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Symmetric generation of finite homomorphic images?

Lee Farber

SYMMETRIC GENERATION OF FINITE HOMOMORPHIC IMAGES

A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Mathematics

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ABSTRACT

We present the technique of double coset enumeration and apply it to construct finite homorphic images of infinite semidirect products. We construct several important homomorphic images including the classical groups $L_3(3)$, the Projective Special Linear group, and the Derived Chevalley group, $G_2(2)' = U_3(3)$.

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CHAPTER ONE

INTRODUCTION

Let G be a group and $T = \{t_1, t_2, \ldots, t_n\} \subseteq G$. Define $\overline{T} = \{T_1, T_2, \ldots, T_n\}$, where $T_i = \langle t_i \rangle$, the cyclic subgoup generated by t_i. Define $N = N_G(\overline{T})$, the set normalizer in G of $\overline{T}$. Under the conditions

1) $G = \langle T \rangle$, and

2) N permutes $\overline{T}$ transitively,

we say that T is a symmetric generating set for G. N is called the control subgroup.

The two conditions imply that G is a homomorphic image of the infinite progenitor $m^{*n} : N$, a free product of n copies of the cyclic group C_m extended by N, where m is the order of t_i. Additional relations required to define G have the form $\pi = w(t_1, t_2, \ldots t_n)$ where $\pi \in N$ and w is a word in the symmetric generators. Thus for $m = 2$ and $n = 3,4\ldots$,

$$\frac{2^{*n} : N}{\pi_1 w_1, \ldots, w_s w_s} \cong \left\langle N, T \middle| t_i^2 = 1, \quad t_i^{\pi} = t_{\pi(i)}, \quad \pi_1 w_1 = \ldots = \pi_s w_s = 1 \right\rangle$$

and relations for other progenitors can be given in a similar manner.

Let i stand for the coset Nt_i, ij for the coset Nt_it_j, etc. Since $i\pi = \pi\pi^{-1}i\pi = \pi i^{\pi}$, the permutations involved in any element of G can be gathered on the left. Thus any element of G can be written as a permutation belonging to N followed by a word in the symmetric generators.

Thus if NxN is a double coset of N in G,

$$NxN = N\pi wN = NwN$$

where $x = \pi w \in G$, with $\pi \in N$ and w is a word in the t_is.

Denote this double coset as $[w]$, e.g. $[01]$ is Nt_0t_1N.

The double coset $NeN = N$, where e is the identity, is denoted by $[*]$.

Also define the single point stabilizer in N as $N^i = C_N(t_i)$, the two point stabilizer in N as $N^{ij} = C_N(\langle t_i, t_j \rangle)$ etc.

We perform a double coset enumeration of G over N to find the index of N in G. The procedure is to find all the double cosets $[w]$ and enumerate the single cosets in each double coset. We will have completed the double coset enumeration when the set of right cosets obtained is closed under right multiplication. We now define the coset stabilizing subgroup,

$$N^{(w)} = \{\pi \in N| \quad Nw\pi = Nw\}$$

for w a word in the symmetric generators, to perform this completion test. Note that $N^{w} \leq N^{(w)}$, and the number of single or right cosets in the double coset $[w] = NwN$ is given by $\frac{|N|}{|N^{(w)}|}$, since

$$\begin{aligned} Nw\pi_1 \neq Nw\pi_2 \quad &\Leftrightarrow \; Nw\pi_1\pi_2^{-1} \neq Nw \\ &\Leftrightarrow \; \pi_1\pi_2^{-1} \notin N^{(w)} \\ &\Leftrightarrow \; N^{(w)}\pi_1 \neq N^{(w)}\pi_2 \end{aligned}$$

The completion test is performed by obtaining the orbits of $N^{(w)}$ on the symmetric generators. We need only identify, for each double coset $[w]$, the double coset to which Nwt_i belongs for one symmetric generator t_i from each orbit.

Example: Generation of $L_2(19)$ Over $L_2(5)$

We consider (see Hasan [7])

$$G \cong \frac{2^{*6} : L_2(5)}{[(\infty,0,1)(2,4,3)t_2]^5}$$
$$\cong \langle N,T \mid N \cong L_2(5), \quad t^2 = 1, \quad [t, N^{\infty}] = 1, \quad [(\infty,0,1)(2,4,3)t_2]^5 = 1 \rangle .$$

Thus G is the homomorphic image of the (infinite) split extension $2^{*6}:L_2(5)$ factored by the relator $[(\infty,0,1)(2,4,3)t_2]^5$ and the action of $N = L_2(5)$ on the six symmetric generators is given by $x \sim (0,1,2,3,4)$ and $y \sim (0,\infty)(1,4)$.

We use the technique of manual double coset enumeration to construct G. We will first show that $|G| = 3420$ and then prove the stronger statement that $G \cong L_2(19)$.

Manual Double Coset Enumeration of G over $L_2(5)$:

Now $[\pi t_2]^5 = 1$, where $\pi = (\infty,0,1)(2,4,3)$ implies

$$\pi t_2 \pi t_2 \pi t_2 \pi t_2 \pi t_2 = 1$$

$$\Rightarrow \pi^2\pi^{-1}t_2\pi t_2\pi^3\pi^{-2}t_2\pi^2\pi^{-1}t_2\pi t_2 = 1$$

$$\Rightarrow \qquad \pi^2 t_2^{\pi} t_2 t_2^{\pi\pi} t_2^{\pi} t_2 = 1$$

$$\Rightarrow \qquad \pi^2 t_4 t_2 t_3 t_4 t_2 = 1$$

$$\Rightarrow \qquad t_4 t_2 t_3 t_4 t_2 = \pi \,.$$

Note, in particular, that all conjugates of the relator $\pi^2 t_4 t_2 t_3 t_4 t_2$, under conjugation by N, are also relators. We now perform the double coset enumeration of the image G over N. First of all $NeN = N$ and we denote this double coset by [*]. Now since N is transitive on $T = \{0,1,2,3,4,\infty\}$, $[\infty] = \{Nt_{Ë}, Nt_0, Nt_2, Nt_3, Nt_4\}$. Next, $N^{Ë} =$

$<(0,1,2,3,4),(1,3)(0,4)> \cong D_{10}$ has orbits $\{\infty\}$ and $\{0,1,2,3,4\}$ on T. Thus we have only the double coset $[\infty 0] = Nt_\infty t_0 N$ to consider at this stage. Now $N^{(\infty 0)} \geq N^{\infty 0} = <(1,4)(2,3)> \cong C_2$. Since $|N^{(\infty 0)}|=2$, the number of single cosets in the double coset $[\infty 0]$ is at most $\frac{|N|}{2} = \frac{60}{2} = 30$.

