ABSTRACT

Title of dissertation: Model-Assisted Estimators for Time-to-Event Data

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In this dissertation, I develop model-assisted estimators for estimating the proportion of a population that experienced some event by time t. I provide the theoretical justification for the new estimators using time-to-event models as the underlying framework. Using simulation, I compared these estimators to traditional methods, then I applied the estimators to a study of nurses health, where I estimated the proportion of the population that had died after a certain period of time. The new estimators performed as well if not better than existing methods. Finally, as this work assumes that all units are censored at the same point in time, I propose an extension that allows units censoring time to vary.

Model-Assisted Estimators for Time-to-Event Data

by

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To

Fane, Avhlee, Ollie, & Willson

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Chapter 1: Introduction

In this dissertation, I use time-to-event models to develop model-assisted estimators that can be used to estimate the proportion, $p(t)$ of the population that have experienced an event by some time t. Many surveys collect the time at which a sampled unit experiences a given event. As an example, consider the National Longitudinal Study of 1972 conducted by the National Center for Education Statistics (https://nces.ed.gov/surveys/nls72/), which surveyed a nationally representative sample of high school 12th graders. One item collected during follow-up interviews was the date after graduation at which each respondent was hired for his or her first full-time job. From this, we can estimate the proportion of people who were 12th graders in 1972 who were hired within five years of graduation. Another example is the Survey of Income and Program Participation (SIPP), which measures how long individuals participate in various government assistance programs like Medicaid, Supplemental Nutrition Assistance Program (SNAP), Housing Assistance, Supplemental Security Income (SSI), and Temporary

Assistance for Needy Families (TANF) (Irving and Loveless, 2015). The Panel Survey of Income Dynamics (PSID, https://psidonline.isr.umich.edu/), conducted by the University of Michigan, is another longitudinal survey that has collected data on employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, since 1968. The Health and Retirement Study (HRS, http://hrsonline.isr.umich.edu/) is also a large longitudinal, panel survey done by the University of Michigan to collect aging, income, biomarker, and other health data. Many different endpoints can be derived from both PSID and HRS that can be used in time-to-event modeling.

The proportion of a given population that has experienced an event by time t can be estimated using a π-estimator[1] (Särndal et al., 1992) as follows:

$$\hat{p}_\pi(t) = N^{-1} \sum_{i \in s} \pi_i^{-1} I_{\{T_i \leq t\}}, \tag{1.1}$$

where N is the size of the finite population, s is the set of units sampled from the population, π_i is the probability of selection for unit i, T_i is the time at which the event happened and $I_{\{T_i \leq t\}}$ is the 0-1 indicator for whether the event happened before time t. If the survey closes out before all units have experienced a given event, then

[1]The π-estimator is sometimes referred to as the Horvitz-Thompson estimator (Horvitz and Thompson, 1952)

T_i is only observed for T_i less than or equal to the time of observation t_o. This means that T_i is right censored for units for which $T_i \geq t_o$ and when $t > t_o$ the π-estimator cannot be used to estimate $p(t)$.

Often covariate data are available on the sampling frame for all population units. When this is the case, the π-estimator only takes advantage of this information if the covariate data are used in the sample design. Model-assisted estimators can leverage covariate data by using models to predict $p(t)$. These models can reduce the sampling variance without the risk of inducing a large amount of bias when the model is misspecified.

I propose new, extended versions of Generalized Difference Estimators (GDEs) and Model Calibrated Estimators (MCEs) to estimate $p(t)$ as follows:

$$\hat{p}_{GDE}(t) = N^{-1}\left(\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) + \sum_{i\in s}\pi_i^{-1}\left[I_{\{T_i\leq t\}} - p(t|z_i,\hat{\theta})\right]\right) \tag{1.2}$$

and

$$\hat{p}_{MCE}(t) = \hat{p}_{\pi}(t) + N^{-1}\left(\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) - \sum_{i\in s}\pi_i^{-1}p(t|z_i,\hat{\theta})\right)\hat{B}, \tag{1.3}$$

where $p(t|z_i,\hat{\theta})$ is the prediction of $p(t)$ from a time-to-event model based on covariates $\mathbf{z}$, and $\hat{B}$ is a calibration adjustment which is a modification of the calibration adjustment proposed by Wu and Sitter (2001). It should be noted that

the z_i's need to be avalible for all members of the finite population. These estimators are doubly-robust in the sense of being consistent if the assisting model is correctly specified or if the inclusion probabilities used in (1.2) and (1.3) provide design-consistent estimators. Thus, the work here is related to and extends the double robustness literature in biostatistics (e.g., see Scharfstein et al. (1999); Van der Laan and Robins (2003)).

This dissertation is laid out as follows. Chapter 2 reviews the literature on to model-assisted estimation, time-to-event models, and models for survey data, which are all fundamental concepts for this dissertation. Chapter 3 shows that $\hat{p}_{GDE}$ and $\hat{p}_{MCE}$ and their respective variance estimators are design consistent. Chapter 4 presents a simulation study that explores the properties of $\hat{p}_{GDE}$ and $\hat{p}_{MCE}$. Chapter 5 presents an application of these new estimators to the Nurses' Health Study. Finally, Chapter 6 provides some concluding remarks as well some possible extensions of this work.

Chapter 2: Literature Review

In this chapter, I review the prerequisite information for developing model-assisted estimators using time-to-event models with applications to complex survey data. Section 2.1 reviews model-assisted estimation, including the Generalized Regression Estimator (GREG), Calibration estimators, GDEs, and MCEs. Section 2.2 reviews time-to-event models, including Proportional Hazard Models (PHMs), Accelerated Failure Time Models (AFTMs) and Threshold Regression Models (TRMs), including how to estimate the cumulative hazard from these models. Finally, Section 2.3 reviews methods for fitting time-to-event models to complex survey data.

2.1 Model Assisted Estimation

For the general model-assisted approach, a model is fit using auxiliary information $\mathbf{x}$, observed for every unit in the population, as predictors of a variable of interest y only observed for the units in sample. This model is then used to

predict y for every unit in the population. Sample values are then used again to protect against model misspecification (Särndal et al., 1992). The classic example of a model-assisted approach is the GREG, which uses a linear model to predict a continuous variable y for the population. The GREG is discussed further in Section 2.1.1. Deville and Särndal (1992) develop a larger class of model-assisted estimators called calibration estimators, which include GREG estimators. Calibration estimators are discussed in Section 2.1.2. Wu and Sitter (2001) consider a few model-assisted approaches using Generalized Linear Models (GLMs) in single-stage sampling. This work is extended by Kennel (2013) to two-stage sampling.

2.1.1 Generalized Difference Estimator (GDE)

The general framework for the GDE presented by Wu and Sitter (2001) for the population mean of variable y_i is as follows:

$$\hat{\bar{Y}}_{GDE} = N^{-1}\left(\sum_{i=1}^{N}\mu(\mathbf{x_i},\hat{\theta}) + \sum_{i\in s}\frac{1}{\pi_i}\left[y_i - \mu(\mathbf{x_i},\hat{\theta})\right]\right), \tag{2.1}$$

where $\mu(\mathbf{x_i}, \hat{\theta})$ is the model prediction for y_i based on a vector of auxiliary variables $\mathbf{x_i}$ using the following common working model:

$$\begin{aligned} E[y_i|\mathbf{x_i}] &= \mu(\mathbf{x_i}, \theta), \\ V[y_i|\mathbf{x_i}] &= v_i^2 \sigma^2 \end{aligned} \tag{2.2}$$

for $i = 1, 2, \ldots, N$. In this model, the y_i's are independent, θ is a $p \times 1$ vector, θ and σ^2 are unknown superpopulation parameters, $\mu(\mathbf{x_i}, \theta)$ is a known function of $\mathbf{x_i}$ and θ, and v_i is some function of $\mathbf{x_i}$. Examples of v_i would be $v_i = x_i$ or x_i^2 where x_i is on of the components of $\mathbf{x_i}$.

If $\mu(\mathbf{x_i}, \hat{\theta})$ is taken to be a standard linear model, then $\hat{\bar{Y}}_{GDE}$ is the classic GREG:

$$\hat{\bar{Y}}_{GDE} = N^{-1} \left(\sum_{i=1}^{N} \mathbf{x_i'} \hat{\theta} + \sum_{i \in s} \pi_i^{-1} \left[y_i - \mathbf{x_i'} \hat{\theta} \right] \right), \tag{2.3}$$

where $\hat{\theta} = (\mathbf{X'\Pi^{-1}QX})^{-1}\mathbf{X'\Pi^{-1}Qy}$, $\mathbf{\Pi}$ is the diagonal matrix of π_i's, and $\mathbf{Q}$ is the diagonal matrix of the $1/v_i^2\sigma^2$'s specified by the working model. A common working model for a ratio estimator is a follows:

$$\begin{aligned} E[y_i|\mathbf{x_i}] &= \beta \mathbf{x_i}, \\ V[y_i|\mathbf{x_i}] &= \sigma^2 \mathbf{x_i}. \end{aligned} \tag{2.4}$$

In this case, $v_i^2 = \mathbf{x_i}$.

Wu and Sitter (2001) consider estimators where $\mu(\mathbf{x_i}, \hat{\theta})$ is a GLM for one-stage sampling fit using a design consistent estimator of θ. Kennel (2013) extends this work for GLMs to two-stage cluster samples.

Wu and Sitter (2001) and Kennel (2013) prove that the GDE is design consistent under the conditions that if for a sequence of populations indexed by j in which both the sample size n_j and the population size N_j approach infinity as $j \to \infty$, then:

(i) $\hat{\theta} = \theta_N + O_p\left(n^{-1/2}\right)$ and $\theta_N \to \theta$, where θ_N is the finite population value for the parameter and θ is its underlying constant value;

(ii) for each $\mathbf{x_i}$, $\partial\mu\left(\mathbf{x_i}, k\right)/\partial k$, where k is one of the components of θ, is continuous in k, $\left|\partial\mu\left(\mathbf{x_i}, k\right)/\partial k\right| \leq h\left(\mathbf{x_i}, \theta\right)$ for all values k in a neighborhood of θ, and $N^{-1}\sum_{i=1}^{N} h\left(\mathbf{x_i}, \theta\right) = O(1)$;

(iii) the basic design weights, $d_i = \pi_i^{-1}$, satisfy that the π-estimator for certain population means are asymptotically normally distributed.

Theorem 1. *If a common working model is used to construct* $\hat{\bar{Y}}_{GDE}$ *and conditions (i)-(iii) hold, then*

$$\hat{\bar{Y}}_{GDE} = \hat{\bar{Y}}_{\pi} + O_p\left(n^{-1/2}\right), \tag{2.5}$$

where $\hat{\bar{Y}}_\pi$ is the π-estimator of the finite population mean $\bar{Y}$. Thus, $\hat{\bar{Y}}_{GDE}$ is design consistent.

Proof. Since (2.1) can be rewritten as

$$\hat{\bar{Y}}_{GDE} = \hat{\bar{Y}}_\pi + \left(N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}) - N^{-1} \sum_{i \in s} d_i \mu(\mathbf{x_i}, \hat{\theta}) \right), \tag{2.6}$$

it suffices to show that

$$\left(N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}) - N^{-1} \sum_{i \in s} d_i \mu(\mathbf{x_i}, \hat{\theta}) \right) = O_p\left(n^{-1/2}\right). \tag{2.7}$$

Now applying a Taylor series approximation to $\mu(\mathbf{x_i}, \hat{\theta})$ at $\hat{\theta} = \theta_N$, we get

$$\mu(\mathbf{x_i}, \hat{\theta}) = \mu\left(\mathbf{x_i}, \theta_N\right) + \left[\frac{\partial \mu\left(\mathbf{x_i}, k\right)}{\partial k} \Big|_{\theta^*} \right]' (\hat{\theta} - \theta_N), \tag{2.8}$$

where $\theta^* \in (\hat{\theta}, \theta_N)$ or $(\theta_N, \hat{\theta})$. Now by (2.8) and conditions (i) and (ii),

$$N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}) = N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \theta_N) + O_p\left(n^{-1/2}\right), \tag{2.9}$$

$$N^{-1} \sum_{i \in s} d_i \mu(\mathbf{x_i}, \hat{\theta}) = N^{-1} \sum_{i \in s} d_i \mu(\mathbf{x_i}, \theta_N) + O_p\left(n^{-1/2}\right). \tag{2.10}$$

Note that because of condition (iii)

$$N^{-1}\sum_{i=1}^{N}\mu(\mathbf{x_i},\theta_N) - N^{-1}\sum_{i\in s} d_i\mu(\mathbf{x_i},\theta_N) = O_p\left(n^{-1/2}\right). \tag{2.11}$$

Now by putting together (2.9), (2.10), and (2.11) we get

$$\left(N^{-1}\sum_{i=1}^{N}\mu_i(\mathbf{x_i},\hat{\theta}) - N^{-1}\sum_{i\in s} d_i\mu_i(\mathbf{x_i},\hat{\theta})\right) = O_p\left(n^{-1/2}\right) \tag{2.12}$$

as desired. □

To show design consistency of the variance estimator of $\hat{\bar{Y}}_{GDE}$ an additional condition is necessary:

(iv) for each $\mathbf{x_i}$, $\partial^2\mu\left(\mathbf{x_i},k\right)/\partial k\partial k'$ where k is one of the components of θ, is continuous in k and $\left|\partial^2\mu\left(\mathbf{x_i},k\right)/\partial k\partial k'\right| \leq g\left(\mathbf{x_i},\theta\right)$ for all k in a neighborhood of θ and $N^{-1}\sum_{i=1}^{N} g\left(\mathbf{x_i},\theta\right) = O(1)$.

Theorem 2. *If a common working model is used to construct* $\hat{\bar{Y}}_{GDE}$ *and conditions (i)-(vi) hold, then the approximate design variance of* $\hat{\bar{Y}}_{GDE}$ *is*

$$V\left(\hat{\bar{Y}}_{GDE}\right) \doteq N^{-2}\sum_{i<j}^{N}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{e_i}{\pi_i} - \frac{e_j}{\pi_j}\right), \tag{2.13}$$

where π_{ij} *is the joint probability of selecting the* i^{th} *and* j^{th} *units, and* $e_i = y_i -$

$\mu(\mathbf{x_i}, \theta_N)$. *This can be estimated by*

$$\hat{V}\left(\hat{\bar{Y}}_{GDE}\right) \doteq N^{-2} \sum_{i<j}^{s} \left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right) \left(\frac{\hat{e}_i}{\pi_i} - \frac{\hat{e}_j}{\pi_j}\right), \tag{2.14}$$

where $\hat{e}_i = y_i - \mu(\mathbf{x_i}, \hat{\theta})$.

Proof. Using (i), (ii), (iv) and applying a Taylor series second order approximation to $\mu(\mathbf{x_i}, \hat{\theta})$ at $\hat{\theta} = \theta_N$, we get

$$\begin{aligned} \mu(\mathbf{x_i}, \hat{\theta}) = \mu\left(\mathbf{x_i}, \theta_N\right) + \left[\frac{\partial \mu\left(\mathbf{x_i}, k\right)}{\partial k}\Big|_{\theta^*}\right]' (\hat{\theta} - \theta_N) \\ + (\hat{\theta} - \theta_N)' \left[\frac{\partial^2 \mu\left(\mathbf{x_i}, k\right)}{\partial k \partial k'}\Big|_{\theta^*}\right] (\hat{\theta} - \theta_N), \end{aligned} \tag{2.15}$$

where $\theta^* \in (\hat{\theta}, \theta_N)$ or $(\theta_N, \hat{\theta})$, and $\left[\frac{\partial^2 \mu(\mathbf{x_i}, k)}{\partial k \partial k'}\Big|_{\theta^*}\right]$ is the $p \times p$ matrix of second derivatives evaluated at θ^*. Now by (2.15) and condition (iv),

$$\begin{aligned} N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}) = & N^{-1} \sum_{i=1}^{N} \mu(\mathbf{x_i}, \theta_N) \\ & + \left\{ N^{-1} \sum_{i=1}^{N} \frac{\partial \mu\left(\mathbf{x_i}, k\right)}{\partial k}\Big|_{\theta^*} \right\}' (\hat{\theta} - \theta_N) + O_p\left(n^{-1}\right), \end{aligned} \tag{2.16}$$

$$N^{-1}\sum_{i\in s} d_i\mu(\mathbf{x_i},\hat{\theta}) = N^{-1}\sum_{i\in s} d_i\mu(\mathbf{x_i},\theta_N) + \left\{N^{-1}\sum_{i\in s} d_i \frac{\partial\mu\left(\mathbf{x_i},k\right)}{\partial k}\Big|_{\theta^*}\right\}' (\hat{\theta}-\theta_N) + O_p\left(n^{-1}\right). \tag{2.17}$$

By conditions (i) and (iii), $(\hat{\theta}-\theta_N) = O_p\left(n^{-1/2}\right)$, and

$$\left\{N^{-1}\sum_{i=1}^{N} \frac{\partial\mu\left(\mathbf{x_i},k\right)}{\partial k}\Big|_{\theta^*}\right\} - \left\{N^{-1}\sum_{i\in s} d_i \frac{\partial\mu\left(\mathbf{x_i},k\right)}{\partial k}\Big|_{\theta^*}\right\} = O_p\left(n^{-1/2}\right). \tag{2.18}$$

Therefore, by combining (2.16) and (2.17) we get

$$N^{-1}\sum_{i=1}^{N}\mu(\mathbf{x_i},\hat{\theta}) - N^{-1}\sum_{i\in s} d_i\mu(\mathbf{x_i},\hat{\theta}) = N^{-1}\sum_{i=1}^{N}\mu(\mathbf{x_i},\theta_N) - N^{-1}\sum_{i\in s} d_i\mu(\mathbf{x_i},\theta_N) + O_p\left(n^{-1}\right). \tag{2.19}$$

Using (2.19) to replace $\hat{\theta}$ with θ_N in

$$\hat{\bar{Y}}_{GDE} = \hat{\bar{Y}}_{\pi} + \left(N^{-1}\sum_{i=1}^{N}\mu_i\left(\mathbf{x}_i,\hat{\theta}\right) - N^{-1}\sum_{i\in s} d_i\mu_i\left(\mathbf{x}_i,\hat{\theta}\right)\right), \tag{2.20}$$

we get

$$\begin{aligned}\hat{\bar{Y}}_{GDE} &= \hat{\bar{Y}}_{\pi} + \left(N^{-1} \sum_{i=1}^{N} \mu_i \left(\mathbf{x}_i, \theta_N \right) - N^{-1} \sum_{i \in s} d_i \mu_i \left(\mathbf{x}_i, \theta_N \right) \right) + O_p \left(n^{-1/2} \right) \\ &= N^{-1} \sum_{i=1}^{N} \mu_i \left(\mathbf{x}_i, \theta_N \right) + N^{-1} \sum_{i \in s} d_i \left[y_i - \mu_i \left(\mathbf{x}_i, \theta_N \right) \right] + O_p \left(n^{-1/2} \right).\end{aligned} \tag{2.21}$$

Finally, by noticing that $N^{-1} \sum_{i=1}^{N} \mu_i \left(\mathbf{x}_i, \theta_N \right)$ is constant, the asymptotic variance of $\hat{\bar{Y}}_{GDE}$ is the asymptotic variance of the π-estimator of the population total i.e., $e_i = y_i - \mu \left(\mathbf{x_i}, \theta_N \right)$. It now follows that the asymptotic variance estimator of $\hat{\bar{Y}}_{GDE}$ is the asymptotic variance estimator of a π-estimator of the estimated total of the residuals, $\hat{e}_i = y_i - \mu(\mathbf{x_i}, \hat{\theta})$. □

Theorems 1 and 2 show that the GDE is design consistent and that the aysmptotic variance and its estimator are equivalent to those of the π-estimator of the total of the residuals.

2.1.2 Model Calibration Estimator (MCE)

Another estimator proposed by Wu and Sitter (2001) is the MCE. This estimator is based on the traditional calibration estimator proposed by Deville and

Särndal (1992). The general form of a calibration estimator is

$$\hat{\bar{Y}}_{Cal} = N^{-1} \sum_{i \in s} w_i y_i, \tag{2.22}$$

where w_i satisfies the constraint

$$\sum_{i \in s} w_i \mathbf{x_i} = \mathbf{X} \tag{2.23}$$

while minimizing the average deviation of the calibration weights w_i from design weights d_i under some distance metric Φ_s. A common distance metric, and the metric discussed by Wu and Sitter (2001), is the Chi-squared distance measure:

$$\Phi_s = \sum_{i \in s} \frac{(w_i - d_i)^2}{d_i q_i} \tag{2.24}$$

for some set of known q_i's which are independent of d_i (Deville and Särndal, 1992; Wu and Sitter, 2001). This distance measure results in the following estimator of $\hat{\bar{Y}}_{Cal}$ also known as the GREG:

$$\begin{aligned} \hat{\bar{Y}}_{Cal} &= N^{-1} \sum_{i \in s} w_i y_i \\ &= \hat{\bar{Y}}_{\pi} + N^{-1} \left(X - \hat{X}_{\pi} \right) \hat{B}, \end{aligned} \tag{2.25}$$

where $\hat{X}_\pi = \sum_{i\in s} d_i \mathbf{x_i}$ and $\hat{B} = [\sum_{i\in s} d_i q_i \mathbf{x_i}\mathbf{x_i}']^{-1} \sum_{i\in s} d_i q_i \mathbf{x_i} y_i$ (Deville and Särndal, 1992; Wu and Sitter, 2001).

The MCE uses model predictions in the constraints; $\hat{\bar{Y}}_{MCE} = N^{-1} \sum_{i\in s} w_i y_i$ is subject to the following constraints:

$$\sum_{i\in s} w_i = N, \tag{2.26}$$

$$\sum_{i\in s} w_i \mu(\mathbf{x_i}, \hat{\theta}) = \sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}). \tag{2.27}$$

The MCE substitutes the X and $\hat{X}_\pi$ into equation (2.25) with model predictions and adjusted $\hat{B}_N$ to include constraint (2.26)

$$\hat{\bar{Y}}_{MCE} = \hat{\bar{Y}}_\pi + N^{-1} \left(\sum_{i=1}^{N} \mu(\mathbf{x_i}, \hat{\theta}) - \sum_{i\in s} d_i \mu(\mathbf{x_i}, \hat{\theta}) \right) \hat{B}_N, \tag{2.28}$$

and

$$\hat{B}_N = \frac{\sum_{i\in s} d_i q_i \left(\mu(\mathbf{x_i}, \hat{\theta}) - \bar{\mu} \right) (y_i - \bar{y})}{\sum_{i\in s} d_i q_i \left(\mu(\mathbf{x_i}, \hat{\theta}) - \bar{\mu} \right)^2}, \tag{2.29}$$

where $\bar{y} = \sum_{i\in s} d_i q_i y_i / \sum_{i\in s} d_i q_i$ and $\bar{\mu} = \sum_{i\in s} d_i q_i \mu(\mathbf{x_i}, \hat{\theta}) / \sum_{i\in s} d_i q_i$. Wu and Sitter (2001) also consider a MCE without constraint (2.26). This new estimator $\hat{\bar{Y}}^*_{MCE}$

replaces $\hat{B}$ with

$$\hat{B}_N^* = \frac{\sum_{i \in s} d_i q_i \mu(\mathbf{x_i}, \hat{\theta}) y_i}{\sum_{i \in s} d_i q_i \left(\mu(\mathbf{x_i}, \hat{\theta})\right)^2}. \tag{2.30}$$

It should be noted that the GDEs are a special case of MCEs where $\hat{B}_N = 1$. Because of this, Theorem 1 can be generalized to show that $\hat{\bar{Y}}_{MCE}$ and $\hat{\bar{Y}}^*_{MCE}$ are design consistent by noting that $\hat{B}_N$ and $\hat{B}^*_N$ are equal to $O_P(1)$. Now by defining B_N and B^*_N as the value of $\hat{B}_N$ and $\hat{B}^*_N$ when s is the entire finite population, Theorem 2 can be generalized by noting that $\hat{B}_N = B_N + o_p(1)$ and $\hat{B}^*_N = B^*_N + o_p(1)$ and substituting $y_i - \mu(\mathbf{x_i}, \theta_N)B_N$ or $y_i - \mu(\mathbf{x_i}, \theta_N)B^*_N$ for e_i in the variance formula and by substituting $y_i - \mu(\mathbf{x_i}, \theta_N)\hat{B}_N$ or $y_i - \mu(\mathbf{x_i}, \theta_N)\hat{B}^*_N$ for $\hat{e}_i$ in the variance estimator.

Wu and Sitter (2001) consider the relationship between $\hat{\bar{Y}}_{GDE}$ and $\hat{\bar{Y}}_{MCE}$ under simple random sampling. They show that the variance of $\hat{\bar{Y}}_{MCE}$ is less than or equal to the variance of the $\hat{\bar{Y}}_{GDE}$. Also, they show that if the relationship between x and y is not strong, then even under the true model $\hat{\bar{Y}}_{GDE}$ could have a larger variance than the π-estimator, and the $\hat{\bar{Y}}_{MCE}$ generally has a variance less than or equal to the variance of the π-estimator.

The GDE and MCE have been extended to include nonparametric and semiparametric models such as neural networks (Montanari and Ranalli, 2005), penalized spline regression (McConville and Breidt, 2013), generalized additive models (Opsomer et al., 2007), and lasso regression (McConville et al., 2017). For

an overview of this work see Breidt and Opsomer (2017).

2.2 Time-to-Event Models

For this dissertation, only continuous time-to-event models will be considered. This section covers three such models:

1. Proportional Hazard Models (PHMs),
2. Accelerated Failure Time Models (AFTMs), and
3. Threshold Regression Models (TRMs).

Both parametric and semiparametric PHMs and AFTMs will be considered. Section 2.2.1 reviews PHMs, Section 2.2.2 reviews AFTMs, and, finally, Section 2.2.3 reviews TRMs.

2.2.1 Proportional Hazard Models (PHMs)

Like many approaches to modeling time-to-event, or survival, data, PHMs model time-to-event data through the hazard function. Based on a set of covariates Z, the hazard function is defined as

$$h\left(t, \mathbf{z}|\theta\right) = h_0\left(t\right) g\left(\theta'\mathbf{z}\right), \tag{2.31}$$

where $h_0(t)$ is the baseline hazard when $z = 0$ and $g(\theta' z)$ is a parametric function where $g(0) = 1$. When $h_0(t)$ also has a parametric specification, the PHM is considered parametric. If $h_0(t)$ is left unspecified, then the PHM is considered semiparametric. Both cases will be discussed later in this section.

Generally, $g(\theta'\mathbf{z})$ is defined as

$$g(\theta'\mathbf{z}) = exp(\theta'\mathbf{z}) = exp\left(\sum_{k=1}^{p} \theta_k z_k\right), \tag{2.32}$$

so the hazard model becomes

$$h(t, \mathbf{z}|\theta) = h_0(t) exp\left(\sum_{k=1}^{p} \theta_k z_k\right). \tag{2.33}$$

This is referred to as a PHM, since the hazard ratio of individuals with covariates $\mathbf{z}$ and $\mathbf{z}^*$ is

$$\begin{aligned} \textit{Hazard Ratio} &= \frac{h\left(t, \mathbf{z}|\theta\right)}{h\left(t, \mathbf{z}^*|\theta\right)} \\ &= \frac{h_0\left(t\right) exp\left(\sum_{k=1}^{p} \theta_k z_k\right)}{h_0\left(t\right) exp\left(\sum_{k=1}^{p} \theta_k z_k^*\right)} \\ &= exp\left(\sum_{k=1}^{p} \theta_k \left(z_k - z_k^*\right)\right). \end{aligned} \tag{2.34}$$

Note that $h_0\left(t\right)$ cancels and the hazard ratio is constant, and thus the hazard

functions are proportional (Klein and Moeschberger, 2003). The proportional hazard assumption is rarely found in nature (Klein and Moeschberger, 2003; Lee and Whitmore, 2010). There are ways to mitigate this by using stratified PHMs or time dependent covariates (Klein and Moeschberger, 2003).

