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The Effects of Changing Running Stride

A Major Qualifying Project Report
Submitted to the Faculty of
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfillment of the requirements for the
Degree of Bachelor of Science



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Submitted to: Brian Savilonis Project Advisor

Abstract

This study explores the effects of changing someone's running stride by analyzing the forces exerted on the knee and ankle joints. Over a 5 week training period, 13 subjects were observed using video and force-plate analysis in order to estimate the landing load on the ankle and knee upon initial foot strike for two different stride forms. Various kinematic variables, such as landing force, joint moments, and landing angles were compared for two different stride forms to determine if stride change is beneficial. Based on the findings of this study, the benefits of stride change are dependent on the individual.

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1. Introduction

There are many different suggested strides and landing techniques in order to help a runner improve. Recently landing mid-foot, also known as Chi running, has become a popular foot striking method that many runners are changing their stride to. One question to ponder may be how changing running stride affects the ankle and knee joints.

Research suggests that the new fad of landing mid-foot is better for runners because there is no longer a breaking force due to landing on the heel of the foot. Instead, it is said that landing mid-foot is very efficient, reduces landing forces, and helps save energy for distance running [1]. Unfortunately, there has been a lack of research into how changing ones stride could affect the runner biomechanically. Therefore this project sought to determine the forces on the knee and ankle joints by changing a person's running stride.

To accomplish this goal, six days of testing and six weeks of training were conducted on a group of 13 participants. Three test days consisted of analyzing the participant's natural running stride. Six weeks of training was then conducted to teach the mid-foot stride. After the six week training period, three days of testing was conducted to on the participant's new trained running style.

During the testing, participant's ground reaction forces were measured using a force plate. High speed video was also taken (in frames per second) in order to conduct a video analysis to determine the accelerations and velocities of the participant's ankle and knee. The landing forces, accelerations, and velocities were then substituted into derived equations to estimate the load on the ankle and knee joint separately. The final load determined during natural stride and mid-foot stride were then compared in order to determine if there is a risk of injury on the knee or ankle joint by changing to a new running stride.

2. Background

The Merriam-Webster Dictionary defines running as: "to go faster than a walk; specifically to go steadily by springing steps so that both feet leave the ground for an instant in each step" [2]. To better understand what running is, the biology and mechanics, or biomechanics, of running must be researched. In addition, for this project running injuries, running studies, and knee and ankle biomechanics must be better understood. These topics will be further discussed in this section.

2.1 Biomechanics of Running

The biomechanics of running, better known as *the running gait cycle*, is the basic unit of measurement used when analyzing a walk, run, or sprint (also known as gait analysis). A single gait cycle is considered to be when one foot is in contact with the ground until that same foot comes in contact with the ground again. The gait cycle is made up of the *stance phase* and *swing phase*. The stance phase is when the foot is in contact with the ground, and the swing phase is when the leg is in the air, swinging forward. The swing phase and stance phase can be seen in Figure 1 and Figure 2 respectively.



Figure 1: Running Swing Phase [3]

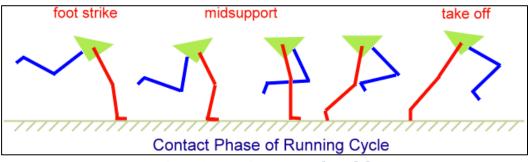


Figure 2: Running Stance Phase [3]

In gait analysis, all forms of gait undergo the swing and stance phases differently. The graph shown in Figure 3 shows the swing and stance phases of walking, running, and sprinting through the course of one gait cycle. In one walking gait cycle, there is an overlap in the stance phase, which means for one moment in time during walking both feet are in contact with the ground. For running, there is a slight overlap in the swing phase. This means that for one moment in time, both feet

are lifted off the ground. This is what separates running from walking; there is no moment where both feet are in contact with the ground during running. Lastly, sprinting acts very similar to running, in which there is an overlap in the swing phase and no overlap in the stance phase. The difference between running and sprinting is the velocity at which the runner is moving. [4]

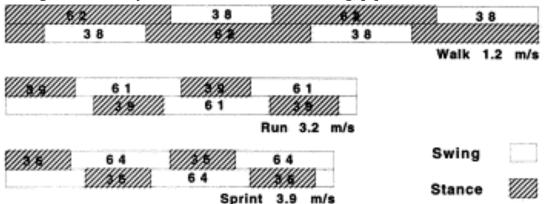


Figure 3: Swing and Stance Phase Time Comparison between Walking, Running, and Sprinting [4]

2.1.1 Running Styles

As previously defined, running is the act of moving steadily by springing steps so both feet leave the ground for an instant in each step, but there are different ways for the foot to return back to the ground. The foot can strike the ground heel first, mid-foot first, or toe first. In addition, the positioning of both the upper and lower body is important to running form. Different running styles provide alternate ways the foot strikes the ground during running.

Heel-toe running is when the front foot is dorsi-flexed; the runner reaches the foot out in front of his/her center of gravity, and lands with their heel striking the ground first. [5] According to running planet, this method of striking the ground is common among beginner runners, and is typically the easiest striking method to pick up and use. [5] Additionally, approximately 80% of distance runners are heel-toe strikers. [4] To better visualize the heel strike landing, view Figure 4.

Mid-Foot strike running is when a runner strikes the ground on the middle of the foot (not on the toe or the heel). The foot will be dorsi-flexed but will not extend out past the runners center of gravity. This type of ground strike will allow the calf muscles to remain stretched, preserving the runner's forward momentum; the preservation of momentum makes ground push-off easier and saves energy. [5] To better visualize the mid-foot strike, view Figure 4.

Forefoot running is a running style where the runner maintains the lifted arch of their foot and the runner's "big toe leaves the ground last and is used as a source of propulsion to move you forward". [6] When a runner lands toe first, it will produce a great deal of up and down motions; this causes more stress on the calf muscle and

therefore makes toe strike running more appropriate for sprinting. [5]To better visualize the forefoot landing, view Figure 4.

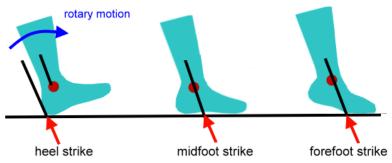


Figure 4: Visualization of Running Styles [3]

2.1.2 Running Body Positioning

Lower Body Positioning:

The proper leg position, as defined by Rick Morris from RunningPlanet.com, is one that is "quick and light" with minimal up and down motion; the runner should feel like they are gliding. Morris states that running quick and light without a bounce puts less stress on the runner's hip, knees, and back, as well as saves energy. [7]

In addition to light and quiet steps, there should not be an exaggerated knee lift; the higher the knee is lifted, the more energy the runner will use to maintain that knee height, in turn wasting energy. Proper knee lift should drive the runner's leg forward, rather than driving the knee and leg upward. [7]

The foot strike is the last factor in proper leg position. The proper foot strike should encourage a smooth and efficient run and avoid a motion that acts as a brake when running. To prevent a braking motion in the foot strike, Morris believes the runner should pull their lead foot off back and off the ground quickly and continue with a constant circular motion similar to the pattern of peddling a bike. [7]

Upper Body Positioning:

Running is a sport that primarily uses the legs but also utilizes the arms to help movement. According to Dr. Ashley Swelin-Worobec, doctor and owner of Active Sport Health Center in Burlington, Ontario, the upper body of a runner is just as important as the runner's lower body. According to Swelin-Worobecs' findings the importance of the upper body is because the body tissue has a crossing connection between a person's pelvis and arms, connecting the left arm and right leg. [6]

In addition to the arms, posture is also important when running. Rick Morris, from runnersplanet.com, states that the proper running posture consists of a slight forward bend at the ankles, and not an upright straight posture. [7] Along with the slight forward bend, according to active.com, an online resource for runners dedicated to training schedules, race registration, workout gear, and community message boards to help athletes, a runners back should be straight, shoulders level,

and head should be up and looking forward. [8]

The arms of the runner should swing loosely with a movement that does not exceed the chest or behind the midline of the runner; proper arm swinging varies from person to person. Elbows should be bent approximately 90-degrees, and arms should casually swing back and forth, brushing past the hips. The swinging arms aids in finding a correct stride length for each runner, as well as providing balance and leg coordination. [7]

According to Morris, the most important aspect of upper body positioning is relaxation. Shoulders, elbows, wrists, and hands should be loose in order to prevent the transfer of tightness all the way up the arm, causing a waste of energy. [7]

2.2 Lower Leg Anatomy: Knee and Ankle

The act of running utilizes muscles, ligaments, tendons and bones throughout the leg in order for a runner to be successful. For the purpose of this study, the anatomy of the leg from the knee down will be looked at in more detail. [9] The picture shown in Figure 5 shows the major muscles that are located within the lower leg.



Figure 5: Major muscles in lower leg [10]

Two common areas of the leg prone injury are the ankle and knee because both the knee and ankle are synovial joints. Synovial joints are joints where the articulating bones are separated by a fluid-containing joint cavity. The six distinguishing features of a synovial joint are the articular cartilage, the joint (articular) cavity, the articular capsule, the synovial fluid, the reinforcing ligaments, and nerves and blood vessels. [11]

2.2.1 Knee

The knee is a synovial modified hinge and a synovial plane joint. The knee is a complex joint that is made up of one end of the femur bone and one end of the shin bone; there are also ligaments and muscles attached to the knee, as well as the a moving bone called the patella (knee cap)(shown in Figure 6) [12]. Additionally, the **meniscus cartilage** is located in the knee joint between the tibia and femur. It

lowers the stress to the articular cartilage in the knee and prevents friction generated between the tibia and femur bones. [11]. Due to the nature of the knee, the knee can swing back and forth going no further than an 180deg straight leg.

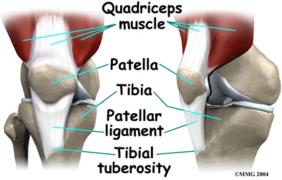


Figure 6: Close up of inside the knee joint [12]

2.2.2 Ankle

The foot is composed of 26 bones connected by joints and with the addition of thick ligaments the foot is designed to withstand the forces involved in daily activities such as walking and running. The ankle joint is formed form the meeting of the talus, fibula and tibia. The bony knob that protrudes on each side of the ankle is called the malleoli and provides additional stability to the joint. The additional stability is needed because during walking, running, jumping, even standing the ankle joint is responsible for weight-bearing. The ankle joint is surrounded by ligaments on each side that tightly strap the lateral and medial malleoli. [13]

Also connected to the tibia is the heel bone or the calcaneus, which provides added stability to the ankle joint. The middle of the foot is composed of the navicular, the cuboid and the three cuneiform bones. The arch of the foot is supported by the plantar fascia, a thick fibrous band of tissue that prevents the bones of the foot from flattening. This band runs from the calcaneus to the metatarsals. The metatarsal bones connect each toe bone or phalange [13]. The diagram in Figure 7 and 8 show a point of reference to each of the previously mentioned bones.

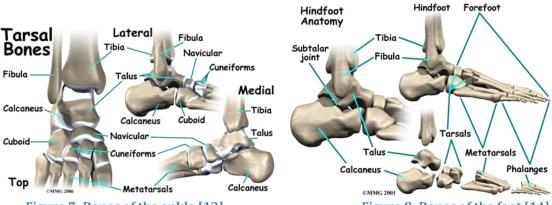


Figure 7: Bones of the ankle [12]

Figure 8: Bones of the foot [14]

The anterior tibialis, extensor digitorum longus, and extensor hallucis longus muscles are activated from heel-strike to toe-off. The extensor digitorum longus is

responsible for the dorsiflexion motion of the foot; the extensor hallucis longus also helps in the dorsiflexion of the foot during gait and is located on the anterior surface of the fibula. The anterior tibialis is located on the lateral condyle of the tibia and assists in both dorsiflexion and inversion of the foot. [15] See Figure 9 to better understand where each muscle is located.



Figure 9: Extensor digitorum longus [left], hallucis longus[center], and anterior tibialis[right] [15]

Since the ankle joint acts a synovial hinge between the foot and leg, it is responsible for the bending and extending of the ankle bone during running. This bending and extending is done mostly through the movement of the tibia and the talus. During foot strike, the compressive weight-bearing force is distributed between the calcaneus and the metatarsal bones. The consistent repetition of stresses can lead to the wearing down and eventual fracture of a bone or joint. [10]

The ankle is commonly injured due to sudden twisting movements of the foot or due to sudden impact. The most common ankle injury is a sprain, which occurs when a ligament is overstretched and partially or completely torn. [16]

2.3 Common Running Injuries and Prevention

Running involves constant repetitive motions of the legs working to move the foot to and from the ground and the foot striking the ground. When the foot contacts the ground, the resulting forces are transmitted up the leg and can cause an injury, especially when the motion is repetitive. [17] According to Dr. Liebentritt, a Board Certified Family Medicine physician specializing in Sports Medicine, approximately 35-45% of people who run suffer from injuries [18], with 70-80% of those injuries occurring at or below the knee. [19] Van Mechelen W, a researcher for the Department of Health Science, Faculty of Human Movement Sciences, University of Amsterdam, The Netherlands, says that each lower body part has a different percentage of injury risk. [19] Table 1 shows the injury risk percentages for different body parts.

Table 1: Percentage of Injuries on Certain Body Parts [19]

Knee	25%
Feet	2 - 22%
Ankle	9 - 20%
Lower Leg	2 - 30%
Shin	6 - 31%
Upper Leg	3 - 18%
Back	3 - 11%
Hip/Pelvis/Groin	2 - 11%

Different running injuries, according to Sport Injury Clinic, can be caused by either intrinsic or extrinsic factors. Intrinsic factors are due to body structure and mechanics of motion, like over pronation. Extrinsic factors are cause by influences outside the body, like worn out running shoes or overuse. The following sections will discuss common intrinsic and extrinsic running injuries. [17]

2.3.1 Knee/Leg Injuries and Prevention

There are many types of injuries that occur in the leg and knee specifically. One example is **patellofemoral Pain Syndrome (PFPS)** is a common injury associated with the knee and is sometimes referred to as anterior knee pain. PFPS is caused when the patella does not move correctly during bending and straightening of the knee. Typically, pain will be experienced to the anterior of the knee. This injury is common for those who participate in many sports, pronate, have weak quadriceps, or frequently run hills. [17]

Iliotibial band syndrome (ITBS), or IT band syndrome, can be caused by a tight or wide IT Band, weak hip muscles, over-pronation, overuse, excessive hill or banked surface running, or difference in leg length. [17] (IT band syndrome can be seen in Figure 10)

Figure 10: Representation of knee with ITBS [17]

Two other common injuries of the leg are **shin splints** and **calf injuries.** *Shin splints* are "throbbing or aching in the shins during or after strenuous activity involved with gait", according to WebMD. [23] *Calf Injuries*, according to Ed and Brenda Lerner, authors of the article *Calf Strain or Pull* in the World of Sports Science Journal, can be in the form of a strain, cramp, or lactic acid build up. [24]

2.3.2 Foot/Ankle Injuries and Prevention

There are many types of injuries that occur in the foot and ankle specifically. One common injury is **Planter Fasciitis.** The Plantar Fascia is a broad, thick band of tissue that runs from under the heel to the front of the foot (see Figure 11). Plantar fasciitis, also called a heel spur, is a bony growth at the attachment of the plantar fascia to the heel bone. [17]

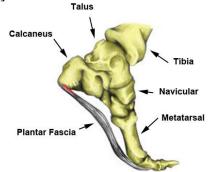


Figure 11: Representation of planter fascia [17]

Achilles Tendonitis is another common injury in the ankle. The Achilles tendon is the large tendon at the back of the ankle connecting the large calf muscles to the heel bone providing power to push off the ground when running. Achilles tendonitis is when tissue degenerates with a loss of normal fiber structure. [17]

Ankle sprains are some of the most common sports injuries according to sports injury clinic. A sprain is the damaging of ligaments from being stretched too far; an ankle sprain is the stretching and/or tearing of ligaments of the ankle. [26] There are two types of ankle sprains, inversion and eversion. An inversion sprain is when the ankle turns over so the sole of the foot faces inwards, damaging the ligaments on the outside of the ankle. An eversion sprain is when the ankle rolls so the sole of the foot faces outwards, damaging the ligaments on the inside of the ankle, typically resulting in a fracture. [17]

Due to running having repetitive forces and impact over a period of time, **stress fractures** are tiny breaks in the bone, are common. (Sports Injury Clinic, 2012) In a study done between 2007 and 2010, researchers analyzed athletes at 57 high schools involving fractures amongst them. Of the 230 fractures, the tibia was affected most, accounting for more than half of all cases. Also, 20 percent of fractures involved the metatarsal bone. (Mozes, 2011)

2.3.3 Fatigue and Fatigue Prevention

After running for an extended period of time fatigue will kick-in. Running fatigue is inevitable and is like aging or sweating says Ed and Brenda Lerner, authors of the article *Fatigue* in the World of Sports Science journal. Fatigue is both a physical and mental state. Lerner says that a running coach will have greater success teaching a strategy or technique earlier in a workout before fatigue signals are sent throughout

the body. Once fatigue hits the body, the athlete will be more focused on completing a run rather than focusing on a specific technique. Fatigue can be the cause of running injuries, and prevention of fatigue will vary from person to person. [27]

2.4 Conducting This Type of Study

In order to conduct a study to analyze how someone runs, certain criteria need to be followed. Studies have been observed in order to analyze runners and their strides.

2.4.1 Determining Volunteer Group, Size of Group, and Study Duration

Through research of similarly conducted studies, subject group sizes ranged anywhere from five to 180 people. Although this is a very large range of people, the most common found group sizes ranged from 5-15 people. For this study students ages 18-25 will be the primary source of volunteers. With the consideration that participants will be completing course work at the same time as this study, the group will look for 10-12 volunteers. The group believes that approximately this many volunteers will be able to commit at least thirty minutes, three days a week for six weeks of training and four weeks of working on their own. [28]

The reason 20 minutes of running was chosen was because Runner's World suggested that an average running workout for a novice runner lasted at least 30 minutes. Due to the insignificant running history of study participants, the length of 30 minutes was shortened to 20 minutes. This will allow for participants to not strain themselves whiling still exercising enough to obtain the best data for this study. [29]

Since participants will be running for 20 minutes a day, the participants would train every other day during the work week for 6 weeks. Although studies like the New York Times "Finding Your Ideal Running Stride" held a 10-week training period. [30]Due to the short terms and student schedules, 6 weeks was the longest amount of time to conduct this study. One study that seemed to be the perfect training period was an article called "The Couch-to-5k Running Plan" which stated that "each [running] session should take about 20 or 30 minutes, three times a week" and if followed "on a regular basis in just two months" running three miles should not be difficult. [31] One woman was even able to accomplish this transformation in just six weeks since she was not a "couch potato", which is the type of participant included in this study. [32]

In addition, one study comparing the gait variability and stableness of stride in trained runners compared to non-runners, suggests that experienced runners have a less variability in the trained movement of running and have a more stable stride. [33] For this reason, non-experienced runners would be better suited for this type of study.

2.4.2 Instruments and Equipment

Similar studies have used force plates and high-speed cameras. The Vernier force plate was used to measure ground reaction forces when a person is running, walking, or sprinting. The specifications for this product can be found in Table 2 below. The force plate will give ground reaction forces at the given moment the person is running over the plate. [34] Another force plate was used to measure the center of pressure (COP). The specifications can be found in Table 3 below. The Casio EX-ZR100 high-speed camera was used to get the velocity the person was moving at, and to capture the person's movements. [35] This camera was used at setting HS240: High Speed 240, 432x320 pixels, 240 frames per second.

Table 2: Vernier Force Plate Specifications

Force Range	-850 to +3500 N
Maximum non-damaging force	4500 N
Resolution	1.2 N
Dimensions	28 cm x 32 cm x 5 cm

Table 3: AccuGait Force Plate Specifications

Table 5. Accudate force flate specifications			
Dimensions	502mm x 502 mm x 44.7mm		
Weight	11.36 kg		
Channels	F_x , F_y , F_z , M_x , M_y , M_z		
Sensing Elements	Hall Effect		
Digital Outputs	6 Channels		
Capacity	F _x : 445 N F _y : 445 N F _z : 1334 N		
	M _x : 169 N-m M _y : 169 N-m M _z : 85 N-m		
Natural Frequency	F _x : 140 Hz F _y : 140 Hz F _z : 150 Hz		

3. Methods

The purpose of this study is to determine the effects of changing someone's running stride by analyzing the forces exerted on the knee and ankle joints. This was done by evaluating the landing forces and observing the flexion and extension angles of a recreational runner's knee and ankle joints before and after changing their stride. To determine landing forces and angles of the knee and ankle, a series of running tests using video observation and force plate analysis were conducted on a group of 13 volunteers, ages 18-25.

3.1 Testing of a New Runner's Comfortable Stride

Before teaching a new stride to the participants, each person was asked to sign a consent form (refer to Appendix B: Participant Agreement Form) and perform a pre-test to determine if they were capable of running for the duration of each testing and training day. The pre-test began with the participants running over an AccuGait force plate to determine the participants "rested" center of pressure and landing force in the x-direction. The last part of the pre-test consisted of a comfortably paced 10-minute treadmill run. The volunteers' heart rate was measured before the run, 5-minutes into the run and 2-minutes after the 10 minutes are completed. Throughout the run, lethargy and breathing were monitored to determine if the volunteer was capable of completing the training and test period without harm. Other factors which would limit someone's participation in the study included people who: are overweight or underweight (based on BMI), have a high heart rate, have irregular heartbeat, have undergone coaching, kick out when they run, and/or are not able to exercise for 20 minutes three times a week. At the end of the pre-test, measurements of the upper and lower leg, height, and weight were recorded in order to aid in the data collection process and help derive force values.