The orbits of $N^{(\infty 0)}$ on T are $\{0\}$, $\{\infty\}$, $\{1,4\}$ and $\{2,3\}$. Thus we must consider the double cosets $[\infty 0 \infty]$, $[\infty 01]$ and $[\infty 02]$. In order to do this, we study our relations in more detail. We now show that $\infty 0 \infty \sim \infty 0$ and $\infty 02 \sim 0\infty$.

Firstly,

$$\begin{aligned}\infty.01401 &= \infty.(1,0,4)(2,3,\infty)\\ &= (1,0,4)(2,3,\infty)\infty^{(1,0,4)(2,3,\infty)}\\ &= (1,0,4)(2,3,\infty)2.\end{aligned}$$

So $\infty 014 = (1,0,4)(2,3,\infty)210$. But $210=(1,2,0)(3,\infty,4)12$. Thus $\infty 014=(1,0,4)(2,3,\infty).(1,2,0)(3,\infty,4)12=(2,\infty,0,3,4)12$ and

$$\infty 01 = (2,\infty,0,3,4)124.$$

Secondly,

$$\begin{aligned}\infty.04104 &= \infty.(1,4,0)(2,\infty,3)\\ &= (1,4,0)(2,\infty,3)\infty^{(1,4,0)(2,\infty,3)}\\ &= (1,4,0)(2,\infty,3)3.\end{aligned}$$

So $\infty.041=(1,4,0)(2,\infty,3)340$. Since $34034=(1,2,\infty)(3,0,4)$, we have $\infty041=(1,4,0)(2,\infty,3).(1,2,\infty)(3,0,4)43$ and

$$\infty04 = (1,3,\infty,0,2)431$$

Now

$$\begin{aligned}
\infty0\infty = \infty01.1\infty &= (2,\infty,0,3,4)124,1\infty \\
&= (2,\infty,0,3,4)12.41\infty41.14 \\
&= (2,\infty,0,3,4)12.(1,4,\infty)(2,3,0).14 \\
&= (2,\infty,0,3,4)(1,4,\infty)(2,3,0)(12)^{(1,4,\infty)(2,3,0)}.14 \\
&= (1,4,3,\infty,2)4314 \\
&= (1,4,3,\infty,2)(1,3,\infty,0,2)^{-1}\infty04.4 \\
&= (1,4)(0,\infty)\infty0.
\end{aligned}$$

So $\infty0\infty \sim \infty0$ and , therefore, $[\infty0\infty] = [\infty0]$.

Also $\infty02\infty0 = (1,3,4)(2,0,\infty) \Rightarrow \infty02 = (1,3,4)(2,0,\infty)0\infty \Rightarrow \infty02\sim0\infty$. Thus $Nt_\infty t_0 t_2 = Nt_0 t_\infty \in [\infty0]$. So $[\infty02]=[\infty0]$.

Next $\infty01 = \infty03.31$ and $\infty03\infty0 = (1,4,2)(3,0,\infty)$. So $\infty01= (1,4,2)(3,0,\infty)0\infty.31 = (1,4,2)(3,0,\infty)0.\infty31$ and $\infty31\infty3=(1,3,\infty)(2,0,4)$. Thus

$$\begin{aligned}
\infty01 &= (1,4,2)(3,0,\infty)0(1,3,\infty)(2,0,4)3\infty \\
&= (1,4,2)(3,0,\infty)(1,3,\infty)(2,0,4)0^{(1,3,\infty)(2,0,4)}3\infty \\
&= (0,1,2,3,4)43\infty.
\end{aligned}$$

So

$$\infty 01 = (0,1,2,3,4)43\infty.$$

This gives

$$\infty 01 = (2,\infty,0,3,4)124 = (0,1,2,3,4)43\infty,$$

and therefore $N^{(\infty 01)} \geq \langle(1,4,\infty)(2,3,0)\rangle \cong C_3$, and it has orbits $\{1,4,\infty\}$ and $\{0,2,3\}$ on T.

Since $01201 = (1,0,2)(3,4,\infty)$,

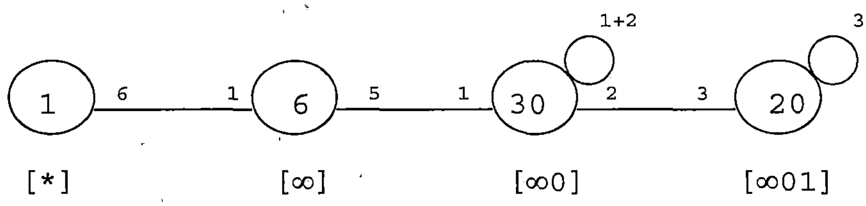
$$\infty 012 = \infty.(1,0,2)(3,4,\infty)10$$

$$= (1,0,2)(3,4,\infty)\infty^{(1,0,2)(3,4,\infty)}10$$

$$= (1,0,2)(3,4,\infty)310.$$

So $Nt_\infty t_0 t_1 t_2 = Nt_3 t_1 t_0$ and $Nt_3 t_1 t_0 \in [\infty 012] = [\infty 01]$.

