Melting transition of an Ising glass driven by a magnetic field

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The quantum critical behavior of the Ising glass in a magnetic field is investigated. We focus on the spin-glass-to-paramagnet transition of the transverse degrees of freedom in the presence of a finite longitudinal field. We use two complementary techniques, the Landau theory close to the $T=0$ transition and the exact diagonalization method for finite systems. This allows us to estimate the size of the critical region and characterize various crossover regimes. An unexpectedly small energy scale on the disordered side of the critical line is found, and its possible relevance to experiments on metallic glasses is briefly discussed.

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I. INTRODUCTION

Understanding disordered systems is one of the main challenges of condensed matter physics, since the presence of disorder is always unavoidable in experiments. When disorder is strong it can dominate the physics and lead to exotic states of matter such as the glassy phases.[1] The most salient properties observed in glassy systems are the slow dynamical relaxation and history dependence of thermodynamics. Research on quantum spin systems is of primary importance because of potential technological applications. Current work in quantum computing and spintronics, where the understanding of relaxation processes is crucial,[2,3] is boosting a renewed interest in basic models of disordered quantum magnets.

The goal of the present work is to consider the random Ising model that displays a quantum paramagnet to spin glass transition driven by fluctuations introduced by an external magnetic field. We tackle the problem by utilizing two different theoretical approaches. We solve the model using the recently introduced technique of exact diagonalization that includes the averaging over an ensemble of disorder realizations in a finite system. The relevant results are then obtained by extrapolation of the data to the thermodynamic limit.[4,5] This method allows for a direct investigation of the $T=0$ behavior in the whole range of parameters, circumventing, thus, the usual technical difficulties encountered in the replica formalism. On the other hand, to investigate in detail the critical behavior,[6] we formulate the Landau theory in the vicinity of the quantum phase transition.[7,8] The consistency check of results obtained using those two approaches allows us not only to confirm their reliability, but also to identify an unexpectedly narrow subregime near the phase boundary, in which the rapid onset of the glassy ordering occurs. We discuss the significance of our findings for the current experiments on metallic glasses.[9]

We consider the random Ising model that is placed in a magnetic field and that has both the transverse and longitudinal components,

$$H=-\sum_{\langle ij\rangle} J_{ij}S_i^z S_j^z-\sum_i \mathbf{h}\cdot\mathbf{S}_i . \tag{1}$$

The random interactions J_{ij} are chosen to be infinite range and Gaussian distributed with variance J, that sets the unit of energy in the model, while $\mathbf{h}=(h^T,0,h^L)$. This model has an experimental realization in the $LiY_{1-x}Ho_xF_4$ compound that has been the subject of recent experiments.[10,11] In this insulating compound, the ground state of the magnetically active Ho ions is the low-energy Ising doublet. In addition to that, the long-range nature of dipolar interactions between the spins enables us to perform the treatment in the large coordination limit. Disorder in the system arises from the fact that the substitutions of the Y atoms by the Ho ions are positionally random. The strong randomness leads to the clear observation of the spin- and ferroglass phases at low concentration x.[12]

To investigate the transition in the system described by the Hamiltonian (1), we employ two methods that complement each other in their scope and range of applicability. The main theoretical tool we use to obtain the detailed analytic behavior is based on the Landau theory approach.[7,8] Though attractive, this method is rigorously valid only close to the quantum critical point, so that the actual range of applicability of this approach is always difficult to assess. Hence, in addition, we also use the exact diagonalization scheme, in which one has to obtain the solution of H for a number of explicit realizations of disorder (typically several tens of thousands). The procedure is implemented on finite systems of up to 17 spins. The physical observables, such as gaps or spectral functions, are obtained along the lines of Ref. 5. In this approach, no *a priori* assumptions are made, and its validity is limited by the reliability of the required extrapolations to the large size limit. The main reason for success of the previous applications of the method is that for high connectivity models the numerical extrapolation to the thermodynamic limit is rather well behaved. Nevertheless, as we shall see and discuss later on, in the present study we find a certain range of parameters,where the previous statement does not hold. Remarkably, this circumstance allows us to gain new insight into the problem.

The work is organized as follows. Sec. II is devoted to the explanation of the technical procedure. Results are presented in Sec. III, and Sec. IV contains the summary of the main results and discussion.

II. TECHNICAL REMARKS

It is useful to characterize the parameter space by h^L and h^T, the longitudinal and transverse components of external magnetic field **h**, respectively. The pure transverse field case was the subject of previous investigations.[4] At $T=0$, the existence of the quantum phase transition was established for a value of $h^T \sim O(J)$. At this point the spin-spin dynamical local susceptibility becomes gapless.[13,14] When the longitudinal field is turned on, the net longitudinal magnetization is immediately generated and the critical point extends into a quantum critical line $h_c^T(h_c^L)$. This line separates the two phases, in which the transverse degrees of freedom of spins are either disordered (large h^T and h^L) or spin-glass ordered (small h^T and h^L). As we shall show, the excitation gap closes at this critical line, becoming very small in some crossover region on the disordered side of the line.