After completing the pre-test without difficulty, injury, or any other significant problem, the runner underwent 3 days of testing, where their running stride was analyzed. During each test day, participants ran over a force plate after each lap of a 20 minute run. A high speed camera was used to record the participant each time they run over the force plate; the ground reaction forces were measured at the same time, by the participant landing on the force plate. For the video analysis, markers were placed 1" above the ankle bone, along the metatarsal bone on the shoe, and on the knee directly in line with the knee bend line. The markers allowed for tracking of the knee and ankle flexion and extension coordinates. The ground reaction forces were measured using a Vernier Force Plate; this force was used to compare the forces found on the AccuGait force plate. The coordinates and ground reaction forces were used to find variables to enable solving for the total load on the knee and ankle joint. All information retrieved from testing was used to compare with the forces and angles after learning a new stride.

3.2 Learning a New, Defined Stride

Participants were coached in a stride for five weeks and asked to focus on learning the new foot strike, ankle and knee positioning. The new defined stride is detailed below:

- Foot strike: participants should run landing on the mid-foot, not heel, and roll to the toe, before pulling the leg forward to strike the ground again.
- Ankle: participants should allow ankle to be underneath or slightly behind the knee while running.
- Knee: knees should not reach out in front of body when striding. Knee should stay underneath the hips, and should bend upwards behind the body after the foot strikes the ground.

3.2.1 Teaching the New Stride

The runners were coached on the mid-foot stride technique over the course of a five-week period. They were coached through running drills and training sessions. Details of the training sessions, running drills, and the mid-foot technique can be found in Appendix A: Participant Training Packet.

3.3 Testing Runner's Using a Newly Learned Stride

After the course of five weeks, the runners endured three days of testing while performing the new stride. During each testing day the runners ran over a force plate throughout a 20 minute period. A high speed camera was used to record the runner while landing on the force plate to record the ground reaction force. The video recording and force plate recordings followed the same procedure used for the first three test days. All information retrieved from testing was used to compare with the forces and angles before learning a new stride.

3.3.1 Observing Stride Change

Video and force plate analysis were done during each of the three final tests. The vertical landing forces on the knee and ankle were found using the Vernier Force Plate. Ankle and knee flexion and extension coordinates were determined at the moment the foot strikes the force plate using the high speed camera.

Using video software, the knee angle was observed to determine if the mid-foot strike provides a smaller flexion angle. It was also determined if mid-foot provides a larger ankle extension angle as well. This information was determined by comparing the results of the new stride to the results from the first data collection period. The information gleaned from video analysis also helped with finding necessary accelerations and velocities.

Vertical forces collected from the Vernier Force Plate were analyzed in order to determine the landing forces. This was used with the assumption that the higher the landing force the greater the risk of injury. The vertical forces were also used to

gauge stride change due to fatigue after an extended run. Additionally, this information was compared to the results from the first data collection period to determine if forces increase or decrease by changing running stride.

The center of pressure and landing force in the x-direction were ascertained using the AccuGait force plate. It was assumed that the participants would retain one center of pressure and x-force during their given stride and another after learning a new defined stride. The center of pressure and x-force allowed for further calculations to be conducted using free body diagrams.

It was believed the flexion angle of the knee would be smaller during the mid-foot stride because the leg will not reach out in front of the body, instead the leg will hang underneath the body. With the leg hanging and not extended outward, more of the landing forces will be absorbed. The ankle extension angle would be larger during the mid-foot stride resulting in less risk of injury to muscles such as the tibial anterior during the mid-foot stride. It was also predicted that the forces of the knee would lessen during the mid-foot stride because the leg should hang below the body absorbing more landing forces. The forces of the ankle should lessen as well during the mid-foot stride and be distributed over the foot instead of directly to the ankle.

4. Analysis

Video analysis, force plate analysis, and free body diagrams and equations were used to analyze the data collected throughout the course of this study. This section will go into more detail the analysis procedures followed in order to properly analyze the data to achieve the desired result.

4.1 Video Analysis

A Vernier force plate was placed in the third lane of the indoor track inside of the recreation center and connected to a laptop to record the vertical force of one step of each participant's run. In order to record the strides of the participants running on the force plate, a high-speed Casio EX-ZR100 digital camera was set on a tripod 74" from the force plate and zoomed at "0.09 foot" onto the lower body of the participants. A detailed procedure can be found in Appendix C: Analysis Procedure Detailed Checklist. The camera was set to film 240 frames per second in order to obtain the most accurate measurements and angles of the leg and foot. The camera recorded only when a participant was stepping onto the force plate from initial foot strike to toe-off.

These videos were then exported off the camera and imported into Adobe AfterEffects for analysis. In Adobe AfterEffects, the video clips could be cut to the time frame of the initial foot strike to toe-off. After cutting the video, the knee, ankle bone, and metatarsal bone, which were marked before testing, were followed using the motion tracking features in order to obtain x and y coordinates. This data was exported into an Excel spreadsheet to compare separate strides of the same participant. Four videos were analyzed to find the values of each test.

Each video was then rendered into a folder to extract separate clips of the video. The instance the foot made contact with the force plate was chosen and used to analyze the angles in Photoshop at the initial foot strike. Angles of the knee and foot were then obtained using linear lines of the upper leg, lower leg and foot. The linear lines were also used to measure the lengths of the upper leg, lower leg and foot. Figure 12 and

Table 4 show a picture and explanation of the different angles measured. Each line in Figure 12 measures an angle from the horizontal the horizontal or x-axis. The angles above the knee measured counterclockwise from the horizontal axis at the knee. The angles below the knee are measured counterclockwise from the horizontal axis created at the force plate. Each angle was made to be positive when added to the equations. This information was exported to an Excel spreadsheet in order to compare angles of each stride.

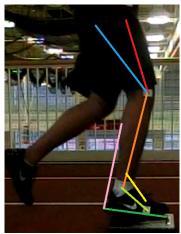


Figure 12: Diagram of measured angles

Table 4: Diagram of Measured Angles Explanation

Red	Femur	
Blue	Hamstring	
Orange	Fibula	
Pink	Achilles Tendon	
Lt Green	Ankle-Metatarsal	
Dk Green	Foot	
Yellow	Tibialis	
Lt Green Dk Green	Ankle-Metatarsal Foot	

4.2 Force Plate Analysis

For the force plate analysis, two force plate tests were conducted. One test was used to determine ground reaction forces of the participant while they are running laps on the track. The second test utilized a different force plate, and was used to determine x- and y-coordinates, center of pressure, and forces in the x- and y-direction. These variables were used in a code written in MATLAB to determine the accelerations and velocities of the foot during ground strike. Both force plate tests will be described further in this section.

4.2.1 Ground Reaction Force Testing

A Vernier force plate was used to obtain the ground reaction force of one of the participants' steps while running laps on the track. In addition to the force plate, a Casio camera was used to record participants running over the force plate. The data from the Vernier force plate was collected in Logger Pro. The data was given in the form of a graph with x- and y-coordinates. The peak of the graph was then analyzed in order to get the maximum ground reaction force. Although the graphs produced show an increasing force throughout the entire stance phase, the point of initial foot-strike on the graph was considered the maximum force. The information gleaned from this force plate was used with video collected data and the data that will be collected from the AccuGait force plate to reach a final result and determine risk of injury.

4.2.2 Computational Bio-Analysis Testing

Participants were also tested on an AccuGait force plate to obtain additional force plate data. The Casio EX-ZR100 digital camera was set up on the same tripod and used to record the participants running over the AccuGait force plate in order to find distances, using a grid on the surface of the force plate, to help obtain the ground reaction force in the x-direction, and center of pressure. This data was exported from the force plate to computer software called AMTI-Net force. Specific information was then selected and exported to another software program called BioAnalysis, which was used to find the COP, normal forces of X and Y, the angular velocity and acceleration in the X and Y directions. This data was used to compare with the Vernier force plate data, as well as used in equations to determine the risk of injury.

4.3 Load Analysis

For the load analysis, the forces on the knee and ankle joint were produced though the use of equations and free body diagrams (FBD). This was done to compare researched numbers to the found numbers in order to determine the risk of injury on the knee and ankle by changing a runner's stride. The FBDs for the ankle and knee joint, and their associated equations, are further described and discussed in this section.

4.3.1 Determining the Forces on the Ankle

The resultant force exerted on the ankle can be found by combining the forces that act in the x- and y-direction separately. In this experiment, the foot was considered to be a lever on an x-y plane; therefore the forces in the z-direction were ignored. In addition to the x- and y- forces, any force that creates a bending moment at the ankle was summed together to determine the resultant moment about the ankle joint. Using the free body diagram (FBD) in Figure 13, one can break down all forces into x- and y-components.

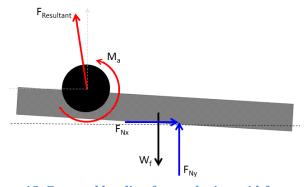


Figure 13: External landing forces during mid-foot strike

The forces acting in the x- and y-direction can then be summed together separately and set equal to mass times acceleration to derive the first two equations. In addition, all forces acting in the y-direction are a moment arm that cause the ankle

to rotate. The last equation is derived from summing all the moments together and setting it equal to mass moment of inertia times angular acceleration. The three equations derived from Figure 13 can be seen below:

$$\begin{split} \sum F_x &= F_{Nx} - F_{R_{Ax}} = ma_x \\ \sum F_y &= F_{Ny} + F_{R_{Ay}} - W_F = ma_y \\ \sum M_A &= W_F \cdot \frac{l}{2} - F_{Ny} \cdot COP = I \propto \end{split}$$

Where:

F_{x}	Force in the x-direction	F_y	Force in the y-direction
F_{N_X}	Normal force x-direction	F_{Ny}	Normal force y-direction
F_{RAx}	Landing force x-direction	F_{RAy}	Landing force y-direction
I	Mass moment of inertia	α	Angular acceleration
M_A	Moment about the ankle joint	$\overline{W_f}$	Weight of the foot
$a_{\scriptscriptstyle X}$	Acceleration in the x-direction	COP	Center of pressure
$\overline{a_{v}}$	Acceleration in the y-direction		•

The answers calculated from the above equations give the overall forces and moment acting on the ankle. This overall force is the combined account for force of muscles, tendons, and bone. The force on the ankle was considered to be equal and opposite to the force of the bone. In order to determine the force that acts only on the ankle, the forces on the muscles, tendons, and bones need to be found. The FBD in Figure 14 shows the different forces acting on muscle, tendons, or bone that were considered in this project.

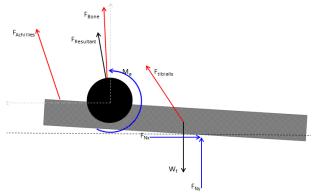


Figure 14: Internal landing forces during mid-foot strike

Again, the forces in the FBD in Figure 14 can be broken down into x- and y-components, and moment arms acting in the y-direction. The equations below were found similar to the first set of equations; they just utilize a more complicated FBD in order to find more specific forces.

$$\sum_{i=1}^{n} F_{i} = F_{i} + F_{i} + F_{i} + F_{i} = F_{i}$$

$$\sum_{i=1}^{n} F_{i} = F_{i} + F_{i} + F_{i} + F_{i} = -F_{i}$$

$$\sum_{i=1}^{n} M_{i} = F_{i} \cdot \frac{l}{2} + F_{i} \cdot d + M_{i} = M_{i}$$

Where:

F_t	Force on the tibialis	F_a	Force on the Achilles
F_b	Force on the bone	M_s	Moment of the ankle due to the ankle acting as a spring
d	Distance of the force from the ankle		

The above equations found the forces and moments from the muscles and bones by solving for the F_t , F_b , and F_a . Since there were three equations and three unknowns, a matrix could be set up, followed by row reduction in order to find the three unknowns. The matrix and final equations are shown below:

$$\begin{bmatrix} F_t \cos \left(\theta_1\right) & F_b \cos \left(\theta_3\right) & F_a \cos \left(\theta_2\right) & 0 & F_{RAx} \\ F_t \sin \left(\theta_1\right) & F_b \sin \left(\theta_3\right) & F_a \sin \left(\theta_2\right) & 0 & -F_{RAy} \\ \frac{F_t \sin \left(\theta_1\right) L_{foot}}{2} & 0 & -F_a \sin \left(\theta_2\right) \cdot d & M_S & M_A \end{bmatrix}$$

Where:

Θ_1	Angle of the tibialis	Θ_3	Angle of the Achilles
Θ_2	Angle of the bone		

After row reduction and solving for individual variables, the following three equations were produced:

$$F_{a} := \begin{bmatrix} \frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} M_{s}\right)}{1 - \frac{0.045 \cdot 2}{L_{foot} \cdot \sin(\theta_{1})}} \\ - \frac{\left(\frac{\left((-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{s} + \sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx}\right)}{2 \cdot \cos(\theta_{1}) \cdot \sin(\theta_{2}) \cdot 0.045 + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right) \cdot \sin(\theta_{3})} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot \sin(\theta_{2}) \cdot 0.045 + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right) \cdot \sin(\theta_{3})}{\cos(\theta_{3})} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot \sin(\theta_{2}) \cdot 0.045 + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right) \cdot \sin(\theta_{3})}{\cos(\theta_{3})} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right) \cdot \sin(\theta_{3})}{\cos(\theta_{3})} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right) \cdot \sin(\theta_{3})}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{3})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{3})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{3})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1})\right)}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a}}{\cos(\theta_{1}) \cdot M_{a}} \right)}{\cos(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a}}{\cos(\theta_{1}) \cdot M_{a}} \right)}{\cos(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a}}{\cos(\theta_{1}) \cdot M_{a}} \right)}{\cos(\theta_{1}) \cdot L_{foot}} \\ - \frac{\left(\frac{(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{a}}{\cos(\theta_{1}) \cdot M_{a}} \right)}{\cos(\theta_{1}) \cdot$$

The three forces determined from the Equation 1, Equation 2, and Equation 3 determined the resultant force on the muscles, tendon and bone. It is important to note that multiple muscles, tendons and bones were lumped together in order to simplify the model. The resultant force on the ankle was considered to be equal and opposite of the resultant force on the bone. (Refer to Appendix G.1 Ankle Free Body Diagrams-G.3 Natural Tired Ankle Equations and Appendix H.1 Ankle Free Body Diagrams-H.3 Mid-Foot Tired Ankle Equations to see full Natural and Mid-foot equation analysis for the ankle respectively)

4.3.2 Determining the Forces on the Knee

The resultant force exerted on the knee was found in a similar way to the forces on the ankle; the forces that act in the x-and y-direction were combined. In this experiment, the leg was considered to be a lever on an x-y plane; therefore the forces in the z-direction were ignored. In addition to the x- and y- forces, any force that creates a bending moment at the knee was summed together to determine the resultant moment about the knee. Using the FBD in Figure 15 one can break down all forces into x- and y-components.

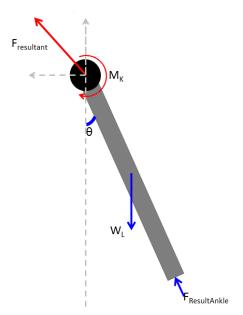


Figure 15: Knee FBD external forces during heel strike

The forces acting in the x- and y-direction can then be summed together separately and set equal to mass times acceleration to derive the first two equations. In addition, all forces acting in the y-direction are a moment arm that cause the knee to rotate. The last equation is derived from summing all the moments together and setting it equal to mass moment of inertia times angular acceleration. The three equations derived from Figure 15 can be seen below:

$$\sum F_{x} = F_{Rax} - F_{Rkx} = ma_{kx}$$

$$\sum F_{y} = F_{Rky} - F_{Ray} - w_{LowerLeg} = ma_{ky}$$

$$\sum M_{K} = w_{LowerLeg} \cdot \frac{l}{2} - F_{Ray} \cdot 1 + M_{K} + M_{A} = I\alpha$$

Where:

$F_{\scriptscriptstyle X}$	Force in the x-direction	F_y	Force in the y-direction
F_{Rkx}	X-resultant force on knee	F_{Rky}	Y-resultant force on knee
F_{Rax}	X-resultant force on ankle	F_{Ray}	Y-resultant force on ankle
I	Mass moment of inertia	α	Angular acceleration
M_K	Moment about the knee joint	$W_{lowerleg}$	Weight of the lower leg
a_{kx}	Knee acceleration in x-direction	a_{ky}	Knee acceleration in y-direction

The answers calculated from the above equations give the overall forces and moment acting on the knee. This overall force is the combined account for force of muscles, bone, and knee joint. In order to determine the resultant knee joint force, the forces on the muscles and bones need to be found. The FBD in Figure 16 shows the different forces acting on muscle or bone that were considered in this project.

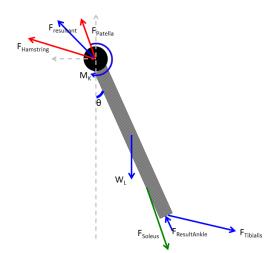


Figure 16: FBD of internal knee force during heel-strike

Again, the forces in the FBD in Figure 16 can be broken down into x- and y-components, and moment arms acting in the y-direction. The equations below were found similar to the first set of equations; they just utilize a more complicated FBD in order to find more specific forces.

$$\sum_{K} F_{x} = F_{Rkx} + F_{Hx} + F_{px} - F_{TAx} - F_{Sx} + F_{Rax} = ma_{kx}$$

$$\sum_{K} F_{y} = F_{Rky} + F_{Hy} + F_{py} - F_{TAy} - F_{Sy} - F_{Ray} = ma_{ky}$$

$$\sum_{K} M_{K} = M_{K} - M_{A} + F_{Rkx} \cdot d + F_{Hx} \cdot d + F_{px} \cdot d - F_{TAx} - F_{Sx} + F_{Rax} = I\alpha$$

$$Where:$$

$$F_{H} \mid Force \ on \ the \ hamstring$$

$$F_{S} \mid Froce \ on \ the \ soleus$$

$$F_{TA} \mid Force \ on \ the \ tibialis$$

The above equations were used to find the forces and moments from the muscles and bones by solving for the F_{Rk} , F_H , and F_p . F_S is found by dividing the found Achilles force by 3. This is because the Achilles tendon force is split between two muscles, 1/3 the Soleus and 2/3 gastrocnemius. Since there were three equations and three unknowns, a matrix could be set up, followed by row reduction in order to find the three unknowns. The matrix and final equations are shown below:

$$\begin{bmatrix} F_{Rk}\cos\left(\delta_{1}\right) & F_{p}\cos\left(\delta_{3}\right) & F_{H}\cos\left(\delta_{2}\right) & -F_{TAx} & -F_{Sx} & F_{RAx} & ma_{kx} \\ F_{Rk}\sin\left(\delta_{1}\right) & F_{p}\sin\left(\delta_{3}\right) & F_{H}\sin\left(\delta_{2}\right) & -F_{TAy} & -F_{Sy} & -F_{RAy} & ma_{ky} \\ \frac{F_{Rk}\cos\left(\delta_{1}\right)L_{LowerLeg}}{d} & \frac{F_{p}\cos\left(\delta_{3}\right)L_{LowerLeg}}{d} & \frac{F_{H}\cos\left(\delta_{2}\right)L_{LowerLeg}}{d} & -F_{TAx} & -F_{Sx} & -F_{RAx} & I_{k}\alpha_{k} \end{bmatrix}$$

Where:

δ_1	Angle of resultant knee force	δ_2	Angle of hamstring
δ_3	Angle of the patella		

After row reduction and solving for individual variables, the following three equations were produced:

Equation 4: Force on the Hamstring

$$F_{H} := \frac{I_{k} \cdot \alpha_{k} + F_{Rax} + F_{Sx} + F_{TAx}}{\left(\text{cos}(\delta_{2}) \cdot \mathbf{d} \right)}$$

Equation 5: Force on the Patella

$$F_p \coloneqq \frac{\left[\left(I_k \cdot \alpha_k\right) - \tan(\theta_1)\right] + F_{RAy} + \tan(\theta_1) + F_{Sy} + \tan(\theta_1) + F_{TAy} - \tan(\theta_1) - \left(F_H \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_2)\right) +$$

Equation 6: Resultant Force on Knee

$$\mathtt{F}_{K} := \frac{m \cdot \mathtt{a}_{kx} - \mathtt{F}_{RAx} + \mathtt{F}_{Sx} + \mathtt{F}_{TAx} - \mathtt{F}_{H} \cdot \mathtt{cos} \left(\delta_{2}\right) - \mathtt{F}_{p} \cdot \mathtt{cos} \left(\delta_{3}\right)}{\mathtt{cos} \left(\delta_{1}\right)}$$

The three forces determined in Equation 4, Equation 5, and Equation 6 determined the resultant force on the muscles, tendon, and bone. It is important to note that multiple muscles, tendons and bones were lumped together in order to simplify the model. The resultant force on the knee, which is a variable in these three equations, is the final resultant force on the knee. (Refer to Appendix G.4 Knee Free Body Diagrams-G.6 Natural Tired Knee Equations and Appendix H.4 Knee Free Body Diagrams-H.6 Mid-Foot Tired Knee Equations to see full Natural and Mid-foot equation analysis for the ankle respectively)

5. Results

Using the aforementioned analysis, results were produced. The following section discusses how the analysis led to the results found.

