3-2016

The Jin and Jang of Quantum Physics Truth Tables

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"We don't see the world as it is, we see it as we are." –Anaïs Nin

Submitted to G4G12

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At the turn of the 20th century, Max Planck uncovered a new physical constant that bears his name and turned Classical Physics upside down. Instead of allowing all possible energy states to be accessed, Max Planck did the unthinkable of transforming an integral over all energy states times the probability of occupation of the energy states of matter into a Geometric Sum of discrete energy states. In the well known experimental but theoretically unexplained results of the Blackbody radiation curve, Planck introduced one constant to the experimental curve of the emitted light intensity versus the frequency of light. Five years later, Albert Einstein was able to explain the Photoelectric Effect by transforming the wave nature of light into a particle description of light. The essence of the Photoelectric Effect is measuring the electrical current of a metal as a function of frequency of the light incident on its surface. The experimental results are linear with a slope of Planck's constant.

The simultaneously discovered quantum theories of Werner Heisenberg and Erwin Schrödinger evolved to explain interactions of light with matter, thus theoretically explaining the line spectra of atoms and molecules. The double slit experiment demonstrates the dual wave/particle nature of light (photons). It is the results of this experiment that defines a Jin and Jang of the Quantum Physics Truth Table.

The results of the experiment are based on a comparison of the Classical Physics results of particle and wave behaviors passing through a single and double slit. If you shoot small paint balls through a fence that has open slots in front of a screen, you will find individual marks on the screen. Those marks would correspond to well defined trajectories (paths) that would lead back to the paint ball gun's angle with respect to the fence and the amount of force with which the paint ball was released from the gun. If you shoot through two slits in the fence you will find two single fence slot patterns of individual paint balls. If you pass a beam of light waves through a single slit, you will observe a diffuse band of light right in the region of the screen where you expect to see the paint balls land. If you pass a monochromatic beam of light through a double slit of the proper geometric proportions, you will find an interference pattern, which is comprised of multiple bands of light with the most intense band lining up with the region halfway between the two slits. Clearly there is a distinction between wave behavior and particle behavior. The particle behavior is described by a trajectory leaving distinct marks on a screen. This gives information about the path between the source of the particle and the point of impact of the particle. Waves, on the other hand, have no defined trajectory and different parts of the wave "interfere" with one another. When the crest (or trough) of one wave reinforces and amplifies the crest (or trough) of another wave, the result is constructive interference. Destructive interference is produced when the crest of one wave meets the trough of another wave, causing them to cancel each other out and leave a node or place with no intensity.

With this introduction, particle and wave behavior are clearly distinguished from one another in Classical Mechanics. Based on Einstein's explanation of the photoelectric effect, experiments have been created in which the single and double slit experiments can be carried out using photons as our light particles. As anticipated, when individual photons pass through a single slit, the screen on the other side shows a single band of individual photons. However, when the both slits are opened, the interference pattern emerges from the pattern of dots showing up on the screen. The eminent physicist, Paul Dirac explained this by stating that the photon interferes with itself as it passes through both slits. The wave/particle duality of quantum-sized entities does not admit of a trajectory when a wave experiment is performed.

First, when setting up truth tables, one can choose to assume that a three valued logic is appropriate in which the Law of the Excluded Middle is set aside. One can create a true, false and maybe table or a true, false, undetermined and indeterminate, thus creating a three or four valued logic table, respectively. Careful examination of these alternatives reveals an imposition of a trajectory on the quantum-sized objects under study. To avoid this inherent implicit assumption, the use of the Heisenberg representation of the state of a system is undertaken. The state of the system is represented as a vector. Since one of the quantum postulates set forth by von Neumann states that the probability of state of the system is the square of the state vector, one can represent the state of an arbitrary vector, $|g\rangle$ in terms of the complete set of vectors describing the pure states of the system. In the case of the truth table, the complete set of vectors is either true or false. Either the particles or photons hit a particular region of the screen or they do not hit other portions of the screen, irrespective of whether there is a trajectory or not.