A. Landau theory

The Landau functional is constructed using the cumulant expansion about the quantum critical point at zero longitudinal field. Both the term with random interactions and the part with longitudinal field in the Hamiltonian (1) are treated as perturbations. This procedure implies that the longitudinal magnetic field h^L is small compared to the primary microscopic energy scale $h^T \sim J$. The derivation is straightforward and leads to the following Ginzburg-Landau action[7]

$$\beta\mathcal{F}=\sum_{a,\omega_n}\left(\frac{r+\omega_n^2}{\kappa}\right)Q^{aa}(\omega_n)+\frac{u}{2\beta}\sum_a\left[\sum_{\omega_n}Q^{aa}(\omega_n)\right]^2 -\frac{\kappa}{3}\sum_{abc}\sum_{\omega_n}Q^{ab}(\omega_n)Q^{bc}(\omega_n)Q^{ca}(\omega_n) -\frac{\beta(h^L)^2}{2}Q^{ab}(\omega_n=0)-\frac{\beta y}{6}\int\int d\tau_1 d\tau_2 \times\sum_{ab}[Q^{ab}(\tau_1-\tau_2)]^4. \tag{2}$$

Here r, being some regular function of h^T/J, is the parameter that governs the transition in the absence of the longitudinal field h^L, while u and y are taken at the critical point $(h^T/J)_c \sim O(1)$. It is important to retain the quartic term, responsible for the replica symmetry breaking (RSB) instability.[8] We must insert then the mean-field ansatz

$$\kappa Q^{ab}(\omega_n)=\begin{cases} D(\omega_n)+\beta q_{\mathrm{EA}}\delta_{\omega_n,0} & a=b\\ \beta q_{ab}\delta_{\omega_n,0} & a\neq b,\end{cases}$$

into Eq. (2) and vary subsequently the free energy with respect to $D(\omega_n)$, q_{EA}, and q_{ab}. The parametrization of q_{ab} depends, however, on the phase under consideration. In the disordered paramagnetic phase (PM) we must use the replica-symmetric ansatz $q_{ab}=q_{EA}$, while in the spin-glass phase (SG) the solution with a broken symmetry should be used.[8,15] The variational procedure is lengthy albeit identical to that performed in the previous works. As a result, we obtain that the equation determining $D(\omega_n)$ is the same in both PM and SG phases, and reads

$$r+\omega_n^2+u\left[\frac{1}{\beta}\sum_{\omega_n}D(\omega_n)+q_{\mathrm{EA}}\right]-D^2(\omega_n)-\frac{2y}{\kappa^2}q_{\mathrm{EA}}^2D(-\omega_n) -\frac{2y}{\kappa^2}\frac{q_{\mathrm{EA}}}{\beta}\sum_{\omega_1}D(\omega_1)D(-\omega_1-\omega_n) -\frac{2y}{3\kappa^2}\frac{1}{\beta^2}\sum_{\omega_1,\omega_2}D(\omega_1)D(\omega_2)D(-\omega_1-\omega_2-\omega_n)=0. \tag{3}$$

This equation must be supplemented by

$$2D(0)q_{\mathrm{EA}}+\frac{2y}{3\kappa^2}q_{\mathrm{EA}}^3+\frac{\kappa(h^L)^2}{2}=0 \tag{4}$$

in the PM phase and

$$q_{\mathrm{EA}}^2=-[D(0)\kappa^2]/y \tag{5}$$

in the SG phase, to comprise the full system to be solved self-consistently. Though the exact treatment of this system is not possible, we can obtain the leading order of the correct solution close to the quantum critical point. We consider here only the case of $T=0$, so that all the sums over Matsubara frequencies are substituted by the corresponding integrals.

We note first that, if $y=0$, the complete solution is easily derived to be[7] $D(\omega_n)=-\sqrt{\omega_n^2+\Delta^2}$. The gap Δ^2, that turns to zero right at the critical point, is determined using the following identity:

$$\int\frac{d\omega}{2\pi}(\omega^2+\Delta^2)^{1/2}=\frac{\Lambda_\omega^2}{2\pi}+\frac{\Delta^2}{2\pi}\ln(c_1\Lambda_\omega/\Delta). \tag{6}$$

In Eq. (6) Λ_ω is the upper frequency cutoff and c_1 is some constant of order unity. Let us assume that for $y\neq 0$ the leading approximation of $D(\omega_n)$ contains the same square-root singularity as for $y=0$, and analyze how the last two terms in Eq. (3) affect the solution in the leading approximation. Simple inspection reveals that in the prelast term it is sufficient to put $\omega_n=0$, $\Delta=0$ while calculating the integral over ω_1. This contributes only to the renormalization of the coefficient u before q_{EA}, so that $uq_{\mathrm{EA}}\rightarrow u_1q_{\mathrm{EA}}$.