5.1 Video Analysis

Through video analysis, specific results were achieved in order to further develop the project. The primary purpose of video analysis was to use video screen shots of initial landing strikes to help form the free body diagrams (FBDs). The snapshots easily showed where the foot met the force plate and the angles in which the legs and feet were positioned.

Through video tracking, x- and y- coordinates of the metatarsal bone, ankle joint and outside profile of the knee were extracted to determine the angular acceleration. The angular acceleration was calculated from the values through the use of a MATLAB code that extracted the x- and y- coordinates in conjunction with time and position vectors in order to determine angular velocity. The MATLAB code then produced a curve fit along the angular velocity points, followed by a derivation of a formula for that curve in order to determine the angular acceleration.

The determined angular accelerations were found to be within 1% of one another. This shows that angular acceleration of each person will not have an effect on the participant's resultant force on the ankle. In addition, the values show that the angular accelerations of the foot do not change extensively based on a runner's stride. The angular accelerations at the moment of impact determined for each participant are shown in Table 5.

Table 5: Participant angular acce	elerations
-----------------------------------	------------

	Natural	Mid-foot
	[rad/s ²]	[rad/s ²]
2	0.77	1.09
6	0.98	0.98
7	1.05	1.30
12	1.26	1.15
4	1.09	0.86
9	1.05	1.19
3	1.54	1.54
13	1.54	1.54

^{**}This table only shows the result of 8 participants due to this analysis being conducted separate from initial and final testing.

These 8 participants' results were the only 8 results that were able to accurately be analyzed. Participant numbers still remain the same.

The video analysis also enabled the use of Photoshop to find the landing angles of each participant while running over the force plate. Each participant's angles were

analyzed at initial foot strike. The angles analyzed for ankle analysis were the tibialis, tibia bone, the landing angle of the foot with the force plate, and the Achilles; for the knee analysis, the hamstring, femur bone, and resultant knee angle were obtained. All measured angles are displayed in Appendix D: Measured Angles. The angles were then used in free body equations to determine loads on the ankle and knee. Finally, the angles were averaged and observed for the natural and mid-foot strides at the beginning of the run and then again at the end of the run to notice any landing posture change. Table 6 shows the vertical forces during the natural and mid-foot strides at the beginning and end of each run. It also has the preferred stride based on landing forces listed.

The results that came from the video analysis were used in conjunction with the force analysis in order to determine overall load on the knee and ankle. One important observation found through video analysis was that participants whose natural stride was heel-toe would sometimes land on the force plate mid-foot. This may have been because the plate was raised off the ground and participants were adjusting their stride to hit the force plate. This was important to notice because it affected the results of the force plate analysis.

5.2 Force Analysis

The force analysis consisted of both force plate data and force equations. In this section the results of each analysis is noted.

5.2.1 Force Plate Analysis

The force analysis consisted of two sets of force plate data, as well as the use of equations. The first data collected was on a Vernier force plate (specifications can be viewed in Table 2) through a series of tests that took place over three days at the beginning and end of the training period on an indoor track. The force plate measured all the forces in the y-direction on the foot throughout the entire stance phase. This force was important to obtain because it showed the change in landing force at the beginning and end of a run during the natural stride and mid-foot stride. These changes can be shown in Table 6. (Forces in Table are not normalized. View Appendix J: Participant Information for participant weight or any other participant information.)

Table 6: Table of participant landing forces for mid-foot and natural stride with recommended stride form

	Natural	Natural	Mid-foot	Mid-foot	Preferred
	Rested [N]	Tired [N]	Rested [N]	Tired [N]	Stride
1	1580	1530	1650	1540	Natural
2	1460	1370	1410	1460	Natural
3	1230	1110	1100	1180	Mid-Foot
4	1550	1570	1540	1450	Mid-Foot
5	1570	1700	1620	1750	Natural
6	1380	1400	1410	1380	Either
7	1410	1560	1610	1430	Either
8	1000	1010	944	908	Mid-Foot
9	1760	1910	1850	1750	Either
10	1480	1400	1530	1470	Natural
11	1550	1530	1410	1550	Either
12	1660	1800	1660	1700	Mid-Foot
13	2120	2190	2100	2110	Mid-Foot

In addition to the Vernier force plate, an AccuGait force plate was also used. The AccuGait force plate was used to determine the loads in the horizontal direction, as well as the center of pressure (COP), the moment about the COP, and, in correspondence with the video data, the angular velocities and accelerations. The forces in the horizontal direction and the COP were then used in the force equations to determine forces on the knee and ankle joints upon landing. Table 7 shows the COP findings.

Table 7: COP data

Subject	Natural Landing COP [cm, from heel]	Mid-Foot Landing COP [cm, from heel]
2	2.4	4.1
6	7.1	4.8
7	4.6	5.6
12	17.2	15.0
4	24.2	3.1
9	16.7	6.0
3	7.1	3.3
13	25.4	10.7

^{**}This table only shows the result of 8 participants due to this analysis being conducted separate from initial and final testing.

These 8 participants' results were the only 8 results that were able to accurately be analyzed. Participant numbers still remain the same.

5.2.3 Force Equation Analysis

Through the use of free body diagrams (FBDs), equations were formulated to analyze the acquired data for the ankle and knee. The equations split the force into components on different muscles and bones. This was done in order to obtain the force acting on only the ankle and knee joint, and not the forces being absorbed by the muscles. For the ankle, the forces on the tibialis, Achilles and shin bone were evaluated from the resultant landing force; for the knee, the resultant knee force, hamstring force, and bone force were evaluated from the resultant landing force.

Through much analysis, and editing, the equations were drastically changed through the course of the study in order to develop a more accurate analysis. Due to time constraints preventing improvement of the knee equations, an analysis of the knee was not completed. Also due to time constraints, the ankle equations could not be made into a form that produced accurate or reasonable answers by the end of the project. Final ankle equations are shown in Figure 17: Final MathCAD Equations for Ankle.

$$\begin{split} F_{a} \coloneqq & \left[\frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} M_{s} \right)}{1 - \frac{0.045 \cdot 2}{L_{foot} \cdot \sin(\theta_{1})}} \right] - \left[\frac{\left(\left(-2 \cdot \cos(\theta_{1}) \cdot M_{a} + 2 \cdot \cos(\theta_{1}) \cdot M_{s} + \sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx} \right) \right)}{\frac{\left(2 \cdot \cos(\theta_{1}) \cdot \sin(\theta_{2}) \cdot 0.045 + \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2}) \right) \cdot \sin(\theta_{3})}{\cos(\theta_{3})}} \right] \\ F_{b} \coloneqq & \frac{-\left[2 \cdot \cos(\theta_{1}) \left(M_{a} - M_{s} + F_{a} \cdot \sin(\theta_{2}) \cdot 0.045 \right) - \sin(\theta_{1}) \cdot L_{foot} \cdot \cos(\theta_{2}) \cdot F_{a} + \sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx} \right]}{\cos(\theta_{3}) \cdot \sin(\theta_{1}) \cdot L_{foot}} \\ F_{t} \coloneqq & \frac{2}{\sin(\theta_{1}) \cdot L_{foot}} \left(M_{a} - M_{s} + F_{a} \cdot \sin(\theta_{2}) \cdot 0.045 \right) \end{split}$$

Figure 17: Final MathCAD Equations for Ankle

The values produced from the equations in Figure 17 were scattered, with most values being unrealistic. This could be due to both insufficient equation that came from a model that was too simple to accurately represent the problem and inaccurate input value due to errors in data collection. The simple model could have come from the lumping of multiple muscles, tendons, and bones together, as well as using a 2-D model to represent a 3-D problem. **Error! Reference source not found.** shows the overall resultant forces on the ankle that were evaluated by the final equations in MathCAD.

Table 8: Resultant for	rces on	ankle
-------------------------------	---------	-------

	Natural Rested [N]	Natural Tired [N]	Mid-foot Rested [N]	Mid-foot Tired [N]
2	2.64×10^4	$4.23\mathrm{^x}10^6$	1.32×10^{5}	4.81×10^5
6	1.46×10^{7}	1.04×10^{7}	8.16×10 ⁴	3.12×10^{3}
7	2.01×10 ⁶	7.50×10^{7}	2.05 x 10 ⁵	5.85×10^{5}
12	5.21×10 ⁶	1.08×10^{5}	5.58×10 ⁸	1.86×10^{5}
4	6055×10 ⁵	21.3	8.19×10^{5}	1.29×10^{5}
9	1.08 x 108	5.83×10 ⁶	1.50×10^{5}	3.79×10^{4}
3	3.56×10 ⁴	5.79×10 ⁶	1.45×10 ⁵	5.1×10^5
13	2.80×10 ⁶	2.72×10^6	7.47 × 10 ⁶	1.05×10^7

**This table only shows the result of 8 participants due to this analysis being conducted separate from initial and final testing. These 8 participants' results were the only 8 results that were able to accurately be analyzed. Participant numbers still remain the same.

One additional finding through the equations was the length of the moment arm from the Achilles tendon. Figure 18, Figure 19, and Figure 20 show the ankle analysis equations for a natural landing participant. The resultant force for on the Achilles tendon is highlighted to show the effects of the moment arm length on the resultant force on the Achilles.

$$F_{a} := \begin{bmatrix} \left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} M_{s} \right) \\ 1 - \frac{0.0254 \cdot 2}{L_{foot} \cdot sin(\theta_{1})} \end{bmatrix} - \begin{bmatrix} \frac{\left(\left(-2 \cdot cos(\theta_{1}) \cdot M_{a} + 2 \cdot cos(\theta_{1}) \cdot M_{s} + sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx} \right) \right)}{\left(2 \cdot cos(\theta_{1}) \cdot sin(\theta_{2}) \cdot 0.0254 + sin(\theta_{1}) \cdot L_{foot} \cdot cos(\theta_{2}) \right) \cdot sin(\theta_{3})} \end{bmatrix}$$

$$F_{a} = 3.173 \times 10^{4}$$
Figure 18: Achilles Distance (L) Equal to 1 in

$$\begin{aligned} F_{a} := & \left[\frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} M_{s} \right)}{1 - \frac{0.0381 \cdot 2}{L_{foot} \cdot sin(\theta_{1})}} \right] - \left[\frac{\left(\left(-2 \cdot cos(\theta_{1}) \cdot M_{a} + 2 \cdot cos(\theta_{1}) \cdot M_{s} + sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx} \right) \right)}{\left(2 \cdot cos(\theta_{1}) \cdot sin(\theta_{2}) \cdot 0.0381 + sin(\theta_{1}) \cdot L_{foot} \cdot cos(\theta_{2}) \right) \cdot sin(\theta_{3})} \right] \\ F_{a} = 8.539 \times 10^{3} \end{aligned}$$

Figure 19: Achilles Distance (L) Equal to 1.5 in

$$\mathbf{F_a} := \left[\frac{\left(-\mathbf{F_{RAy}} - \frac{2}{\mathbf{L_{foot}}} \cdot \mathbf{M_a} + \frac{2}{\mathbf{L_{foot}}} \mathbf{M_s} \right)}{1 - \frac{0.0127 \cdot 2}{\mathbf{L_{foot}} \cdot \sin(\theta_1)}} \right] - \left[\frac{\left(\left(-2 \cdot \cos(\theta_1) \cdot \mathbf{M_a} + 2 \cdot \cos(\theta_1) \cdot \mathbf{M_s} + \sin(\theta_1) \cdot \mathbf{L_{foot}} \cdot \mathbf{F_{RAx}} \right) \right)}{\left(2 \cdot \cos(\theta_1) \cdot \sin(\theta_2) \cdot 0.0127 + \sin(\theta_1) \cdot \mathbf{L_{foot}} \cdot \cos(\theta_2) \right) \cdot \sin(\theta_3)} \right] \\ \mathbf{F_a} = 6.201 \times 10^4$$

Figure 20: Achilles Distance (L) Equal to 0.5 in

5.3 Comparisons

After completing all video and force analysis, the data was compiled into 23 different variables. The most influential variables used throughout the course of the study were used to make comparisons in order to come to additional conclusions. The variables considered are listed in Table 9.

Table 9: Table of Significant Variables Compared

F_{y}	Vertical force						
$F_{\rm r}$	Resultant force						
Θ_1 , Θ_2 , Θ_3 ,	Angles of different muscles and						
δ_1 , δ_2 , δ_3	joints upon initial foot strike.						
Ma	Moment about the ankle						
COP	Center of pressure						

First, the center of pressure (COP) data collected by the AccuGait force plate was compared to the force data that led to a recommended stride form (Table 6). The COP was used to determine whether or not a person pronates or supinates while running. The COP graph shown in Figure 21 shows a change of -1cm to -4cm over time. This means that the participant landed on the force plate at -1cm, and as they ran across the plate, they adjusted to -4cm. This participant was given the recommendation to run with their natural stride, but was also a pronator. This correlation was found with all pronators (All COP graphs and data can be found in Appendix I: Compared Data). Due to this trend, it was recognized that pronators should continue to run with their natural stride.

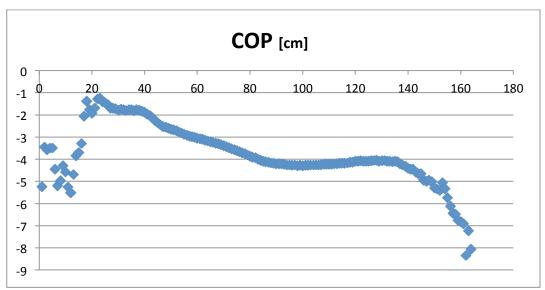
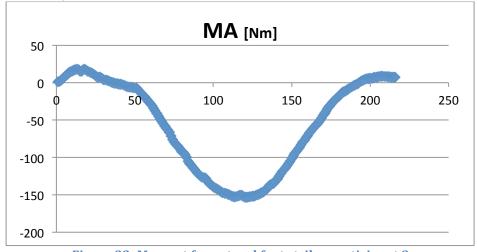


Figure 21: COP data, stride change not recommended

In addition to the center of pressure data, the moment about the ankle was analyzed. The moment was calculated throughout the entire stance phase using a MATLAB code; the data points collected were plotted and the graphs for natural and mid-foot strike were analyzed against one another. The moment for natural foot strike is less than the moment for the mid-foot strike overall. Appendix I.4 shows the comparisons for all participants observed. In addition, some of the natural moment graphs have a small spike in the positive x- direction at the beginning of the graph. This is because the foot is dorsi-flexed at the beginning of the foot strike. Figure 22 and Figure 23 show the moment about the ankle for mid-foot and natural foot strike. To view all graphs refer to Appendix I: Compared Data.



Figure~22: Moment~for~natural~foot~strike, participant~3

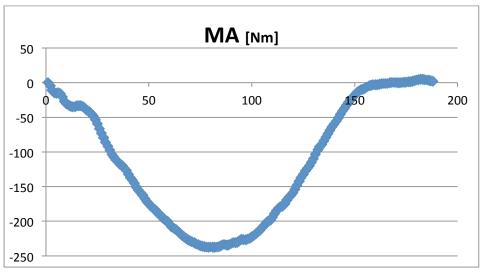


Figure 23: Moment for mid-foot strike, participant 3

The last comparison conducted with the collected data was landing force versus landing angles. The landing angles at the beginning and end of a run were compared to the forces produced. There was no clear correlation between the landing angle and the landing force. This result coincides with the overall conclusion that benefits of stride change are dependent upon the individual. Table 10 shows a comparison chart for natural landing angles vs. natural landing forces, and Table 11 shows the mid-foot landing angles vs. mid-foot landing forces. To see the landing force compared to all foot and leg angles upon landing, view Appendix I.5 Force-Angle ComparisonFrom the table, it can be seen that there is no specific data correlation.

Table 10: Comparison of Natural Stride Landing Forces and Landing Angle

Participant #	Rested Landing Force [N]	Rested Landing Ankle Angle [deg]	Tired Landing Force [N]	Tired Landing Ankle Angle [deg]			
1	1580	31	1530	34			
2	1460	32	1370	36			
3	1230	34	1110	37			
4	1550	25	1570	34			
5	1570	36	1700	33			
6	1380	37	1400	33			
7	1410	39	1560	38			
8	1000	43	1010	36			
9	1760	52	1910	51			
10	1480	35	1400	25			
11	1550	35	1530	33			
12	1660	38	1800	36			
13	2120	42	2190	45			

Table 11: Comparison of Natural Stride Landing Forces and Landing Angle

10010 221		turar Striac Editari	8	88				
Participant #	Rested Landing Force [N]	Rested Landing Ankle Angle [deg]	Tired Landing Force [N]	Tired Landing Ankle Angle [deg]				
1	1650	134	1540	132				
2	1410	130	1460	125				
3	1100	125	1180	129				
4	1540	136	1450	135				
5	1620	134	1750	131				
6	1410	132	1380	127				
7	1610	130	1430	133				
8	944	132	908	133				
9	1850	122	1750	123				
10	1530	135	1470	139				
11	1410	134	1550	137				
12	1660	133	1700	144				
13	2100	127	2110	130				

6. Conclusions and Recommendations

After completion of data collecting and analysis on the benefits of changing running stride, the following conclusions were determined and recommendations deemed valuable to enhancing future studies.

6.1 Conclusions from Findings

The benefits of stride change are dependent upon the individual. Based on the findings, there was no clear correlation in results that could determine whether or not stride change is beneficial. Instead, the benefit of stride change depends on the individual.

Pronators should not change their running stride. From the comparisons, participants that pronated were also the participants who were recommended to maintain their natural running stride. A possible explanation could be that since pronators are already adjusting to the pronation when running, a change in their stride might cause the body to respond with higher landing forces. Over time a pronator may be able adjust to a different stride, but it is not clear exactly how long it would take. *It is recommended for pronators to not adjust their running stride*.

6.2 Overall Recommendations to Improve Future Studies

The first recommendation is for **further study into the muscles that alleviate forces from the ankle.** With more focus being put on the muscles that alleviate the overall force, a more accurate resultant force on only the ankle joint can be found.

Further study into the equation analysis is recommended. In this study the model used to estimate forces on the joint may have been too simple to produce accurate results. In the future it is recommended to develop a more sophisticated model to analyze the collected data.

If further study is conducted on this topic, additional recommendations have been made in order to eliminate potential errors in data. The recommendations are listed below: **Eliminate the use of two force plates, and use the AccuGait force plate, or force plates similar, for all analysis.** For the purposes of this study, it was not possible to use an AccuGait force plate for the entirety of the study. Due to this, there was an error factor caused by having two different force plate readings as well as a certain level of assumptions that had to be made. If an AccuGait force plate is used for the full analysis, results will be more consistent and accurate.

Remove participant error, including fitness, landing on force plate, running shoes. To eliminate the inconsistency caused by increased participant fitness, a two-week training period could be required for each participant before they begin the study. Another option is to have a control group to use for comparison. Additionally, the style of the shoe may affect results and forces, therefore it is recommended to split participants up into groups by shoe type or make sure they are all wearing similar shoes. Lastly, participants tend to change

their stride when they have a small step up to the force plate. It is recommended to use either a runway up to the plate, or use a plate that is sunk into the ground.

7. Discussion

After obtaining all necessary data, there are various situations that could have affected the final results achieved. In this study, two different group members were responsible for analyzing video. This led to some potential error due to differences in measuring and tracking style. In addition to tracking errors, the equations that were developed through the use of free body diagrams may have been too simple of a model to develop accurate data. Also, a group member did not write the MATLAB code used to further develop values and equations; due to this, the group cannot justify every aspect of the code. Along with the model being too simple, there was a chance of error due to the use of two different force plates and the assumptions that went along with them. One force plate was used to measure the vertical force, and a different force plate was used to measure horizontal force and center of pressure (COP). The assumptions made were that the COP would remain the same for all recorded tests. Additionally, the horizontal force was assumed to be the same for each individual stride.

One other problem that was encountered with the Vernier force plate was that it was elevated off the ground, requiring participants to step up a small amount to strike the force plate. This sometimes caused the participant to adjust their landing when hitting the force plate, which caused some error in data. Lastly, throughout the course of testing there were two different people responsible for data collection and set-up of equipment. It is possible that having two different people in charge of testing on different days could have caused some error in the data.

