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A First-Principles Analysis of Alternative Marine Propulsion Mechanisms

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A First Principles Analysis of Alternative Marine Propulsion Mechanisms

A Major Qualifying Project Report
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By

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Abstract

The marine propulsion industry is constantly looking for ways to improve propulsion efficiency. Since the industry is dominated by propellers, it begs the question of whether propellers are the best options for every task. Therefore first-principles analyses of three alternative mechanisms were conducted: feathered paddlewheels, caterpillar drives, and jellyfish actuators. Equations predicting the thrust and power generated by each mechanism were developed and used to compute efficiency. Efficiency comparisons were made to ideal propellers attempting similar tasks. The Kramer diagram was used to determine the efficiency of ideal propellers. From these results, recommendations were made to theorists and experimentalists hoping to build off the theory developed here. Results indicate that caterpillar drives (a) have potential due to their ability to maintain efficiency after significant internal friction losses and (b) may be more efficient than propellers in non-ideal speeds.

Acknowledgements

I would like to thank my advisor, Professor Popovich, for his support, patience, and commitment to excellence.

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Chapter 1: Introduction

1.1: Project Motivation and Overall Approach

This project began as a discussion on the marine propulsion industry. Since it is large, competitive, and motivated by cost reduction, both academia and industry are invested in maximizing the efficiency of marine propulsion systems. If propulsion efficiency can be improved by even a small percent, it could reduce fuel costs for entire shipping companies and navies.

One way to maximize efficiency is to optimize the propulsion mechanism. Since the industry is dominated by propellers, it begs the question of whether propellers are the best option for every task. Specifically: can other mechanisms compare to propellers for certain tasks? Also, given a task, at what point does a mechanism become better than propellers? These are the motivating questions behind this project.

Regarding the first question: one way to compare non-propellers against propellers is to compare theoretical models. If a model is developed which can predict the thrust and resistive forces of non-propeller system, its efficiency could be calculated and compared against propeller efficiency. That is the approach taken here. In this case, three alternative mechanism will be analyzed: the feathered paddle wheel, the caterpillar drive, and the jellyfish actuator. The details of these mechanisms will be explained in chapters 3, 4, and 5, respectively. Mathematical descriptions of these mechanisms will be developed, explored, and compared against propellers.

Regarding the second question: a mechanism becomes better than a propeller when, for a certain task, its ideal efficiency is higher than the ideal propeller efficiency. The ideal efficiency of propellers performing a certain task will be determined by the Kramer diagram.

1.2: Information on Kramer Diagram

The Kramer diagram computes the efficiency of propellers from the torque imparted to the fluid by the shaft. In the case of a constant torque and boat speed, energy is imparted to the fluid at a constant rate. This information is combined with the power required to move the vessel at a constant speed, and that is enough information to compute the efficiency of an ideal propeller. It's worth noting that the Kramer diagram does not include viscous losses and "gives the maximum achievable efficiency for a real propeller in uniform inflow."[1] Thus it can be used to provide high-end propeller efficiency for a given task.

The Kramer diagram is shown on the next page. To use the diagram, certain quantities must be known:

- V_b, the speed of the boat relative to the water (is V_A in the diagram)
- n, the rotational speed of the propeller in revolutions per second
- D, the diameter of the propeller
- C_T, the thrust coefficient (or the thrust required to push the bat), defined as

$$T = \frac{1}{2} C_T \rho A V_b^2$$

Where T is the thrust and A is the propeller area (πR^2).

- λ, the absolute advance coefficient, defined as

$$\lambda = \frac{V_b}{\pi n D}$$

[1] Techet

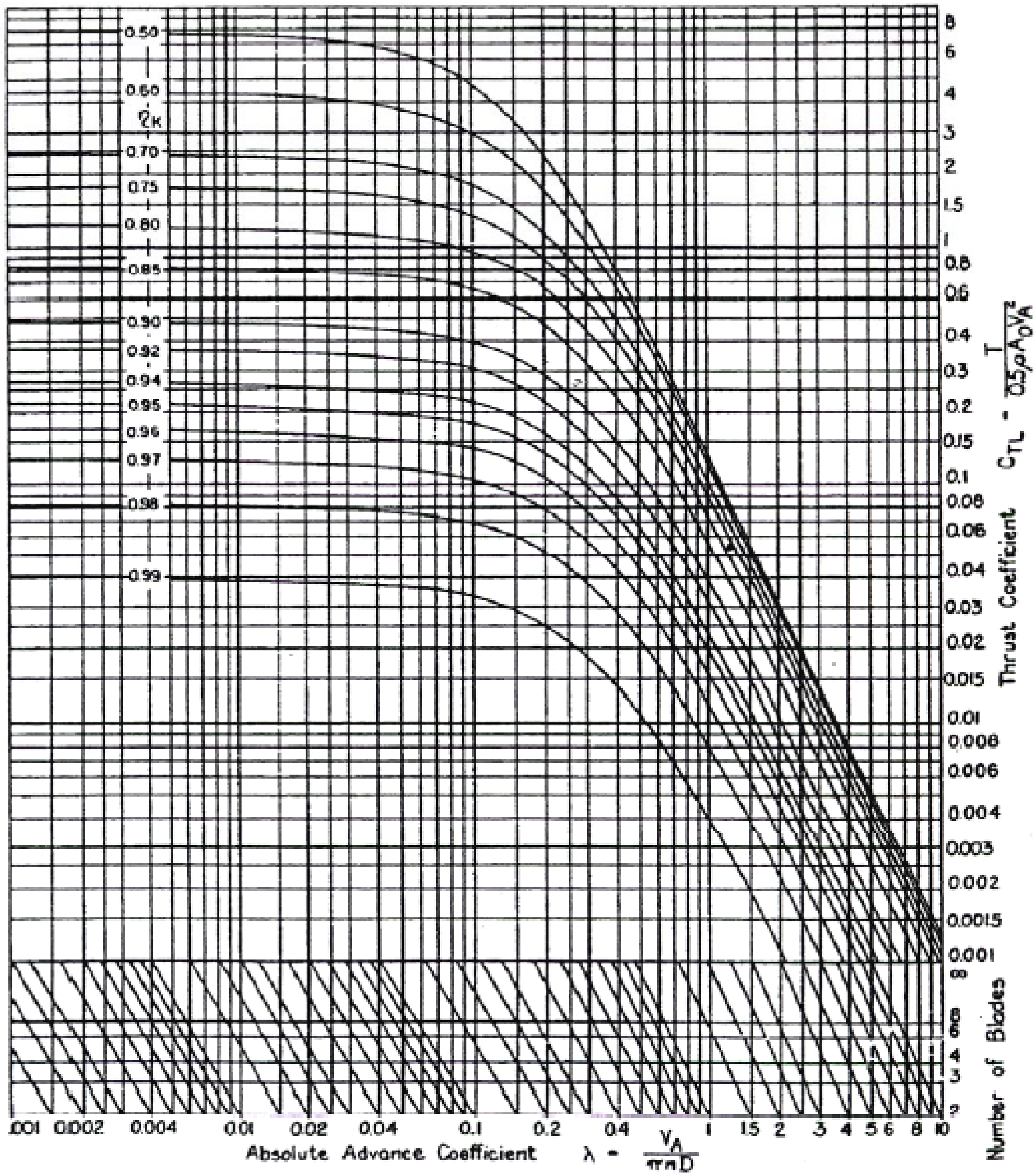

Figure 1: Kramer Diagram[2]

[2] Techet

The algorithm for determining propeller efficiency is:

1. Using propeller specifications, calculate λ and obtain C_T by calculation or measurement
2. Select the number of blades
3. Start at the calculated λ value and move up the diagonals until the diagonal intersects with the number of blades
4. From that intersection point, go horizontally up the graph to the thrust coefficient
5. The efficiency curve going through that point is the ideal efficiency of the propeller

It's worth mentioning why the Kramer method was selected over other methods.

One option was to develop a first-principles approach which could be used to derive optimal propeller efficiency curves. This was attempted by the author and did not prove successful.

Another was to use established vortex theory to calculate the efficiency of propellers directly. The dominant analytical approach combines blade element theory with vortex theory. Recent work by Moffitt, et al has established this combination as a reliable method of predicting the performance of screw propellers.[3] In their own words, "The vortex theory of screw propellers is based on a lifting line approximation of the blades of the propeller. This implies that the propeller is approximated by a lifting surface about which there is bound circulation. The total circulation is associated both with vorticity bound to the propeller and with the free vorticity that is continuously shed from the propeller in the form of a helical sheet."[4] The circulation in question could be quantified for each segment of the blade, and the net effects could be predicted with

[3] Moffitt, et al
[4] Ibid

computational methods. After developing and testing the theory, the paper concluded that vortex theory can accurately predict the performance of screw propellers in a “variety of conditions.”[5]

Unfortunately it is not clear how to use this method to determine optimal propellers except by guess-and-check. The research was concerned with predicting actual propeller behavior rather than optimizing propeller specifications to perform a certain task. Therefore the use of their methods proves unnecessarily tedious when it comes to identifying ideal propeller efficiency for a task.

Since the Kramer diagram provides a relatively simple method of going from a task to an ideal propeller efficiency, it was selected as the standard against which other mechanisms will be compared. Although the efficiencies of the ideal propellers will be high, so will the efficiencies of the other idealized mechanisms. Therefore the comparison between them will be meaningful.

[5] Ibid

Chapter 2: Project Strategy

This chapter more clearly defines the project goal and methodology used to achieve it.

2.1: Original Project Goals

It's worth mentioning that the project goals were changed late in the year. Early in the academic year, the caterpillar drive was imagined. It was designed specifically to avoid introducing a torque into the water – which screw propellers necessarily do. To put it formally, there are efficiency losses intrinsic to the propeller paradigm, and the caterpillar drive was designed to avoid these specific losses. This begged two follow-up questions. First, what are propeller-intrinsic losses? In other words, what efficiency losses necessarily come with the propeller paradigm? How can they be measured? Second, when we design an alternate mechanism to eliminate those losses, how do we know that the new mechanism can out-perform propellers? And if it can, by how much?

So the original goals involved (a) developing the theory needed to compare caterpillar drives against propellers and (b) building and testing a caterpillar prototype. It's worth mentioning that caterpillar drive theory is simpler than propeller theory. In caterpillar drives, the control volume giving boundaries to the water flux are well-defined. Using the control volume technique to analyze propeller dynamics is avoided except for very simplistic cases, and serious analysis of ideal propellers has been done with vortex theory. This is because defining a rigorous control volume (around the individual blades) is technically challenging. But the caterpillar drive would be analyzed with the control volume technique, so this was attempted for propellers (to compare "apples to apples," mathematically). Because of the technical issues involved with the more rigorous control volume approach to propellers, that analysis was never completed. However, that process generated an interest in propulsion theory. And since a caterpillar prototype would not give theoretically meaningful data (i.e., more serious experimentalist approaches would be needed before the success/ failure of the Caterpillar paradigm would be taken seriously), the goals were switched to a completely theoretical project.

2.2: Defining the Project Goal

As mentioned earlier, this project analyzes three mechanisms: the feathered paddle wheel, caterpillar drive, and jellyfish actuator. It is desirable to understand the performance of these mechanisms and the scales at which they are best suited. Therefore the project goal becomes clear: *"To devise theoretical models of three alternative, marine propulsion mechanisms, describe their utility for a variety of tasks, compare their performance against the traditional screw propeller, and make recommendations based on those findings."*

2.3: Project Strategy

To achieve the project goal, the following methodology will be used for each alternative mechanism:

1. Develop equations predicting the thrust and the power required to maintain that thrust for each mechanism
2. Use those equations to generate the efficiency curves for those mechanisms attempting different tasks
3. Compare those efficiencies against ideal screw propellers attempting similar tasks
4. Based on 1-3, make recommendations. Recommendations will be made for theorists hoping to build off this foundation as well as experimentalists hoping to design prototypes.

Chapter 3 will use this methodology to analyze feathered paddle wheels. Chapter 4 will analyze caterpillar drives, and Chapter 5 will analyze the jellyfish actuator.

Chapter 3: Feathered Paddlewheel

This chapter performs the analysis outlined in Chapter 2 on feathered paddle wheels.

3.1: Mechanism Description

The feathered paddle wheel is a paddle wheel whose blades are angled to remain perpendicular to the water when wet:

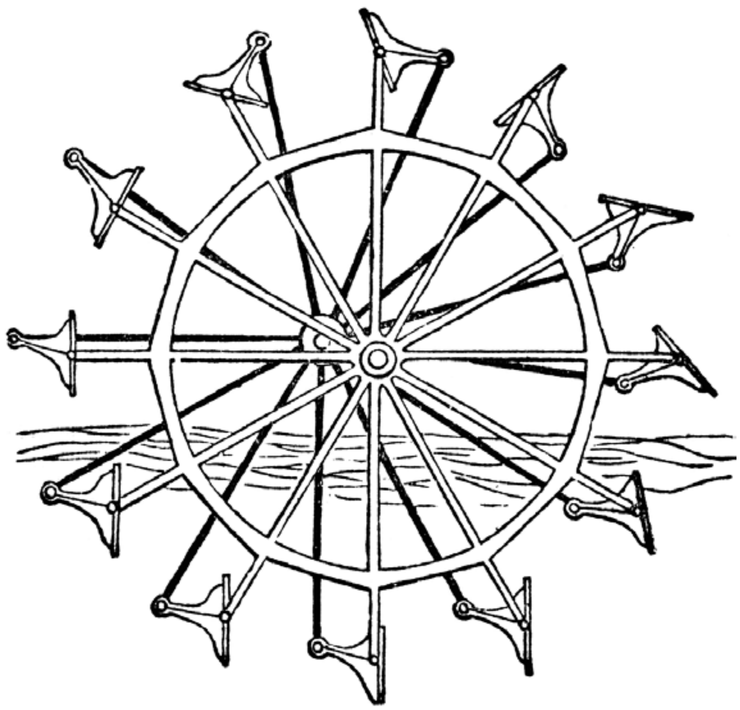

Figure 2: The Feathered Paddle Wheel[6]

This mechanism was patented in England in 1829 and is considered “the most successful attempt to improve the efficiency of the conventional paddlewheel.”[7] Feathered paddle wheels were prevalent in the marine propulsion industry through the 19th century. They became less common as screw propellers gained popularity in the second half of that century, although paddlewheels were still used in active service after World War II.[8]

[6] William & Robert Chambers
[7] *The Feathering Sidewheel*
[8] Ibid

3.2: Mathematical Characterization

Consider the single blade case:

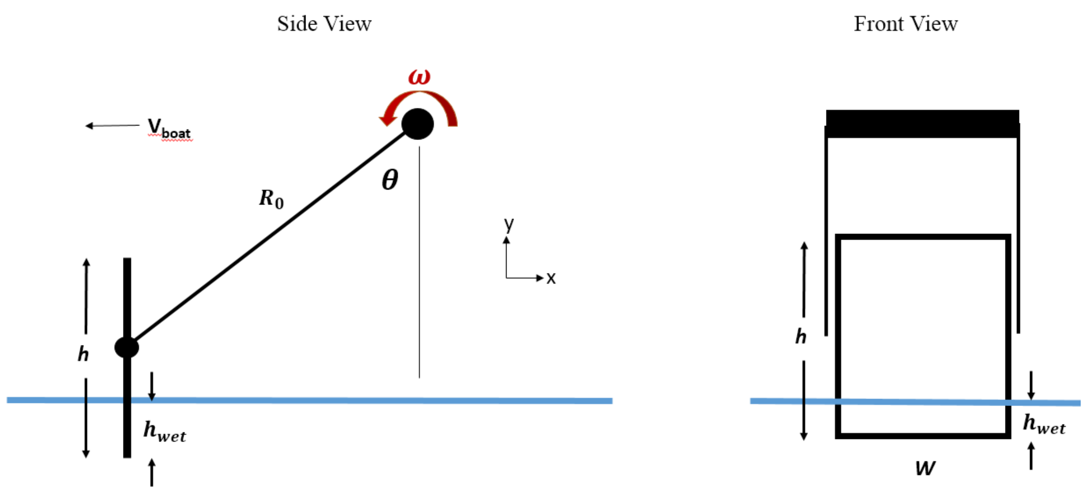

Figure 3: Feathered Paddlewheel Diagram

Here, R_0 is the distance from the center of rotation to the middle of the blade, h is the height of the blade, h_{wet} (or h_w) is the height of blade under water, ω is the rotational speed in radians per second, θ is the angle of $\mathbf{R_0}$ relative to the y-axis (defined in the diagram), V_{boat} (or V_b) is the boat speed, and W is the width of the paddle.

The force diagram on the blade becomes:

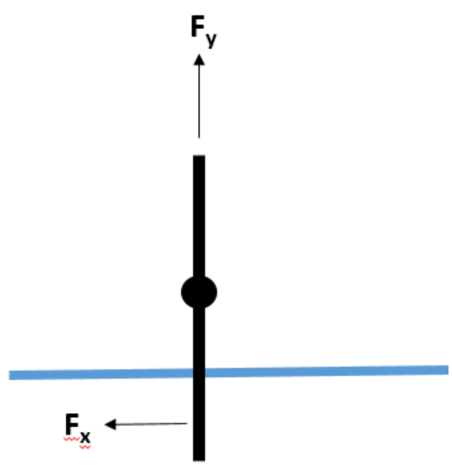

Figure 4: Forces on Single, Feathered Paddlewheel Blade

Here, F_x is the drag force associated with water flowing perpendicular to the blade:

$$F_x = \frac{1}{2} C_D \rho V_{rel}^2 A_{wet}$$

In this case:

- C_D is the drag coefficient associated with perpendicular flow:

$$C_D = 1.28$$[9]

- V_{rel} is the speed of the water relative to the blade. See Appendix A for the derivation.

$$V_{rel} = \omega R_0 - V_b$$

- A_{wet} is the submerged area of the blade. See Appendix A for derivation.

$$A_{wet} = W[R_0(cos\theta - 1) + h]$$

- F_x is approximated as acting on the point of the blade halfway between the blade tip and water surface.

From the same figure, F_y is the drag force associated with water flowing parallel to the surface of the blade:

$$F_y = \frac{1}{2} C_D \rho V_y^2 A_{wet}$$

In this case:

- C_D is the skin drag coefficient associated with perpendicular flow and is very small (typically less than 0.002 for turbulent flow, 0.005 for laminar flow).[10]
- V_y is the speed of the blade in the y direction [see Appendix A for derivation]:

$$V_y = \omega R_0 sin\theta$$

- Note that V_y is small for small angles.