The last term requires, however, the calculation of the integral

$$K(\Delta,\omega_n)=\int\frac{d\omega_1}{2\pi}\int\frac{d\omega_2}{2\pi}\sqrt{\omega_1^2+\Delta^2}\sqrt{\omega_2^2+\Delta^2} \times\sqrt{(\omega_1+\omega_2+\omega_n)^2+\Delta^2} \tag{7}$$

that is difficult to perform exactly for arbitrary ω_n and Δ^2. We need, however, only the leading behavior of this integral provided $\omega_n,\Delta\ll 1$. A simple estimate yields

$$K(\Delta,\omega_n)=A+B\omega_n^2+C_1\Delta^2\ln(C_2/\Delta), \tag{8}$$

where the constants A, B, C_1, and C_2 are some cutoff Λ_ω dependent functions. We see that the first term in the above expression renormalizes the critical value r_c (equal to $u\Lambda_\omega^2/2\pi$ for $y=0$), while the contribution from the second one can be simply absorbed by the appropriate rescaling of temperature T in ω_n^2. The third term in Eq. (8) leads to the renormalization of the coefficient before the Δ-dependent part of Eq. (6).

Similarly as in Ref. 8, we obtain that in the PM phase

$$D(\omega_n) = -yq_{\rm EA}^2/\kappa^2 - \sqrt{\omega_n^2+\Delta^2},$$

$$\Delta^2 = \frac{r-r_c+u_1q_{\rm EA}}{u_2\ln[Cu_2/(r-r_c+u_1q_{\rm EA})]}, \qquad (9)$$

where C, u_1, and u_2 are again some Λ_ω dependent functions of the order unity.

B. Numerical diagonalization

The general strategy is to take samples from the random ensemble of systems of size N and exactly diagonalize the ensuing Hamiltonians (1). The different physical quantities are computed for each realization and then averaged over the number of samples. Finite size effects are analyzed and results are extrapolated to the thermodynamic limit ($N\to\infty$). Typically, systems with up to $N=17$ spins can be dealt with. Averages are performed over several thousands to hundreds of thousands of disorder realizations. A typical run demands up to a week for the larger systems on an eight-node parallel cluster. The ground state and the dynamical correlation functions at $T=0$ are calculated by the Lanczos method.[16]

The local spin susceptibility is obtained from

$$\chi_{loc}^{zz}(\omega) = \frac{1}{M}\sum_{m=1}^{M}\frac{1}{N}\sum_{i=1}^{N}\left[\langle\Phi_0^{(m)}|S_i^z\frac{1}{\omega-H^{(m)}}S_i^z|\Phi_0^{(m)}\rangle\right], \qquad (10)$$

where M is the number of realizations of disorder and $|\Phi_0^{(m)}\rangle$ denotes the ground state for the J_{ij} set corresponding to the mth realization. Although we deal with systems having a finite number of poles for each realization, the average over disorder naturally produces smooth response functions without the need of introducing an artificial broadening as in usual exact diagonalization methods. In some cases, we found that it is useful to use a logarithmic discretization of the ω axis to obtain accurate results due to the large number of poles occurring at low frequencies.

Two criteria can be used to obtain the boundary of the quantum transition from the paramagnetic to the spin-glass phase.

(i) The onset for spin-glass order is naturally signaled by the divergence of the spin-glass susceptibility χ_{SG}, which is related to the local-spin susceptibility by[1]

$$\chi_{SG} = \frac{\langle[\chi_{loc}^{zz}]^2\rangle}{1-J^2\langle[\chi_{loc}^{zz}]^2\rangle}, \qquad (11)$$