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Appendix

Appendix A: Participant Training Packet

This packet includes a calendar with the training schedule until the end of this study. The days that you will be training with us, we will explain what drills you will do and how long you will run for. It is expected that you will run at most 20 minutes per day, 3 times a week, for 6 consecutive weeks, in the Rec center. You are not expected to train on your own, only in the allotted times on the schedule below with study conductors. Your lower body will be video taped only to be viewed by the conductors of the study. The footage will be used, along with the force plate data collected as you run, to find angles and forces of the legs and feet. The drills and fitness test are described after the calendars. If there are any questions or concerns, please contact Chelsea, Heather, or Alicia.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5
		CHRISTMAS BREAK			
7	8	6	10	=	12
	CHRISTMAS BREAK			Pre-Training Fitness Test	
				Test #1: Natural Stride Running	
14	15	16	17	18	19
Test #2: Natural Stride Running		Test #3: Natural Stride Running		Drills: Calf raises, High Knees, Weight Shiffing, Falling Drill, Foot Tapping, and Arm swinging	
21	22	23	24	25	26
NO CLASSES	Drills: leaning, foot tapping, arm swinging.		Drills: Hip, arm swinging, high knees, calf raises (3 x 15)		
	Run: 3 x 5 minutes.		Run: 3 x 5 minute barefoot		
28	29	30	31		
Drills: High hopping, heel slides, marching		Drills: high knees, calf raises (3 x 20)			
Run: 10 minutes, BREAK to adjust stride, 5 minutes		Run: 10 minutes, BREAK to adjust stride, 5 minutes			

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				_	2
				Run: 10 minutes barefoot	
	5	9	7	8	6
Drills: weight changing, hips, calf raises (3 x 20). Run: 3 x 7 minutes.		Drills: leaning, foot tapping Run: 15 minutes		Run: 15 minutes, BREAK to adjust stride, 5 minutes	
	12	13	14	15	16
Drills: review of drills Run: 20 minutes (adjusting as needed)		Drills: review of drills Run: 20 minutes	NO CLASSES	Post-Training Fitness Test Test #1: Mid-foot Stride	
	19	20	21	22	23
Test #2: Mid- ot Stride		Test #3: Mid-foot Stride			
	26	27	28		

Pre- and Post-Training Fitness Test

- 10 minutes run at comfortable pace on treadmill. Timer will start once the runner finds his/her comfortable speed.
- Heart-rate will be checked
 - o Before starting 10-minute run
 - o Approximately 5-minutes into run
 - o After 1.5-minutes
- Measurements will be taken:
 - o Length of leg, ankle to knee
 - Length of foot, toe to ankle
 - Shoe size
 - o Height
 - o Weight

Test Day (Natural and New Stride

- 20 minutes running consistently, at comfortable pace. (DOES NOT HAVE TO BE FAST)
 - This will be run on the indoor track
- During every lap you will run over a force plate, which will be in lane 3 of the track.
- When running over the force plate your legs will be videotaped using a high-speed camera, and the forces you strike the ground with will be recorded using the force plate.
- You will be asked to run over the force plate every time you run a lap, but you do not need to stay in lane 3 the entire lap; you just need to hit the force plate!
- After the 20 minute run, for two of the six test days, you will be asked to go to Goddard hall to run over an additional force plate. This will be a short run of about 30 ft. MAX (this is just to help us further analyze the way you are landing, but this will require very minimal effort, and should not add more than 10 minutes to the test day).

Drill Description

• **Weight Shifting**: The purpose of this drill is to focus on shifting body weight from the heel to the forefoot. This will help to understand how there body should be when running correctly. This drill is shown in Figure 24.



Figure 24: Weight Shifting Drill

• **Falling Forward**: For this drill, participants will fall forward in front of a wall while maintaining a running position. Participants will start close to the wall and move farther from the wall as comfort level increases. This drill is shown in Figure 25



Figure 25: Falling Forward Drill

• **Foot Tapping**: Participants will pull their foot from the ground using the hamstrings and allows the foot to fall back to the ground using gravity; the foot should not actively be lowered. In addition, the foot should not extend out in front of the participant. The foot should remain under the body. This drill is shown in Figure 26.



Figure 26: Foot Tapping Drill

- **Calf Raises:** Participant will stand with feet hip distance apart, then slowly rise to the toes of their feet and then slowly lower him/herself down. This motion will help strengthen calf muscles in order to prevent strained muscles.
- **High Knees:** Participant will jog in place lifting one knee at a time to about hip level. The ankle should remain under the knee and toe should be pointed up. This will focus directly on the proper leg motion and foot landing.
- Barefoot running: Running barefoot will help to strike the ground more "natural".
 **FOR THIS YOU MAY RUN WITH SOCKS ON, AND WILL NOT HAVE TO RUN LONGER THAN 10 MINUTES!
- Marching: Participant will start walking slowly forward on the balls of their feet (heels should not touch the ground). The participant will then raise their knee until their thigh is parallel to the ground, or hip level, on each stride. The participant should rise on the toe of the opposite foot, as their heel moves upward along the inseam of their pants. The ankle should be directly under or slightly behind the lifted knee, as well as toe pointing upward. The chin and torso should be upright, and the participant should not be leaning backward.
- **HEEL SLIDES:** Begin by performing a slow jog. Using a short stride and bouncing on

your toes, raise your heels as high as possible but do not allow your heels to travel behind your body. Imagine a wall at your back. Bring your heels back so that your feet are flat against the imaginary wall and bring them up as high as possible. Your heels should nearly reach your buttocks. Both upper and lower leg action is involved in this exercise. There should be little forward distance covered, but keep moving forward.

Hip Drill

• **Hips Tall Position**: The participant will stand with their feet a comfortable distance apart. They will slowly rise, supporting their body weight on the balls of their feet while squeezing their abdominals. This motion will help with hip positioning while running.

Arm Swinging Drills

- **Side Brush**: The participant will move their arms in a motion where the palm of their hands gently brushes the area between ribcage and hips. Hands should be relaxed.
- **Pendulum**: Participants will swing their arms loosely front to back with a 90degree bend in the elbow. The key to this drill is that the shoulders are relaxed, hands are relaxed, and shoulders are not rotating. After a period of time, participant should be able to increase the speed for the swing while maintaining the same relaxed composure.

Appendix B: Participant Agreement Form

Informed Consent Agreement for Participation in a Research Study

Purpose: The purpose of this study is to determine the forces and stresses on the knee and ankle associated with changing running stride. This will possibly allow for a reduction in injury to the knee and ankle joints.

Procedure: Testing will last for approximately 6-weeks throughout the course of C-term (ending before finals week). You will be expected to run 3 days per week for the 6 week period, for no more than 20 minutes (some days less). You will also be asked to run barefoot for 1 or 2 days of the training for no more than 10 minutes each time. In addition, hefore each testing week you will be expected to participate in a pro-test to door your b a S d

fitness level and reduce your risk for injury throughout the study. Two of the 6 weeks will be saved for testing. During testing you will be videotaped from the hip down, in order for analysis of your knee, ankle, and foot strike to be completed. In addition, the landing forces during your run will be recorded through the use of a force plate.
☐ "I have been given a copy of the training calendar and packet that describes study activities and understand what I am expected to do to complete this study."
"I have been warned about running barefoot on the track in the rec center for no more than 10 minutes for a maximum of 2 days during training and I am willing to comply. I also understand that barefoot running may lead to blisters, foot abrasion, and/or joint pain and barefoot running can be more uncomfortable than running with shoes; therefore I will stop running if experiencing pain level 7 or greater."
Risks to participants: Same risk as if training of your own.
In the event of injury: If an injury does happen as a result of this study, the investigators will take proper measures to ensure your safety. We will always encourage rest when discomfort arises, recommend ice when necessary, and consult the University athletic trainers if a severe injury occurs.
\square "I understand the risk involved with this study, and agree to tell Chelsea, Heather, or Alicia the moment an injury, pain, or discomfort arises throughout the course of this study."
Benefits to research participants and others: Improved fitness as well as gaining an understanding for the effects on Impact and applies when changing running strides to help

understanding for the effects on knees and ankles when changing running strides to help decrease injury.

Record keeping and confidentiality: All recorded data from these tests will be maintained by the members of the MQP group until the conclusion of the project. At the conclusion of the project, all data will be transferred to Professor Savilonis. No data allowing for personal identification of the subjects will be required.

Ι t.

Your participation in this research is volunt result in any penalty to you or any loss of bene You may decide to stop participating in the res other benefits. The project investigators retain experimental procedures at any time they see it	fits to which you may otherwise be entitled. earch at any time without penalty or loss of the right to cancel or postpone the
"I understand that all video, or additional understand that my participation in this research	
For more information about this research, to in case of research-related injury, please co Turner at gait13@wpi.edu.	
If additional assistance is required, please cont Review Board at <u>kjr@wpi.edu</u> or ext. +5296, or advisor, at <u>bjs@wpi.edu</u> or ext. +5686.	
By signing below, you acknowledge that you has a participant in the study described above. Makeyour satisfaction before signing. You are entitled	ke sure that your questions are answered to
	Date:
Study Participant Signature	
Study Participant Name (Please print)	
	Date:
Signature of Person who explained this study	

Appendix C: Analysis Procedure Detailed Checklist Collecting Data ____Ask participants pre-running questions ____Mark participants for video recording ____Place large black dot 1" above the anklebone. ____Wrap tape around the participant's right hoe at their metatarsal bone, and place a black dot on the tape at the point of the metatarsal. (Should be on shoe) ____Place a large black dot on the knee. Have the participant bend there knee fully and draw the dot along the straight line from the line bend.

a black dot on the tape at the point of the metatarsal. (Should be on shoe) ____Place a large black dot on the knee. Have the participant bend there knee fully and draw the dot along the straight line from the line bend. ____Record force using force plate FORCE PLATE SET UP ____Place force plate in lane 3 of track, 25" from the white pole (measure from front edge of f.p.) ____Place force plate into lane three, 10" into lane 3 from the outside white line (measure from the right side edge of f.p.) ____Video record running with light CAMERA SET UP ____Set tripod up with one leg forward, and two legs back; the two back legs should be in a straight line. ____The back right leg of the camera tripod is 74" from the top right edge corner of force plate

____Zoom on the camera should be exactly 0.09foot. The setting on the top is on "S", and the quality is "HS 240" _____Align force plate in the middle, and bottom window is lined up with the inside white line of lane 3. Video Analysis

____Import video file (make sure this corresponds with the force reading) ____Cut video down to be heel strike to toe-off ____Put motion trackers on all three markings ____Export the data to excel ____Use this data with the AccuGait force plate data to calculate the angular velocity,

angular acceleration x and y

AccuGait Force Plate Procedure and Analysis

_Set up force plate
Get USB from office on first floor of Goddard
Plug Ethernet cord into "A" outlet
Plug in white chord, which connects computer to force plate
Open AMTI-Net force *top right is center of pressure
Go to "Start up" in AMTI and zero the program
Check acquisition in "settings" and put it to 500
Check trigger in "Settings" (possible to set delay)
All analysis is save to local C drive \rightarrow AMTI-Net force folder \rightarrow Data folder \rightarrow force
plate data

i

To save hit "save" button and it goes to the C driveTo review, go to BioAnalysisPut into excel; Bioanalysis → view → data → COP data; Copy and paste into excelHave participants run over force plateView data, copy and paste COP x- and y- and Fx, Fy, Fz into excel, along with the time of	
those forces Calculate x-distance, y-distance Use MATLAB functions to calculate angular velocity, angular acceleration x and angula acceleration y.	
EquationsInsert any new known numbers into MATHCADSolve for the desired variablesCompare results	

Appendix D: Measured Angles

	-89.72 Knee (Delta 1)	133.59 Ankle (Theta 2)	-17.05 meta(Theta i)	114.79 Patella (Delta 3)	139.67 Hamstring (Delta 2)	89.64 Achillies (Theta 3)	142.43 136.27 Tibialis (Theta1)		-90.10 Knee (Delta 1)	134.24 131.45 Ankle (Theta 2)	-12.17 -13.42 meta(Theta i)	110.76 Patella (Delta 3)	139.16 Hamstring (Delta 2)	87.51 Achillies (Theta 3)	133.87 Tibialis (Theta1)		-93.48 Knee (Delta 1)	-35.52 Ankle (Theta 2)	meta(Theta i)	112.60 Patella (Delta 3)	140.19 137.99 Hamstring (Delta 2)	92.53 Achillies (Theta 3)	Tibialis (Theta1)		-91.85 Knee (Delta 1)		Ankle (Theta 2)	2.58 Ankle (Theta 2) 4.33 meta(Theta i)	-28.14 -32.58 Ankle (Theta 2) 10.46 4.33 meta(Theta i) 112.17 116.86 Patella (Delta 3)	-28.14 -32.58 Ankle (Theta 2) 10.46 4.33 meta(Theta i) 112.17 116.86 Patella (Delta 3) 139.16 140.61 Hamstring (Delta 2)
	.72 Knee	.59 Ankle	.05 meta	.79 Patel	.67 Ham	.64 Achil	.27 Tibia		.10 Knee	.45 Ankle	.42 meta	.76 Patel	.16 Ham	.51 Achil	.87 Tibia		.48 Knee	.52 Ankle	5.03 meta	.60 Pate	.99 Ham	.53 Achil	.88 Tibia		.85 Knee	58 Ankly		.33 meta	.33 meta .86 Patel	.33 meta .86 Patel .61 Ham
			54 -17			92.15 89.	43 136			24 131	17 -13		24 139	81.00 87.					-3.27 5.	13 112	19 137		58 142.88			14 -32.58	-1		46 4. 17 116.	46 4. 17 116. 16 140.
2	-86.52	140.06	-13.54	113.88	138.37	92.	142.	2	-93.71	134.	-12	108.31	139.24	81.	133.30	2	-101.09	7 -38.29		5 109.13		0 86.31	6 138.58	2	1 -88.90	7 -28.14				
		2	2	0	3	1	0		0	5	3	0	7	3	et		9 -88.26	-33.37	7 15.10	114.25	137.91	98.90	147.46		1 -90.61	3 -35.27		0 2.53	12	0.00 2.53 115.90 122.51 137.47 145.20
	-92.92	127.12	-20.56	115.70	140.98	87.14	130.10		-86.50	128.66	-14.68	113.20	139.07	94.03	134.44		-91.09	-34.90	3.27	114.42	135.88	92.39	142.59		-96.04	-34.33		0.00		0.00
	-82.32	136.82	-12.16	120.59	142.10	-22.51	138.43		-96.39	134.63	-13.80	115.68	134.38	89.37	133.29		-84.79	-25.07	18.96	113.91	137.42	95.39	144.26		-89.11	-34.04		9.99	11	17.04 9.99 114.85 113.67 136.71 135.00
	-85.29	135.00	-11.31	120.64	143.67	93.05	135.92		-82.57	135.00	-15.78	121.18	142.33	96.01	141.16		-84.17	-25.41	19.54	116.27	137.82	99.83	146.56		-86.15	-27.76		17.04	17.04	17.04 114.85 136.71
4	-82.36	135.76	-14.32	123.60	140.37	-83.83	135.52	4	-99.69	129.81	-15.95	114.24	133.49	82.09	130.19	4	-83.53	-20.07	22.50	119.30	141.88	95.64	139.72	4	-87.73	-31.61		11.31	11.31	11.31 112.64 133.08
	-79.30	139.69	-10.84	117.54	142.27	-76.76	143.86 135.52		-106.93	139.09 129.81	-9.66	111.61	127.34	90.00	128.52		-86.67	-29.74	14.83	106.15	132.56	90.71	146.51		-93.43	-42.77	ı	1.61		
	-93.39	125.28	21.15	105.73	131.91	79.47			-85.25		18.40	110.61	137.19	92.13			-91.83	-34.17	11.25	112.89	137.86	86.29	138.61		-93.49	-36.93		7.34		
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3	-95.64	123.31	25.28	102.00	125.94	84.12	134.19 132.04 134.29	3	- 86.59	131.31	18.82	111.80	135.88 139.84	90.00 103.52	141.19	3	- 86.59	-29.58	15.80	112.05 1	136.97	89.27	142.82	3	-89.29	-32.97		14.74	14.74	14.74 114.71 137.12
	-88.77	126.87	10.62	112.43	137.55	78.33	136.64		-89.42	128.23	9.27	111.20	135.86 1	82.87	138.01		-91.85	-38.05	6.05	115.40	136.29 1	84.62	135.00		-96.10	-42.27		-0.97	7	
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1	-79.99	133.15	11.59	107.86	136.09	82.45	133.03	1	-81.35	135.83	9.46	116.40	140.54	85.24	131.03	1	-83.93	158.75	15.46	113.96	136.03	102.24	147.99	1	-81.06	145.12		1.19	11	
	-82.87	31.19	10.78	15.09	138.81	86.22	31.63		89.31	31.19	6.48	15.24	133.54	82.54	126.03		-80.54	140.01	17.74	114.87	138.48	98.13	143.28		-81.30	145.56		9.27	9.27	9.27 116.40 140.19

-45.39 Ankle (Theta 2)	-13.32 meta(Theta i)	Patella (Delta 3)	137.75 Hamstring (Delta 2)	118.80 Achillies (Theta 3)	Tibialis (Theta1)		Knee (Delta 1)	-41.37 Ankle (Theta 2)	meta(Theta i)	Patella (Delta 3)	140.80 Hamstring (Delta 2)	Achillies (Theta 3)	Tibialis (Theta1)		-88.31 Knee (Delta 1)	-34.65 Ankle (Theta 2)	10.53 meta(Theta i)	105.41 Patella (Delta 3)	143.84 153.31 Hamstring (Delta 2)	119.50 Achillies (Theta 3)	Tibialis (Theta1)		Knee (Delta 1)	-24.86 Ankle (Theta 2)	meta(Theta i)	110.50 110.03 Patella (Delta 3)	154.20 156.01 Hamstring (Delta 2)	119.29 121.18 Achillies (Theta 3)	138.79 141.74 Tibialis (Theta1)
-45.39	-13.32	97.07	137.75	118.80	138.26		-92.07	-41.37	-5.84	96.66	140.80	119.94	139.94		-88.31	-34.65	10.53	105.41	153.31	119.50	140.37		-85.09	-24.86	17.41	110.03	156.01	121.18	141.74
-45.00	-10.39	91.74	135.67	119.03	137.66		-93.58	-40.91	-5.53	97.20	144.38	120.58 119.94	141.00		-92.17	-38.66	86.9	98.04	143.84	119.98	140.10 140.37		-84.81	-24.57	17.10	110.50	154.20	119.29	138.79
-45.78	-16.26	102.41	139.83	118.57	138.87	10	-90.57	-41.82	-6.15	96.12	137.22	119.31	138.87	10	-83.35	-31.18	15.95	111.70	58.20	121.78	140.95	10	-87.25	-28.22	17.35	05.15	152.53	121.64	41.72
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-58.41	-21.80	96.55	130.09	115.35	137.63		-92.89	-56.75	-25.75	99.11	125.24	17.42	138.81		-94.27	-51.87	-17.42	91.85		116.96	138.56		-91.83	-50.53	-15.77	88.59	32.59	116.54	37.36 1
- 57.09	-25.50	98.79	129.17	112.89	132.01		-100.30	-63.43	-31.83	87.95	118.01	116.42 117.42	135.44		-92.97	-52.35	-16.70	90.00	127.30 131.27	119.93	140.01		-95.13	-54.90	-19.44	76.18	126.14 132.59 161.30	115.52	136.00 137.36 144.70 141.72
-53.70	-17.22	92.83	132.27	117.36	134.34	6	-91.13 -:	-57.99	-27.30	97.56	123.55		139.71	6	-92.86	-52.13	-19.98	95.19	135.00	117.78	139.23	6	-92.78	-53.33	-15.83	93.39	131.38	115.49	139.37
-64.44	-22.69	98.03	128.83	115.79	146.54		-87.24	-48.81	-18.12	111.80	134.16	119.01 116.84	141.28		- 6.96	-51.15	-15.57	90.35	131.50	113.17 1	136.45		- 87.59	-43.36	-12.03	96.20	140.25 1	118.63 1	136.73 1
-48.54	-15.04	90.78	131.19	117.34			-89.91	-46.96	-10.43	94.76	137.21		139.50		-97.61	-43.12	-0.84	92.00		118.67	138.04		-84.07	-35.48	3.14	95.83	146.86		138.06
-47.23	-13.19	90.88	130.19	117.87	134.20 135.82		-85.08	-43.45	-10.12	96.09	140.50	118.47 117.34 116.83	141.40		-95.63	-45.00	-2.54	94.64	140.06 140.25	120.44	138.98		-83.57	-32.74	4.97	96.34	148.39	116.35 116.01	138.96
-47.23	-13.19 -1	5 29.06	132.44	115.77 11	137.73	8	-82.08	-46.64	-9.46	99.46	140.53 14	8.47 11	140.85 14	8	98.97	-42.18	1.02	90.70	135.63 14	118.46 12	136.29 13	8	-83.66 -8	-42.27	-2.01	95.96	143.75 14	115.83 11	137.06
-51.15 -4	-18.74 -1	90.81	130.95	118.39 11	135.54 13		-99.57	62.05-	-11.69	88.74 9	130.60 14	114.70 11	136.26 14		-98.23 -9	-42.18	-1.01	90.65	145.07 13	117.11	138.86 13		-84.98 -8	-31.43 -4	6.46	95.19	148.45 14	115.84 11	138.18
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-49.59	-14.34	117.23	135.20	92.00	134.72		-96.16	-46.85	-12.04	112.94	130.24	86.44	135.16		-88.95	-38.68		116.31	135.82	92.98	143.46		-90.63	-38.02		115.63	135.78	93.22	141.39
-49.09	-14.28	116.11	135.40 133.99	87.36	133.15		-97.20	-50.63	-12.34	115.82	131.99	86.78	133.39	7	-91.81	-42.44	2.77	114.99	135.47	87.14	136.74	7	-88.73	-39.21	-4.47	115.02	133.09	97.21	145.78
-52.03	-15.33	117.44	135.40	94.03	137.53		-97.05	-47.49	-14.04	113.31	130.06	88.55	137.08		-91.32	-42.18	0.95	118.83	135.45	97.47	146.06		-92.57	-40.46	3.88	116.57	136.90	93.45	138.08
-47.64	-13.39	118.14	136.22	94.61	133.48		-94.24	-42.44	-9.75	109.68	128.66	83.99	135.00		-83.73	-31.43	15.10	115.11	136.53	94.33	147.60		-90.60	-34.38	7.59	115.30	137.35	88.99	140.30
132.28	-15.40	116.40	140.47	91.84	134.12		-91.62	126.47	-17.51	117.39	139.54	87.89	132.12		-82.41	-37.24	-4.63	115.22		91.49	144.48		-87.29	-32.54	6.93	119.60	138.77	96.26	140.65
132.27 132.28	-18.89	113.03	139.01	92.39	129.70		-100.62	126.38	-17.28	119.21	139.60	88.33	128.52		-84.05	-20.85	14.93	115.60	137.08 138.99	102.23	158.03		-85.70	-27.15	12.63	125.89	143.19	95.40	143.81
30.36	11.04	17.77	42.45	92.27	36.19	9	85.91	26.38	.17.65	15.80	40.40	84.67	33.96	9	.78.69	46.68	17.18	13.33	36.40	88.29	41.48	9	93.73	41.19	-2.96	17.37	35.43	87.32	32.06