2.2.1.1 Parametric PHM

PHMs can be fit using a traditional maximum likelihood estimation (MLE) framework if both $h_0(t|\alpha)$ and $g(\theta'\mathbf{z})$ are parametric (Lawless, 2003b). In the general case, time-to-event with right censoring has a likelihood that can be expressed as

$$L(\theta|\mathbf{c},\mathbf{x}) = \prod_{i=1}^{n} f(t_i,\mathbf{x_i}|\theta)^{c_i} S(t_i,\mathbf{x_i}|\theta)^{1-c_i}. \tag{2.35}$$

Or equivalently since we know $f(t) = h(t)S(t)$,

$$L(\theta|\mathbf{c},\mathbf{x}) = \prod_{i=1}^{n} h(t_i,\mathbf{x_i}|\theta)^{c_i} S(t_i,\mathbf{x_i}|\theta), \tag{2.36}$$

where $f(t_i,\mathbf{x_i}|\theta)$ is the probability density function (p.d.f), $F(t_i,\mathbf{x_i}|\theta) = \int_0^{t_i} f(t_i,\mathbf{x_i}|\theta)\,dt$ is the cumulative density function (c.d.f.), and $S(t_i,\mathbf{x_i}|\theta) = 1 - F(t_i,\mathbf{x_i}|\theta)$ is the survival function of the distribution (Klein and Moeschberger, 2003). The term c_i is the event indicator and is equal to 1 if the

event is observed for the i^{th} and 0 if the case is right censored. Considering the formulation of the PHM seen in equation (2.33), now it can be shown that

$$S\left(t, z|\theta\right) = \left[S_0\left(t\right)\right]^{\exp\theta' z}, \tag{2.37}$$

where $S_0\left(t\right)$ is the baseline survival function (Hosmer et al., 2008). It now follows that the log-likelihood is

$$\ln\left[L\left(\theta\right)\right] = \sum_{i=i}^{n} c_i \ln\left[h_0\left(t_i\right)\right] + c_i\theta' z_i + e^{\theta' z_i} \ln\left[S_0\left(t_i\right)\right]. \tag{2.38}$$

Using this log-likelihood, θ and the variance covariance matrix of $\hat{\theta}$ can be estimated using the standard MLE procedures discussed in Section 2.3.1 (Hosmer et al., 2008). An example of a parametric PHM is the following Weibull distribution specification:

$$h\left(t, \mathbf{z}|\theta\right) = \frac{\delta}{g\left(\theta'\mathbf{z}\right)}\left[\frac{t}{g\left(\theta'\mathbf{z}\right)}\right]^{\delta-1} = \left(\delta t^{\delta-1}\right) g\left(\theta'\mathbf{z}\right)^{-\delta}, \tag{2.39}$$

where δ is known as the *shape parameter* (Hosmer et al., 2008). One common way of specifying $g\left(\theta'\mathbf{z}\right)$ in a proportional hazard context is to let $g\left(\theta'\mathbf{z}\right) = \exp(\theta'\mathbf{z})$ (Lawless, 2003b). To put this in the standard formulation of the PHM seen in equation (2.31),

$$h\left(t, \mathbf{z}|\theta\right) = \delta t^{\delta-1} e^{-\delta\theta'\mathbf{z}} = h_0\left(t\right) e^{\theta'^{*}\mathbf{z}}, \tag{2.40}$$

so $\theta^* = -\delta\theta$ and $h_0(t) = \exp(-\delta\theta_0)t^{\delta-1}$. It should be noted that a Weibull regression can be specified so that it is an AFTM, which is discussed in section 2.2.2.

2.2.1.2 Semiparametic PHM

The semiparametic version of a PHM is one of the most widely used time-to-event models because of the flexibility gained by not needing to specify the distribution of the baseline hazard. These are sometimes referred to as Cox models since they were first proposed by Cox (1972). Standard MLE methods cannot be used if the baseline hazard does not have a parametric specification, since maximizing the log-likelihood as seen in (2.38) requires the baseline hazard to be defined (Hosmer et al., 2008). Because of this, an alternative method for estimating θ was developed. These models are fit using partial likelihood (Cox, 1972, 1975).

The partial likelihood for the standard formulation of the PHM seen in equation (2.32) is

$$l_p(\theta) = \prod_{i=1}^{n} \left[\frac{e^{\theta' z_i}}{\sum_{j=1}^{n} Y_j(\theta, t_i)\, e^{\theta' z_j}} \right]^{c_i}, \tag{2.41}$$

where $Y_j(\theta, t_i) = 1_{\{e_i \geq t\}}$. $Y_i(\theta, t)$ indicates if the i^{th} case is at risk at time t, then the log partial likelihood is

$$\ln\left[l_p(\theta)\right] = \sum_{i=1}^{n} c_i \left\{ \theta' z_i - \ln\left[\frac{e^{\theta' z_i}}{\sum_{j=1}^{n} Y_j(\theta, t_i)\, e^{\theta' z_j}} \right] \right\}. \tag{2.42}$$

Using this log partial likelihood θ, the variance covariance matrix of $\hat{\theta}$ can be estimated by maximizing the partial likelihood (Hosmer et al., 2008), which is discussed in Section 2.3.2.1. It should be noted that this formulation of the partial likelihood only works if there are not ties in the t_i's (Hosmer et al., 2008). In the case of ties, approximations of the partial likelihood have been proposed by Breslow (1974) and Efron (1977).

Using counting process theory, Andersen and Gill (1982) provide asymptotic results for θ estimated through the maximum partial likelihood for PHMs.

2.2.2 Accelerated Failure Time Models (AFTMs)

One straight-forward way to consider modeling the time-to-event T is to consider a log-linear formulation

$$\ln(T) = \theta'\mathbf{z} + \epsilon. \tag{2.43}$$

A model that can be expressed in this form is called an AFTM [1], because the effect of covariates is to accelerate or decelerate the time-to-event (Hosmer et al., 2008). Wei (1992) argues that AFTMs are easily interpreted since covariates have a direct effect

[1] Accelerated Failure Time Models are also sometimes refered to as Accelerated Life Models (Cox and Oakes, 1984) of Log-Location-Scale Regression Models (Lawless, 2003b).

on failure times. Their ease of interpretation may make AFTMs preferable to PHMs. Cox also has stated that a parametric approach such as a Weibull model, which is typically expressed as an AFTM, is preferable to the semiparametric version of the PHM he developed, especially when predicting a single patient's outcome (Reid, 1994).

2.2.2.1 Parametric AFTM

Most of the commonly used parametric time-to-event models are AFTMs. The exponential,Weibull, Log-normal, Log-logistic, gamma, inverse Gaussian, and generalized gamma models are all AFTMs. Equation (2.43) is usually generalized to include a shape parameter σ such that

$$\ln(T) = \theta'\mathbf{z} + \sigma\epsilon. \tag{2.44}$$

The models differ based on the distribution assumed for ϵ. For example, if ϵ follows a logistic distribution, then the model in (2.44) becomes a log-logistic model (Hosmer et al., 2008). Parametric AFTMs can be fit, under right censoring, using the same formulation of the likelihood used for the parametric PHM in (2.35).

For a log-logistic model the survival function can be written as

$$S\left(t, z|\sigma, \theta\right) = \left[1 + \exp\left(\frac{e\left(t, \mathbf{z}|\theta\right)}{\sigma}\right)\right]^{-1}, \tag{2.45}$$

where $e\left(t, \mathbf{z}|\theta\right) = \ln\left(t\right) - \theta' z$ is the residual. Using this survival function we now can see that the log-likelihood is

$$\begin{aligned} \ln\left[L\left(\sigma, \theta\right)\right] = \sum_{i=1}^{n} c_i \left\{ -\ln\left(\sigma\right) - \left(\frac{e_i\left(t, \mathbf{z}|\theta\right)}{\sigma}\right) + 2\ln\left[S\left(t_i, z_i|\sigma, \theta\right)\right] \right\} \\ + \left(1 - c_i\right) \ln\left[S\left(t_i, z_i|\sigma, \theta\right)\right]. \end{aligned} \tag{2.46}$$

Using this log likelihood, θ, σ, and the variance covariance matrix of $\hat{\theta}$ and $\hat{\sigma}$ can be estimated using the standard procedures used in MLE as discussed in section 2.3.1 (Hosmer et al., 2008).

2.2.2.2 Semiparametric AFTM

After the development of the semiparametric PHM (Cox, 1972) an analogous form of an AFTM was developed. Louis (1981) first developed a semiparametric formulation of the AFTM for a single treatment variable. Later Tsiatis (1990) and Wei (1992) generalized this to a multiple random variables setting.

Semiparametric AFTMs were put into a rank base inference framework by Jin

et al. (2003). To do this we need to use the counting process formulation of AFTMs. A counting process is stochastic process $\{\mathcal{N}(t), t \geq 0\}$ for which meets the following criteria:

1. $\mathcal{N}(t) \geq 0$,

2. $\mathcal{N}(t)$ is an integer for all values of t,

3. If $s \leq t$ then$\mathcal{N}(s) \leq \mathcal{N}(t)$.

We define $\mathcal{N}_i^{(e)}(\theta, t) = c_i 1_{\{e_i \leq t\}}$, and $Y_i^{(e)}(\theta, t) = 1_{\{t_i \geq t\}}$, the counting process and the risk indicator on the residual scale. It now follows that

$$S^{(0)}(\theta, t) = n^{-1} \sum_{i=1}^{n} Y_i^{(e)}(\theta, t), \quad S^{(1)}(\theta, t) = n^{-1} \sum_{i=1}^{n} Y_i^{(e)}(\theta, t) z_i, \tag{2.47}$$

and the estimating equations for θ take the form

$$U_\phi(\theta) = \sum_{i=1}^{n} c_i \phi\{\theta, e_i(\theta)\} \left[z_i - \frac{S^{(1)}(\theta, t)}{S^{(0)}(\theta, t)} \right] = 0, \tag{2.48}$$

or

$$U_\phi(\theta) = \sum_{i=1}^{n} \int_{-\infty}^{\infty} \phi\{\theta, e_i(\theta)\} \left[z_i - \frac{S^{(1)}(\theta, t)}{S^{(0)}(\theta, t)} \right] d\mathcal{N}_i^{(e)}(\theta, t) = 0. \tag{2.49}$$

Two standard choices of the weight function $\phi\{\cdot\}$ are $\phi\{\cdot\} = 1$, which results in the log-rank statistic (Mantel, 1966), or $\phi\{\cdot\} = S^{(0)}$, which results in a Gehan statistic

(Gehan, 1965; Jin et al., 2003). Setting these estimating equations to zero and solving for θ results in an estimate of θ, denoted as $\hat{\theta}_\phi$.

It can be shown that the vector $n^{\frac{1}{2}}\left(\hat{\theta}_\phi - \theta\right)$ is asymptotically normal with mean zero and a covariance matrix $A_\phi^{-1} B_\phi A_\phi^{-1}$, where

$$A_\phi = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \phi\left\{\theta, e_i\left(\theta\right)\right\} \left[z_i - \frac{S^{(1)}\left(\theta, t\right)}{S^{(0)}\left(\theta, t\right)}\right]^{\otimes 2} \left\{\frac{\lambda'\left(t\right)}{\lambda\left(t\right)}\right\} d\mathcal{N}_i^{(e)}\left(\theta, t\right), \quad (2.50)$$

and

$$B_\phi = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \phi^2\left\{\theta, e_i\left(\theta\right)\right\} \left[z_i - \frac{S^{(1)}\left(\theta, t\right)}{S^{(0)}\left(\theta, t\right)}\right]^{\otimes 2} d\mathcal{N}_i^{(e)}\left(\theta, t\right). \quad (2.51)$$

In these equations, $\lambda\left(\cdot\right)$ is the common hazard function of the error terms and $\lambda'\left(t\right) = d\lambda\left(t\right)/dt$ (Jin et al., 2003). Jin et al. (2003) go on to show how simplifications arise when a Gehan weight function is used.

2.2.3 Threshold Regression

Unlike the PHM and AFTM which directly estimate the hazard function, the TRM comes from the stochastic process of survival and time-to-event analysis (Aalen et al., 2008). The TRM as laid out by Lee and Whitmore (2006) is based on a latent Wiener process and an Inverse-Gaussian First-Hitting-Time Model

(FHTM)[2]. It should be noted that threshold regression can be based on other underlying stochastic processes, but for this research is limited to the Wiener process formulation.

TRMs are useful in the analysis of time-to-event data, because unlike other methods they attempt to model an underlying latent stochastic process, and they separate the association of covariates on initial health status from the association of covariates on the change in health status over time.

2.2.3.1 First-Hitting-Time Models (FHTMs)

FHTMs are defined by a stochastic process $\{X(t), t \in \mathcal{T}, x \in \mathcal{X}\}$ with an initial value $X(0) = x_0$, where $\mathcal{T}$ is the time space, X is the state space of the process, and a subset $\mathcal{B}$ of the state space is called the boundary. If it is assumed that the x_0 is not in $\mathcal{B}$, then the FHTM is

$$S = \inf \{t : X(t) \in \mathcal{B}\}. \tag{2.52}$$

In other words, S is the first instance when the stochastic process encounters the boundary. S is referred to as the threshold state, from which threshold regression gets its name (Lee and Whitmore, 2006).

[2]First-Hitting-Time Models are also sometimes referred to as first-passage-time models.

An example of a FHTM is a Bernoulli process with a negative binomial first-hitting-time. This is the number of independent trials that it takes to get m successes when there is a p probability of success on each independent trial. In this case, the stochastic process $X(t)$ is the number of successes in t independent trials. The boundary in this case is $\mathcal{B} = \{x : x > m - 1\}$ and $x_0 = 0$. The number of coin flips needed to get m heads is an example of a model that has this type of FHTM. For additional examples of FHTMs see Section 3 of Lee and Whitmore (2006) and Section 11.5.1 of Lawless (2003b). For a general overview of FHTMs in the context of survival analysis see Aalen and Gjessing (2001).

2.2.3.2 Wiener Process

A Wiener process, also sometimes called Brownian motion, is a stochastic process $W(t)$ for $t \in [0, \infty)$, which takes on values from the real numbers and meets the following criteria:

1. $W(0) = 0$,
2. $W(t)$ has independent increments (i.e., $W(t_2) - W(t_1)$ and $W(t_4) - W(t_3)$ are independent random variables if $0 \leq t_1 < t_2 \leq t_3 < t_4$), and
3. The increment $W(t_2) - W(t_1)$ is normally distributed with

(a) $E\left[W\left(t_2\right)-W\left(t_1\right)\right]=0$,

(b) $Var\left[W\left(t_2\right)-W\left(t_1\right)\right]=t_2-t_1$.

It follows that $W\left(t\right)\sim N\left(0,t\right)$.

Wiener processes can be generalized to incorporate a parameter for an initial starting value x_0, a drift μ, and a diffusion coefficient σ^2 defined as

$$X\left(t\right)=x_0+\mu t+\sigma W\left(t\right). \tag{2.53}$$

Using the criteria above, $E\left[X\left(t\right)\right]=x_0+\mu t$ and $Var\left[X\left(t\right)\right]=\sigma^2 t$. So μ can be seen as the mean change in $X\left(t\right)$ and σ is the variance of $X\left(t\right)$ from time t to $t+1$ (Cox and Miller, 1965). The terms μ and σ^2 are referred to as the infinitesimal mean and variance, respectively. This formulation of a Wiener process is referred to as a generalized Wiener process.

If $x_0>0$, then the FHTM distribution is an Inverse Gaussian distribution. That is, at time k when $X\left(k\right)\geq 0$ the distribution of k has the p.d.f.

$$f\left(k|\mu,\sigma^2,x_o\right)=\frac{x_0}{\sqrt{2\pi\sigma^2k^3}}\exp\left[\frac{\left(x_0+\mu k\right)^2}{2\sigma^2k}\right] \tag{2.54}$$

and the c.d.f.

$$F\left(k|\mu,\sigma^2,x_0\right)=\Phi\left[\frac{x_0+\mu k}{\sqrt{2\sigma^2 k}}\right]+\exp\left(\frac{-2x_0\mu}{\sigma^2}\right)\Phi\left[\frac{\mu k-x_0}{\sqrt{\sigma^2 k}}\right] \tag{2.55}$$

for $-\infty<\mu<\infty$, $\sigma^2>0$, $x_0>0$, where $\Phi(\cdot)$ is the c.d.f. of the standard normal. It should be noted that although there are three parameters, there are only two free parameters, which can be written as μ/σ and x_0/σ (Chhikara, 1988).

If $\mu\leq 0$ it has been shown that $P(k<\infty)=1$. Now if $\mu>0$, then the c.d.f. above is improper with $P(k=\infty)=1-\exp\left(\left(-2x_0\mu\right)/\sigma^2\right)$ (Cox and Miller, 1965). This becomes important for threshold regression as it acknowledges the possibility that some cases may be cured. In other words, a case can have a positive probability that it will not die (or experience whatever event is being modeled). These are sometimes referred to as cure-rate models.

2.2.3.3 Threshold Regression Models (TRMs)

A TRM is based on a FHTM. As mentioned earlier, the standard formulation of threshold regression is based on a generalized Wiener process and an inverse Gaussian FHTM although other threshold regression models can be formulated using other underlying stochastic processes such as a gamma process (Lawless and Crowder, 2004) or an Ornstein-Uhlenbeck process (Aalen and Gjessing, 2004; Erich

and Pennell, 2015). In the standard formulation of threshold regression, the underlying Wiener process is assumed latent. This assumption allows μ and x_0 to be scaled such that $\sigma = 1$. This simplifies the p.d.f. and c.d.f. in (2.54) and (2.55) to

$$f\left(k|\mu, x_o\right) = \frac{x_0}{\sqrt{2\pi k^3}} \exp\left[\frac{\left(x_0 + \mu k\right)^2}{2k}\right] \tag{2.56}$$

and

$$F\left(k|\mu, x_0\right) = \Phi\left[\frac{x_0 + \mu k}{\sqrt{2k}}\right] + \exp\left(-2x_0\mu\right)\Phi\left[\frac{\mu k - x_0}{\sqrt{k}}\right] \tag{2.57}$$

for $-\infty < \mu < \infty$, $x_0 > 0$, where $\Phi\left(\cdot\right)$ is the c.d.f of the standard normal. Both μ and x_0 are linked to the regression covariates z and u, respectively. The identity link function is used for μ as follows:

$$\mu = \beta'\mathbf{z} = \beta_0 + \beta_1 z_1 + \ldots + \beta_h z_h. \tag{2.58}$$

A log-link function is used for x_0 as follows:

$$\ln\left(x_0\right) = \zeta'\mathbf{u} = \zeta_0 + \zeta_1 u_1 + \ldots + \zeta_p u_p. \tag{2.59}$$

It should be noted that $\mathbf{z}$ and $\mathbf{u}$ need not have the same covariates but may overlap. This allows for the capture of different associations for the initial health status and

the drift toward death (or away from death if there is a positive probability of being cured). For the i^{th} case μ, x_0, and k are identified as $\mu^{(i)}$, $x_0^{(i)}$, and $k^{(i)}$. Since not all cases have an observed death, $k^{(i)}$ is right censored. In light of this, each case $i = 1, \ldots, n_1$ when death is observed contributes $f\left(k^{(i)}|\mu^{(i)}, x_0^{(i)}\right)$ to the sample likelihood and each case that is censored $i = n_1 + 1, \ldots, n_1 + n_2$ contributes the survival probability $S\left(k^{(i)}|\mu^{(i)}, x_0^{(i)}\right) = 1 - F\left(k^{(i)}|\mu^{(i)}, x_0^{(i)}\right)$ to the sample likelihood. This gives us the following log-likelihood

$$\ln\left[L\left(\beta, \zeta\right)\right] = \sum_{i=1}^{n_1} \ln\left[f\left(k^{(i)}|\mu^{(i)}, x_0^{(i)}\right)\right] + \ln\left[S\left(k^{(i)}|\mu^{(i)}, x_0^{(i)}\right)\right]. \tag{2.60}$$

Finally, likelihood estimates of β and ζ are obtained as explained in Section 2.3.1.

2.2.4 Cumulative Hazard Estimation

The cumlative hazard $\Lambda\left(t\right)$ has to be estimated for every unit i in the population for a specific time t, where:

$$\Lambda\left(t\right) = \int_0^t \lambda\left(k\right) dk. \tag{2.61}$$

When a parametric time-to-event model, such as Weibull, Lognormal, or Wiener-Process-Based Threshold Regresson, is used, then $\hat{\lambda}_i\left(t|z_i, \hat{\theta}\right)$ is known, and

$\hat{\Lambda}_i\left(t|z_i,\hat{\theta}\right)$ can be estimated as follows:

$$\hat{\Lambda}_i\left(t|z_i,\hat{\theta}\right)=\int_0^k \hat{\lambda}_i\left(k|z_i,\hat{\theta}\right)dk. \tag{2.62}$$

A standard way to estimate the baseline hazard is to use a Breslow type estimator (Breslow, 1972, 1974; Lin, 2007). The Breslow type estimator was first proposed for the PHM by Breslow (1972, 1974), then it was adapted to the AFTM by Tsiatis (1990).

The Breslow type estimator is based on the Nelson-Aalen estimator (Aalen et al., 2008). The Nelson-Aalen estimator is a nonparametric estimator of the cumulative hazard.

$$\hat{\Lambda}\left(t\right)=\sum_{i=1}^{n}\frac{1_{(T_i\leq t)}\left(1-c_i\right)}{\sum_{j=1}^{n}1_{(T_j\geq T_i)}}, \tag{2.63}$$

where T_i is the time of event, or the censoring time for unit i, and c_i is the censoring indicator (Nelson, 1969, 1972). The Nelson-Aalen estimator can now be expressed in counting process notation as

$$\hat{\Lambda}\left(t\right)=\sum_{i=1}^{n}\int_0^t\frac{d\mathcal{N}_i(k)}{\sum_{j=1}^{n}Y_j(k)}, \tag{2.64}$$

where $\mathcal{N}_i(t)$ is the counting process which counts the number of events observed by

the i^{th} unit by time t, and $Y_i(t)$ is an indicator that unit i is at risk at time t (Aalen, 1975, 1978).

This section only focuses on the PHM. For the PHM, $\hat{\Lambda}_i(t|z_i, \hat{\theta})$ now simplifies to

$$\begin{aligned}\hat{\Lambda}_i\left(t|z_i,\hat{\theta}\right) &= \int_0^t \hat{\lambda}_i\left(k|z_i,\hat{\theta}\right)dk \\ &= \int_0^t \lambda_0\left(k\right)e^{z_i\hat{\theta}}dk \\ &= e^{z_i\hat{\theta}}\int_0^t \lambda_0\left(k\right)dk \\ &= e^{z_i\hat{\theta}}\hat{\Lambda}_{i,0}\left(t\right).\end{aligned} \tag{2.65}$$

Estimate θ using the partial likelihood method proposed by Cox (1972, 1975), i.e., maximizing

$$l_p\left(\theta\right) = \prod_{i=1}^{n}\left[\frac{e^{\theta' z_i}}{\sum_{j=1}^{n} Y_j\left(\theta, t_i\right)e^{\theta' z_j}}\right]^{(1-c_i)}. \tag{2.66}$$

Using the $\hat{\theta}$ which maximizes the partial likelihood, we get the Breslow type estimator of

$$\hat{\Lambda}_{i,0}\left(t\right) = \sum_{i=1}^{n}\int_0^t \frac{d\mathcal{N}_i(k)}{\sum_{j=1}^{n} Y_j(k)e^{z_i\hat{\theta}}}. \tag{2.67}$$

If θ and Λ_0 are estimated simultaneously using the maximum likelihood framework,

then the joint likelihood of θ and Λ_0 is

$$L(\theta, \Lambda_0) = \prod_{i=1}^{n} \left[e^{z_i \theta} \lambda_0 (T_i) \right]^{(1-c_i)} \exp \left[- \int_0^{T_i} e^{z_i \theta} \lambda_0 (t) \, dt \right]. \tag{2.68}$$

If $\lambda_0(\cdot)$ is assumed to be piecewise constant between uncensored failure times, then θ and Λ_0 are maximized by the partial maximum likelihood estimates and the Breslow type estimator in equation (2.67) (Tsiatis, 1981; Andersen and Gill, 1982; Lin, 2007).

2.3 Models for Survey Data

The methods presented in Section 2.2 will not provide design consistent estimators of θ. This section will review the adjustments that need to be made to produce design consistent parameter estimates for the time-to-event models presented in Section 2.2.

2.3.1 Pseudo Maximum Likelihood Estimation (PMLE)

Pseudo Maximum Likelihood Estimation (PMLE) is the standard method in the complex survey literature to estimate design consistent estimates of regression parameters. In the context of survival data, the PMLE method has been used for the Weibull AFTM model (Lawless, 2003a) and for TRMs (Li et al., 2015).

2.3.1.1 Parameter Estimation Using PMLE

It is useful to discuss MLE where the likelihood function can be constructed from a p.d.f. for a finite population as follows:

$$L(\theta) = \prod_{i=1}^{N} f(\mathbf{x_i}|\theta) \tag{2.69}$$

The full sample MLEs can be obtained by maximizing the log-likelihood, which converts the product of the p.d.f.'s above into a sum of log p.d.f.'s:

$$\ln[L(\theta)] = \sum_{i=1}^{N} \ln[f(\mathbf{x_i}|\theta)] \tag{2.70}$$

The maximization of the log-likelihood can be achieved by solving estimating equations. If we define $u_i(\theta) = \frac{\partial}{\partial\theta} \ln[f(\mathbf{x_i}|\theta)]$, then the full finite population estimating equations are defined as

$$U(\theta) = \sum_{i=1}^{N} u_i(\theta) = 0. \tag{2.71}$$

Note that the p.d.f. needs to be differentiable for these estimating equations to exist. The solution to these estimating equations that maximize $L(\theta)$ is referred to as the population MLE ($\hat{\theta}_{\mathbf{MLE}}$). This maximization might be found using the

Newton-Raphson method when second moments of $U(\theta)$ exist.

For a probability sample selected from a finite population with the selection probability π_i for the i^{th} unit, PMLEs can be obtained. This is done by constructing a likelihood function from the sample as follows:

$$L_{\pi_i}(\theta) = \prod_{i=1}^{n} f(\mathbf{x_i}|\theta)^{\pi_i^{-1}}, \tag{2.72}$$

because conceptually the i^{th} unit represents π_i^{-1} units in the finite population. The contribution of the i^{th} unit to the likelihood is $f(\mathbf{x_i}|\theta)^{\pi_i^{-1}}$. It follows that the log-likelihood becomes

$$\ln[L_\pi(\theta)] = \sum_{i=1}^{n} \pi_i^{-1} \ln[f(\mathbf{x_i}|\theta)], \tag{2.73}$$

and the estimating equations are now

$$\hat{U}(\theta) = \sum_{i=1}^{n} \pi_i^{-1} u_i(\theta) = 0. \tag{2.74}$$

The solution to this system of estimating equations $\hat{U}(\theta)$ is the PMLE ($\hat{\theta}_{\mathbf{PMLE}}$) (Skinner, 1989; Fuller, 2011). It should be noted that the estimating equations are π-estimators of totals (Särndal et al., 1992), So, for sample designs with design-consistent π-estimators of totals, $\hat{U}(\theta)$ is a consistent estimator for $U(\theta)$. Note that this can be generalized to incorporate weights other than π_i^{-1} which induce

design-consistent estimators of totals and thus $\hat{U}(\theta)$ is a consistent estimator for $U(\theta)$ (Fuller, 2011). It also should be noted that the $\hat{\theta}_{\mathbf{PMLE}}$ is not an exact MLE in that it does not share some of the asymptotic properties of MLE, such as efficiency. Additionally, $\hat{\theta}_{\mathbf{PMLE}}$ is not generally unique like a MLE, since there can be more than one consistent estimator of $U(\theta)$ (Skinner, 1989).