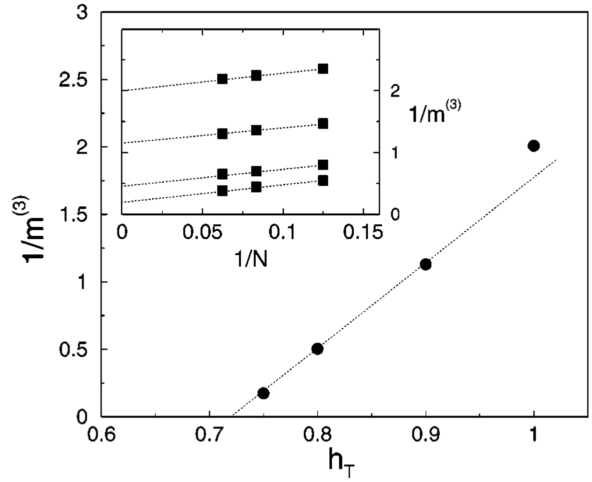

FIG. 1. Inverse of the third inverse moment as a function of h^T. The intercept of the dashed line with the horizontal axis gives an estimate of the critical h_c^T. The inset shows the same quantity as a function of the inverse of the system size for $h^T=0.75,0.8,0.9,1$. The dotted lines indicate the linear fits for the extrapolation of this quantity to the thermodynamic limit.

where $\langle[\chi_{loc}^{zz}]^2\rangle$ denotes the site and realization average of the quantity between square brackets at $\omega=0$ in Eq. (10). Thus, the condition

$$J^2\langle[\chi_{loc}^{zz}]^2\rangle = 1 \qquad (12)$$

indicates the instability of the system toward a spin-glass state.

In previous papers[4,17] the accuracy of the method was demonstrated by reproducing several known results for the infinite-range Ising model with random exchange interactions and transverse magnetic field h^T[14] (i.e., the present model with $h^L=0$). In particular, an accurate estimate for the critical value of the transverse field h_c^T, at which the quantum transition between the spin-glass and the paramagnetic phases takes place, was obtained.

(ii) A second criterion that signals the instability toward a spin-glass phase is the closing of the gap of the dynamical susceptibility. This criterion is called the *marginality condition* or *replicon criterion* and has already been discussed in the context of related models.[15,18]

Under the reasonable assumption of a clean gap in the paramagnetic phase and a lower-frequency edge of the spectral function $\chi''(\omega) = -2\,\mathrm{Im}[\chi_{loc}^{zz}(\omega)]$ that grows faster than quadratic, it is easy to see that the closing of the gap implies a divergence in the third inverse moment of $\chi''(\omega)$.

Thus, we establish the closing of the gap by computing the quantity

$$m^{(-3)} = \int_0^\infty \frac{d\omega}{2\pi}\frac{\chi''(\omega)}{\omega^3} \qquad (13)$$

at given values of h^L and system size N, and then looking for the vanishing of the extrapolations of $[1/m^{(-3)}]$ to the limit of large system.[19] The procedure is illustrated in Fig. 1. The estimate of $[1/m^{(-3)}]$ in the thermodynamic limit is obtained

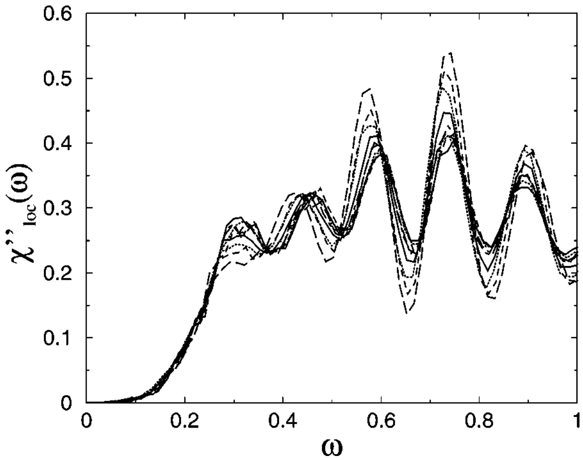

FIG. 2. Spectral function $\chi''_{loc}(\omega)$ for $N=12$, $h^L=0$, and $h^T=0.71,0.72,0.73,0.74,0.75,0.8,0.9,1,1.5$. Data for $h^T \neq 0.71$ were rigidly shifted down in frequency along the ω axis in order to overlay all the low-frequency edges. The shifts provide the estimates for the magnitude of the gap $\Delta(h^T-h^T_c)$ for the given system size.

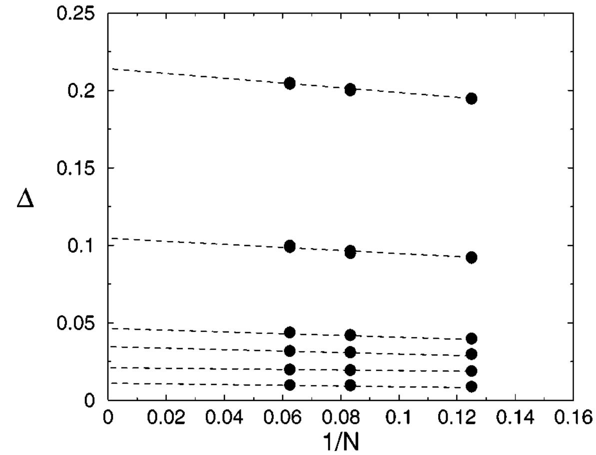

FIG. 3. The estimates of $\Delta(h^T-h^T_c)$ at $h^L=0$ and $h^T=0.72,0.73,0.74,0.75,0.8,0.9$ (bottom to top) for different system sizes, $N=8,12,16$. The dotted lines indicate the linear fits for the extrapolation of the gap to the thermodynamic limit. The good quality of the linear fit does not require the use of intermediate system sizes.

