph, and the guess and slip rates of the items tagged with skills in the graph have an impact on the search. Further, the size of the evaluation data set impacts the search. The more data there is for the search, the more predictive the learned skill graph.

Additionally, PLACEments, an adaptive testing feature of ASSISTments, has been found to be useful for refining skill graphs by detecting the strengths of prerequisite links between skills within a graph.

Acknowledgements

I wish to give the greatest of thanks to God for assisting me to complete this thesis. Without him, I could not have done this.

I wish to acknowledge the understanding of my wife, Esther and my children Bryan and Kayla Adjei. They have been tremendous in their support of my work. I know I have been away many times, but they have all shown their understanding as we have gone through these stressful times.

Professor Neil Heffernan, my advisor, is one professor in the Computer Science (CS) department who deserves mention in this acknowledgement. I appreciate his understanding of my circumstances and his willingness to assist even when the going was tough.

I cannot but mention Corey Belhemeur and Skyler Whorton, former students of WPI, for initiating the development of PLACEments. This is one of the tools that has been very useful in the studies I have conducted so far and which I report in this thesis. I acknowledge the assistance of Douglas Selent, a colleague graduate student in the Computer Science department as well as Bohao Li, an undergraduate student also in the Computer Science Department for their assistance in developing tools for this thesis.

Table of Contents

List of Figures

List of Tables

Chapter 1: Introduction

Several learning maps, representing a set of skills in a domain and the prerequisite skill relationship between the skills, have been developed to represent what a person knows. The learning maps are sometimes called skill graphs or prerequisite skill graphs (in view of this these three terms will be used interchangeably). Many of these learning maps are hand designed by experts in the domain for which the map is made. However, can we detect any defects in these learning maps with data collected from students? Can these defects be removed? What methods are effective for determining these defects and for improving the search? The Learning Factors Analysis method has been proposed to answer some of these questions, however, not much has been reported on its effectiveness. Additionally it does not present any information the strength of the prerequisite skill links. In this thesis an attempt has been made to answer these questions.

This thesis presents a number of approaches that I have taken to solve this problem. The approaches vary from the use of data-mining techniques to empirical studies performed to detect problems with skill graphs. One of the approaches proposed is a greedy search algorithm that applies a portion of the Learning Factors Analysis method to the search. The other approach involves the use of an adaptive testing system to augment the search for more predictive data fitting prerequisite skill graphs, or learning maps.

The thesis is organized as follows: The first chapter presents a description of the main problem that my work has attempted to solve. Chapter 2 presents one approach that was used to refine a leaning map. The approach involves the use of a greedy search algorithm together with Bayesian networks to improve learning maps. Chapter three presents a set of factors that need to be considered if the algorithm proposed and used in Chapter 2 is used to search for learning maps. In other words, the factors that the algorithm is susceptible to are presented and analyzed. In Chapter 3, a completely different approach is presented. This approach uses an adaptive testing system, PLACEments (Belhumeur, 2013), which was developed in ASSISTments for testing students' knowledge of the certain skills in a given domain. We report on a few modifications of this tool and how it was used to detect parts of a skill graph that needs improvement.

Chapter 2: Refining Learning Maps with Data Fitting Techniques: Searching for Better Fitting Learning Maps

The learning sciences need quantitative methods for comparing alternative theories of what students are learning, represented as learning maps. This study investigated the accuracy of a learning map and its utility to predict student responses. Our data included a learning map detailing a hierarchical prerequisite skill graph and student responses to questions developed specifically to assess the concepts and skills represented in the learning map. Each question was aligned to one skill in the map, and each skill had one or more prerequisite skills. Our research goal was to test if the knowledge representation in the learning map could be improved upon in an iterative way. To this end we applied a greedy iterative search algorithm to simplify the learning map by merging nodes together. Each successive merge resulted in a model with one skill less than the previous model. We share the results of the revised model, its reliability, reproducibility, and discuss the validity of the most significant merges.

Portions of this work were published at the following venue:

Adjei, S. A., Selent, D., Heffernan, N. T., Broadus, A, Kingston, N. (2014) Refining Learning Maps with Data Fitting Techniques: Searching for Better Fitting Learning Maps. In Stamper, J., Pardos, Z., Mavrikis, M., McLaren, B.M. (eds.) Proceedings of the 7th International Conference on Educational Data Mining pp. 413-414

Introduction

Cognitive models are used to represent how a person's knowledge may be organized (Gierl, Wang, & Zhou, 2008). They contain descriptions of component pieces of knowledge and connections among the components to indicate how understanding develops in a specified domain (Gierl, Wang, & Zhou, 2008). Different authors have described various cognitive models, including learning maps (Popham, 2011), learning trajectories (Clements & Sarama, 2004), and learning hierarchies (Gagné, 1968). Learning maps use linear sequences of learning goals and are useful for instructional planning (Popham, 2011). A learning trajectory includes a learning goal, a developmental progression defining the levels of thinking that students pass through as they work toward the defined goal, and a set of learning activities or experiences that assist students in reaching the defined goal (Clements & Sarama, 2004). As their name implies, learning hierarchies model prerequisite knowledge components in hierarchies, allowing multiple pathways to extend from one prerequisite skill to multiple learning goals (Gagné, 1968).

The learning map extends the notion of a learning hierarchy by representing domain knowledge as a network of component skills and connections, allowing for multiple paths from prerequisites to learning goals. While multiple paths add complexity to the cognitive model, they allow the learning map to represent the potential learning of a broad range of individuals who may experience difficulties traversing certain pathways due to disabilities or particular learning preferences. As such, the learning map provides a flexible model of learning that is consistent with recent advances in universal design for learning (Center for Applied Special Technology [CAST], 2012).

In the present study we examine a small section of a learning map and investigate the effects of permuting the topology of the hierarchy. Skills and concepts are represented by latent nodes in the learning map, while directed edges represent the prerequisite relationship among latent nodes and between those nodes and their associated test items. We present a simple method for improving the predictive power of the learning map by combining latent nodes, our initial results on the fit improvement, stability of the resulting map, and interpretation of the algorithms chosen node combinations.

This work connects with literature on searching for better fitting cognitive models. Several non-hierarchical cognitive models have been developed to represent the relationship between knowledge components (KCs) in the form of prerequisite skill maps. These cognitive models have been developed to help intelligent tutors, as well as experts, determine student mastery of KCs. A number of technical approaches have been developed to evaluate cognitive models developed by domain experts. One approach is Learning Factors Analysis (LFA), developed by Cen, Koedinger and Junker (2006) to help the Educational Data Mining (EDM) community evaluate different cognitive models.

There are several different methods for analyzing skills. Tatsuoka (1983) introduced the rule space method for representing and determining how well students understood the underlying skills (or rules as termed in the paper) for test items. Additionally, the method is used to identify any erroneous classification or misconceptions of students in responding to test items. Barnes (2005) utilized the Q-matrix method from Tatsuoka's rule space method to organize combinations of skills into distinct latent classes and assign students to latent classes based on level of mastery. Additive Factor Models (AFM) also utilize the Q-matrix but with a multiple logistic regression model which predicts student performance based on a number of factors, primarily the number of opportunities a students has to demonstrate a particular skill. However Cen (2009) reported that AFM did not accurately predict items involving conjunctive skills and hence introduced the Conjunctive Factor Model (CFM) to improve predictions in this area. In addition to latent skill cognitive models, item to item knowledge structures have also been learned from empirical data using Bayesian Network structure learning and partial order knowledge structures (Desmarais, Gagnon 2006).

Our approach to simple merging of skills was inspired by Learning Factors Analysis, which uses a combinatorial search to determine which model best fits student data. The combinatorial search consists of three different types of operations: splitting, merging, or adding existing KCs. Splits occur when a knowledge component is determined to be composed of more than one skill, and hence splits into multiple skills. One or more skills are merged if they are determined to be inseparable skills, given student data. The add operation involves the inclusion of a completely new skill to the original map (Cen, 2009).

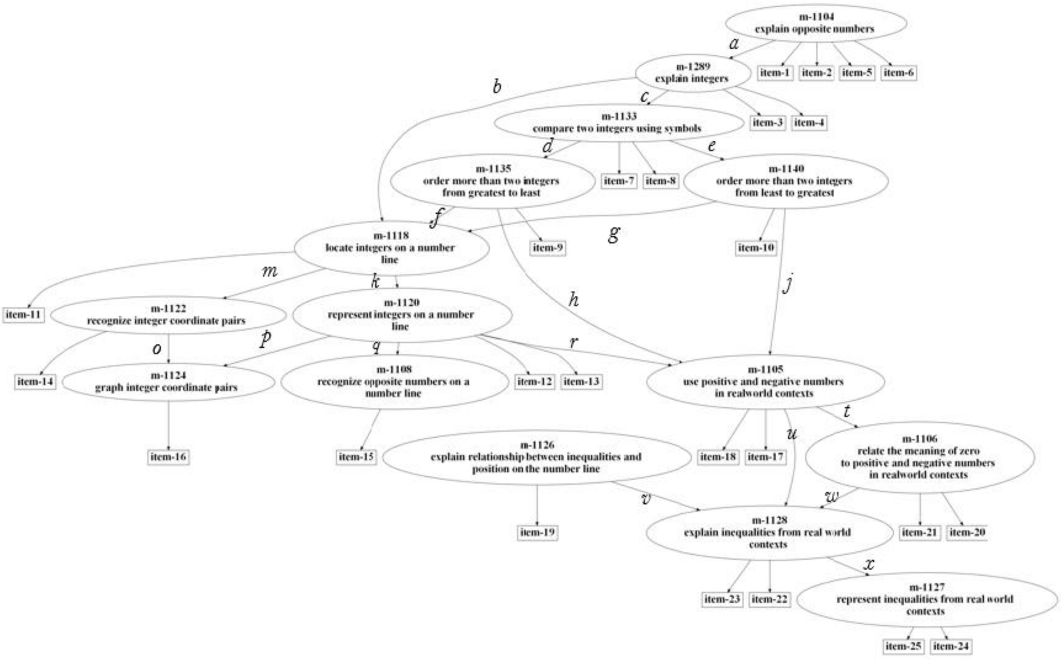
Other researchers have tried to extend LFA to other subject domains. Leszczenski and Beck (2007) introduced a scalable application of the LFA framework in the context of reading knowledge transfer. The problem with this approach is that the search was unstable and could give different results each time the search was run. Instead of determining a student model given an initial human generated model, Li, Cohen, Noboru, and Koedinger (2011) proposed a method for automatically generating the KCs from student responses to individual items. Although their method resulted in the best fit among the other candidates, it may not generalize for models with less coarse grained KCs.

Other models have focused on the determination of a student's knowledge of certain skills. Logistic regression has been used to trace multiple sub-skills of a given skill (Xu and Mostow, 2011). Pavlik, Cen, and Koedinger (2009) proposed a method for automatically deriving a cognitive model by generating a Q-matrix, which provides a representation of the KCs required for each test item.

In this work we follow the combinatorial search described by Cen, Koedinger and Junker (2006). This technique can be used to analyze hypothesized learning maps and consider whether small improvements to the model result in a better fit to the data. Cen, Koedinger, and Junker (2006) suggested three types of operations, i.e., merges, splits, and adds. However, in this study, we used only merge operations because the initial learning map had high granularity.

Initial Learning Map

This study examined a section of the learning map containing 15 concepts and skills related to understanding integers. The map was developed using educational literature describing how students learn to understand and operate with integers. The set of integers includes whole numbers and their opposites, presenting many students their first exposure to negative numbers (Van de Walle, Bay-Williams, Karp, & Lovin, 2014). Although many students have prior knowledge of negative values within contexts such as debt or temperatures below freezing, they often struggle when first learning to work with negative numbers. Proficiency with integers includes understanding opposite numbers, comparing integers, representing integers on number lines and graphs, and using integers in real world problem contexts. The learning map shown in Figure 1 illustrates the component concepts and skills that comprise such understanding. This map suggests that students should learn to identify opposite numbers (M-1104) and integers (M-1289) in preparation for comparing and ordering integers (M-1133, M-1135, M-1140) as well as representing integers on number lines (M-1118, M-1120, M-1108, M-1126) and coordinate planes (M-1122, M-1124). Because integers challenge the initial counting strategies students learned for positive numbers, it is beneficial for students to work with integers in real-world contexts (M-1106, M-1105, M-1127, M-1128) (Van de Walle, Bay-Williams, Lovin, & Karp, 2014)


m-1104
explain opposite numbers
a
m-1289
explain integers
item-1
item-2
item-5
item-6
b
c
m-1133
compare two integers using symbols
item-3
item-4
d
e
m-1135
order more than two integers
from greatest to least
item-7
item-8
m-1140
order more than two integers
from least to greatest
f
g
m-1118
locate integers on a number
line
item-9
item-10
m
k
j
h
item-11
m-1122
recognize integer coordinate pairs
m-1120
represent integers on a number
line
o
p
q
r
item-14
m-1124
graph integer coordinate pairs
m-1108
recognize opposite numbers on a
number line
item-12
item-13
m-1105
use positive and negative numbers
in realworld contexts
t
u
item-16
item-15
m-1126
explain relationship between inequalities and
position on the number line
item-18
item-17
m-1106
relate the meaning of zero
to positive and negative numbers
in realworld contexts
v
w
item-19
m-1128
explain inequalities from real world
contexts
item-21
item-20
x
item-23
item-22
m-1127
represent inequalities from real world
contexts
item-25
item-24

.

