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Applets and Groupwork in Intro. Electromagnetism

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Applets and Groupwork in Introductory Electromagnetism

An Interactive Qualifying Project Report

submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Bachelor of Science

by

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Warren Schudy

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Professor Carolann Koleci, Advisor

Abstract

The effect of applets and groupwork on exam grades in introductory college electromagnetism was measured experimentally. Three conference sections (class meetings of about 30 people to digest material presented in lecture) were used in the study. During a conference meeting a few days before each test, the conference sections were given applets, groupwork, or a traditional conference (for control). To correct for differences in the difficulty of the tests and the preparation of the students in different sections, each section was given groupwork, applets, and control in a different order.

The study (involving 95 students) was dominated by statistical noise of about ±2 percentage points for each test/section pair. The modules consisted of about 5% of the total class-time of thirty-five 50-minute classes, so a large increase in educational productivity would have been detected by this study.

It was determined that the population available was too small to get statistically significant results, so a qualitative study was conducted to improve the applets and groupwork.

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Acknowledgements

I would like to thank my advisor (Professor Koleci), the conference instructors who loaned me their conference time, the students who involuntarily gave me their conference time, and the students who attended my review session and endured me interviewing them. I would also like to thank Davidson physics for making the applets available that I used to construct the visualization portion, and MIT TEAL for offering some electromagnetism movies I used.

Introduction

Professors are increasingly departing from the standard lecture model of teaching where the professor writes on the blackboard and the students take notes. At WPI, professors occasionally use computerized applets to demonstrate physics concepts, and problem solving in groups is quite common. To a scientist, it is natural to ask to what extent these techniques help student understanding.

A study was designed to answer this question in the context of introductory electromagnetism at WPI, an engineering college with 2800 undergraduates and 950 graduate students [8]. The treatments were half-hour sessions in conference with applets, groupwork, or a traditional conference (at the discretion of the instructor) for the control. Each conference section was given each treatment over the course of the four exams in the course, so differences between the students in different sections and between exams could be corrected for.

The quantitative study revealed no statistically significant effect on student exam performance from the half-hour sessions. A follow-up, qualitative study was designed to gather information on how to improve the treatments, hopefully allowing statistical significance in a future study.

Literature Review

Dimensional analysis is the process of analyzing the dimensions (length, mass, time, charge) of equations. For an equation to give the same physical predictions regardless of the system of units used, the dimensions of all terms in an equation should be the same. Schmidt and Housen [7] describe how dimensional analysis can be used to reduce the number of apparent parameters in a problem by forming dimensionless ratios, reducing the number of parameters by the number of distinct dimensions in the problem. For example, a ball falling from rest involves a position, time, and acceleration, with two dimensions, length and time. This means that one dimensionless parameter characterizes the motion – the rational number 2.

Chi, Feltovich and Glaser describe the categorization of physics problems by novice (freshmen) and expert (graduate student) problem solvers [1]. They experimentally determined that freshmen generally classify problems based on surface features, such as rotation, springs, and gravity, while graduate students classify problems based on the preferred solution method, such as conservation of energy.

The University of Minnesota uses Cooperative Group Problem Solving to teach problem solving techniques [3]. They recommend groups of three students, fearing that two students are likely to have insufficient knowledge to solve the problem, but four students may not fully involve everyone. They recommend choosing groups with a mix of student abilities in each group. They offer students a problem solving strategy consisting of "Comprehend the problem, Represent the problem in formal terms, Plan a solution, Execute the plan, Interpret and evaluate the solution."

Maloney et al created a Conceptual Survey of Electromagnetism (CSEM) to measure student understanding the qualitative aspects of electromagnetism [1]. When given to students before taking an introductory college-level electromagnetism class, average scores varied from around 20 to 40%. When given after taking the class, average scores had only raised to around 30 to 70%. When given to an unspecified set of physics majors and graduate students, an average of 70% was obtained, and two-year college physics professors averaged 77%. A study of "two honors calculus-based engineering physics courses at a large research university" that used an "interactive engagement approach" yielded a 69% posttest average – almost the same score as the physics majors, and nearly as good as the two-year professors! One wonders if some students evidently understand the material poorly partly because some of the professors teaching them know the material less well than an average honors student.

The Force Concepts Inventory (FCI) tests students' qualitative understanding of Newtonian Mechanics. Hake [3] collects the results of the FCI being given to students before and after various high-school and university courses. Some courses were classified as using *Interactive-Engagement* methods, while some used traditional teaching methods. Hake found a gain from the pre-test to the post-test of 23%±4% for traditional classes, and 48±14% for interactive-engagement. These distributions barely overlap, with the interactive-engagement courses being over twice as effective on average. Further, in many of the classes a more quantitative post-test was also given (Mechanics Baseline), and the MB and FCI post-test scores were strongly positively correlated with r=0.91. Interactive-engagement appears to improve both qualitative and quantitative student performance.

Davidson's Physics Department has created a number of Java applets (which Davidson terms Physlets®) demonstrating a wide variety of physics concepts, including mechanics, electromagnetism, relativity, quantum mechanics, and statistical mechanics [7]. Each applet can be configured to be used in a variety of physical scenarios. These applets may be used freely for non-profit educational use, subject to crediting Davidson. A book, *Physlets: Teaching Physics with Interactive Curricular Material* by Wolfgang Christian and Mario Belloni, describes their recommended use of the applets. The present project used several of these applets, but did not use the recommended problem, for reasons which will be discussed later.

Massachusetts Institute of Technology (MIT) is investigating ways to integrate experimentation and visualizations into introductory physics courses in the TEAL (Technology Enabled Active Learning) project. The TEAL project includes a number of videos and applets designed to showcase electromagnetism concepts. One of these videos was used in the present project. Like the Davidson Physlets®, the TEAL animations may be used freely for educational use if credited.

Experimental Design 1

WPI has four 7-week terms per academic year, with an optional fifth summer term. Students normally take three courses per term. The introductory physics sequence consists of four courses, covering respectively mechanics, electromagnetism, relativity/quantum mechanics, and waves. The first two courses are offered in two versions, assuming different levels of mathematical background and rigor. The lower-level mechanics course, PH1110, requires concurrent study of differential calculus. The higher-level course, PH1111, requires concurrent study of vector calculus. The two versions cover similar topics, but PH1110 lacks the mathematical background to prove important theorems such as work-energy. PH1111 covers rotation in three dimensions, while PH1110 is confined to two dimensions.

I decided to concentrate on teaching methods for electromagnetism, not mechanics, since electromagnetism involves concepts which are relatively unfamiliar and therefore hard to understand and visualize. Since learning electromagnetism is harder, more might be gained by improving electromagnetism education.

The electromagnetism courses are similarly divided into a section requiring concurrent study of integral calculus (PH1120) and vector calculus (PH1121). PH1120 does not use much calculus since the students do not have adequate exposure. Gauss's and Ampere's laws are not taught to PH1120 students B-term (the second fall term) since the class contains many freshmen who are concurrently taking integral calculus. When

PH1120 is taught the second spring term (D-term), most students have been exposed to integrals previously so the integral laws (Gauss's and Ampere's) are taught.

This study concentrated on PH1120 since the number of students is much larger, around 300 vs. around 60 for PH1121.

PH1120 has 4 exams. The first exam covers electric force and Coulomb's law. The second covers potential and capacitors. The third covers circuits and the magnetic force law. The fourth exam covers the Biot-Savart law, solenoids, infinite wires, and Faraday's Law of Induction. There is no cumulative final exam.

A study was conducted in D-term 2004, investigating whether exposure to applets or groupwork would improve student performance on exams. An applet is a small computer program viewable with a web browser allowing students to visualize electromagnetic fields.

I elected to use the exams to measure student performance. One could potentially use a well-known test, such as the Conceptual Survey on Electricity and Magnetism [1]), facilitating comparison with other work. Exams have the major advantage that using them requires no additional effort on the part of students (to take another exam) or me (to grade another exam). Furthermore, the existing instruments focus on concepts, unlike most exams which emphasize problem-solving. I defer to most professors' implicit decision to emphasize problem-solving ability as a good measure of students understanding of the material. Many problem-solving type questions cannot be answered without some conceptual knowledge, so exams measure, to some extent, both types of understanding. This choice also eliminates a possible conflict of interest where I might be tempted to teach the concepts that I would be testing.

Several conference instructors (called recitation sections at some colleges) graciously agreed to let me use part of one meeting a few days before each exam to do applets or groupwork. Another treatment consisted of the conference instructor doing whatever he or she thought appropriate. Several conference instructors used groupwork frequently in their normal conference instruction, which would be expected to reduce the difference between the groupwork and control treatments. The conference instructors did not make use of applets during the control treatments.

To correct for variation in the difficulty of exams and the background of the students in each section, every section was exposed to every type of instruction, and every exam had a section doing each type of instruction.