We have found all the double cosets, since the set of right cosets of N is closed under right multiplication by the symmetric generators. These results are summarized in Appendix A, Table 1 and the indicated Cayley diagram is:



The Cayley diagram of

$$G \cong \frac{2^{*6} : L_2(5)}{[(\infty,0,1)(2,4,3)t_2]^5}$$

over $L_2(5)$ has the set of right cosets $\{Nw \mid w$ a word in the symmetric generators $t_i\}$ as its vertices. Each vertex Nw is joined to the vertex Nwt_i, for $i \in \{1,2,...,n\}$. The graph has five nodes. Each node, a double coset of N in G, is an orbit, $\{Nw\pi \mid w$ a word in the symmetric generators and $\pi \in N\}$, of N in its action on the vertices by right multiplication. All nodes are labelled by the number of right cosets that they contain. Lines joining any two nodes are labelled with integers to indicate how many edges from a vertex of one node lead to verticies of the other. For example, the node $Nt_\infty t_0 t_1$ is labelled with 20, since $Nt_\infty t_0 t_1 = Nt_1 t_1 t_4 = Nt_4 t_3 t_\infty$ and the integers 2 and 3 on the line joining the nodes labelled with 30 and 20 indicate that 2 of the symmetric generators take each coset from the double coset [∞0] to a coset in the double coset [∞01] and 3 of the symmetric generators take each coset from the double coset [∞01] to a coset in the double coset [∞0]. This particular graph does not have multiple edges but has two loops. The integer 1+2 at the top of the loop over the node labelled 30 indicates that $[\infty 0\infty] = [\infty 0] = [\infty 02] = [\infty 0]$, with ∞ in the 1-orbit $\{\infty\}$ and 2 in the 2-orbit $\{2,3\}$.

Our argument shows that the maxiumum possible index of N in G is $\frac{|N|}{|N|} + \frac{|N|}{|N^{(\infty)}|} + \frac{|N|}{|N^{(\infty 0)}|} + \frac{|N|}{|N^{(\infty 01)}|} = 1 + \frac{|N|}{|D_{10}|} + \frac{|N|}{|C_2|} + \frac{|N|}{|C_3|}$ $= 1 + 6 + 30 + 20 = 57$. Thus $|G| \leq 57 \times |N| = 57 \times 60 = 3420$. We will prove below that $G \cong L_2(19)$. However, in general, even if the target group is not obvious, it is still possible to prove that the maximum possible order of the group is the actual order. We show this for the current example; that is, we prove that $|G|$ is exactly 3420. We relabel our fifty seven cosets

*,1,2,3,4,0,∞,∞0,∞1,∞2,∞3,∞4,01,02,03,04,0∞,10,12,13,14,1∞,20,21,23,24,2∞,30,31,32,34,3∞,40,41,42,43,4∞,∞01,∞04,∞10,∞12,∞21,∞23,∞32,∞34,∞40,∞43,023,024,031,032,0∞1,0∞4,203,204,2∞1, and 2∞3 by the numbers 1,2,3,...,56, and 57, respectively. The actions of the generators x and y of N and t on the fifty seven cosets are given by:

x:(2,7,18,23,3)(5,8,24,12,4)(33,16,29,21,9)(37,22,10,51,28)(17,30,49,34,27)(13,48,52,35,36)(31,40,54,38,11)

(26,47,14,45,25)(46,42,55,39,41)(19,20,32,43,15)

(50,44,56,57,53),

y:(6,2)(7,3)(5,33)(8,37)(4,17)(24,13)(16,27)(9,22)(12,19)

(31,46)(28,48)(29,32)(21,35)(30,15)(26,50)(36,20)(10,52)

(40,14)(11,39)(25,38)(34,43)(47,56)(54,44)(42,55)(41,45)

(53,57) and

t:(1,6)(2,5)(7,8)(3,4)(18,24)(23,12)(37,31)(17,26)(13,32)

(22,40)(28,11)(27,25)(30,47)(19,35)(48,43)(36,20)(10,54)

(46,50)(51,38)(34,45)(49,14)(15,52)(42,44)(41,53)(39,57)

(56,55).

It readily checks that the order of $<x,y,t>$, a subgroup of the symmetric group S_{57} acting on the fifty seven right cosets of N in G, is 3420. Since the order of xy is 3, $N=<x,y> \cong L_2(5)$. Now t has exactly six conjugates under conjugation by N; namely $t=t_\infty$, $t_0=t^y$, $t_1=t_0^x$, $t_2=t_0^{x^2}$, $t_3=t_0^{x^3}$, and $t_4=t_0^{x^4}$, we conclude that $<x,y,t>$ is a homomorphic image of the progenitor $2^{*6}:L_2(5)$.

Thus, if the additional relation $[(\infty,0,1)(2,4,3)t_2]^5=1$ is satisfied in $<x,y,t> \leq S_{57}$, then $<x,y,t>$ is a homomorphic image of G and this will give $|G| \geq |<x,y,t>| = 3420$.

Now $[(\infty,0,1)(2,4,3)t_2]^5=1$ is equivalent to $t_4t_2t_3t_4t_2 = (\infty,0,1)(2,4,3)$ and indeed, $t_4t_2t_3t_4t_2 = (t^y)^{x^4}(t^y)^{x^2}(t^y)^{x^3}(t^y)^{x^4}(t^y)^{x^2} \in S_{57}$ acts as $(t_\infty,t_0,t_1)(t_2,t_4,t_3)$ on the six symmetric generators, t_∞, t_0, t_1, t_2, t_3, t_4 by conjugation.

Alternatively, we note that $[(\infty,0,1)(2,4,3)t_2]^5=1 \Leftrightarrow \left[(xy)^{y^{x^3}yx^3} t_2\right]^5=1$ and, in S_{57}, the order of $(xy)^{y^{x^3}yx^3} t_2 = (xy)^{y^{x^3}yx^3}\left(t^y\right)^{x^2}$ = (1,18,52,34,23)(2,10,4,42,21)(3,49,15,28,51)(5,22,56,14,8)(6,32,57,37,43)(7,29,38,40,35)(9,46,39,19,30)(11,16,33,17,25)(12,13,53,20,55)(24,47,26,27,48)(31,45,44,50,36), is 5.