by recourse to linear fits as indicated in the inset of this figure. The accuracy of this alternative method was tested for the case $h^L=0$ where good estimates of h^T_c are available. By a linear fit of the points closer to h^T_c (cf. Fig. 1), we obtained $h^T_c=0.72\pm0.01$, which is in agreement with previously reported values.[4,13,14]

Finally, we also computed the dependence of the gap on the transverse field. Numerically, the calculation of the size of the gap is more challenging than the calculation of integrated quantities such as χ^{zz}_{loc}. The origin of the difficulties is that for any given realization of J_{ij}, poles in $\chi''(\omega)$ may appear at frequencies substantially smaller than the actual value of the gap but with a very small weight. Averaged over disorder, these poles will contribute with no significant statistical weight to the line shape of $\chi''(\omega)$. However, a naive determination of the value of the gap through the criterion of the average position of the lowest-frequency pole, would lead to a substantial underestimate of the position of the gap, since no information on the spectral weight is used. Therefore, we need a more accurate method for the determination of the gap. We used the following procedure. First, we get an accurate estimate of the critical transverse field $h^T_c(h^L)$. Then, at any given system size and h^L, we obtain the dynamical response for various values of $h^T>h^T_c$. We then rigidly *shift* the spectra down in frequency until we get the collapse of the low-frequency edges. As illustrated in Fig. 2 there is a very weak dependence of the shape of the edge with h^T, that makes this procedure sound. Then, simply from the energy shift we get estimates of the gap at the given h^L and system size N. Finally, a $N\to\infty$ extrapolation of the gaps is made assuming a simple linear in $1/N$ behavior,[14] as shown in Fig. 3.

III. RESULTS

As a result of solution of Eqs. (3) and (4), one can distinguish the following regimes on a (h^T,h^L) plane (see Fig. 4).

(A) In this regime, in which $h^L \ll (r-r_c)^{3/4}$, $q_{\rm EA}$ is the smallest parameter and can be treated as a perturbation. As a result, we obtain with the logarithmic accuracy, that $q_{\rm EA}=(\kappa(h^L)^2)/4\Delta$, $\Delta\approx\{(r-r_c)/u_2 \ln(1/(r-r_c)\}^{1/2}$. This equation shows that when h^L becomes nonzero, $q_{\rm EA}$ also becomes finite even in the PM phase due to the finite magnetization along the longitudinal axis.

The expression for the gap was first obtained in Refs. 13 and 20 that considered the $h^L=0$ case. To answer the question of the region of validity of the Landau approach, we use the exact diagonalization method to obtain the gap as a function of h^T at $h^L=0$. The results are shown in Fig. 5. The agreement at small values of Δ demonstrates the reliability of our methods and gives an indication of the size of the critical region.

(B) This region is characterized by the condition $|r-r_c|^{3/4} \ll h^L$. In the leading approximation Δ

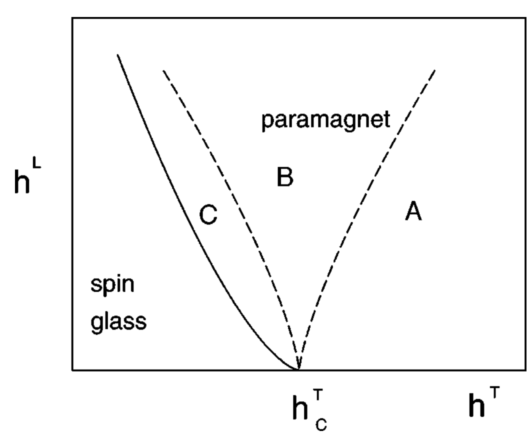

FIG. 4. Schematic phase diagram predicted by the Landau theory. The dashed lines denote crossovers while the full line is a critical line.

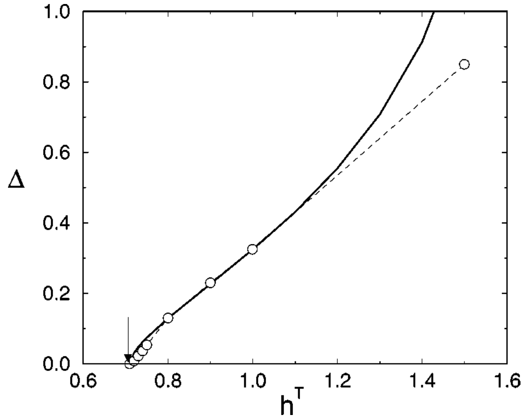

FIG. 5. Gap vs transverse field h^T at $h^L=0$ (open circles). The fitting function from Eq. (9) with $q_{\rm EA}=0$ is plotted in the solid line. The arrow indicates the critical field.