The conference sections were Professor Koleci's 9:00 section, and Professor Turner's 12:00 section, and Professor Quimby's 2:00 section (Professor Quimby was also the lecturer). I will refer to these as section 2, 3 and 6 respectively.

	Section		
	2	**3**	**6**
Test 1 (Electric forces, fields & Gauss's law)	Control	Applet	Group
Test 2 (Electric Potential)	Applet	Group	Control
Test 3 (Circuits & Magnetic forces)	Group	Control	Applet
Test 4 (Biot-Savart, Ampere, Lenz laws)	Control	Applet	Control

For example, group 1 had extra conferences for test 1, group work for test 3, and applets for test 4.

Design of Groupwork

The groupwork was designed to cover the topics covered in previous years' exams. Mirroring the exams, most of the problems were quantitative, with a few qualitative questions. To mitigate the possibility that an observed difference in student performance would be caused by a difference in the topics presented, not the method of instruction, one of each set of groupwork problems was designed so that it could be incorporated into the applet treatment. The selection of questions was also guided by my experience as a tutor, emphasizing concepts that students were likely to have difficulty with.

The groupwork exercises were designed so that most students would not finish in the half-hour allotted. This ensured that no one would run out of things to do. The actual groupwork exercises are provided in the sections that follow.

Groupwork for exam 1:

Three point charges are placed at the corners of an imaginary square. The charges are +2 microcoulombs at (1 cm, 1 cm), -3 microcoulombs at (1 cm, -1 cm), and -1 microcoulombs at (-1 cm, 1 cm). To make your work more readable, choose variables for the square's side-length and the values of the three charges, and substitute numbers at the end.

1. *Find the electric field (a vector) at the -3 microcoulomb charge, due to the two other charges.*
2. *Find the net electrostatic force (a vector) on the -3 microcoulomb charge.*
3. *Find the electric flux through an imaginary spherical shell centered at the origin with radius 3 cm.*
4. *Can the result from #3 be used to determine the electric field at an arbitrary point on the sphere, such as (3 cm, 0 cm)? Explain.*
5. *A conducting spherical shell with net charge of +2 microcoulombs is now placed around the point charge. The spherical shell has a radius of 2 cm and is centered at the origin. Determine the net charge on the inside and outside surfaces of the spherical shell. Also sketch the resulting electric field lines.*
6. *(Extra Credit) Repeat questions 3 and 4, but including the shell that was introduced in question 5.*

Groupwork for exam 2:

1. *A point charge of charge 5 μC is placed at the origin. A second point charge -4 μC is placed at (2, 3) meters. What is the electrostatic potential at (-4, 5) meters?*
2. *A third charge (-7 μC) is brought in from infinity to (-4, 5) meters. How much work must an external agent provide to accomplish this?*
3. *A 5 μF capacitor, a 3 μF capacitor, and the parallel combination of three capacitors (3 μF, 6 μF, and 9 μF) are in series. All the capacitors start*

discharged, and a battery is connected to the circuit. When the circuit reaches equilibrium, the 6uF capacitor has a charge of 12 μC. What is the voltage of the battery? Find the charge of and voltage across the 5 μF and 9 μF capacitors.

4. *An electrostatic potential is given by V = g(x) (e.g., V = 1/x). Describe how to find the electric field as a function of position.*
5. *An electric field is given by E = f(x)*i (e.g., E = x*x*i). Describe how to find the potential as a function of position.*

Groupwork for exam 3:

1. *A 1500W, 120V appliance has a 2-meter power cord, and each wire is copper and 3 mm in diameter. What is the resistance of each 2-meter piece of wire?*
2. *How much power is dissipated in the power cord? Give the total power dissipated in both wires.*
3. *A 5V battery, 1 Ohm resistor, 4 Ohm resistor, 7 μF capacitor, and 5 μF capacitor are connected in series to make a closed loop. The capacitors are initially uncharged. Find the current through the circuit, voltage across the 1 Ohm resistor, and charge on each capacitor as a function of time.*
4. *What do the "C"s stand for in the equations "Q = CV" and "Q = 5 mC" ?*

Groupwork for exam 4:

1. *An infinite wire with current 1A is bent into a "U"-shape, where the bottom of the "U" is a semi-circle of radius R, and the legs are parallel. Find the magnetic field at the center of the semi-circle.*
2. *Find the magnetic field of an infinite straight wire of non-zero radius R inside and outside the wire. Assume the current is uniformly distributed throughout the wire. Use ampere's law, not a memorized formula.*
3. *Two parallel wires a distance "d" apart have a third wire placed across them and slid along by an external agent at speed "v". The region between the wires has a uniform magnetic field of magnitude B. Find the induced emf twice: by F = qvxB, and by the universal flux rule.*
4. *(Extra credit) An infinite straight wire with current 2A is added to the configuration of question 1, perpendicular to the plane of the "U" and passing through the center of the semicircle. Find the net magnetic force on this wire. (Hint: little calculations and no integrals required).*

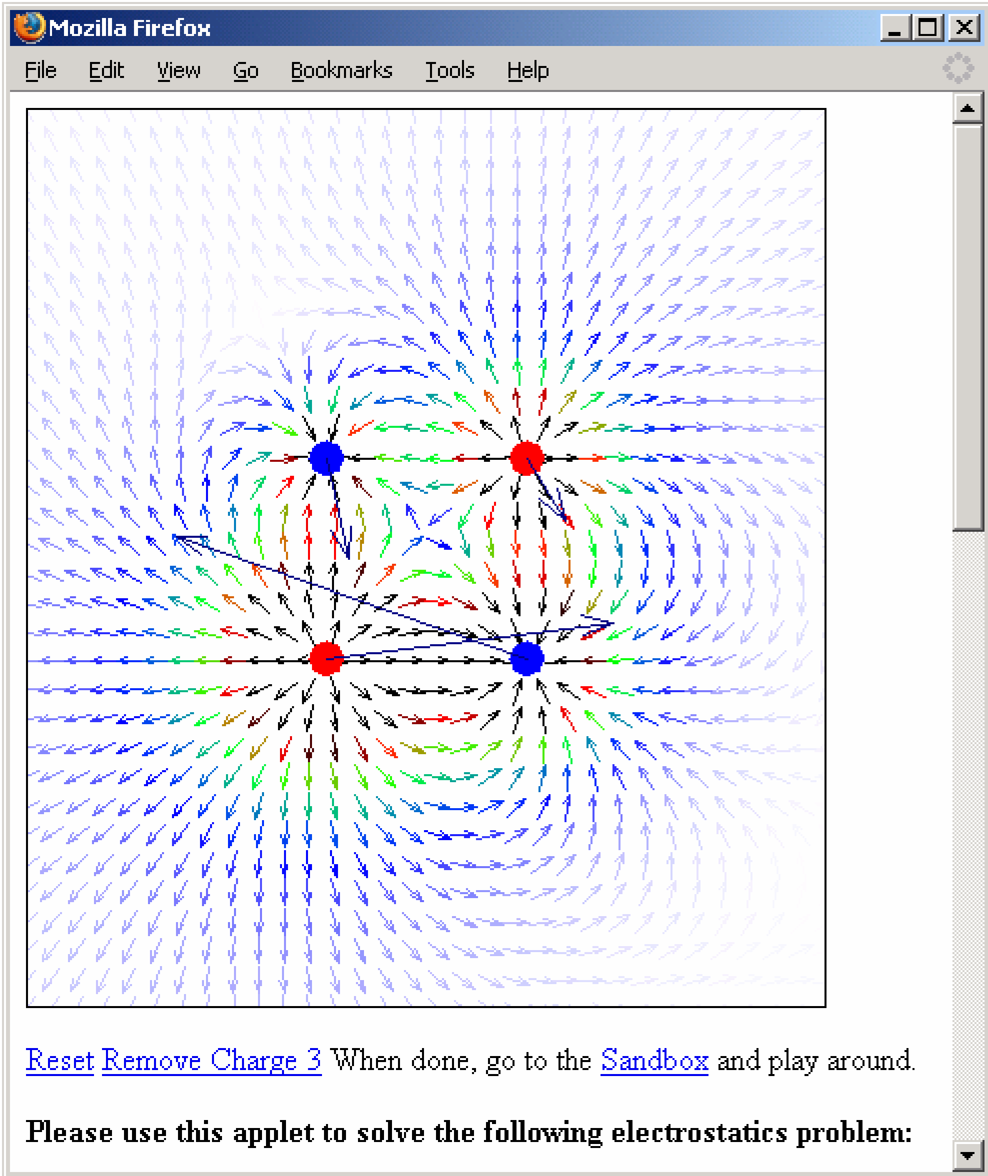
Design of Computerized Applets

Davidson's Physics Department has made a series of applets that simulate various aspects of mechanics, electromagnetism, relativity, and other topics in introductory physics. The applets are highly customizable, allowing an arbitrary configuration of charges and currents to be selected by the designer of the webpage that contains the applet. I used two of Davidson's electrostatics applets, a circuit applet, and a magnetostatics applet in this study.