We now show that G is isomorphic to $L_2(19)$. We construct a homomorphism from the progenitor $2^{*6}:L_2(5)$ to $L_2(19)$ by defining $x \equiv \left(\frac{8\eta+15}{\eta}\right)$ = (1,4,7,2,6)(3,13,15,9,16)(5,11,18,12,14)(8,17,10,0,∞), and

$y \equiv \left(\frac{-\eta-1}{2\eta+1}\right)$ = (1,12)(2,7)(3,13)(4,10)(5,15,)(6,17)(8,14)(9,∞)(11,16)(18,0).

Since the orders of x,y, and xy are 5,2 and 3 respectively, $N = \langle x,y\rangle \cong A_5$.

We now let

$$t_\infty \equiv \left(\frac{4\eta+15}{\eta-4}\right) = (1,0)(2,17)(3,11)(4,\infty)(5,16)(6,10)(7,8)(9,14)$$

(12,15)(13,18), and find that $|t_\infty^N|$ = 6 and

$t = t_\infty =$ (1,19)(2,17)(3,11)(4,∞)(5,16)(6,10)(7,8)(9,14)

(12,15)(13,18),

$t^y = t_0 =$ (1,5)(2,14)(3,19)(4,17)(6,7)(8,20)(9,10)(11,15)

(12,18)(13,16),

$t_0^x = t_1 =$ (1,2)(3,15)(4,11)(5,6)(7,10)(8,17)(9,18)(12,14)

(13,20)(16,19),

$t_0^{x^2} = t_2 =$ (1,11)(2,19)(3,20)(4,6)(5,14)(7,18)(8,15)(9,13)

(10,17)(12,16),

$t_0^{x^3} = t_3 =$ (1,7)(2,12)(3,14)(4,18)(5,11)(6,20)(8,13)(9,17)

(10,19)(15,16),

$t_0^{x^4} = t_4 =$ (1,8)(2,4)(3,9)(5,13)(6,14)(7,12)(10,16)(11,18)

(15,17)(19,20), and that N permutes the six images of t_∞, by conjugation, as the group $L_2(5)$ given by:

$$x:\ (t_o,t_1,t_2,t_3,t_4)(t_\infty) \text{ and}$$

$$y:\ (t_\infty,t_0)(t_1,t_4)(t_2)(t_3).$$

Since $N \cong A_5$ is maximal in $L_2(19)$ (see ATLAS [5]) and $t_\infty \notin N$, $L_2(19)$ is a homomorphic image of the progenitor $2^{*6}:L_2(5)$.

The additional relation given by $[\pi t_2]^5 = 1 \Leftrightarrow t_4t_2t_3t_4t_2 = \pi$ $= (\infty,0,1)(2,4,3)$ is satisfied in $L_2(19)$, because $t_4t_2t_3t_4t_2 =$ $(1,17,14)(3,11,15)(4,12,19)(5,8,9)(6,7,10)(16,20,18)$ acts as $(t_\infty,t_0,t_1)(t_2,t_4,t_3)$, by conjugation, on the six symmetric generators. This shows that $L_2(19)$ is an image of G. Thus $|G| \geq |L_2(19)|$; but $|G| \leq 3420 = |L_2(19)|$, and so the equality holds and $G \cong L_2(19)$.

The Cayley graph of $L_2(19)$ over $L_2(5)$ suggests that every element of $L_2(19)$ can be written, or symmetrically represented, as a permutation of $L_2(5)$, on six letters, followed by a word in terms of the symmetric generators of length at most three, and that the representation is unique when the length of the word is two or less.

Role of MAGMA

The computer algebra system MAGMA (see Cannon [4]) is designed to provide a software environment for computing with the structures which arise in areas such as algebra, number theory, algebraic geometry and (algebraic) combinatorics. It was used in this thesis to analyze presentations of the finite groups including the orbits of their double cosets. It was also used in Chapters 3-6 to numerically label the single cosets detailed in the appendices. The script that I programmed to do so is reproduced in Appendix G.

CHAPTER TWO

GENERATION OF S_6 OVER S_4

We consider the infinite semidirect product called a progenitor $3^{*4}:S_4$, of which a symmetric presentation is

$$< x,y,t \mid x^4,\ y^2,\ (y*x)^3,\ t^3,\ (t,y),\ (t^x,y) >$$

and factor it by the relations,

$$t_0^3=[(012)t_0]^5, \quad [(01)t_0]^4, \quad [(012)t_0t_1^{-1}]^2.$$

Thus $G \cong \dfrac{3^{*4}:S_4}{((0,1,2)t_0)^5,\ ((0,1)t_0)^4,\ ((0,1,2)t_0t_1^{-1})^2}$

$\cong <N,T \mid N \cong S_4,\ t_i^{\pi}=t_{\pi(i)},\ t_0^3=[(012)t_0]^5=[(01)t_0]^4=[(012)t_0t_1^{-1}]^2=1>$

A table in Bray[2] lists the homomorphic image as a group isomorphic to S_6. We will first show that $|G|=720$ and then the stronger statement that $G \cong S_6$. The action of the control group $S_4=$ <x,y>, on the symmetric generators is

$$x\sim(0,1,2,3) \text{ and } y\sim(2,3).$$

$((0,1,2)t_0)^5$ translates to $((x^{-3}yxyx^2)t)^5$,

$((0,1)t_0)^4$ translates to $((x^{-2}yx^2t)^4$,

$((0,1,2)t_0t_1^{-1}))^2$ translates to $((x^{-3}yxyx^2)tt^{-x})^2$.