	-93.36	-52.56	-6.83	87.86	130.36	112.04	131.89		-94.91	-49.86	-5.61	88.33	130.19	112.12	137.25		-93.77	-42.37	2.61	86.67	133.87	117.67	120.99		-93.96	-44.75	0.20	88.21	129.95	112.55	134.23
								_								_	-90.00	-38.00	2.08	95.57	144.95	111.25	132.69	_	-98.83	-47.05	0.86	82.67	124.47	109.20	130.60 130.97 134.23
13	-93.93	-54.55	-9.07	86.96	127.38	113.53	131.19	13	-100.27	-48.81	-5.64	85.14	128.83	111.80	136.40	13	-90.00	-47.94	7.31	86.51	130.76	110.85	123.92	13	-92.56	-48.53	-2.05	89.64	124.36	108.06	130.60
	-92.79	-50.57	-4.60	88.76	133.34	110.56	132.59		-89.55	-50.91	-5.57	91.52	131.55	112.44	138.11		-101.31	-41.19	-1.55	77.94	125.91	130.90	106.36		-90.49	-38.66	1.79	92.33	141.01	120.40	141.12
	-92.89	-46.86	-11.41	89.49	128.89	109.81	128.76		-86.07	-35.79	-5.55	93.63	133.85	112.60	127.93		-88.97	-37.64	1.40	91.92	138.77	108.48	127.84		-86.37	-36.08	3.37	92.89	140.44	111.22	128.21
12	-93.81	-42.93	-11.15	91.25	131.03	104.91	124.66	12	-86.07	-35.79	-5.55	93.63	133.85	112.60	127.93	12	-87.43	-36.61	-1.04	94.62	144.22	110.24 114.10	130.97	12	-88.13	-36.38	0.00	91.36	145.76	112.59	131.63
,								1									-90.67	-39.21	1.88	88.70	134.56		129.02	1	-84.03	-35.68	4.76	93.99	137.29	110.56	132.90 132.69 125.78 127.22 131.63
	-91.97	-50.79	-11.66	87.73	126.75	114.70	132.86										-88.82	-37.09	3.37	92.44	137.53	101.09	123.52		-86.96	-36.19	5.36	93.30	138.27	110.52	125.78
	-90.49	-45.78	-12.70	95.67	135.42	114.07	128.96		-84.54	-42.51	-7.88	96.91	139.90	116.82	132.09		-89.82	-34.75	9.99	98.89	143.85	112.87 109.64	129.00		-87.19	-32.86	13.93	103.16	146.70	119.03	132.69
11								11								11	-94.13	-39.81	6.46	94.40	144.13	112.87	131.52	11	-93.37	-36.87	10.12	94.13	140.95	121.22	
1								1								1	-87.78	-33.69	9.46	102.09	146.47	108.75	126.71	1	-85.10	-30.96	9.87	104.36	146.67	117.25	131.35
	-90.49	-45.78	-12.70	95.67	135.42	114.07	128.96		-84.54	-42.51	-7.88	96.91	139.90	116.82	132.09		-87.56	-30.76	14.04	100.19	140.94	107.29	128.77		-83.11	-30.74	21.80	111.00	152.47	118.61	133.83
Participant #	Knee (Delta 1)	Ankle (Theta 2)	meta(Theta i)	Patella (Delta 3)	Hamstring (Delta 2)	Achillies (Theta 3)	Tibialis (Theta1)	Participant #	Knee (Delta 1)	Ankle (Theta 2)	meta(Theta i)	Patella (Delta 3)	Hamstring (Delta 2)	Achillies (Theta 3)	Tibialis (Theta1)	Participant #	Knee (Delta 1)	Ankle (Theta 2)	meta(Theta i)	Patella (Delta 3)	Hamstring (Delta 2)	Achillies (Theta 3)	Tibialis (Theta1)	Participant#	Knee (Delta 1)	Ankle (Theta 2)	meta(Theta i)	Patella (Delta 3)	Hamstring (Delta 2)	Achillies (Theta 3)	Tibialis (Theta1)
4			Landing	Angles	first first Pe Pa Angles Midfoot Last				4			Landing	Angles	first			4			Landing	Angles	Last									

Appendix E: Video Analysis of Participants

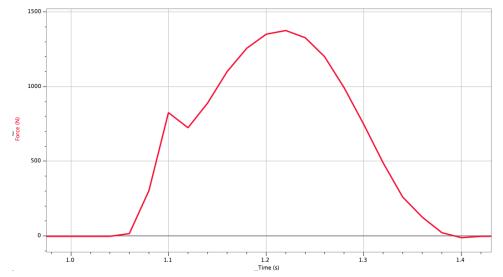
E.1 Exported Data from Adobe AfterEffects

4 A	Exported Da	С	D	Е	F	G	Н	-	J	K	L	M	N	0	Р	Q
1 BK						AK						Ankle				
2 Adobe A	After Effects 8.0 Keyframe Da	ta				Adobe A	fter Effects 8.0 Keyframe Da	ta				Adobe A	fter Effects 8.0 Keyframe Da	ata		
3																
4	Units Per Second	29.97					Units Per Second	29.9	7				Units Per Second	29.97	7	
5	Source Width	432					Source Width	43	2				Source Width	432	2	
6	Source Height	320					Source Height	32	0				Source Height	320)	
7	Source Pixel Aspect Ratio	1					Source Pixel Aspect Ratio		1				Source Pixel Aspect Ratio	1		
8	Comp Pixel Aspect Ratio	1					Comp Pixel Aspect Ratio		1				Comp Pixel Aspect Ratio	1		
9																
10 Motion	Tr Tracker #1	Track Poir	Feature Ce	enter		Motion 1	r Tracker #1	Track Po	in Feature	Center		Motion 1	r Tracker #1	Track Poi	n Confiden	ce
11	Frame	X pixels	Y pixels				Frame	X pixels	Y pixels				Frame	percent		
12	0	215.5	62				(21	9 2	2				100)	
13	1	218.117	62.3789				1	221.8	2 22.308	5						
14	2	220.73	62.8789				2	225.13	3 22.867	2		Motion 1	r Tracker #1	Track Poi	n Feature C	enter
15	3	223.641	65.8633				3	228.50	4 23,550	3			Frame	X pixels	Y pixels	
16	4	226.25	67.3633				4	232.00	4 24.855	5				213	190.5	
17	5	228.746	68.3633				5	235.41	8 26.156	3						
18	6	230.016	69.7305				6	238.62	5 27.546	9		Motion 1	r Tracker #1	Track Poi	n Attach Po	int
19	7	234.574	73.2305				7	241.75	4 28.33	2			Frame	X pixels	Y pixels	
20	8	237.168	74.543				3		4 29.70	7				212		
21	9	240.758	78.0977				9	248.	5 31.20	7				1 212.5	190.461	
22	10						10							2 213.246	190.383	
23	11						11							3 213.176		
24	12						12							4 213.5		
25	13						13							5 213.469		
26	14						14							5 213.344		
27	15						15							7 213.5		
28	16						16							3 213.57		
29	17						17							9 214.152		
30	18						18						1			
31	19						19						1			
32	20						20						1			
33	21		12.22.				21						1			
34	22						22						1			
35	23						23						1			
36	24						24						1			
37	25						25						1			
38	26						26						1			
39	27						27						1			
10	28						28						2			
+0 1 1	29						20						2			
	ap 2 / Lap 3 / Lap 14 / Lap	-	13:1011				2:	230.01	U 47,103	,				217.00	170,002	

E.2 Exported Data from Photoshop

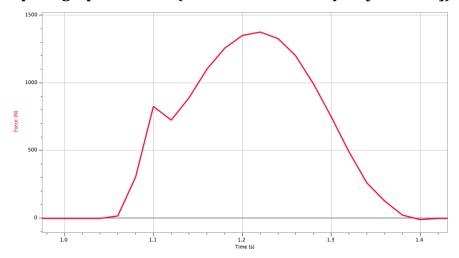
R	S	T	U	V	W	X	Y	Z	AA
BK									
Label	Date and Time	Document	Source	Scale	Scale Unit	Scale Fact	Count	Length	Angle
Ruler 2	2012-11-15T12:53:07-05:00	CIMG0926 (1) 2_00314.tif	Ruler Too	1 pixel(s)	pixels	1	1	132.4392	-88.3572
AK									
Label	Date and Time	Document	Source	Scale	Scale Unit	Scale Fact	Count	Length	Angle
Ruler 3	2012-11-15T12:54:20-05:00	CIMG0926 (1) 2_00314.tif	Ruler Too	1 pixel(s)	pixels	1	1	32.16494	124.332
Ankle									
Label	Date and Time	Document	Source	Scale	Scale Unit	Scale Fact	Count	Length	Angle
Ruler 1	2012-11-15T12:52:57-05:00	CIMG0926 (1) 2_00314.tif	Ruler Too	1 pixel(s)	pixels	1	1	80.64869	-2.3969

Appendix F: Force Plate Analysis of Participants Vernier force plate graph and data (from beginning of run analysis [~1 min])



	Late	est
	Time	Force
	(s)	(N)
53	1.04	-2
54	1.06	15
55	1.08	304
56	1.10	827
57	1.12	725
58	1.14	889
59	1.16	1099
60	1.18	1255
61	1.20	1351
62	1.22	1376
63	1.24	1326
64	1.26	1200
65	1.28	993
66	1.30	750
67	1.32	488
68	1.34	261
69	1.36	126
70	1.38	22
71	1.40	-11
72	1.42	-2

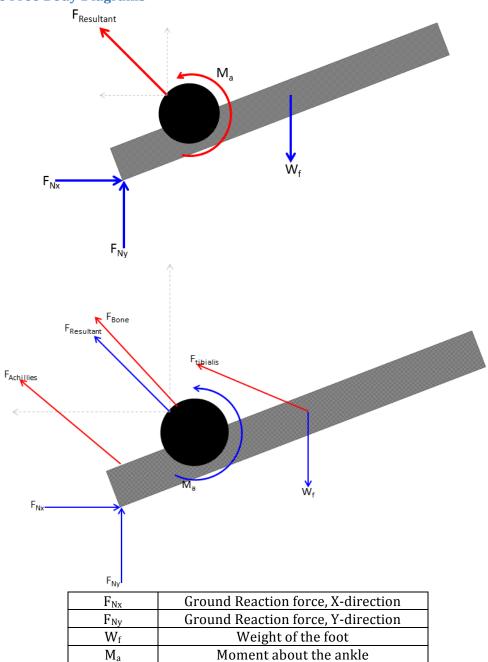
Vernier force plate graph and data (from end of run analysis [~18 mins])



	Lat	est
	Time	Force
	(s)	(N)
53	1.04	-2
54	1.06	15
55	1.08	304
56	1.10	827
57	1.12	725
58	1.14	889
59	1.16	1099
60	1.18	1255
61	1.20	1351
62	1.22	1376
63	1.24	1326
64	1.26	1200
65	1.28	993
66	1.30	750
67	1.32	488
68	1.34	261
69	1.36	126
70	1.38	22
71	1.40	-11
72	1.42	-2

Appendix G: Natural Strike Free Body Diagrams and Equations

G.1 Ankle Free Body Diagrams



G.2 Natural Rested Ankle Equations

To find the moment about the ankle

$$\sum F_{x} = F_{nx} - F_{RAx} = m \cdot a_{a}$$

$$F_{RAx} := m \cdot a_{ax} + F_{nx}$$

$$F_{RAx} = 1.535 \times 10^{3}$$

$$\sum F_{y} = F_{ny} + F_{RAy} - w_{foot} = m \cdot a_{a}$$

$$F_{RAy} := (m \cdot a_{ay}) + w_{foot} - F_{ny}$$

$$F_{RAy} = -363.091$$

$$\sum M_{a} = W_{foot} \cdot \frac{L}{2} - F_{ny} \cdot COP = I_{a} \cdot \alpha_{a}$$

$$M_{a} := I_{a} \cdot \alpha_{a} - \left(w_{foot} \cdot \frac{L_{foot}}{2} - F_{ny} \cdot COP \right)$$

$$M_{a} = 296.988$$

$$F_{RA}^{2} = F_{RAx}^{2} + F_{RAy}^{2}$$

$$F_{RA} := \sqrt{F_{RAx}^{2} + F_{RAy}^{2}} = 1.578 \times 10^{3}$$

$$F_{RA} = 1.578 \times 10^{3}$$

To find the force on the Tibialis Anterior, Achillies, and Bone

$$\sum F_x = F_{tx} + F_{bx} + F_{ax} = F_{RAx}$$

$$\sum F_y = F_{ty} + F_{by} + F_{ay} = -F_{RAy}$$

$$\sum M_y = F_{ty} \cdot \frac{L}{2} + F_{ay} \cdot 0.0254 + M_s = M_a$$

$$\begin{bmatrix} F_t \cdot cos(\theta_1) & F_b \cdot cos(\theta_3) & F_a \cdot cos(\theta_2) & 0 & F_{RAx} \\ F_t \cdot sin(\theta_1) & F_b \cdot sin(\theta_3) & F_a \cdot sin(\theta_2) & 0 & -F_{RAy} \\ \\ \frac{F_t \cdot sin(\theta_1) \cdot L_{foot}}{2} & 0 & -F_a \cdot sin(\theta_2) \cdot 0.0254 & M_s & M_a \end{bmatrix}$$

$$\mathbf{F_{t} \cdot \cos(\theta_1)} + \mathbf{F_{b} \cdot \cos(\theta_3)} + \mathbf{F_{a} \cdot \cos(\theta_2)} + \mathbf{0} = \mathbf{F_{RAx}}$$

$$\left(\mathtt{F}_{b} \cdot \sin(\theta_{3}) - \cos(\theta_{3}) \cdot \tan(\theta_{1})\right) + \left(\mathtt{F}_{t} \cdot \sin(\theta_{2}) - \cos(\theta_{2}) \cdot \tan(\theta_{1})\right) + 0 = -\mathtt{F}_{RAy} - \tan(\theta_{1})$$

$$F_{a} := \left[\frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} \cdot M_{s} + \frac{2}{L_{foot}} \cdot M_{s}\right)}{1 - \frac{0.06 \cdot 2}{L_{foot} \cdot sin(\theta_{1})}}\right] - \left[\frac{\left(\left(-2 \cdot cos(\theta_{1}) \cdot M_{a} + 2 \cdot cos(\theta_{1}) \cdot M_{s} + sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx}\right)\right)}{\left(2 \cdot cos(\theta_{1}) \cdot sin(\theta_{2}) \cdot 0.06 + sin(\theta_{1}) \cdot L_{foot} \cdot cos(\theta_{2})\right) \cdot sin(\theta_{3})}\right]$$

$$F_a = 2.269 \times 10^4$$

$$F_b := \frac{-\left[2 \cdot cos\left(\theta_1\right)\left(\mathbf{M_a} - \mathbf{M_s} + F_a \cdot sin\left(\theta_2\right) \cdot 0.06\right) - sin\left(\theta_1\right) \cdot L_{foot} \cdot cos\left(\theta_2\right) \cdot F_a + sin\left(\theta_1\right) \cdot L_{foot} \cdot F_{RAx}\right]}{cos\left(\theta_3\right) \cdot sin\left(\theta_1\right) \cdot L_{foot}}$$

$$F_b = -3.562 \times 10^4$$

$$\mathbf{F_t} \coloneqq \frac{2}{\sin(\theta_1) \cdot \mathbf{L_{foot}}} \big(\mathbf{M_a} - \mathbf{M_s} + \mathbf{F_a} \cdot \sin(\theta_2) \cdot \mathbf{0.06} \big)$$

$$F_t = 3.794 \times 10^4$$

ResultantForceOnAnkle := $-F_b = 3.562 \times 10^4$

G.3 Natural Tired Ankle Equations

To find the moment about the ankle

$$\sum F_{x} = F_{nx} - F_{RAx} = m \cdot a_{a}$$

$$F_{RAx} := m \cdot a_{ax} + F_{nx}$$

$$F_{RAx} = 1.535 \times 10^3$$

$$\sum F_y = F_{ny} + F_{RAy} - w_{foot} = m \cdot a_a$$

$$\mathtt{F}_{\texttt{RAy}} \coloneqq \left(\mathtt{m} \cdot \mathtt{a}_{\texttt{ay}} \right) + \mathtt{w}_{\texttt{foot}} - \mathtt{F}_{\texttt{ny}}$$

$$F_{RAy} = -243.751$$

$$\sum M_a = W_{foot} \cdot \frac{L}{2} - F_{ny} \cdot COP = I_a \cdot \alpha_a$$

$$M_{a} := I_{a} \cdot \alpha_{a} - \left(w_{foot} \cdot \frac{L_{foot}}{2} - F_{ny} \cdot COP\right)$$

$$M_a = 2.691 \times 10^4$$

$$F_{RA}^2 = F_{RAx}^2 + F_{RAy}^2$$

$$F_{RA} := \sqrt{F_{RAx}^2 + F_{RAy}^2} = 1.554 \times 10^3$$

$$F_{RA} = 1.554 \times 10^3$$

To find the force on the Tibialis Anterior, Achillies, and Bone

$$\sum F_x = F_{tx} + F_{bx} + F_{ax} = F_{RAx}$$

$$\sum F_y = F_{ty} + F_{by} + F_{ay} = -F_{RAy}$$

$$\sum M_y = F_{ty} \cdot \frac{L}{2} + F_{ay} \cdot 0.0254 + M_s = M_a$$

$$\begin{bmatrix} F_{t} \cdot cos(\theta_{1}) & F_{b} \cdot cos(\theta_{3}) & F_{a} \cdot cos(\theta_{2}) & 0 & F_{RAx} \\ F_{t} \cdot sin(\theta_{1}) & F_{b} \cdot sin(\theta_{3}) & F_{a} \cdot sin(\theta_{2}) & 0 & -F_{RAy} \\ \frac{F_{t} \cdot sin(\theta_{1}) \cdot L_{foot}}{2} & 0 & -F_{a} \cdot sin(\theta_{2}) \cdot 0.0254 & M_{s} & M_{a} \end{bmatrix}$$

$$\mathbf{F_{t} \cdot \cos(\theta_1)} + \mathbf{F_{b} \cdot \cos(\theta_3)} + \mathbf{F_{a} \cdot \cos(\theta_2)} + \mathbf{0} = \mathbf{F_{RAx}}$$

$$\left(\mathtt{F}_{b} \cdot \sin(\theta_{3}) - \cos(\theta_{3}) \cdot \tan(\theta_{1})\right) + \left(\mathtt{F}_{t} \cdot \sin(\theta_{2}) - \cos(\theta_{2}) \cdot \tan(\theta_{1})\right) + 0 = -\mathtt{F}_{RAy} - \tan(\theta_{1})$$

$$F_{a} := \left[\frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{a} + \frac{2}{L_{foot}} M_{s}\right)}{1 - \frac{0.06 \cdot 2}{L_{foot} \cdot sin(\theta_{1})}}\right] - \left[\frac{\left(\left(-2 \cdot cos(\theta_{1}) \cdot M_{a} + 2 \cdot cos(\theta_{1}) \cdot M_{s} + sin(\theta_{1}) \cdot L_{foot} \cdot F_{RAx}\right)\right)}{\left(\frac{2 \cdot cos(\theta_{1}) \cdot sin(\theta_{2}) \cdot 0.06 + sin(\theta_{1}) \cdot L_{foot} \cdot cos(\theta_{2})\right) \cdot sin(\theta_{3})}{cos(\theta_{3})}\right]$$

$$F_a = 1.072 \times 10^5$$

+

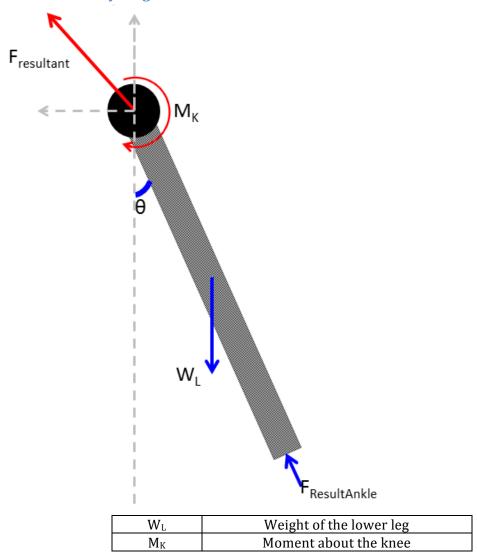
$$F_b := \frac{-\left[2 \cdot cos\left(\theta_1\right)\!\left(M_a - M_s + F_a \cdot sin\!\left(\theta_2\right) \cdot 0.06\right) - sin\!\left(\theta_1\right) \cdot L_{foot} \cdot cos\!\left(\theta_2\right) \cdot F_a + sin\!\left(\theta_1\right) \cdot L_{foot} \cdot F_{RAx}\right]}{cos\!\left(\theta_3\right) \cdot sin\!\left(\theta_1\right) \cdot L_{foot}}$$