Figure 1: The initial learning map that researchers created. Each ellipse represents a "skill" and each rectangle represents a test item. For easy reference, the links are labeled. The labels do not have any specific meaning.

The data for this study was gathered from student responses to 25 test items aligned to the 15 skills shown in the learning map in Figure 1. All of the test items were multiple choice questions, with four answer options per question. Each skill was assessed by at least one test item, in many cases two items. As part of the test development process, subject matter experts confirmed the alignment of each item to its associated skill, meaning that the item was judged by experts to evoke the intended skill. Therefore, when a student answered a test item correctly, we assumed in this study that the student had mastered the skill associated with that test item. Furthermore, due to the hierarchical structure of the learning map, items associated with skills lower in the learning map were assumed to be more difficult, i.e., require more skills, than items associated with skills higher in the learning map.

In addition to the graph, we utilized a data-set containing the responses of 2,846 students answering the same sequence of 25 items in the learning map. All the students were chosen from middle schools in a mid-western state from grades 6 (8%), 7 (49%), 8 (39%) and 10 (4%). The students' responses were dichotomous, '1' for correct and '0' otherwise.

Methodology

This section describes the methods used for the study. We describe the merge operation and the evaluation procedure used in the search. We subsequently describe the search algorithm.

Merge Operation

In all the experiments described in this chapter, our sole change made to the map was to merge latent nodes. A merge operation occurred when two skills adjacent to each other in the map were combined into one skill. Items from both skills that were merged were reattached to the new single skill. The prerequisites of the constituent skills became prerequisites of the merged skill and the same applied to the post-requisites. An example of the merge operation on a section of the skill map is shown below in figure 2 and 3. The figures show a skill map before and after the merge operation. M-1289 and M-1133 are the skills that were merged into a single skill, named “M-1289XM-1133”. Note that the names of the skill hold no meaning of their own, just as the labels of the arcs between the skills.

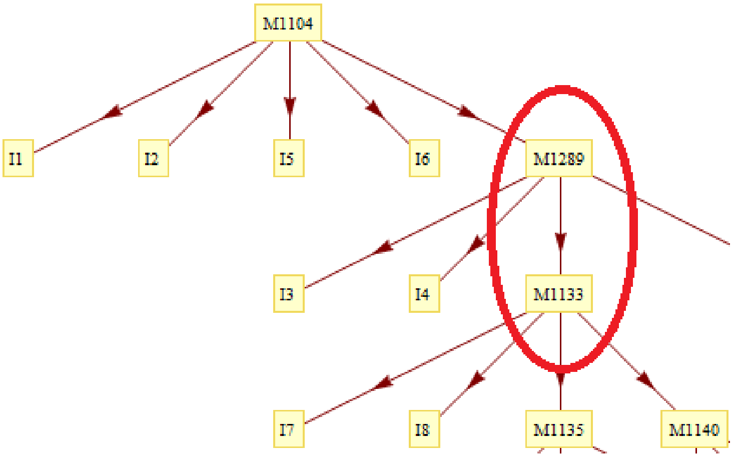


Figure 2 The graph before the merge of skills M1289 and M1133

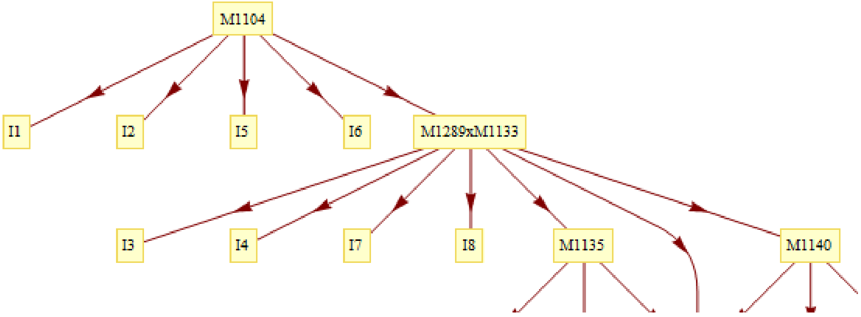


Figure 3 After the merge of the arc between M-1289 and M1133. Note that after the merge, all the items mapped to both M1289 and M1133 are now mapped to the joint skill labeled “M1289xM1133”.

Evaluation Procedure

Each graph that is to be evaluated is modeled as a Bayesian Network of nodes. Each latent node in the network represents a skill in the graph, and the observables represent the items that are tagged with the skills in the graph. Expectation Maximization was used to learn the parameters for the Bayesian Network representation of the skill graph. These parameters were then used to predict unseen student data. We used student-item cross validation with 5 student folds and 3 item folds for evaluating the models. Our student and item folds were chosen randomly for the evaluation. More details about how the cross-validation was done can be found in the technical document[1]. We used the Root Mean Squared Error (RMSE) metric to evaluate the results of the experiments. RMSE is calculated by squaring the differences between each actual value and predicted value and then finding the average value of the

[1] The dataset, evaluation algorithm, and a technical report describing the algorithm in detail can be found at https://sites.google.com/site/assistmentsdata/kansas-project

differences. Taking the square root of the average will give the RMSE value for the model. The closer the RMSE is to 0, the better the model.

Experiment 1: The Search Algorithm

This section describes the greedy search algorithm as well as the experiment that was run to evaluate the algorithm. Using Bayesian networks, a model of an initial skill graph is created. Starting with this original graph, we programmatically find all possible skill pairs that can be merged. We consider merging adjacent skills as described in the section describing the merge operation. Each possible merge is evaluated with the evaluation procedure just described and the best merge is chosen. We apply the best possible merge to the skill graph to create a new skill graph, which will have one less skill than the graph from the previous iteration. The new skill graph is used as the input to the next iteration of the algorithm. This technique is iteratively applied until all the skills are merged into a single skill. Figure 4 is a listing of the algorithm designed for the study.

```
Input: initialGraph, dataSet
Output: bestGraph
Algorithm:
currentIterationBest = initialGraph;
currentBestGraphs = empty;
for noOfNodes = noOfNodesInInitialGraph-1 downto 1;
        currentGraphs = generate all possible graphs with noOfNodes from the currentIterationBest;
        curentIterationBest = select the best from the currentGraphs by fitting each to dataSet;
        currentBestGraphs << currentIterationBest;
```

Figure 4 Greedy Search Algorithm

Additional details of the search algorithm can be found in the technical documents referenced above.

Results and Analysis

Figures 5 and 6 respectively show a table and a graph of the results from the iterative search. The search started at iteration 0, which was the initial skill map consisting of 15 skills before any merges were applied. The search ended at iteration 14, which is a graph consisting of just one skill with all items attached to that one skill. The best models from each iteration are shown below. We recorded the Area under ROC, Root Mean Square Error (RMSE) and accuracy metrics, although we only used RMSE to choose the best models at each iteration and to guide our search. Ultimately, we chose RMSE as the deciding metric since this metric penalizes more for errors in prediction and is a more accurate measure of the goodness of a model.

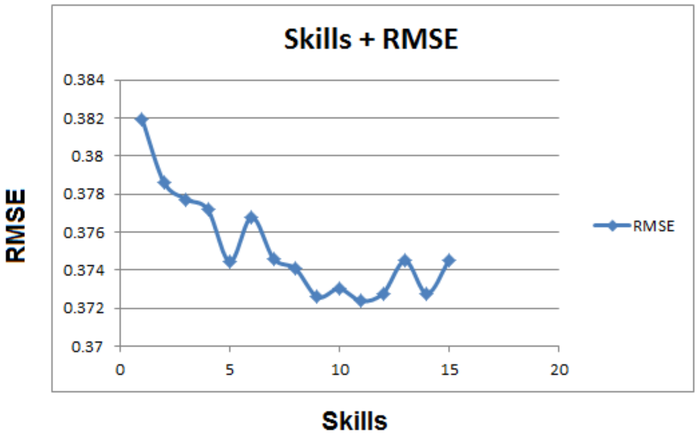


Figure 5 Search Results

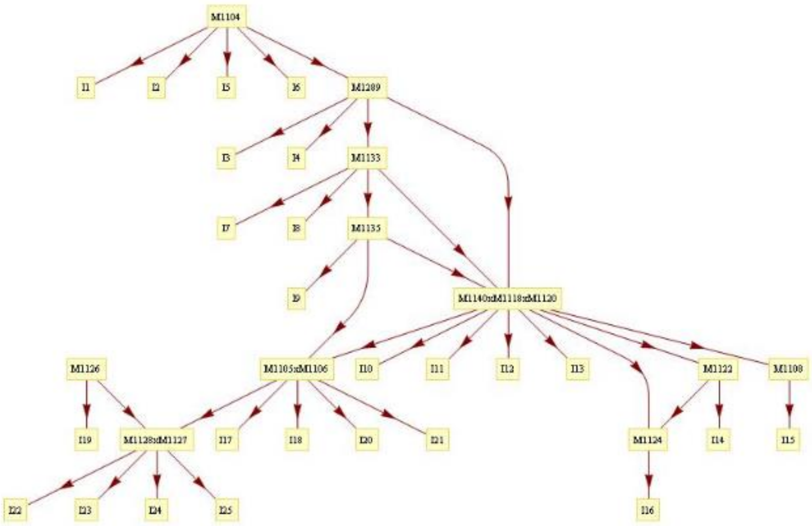


Figure 6 The graph of the best skill model

The results show that the best RMSE obtained was from the 11-skill map (shown in Figure 6) at iteration 4 with an RMSE of 0.37238. This is slightly better than the original skill map with RMSE of 0.37451. The 11-skill map has a small but significant improvement (*p* <= 0.001) from the original skill map. The graph shown in Figure 5 also shows that the models consisting of between 9 and 12 skills have similar RMSE values and are alternative choices for a best model depending on the level of skill granularity desired. Those models are also significantly better than the original model.

In addition to looking at which model best predicted actual responses, we examined which skills were being merged throughout our iterative search to see if we could find any general trends. The individual skills are represented by their original numbers and a merged skill is represented by the numbers of each skill concatenated with an 'x'. The numbering is in topological order, meaning that the skill highest up on the skill map was listed first for a merged skill. The first merge occurred for skills M-1128 and M-1127. Since skill M-1128 was a parent of skill M-1127, it is listed first in the combined skill name M-1128xM-1127. The listing in Figure 7 shows the order in which the merges occurred for all the iterations and the skills involved in the merge for each iterations.

```
Merge 1:  1128 and 1127

Merge 2:  1140 and 1118

Merge 3:  1140x1118 and 1120

Merge 4:  1105 and 1106

Merge 5:  1122 and 1124

Merge 6:  1140x1118x1120 and 1105x1106

Merge 7:  1135 and 1140x1118x1120x1105x1106

Merge 8:  1289 and 1233

Merge 9:  1135x1140x1118x1120x1105x1106 and 1108

Merge 10:  1126 and 1128x1127

Merge 11:  1135x1140x1118x1120x1105x1106x1108 and 1122x1124

Merge 12:  1104 and 1289x1233

Merge 13:  1135x1140x1118x1120x1105x1106x1108x1122x1124 and 1126x1128x1127

Merge 14:  1104x1289x1233 and 1135x1140x1118x1120x1105x1106x1108x1122x1124x1126x1128x1127
```

Figure 7 The Sequence of Merges performed for the best skill graph

One observation is that the skills that are chosen to be merged in the first few iterations tend to be near the bottom of the skill graph. This suggests that the skills near the top of the skill graph are really separate skills compared to the skills near the bottom of the skill graph. Since the skills near the bottom are merged first based on best RMSE, those skills are better predicted with one skill parameter. Therefore those skills are not really distinct, as they are better modeled with one skill parameter compared to skills near the top of the skill graph.

The last skills to be merged are more likely to be distinct skills from the other skills. Our last merge was the merge between the skill group of 1104x1289x1233 and the rest of the skills in the skill graph. Since this was the last merge chosen resulting in the worst RMSE, it is likely that the group of skills 1104x1289x1233 is a separate group from the rest of the skills.