Expanding $(\pi t_0)^5=1$, where $\pi=(0,1,2)$, we have

$$\pi t_0\pi t_0\pi t_0\pi t_0\pi t_0 = \pi^2 t_0^{\pi} t_0 t_0^{\pi^2} t_0^{\pi} t_0 = \pi^2 t_1 t_0 t_2 t_1 t_0 = 1$$

$$\Rightarrow \quad \pi^2 t_1 t_0 t_2 = t_0^{-1} t_1^{-1} \quad \text{(relation 1)}.$$

Expanding $(\pi t_0)^4 = 1$, where $\pi = (0,1)$, we have

$$\pi t_0 \pi t_0 \pi t_0 \pi t_0 = t_0^{\pi} t_0 t_0^{\pi} t_0 = t_1 t_0 t_1 t_0 = 1$$

$$\Rightarrow \quad t_1 t_0 = t_0^{-1} t_1^{-1} \quad \text{(relation 2)}.$$

Expanding $(\pi t_0 t_1^{-1})^2 = 1$, where $\pi = (0,1,2)$

$$\pi t_0 t_1^{-1} \pi t_0 t_1^{-1} = \pi^2 (t_0 t_1^{-1})^{\pi} t_0 t_1^{-1} = (0,2,1) t_1 t_2^{-1} t_0 t_1^{-1} = 1.$$

$$(0,2,1) t_1 t_2^{-1} = t_1 t_0^{-1} \quad \text{(relation 3)}.$$

A weaker statement is $N t_1 t_2^{-1} = N t_1 t_0^{-1}$.

We have found 30 single (right) cosets of N in G through our double coset enumeration. We now explain how actions of N and the symmetric generators t_0, t_1, t_2, and t_3 are computed on these cosets. We relabel the thirty cosets sequentially from 1 through 30.(see Appendix B, Table 2).

$$x : (N, Nx = N)(Nt_0, Nt_0^{x}, Nt_0^{x^2}, Nt_0^{x^3}) \ldots$$

We use the technique of manual double coset enumeration to construct S_6. Let N=S_4. First NeN={N}. Thus the double coset NeN=[*] contains exactly one single coset. We note that N is transitive on {0,1,2,3} and therefore on their inverses $\{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$. Thus we must consider the double cosets Nt_0N=N0N=[0] and $Nt_0^{-1}N = N\bar{0}N = [\bar{0}]$.

We note that $[0]=\{Nt_0, Nt_1, Nt_2, Nt_3\}$ and $[\bar{0}]=\{Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}, Nt_3^{-1},\}$. Now $N^{(0)} \geq N^0 = <(1,2,3),(1,2)> \cong S_3$. Thus $N^{(0)}$ has orbits $\{0\}$, $\{1,2,3\}$ and $\{\bar{0}\},\{\bar{1},\bar{2},\bar{3}\}$ on the symmetric generators and their inverses respectively. So we need to consider the double cosets $[00]$, $[01]$, $[0\bar{0}]$, $[0\bar{1}]$ and $[\bar{0}0]$, $[\bar{0}1]$, $[\bar{0}\bar{0}]$, $[\bar{0}\bar{1}]$. However,

$$00=\bar{0} \Rightarrow [00]= [\bar{0}],$$

$$0\bar{0}=e=\bar{0}0 \Rightarrow [0\bar{0}]=[*]=[\bar{0}0] \text{ and}$$

$$\bar{0}\bar{0}=0 \Rightarrow [\bar{0}\bar{0}]=[0].$$

Also by relation 2, $[\bar{0}\bar{1}]=[10]$. But $N(01)^{(0,1)} \in [01] \Rightarrow N10 \in [01]$. Since $N10 = N\bar{1}\bar{0} \Rightarrow \bar{1}\bar{0} \in [01]$,

$$[\bar{0}\bar{1}]=[01] \text{ by relation 2.}$$

Thus the double cosets that we must consider are $[01]$, $[0\bar{1}]$ and $[\bar{0}1]$. Now $N^{(01)} \geq N^{01} = <(2,3)>$ has orbits $\{0\}$, $\{1\}$, $\{2,3\}$ and $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2},\bar{3}\}$ on the symmetric generators and their inverses. So we need to analyze the double cosets

$[010],[011],[012],[01\bar{0}],[01\bar{1}],[01\bar{2}]$

$[0\bar{1}0],[0\bar{1}1],[0\bar{1}2],[0\bar{1}\bar{0}],[0\bar{1}\bar{1}],[0\bar{1}\bar{2}]$ and

$[\bar{0}10],[\bar{0}11],[\bar{0}12],[\bar{0}1\bar{0}],[\bar{0}1\bar{1}],[\bar{0}1\bar{2}]$.

Now $010=\bar{1}$ by relation 2 $\Rightarrow [010]=[\bar{1}]=[\bar{0}]$. Also, to be considered in a moment, $011=0\bar{1} \Rightarrow [011]=[0\bar{1}]$. Finally

$(0,2,1)102=\bar{0}\,\bar{1}$ by relation 1.

$((0,2,1)102)^{(0,1)}=(10)^{(0,1)}\Rightarrow(1,2,0)012=\bar{1}\,\bar{0}=01$ by relation 2 $\Rightarrow 012\sim01\Rightarrow[012]=[01]$.

Next, $01\bar{0}=\bar{1}\,\bar{0}\,\bar{0}$ (relation 2) $\Rightarrow 01\bar{0}=\bar{1}\,0\in[0\bar{1}]\Rightarrow [01\bar{0}]=[\bar{0}\,1]$. Also, trivially, $01\bar{1}=0\Rightarrow[01\bar{1}]=[0]$. Next, because relation 3 gives $1\bar{2}=(0,1,2)1\bar{0}$, then $01\bar{2}=0(0,1,2)1\bar{0}\Rightarrow 01\bar{2}=(0,1,2)11\bar{0}\Rightarrow 01\bar{2}=(0,1,2)\bar{1}\,\bar{0}=(0,1,2)01$ by relation 2 $\Rightarrow[01\bar{2}]=[01]$.