[9] "Shape Effects on Drag."
[10] Young, table 1

F_y is negligible when the blade is submerged, since V_y is small for small θ and C_D is small. F_y is also negligible when the blade is partly submerged because the drag forces are acting on a small area. This neglecting of forces in the y-direction was used in an analysis of regular paddlewheels and confirmed with CFD.[11] Since the effect is even less significant in the feathered case, it will be ignored here.

The torque required to maintain ω through a single stroke is

$$\tau = \overline{R_{eff}} \times \overline{F_x} = R_{eff} F_x cos\theta$$

Where R_{eff} is the vector from the point of rotation to the point where F_x is acting on the blade. Recall that F_x is acting on the point halfway between the water surface the blade tip.

The magnitude of R_{eff} is given by the following expression. See Appendix A for derivation.

$$R_{eff} = R_0[0.25(cos\theta + 1)^2 + sin^2\theta]^{1/2}$$

The power required to push the blade through one stroke at a constant angular speed is

$$P = \omega\tau = \omega R_{eff} F_x cos\theta$$

Substituting the expressions into the power equation gives

$$P(\theta) = \frac{1}{2}\omega W C_D \rho R_0(\omega R_0 - V_b)^2[R_0(cos\theta - 1) + h][0.25(cos\theta + 1)^2 + sin^2\theta]^{1/2}cos\theta$$

Note that C_D is the drag coefficient of water flowing perpendicular to a flat surface.

This power equation is valid for angles where the paddle is at least partially submerged. In mathematical terms, the power equation is valid between θ_{max} and θ_{min}, where

$$\theta_{max} = cos^{-1}(1 - \frac{h}{R_0})$$

And

$$\theta_{max} = -\theta_{min}$$

---

[11] Liu

The power is defined as 0 otherwise. See Appendix A for the derivation of these angles.

Note that the speed of the boat is assumed to be constant, even though the thrust and power are functions of the angle. V_b is assumed constant because there will be more than one blade in the water. Therefore the thrust is

$$T = F_{x1}(\theta) + F_{x2}(\theta + \varphi) + F_{x3}(\theta + 2\varphi) + \cdots + F_{xN}(\theta + (N-1)\varphi)$$

Where N is the number of blades and φ is the angle between blades. Note that the force $F_x(\theta)$ is $\frac{1}{2}C_D\rho V_{rel}^2 A_{wet}$ between θ_{max} and θ_{min} and zero otherwise. Written in the system parameters, this becomes

$$F_{x1}(\theta) = \frac{1}{2}C_D\rho W(\omega R_0 - V_b)^2[R_0(cos\theta - 1) + h]$$

Likewise, the power required to maintain the angular speed of the mechanism is

$$P = P_1(\theta) + P_2(\theta + \varphi) + P_3(\theta + 2\varphi) + \cdots + P_N(\theta + (N-1)\varphi)$$

The power required to push each blade through a stroke is given by the formula presented earlier:

$$P_1(\theta) = \frac{1}{2}\omega W C_D\rho R_0(\omega R_0 - V_b)^2[R_0(cos\theta - 1) + h][0.25(cos\theta + 1)^2 + sin^2\theta]^{1/2}cos\theta$$

These expressions are oscillations whose amplitude decreases as N increases. Therefore a reasonable approach is to assume a constant V_b for simplicity and use the average T and P values to determine efficiency.

The efficiency of this mechanism is defined as

$$\eta_p = \frac{V_b \cdot Average\ T}{Average\ P + k_p}$$

Note the term k_p. Since the paddlewheel has moving components, power will be required to maintain its rotation without resistance. In other words, if you were to remove the paddlewheel from the water and spin it, the wheel will slow down over time. This term represents the internal friction losses of the system, where k_p is the internal friction coefficient and has units of power. This coefficient will have to be determined experimentally and will vary for each paddlewheel.

Instead of using calculus to determine average T and P, MATLAB will be used because it is easier to change parameters and quickly compare results. These computations will be described in the next section.

3.3: Efficiency for Different Tasks

To identify the tasks which feathered paddle wheels are most efficient at, the following process will be used:

1. Identify an ideal cruise speed
2. Pick a corresponding R_0, h, and W
3. Starting at $\omega = \frac{V_b}{R_0}$, calculate the efficiency of the mechanism for that value of ω. Repeat the process for higher ω until an ideal ω becomes apparent.
4. Using the ideal ω identified from (3), generate the efficiency vs V_b curve
5. Repeat this process for different size boats and cruising speeds

Here, three boat sizes will be used: an RC toy boat, a canoe, and a tugboat.

3.3.1: RC-Size Boat Calculations

The following specs are used for RC boats:[12]

- Cruise speed = 15 mph = 6.7 m/s
- R_0 = 0.2 meters

[12] Specs informed by Preece

- h = 0.08 meters
- W = 0.08 meters

Note: for this case, 8 blades were selected. This implies that each blade is 45 degrees from the next. Therefore five blades will interact with the water in a single stroke, and the blades can be labeled:

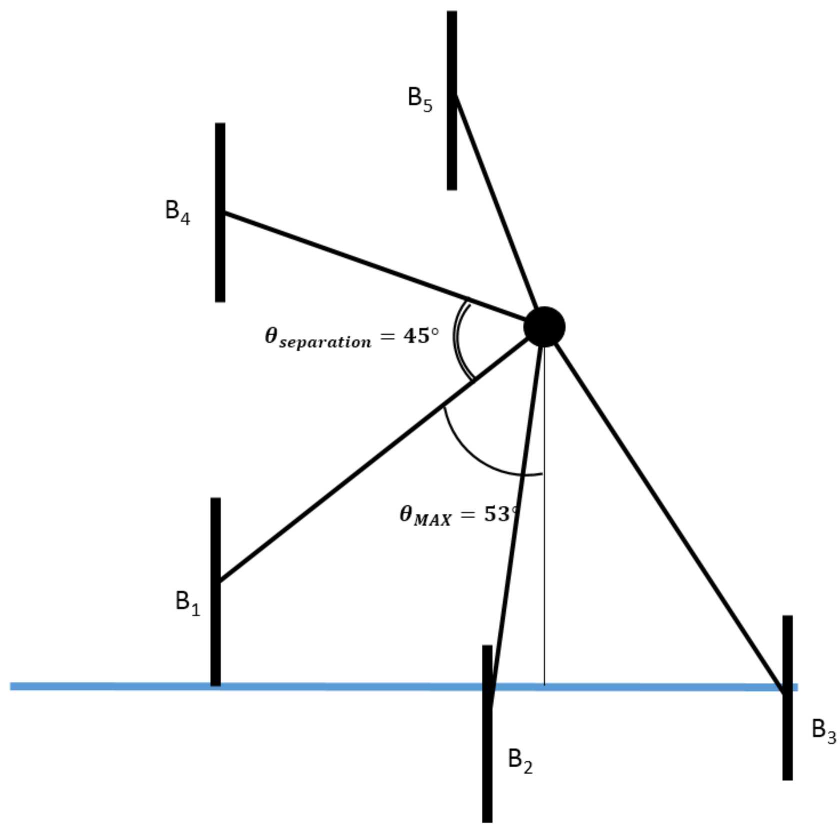

Figure 5: Feathered Paddle Blade Labels, 8 Blades

The stroke (or “rotation”) is defined as Blade 1 going from θ_{max} to θ_{min}:

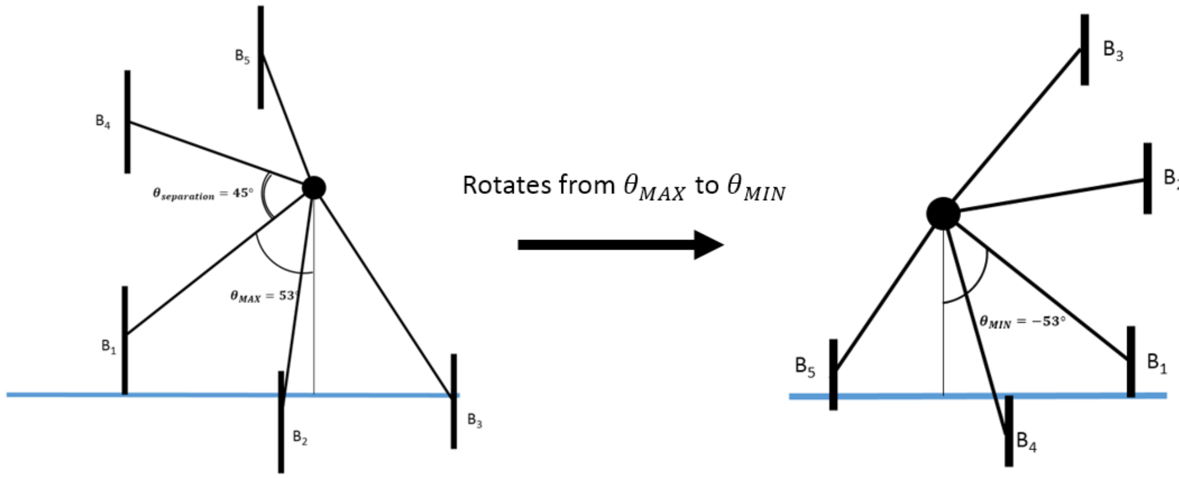

Figure 6: A Single Stroke of the Feathered Paddlewheel

Using the equations derived in the previous section, the thrust corresponding to each blade segment could be graphed in MATLAB as a function of the angle of B_1. The MATLAB code can be found in Appendix A.

The thrust curves are given below, on the assumption that $\omega = 1\, rads/sec$. This value was chosen arbitrarily – the graphs merely illustrate that the equations behave in an expected manner. Furthermore, the graphs are functions of an index "i." The angles between θ_{max} and θ_{min} were divided into 186 equally-spaced segments. The indices "i" are integers from 1 to 186 and correspond to each segment. Therefore the graphs generated by MATLAB are on a scale from 1 to 186 instead of angles between θ_{max} and θ_{min}.

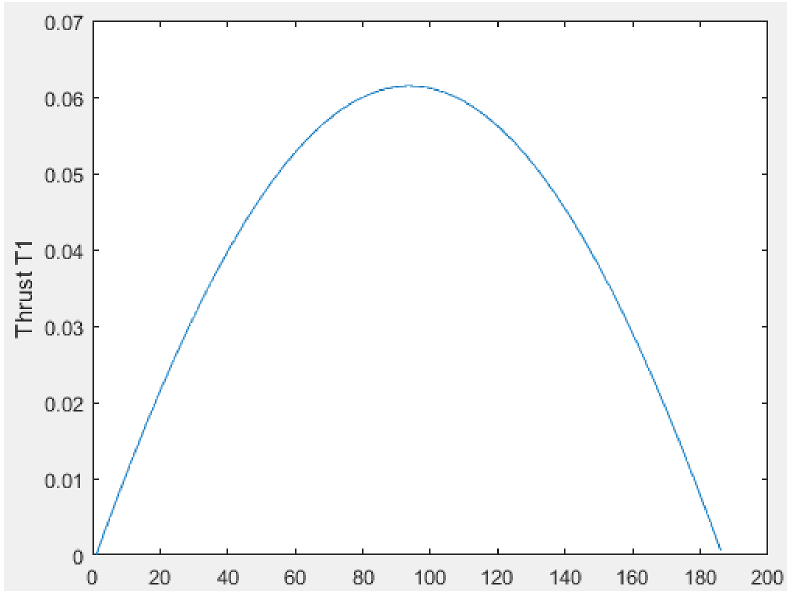

Figure 7: Thrust Generated by Blade 1

Recall that blade 1 is the blade which starts at the θ_{min} (which corresponds to index i=1 in the graph), the angle where it enters the water. Therefore the wetted area starts off as zero, and the thrust generated is also zero. As more of the blade dips into the water, the thrust increases. The thrust dips back to zero as the blade exits the water at θ_{max} (corresponding to index i=186).

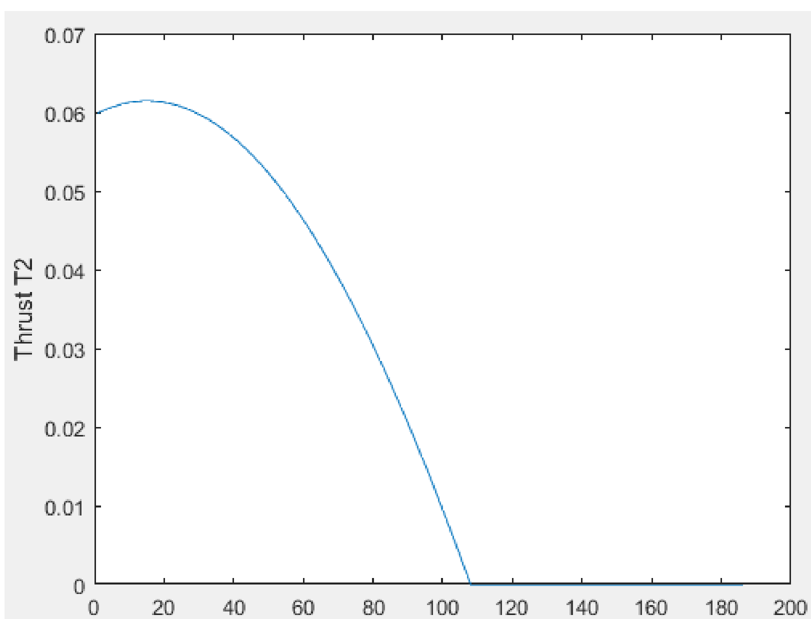

Figure 8: Thrust Generated by Blade 2

Recall that blade 2 starts off in the water and exits. Therefore it starts off generating thrust, then leaves the water and generates zero thrust for the rest of the rotation.

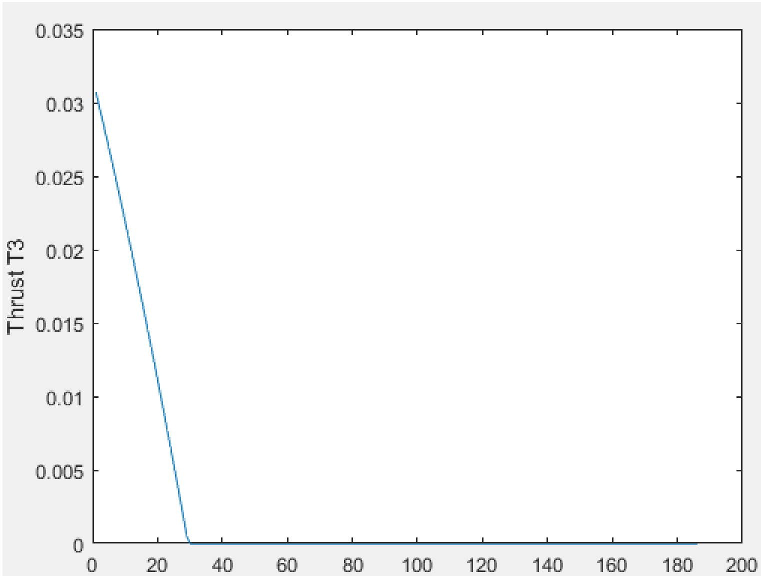

Figure 9: Thrust Generated by Blade 3

At the beginning of the stroke, blade 3 is almost out of the water. Therefore it generates a small amount of thrust before exiting the water.

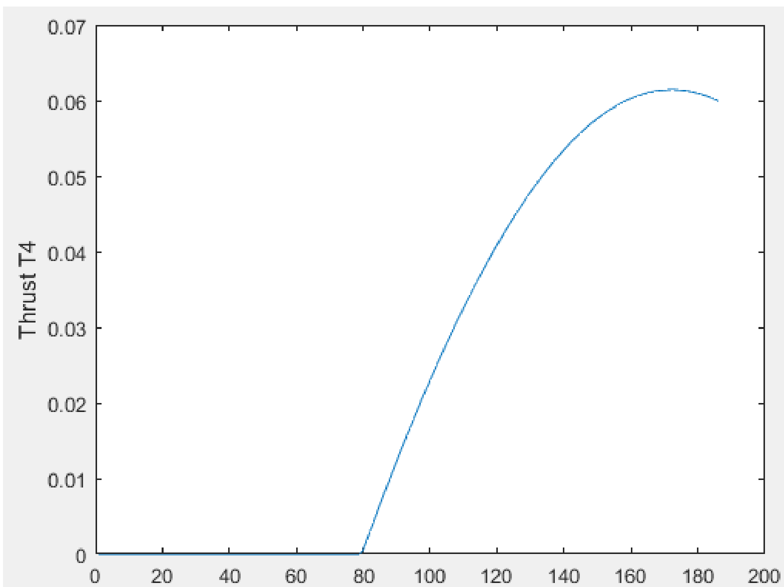

Figure 10: Thrust Generated by Blade 4

Recall that blade 4 starts outside of the water and enters after blade 1. Therefore at the beginning of the stroke, it does not generate any thrust, then generates thrust after it enters the water.

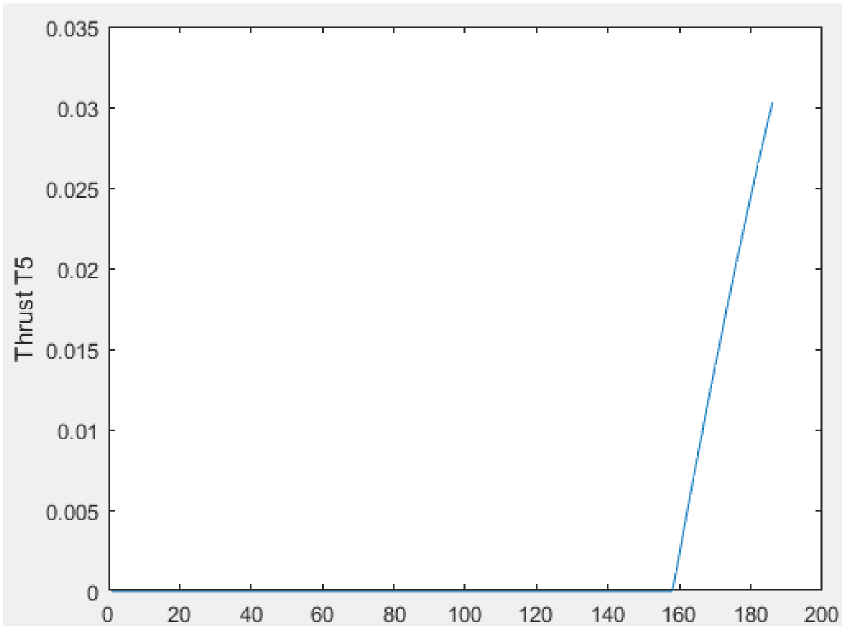

Figure 11: Thrust Generated by Blade 5

This is the “inverse” of blade 3 – it starts off outside the water, then at the end of the stroke it enters and generates a bit of thrust.