2.3.1.2 Variance Estimation Using PMLE

Again, it is useful to first discuss MLE. The standard asymptotic estimator $V\left(\hat{\theta}_{\mathbf{MLE}}\right)$, the covariance matrix of $\hat{\theta}_{MLE}$, is the inverse of the full sample information matrix $I\left(\hat{\theta}_{\mathbf{MLE}}\right)^{-1}$, where

$$I(\theta) = -\frac{\partial}{\partial\theta}\ln\left[L(\theta)\right] = \frac{\partial U(\theta)}{\partial\theta}. \tag{2.75}$$

An estimator for $V\left(\hat{\theta}_{\mathbf{MLE}}\right)$ that is more robust to some types of model misspecification is the sandwich estimator defined as

$$\hat{V}\left(\hat{\theta}_{\mathbf{MLE}}\right) = \hat{I}\left(\hat{\theta}_{\mathbf{MLE}}\right)^{-1}\hat{V}_L\left(U\left(\hat{\theta}_{\mathbf{MLE}}\right)\right)\hat{I}\left(\hat{\theta}_{\mathbf{MLE}}\right)^{-1}, \tag{2.76}$$

where $\hat{V}_L\left(U\left(\hat{\theta}_{\mathbf{MLE}}\right)\right)$ is the linearized variance estimator of $U(\hat{\theta})$, which is calculated

as follows

$$\hat{V}_L\left(U\left(\hat{\theta}_{\mathbf{MLE}}\right)\right) = N\sum_{i=1}^{N} u_i\left(\hat{\theta}_{\mathbf{MLE}}\right)u_i\left(\hat{\theta}_{\mathbf{MLE}}\right)', \tag{2.77}$$

Note that both of these estimators assume independent observations (Skinner, 1989). This can be generalized to PMLE. In the case of $\hat{\theta}_{\mathbf{PMLE}}$, Taylor series linearization is needed to incorporate the complex sample design into the estimation of the covariance matrix $\hat{V}\left(\hat{\theta}_{\mathbf{PMLE}}\right)$ using the sandwich estimator. This is done by substituting $\hat{U}\left(\theta\right)$ for $U\left(\theta\right)$ to get

$$\hat{V}\left(\hat{\theta}_{\mathbf{PMLE}}\right) = \hat{I}\left(\hat{\theta}_{\mathbf{PMLE}}\right)^{-1}\hat{V}_L\left(\hat{U}\left(\hat{\theta}_{\mathbf{PMLE}}\right)\right)\hat{I}\left(\hat{\theta}_{\mathbf{PMLE}}\right)^{-1}, \tag{2.78}$$

where

$$\hat{I}\left(\theta\right) = -\frac{\partial}{\partial\theta}\hat{U}\left(\theta\right), \tag{2.79}$$

and $\hat{V}_L(\hat{U}(\hat{\theta}_{\mathbf{PMLE}}))$ is the linearized variance estimator of $\hat{U}\left(\theta\right)$ given the complex design. Using sample estimates in (2.79) when computing the Fisher information has better conditional properties when compared to using expected values (Efron and Hinkley, 1978). It also should be noted that this is not the only way to obtain design-consistent variance estimates (Skinner, 1989). Replication methods could also be used. For more information about replication methods, see (Wolter, 1985).

The design consistency of $\hat{\theta}_{PMLE}$, asymptotics and design consistency of the

sandwich estimator are discussed by Binder (1983) and Fuller (2011).

2.3.2 Semiparametric Time-to-Event Models

The estimation of design consistent estimators of θ is fairly straightforward and similar to PMLE, but the steps to show design consistency are more involved than PMLE.

2.3.2.1 PHM

The estimating equations for the PHM discussed in section 2.2.1 can be expressed for a finite population with N cases as

$$U(\theta) = \sum_{i=1}^{N} c_i \left[z_i - \frac{S^{(1)}(\theta, t)}{S^{(0)}(\theta, t)} \right], \tag{2.80}$$

where

$$S^{(0)}(\theta, t) = N^{-1} \sum_{i=1}^{N} Y_i(\theta, t)\, e^{\theta' z_i}, \quad S^{(1)}(\theta, t) = N^{-1} \sum_{i=1}^{N} Y_i(\theta, t)\, e^{\theta' z_i} z_i. \tag{2.81}$$

Within the PMLE context, probabilities of selection, π_i, can be inserted to adjust for only observing the sample

$$\hat{U}(\theta) = \sum_{i=1}^{n} \pi_i^{-1} c_i \left[z_i - \frac{\hat{S}^{(1)}(\theta, t)}{\hat{S}^{(0)}(\theta, t)} \right], \tag{2.82}$$

where

$$\hat{S}^{(0)}(\theta, t) = N^{-1} \sum_{i=1}^{n} \pi_i^{-1} Y_i(\theta, t) e^{\theta' z_i}, \quad \hat{S}^{(1)}(\theta, t) = N^{-1} \sum_{i=1}^{n} \pi_i^{-1} Y_i(\theta, t) e^{\theta' z_i} z_i \tag{2.83}$$

The solution to the system of equations $\hat{U}(\theta) = 0$, where $\hat{U}(\theta)$ is defined in (2.82), is the design consistent estimator of θ. The design consistency of $\hat{\theta}$ does not follow from the PMLE case.

Equations (2.81) and (2.82) can be written in terms stochastic integrals as follows:

$$U(\theta) = \sum_{i=1}^{N} \int_{-\infty}^{\infty} \left[z_i - \frac{S^{(1)}(\theta, t)}{S^{(0)}(\theta, t)} \right] d\mathcal{N}_i(t), \tag{2.84}$$

and

$$\hat{U}(\theta) = \sum_{i=1}^{N} \int_{-\infty}^{\infty} \frac{\xi_i}{\pi_i} \left[z_i - \frac{\hat{S}^{(1)}(\theta, t)}{\hat{S}^{(0)}(t)} \right] d\hat{\mathcal{N}}_i(t), \tag{2.85}$$

where $\mathcal{N}_i(t) = c_i 1_{\{t_i \leq t\}}$ is the standard counting process, $\hat{\mathcal{N}}(t) = \sum_{i=1}^{n} \pi_i^{-1} \mathcal{N}_i(t)$ is the π-estimator of $\mathcal{N}(t) = \sum_{i=1}^{N} \mathcal{N}_i(t)$, and ξ_i is 1 if the i^{th} unit is in sample and 0

if it is not.

It is necessary to define the following terms:

$$\begin{aligned} s^{(0)}(\theta,t) &= \lim_{N\to\infty} S^{(0)}(\theta,t), \quad s^{(1)}(\theta,t) = \lim_{N\to\infty} S^{(1)}(\theta,t) \\ g(t) &= \lim_{N\to\infty} \mathcal{N}(t), \quad a = \lim_{N\to\infty} N^{-1} \sum_{i=1}^{N} \int_0^{\infty} z_i(t) d\hat{\mathcal{N}}_i(t). \end{aligned} \tag{2.86}$$

Now $N^{-1}U(\theta)$ converges to

$$u(\theta) = a - \int_0^{\infty} \frac{s^{(1)}(\theta,t)}{s^{(0)}(\theta,t)} dg(t), \tag{2.87}$$

which is also the probability limit of $N^{-1}\hat{U}(\theta)$ (Lin, 2000). It now can be shown using Lemma 3.1 in Andersen and Gill (1982) that $\hat{\theta}$ is a consistent estimator for θ and that $\hat{\theta}$ and θ converge to the same limit (Lin, 2000).

Lin (2000) also provides a design consistent estimator of the baseline hazard:

$$\hat{\Lambda}_{i,0}\left(t,\hat{\theta}\right) = \sum_{i=1}^{n} \int_0^{t} \frac{d\hat{\mathcal{N}}_i(t)}{\hat{S}^{(1)}(\theta,t)}. \tag{2.88}$$

The consistency of this estimator follows given the consistency of $\hat{\theta}$, $\hat{S}^{(1)}(\theta,t)$,and $\hat{\mathcal{N}}(t)$ Lin (2000).

2.3.2.2 AFTM

Design consistent estimation of semiparametric AFTMs has not appeared in the survey literature. There has been work in the case control literature (Kong et al., 2004; Kong and Cai, 2009; Chiou et al., 2014, 2015) for a simple random sample, stratified simple random sample, and stratified simple random cluster sample. These methods use weighted estimating equations similar to the method used by Binder (1992) and Lin (2000). A weighted sandwich estimator is proposed for estimating the variance covariance matrix. The asymptotics are only worked out for the superpopulation based on the work of Hájek (1960, 1964) and not for design consistency.

Although these methods could possibly be altered for the complex survey context, additional theory will need to be developed to show design consistency akin to the work of Binder (1983, 1992) and Lin (2000).

Chapter 3: Theory

In this chapter, I develop the theory for model-assisted GDEs and MCEs for time-to-event data. This chapter provides the asymptotic results for GDEs and MCEs estimators constructed using time-to-event models.

As noted earlier the proportion of a given population that has experienced an event by time t can be estimated using a π-estimator as follows:

$$\hat{p}_{\pi}(t) = N^{-1} \sum_{i \in s} \pi_i^{-1} I_{\{T_i \leq t\}}, \tag{3.1}$$

where N is the size of the finite population, s is the set of units sampled from the population, π_i is the probability of selection for unit i, and T_i is the time at which the event happened. As previously mentioned, the survey closes out before all units have experienced a given event, then T_i is only observed for T_i less than or equal to the time of observation t_o. This means that T_i is right censored for units for which $T_i \leq t_o$, and, when $t > t_o$, the π-estimator cannot be used to estimate $p(t)$.

Additionally this estimator cannot be used if any units are censored before time t.

From a modeling perspective, if auxiliary variables are available, then there are two approaches that can be used to estimate $p(t)$. One approach is to fit a model to predict $I_{\{T_i \leq t\}}$ directly using a binary response model such as a logistic model. However, when T is right censored, this approach will only work for $t \leq t_o$. Additionally, it estimates $p(t)$ for only one value of t, thus a model needs to be estimated for each desired t. Another approach is to use a model to predict T_i, then estimate $p(t)$ using the predicted T_i's. This can be done by noting that $p(t)$ is the cumulative distribution function of T ($F(t)$), so estimating $p(t)$ is equivalent to estimating $F(t)$. Thus $p(t)$ can be estimated by the empirical distribution function of the predicted T_i's. Using this approach of modeling T when T is censored requires the use of time-to-event models which can account for the censoring. There are two benefits of using this approach instead of modeling $I_{\{T_i \leq t\}}$ directly. The first is that only one model needs to be fit to obtain an estimate of $p(t)$ for a $t \geq 0$. The second is that the estimation of $p(t)$ is not limited to cases where $t \leq t_o$.

These two modeling approaches can be used to construct GDE and Model Calibrated (MCE) estimators (Wu and Sitter, 2001) to estimate $p(t)$ for a given $t \leq t_o$. The work of Wu and Sitter (2001) and Kennel (2013), discussed in Chapter 2, can be used to construct GDE and MCE estimators of $p(t)$ when GLMs are used

to model $I_{\{T_i \leq t\}}$ or T. As mentioned above, a GLM cannot be used to predict T and estimate $p(t)$ with the empirical distribution function of the T_i's if T is censored.

Standard time-to-event models such as a PHMs can be used to develop GDEs and MCEs for predicting $p(t)$ for $t \leq t_o$. Here, AFTMs, THMs based on an Inverse-Gaussian FHTMs, and both parametric and semiparametric PHMs are considered. These models can be used to directly estimate $p(t)$. The PHMs and AFTMs model the hazard function $\lambda(t)$, then a standard transformation is used to estimate $p(t)$. In the case of the TRMs, the underlining stochastic process which generates T is modeled to estimate $p(t)$. More about these models can be found in Chapter 2.

This chapter is laid out as follows: Section 3.1 reviews how $p(t)$ can be estimated using PHMs, AFTMs, and TRMs; Section 3.2 presents the Generalized Difference and Model Calibrated point and variance estimators constructed using time-to-event models; finally, Section 3.3 provides the asymptotic results for the Generalized Difference and Model Calibrated point and variance estimators.

3.1 Estimating $p(t|Z)$

A standard use of time-to-event models is to predict the failure probability $p(t)$ for an individual at some time t given some vector of covariates Z. The survival failure probability can be estimated using the time-to-event models discussed in

Chapter 2. PHMs and AFTMs model time-to-event data through the hazard function $\lambda(t|\theta, Z)$. Once the hazard function is estimated, since we know that $p(t) = F(t)$, $F(t) = 1 - S(t)$, $S(t) = \exp(-\Lambda(t))$, and $\Lambda(t) = \int_0^t \lambda(t)$, $p(t|Z)$ can be estimated as follows:

$$p(t|\hat{\theta}, Z) = 1 - \exp\left(-\int_0^t \lambda(t|\hat{\theta}, Z)dt\right). \tag{3.2}$$

For TRMs based on an Inverse-Gaussian FHTM, $p(t)$ can be estimated using the c.d.f. of the Inverse-Gaussian distribution as follows:

$$p(t|\hat{\mu}, \hat{x}_0, M, U) = \Phi\left[\frac{\hat{x}_0 + \hat{\mu}t}{\sqrt{2t}}\right] + \exp\left(-2\hat{x}_0\hat{\mu}\right)\Phi\left[\frac{\hat{\mu}t - \hat{x}_0}{\sqrt{t}}\right], \tag{3.3}$$

where $\hat{\mu} = \hat{\beta}'\mathbf{m}$ and $\hat{x_0} = \exp(\hat{\zeta}'\mathbf{u})$. In this formulation $\theta = (\beta, \zeta)$ and $Z = [M, U]$. As noted in Chapter 2, $\mathbf{m}$ and $\mathbf{u}$ need not have the same covariates but may overlap. Details on how β and ζ are estimated can be found in Chapter 2, Section 2.2.3.

3.2 Time-to-Event GDEs and MCEs

Following the formulation found in Wu and Sitter (2001), GDEs and MCEs can be constructed using the estimates of $p(t)$ found in Section 3.1 as follows:

$$\hat{p}_{GDE}(t) = N^{-1}\left(\sum_{i=1}^{N} p(t|z_i, \hat{\theta}) + \sum_{i\in s} \pi_i^{-1}\left[I_{\{T_i \le t\}} - p(t|z_i, \hat{\theta})\right]\right) \tag{3.4}$$

and

$$\hat{p}_{MCE}(t) = \hat{p}_{\pi}(t) + N^{-1}\left(\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) - \sum_{i\in s}\pi_i^{-1}p(t|z_i,\hat{\theta})\right)\hat{B}, \tag{3.5}$$

where $\hat{B}$ is a calibration adjustment similar to what is proposed by Wu and Sitter (2001) and discussed in Chapter 2.

Two calibration adjustments will be considered which are adapted from Wu and Sitter (2001). The first adjustment, $\hat{B}$, is derived subject to the following constraints:

$$\sum_{i\in s} w_i = N, \text{ and} \tag{3.6}$$

$$\sum_{i\in s} w_i p(t|z_i,\hat{\theta}) = \sum_{i=1}^{N} p(t|z_i,\hat{\theta}). \tag{3.7}$$

$\hat{B}$ can be calculated as

$$\hat{B} = \frac{\sum_{i\in s}\pi_i^{-1}\left(p(t|z_i,\hat{\theta}) - \bar{p}\right)\left(I_{\{T_i\le t\}} - \bar{I}\right)}{\sum_{i\in s}\pi_i^{-1}\left(p(t|z_i,\hat{\theta}) - \bar{p}\right)^2}, \tag{3.8}$$

where $\bar{I} = \sum_{i\in s}\pi_i^{-1}I_{\{T_i\le t\}}/\sum_{i\in s}\pi_i^{-1}$, and $\bar{p} = \sum_{i\in s}\pi_i^{-1}p(t|z_i,\hat{\theta})/\sum_{i\in s}\pi_i^{-1}$. The second adjustment, $\hat{B}^*$, which can also be used in (3.5), is derived subject to only one constraint:

$$\sum_{i\in s} w_i p(t|z_i,\hat{\theta}) = \sum_{i=1}^{N} p(t|z_i,\hat{\theta}). \tag{3.9}$$

$\hat{B}^*$ can be calculated as

$$\hat{B}^* = \frac{\sum_{i \in s} \pi_i^{-1} p(t|z_i, \hat{\theta}) I_{\{T_i \leq t\}}}{\sum_{i \in s} \pi_i^{-1} \left(p(t|z_i, \hat{\theta}) \right)^2}. \tag{3.10}$$

Similar to Theorem 2 in Chapter 2, the asymptotic design variance of $\hat{p}_{GDE}(t)$ is

$$V\left(\hat{p}_{GDE}(t)\right) \doteq N^{-2} \sum_{i<j}^{N} \left(\pi_i \pi_j - \pi_{ij}\right) \left(\frac{e_i}{\pi_i} - \frac{e_j}{\pi_j} \right), \tag{3.11}$$

where π_{ij} is the joint probability of selecting the i^{th} and j^{th} units and $e_i = I_{\{T_i \leq t\}} - p(t|z_i, \theta_N)$. This can be estimated by

$$\hat{V}\left(\hat{p}_{GDE}(t)\right) \doteq N^{-2} \sum_{i<j}^{N} \left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \right) \left(\frac{\hat{e}_i}{\pi_i} - \frac{\hat{e}_j}{\pi_j} \right), \tag{3.12}$$

where $\hat{e}_i = I_{\{T_i \leq t\}} - p(t|z_i, \hat{\theta})$. The asymptotic variance $\hat{p}_{MCE}(t)$ is obtained by setting $e_i = I_{\{T_i \leq t\}} - (p(t|z_i, \theta_N))B_N$ in equation (3.11), and the estimated asymptotic variance is obtained by setting $\hat{e}_i = I_{\{T_i \leq t\}} - (p(t|z_i, \hat{\theta}))\hat{B}$ in equation (3.12), where $\hat{B}$ is given by either (3.8) or (3.10). Theory for these estimators is provided in the next section.

3.3 Theoretical Results

This section provides asymptotic results for both the $\hat{p}_{GDE}(t)$ and $\hat{p}_{MCE}(t)$ estimators and their respective variance estimators, where the underlying model is a time-to-event model. Specifically, results are shown for parametric AFTM, TRM based on an Inverse-Gaussian FHTM, and both parametric and semiparametric PHMs. For the semi-parametric PHMs, I address the case where the baseline hazard is estimated using a Breslow type estimator (Breslow, 1972, 1974; Lin, 2007). It will be shown that both $\hat{p}(t)_{GDE}$ and $\hat{p}(t)_{MCE}$ are design consistent, and that the asymptotic variance estimators $V[\hat{p}_{GDE}(t)]$ and $V[\hat{p}_{MCE}(t)]$ are design consistent.

3.3.1 Design Consistency of $\hat{p}_{GDE}(t)$ and $\hat{p}_{MCE}(t)$

To prove design consistency of $\hat{p}_{GDE}(t)$ for a fixed t, assume that if, for a sequence of populations indexed by j in which both the sample size n_j and the population size N_j approach infinity as $j \to \infty$, then:

(i) $\hat{\theta} = \theta_N + O_p\left(n^{-1/2}\right)$ and $\theta_N \to \theta$, where θ_N is the finite population values of the parameter, and θ is its underlying constant value;

(ii) for each z_i and a fixed t, $\partial p(t|z_i, \gamma)/\partial\gamma$, where γ is one of the components of

θ, is continuous in γ, and $|\partial p(t|z_i,\gamma)/\partial\gamma| \leq g(t,z_i,\theta)$ for all values γ in a neighborhood of θ and $N^{-1}\sum_{i=1}^{N} g(t,z_i,\theta) = O(1)$; and

(iii) the basic design weights, $d_i = \pi_i^{-1}$, satisfy that the π-estimators for certain population means are asymptotically normally distributed.

These are similar to the assumptions made by Wu and Sitter (2001) and Kennel (2013).

In the context of the parametric PHM, AFTM, and TRM, assumption (i) follows from the same argument as found in Wu (1999) since $\hat{\theta}$ is estimated using PMLE. For the semiparametric form of the PHM, where $\hat{\theta}$ is estimated using the method found in Binder (1992) and Lin (2000), (i) follows from the fact that $N^{1/2}(\hat{\theta}-\theta)$ is asymptotically zero-mean normal (Lin, 2000). It should be noted that the normality of $N^{1/2}(\hat{\theta}-\theta)$ has only been shown for one-stage Bernoulli and stratified simple random sample designs (Lin, 2000).

Asumption (ii) that for each z_i and a fixed t $\partial p(t|z_i,\gamma)/\partial\gamma$ is continuous in γ is also reasonable. For the TRM model, it is clear that $\partial p(t|z_i,\gamma)/\partial\gamma$ is continuous in γ in a neighborhood of $\theta = (\beta,\zeta)$ since

$$p(t|\hat{\mu},\hat{x}_0) = \Phi\left[\frac{\hat{x}_0+\hat{\mu}t}{\sqrt{2t}}\right] + \exp(-2\hat{x}_0\hat{\mu})\,\Phi\left[\frac{\hat{\mu}t-\hat{x}_0}{\sqrt{t}}\right], \tag{3.13}$$

where $\hat{\mu} = \hat{\beta}'\mathbf{z}$ and $\hat{x_0} = \exp(\hat{\zeta}'\mathbf{u})$.

For the parametric AFTM, since $\lambda(t|z,\hat{\theta}) = \lambda_0(te^{\hat{\theta}'z})e^{\hat{\theta}'z}$,

$$\begin{aligned} p(t|\hat{\theta}, Z) &= 1 - \exp\left(-\int_0^t \lambda(k|\hat{\theta}, Z)dk\right) \\ &= 1 - \exp\left(-\int_0^t \lambda_0(ke^{\hat{\theta}'z})e^{\hat{\theta}'z}dk\right) \\ &= 1 - \exp\left(-\Lambda_0(te^{\hat{\theta}'z})\right), \end{aligned} \tag{3.14}$$

where $\Lambda_0(t) = \int_0^t \lambda_0(k)dk$ is the cumulative hazard function. So for $\partial p(t|z_i,\gamma)/\partial\gamma$ to be continuous in γ, $\partial\Lambda_0(ke^{\hat{\gamma}'z})/\partial\gamma$ must be continuous in γ, which is true for most standard parametric AFTM model formulations.

For parametric PHMs, $\lambda(t|z,\hat{\theta}) = \lambda_0(t)e^{\hat{\theta}'z}$, and

$$\begin{aligned} p(t|\hat{\theta}, Z) &= 1 - \exp\left(-\int_0^t \lambda(k|\hat{\theta}, Z)dk\right) \\ &= 1 - \exp\left(-\int_0^t \lambda_0(k)e^{\hat{\theta}'z}dk\right) \\ &= 1 - \exp\left(-\Lambda_0(t)e^{\hat{\theta}'z}\right). \end{aligned} \tag{3.15}$$

Since $\Lambda_0(t)$ does not depend on $\hat{\theta}$, it is generally true that $\partial p(t|\hat{\gamma}, Z)/\partial\gamma$ is continuous in γ.

For a semiparametric PHM estimating $\Lambda_0(t)$ using the Breslow estimator, $\Lambda_0(t)$

is now dependent on θ. The Breslow estimator is:

$$\Lambda_0(t,\hat{\theta}) = \sum_{i\in s} \frac{\pi_i^{-1}\Delta_i I_{\{T_i\leq t\}}}{\sum_{j\in s}\pi_j^{-1}Y_j(t)e^{\hat{\theta}'z_j}}, \tag{3.16}$$

where $Y_j(t)$ indicates if the j^{th} unit is at risk at time t. Now $\partial\Lambda_0(t,\hat{\gamma})/\partial\gamma$ is continuous in γ, since $\Lambda_0(t,\hat{\gamma})$ is the sum of a known number of fractions in which the numerator is constant and the denominator is the sum $\sum_{j\in s}\pi_j^{-1}Y_j(t)e^{\gamma' z_j}$, which is differentiable with respect to γ.

Since $\partial\Lambda_0(t,\hat{\gamma})/\partial\gamma$ is continuous in γ, it is clear that $\partial p(t|\hat{\gamma},Z)/\partial\gamma$ is continuous in γ, since

$$\begin{aligned} p(t|\hat{\theta},Z) &= 1-\exp\left(-\int_0^t \lambda(k|\hat{\theta},Z)dk\right) \\ &= 1-\exp\left(-\int_0^t \lambda_0(k)e^{\hat{\theta}'z}dk\right) \\ &= 1-\exp\left(-\Lambda_0(t,\hat{\theta})e^{\hat{\theta}'z}\right). \end{aligned} \tag{3.17}$$

Finally, assumption (iii) that the basic design weights, $d_i = \pi^{-1}$, satisfy that the π-estimators for certain population means are asymptotically normally distributed, is true for common sample designs, including simple random sample and stratified simple random sampling with or without replacement, and multistage designs in which the first-stage units are selected with replacement.

Theorem 3. *If $\hat{p}(t)_{GDE}$ is constructed using a time-to-event model where (i)-(iii) hold, then for a fixed time t*

$$\hat{p}(t)_{GDE} = \hat{p}_{\pi}(t) + O_p\left(n^{-1/2}\right), \tag{3.18}$$

where $\hat{p}_{\pi}(t)$ is the π-estimator of the finite population proportion $p_N(t)$. Thus $\hat{p}_{GDE}(t)$ is design consistent.

Proof. Since (3.4) can be rewritten as

$$\hat{p}_{GDE}(t) = \hat{p}_{\pi}(t) + N^{-1}\left(\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) - \sum_{i\in s} d_i p(t|z_i,\hat{\theta})\right), \tag{3.19}$$

it suffices to show that

$$\left(N^{-1}\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) - N^{-1}\sum_{i\in s} d_i p(t|z_i,\hat{\theta})\right) = O_p\left(n^{-1/2}\right). \tag{3.20}$$

Now using assumptions (i) and (ii) and applying a Taylor series approximation to $p(t|z_i,\hat{\theta})$ at $\hat{\theta} = \theta_N$, we get

$$p(t|z_i,\hat{\theta}) = p(t|z_i,\theta_N) + \left[\frac{\partial p(t|z_i,\gamma)}{\partial\gamma}|_{\theta^*}\right]'(\hat{\theta} - \theta_N), \tag{3.21}$$

where $\theta^* \in (\hat{\theta}, \theta_N)$ or $(\theta_N, \hat{\theta})$. Now by (2.8) and assumptions (i) and (ii),

$$N^{-1} \sum_{i=1}^{N} p(t|z_i, \hat{\theta}) = N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N) + O_p\left(n^{-1/2}\right), \tag{3.22}$$

and

$$N^{-1} \sum_{i \in s} d_i p(t|z_i, \hat{\theta}) = N^{-1} \sum_{i \in s} d_i p(t|z_i, \theta_N) + O_p\left(n^{-1/2}\right). \tag{3.23}$$

Note that because of condition (iii)

$$N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \theta_N) = O_p\left(n^{-1/2}\right) \tag{3.24}$$

Now by putting together (3.22), (3.23), and (3.24), we get

$$\left(N^{-1} \sum_{i=1}^{N} p(t|z_i, \hat{\theta}) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \hat{\theta}) \right) = O_p\left(n^{-1/2}\right), \tag{3.25}$$

as desired. □

It should be noted that $\hat{p}_{GDE}(t)$ is a special case of $\hat{p}_{MCE}(t)$, where $\hat{B}_N = 1$. Because of this, Theorem 3 can be generalized to show that $\hat{p}_{MCE}(t)$ and $\hat{p}^*_{MCE}(t)$ are design consistent by noting that $\hat{B}_N$ and $\hat{B}^*_N$ are both $O_P(1)$.

3.3.2 Design Consistency of $\hat{V}[\hat{p}_{GDE}(t)]$ and $\hat{V}[\hat{p}_{MCE}(t)]$

To show design consistency of the variance estimators an additional is necessary condition:

(iv) for each z_i, $\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'$, where γ is one of the components of θ, is continuous in γ, $|\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'| \leq h(z_i,\theta)$ for γ in a neighborhood of θ, and $N^{-1}\sum_{i=1}^{N} h(x_i,\theta) = O(1)$.

The assumption that for each z_i $\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'$ is continuous in γ is reasonable. For the TRM it is clear that $\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'$ is continuous in γ since

$$p(t|\hat{\mu},\hat{x}_0) = \Phi\left[\frac{\hat{x}_0+\hat{\mu}t}{\sqrt{2t}}\right] + \exp(-2\hat{x}_0\hat{\mu})\,\Phi\left[\frac{\hat{\mu}t-\hat{x}_0}{\sqrt{t}}\right], \tag{3.26}$$

where $\hat{\mu} = \hat{\beta}'\mathbf{z}$ and $\hat{x_0} = \exp(\hat{\zeta}'\mathbf{u})$. For the parametric AFTM, $\lambda(t|z,\hat{\theta}) = \lambda_0(te^{\hat{\theta}'z})e^{\hat{\theta}'z}$ and $p(t|\hat{\theta},\mathbf{z})$ is given by (3.14). So for $\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'$ to be continuous in γ, $\partial^2\Lambda_0(ke^{\hat{\gamma}'z})/\partial\gamma\partial\gamma'$ must be continuous in γ, which is true for most standard parametric AFTM formulations. For parametric PHMs $\lambda(t|z,\hat{\theta}) = \lambda_0(t)e^{\hat{\theta}'z}$ and $p(t|\hat{\theta},\mathbf{z})$ is given by (3.15). Since $\Lambda_0(t)$ does not depend on $\hat{\theta}$, it is generally true that $\partial^2 p(t|z_i,\gamma)/\partial\gamma\partial\gamma'$ is continuous in γ. Finally, for semiparametric PHMs, estimating $\Lambda_0(t)$ using the Breslow estimator (3.16), $\Lambda_0(t)$ is now dependent on θ. Now $\partial^2\Lambda_0(t,\hat{\gamma})/\partial\gamma$ is continuous in γ since $\Lambda_0(t,\hat{\gamma})$ is the sum

of a known number of fractions for which the numerator is constant and the denominator is the sum $\sum_{j\in s}\pi_j^{-1}Y_j(t)e^{\gamma' z_j}$, which is twice differentiable with respect to θ.