We believe the structure of the initial skill graph also has influence on the distinct skill groups. Since our merge operation only merges adjacent skills, it takes several merges for a skill at the top of the graph to merge with a skill at the bottom of the graph. A separation will naturally occur between the different levels of the graph. This implies that the original skill graph would need to be somewhat correct in terms of network topology. If non-adjacent skills were in fact the same or similar skills, there would be no easy way for our iterative algorithm to merge them, because our merge operation can only merge adjacent skills. Three distinct skill groups can be seen after iteration 12 in a 3-skill graph (See figure 8). These skills show the influence of the original network topology. The skills tend to group by their locations in the original network. The topmost skill group consists of the skills at the top of the graph, the middle group of skills consists of the skills in the middle of the graph, and the bottommost group of skills consists of the skills at the bottom of the original skill graph.

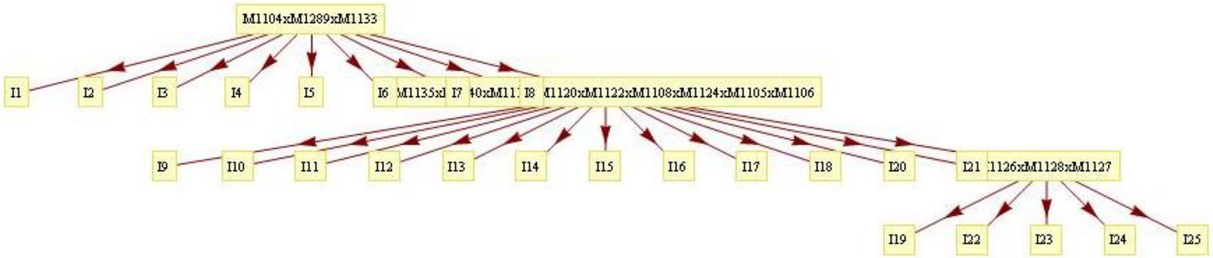


Figure 8 Skill Graph after Merge 12 (3 skills)

An additional observation is that, some of the skills tend to merge by pairing up with one and only one adjacent skill before RMSE starts to decline. Before merge 5, the merges are all pairwise with the exception of merge 3. After merge 5, the skills tend to keep merging into the same skill. The graph generated after merge 4 corresponds to the best skill graph. This suggests that the adjacent skills tend to be similar skills. It also suggests that skills 1140 and 1120 are similar although they are not adjacent. This is a stronger relationship for several reasons. Firstly, the merges that culminated in the merger of M1140, M1118, and M1120 all took place before the best skill graph was reached. This indicates that those three skills give better predictive performance when represented as one skill. Secondly, this was the first 3-skill group to be merged and the only 3-skill group in the best model before RMSE declines. Lastly the three skills took two iterations of the search algorithm to merge together because skills 1140 and M1120 were not adjacent skills. Despite the initial graph topology our search decided to merge these three skills. The combination of all these factors give stronger reasoning that the three skills M1140, M1118, and M1120 are not really distinct skills.

Experiment 2: Stability Experiment

In the previous experiment, every model was evaluated once and only once, which lead to the question of whether or not our results were stable. Our model evaluation used the Expectation Maximization (EM) algorithm, which is known to be affected by the starting value. In general, the EM algorithm does converge to the correct value, but there are cases where it can converge to incorrect values or to the "opposite" value. Considering the range to be between 0-1, if the actual true value of a parameter was 0.3, EM could converge to (1 - 0.3) = 0.7 instead, if the initial starting point was too far from the true value.

Our research question was: if we were to run the iterative search experiment several times would we end up with the same results using different starting values for EM? Since it takes several hours just to evaluate a single model, running the entire search consisting of over 100 models to evaluate would take too long. Therefore the purpose of this experiment was to evaluate just the first iteration of the search ten times to see if the results converged to a single best graph.

For the first iteration of the algorithm there were sixteen possible merges that could occur. For each of these possible merges we evaluated the resulting model ten times. The evaluation used was the same evaluation as the iterative search experiment for which we tested stability. For each of the ten runs we set the random seed in MatLab to correspond to the run number. This gave us a different set of random numbers for each run of the 16 possible merges, where each merge got the same random seed within a run. Manually setting the random seed also meant our results for the stability experiment would be reproducible

Results and Analysis

After evaluating all sixteen models from the first iteration ten times, we kept a count of how many times a model was the best model and how many times a model was in the top 3 best models. RMSE was used to choose the best models since it was used to determine the best model in the iterative search experiment (figure 4). The results are shown in the figure 9 below.

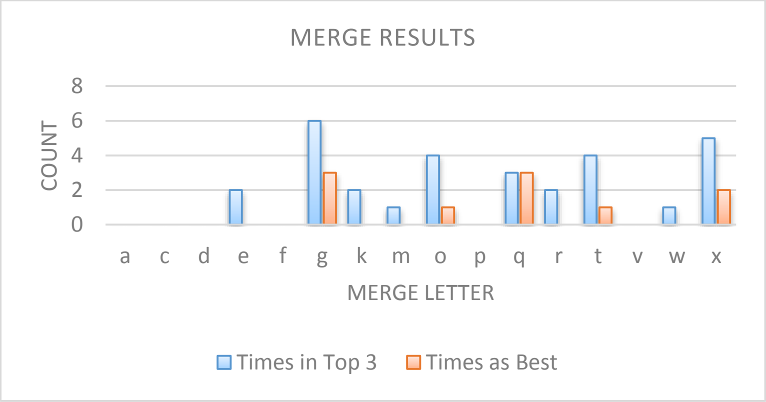


Figure 9 Stability Results

Merge ‘g’[2] was in the top 3 the most times (6) and was also the best model the most times (3). Merge ‘x’ and merge ‘q’ also did well. Merge ‘x’ was in the top 3, five times and was the best model two times. Merge ‘q’ was in the top 3, three times and was the best model three times. Merges ‘t’ and ‘o’ also did well. The general observation was that there was separation between good and bad merges but the best merge was not stable and did not converge.

We compared the graphs to our original iterative search experiment. In the original iterative search the first two skills that were merged were skills M-1128 and M-1127, corresponding to merge ‘x’ in our stability experiment. The second two skills that were merged in the iterative search experiment were skills M-1140 and M-1118, corresponding to merge ‘g’ in the stability experiment. Both merges ‘x’ and ‘g’ were the best two graphs in the stability experiment. Although merge ‘g’ did slightly better in the stability experiment, the order in which the merges took place did not matter. The best model in the iterative search took place after 4 merges, which included merges ‘x’ and ‘g’. Although we could not run the stability experiment 10 times for all possible merges and merge paths, we believe that it has a decent chance to converge to the same best model, which occurred after the fourth merge in the iterative search.

Discussion

When analyzing each merge, we considered the skills or concepts described by the affected skills as well as the test items associated to those skills. The descriptions below discuss the three groups of skills merged in experiment 1 and shown in the Best Model Skill Map (Figure 3b) and the two additional pairs of skills merged in experiment 2. In each case, the merges point to commonalities in the skills themselves or among the test items used to assess different skills.

Merge ‘x’ affected skills M-1127 and M-1128. These skills represent “the ability to represent inequalities from real world contexts” and “explain inequalities from real-world contexts,” respectively. The test items associated with these skills required students to read problems and identify inequality statements that matched the problems. In this case the test items did not distinguish between two unique skills, i.e., representing a problem or explaining a problem, as was suggested by the two skills.

[2] See Figure 1 for the edges that correspond to the named merges.

Merges ‘g’ and ‘k’ affected skills M-1118xM-1140 and M-1120. These skills represent the abilities to “locate integers on a number line,” “represent integers on a number line,” and “order integers from least to greatest,” respectively. The test items associated with these skills required students to select lists of correctly ordered integers or identify the correct number line graph of a particular integer. In this case the test items did not adequately distinguish between locating and representing integers on a number line (i.e., M-1118 and M-1120) because all of the items were multiple choice, and none provided students the opportunity to construct their own number line representations of integers. The inclusion of ordering integers from least to greatest (i.e., M-1140) with the other two skills is possibly due to the fact that using a number line is inherently, cognitively connected to ordering numbers from least to greatest.

Merge ‘t’ affected skills M-1105 and M-1106. These skills represent the abilities to “use positive and negative numbers in real-world contexts” and “relate the meaning of zero to positive and negative numbers in real-world contexts,” respectively. The test items associated with these skills required students to interpret problems involving integers and choose integer answers or verbal statements about integers. Two of the four test items included references to zero either as freezing point or sea level. In this case the items were designed to distinguish between the two skills, i.e., using integers and relating integers to zero. However, the relationship between zero and positive or negative numbers is so critical for understanding integers, it is likely that one cannot compare integers without considering their values in relation to zero.

Merge ‘q’ affected skills M-1120 and M-1108. These skills represent the abilities to “represent integers on a number line” and “recognize opposite numbers on a number line,” respectively. The test items associated with these skills required students to identify the correct number line graph of a particular integer or the opposite of a given integer. In this case, the two skills are inherently connected by the very definition of an integer as the opposite of a whole number. Consequently, it is likely that once students understand the definitions of integers and opposites and can use a number line, the act of graphing an integer is the same as graphing an opposite.

Merge ‘o’ affected skills M-1122 and M-1124. These skills represent the abilities to “recognize integer coordinate pairs” and “graph integer coordinate pairs,” respectively. The test items associated with these skills required students to identify the graph of a given integer ordered pair or to select the description of how to graph a given ordered pair on a coordinate plane. In this case, the items did not clearly distinguish between the two skills because the items associated with recognizing integer coordinate pairs included graphs. Furthermore, the skills themselves are difficult to distinguish in a practical sense because when students learn to graph integer ordered pairs, they routinely associate the numerical representation (i.e., the ordered pair) with its graphical representation (i.e., the point graphed in the coordinate plane).

An additional observation is that some of the skills tended to merge by pairing up with one and only one adjacent skill before RMSE started to decline. Before merge 5, the merges were all pairwise with the exception of merge 3. After merge 5, the skills tended to keep merging into the same skill. The best skill map was generated after merge 4, suggesting that adjacent skills tended to be similar skills and skills M-1140 and M-1120 were similar although they were not adjacent. This was a stronger relationship for several reasons. The merges that culminated in the merger of M-1140, M-1118, and M-1120 all took place before the best skill map was reached. This indicated that those three skills give better predictive performance when represented as one skill. This was also the first and only 3-skill group to be merged in the best model before RMSE declines. Finally, the three skills took two iterations of the search algorithm to merge together because skills M-1140 and M-1120 were not adjacent skills. Despite the initial graph topology, our search decided to merge these three skills. The combination of all these factors provided strong reasoning that the three skills M-1140, M-1118, and M-1120 were not really distinct skills.

Contributions, Conclusions and Future Work

Our contribution is that we provide a search algorithm to reduce the complexity of a given learning map while improving its fit to real student data. Since merging skills increased accuracy, these results suggest that the original skill map was too fine-grained (given the amount of questions per skill and the amount of students who took the test.) In some cases the test items did not adequately distinguish between the skills that were merged; hence such skills were merged. The results of algorithms like this can help content experts that are creating such skill maps and questions either reconsider thinking of the two skills as separate, or spur them to write some new questions that might better distinguish students that have one of the skills mastered but not the subsequent skill. In this work, those that created the learning map thought that item 11 was a prerequisite for items 12 and 13, but our stability results presented suggest that of all the arcs, this arc is the most unsupported by the data (see Figure 4, arc "g"). In fact, due to this work, we asked an unbiased teacher who did know what our mapping was, to create a hierarchy between items 11, 12 and 13. Surprisingly, she suggested that 12 and 13 were prerequisites to item 11, suggesting that the arc should point in the exact opposite direction. We think that this means that our method is helpful in using the data to suggest places in the skill graph that require additional attention.

This work relates to additional research by our team. Heffernan's ASSISTments project is attempting to track and improve students' knowledge across middle school mathematics. About a decade ago we had a learning map with over 300 skills but we now have reduced that complexity to 147 skills. Curriculum designers are correctly thinking about the subtle ways in which problems are different from one another, which cause them to want to add skills to the skill maps to make more subtle distinctions between questions. However if you also want to use the hierarchy to track knowledge, having more skills is bad, as fewer questions remain for each skill making fitting quantitative models harder.

All of the work we have done in this paper has a very small number of questions per skill. This naturally would cause us to think that many merges would be necessary, but if we had a large number of questions, and all students answered that large number of questions, we could probably justify more complicated models.

In our experiments we examined the effects of merging skills on an existing learning map. There are many other ways we could have used the existing map to create alternatives. For instance, Cen, Koedinger and Junker (2006) have explored ways of splitting skills or adding new skills, but all of those make more complicated models. What was not examined were the split and add operations. Possible future work could examine those operations to see if a better model can be obtained with them.