These thrust curves are summed to give the thrust generated by system as a function of Blade 1’s angle:

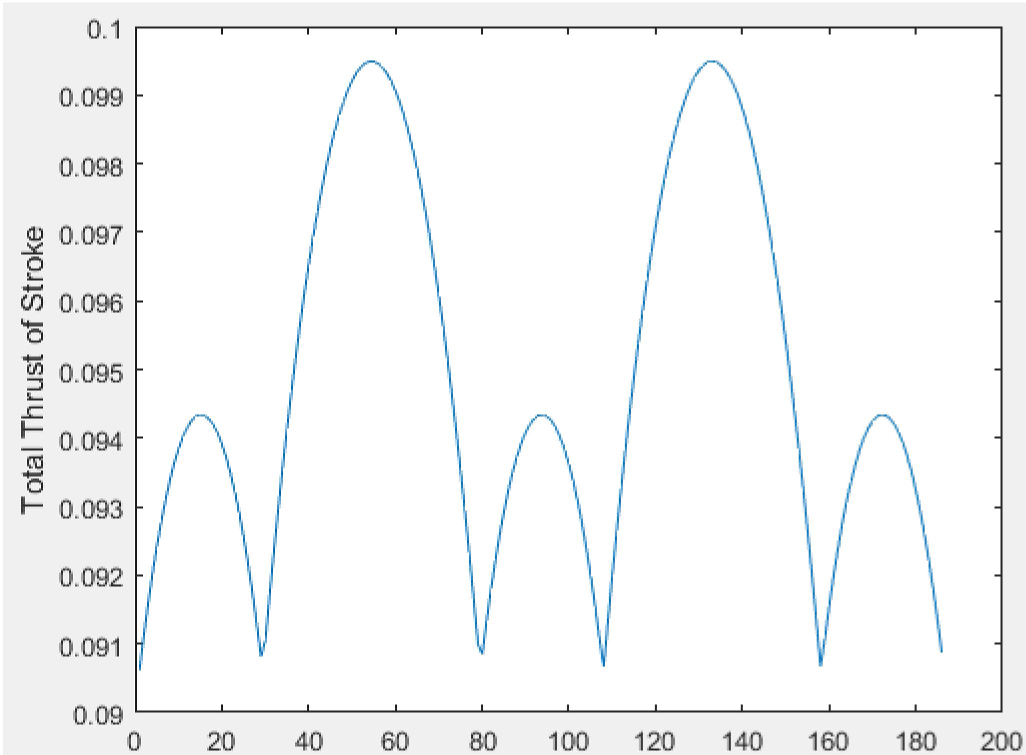

Figure 12: Total Thrust Generated by Single Paddlewheel Stroke

Notice the upper and lower bounds on the thrust: the values stay between 0.09 and 0.1. This validates the assumption that the average thrust can be a meaningful characterization of the system.

The power curves have the same overall shape and pattern, so listing them here is redundant.

The efficiency of the paddlewheel can now be computed. The efficiency is defined as

$$\eta_p = \frac{V_b \cdot Average\ T}{Average\ P + k_p}$$

Where Average T is the average thrust and Average P is the average power generated over the course of the stroke. This value is computed at different values of ω. These calculations made on the assumption that $k_p = 7 \cdot 10^{-6}\ Watts$. If that coefficient were zero (the frictionless case), the shape of the curves would be different. Therefore a small value was inserted to give the near-frictionless results and give more realistic efficiency curves later. Recall the specs used for the RC boat:

- Cruise speed = 15 mph = 6.7 m/s
- R_0 = 0.2 meters
- h = 0.08 meters
- W = 0.08 meters

Again, these numbers were plugged into the equations for thrust and power outlined in the last section and the angular speed was varied. The results are:

Angular Speed	Efficiency
34	0.622
35	0.938
36	0.956
37	0.943
38	0.924
39	0.903
40	0.882
41	0.8611

Table 1: Efficiency vs Angular Speed, Small Paddlewheel

These results can be illustrated graphically:

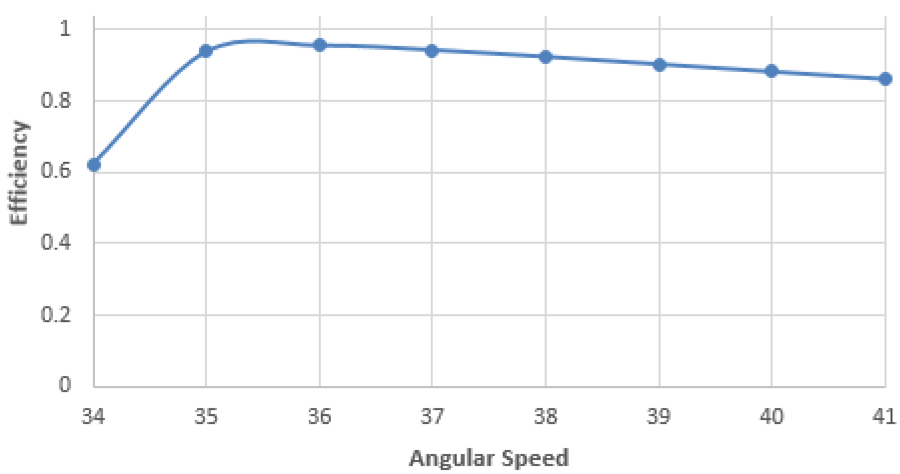

Figure 13: Efficiency vs Angular Speed, Small Paddlewheel

Notice that the efficiency never goes above 1: it just comes very close. From this graph, assume an angular speed of 35.5 s^{-1} as the optimal speed.

To generate the efficiency curve, maintain the paddlewheel angular speed and change the boat speed. Using the equation without the k_P component, the results are:

Boat Speed	Efficiency
0	0
1	0.149
2	0.298
3	0.447
4	0.596
5	0.744
6	0.889
7	0.6

Table 2: Boat Speed vs Efficiency, Small Paddlewheel

These results can be illustrated graphically:

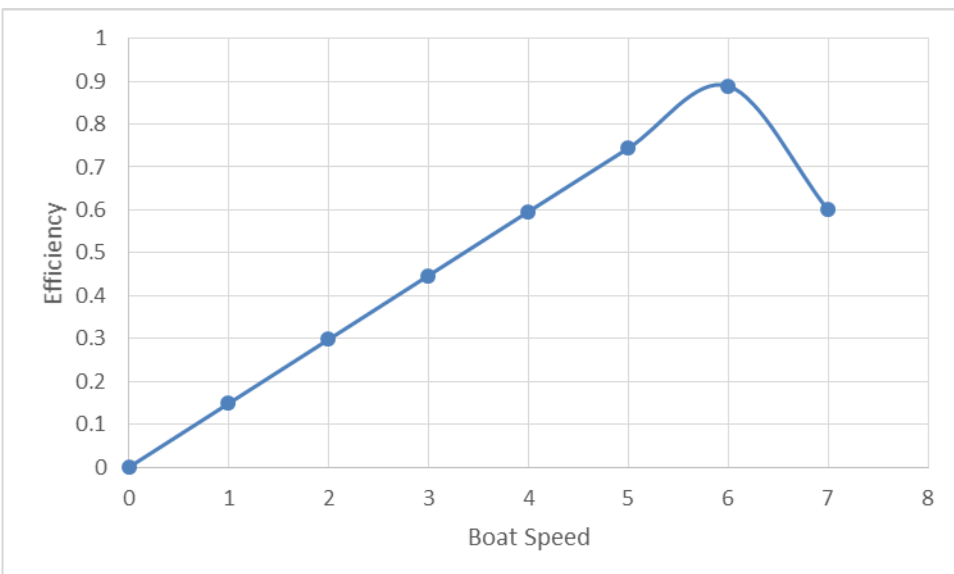

Figure 14: Efficiency vs Speed Graph, Small Paddlewheel

The same MATLAB code was used to generate these graphs, but different values were used and written in an excel sheet to generate these curves. See Appendix A for the code.

3.3.2: Canoe-Size Boat Calculations

The following specs are used for canoe-sized boats:[13]

- Cruise speed = 3 mph = 1.34 m/s
- R_0 = 4 feet = 1.22 meters
- h = 1.6 feet = 0.488 meters
- W = 0.488 meters

Again, 8 blades were selected. To maintain the geometry, the h/R_0 and h/W ratios were kept constant. The graphs have similar shapes, so there is no need to put them here. The internal friction coefficient used was k_c = 0.15 Watts and the starting angular speed was 1.098 s^{-1}.

The angular speed vs efficiency curves were generated in a similar fashion:

Angular Speed	Efficiency
1.1	0.006
1.15	0.854
1.2	0.921
1.25	0.907
1.3	0.881
1.35	0.852
1.4	0.824

Table 3: Efficiency vs Angular Speed, Canoe Paddlewheel

These results can be displayed graphically:

[13] Specs informed by BCWA

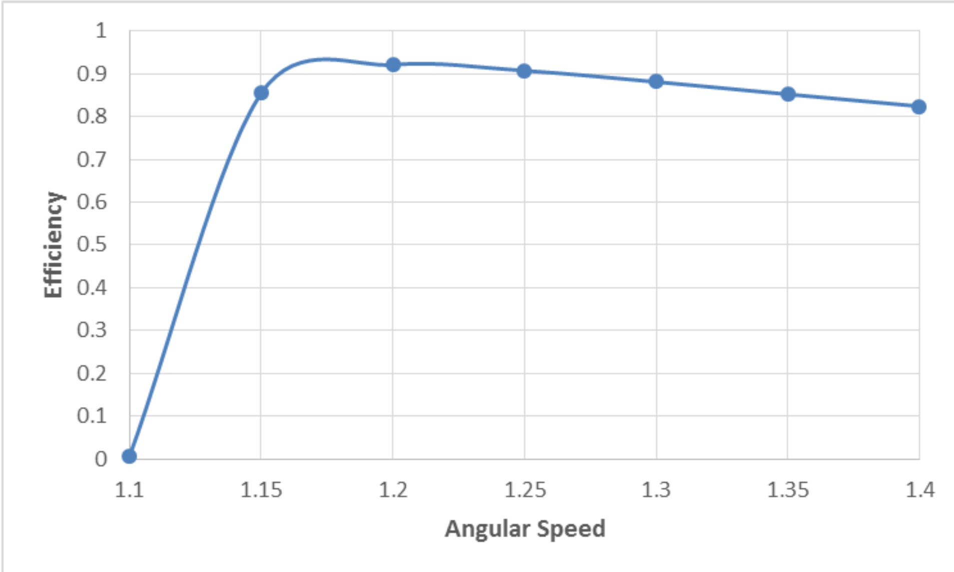

Figure 15: Efficiency vs Angular Speed, Canoe-Sized Paddlewheel

From this data, the optimal angular speed is 1.2 s^{-1}. The efficiency vs boat speed curve can be generated like before:

Angular Speed	Efficiency
1.1	0.006
1.15	0.854
1.2	0.921
1.25	0.907
1.3	0.881
1.35	0.852
1.4	0.824

Table 4: Boat Speed vs Efficiency, Canoe-Sized Paddlewheel

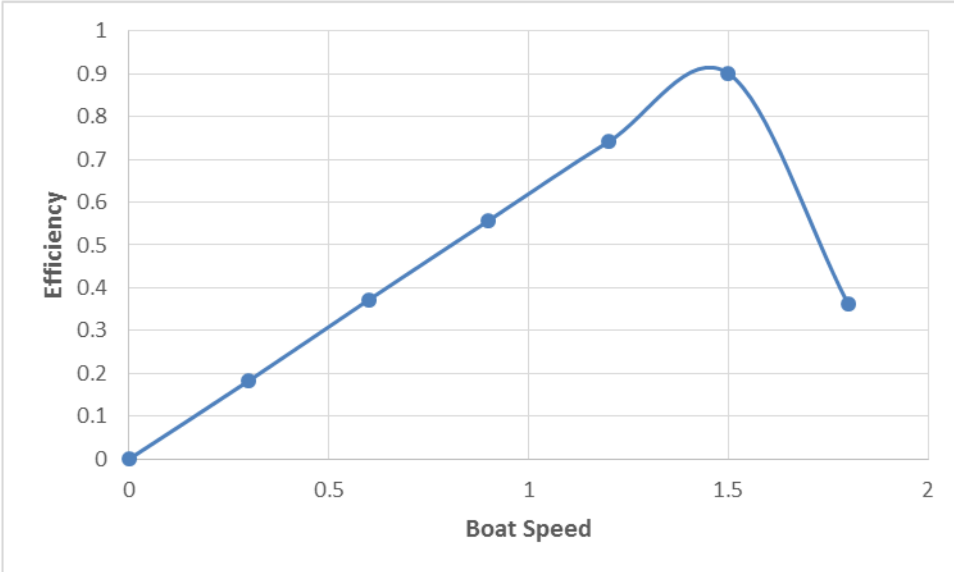

Figure 16: Boat Speed vs Efficiency, Canoe-Sized Paddlewheel

3.3.3: Tugboat-Size Paddlewheel Calculations

The following specs are used for Tugboat-size paddlewheels:[14]

- Cruise speed = 12 knots = 6 m/s
- R_0 = 4.5 meters
- h = 1.8 meters
- W = 1.8 meters

As before, 8 blades were chosen and the R_0:h:W ratios were maintained to preserve geometry. The starting angular speed is V_b/R_0 = 1.33 s^{-1}. The internal friction coefficient selected is $k_c = 125\ Watts$. The angular speed vs efficiency curves at cruise speed were generated as before:

[14] Specs informed by Hoffman

Angular Speed	Efficiency
1.35	0.263
1.4	0.84
1.45	0.91
1.5	0.908
1.55	0.89
1.6	0.868
1.65	0.845

Table 5: Efficiency vs Angular Speed, Tugboat-Size Paddlewheel at Cruise Speed

These results were visualized:

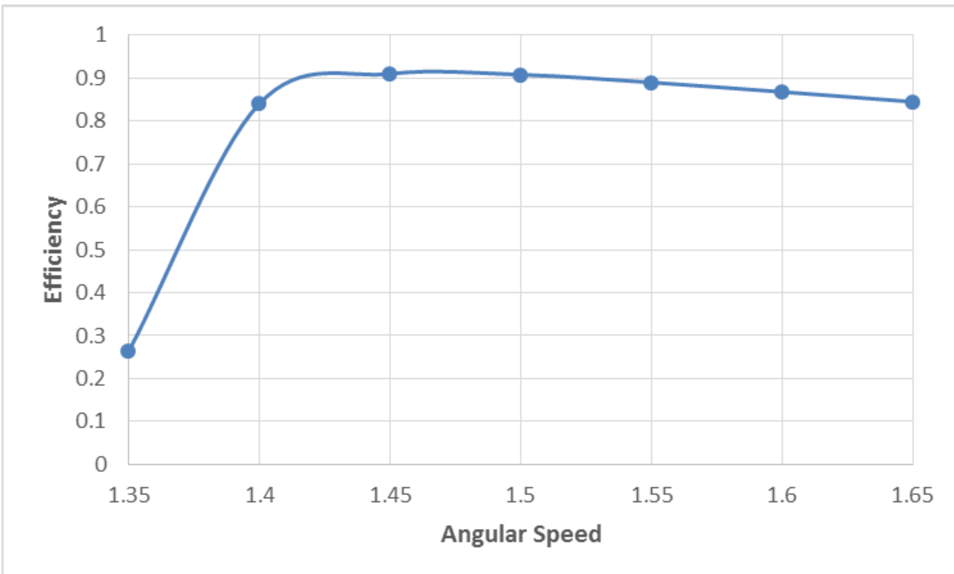

Figure 17: Efficiency vs Angular Speed, Tugboat-Size Paddlewheel at Cruise Speed

From this graph, the optimal angular speed is 1.45 s^{-1}. Keeping this speed constant and changing the boat speed can be used to generate the efficiency vs boat speed curves as before:

Boat Speed	Efficiency
0	0
1	0.142
2	0.285
3	0.427
4	0.569
5	0.71
6	0.845
7	0.876
8	0.256

Table 6: Efficiency vs Boat Speed, Tugboat-Sized Paddlewheel, $\omega = 1.74$

Graphically:

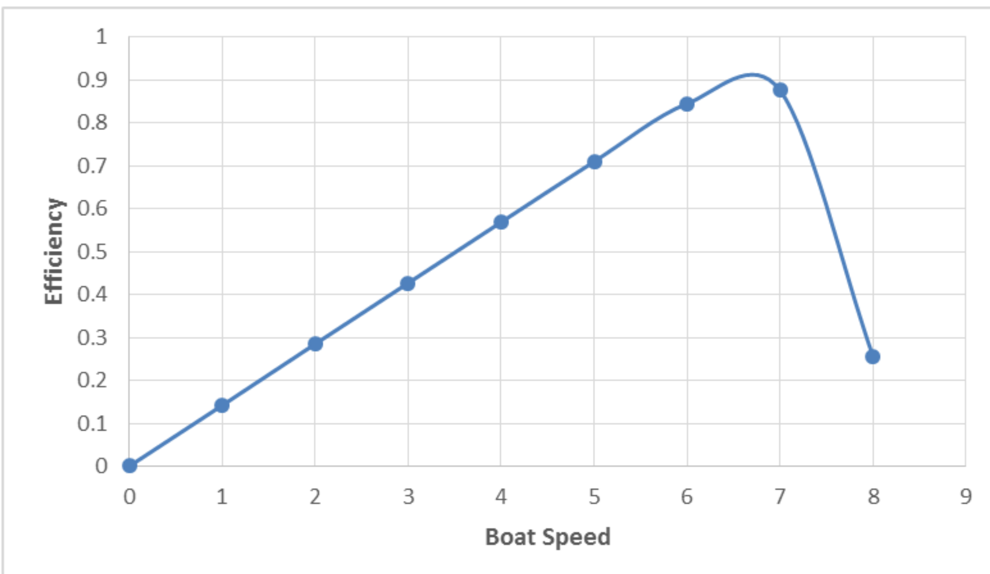

Figure 18: Efficiency vs Boat Speed, Tugboat-Sized Paddlewheel, $\omega = 1.74$

3.4: Comparison to Ideal Propellers

The graphs in the previous section suggest that the efficiency of ideal feathered paddlewheels consistently reach 90% at ideal cruise speeds. However, this was based on arbitrary assumptions about what the internal loss coefficients k_c are for different mechanisms. Therefore to provide a more meaningful comparison with ideal propellers, the following process will be used:

1. Determine the thrust generated by the paddlewheel to perform the task
2. Assume the propeller has to generate that thrust, and use propeller specs which are standard for that task
3. Use the Kramer diagram to determine the efficiency of ideal propellers performing that task
4. Going back to the MATLAB code used to generate the efficiency vs angular speed curves, determine the coefficient k_c required to match the efficiency of ideal propellers determined from (3)

3.4.1: Comparison with ideal RC Propellers

The RC-sized boat was generating a thrust of about 130 N at a boat speed of 6.7 m/s and angular speed of 35.5 s^{-1}. This thrust force might seem high, but realize that the RC boat is traveling almost 7 m/s and the specs were based off boats which were 0.5 meters long. The thrust can be easily calculated by using the thrust equation developed in section 3.2 and the specs from section 3.3.1:

$$T = F_{x1}(\theta) + F_{x2}(\theta+\varphi) + F_{x3}(\theta+2\varphi) + \cdots + F_{xN}(\theta+(N-1)\varphi)$$

$$F_{x1}(\theta) = \frac{1}{2} C_D \rho W (\omega R_0 - V_b)^2 [R_0(cos\theta - 1) + h]$$

The forces F_{xi} are averaged over the course of the stroke, defined as Blade 1 going from θ_{MAX} to θ_{MIN}, where

$$\theta_{max} = cos^{-1}(1 - \frac{h}{R_0})$$

$$\theta_{max} = -\theta_{min}$$

For a 3-blade, 4-inch diameter propeller rotating at 9000 RPM[15], the Kramer diagram efficiency comes out to about 70% efficient.