The classic truth table can be constructed in the following way:

p	q		p∧q	p∨q	p→q	p←q
T	T		T	T	T	T
T	F		F	T	F	T
F	T		F	T	T	F
F	F		F	F	T	T

The p∧q column has only one T value and the rest are F. The p∨q column reveals that there is only one F value and the rest are T. If the particular trajectory from the p (source) to the q (screen) is ignored, p∧q is only T when both p and q are both T. Likewise p∨q is only F when both p and q are F. These are the only necessary rows of the classic truth table. The other two columns of the truth table are only F when p→q has p = F and p←q has q = F. Once again only two rows of the truth table are needed to describe the probability of the truth of the outcome. Those two rows are two different rows than the ∧ and ∨ rows. Further more, the p→q column can be replaced by its equivalent of p or **not** q and p←q can be replaced by **not** p or q. The following postulates are set forth. Another quantum physics postulate is that the expectation value of a particular operation is equal to the following: $\langle f|\hat{O}|g\rangle$ = scalar value dependent on the particular operator "$\hat{O}$". The notation $|g\rangle$ is called a ket vector representing the state g and because the vectors are in general representative of complex functions. The other notation of $\langle g|$ (the bra vector) is the complex conjugate of the ket vector. The particular representation that is adopted here is the following. In the case of the double slit experiment, the representation of the state in which the particle passes through the right slit will be: $|R\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\langle R| = [1\ 0]$. A particle passing through the left slit will be: $|L\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\langle L| = [0\ 1]$. Note the following properties of these two representations of classical particles:

$$\langle R_C|R_C\rangle = \langle L_C|L_C\rangle = 1 \text{ and } \langle R_C|L_C\rangle = \langle L_C|R_C\rangle = 0$$

where the subscript C is added to make a distinction between Classical Physics and Quantum Physics. This follows another postulate of von Neumann for quantum physics and that is that the square of the state vector equals the probability of finding a particle in a particular state. The probability of a particle aimed at the right slit and showing up at the right region of the screen is certain, but the probability of a particle aimed at the left slit and showing up at the right region of the screen is zero.

To complete the description the quantum mechanical state, one has to evoke the notion that solutions are most readily described as vectors in the complex plane. The pure states that are orthogonal to the real axis are:

$$|R_Q\rangle = \begin{bmatrix} e^{i\pi/2} \\ 0 \end{bmatrix} \text{ and } |L_Q\rangle = \begin{bmatrix} 0 \\ e^{-i\pi/2} \end{bmatrix}$$

Note, once again, that The subscript Q refers to a quantum state, but as is readily noted the designation of right and left are completely arbitrary. As in the classical physics representation the following results are:

$$\langle R_Q|R_Q\rangle = \langle L_Q|L_Q\rangle = 1 \text{ and } \langle R_Q|L_Q\rangle = \langle L_Q|R_Q\rangle = 0 .$$

Based on the experimental results of the double slit experiment, the trajectory is undefined for photons and photons can only be represented as linear combinations of their pure states, referred to as mixed states. The mixed quantum states in this representation would be:

$$\left|R_Q^m\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\pi/2} \\ e^{-i\pi/2} \end{bmatrix} \text{ and } \left|L_Q^m\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\pi/2} \\ -e^{-i\pi/2} \end{bmatrix}.$$

As before, these mixed states are orthogonal to one another, but a factor of the square root of 2 is needed to keep the square of the vector equal to unity.

Having the states of the system defined, the definition of the operators of ∧, ∨, → and ← need to be defined. Once again, a quantum projection operator technique is going to be used that is based on the classical state vectors. Since the experiment uses a classical sized experimental apparatus (i.e., the double slit), the projection operators for each of the four symbols of ∧, ∨, → and ← will be based on the classical physics pure left and right slit vectors. The projection operators for each of the operators are defined as:

$$\hat{P}_C(\wedge) = |R_C\rangle\langle R_C| + |L_C\rangle\langle L_C| = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{P}_C(\vee) = |R_C\rangle\langle R_C| \oplus |L_C\rangle\langle L_C| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{P}_C(\rightarrow) = |\bar{R}_C\rangle\langle R_C|(-) \oplus |\bar{L}_C\rangle\langle L_C|(-) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}(-) \oplus \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}(-)$$

$$\hat{P}_C(\leftarrow) = (-)|R_C\rangle\langle \bar{R}_C| \oplus (-)|L_C\rangle\langle \bar{L}_C| = (-)\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \oplus (-)\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

The plus sign in the ∧ operator means the arithmetic plus. The plus within a circle is a designation like the plus sign linking the real and imaginary parts of a complex number. The classic projection operator created by the $\langle \bar{R}_C| = [-1\ 0]$ or $|\bar{R}_C\rangle = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ produces the matrices with negative ones on the diagonal element. The (−) to the left or the right of the projection operators means to change the phase of the vector that is multiplying either from the left or right of the operator. In terms of the classical mechanics particles this has no effect on the outcome of the experimental results, but in the case of the mixed quantum states representing the wave/particle duality of particles there is an effect when using the classical mechanics projection operator.