To verify that our algorithm works correctly, applying it to synthetic learning maps and synthetic data could be useful to determine if our algorithm converges to a true learning map.

Chapter 3: Refining Learning Maps with Data Fitting Techniques: What Factors Matter?

Cognitive models/Learning maps (skill graphs) have been identified as potentially improvable through the use of data mining techniques. However the factors that affect this improvement/refining process are not so clear. In an earlier paper we presented a method for improving these cognitive models. The purpose of this paper is to present the factors to consider when using our initial algorithm to refine learning maps. We present a simulation study that shows how important each of the factors are for this refinement process.

Introduction

Learning maps have been used as a tool to depict the set of skills in a cognitive domain and the relationship between these skills. A number of studies have been conducted to find and represent the relationships between the skills (Embretson, 1998; Mislevy, Steinberg, & Almond, 2002; Sheehan, 1997; Tatsuoka, 1983). Tatsuoka introduced the Rule-space method for identifying skills/knowledge components in a given cognitive domain whereas (Leighton, Gierl & Hunka, 2004) present another approach called the Attribute Hierarchy Method (AHM). The rule-space method (RSM) does not present the relationship of the skills/knowledge components as a hierarchy. However AHM, which is a variation of Tatsuoka's RSM, considers the hierarchical relationship between the components (Tatsuoka, 1995). Gierl et. al. (2008) used the AHM approach to make inferences of students' cognitive assessment. None of these approaches dealt with methods for improving the item response models developed using the methods proposed. The Learning Factors Analysis (LFA) method by Cen, and colleagues (2006) was introduced to deal with this problem. In that paper three different operations for improving the predictive abilities of learning maps or cognitive models are introduced. In (Adjei, et al. 2014) an attempt was made to solve this problem by presenting the results of a number of experiments that showed that learning maps can be refined using just one (the merge operation) of the three possible operations within the LFA method. It was shown that there were significant improvements in RMSE for the best model chosen, starting off with a pre-defined learning map.

We realize that, to generalize the method for refining learning maps, there are a number of questions that still need to be answered. These include: "What are the factors that can determine when a model can be best refined?" and "Do the number of skills, the number of items per skills, the number of levels in the skill hierarchy and the number of data points have any effect in determining the best refined model?" Whilst the LFA methods use a set of factors to determine whether to merge, add or split skills to generate better models from an existing one, all the factors used are based on expert knowledge and are independent of data. In order to answer the above questions, we present a number of simulation experiments.

Problem Statement

The LFA model uses three operations (splits, merges, and adds) to refine knowledge components. In each of the operations, learning factors were included in the model refinement process. These factors did not include the number of skills in the model, the levels in the hierarchy of skills in the model, the number of items per skill and the guess and slip parameter values for the items. We hypothesize that these factors are important in generating an optimal model from a given learning map (pre-requisite skill hierarchy). Hence we set out in this paper to present a series of experiments that help in determining the impact of the above mentioned factors in refining a given learning map or cognitive model.

Methodology

To be able to answer our research questions, we started off with a 3-skill graph. We inserted a fake skill at different locations of the graph and ran our evaluation code to determine when the original skill-graph is learned back and what factors determine when this occurs. We examined the following factors and determined which of these factors have the most impact on using the greedy algorithm presented in the earlier paper to refine a given model: guess and slip parameter values, the number of levels in the skill graph hierarchy, and the number of data points (i.e. students and items). For each randomly chosen skill graph we generate a set of simulated data, one each for the number of student and item pairs used. We then evaluate the models using Expectation Maximization to determine the factors that have the most impact. The following section presents the random graph generation, Bayesian network creation, fake skill creation, and the evaluation code.

Random Skill Hierarchy Generation.

To generate a skill graph randomly we began by choosing a random skill hierarchy. Our algorithm to generate the skill hierarchy took a range of skills and a graph depth as input parameters. The output of the algorithm was a valid skill hierarchy where the number of vertices was within the skill range and the number of levels was within the depth range. We order our vertices from 1 to N and use the constraint that a vertex cannot have a directed edge pointing to a smaller numbered vertex. We also enforce the constraint that a vertex cannot have any self-edges.

To generate a random graph we chose a random number within the range of possible graphs. We then converted this number to binary form and added the correct number of leading zeros (we knew the number of skills from the random number chosen). Then we simply inserted the bits of the binary number into the varying spots of the matrix form of the graph in order.

The result was a directed acyclic graph with no self-edges. It will not necessarily be connected. The final step was to check if the graph is connected. If the graph was connected, we kept it; otherwise we discarded it and repeated the generation process. This method allowed us to instantly generate valid graphs. An example is shown in Table 1 and Fig 10 for a graph with three skills.

Table 1. Example Matrix. Matrix generated by the random number 5. A ‘Y’ represents that this cell is ignored because it must be a zero since a vertex cannot have directed edges pointing to vertices with larger numbers. An ‘X’ indicates that this cell is ignored because it must be a zero since a vertex cannot have self-edges.

Vertex / Vertex	1	2	3
1	X (0)	1	0
2	Y (0)	X (0)	1
3	Y (0)	Y (0)	X (0)

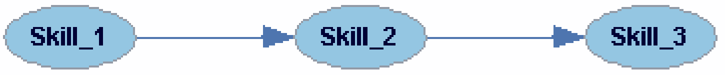


Figure 10 A Sample 3-skill graph generated for the experiments

Create Bayesian Network

The Bayesian network used for the analysis was generated from the skill graph selected from the previous step. To generate the items for the skills an item range was specified. A random number of items was chosen within the item range for each skill. In our experiments we restricted our range to be a single value so all skills would have an equal number of items. We set our Bayesian network up like knowledge tracing, where every skill had one or more items and every item had a guess and slip node (Colbert and Anderson, 1995). An item was tagged with only one skill. The skill nodes were latent nodes since we could not observe whether or not a student knows the skill. Each item node was an observable node, which is a '1' if the student answered the item correctly and a '0' if the student did not answer the item correctly. Both the guess and slip nodes were also latent nodes representing whether or not the student guessed or slipped on the item. A student was considered to have guessed when she answered correctly but did not know the skill. A student was considered to have slipped when she answered incorrectly but knew the skill. Using the previous skill graph example we added the item, guess, and slip nodes to the graph.

The final step to create the Bayesian network was to create the conditional probability tables (CPT) for the nodes. For our experiments we defined each skill node as AND nodes. This means that a student could only know a post-requisite skill if the student knew all of the prerequisite skills. Therefore if a student does not know one of the prerequisite skills then the student cannot know the post-requisite skill. If the student does know all the prerequisite skills (or there are no prerequisite skills), we picked a random probability that the student will know the post-requisite skill between 0.3 – 0.7. Our guess and slip parameters had varying probabilities since that was one of the parameters we experimented with. All the item nodes had a deterministic (0% chance or 100% chance of correctness) CPT based off of the skill, guess, and slip nodes (which were not deterministic).

Creation of Fake Skill

We exported our Bayesian network to MATLAB and used Kevin Murphy's Bayes Net Toolkit (Murphy, 2001) to generate the ground truth data. Once the ground truth data was generated we randomly generated "fake" skills from the original graph. We defined a fake skill as one that does not belong in the graph. The intention was to test the algorithm's ability to detect that fake skill. A fake skill was generated by randomly choosing a real skill. Once a real skill was chosen, a random number of items was chosen from the real skill. These items were then detached from the real skill and attached to the fake skill. The fake skill was then randomly chosen to be either a parent or a child of the real skill. Fig 11 shows the creation of a fake skill.

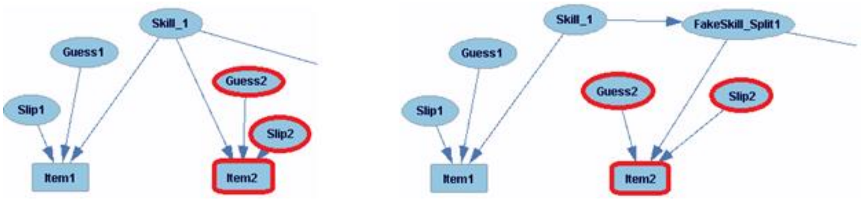


Figure 11 Creation of Fake Skill. The left skill graph shows the original skill graph before the creation of the fake skill. The skill graph on the right shows the skill graph after the creation of the fake skill. The fake skill was created from Skill_1 where item two was removed from skill 1 and attached to the fake skill.

Evaluation

In order to evaluate our Bayesian Network we used a similar process as that found in (Adjei, et. al. 2014). We used Expectation Maximization (EM) to learn parameters and fit our model. To evaluate our model we used per student per item cross validation with 5 student folds and 3 item folds. Our student and item folds were chosen randomly for our evaluation. In (Adjei, et. al. 2014) the item folds were chosen randomly but kept the same for each student. The only difference between the evaluations in this experiment is that each student is assigned a different set of random item folds instead of all students having the same set of random item folds.

Experiments

This section describes the set of experiments that we run to determine the factors that impact the search of better predictive skill graphs. These include determining the factors that matter, investigating the impact of student and item numbers on the search, and a final experiment that checked the stability of the search after fixing a number of factors.

Experiment 1: Determining the Factors that Matter

In this first experiment, we started with a set of skill graphs randomly generated from a set of chosen factors. These factors included the number of fake skills, the number of original skills, the number of items attached per shill, and the guess and slip rates for the items used in the skill graph. For each of the graphs, we insert a number of fake skills. We define a fake skill as one that is broken off of an existing skill. The fake skill has a random number of items chosen from the original skill and the fake skill is either a pre-requisite or post-requisite of the original skill. If the fake skill is a pre-requisite of the original skill, all the previous pre-requisites of the original skill become the pre-requisites of the new fake skill and the original skill becomes the post-requisite of the fake skill. The idea is to figure out if this fake skill will be easily identified and merged with the skill from which it was created from. This is to validate our merge operations and to determine what factors influence the determination of a better skill-model /skill map than the original.

Analysis

We analyzed the results of the experiment and looked at how the number of students, number of items, guess/slip values, and the number of fake skills impacted RMSE of our predictions and the percent of correct graphs learned back. Figure 12 shows the relationship between the probability a student guessed/slipped and the RMSE as well as the percent of the correct skill graph being learned back. We paired guess and slip values to lower the number of variables in our experiment. Our guess/slip pairings are as follows {(0, 0), (0.1, 0.08), (0.3, 0.16), (0.5, 0.25)}. It shows that the higher chance the student has to guess the answer the less accurate and harder it is to learn back the true original graph. The percent of graphs learned back with a guess/slip probability of 0 is significantly better than the percent of graphs learned back with a guess probability of .5 ($p < .001$). A realistic guess probability is around 0.14 calculated by Pardos and Heffernan (2010). At this point the percentage of graphs learned is somewhere between 0.25 and 0.33. These are not great percentages to learn back a correct graph under realistic guess and slip values. Not much can be done to lower the guess probability on typical questions middle school math students would see. However more student data can be used to increase model performance.

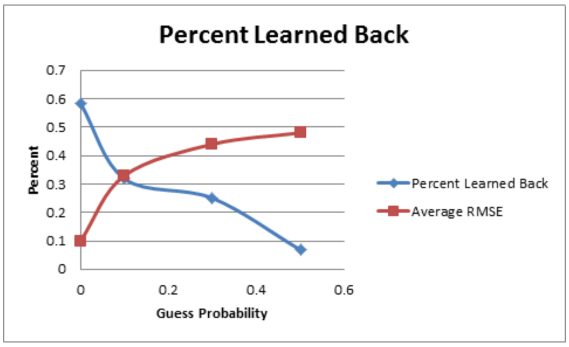


Figure 12 Effect of guess/slip on learning back the original graph

The guess/slip probability is the biggest factor that affects model accuracy followed by the number of students. Table 2 shows how both the guess/slip probability and the number of students affects the percentage of correct graphs learned back and average RMSE. A cell is broken up into two columns where the first column in the cell is the percentage of correct graphs learned back and the second column in the cell in the average RMSE value.

Table 2. Student/Guess Impact on Evaluation

	Students							
Guess	**50**		**100**		**150**		**200**	
	P_{LB}	**RMSE**	**P_{LB}**	**RMSE**	**P_{LB}**	**RMSE**	**P_{LB}**	**RMSE**
0	0.33	0.09	0.64	0.08	0.7	0.1	0.67	0.02
0.1	0.25	0.36	0.33	0.33	0.33	0.32	0.38	0.31
0.3	0.25	0.46	0.33	0.44	0.08	0.44	0.38	0.43
0.5	0.08	0.49	0.08	0.48	0.08	0.48	0	0.46

For a guess probability of 0.3, the percentage of correct graphs (P_{LB}) increases from 25% for 50 students to 38% for 200 students ($p = 0.2$). This shows that under a realistic worst case guess probability, increasing the number of students can increase the percentage of correct skill graphs learned back. The number of fake skills seemed to have little effect on RMSE, however a large effect on learning the correct graph. With more than one fake skill the percentage of correct graphs drops significantly from 0.6 for 1 fake skill to 0 for 3 fake skills ($p < .002$) for guess values of 0.1. This can be seen for the three points with an x-axis value of 0.1 in Fig 4. No additional findings are presented here as there were no strong trends.