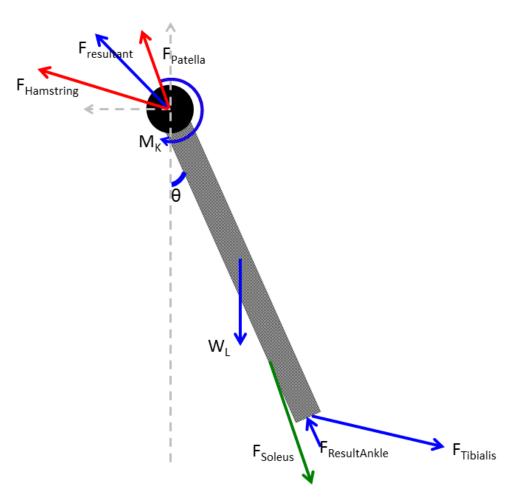
$$F_{t} := \frac{F_{b} = 5.786 \times 10^{6}}{\sin(\theta_{1}) \cdot L_{foot}} (M_{a} - M_{s} + F_{a} \cdot \sin(\theta_{2}) \cdot 0.06)$$

$$F_{t} = 6.526 \times 10^{5}$$

ResultantForceOnAnkle := $-F_b = -5.786 \times 10^6$

G.4 Knee Free Body Diagrams





W_{L}	Weight of the lower leg
M_{K}	Moment about the knee

G.5 Natural Rested Knee Equations

To find the moment about the knee

$$\sum F_{x} = F_{Rax} - F_{Rkx} = m \cdot a_{k}$$

$$F_{Rkx} := F_{Rax} - m \cdot a_{kx}$$

$$F_{Rkx} = 3.549 \times 10^{4}$$

$$\sum F_y = F_{Rky} - F_{Ray} - W_L = m \cdot a_k$$

$$F_{Rky} := m \cdot a_{ky} + F_{Ray} + w_{LowerLeg}$$

$$F_{Rky} = 3.016 \times 10^{3}$$

$$\sum M_y = W_L \cdot \frac{L}{2} - F_{Ray} \cdot \frac{L}{1} + M_K + M_a = I_k \cdot \alpha_k$$

$$M_K := I_k \cdot \alpha_k - w_{LowerLeg} \cdot \frac{L_{LowerLeg}}{2} + F_{Ray} \cdot \frac{L_{LowerLeg}}{1} - M_a$$

$$M_K = 837.368$$

$$F_{Rk}^2 = F_{Rkx}^2 + F_{Rky}^2$$

$$F_{Rk} := \sqrt{F_{Rkx}^2 + F_{Rky}^2} = 3.562 \times 10^4$$

$$F_{Rk} = 3.562 \times 10^4$$

$$\sum F_{x} = F_{Rk} \cdot \cos(\delta_{1}) + F_{H} \cdot \cos(\delta_{2}) + F_{p} \cdot \cos(\delta_{3}) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$\sum F_y = F_{RK} \cdot sin(\delta_1) + F_{H} \cdot sin(\delta_2) + F_p \cdot cos(\delta_3) - F_{TAy} - F_{Sy} - F_{RAy} = m \cdot a_{ky}$$

$$\sum \mathbf{M}_y = \mathbf{M}_K - \mathbf{M}_a + \mathbf{F}_{Rk} \cdot \frac{1}{d} + \mathbf{F}_p \cdot \frac{1}{d} + \mathbf{F}_{H} \cdot cos(\delta_2) \cdot \frac{1}{d} - \mathbf{F}_{Tx} - \mathbf{F}_{Sx} + \mathbf{F}_{RAx} = \mathbf{I}_k \cdot c_k$$

$$\begin{pmatrix} F_{Rk} \cdot \cos(\delta_1) & F_p \cdot \cos(\delta_3) & F_{H} \cdot \cos(\delta_2) & -F_{TAx} & -F_{Sx} & F_{RAx} & m \cdot a_{kx} \\ F_{Rk} \cdot \sin(\delta_1) & F_p \cdot \sin(\delta_3) & F_{H} \cdot \cos(\delta_2) & -F_{TAy} & -F_{Sy} & -F_{RAy} & m \cdot a_{ky} \\ F_{Rk} \cdot \cos(\delta_1) \cdot \frac{1}{d} & F_p \cdot \cos(\delta_3) \cdot \frac{1}{d} & F_{H} \cdot \cos(\delta_2) \cdot \frac{1}{d} & -F_{TAx} & -F_{Sx} & -F_{RAx} & I_k \cdot \alpha_k \end{pmatrix}$$

$$\begin{pmatrix} \cos(\delta_1) & \cos(\delta_3) & \cos(\delta_2) & -1 & -1 & 1 \\ \sin(\delta_1) & \sin(\delta_3) & \cos(\delta_2) & -1 & -1 & -1 & 1 \\ \\ 0 & 0 & \cos(\delta_2) \cdot \frac{L_{LowerLeg}}{d} & -1 & -1 & -1 & 1 \end{pmatrix}$$

$$\mathbf{F_K} = \mathbf{F_K} \cdot \cos(\delta_1) + \mathbf{F_p} \cdot \cos(\delta_3) + \mathbf{F_H} \cdot \cos(\delta_2) - \mathbf{F_{TAx}} - \mathbf{F_{Sx}} + \mathbf{F_{RAx}} = \mathbf{m} \cdot \mathbf{a_{kx}}$$

$$F_{p} = \left(F_{p} \cdot \sin(\theta_{3}) - \cos(\theta_{3}) \cdot \tan(\theta_{1})\right) + \left(F_{H} \cdot \sin(\theta_{2}) - \cos(\theta_{2}) \cdot \tan(\theta_{1})\right) - F_{TAy} + \tan(\theta_{1}) - F_{Sy} + \tan(\theta_{1}) - F_{RAy} - \tan(\theta_{1}) = \left(I_{k} \cdot \alpha_{k}\right) - \tan(\theta_{1}) + \left(I_{k} \cdot \alpha_{k}\right) - \tan(\theta_{1}) - \left(I_{k} \cdot \alpha_{k}\right) - \tan(\theta_{1}) + \left(I_{k} \cdot \alpha_{k}\right) + \tan(\theta_{1}) + \left(I_{k} \cdot \alpha_{k}\right) + \tan(\theta_{1}) + \left(I_{k} \cdot \alpha_{k}\right) + \tan(\theta_{1}) + \tan(\theta_{1})$$

$$\mathbf{F}_{H} = \frac{\mathbf{F}_{H} \cdot \cos(\delta_{2}) \cdot \mathbf{1}}{\mathbf{d}} - \mathbf{F}_{TAx} - \mathbf{F}_{Sx} - \mathbf{F}_{RAx} = \mathbf{I}_{k} \cdot \alpha_{k}$$

$$F_{H} := \frac{I_{k} \cdot c_{k} + F_{Rax} + F_{Sx} + F_{TAx}}{\left(\frac{cos(\delta_{2}) \cdot L_{LowerLeg}}{d}\right)} = 3.013 \times 10^{4}$$

$$F_{\mathbf{p}} \coloneqq \frac{\left[\left(I_{\mathbf{k}} \cdot \mathbf{c}_{\mathbf{k}}\right) - \tan(\theta_1)\right] + F_{\mathbf{R}\mathbf{A}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{S}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{T}\mathbf{A}\mathbf{y}} - \tan(\theta_1) - \left(F_{\mathbf{H}} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_2)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_2)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_3)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_3)\right)$$

$$F_{K} := \frac{m \cdot a_{kx} - F_{RAx} + F_{Sx} + F_{TAx} - F_{H} \cdot cos(\delta_{2}) - F_{p} \cdot cos(\delta_{3})}{cos(\delta_{1})} = -6.458 \times 10^{4}$$

G.6 Natural Tired Knee Equations

To find the moment about the knee

$$\sum F_{x} = F_{Rax} - F_{Rkx} = m \cdot a_{k}$$

$$F_{Rkx} := F_{Rax} - m \cdot a_{kx}$$

$$F_{Rkx} = -260.958$$

$$\sum F_y = F_{Rky} - F_{Ray} - W_L = m \cdot a_k$$

$$F_{Rky} := m \cdot a_{ky} + F_{Ray} + w_{LowerLeg}$$

$$F_{Rky} = -243.74$$

$$\sum \mathbf{M_y} = \mathbf{W_L} \cdot \frac{\mathbf{L}}{2} - \mathbf{F_{Ray}} \cdot \frac{\mathbf{L}}{1} + \mathbf{M_K} + \mathbf{M_a} = \mathbf{I_k} \cdot \alpha_k$$

$$\mathbf{M}_{K} := \mathbf{I}_{k} \cdot \alpha_{k} - \mathbf{w}_{LowerLeg} \cdot \frac{\mathbf{L}_{LowerLeg}}{2} + \mathbf{F}_{Ray} \cdot \frac{\mathbf{L}_{LowerLeg}}{1} - \mathbf{M}_{a}$$

$$M_{K} = -117.288$$

$$F_{Rk}^2 = F_{Rkx}^2 + F_{Rky}^2$$

$$F_{Rk} := \sqrt{F_{Rkx}^2 + F_{Rky}^2} = 357.083$$

$$F_{Rk} = 357.083$$

$$\sum F_{x} = F_{Rk} \cdot \cos\left(\delta_{1}\right) + F_{H} \cdot \cos\left(\delta_{2}\right) + F_{p} \cdot \cos\left(\delta_{3}\right) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$\sum F_y = F_{Rk} \cdot sin(\delta_1) + F_H \cdot sin(\delta_2) + F_p \cdot cos(\delta_3) - F_{TAy} - F_{Sy} - F_{RAy} = m \cdot a_{ky}$$

$$\sum \mathbf{M_y} = \mathbf{M_K} - \mathbf{M_a} + \mathbf{F_{Rk}} \cdot \frac{1}{d} + \mathbf{F_p} \cdot \frac{1}{d} + \mathbf{F_{H^*}cos}(\delta_2) \cdot \frac{1}{d} - \mathbf{F_{Tx}} - \mathbf{F_{Sx}} + \mathbf{F_{RAx}} = \mathbf{I_k \cdot \alpha_k}$$

$$\begin{pmatrix} F_{Rk} \cdot cos(\delta_1) & F_p \cdot cos(\delta_3) & F_{H} \cdot cos(\delta_2) & -F_{TAx} & -F_{Sx} & F_{RAx} & m \cdot a_{kx} \\ F_{Rk} \cdot sin(\delta_1) & F_p \cdot sin(\delta_3) & F_{H} \cdot cos(\delta_2) & -F_{TAy} & -F_{Sy} & -F_{RAy} & m \cdot a_{ky} \\ F_{Rk} \cdot cos(\delta_1) \cdot \frac{1}{d} & F_p \cdot cos(\delta_3) \cdot \frac{1}{d} & F_{H} \cdot cos(\delta_2) \cdot \frac{1}{d} & -F_{TAx} & -F_{Sx} & -F_{RAx} & I_k \cdot \alpha_k \end{pmatrix}$$

$$\begin{pmatrix} \cos(\delta_1) & \cos(\delta_3) & \cos(\delta_2) & -1 & -1 & 1 \\ \sin(\delta_1) & \sin(\delta_3) & \cos(\delta_2) & -1 & -1 & -1 & 1 \\ \\ 0 & 0 & \cos(\delta_2) \cdot \frac{L_{LowerLeg}}{d} & -1 & -1 & -1 & 1 \end{pmatrix}$$

$$F_K = F_K \cdot \cos(\delta_1) + F_p \cdot \cos(\delta_3) + F_H \cdot \cos(\delta_2) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$F_{\mathbf{p}} = \left(F_{\mathbf{p}} \cdot \sin(\theta_3) - \cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(F_{H} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) - F_{TAy} + \tan(\theta_1) - F_{Sy} + \tan(\theta_1) - F_{RAy} - \tan(\theta_1) = \left(I_k \cdot \alpha_k\right) - \tan(\theta_1) + \left(I_k \cdot \alpha_k\right) + \tan(\theta_1) +$$

$$F_{H} = \frac{F_{H} \cdot \cos(\delta_{2}) \cdot 1}{d} - F_{TAx} - F_{Sx} - F_{RAx} = I_{k} \cdot \alpha_{k}$$

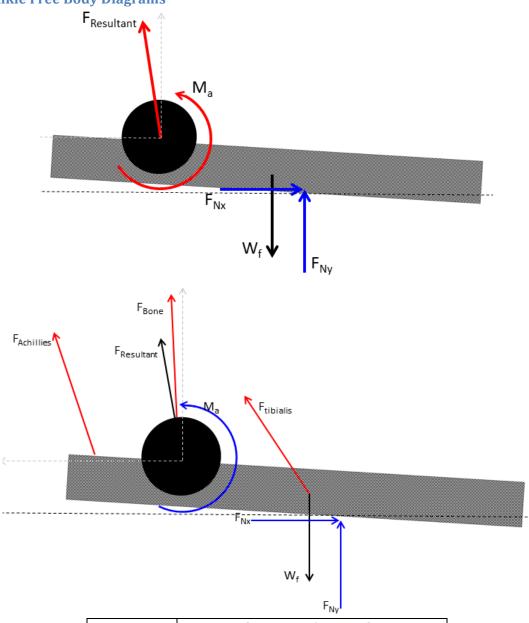
$$F_{H} := \frac{I_k \cdot \alpha_k + F_{Rax} + F_{Sx} + F_{TAx}}{\left(\frac{\cos(\delta_2) \cdot L_{LowerLeg}}{d}\right)} = -386.242$$

$$F_p := \frac{\left[\left(I_k \cdot \alpha_k\right) - \tan(\theta_1)\right] + F_{RAy} + \tan(\theta_1) + F_{Sy} + \tan(\theta_1) + F_{TAy} - \tan(\theta_1) - \left(F_{H} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_2)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_2)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_3)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_3)\right)$$

$$F_{K} := \frac{m \cdot a_{kx} - F_{RAx} + F_{Sx} + F_{TAx} - F_{H} \cdot \cos(\delta_{2}) - F_{p} \cdot \cos(\delta_{3})}{\cos(\delta_{1})} = -423.607 + 423.607$$

Appendix H: Mid-Foot Strike Free Body Diagrams and Equations

H.1 Ankle Free Body Diagrams



F_{Nx}	Ground Reaction force, X-direction
F_{Ny}	Ground Reaction force, Y-direction
W_{f}	Weight of the foot
Ma	Moment about the ankle

H.2 Mid-Foot Rested Ankle Equations

To find the moment about the ankle

$$\sum F_{x} = F_{nx} - F_{RAx} = m \cdot a_{a}$$

$$F_{RAx} := m \cdot a_{ax} + F_{nx}$$

$$F_{RAx} = 1.645 \times 10^3$$

$$\sum F_y = F_{ny} + F_{RAy} - w_{foot} = m \cdot a_a$$

$$\begin{aligned} F_{RAy} &:= \left(m \cdot a_{ay}\right) + w_{foot} - F_{ny} \\ \hline F_{RAy} &= -4.243 \times 10^3 \end{aligned}$$

$$\sum M_y = M_a + W_{foot} \cdot \frac{L}{2} - F_{ny} \cdot \frac{L}{COP} = I_a \cdot \alpha_a$$

$$M_a := F_{ny} \cdot \frac{L_{foot}}{COP} - w_{foot} \cdot \frac{L_{foot}}{2}$$

$$-M_a = -23.263$$

$$F_{RA}^2 = F_{RAx}^2 + F_{RAy}^2$$

$$F_{RA} := \sqrt{F_{RAx}^2 + F_{RAy}^2} = 4.551 \times 10^3$$

$$F_{RA} = 4.551 \times 10^3$$

To find the force on the Tibialis Anterior, Achillies, and Bone

$$\sum F_x = F_{tx} + F_{tx} + F_{ax} - F_{RAx} = m \cdot a_{ax}$$

$$\sum F_{y} = F_{ty} + F_{by} + F_{ay} + F_{RAy} = m \cdot a_{ay}$$

$$\sum M_y = F_{ty} \cdot \frac{L}{2} - F_{Ry} \cdot 0 + F_{ay} \cdot \frac{L}{0.5} + F_{by} \cdot 0 + M_a = I_a \cdot \alpha_a$$

$$\begin{pmatrix} F_{t} \cdot \cos(\theta_{1}) & F_{b} \cdot \cos(\theta_{3}) & F_{a} \cdot \cos(\theta_{2}) & -F_{RAx} & m \cdot a_{ax} \\ F_{t} \cdot \sin(\theta_{1}) & F_{b} \cdot \sin(\theta_{3}) & F_{a} \cdot \sin(\theta_{2}) & F_{RAy} & m \cdot a_{ay} \\ \\ \frac{F_{t} \cdot \sin(\theta_{1}) \cdot L_{foot}}{2} & 0 & \frac{-F_{a} \cdot \sin(\theta_{2}) \cdot L_{foot}}{0.5} & M_{a} & I_{a} \cdot \alpha_{a} \end{pmatrix}$$

$$F_t \cdot \cos(\theta_1) + F_b \cdot \cos(\theta_3) + F_a \cdot \cos(\theta_2) - F_{RAx} = m \cdot a_{ax}$$

$$-\left(F_{b} \cdot cos(\theta_{3}) - \frac{tan(\theta_{1}) \cdot L_{foot}}{2}\right) - \left[\left(\frac{F_{a} \cdot sin(\theta_{2}) \cdot L_{foot}}{0.5}\right) - \left(cos(\theta_{2}) - \frac{tan(\theta_{1}) \cdot L_{foot}}{2}\right)\right] + \left(F_{RAy} + \frac{tan(\theta_{1}) \cdot L_{foot}}{2}\right) = m \cdot a_{ay} - \frac{tan(\theta_{1}) \cdot L_{foot}}{2}$$

$$F_a \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1) - \left\lceil \cos(\theta_3) \left(\tan(\theta_3) + \tan(\theta_1) \right) \right\rceil + \left\lceil M_a + \tan(\theta_1) + \left(\tan(\theta_3) + \tan(\theta_1) \right) \right\rceil = \left(I_a \cdot \alpha_a \right) - \tan(\theta_1) - \left(\tan(\theta_3) + \tan(\theta_1) \right) - \left(\tan(\theta_3) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) \right) - \left(\tan(\theta_3) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) \right) - \left(\tan(\theta_3) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) \right) - \left(\tan(\theta_3) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) + \tan(\theta_1) \right) - \left(\tan(\theta_3) + \tan(\theta_1) +$$

$$F_{\mathbf{a}} := \frac{\left[\left(I_{\mathbf{a}} \cdot \alpha_{\mathbf{a}}\right) - \tan(\theta_{3})\right] - \left(-M_{\mathbf{a}} + 2\tan(\theta_{1}) + \tan(\theta_{3})\right) + \left[\cos(\theta_{3})(\tan(\theta_{3}) + \tan(\theta_{1}))\right] + \left(\cos(\theta_{2}) \cdot \tan(\theta_{1})\right) + \left(\sin(\theta_{1}) \cdot \tan(\theta_{1})\right) + \left(\cos(\theta_{1}) \cdot \tan(\theta$$

$$F_a = -57.424$$

$$F_b := \frac{\left(m \cdot a_{ay} - \frac{tan(\theta_1) \cdot L_{foot}}{2}\right) - \left(F_{RAy} + \frac{tan(\theta_1) \cdot L_{foot}}{2}\right) + \left[\left(\frac{F_a \cdot sin(\theta_2) \cdot L_{foot}}{0.5}\right) - \left(cos(\theta_2) - \frac{tan(\theta_1) \cdot L_{foot}}{2}\right)\right] + \left(\frac{tan(\theta_1) \cdot L_{foot}}{2}\right) - \left(cos(\theta_2) - \frac{tan(\theta_1) \cdot L_{foot}}{2}\right) - \left(cos(\theta_2) - \frac{tan(\theta_1)$$

$$-F_b = 1.846 \times 10^3$$

$$\mathbf{F_t} \coloneqq \mathbf{m} \cdot \mathbf{a_{ax}} + \mathbf{F_{RAx}} - \mathbf{F_b} \cdot \mathbf{cos} \big(\mathbf{\theta_3} \big) - \mathbf{F_a} \cdot \mathbf{cos} \big(\mathbf{\theta_2} \big)$$

$$F_t = 1.08 \times 10^3$$

$$F_{tot} := F_t + (-F_b) + F_a = 2.869 \times 10^3$$

ResultantForceOnAnkle :=
$$F_{RA} - |F_{tot}| = 1.682 \times 10^3$$

H.3 Mid-Foot Tired Ankle Equations

To find the moment about the ankle

$$\sum F_{x} = F_{nx} - F_{RAx} = m \cdot a_{a}$$

$$F_{RAx} := m \cdot a_{ax} + F_{nx}$$

$$F_{RAx} = 1.645 \times 10^3$$

$$\sum F_y = F_{ny} + F_{RAy} - w_{foot} = m \cdot a_a$$

$$\begin{aligned} F_{RAy} &:= \left(m \cdot a_{ay} \right) + w_{foot} - F_{ny} \\ \hline F_{RAy} &= -4.316 \times 10^3 \end{aligned}$$

$$\sum M_a = W_{foot} \cdot \frac{L}{2} - F_{ny} \cdot COP = I_a \cdot \alpha_a$$

$$M_{\mathbf{a}} := I_{\mathbf{a}} \cdot \alpha_{\mathbf{a}} - \left(w_{\mathbf{foot}} \cdot \frac{L_{\mathbf{foot}}}{2} - F_{\mathbf{ny}} \cdot COP \right)$$
$$M_{\mathbf{a}} = 1.258 \times 10^{4}$$

$$F_{RA}^{2} = F_{RAx}^{2} + F_{RAy}^{2}$$

$$F_{RA} := \sqrt{F_{RAx}^{2} + F_{RAy}^{2}} = 4.619 \times 10^{3}$$

$$F_{RA} = 4.619 \times 10^{3}$$

To find the force on the Tibialis Anterior, Achillies, and Bone

$$\sum F_x = F_{tx} + F_{bx} + F_{ax} = F_{RAx}$$

$$\sum F_y = F_{ty} + F_{by} + F_{ay} = -F_{RAy}$$

$$\sum M_y = F_{ty} \cdot \frac{L}{2} + F_{ay} \cdot 0.0254 + M_s = M_a$$

$$\begin{bmatrix} F_{t} \cdot \cos(\theta_{1}) & F_{b} \cdot \cos(\theta_{3}) & F_{a} \cdot \cos(\theta_{2}) & 0 & F_{RAx} \\ F_{t} \cdot \sin(\theta_{1}) & F_{b} \cdot \sin(\theta_{3}) & F_{a} \cdot \sin(\theta_{2}) & 0 & -F_{RAy} \\ \frac{F_{t} \cdot \sin(\theta_{1}) \cdot L_{foot}}{2} & 0 & -F_{a} \cdot \sin(\theta_{2}) \cdot 0.0254 & M_{s} & M_{a} \end{bmatrix}$$

$$F_{t} \cdot \cos(\theta_{1}) + F_{b} \cdot \cos(\theta_{3}) + F_{a} \cdot \cos(\theta_{2}) + 0 = F_{RAx}$$

$$\left(\mathtt{F}_{\mathbf{b}} \cdot \sin(\theta_3) - \cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\mathtt{F}_{\mathbf{t}} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + 0 = -\mathtt{F}_{\mathbf{RA_V}} - \tan(\theta_1)$$

