BOOTSTRAPPED INTERSECTING REMAGNETIZATION GREAT CIRCLES AND THE SUBSEQUENT EMPIRICAL CONFIDENCE REGION

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ABSTRACT

Given the already documented bias in the determination of intersecting remagnetization great circles as a function of the parallelism of the circles, resulting in an elongated bias in that direction, we found that rotational symmetry assumptions around the intersection would seem to be insufficient. In this contribution, we address the intersection and its inherent bias by doing bootstrap. Repeated calculations explore possible outcomes numerically; elongated distributions of the bootstrapped intersections are regarded as biased. The method calculates a more realistic confidence region based on the empirical distribution of the bootstrapped intersections. A preliminary version of the program implementing the method described here is available from L. Gallo on request.

Keywords: Paleomagnetism, remagnetization great circles, intersecting great circles, moment of inertia analysis, geostatistics.

RESUMEN

Dado el sesgo documentado en el cálculo de la intersección de círculos máximos de remagnetización como función del paralelismo de los mismos, y que el mismo es elongado en la dirección de paralelismo, se encuentra que la asunción de simetría rotacional (*i.e.* estadística de Fisher) alrededor de la intersección medida, parecería ser insuficiente para abordarlo. En esta contribución, se aborda la intersección de círculos máximos a partir del método "bootstrap" de las mismas. Cálculos repetidos exploran los posibles resultados numéricamente; intersecciones sesgadas debido al paralelismo de los círculos máximos tendrán una distribución elongada de las intersecciones re-muestreadas. El método calcula una región de confianza basada en la distribución empírica de las intersecciones re-muestreadas. Una versión preliminar del programa descripto se encuentra disponible a pedido.

Palabras Clave: Paleomagnetismo, círculos máximos de remagnetización, intersección de círculos máximos, análisis de momentos de inercia, geoestadística.

1. Introduction

Paleomagnetic studies may identify discrete demagnetization lines as an estimate of the direction of a discrete magnetic component. Nevertheless, the demagnetization of one component in rocks with more than one direction of magnetization does not lie on a line. The resulting vectors would move along a great circle, which on a stereonet defines the so-called remagnetization circle (*e.g.* Buchan and Dunlop, 1976).

Essentially, there are two methods to establish the intersection point of remagnetization circles. The first one (Halls, 1976), first define the best fit of remagnetization great circles, by the sum of the deviations of the least squared, and then repeat the process for the normals of those calculated planes. The normal for the plane containing the normals for the remagnetization circles determines their intersection. The least-square plane fit is calculated by finding the eigensolution of their associated covariance matrix (Scheidegger, 1965).

On the other hand, the second option allows the incorporation of any observed end-points determined from linear segments near the origin of a Zijderveld plot (McFadden and McElhinny, 1988). The method includes an iterative process where the vectors constrained to lie within a great circle are increased until a maximum vector resultant is obtained.

Schmidt (1985) has raised the problem of bias in converging great circle methods. The unconditional application of intersecting remagnetization great circles fitting routines should however be viewed with caution, with a degenerate case being identifiable. He found that biasing in these depends upon the distribution parameters; a departure from anti-parallelism of the parent great circle distribution contributes to increasing the accuracy.

The reliability of an intersection depends upon the distribution of the normals to least-square great circles fits. Intuitively, the closer the normals are to being parallel (*i.e.* towards a collinear configuration), the lower shall be the precision of the fitted great circle/plane and thus, his normal which, in turn, is the intersection. Therefore, normals show ideally a girdle distribution or, in other words, great circles must be intersected at not-too-acute angles so that the inherent bias of the intersection is kept to a minimum.

In both, Halls (1978) and Mcfadden and McElhinny (1988) methods, Fisher distributions are assumed and errors with rotational symmetry around the mean direction calculated. As the bias lie along the parallelism of great circles, an elongated confidence region should be expected and the assumption of rotational symmetry would seem to be insufficient. Taking advantage of computer-intensive statistical methods such as bootstrapping (Efron, 1979), it is not necessary to assume a prior mathematical form for a given distribution at all. Instead, we can use our existing set of directions/great circles as an approximation of the underlying population.

2. Procedure

Bootstrap techniques are used in situations where it is difficult to use an analytical method to provide estimates of variability; repeated calculations explore possible outcomes numerically, and are used in place of complicated or intractable theoretical development.