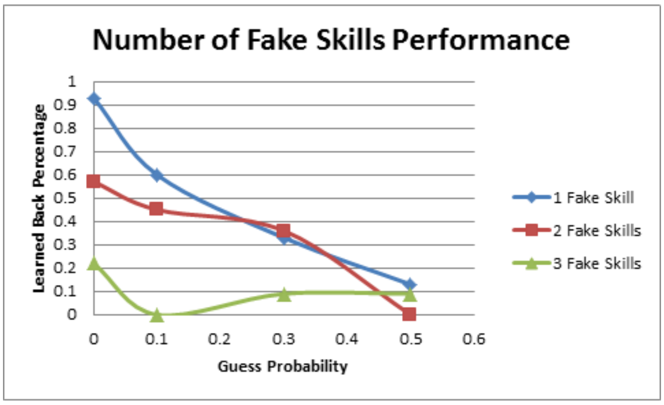


Figure 13 Effect of Number of Fake Skills on model improvements

Experiment 2: Impact of Item/Student counts on Recoverability

In experiment 1 multiple randomly chosen graphs were used as the ground truth. In this experiment we chose to try each possible 3-skill graph to see if the graph structure had an effect on whether or not the correct skill graph was learned back. The methodology was the same as experiment 1 except instead of randomly choosing graphs we ran each of the four graphs for each possible number of students and items per skill. Figure 12 shows all four possible 3-skill graphs. After determining that the major factor impacting performance were guess/slip values, a reasonable pair of values were chosen for the guess and slip values (guess=0.1 and slip=0.08). Additionally we fixed the number of fake skills to one in order to reduce the variability of the factors.

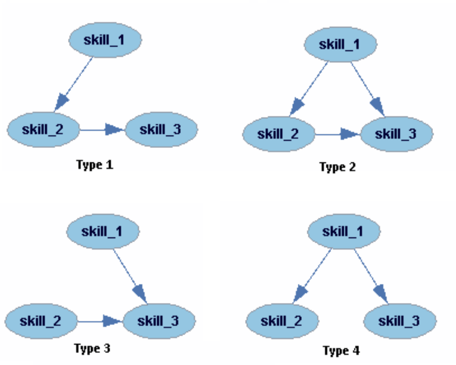


Figure 14 Different graph types for experiment 2

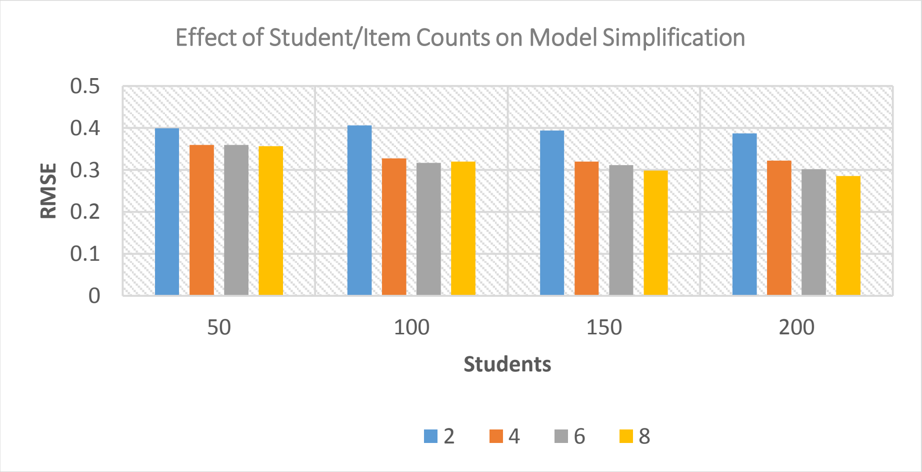


Figure 15 Effect of students/items on the model simplification.

The general observation from this experiment is clear from Fig 15 above. As the number of data points increases, the level of accuracy in recovering the original graph increases. This is despite the fact that the location of the fake skill was not fixed. Moreover, for any given number of students, an increase in the number of items results in a slight decrease in RMSE and hence better chance of learning back the original graph. This experiment shows that the data points (i.e. student and item numbers) have an impact on improving on the determination of the best model from a given model. The more data points there are, the more accurate the search.

Experiment 3: Stability Experiment

The main purpose of this experiment was to determine whether the results we found in the earlier experiments were stable. In this experiment we fixed all the variables except for the number of students and the number of items per skill. We wanted to see how stable our search was and how well it performed for a small example with reasonable parameter values. We fixed guess at 0.10 and slip at 0.08 with three skills and one fake skill. For the fake skill we took the first half of items from the original skill. We ran our algorithm for 50, 100, 150, and 200 students for 2 and 8 items per skill. For each pair of parameters we ran the experiment 10 times with different random seeds and took an average of the number times the correct graph was learned back. Figure 14 shows the results of this experiment. We found that the results are very stable for graphs that had two items per skill. The results were less stable for graphs with eight items per skill although the percent of graphs learned back was much batter. The graphs that had two items per skill were learned back correctly 8% of the time, where graphs with eight items per skill were learned back correctly 43% of the time, which is a significant improvement (n=40, p<.001).

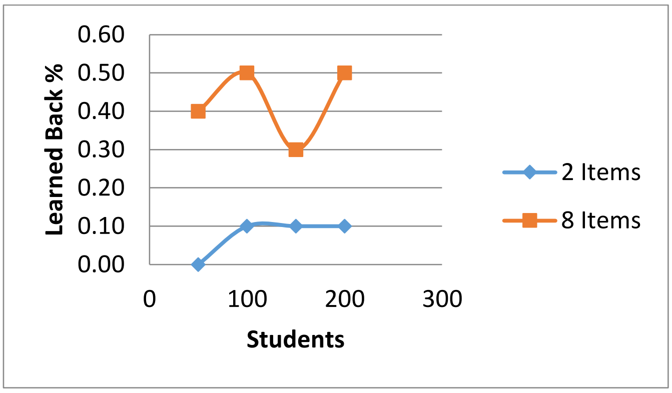


Figure 16 Percent of graphs learned back for student ranges 50-200 and 2+8 items per skill

Experiment 4: Impact of Student Numbers on Initial Graph Recoverability

We ran experiment 4 to confirm that the number of students has an impact on the recoverability of the original graph, fixing all other parameters at reasonable values and varying the number of students. For this experiment, guess and slip values were set at 0.1 and 0.08 respectively. We used graph type 4 (Figure 14), set the number of items to 4 and fake skills at 1, varying the location of the fake skill. The student numbers were varied from 10 to 100. For each student number, the evaluation was run 10 times. The results in Figure 17 show that as we intuitively assumed, the number of students has a huge impact on the algorithm’s ability to learn back the true graph. The results show that as the number of students increases the probability of a skill graph being learned back increases whilst the RMSE reduces. These results are significant with p-values below 0.01. This finding confirms that student numbers is an important factor that needs to be considered when refining learning maps.

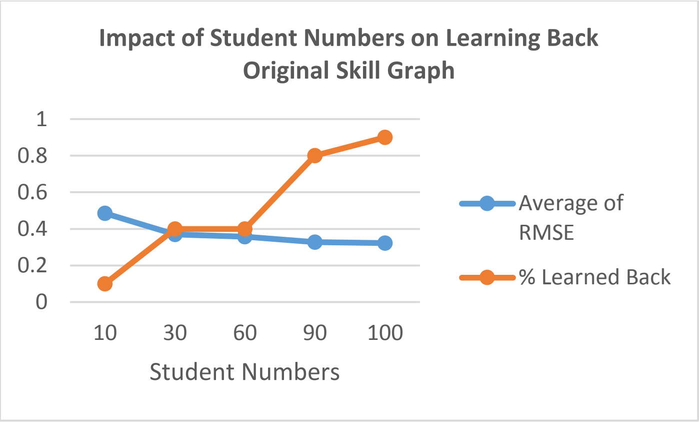


Figure 17 Impact of Student Numbers

Conclusion

Many learning maps/cognitive models are built from expert knowledge. With the production of a plethora of educational data on student performance, it has become imperative to find data centered methods of improving upon these expert-designed learning maps. In our earlier studies we designed and presented an algorithm for simplifying/improving the predictive accuracy of these models. In this paper we have presented a number of factors that influence the data centered model improvement process we initially published. We have shown with our simulation studies that the guess/slip values, number of items per skill, the number of students and the number of fake skills in the graph affect the simplification of the skill models. We also explored many parameters to see how much data is needed to recover the true learning maps. For future work we plan to continue to evaluate our algorithm on larger examples to see how well our algorithm can scale up and test it on well-known real data sets.

Chapter 4: Searching for Learning Maps using Adaptive Testing System: Placements

In our earlier work in the search for better fitting models, we showed that skill graphs can easily be improved by considering just one of the three operations proposed in the Learning Factors Analysis. (Adjei, et. al. 2014) In chapter one, we proposed a greedy search algorithm for searching data fitting models with equally accurate predictive power as the original skill graph but with fewer nodes in the graph. Additionally it has been shown that a number of factors need to be considered in order to improve upon the search. In chapter 3, it was explained that the number of students and items also has an impact on the search. Similarly the choice of the problem with which the skills are tagged also impacts the search.

However, this chapter is meant to answer the following research question: are there more dynamic ways of determining the strength of the relationship between skills in the skill graph? In this chapter, PLACEments, an adaptive testing system is presented and used to determine different portions of a skill graph that need to be looked at. The tool identifies the prerequisite relationships in a skill graph that need to be maintained and those that should not exist.

Introduction

In order to improve upon student learning, a number of approaches have been studied to determine ways of improving the effectiveness of the skills teachers transfer to students and the order in which these skills need to be taught. Learning Factors Analysis, Q-Matrices and a few other methods have been proposed and used to improve upon learning maps. Some have shown promising results, though many have not been applied to different subject domains.

However, none of the approaches that have been used have studied the ways in which to determine the strength of the relationships in a prerequisite skill graph. In our quest to find the best methods for improving upon learning maps, could we use empirical studies to find the best order in which students must learning certain skills? Can we determine the strength of the relationships between learning maps are their effectiveness? This chapter presents an adaptive testing system that traverses a prerequisite skill graph based on a student's performance. We present a brief description of how the system works, the design of the study, and the results we found. The chapter concludes with a discussion of the findings of the paper as well as the limitations of this approach.

PLACEments, an Adaptive Testing Systems

PLACEments is a computer aided adaptive testing system. This system is a feature of an intelligent tutoring system, ASSISTments, that mainly provides teachers a means of creating and assigning exercises and tests to their students. (Razzaq, et. al., 2009) ASSISTments also has a feature that allows students to do the exercises and tests assigned by their teachers.

The PLACEments system has a number of components: item pool, item selection, termination rules (a skill graph for the knowledge domain in which the students' knowledge will be assessed and three modules (test creation, test taking (Tutor), and remediation creation)). The item pool for placements is chosen from a list of skill builders used extensively in ASSISTments. The choice of problems for the placements test was made based on the difficulty of the item. The difficulty was determined by calculating the percent correct for all responses of students in ASSISTments to that item and subtracting that value from 1. (see equation 1 below) To ensure that PLACEments does not present overly easy or overly difficult problems, the items for each of the skills tested were chosen such that their difficulty is between .4 and .6 (the smaller the number, the more difficult the item is).

$$difficulty_i = 1 - \Pr(item_i = 1) \qquad (1)$$

As noted earlier, PLACEments uses a predefined skill graph to guide test item selection. Though we currently use a prerequisite skill graph developed based on the Massachusetts Common Core State Standards for Mathematics (2010), the system is designed such that it can use any prerequisite skill graph from which tests can be drawn. The initial set of problems is chosen from the initial set of skills. Each skill has one problem chosen from the item pool. When students get an item for a skill incorrect, implying that the student does not have that cognitive skill, the test is expanded by including the problems from the prerequisite skills of each of the skills the student has gotten incorrect. The test bank increases until the grade boundaries chosen at test creation are reached. For a given student, the test terminates when all the skills in the initial set of skills have been tested, and the student gets all the items for that skill correct. If the student is not able to answer any of the initial problems correctly, then the test terminates when there are no prerequisites remaining to be tested.