Since $\partial^2\Lambda_0(t,\hat{\gamma})/\partial\gamma\partial\gamma'$ is continuous in γ, it is clear that $\partial^2 p(t|\hat{\gamma},Z)/\partial\gamma\partial\gamma'$ is continuous in γ considering the form of $p(t|\hat{\theta},\mathbf{z})$ in (3.16).

Theorem 4. *If $\hat{p}_{GDE}(t)$ is constructed using a time-to-event model where (i) $\sim$ (iv) hold then for a fixed time t, then the approximate design variance estimator of $\hat{p}_{GDE}(t)$ is*

$$V\left(\hat{p}_{GDE}(t)\right) \doteq N^{-2}\sum_{i<j}^{N}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{e_i}{\pi_i}-\frac{e_j}{\pi_j}\right), \tag{3.27}$$

where π_{ij} is the joint probability of selecting the i^{th} and j^{th} units and $e_i = I_{\{T_i\le t\}} - p(t|z_i,\theta_N)$. This can be estimated by

$$\hat{V}\left(\hat{p}_{GDE}(t)\right) \doteq N^{-2}\sum_{i<j}^{s}\left(\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\right)\left(\frac{\hat{e}_i}{\pi_i}-\frac{\hat{e}_j}{\pi_j}\right), \tag{3.28}$$

where $\hat{e}_i = I_{\{T_i\le t\}} - p(t|z_i,\hat{\theta})$.

Proof. Using assumptions (i), (ii), (iv) and applying a Taylor series second order

approximation to $p(t|z_i,\hat{\theta})$ at $\hat{\theta}=\theta_N$, we get

$$\begin{aligned} p(t|z_i,\hat{\theta}) = p(t|z_i,\theta_N) &+ \left[\frac{\partial p(t|z_i,\gamma)}{\partial\gamma}\Big|_{\theta^*}\right]'(\hat{\theta}-\theta_N) \\ &+ (\hat{\theta}-\theta_N)'\left[\frac{\partial^2 p(t|z_i,\gamma)}{\partial\gamma\partial\gamma'}\Big|_{\theta^*}\right](\hat{\theta}-\theta_N), \end{aligned} \tag{3.29}$$

where $\theta^* \in (\hat{\theta},\theta_N)$ or $(\theta_N,\hat{\theta})$ and $\left[\frac{\partial^2 p(t|z_i,\gamma)}{\partial\gamma\partial\gamma'}\Big|_{\theta^*}\right]$ is the $p\times p$ matrix of second derivatives evaluated at θ^*. Now, by (3.29) and assumption (iv),

$$\begin{aligned} N^{-1}\sum_{i=1}^{N} p(t|z_i,\hat{\theta}) =& N^{-1}\sum_{i=1}^{N} p(t|z_i,\theta_N) \\ &+ \left\{N^{-1}\sum_{i=1}^{N}\frac{\partial p(t|z_i,\gamma)}{\partial\gamma}\Big|_{\theta^*}\right\}'(\hat{\theta}-\theta_N) \\ &+ O_p\left(n^{-1}\right) \end{aligned} \tag{3.30}$$

and

$$\begin{aligned} N^{-1}\sum_{i\in s} d_i p(t|z_i,\hat{\theta}) =& N^{-1}\sum_{i\in s} d_i p(t|z_i,\theta_N) \\ &+ \left\{N^{-1}\sum_{i\in s} d_i\frac{\partial p(t|z_i,\gamma)}{\partial\gamma}\Big|_{\theta^*}\right\}'(\hat{\theta}-\theta_N) \\ &+ O_p\left(n^{-1}\right). \end{aligned} \tag{3.31}$$

By assumptions (i) and (iii), $(\hat{\theta} - \theta_N) = O_p\left(n^{-1/2}\right)$, and

$$\left\{ N^{-1} \sum_{i=1}^{N} \frac{\partial p(t|z_i, \gamma)}{\partial \gamma}\Big|_{\theta^*} \right\} - \left\{ N^{-1} \sum_{i \in s} d_i \frac{\partial p(t|z_i, \gamma)}{\partial \gamma}\Big|_{\theta^*} \right\} = O_p\left(n^{-1/2}\right). \tag{3.32}$$

Therefore, by combining (3.30) and (3.31) we get

$$\begin{aligned} N^{-1} \sum_{i=1}^{N} p(t|z_i, \hat{\theta}) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \hat{\theta}) \\ = N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \theta_N) + O_p\left(n^{-1}\right). \end{aligned} \tag{3.33}$$

Using (3.33) to replace $\hat{\theta}$ with θ_N in

$$\hat{p}_{GDE}(t) = \hat{p}_{\pi}(t) + \left(N^{-1} \sum_{i=1}^{N} p(t|z_i, \hat{\theta}) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \hat{\theta}) \right), \tag{3.34}$$

we get

$$\begin{aligned} \hat{p}_{GDE}(t) &= \hat{p}_{\pi}(t) + \left(N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N) - N^{-1} \sum_{i \in s} d_i p(t|z_i, \theta_N) \right) + O_p\left(n^{-1/2}\right) \\ &= N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N) + N^{-1} \sum_{i \in s} d_i \left[I_{\{T_i \le t\}} - p(t|z_i, \theta_N) \right] + O_p\left(n^{-1/2}\right). \end{aligned} \tag{3.35}$$

Finally, by noticing that $N^{-1} \sum_{i=1}^{N} p(t|z_i, \theta_N)$ is constant, the asymptotic

variance of $\hat{p}_{GDE}(t)$ is the asymptotic variance of the π-estimator of the population total of the $e_i = I_{\{T_i \leq t\}} - p(t|z_i, \theta_N)$. It now follows that the asymptotic variance estimator of $\hat{p}_{GDE}(t)$ is the asymptotic variance estimator of a π-estimator of the estimated total of the $\hat{e}_i = I_{\{T_i \leq t\}} - p(t|z_i, \hat{\theta})$. □

Further, Theorem 4 can be generalized by noting that $\hat{B}_N = B_N + o_p(1)$ and $\hat{B}^*_N = B^*_N + o_p(1)$ and substituting $I_{\{T_i \leq t\}} - p(t|z_i, \theta_N)B_N$ or $I_{\{T_i \leq t\}} - p(t|z_i, \theta_N)B^*_N$ for e_i in the variance formula and $I_{\{T_i \leq t\}} - p(t|z_i, \hat{\theta})B_N$ or $I_{\{T_i \leq t\}} - p(t|z_i, \hat{\theta})B^*_N$ into the variance estimator.

Chapter 4: Simulation Study

In the previous chapter, I developed GDEs and MCEs using time-to-event models. In this chapter, I conduct a simulation study to evaluate the performance of these estimators by manipulating the:

- Correlation between $ln(T)$ and predictor Z,
- Distribution of T, the time to an event,
- Amount of censoring, $\%C$,
- Sample size, n,
- Prevalence of the event at time t in the finite population, $p_N(t)$.

The simulations are limited to GDEs and MCEs constructed using Lognormal, Wiebull and semiparametric PH models. These estimators are compared to traditional estimators: the π-estimator, the GREG, GDE and MCEs constructed from a Logistic model.

This chapter is organized as follows: Section 4.1 describes how the finite populations were generated. Section 4.2 describes the sample design used in this simulation study. Section 4.3 describes how the estimators used in the simulation study were constructed. Section 4.4 presents the criteria used to evaluate the estimators, Section 4.5 reviews the results of the simulation study. Finally, Section 4.6 discusses the results of the simulation study.

4.1 Populations

Four types populations were generated: Lognormal (LOG1), Lognormal with a squared term (LOG2), Weibull with a common baseline hazard (WCB), and Weibull with a mixture of two baseline hazards (WMB). Finite populations with $N = 100,000$ were generated from independent-identically distributed samples from:

$$ln(T) = \theta_0 + \theta_1 X + \theta_2 Z + \theta_3 Z^2 + W, \tag{4.1}$$

where $\theta_0 = \theta_1 = \theta_2 = 1$, and Z was generated from a gamma distribution with shape and scale parameters equal to one. For the LOG1 and LOG2 populations, W was drawn from a normal distribution with mean zero, standard deviation σ, and $X = 0$. For the LOG1 populations $\theta_3 = 0$ and for the LOG2 populations $\theta = 4$. For the

WCB and the WMB populations, W was drawn from a generalized extreme value distribution with the location parameter and shape parameters set to zero and shape parameter σ. For the populations which have a common baseline hazard $X = 0$. For populations with a mixture of two baseline hazards, X was drawn from a Bernoulli distribution with $p = 0.4$.

In all four cases, σ was set to generate finite populations in which the correlation between $ln(T)$ and Z was a given ρ. Nine populations were generated by crossing the LOG1,LOG2, WCB, and WMB distributions with the correlations $\rho = 0.8, 0.6, 0.4$.

For each population, three sets of censored values of T and censor indicators were derived as follows:

$$\tilde{T}_i^{(j)} = min(T_i, Q_j), \tag{4.2}$$

$$c_i^{(j)} = I_{\{T_i \leq Q_j\}} \tag{4.3}$$

for $j = 1, 2, 3$, where Q_j is the j^{th} finite population quartile. This generated censored values of T such that 75%, 50%, or 25% of the cases in the population were censored in the sense that there is no observation after time $t_o = Q_j$.

4.2 Sample Design

For this simulation a stratified simple random sample design was used. Units were stratified based on the value of Z. The units were sorted in ascending order based on Z, then the first 10,000 were assigned to stratum 1, the next 20,000 were assigned to stratum 2, the next 30,000 were assigned to stratum 3, and the last 40,000 were assigned to stratum 4. Two sample sizes were used: $n = 200, 1000$. Sample was allocated equally to each strata, i.e., $n_k = n/4$ for all k. For each population-sample size combination, $L = 10,000$ samples were drawn.

4.3 Estimators

For each sample, nine time-to-event models were fit, one for each of the Lognormal, Weibull, and semiparametric PH models with the censoring conditions 75%, 50%, and 25%. All of the models were fit with an intercept and one predictor, Z. For each model, three types of model assisted estimates of $p(t)$ were calculated, a GDE and the two MCEs presented in Chapter 3, Section 3.2. These are denoted as GD, MC1, and MC2 for the remainder of this chapter. MC1 is the MCE with one constraint defined by (3.9) in Section 3.2, and MC2 is the MCE estimator with two constraints defined by (3.6) and (3.7) in section 3.2. With all of the

combinations of models and types of model assisted estimators, this results in 9 estimators of $p(t)$.

Estimates were then generated using each of these 9 estimators of $p(t)$ for three values of t. The three values of t were selected such that the finite population value of $p(t)$ was 0.75, 0.50, and 0.25. It should be noted that for 75% censoring only $p(t) = 0.25$ could be estimated. Likewise for 50% censoring, only $p(t) = 0.50$ or $p(t) = 0.25$ could be estimated. This resulted in 54 estimates for each sample.

Additionally, to compare these methods with existing methods, five other estimators were used: the π-estimator; a GREG; and GD, MC1 and MC2 based on a Logistic model with an intercept and one predictor, Z. This resulted in another 30 estimates for each sample and a total of 84 estimates per simulated sample.

4.4 Evaluation Critera

The following criteria were used to evaluate the performance of the time-to-event based GDE and MCE: efficiency, bias, and performance of variance estimators.

4.4.1 Efficiency

To evaluate the efficiency of time-to-event based GDE and MCE the root mean squared error (RMSE) of these estimators was compared to the π-estimator. The

simulated RMSE at a fixed time t was estimated as follows:

$$RMSE = \left(L^{-1} \sum_{k=1}^{L} [\hat{p}_k(t) - p_N(t)]^2 \right)^{\frac{1}{2}}, \tag{4.4}$$

where L represents the 10,000 simulations, $\hat{p}_k(t)$ is the estimate of $p_N(t)$ for the k^{th} simulation, and $p_N(t)$ is the finite population value of $p(t)$. To compare the simulated RMSE of an estimator A with the RMSE of the π-estimator, the percent reduction in RMSE (Δ_{RMSE}) was calculated as follows:

$$\Delta_{RMSE} = 1 - \left(\frac{RMSE_A}{RMSE_\pi} \right), \tag{4.5}$$

4.4.2 Bias

Two measures were calculated to evaluate the bias of GDE and MCE that were derived from time-to-event models. The first measure is the simulated Relative Bias (RB). The RB compares the magnitude of the simulated bias of an estimator relative to the finite population value that is being estimated. The RB was calculated as follows:

$$RB = \frac{1}{L} \sum_{k=1}^{L} \left(\frac{\hat{p}_k(t) - p_N(t)}{p_N(t)} \right). \tag{4.6}$$

The second measure is the Bias Ratio (BR). The BR compares the magnitude

of the simulated bias of an estimator to the magnitude of the simulated standard error of the same estimator. The BR is calculated as follows:

$$BR = \frac{L^{-1}\sum_{k=1}^{L}\left[\hat{p}_k(t) - p_N(t)\right]}{\left(L^{-1}\sum_{k=1}^{L}\left[\hat{p}_k(t) - \bar{p}(t)\right]^2\right)^{\frac{1}{2}}}, \tag{4.7}$$

where $\bar{p}(t) = L^{-1}\sum_{k=1}^{L}\hat{p}_k(t)$. For confidence intervals to cover at the desired rate, BR must converge to 0 with increasing sample size in addition to $\hat{p}_k(t) - p_N(t)$ converging to 0.

4.4.3 Performance of Variance Estimators

Two measures were calculated to evaluate the performance of the Generalized Difference and Model Calibrated variance estimators presented in Chapter 3. The first measure is the Variance Ratio (VR), which is the ratio of the simulation mean of the estimated sampling variance to the simulated variance of the estimator. The VR evaluates on average how well the variance estimator estimates the simulated variance of the estimator. This was calculated as follows:

$$VR = \frac{L^{-1}\sum_{k=1}^{L}\hat{V}_k(\hat{p}(t))}{L^{-1}\sum_{k=1}^{L}\left[\hat{p}_k(t) - \bar{p}(t)\right]^2}. \tag{4.8}$$

The variance estimator for each $\hat{p}_k(t)$ was defined in Section 3.2. The second measure is confidence interval coverage. For each simulation, the 95% normal approximation confidence interval was calculated as

$$CI_i = \left(\hat{p}_i(t) - 1.96\sqrt{\hat{V}_i(\hat{p}_i(t))}, \quad \hat{p}_i(t) + 1.96\sqrt{\hat{V}_i(\hat{p}_i(t))} \right). \tag{4.9}$$

From this, a binary variable C_i was calculated as follows:

$$C_i = \begin{cases} 1 & \text{if} \quad p_N(t) \in CI_i \\ \\ 0 & \text{otherwise} \end{cases} . \tag{4.10}$$

Finally, the confidence interval coverage rate (CR) was calculated as

$$CR = L^{-1} \sum_{k=1}^{L} C_k. \tag{4.11}$$

4.5 Results

The simulation results for the LOG1, LOG2, WCB, and WCM were strikingly similar. As such, I only show the LOG1 populations results here. Tables with the results for the LOG2, WCB, and WCM populations are located in Appendix A.

4.5.1 Bias

Table 4.1 and Table 4.2 show the simulated percent RBs for the Lognormal populations with $n = 200$ and $n = 1,000$, respectively. There are a few general conclusions that can be drawn from these tables:

1. The percent RBs were small (at most 0.5%);

2. The estimators based on time-to-event models had RBs similar to the π-estimator; and

3. For $n = 200$, the Logistic model based Generalized Difference Estimator (LG-GD) tended to have larger RB than the other estimators, but this difference is not meaningful.

Table 4.3 and Table 4.4 show the simulated BRs for the Lognormal populations with $n = 200$ and $n = 1,000$, respectively. There are a a few general conclusions that can be drawn from these tables:

1. The BRs were small (at most 0.079);

2. The estimators based on time-to-event models had BRs similar to the π-estimator;

3. The BRs indicate that variance is the major driver of MSE for time-to-event model-based estimators.

There was only a small amount of bias in any of the estimators used in this simulation. The RBs were much smaller than those seen in the simulation performed by Wu and Sitter (2001) which reported relative biases as high as 5.71%. Because all estimators were essentially unbiased, selection of an estimator can be based on RMSE and confidence interval coverage, at least in the circumstances studied here

Table 4.1: Simulated Percent Relative Bias: Lognormal Population, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.08	0.11	0.26	0.15	0.17
		0.50	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.09	0.11	0.26	0.15	0.17
		0.75	0.13	-0.36	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.26	0.15	0.17
	0.5	0.25	0.03	-0.31	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.07	0.09	0.08
		0.50	0.03	-0.31	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.07	0.09	0.08
	0.75	0.25	0.06	-0.09	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.32	0.30	0.30
0.6	0.25	0.25	0.21	0.12	0.18	0.16	0.19	0.18	0.18	0.19	0.19	0.16	0.19	0.50	0.08	0.12
		0.50	0.21	0.12	0.18	0.17	0.18	0.18	0.19	0.19	0.18	0.17	0.18	0.50	0.08	0.12
		0.75	0.21	0.12	0.18	0.18	0.18	0.19	0.19	0.19	0.18	0.18	0.18	0.50	0.08	0.12
	0.5	0.25	0.07	-0.13	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.20	0.12	0.15
		0.50	0.07	-0.13	0.05	0.06	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.20	0.12	0.15
	0.75	0.25	0.03	-0.04	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.27	0.21	0.23
0.4	0.25	0.25	0.24	0.06	0.23	0.21	0.23	0.23	0.23	0.23	0.23	0.21	0.23	0.35	0.06	0.06
		0.50	0.24	0.06	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.35	0.06	0.06
		0.75	0.24	0.06	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.35	0.06	0.06
	0.5	0.25	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.23	0.07	0.12
		0.50	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.13	0.12	0.23	0.07	0.12
	0.75	0.25	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.15	0.12	0.12

Table 4.2: Simulated Percent Relative Bias: Lognormal Population, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal		Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.75	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.5	0.25	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
		0.50	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
0.6	0.25	0.25	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.50	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.75	-0.04	-0.11	-0.05	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.05	-0.01	0.00	0.00
	0.5	0.25	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
		0.50	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	0.75	0.25	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.05	0.04
0.4	0.25	0.25	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.50	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.75	0.01	-0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.06	0.02	0.02
	0.5	0.25	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.02	0.02

Table 4.3: Simulated Bias Ratio: Lognormal Population, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.08	0.11	0.26	0.15	0.17
		0.50	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.09	0.11	0.26	0.15	0.17
		0.75	0.13	-0.36	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.26	0.15	0.17
	0.5	0.25	0.03	-0.31	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.07	0.09	0.08
		0.50	0.03	-0.31	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.07	0.09	0.08
	0.75	0.25	0.06	-0.09	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.32	0.30	0.30
0.6	0.25	0.25	0.21	0.12	0.18	0.16	0.19	0.18	0.18	0.19	0.19	0.16	0.19	0.50	0.08	0.12
		0.50	0.21	0.12	0.18	0.17	0.18	0.18	0.19	0.19	0.18	0.17	0.18	0.50	0.08	0.12
		0.75	0.21	0.12	0.18	0.18	0.18	0.19	0.19	0.19	0.18	0.18	0.18	0.50	0.08	0.12
	0.5	0.25	0.07	-0.13	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.20	0.12	0.15
		0.50	0.07	-0.13	0.05	0.06	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.20	0.12	0.15
	0.75	0.25	0.03	-0.04	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.27	0.21	0.23
0.4	0.25	0.25	0.24	0.06	0.23	0.21	0.23	0.23	0.23	0.23	0.23	0.21	0.23	0.35	0.06	0.06
		0.50	0.24	0.06	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.35	0.06	0.06
		0.75	0.24	0.06	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.35	0.06	0.06
	0.5	0.25	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.23	0.07	0.12
		0.50	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.13	0.12	0.23	0.07	0.12
	0.75	0.25	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.15	0.12	0.12

Table 4.4: Simulated Bias Ratio: Lognormal Population, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal		Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.75	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.5	0.25	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
		0.50	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
0.6	0.25	0.25	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.50	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.75	-0.04	-0.11	-0.05	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.05	-0.01	0.00	0.00
	0.5	0.25	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
		0.50	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	0.75	0.25	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.05	0.04
0.4	0.25	0.25	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.50	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.75	0.01	-0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.06	0.02	0.02
	0.5	0.25	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.02	0.02

4.5.2 Efficiency

An estimators reduction in RMSE when compared to the π-estimator was affected by three conditions: the correlation between $ln(T)$ and predictor Z, sample size, and prevalence of the event in the finite population.

Generally, the distribution of T did not have a noticeable effect on the any of estimators tested in this simulation, the exception being percent reduction in RMSE when compared to the π-estimator when $p(t) = .75$ and $\rho = 0.8$, which was as high as 16.99%. Although the magnitude of the reductions were larger in some cases, the conclusions drawn are the same.

Table 4.5 and 4.6 provide the simulated reductions in RMSE when compared to the π-estimator for the Lognormal populations with $n = 200$ and $n = 1,000$, respectively. There are a number of conclusions that can be drawn from these tables:

1. The time-to-event model-based estimators never underperformed the π-estimator.

2. The time-to-event model-based estimators never underperformed, and in many cases outperformed, the GREG and Logistic-based GD, MC1, and MC2 estimators (LG-GD, LG-MC1, LG-MC2).

3. The reductions in RMSE for the nine estimators based on time-to-event models

were similar.

4. The reduction in RMSE for the nine estimators based on time-to-event models and the GREG were positively correlated with $p(t)$.

5. The reduction in RMSE for the nine estimators based on time-to-event models, the GREG, and the MC1-LG were positively correlated with ρ.

6. The reduction in RMSE for the nine estimators based on time-to-event models and the GREG were not negatively affected by small sample sizes.

7. Reduction in RMSE for GD, MC1, and MC2 were substantially reduced when prevalences were estimated using the logistic model when $n = 200$.

8. The amount of censoring did not affect the performance of any of the estimators.

Table 4.5: Simulated Percent Reduction of RMSE Relative to the π-estimator: Lognormal Population, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.08	0.11	0.26	0.15	0.17
		0.50	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.09	0.11	0.26	0.15	0.17
		0.75	0.13	-0.36	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.26	0.15	0.17
	0.5	0.25	0.03	-0.31	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.07	0.09	0.08
		0.50	0.03	-0.31	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.07	0.09	0.08
	0.75	0.25	0.06	-0.09	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.32	0.30	0.30
0.6	0.25	0.25	0.21	0.12	0.18	0.16	0.19	0.18	0.18	0.19	0.19	0.16	0.19	0.50	0.08	0.12
		0.50	0.21	0.12	0.18	0.17	0.18	0.18	0.19	0.19	0.18	0.17	0.18	0.50	0.08	0.12
		0.75	0.21	0.12	0.18	0.18	0.18	0.19	0.19	0.19	0.18	0.18	0.18	0.50	0.08	0.12
	0.5	0.25	0.07	-0.13	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.20	0.12	0.15
		0.50	0.07	-0.13	0.05	0.06	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.20	0.12	0.15
	0.75	0.25	0.03	-0.04	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.27	0.21	0.23
0.4	0.25	0.25	0.24	0.06	0.23	0.21	0.23	0.23	0.23	0.23	0.23	0.21	0.23	0.35	0.06	0.06
		0.50	0.24	0.06	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.35	0.06	0.06
		0.75	0.24	0.06	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.35	0.06	0.06
	0.5	0.25	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.23	0.07	0.12
		0.50	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.13	0.12	0.23	0.07	0.12
	0.75	0.25	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.15	0.12	0.12

Table 4.6: Simulated Percent Reduction of RMSE Relative to the π-estimator: Lognormal Population, N=100,000, n=1,000, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.75	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.5	0.25	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
		0.50	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
0.6	0.25	0.25	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.50	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.75	-0.04	-0.11	-0.05	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.05	-0.01	0.00	0.00
	0.5	0.25	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
		0.50	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	0.75	0.25	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.05	0.04
0.4	0.25	0.25	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.50	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.75	0.01	-0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.06	0.02	0.02
	0.5	0.25	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.02	0.02

As mentioned above, because of the similarity in the performance of the time-to-event based estimators, only the Lognormal GD (LN-GD) is presented in Figures 4.1-4.6. In Figure 4.2, only the LN-GD and GREG are presented, because the LN-GD, Logistic GD (LG-GD), Logistic MC1 (LG-MC1), and Logistic MC2 (LG-MC2) performed similarly.

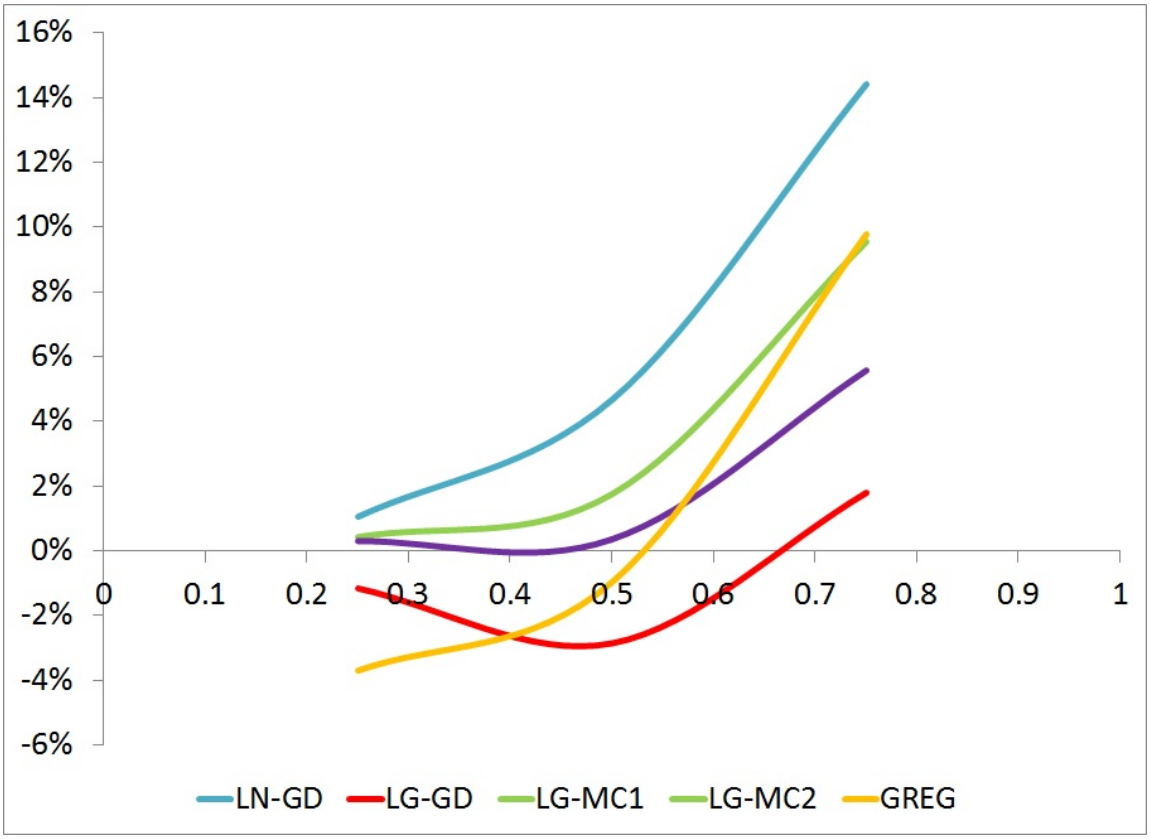

Figure 4.1: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.8$, 25% censoring, $n = 200$

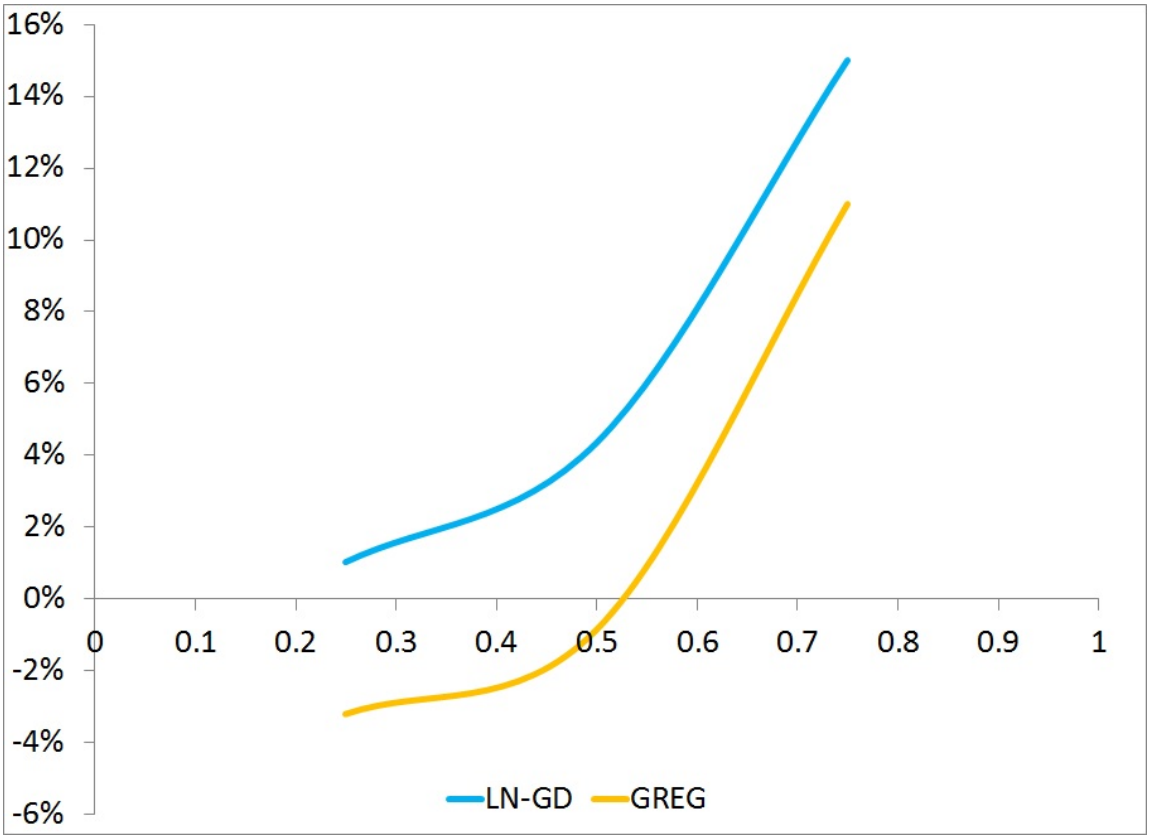

Figure 4.2: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.8$, 25% censoring, $n = 1,000$

In Figure 4.1 and 4.2 it can be seen that for $\rho = 0.8$ and $n = 200$ or $n = 1,000$ the LN-GD outperformed the GREG by about 4 percentage points for all of $p(t)$, and the GREG estimator performed worse than the π-estimator for smaller values of $p(t)$. Additionally, while the LN-GD, LG-GD, LG-MC1, and LG-MC2 performed similarly for $n = 1,000$, for $n = 200$ the LN-GD outperformed the LG-GD, LG-MC1, and LG-

MC2, with the LG-MC1 performing the best. For $n = 200$, the LG-GD performed worse than the π-estimator for some values of $p(t)$.