So at what point does the feathered paddlewheel beat the propeller for this RC example?

Recall that the term k_P was arbitrarily selected to give more realistic curves than the frictionless case, and the efficiency was close to 90% at cruise speed. Cruise speed efficiencies of 70% are generated when k_P is 29 W. This can easily be determined by changing the value of k_c in the MATLAB code until the curve generates a maximum efficiency of 70%. Therefore to beat the ideal propeller, a caterpillar drive with the design specs from section 4.3.1 and a k_c of 29 W or less should perform better than any propeller attempting the same task.

3.4.2: Comparison with ideal Canoe Propellers

The canoe-sized feathered paddlewheel was generating a thrust of 53 N according to the thrust equation. The boat speed used was 1.34 m/s and the angular speed was 1.2 s^{-1}. For a 3-blade, 0.36 m diameter propeller spinning at 4000 RPM[16] the efficiency of ideal propellers is about 90%.

As in the previous section, the term k_P was arbitrarily selected. Cruise speed efficiencies of 90% are generated when k_c is about 0.38 W. This can be determined in a similar manner. Therefore to beat the ideal propeller, a feathered paddlewheel with the design specs from section 3.3.2 and a k_c of 0.38 W or less should perform better than propellers attempting a similar task.

[15] Informed by Preece and RC Boat Calculator
[16] Specs informed by Marine Engine RPM Chart

3.4.3: Comparison with ideal Tugboat Propellers

The tugboat-sized caterpillar drive was generating a thrust of 138,000 N according to the thrust equation. The boat speed used was 6 m/s and the angular speed was 1.45 s^{-1}. Typical tugboat specs are 2.4 m diameter, 4-blade propellers rotating at 24 radians per second.[17] From the Kramer diagram, propellers with these specs providing a thrust of 140,000 N are about 80% efficient.

As in the previous sections, the k_P term was arbitrarily selected, and the efficiency was close to 1 at cruise speed. Cruise speed efficiencies of 80% are generated when k_c is about 2028 W. This can be determined in a similar manner. Therefore to beat the ideal propeller, a caterpillar drive with the design specs from section 3.3.3 and a k_c of 2028 W or less should perform better than propellers attempting a similar task.

3.5: Feathered Paddlewheel Discussion and Recommendations

First, a statement on what was accomplished in this section. A first-principles approach to feathered paddlewheels was developed and several graphs were generated to give an intuition for how paddlewheels behave. Then friction coefficients were assumed to give a sense of paddlewheel performance. The graphs suggest that feathered paddlewheels only perform well near optimal cruise speeds. Comparing these graphs against propellers suggest that screw propellers can perform more efficiently at non-optimal cruise speeds. That being said, feathered paddlewheels move through the water more intuitively – pushing straight back instead of slicing through. Because of that, feathered paddlewheels can out-perform screw propellers in some cases, provided their internal friction losses are not too significant. A methodology for determining the maximum friction losses to beat ideal propellers was outlined.

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[17] Informed by Hoffman and Argyriadis

An important omission was alternating geometries. The number of blades was standardized at 8 and the R_0:h:W ratios were maintained throughout. Regarding the number of propellers: according to the framework developed here, the thrust and power will increase at the same rate, and no difference will be noticed for larger numbers of propellers. A full CFD or Naiver-Stokes analysis is not necessary here, since this paper is focused on developing a first-principles framework which can be easily modified for future researchers. Regarding the ratios: since the focus here was developing the framework, research into the effect of different ratios is left to the reader. The framework has been developed and recommendations are listed below. The MATLAB code is attached in Appendix A and can be easily modified for different ratios.

The following are recommendations for future researchers using this framework:

1. Experimentalists should measure the k term for different designs in order to make more accurate predictions.
2. The k term will be more significant for larger, faster paddlewheels. Therefore the results of this paper suggest using paddlewheels for slower, smaller and/or medium sized vessels.
3. Theorists should take different geometries into account. Here reasonable geometries were assumed to focus on analyzing the effects of ω at constant V_b. However it cannot be assumed that these were the optimal geometries. Researchers interested in the effect of R_0:h:W ratios on paddlewheel design should keep ω constant and vary the geometry. This should be repeated for different ω and different scales.
4. Theorists should also use CFD to examine the effects of different numbers of blades on efficiency. The results can be used to modify this first-principles analysis and incorporate better assumptions.

Chapter 4: Caterpillar Drive

4.1: Mechanism Description

The modified caterpillar drive analyzed here was conceptualized by the author. It is based on the classic caterpillar drive, which is similar to a paddlewheel. But instead of rotating around a cylinder, the blades are rotated with a belt:

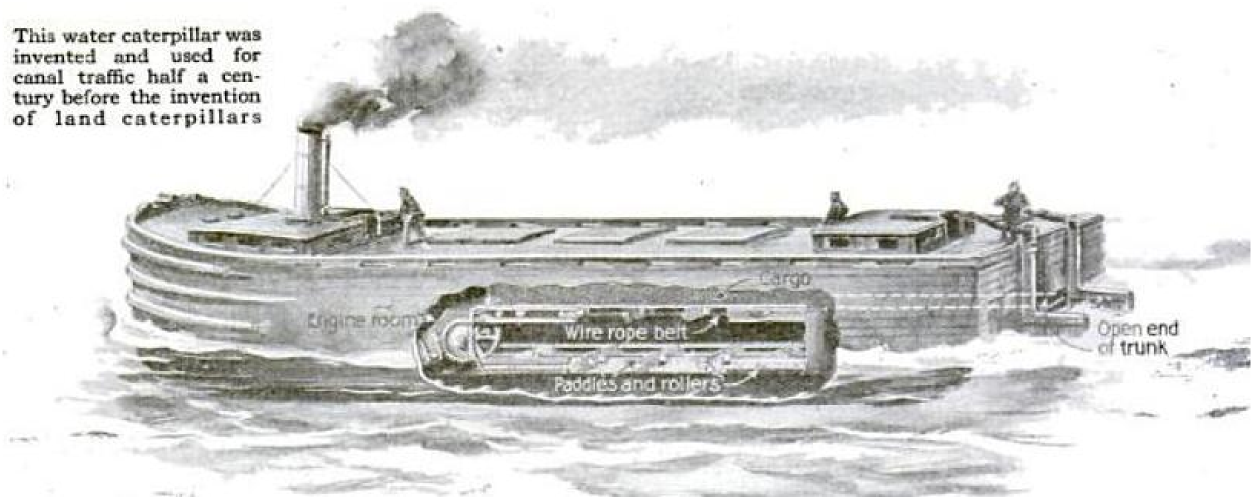

Figure 19: Classic Caterpillar Drive[18]

The Classic Caterpillar Drive (or "Water Caterpillar") was developed in 1782 in France and received a US patent in 1839.[19] It operated under the same principles as the feathered paddlewheel: the drag forces against the blades as they pass through the water provide the thrust which moves the boat. Unlike the paddlewheel, the water caterpillar required a belt. This was less efficient than its paddlewheel contemporaries and lost popularity.

Explored in this chapter is a modification of the water caterpillar. In this case, the blades are surrounded by a moat which prevents the water from passing around them. This means that the traditional drag coefficient method of calculating thrust is not useful here, since the drag coefficient

[18] Figure from "Marine Propulsion," Wikipedia
[19] Information from Ibid

assumes the water's freedom to move around objects. Rather, the resistive forces in play here will be the skin friction of the water moving past the walls of the moat. The modified caterpillar drive (or "caterpillar drive") is pictured below:

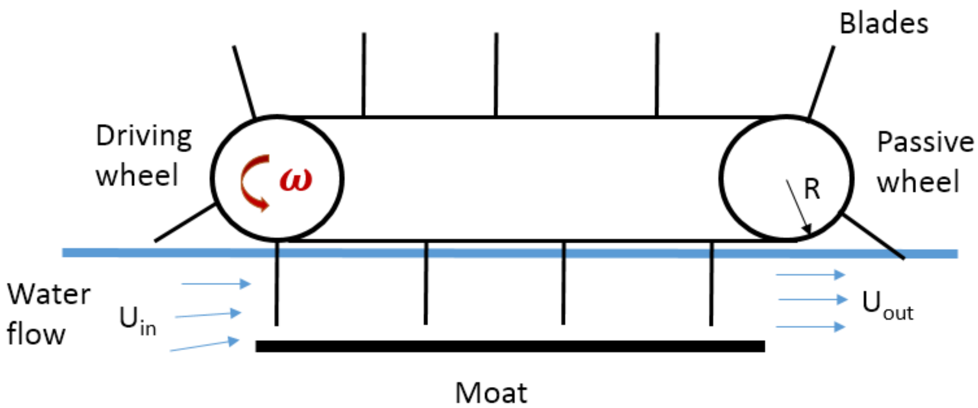

Figure 20: Caterpillar Drive

Assume the length of the moat is L, the width of the blades is W, and the height of each blade is H. Therefore the dimensions of the moat are approximately:

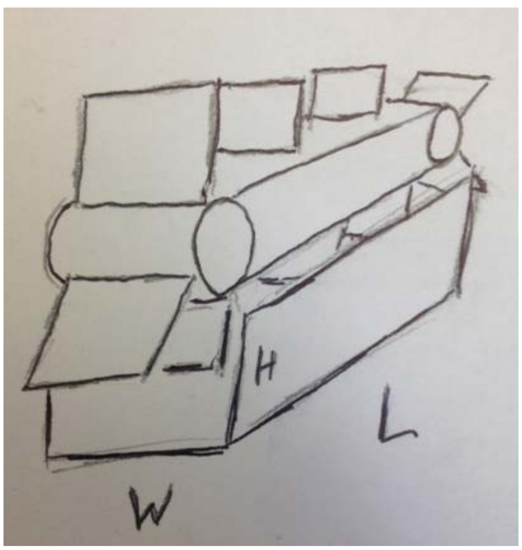

Figure 21: Caterpillar Drive Dimensions

In reality, there will be some clearance between the blades and the moat. However, this clearance will be small relative to the moat dimensions and can be ignored for purposes here. It will be made clear which dimensions are being used in the calculations so future calculations on prototypes/ designs can be made with more precise dimensions.

4.2: Mathematical Characterization

When deriving expressions for the power and thrust corresponding with the caterpillar drive, two factors will be taken into account.

The first is the momentum change of the water as it passes through the mechanism. There is a force required to maintain that momentum change in the steady state case, and that force will correspond to the thrust. The water molecules also experience a change of kinetic energy. To maintain that energy change for the constant influx of water, power is required. This is part of the power required by the mechanism.

The second is the resistive forces intrinsic to this mechanism. Inside the moat, skin friction drag will come into play as water is pushed through the moat. Outside the moat, skin friction drag will detract from the thrust. Lastly, the mechanism itself is a belt. Therefore power will be required to maintain a constant angular speed unless the belt is frictionless.

From this, expressions for thrust and power become apparent:

$$Thrust = [force\ corresponding\ to\ momentum\ change\ of\ water]$$
$$- [drag\ force\ on\ exterior\ face\ of\ moat]$$

$$Power = [power\ corresponding\ to\ changing\ kinetic\ energy\ of\ water]$$
$$+ [power\ to\ overcome\ skin\ friction\ on\ internal\ face\ of\ moat]$$
$$+ [power\ required\ to\ maintain\ system's\ rotation\ at\ constant\ \omega]$$

The expressions for each of these components are presented below. Further explanation can be found in Appendix B.

- The force corresponding to momentum change of water:

$$F_w = \dot{m}(V_{out} - V_{in})$$

Re-written in terms of the caterpillar drive parameters,

$$F_W = \rho \mathrm{R}\omega LH(\mathrm{R}\omega - V_b)$$

- The drag force on the moat exterior:

$$F_d = \frac{1}{2}\rho C_d V_b^2 A_{moat}$$

Re-written in terms of the caterpillar drive parameters,

$$F_d = \frac{1}{2}\rho C_d V_b^2 (2HL + WL)$$

- Power due to changing kinetic energy of the water:

$$P = \frac{1}{2}\dot{m}(V_{out}^2 - V_{in}^2)$$

Re-written in terms of the caterpillar drive parameters,

$$P_w = \frac{1}{2}\mathrm{R}\omega LH((\mathrm{R}\omega)^2 - V_b^2)$$

- Power to overcome skin friction drag on internal face of the moat:

The same formula for skin friction drag is used, multiplied by the relative speed:

$$P_d = (\frac{1}{2}\rho C_d V_{relative}^2 A_{moat})V_{relative}$$

Here, A_{moat} is the interior area of the moat walls. It is assumed to be the same as the outer wall area. The relative speed of the water is not V_b but ωR, which give the relative speed of the blades to the interior walls. The water is assumed to be traveling at the same speed as the blades. The equation becomes

$$P_d = \frac{1}{2}\rho C_d (\omega R)^3 (2HL + WL)$$

- Power required to maintain system's angular rotation:

$$P_{\omega} = k_c$$

Where k_c must be experimentally determined and are different for each mechanism.

When expanded, the equations for thrust and the power required to maintain ω are:

$$T = \rho \mathrm{R}\omega LH(\mathrm{R}\omega - V_b) - \frac{1}{2}\rho C_d V_b^2(2HL + WL)$$

$$P = \frac{1}{2}\mathrm{R}\omega LH((\mathrm{R}\omega)^2 - V_b^2) + \frac{1}{2}\rho C_d(\omega R)^3(2HL + WL) + k_c$$

The efficiency of the mechanism is the thrust power divided by input power:

$$\eta_c = \frac{V_b T}{P}$$

Since the term k_c must be experimentally determined, calculations presented here will assume that the mechanism is near frictionless for the same reasons as before: to give a sense of how high the efficiencies can be in theory while maintaining realistic efficiency curve shapes.

4.3: Efficiency for Different Tasks

To identify the tasks for which caterpillar drives are best suited, the following process will be used:

1. Identify a boat size and ideal cruise speed
2. Identify Reynold's Number for the system
3. Based (1) and (2), select values for R, L, H, W, and Cd
4. Calculate the efficiency for varying values of ω, beginning at $\omega = V_b/R$ and increasing the value from there.
5. When an ideal ω is identified, generate the efficiency vs boat speed curves using that value of ω

As before, this process will be repeated for three different boats: RC, canoe, and tugboat.

For simplicity, the same boats and cruise speeds as before will be used.

4.3.1: RC-Sized Caterpillar Drive

For an RC-sized boat, a cruise speed of 6.7 m/s is assumed.

The Reynold's number for a system is given by

$$Re = \frac{\rho V L}{\mu}$$

For water, the dynamic viscosity is $8.9 \times 10^{-4} Pa \cdot s$. Therefore Re for water is

$$Re = 1.124 \cdot 10^{6} \cdot VL$$

For the RC boat, V is around 7 m/s and L is about 0.5 m. Therefore for RC boats,

$$Re_{RC} = 4 \cdot 10^{6}$$

This implies turbulent flow about the moat. For turbulent flow, the skin friction drag coefficient is given by

$$C_{dT} = \frac{0.0594}{Re^{0.2}}$$

In this case, the following values will be used:

- $C_{dT} = 0.003$
- R = 0.05 m
- H = 0.1 m
- W = 0.15 m
- L = 0.16 m
- Starting ω = 137 s^{-1}

From this data, the relationship between efficiency and angular speed is determined by the formula

$$\eta_c = \frac{V_b[\rho \mathrm{R}\omega LH(\mathrm{R}\omega - V_b) - \frac{1}{2}\rho C_d V_b^2(2HL + WL)]}{\frac{1}{2}\mathrm{R}\omega LH((\mathrm{R}\omega)^2 - V_b^2) + \frac{1}{2}\rho C_d(\omega R)^3(2HL + WL) + k_c}$$

As described in the previous section. The assumption k_c=0.005 W was made. The resulting relationship between efficiency and angular speed becomes:

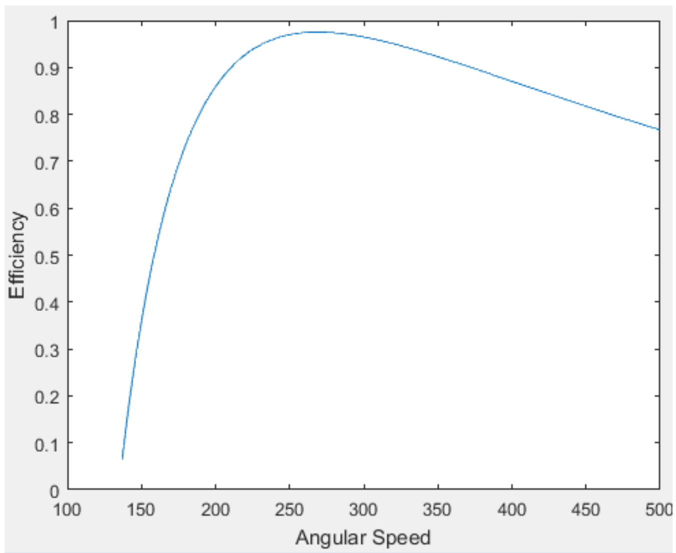

Figure 21: Efficiency vs Angular Speed, RC-Sized Caterpillar Drive

From this curve, the optimal efficiency is found at an angular speed of 254 s^{-1}. For the rest of the calculations, assume that the angular speed is 254 s^{-1}. The efficiency vs boat speed curve can be generated using the same formula but by changing the boat speed:

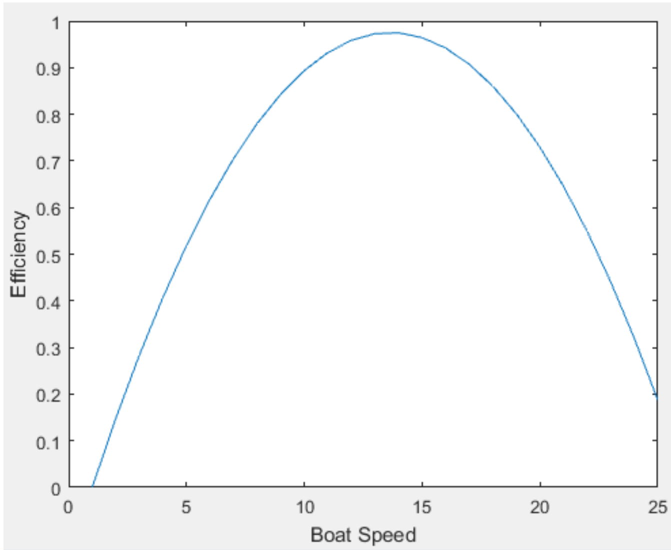

Figure 22: Efficiency vs Boat Speed, RC-Sized Caterpillar Drive

The MATLAB code used to generate these figures can be found in Appendix B.