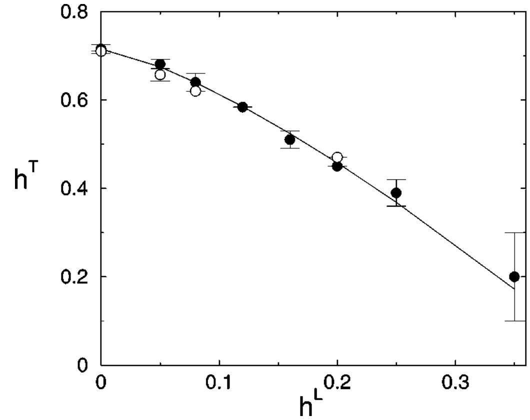

FIG. 6. SG-PM phase boundary obtained with exact diagonalization. Filled and open circles correspond to the two different criteria (i) and (ii), respectively (see text). The solid line corresponds to the fitting function $h^T=h_c^T-2.2(h^L)^{4/3}$.

$\approx\{(u_1\kappa(h^L)^2)/4u_2\ln(1/(h^L)^{4/3})\}^{1/3}$, while $q_{\rm EA}\approx\{(\kappa(h^L)^2/4)\sqrt{(u_2/u_1)\ln(1/(h^L)^{4/3})}\}^{2/3}$.

(C) This regime, in which $(r_c-r)^{3/4}\gg h^L$, is the closest to the $T=0$ critical boundary. The Edwards-Anderson (EA) order parameter, that crosses over to its value in the glassy phase, is given by $q_{\rm EA}=[(r_c-r)/u_1]+(u_2\Delta^2/u_1)\ln[1/\Delta^2]$, with $\Delta\approx[\kappa u_1(h^L)^2/4(r_c-r)]-[2y(r_c-r)^2/3u_1^2\kappa^2]$. From this expression it is easily seen that Δ vanishes at the critical line given by

$$h^L=(8y/3)[(r_c-r)/u_1\kappa]^{3/2}. \tag{14}$$

Since r is a regular function of h^T/J, so that $(r_c-r)\propto(h_c^T-h^T)$, we see that the gap vanishes at the line that is in fact determined by $h_c^L\propto(h_c^T-h^T)^{3/2}$.

Finally, in the SG phase

$$D(\omega_n)=-yq_{\rm EA}^2/\kappa^2-|\omega_n|,\quad q_{\rm EA}=(r_c-r)/u_1, \tag{15}$$

resulting in a gapless form of the spectral density $\chi''(\omega)\propto\omega$.

We would like now to discuss the nature of the crossover between subregimes (B) and (C) in more detail. A rather surprising result, one obtains from the exact diagonalization method, is that in fact the freezing transition of the transverse degrees of freedom takes place at the critical boundary line given by $h_{c\rm ED}^L\propto(h_c^T-h^T)^{3/4}$ (see Fig. 6). This result was verified by the two different criteria discussed in Sec. II B, namely, the divergence of the spin-glass susceptibility given by $J^2\langle[\chi_{loc}^{zz}]^2\rangle=1$, and the vanishing of the excitation energy gap of the regular part of the dynamical spin susceptibility. It is notable that the extrapolations to the thermodynamic limit for these two different freezing transition criteria do agree well. However, these results seem paradoxical since the Landau theory (14) predicts a phase transition boundary with a different functional form, namely, $h_c^L\propto(h_c^T-h^T)^{3/2}$ (and different curvature, see Fig. 4).

At this stage we are faced with two possibilities to solve this paradox. (i) The Landau theory fails to properly describe the correct boundaries between the different phases of the model. (ii) The numerical results suffer from severe finite-size effects. We analyze below each of these possibilities, trying to understand their origins and their physical implications.