The following diagram, figure 18 shows a hypothetical graph that explains how the test proceeds. The correctness indicator attached to each node in the graph is a particular representation of a given student's performance. The nodes in the graph represents the skills, the arrows between the skills represents the prerequisite relationship between the skills (thus, skill 'D' is one of the prerequisites of skill 'A'). In this configuration, the students are assigned skills 'A', 'B' and 'C' as the initial skills. This student is adaptively assigned questions D, E, and H in that order, because he performed poorly on A and E.

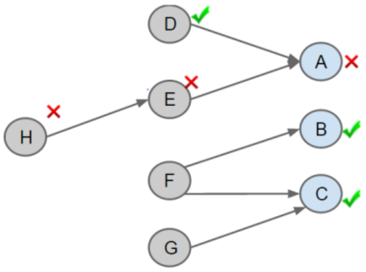


Figure 18 A sample skill graph and a sample student's response configuration

The size of the test is affected by the students' performance as well as the structure of the skill graph.

Once the students complete the test, remediation assignments are created based on the skills they performed poorly on. Each student is assigned a different set of remediation. The remediation assignments on the lowest grade level skills are released before those of the higher grade levels. In the example shown in fig 18, the student will be assigned a skill builder assignment in the following sequence: H, E and then A. Once the prerequisite skill is completed, the next skill inline is released.

Research Question

As was stated earlier, this study is meant to determine whether a skill graph can be improved using an empirical studies. To be specific we want to determine the strength of prerequisite skill relationships between skills and hence determine which of such relationships to remove or maintain in a skill graph.

Methodology

To answer our research question, we run a study in which the navigation of a skill graph in a PLACEments test is modified for a random sample of students. Figure 19 demonstrates the modifications made to PLACEments in order to answer this question. For those randomly chosen students, a random initial skill (skill ‘A’ in figure 19) is selected and the students get to answer questions from the prerequisite skills (‘B’ and ‘C’) of the chosen skill if the students get the initial skill correct.

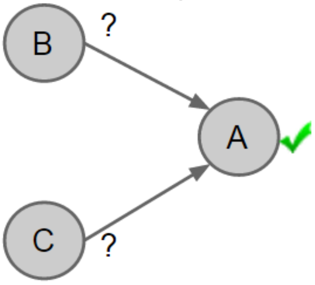


Figure 19 Sample navigation of the graph for this study

In order not to overload the chosen students with work, in a given assignment and for those students, only one initial skill is used in the study. Additionally, if any of the chosen students gets the prerequisite skill incorrect, they are not assigned remediation assignments as is the case with all the other assignments, and the navigation does not continue to the second level of prerequisite skills (i.e. those of ‘B’ and ‘C’ in figure 19) for the chosen initial skill.

It is hoped that if a higher percentage of the students in the study answer the prerequisite skills of a given skill correctly, this would suggest a strong relationship between the skills, and hence maintain the skill in the graph. On the other hand, if the percentage is low then it would suggest that that prerequisite link in the graph would either require further scrutiny or must be removed.

Dataset

The dataset includes a prerequisite skill graph developed by a Mathematics domain expert. This graph contains skills from the Common Core Standards (2010) spanning grades K-9. The graph, which is the graph used in PLACEments, has a total of 495 prerequisite relationship. A portion of the graph is shown in Figure 20 below. The green lines in the graph indicate that the prerequisite skills are in a lower grade level, whiles the black arrows between the nodes show prerequisite link between skills of the same grade level as the post-requisite skill. The node names represent the skill codes from the Common Core Standards for Mathematics. Each of these nodes has a complete description and examples of what students need to be taught. See http://www.corestandards.org/Math/ for a complete listing and a detailed description of the standards.

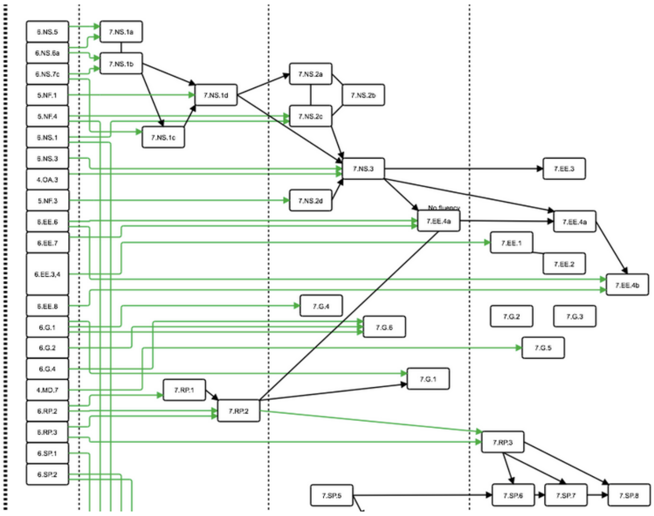


Figure 20 A portion of the prerequisite skill graph designed by a math expert and based on standards from the Common Core Mathematics Standards

The dataset additionally includes 1272 problem logs from ASSISTments. Each row in the dataset represents a student's response to a placements test item. That dataset also includes a matrix of item to skill tagging. These logs were from 601 distinct students whose grades ranged between 6 and 12. Each of these students was assigned at least one of the 119 placements assignments used in the study. The data set represented 60 of the 495 prerequisite relationships in the skill graph.

Results and Analyses

As of the time of reporting this study, data had been collected on 60 of the 495 relationships/prerequisite skill links. Of these 60, 35 had at least 10 responses. (See table 3 for the complete list of 35 links) We limit the number of responses per relationship to 10 in order to achieve some generalization of the results. The graph in figure 21 shows that three (3) of the relationships had link strength of 0, since none of the students had the prerequisite questions correct even though the new the post-requisite. Two (2) were of the maximum strength (1). A larger proportion of the links examined so far has strengths ranging between 0 and 1. As many as 24 of the links have a significantly low link strength as the figure shows.

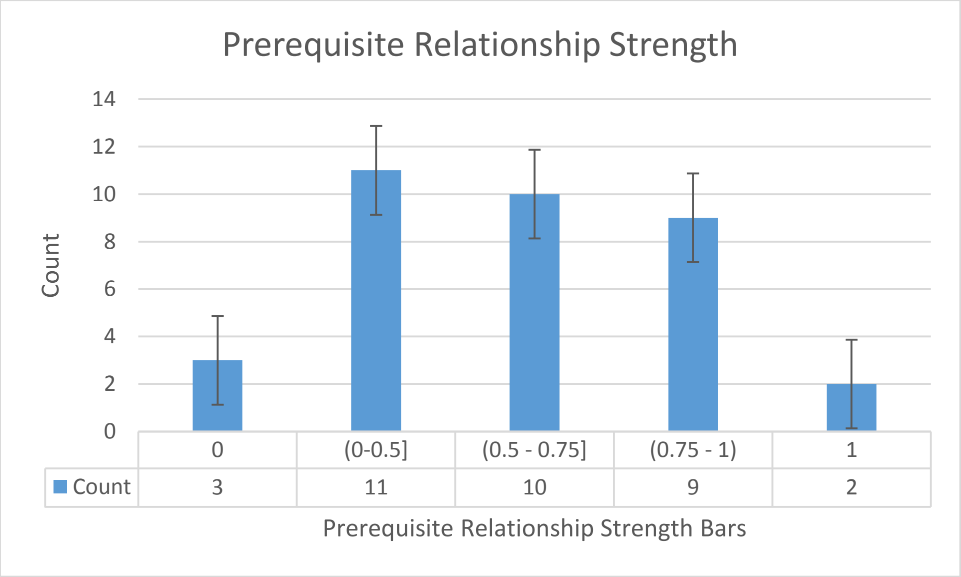


Figure 21 Prerequisite Link Strength

As the graph indicates, we can make general statements about the relationships. For three of the relationships, none of the students knew the prerequisite skills even though they performed well on the post-requisite skill. Similarly two of the links can be believed since all the students who knew the post-requisite skills also knew the pre-requisite, suggesting that the link belongs in the graph. There was a larger number of the links for which strengths were inconclusive. Of particular interest are the skill links with strength below 0.5. Those strength values show that a big percentage of students did not know the prerequisite skill even though they all got the post-requisite skills correct. These low numbers suggest that the prerequisite relationship between the skills need to be looked at extensively, and may warrant a removal from the skill graph. It may be safe to assume that the skills with a link strength above 0.5 may be valid and the reason for which the strength is not 1 may be because the items used in the test have high slip rates. However this assertion needs further studies to ascertain.

To help us understand how the link strength is affected by the knowledge level of the students who participated in the study, the results were sub-divided into the different knowledge levels. The knowledge level of a student was determined by the student's prior percent correct, i.e. the percent of correctness of a student's previous performance on problems in ASSISTments. All students with a percent correct value below 0.5 were assigned to the low knowledge group, while those students with percent correct values between 0.5 and 0.75 were tagged as medium level students. Any student whose prior percent correct was above 0.75 was tagged as a high knowledge student.

Table 3 A Subset of the List of Skill Links in the prerequisite skill graph.

Link Code	**Skill**	**Prerequisite Skill**
A	Ordering Fractions	Equivalent Fractions
B	Subtracting fractions like denominator	Adding mixed numbers like denominator
C	Comparing Positive Decimals	Read & write decimals
D	Subtraction Mixed Numbers	Addition Mixed Numbers
E	Word problems with fractions as division	Multiplication Fractions
F	Multiplication Fractions	Area of rectangle word problems
G	Line Plot	Real world fraction multiplication
H	Line Plot	Line Plot with fractions
I	Expressing unit rate in words	Finding The Ratio
J	Expressing unit rate in words	Word problems with fractions as division
K	Solve unit rate problems	Expressing unit rate in words
L	Percent of	Expressing unit rate in words
M	Divide multi-digit numbers	Division Whole Numbers
N	Division of Positive Decimals	Multiplication Positive Decimals
O	Comparing integers on number line	Plot on coordinate plane
P	Evaluate exponents	Multiply by Powers of 10 (number of zeros)
Q	Deviations in measures of center & spread	Median
R	Identify constant of proportionality	Unit Conversions with ratios
S	Identify constant of proportionality	Unit rate with fractions
T	Identify constant of proportionality	Solve unit rate problems
U	Identify constant of proportionality	Percent of
V	Identify constant of proportionality	Expressing unit rate in words
W	Identify constant of proportionality	Percent- finding whole
X	Divide Integers	Multiply Integers
AA	Word problems all operations w/ integers	Word problems with fractions as division
AB	Word problems all operations w/ integers	Divide Integers
Y	Word problems all operations w/ integers	Multiply and Divide non integer rationals
Z	Word problems all operations w/ integers	Multiply Integers
AC	Combining Like Terms	Distributive Property
AD	Equation Solving Two or Fewer Steps	Word problems all operations w/ integers
AE	Scale drawings	Identify constant of proportionality
AF	Operations with scientific notation	Dividing Monomials
AG	Operations with scientific notation	Power of Powers
AH	Operations with scientific notation	Multiplying Monomials
AI	Transversal	Sum of angles

Figures 22 and 23 present a breakdown of the results by knowledge level. Table 3 lists the 35 links considered in this study. Of the 35 skill links studied, there were 12 of the skills for which we had data for all three knowledge levels. Twenty-three (23) of these links were examined for both medium and high knowledge students.

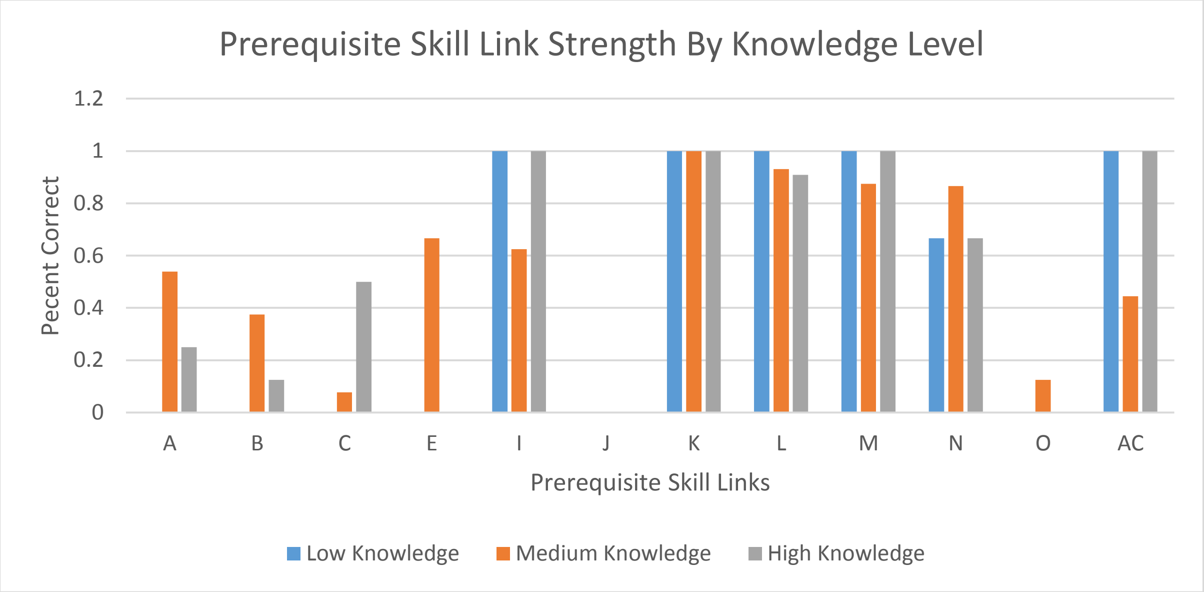


Figure 22. Prerequisite Skill Link Strength by knowledge level

The results show some variations in the link strength when the data is split into different knowledge levels. The results for links J and K clearly show an agreement in the results across knowledge levels. The results for J suggest a very weak link and hence that link must be removed from the skill graph. For all knowledge groups, link K is strong, suggesting that the link is believable and hence belong in the graph.