The applets were designed to exploit their benefits for visualization. When possible, the applets paralleled the material covered in the groupwork to ensure that the difference was in the method of instruction, not the material instructed.

The applets were designed to offer the students flexibility to observe field lines of various configurations of charges and currents in a pseudo-experimental fashion. I thought that sort of applet would be interesting to me (and hopefully also for the students). This takes advantage of the flexibility of computer over static visualizations.

The first applet:



The text:

> *Four point charges are placed at the corners of an imaginary square. The charges are +2 microcoulombs at (1 cm, 1 cm), -3 microcoulombs at (1 cm, -1 cm), -1 microcoulombs at (-1 cm, 1 cm), and +5 microcoulombs at (-1 cm, -1 cm). Find the electric field (a vector) at the -3 microcoulomb charge, due to the three other charges. Also find the net electrostatic force (a vector) on the -3 microcoulomb charge.*

Walkthrough

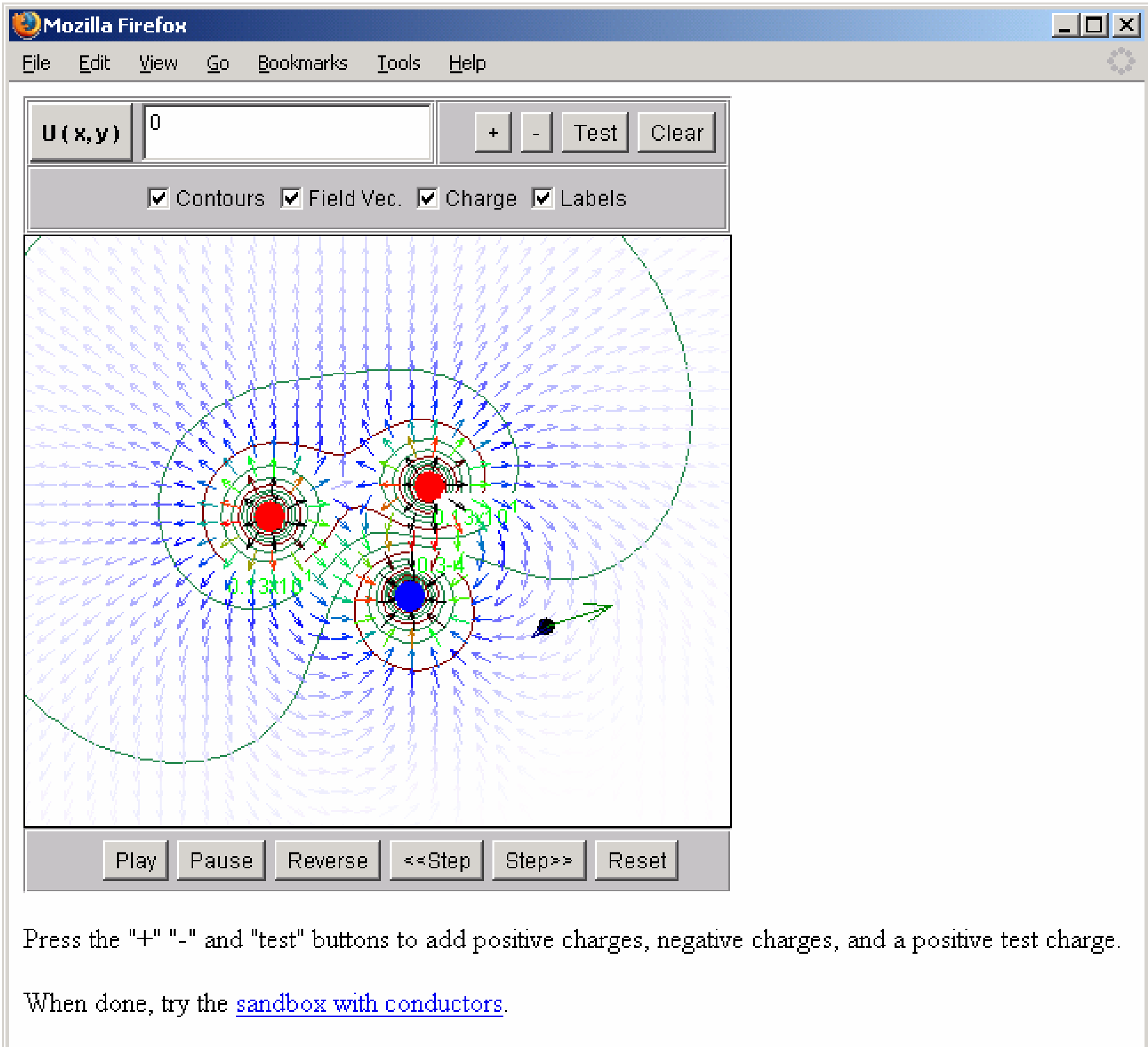
The blue and red colored dots correspond to the point charges described in the problem. The Long black arrows correspond to the force on each point charge. The small black dot is an additional test charge of negligible charge that can be used to measure the field. The smaller colored arrows indicate the strength and direction of the electric field. The arrows point in the direction of the field, and the color indicates the strength. Be aware that an electric field value given by the applet of 1 actually corresponds to a field strength of 10 to the 8 Newtons per Coulomb!

Try clicking the mouse on the applet and dragging around. The position and electric field strength will be shown in a yellow box in the lower-left corner of the applet. Try clicking on the charge in the lower-right (the -3 microcoulomb one the question asks about). The simulation refuses to give the value of the field at that point because the field goes to infinity near a point charge. To calculate the field at that point due to the other charges, one needs to remove the offending -3 microcoulomb charge. Click on the link to remove that charge, and click-drag until you find the field where that charge used to be. Write down the magnitude and direction of the electric field at that point. Now click the "reset" link to get all four charges back. Does the direction of the force on the -3 microcoulomb charge make sense, given the direction of the field noted earlier?

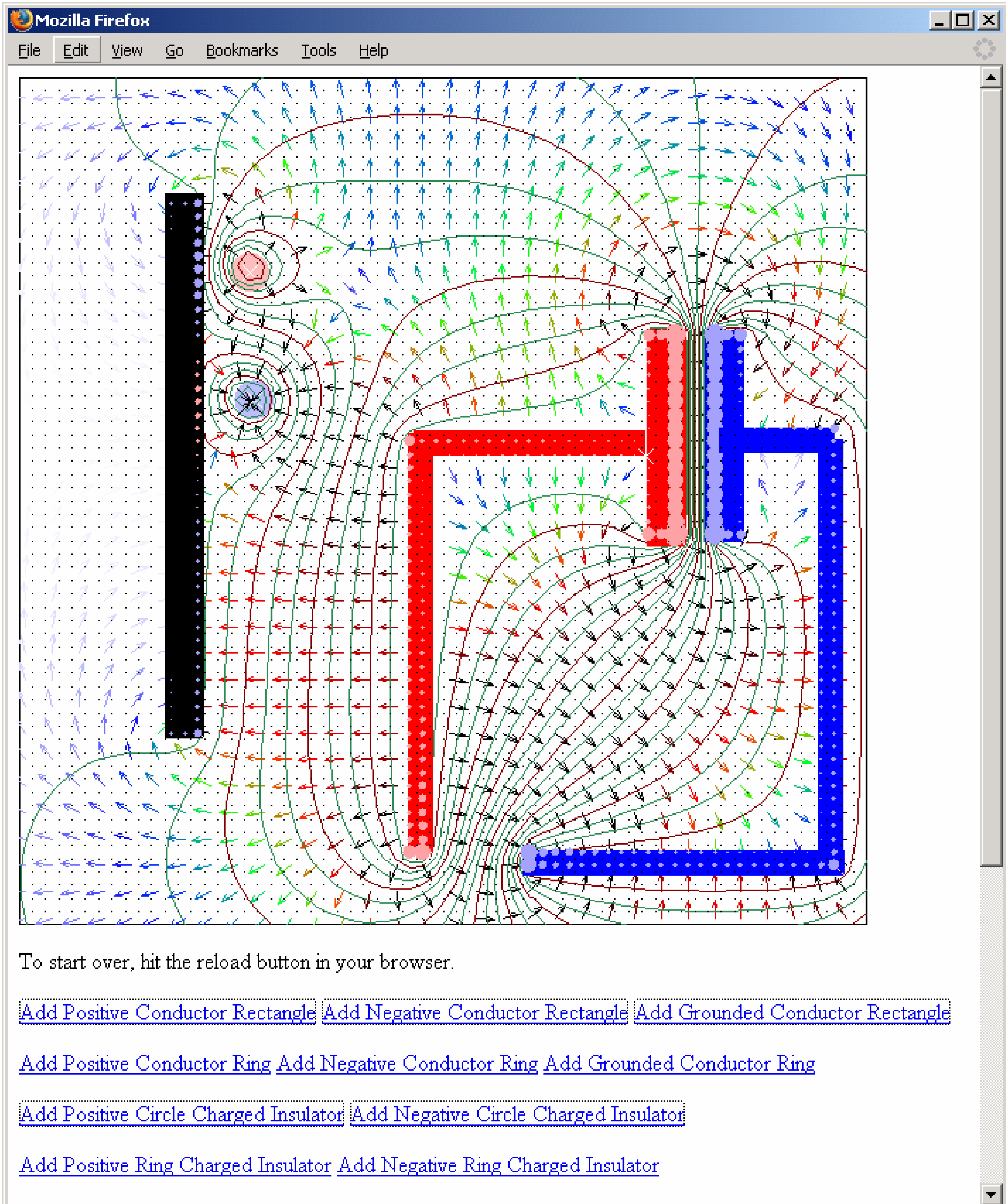
When done, go play around with an empty sandbox.