The procedure designed for the bootstrapped intersection proceeds as follows:

1. Following Halls (1978), a great circle is calculated for any given specimen based on the orientation matrix T or moment of inertia analysis. Given *n* unit vectors observations containing its *x, y, z* direction cosines $[x_i, y_i, z_i]$, the orientation matrix is defined as follows:

$$T = \frac{i}{n}\begin{bmatrix} \sum_{i=1}^{n} x^2{}_i & \sum_{i=1}^{n} x_i . y_i & \sum_{i=1}^{n} x_i . z_i \\ \sum_{i=1}^{n} x_i . y_i & \sum_{i=1}^{n} y^2{}_i & \sum_{i=1}^{n} y_i . z_i \\ \sum_{i=1}^{n} x_i . z_i & \sum_{i=1}^{n} y_i . z_i & \sum_{i=1}^{n} z^2{}_i \end{bmatrix}$$

   T is a symmetrical matrix and can therefore be solved to obtain its eigenvalues $[\tau_1, \tau_2, \tau_3]$ and eigenvectors $[\gamma_1, \gamma_2, \gamma_3]$. The eigenvector corresponding to the eigenvalue minimum $[\gamma_3 \equiv \tau_3]$ or maximum moment of inertia is the pole to its best-fit plane, which in turn contain γ_1 and γ_2.

2. Assuming that we have m great circles and m γ3 (minimum eigenvectors), the procedure draws a random sample (with replacement) from the original data of the same size (m) to serve as a pseudosample.
3. Repeat the process until have a large number of pseudosamples (>1000).
4. Calculate the eigenparameters of the orientation matrix for each pseudosample and separate γ3 (intersection of the great circles) from each.
5. Calculate the eigenparameters of the orientation matrix of the bootstrapped intersections (γ3).
6. Calculate the approximate ellipse of 95% confidence for the set of bootstrap by assuming that they are Kent distributed (Tauxe et al., 1991).

3. Degree of biasing and reliability for the analysis

As pointed out above, the more parallel the great circles are, the more elongated the distribution of the bootstrapped intersections. For purposes of illustration, we have chosen random datasets. Figure 1 shows the results of 1000 bootstrapped intersections of three different underlying distributions (n = 10). In order to discriminate the type of distribution and quantify the bias, we use the shape criterion of the orientation matrix (Woodcock, 1977) which is defined as

$$K = \frac{\log\left(\frac{V_3}{V_2}\right)}{\log\left(\frac{V_2}{V_1}\right)}$$