Links I, L, M, N and AC show that the different knowledge levels contributed differently to the strength. While for links I, M and AC, both high and low knowledge students demonstrate that those links are strong, there is a different result for the medium knowledge students. Apart from links I and AC, all the other links in that group have a link strength above 0.8. The other interesting link is C. Though the results shows link C as a weak link, it is a much weaker link for medium knowledge students than for high knowledge students, and worse for low knowledge students. Another set of interesting results was that for links A, B, E and O. In all of these, it would be expected that medium knowledge students will do poorly on a prerequisite skill than high knowledge students, however the results show that this is not the case for those skills. The medium knowledge students performed better than expected. Link O is even more interesting. The data for both low and high knowledge students suggest that the link should be removed from the skill graph. However, this is a much weaker statement to make for medium knowledge students, suggesting that there should be different prerequisite skill graphs for students with different knowledge levels.

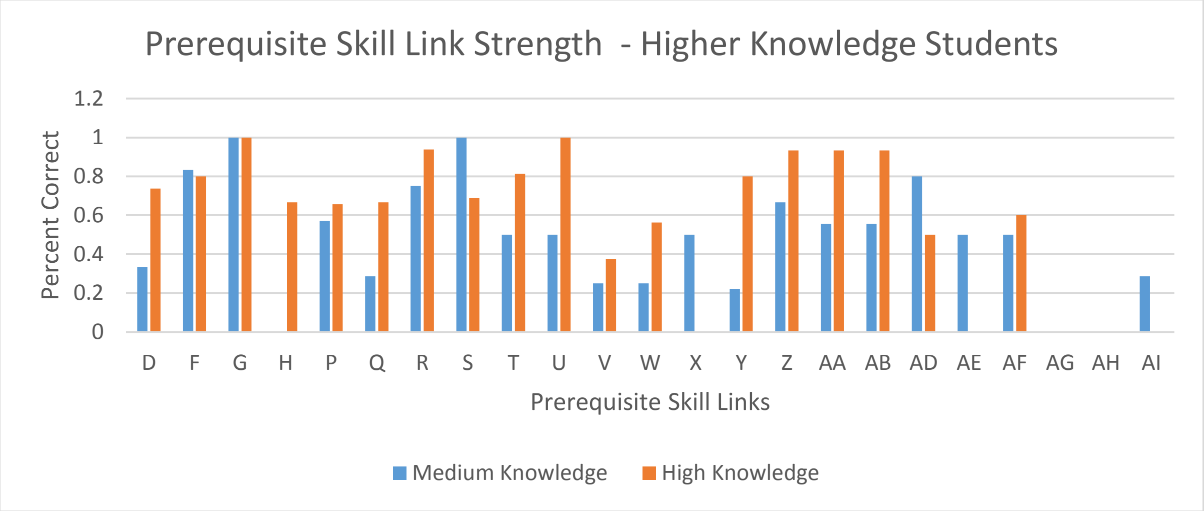


Figure 23 Medium and High Knowledge Students' contribution to link strength

In the data set, there were 23 of the links for which we did not have responses from low knowledge skills. The medium and high knowledge level students are compared in Figure 23 and we can see that each of the two knowledge groups contributed differently to the results of the study. Links AG and AH appear to be non-existent since the results show that none of the students in the two knowledge groups could know the prerequisites in that link even though they knew the post requisite skill. These links appear to be other candidates for removal from the skill graph. The results in figure 23 show a set of interesting variations in the link strengths across the knowledge levels.

Overall, these results suggest a number of the prerequisite skills that need to be assessed in the graph: some may require complete removal from the skill graph and others suggest a different skill graph for different students with different knowledge levels. Additionally breaking the results down into the different knowledge levels has resulted in one minor finding: Students with different knowledge levels have different representation of knowledge and hence different skill graphs need to be designed for student with different knowledge levels.

Limitations of the Approach

The approach described in this chapter has a number of limitations. The first limitation is about the choice of questions for the skills in the skill graph. Since the current implementation of PLACEments uses just one item (or question) to represent a skill in the test, a poorly chosen problem will affect the performance of the search. If a problem chosen to represent a skill in the test has a very high slip or guess rate, the performance of the students (the basis of which is used to determine the strength of links in the graph) will be affected. In other words, if a problem has a high slip rate, PLACEments will assume that students do not know a skill because most of the students will not perform well on that problem, even though there might be a high probability of the students knowing the skill tagged by the problem. Another limitation that relates to the choice of the questions for skills in the graph is about the number of questions used in the test to assess a student’s knowledge of a skill. Since PLACEments currently uses just one question per skill, the choice of problems has to be such that it is a good representation of problems tagged by that skill. In other words it has to be an almost perfect determinant of a student’s knowledge of the skill. One way to deal with this limitation is to use multiple problems with varying guess and slip rates for a given skill.

Another limitation of this approach is that the fact that we start the search by believing the initial set of skills and their ordering. In fact the approach does not help in determining whether the ordering of the skills in a skill graph is problematic or not. A new ordering of skills cannot be suggested by using this approach. Finally, a large number of student data is needed in order to make reasonable conclusions.

Contribution

In this paper we have proposed an intuitive but novel method for improving prerequisite skill graphs. The freely available adaptive testing system, PLACEments, can be used to collect and analyze student performance data on the items tagged by the skills in a skill graph in order to determine the appropriateness of some of the skill links in a given prerequisite skill graph. Of course with the limitations mentioned earlier, we think that the educational data mining community can take a good look at this process and augment the search for better fitting models with this new technique.

We have also shown that students of different knowledge levels learn differently and as such there should be a different representation of the skills they have in a given domain. Our results suggest that curriculum designers may want to think about the needs of these different knowledge levels in the design of curricula.

Conclusion and Future Work

Several methods have been proposed and used to improve upon the predictive power of learning maps. These methods include the Learning Factors Analysis, Q-Matrices and the greedy search algorithm proposed and reported upon by the authors in an earlier paper. Many of them have shown promise, especially with the greedy search algorithm showing that an equally predictive model can be found which is different from the initial skill graph but with fewer skill-nodes. However, we have not found any empirical studies that have been conducted with the aim of determining the flaws in a given skill graph/learning map and fixing them. The present study set out to solve this issue. The initial research question was: Can we do a better job at determining the deficiencies of a skill graph with empirical data and improving upon the skill graph?

To answer that question, we built an adaptive testing feature, PLACEments, in ASSISTments and used that to collect data on prerequisite skill graphs. The results of the study showed that deficiencies of a skill graph can be determined. Some relationships can be identified as un-necessary and hence be removed from the system. This finding is true irrespective of the knowledge level of the students involved. Additionally, we found that students with different knowledge levels of a given domain require different skill graphs. This was from the fact that some of the relationships appeared to be stronger for students of one knowledge level than for those of another.

There were a few limitations of this approach requiring further studies to make the method more robust. It was mentioned that just one item was used to test a students' knowledge of a skill, and so our estimate of the students' knowledge of a skill may be biased by either the difficulty of the item or the guess and slip rates of the item. This requires further studies. One particular future study to perform in this regard is to vary the number and difficulty of the items used to estimate a student's knowledge of the skill.

The results would be made even stronger with more student data. So far we have looked at only 35 of the over 400 different links in the graph. More data is needed to test different links in the graph. We have therefore proposed an alternative method for improving upon prerequisite skill graphs. This method can be used to augment the results from the other three methods mentioned earlier.

References

Adjei, S. A., Selent, D., Heffernan, N. T., Broadus, A, Kingston, N. (2014) Refining Learning Maps with Data Fitting Techniques: Searching for Better Fitting Learning Maps. In Stamper, J., Pardos, Z., Mavrikis, M., McLaren, B.M. (eds.) Proceedings of the 7th *International Conference on Educational Data Mining* pp. 413-414

Barnes T. (2005) The Q-matrix Method: Mining Student Response Data for Knowledge. In: *Proceedings of AAAI 2005 Educational Data Mining Workshop*.

Belhumeur, Corey (2013), An Empirical Evaluation of Student Learning by the Use of a Computer Adaptive System; *A Masters' Thesis* can be referenced at https://www.wpi.edu/Pubs/ETD/Available/etd-041913-085542/unrestricted/cbelhumeur.pdf

CAST (2011). *Universal Design for Learning Guidelines version 2.0.* Wakefield, MA: Author.

Cen, H. (2009). Generalized Learning Factor Analysis: Improving Cognitive Models with Machine Learning. *Doctoral Thesis. Carnegie Mellon University*.

Cen, H., Koedinger, K. R., & Junker, B. (2006). Learning Factors Analysis: A general method for cognitive model evaluation and improvement. In M. Ikeda, K. D. Ashley, T.-W. Chan (Eds.) *Proceedings of the 8th International Conference on Intelligent Tutoring Systems*, 164-175. Berlin: Springer-Verlag.

Cen, H., Koedinger, K.R., & Junker, B. (2007). Is over practice necessary? – improving learning efficiency with the Cognitive Tutor through educational data mining. In Luckin, R., Koedinger, K. R. & Greer, J. (Eds.). *Proceedings of 13th International Conference on Artificial Intelligence in Education (AIED2007)*, 511-518. Amsterdam, IOS Press.

Clements, D, and Sarama, J. (2004). "Learning Trajectories in Mathematics Education." *Mathematical Thinking and Learning An International Journal* 6:81-89.

Corbett, A.T., and Anderson, J.R. (1994) Knowledge tracing: Modeling the acquisition of procedural knowledge, *User modeling and user-adapted interaction* 4.4 (1994): 253-278.

Desmarais, M. C. and Gagnon M. (2005) Bayesian Student Models BAsed on Item to Item Knowledge Structures. *Artificial Intelligence in Education*. Lecture Notes in Computer Science. 4227:111-124

Embretson, S. E. (1998). A cognitive design system approach to generating valid tests: Application to abstract reasoning. Psychological Methods, 3(3), 380-396

Freyberger, J., Heffernan, N., Ruiz, C., (2004). Using association rules to guide a search for best fitting transfer models of student learning. *In Workshop on Analyzing Student-Tutor Interactions Logs to Improve Educational Outcomes* at ITS Conference, Alagoas, Brazil. (pp. 1-10).

Gagné, R.. (1968). "Learning Hierarchies." *Educational Psychologist* 6:1-9.

Gierl, M., Wang, C., and Zhou, J. (2008). "Using the Attribute Hierarchy Method to Make Diagnostic Inferences about Examinees' Cognitive Skills in Algebra on the SAT." *The Journal of Technology, Learning, and Assessment*, 6(6):4-49.

Leighton, J. P., Gierl, M. J., & Hunka, S. M. (2004). The attribute hierarchy model for cognitive assessment: A variation on Tatsuoka's rule-space approach. *Journal of Educational Measurement*, 41, 205-237.

Leszczenski, J.M., Beck, J.E. (2007). What's in a Word? Extending Learning Factors Analysis to Model Reading Transfer. *13th International Conference on Artificial Intelligence in Education, Educational Data Mining Workshop*, 2007. Los Angeles, CA.

Li N., Cohen W. W., Noboru M. and Koedinger K. R. (2011) 'A machine learning approach for automatic student model discovery', in *Proceedings of the 4th International Conference on Educational Data Mining*, pp. 31–40.

Lobo, J. M., Jiménez-Valverde, A. and Real, R. (2008), AUC: a misleading measure of the performance of predictive distribution models. *Global Ecology and Biogeography*, 17: 145–151. doi: 10.1111/j.1466-8238.2007.00358.x00

Mislevy, R. J., Steinberg, L. S., & Almond, R. G. (2002). Design and analysis in task-based language assessment. Language Testing, 19(4), 477-496.