The applet used in this webpage was designed and programmed by the Davidson Physics department. **Error! Hyperlink reference not valid.**The scripting that makes use of the applet, and the accompanying text, is by Warren Schudy, of WPI.

The “sandbox” is a relatively unstructured applet that encourages students to build various configurations of charges and observe the resulting field lines:



The sandbox with conductors:



The instructions:

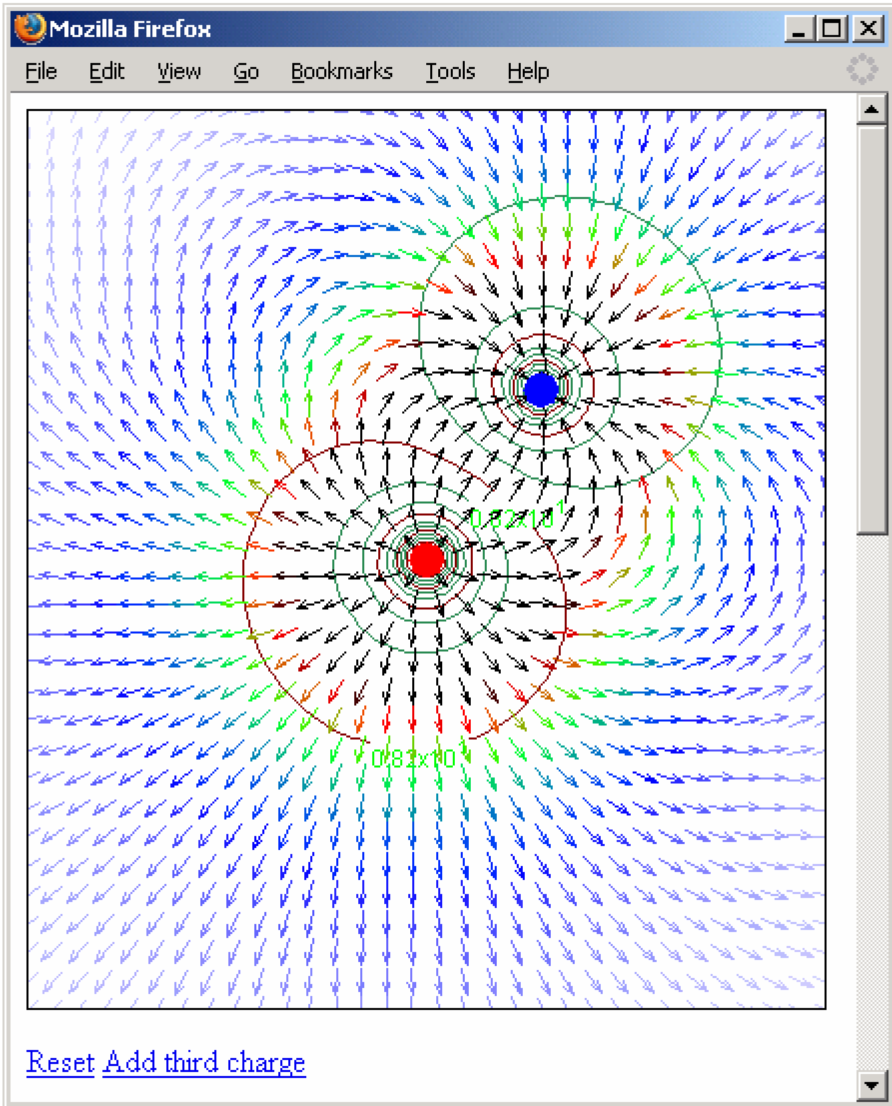
> Click one of the links to add conducting or insulating objects. The conductors each have a fixed potential (+ or - 10 volts), and the insulators each have a constant charge density, though the rings have a lower charge density than the circles.

You can drag the charges and conductors around, and resize them. The tiny light red/blue circles that appear on conductors correspond to the induced surface charge. Be aware that the surface charge may not quite appear to be on the surface of the conductor due to simulation error.

Try making things such as a parallel plate capacitor, a coaxial wire (positive conducting circle inside grounded conducting ring) and a circuit with a resistor and a battery (use conducting rectangles for the wires, leaves spaces for the battery and resistor). Also try placing a positive conductor inside a conducting box made of four grounded rectangles. Observe how the electric field outside the box is zero. Next shrink the rectangles so that there are gaps in the corners of the box and observe the results.

In all cases, observe how the electric field is always perpendicular to the surface of conductors, and the potential lines are parallel to the surface.

The second applet:



Please use this applet to solve the following electrostatics problem:

> *A point charge of charge 5 nC is placed at the origin. A second point charge -4 nC is placed at (2,3) meters. What is the electrostatic potential at (-4,5) meters? A third charge (-7 nC) is brought in from infinity to (-4, 5) meters. How much work must an external agent provide to accomplish this?*

Walkthrough

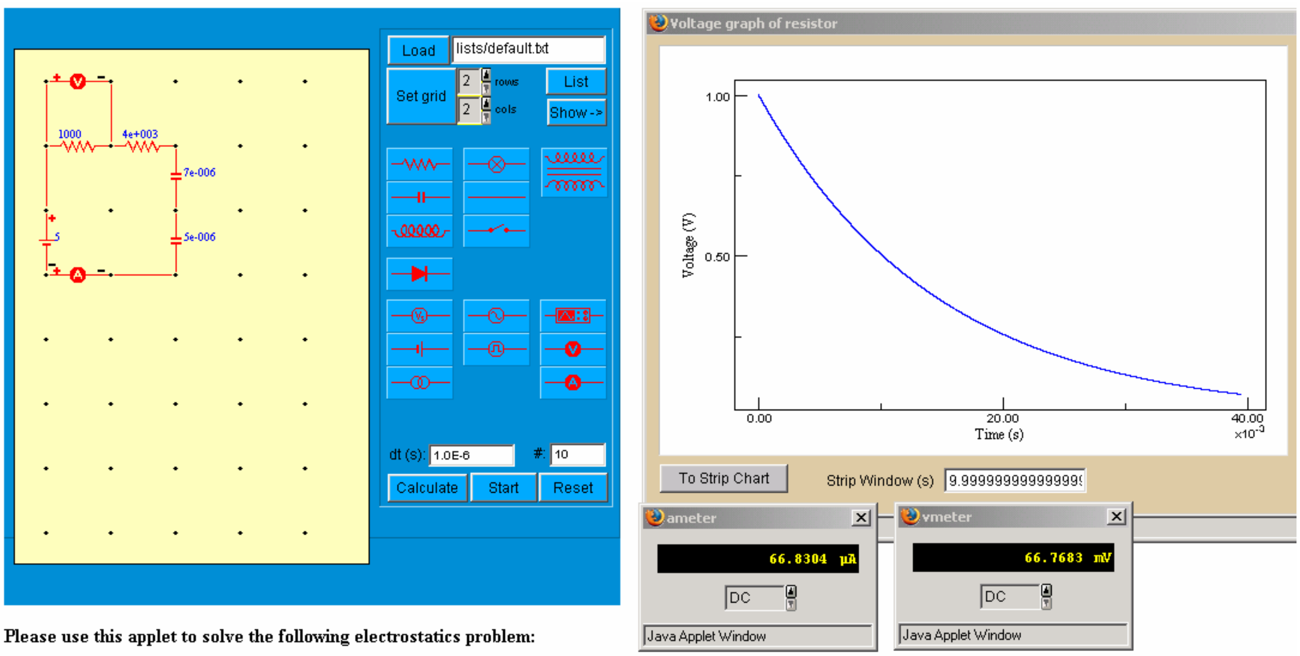
The blue and red colored dots correspond to the point charges described in the problem. The small colored arrows indicate the strength and direction of the electric field. The arrows point in the direction of the field, and the color indicates the strength. The applet gives x and y values in meters, potential values in Joules per Coulomb, and field values in Newtons per coulomb.

Try clicking the mouse on the applet and dragging around. The position, potential, and electric field will be shown in a yellow box in the lower-left corner of the applet.