Now consider $[0\bar{1}]$. We know $(0,2,1)t_1t_2^{-1}=t_1t_0^{-1}$ (relation 3)$\Rightarrow((0,2,1)t_1t_2^{-1})^{(2,3)}=(t_1t_0^{-1})^{(2,3)}\Rightarrow(0,3,1)1\bar{3}=1\bar{0}$. So $1\bar{0}=(0,2,1)1\bar{2}=(0,3,1)1\bar{3}$ and therefore $1\bar{0}\sim1\bar{2}\sim1\bar{3}$.

Now $N^{(01)}\geq\langle(2,3),(0,2),(0,3)\rangle\cong S_3$ has orbits $\{1\}$, $\{0,2,3\}$ and $\{\bar{1}\}$, $\{\bar{0},\bar{2},\bar{3}\}$. Noting that $[0\bar{1}1]=[0]$ and $[0\bar{1}\,\bar{1}]=[01]$, we have left $[0\bar{1}0]$ and $[0\bar{1}\,\bar{0}]$. But $0\bar{1}\,\bar{0}=001$ (relation 2)$=\bar{0}1$. So the double cosets left to be considered are $[\bar{0}1]$ and $[0\bar{1}0]$.

Let us prove $0\bar{1}0=1\bar{0}1$. We know $010=\bar{1}$ by relation 2. So $0\bar{1}0=\bar{0}1\bar{0}$ since $0\bar{1}0=00100$. Also $\bar{0}1\bar{0}=1\bar{0}1$ since $\bar{1}=010\Rightarrow\bar{1}^{(0,1)}=(010)^{(0,1)}\Rightarrow\bar{0}=101$. The result is proved.

So now consider $[\bar{0}1]$. By relation 3,

$(0,2,1)1\bar{2}=1\bar{0} \Rightarrow (0,2,1)1=1\bar{0}2 \Rightarrow \bar{1}(0,2,1)1=\bar{1}1\bar{0}2 \Rightarrow (0,2,1)\bar{1}^{(0,2,1)}1=\bar{0}2$

$\Rightarrow (0,2,1)\bar{0}1=\bar{0}2$. Now conjugate by $(2,3) \in N$ and we see

$(0,3,1)\bar{0}1=\bar{0}3$. Since $(0,2,1)\bar{0}1=(0,2,1)(0,1,3)\bar{0}3$ we have

$(0,2,1)\bar{0}1=\bar{0}2=(0,2,3)\bar{0}3$. Now $N^{(01)} \geq N^{01} \cong \langle (2,3) \rangle$ and

$N\bar{0}1^{(1,2)}=N\bar{0}2=N\bar{0}1 \Rightarrow (1,2)$ is in the coset stabilizing group.

So $N^{(01)} \geq \langle (2,3),(1,2) \rangle \cong S_3$ has orbits $\{0\}$, $\{1,2,3\}$, $\{\bar{0}\}$, and

$\{\bar{1},\bar{2},\bar{3}\}$. Finally, $\bar{0}10=\bar{0}\bar{0}\bar{1}$ (relation 2) $=0\bar{1} \Rightarrow [\bar{0}10]=[0\bar{1}]$.

Also $\bar{0}11=\bar{0}\bar{1}=10 \in [01]$ (relation 2) $\Rightarrow [01]=[\bar{0}11]$. Next

$\bar{0}1\bar{0}=0\bar{1}0 \Rightarrow [\bar{0}1\bar{0}]=[0\bar{1}0]$ by the previous proof. And $\bar{0}1\bar{1}=\bar{0}$

$\Rightarrow [\bar{0}1\bar{1}]=[\bar{0}]$.

Finally we consider $[0\bar{1}0]$. We know $0\bar{1} \sim 0\bar{2} \sim 0\bar{3} \Rightarrow$

$0\bar{1}0 \sim 0\bar{2}0 \sim 0\bar{3}0$. By conjugating respectively by

$(0,1),(0,2),(0,3) \in N$ we want to show that (a),(b),(c),(d)

given below;

(a) $0\bar{1}0 \sim 0\bar{2}0 \sim 0\bar{3}0$,

(b) $1\bar{0}1 \sim 1\bar{2}1 \sim 1\bar{3}1$,

(c) $2\bar{1}2 \sim 2\bar{0}2 \sim 2\bar{3}2$ and

(d) $3\bar{1}3 \sim 3\bar{2}3 \sim 3\bar{0}3$

satisfy (a) = (b) = (c) = (d). However $0\bar{1}0 = 1\bar{0}1$ by our previous proof $\Rightarrow$ (a) = (b). Conjugation by (0,2) giving $2\bar{1}2 = 1\bar{2}1$ $\Rightarrow$ (b) = (c). Conjugation by (0,3) giving $3\bar{1}3 = 1\bar{3}1$ $\Rightarrow$ (b) = (d). Now $0\bar{1}0 = 2\bar{1}2 = 3\bar{1}3 \Rightarrow (0,1), (0,2), (0,3) \in N^{(01)}$. Therefore $N^{(01)} \geq \langle (0,1), (0,2), (0,3) \rangle \cong S_4$. Since $N^{(01)} \leq S_4$, $N^{(01)} = S_4$.

Now $N^{(01)}$ is transitive on $\{0,1,2,3\}$ and $\{\bar{0},\bar{1},\bar{2},\bar{3}\}$. So we need to consider $[0\bar{1}00]$ and $[0\bar{1}0\bar{0}]$. However, $0\bar{1}\bar{0} = 001$ (by relation 2) $= \bar{0}1 \Rightarrow [0\bar{1}00] = [\bar{0}1]$. Also $0\bar{1}0\bar{0} = 0\bar{1} \Rightarrow [0\bar{1}0\bar{0}] = [0\bar{1}]$. Our double coset enumeration must be completed since the set of right cosets is closed under right multiplication by the symmetric generators.