4.3.2: Canoe-Sized Caterpillar Drive

For a canoe-sized boat, a cruise speed of 1.34 m/s is assumed.

For the canoe, L is about 4.5 m. Therefore for canoe-sized boats,

$$Re_{RC} = 6.8 \cdot 10^6$$

This implies turbulent flow about the moat. For turbulent flow, the skin friction drag coefficient is given by

$$C_{dT} = \frac{0.0594}{Re^{0.2}} = 0.0026$$

In this case, the following values will be used:

- C_{dT} = 0.0026
- R = 0.15 m
- H = 0.3 m
- W = 0.45 m
- L = 1.5 m

- Starting $\omega = 9\ s^{-1}$

Recall the same efficiency formula as used previously:

$$\eta_c = \frac{V_b[\rho R\omega LH(R\omega - V_b) - \frac{1}{2}\rho C_d V_b^2(2HL + WL)]}{\frac{1}{2}R\omega LH((R\omega)^2 - V_b^2) + \frac{1}{2}\rho C_d(\omega R)^3(2HL + WL) + k_c}$$

In this case, k_c was assumed to be $0.4\,W$. The relationship between angular speed and efficiency at cruise speed is:

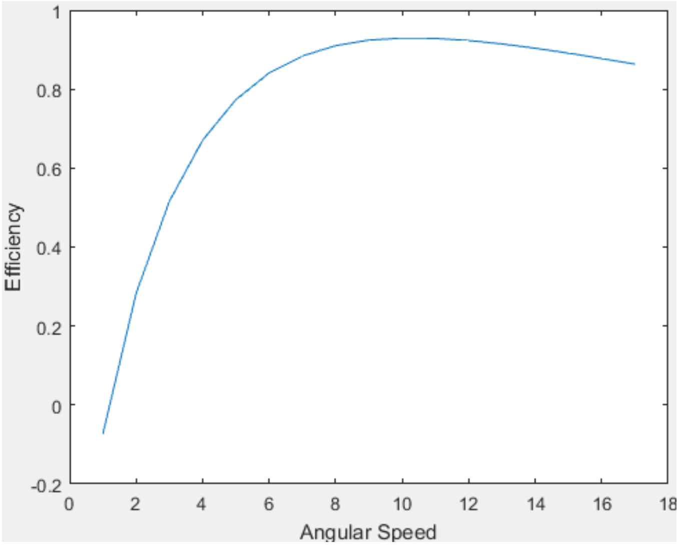

Figure 23: Efficiency vs Angular Speed, Canoe-Sized Caterpillar Drive

From this curve, the optimal angular speed is 10 s^{-1}. Assuming that the caterpillar drive rotates at this speed for different boat speeds, the efficiency vs boat speed graph can be generated:

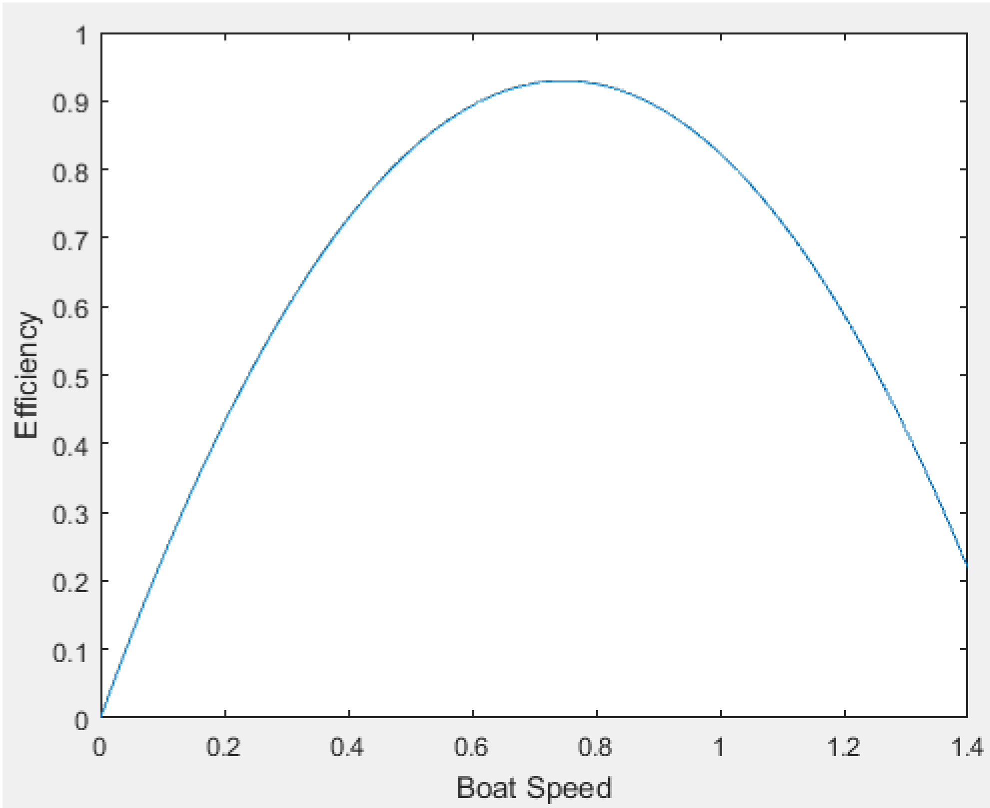

Figure 24: Efficiency vs Boat Speed, Canoe-Sized Caterpillar Drive

The code used to generate these figures can be found in Appendix B.

4.3.3: Tugboat-Sized Caterpillar Drive

For a tugboat-sized boat, a cruise speed of 6 m/s is assumed.

For the tugboat, L is about 10 m. Therefore for tugboats,

$$Re_{RC} = 6.7 \cdot 10^{7}$$

This implies turbulent flow about the moat. For turbulent flow, the skin friction drag coefficient is given by

$$C_{dT} = \frac{0.0594}{Re^{0.2}} = 0.0016$$

In this case, the following values will be used:

- $C_{dT} = 0.0016$
- $R = 1$ m

- $H = 2$ m
- $W = 3$ m
- $L = 10$ m
- Starting $\omega = 6$ s^{-1}

Recall the same efficiency formula as used previously:

$$\eta_c = \frac{V_b[\rho \mathrm{R}\omega LH(\mathrm{R}\omega - V_b) - \frac{1}{2}\rho C_d V_b^2(2HL + WL)]}{\frac{1}{2}\mathrm{R}\omega LH((\mathrm{R}\omega)^2 - V_b^2) + \frac{1}{2}\rho C_d(\omega R)^3(2HL + WL) + k_c}$$

In this case, k_c was assumed to be 5000 W. The relationship between angular speed and efficiency at cruise speed is:

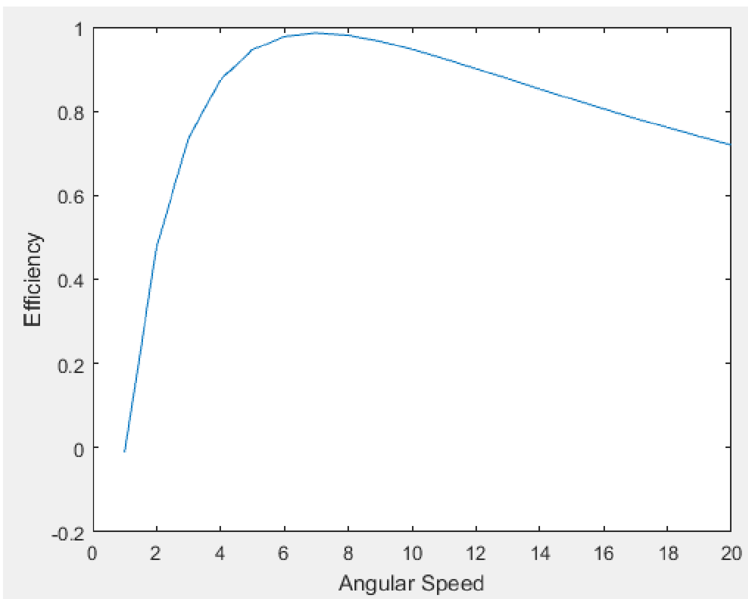

Figure 25: Efficiency vs Angular Speed, Tugboat-Sized Paddlewheel

From this curve, the optimal angular speed for this system is 7 s^{-1}. Assuming that the caterpillar drive rotates at this speed for different boat speeds, the efficiency vs boat speed graph can be generated as before:

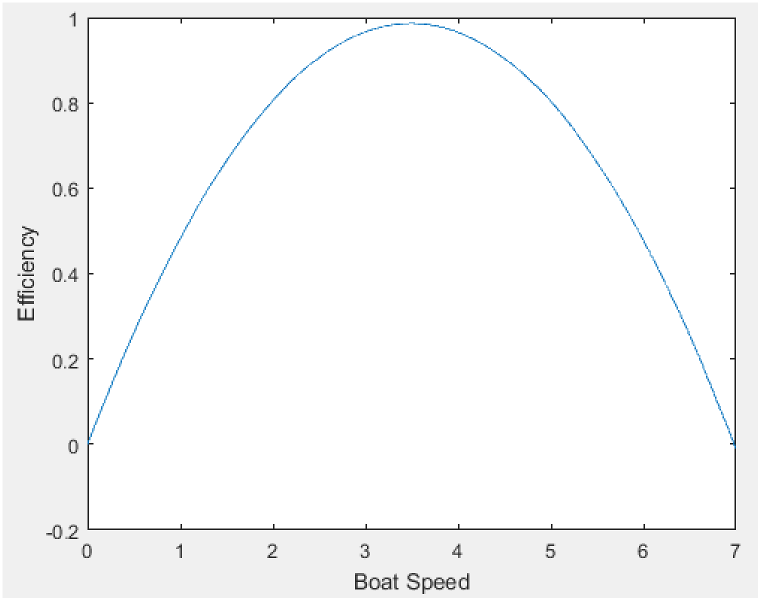

Figure 26: Efficiency vs Boat Speed, Tugboat-Sized Caterpillar Drive

The code used to generate these figures can be found in Appendix B. Note that the curve here does not actually go below zero.

4.4: Comparison to Ideal Propellers

The graphs in the previous section suggest that the efficiency of ideal caterpillar drives is approximately 1 at ideal cruise speeds. However, this was based on arbitrary assumptions about what the internal loss coefficients k_c are for different mechanisms. Therefore to provide a more meaningful comparison with ideal propellers, the following process will be used:

5. Determine the thrust generated by the caterpillar drive to perform the task
6. Assume the propeller has to generate that thrust, and use propeller specs which are standard for that task
7. Use the Kramer diagram to determine the efficiency of ideal propellers performing that task

8. Going back to the MATLAB code used to generate the efficiency vs angular speed curves, determine the coefficient k_c required to match the efficiency of ideal propellers determined from (3)

4.4.1: Comparison with ideal RC Propellers

The RC-sized boat was generating a thrust of about 120 N at a boat speed of 6.7 m/s and angular speed of 254 s^{-1}. This thrust force might seem high, but realize that the RC boat is traveling almost 7 m/s and the specs were based off boats which were 0.5 meters long. The thrust can be easily calculated by using the thrust equation generated in section 4.2 and the specs from section 4.3.1:

$$T = \rho \mathrm{R}\omega LH(\mathrm{R}\omega - V_b) - \frac{1}{2}\rho C_d V_b^2(2HL + WL)$$

For a 3-blade, 4-inch diameter propeller rotating at 9000 RPM[20], the Kramer diagram efficiency comes out to about 70% efficient.

So at what point does the caterpillar drive beat the propeller for this RC example?

Recall that the term k_c was arbitrarily selected, and the efficiency was close to 1 at cruise speed. Cruise speed efficiencies of 70% are generated when k_c is $1.1 \cdot 10^4$ W. This number seems very large – it is big because the angular speed of the mechanism is very high. It may be that the numbers selected here were not realistic. Nonetheless, the process for determining it still holds; the value for k_c can easily be determined by changing the value of k_c in the MATLAB code until the curve generates a maximum efficiency of 70%. Therefore to beat the ideal propeller, a caterpillar drive with the design specs from section 4.3.1 and a k_c of $1.1 \cdot 10^4$ W or less should perform better than any propeller attempting the same task.

[20] Specs informed by Preece and RC Boat Calculator

4.4.2: Comparison with Ideal Canoe Propellers

The canoe-sized caterpillar drive was generating a thrust of 84 N according to the thrust equation. The boat speed used was 1.34 m/s and the angular speed was 10 s^{-1}. Using the medium-size boat from section 3.4, the efficiency of ideal propellers is about 90%.

As in the previous section, the term k_c was arbitrarily selected, and the efficiency was close to 1 at cruise speed. Cruise speed efficiencies of 90% are generated when k_c is about 420 W. This can be determined in a similar manner. Therefore to beat the ideal propeller, a caterpillar drive with the design specs from section 4.3.2 and a k_c of 420 W or less should perform better than propellers attempting a similar task.

4.4.3: Comparison with Ideal Tugboat Propellers

The tugboat-sized caterpillar drive was generating a thrust of 140,000 N according to the thrust equation. The boat speed used was 6 m/s and the angular speed was 7 s^{-1}. Typical tugboat specs are 2.4 m diameter, 4-blade propellers rotating at 24 radians per second.[21] From the Kramer diagram, propellers with these specs providing a thrust of 140,000 N are about 80% efficient.

As in the previous sections, the k_c term was arbitrarily selected, and the efficiency was close to 1 at cruise speed. Cruise speed efficiencies of 80% are generated when k_c is about $2.1 \cdot 10^6$ W. This can be determined in a similar manner. Therefore to beat the ideal propeller, a caterpillar drive with the design specs from section 4.3.3 and a k_c of $2.1 \cdot 10^6$ W or less should perform better than propellers attempting a similar task. As in section 4.4.1, this number is large because of the high angular speed of the mechanism. It may be that the dimensions selected here are not realistic.

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[21] Informed by Hoffman and Argyriadis

4.5: Caterpillar Drive Discussion and Recommendations

In this chapter, a first-principles approach to modified caterpillar drives was developed. Expressions for the thrust generated and power required to maintain some constant angular rotation were developed. These equations were used to determine optimal angular speed for different tasks. From those optimal angular speeds, efficiency vs boat speed curves were generated. For different-sized boats, the values for internal friction loss coefficients were determined to beat the ideal propeller. The comparison of these coefficients against the feathered paddlewheel will be discussed in Chapter 6.

From these results, the following recommendations can be made:

- For theorists:
    - Use CFD to predict the thrust required to maintain a boat's cruise speed. This can be used to more precisely predict the performance of ideal propellers and therefore allow more precise calculations of the k_c required by caterpillar drives to out-perform ideal propellers
    - Since water is incompressible, the water rushing into the system will come from a larger area than HW (since $A_1V_1 = A_2V_2$). The effects of water rushing in from a larger area are not taken into account here. CFD may be able to inform theorists of the effects this "rushing in" has on the system's efficiency.
- For experimentalists and engineers:
    - Determine the internal friction coefficient k_c for different caterpillar drive designs in order to make more accurate predictions of the system's performance
    - The internal friction coefficient will be larger for bigger and faster systems, so the recommendation of the author is to use caterpillar drives for slower systems.

Chapter 5: Jellyfish Actuator

5.1: Mechanism Description

The jellyfish actuator was inspired by the marine animal, which moves with a two-stage cycle:

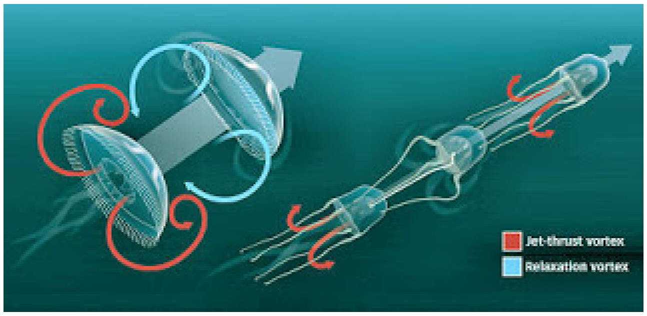

Figure 27: Jellyfish Propulsion[22]

The intake cycle involves the expansion of the jellyfish cavity. A volume of water rushes in, and in the thrust cycle the cavity contracts, shooting a water jet out of the cavity. This pushes the animal forward in a very efficient manner.[23]Because of its success, an actuator mechanism can be proposed which represents a variant of the jellyfish mechanism.