When done, go play around with an empty sandbox of point charges or conductors and charges.

The applet used in this webpage was designed and programmed by the Davidson Physics department. **Error! Hyperlink reference not valid.**The scripting that makes use of the applet, and the accompanying text, is by Warren Schudy, of WPI.

The third applet:



Please use this applet to solve the following electrostatics problem:

> *A 5V battery, 1 kilohm resistor, 4 kilohm resistor, 7 microfarad capacitor, and 5 microfarad capacitor are connected in series to make a closed loop. The capacitors are initially uncharged. Find the current through the circuit, voltage across the 1 kilohm resistor, and charge on each capacitor as a function of time.*

Instructions

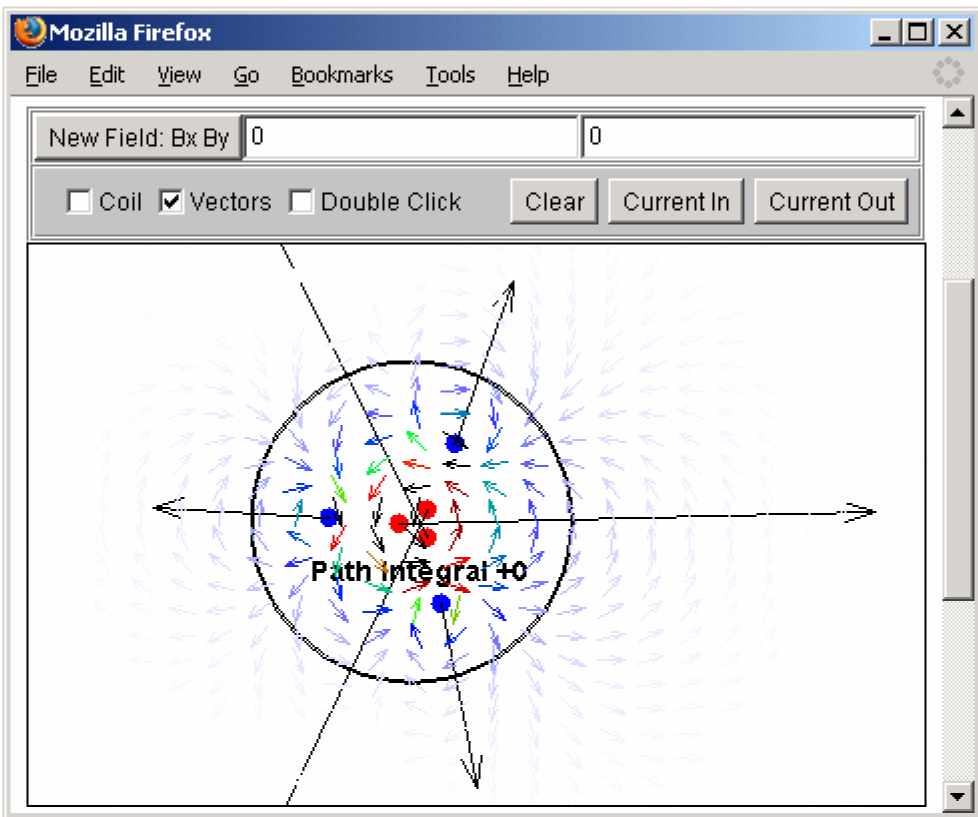
Watch the demonstration on the overhead for instructions on how to use the applet. Be aware that there isn't a way to get the time constant from the applet explicitly, so you'll have to be slightly clever and extract the time constant from a graph.

When done, play with different circuits.

The applet used in this webpage was designed and programmed by the Davidson Physics department. **Error! Hyperlink reference not valid.**The scripting that makes use of the applet, and the accompanying text, is by Warren Schudy, of WPI.

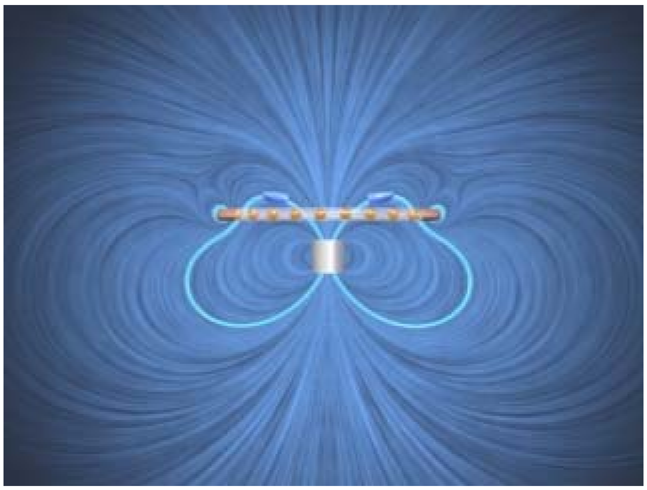
The fourth applet:

The following applet lets you draw infinite straight wires with current, and calculate the path integral, to verify ampere's law. Hit the current in/out buttons to add currents into and out of the page. The black arrows connected to each wire represent the force on that wire. The small colored arrows represent the magnetic field. Try making a coaxial cable, by putting 6 wires of one type in a clump with 6 of the other type in a circle around them. Notice how the magnetic field is very weak outside the cable.



When done with the applet, please watch a few animations from this page: the levitating ring and the falling ring with finite resistance. Each demonstrates Faraday's law of induction. As the rings fall, the magnetic flux through the ring changes, inducing an emf which produces a current in the ring, which, according to Lenz's law, partially counteracts the change in the magnetic flux through the ring. In fact, in the case of the perfectly conducting ring, the magnetic flux through the ring never changes. Look at the animations, and try to understand why the ring moves the way it does.

This is a shot of one of the animations, "Falling Ring with finite resistance". This animation is from MIT's TEAL project [6].



Experiment 1 Results

The data was first analyzed in an ad-hoc manner by adding a constant to each test and conference section so that the average grade for each conference and each test was zero. This eliminated the effect due to differences in the background of the students and test difficulty, (if performance is a linear combination of conference ability, test difficulty, and instruction type) leaving the variation caused by noise and the instruction type. The data is shown below.

Original Data:

		Exam 1	2	3	4	Ave
Section	**2**	64.58	56.30	55.21	57.50	58.40
	3	54.06	51.49	48.91	51.97	51.60
	6	49.21	49.17	47.18	48.05	48.40
	Ave	55.95	52.32	50.43	52.50	52.80

After subtracting section mean from every datum:

	1	2	3	4	Ave
2	6.18	-2.09	-3.19	-0.90	0.00
3	2.45	-0.12	-2.70	0.36	0.00
6	0.81	0.77	-1.22	-0.35	0.00
Ave	3.15	-0.48	-2.37	-0.30	0.00

After subtracting exam mean from each datum:

	1	2	3	4	Ave
2	3.03	**-1.61**	*-0.82*	-0.60	0.00
3	**-0.69**	*0.36*	-0.33	**0.66**	0.00
6	*-2.34*	1.25	**1.15**	-0.06	0.00
Ave	0.00	0.00	0.00	0.00	0.00

Average by treatment:

applets -0.499 *group* -2.793

In the last table, the section/exam combinations that were given applets are bolded, groupwork italicized, and control normal font. The average by treatment is shown at the bottom of the table.

The data suggests that groupwork helped student performance on exam 2 and hurt for exam 1 and 3, while applets helped for exams 3 and 4 while hurting performance on exams 1 and 2. The noisiness in this data is supported by the fact that the third applet seemed to improve student performance more than the others, yet that applet involved circuits, and was presumed to be relatively ineffective due to poor reliability and little visualization advantage.

A rigorous analysis was conducted using the general linear model facilities of SPSS 11 (Statistical Package for the Social Sciences). This analysis supports the conclusion that the data is consistent with the null hypothesis that instruction type has no effect on exam performance.

Experimental Design 2

It was determined that a repeat of the experiment with a somewhat larger population the next term would be unlikely to produce statistically significant results. I decided to do a qualitative study, using interviews to determine the students' attitude towards the modules and solicit recommendations for improving the modules.

For a qualitative study, a self-selected sample would not be quite as damaging as it would be in a quantitative study. The sessions for the qualitative study were held outside of class, in the evenings for 90 minutes. Each session included a groupwork activity, applets, and worked solutions for some of the problems.

I prepared sessions for the first and fourth test, focusing on electrostatics and magnetostatics respectively. Unfortunately, no students responded to my emailed request for preferred meeting times for the magnetostatics module, so that module was canceled.

Applet Modifications

Several changes were made to the applets. Since a qualitative comparison does not require a one-to-one correspondence between the applets and the groupwork, the emphasis was shifted a bit towards visualizations. The students did the groupwork before seeing the applets, so the applet that visualized the problem in the groupwork was reinterpreted in that light. The students were directed to concentrate on the sandbox with conductors.