The final step in the construction of our truth table for classical and quantum particles is to apply the four operators to combinations of the two pure classical particle states and to the two mixed quantum particle states. The first test is to see if the projection operator for a single slit (either the right or left slit only) would give the experimental results for the small paint ball and photon passing through a single slit. In this case, the projection operators are:

$$\hat{P}_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \hat{P}_L = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The results of the experiments based on the expectation values produced by these operators follows:

$$\langle R_C|\hat{P}_R|R_C\rangle = \langle L_C|\hat{P}_L|L_C\rangle = 1;\ \langle R_C|\hat{P}_R|L_C\rangle = \langle R_C|\hat{P}_L|L_C\rangle = \langle R_C|\hat{P}_L|R_C\rangle = \langle L_C|\hat{P}_R|L_C\rangle = 0$$

Also:

$$\langle R_Q^m|\hat{P}_R|R_Q^m\rangle = \langle L_Q^m|\hat{P}_L|L_Q^m\rangle = \langle R_Q^m|\hat{P}_R|L_Q^m\rangle = \langle R_Q^m|\hat{P}_L|L_Q^m\rangle = 1;\ \langle R_Q^m|\hat{P}_L|R_Q^m\rangle = \langle L_Q^m|\hat{P}_R|L_Q^m\rangle = 0$$

The last result may seem surprising, because of the labeling of L and R on the projection operator and the state vectors not matching. As noted earlier, the labeling of the quantum state vector is arbitrary and irrespective of the labeling the projection operator, the expectation value for the probability of the L_Q mixed state going through the projection operator for the right slit is the same as the probability of the R_Q mixed state going through the projection operator for the left slit. Otherwise, the passage of a R_Q mixed state and a L_Q mixed state through the same L or R projection operator causes a destructive interference of the two out of phase states. These results are in complete agreement with the experimental results.

Applying the right and left projection operators for the double slit experiment to the classical mechanical particles, yield the following results:

$$\langle R_C|\hat{P}_C(\wedge)|R_C\rangle = 1;\ \langle L_C|\hat{P}_C(\wedge)|L_C\rangle = 1;\ \langle R_C|\hat{P}_C(\wedge)|L_C\rangle = \langle L_C|\hat{P}_C(\wedge)|R_C\rangle = 0$$

$$\langle R_C|\hat{P}_C(\vee)|R_C\rangle = 1\oplus 0;\ \langle L_C|\hat{P}_C(\vee)|L_C\rangle = 0\oplus 1;\ \langle R_C|\hat{P}_C(\vee)|L_C\rangle = \langle L_C|\hat{P}_C(\vee)|R_C\rangle = 0\oplus 0$$

$$\langle R_C|\hat{P}_C(\rightarrow)|R_C\rangle = 1\oplus 0;\ \langle L_C|\hat{P}_C(\rightarrow)|L_C\rangle = 0\oplus 1;\ \langle R_C|\hat{P}_C(\rightarrow)|L_C\rangle = \langle L_C|\hat{P}_C(\rightarrow)|R_C\rangle = 0\oplus 0$$

$$\langle R_C|\hat{P}_C(\leftarrow)|R_C\rangle = 1\oplus 0;\ \langle L_C|\hat{P}_C(\leftarrow)|L_C\rangle = 0\oplus 1;\ \langle R_C|\hat{P}_C(\leftarrow)|L_C\rangle = \langle L_C|\hat{P}_C(\leftarrow)|R_C\rangle = 0\oplus 0$$

The interpretation of these equations is that if the probability of the classical particle is certain to go through the right side of the double slit, then it will end up on the right side of the screen. But, if one shoots our tiny paint ball toward the left slit, then it will go only show up on the left side of the screen.