The alternate mechanism consists of an actuator pushing two flaps. The flaps begin close to the body in an open position. They are thrust in the opposite direction of locomotion. At the end of the thrust, the flaps collapse and return to the starting point. The following graphic illustrates the process:

[22] Image from Margaret
[23] Fischman

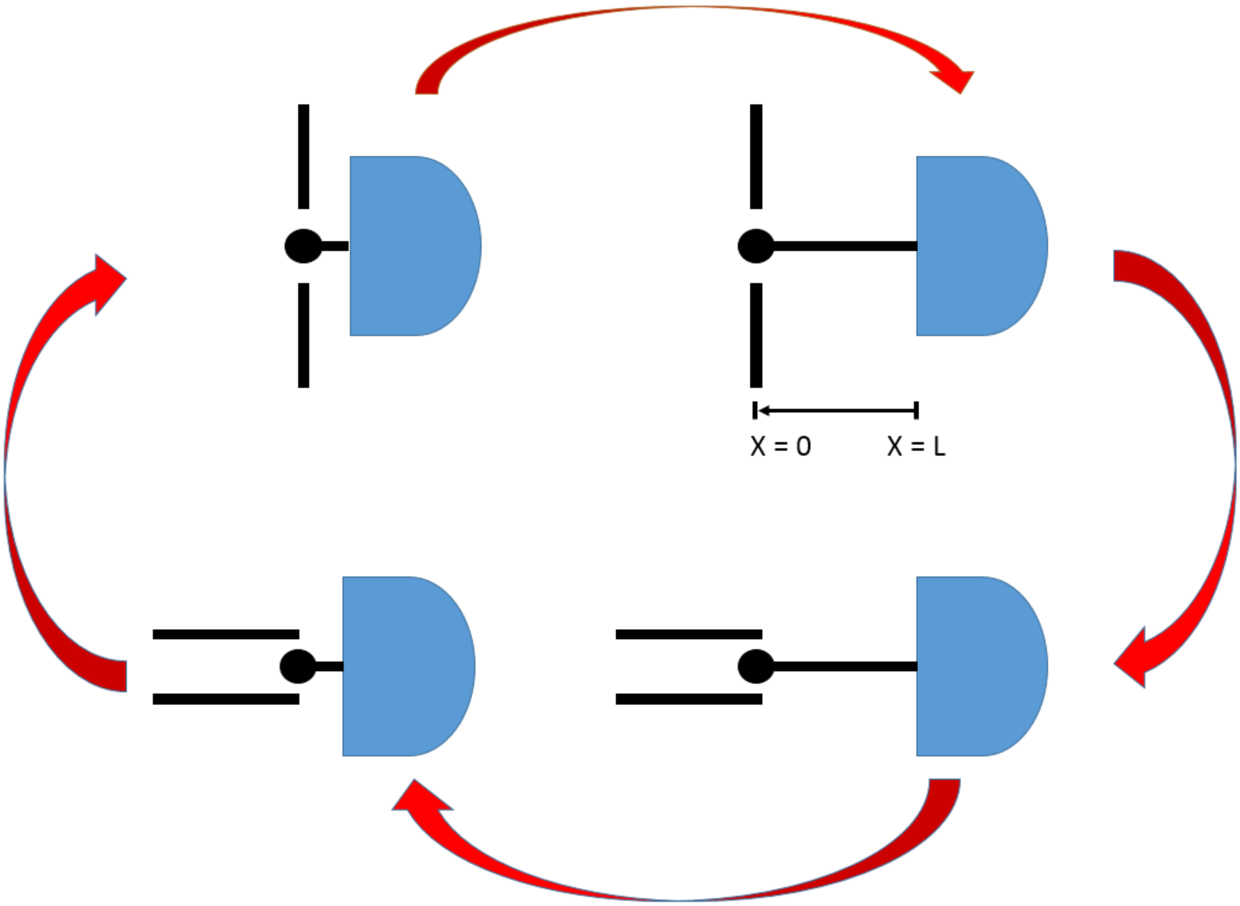

Figure 28: The Jellyfish Actuator

The width of the flaps is W, and the height is H:

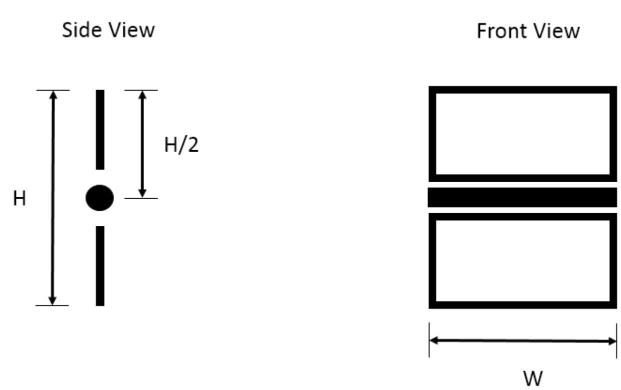

Figure 29: Jellyfish Actuator Flap Dimensions

Assume that the entire mechanism is moving at V_b, to keep the notation that was used for the other mechanisms.

The flaps are thrust at a speed V_E relative to the mechanism and are retracted at speed V_C relative to the mechanism:

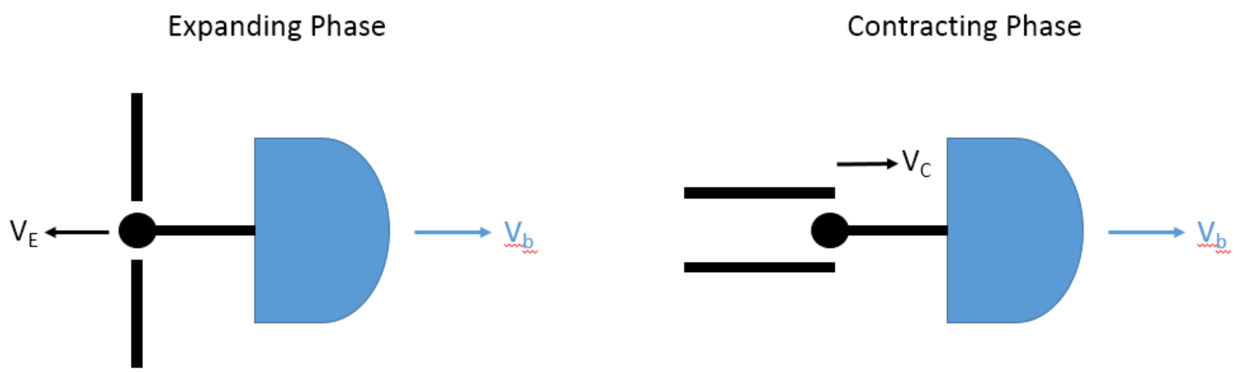

Figure 30: Expanding and Contracting Phases of Jellyfish Actuator

For simplicity, no mechanisms by which (a) the flaps are expanded and contracted and (b) the flaps are opened and closed are not specified. It is assumed that the flaps are opened at t=0, are thrust at a constant speed V_E until time t = L/V_E. The flaps are then closed instantly and are contracted at constant speed V_C. The flaps reach their original point at time t = $L/V_E + L/V_C$ and open instantly, repeating the cycle. These assumptions are being made to develop a baseline mathematics that can be used to analyze the mechanism. Specific designs can build off the mathematical framework outlined in the next section.

5.2: Mathematical Characterization

The drag force acting on an object is given by the same equation used in previous sections:

$$F_d = \frac{1}{2}\rho V^2 C_d A_c$$

Where V is the speed of the object relative to the surrounding fluid, C_d is the drag coefficient of the object, and A_c is the characteristic area of the object.

For the expanding phase of the jellyfish actuator, the drag force acting on the flaps is

$$F_E = \frac{1}{2}\rho(V_E - V_b)^2 C_E W H$$

In this case, C_E is the drag coefficient associated with flow moving normal to a flat surface. As used in previous sections, this coefficient is $C_E = 1.28$. Again, W and H are the width and height of the flaps, respectively. This drag force will provide the thrust which pushes against the mechanism.

The power required to maintain this force is

$$P_E = F_E V_E$$

By the same logic, the force acting on the flaps during the contracting phase is

$$F_C = \frac{1}{2}\rho(V_C + V_b)^2 C_C W H$$

Here, C_C is the drag coefficient associated with flow parallel to a flat surface. For low Reynold's numbers, this is approximately 0.001[24] (the reason for assuming low Reynold's numbers will be discussed in the next section). The characteristic area WH is used instead of $\frac{1}{2}$WH because water is flowing over both sides of the half-flaps.

Notice that both F_E and F_C are dependent on V_b. Unlike the previous mechanisms, there is only one effective "blade" providing periodic thrust. In the other mechanisms, when one blade finished providing its thrust for the cycle, another blade or two were already providing thrust behind it. That is not the case here, and V_b will oscillate in a more pronounced manner than in the paddlewheel case. Therefore V_b cannot be assumed constant throughout the course of the cycle.

To accommodate the shifting V_b, a force balance must be made on the entire system:

[24] "Drag Coefficient," The Engineering ToolBox

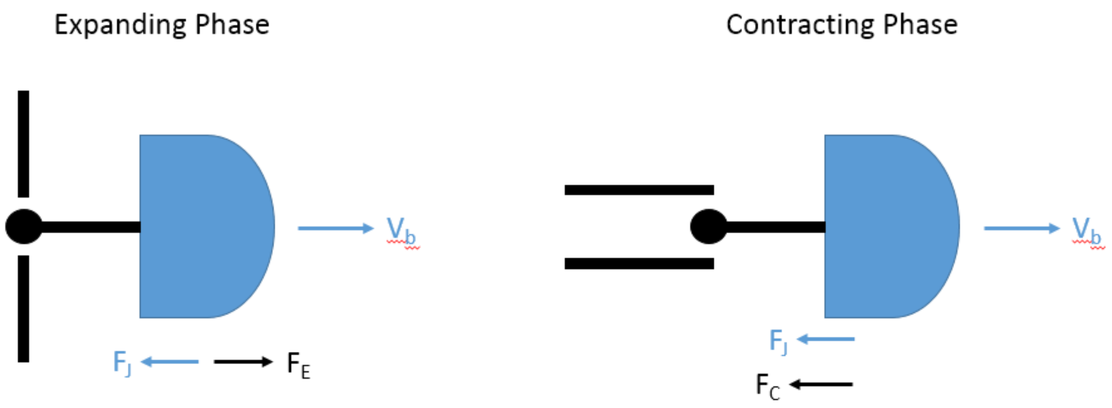

Figure 31: Forces on Jellyfish Actuator System

Again, F_E and F_C are the drag forces associated with moving the flaps through the water in the expanding and contracting phases, respectively. F_J is the drag force on the jellyfish body, and is given by

$$F_J = \frac{1}{2}\rho V_b^2 C_J A_J$$

Here, C_J is the drag coefficient of the jellyfish mechanism and A_J is its characteristic area. This equation holds for both the expanding and contracting phases.

Assuming the mass of the jellyfish mechanism is m_J, the force balance can be written mathematically as:

Expanding Phase:

$$m_J \dot{V}_b = F_E - F_J$$

Contracting Phase:

$$m_J \dot{V}_b = -F_C - F_J$$

Using the expressions obtained earlier in the section and substituting them into the expressions above, they reduce to

Expanding Phase:

$$\dot{V}_b = V_b^2\left[\frac{1}{2m_J}\rho\left(C_E WH - C_J A_J\right)\right] - V_b\left[\frac{\rho C_E WHV_E}{m_J}\right] + \frac{\rho V_E^2 C_E WH}{2m_J}$$

Contracting Phase:

$$\dot{V}_b = V_b^2\left[\frac{1}{2m_J}\rho\left(C_C WH + C_J A_J\right)\right] + V_b\left[\frac{\rho C_C WHV_C}{m_J}\right] + \frac{\rho V_C^2 C_C WH}{2m_J}$$

The derivations of these expressions can be found in Appendix C.

Notice that these are nonlinear differential equations of the form

$$\dot{V}_b = AV_b^2 - BV_b + C$$

and

$$\dot{V}_b = -DV_b^2 - EV_b - F$$

For the expanding and retracting phases, respectively, where

$$A = \frac{1}{2m_J}\rho\left(C_E WH - C_J A_J\right)$$

$$B = \frac{\rho C_E WHV_E}{m_J}$$

$$C = \frac{\rho V_E^2 C_E WH}{2m_J}$$

$$D = \frac{1}{2m_J}\rho\left(C_C WH + C_J A_J\right)$$

$$E = \frac{\rho C_C WHV_C}{m_J}$$

$$F = \frac{\rho V_C^2 C_C WH}{2m_J}$$

The next step is to use this information to calculate efficiency. The first step is to select parameters which give the values of A through F. The differential equations can then be solved.

As time goes on, an average boat speed (V_{bA}) becomes apparent. The efficiency is the thrust power required to maintain that speed divided by the power provided by the average power in a stroke:

$$\eta_J = \frac{V_{bA} F_{JA}}{F_E V_E}$$

Where F_{JA} is the drag force F_J required to pull the mechanism through the water, using the average boat speed V_{bA} instead of V_b. Substituting the formulas from above, this equation becomes:

$$\eta_J = \frac{V_{bA}[\frac{1}{2}\rho V_{bA}^2 C_J A_J]}{[\frac{1}{2}\rho (V_E - V_b)^2 C_E W H] V_E}$$

It simplifies to

$$\eta_J = \frac{V_{bA}^3 C_J A_J}{(V_E - V_{bA})^2 C_E W H V_E}$$

5.3: Efficiency for Different Tasks

It is clear that this mechanism will not be practical for large boats. Imagine a single jellyfish actuator powering a large boat. Its flaps will have to be large. Therefore opening and closing them will displace lots of water and require significant amounts of power, affecting the calculations. If the flaps are to be opened and closed passively, then large flaps will require large displacements to open – they will not be fully open (if at all) until the end of the stroke. It's conceivable that many small jellyfish actuators can be used to power larger boats, but that would not change the calculations of the individual, small ones. Since the only practical jellyfish actuators are small ones, only small ones will be analyzed here. This is why the Reynold's numbers used to calculate drag coefficients, since the speed and size of the system will always be relatively low.

In this chapter, three jellyfish actuators of the same size but different speeds will be analyzed. This could give a sense of whether they are more efficient for faster or slower mechanisms.

The methodology used here for determining the efficiency of jellyfish actuators is:

1. Select values for m_J, A_J, W, H, L, V_E, and V_C
2. Solve the differential equations to determine the speed of the mechanism as a function of time
3. As time goes on, the speed will oscillate about some average speed. Use that average speed as V_{bA} in the efficiency formula for η_J presented in the previous section
4. Repeat this process for different values of V_E and V_C to get a sense of how the mechanism performs at different speeds.

5.3.1: Slow Jellyfish Actuator

In all three cases, the mechanism will be approximately the size of a box jellyfish. The mass m_J is 2 kg and its characteristic area A_J is 0.1 m^2.[25] The "boat" here is assumed to be a half-sphere, which has a drag coefficient C_J of 0.42.[26]

For a slow jellyfish actuator will have a cycle time of 2 seconds. Assuming L is 0.25 m and V_E=V_C, both V_E and V_C are 0.25 m/s. Assume W = H = 0.3 m.

Recall the differential equations describing the system:

Expanding:

$$\frac{dV_b}{dt} = AV_b^2 - By + C$$

[25] Specs taken from "Box Jellyfish." Wikipedia
[26] "Drag Coefficient," The Engineering ToolBox

Contracting:

$$\frac{dV_b}{dt} = -DV_b^2 - Ey - F$$

Where

$$A = \frac{1}{2m_J}\rho\left(C_E WH - C_J A_J\right) \qquad D = \frac{1}{2m_J}\rho\left(C_C WH + C_J A_J\right)$$

$$B = \frac{\rho C_E WHV_E}{m_J} \qquad E = \frac{\rho C_C WHV_C}{m_J}$$

$$C = \frac{\rho V_E^2 C_E WH}{2m_J} \qquad F = \frac{\rho V_C^2 C_C WH}{2m_J}$$

Using the values listed above, the equations become

Expanding (from time 0 < t < 1, 2 < t < 3, 4 < t < 5, etc.)

$$\frac{dV_b}{dt} = 18.3V_b^2 - 14.4y + 1.8$$

Contracting (from time 1 < t < 2, 3 < t < 4, etc.)

$$\frac{dV_b}{dt} = -10.5525V_b^2 - 0.1125y - 0.00140625$$

The exact values for the coefficients were used to obtain an exact numerical solution.

These differential equations were solved using SIMULINK. The block diagram can be found in Appendix C. The results are graphed below:

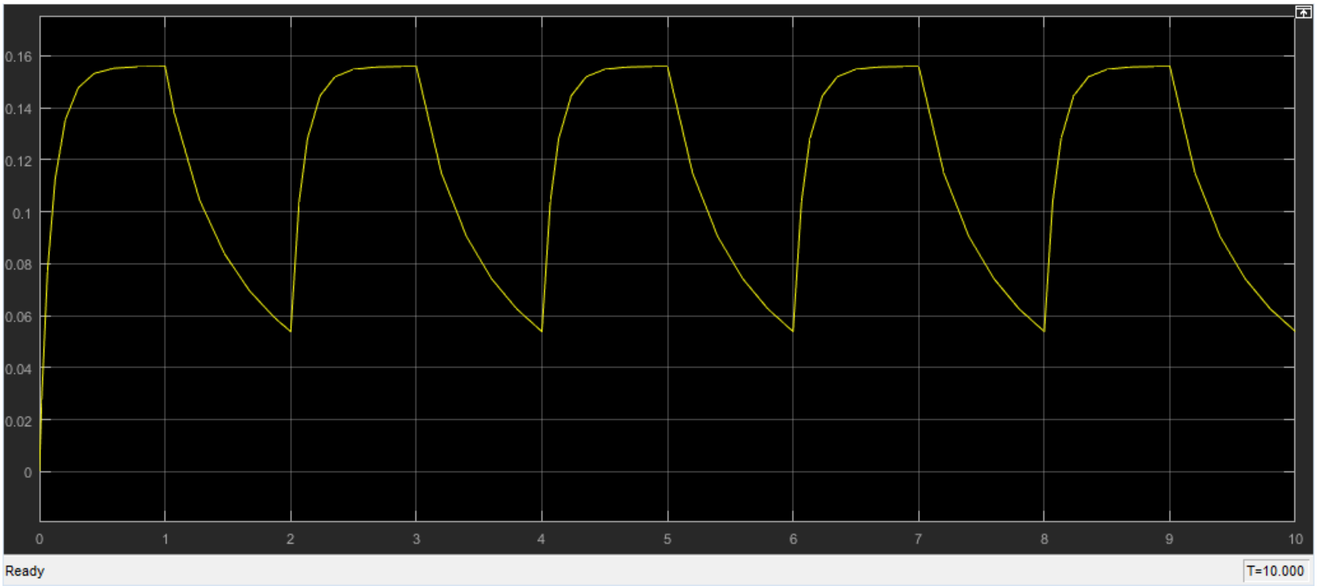

Figure 32: Slow Jellyfish V_b vs time graph, 0-10 Seconds, $V_b(0) = 0$

Interestingly, the jellyfish reaches its peak speed (about 0.16 m/s) at the end of the first stroke. The signal statistics were generated from SIMULINK:

Signal Statistics

	Value	Time
Max	1.559e-01	5.000
Min	0.000e+00	0.000e+00
Peak to Peak	1.559e-01	
Mean	1.124e-01	
Median	1.236e-01	
RMS	1.210e-01	

From the statistics, the mean V_{bA} was 0.1124 m/s. Therefore the efficiency of the mechanism is

$$\eta_J = \frac{V_{bA}^3 C_J A_J}{(V_E - V_{bA})^2 C_E W H V_E} = 10.9\%$$

This is not very efficient. Although that is to be expected: for half the cycle, power is expended to retract the flaps. This is significant power which is not providing thrust.