The applet for circuits was unreliable. Circuits are easier to visualize with pencil and paper than electric fields, so the applet was also less useful. The applet for circuits was therefore not given to the students during the second run of the experiment.

Problem Solving Document

To encourage attendance, participants were given a document I wrote describing tricks of the trade in physics problem solving, including counting equations and unknowns, dimensional analysis, and checking limiting cases. The complete document is given in an appendix. This document was developed from my experience as a physics undergraduate as well as experience tutoring students as part of WPI's Math and Science Help (MASH) program for three years. Student feedback on the document is described in the interview section.

Experiment 2 results

Eight students showed up to the first session. The session worked according to plan. In response to student request, I gave the solutions to several groupwork problems on the board immediately after the scheduled session. I also posted solutions online.

For the magnetism module, no students responded to an emailed request for times to meet. Therefore, no results could be obtained.

Interview Methodology

Interviews were 20 minutes and audiotaped. Eight students showed up to the session. I successfully arranged an interview with five out of the eight students.

A set of questions was prepared, but questions were omitted or added as appropriate for the flow of the interview. The major topics of the interview were how the students thought the groupwork, applets and problem solving document affected their understanding of electromagnetism, and how it could be improved. The note sheet used in the interviews follows. Questions were not read verbatim.

- Year, major
- Before the review session, what concepts in E&M did you think you understood? What concepts did you think you did not understand?
- Did the group problems affect what you thought you understood? If so, how?
- What did later experiences (such as the exam) reveal about what you actually understood compared to what you thought you understood?
- Did the group problems affect your understanding of E&M?
- In general, do you find working in groups to be more or less helpful than doing similar problems by yourself? How about this group problem solving exercise?
- Did you look at the solutions to the group problem I posted? Any comments?
- Recommend a way to improve the group problem solving activity.
- Do you recommend groupwork as a good way to help one study?
- How familiar were you with the problem solving techniques I described in the problem solving document from past experience?
- Have you used it to solve problems? Was it helpful?
- Suggest a way to improve the problem solving methodology document.
- What experience have you had in the past with applets?
- Did the applet Wednesday affect your understanding of E&M? How?
- Did you refer to the applet after the review session?
- Recommend a way to improve the applets.
- Do you recommend applets as a good way to help one study?

Since some questions might be embarrassing to the students, this part of the experiment was not exempt from Institutional Review Board (IRB) approval. WPI's IRB approved the experimental design and interview methodology for human subjects. The email granting approval is given in an appendix.

Interview Results

Of the eight participants in the first session, I managed to interview five. Interviews were conducted in various unoccupied public places, including lounges on the first floor of Salisbury Labs and a meeting room in the library.

Of the five students interviewed, four were freshmen and one was a junior. Two were mechanical engineers and three were biomedical engineers. The proportion of freshmen is expected for an introductory class, but the proportion of biomedical engineering majors does not reflect the distribution of majors at WPI. This is presumably caused by a combination of statistical noise and the self-selected sample. Three of the five interviewees were female. This is a much greater proportion that the WPI undergraduate body as a whole, which is about 20% female. To protect the anonymity of the interviewees, I will refer to the interviewees using the male pronoun "he" regardless of actual gender.

All of the interviewees thought they understood some of the material, but were not comfortable with some or all of it. Two of the interviewees realized after the groupwork that they needed to study more, one realized he knew more than he thought, and two did not notice a change in what they thought they understood. Three of the interviewees stated that they thought they understood the material before taking the exam, but realized when taking it that they did not.

All of the interviewees were familiar with dimensional analysis from prior experience. Four of the five interviewees had seen most of the problem solving techniques before, two of which mentioned that they had been exposed to them in high school. Two of the interviewees found the document helpful, two were already using the techniques, and one did not spend the time to review it. Two students recommended more electromagnetism examples (the document uses a mechanics example).

Two of the interviewees had no prior experience to applets. The remaining interviewees had seen applets a few times, including one that mentioned Mastering Physics in PH 1110. Four of the interviewees thought the applet affected their understanding of electromagnetism, all in positive ways. The other interviewee spent too much time understanding the applet to learn any electromagnetism. Three of the interviewees did not look at the applet after the review session, one looked at it for about 10 minutes, and another looked at it intermittently for three hours while studying.

Conclusions and Future Work

First, a quantitative study of the effects of groupwork and applets was done by giving the students in different conference sections either groupwork, applets, or ordinary conference for about half of a 50-minute conference a few days before each exam. The students' exam scores were compared between sections and exams, revealing that method of instruction for half an hour out of about 10 hours of in-class preparation per exam is insufficient to overcome statistical noise in student performance of a few percent. This result is consistent with Hake's findings [3] that interactive-engagement techniques in mechanics provide similar results as traditional methods if implemented inexpertly or incompletely.

Having learned first-hand the difficulty in successfully doing experimental educational research, I conducted a second, qualitative study the following fall, with out of class study sessions with both applets and groupwork, and post-experience interviews. Five of the eight participants were interviewed. The interviewees were primarily happy with the experience. Some suggestions for improvements were made, however the suggestions were relatively minor.

In the future, it would be useful to do a quantitative experiment similar to this one but using a more significant fraction of the total class time. This would hopefully increase the effect and produce statistically significant results. It would be useful to do Conceptual Survey of Electromagnetism and Force Concepts Inventory pre and post tests at WPI to calibrate the student population relative to the literature.

Professors should reconsider the mix of conceptual and problem-solving questions on exams, to ensure that the questions accurately reflect what they want the students to learn from the course. A typical exam measures problem-solving ability almost exclusively, possibly encouraging memorization of formulae at the expense of basic principles.

The literature describes many techniques available for teaching that have been shown to improve student performance on qualitative and quantitative questions as much as or more than traditional instruction. Since these methods have been shown to do no worse than traditional instruction, instructors should use these techniques extensively.

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Appendix A: IRB Approval

Here is the final email granting approval. The document he approved is given after the email.

Date: Thu, 18 Nov 2004 09:14:36 -0500
From: "Shannon, Thomas A." <tshannon@WPI.EDU>
To: "Schudy, Warren J" <wschudy@WPI.EDU>
Subject: RE: IRB

This is fine. Thanks for your cooperation. Tom

-----Original Message-----
From: Warren Schudy [mailto:wschudy@WPI.EDU]
Sent: Wednesday, November 17, 2004 12:01 AM
To: Shannon, Thomas A.
Cc: Koleci, Carolann
Subject: RE: IRB

I have attached the updated consent form and interview. I may ask additional questions as necessary to elicit complete responses to these.

-wjs

On Mon, 8 Nov 2004, Shannon, Thomas A. wrote:

> I think numbering the consent forms and keeping the list with names
and
> numbers on a separate sheet in a relatively secure place will be
fine.
Also
> I would suggest changing a sentence in the form. You say " I hereby
hold
the
> experimenter harmless," Perhaps a better phrasing would be "I will
not
hold
> the experimenter responsible for any ....."
>
> Othewise ok
>
> Please send me a final copy of the questionnaire, the consent form
and if
> convenient, an abstract of the research. I'll then draft a formal
response.
> But this is fine as it. Thanks Tom
>
> -----Original Message-----
> From: Warren Schudy [mailto:wschudy@WPI.EDU]
> Sent: Friday, November 05, 2004 11:14 PM
> To: Shannon, Thomas A.
> Cc: Koleci, Carolann
> Subject: RE: IRB
>

> I have attached a brief description of the project and a tentative
list of
> interview questions. It is in MS Word format. Let me know if you have
> further questions.
>
> -wjs
>
> On Mon, 25 Oct 2004, Shannon, Thomas A. wrote:
>
> > Send me a copy of the proposal and particularly the interview
section.
In
> > particular will you keep any personal identifiers with the
interview
> > material? Will you code it. What we need to know is how you will
protect
> > their privacy. After I review the proposal and you tell me more
about
the
> > interview part, it may be necessary for you to draft a consent form
for
the
> > students to sign--but we will cross that bridge is you need to.
TAS
> >
> > ________________________________
> >
> > From: Warren Schudy [mailto:wschudy@WPI.EDU]
> > Sent: Mon 10/25/2004 4:15 PM
> > To: Shannon, Thomas A.
> > Cc: Koleci, Carolann
> > Subject: IRB
> >
> >
> >
> > As part of my IQP on education methods, I plan to offer students
extra
> > help sessions using alternate educational techniques such as
computerized
> > applets and groupwork. I also plan to interview a subset of the
students
> > about their experiences. From my reading of
> > http://www.wpi.edu/Academics/Projects/humansubjects.html, the
educational
> > portion of the project is exempt from the regulations, but the
interview
> > is not, because students may tell me sensitive information (about
their
> > performance in the class or opinion of professors) as described by
> > exemption 2b.
> >
> > According to Professor Koleci you are (or were recently) the head
of the
> > IRB. What do I need to do to get approval to collect moderately
sensitive
> > information?
> >

> > Thanks, wjs

Here's the description of the project I gave the students:

I, Warren Schudy, a physics major, am studying (as part of my IQP) ways to help you learn electromagnetism more effectively. I need volunteers to try alternate learning methods and comment on how well they work. As a bonus, participants will receive a guide describing how to solve physics problems and check whether answers are correct.