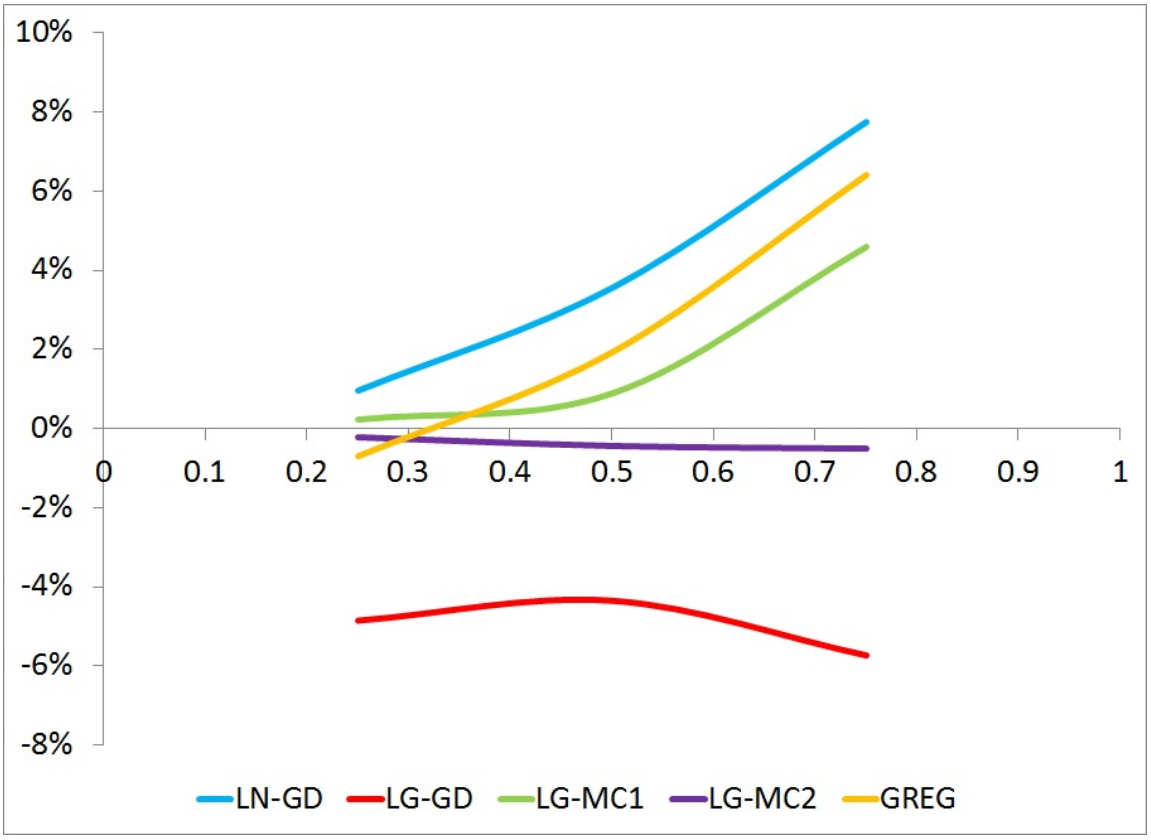

Figure 4.3: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.6$, 25% censoring, $n = 200$

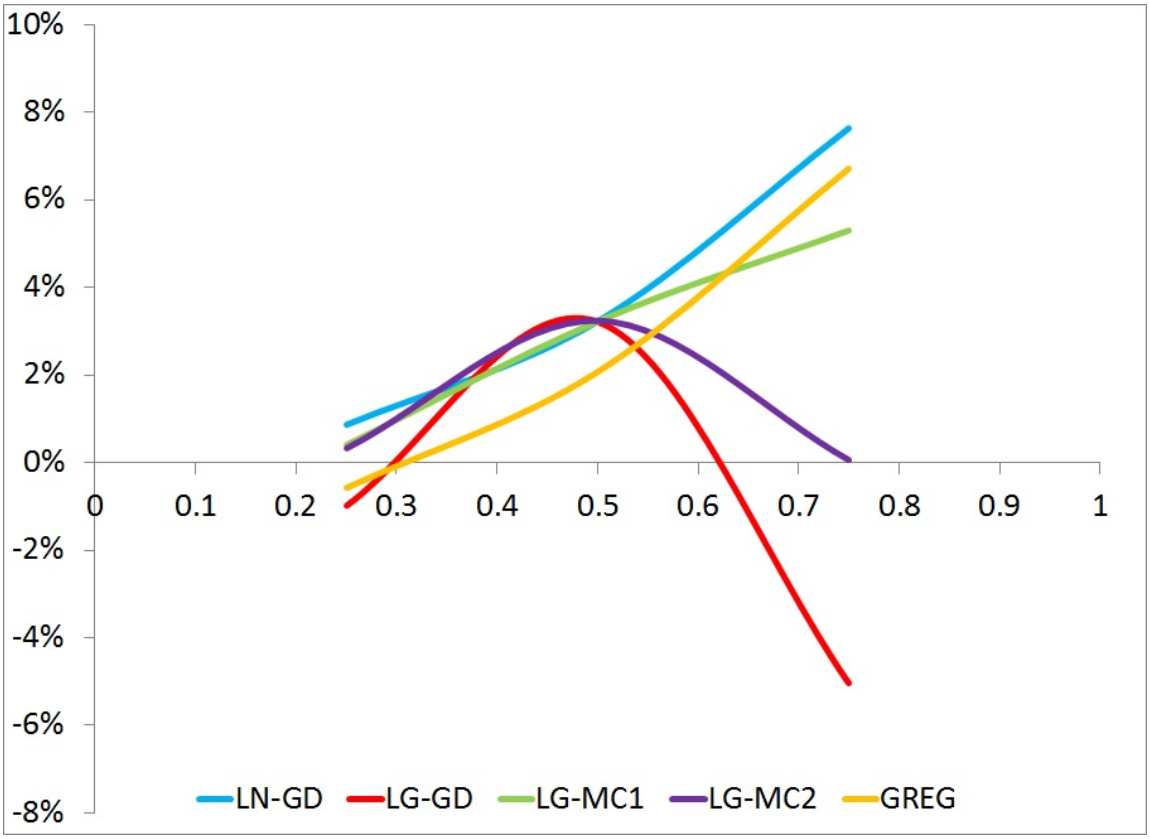

Figure 4.4: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.6$, 25% censoring, $n = 1,000$

In figures 4.3 and 4.4, it can be seen that, for $\rho = 0.6$ and $n = 200$ or $n = 1,000$, once again the LN-GD outperforms the GREG by about 1.5 percentage points for all of $p(t)$. Once again, for $n = 200$, the LN-GD outperformed the LG-GD, LG-MC1, and LG-MC2 with the LG-MC1 performing the best. For $n = 200$, LG-GD performed worse then the π-estimator for all values of $p(t)$. For $n = 1,000$, the LN-

GD performed at least as well as the LG-GD, LG-MC1 and LG-MC2 for all $p(t)$, and outperformed all three estimators for $p(t) = .75$, with the LG-GD underperforming the π-estimator by almost 6%.

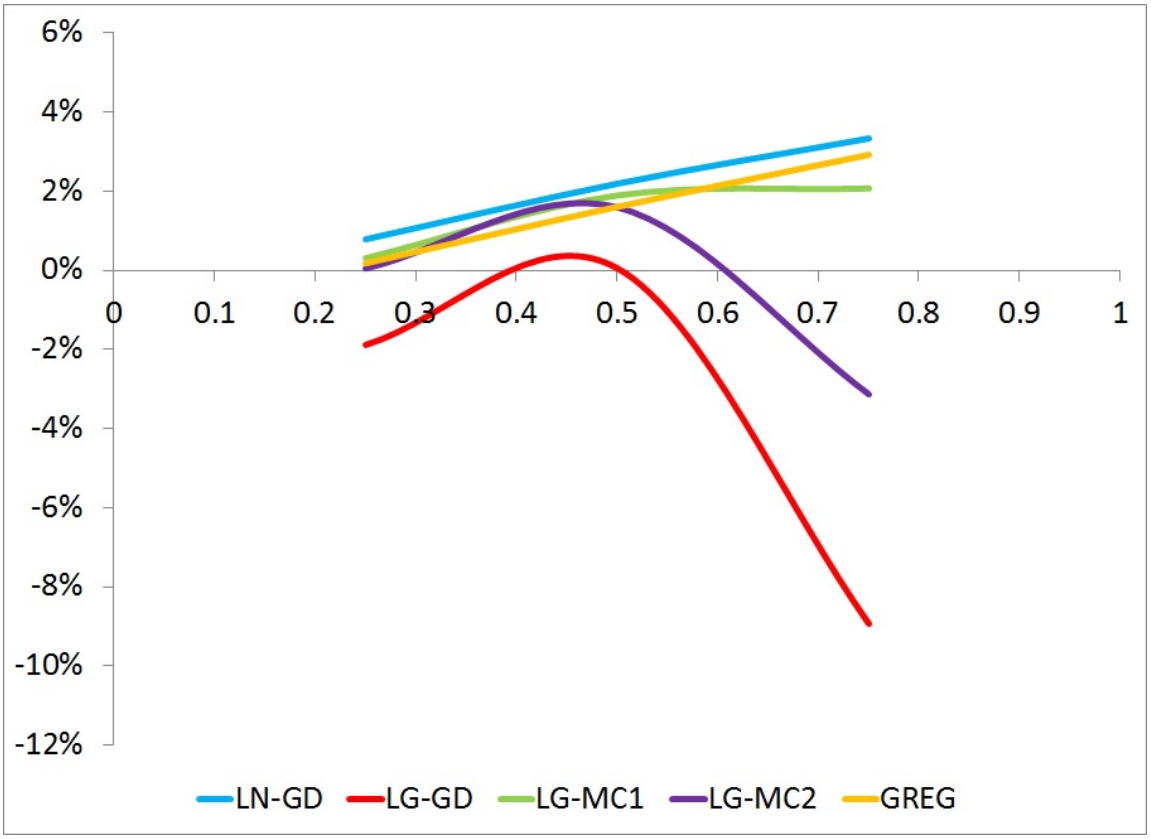

Figure 4.5: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.4$, 25% censoring, $n = 200$

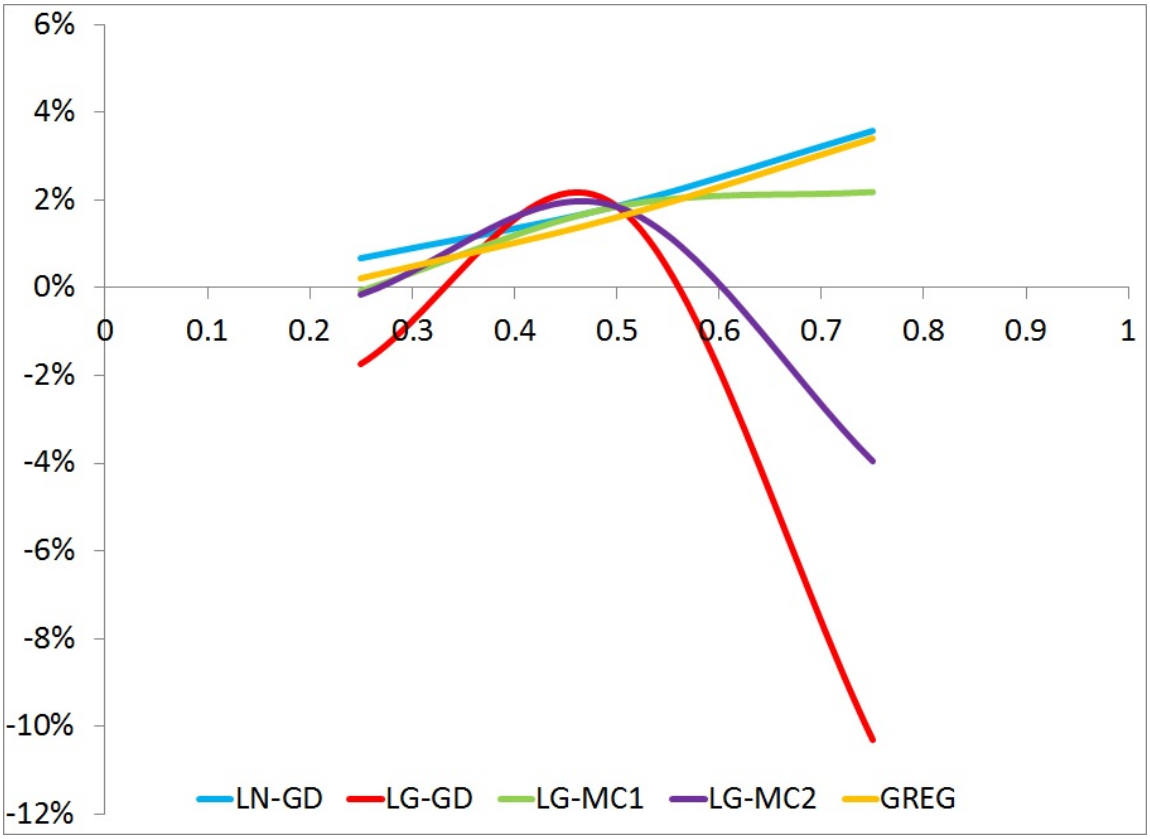

Figure 4.6: Percent reduction in RMSE as a function of $p(t)$: Lognormal population, $\rho = 0.4$, 25% censoring, $n = 1,000$

In Figures 4.5 and 4.6, it can be seen that for $\rho = 0.4$ and $n = 200$ or $n = 1,000$ the LN-GD and GREG performed similarly. For $n = 200$ and $n = 1,000$, the LN-GD performed at least as well as the LG-GD, LG-MC1 and LG-MC2. For $n = 200$ and $n = 1,000$, the LN-GD performed at least as well as the LG-GD, LG-MC1 and LG-MC2 for all $p(t)$ and outperformed all three estimators for $p(t) = .75$, with

the LG-GD underperforming the π-estimator by more than 8% and the LG-MC2 by more than 3%.

Overall, the LN-GD outperformed the GREG for larger values of ρ. The LN-GD performed at least as well as, and in many cases better than, the LG-GD, LG-MC1, and LG-MC2. The LN-GD performed better for smaller sample sizes than the LG-GD, LG-MC1, and LG-MC2. Frequently, the LG-MC2 and LG-GD did not perform as well as the other estimators and performed worse than the π-estimator.

It should be noted that we did not explore wheither ther RMSEs were statistically different from each other. To do this the standard error of the RMSE's would need to be calculated. To do this we could estimate a standard error by taking bootstrap samples of the simulates.

4.5.3 Performance of Variance Estimators

Table 4.7 and Table 4.8 show the simulated VRs for the Lognormal populations with $n = 200$ and $n = 1,000$, respectively. These tables show VRs close to one for all of the estimators. This tells us that on average the asymptotic variance estimate was equivalent to the empirical variance of the estimators seen in the simulation.

Table 4.9 and Table 4.10 show the simulated 95% confidence interval coverage for the Lognormal populations with $n = 200$ and $n = 1,000$, respectively. These

tables show that all of the estimators provided nominal coverage. Additionally, for all the estimators, the coverage was similar to the π-estimator's coverage. Given that the VRs were near 1 and each $\hat{p}(t)$ estimator was approximately unbiased, the fact that the confidence intervals covered $p_N(t)$ at the desired rate is not surprising.

Table 4.7: Simulated Variance Ratio: Lognormal Population, N=100,000, n=200, L=10,000

						Weibull			Lognormal		Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.01	0.99	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00
		0.50	1.01	0.99	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00
		0.75	1.01	0.99	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00
	0.5	0.25	1.01	1.00	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.50	1.01	1.00	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
	0.75	0.25	1.00	0.96	0.99	0.98	0.99	0.99	0.98	0.98	0.99	0.98	0.99	1.02	0.99	1.01
0.6	0.25	0.25	1.01	1.00	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.00	1.01	0.97	1.01	1.01
		0.50	1.01	1.00	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.00	1.01	0.97	1.01	1.01
		0.75	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.01	1.01
	0.5	0.25	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	0.99	0.99
		0.50	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	0.99	0.99
	0.75	0.25	0.99	0.98	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.03	0.99	1.02
0.4	0.25	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.02	1.01
		0.50	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.01
		0.75	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.02	1.01
	0.5	0.25	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.01	1.02
		0.50	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.01	1.02
	0.75	0.25	1.03	1.02	1.03	1.02	1.03	1.03	1.02	1.03	1.03	1.02	1.03	1.05	1.02	1.05

Table 4.8: Simulated Variance Ratio: Lognormal Population, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal		Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.50	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.75	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
	0.5	0.25	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.50	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
	0.75	0.25	0.98	0.98	0.99	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.98	0.99	0.99	0.99
0.6	0.25	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
		0.50	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
		0.75	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
	0.5	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.75	0.25	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.02	1.00	1.02
0.4	0.25	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.5	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.75	0.25	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	1.01	0.98	1.00

Table 4.9: Simulated 95% Confidence Interval Coverage: Lognormal Population, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.948	0.944	0.947	0.947	0.946	0.947	0.947	0.947	0.946	0.947	0.947	0.946	0.946	0.946
		0.50	0.948	0.944	0.946	0.946	0.946	0.947	0.947	0.947	0.946	0.946	0.946	0.946	0.946	0.946
		0.75	0.948	0.944	0.946	0.946	0.946	0.947	0.947	0.947	0.946	0.946	0.946	0.946	0.946	0.946
	0.5	0.25	0.947	0.949	0.952	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.949	0.951	0.951
		0.50	0.947	0.949	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.949	0.951	0.951
	0.75	0.25	0.947	0.943	0.943	0.943	0.943	0.944	0.943	0.943	0.943	0.942	0.943	0.939	0.937	0.939
0.6	0.25	0.25	0.946	0.946	0.948	0.948	0.949	0.948	0.948	0.948	0.948	0.948	0.949	0.943	0.946	0.947
		0.50	0.946	0.946	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.949	0.948	0.943	0.946	0.947
		0.75	0.946	0.946	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.943	0.946	0.947
	0.5	0.25	0.946	0.948	0.947	0.948	0.947	0.948	0.948	0.948	0.947	0.948	0.948	0.946	0.947	0.948
		0.50	0.946	0.948	0.948	0.948	0.948	0.947	0.948	0.948	0.948	0.948	0.948	0.946	0.947	0.948
	0.75	0.25	0.945	0.942	0.944	0.944	0.945	0.944	0.943	0.944	0.945	0.944	0.945	0.944	0.943	0.944
0.4	0.25	0.25	0.946	0.944	0.945	0.945	0.945	0.946	0.945	0.946	0.945	0.945	0.945	0.946	0.946	0.946
		0.50	0.946	0.944	0.950	0.949	0.950	0.950	0.950	0.950	0.950	0.949	0.950	0.946	0.946	0.946
		0.75	0.946	0.944	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.946	0.946	0.946
	0.5	0.25	0.946	0.945	0.946	0.946	0.946	0.946	0.945	0.945	0.946	0.945	0.946	0.951	0.946	0.946
		0.50	0.946	0.945	0.945	0.946	0.945	0.945	0.945	0.945	0.945	0.946	0.945	0.951	0.946	0.946
	0.75	0.25	0.947	0.946	0.948	0.947	0.948	0.947	0.947	0.947	0.948	0.947	0.948	0.952	0.947	0.951

Table 4.10: Simulated 95% Confidence Interval Coverage: Lognormal Population, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.951	0.954	0.952	0.953	0.953	0.952	0.953	0.952	0.952	0.953	0.953	0.952	0.952	0.952
		0.50	0.951	0.954	0.953	0.953	0.952	0.952	0.953	0.952	0.953	0.953	0.953	0.952	0.952	0.952
		0.75	0.951	0.954	0.952	0.952	0.951	0.953	0.953	0.953	0.952	0.952	0.951	0.952	0.952	0.952
	0.5	0.25	0.953	0.950	0.951	0.951	0.951	0.952	0.952	0.952	0.952	0.951	0.952	0.951	0.951	0.951
		0.50	0.953	0.950	0.951	0.951	0.951	0.952	0.951	0.952	0.951	0.951	0.951	0.951	0.951	0.951
	0.75	0.25	0.948	0.946	0.947	0.947	0.947	0.947	0.946	0.947	0.947	0.947	0.947	0.947	0.947	0.947
0.6	0.25	0.25	0.951	0.949	0.952	0.952	0.953	0.952	0.952	0.952	0.952	0.952	0.953	0.951	0.950	0.950
		0.50	0.951	0.949	0.952	0.952	0.952	0.952	0.952	0.952	0.953	0.952	0.953	0.951	0.950	0.950
		0.75	0.951	0.949	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.951	0.950	0.950
	0.5	0.25	0.950	0.950	0.951	0.950	0.950	0.950	0.950	0.950	0.951	0.951	0.951	0.950	0.950	0.950
		0.50	0.950	0.950	0.951	0.951	0.951	0.950	0.950	0.950	0.951	0.951	0.951	0.950	0.950	0.950
	0.75	0.25	0.949	0.948	0.947	0.946	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.948	0.948	0.948
0.4	0.25	0.25	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.949	0.950	0.950
		0.50	0.950	0.950	0.950	0.949	0.950	0.950	0.950	0.950	0.950	0.949	0.950	0.949	0.950	0.950
		0.75	0.950	0.950	0.950	0.950	0.949	0.950	0.950	0.950	0.949	0.950	0.949	0.949	0.950	0.950
	0.5	0.25	0.949	0.950	0.950	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951
		0.50	0.949	0.950	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951
	0.75	0.25	0.947	0.949	0.948	0.949	0.949	0.948	0.949	0.948	0.949	0.949	0.949	0.949	0.947	0.949

4.5.4 Computational Problem with the Logistic Estimators

The LG-MC1 and LG-MC2 had some simulated samples excluded from the analysis, because for a small portion of the samples the estimate of $p(t)$ was greater than 1 or less than 0. Knowing that $p(t)$ ranges from 0 to 1 these estimates did not seem reasonable. Moreover, these estimates were not just slightly outside this range, they were usually greater than 10^{10} or less than 10^{-10}. This was caused by the calibration adjustment, $\hat{B}$ diverging to either positive or negative infinity. This was not an issue with any of the other estimators used in these simulations, even the time-to-event MCE. Table 4.11 shows how many samples were thrown out for each population. The most severe problem occurred for the Weibull Population 2 with $\rho = 0.6$ where 331 (or 3.31%) of the samples could not be used. Although the computational problems were rare, the fact that they occur at all is another reason to avoid estimating $p_N(t)$ using a logistic model. The problems with the logistic model-calibrated approach are caused by some combinations of covariates all having the event or not having the event. The fitting alogrithm sends one or more of the parameter estimates to $\pm\infty$. In this case we have a single continuous predictor. A potential fix is to combine levels of factors or in this case discretize the continuous predictor to create mixtures of events and non-events.

Table 4.11: Number of Simulated Samples, Out of 10,000, Where the Model Calibrated Logistic Estimate was Greater that 1 or Less than 0

			Lognormal Population 1		Lognormal Population 2		Weibull Population 1		Weibull Population 2	
ample Size	ρ	p(t)	MC1	MC2	MC1	MC2	MC1	MC2	MC1	MC2
200	0.8	0.25	99	199	268	311	2	3	33	46
		0.50	14	15	18	23	7	7	7	8
		0.75	0	0	0	0	0	0	1	1
	0.6	0.25	222	257	130	147	231	276	285	331
		0.50	32	38	42	49	22	26	22	27
		0.75	1	3	5	5	2	3	2	2
	0.4	0.25	163	190	175	192	192	217	195	222
		0.50	56	66	49	59	68	80	51	63
		0.75	1	2	9	9	8	8	195	222
1,000	0.8	0.25	0	0	135	136	0	0	0	0
		0.50	0	0	0	0	0	0	0	0
		0.75	0	0	0	0	0	0	0	0
	0.6	0.25	123	125	26	26	0	0	0	0
		0.50	0	0	0	0	0	0	0	0
		0.75	2	2	5	5	4	4	6	6
	0.4	0.25	38	39	165	168	139	139	122	122
		0.50	0	0	0	0	0	0	0	0
		0.75	8	8	11	11	10	10	8	8

4.6 Discussion

This simulation study shows that the time-to-event based MCE and GDE performed just as well, if not better than, the current methods for all of the conditions tested. The time-to-event based MCE and GDE provided reductions in RMSE without inducing much, if any, bias, and the asymptotic variance estimator performed well. The time-to-event MCE did not perform any better than the GDE, even when the relationship between Z and $ln(T)$ was weak. This is contrary to the results in Wu and Sitter (2001), where MCE outperformed GDE for all ρ. The reductions in RMSE, compared to the π-estimator, were positively correlated with $p(t)$, which is consistent with the results in Wu and Sitter (2001). There were meaningful reductions in RMSE compared to the π-estimator for larger values of t.