The Landau theory has been successful to describe the critical behavior of the model with $h_L=0$. The independent methods, such as one-loop expansion,[13] the exact solution of a related rotor model[20] as well as numerical methods, similar to the ones considered in the present work,[4] lead to the same picture. In addition, no doubts seem to rise upon its validity to describe the critical region of the model with $h^T=0$, the classical Sherrington-Kirkpatrik (SK) model in a longitudinal field at finite temperature T. In the latter case, Landau theory predicts the so-called de Almeida Thouless (AT) line, dividing the paramagnetic phase from the ordered one $h_c^L\propto(T_c-T)^{3/2}$, being T_c the critical temperature.[1] On the other hand, at $h^L=0$ the numerical method shows no problem and is consistent with the Landau theory (cf. Fig. 5 and Ref. 4). So, as a further test of the finite-size effects in the numerical method, we decided to investigate the behavior along a different axis, namely, the temperature axis that allows for additional comparison to the Landau theory in the (h^L,T) plane. Thus, we evaluated $J^2\langle[\chi_{loc}^{zz}]^2\rangle$ for the SK model as a function of T for systems of the same size as those used in the quantum case, and performed the extrapolations to the thermodynamic limit with the same criteria. Details of the numerical procedure to obtain the behavior of the latter quantity in the thermodynamic limit are shown in Fig. 7. At a given h_L the critical temperature then is determined from the condition $J^2\langle[\chi_{loc}^{zz}]^2\rangle=1$. The resulting critical line is shown in Fig. 8. Interestingly, we found similar discrepancies as before, since numerical calculations suggest that the critical temperature is $h_c^L\propto(T_c-T)^{3/4}$, instead of the correct result with a critical exponent 3/2. This fact led us to suspect that the numerical procedure fails to capture the correct boundary at finite h_L. Furthermore, very large finite-size effects have been recently reported also in the classical model.[21] Note that in the latter work, special methods valid only for the classical model allow for the numerical solution of systems much larger than the ones considered here. Despite the large sizes of the samples, the correct transition line

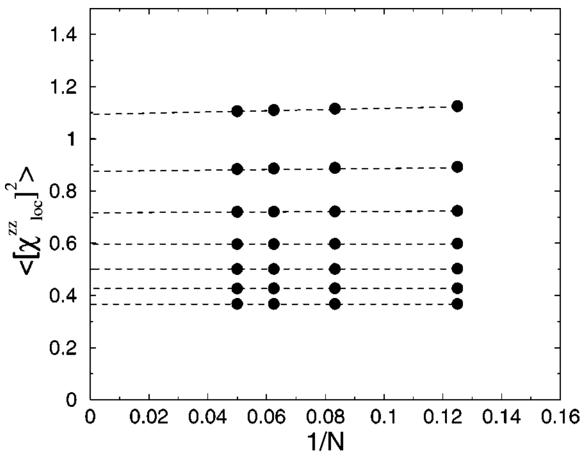

FIG. 7. $\langle[\chi_{loc}^{zz}]^2\rangle$ for the SK model at $h_L=0.05$ and $T=0.23,0.26,0.29,0.32,0.35,0.38,0.41$ as a function of the inverse of the system size. The linear fits to perform the extrapolations to the thermodynamic limit are indicated in dashed lines. The quality of the fits is similar for all h_L and T.

is still not properly captured. Thus, it seems that within the present state of the art of numerical methods and the available computing power there is no hope to overcome the problem of the finite-size effects near the critical line at finite longitudinal field.

In light of these results let us further scrutinize the predictions of the Landau theory. In particular, it is important to note that, in the presence of the nonzero longitudinal field, the critical behavior of the gap is different than at $h^L=0$. It takes a much slower, linear form $\Delta(\delta r)\sim\delta r$ (δr is the distance to the critical line), becoming the new effective small energy scale that characterizes the region (C). This linear regime of $\Delta(\delta r)$ crosses over to the regime (B), at values of

$$r_c-r\approx[\kappa u_1(h^L)^2/4]^{2/3}u_2^{1/3}\ln^{1/3}[1/(h^L)^{4/3}], \qquad (16)$$

rendering $h^L\propto(h_c^T-h^T)^{3/4}$ up to an inessential logarithmic prefactor. Remarkably, this is precisely the functional form obtained for the critical line (and gap closure) from the numerical calculation. Therefore, we are led to conclude that

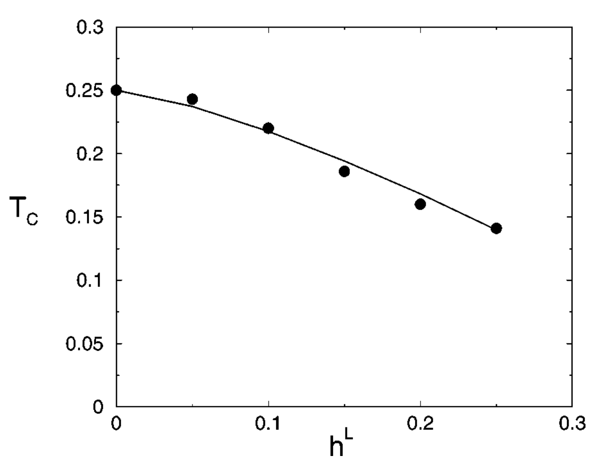

FIG. 8. T_c as a function of the longitudinal field h_L in the SK model. The fitting function is $T_c=T_c(h_L=0)-0.7h_L^{4/3}$.

for systems of the size that one can diagonalize, the physics of the small gap is masked by the finite-size effects, affecting, thus, the validity of extrapolations.