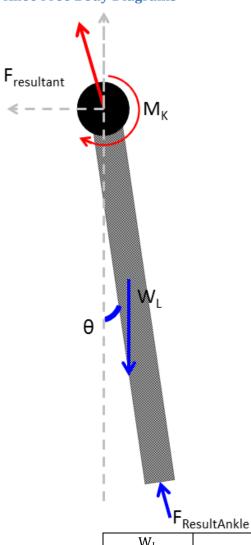
$$\begin{aligned} F_{\mathbf{a}} &:= \left[\frac{\left(-F_{RAy} - \frac{2}{L_{foot}} \cdot M_{\mathbf{a}} + \frac{2}{L_{foot}} M_{\mathbf{s}} \right)}{1 - \frac{0.06 \cdot 2}{L_{foot} \cdot \sin(\theta_1)}} \right] - \left[\frac{\left(\left(-2 \cdot \cos(\theta_1) \cdot M_{\mathbf{a}} + 2 \cdot \cos(\theta_1) \cdot M_{\mathbf{s}} + \sin(\theta_1) \cdot L_{foot} \cdot F_{RAx} \right) \right)}{\left(2 \cdot \cos(\theta_1) \cdot \sin(\theta_2) \cdot 0.06 + \sin(\theta_1) \cdot L_{foot} \cdot \cos(\theta_2) \right) \cdot \sin(\theta_3)} \right] \\ F_{\mathbf{a}} &= -5.298 \times 10^5 \end{aligned}$$

$$\begin{split} F_{b} &:= \frac{-\left[2 \cdot cos\left(\theta_{1}\right)\left(M_{a} - M_{s} + F_{a} \cdot sin\left(\theta_{2}\right) \cdot 0.06\right) - sin\left(\theta_{1}\right) \cdot L_{foot} \cdot cos\left(\theta_{2}\right) \cdot F_{a} + sin\left(\theta_{1}\right) \cdot L_{foot} \cdot F_{RAx}\right]}{cos\left(\theta_{3}\right) \cdot sin\left(\theta_{1}\right) \cdot L_{foot}} \\ F_{b} &= -5.1 \times 10^{5} \end{split}$$

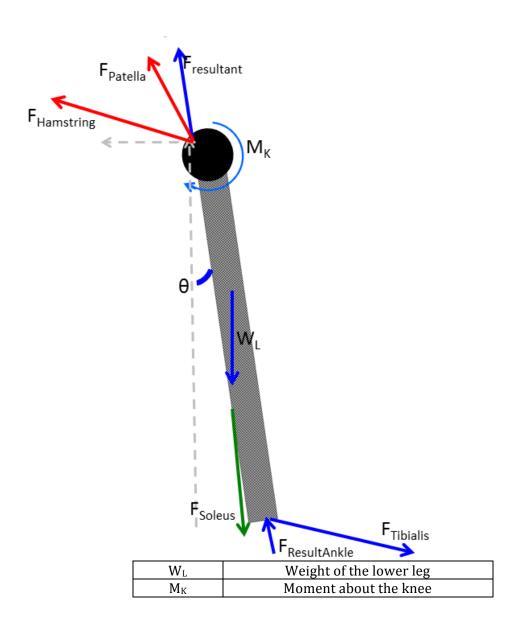
$$\begin{aligned} F_{t} &\coloneqq \frac{2}{\sin(\theta_{1}) \cdot L_{foot}} \big(\mathbf{M}_{a} - \mathbf{M}_{s} + F_{a} \cdot \sin(\theta_{2}) \cdot 0.06 \big) \\ F_{t} &= 8.955 \times 10^{4} \end{aligned}$$

ResultantForceOnAnkle :=
$$-F_b = 5.1 \times 10^5$$

H.4 Knee Free Body Diagrams



W_{L}	Weight of the lower leg
M_{K}	Moment about the knee



H.5 Mid-Foot Rested Knee Equations

To find the moment about the knee

$$\sum F_{x} = F_{Rax} - F_{Rkx} = m \cdot a_{k}$$

$$F_{Rkx} := F_{Rax} - m \cdot a_{kx}$$

$$F_{Rkx} = -629.62$$

$$\sum F_y = F_{Rky} - F_{Ray} - W_L = m \cdot a_k$$

$$F_{Rky} := m \cdot a_{ky} + F_{Ray} + w_{LowerLeg}$$

$$F_{Rky} = 1.585 \times 10^{3}$$

$$\sum M_y = W_L \cdot \frac{L}{2} - F_{Ray} \cdot \frac{L}{1} + M_K + M_a = I_k \cdot \alpha_k$$

$$\mathbf{M}_{\mathbf{K}} \coloneqq \mathbf{I}_{\mathbf{k}} \cdot \alpha_{\mathbf{k}} - \mathbf{w}_{\mathbf{LowerLeg}} \cdot \frac{\mathbf{L}_{\mathbf{LowerLeg}}}{2} + \mathbf{F}_{\mathbf{Ray}} \cdot \frac{\mathbf{L}_{\mathbf{LowerLeg}}}{1} - \mathbf{M}_{\mathbf{a}}$$

$$M_{K} = 566.003$$

$$F_{Rk}^2 = F_{Rkx}^2 + F_{Rky}^2$$

$$F_{Rk} := \sqrt{F_{Rkx}^2 + F_{Rky}^2} = 1.706 \times 10^3$$

$$F_{Rk} = 1.706 \times 10^3$$

$$\sum F_x = F_{Rk} \cdot \cos(\delta_1) + F_{H} \cdot \cos(\delta_2) + F_{p} \cdot \cos(\delta_3) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$\sum F_{\mathbf{y}} = F_{\mathbf{R}\mathbf{k}} \cdot \sin(\delta_1) + F_{\mathbf{H}} \cdot \sin(\delta_2) + F_{\mathbf{p}} \cdot \cos(\delta_3) - F_{\mathbf{T}\mathbf{A}\mathbf{y}} - F_{\mathbf{S}\mathbf{y}} - F_{\mathbf{R}\mathbf{A}\mathbf{y}} = m \cdot a_{\mathbf{k}\mathbf{y}}$$

$$\sum M_{y} = M_{K} - M_{a} + F_{Rk} \cdot \frac{1}{d} + F_{p} \cdot \frac{1}{d} + F_{H} \cdot \cos(\delta_{2}) \cdot \frac{1}{d} - F_{Tx} - F_{Sx} + F_{RAx} = I_{k} \cdot c_{k}$$

$$\begin{pmatrix} F_{Rk} \cdot \cos(\delta_1) & F_p \cdot \cos(\delta_3) & F_H \cdot \cos(\delta_2) & -F_{TAx} & -F_{Sx} & F_{RAx} & m \cdot a_{kx} \\ F_{Rk} \cdot \sin(\delta_1) & F_p \cdot \sin(\delta_3) & F_H \cdot \cos(\delta_2) & -F_{TAy} & -F_{Sy} & -F_{RAy} & m \cdot a_{ky} \\ F_{Rk} \cdot \cos(\delta_1) \cdot \frac{1}{d} & F_p \cdot \cos(\delta_3) \cdot \frac{1}{d} & F_H \cdot \cos(\delta_2) \cdot \frac{1}{d} & -F_{TAx} & -F_{Sx} & -F_{RAx} & I_k \cdot \alpha_k \end{pmatrix}$$

$$\begin{pmatrix} \cos(\delta_1) & \cos(\delta_3) & \cos(\delta_2) & -1 & -1 & 1 \\ \sin(\delta_1) & \sin(\delta_3) & \cos(\delta_2) & -1 & -1 & -1 & 1 \\ 0 & 0 & \cos(\delta_2) \cdot \frac{L_{LowerLeg}}{d} & -1 & -1 & -1 & 1 \end{pmatrix}$$

$$F_K = F_K \cdot \cos(\delta_1) + F_D \cdot \cos(\delta_3) + F_H \cdot \cos(\delta_2) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$F_{\mathbf{p}} = \left(F_{\mathbf{p}} \cdot \sin(\theta_3) - \cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(F_{\mathbf{H}} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) - F_{\mathbf{T} \mathbf{A} \mathbf{y}} + \tan(\theta_1) - F_{\mathbf{S} \mathbf{y}} + \tan(\theta_1) - F_{\mathbf{R} \mathbf{A} \mathbf{y}} - \tan(\theta_1) = \left(I_{\mathbf{k}} \cdot \alpha_{\mathbf{k}}\right) - \tan(\theta_1) + \left(I_{\mathbf{k}} \cdot \alpha_{\mathbf{k}}\right) - \left(I_{\mathbf{k$$

$$F_{H} = \frac{F_{H} \cdot \cos\left(\delta_{2}\right) \cdot I}{d} - F_{TAx} - F_{Sx} - F_{RAx} = I_{k} \cdot \alpha_{k}$$

$$F_{H} := \frac{I_{k} \cdot \alpha_{k} + F_{Rax} + F_{Sx} + F_{TAx}}{\left(\frac{\cos(\delta_{2}) \cdot L_{LowerLeg}}{d}\right)} = -551.943$$

$$F_{\mathbf{p}} := \frac{\left[\left(I_{\mathbf{k}} \cdot \alpha_{\mathbf{k}}\right) - \tan(\theta_1)\right] + F_{\mathbf{R}\mathbf{A}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{S}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{T}\mathbf{A}\mathbf{y}} - \tan(\theta_1) - \left(F_{\mathbf{H}} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1) - \cos(\theta_3) \cdot \tan(\theta_3)\right) + \left(\cos(\theta_3) \cdot \sin(\theta_3) - \cos(\theta_3) \cdot \sin(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3) - \cos(\theta_3) \cdot \sin(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3) - \cos(\theta_3) \cdot \cos(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3) - \cos(\theta_3) \cdot \cos(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3) - \cos(\theta_3) \cdot \cos(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3) - \cos(\theta_3) - \cos(\theta_3)\right) + \left(\cos(\theta_3) \cdot \cos(\theta_3)\right) +$$

$$F_{K} := \frac{m \cdot a_{kx} - F_{RAx} + F_{Sx} + F_{TAx} - F_{H} \cdot \cos(\delta_{2}) - F_{p} \cdot \cos(\delta_{3})}{\cos(\delta_{1})} = -6.123 \times 10^{3}$$

H.6 Mid-Foot Tired Knee Equations

To find the moment about the knee

$$\sum F_{x} = F_{Rax} - F_{Rkx} = m \cdot a_{k}$$

$$\mathsf{F}_{Rkx} := \mathsf{F}_{Rax} - \mathsf{m} \!\cdot\! \mathsf{a}_{kx}$$

$$F_{Rkx} = 7.384 \times 10^4$$

$$\sum F_y = F_{Rky} - F_{Ray} - W_L = m \cdot a_k$$

$$F_{Rky} := m \cdot a_{ky} + F_{Ray} + w_{LowerLeg}$$

$$F_{Rky} = -5.046 \times 10^5$$

$$\sum M_y = W_L \cdot \frac{L}{2} - F_{Ray} \cdot \frac{L}{1} + M_K + M_a = I_k \cdot \alpha_k$$

$$\begin{aligned} \mathbf{M_{K}} &:= \mathbf{I_{k}} \cdot \alpha_{k} - \mathbf{w_{LowerLeg}} \cdot \frac{\mathbf{L_{LowerLeg}}}{2} + \mathbf{F_{Ray}} \cdot \frac{\mathbf{L_{LowerLeg}}}{1} - \mathbf{M_{a}} \\ \mathbf{M_{K}} &= -2.048 \times 10^{5} \end{aligned}$$

$$F_{Rk}^2 = F_{Rkx}^2 + F_{Rky}^2$$

$$F_{Rk} := \sqrt{F_{Rkx}^2 + F_{Rky}^2} = 5.099 \times 10^5$$

$$F_{RL} = 5.099 \times 10^5$$

$$\sum F_x = F_{Rk} \cdot \cos\left(\delta_1\right) + F_{H^*} \cos\left(\delta_2\right) + F_p \cdot \cos\left(\delta_3\right) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$\sum F_y = F_{Rk} \cdot sin(\delta_1) + F_H \cdot sin(\delta_2) + F_p \cdot cos(\delta_3) - F_{TAy} - F_{Sy} - F_{RAy} = m \cdot a_{ky}$$

$$\sum M_y = M_K - M_a + F_{Rk} \cdot \frac{1}{d} + F_p \cdot \frac{1}{d} + F_{H} \cdot \cos(\delta_2) \cdot \frac{1}{d} - F_{Tx} - F_{Sx} + F_{RAx} = I_k \cdot c_k$$

$$\begin{pmatrix} F_{Rk} \cdot \cos(\delta_1) & F_p \cdot \cos(\delta_3) & F_{H} \cdot \cos(\delta_2) & -F_{TAx} & -F_{Sx} & F_{RAx} & m \cdot a_{kx} \\ F_{Rk} \cdot \sin(\delta_1) & F_p \cdot \sin(\delta_3) & F_{H} \cdot \cos(\delta_2) & -F_{TAy} & -F_{Sy} & -F_{RAy} & m \cdot a_{ky} \\ F_{Rk} \cdot \cos(\delta_1) \cdot \frac{1}{d} & F_p \cdot \cos(\delta_3) \cdot \frac{1}{d} & F_{H} \cdot \cos(\delta_2) \cdot \frac{1}{d} & -F_{TAx} & -F_{Sx} & -F_{RAx} & I_k \cdot \alpha_k \end{pmatrix}$$

$$\begin{pmatrix} \cos(\delta_1) & \cos(\delta_3) & \cos(\delta_2) & -1 & -1 & 1 \\ \sin(\delta_1) & \sin(\delta_3) & \cos(\delta_2) & -1 & -1 & -1 & 1 \\ \\ 0 & 0 & \cos(\delta_2) \cdot \frac{L_{LowerLeg}}{d} & -1 & -1 & -1 & 1 \end{pmatrix}$$

$$F_{K} = F_{K} \cdot \cos(\delta_{1}) + F_{p} \cdot \cos(\delta_{3}) + F_{H} \cdot \cos(\delta_{2}) - F_{TAx} - F_{Sx} + F_{RAx} = m \cdot a_{kx}$$

$$\textbf{F}_{\textbf{p}} = \left(\textbf{F}_{\textbf{p}} \cdot \sin(\theta_3) - \cos(\theta_3) \cdot \tan(\theta_1)\right) + \left(\textbf{F}_{\textbf{H}} \cdot \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) - \textbf{F}_{\textbf{TAy}} + \tan(\theta_1) - \textbf{F}_{\textbf{Sy}} + \tan(\theta_1) - \textbf{F}_{\textbf{RAy}} - \tan(\theta_1) = \left(\textbf{I}_{\textbf{k}} \cdot \textbf{c}_{\textbf{k}}\right) - \tan(\theta_1) + \left(\textbf{I}_{\textbf{k}} \cdot \textbf{c}_$$

$$F_{H} = \frac{F_{H} \cdot \cos(\delta_2) \cdot 1}{d} - F_{TAx} - F_{Sx} - F_{RAx} = I_k \cdot \alpha_k$$

+

$$F_{H} := \frac{I_{k} \cdot o_{k} + F_{Rax} + F_{Sx} + F_{TAx}}{\left(\frac{cos(\delta_{2}) \cdot L_{LowerLeg}}{d}\right)} = 2.388 \times 10^{5}$$

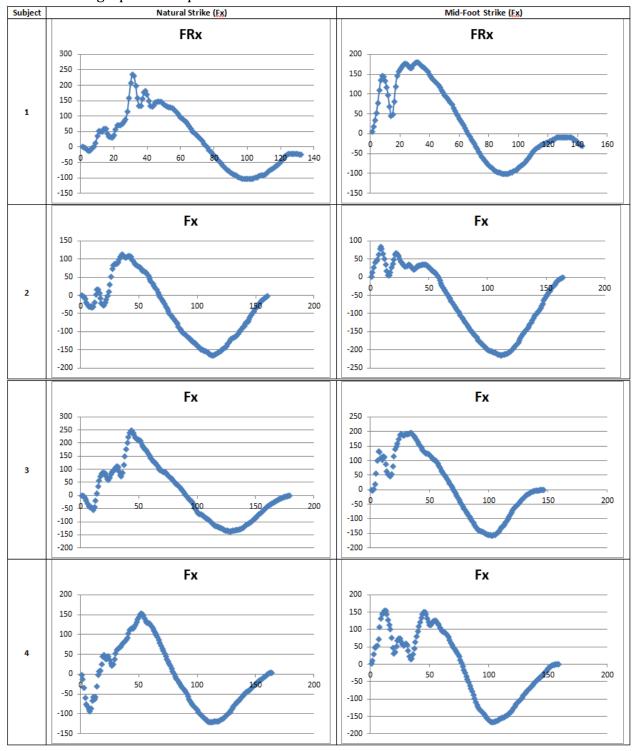
$$F_{\mathbf{p}} := \frac{\left[\left(I_{\mathbf{k}} \cdot \alpha_{\mathbf{k}}\right) - \tan(\theta_1)\right] + F_{\mathbf{R}\mathbf{A}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{S}\mathbf{y}} + \tan(\theta_1) + F_{\mathbf{T}\mathbf{A}\mathbf{y}} - \tan(\theta_1) - \left(F_{\mathbf{H}} \sin(\theta_2) - \cos(\theta_2) \cdot \tan(\theta_1)\right) + \left(\cos(\theta_3) \cdot \tan(\theta_1)\right) - \sin(\theta_3) + \cos(\theta_3) \cdot \sin(\theta_3)}{\sin(\theta_3)}$$

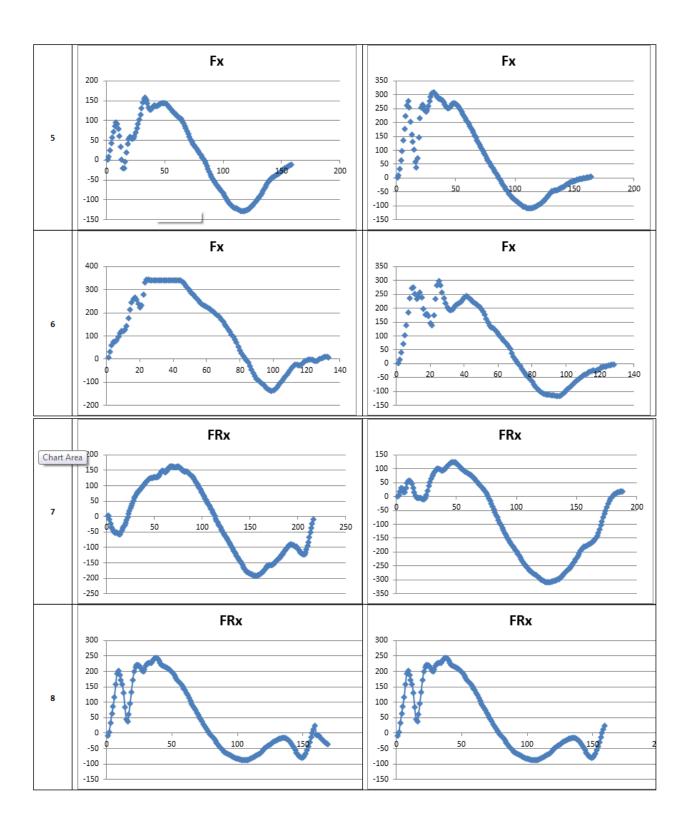
$$F_{K} := \frac{m \cdot a_{kx} - F_{RAx} + F_{Sx} + F_{TAx} - F_{H} \cdot cos(\delta_{2}) - F_{p} \cdot cos(\delta_{3})}{cos(\delta_{1})} = -1.144 \times 10^{5}$$

Appendix I: Compared Data

I.1 Resultant Horizontal Force Comparison

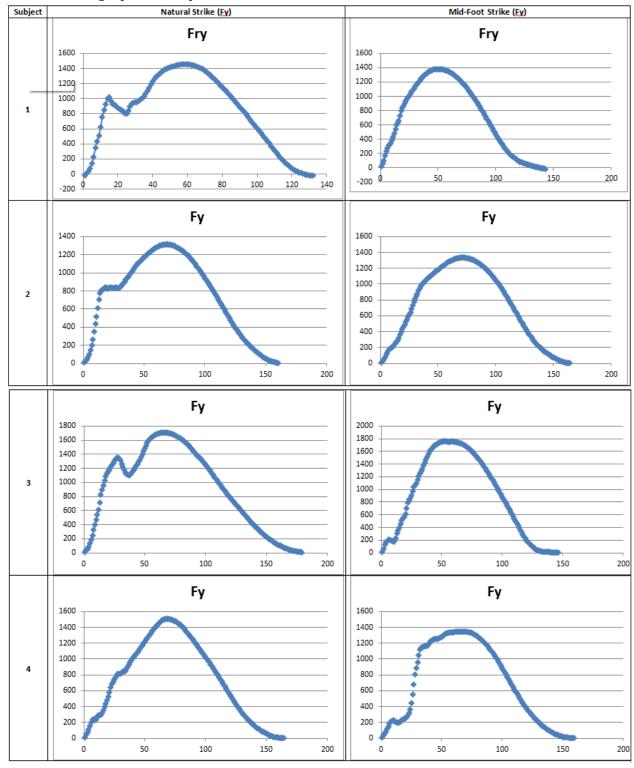
Forces on all graphs are reported in NEWTONS.