The coset enumeration is summarized in Appendix B, Table 3 from which Cayley Diagram 2 (Appendix H) is obtained.

Proof of Isomorphism

Since $\frac{|N|}{|N|} + \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(00)}|} + \frac{|N|}{|N^{(01)}|} + \frac{|N|}{|N^{(011)}|} + \frac{|N|}{|N^{(001)}|} + \frac{|N|}{|N^{(0110)}|} = \frac{24}{24} + \frac{24}{6} + \frac{24}{6} + \frac{24}{2} + \frac{24}{6} + \frac{24}{6} + \frac{24}{24} = 30$ (see Appendix B, Table 3), the maximum possible index of N in G is 30. It follows that the order of the image group G is at most

$|N|*30=24*30=720$. In order to show that $|G|$=720, we consider G as a subgroup of S_{30} acting on the 30 symbols that we have found. For this purpose we compute the action of the control group N as well as the action of t on the 30 cosets. These permutations are given in Appendix B, Table 4.

It suffices to show that; 1) $|\langle x,y,t\rangle|=720$, 2) $\langle x,y\rangle \cong S_4$, 3) t has exactly four conjugates under conjugation by N and 4) additional relations hold in $\langle x,y,t\rangle$.

It is verified that $|\langle x,y,t\rangle|=720$. Since the order of xy is 3, $N=\langle x,y\rangle \cong S_4$.

The action of x and y, on the symmetric generators is given by x: (t_0, t_1, t_2, t_3) and y: (t_2, t_3) where $t_0=t$, $t_1=t_0^x$, $t_2=t_0^{x^2}$, and $t_3=t_0^{x^3}$ (see Appendix B, Table 5). Note that this implies $\langle x,y,t\rangle$ is a homomorphic image of $3^{*4}:S_4$.

Verify relation (1) (0,2,1) $t_1t_0t_2t_1t_0 = 1$ by conjugating the four symmetric generators as follows;

$t_0^{t_1t_0t_2t_1t_0} = t_2$, $t_1^{t_1t_0t_2t_1t_0} = t_0$,

$t_2^{t_1t_0t_2t_1t_0} = t_1$, $t_3^{t_1t_0t_2t_1t_0} = t_3$

So $t_1t_0t_2t_1t_0$ acts as the permutation (0,2,1). Note that we have used t_0, t_1, t_2, t_3 in their action on the 30 cosets given in Appendix B, Table 4.

Verify relation (2) $t_1t_0t_1t_0 = 1$ by conjugating the generators as follows;

$$t_0^{t_1t_0t_1t_0} = t_0 , \qquad t_1^{t_1t_0t_1t_0} = t_1 ,$$

$$t_2^{t_1t_0t_1t_0} = t_2 , \qquad t_3^{t_1t_0t_1t_0} = t_3 ,$$

So $t_1t_0t_1t_0$ acts as the identity.

Verify relation (3) $(0,2,1)t_1t_2^{-1}t_0t_1^{-1} = 1$ by conjugating the generators as follows;

$$t_0^{t_1t_2t_2t_0t_1t_1} = t_1 , \qquad t_1^{t_1t_2t_2t_0t_1t_1} = t_2 ,$$

$$t_2^{t_1t_2t_2t_0t_1t_1} = t_0 , \qquad t_3^{t_1t_2t_2t_0t_1t_1} = t_3 ,$$

So $t_1t_2^{-1}t_0t_1^{-1}$ acts as the permutation (0,1,2) on the symmetric generators. Thus $G/_{Ker\phi} \cong \langle x,y,t\rangle \Rightarrow |G| \geq |\langle x,y,t\rangle| = 720$.

The symmetric generators, do not generate our group since our relations allow only the even permutations to be written in terms of the t_is. In fact, $\langle t_0,t_1,t_2,t_3\rangle$ is a subgroup of G of index 2 in G. We know that $S_6 \cong \langle a,b \mid a^6, b^2, (ab)^5, (a^4(ab)^2)^3, (a^4ba^2b)^2\rangle$ and note that $a,b \in G$ (see

Appendix B, Table 6) satisfy this presentation of $S_6 \Rightarrow$

$S_6 \leq G$. However, $|S_6| = 720 = |G| \Rightarrow G \cong S_6$.

CHAPTER THREE

GENERATION OF $S_6 \times 3$ OVER S_4

We consider the infinite semidirect product $3^{*4}:S_4$, of which a symmetric presentation is

$$< x, y, t \mid x^4, y^2, (y*x)^3, t^3, (t,y), (t^x,y) >$$

and factor it by the relation, $t_0^3 = [(012)t_0t_1^{-1}]^2$.

$$G \cong \frac{3^{*4}:S_4}{((0,1,2)t_0t_1^{-1})^2}$$

$\cong < N, T \mid N \cong S_4,\ t_i^{\pi} = t_{\pi(i)},\ t_0^3 = [(012)t_0t_1^{-1}]^2 = 1 >$.

A table in Bray[2] lists the homomorphic image as a group isomorphic to $S_6 \times 3$, the direct product of S_6 and C_3, the cyclic group of order 3. We will first show that $|G|=2160$ and then the stronger statement that $G \cong S_6 \times 3$. The action of the control group $S_4 = <x,y>$, on the symmetric generators is

$$x \sim (0,1,2,3) \text{ and } y \sim (2,3).$$

$((0,1,2)t_0t_1^{-1}))^2$ translates to $((x^{-3}yxyx^2)tt^{-x})^2$.