I have scheduled a review session for this Wednesday evening (November 3rd) from 8:30-10:00 pm. Meet in Higgins Labs 202. Participants will practice physics problems in groups and use a computer program to visualize electric fields. Be aware that by attending you are *not* committing to attend any future review sessions I may hold. I will randomly select some participants to be interviewed for 20 minutes about their experience. Interviewees will receive a small monetary compensation for their time, so I expect participants in the review session to be interviewed if selected.

(End of description given to students)

The purpose of the study is to determine student perceptions of applets and groupwork.

I plan to record an audiotape of the interview sessions. I don't see an immediate need to associate the name of the subject with the interview, but I might want to know later to ask for clarifications. I am inclined to number the interviewees and label the interview by number, but make a separate note of the name/number association in case follow-ups are desirable. If the existence of the number/name mapping is problematic, I can easily do without.

I will keep the tapes strictly confidential. Written excerpts of the interviews will be shared with my advisor, possibly allowing my advisor to infer the identity of the subjects. Written excerpts will also appear in the IQP report, after me and my advisor have ensured that the subjects identity cannot be inferred from the excerpts.

Here's the consent form:

Warren Schudy (the experimenter) is conducting a study of different educational techniques in introductory electromagnetism (PH 1120) at WPI. The experimenter will be offering 90-minute study sessions about 3 days before exams 1 and 4, utilizing a variety of techniques, including groupwork and computerized applets to visualize electric and magnetic fields.

I hereby grant the experimenter the non-exclusive right to reproduce written descriptions or transcripts of my participation in the review sessions and interview. I understand that my name and personally identifiable information will be kept confidential.

I will not hold the experimenter responsible for any possible decreased performance in the course or anxiety caused by being reminded about past events during the interview. I am aware that my participation is voluntary and I may leave the review sessions or interview at any time. I may also decline to answer any interview questions.

(End of consent form)

A list of interview questions follows, though I may ask other questions in response to subject responses.

- Year, major
- Before the review session, what concepts in E&M did you think you understood? What concepts did you think you did not understand?
- Did the group problems affect what you thought you understood? If so, how?
- What did later experiences (such as the exam) reveal about what you actually understood compared to what you thought you understood?
- Did the group problems affect your understanding of E&M?
- In general, do you find working in groups to be more or less helpful than doing similar problems by yourself? How about this group problem solving exercise?
- Did you look at the solutions to the group problem I posted? Any comments?
- Recommend a way to improve the group problem solving activity.
- Do you recommend groupwork as a good way to help one study?
- How familiar were you with the problem solving techniques I described in the problem solving document from past experience?
- Have you used it to solve problems? Was it helpful?
- Suggest a way to improve the problem solving methodology document.
- What experience have you had in the past with applets?
- Did the applet Wednesday affect your understanding of E&M? How?
- Did you refer to the applet after the review session?
- Recommend a way to improve the applets.

- Do you recommend applets as a good way to help one study?

Appendix B: Problem Solving Document

Problem Solving in Introductory Physics

By Warren Schudy

Contents

Introduction

Purpose: Teach students how to learn physics and solve physics problems in an organized manner. This document is not intended to teach physical laws.

Audience: Introductory physics students. Knowledge of mechanics is helpful but not required. This document is designed for college freshmen, but high-school physics students may also benefit.

How to use: Read once. Then solve problems using my procedure, referring to the list as you go.

Learning

Learning physics often takes concentrated study and hard work. Here are a few general hints on things you can do to improve your chances of learning what you need to learn:

- **Attend class** – professors will tell you what material they expect you to learn.
- **Do homework** – humans need practice to learn problem solving.
- **Know homework solutions before test** – do not memorize the solutions, but determine what you did wrong with each homework and what the correct solution is. Test problems usually cover similar concepts to homework problems, so homework problems you don't understand are a great place to start studying.

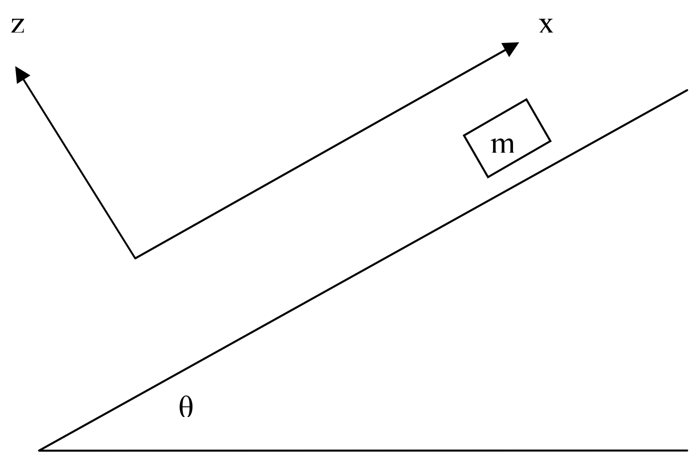
Problem Solving

This section describes a physicist's problem solving methodology. Few problems are identical, but most problems include most of the following steps. To clarify the exposition, I will frequently refer to the following example problem. Throughout the problem solving section, wavy lines in the left margin indicate text related to the example. This example involves mechanics, but this problem solving methodology is applicable to many other fields, such as electromagnetism and engineering.

A block is sitting on a frictionless inclined plane with an angle of theta=27 degrees. The block starts at rest. Find the time required for it to travel 1 meter down the inclined plane. Also find the position on the incline when the block is traveling at a speed of 3 meters per second.

1. **Sketching the problem**

Sketching the problem helps sort out the relevant information from the fluff. When sketching, label the known and unknown variables.



2. **Identifying Events, Boundary Conditions and Other Important Points**

Most problems that involve time contain one or more events which are important to the problem. Example events include the instant a ball is thrown and the instant a ball hits the ground. Events need not be dramatic; the time when a particle reaches a given speed is a legitimate event. Events are important because many physical laws are most usefully applied at events.

The concept of an event can be generalized to include any piece of space or time that is more important that others. If the system behaves according to a certain pattern in one area of space-time but a different pattern in a different area, the boundary is important. For example, if a car is traveling at constant speed and suddenly slams on the breaks, the time when it does so is the boundary between a zero-acceleration regime and a constant, non-zero acceleration regime. Points in space-time can also be interesting because the problem you are trying to solve involves a physical quantity measured at that point.

In the example problem, there are three events.

- When the block is released from rest

- When the block travels 1 meter down the inclined plane
- When the block reaches a speed of 3 meters per second

Summarize the events in a table.

Event	x (position)	V (velocity)	T (time)
Released from rest	0	0	0
1-meter	X1=1 meter	V1	T1=?
3 meters/second	X2=?	V2=3 meters/sec	T2

3. **Choosing physical laws**

Once you understand the problem, choose one or more physical laws and/or mathematical theorems to use to relate the unknowns and known quantities.

For introductory mechanics (physics 1), physical laws include:

- Newton's Laws
- The Work-Energy theorem
- Conservation of energy
- Conservation of momentum.

Conservation laws are usually much easier to use, but are not universally applicable. In particular, conservation laws do not determine times easily.

When using Newton's laws, draw a force diagram, showing the forces acting on each object. Also draw a set of axes for each object.

For the example problem, I use Newton's laws for the first part, since time is involved. For the second part, I use conservation of energy to illustrate its use. In practice, after doing all the work to set up Newton's laws for the first part, you would use Newton's laws for the second part as well.

4. **Applying laws to knowns/unknowns**

Once you have determined which laws to use, you need to apply the laws to the problem at hand. Be aware that in many circumstances, you need to use a law more than once. Here are guidelines for applying laws to unknowns

- **Constant acceleration** – write $x = x_0+v_0t + at^2/2$ and $v=v_0+at$ once for each event that you identified in step two. Substitute into those equations the variables for the time when each event occurs and the variable for the position or velocity at that point in time.
- **Newton's laws** – convert the vector equation $F = ma$ into a number of scalar equations, one for each relevant dimension in the problem (usually 1 or 2). Split each force into its components, and put the components of the force into the appropriate equations.

- **Conservation laws** – write an expression for the energy or momentum in terms of the positions and velocities of the particles. Write this expression once for each event. If the event involves a collision, you need to do this twice: once before and once after the collision. Equate the expressions.