5.3.2: Medium-Speed Jellyfish Actuator

For the medium-speed actuator, the entire expansion/ contraction cycle takes 1 second instead of 2. Therefore V_E and V_C are 0.5 m/s instead of 0.25 m/s. All of the other parameters stay the same. Using the same formulas, the equations become

Expanding (from time 0 < t < .5, 1 < t < 1.5, 2 < t < 2.5, etc.)

$$\frac{dV_b}{dt} = 18.3V_b^2 - 28.8y + 7.2$$

Contracting (from time 0.5 < t < 1, 1.5 < t < 2, etc.)

$$\frac{dV_b}{dt} = -10.5525V_b^2 - 0.0225y - 0.0056$$

These differential equations were solved using SIMULINK. The results are graphed below:

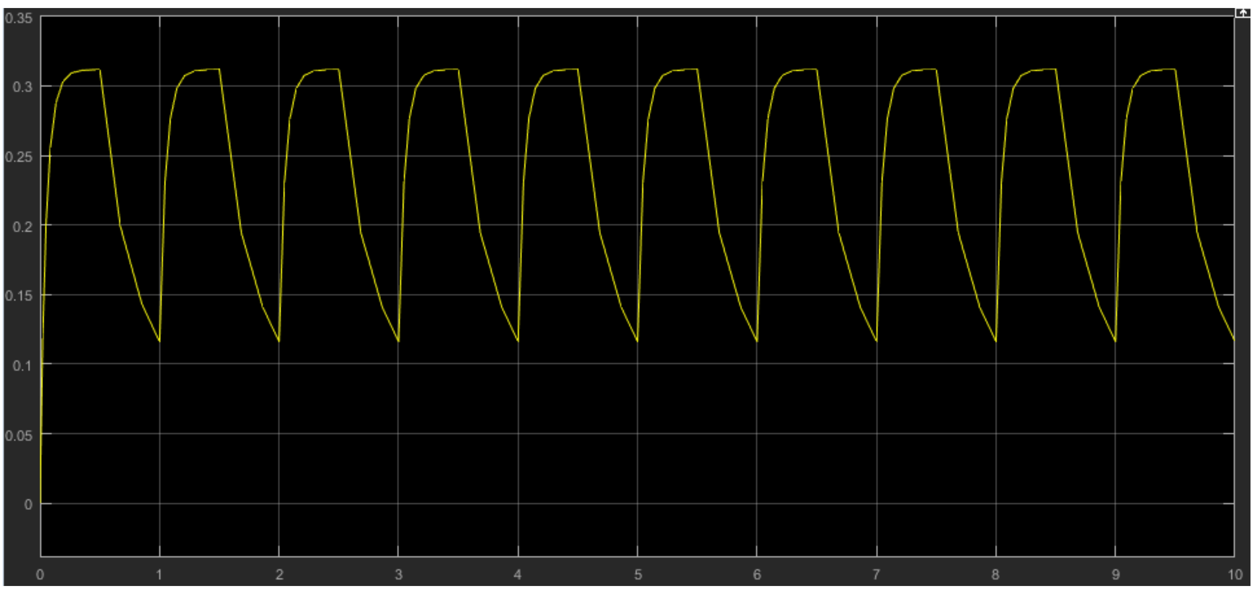

Figure 33: Medium-Speed Jellyfish V_b vs time graph, 0-10 Seconds, $V_b(0) = 0$

Once again, the jellyfish reaches its peak speed on the first thrust. This time the peak speed is about 0.3 m/s. The graph's statistics are:

Signal Statistics

	Value	Time
Max	3.117e-01	9.500
Min	0.000e+00	0.000e+00
Peak to Peak	3.117e-01	
Mean	2.374e-01	
Median	2.766e-01	
RMS	2.527e-01	

Therefore V_{bA} for this system is 0.2374 m/s. Using the same formula to calculate efficiency:

$$\eta_J = \frac{V_{bA}^3 C_J A_J}{(V_E - V_{bA})^2 C_E W H V_E} = 14.2\%$$

This is better than the slower case but is still bad.

5.3.3: Fast Jellyfish Actuator

For the high-speed actuator, the entire expansion/ contraction cycle takes 0.5 seconds. Therefore V_E and V_C are 1 m/s. All of the other parameters stay the same. Using the same formulas, the equations become

Expanding (from time 0 < t < .25, 0.5 < t < 0.75, etc.)

$$\frac{dV_b}{dt} = 18.3V_b^2 - 57.6y + 28.8$$

Contracting (from time 0.25 < t < 0.5, 0.75 < t < 1, etc.)

$$\frac{dV_b}{dt} = -10.5525V_b^2 - 0.045y - 0.0225$$

These differential equations were solved using SIMULINK. The results are graphed below:

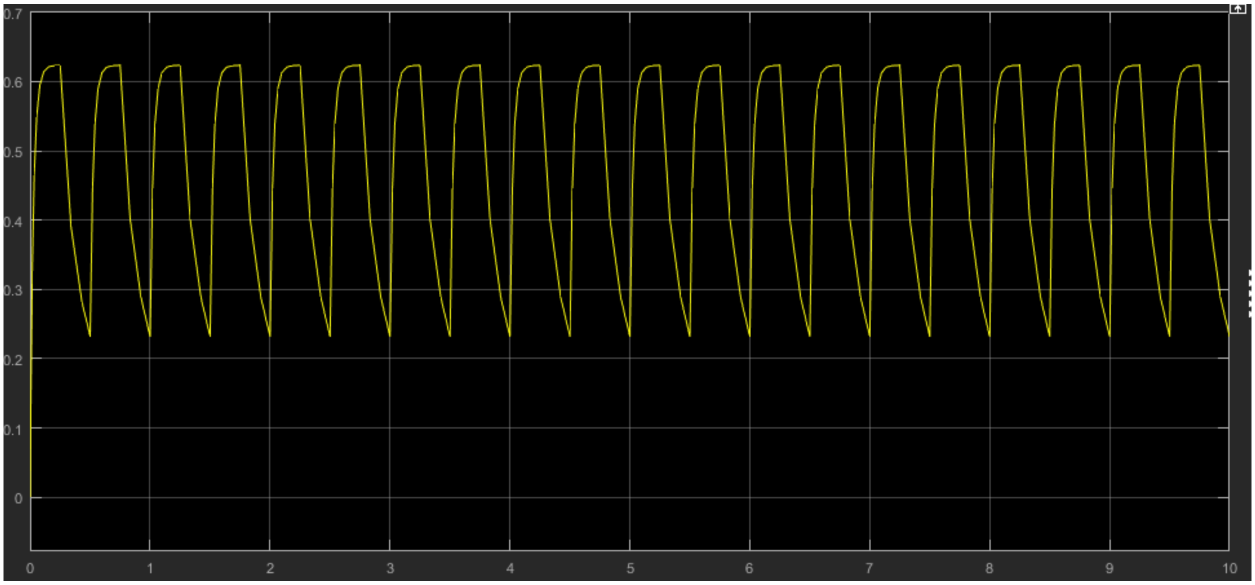

Figure 34: Fast-Speed Jellyfish V_b vs time graph, 0-10 Seconds, $V_b(0) = 0$

Once again, the jellyfish reaches its peak speed on the first thrust. This time, the peak speed is 0.6 m/s. The graph's statistics are:

Signal Statistics

	Value	Time
Max	6.234e-01	0.250
Min	0.000e+00	0.000e+00
Peak to Peak	6.234e-01	
Mean	4.834e-01	
Median	5.389e-01	
RMS	5.089e-01	

Therefore V_{bA} for this system is 0.4834 m/s. Using the same formula to calculate efficiency:

$$\eta_J = \frac{V_{bA}^3 C_J A_J}{(V_E - V_{bA})^2 C_E W H V_E} = 9.7\%$$

This is worse than either of the two previous cases.

5.3.4: Medium Speed Jellyfish Actuator, Fast Contraction Speed

In the previous cases, half of the cycle was spent retracting the flaps. What if less time was spent on retracting the flaps? Therefore most of the cycle is spent providing thrust.

Assume that the total cycle takes 0.67 seconds, the expanding speed V_E = 0.5 m/s and the contracting speed V_C = 1.5 m/s. Using the same formulas as before, the equations of motion become

Expanding:

$$\frac{dV_b}{dt} = 18.3V_b^2 - 28.8y + 7.2$$

Contracting:

$$\frac{dV_b}{dt} = -10.5525V_b^2 - 0.0675y - 0.0506$$

In the SIMULINK pulse generator, set the period to 0.6667 seconds and the pulse width to 75% of the period. The results are:

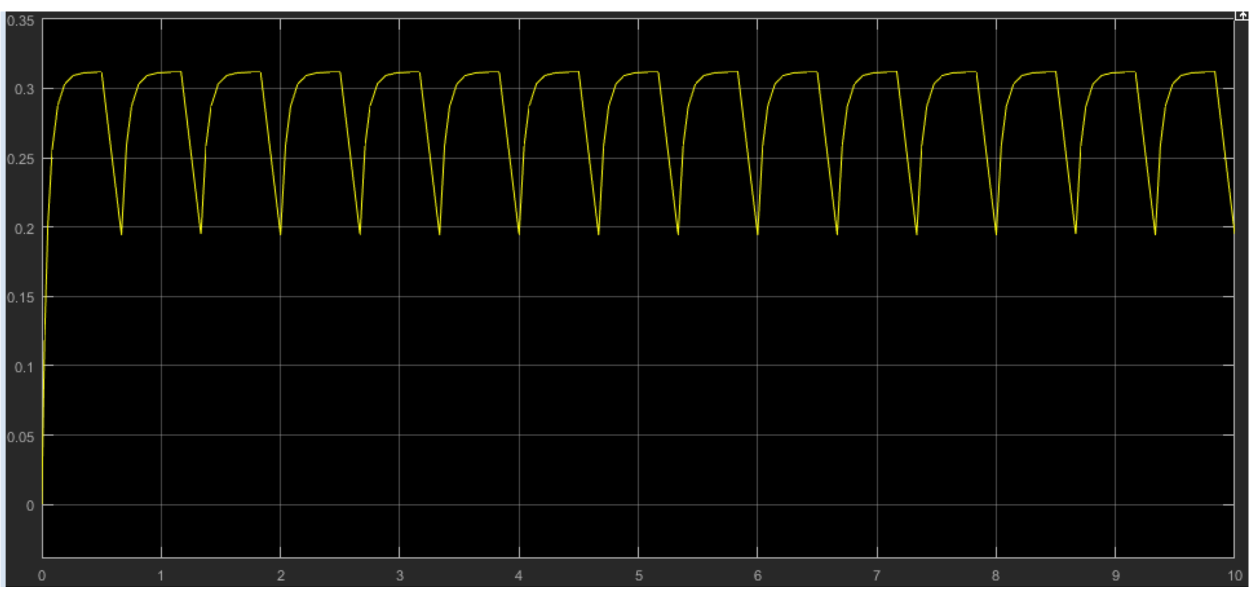

Figure 34: Fast-Contraction Jellyfish V_b vs time graph, 0-10 Seconds, $V_b(0) = 0$

The graph statistics are

Signal Statistics

	Value	Time
Max	3.117e-01	3.167
Min	0.000e+00	0.000e+00
Peak to Peak	3.117e-01	
Mean	2.730e-01	
Median	3.032e-01	
RMS	2.812e-01	

Here the mean speed is 0.273 m/s. Therefore the efficiency becomes

$$\eta_J = \frac{V_{bA}^3 C_J A_J}{(V_E - V_{bA})^2 C_E W H V_E} = 28.8\%$$

This is still low, but much more efficient than the other scenarios.

5.4: Comparison to Ideal Propellers

There is a lot of available data on propeller performances for RC boats, canoe-sized boats, and tugboats. Unfortunately, there is not a lot of data on propellers used to move box jellyfish. Therefore the propeller specs will be determined by an educated guess and a range of efficiencies will be given.

For a slow box jellyfish, the average speed V_{bA} was 0.1124 m/s. The drag coefficient was 0.42, and since its diameter was about 0.35 m, assume a propeller diameter of 0.18 m (propeller area 0.25 m^2). If the propeller is spinning between 10 and 50 radians per second, the Kramer diagram gives ideal propeller efficiencies between 96% and 94%.

For a medium-speed box jellyfish, the average speed V_{bA} was 0.2374 m/s. Using the same propeller specs except the propeller is rotating between 50 and 100 s^{-1}, the Kramer efficiency is also between 94% and 96%.

The fast box jellyfish has an average speed of 0.4384 m/s. Using the same propeller specs except the propeller is rotating between 50 and 100 s^{-1}, the Kramer efficiency is also between 94% and 96%.

In all cases, ideal propellers used by jellyfish-sized mechanisms are very efficient. The data from earlier in the chapter does not indicate that jellyfish actuators are capable of matching propellers. More computational work should be done to identify the upper performance limits of jellyfish actuators.

5.5: Jellyfish Actuator Discussion and Recommendations

There are two major differences in the analysis of this mechanism. First, the mechanical design of the system was not laid out in detail. Therefore no internal friction terms were considered. Second, the drag of the boat played a role in the efficiency of the mechanism itself. The other mechanisms were considered in isolation: "assume the boat is going at this speed, what will be the thrust produced and the power required to maintain that thrust?" Since the speed of this jellyfish "boat" is not close to constant, that approach was not meaningful here. That distinction gives rise to methodological differences.

For the other mechanisms, it was easy to find the optimal performance given a design. That was because a task could be assumed, and it could be asked "what rotational speed optimizes this paddlewheel or caterpillar drive at that boat speed?"

That is not the case here because a task cannot be assumed. Since the speed of the boat oscillates significantly at different stages of the actuator's cycle, one cannot ask "what speeds V_C and V_E work best for this V_b?" because V_b depends on V_E and V_C. That would have to be determined by guess-and-check. This conundrum was avoided in the other mechanisms because the relationship between efficiency and boat speed was not buried in a pair of nonlinear differential equations.

From this chapter, the following recommendations are made:

- For theorists: attempt to determine the upper performance limits of jellyfish actuators by examining the relationship between V_E, V_C, and V_b for constant geometries. Once a relationship becomes apparent, repeat that process for different flap sizes to determine optimal geometries.
- For theorists: propose mechanisms by which flaps are extended/ contracted and opened/ closed. From those assumptions, analytical and computational methods should give a better sense of actual jellyfish actuator performance.
- For experimentalists: if jellyfish actuators will be useful, they will only be useful in small, slow mechanisms. This is because the flaps will have to open and close, and doing that for larger flaps will prove difficult.
- For experimentalists: the results here show that the best jellyfish mechanism contract at a higher rate than they expand.
- For experimentalists: the initial results here give low efficiencies. It is possible that this is not the optimal design, and that this design does not mimic actual jellyfish. Jellyfish fill a cavity which they contract and shoot out a water jet. This mechanism merely pushed a flap through the water. It may be worth experimenting with designs that better mimic actual jellyfish, since their method of propulsion has proven successful.

Chapter 6: Conclusion

This paper accomplished the following:

1. A first-principles analysis of feathered paddlewheels, caterpillar drives, and jellyfish actuators. Equations determining the thrust generated by the mechanism and the power required to maintain that thrust were derived, and those equations were used to determine efficiency.
2. The efficiency of these mechanisms were determined for several boat speeds and thrust outputs. This information gave a sense of which tasks the mechanisms are best suited. In the case of jellyfish actuators, the actuator could be performing the same task in a variety of ways (by adjusting expansion and contraction rates). This added a level of complexity to the problem, so instead of identifying ideal tasks, ideal ways of performing the task were identified. The numerical values for best performance behaviors were not determined. This is because the question "what is the best jellyfish actuator" is not a reasonable question – a more reasonable question is "what is the best jellyfish actuator for this task." The same can be said for caterpillar drives and paddlewheels. Therefore the emphasis was placed on methodology – on using the equations from (1) to determine best performances for a task.
3. For the variety of tasks analyzed, the idealized mechanisms were compared against ideal propellers. In the case of the caterpillar drive and feathered paddlewheel, a methodology for determining the minimum intrinsic losses required to beat propellers for a given task was outlined. Again, since the question "at what point do paddlewheels and caterpillar drives beat propellers?" is not reasonable because it varies with the task. Therefore a methodology of answering that question was proposed, so it could be answered for tasks deemed relevant to the reader.

4. Based on the results from (1) – (3), recommendations were made for theorists and experimentalists hoping to pursue further research which builds off first-principles analyses of these mechanisms.

Notable findings:

1. The shape of caterpillar drive efficiency curves indicates that it may out-perform propellers in non-ideal speeds. More precisely, given the same maximum efficiency, the efficiency vs boat speed curve is more elongated for caterpillar drives than propellers, indicating that caterpillar drives may be more efficient for boats which need bursts of high speed (above cruise speed).
2. The internal loss coefficient required to beat propellers was significantly higher for caterpillar drives than feathered paddlewheels in all cases. This indicates a higher potential for caterpillar drives, since it can afford to have higher amounts of internal loss to maintain the same efficiency.
3. The jellyfish actuator was highly inefficient for the cases analyzed. This could be due to a poor design choice, and efforts to better mimic actual jellyfish can improve efficiency. Computational work should be done to identify the upper performance of the jellyfish mechanism for a variety of tasks

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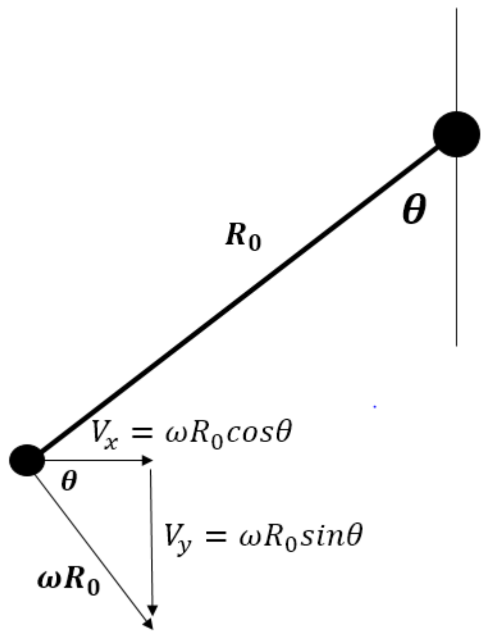
Appendix A: Feathered Paddlewheel Supplements

<u>Derivation of V_{rel} and V_y</u>:

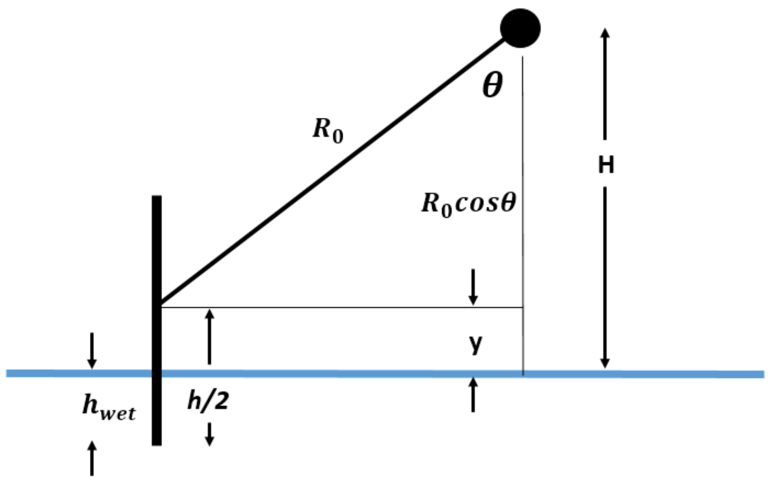


Since V_{rel} is the speed of water in the x-direction relative to the speed of the paddle,

$$V_{rel} = \omega R_0 cos\theta - V_b$$

In the previously mentioned analysis of normal paddlewheels, V_{rel} was approximated as $V_{rel} = \omega R_0 - V_b$. This small angle approximation neglects the slightly negative thrusts generated when $\omega R_0 cos\theta < V_b$. Since V_{rel} will be squared when calculating the force F_x, being able to ignore these negative thrust contributions is useful when performing computations, since it avoids the extra bookkeeping required to identify when the thrust goes from slightly negative to slightly positive (since it will not appear as such). Since this is only relevant in cases where $\omega R_0 cos\theta \approx V_b$ and therefore only in cases where the thrust is small, ignoring these contributions is reasonable.