Applying the classical mechanics projection operators for the double slit experiment to the quantum mechanical wave/particle properties of photons, yield the following results.

$$\langle R_Q^m|\hat{P}_C(\wedge)|R_Q^m\rangle = 1;\langle L_Q^m|\hat{P}_C(\wedge)|L_Q^m\rangle = 1;\langle R_Q^m|\hat{P}_C(\wedge)|L_Q^m\rangle = \langle L_Q^m|\hat{P}_C(\wedge)|R_Q^m\rangle = 0$$

$$\langle R_Q^m|\hat{P}_C(\vee)|R_Q^m\rangle = 1\oplus 0;\langle L_Q^m|\hat{P}_C(\vee)|L_Q^m\rangle = 0\oplus 1;\ \langle R_Q^m|\hat{P}_C(\vee)|L_Q^m\rangle = \langle L_Q^m|\hat{P}_C(\vee)|R_Q^m\rangle = \frac{1}{2}\oplus\frac{1}{2}$$

$$\langle R_Q^m|\hat{P}_C(\rightarrow)|R_Q^m\rangle = 1\oplus 0;\langle L_Q^m|\hat{P}_C(\rightarrow)|L_Q^m\rangle = 0\oplus 1;\langle R_Q^m|\hat{P}_C(\rightarrow)|L_Q^m\rangle = \frac{1}{2}\oplus -\frac{1}{2};\langle L_Q^m|\hat{P}_C(\rightarrow)|R_Q^m\rangle = -\frac{1}{2}\oplus\frac{1}{2}$$

$$\langle R_Q^m|\hat{P}_C(\leftarrow)|R_Q^m\rangle = 1\oplus 0;\langle L_Q^m|\hat{P}_C(\leftarrow)|L_Q^m\rangle = 0\oplus 1;\langle R_Q^m|\hat{P}_C(\leftarrow)|L_Q^m\rangle = -\frac{1}{2}\oplus\frac{1}{2};\langle L_Q^m|\hat{P}_C(\leftarrow)|R_Q^m\rangle = \frac{1}{2}\oplus -\frac{1}{2}$$

In some respects, the results look similar to the classical mechanics particle vector results. However, the quantum mixed vectors carry both of the pure imaginary states. The results of the $\langle X_Q^m | \hat{P}_C(\wedge) | Y_Q^m \rangle = 1$ when $X_Q = Y_Q$ and $\langle X_Q^m | \hat{P}_C(\wedge) | Y_Q^m \rangle = 0$ when $X_Q \neq Y_Q$ does not have the same interpretation as the classical double slit results. In the classical regime, the right and left vectors are pure states; whereas, in the quantum regime, the right and left vectors are not pure quantum states, but mixed states, which makes the labels of R_Q^m and L_Q^m meaningless in terms of trying to put a Newtonian label on the quantum particles that have no trajectories. Viewing the totality of both the classical particle and the quantum particle, gives a quite different interpretation of the quantum regime. The $\langle X_Q^m | \hat{P}_C(\wedge) | Y_Q^m \rangle = 1$ when $X_Q = Y_Q$ is interpreted as a linear superposition principle of both pure states that constructively interfere with one another. The $\langle X_Q^m | \hat{P}_C(\wedge) | Y_Q^m \rangle = 0$ when $X_Q \neq Y_Q$ means that the interference of these two different, orthogonal, linear superpositions destructively interfere with one another, thus, setting up an interference pattern.

The same argument applies to the other three projection operators for the case where $X_Q = Y_Q$. Moving on to the cases where $X_Q \neq Y_Q$, the $\hat{P}_C(\vee)$ projection operator shows that going through the double slit with two orthogonal mixed states, is equally probable for these two different, orthogonal, mixed states to constructively interfere with one another. This is another way of expressing constructive interference of the two pure states as they pass through the double slit. Now for the $\hat{P}_C(\rightarrow)$ and $\hat{P}_C(\leftarrow)$ projection operators. Their results not only show the equal probability but the relative phases of the two pure states.

Classical Mechanical Truth Table

$\langle p\|$	$\|q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}_C(\vee)$	$\hat{P}_C(\rightarrow)$	$\hat{P}_C(\leftarrow)$
$\langle R_C\|$	$\|R_C\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_C\|$	$\|L_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\|$	$\|R_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\|$	$\|L_C\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$

Quantum Mechanical Truth Table

$\langle p\|$	$\|q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}(\vee)$	$\hat{P}(\rightarrow)$	$\hat{P}(\leftarrow)$
$\langle R_Q^m\|$	$\|R_Q^m\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_Q^m\|$	$\|L_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$
$\langle L_Q^m\|$	$\|R_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$
$\langle L_Q^m\|$	$\|L_Q^m\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$