In the example, I write down equations that aren't really relevant, to show how to be systematic. Equations ii and vi each add an unknown that one needn't find to solve the problem, and an extra equation.

$$(i): F_x = mg\sin\theta = ma$$

$$(ii): F_z = N - mg\cos\theta = 0$$

$$(iii): x(0) = 0 = x_0 + v_0 0 + \frac{1}{2}a0^2$$

$$(iv): v(0) = 0 = v_0 + a0$$

$$(v): x(T) = X_1 = x_0 + v_0 T + \frac{1}{2}aT^2$$

$$(vi): v(T) = V_1 = v_0 + aT$$

$$(vii): E_i = 1/2m0^2 - mg0\sin\theta$$

$$(viii): E_f = 1/2mV_2^{\ 2} - mgX_2\sin\theta$$

$$(ix): E_i = E_f$$

Be aware that Fx, Fy, x(t), and v(t) appear in these equations to label it – they are not intended to be part of the equations per se.

5. Counting unknowns and equations

Usually, a system of equations (i.e., several simultaneous equations) is solvable if the number of unique equations equals the number of unknowns. In the example, there are 9 equations and 10 unknowns: x_0, v_0, a, t_1, v_1,*x1*, *N*, x_2, *v2,* and *m*. Read below about having too few or too many unknowns, figure out why there are too many unknowns in the example, and read the solution.

a. Too many unknowns

There are two causes of excess unknowns.

- **Forgetting a physical law or initial condition** - the most common cause. Look for statements implicit in the problem, such as a dropped ball having initial velocity zero, or two objects being in the same location when they collide.
- **Inclusion of irrelevant information**. In falling object problems, Newton's second law is $mg = ma$. This equation has two unknowns (*m* and *a*), so one cannot solve for both m and a. However, dividing by m eliminates one of the unknowns.

It is also possible that the problem gives insufficient information for you to solve.

b. Too many equations

It is possible but uncommon to have more equations than unknowns. There are two causes of excess equations:

- **Accidental duplication of equations** – In the example problem, if you use both Newton's laws and energy conservation to find the position where the block reaches the desired speed, you will get redundant equations. This is ok, but confuses the algebra and should be avoided.
- An alternative cause is the designer of the problem might give **excess, potentially inconsistent, information**. For example, if a problem states that a particle in free-fall starts at rest, you can calculate its velocity at any later time. If the problem tells you that 3 seconds later, the particle is traveling at 45 meters per second, that is excess information since you could have calculated that yourself.

In the example, equations *iv* and *v* each introduce a new unknown whose value is not needed. Eliminating these equations from consideration leaves 5 equations and 6 unknowns. Intuitively, the mass of the object is irrevelant to the problem. To show this mathematically, divide both sides of equations *vi* and *vii* by the mass of the object, eliminating the last occurrence of that variable, leaving 5 equations and 5 unknowns.

6. Doing Algebra

Use your favorite method to solve the system of equations.

$$(i \rightarrow x): a = g \sin\theta$$

$$(iii \rightarrow xi): x_0 = 0$$

$$(iv \rightarrow xii): v_0 = 0$$

$$(v \rightarrow xiii): t_1 = \sqrt{2x/a}$$

$$(vii, ix \rightarrow xiv): E_i = 0 = E_f$$

$$(xiv, viii -> xv): 0 = 1/2V_2^{\ 2} - gx_2 \sin\theta$$

$$(xv \rightarrow xvi): x_2 = \frac{V_2^{\ 2}}{2g\sin\theta}$$

7. Checking

Once you have an algebraic solution, you are not done. Even experienced physicists make mistakes with algebra. There are a number of ways of checking your answers that will uncover most mistakes. Be aware that many of these checks can only be applied to algebraic answers. This is why one must not substitute numbers until the last possible moment.

c. Checking algebra

To check that you did the algebra correctly, substitute your final answers into the original equations, and verify that the equations are true (simplify to 0 = 0). If the algebra gets too messy, you can use numerical values (calculated in the final step) instead of algebraic expressions. I do not recommend doing this step when takings exams unless you have extra time, since this step is time-consuming and detects mistakes in your algebra but not in your physics. The other checking techniques can detect errors in both algebra and physics.

d. Checking units (Dimensional Analysis)

The easiest way to find some physics or algebra errors is to check the units of your answers for consistency. If you solve for a velocity, it should be equal to an expression with dimensions of length per time, for example meters per second. With practice, a unit check should only take about 5 seconds per equation. If you discover a unit problem, check your intermediate equations for unit consistency, looking for the equation where the inconsistency appeared. If under exam time pressure, write down that the units are inconsistent and go on to another problem. Find and fix the error when you have time.

The fundamental dimensions are length (L), mass (M), and time (T). For t_1, the expression within the square root is a distance divided by an acceleration, or $L^1M^0T^0 / L^1M^0T^{-2} = L^0M^0T^2$. Taking the square root yields $L^0M^0T^1$, which is the correct dimensions for a time. *g* is an acceleration, so the equation for *a* is dimensionally correct. For x_2, $(L^1M^0T^{-1})^2 / L^1M^0T^{-2} = L^1M^0T^0$, the correct dimensions for a distance.

e. Checking limiting cases

A second checking method is considering limiting cases. If the problem involves angles, consider what would happen if an angle were 0 or 90 degrees. Usually you can solve these limiting cases instantly – for example, a block sliding down an inclined plane with an angle of 90 degrees accelerates at g. If two or more knowns have the same dimensions (especially masses), consider what would happen if one were much larger or smaller than another. Also consider when a square root is taken of a negative number, and verify that the problem should be physically unsolvable in that case. With practice, these checks should take about 15 seconds per equation.

This table shows some of the limiting cases of the example problem. These checks are usually best done in your head, since writing them down takes much longer than doing them.

$\theta = 0$	$a = 0$, $t_1 = \infty$, $x_2 = \infty$	The plane is not actually inclined, so the block never moves.
$\theta = 90$	$a = -g$, $t_1 = \sqrt{(-2 x_1 / g)}$, $x_2 = -v^2 / (2 g)$	Free fall.
$x_1 > 0$		The equation for t_1 includes the square root of a negative number, since the block never reaches points higher on the plane than it started.

f. Check boundary conditions

If a problem involves a boundary of some sort as discussed in step 2, it is useful to verify that physical quantities that should be continuous across that boundary actually are. For example, velocity is always continuous when a particle is subject to finite forces. (On the other hand, when modeling a collision one assumes that the collision forces are essentially infinite, so velocity is not continuous). To verify continuity substitute the boundary value into the equation for the physical quantity at each side of the boundary, and verify that the physical quantity is the same on both sides.

8. **Finishing**

g. **Converting knowns to consistent set of units**

Problems may give quantities in a non-traditional set of units, such as giving lengths in centimeters but masses in kilograms. It is usually easiest to convert everything into one set of units, such as the meter-kilogram-second (SI) system.

h. **Calculating answers, with units**

Plug the numerical values calculated in the previous subpart into the solved equations and do the arithmetic (Use a calculator, if convenient). Check that your answers have the correct sign (positive or negative), and that they compare to other answers and givens in a reasonable way. For example, if you determine that a ball was caught before it was thrown, something must be wrong.

i. **Boxing algebraic and numeric answers**

If a grader is reading your answers, s/he will save time if your answers are clearly boxed. A fast grader is a happy grader is a generous grader.

Concluding Remarks

I do not recommend explicitly memorizing this methodology. Instead, do several problems while referencing it. With practice, it should become second nature. If you want to memorize something, train yourself to remember the most commonly forgotten steps: counting unknowns, and checking your results.

Do not feel bound to follow the steps in precisely the order given. It is normal to jump back and forth from one step to another. However, I still discourage doing any algebra before counting knowns and unknowns.

Some problems require proofs of a statement. Proofs can be quite variable. Some things to keep in mind:

- When trying to prove two things are equal, try simplifying one or both sides. Sometimes a proof can be easier to think about if you imagine “finding” what the one side of the equation is equal to.
- Remember that if you are given that a certain fact is true, you probably need to use the fact that it is true to do your proof. Often one can discover much of a proof by simply rearranging things into a form where given information is usable.

Application to Electromagnetism and other fields

Much of this guide is applicable to problem solving in math, physics, chemistry, or engineering. The physical laws change, but most of the steps remain the same.

Counting unknowns gets trickier when more complicated math, such as calculus, is involved. For example, one vector unknown is equivalent to three scalar unknowns. A complex number is equivalent to two real numbers. A matrix is equivalent to many scalars.

A differential equation is valid for a range of times. This introduces a catch into the one equation per unknown rule. It turns out that to solve for one function of time, one needs a differential equation that is valid at all times, plus one equation valid at one point in time per derivative.

Dimensional analysis is applicable to almost every problem in physics. This check is even useful for math problems where there aren't explicit units. What you can do is invent units for variables, and verify that your answers have the right units that you invented. For example, when integrating x*sin(a*x) from x is 0 to L, suppose that x and L have units of length, and "a" has units of inverse length. The result should have units of length squared—one from the x in the integrand, and one from integrating over x.

Limiting cases vary somewhat from problem to problem, but one can almost always find one or more limiting cases to check ones answer. Boundary condition checks are common but not universal.