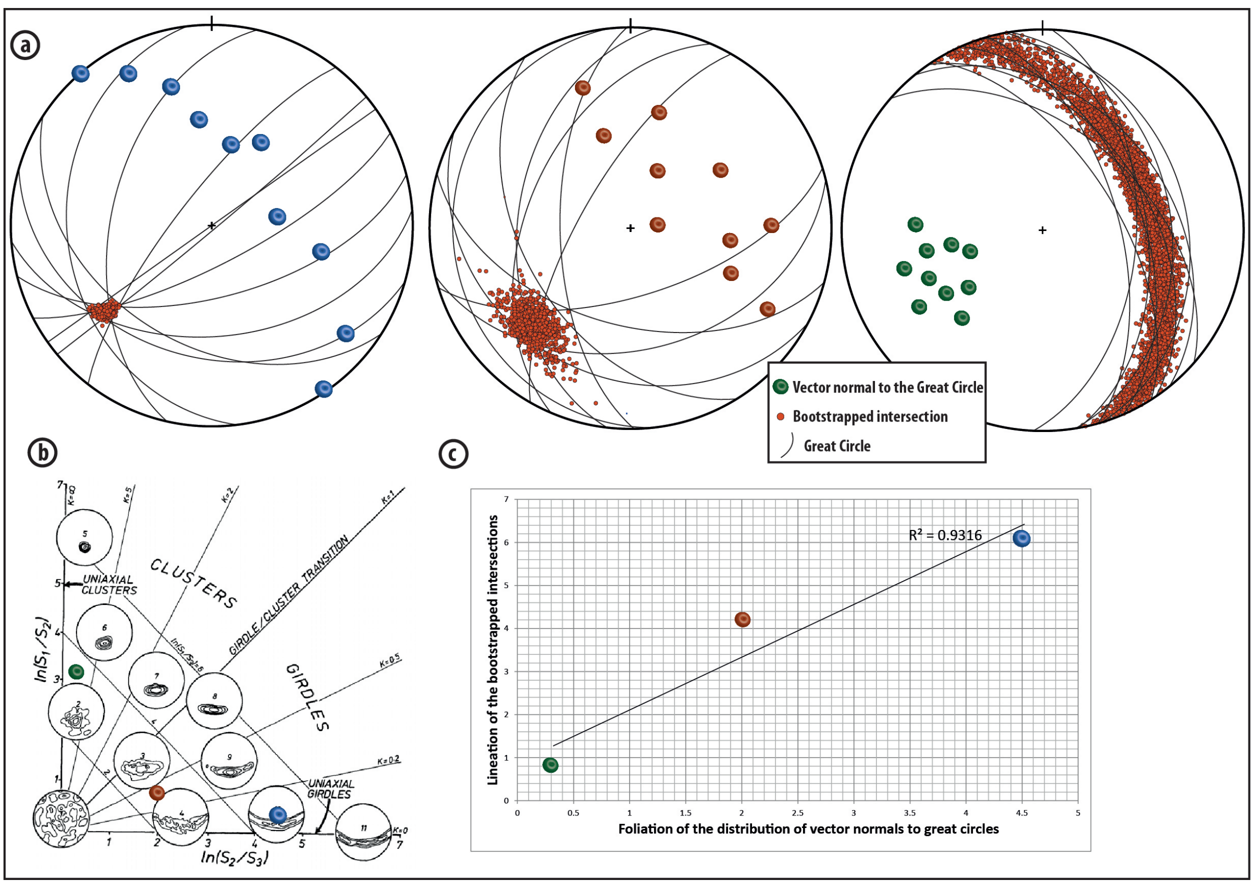

Figure 1. **a)** Three different examples of great circle intersections showing a broad spectrum of vector normal distribution (*i.e.* great circle parallelism). **b)** Spatial distribution of point data according to the different ratios between eigenvalues (Woodcock, 1977). **c)** Linear correlation between foliation of the distribution of normals to great circles and lineation of the bootstrapped intersections.

We then address biasing by calculating the foliation (ln τ_2/τ_3) of the underlying distribution of vector normals to the least-square great circles and the corresponding lineation (ln τ_2/τ_3) of the bootstrapped intersections (taken from Step 5).

Figure 1c shows a linear correlation between the foliation of the distribution and the lineation of the bootstrapped intersections. This correlation quantifies our previous assumptions: the more foliated the underlying distribution is, the more lineation the bootstrapped intersection are, and thus, the more accurate and precise the intersection is.

4. Confidence Regions

The goal of this analysis is to determine some measure of the confidence regions in the minimum eigenvectors (*i.e.* intersections of great circles). There are two choices here, assume a distribution for the underlying data to constrain the confidence region under parametric models (Tauxe *et al.*, 1991) or to take a non-parametric approach to estimate confidence region drawing a contour enclosing 95% of the given directions (Fisher and Hall, 1989; Gallo *et al.*, 2017). Following Tauxe *et al.* (1991), for the parametric model one can calculate approximate ellipses of 95% confidence for the set of bootstrapped directions by assuming that they are Kent distributed.

5. Conclusions

By using numerical simulations we address some shortcomings of the conventional approach to assessing confident in the direction of the intersection of great circles. A bootstrap method is used to provide accurate estimates of that intersection. Our approach is more realistic as no rotational symmetry is assumed. We found empirically that the bigger the foliation parameter of the underlying distribution of vector normals to great circles is (girdle shape tendency) the bigger shall be the lineation of the bootstrapped intersections (*i.e.*, lesser bias). The method calculates a more realistic confidence region based on the empirical distribution of the bootstrapped intersections.

Ackowledgements

We thank the Laboratorio de Paleomagnetismo "D. A. Valencio" at the FCEyN, Universidad de Buenos Aires (UBA) and IGEBA-CONICET. Financial support for this research was provided by the Technology Vinculation Projects "Ingeniero Enrique Mosconi" and "J. A. Sábato".

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