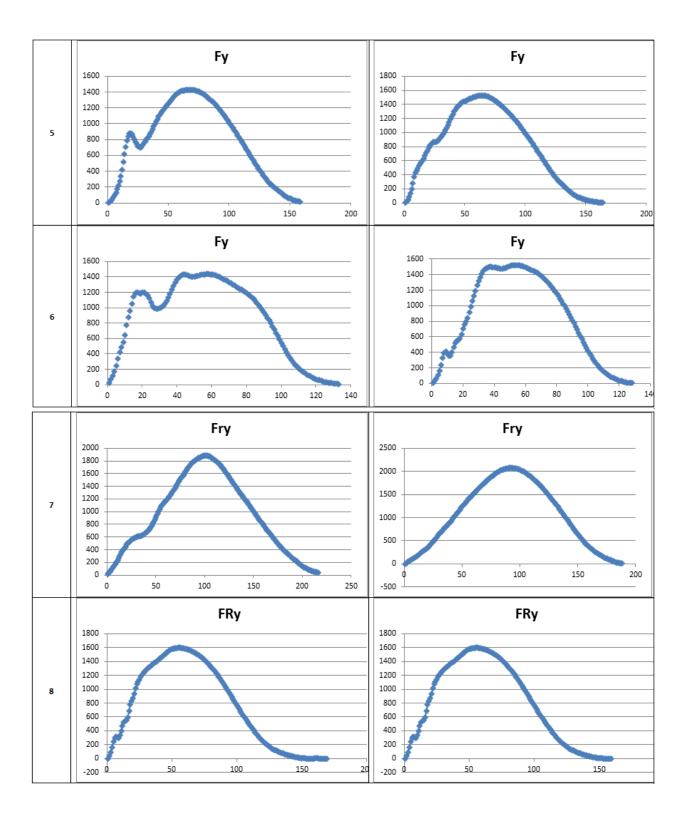




I.2 Vertical Resultant Force Comparison

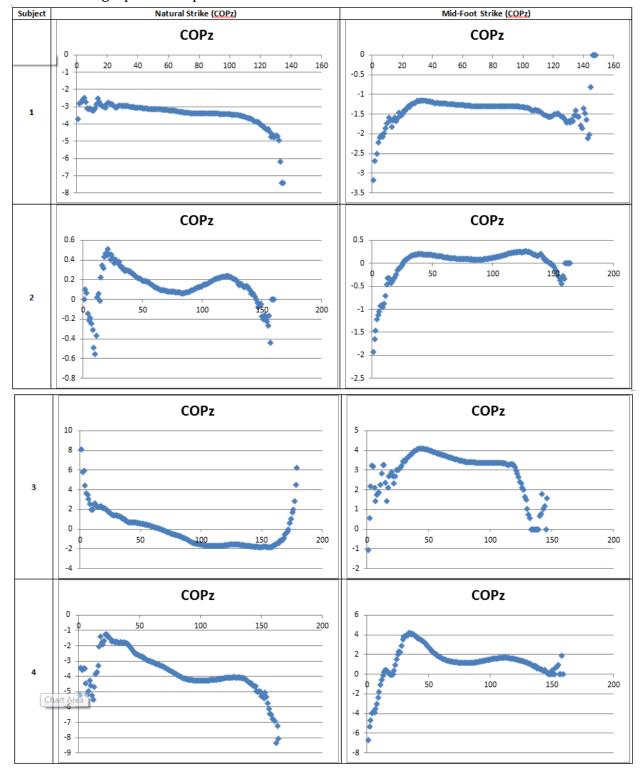
Forces on all graphs are reported in NEWTONS.

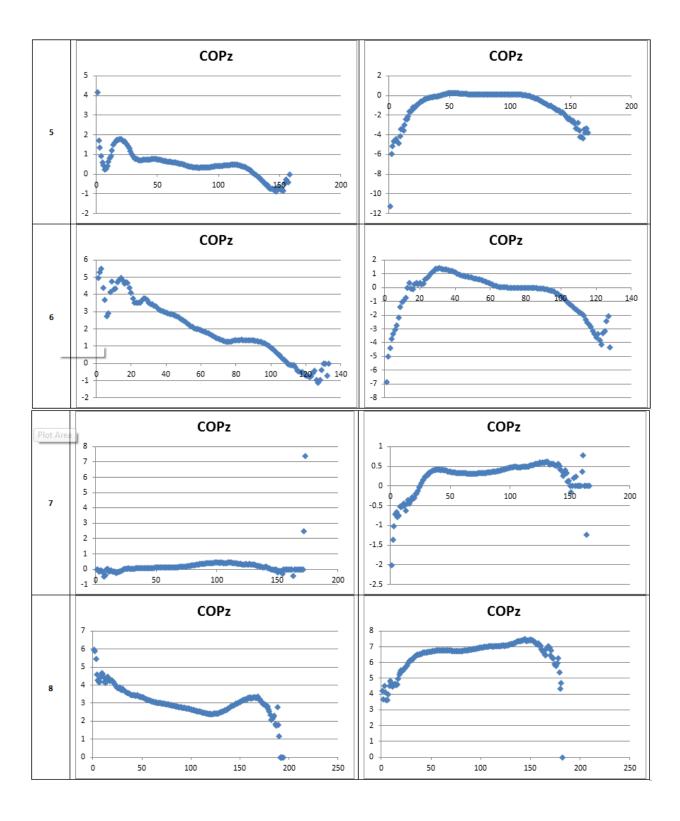




I.3 Center of Pressure in the Z-direction Comparison

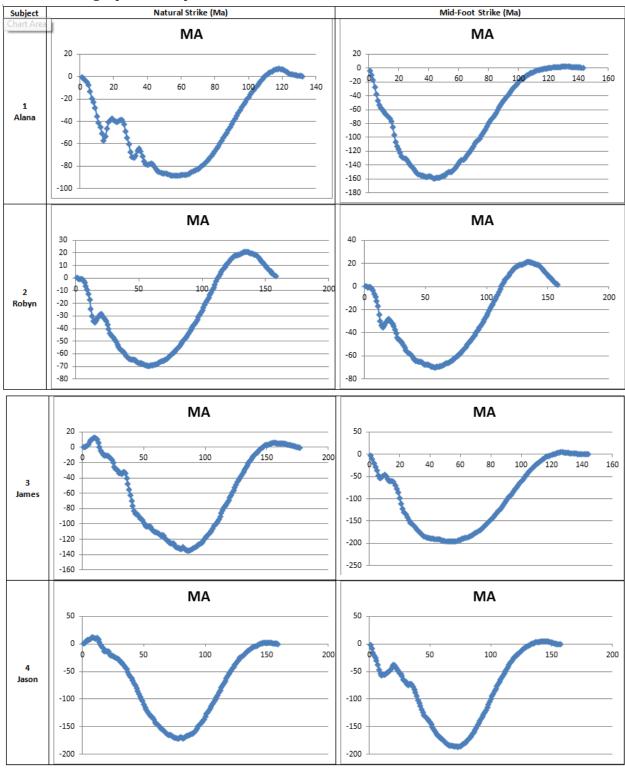
Forces on all graphs are reported in CENTIMETERS.

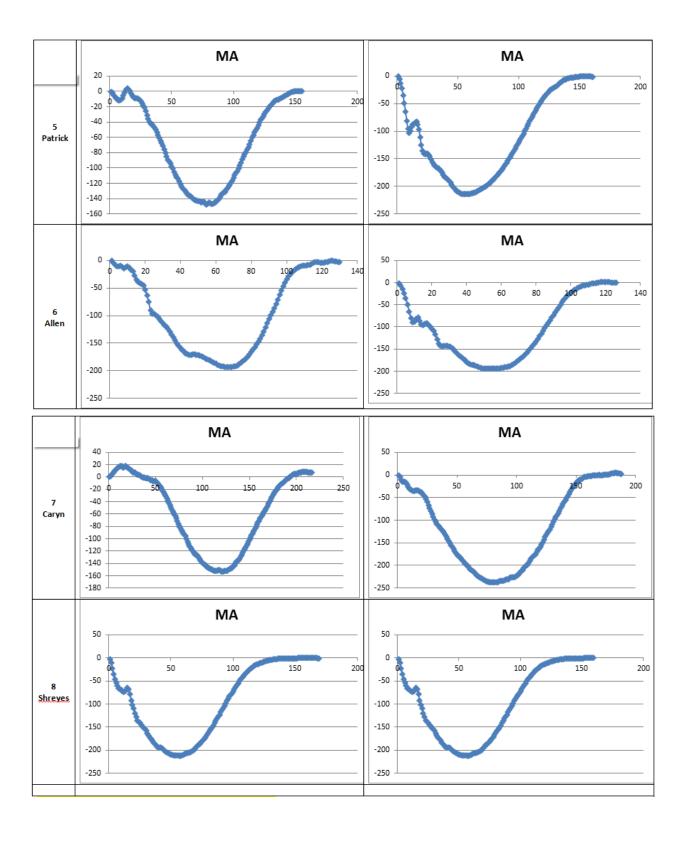




I.4 Moment About the Ankle Comparison

Forces on all graphs are reported in NEWTON-METERS.





I.5 Force-Angle Comparison

Blue= Mid-foot Stride Purple=Natural Stride

135.82

143.46

-90.63

-38.02

1011 N

2.34 115.63 135.78 93.22 141.39

1561 N

140.25

118.67

138.04

-84.07

-35.48

1907 N

3.14 95.83 146.86 116.01 138.06

131.27

116.96

138.56

-91.83

-50.53 -15.77 88.59 132.59

116.54

1396 N

		1		2		3			4			5		6	
Knee		-83.4	19				-93.39)		-82.32	2		-89.72		-86.06
Ankle	1	134.4	17				125.28	3	- [136.82	2		133.59		132.28
meta	1	8.8	37				21.15	5	- [-12.16	5		-17.05		-15.40
Patella	1653 N	111.9	99 1410 N		1102	2 N	105.73	15	37 N	120.59	16	522 N	114.79	1411 N	116.40
Hamstring	1	137.6	55				131.91	1	1	142.10)		139.67		140.47
Achillies	1	84.6	50				79.47	7	1	-22.51	1		89.64		91.84
Tibialis	1	133.3	37				134.29	9	1	138.43	3		136.27		134.12
Knee		-85.3	37				-85.25	5		-96.39)		-90.10		-91.62
Ankle	1	131.8	39				128.66	5	ı	134.63	3		131.45		126.47
meta	1	7.3	84				18.40)	Ī	-13.80	0		-13.42		-17.51
Patella	1536 N	115.8	1457 N		1179	5 N	110.61	14	54 N	115.68	17	748 N	110.76	1378 N	117.39
Hamstring	1	138.5	52				137.19	9	ı	134.38	3		139.16		139.54
Achillies	1	84.2	26				92.13	3	ı	89.37	7		87.51		87.89
Tibialis	1	129.5	55				139.09	9		133.29	9		133.87		132.12
Knee		-82.2	23				-91.83	3		-84.79)		-93.48		-82.41
Ankle	i	149.3				H	-34.17		ŀ	-25.07	_	h	-35.52		-37.24
meta	1	16.6				- 1	11.25	_	ı	18.96	_	ı	5.03		-4.63
Patella	1581 N	114.4			1234	ın l	112.89	_	48 N	113.91	_	568 N	112.60	1377 N	115.22
Hamstring	1	137.2					137.86			137.42	_		137.99		138.99
Achillies	1	100.1	_				86.29	-	ı	95.39	_	ı	92.53		91.49
Tibialis	1	145.6					138.61		ı	144.26		ı	142.88		144.48
Knee		-79.3					-93.49	_		-89.11			-91.85		-87.29
Ankle	1	146.2					-36.93		ı	-34.04	_		-32.58		-32.54
meta	1	10.5	52				7.34	1	ı	9.99)		4.33		6.93
Patella	1528 N	116.6	1366 N		1114	ı N	113.86	15	66 N	113.67	7 17	701 N	116.86	1398 N	119.60
Hamstring	1	137.2	28				138.32	2	ı	135.00)		140.61		138.77
Achillies	1	95.4	16				91.27	7	İ	34.75	5		91.13		96.26
Tibialis	1	144.8	36				140.95	5	1	145.68	3		145.95		140.65
7		8	1		9		10			11			12		13
	-87.70		-94.22		-86.33			-88.55			90.49		-92.89	9	-93.36
	-49.59		-48.54		-58.41	1		134.61		-4	15.78		-46.86	5	-52.56
	-14.34		-15.04		-21.80			-13.32		-1	12.70		-11.43	1	-6.83
1610 N	117.23	944 N	90.78	1853 N	96.55	153		97.07	1413		95.67	1661 N	89.49		87.86
	135.20		131.19		130.09			137.75			35.42		128.89		130.36
	92.00		117.34		115.35			118.80			4.07		109.83		112.04
\vdash	134.72		135.82		137.63			138.26			8.96		128.76		131.89
	-96.16 -46.85		-89.91 -46.96		-92.89 -56.75			-92.07 -41.37			34.54 42.51		-86.07 -35.79		-94.91 -49.86
I -	-46.85		-46.96		-25.75			-5.84			-7.88		-5.55		-49.86 -5.61
1430 N	112.94	908 N	94.76	1755 N	99.11	147	0 N	96.66	1546		6.91	1704 N	93.63		88.33
1430 N	130.24	300 N	137.21	1/33 N	125.24	14/		140.80	1340		9.90	1704 N	133.85		130.19
H	86.44		116.83		117.42			119.94			6.82		112.60		112.12
	135.16		139.50		138.81			139.94			32.09		127.93		137.25
	-88.95		-97.61		-94.27			-88.31			39.82		-88.97		-93.77
	-38.68		-43.12		-51.87	1		-34.65			34.75		-37.64		-42.37
	6.28		-0.84		-17.42	1		10.53			9.99		1.40	_	2.61
1414 N	116.31	1002 N	92.00	1755 N	91.85	148	1 N	105.41	1545	N 9	98.89	1661 N	91.92	2115 N	86.67
1 1	135.82	1	140.25		131 27	1		153 31		1/	13 85		138 7	7	133 87

153.31

119.50

140.37

-85.09

-85.09 -24.86 17.41 110.03 156.01 121.18 141.74

1531 N

143.85

109.64

129.00

-87.19

-32.86

13.93 103.16 146.70 119.03

132.69

1796 N

138.77

108.48

127.84

-86.37

-36.08

3.37 92.89 140.44

111.22 128.21

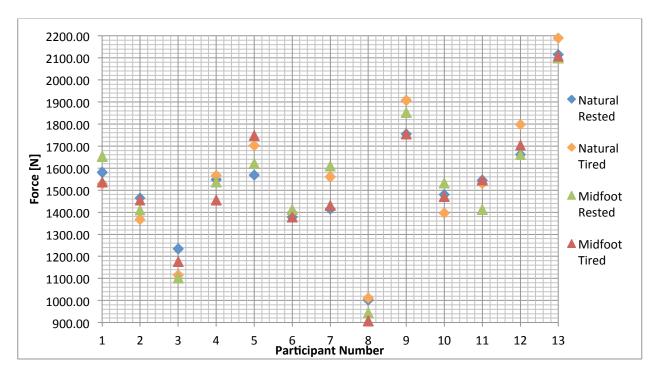
2188 N

133.87 117.67 120.99

-93.96

-93.96 -44.75 0.20 88.21 129.95 112.55

I.6 Landing Force Comparison: Natural Stride VS Mid-Foot Stride



	Natural	Natural Mid-foot Mid-foot				
	Rested [N]	Tired [N]	Rested [N]	Tired [N]	Stride	
1	1580	1530	1650	1540	Natural	
2	1460	1370	1410	1460	Natural	
3	1230	1110	1100	1180	Mid-Foot	
4	1550	1570	1540	1450	Mid-Foot	
5	1570	1700	1620	1750	Natural	
6	1380	1400	1410	1380	Either	
7	1410	1560	1610	1430	Either	
8	1000	1010	944	908	Mid-Foot	
9	1760	1910	1850	1750	Mid-Foot	
10	1480	1400	1530	1470	Natural	
11	1550	1530	1410	1550	Either	
12	1660	1800	1660	1700	Mid-Foot	
13	2120	2190	2100	2110	Mid-Foot	

Appendix J: Participant Information

BMI	25.1	25.7	22.9	21.6	22.2	23.8	19.4	22.9	28.8	29.2	22.8	25.1	24.9
HR After	56 2	71 2	73 2	90 2	88 2	71 2	83 1	92 2	60 2	78 2	62 2	120 2	85 2
HR After (5 min)	166	163	157	185	182	196	192	173	138	176	136	197	194
Resting HR	42	40	41	62	20	46	20	46	45	45	31	72	99
Ankle to Resting Meta HR	.9	5 1/2"	2"	5 1/2"	.9	.9	6"	5"	.9	5"	5 1/4"	6"	9
calf Length	10 1/2"	16"	15"	17"	18"	15 1/2"	16"	15"	16"	16"	16"	16"	19 1/2"
Thigh Length	15"	19"	18 1/2"	21"	21"	19"	18"	17"	18 1/2"	18"	19 1/2"	19 1/2"	7.3mph 21 1/4"
Avg. Speed	5.5mph	5.8mph	9 emph	6.8mph	6.5mph	6.8mph	6.5mph	5.7mph	5.0mph	5.5mph	5.8mph	6.2mph 19 1/2"	7.3mph
Shoe Type	Nike	Reebok	Adidas		Joma Lozano 6.5mph	Asics	Mizuno	Champion	Adidas	Nike	Nike	Avia	
Shoe Size	10	7 1/2	7	10	10 1/2	6	9 1/2	7 1/2	10 1/2	6	9 1/2	12	15
Weight	160lbs	152lbs	125lbs	155lbs	155lbs	145lbs	135lbs	125lbs	195lbs	165lbs	150lbs	160lbs	210lbs
Height		Female 5'4 1/2"		11,5	2,10,,	Female 5'5 1/2"	2,10,,		6,5	5'3"	8,5	"L,S	"S,9
Gender	Female	Female	Female	Male	Male	Female	Male	Female	Male	Female	Male	Male	Male
Participant Gender Height	1	2	3	4	5	9	7	8	6	10	11	12	13