The GREG, LG-GD, or LG-MC2 often had a RMSE larger than the π-estimator. The Logistic based estimators were negatively affected by the small sample size of $n = 200$, especially the LG-GD. Wu and Sitter (2001) assert that the MCE should never have RMSE greater than the π-estimator. These simulations showed that the LG-MC2 can have RMSE greater than the π-estimator. It should be noted that the MC estimators seemed more robust in protecting against larger RMSE than the π-estimator, especially LG-MC1. When the LG-GD was less

efficient than the π-estimator the LG-MC1 and LG-MC2 were more efficient than the LG-GD under every condition. This was especially true for $n = 200$. In addition, the LG-MC1 and LG-MC2 occasionally provided estimates that were greater than 1 or less than 0, which were not valid values for $p(t)$. This was not seen for any of the other estimators in the simulation study.

In sum, these simulations showed that time-to-event based MCE and GDE can provide reductions in RMSE over the π-estimator for large values of $p(t)$ without causing any issues with bias. These reductions were at least as large as, if not larger than, current methods, and the estimators generally outperformed current methods for small sample sizes. In addition, the variance estimator presented in this dissertation performed well and provided nominal coverage.

Chapter 5: Nurses' Health Study Application

In this chapter, I apply the previously developed GD and MC estimators to data from the Nurses' Heath Study (NHS). This application shows how these estimators can be used to estimate the proportion of a population who have experienced an event, in this case death, using only a sample of the population. This application uses the same estimators and evaluation criteria used in Chapter 4. A subset of the NHS population is used as a finite population from which samples are repeatedly selected for a simulation study.

This chapter is laid out as follows: Section 5.1 provides background about the NHS, Section 5.2 discusses how the finite population was constructed using NHS data, Section 5.3 discusses the sample design used for this application, Section 5.4 discusses the models used in this application, Section 5.5 presents the results from, and lastly, Section 5.6 discusses the results.

5.1 About the Nurses' Health Study

The NHS is based on a panel of over 120,000 female nurses that has been followed since the mid-1970s. Originally, the NHS focused on the long-term effects of oral contraceptives. Although this is still a main focus of the NHS, the NHS now also focuses on smoking, cancer, and heart disease. It asks about lifestyle factors, such as nutrition and quality of life. It also collects information on more than 30 diseases.

The target population for the NHS is female registered nurses in the 11 most populated states who were married and ages 30-55 in 1976. The frame was constructed using membership roles from nursing boards who agreed to participate in the NHS. In 1976, the 238,026 nurses on the frame were mailed an initial questionnaire. Of these, 121,700 nurses returned a completed questionnaire and were enrolled in the study. Every other year since 1976, study participants have received a follow-up questionnaire to collect information about disease and health-related topics. In addition, biological samples have been collected from subsamples of the panel. More information about the NHS can be found at http://www.nurseshealthstudy.org.

5.2 Finite Population Creation

The finite population used in this application represents a subset of the NHS population. The population is similar to other studies that used time-to-event models to study the incidence of lung disease. One used semiparametric PH models (Bain et al., 2004) and one used TR models (Lee et al., 2010). This extract contained information from 1986 through 2012. To be eligible for the population, a panel participant had to meet the following criteria:

- Alive in 1986,
- Not diagnosed with cancer prior to 1986 (with the exception of non-melanoma skin cancer)
- Known smoking status in 1986,
- Known pack years in 1986,
- Known body mass index (BMI) for at least one year during 1986 to 2012

Pack years is calculated by multiplying the packs of cigarettes smoked per day by the number of years that a person smoked. One pack year is equal to smoking 20 cigarettes per day for one year. BMI is equal to a person's weight in kilograms

divided by the square of the person's height in meters. This resulted in a finite population of 103,878 nurses. The following variables were retained on the file:

- Death indicator (died between 1986 and 2012)
- Age at death (in years, to the tenth of a year)
- Age in 1986 (in years, to the tenth of a year)
- BMI for every observation between 1986 and 2012 (based on height reported in 1976)
- Smoking status in 1986 (Current Smoker, Past Smoker, Never Smoked)
- Pack years smoked as of 1986 (365 packs to a pack year)

The following variables were derived from these variables:

- BMI in 1986, where missing values of BMI were imputed using the BMI closest to 1986 that was observed
- A six level classification of BMI (Underweight, Normal, Overweight, Class 1 Obesity, Class 2 Obesity, Class 3 Obesity)
- A four level classification of BMI, which groups all three levels of obesity into one category (Underweight, Normal, Overweight, Obese)

- A three level classification of age in 1986 (<50, 50 to 60, >60)
- Years to death after 1986 calculated to the tenth of a year (with a value of 26 if alive in 2012)

5.3 Sample Design

Two stratified simple random sample designs were used in this simulation study. The first had three strata based on the three levels of smoking status. The second had 36 strata formed by crossing smoking status, age group, and four level BMI. Both of these designs used strata that are related to death, with the 36 strata design expected to be more effective in reducing variance for estimates of the proportion of persons experiencing the event. Tables 5.1-5.3 show the counts and row percentages of smoking status, age group, and six level BMI crossed with the death by 2012 indicator in the finite population.

Table 5.1: Smoking Status by Death Indicator: Counts and Row Percentages (as of 2012)

	Alive		Deceased	
Status	Count	%	Count	%
Never Smoked	37,789	80.00	9,445	20.00
Current Smoker	13,698	61.81	8,463	38.19
Past Smoker	26,277	76.20	8,206	23.80

Table 5.2: Age Group by Death Indicator: Counts and Row Percentages (as of 2012)

	Alive		Deceased	
Age	Count	%	Count	%
<50	36,077	91.09	3,531	8.91
50-60	31,286	61.81	11,243	26.44
>60	10,401	47.84	11,349	52.16

Table 5.3: BMI by Death Indicator: Counts and Row Percentages (as of 2012)

		Alive		Deceased	
BMI	Classification	Count	%	Count	%
<18.5	Underweight	816	56.78	621	43.22
18.5-24.9	Normal Weight	42,302	77.79	12,079	22.21
25.0-29.5	Overweight	22,991	74.08	8,043	25.92
30.0-34.9	Class 1 Obesity	8,133	70.57	3,392	29.43
35.0-39.9	Class 2 Obesity	2,532	66.21	1,292	33.79
$\geq$ 40.0	Class 3 Obesity	990	59.03	687	40.97

For all three variables, the Chi-squared test of independence rejected the null hypothesis of independence for $\alpha = 0.01$. The finite population sample size was large, meaning that very small differences could be detected. Because of this, the Chi-squared test might not be the best way to evaluate the usefulness of these variables for stratification. There is variation in the percentage of nurses who have died across subgroups, which suggests that these variables have some predictive value in predicting death by 2012 and, thus, also time to death.

Two sample sizes were used to mimic the simulation study in Chapter 4. For each sample design, samples of 216 and 1,008 were selected. These total samples were allocated equally to each of the 36 strata. For the total sample size of 216, simple random samples of 6 persons were selected without replacement from each stratum. For the total sample size of 1008, 28 persons were selected per stratum.

5.4 Model Development

As with the simulation study, five different models were fit to calculate $\hat{p}(t)$, the proportion of the population who had died at or before time t, which in this study was the year 2012 or 26 years after the recruitment of the nurses population. As in Chapter 4, the five models were a Linear model, Logistic model, Weibull model, Lognormal model, and semiparametric proportional hazard model. All five models were fit using the same set of predictor variables: smoking status, BMI, BMI squared, age, pack years, and pack years squared. The squared term for BMI was used to account for the fact that both small and large values of BMI result in higher risk of death. In an attempt to reduce collinearity between BMI and BMI squared, mean BMI was subtracted from BMI before it was squared.

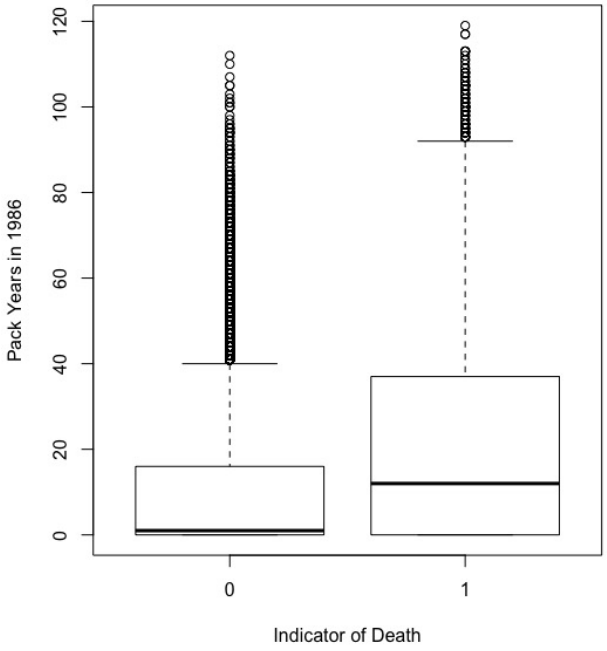

Figure 5.1: Pack Years by Death Indicator

The box plot of pack years is displayed in Figure 5.1. This box plot shows that death generally seems more likely among nurses with more pack years by 1986. A squared term was introduced, because in similar studies it was thought that an increase in smoking has a negative effect on time to death, but this effect moderates for higher levels of pack years (Lee et al., 2010). As with BMI squared, mean pack years was subtracted from pack years before it was squared to reduce collinearity between pack years and pack years squared.

The use of variable selection procedures might be a refinement that was not

explored in this dissertation. This would add complications to variance estimation since reflecting the covariate selection variability is difficult. Replication where covariate selection is done separately for every replicate would be a possible way to capture the sample-to-sample variation in which covariates are selected.

5.5 Results

For this application, as with the simulations in Chapter 4, 10,000 samples were drawn for each of the four sample design-sample size combinations. The same 14 estimators used in Chapter 4 were used here to estimate the percentage of the population that had died by the end of 2012, i.e., $p_N(26) \approx 0.25$. Table 5.4 shows the results using the same five metrics used in Chapter 4 for each estimator and sample design - sample size combination.

5.5.1 Bias

Similar to the findings in Chapter 4, all of the estimators were approximately unbiased. (See the rows in Table 5.4 for %RB) Also, the RBs in this application were much smaller that those in Wu and Sitter (2001), who reported RBs as high as 5.71%. A few general conclusions can be drawn from Table 5.4:

1. % RBs for all of the estimators were small (at most 0.9%).

2. Estimators based on time-to-event models had % RB similar to the π-estimator.

3. BRs for all of the estimators were small (at most 0.067).

4. Estimators based on time-to-event models had BRs similar to the π-estimator.

5. Estimators based on logistic models had larger % RBs and BRs than all of the other estimators, with the LG-GD having the largest % RB and BR, but the difference is not meaningful.

Because all estimators were essentially unbiased, selection of an estimator can be based on RMSE and confidence interval coverage, at least in this application.

5.5.2 Efficiency

Table 5.4 shows that the performance of the nine time-to-event model based estimators was similar. (See the rows in Table 5.1 for VR) Therefore, for simplicity, only the LN-GD is compared to the current methods when examining efficiency.

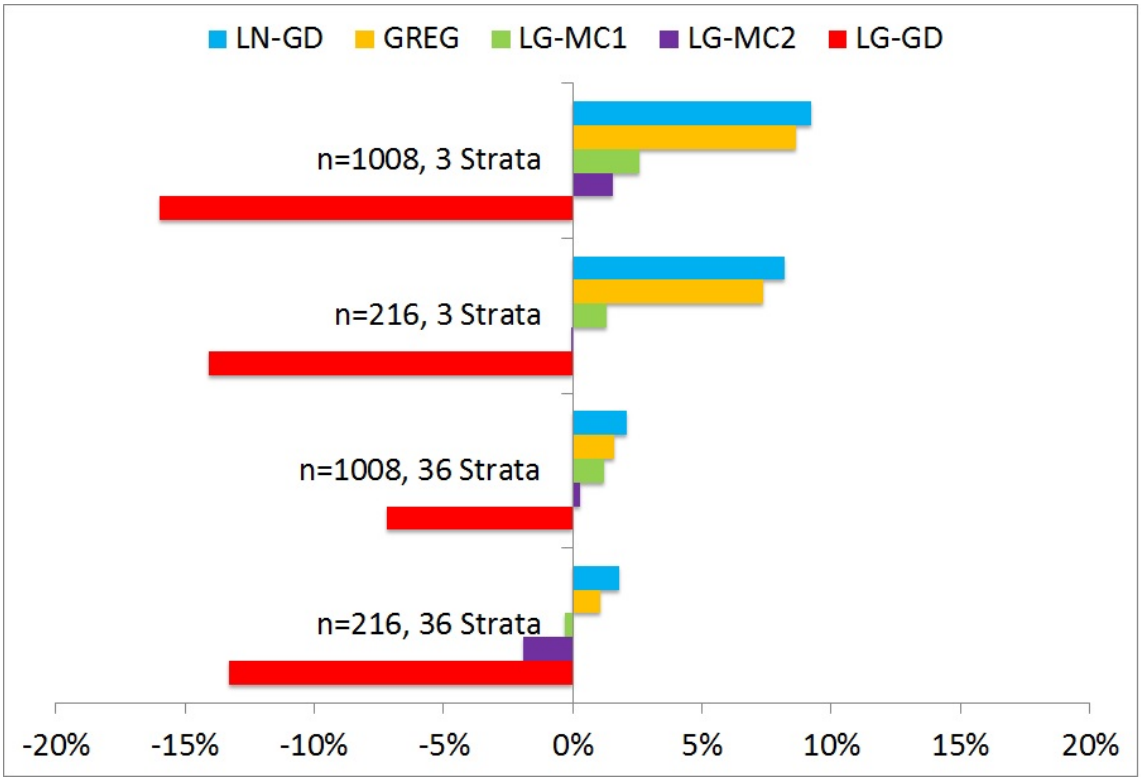

Figure 5.2: Simulated Percent Reduction of RMSE Relative to the π-estimator by Sample Size and Strata

Figure 5.2 shows the percent reduction in RMSE of each of the estimators compared to the π-estimator. Negative values mean that an estimator had a larger RMSE than the π-estimator. The LN-GD and GREG outperformed the estimators based on logistic models for every condition. The LG-GD estimator had significantly larger RMSEs than the π-estimator. Similar to the Chapter 4 simulation study, the LG-MC2 underperformed the π-estimator for one combination ($n = 216$, 36 strata)

and had little if any gains for the other combinations. This finding is contrary to the assertions of Wu and Sitter (2001).

Figure 5.2 shows the importance of number of strata on the LN-GD and the GREG. For both estimators, the percent reduction in RMSE for samples with three strata is four times larger than the RMSE for samples with 36 strata. Hence, for the GREG and LN-GD to see significant reductions in RMSE covariates need to be used that are not in the sample design. In the 36 strata design, BMI, age, and smoking status were used to define the strata. Besides that fact that continuous versions of BMI and age were used in the model, pack years was the only new information. In the 3 strata design, only smoking status was used to define the strata. This means that the BMI, age, and pack years were all providing new information that was not part of the sample design.

5.5.3 Performance of the Variance Estimator

Table 5.4 shows that when n=1008 the VRs were close to 1 for all of the estimators. This was also seen in Chapter 4, and it tells us that on average the asymptotic variance estimator was equivalent to the empirical variance of the estimator. Additionally, the simulated 95% confidence interval provided nominal coverage. This is not surprising, since all of the estimators were approximately

unbiased and the VRs were close to 1.

For n=218, the VRs were somewhat less than 1 for all of the estimators, except the π-estimator. The VRs were around 0.95. This means that the variance estimators on average underestimated the empirical variance by about 5%. Therefore, the simulated 95% confidence interval coverage was slightly less than nominal coverage. The undercoverage was not large. The coverage for all of the estimators other than the π-estimator was around 0.94, suggesting that the variance estimator might have been sensitive to sample size.

Table 5.4: Nurses' Health Study Simulation Results

				Weibull			Lognormal			Proportional Hazard			Logistic		
Metric	n	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
Δ_{RMSE}	180	0.00%	1.15%	1.77%	1.74%	1.76%	1.72%	1.67%	1.69%	1.77%	1.75%	1.76%	-11.54%	-0.56%	-1.68%
	1080	0.00%	1.62%	2.07%	2.08%	2.08%	1.98%	2.02%	2.02%	2.08%	2.08%	2.08%	1.23%	1.97%	1.89%
% RB	180	0.02%	-0.06%	-0.11%	-0.11%	-0.11%	-0.04%	-0.04%	-0.04%	-0.11%	-0.11%	-0.11%	0.29%	0.14%	0.19%
	1080	0.02%	0.02%	0.02%	0.02%	0.02%	0.03%	0.03%	0.03%	0.02%	0.02%	0.02%	0.03%	0.03%	0.03%
BR	180	0.0010	-0.0042	-0.0076	-0.0077	-0.0077	-0.0076	-0.0077	-0.0077	-0.0074	-0.0074	-0.0074	0.0174	0.0096	0.0126
	1080	0.0038	0.0034	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0056	0.0046	0.0048
VR	180	0.99	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.98	0.96	0.96
	1080	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
CR	180	0.936	0.933	0.932	0.932	0.932	0.933	0.932	0.932	0.932	0.932	0.932	0.935	0.933	0.933
	1080	0.950	0.949	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950

5.5.4 Computational Problems with the Model Calibrated Logistic Estimators

Similar to the Chapter 4 simulation study, the LG-MC1 and LG-MC2 had some simulated samples were excluded from analysis, because $\hat{p}(t)$ was less than 0 or greater than 1. This affected only a small proportion of the samples. Since it was known that $p(t)$ ranges from 0 to 1, these estimates were not reasonable. As was seen previously, these estimates were not just slightly outside of range, they were usually greater than 10^{10} or -10^{10}. This was caused by the calibration adjustment, $\hat{B}$, diverging to either positive or negative infinity. As in the Chapter 4 simulation study, this issue did not affect any of the time-to-event model based MCEs. Table 5.5 shows the number of simulates thrown out for each set of conditions. The problems with the logistic model-calibrated approach are caused by some combinations of covariates all having the event or not having the event. The fitting alogrithm sends one or more of the parameter estimates to $\pm\infty$. A potential fix is to combine levels of factors to create combos where there is a mixture of events and non-events

The number of simulates excluded was influenced by number of strata and sample size. A smaller sample size and fewer strata resulted in more excluded simulates, i.e.,

Table 5.5: Number of Simulations out of 10,000 where the Model Calibrated Logistic Estimate was Greater than 1 or Less than 0

n	Strata	MC1	MC2
216	3	146	146
216	36	81	81
1,008	3	12	12
1,008	36	6	6

a less efficient design, resulted in more simulates beingexcluded. The most severe problem was with $n = 216$ and 3 strata, where 146 (or 1.46%) of the simulates could not be included. Although this computational problem was rare, the fact that it happened at all is another reason not to use LG-MC1 and LG-MC2 to estimate $p_N(t)$.

5.6 Discussion

The application of the time-to-event based GDEs and MCEs to the NHS data showed that these estimators performed better than current methods under all of the conditions tested. The GREG performed almost as well as the time-to-event based GDEs and MCEs. Both the GREG and the estimators based on time-to-event models were approximately unbiased and outperformed the π-estimator. For this application, these estimators were clearly the best options with a slight edge to the time-to-event GDEs and MCEs. The only drawback was the slight confidence

interval undercoverage when using the 36 strata design with a sample size of $n = 216$.

The logistic-based estimators performed poorly under every condition, with the LG-GD underperforming the π-estimator by more than 7 percentage points with respect to RMSE. With the estimators based on a logistic model performing poorly with respect to the reduction in RMSE, and the computational issues with the LG-MC1 and LG-MC2, it is clear that estimators based on a logistic model were not a good choice for the NHS data.

Perhaps the most important finding from this application is that the time-to-event based GDEs and MCEs performed particularly well when model information was not also used in the sample design. Therefore, for these estimators to perform well generally, covariates are needed that are both predictive of time-to-event and not used in the sample design. This might happen as in this application, when the sample was poorly designed creating an inefficient sample. It might also happen when good covariate information is not available at the time of data collection but is available after data collection. For example, when covariate information is obtained from administrative records, the lag time between the survey data collection and the acquisition, preparation, and linking of administrative data can be lengthy. Another example is a longitudinal survey where the sample is drawn at the beginning of a panel and covariates are collected

sometime after the panel is originally fielded. Additionally, it might happen when the survey was designed to estimate some outcome other than death.

Chapter 6: Conclusions

In this dissertation, I proposed a new class of model-assisted estimators for estimating the proportion of the population that has experienced an event by some time t. I used time-to-event models to develop GDEs and MCEs. These estimators are an extension of the model-calibrated estimators proposed by Wu and Sitter (2001). When constructing a GDE or MCE, the probability that an event has occurred at or before time t must be estimated for each sampled and nonsampled unit. These estimates, $\{\hat{p}(t|\mathbf{z_i})\}_{i=1}^{N}$, depend on fitting a model to predict the event probability for each combination of covariates, $\mathbf{z_i}$, that occurs in the population.

It was proved that under some general regularity conditions both the GDEs and MCEs are design consistent. Additionally, it was proved that the proposed asymptotic variance estimators for the GDEs and MCEs are also design consistent.

Through simulation, it was shown that time-to-event model-based GDEs and MCEs were approximately unbiased for all conditions tested. Additionally, these estimators were at least as efficient as the existing model-assisted estimators and

the π-estimator and in many situations outperformed existing methods. As expected, the reductions in RMSE increased as the correlation between the predictors in the model and the time to event increased. The reduction in RMSE was positively correlated with the proportion of the population that experienced the event. Finally, the asymptotic variance estimator for the proposed estimators was on average equal to the empirical variance of the estimators seen in the simulation. Because of this and the fact that the GDEs and MCEs were approximately unbiased, the confidence intervals provided nominal coverage. These simulations also showed that using logistic model-based GDEs and MCEs can caused both computational issues as well as poor performing estimators. Estimators based on lognormal, Weibull, and semiparametric PH models were not vulnerable to these problems.

An application to the NHS confirmed the findings from the simulation study that these estimators are approximately unbiased, at least as efficient as the existing methods, and in many situations outperform existing methods. The application showed that there is a relationship between the sample design and the time-to-event model-based GDEs and MCEs for reducing RMSE. When covariates used in the model were not used in the sample design, the time-to-event model based GDEs and MCEs provided larger reductions in RMSE.

This dissertation adds to the toolkit available to survey practitioners for estimating the proportion of the population that has experienced an event after a specified amount of time has elapsed. This is especially important for panel surveys in that a these estimators can be used to leverage covariate information from administrative sources. Since they have been shown to be design consistent and approximately unbiased, these estimators can be used without the risk of introducing bias, and the effectiveness of these estimators can be evaluated based on reductions in variance. Because of this, the effectiveness of these estimators can be evaluated during post data collection processing.

There are some important considerations when using time-to-event model-based GDEs and MCEs. First, these estimators can only be used if covariates are available for each unit in the entire population. Second, to see the most gains in efficiency, the covariates need to be predictive of time-to-event and not used in the sample design. Third, the expected proportion of the population that has experienced the event needs to be large. Finally, the time to event needs to be censored at the same time t for all of units.

There are many ways that this work could be expanded. It could be extended for left or interval censoring; this dissertation focused on commonly used time-to-event models under right censoring. The simulations could be expanded to include

TRMs, models with time-varying covariates, and semiparametric AFTMs, to name a few. Although the design consistency of the GDEs and MCEs and asymptotic variance estimators covers a wide range of models, the simulations were limited to only a few models. Additionally, the simulations could be expanded to explore the effect of model misspecification on these estimators. Here, the time-to-event model-based GDEs and MCEs performed similarly. Model misspecification might cause the estimators to perform differently. There is some evidence to support this given the logistic model based GDEs and MCEs findings.

One of the issues with using $\hat{p}_{GDE}(t)$ and $\hat{p}_{MCE}(t)$ is that the censoring has to happen at a given t for all units. This can be restrictive. In some applications censoring might happen at different times. For example, when estimating the proportion of first time mothers who stopped breastfeeding by one year after birth. The 1979 Panel of the National Longitudinal Survey of Youth collected information on pregnancies and breastfeeding from female respondents. Since all of the women in this survey did not give birth on the same day, the time at which a woman could be censored, i.e., still breastfeeding at time of data collection, varies (Klein and Moeschberger, 2003).

To deal with this variation in censoring time, $\hat{p}_{GDE}(t)$ and $\hat{p}_{MCE}(t)$ need to be modified. I propose the following modification.

One way to calculate survival time, and thus failure time, is to use a Kaplan-Meier estimator of $S(t)$ (Kaplan and Meier, 1958).

$$S_{KM}(t) = \begin{cases} 1 & \text{if } t < t_1 \\ \prod_{t_i \leq t} \left[1 - \frac{d_i}{r_i}\right] & \text{if } t_1 \leq t \end{cases}, \tag{6.1}$$

where $t_1 < t_2 < \ldots$ are the distinct event times, d_i is the number of events at time t_i, and r_i is the number of units who have not experienced the event or have not been censored at time t_i. Since we know that $p(t) = F(t) = 1 - S(t)$, a Kaplan-Meier based estimate of $p(t)$ can be written as follows:

$$p_{KM}(t) = \begin{cases} 0 & \text{if } t < t_1 \\ 1 - \prod_{t_i \leq t} \left[1 - \frac{d_i}{r_i}\right] & \text{if } t_1 \leq t \end{cases}. \tag{6.2}$$

In the case of sample surveys, $S_{KM}(t)$, can be calculated by estimating d_i and r_i using π-estimators (Korn and Graubard, 1999), giving us the following estimator of $p_N(t)$

$$\hat{p}_{KM}(t) = \begin{cases} 0 & \text{if } t < t_1 \\ 1 - \prod_{t_i \leq t} \left[1 - \frac{\hat{d}_i}{\hat{r}_i}\right] & \text{if } t_1 \leq t \end{cases}, \tag{6.3}$$

where $\hat{d}_i$ and $\hat{r}_i$ are the π-estimators of d_i and r_i, respectively. Since $\sum_{i=1}^{n} \pi_i^{-1} = \hat{N}$,

$$\hat{p}_{KM}(t) = \hat{N}^{-1} \sum_{i=1}^{n} \pi_i^{-1} \hat{p}_{KM}. \tag{6.4}$$

From this we can form new GDEs and MCEs:

$$\hat{p}^*_{GDE}(t) = \hat{N}^{-1} \left(\sum_{i=1}^{N} p(t|z_i, \hat{\theta}) + \sum_{i \in s} \pi_i^{-1} \left[\hat{p}_{KM}(t) - p(t|z_i, \hat{\theta}) \right] \right), \tag{6.5}$$

and

$$\hat{p}^*_{MCE}(t) = \hat{p}_{KM}(t) + \hat{N}^{-1} \left(\sum_{i=1}^{N} p(t|z_i, \hat{\theta}) - \sum_{i \in s} \pi_i^{-1} p(t|z_i, \hat{\theta}) \right) \hat{B}, \tag{6.6}$$

where $\hat{B}$ is a calibration adjustment similar to what is proposed by Wu and Sitter (2001) and discussed in Chapter 2. Since $\hat{p}_{KM}(t)$ provides a value for every case at a fixed time t, censored or not, $\hat{p}^*_{GDE}(t)$ and $\hat{p}^*_{MCE}(t)$ can accommodate varying censoring times. Another advantage of using $\hat{p}^*_{GDE}(t)$ and $\hat{p}^*_{MCE}(t)$ is that these estimators can handle panel attrition. Traditionally, this would be handled via a weighting adjustment.

Some of the theoretical work presented in Chapter 3 could possibly be modified to incorporate $\hat{p}^*_{GDE}(t)$ and $\hat{p}^*_{MCE}(t)$, but this will not be straightforward. In addition, simulation work would need to be done to explore the properties of these new estimators.

In summation, time-to-event model-based GDE and MCE increase efficiency without inducing much bias. As with other model-assisted approaches these estimators effectively use models to improve efficiency over the π-estimator with little risk of inducing bias. They are an effective way to leverage covariate information for surveys that collect time-to-event data.