Derivation of A_{wet}:



From this diagram, three relationships are apparent:

$R_0 cos\theta + y = H$

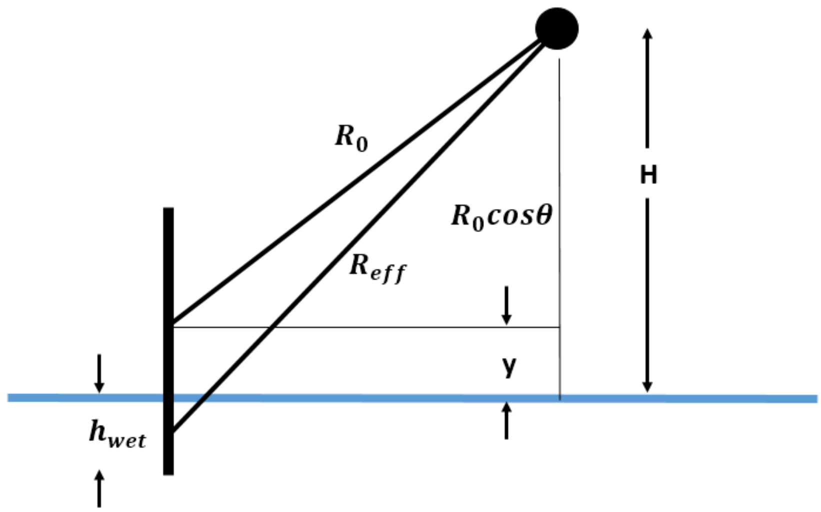
$H = R_0 - \frac{h}{2}$ (think of the case where θ = 0)

$\frac{h}{2} = y + h_{wet}$

After using the first two equations to eliminate H, that expression can be combined with the third to eliminate y. This gives a relationship between the remaining variables, which reduces to

$$A_{wet} = W[R_0(cos\theta - 1) + h]$$

Derivation of R_{eff}:



From the Pythagorean theorem,

$$R_{eff} = [(R_0 cos\theta + y + 0.5h_{wet})^2 + (R_0 sin\theta)^2]^{1/2}$$

Using the three equations from the previous derivation, y and h_{wet} can be re-written in terms of R_0 and θ. After substituting, the expression reduces to:

$$R_{eff} = R_0[0.25(cos\theta + 1)^2 + sin^2\theta]^{1/2}$$

θ_{max} and θ_{min} Derivation:

The angles are the minimum and maximum values when y = h/2. There the equations from the derivation of A_{wet} give

$$R_0 cos\theta_{max} + \frac{h}{2} = H = R_0 - \frac{h}{2}$$

Solving for the angle gives

$$\theta_{max} = cos^{-1}(1 - \frac{h}{R_0})$$

<u>Feathered Paddlewheel MATLAB Code</u> (specs used for RC-Size Paddlewheels):

```
clear all;
clc;
close all;

Cd = 1.28;                % Drag coefficient of water perpendicular to blade
Rho = 1000;               % Density of water
Vb = 6.7;                 % Boat Speed
Ro= .1;                   % Radius of paddlewheel R_0
h = .4*Ro;                % Height of blade
W = h*3/2;                % Width of blade
MAX = acos(1-h/Ro);       % Minimum angle where blade first contacts water
MIN = -acos(1-h/Ro);      % Max angle where blade leaves water
t = MIN:.01:MAX;          % Angles where blade 1 is contacting water
w = Vb/Ro+.1;             % Angular speed of mechanism
kp = 11000;               % Internal Drag Coefficient


for i = 1:length(t)

      % Create T1, thrust curve for blade 1
      T1(1,i) = 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i))-1)+h);

            if 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i))-1)+h) < 0;
                  T1(1,i) = 0;
            end

      % Create T2 thrust curve, offset 45 degrees, starts in the water
      T2(1,i) = 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)+pi/4)-1)+h);

            if 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)+pi/4)+h)) < 0;
                  T2(1,i) = 0;
            end

            if i > 107; %corrects for the curve going back above zero when it
            should be defined as zero here
                  T2(1,i) = 0;
            end

      % Create T3 curve, offset 90 degrees, starts in the water
      T3(1,i) = 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)+pi/2)-1)+h);

            if 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)+pi/2)+h)) < 0;
                  T3(1,i) = 0;
            end

            if i > 29; %corrects for the curve going back above zero when it
            should be defined as zero here
                  T3(1,i) = 0;
            end

      % Create T4 curve, offset 45 degrees, starts outside the water
      T4(1,i) = 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)-pi/4)-1)+h);
```

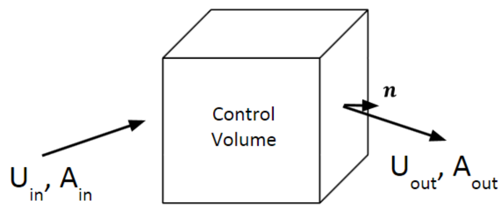
```
      if 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)-pi/4)+h)) < 0;
            T4(1,i) = 0;
      end

      if i < 80; %corrects for the curve going back above zero when it
      should be defined as zero here
            T4(1,i) = 0;
      end

% Create T5, offset 90 degrees, starts outside the water
T5(1,i) = 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)-pi/2)-1)+h);

      if 0.5*Cd*Rho*((w*Ro-Vb)^2)*W*(Ro*(cos(t(i)-pi/2)+h)) < 0;
            T5(1,i) = 0;
      end

      if i < 159; %corrects for the curve going back above zero when it
      should be defined as zero here
            T5(1,i) = 0;
      end

% Create P1, the power curve for blade 1
P1(1,i)          =          w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i))-
1)+h)*W*Ro*((0.25*((cos(t(i))+1)^2)+(sin(t(i))^2))^(.5))*cos(t(i));

      if                    w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i))-
      1)+h)*W*Ro*((0.25*((cos(t(i))+1)^2)+(sin(t(i))^2))^(.5))*cos(t(i)
      ) < 0;
            P1(1,i)=0;
      end

% Create P2 curve, offset 45 degrees, starts in the water
P2(1,i)=                w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)+pi/4)-
1)+h)*W*Ro*((0.25*((cos(t(i)+pi/4)+1)^2)+(sin(t(i)+pi/4)^2))^(.5))*cos(
t(i)+pi/4);

      if                w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)+pi/4)-
      1)+h)*W*Ro*((0.25*((cos(t(i)+pi/4)+1)^2)+(sin(t(i)+pi/4)^2))^(.5)
      )*cos(t(i)+pi/4) < 0;
            P2(1,i)=0;
      end

      if i > 160; %corrects for the curve going back above zero when it
      should be defined as zero here
            P2(1,i)=0;
      end

% Create P3 curve, offset 90 degrees, starts in the water
P3(1,i)=                w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)+pi/2)-
1)+h)*W*Ro*((0.25*((cos(t(i)+pi/2)+1)^2)+(sin(t(i)+pi/2)^2))^(.5))*cos(
t(i)+pi/2);

      if                w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)+pi/2)-
      1)+h)*W*Ro*((0.25*((cos(t(i)+pi/2)+1)^2)+(sin(t(i)+pi/2)^2))^(.5)
      )*cos(t(i)+pi/2) < 0;
            P3(1,i)=0;
```

```
            end

            if i > 80; %corrects for the curve going back above zero when it
            should be defined as zero here
                  P3(1,i)=0;
            end

      % Create P4 curve, offset 45 degrees, starts outside the water
      P4(1,i)=                 w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)-pi/4)-
      1)+h)*W*Ro*((0.25*((cos(t(i)-pi/4)+1)^2)+(sin(t(i)-
      pi/4)^2))^(.5))*cos(t(i)-pi/4);

            if                 w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)-pi/4)-
            1)+h)*W*Ro*((0.25*((cos(t(i)-pi/4)+1)^2)+(sin(t(i)-
            pi/4)^2))^(.5))*cos(t(i)-pi/4) < 0;
                  P4(1,i)=0;
            end

            if i < 60; %corrects for the curve going back above zero when it
            should be defined as zero here
                  P4(1,i)=0;
            end

      % Create P5 curve, offset 90 degrees, starts outside the water
      P5(1,i)=                 w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)-pi/2)-
      1)+h)*W*Ro*((0.25*((cos(t(i)-pi/2)+1)^2)+(sin(t(i)-
      pi/2)^2))^(.5))*cos(t(i)-pi/2);

            if                 w*0.5*Cd*Rho*((w*Ro-Vb)^2)*(Ro*(cos(t(i)-pi/2)-
            1)+h)*W*Ro*((0.25*((cos(t(i)-pi/2)+1)^2)+(sin(t(i)-
            pi/2)^2))^(.5))*cos(t(i)-pi/2) < 0;
                  P5(1,i)=0;
            end

            if i < 140; %corrects for the curve going back above zero when it
            should be defined as zero here
                  P5(1,i)=0;
            end
end


Efficiency = Vb*mean(T1+T2+T3+T4+T5)/(mean(P1+P2+P3+P4+P5)+kp)
```

Appendix B: Caterpillar Drive Supplements

Force corresponding to momentum change of water:

Reynold's Transport Theorem gives the force corresponding to momentum flux across a control volume:



$$F = \int_{CS} \bar{U}(\rho \bar{U} \cdot \hat{n}) dA$$

For the case when U is perpendicular to the surface, the integral reduces to

$$F_w = \dot{m}(V_{out} - V_{in})$$

Where F_w is the force required to maintain the velocity gradient in the steady state. This corresponds to the thrust generated by the "black box" mechanism.

In the case where the box dimensions are $L \times H \times W$, the mass flow rate is

$$\dot{m} = \rho V_{out} A_{opening}$$

In the caterpillar drive case, $A_{opening}$ = LH and V_{out} = Rω. Using these values,

$$F_W = \rho \mathrm{R} \omega L H (\mathrm{R}\omega - V_b)$$

Power due to changing kinetic energy:

The kinetic energy going into the control volume is

$$KE_{in} = \frac{1}{2} m V_{in}^2$$

The kinetic energy going out of the control volume is

$$KE_{out} = \frac{1}{2} m V_{out}^2$$

The power is the first time derivative of kinetic energy, $\frac{\Delta KE}{\Delta t}$. This reduces to

$$P_w = \frac{1}{2}\dot{m}(V_{out}^2 - V_{in}^2)$$

Therefore using the caterpillar drive parameters,

$$P_w = \frac{1}{2}\mathrm{R}\omega LH((\mathrm{R}\omega)^2 - V_b^2)$$

RC-Sized Caterpillar Drive MATLAB Code:

```
% Finding optimal angular speed at cruise speed

      Cd = 0.003; R = 0.05; H = 0.1; W = 0.15; L = .16; Vb = 6.7; w = 137:500;

      T = 1000*R*L*H*w.*(w.*R-Vb)-0.5*1000*Cd*(Vb^2)*(2*H*L+W*L);

      P = 0.5*R*L*H*w.*(((R*w).^2) -Vb^2)+0.5*1000*Cd*((w*R).^3) *(2*H*L+W*L)
      + 29;

      Efficiency = Vb*T.*P.^(-1);

      plot(Efficiency)
      xlabel(‘Angular Speed’)
      ylabel(‘Efficiency’)

% Generating efficiency vs boat speed curve

      Cd = 0.003; R = 0.05; H = 0.1; W = 0.15; L = .16;  w = 254; Vb = 0:.5:12;

      T = 1000*R*L*H*w*(w*R-Vb)-0.5*1000*Cd*(Vb.^2)*(2*H*L+W*L);

      P = 0.5*R*L*H*w*(((R*w)^2)-Vb.^2)+0.5*1000*Cd*((w*R)^3)*(2*H*L+W*L)+29;

      Efficiency = Vb.*T.*P.^(-1);

      plot(Efficiency)
      xlabel(‘Boat Speed’)
      ylabel(‘Efficiency’)
```

Canoe-Sized Caterpillar Drive MATLAB Code:

```
% Finding optimal angular speed at cruise speed

      Cd = 0.0026; R = 0.15; H = 0.3; W = 0.45; L = 1.5; Vb = 1.37; w = 9:25;

      T = 1000*R*L*H*w.*(w.*R-Vb)-0.5*1000*Cd*(Vb^2)*(2*H*L+W*L);

      P = 0.5*R*L*H*w.*(((R*w).^2)-
      Vb^2)+0.5*1000*Cd*((w*R).^3)*(2*H*L+W*L)+420;
```

```
    Efficiency = Vb*T.*P.^(-1);

    plot(Efficiency)
    xlabel('Angular Speed')
    ylabel('Efficiency')

% Generating efficiency vs boat speed curve

    Cd = 0.0026; R = 0.15; H = 0.3; W = 0.45; L = 1.5; w = 10; Vb = 0:.01:1.4;

    T = 1000*R*L*H*w*(w*R-Vb)-0.5*1000*Cd*(Vb.^2)*(2*H*L+W*L);

    P = 0.5*R*L*H*w*(((R*w)^2)-Vb.^2)+0.5*1000*Cd*((w*R)^3)*(2*H*L+W*L)+420;

    Efficiency = Vb.*T.*P.^(-1);

    plot(Efficiency)
    xlabel('Boat Speed')
    ylabel('Efficiency')
```

<u>Tugboat-Sized Caterpillar Drive MATLAB Code</u>:

```
% Finding optimal angular speed at cruise speed

    Cd = 0.0016; R = 1; H = 2; W = 3; L = 10; Vb = 6; w = 6:25;

    T = 1000*R*L*H*w.*(w.*R-Vb)-0.5*1000*Cd*(Vb^2)*(2*H*L+W*L);

    P = 0.5*R*L*H*w.*(((R*w).^2)-
    Vb^2)+0.5*1000*Cd*((w*R).^3)*(2*H*L+W*L)+2100000;

    Efficiency = Vb*T.*P.^(-1);

    plot(Efficiency)
    xlabel('Angular Speed')
    ylabel('Efficiency')

% Generating efficiency vs boat speed curve

    Cd = 0.0016; R = 1; H = 2; W = 3; L = 10; w = 7; Vb = 0:.01:7;

    T = 1000*R*L*H*w*(w*R-Vb)-0.5*1000*Cd*(Vb.^2)*(2*H*L+W*L);

    P = 0.5*R*L*H*w*(((R*w)^2)-Vb.^2)+0.5*1000*Cd*((w*R)^3)*(2*H*L+W*L)+
    2100000;

    Efficiency = Vb.*T.*P.^(-1);

    plot(Efficiency)
    xlabel('Boat Speed')
    ylabel('Efficiency')
```

Appendix C: Jellyfish Actuator Supplements

Expanding Phase Equation Derivation:

Start with the expression given in section 5.2:

$$m_J \dot{V}_b = F_E - F_J$$

Substitute in the expressions for F_E and F_J:

$$F_E = \frac{1}{2}\rho(V_E - V_b)^2 C_E W H$$

$$F_J = \frac{1}{2}\rho V_b^2 C_J A_J$$

This becomes

$$m_J \dot{V}_b = \frac{1}{2}\rho(V_E - V_b)^2 C_E W H - \frac{1}{2}\rho V_b^2 C_J A_J$$

This can be simplified:

$$m_J \dot{V}_b = \frac{1}{2}\rho C_E W H(V_E^2 + V_b^2 - 2V_E V_b) - \frac{1}{2}\rho V_b^2 C_J A_J$$

Combining like terms and dividing both sides by m_J, the expression given in section 5.2 is obtained:

$$\dot{V}_b = V_b^2\left[\frac{1}{2m_J}\rho\big(C_E W H - C_J A_J\big)\right] - V_b\left[\frac{\rho C_E W H V_E}{m_J}\right] + \frac{\rho V_E^2 C_E W H}{2m_J}$$

<u>Contracting Phase Equation Derivation</u>:

Start with the expression given in section 5.2:

$$m_J \dot{V}_b = -F_C - F_J$$

Substitute in the expressions for F_C and F_J:

$$F_C = \frac{1}{2}\rho(V_C + V_b)^2 C_C W H$$

$$F_J = \frac{1}{2}\rho V_b^2 C_J A_J$$

This becomes

$$m_J \dot{V}_b = -\frac{1}{2}\rho(V_C + V_b)^2 C_C W H - \frac{1}{2}\rho V_b^2 C_J A_J$$

After expanding the $(V_C + V_b)^2$ term, combining like terms, and dividing both sides by m_J, the equation given in section 5.2 is obtained:

$$\dot{V}_b = V_b^2\left[\frac{1}{2m_J}\rho\left(C_C W H + C_J A_J\right)\right] + V_b\left[\frac{\rho C_C W H V_C}{m_J}\right] + \frac{\rho V_C^2 C_C W H}{2m_J}$$

SIMULINK Block Diagram:

Note: the values of A-F are used for the slow jellyfish actuator