This kind of effects is well known in the context of one-dimensional models. The latter are excellent laboratories to test the reliability of numerical methods since independent results by conformal field theory and Luttinger liquid theory are available. In some cases, even the exact solution is possible by recourse to the Bethe Ansatz technique. For this reason, let us make a brief review of some features observed in the one-dimensional Hubbard model, which might help as a reference and illustration of a similar situation as the one we have described above. The Hubbard model is exactly solvable with Bethe Ansatz and is known to be an insulator at half-filling for all positive values of the Coulomb repulsion U. A detailed study of the exact solution[22] reveals that the charge gap is exponentially small for small U, while it grows linearly with U when this parameter overcomes a critical value. A finite system is characterized by discrete levels separated by finite-energy gaps. In particular, there is a finite-energy gap between the ground state and the lowest charge excitation, which is the relevant excitation to evaluate the charge gap. The charge gap is related to the coherence length for the charge propagation. Extrapolations of the gap based on data corresponding to systems with lengths smaller than the coherence length (which can be very large within the small U regime) lead to the prediction of a vanishing gap in the thermodynamic limit, thus concluding (incorrectly) that the system is metallic. Similar difficulties are encountered in extended Hubbard models with a metal-insulator transition.[23]

More interesting are the implications of such a small energy scale. In case of the Hubbard model, slight departures from the ideal situation of perfect nesting of the Fermi surface due to disorder, additional interactions or geometrical frustration, result in the occurrence of a true metallic phase at half-filling within the small U region. At the same time, the insulating phase at larger U is more robust and survives such perturbations. In the Ising model with two fields studied in this work, an analogous behavior can be expected, and it is likely that the small gap within the region (C) may be also difficult to observe in experiments as well as in numerical calculations. In the classical case, the numerical results can be an indication that the free energies of the ordered and paramagnetic phases are actually very close within a crossover region in the (h^L,T) diagram, equivalent to the region (C) of Fig. 4. This may be also a possible explanation to the anomalous behavior observed in experimental studies of the AT line.[24] In contrast, in regions (A) and (B), the r dependence of the gap assumes a form similar to the zero-field limit [see Eq. (9), except that the variable r is shifted by the quantity $u_1q_{\rm EA}$]. Since in region (B) (dropping logarithmic corrections) $q_{\rm EA}\sim(h^L)^{4/3}$, we conclude that the crossover line separating regions (B) and (C) may play a role of an *apparent* critical line, below which the gap, although finite, may assume unobservable small values.

IV. SUMMARY AND CONCLUSIONS

We have investigated the $T=0$ phase diagram of the fully connected Ising model with random exchange interactions in the presence of longitudinal and transverse magnetic fields. We have used the complementary techniques: the Landau theory and exact diagonalization to determine the phase boundaries in the plane (h^L,h^T), as well as the behavior of the spin gap close to the transition. We found that while both methods fully agree for vanishing small longitudinal fields, a different critical boundary is predicted at finite h^L. We have also employed the same methods to determine the de Almeida-Thouless transition line in the classical (Sherrington-Kirpatrik) model as a function of temperature finding the same kind of discrepancy. Since the Landau theory predicts in this case the expected correct behavior, we conclude that the numerical results are likely to be affected by finite-size effects. A careful analysis of the behavior of the gap given by the Landau theory, points towards the possibility that the transition line observed by the numerical method is actually a crossover line at which the gap changes its behavior as a function of the transverse field, assuming very small values.

This outstanding feature, which was overlooked in previous works, may have important consequences. For instance, it may be responsible for the peculiar observation of the quenching of the nonlinear susceptibility at the quantum critical point of the $LiY_{1-x}Ho_xF_4$ series.[25] Another example is the electron glass model that was recently described in Ref. 8 and for which essentially identical arguments apply. In this case the dynamical exponent is $z=1$, and we find that the crossover energy scale (corresponding to the gap in the Ising case) behaves as $\Delta \sim \delta r^2$ and corresponds to a crossover temperature separating the Fermi liquid regime (at low T) from the quantum critical regime (at high T). The second power in δr indicates an even broader quantum critical regime than in the Ising case. Such an extended quantum critical region may result in enhanced dissipation at low temperatures, a possibility which may bear relevance for the puzzling absence of weak localization (interference) corrections in certain two-dimensional electron gases in the low-density regime.

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