Appendix A: Simulation Tables

A.1 Weibull Populations with a Common Baseline Hazard

A.1.0.1 Relative Bias

Table A.1: Simulated Percent Relative Bias: Weibull Population with a Common Baseline Hazard, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.08	0.11	0.26	0.15	0.17
		0.50	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.09	0.11	0.26	0.15	0.17
		0.75	0.13	-0.36	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.26	0.15	0.17
	0.5	0.25	0.03	-0.31	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.07	0.09	0.08
		0.50	0.03	-0.31	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.07	0.09	0.08
	0.75	0.25	0.06	-0.09	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.32	0.30	0.30
0.6	0.25	0.25	0.21	0.12	0.18	0.16	0.19	0.18	0.18	0.19	0.19	0.16	0.19	0.50	0.08	0.12
		0.50	0.21	0.12	0.18	0.17	0.18	0.18	0.19	0.19	0.18	0.17	0.18	0.50	0.08	0.12
		0.75	0.21	0.12	0.18	0.18	0.18	0.19	0.19	0.19	0.18	0.18	0.18	0.50	0.08	0.12
	0.5	0.25	0.07	-0.13	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.20	0.12	0.15
		0.50	0.07	-0.13	0.05	0.06	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.20	0.12	0.15
	0.75	0.25	0.03	-0.04	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.27	0.21	0.23
0.4	0.25	0.25	0.24	0.06	0.23	0.21	0.23	0.23	0.23	0.23	0.23	0.21	0.23	0.35	0.06	0.06
		0.50	0.24	0.06	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.35	0.06	0.06
		0.75	0.24	0.06	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.35	0.06	0.06
	0.5	0.25	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.23	0.07	0.12
		0.50	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.13	0.12	0.23	0.07	0.12
	0.75	0.25	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.15	0.12	0.12

Table A.2: Simulated Percent Relative Bias: Weibull Population with a Common Baseline Hazard, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.75	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.5	0.25	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
		0.50	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
0.6	0.25	0.25	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.50	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.75	-0.04	-0.11	-0.05	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.05	-0.01	0.00	0.00
	0.5	0.25	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
		0.50	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	0.75	0.25	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.05	0.04
0.4	0.25	0.25	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.50	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.75	0.01	-0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.06	0.02	0.02
	0.5	0.25	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.02	0.02

A.1.0.2 Bias Ratio

Table A.3: Simulated Bias Ratio: Weibull Population with a Common Baseline Hazard, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.08	0.11	0.26	0.15	0.17
		0.50	0.13	-0.36	0.11	0.09	0.11	0.11	0.11	0.11	0.11	0.09	0.11	0.26	0.15	0.17
		0.75	0.13	-0.36	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.26	0.15	0.17
	0.5	0.25	0.03	-0.31	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.07	0.09	0.08
		0.50	0.03	-0.31	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.07	0.09	0.08
	0.75	0.25	0.06	-0.09	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.32	0.30	0.30
0.6	0.25	0.25	0.21	0.12	0.18	0.16	0.19	0.18	0.18	0.19	0.19	0.16	0.19	0.50	0.08	0.12
		0.50	0.21	0.12	0.18	0.17	0.18	0.18	0.19	0.19	0.18	0.17	0.18	0.50	0.08	0.12
		0.75	0.21	0.12	0.18	0.18	0.18	0.19	0.19	0.19	0.18	0.18	0.18	0.50	0.08	0.12
	0.5	0.25	0.07	-0.13	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.20	0.12	0.15
		0.50	0.07	-0.13	0.05	0.06	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.20	0.12	0.15
	0.75	0.25	0.03	-0.04	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.27	0.21	0.23
0.4	0.25	0.25	0.24	0.06	0.23	0.21	0.23	0.23	0.23	0.23	0.23	0.21	0.23	0.35	0.06	0.06
		0.50	0.24	0.06	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.35	0.06	0.06
		0.75	0.24	0.06	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.35	0.06	0.06
	0.5	0.25	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.23	0.07	0.12
		0.50	0.13	0.04	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.13	0.12	0.23	0.07	0.12
	0.75	0.25	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.15	0.12	0.12

Table A.4: Simulated Bias Ratio: Weibull Population with a Common Baseline Hazard, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.75	-0.01	-0.11	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.5	0.25	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
		0.50	-0.01	-0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	-0.03	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
0.6	0.25	0.25	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.50	-0.04	-0.11	-0.04	-0.05	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.01	0.00	0.00
		0.75	-0.04	-0.11	-0.05	-0.04	-0.05	-0.04	-0.04	-0.04	-0.05	-0.05	-0.05	-0.01	0.00	0.00
	0.5	0.25	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
		0.50	-0.04	-0.08	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	0.75	0.25	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.05	0.04
0.4	0.25	0.25	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.50	0.01	-0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.06	0.02	0.02
		0.75	0.01	-0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.06	0.02	0.02
	0.5	0.25	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.02	0.02

A.1.0.3 Percent Reduction of RMSE Relative to the π-estimator

Table A.5: Simulated Percent Reduction of RMSE Relative to the π-estimator: Weibull Population with a Common Baseline Hazard, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-5.66	0.97	0.61	0.91	1.39	1.36	1.40	0.86	0.45	0.81	1.22	1.37	1.36
		0.50	0.00	-5.66	1.24	1.18	1.25	1.44	1.43	1.46	1.21	1.12	1.20	1.22	1.37	1.36
		0.75	0.00	-5.66	1.33	1.35	1.35	1.37	1.39	1.39	1.34	1.35	1.35	1.22	1.37	1.36
	0.5	0.25	0.00	-1.73	4.43	4.43	4.54	4.75	4.65	4.77	4.54	4.46	4.53	-0.83	2.03	1.14
		0.50	0.00	-1.73	4.45	4.37	4.41	4.65	4.58	4.63	4.42	4.35	4.40	-0.83	2.03	1.14
	0.75	0.25	0.00	12.39	16.21	16.11	16.10	16.84	16.63	16.83	15.85	15.68	15.85	4.77	11.55	8.27
0.6	0.25	0.25	0.00	-1.47	1.02	0.78	1.05	1.14	1.06	1.16	1.03	0.74	1.03	-3.85	0.08	-0.21
		0.50	0.00	-1.47	1.08	1.03	1.12	1.15	1.10	1.16	1.12	1.03	1.11	-3.85	0.08	-0.21
		0.75	0.00	-1.47	1.10	1.09	1.10	1.13	1.10	1.13	1.10	1.09	1.10	-3.85	0.08	-0.21
	0.5	0.25	0.00	2.16	3.68	3.83	3.88	4.05	3.96	4.11	3.86	3.91	3.87	-2.18	1.63	0.77
		0.50	0.00	2.16	3.91	3.80	3.86	4.06	3.97	4.04	3.87	3.79	3.85	-2.18	1.63	0.77
	0.75	0.25	0.00	7.76	8.48	8.32	8.40	8.76	8.58	8.75	8.36	8.13	8.35	-4.43	5.18	0.63
0.4	0.25	0.25	0.00	0.12	0.95	0.84	1.02	1.07	0.95	1.11	0.99	0.83	1.00	-2.87	0.44	0.09
		0.50	0.00	0.12	0.99	0.98	1.07	1.09	1.01	1.12	1.05	0.98	1.06	-2.87	0.44	0.09
		0.75	0.00	0.12	1.05	1.02	1.05	1.07	1.03	1.07	1.05	1.02	1.05	-2.87	0.44	0.09
	0.5	0.25	0.00	2.08	2.28	2.42	2.44	2.57	2.47	2.62	2.41	2.47	2.42	-0.62	2.14	1.63
		0.50	0.00	2.08	2.52	2.45	2.50	2.58	2.51	2.58	2.50	2.43	2.49	-0.62	2.14	1.63
	0.75	0.25	0.00	3.51	3.81	3.67	3.78	3.99	3.79	3.99	3.80	3.60	3.80	-7.80	2.28	-2.32

Table A.6: Simulated Percent Reduction of RMSE Relative to the π-estimator: Weibull Population with a Common Baseline Hazard, N=100,000, n=1,000, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-4.84	1.00	0.75	0.97	1.42	1.42	1.44	0.92	0.60	0.88	1.49	1.49	1.49
		0.50	0.00	-4.84	1.26	1.25	1.29	1.46	1.47	1.48	1.25	1.20	1.25	1.49	1.49	1.49
		0.75	0.00	-4.84	1.45	1.45	1.45	1.49	1.49	1.49	1.45	1.44	1.45	1.49	1.49	1.49
	0.5	0.25	0.00	-2.08	4.19	4.26	4.26	4.52	4.50	4.53	4.24	4.20	4.24	4.45	4.45	4.45
		0.50	0.00	-2.08	4.29	4.26	4.27	4.50	4.47	4.49	4.27	4.23	4.26	4.45	4.45	4.45
	0.75	0.25	0.00	12.28	16.39	16.30	16.28	16.99	16.89	16.97	16.04	15.90	16.03	16.85	16.87	16.85
0.6	0.25	0.25	0.00	-1.13	1.05	0.94	1.07	1.18	1.19	1.20	1.05	0.89	1.05	1.18	1.17	1.18
		0.50	0.00	-1.13	1.12	1.15	1.17	1.19	1.20	1.21	1.15	1.13	1.15	1.18	1.17	1.18
		0.75	0.00	-1.13	1.17	1.16	1.16	1.18	1.18	1.18	1.17	1.16	1.16	1.18	1.17	1.18
	0.5	0.25	0.00	2.05	3.56	3.81	3.73	3.89	3.90	3.93	3.72	3.84	3.72	3.88	3.87	3.88
		0.50	0.00	2.05	3.81	3.73	3.77	3.92	3.88	3.91	3.78	3.72	3.76	3.88	3.87	3.88
	0.75	0.25	0.00	7.61	8.60	8.45	8.53	8.78	8.70	8.77	8.49	8.31	8.47	-1.71	6.64	2.60
0.4	0.25	0.25	0.00	0.26	0.87	0.90	0.92	0.96	0.97	0.98	0.90	0.88	0.91	-1.88	0.16	0.03
		0.50	0.00	0.26	0.92	0.97	0.97	0.98	0.98	0.99	0.96	0.97	0.97	-1.88	0.16	0.03
		0.75	0.00	0.26	0.97	0.96	0.97	0.98	0.97	0.98	0.97	0.96	0.97	-1.88	0.16	0.03
	0.5	0.25	0.00	2.10	2.24	2.45	2.38	2.48	2.49	2.51	2.35	2.48	2.35	2.50	2.49	2.50
		0.50	0.00	2.10	2.47	2.42	2.46	2.52	2.49	2.52	2.46	2.42	2.45	2.50	2.49	2.50
	0.75	0.25	0.00	3.57	3.78	3.68	3.76	3.81	3.76	3.82	3.76	3.64	3.76	-9.92	2.26	-3.66

A.1.0.4 Variance Ratio

Table A.7: Simulated Variance Ratio: Weibull Population with a Common Baseline Hazard, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.75	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.5	0.25	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.96	0.97
		0.50	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.97	0.96	0.96	0.97	0.96	0.97
	0.75	0.25	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	0.99	1.01
0.6	0.25	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
		0.75	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
	0.5	0.25	0.98	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97
		0.50	0.98	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97
	0.75	0.25	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	0.99	1.02
0.4	0.25	0.25	1.00	1.00	1.01	1.00	1.01	1.01	1.00	1.01	1.01	1.00	1.01	0.99	1.01	1.01
		0.50	1.00	1.00	1.00	1.00	1.01	1.01	1.00	1.01	1.00	1.00	1.00	0.99	1.01	1.01
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
	0.5	0.25	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.99	0.98	0.98
		0.50	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.99	0.98	0.98
	0.75	0.25	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.02

Table A.8: Simulated Variance Ratio: Weibull Population with a Common Baseline Hazard, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.50	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.75	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
	0.5	0.25	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
		0.50	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
	0.75	0.25	0.98	0.98	0.99	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.98	0.99	0.99	0.99
0.6	0.25	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
		0.50	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
		0.75	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.01
	0.5	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.75	0.25	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.02	1.00	1.02
0.4	0.25	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.5	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.75	0.25	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	1.01	0.98	1.00

A.1.0.5 Confidence Interval Coverage

Table A.9: Simulated 95% Confidence Interval Coverage: Weibull Population with a Common Baseline Hazard, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.946	0.946	0.946	0.945	0.946	0.945	0.945	0.946	0.946	0.946	0.946	0.945	0.945	0.945
		0.50	0.946	0.946	0.946	0.945	0.945	0.945	0.945	0.945	0.946	0.946	0.946	0.945	0.945	0.945
		0.75	0.946	0.946	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945
	0.5	0.25	0.943	0.943	0.942	0.943	0.942	0.942	0.943	0.942	0.942	0.942	0.942	0.945	0.944	0.944
		0.50	0.943	0.943	0.942	0.941	0.941	0.941	0.940	0.940	0.941	0.941	0.941	0.945	0.944	0.944
	0.75	0.25	0.946	0.943	0.943	0.942	0.943	0.942	0.942	0.942	0.944	0.943	0.944	0.940	0.938	0.940
0.6	0.25	0.25	0.946	0.944	0.947	0.947	0.947	0.946	0.947	0.947	0.947	0.947	0.947	0.943	0.945	0.944
		0.50	0.946	0.944	0.946	0.946	0.946	0.946	0.947	0.946	0.946	0.946	0.946	0.943	0.945	0.944
		0.75	0.946	0.944	0.946	0.946	0.946	0.946	0.945	0.946	0.946	0.946	0.946	0.943	0.945	0.944
	0.5	0.25	0.944	0.943	0.945	0.944	0.944	0.944	0.944	0.944	0.945	0.944	0.945	0.945	0.945	0.944
		0.50	0.944	0.943	0.944	0.944	0.943	0.944	0.944	0.944	0.943	0.943	0.943	0.945	0.945	0.944
	0.75	0.25	0.945	0.948	0.947	0.946	0.947	0.947	0.946	0.947	0.948	0.947	0.947	0.948	0.945	0.948
0.4	0.25	0.25	0.945	0.946	0.946	0.947	0.945	0.946	0.946	0.946	0.945	0.947	0.945	0.944	0.945	0.945
		0.50	0.945	0.946	0.946	0.947	0.946	0.946	0.946	0.946	0.946	0.947	0.946	0.944	0.945	0.945
		0.75	0.945	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.944	0.945	0.945
	0.5	0.25	0.943	0.942	0.943	0.941	0.942	0.942	0.942	0.942	0.942	0.941	0.942	0.945	0.944	0.943
		0.50	0.943	0.942	0.941	0.941	0.941	0.942	0.942	0.942	0.941	0.941	0.941	0.945	0.944	0.943
	0.75	0.25	0.944	0.944	0.945	0.944	0.945	0.945	0.944	0.944	0.945	0.944	0.945	0.944	0.940	0.943

Table A.10: Simulated 95% Confidence Interval Coverage: Weibull Population with a Common Baseline Hazard, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.946	0.948	0.947	0.948	0.947	0.945	0.945	0.945	0.947	0.948	0.948	0.946	0.946	0.946
		0.50	0.946	0.948	0.947	0.946	0.946	0.945	0.945	0.946	0.946	0.947	0.946	0.946	0.946	0.946
		0.75	0.946	0.948	0.945	0.945	0.945	0.946	0.946	0.946	0.945	0.945	0.945	0.946	0.946	0.946
	0.5	0.25	0.948	0.948	0.948	0.948	0.949	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948
		0.50	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948
	0.75	0.25	0.949	0.952	0.952	0.951	0.952	0.951	0.952	0.951	0.952	0.951	0.952	0.952	0.951	0.951
0.6	0.25	0.25	0.948	0.947	0.948	0.948	0.948	0.949	0.949	0.949	0.949	0.948	0.948	0.948	0.948	0.948
		0.50	0.948	0.947	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.948	0.948	0.948
		0.75	0.948	0.947	0.949	0.948	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.948	0.948	0.948
	0.5	0.25	0.946	0.947	0.946	0.946	0.947	0.945	0.946	0.945	0.946	0.945	0.947	0.945	0.945	0.945
		0.50	0.946	0.947	0.946	0.947	0.946	0.945	0.945	0.945	0.946	0.947	0.946	0.945	0.945	0.945
	0.75	0.25	0.948	0.946	0.948	0.948	0.948	0.949	0.949	0.949	0.948	0.948	0.948	0.951	0.948	0.949
0.4	0.25	0.25	0.946	0.946	0.944	0.945	0.944	0.945	0.945	0.944	0.944	0.944	0.944	0.946	0.946	0.946
		0.50	0.946	0.946	0.944	0.945	0.945	0.944	0.944	0.944	0.945	0.945	0.945	0.946	0.946	0.946
		0.75	0.946	0.946	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.946	0.946	0.946
	0.5	0.25	0.949	0.949	0.947	0.947	0.947	0.947	0.948	0.947	0.947	0.948	0.947	0.948	0.948	0.948
		0.50	0.949	0.949	0.948	0.947	0.947	0.948	0.947	0.948	0.947	0.947	0.947	0.948	0.948	0.948
	0.75	0.25	0.952	0.949	0.950	0.950	0.950	0.949	0.950	0.949	0.950	0.950	0.950	0.949	0.951	0.950

A.2 Weibull Populations with a mixture of two Baseline Hazards

A.2.0.1 Relative Bias

Table A.11: Simulated Percent Relative Bias: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.20	-0.38	0.16	0.13	0.16	0.17	0.17	0.17	0.16	0.12	0.16	0.29	0.23	0.25
		0.50	0.20	-0.38	0.16	0.14	0.16	0.17	0.17	0.17	0.16	0.14	0.16	0.29	0.23	0.25
		0.75	0.20	-0.38	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.29	0.23	0.25
	0.5	0.25	0.02	-0.36	-0.02	-0.02	-0.03	-0.03	-0.02	-0.03	-0.03	-0.04	-0.03	-0.08	0.10	0.09
		0.50	0.02	-0.36	-0.04	-0.03	-0.03	-0.04	-0.03	-0.04	-0.03	-0.03	-0.03	-0.08	0.10	0.09
	0.75	0.25	-0.01	-0.17	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.05	-0.06	-0.19	0.18	0.18
0.6	0.25	0.25	0.13	-0.31	0.09	0.05	0.09	0.09	0.09	0.09	0.09	0.04	0.09	0.43	0.04	0.10
		0.50	0.13	-0.31	0.09	0.07	0.09	0.09	0.09	0.09	0.09	0.06	0.09	0.43	0.04	0.10
		0.75	0.13	-0.31	0.09	0.09	0.09	0.08	0.09	0.08	0.09	0.09	0.09	0.43	0.04	0.10
	0.5	0.25	0.00	-0.25	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.05	-0.04	-0.15	0.06	0.10
		0.50	0.00	-0.25	-0.05	-0.04	-0.05	-0.05	-0.04	-0.05	-0.05	-0.04	-0.05	-0.15	0.06	0.10
	0.75	0.25	0.03	-0.04	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.01	-0.01	-0.20	0.18	0.18
0.4	0.25	0.25	-0.04	-0.34	-0.07	-0.13	-0.07	-0.08	-0.09	-0.08	-0.07	-0.13	-0.07	-0.10	-0.23	-0.21
		0.50	-0.04	-0.34	-0.07	-0.13	-0.07	-0.08	-0.09	-0.08	-0.07	-0.13	-0.07	-0.10	-0.23	-0.21
		0.75	-0.04	-0.34	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.10	-0.23	-0.21
	0.5	0.25	0.03	-0.10	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.09	-0.02	0.01
		0.50	0.03	-0.10	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.09	-0.02	0.01
	0.75	0.25	0.05	0.04	0.04	0.05	0.04	0.03	0.05	0.03	0.04	0.06	0.04	0.14	0.15	0.12

Table A.12: Simulated Percent Relative Bias: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=1,000, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.08	-0.02	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		0.50	0.08	-0.02	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		0.75	0.08	-0.02	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
	0.5	0.25	0.09	0.02	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
		0.50	0.09	0.02	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
	0.75	0.25	0.01	-0.02	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.25	0.25	0.16	0.08	0.15	0.14	0.15	0.15	0.15	0.15	0.15	0.14	0.15	0.15	0.15	0.15
		0.50	0.16	0.08	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
		0.75	0.16	0.08	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	0.5	0.25	0.06	0.02	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
		0.50	0.06	0.02	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	0.75	0.25	0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.07	0.07	0.07
0.4	0.25	0.25	0.14	0.09	0.14	0.13	0.14	0.14	0.14	0.14	0.14	0.13	0.14	0.18	0.16	0.17
		0.50	0.14	0.09	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.18	0.16	0.17
		0.75	0.14	0.09	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.18	0.16	0.17
	0.5	0.25	0.09	0.07	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
		0.50	0.09	0.07	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
	0.75	0.25	0.04	0.04	0.05	0.05	0.04	0.04	0.05	0.04	0.04	0.05	0.04	0.08	0.16	0.17

A.2.0.2 Bias Ratio

Table A.13: Simulated Bias Ratio: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.019	-0.034	0.015	0.012	0.015	0.015	0.013	0.015	0.016	0.012	0.015	0.017	0.022	0.024
		0.50	0.019	-0.034	0.016	0.014	0.015	0.016	0.014	0.015	0.015	0.013	0.015	0.017	0.022	0.024
		0.75	0.019	-0.034	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.022	0.024
	0.5	0.25	0.003	-0.056	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.005	-0.006	-0.005	-0.005	0.016	0.015
		0.50	0.003	-0.056	-0.006	-0.005	-0.005	-0.006	-0.005	-0.005	-0.005	-0.004	-0.005	-0.005	0.016	0.015
	0.75	0.25	-0.003	-0.047	-0.018	-0.018	-0.018	-0.018	-0.018	-0.018	-0.017	-0.015	-0.017	-0.016	0.048	0.045
0.6	0.25	0.25	0.011	-0.027	0.008	0.004	0.008	0.008	0.004	0.008	0.008	0.003	0.008	0.007	0.004	0.009
		0.50	0.011	-0.027	0.008	0.006	0.008	0.008	0.006	0.008	0.008	0.006	0.008	0.007	0.004	0.009
		0.75	0.011	-0.027	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.004	0.009
	0.5	0.25	0.000	-0.036	-0.005	-0.007	-0.006	-0.005	-0.007	-0.006	-0.006	-0.008	-0.006	-0.006	0.009	0.014
		0.50	0.000	-0.036	-0.007	-0.006	-0.007	-0.007	-0.006	-0.007	-0.007	-0.006	-0.007	-0.006	0.009	0.014
	0.75	0.25	0.006	-0.010	-0.001	0.001	-0.001	-0.001	0.001	-0.001	-0.001	0.002	-0.001	-0.001	0.042	0.040
0.4	0.25	0.25	-0.004	-0.028	-0.006	-0.011	-0.006	-0.006	-0.011	-0.006	-0.006	-0.011	-0.006	-0.007	-0.019	-0.017
		0.50	-0.004	-0.028	-0.006	-0.011	-0.006	-0.006	-0.011	-0.006	-0.006	-0.011	-0.006	-0.007	-0.019	-0.017
		0.75	-0.004	-0.028	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.019	-0.017
	0.5	0.25	0.004	-0.013	0.002	0.000	0.001	0.002	0.000	0.001	0.001	0.000	0.002	0.000	-0.003	0.002
		0.50	0.004	-0.013	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	-0.003	0.002
	0.75	0.25	0.011	0.010	0.009	0.012	0.009	0.009	0.012	0.009	0.009	0.013	0.009	0.008	0.033	0.027

Table A.14: Simulated Bias Ratio: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.017	-0.004	0.016	0.014	0.016	0.016	0.014	0.016	0.016	0.014	0.016	0.015	0.015	0.015
		0.50	0.017	-0.004	0.015	0.014	0.015	0.015	0.014	0.015	0.015	0.014	0.015	0.015	0.015	0.015
		0.75	0.017	-0.004	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
	0.5	0.25	0.031	0.008	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.029	0.029	0.029
		0.50	0.031	0.008	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	0.75	0.25	0.003	-0.013	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002
0.6	0.25	0.25	0.031	0.015	0.030	0.028	0.030	0.030	0.028	0.030	0.030	0.028	0.030	0.029	0.029	0.029
		0.50	0.031	0.015	0.030	0.029	0.030	0.030	0.029	0.030	0.030	0.029	0.030	0.029	0.029	0.029
		0.75	0.031	0.015	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	0.5	0.25	0.020	0.006	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.019	0.020	0.019	0.020	0.019
		0.50	0.020	0.006	0.019	0.020	0.019	0.019	0.020	0.019	0.019	0.020	0.019	0.019	0.020	0.019
	0.75	0.25	0.008	0.003	0.009	0.009	0.009	0.009	0.009	0.009	0.008	0.010	0.008	0.008	0.036	0.033
0.4	0.25	0.25	0.026	0.017	0.026	0.025	0.026	0.026	0.025	0.026	0.026	0.025	0.026	0.025	0.030	0.031
		0.50	0.026	0.017	0.026	0.025	0.026	0.026	0.025	0.026	0.026	0.025	0.026	0.025	0.030	0.031
		0.75	0.026	0.017	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.030	0.031
	0.5	0.25	0.027	0.021	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
		0.50	0.027	0.021	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027
	0.75	0.25	0.022	0.022	0.023	0.024	0.023	0.023	0.024	0.023	0.023	0.024	0.023	0.020	0.035	0.035

A.2.0.3 Percent Reduction of RMSE Relative to the π-estimator

Table A.15: Simulated Percent Reduction of RMSE Relative to the π-estimator: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=200, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-4.17	1.00	0.86	0.99	1.13	1.10	1.13	0.96	0.78	0.94	-0.47	0.76	0.69
		0.50	0.00	-4.17	1.06	1.02	1.06	1.10	1.08	1.10	1.06	1.00	1.05	-0.47	0.76	0.69
		0.75	0.00	-4.17	1.00	1.01	1.01	0.99	1.00	1.01	1.01	1.01	1.01	-0.47	0.76	0.69
	0.5	0.25	0.00	0.10	5.13	5.22	5.26	5.52	5.43	5.53	5.33	5.40	5.33	-1.47	2.06	0.94
		0.50	0.00	0.10	5.24	5.14	5.18	5.50	5.40	5.46	5.18	5.11	5.17	-1.47	2.06	0.94
	0.75	0.25	0.00	10.58	14.70	14.57	14.65	14.92	14.76	14.93	14.51	14.36	14.50	1.13	9.06	5.08
0.6	0.25	0.25	0.00	-0.66	1.39	1.44	1.49	1.55	1.56	1.62	1.44	1.42	1.46	-4.54	0.27	-0.20
		0.50	0.00	-0.66	1.44	1.54	1.56	1.55	1.55	1.61	1.54	1.55	1.55	-4.54	0.27	-0.20
		0.75	0.00	-0.66	1.49	1.45	1.47	1.54	1.49	1.52	1.48	1.45	1.47	-4.54	0.27	-0.20
	0.5	0.25	0.00	2.11	3.72	3.87	3.90	4.10	4.01	4.15	3.90	3.97	3.91	-3.31	1.58	0.32
		0.50	0.00	2.11	3.97	3.86	3.92	4.14	4.04	4.12	3.93	3.84	3.91	-3.31	1.58	0.32
	0.75	0.25	0.00	6.92	7.99	7.85	7.94	8.18	8.02	8.19	7.92	7.71	7.91	-6.11	4.37	-0.76
0.4	0.25	0.25	0.00	0.42	1.10	1.20	1.19	1.30	1.30	1.37	1.15	1.20	1.17	-3.45	0.61	0.17
		0.50	0.00	0.42	1.10	1.20	1.19	1.30	1.30	1.37	1.15	1.20	1.17	-3.45	0.61	0.17
		0.75	0.00	0.42	1.31	1.28	1.31	1.36	1.32	1.35	1.31	1.28	1.31	-3.45	0.61	0.17
	0.5	0.25	0.00	1.93	2.34	2.47	2.48	2.67	2.56	2.69	2.45	2.51	2.46	-0.68	2.11	1.56
		0.50	0.00	1.93	2.58	2.49	2.56	2.68	2.58	2.67	2.56	2.48	2.55	-0.68	2.11	1.56
	0.75	0.25	0.00	2.87	3.28	3.17	3.29	3.38	3.18	3.41	3.32	3.13	3.32	-7.90	2.22	-2.42

Table A.16: Simulated Percent Reduction of RMSE Relative to the π-estimator: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=1,000, L=10,000

					Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-3.85	1.06	0.97	1.06	1.21	1.22	1.22	1.03	0.89	1.01	1.18	1.18	1.18
		0.50	0.00	-3.85	1.14	1.13	1.15	1.20	1.20	1.21	1.15	1.12	1.14	1.18	1.18	1.18
		0.75	0.00	-3.85	1.14	1.14	1.14	1.16	1.17	1.17	1.14	1.14	1.14	1.18	1.18	1.18
	0.5	0.25	0.00	0.31	4.69	4.82	4.79	5.06	5.03	5.06	4.85	4.93	4.84	4.98	4.97	4.98
		0.50	0.00	0.31	4.78	4.71	4.74	5.06	5.00	5.03	4.74	4.69	4.72	4.98	4.97	4.98
	0.75	0.25	0.00	11.12	14.30	14.23	14.25	14.51	14.46	14.51	14.14	14.07	14.13	14.45	14.45	14.45
0.6	0.25	0.25	0.00	-0.86	1.15	1.14	1.20	1.29	1.31	1.32	1.17	1.11	1.18	1.19	1.23	1.23
		0.50	0.00	-0.86	1.20	1.27	1.27	1.30	1.31	1.32	1.25	1.27	1.26	1.19	1.23	1.23
		0.75	0.00	-0.86	1.23	1.21	1.22	1.28	1.26	1.28	1.23	1.21	1.22	1.19	1.23	1.23
	0.5	0.25	0.00	-0.86	3.85	4.11	4.04	4.24	4.24	4.28	4.04	4.18	4.04	4.23	4.22	4.23
		0.50	0.00	-0.86	4.12	4.02	4.07	4.29	4.23	4.27	4.08	4.01	4.06	4.23	4.22	4.23
	0.75	0.25	0.00	7.50	8.26	8.13	8.20	8.45	8.39	8.44	8.14	7.99	8.13	-3.32	5.63	1.17
0.4	0.25	0.25	0.00	0.22	0.92	0.95	0.98	1.06	1.06	1.09	0.95	0.94	0.97	-1.21	0.35	0.32
		0.50	0.00	0.22	0.99	1.05	1.05	1.09	1.09	1.11	1.03	1.04	1.04	-1.21	0.35	0.32
		0.75	0.00	0.22	1.07	1.05	1.06	1.10	1.09	1.10	1.07	1.06	1.06	-1.21	0.35	0.32
	0.5	0.25	0.00	2.36	2.45	2.68	2.60	2.75	2.76	2.78	2.57	2.72	2.57	2.76	2.76	2.76
		0.50	0.00	2.36	2.72	2.66	2.70	2.81	2.77	2.80	2.70	2.65	2.69	2.76	2.76	2.76
	0.75	0.25	0.00	3.32	3.48	3.42	3.47	3.53	3.53	3.55	3.46	3.38	3.45	-10.47	2.20	-4.00

A.2.0.4 Variance Ratio

Table A.17: Simulated Variance Ratio: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
		0.50	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
		0.75	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
	0.5	0.25	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98
		0.50	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98
	0.75	0.25	1.02	1.00	1.02	1.02	1.02	1.02	1.02	1.02	1.03	1.02	1.02	1.04	1.01	1.03
0.6	0.25	0.25	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	0.99	1.01	1.00
		0.50	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	0.99	1.01	1.00
		0.75	1.01	1.01	1.02	1.01	1.01	1.02	1.01	1.02	1.02	1.01	1.01	0.99	1.01	1.00
	0.5	0.25	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.98	0.97	0.97	0.97
		0.50	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.97
	0.75	0.25	1.00	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.98	0.99
0.4	0.25	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	1.02	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	1.02	1.01
		0.75	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	1.02	1.01
	0.5	0.25	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.01	1.00	1.00
		0.50	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.01	1.00	1.00
	0.75	0.25	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.99	0.98	0.99	1.02	0.99	1.01

Table A.18: Simulated Variance Ratio: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01
		0.50	1.01	1.01	1.02	1.02	1.02	1.01	1.01	1.01	1.02	1.02	1.02	1.01	1.01	1.01
		0.75	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.5	0.25	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
		0.50	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
	0.75	0.25	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
0.6	0.25	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.5	0.25	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.75	0.25	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.01	0.99	1.00
0.4	0.25	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.75	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.5	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.75	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.01	1.02

A.2.0.5 Confidence Interval Coverage

A.3 Lognormal Populations with Z^2 term

A.3.0.1 Relative Bias

Table A.19: Simulated 95% Confidence Interval Coverage: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.948	0.945	0.948	0.948	0.947	0.947	0.947	0.947	0.948	0.948	0.947	0.946	0.946	0.946
		0.50	0.948	0.945	0.948	0.948	0.948	0.947	0.947	0.947	0.948	0.948	0.948	0.946	0.946	0.946
		0.75	0.948	0.945	0.947	0.946	0.946	0.946	0.945	0.945	0.947	0.946	0.947	0.946	0.946	0.946
	0.5	0.25	0.944	0.944	0.947	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.947	0.949	0.947
		0.50	0.944	0.944	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.947	0.949	0.947
	0.75	0.25	0.950	0.946	0.948	0.947	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.941	0.942	0.942
0.6	0.25	0.25	0.946	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.950	0.949	0.943	0.946	0.945
		0.50	0.946	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.943	0.946	0.945
		0.75	0.946	0.949	0.948	0.948	0.947	0.948	0.948	0.948	0.948	0.948	0.948	0.943	0.946	0.945
	0.5	0.25	0.945	0.947	0.948	0.948	0.947	0.948	0.948	0.947	0.947	0.947	0.947	0.946	0.946	0.946
		0.50	0.945	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.946	0.946	0.946
	0.75	0.25	0.948	0.947	0.945	0.946	0.945	0.945	0.945	0.945	0.946	0.946	0.946	0.941	0.943	0.942
0.4	0.25	0.25	0.946	0.948	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.948	0.947	0.943	0.947	0.946
		0.50	0.946	0.948	0.947	0.947	0.947	0.947	0.947	0.947	0.947	0.948	0.947	0.943	0.947	0.946
		0.75	0.946	0.948	0.948	0.947	0.948	0.948	0.948	0.948	0.948	0.947	0.948	0.943	0.947	0.946
	0.5	0.25	0.946	0.946	0.947	0.946	0.946	0.946	0.945	0.945	0.946	0.946	0.946	0.949	0.947	0.948
		0.50	0.946	0.946	0.945	0.945	0.945	0.946	0.945	0.946	0.945	0.945	0.945	0.949	0.947	0.948
	0.75	0.25	0.943	0.944	0.946	0.944	0.946	0.948	0.945	0.947	0.946	0.944	0.946	0.947	0.944	0.949

Table A.20: Simulated 95% Confidence Interval Coverage: Weibull Populations with a mixture of two Baseline Hazards, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.951	0.951	0.950	0.949	0.949	0.949	0.948	0.949	0.950	0.950	0.950	0.949	0.949	0.949
		0.50	0.951	0.951	0.949	0.949	0.949	0.948	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949
		0.75	0.951	0.951	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949
	0.5	0.25	0.948	0.948	0.947	0.947	0.947	0.948	0.948	0.948	0.948	0.947	0.948	0.947	0.948	0.947
		0.50	0.948	0.948	0.947	0.947	0.947	0.948	0.948	0.948	0.947	0.947	0.947	0.947	0.948	0.947
	0.75	0.25	0.950	0.948	0.948	0.948	0.949	0.948	0.948	0.948	0.949	0.949	0.949	0.948	0.948	0.948
0.6	0.25	0.25	0.951	0.955	0.953	0.954	0.953	0.953	0.953	0.953	0.953	0.954	0.953	0.953	0.953	0.952
		0.50	0.951	0.955	0.953	0.953	0.953	0.952	0.953	0.953	0.953	0.953	0.953	0.953	0.953	0.952
		0.75	0.951	0.955	0.953	0.953	0.952	0.952	0.952	0.952	0.952	0.953	0.952	0.953	0.953	0.952
	0.5	0.25	0.951	0.952	0.950	0.950	0.950	0.951	0.951	0.951	0.950	0.950	0.950	0.951	0.951	0.951
		0.50	0.951	0.952	0.951	0.950	0.950	0.951	0.951	0.951	0.950	0.950	0.950	0.951	0.951	0.951
	0.75	0.25	0.949	0.948	0.947	0.947	0.947	0.947	0.946	0.946	0.947	0.947	0.947	0.947	0.947	0.948
0.4	0.25	0.25	0.951	0.949	0.950	0.950	0.950	0.951	0.950	0.951	0.950	0.950	0.950	0.951	0.951	0.951
		0.50	0.951	0.949	0.951	0.950	0.951	0.951	0.950	0.951	0.951	0.950	0.950	0.951	0.951	0.951
		0.75	0.951	0.949	0.950	0.950	0.950	0.951	0.950	0.951	0.950	0.950	0.950	0.951	0.951	0.951
	0.5	0.25	0.949	0.951	0.950	0.949	0.949	0.950	0.950	0.950	0.949	0.949	0.949	0.950	0.949	0.950
		0.50	0.949	0.951	0.949	0.949	0.950	0.949	0.950	0.950	0.950	0.949	0.950	0.950	0.949	0.950
	0.75	0.25	0.952	0.952	0.953	0.952	0.953	0.953	0.953	0.953	0.952	0.952	0.953	0.946	0.950	0.948

Table A.21: Simulated Percent Relative Bias: Lognormal Populations with Z^2 term, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.24	-0.09	0.21	0.19	0.21	0.19	0.20	0.19	0.21	0.19	0.21	0.48	0.00	0.04
		0.50	0.24	-0.09	0.22	0.20	0.21	0.20	0.21	0.20	0.22	0.20	0.21	0.48	0.00	0.04
		0.75	0.24	-0.09	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.48	0.00	0.04
	0.5	0.25	0.04	-0.19	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.12	0.09	0.10
		0.50	0.04	-0.19	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.12	0.09	0.10
	0.75	0.25	0.03	-0.08	0.02	0.00	0.03	0.03	0.01	0.03	0.02	0.01	0.02	0.28	0.24	0.25
0.6	0.25	0.25	0.23	0.06	0.21	0.20	0.21	0.19	0.20	0.18	0.21	0.20	0.21	0.32	0.05	0.06
		0.50	0.23	0.06	0.21	0.20	0.21	0.20	0.21	0.20	0.21	0.20	0.21	0.32	0.05	0.06
		0.75	0.23	0.06	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.32	0.05	0.06
	0.5	0.25	0.13	0.04	0.12	0.12	0.12	0.12	0.11	0.12	0.13	0.12	0.13	0.14	0.06	0.09
		0.50	0.13	0.04	0.12	0.12	0.12	0.13	0.12	0.13	0.12	0.12	0.12	0.14	0.06	0.09
	0.75	0.25	0.03	0.01	0.04	0.03	0.04	0.05	0.03	0.05	0.04	0.04	0.04	0.20	0.16	0.17
0.4	0.25	0.25	0.28	0.22	0.29	0.28	0.29	0.27	0.28	0.27	0.29	0.27	0.29	0.33	0.05	0.05
		0.50	0.28	0.22	0.29	0.28	0.29	0.28	0.28	0.28	0.29	0.28	0.29	0.33	0.05	0.05
		0.75	0.28	0.22	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.33	0.05	0.05
	0.5	0.25	0.11	0.10	0.13	0.13	0.13	0.13	0.12	0.13	0.13	0.13	0.13	0.19	0.13	0.16
		0.50	0.11	0.10	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.19	0.13	0.16
	0.75	0.25	0.02	0.05	0.04	0.04	0.04	0.05	0.04	0.05	0.04	0.04	0.04	0.14	0.13	0.12

Table A.22: Simulated Percent Relative Bias: Lognormal Populations with Z^2 term, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.05	-0.12	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	0.00	-0.02	-0.01
		0.50	-0.05	-0.12	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	0.00	-0.02	-0.01
		0.75	-0.05	-0.12	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	0.00	-0.02	-0.01
	0.5	0.25	-0.03	-0.08	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
		0.50	-0.03	-0.08	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
	0.75	0.25	0.00	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
0.6	0.25	0.25	-0.01	-0.05	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02	0.01	-0.01	-0.01
		0.50	-0.01	-0.05	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	0.01	-0.01	-0.01
		0.75	-0.01	-0.05	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	0.01	-0.01	-0.01
	0.5	0.25	-0.01	-0.03	-0.01	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.02	-0.01	-0.02	-0.02	-0.02
		0.50	-0.01	-0.03	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02
	0.75	0.25	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.02	0.01	0.01	0.02	0.01	0.04	0.02
0.4	0.25	0.25	-0.04	-0.06	-0.04	-0.04	-0.04	-0.04	-0.04	-0.05	-0.04	-0.04	-0.04	0.01	-0.07	-0.07
		0.50	-0.04	-0.06	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	0.01	-0.07	-0.07
		0.75	-0.04	-0.06	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	0.01	-0.07	-0.07
	0.5	0.25	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01
		0.50	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	0.75	0.25	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	-0.01	0.02	-0.01

A.3.0.2 Bias Ratio

Table A.23: Simulated Bias Ratio: Lognormal Populations with Z^2 term, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.021	-0.008	0.018	0.016	0.018	0.018	0.016	0.018	0.018	0.016	0.018	0.039	0.000	0.004
		0.50	0.021	-0.008	0.018	0.017	0.018	0.018	0.017	0.018	0.018	0.017	0.018	0.039	0.000	0.004
		0.75	0.021	-0.008	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.039	0.000	0.004
	0.5	0.25	0.006	-0.028	0.002	0.002	0.001	0.002	0.002	0.001	0.002	0.001	0.002	0.017	0.013	0.015
		0.50	0.006	-0.028	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.017	0.013	0.015
	0.75	0.25	0.007	-0.022	0.006	0.001	0.007	0.006	0.001	0.007	0.006	0.002	0.006	0.068	0.064	0.065
0.6	0.25	0.25	0.018	0.005	0.017	0.016	0.017	0.017	0.016	0.017	0.017	0.016	0.017	0.025	0.004	0.005
		0.50	0.018	0.005	0.017	0.016	0.017	0.017	0.016	0.017	0.017	0.016	0.017	0.025	0.004	0.005
		0.75	0.018	0.005	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.025	0.004	0.005
	0.5	0.25	0.017	0.006	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.017	0.019	0.009	0.012
		0.50	0.017	0.006	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.019	0.009	0.012
	0.75	0.25	0.006	0.004	0.010	0.008	0.010	0.010	0.008	0.010	0.009	0.009	0.010	0.043	0.039	0.038
0.4	0.25	0.25	0.022	0.017	0.022	0.021	0.022	0.022	0.021	0.022	0.022	0.021	0.022	0.025	0.004	0.004
		0.50	0.022	0.017	0.022	0.022	0.022	0.022	0.022	0.022	0.023	0.022	0.023	0.025	0.004	0.004
		0.75	0.022	0.017	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.025	0.004	0.004
	0.5	0.25	0.015	0.014	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.025	0.017	0.021
		0.50	0.015	0.014	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.025	0.017	0.021
	0.75	0.25	0.003	0.011	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.029	0.029	0.025

Table A.24: Simulated Bias Ratio: Lognormal Populations with Z^2 term, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	-0.010	-0.022	-0.011	-0.012	-0.011	-0.011	-0.012	-0.011	-0.011	-0.012	-0.011	0.000	-0.003	-0.001
		0.50	-0.010	-0.022	-0.011	-0.012	-0.011	-0.011	-0.012	-0.011	-0.011	-0.012	-0.011	0.000	-0.003	-0.001
		0.75	-0.010	-0.022	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	0.000	-0.003	-0.001
	0.5	0.25	-0.011	-0.027	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014	-0.014
		0.50	-0.011	-0.027	-0.014	-0.013	-0.013	-0.014	-0.014	-0.014	-0.014	-0.014	-0.013	-0.014	-0.014	-0.014
	0.75	0.25	0.002	-0.012	0.000	-0.002	0.000	0.000	-0.003	0.000	0.000	-0.002	0.000	-0.003	-0.004	-0.003
0.6	0.25	0.25	-0.003	-0.009	-0.003	-0.004	-0.004	-0.003	-0.004	-0.004	-0.003	-0.004	-0.003	0.001	-0.002	-0.002
		0.50	-0.003	-0.009	-0.003	-0.004	-0.004	-0.003	-0.004	-0.004	-0.003	-0.004	-0.003	0.001	-0.002	-0.002
		0.75	-0.003	-0.009	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	0.001	-0.002	-0.002
	0.5	0.25	-0.004	-0.010	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.004	-0.005	-0.004	-0.005	-0.005	-0.005
		0.50	-0.004	-0.010	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005
	0.75	0.25	0.007	0.005	0.008	0.007	0.008	0.008	0.007	0.008	0.008	0.007	0.008	-0.005	-0.005	-0.005
0.4	0.25	0.25	-0.007	-0.010	-0.007	-0.008	-0.007	-0.007	-0.008	-0.007	-0.007	-0.008	-0.007	0.002	-0.012	-0.011
		0.50	-0.007	-0.010	-0.007	-0.007	-0.007	-0.007	-0.008	-0.007	-0.007	-0.008	-0.007	0.002	-0.012	-0.011
		0.75	-0.007	-0.010	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	0.002	-0.012	-0.011
	0.5	0.25	-0.002	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.002	-0.002	-0.002
		0.50	-0.002	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	0.75	0.25	-0.001	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	-0.004	0.008	-0.003

A.3.0.3 Percent Reduction of RMSE Relative to the π-estimator

Table A.25: Simulated Percent Reduction of RMSE Relative to the π-estimator: Lognormal Populations with Z^2 term, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-0.48	1.09	1.07	1.13	1.17	1.09	1.19	1.12	1.08	1.12	-4.29	0.81	0.30
		0.50	0.00	-0.48	1.07	1.05	1.11	1.17	1.12	1.19	1.11	1.07	1.11	-4.29	0.81	0.30
		0.75	0.00	-0.48	1.06	1.05	1.06	1.14	1.12	1.14	1.06	1.05	1.06	-4.29	0.81	0.30
	0.5	0.25	0.00	2.94	4.69	4.71	4.85	5.32	5.11	5.33	4.86	4.89	4.88	0.01	2.91	2.25
		0.50	0.00	2.94	4.91	4.85	4.87	5.24	5.22	5.24	4.85	4.83	4.85	0.01	2.91	2.25
	0.75	0.25	0.00	11.78	14.74	15.19	14.66	15.35	15.68	15.39	14.38	14.90	14.46	4.52	11.38	8.14
0.6	0.25	0.25	0.00	0.73	1.11	1.14	1.17	1.29	1.17	1.31	1.14	1.15	1.15	-1.76	0.64	0.42
		0.50	0.00	0.73	1.10	1.12	1.17	1.26	1.16	1.27	1.15	1.13	1.16	-1.76	0.64	0.42
		0.75	0.00	0.73	1.14	1.10	1.13	1.19	1.16	1.19	1.13	1.10	1.13	-1.76	0.64	0.42
	0.5	0.25	0.00	2.97	3.15	3.19	3.28	3.57	3.41	3.57	3.23	3.27	3.24	2.02	3.07	3.06
		0.50	0.00	2.97	3.28	3.22	3.25	3.46	3.42	3.45	3.25	3.21	3.24	2.02	3.07	3.06
	0.75	0.25	0.00	7.00	7.59	7.68	7.51	7.81	7.95	7.82	7.45	7.57	7.47	-3.76	5.01	1.21
0.4	0.25	0.25	0.00	0.56	0.73	0.73	0.77	0.87	0.76	0.88	0.79	0.74	0.74	-0.88	0.86	0.74
		0.50	0.00	0.56	0.72	0.73	0.76	0.83	0.76	0.83	0.78	0.74	0.75	-0.88	0.86	0.74
		0.75	0.00	0.56	0.74	0.72	0.74	0.77	0.75	0.77	0.74	0.72	0.74	-0.88	0.86	0.74
	0.5	0.25	0.00	1.87	1.93	1.91	2.02	2.17	2.01	2.17	1.97	1.94	1.98	1.85	2.27	2.48
		0.50	0.00	1.87	2.00	1.94	1.98	2.08	2.02	2.08	1.98	1.94	1.98	1.85	2.27	2.48
	0.75	0.25	0.00	3.49	3.68	3.64	3.64	3.74	3.73	3.74	3.64	3.60	3.64	-6.63	2.23	-1.52

Table A.26: Simulated Percent Reduction of RMSE Relative to the π-estimator: Lognormal Populations with Z^2 term, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.00	-0.26	1.13	1.14	1.15	1.16	1.09	1.15	1.15	1.15	1.16	-0.67	0.76	0.70
		0.50	0.00	-0.26	1.12	1.13	1.15	1.19	1.14	1.19	1.16	1.14	1.16	-0.67	0.76	0.70
		0.75	0.00	-0.26	1.14	1.13	1.14	1.20	1.19	1.20	1.14	1.13	1.14	-0.67	0.76	0.70
	0.5	0.25	0.00	3.15	4.54	4.63	4.69	5.07	4.95	5.09	4.73	4.79	4.74	5.04	5.02	5.04
		0.50	0.00	3.15	4.79	4.74	4.76	5.09	5.10	5.10	4.74	4.73	4.74	5.04	5.02	5.04
	0.75	0.25	0.00	12.31	14.93	15.36	14.85	15.56	15.93	15.61	14.57	15.08	14.64	16.11	16.18	16.12
0.6	0.25	0.25	0.00	0.84	1.07	1.12	1.11	1.18	1.09	1.18	1.10	1.13	1.11	-0.60	0.17	0.16
		0.50	0.00	0.84	1.07	1.10	1.12	1.18	1.12	1.18	1.12	1.11	1.12	-0.60	0.17	0.16
		0.75	0.00	0.84	1.13	1.10	1.12	1.18	1.16	1.17	1.12	1.10	1.12	-0.60	0.17	0.16
	0.5	0.25	0.00	3.19	3.12	3.21	3.24	3.49	3.40	3.50	3.22	3.29	3.23	3.43	3.40	3.42
		0.50	0.00	3.19	3.29	3.24	3.27	3.44	3.45	3.45	3.26	3.24	3.26	3.43	3.40	3.42
	0.75	0.25	0.00	7.26	7.58	7.72	7.51	7.82	8.06	7.84	7.43	7.63	7.45	-0.94	6.45	3.26
0.4	0.25	0.25	0.00	0.74	0.74	0.77	0.77	0.85	0.76	0.85	0.81	0.78	0.76	-1.58	0.13	0.03
		0.50	0.00	0.74	0.75	0.77	0.78	0.84	0.78	0.84	0.82	0.78	0.78	-1.58	0.13	0.03
		0.75	0.00	0.74	0.79	0.77	0.78	0.81	0.80	0.81	0.78	0.77	0.78	-1.58	0.13	0.03
	0.5	0.25	0.00	1.99	1.86	1.89	1.93	2.07	1.99	2.07	1.91	1.93	1.91	2.00	1.99	2.00
		0.50	0.00	1.99	1.95	1.91	1.93	2.02	2.01	2.02	1.93	1.91	1.93	2.00	1.99	2.00
	0.75	0.25	0.00	3.91	3.95	4.00	3.91	4.04	4.19	4.04	3.88	3.95	3.88	-5.36	3.29	-0.79

A.3.0.4 Variance Ratio

Table A.27: Simulated Variance Ratio: Lognormal Populations with Z^2 term, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	1.02	1.02
		0.50	1.01	1.00	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.00	1.01	0.98	1.02	1.02
		0.75	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.02	1.02
	0.5	0.25	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99
		0.50	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99
	0.75	0.25	0.99	0.97	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	1.01	0.98	1.00
0.6	0.25	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
	0.5	0.25	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.03	1.00	1.01
		0.50	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.03	1.00	1.01
	0.75	0.25	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.03	1.00	1.02
0.4	0.25	0.25	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
		0.50	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01
		0.75	1.00	0.99	1.00	1.00	1.00	0.99	1.00	0.99	1.00	1.00	1.00	0.99	1.01	1.01
	0.5	0.25	1.00	0.99	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.03	1.01	1.01
		0.50	1.00	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.03	1.01	1.01
	0.75	0.25	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.03	1.00	1.03

Table A.28: Simulated Variance Ratio: Lognormal Populations with Z^2 term, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.75	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.5	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.75	0.25	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
0.6	0.25	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01
		0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01
	0.5	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.75	0.25	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	1.00	0.98	1.00
0.4	0.25	0.25	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
		0.50	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
		0.75	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	0.5	0.25	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		0.50	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	0.75	0.25	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.99	0.98	0.99

A.3.0.5 Confidence Interval Coverage

Table A.29: Simulated 95% Confidence Interval Coverage: Lognormal Populations with Z^2 term, N=100,000, n=200, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic		
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.949	0.946	0.948	0.948	0.948	0.947	0.948	0.947	0.948	0.949	0.948	0.946	0.949	0.948
		0.50	0.949	0.946	0.948	0.948	0.948	0.947	0.948	0.947	0.948	0.948	0.948	0.946	0.949	0.948
		0.75	0.949	0.946	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.946	0.949	0.948
	0.5	0.25	0.946	0.947	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.948	0.947	0.947	0.947
		0.50	0.946	0.947	0.948	0.948	0.948	0.947	0.947	0.947	0.948	0.948	0.948	0.947	0.947	0.947
	0.75	0.25	0.944	0.944	0.943	0.943	0.943	0.944	0.943	0.943	0.943	0.943	0.943	0.939	0.938	0.939
0.6	0.25	0.25	0.946	0.944	0.944	0.944	0.943	0.944	0.944	0.944	0.943	0.943	0.944	0.946	0.947	0.946
		0.50	0.946	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.944	0.946	0.947	0.946
		0.75	0.946	0.944	0.943	0.944	0.943	0.944	0.944	0.944	0.943	0.943	0.943	0.946	0.947	0.946
	0.5	0.25	0.949	0.947	0.949	0.948	0.949	0.948	0.948	0.948	0.948	0.948	0.948	0.952	0.949	0.949
		0.50	0.949	0.947	0.948	0.948	0.948	0.948	0.947	0.948	0.948	0.948	0.948	0.952	0.949	0.949
	0.75	0.25	0.946	0.944	0.947	0.946	0.947	0.946	0.946	0.946	0.947	0.946	0.947	0.948	0.944	0.947
0.4	0.25	0.25	0.944	0.941	0.942	0.942	0.942	0.942	0.943	0.942	0.942	0.942	0.942	0.945	0.944	0.944
		0.50	0.944	0.941	0.943	0.943	0.942	0.942	0.943	0.942	0.942	0.942	0.942	0.945	0.944	0.944
		0.75	0.944	0.941	0.943	0.943	0.943	0.942	0.942	0.942	0.942	0.943	0.942	0.945	0.944	0.944
	0.5	0.25	0.948	0.948	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.955	0.951	0.952
		0.50	0.948	0.948	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.955	0.951	0.952
	0.75	0.25	0.947	0.947	0.947	0.947	0.947	0.947	0.946	0.947	0.948	0.947	0.948	0.947	0.947	0.948

Table A.30: Simulated 95% Confidence Interval Coverage: Lognormal Populations with Z^2 term, N=100,000, n=1,000, L=10,000

						Weibull			Lognormal			Proportional Hazard			Logistic	
ρ	p(t)	% censored	π	GREG	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2	GD	MC1	MC2
0.8	0.25	0.25	0.950	0.950	0.949	0.949	0.949	0.951	0.950	0.950	0.949	0.950	0.950	0.950	0.950	0.951
		0.50	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.951
		0.75	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.951
	0.5	0.25	0.950	0.952	0.951	0.951	0.951	0.950	0.950	0.951	0.951	0.951	0.951	0.951	0.951	0.951
		0.50	0.950	0.952	0.951	0.951	0.951	0.950	0.950	0.951	0.951	0.951	0.951	0.951	0.951	0.951
	0.75	0.25	0.948	0.945	0.946	0.945	0.946	0.945	0.944	0.945	0.946	0.946	0.946	0.944	0.944	0.944
0.6	0.25	0.25	0.950	0.949	0.950	0.950	0.950	0.950	0.950	0.950	0.949	0.950	0.950	0.950	0.950	0.950
		0.50	0.950	0.949	0.949	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950
		0.75	0.950	0.949	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950
	0.5	0.25	0.951	0.949	0.951	0.951	0.951	0.951	0.951	0.950	0.952	0.951	0.952	0.951	0.951	0.951
		0.50	0.951	0.949	0.951	0.952	0.952	0.951	0.951	0.951	0.951	0.952	0.951	0.951	0.951	0.951
	0.75	0.25	0.947	0.949	0.949	0.949	0.949	0.949	0.948	0.949	0.948	0.949	0.948	0.949	0.949	0.950
0.4	0.25	0.25	0.949	0.949	0.948	0.949	0.949	0.949	0.948	0.949	0.949	0.949	0.949	0.948	0.949	0.949
		0.50	0.949	0.949	0.948	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.948	0.949	0.949
		0.75	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.948	0.949	0.949
	0.5	0.25	0.951	0.952	0.952	0.952	0.952	0.952	0.953	0.952	0.952	0.952	0.952	0.953	0.952	0.953
		0.50	0.951	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.953	0.952	0.953
	0.75	0.25	0.948	0.949	0.949	0.948	0.949	0.949	0.949	0.949	0.949	0.948	0.949	0.945	0.950	0.947

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