Murphy, K. P. (2001) The Bayes Net Toolbox for MATLAB, *Computing Science and Statistics*, 2001, vol. 33 page 2001

Common Core State Standards (2010). National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

Pardos, Z. & Heffernan, N. (2010). Navigating the parameter space of Bayesian Knowledge Tracing models: Visualization of the convergence of the Expectation Maximization algorithm. In Baker, R.S.J.d., Merceron, A., Pavlik, P.I. Jr. (Eds.) *Proceedings of the 3rd International Conference on Educational Data Mining*.Pages 161-170.

Pavlik JR., P.I., Cen, H. and Koedinger, K.R. (2009). Learning factors transfer analysis: Using learning curve analysis to automatically generate domain models. In *Proceedings of the 2nd International Conference on*

Educational Data Mining, Cordoba, Spain, 2009a, T. BARNES, M. DESMARAIS, C. ROMERO and S. VENTURA, Eds., 121-130.

Popham, W. James. (2011). Transformative Assessment in Action: An Inside Look at Applying the Process. Alexandria,VA: ASCD.

Sheehan, K. M. (1997). A tree-based approach to proficiency scaling and diagnostic assessment. Journal of Educational Measurement, 34(4), 333-352

Tatsuoka, K. K. (1983) Rule Space: An approach for dealing with misconceptions based on item response theory. *Journal of Educational Measurement*, Vol 20, No. 4

Tatsuoka, K. K. (1995). Architecture of knowledge structures and cognitive diagnosis. In P. D. Nichols, S. F. Chipman & R. L. Brennan (Eds.), *Cognitively diagnostic assessment* (pp. 327-359). Hillsdale, NJ: Erlbaum

Van de Walle, J. A., Bay-Williams, J. M., Karp, K. S., Lovin, L. H. (2014). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6-8* (2nd ed.). Upper Saddle River, NJ: Pearson Inc.

Xu, Y. and Mostow, J. (2011). Logistic Regression in a Dynamic Bayes Net Models Multiple Subskills Better! In *4th International Conference on Educational Data Mining* Eindhoven, Netherlands, July 6-8, 2011.

Appendix: Questions Used for Experiments in Chapter 1

Questions

1. Which statement about 3+ (-2) and 3 2 best describes the meaning of the symbols "+" and " "?
A) 3 + (-2) and 3 2 both mean subtracting 2 from 3
B) 3 + (-2) and 3 2 both mean adding 3 to the opposite of 2
C) 3 + (-2) means subtracting 2 from 3, and 3 2 means adding 3 to the opposite of 2
D) X 3 + (-2) means adding 3 to the opposite of 2, and 3 2 means subtracting 2 from 3
QuestionId: 50049, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

2. Which statement about 6 (-5) and 6 + 5 best describes the meaning of the symbols "+" and " "?

A) 6 (-5) and 6 + 5 both mean adding 6 and 5
B) 6 (-5) and 6 + 5 both mean subtracting the opposite of 5 from 6
C) X 6 (-5) means subtracting the opposite of 5 from 6, and 6 + 5 means adding 6 and 5
D) 6 (-5) means adding 6 and 5, and 6 + 5 means subtracting the opposite of 5 from 6
QuestionId: 50050, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

3. Which statement about integers is true?
A) X -3 is an integer because 3 is an integer
B) is an integer because both 1 and 2 are integers
C) 0.75 is an integer because 75 is an integer
D) -1.26 is an integer because both -1 and 26 are integers
QuestionId: 50051, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

4. Which is always an integer?
A) positive fraction
B) negative decimal
C) improper fraction
D) X zero
QuestionId: 50052, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

5. Which correctly explains why -5 is the opposite of 5?
A) X -5 added to 5 equals 0
B) 0 is to the left of 5 on a number line
C) -5 is to the left of 5 on a number line
D) they are located on different sides of 0
QuestionId: 50053, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

6. Which choice contains a pair of opposite numbers?

A) -5,
B) -5,
C) -5, -5
D) X -5, 5
QuestionId: 50054, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

7. Which statement comparing two integers is correct?

A) -1,537 > 1,576
B) -1,537 < -1,576
C) 1,537 < -1,576
D) X -1,537 > -1,576
QuestionId: 50055, Standard 6 "NS", Benchmark 7 "7", Indicator "a", Sub Indicator ""

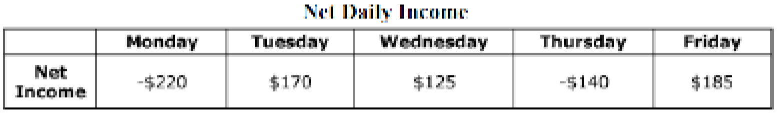
8. Which expression comparing two integers is correct?

A) -2,789 > 2,980
B) X -2,789 > -2,980
C) 2,789 < -2,980
D) -2,789 < -2,980
QuestionId: 50056, Standard 6 "NS", Benchmark 7 "7", Indicator "a", Sub Indicator ""

9. Which list of temperatures is in order from **highest** to **lowest**?

A) X 45 degrees F, 2 degrees F, -32 degrees F, -38 degrees F, -40 degrees F
B) 45 degrees F, -40 degrees F, -38 degrees F, -32 degrees F, 2 degrees F
C) 2 degrees F, -32 degrees F, -38 degrees F, -40 degrees F, 45 degrees F
D) -40 degrees F, -38 degrees F, -32 degrees F, 2 degrees F, 45 degrees F
QuestionId: 50057, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""

10. A business's net daily income for five days is shown in the table below.



Net Daily Income

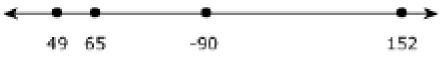
	Monday	Tuesday	Wednesday	Thursday	Friday
Net Income	-$220	$170	$125	-$140	$185

Which list of incomes is in order from **least** to **greatest**?

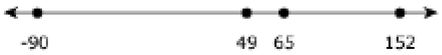
A) -$220, $185, $170, -$140, $125
B) $125, -$140, $170, $185, -$220
C) X -$220, -$140, $125, $170, $185
D) $185, $170, $125, -$140, -$220
QuestionId: 50058, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""

11. The school's band members are selling cookies to raise money. The band members need to pay for unsold boxes. The table below shows the total sales, including fees for unsold boxes of each member.

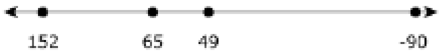
A)



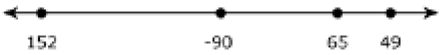
B) X



C)



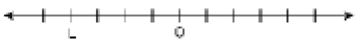
D)



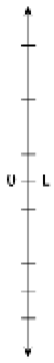
QuestionId: 50059, Standard 6 "NS", Benchmark 6 "6", Indicator "c", Sub Indicator ""

12. Each graph displays points L and 0 on a number line. On which number line is point L positioned at the value of

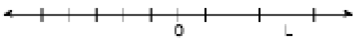
A)



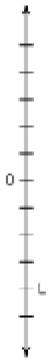
B)



C) X



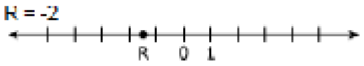
D)



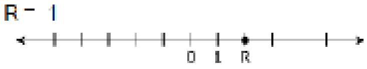
QuestionId: 50060, Standard 6 "NS", Benchmark 6 "6", Indicator "c", Sub Indicator ""

13. Which graph correctly represents the integer shown?

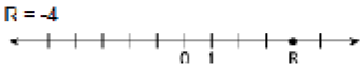
A)



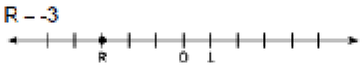
B)



C)

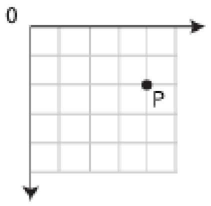


D) X



QuestionId: 50061, Standard 6 "NS", Benchmark 6 "6", Indicator "c", Sub Indicator ""

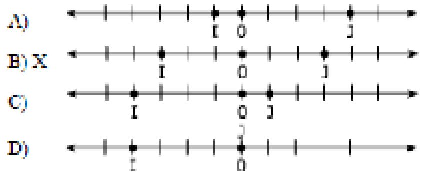
14. Point P is plotted in the graph shown below. Which coordinate pair lists the correct values for P?



A) (-2, 4)
B) (-5, 4)

C) X (4, -2)
D) (4, -5)
QuestionId: 50062, Standard 6 "NS", Benchmark 6 "6", Indicator "c", Sub Indicator ""

15. Which number line shows that I is the opposite of J?



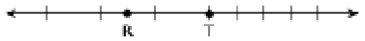
QuestionId: 50063, Standard 6 "NS", Benchmark 6 "6", Indicator "a", Sub Indicator ""

16. Which procedure describes correctly how to graph the ordered pair (-2, 3) on the coordinate plane?
A) X Starting from the origin, move 2 units to the left, and move 3 units up.
B) Starting from the origin, move 2 units to the right, and move 3 units up.
C) Starting from the origin, move 2 units to the left, and move 3 units down.
D) Starting from the origin, move 2 units to the right, and move 3 units down.
QuestionId: 50064, Standard 6 "NS", Benchmark 6 "6", Indicator "c", Sub Indicator ""

17. The freezing point of water is 0 degrees C. Which temperature is 10 degrees below the freezing point of water?
A) 10 degrees C
B) 100 degrees C
C) X -10 degrees C
D) -100 degrees C
QuestionId: 50065, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

18. Antonio's account had a $5 balance before he went to the bank to make a deposit. What is his balance after putting $25 into this account?
A) $0
B) -$20
C) $25
D) X $30
QuestionId: 50066, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

19. Points R and T are plotted on the number line shown below.



Which sentence correctly explains the relationship between R and T?

A) X R is less than T, because R is to the left of T on the number line
B) T is less than R, because T is to the right of R on the number line
C) R is greater than T, because the number line extends to the left and right of R
D) T is greater than R, because the number line extends to the left and right of T
QuestionId: 50067, Standard 6 "NS", Benchmark 7 "7", Indicator "a", Sub Indicator ""

20. Bob owed $50 to a business. Then Bob paid $50 to the business. Which statement is true?
A) X Bob now owes the business $0.
B) Bob now owes the business $100.
C) The business now owes Bob $50.
D) The business now owes Bob $100.
QuestionId: 50068, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

21. The elevation of Village A is 5 meters below sea level. Village B is 4 meters above Village A. What is Village B's elevation, in relation to sea level?
A) 1 meter above sea level
B) X 1 meter below sea level
C) 9 meters above sea level
D) 9 meters below sea level

QuestionId: 50069, Standard 6 "NS", Benchmark 5 "5", Indicator "", Sub Indicator ""

22. Which sentence correctly explains why a temperature of 3 degrees below 0 is warmer than a temperature of 7 degrees below 0?
A) X -3 is greater than -7
B) -3 is not equal to -7
C) -3 is equal to -7
D) -3 is less than -7
QuestionId: 50070, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""

23. Which sentence below correctly explains why 6 meters below sea level is lower than 2 meters below sea level?
A) -6 is greater than -2
B) -6 is not equal to -2
C) X -6 is less than -2
D) -6 is equal to -2
QuestionId: 50071, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""

24. Family J spent 3 $^{1}/_{2}$ hours traveling to a campsite. Departing from the same place, Family K spent 3 $^{5}/_{6}$hours traveling to the same campsite. Which statement correctly compares the two families' traveling time?

A) Family J spent more time traveling because 3 $^{1}/_{2}$ > 3 $^{5}/_{6}$.
B) X Family J spent less time traveling because 3 $^{1}/_{2}$ < 3 $^{5}/_{6}$.
C) Family K spent more time traveling because 3 $^{1}/_{2}$ > 3 $^{5}/_{6}$.
D) Family K spent less time traveling because 3 $^{1}/_{2}$ < 3 $^{5}/_{6}$.
QuestionId: 50072, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""

25. Two gas pipelines are being placed in a town. Pipeline A is placed at a depth of 2 $^{3}/_{7}$meters below the ground and pipeline B is placed at a depth of 3 $^{1}/_{4}$ meters below the ground. Which statement is correct?
A) Pipeline A is deeper than pipeline B because -2 $^{3}/_{7}$ < -3 $^{1}/_{4}$.
B) X Pipeline B is deeper than pipeline A because -2 $^{3}/_{7}$ > -3 $^{1}/_{4}$.
C) Pipeline A is deeper than pipeline B because -2 $^{3}/_{7}$ > -3 $^{1}/_{4}$.
D) Pipeline B is deeper than pipeline A because -2 $^{3}/_{7}$ < -3 $^{1}/_{4}$.
QuestionId: 50073, Standard 6 "NS", Benchmark 7 "7", Indicator "b", Sub Indicator ""