Expanding $(\pi t_0 t_1^{-1})^2$ where $\pi = (0,1,2)$

$$\pi t_0 t_1^{-1} \pi t_0 t_1^{-1} = \pi^2 (t_0 t_1^{-1})^{\pi} t_0 t_1^{-1} = \pi^2 t_1 t_2^{-1} t_0 t_1^{-1} = 1$$

$$\Rightarrow (0,2,1)\, t_1 t_2^{-1} = t_1 t_0^{-1} \quad \text{(relation 1)}$$

We have found 90 single (right) cosets of N in G using MAGMA. The actions of N and the symmetric generators t_0, t_1, t_2, and t_3 are computed on these cosets. We relabel the ninety cosets sequentially from 1 through 90. (see Appendix C, Table 7).

$$x:(N, Nx = N)(Nt_0, Nt_0^x, Nt_0^{x^2}, Nt_0^{x^3})\ldots$$

The coset enumeration is summarized in Appendix C, Table 8 from which Cayley Diagram 3 (Appendix H) is obtained.

Proof of Isomorphism

Since $\frac{|N|}{|N|} + \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(00)}|} + \frac{|N|}{|N^{(01)}|} + \frac{|N|}{|N^{(011)}|} + \frac{|N|}{|N^{(001)}|} +$

$$\frac{|N|}{|N^{(0011)}|} + \frac{|N|}{|N^{(010)}|} + \frac{|N|}{|N^{(0100)}|} + \frac{|N|}{|N^{(012)}|} + \frac{|N|}{|N^{(0110)}|} + \frac{|N|}{|N^{(01100)}|} + \frac{|N|}{|N^{(0010)}|} +$$

$$\frac{|N|}{|N^{(00100)}|} + \frac{|N|}{|N^{(00110)}|} + \frac{|N|}{|N^{(001100)}|} + \frac{|N|}{|N^{(0101)}|} + \frac{|N|}{|N^{(01011)}|} + \frac{|N|}{|N^{(010022)}|} +$$

$$\frac{|N|}{|N^{(00110011)}|} + \frac{|N|}{|N^{(0121)}|}$$

$$= \frac{24}{24} + \frac{24}{6} + \frac{24}{6} + \frac{24}{2} + \frac{24}{6} + \frac{24}{6} + \frac{24}{2} + \frac{24}{6} + \frac{24}{6} + \frac{24}{2} + \frac{24}{24} + \frac{24}{6} + \frac{24}{6} + \frac{24}{24} + \frac{24}{6} + \frac{24}{6} + \frac{24}{24}$$

$+ \frac{24}{6} + \frac{24}{24} + \frac{24}{24} + \frac{24}{6}$ =90 (see Appendix C, Table 8), the maximum possible index of N in G is 90. It follows that the order of the image group G is at most $|N|*90 = 24*90 = 2160$. In order to show that $|G|$=2160, we consider G as a subgroup of S_{90} acting on the 90 symbols that we have found. For this purpose we examine the action of the control group N as well as the action of t on the 90 cosets. These permutations are given in Appendix C, Table 9.

It suffices to show that; 1) $|\langle x,y,t\rangle| = 2160$, 2) $\langle x,y\rangle \cong S_4$, 3) t has exactly four conjugates under conjugation by N and 4) additional relations hold in $\langle x,y,t\rangle$.

It is verified that $|\langle x,y,t\rangle| = 2160$. Since the order of xy is 3, $N = \langle x,y\rangle \cong S_4$.

The action of x and y, on the symmetric generators is given by x:(t_0, t_1, t_2, t_3) and y:(t_2, t_3) where $t_0 = t$, $t_1 = t_0^x$, $t_2 = t_0^{x^2}$, and $t_3 = t_0^{x^3}$ (see Appendix C, Table 10). Note that this implies $\langle x,y,t\rangle$ is a homomorphic image of $3^{*4}:S_4$.

Verify relation (1) $t_1t_2t_2t_0t_1t_1$ = (0,1,2) by conjugating the four symmetric generators as follows;

$t_0^{t_1t_2t_2t_0t_1t_1} = t_1$, $t_1^{t_1t_2t_2t_0t_1t_1} = t_2$,

$t_2^{t_1t_2t_2t_0t_1t_1} = t_0$, $t_3^{t_1t_2t_2t_0t_1t_1} = t_3$, So $t_1t_2t_2t_0t_1t_1$ acts as the permutation (0,1,2). Thus

$G/_{Ker\phi} \cong \langle x,y,t \rangle \Rightarrow |G| \geq |\langle x,y,t \rangle| = 2160$.

The symmetric generators, do not generate our group since our relations allow only the even permutations to be written in terms of the t_is. In fact, $\langle t_0,t_1,t_2,t_3 \rangle$ is a subgroup of G of index 2 in G. We know that

$S_6 \times 3 \cong \langle a,b,c \mid a^6,b^2,(ab)^5,(a^4(ab)^2)^3,(a^4ba^2b)^2,(a,c),(b,c) \rangle$ and note that $a,b,c \in G$ (see Appendix C, Table 11) satisfy this presentation of $S_6 \times 3 \Rightarrow S_6 \times 3 \leq G$. However, $|S_6 \times 3| = 2160 = |G| \Rightarrow$

$G \cong S_6 \times 3$.

CHAPTER FOUR

GENERATION OF $L_3(3)$ OVER S_4

The general linear group $GL_n(q)$ is the group of all $n \times n$ non-singular matrices over the field F_q, q being a prime power. The special linear group $SL_n(q)$ is the normal subgroup of $GL_n(q)$, consisting of matrices of determinant 1. The center of either of these groups consists of scalar matrices over F_q. We obtain the corresponding projective groups $PGL_n(q)$ and $PSL_n(q)$ by factoring out these centers. Artin's abbreviation for $PSL_n(q)$ is $L_n(q)$. Moreover, $L_n(q)$ is simple for $n \geq 2$, except for the two cases n=2 and q=2 or 3. Thus $L_3(3)$ is a simple group.

We consider the infinite semidirect product $3^{*4}:S_4$, of which a symmetric presentation is

$$< x,y,t \mid x^4,\ y^2,\ (y*x)^3,\ t^3,\ (t,y),\ (t^x,y) >$$

and factor it by the relation, $[t_0t_1]^2=(2,3)$.

$$G \cong \frac{3^{*4}:S_4}{(t_0t_1)^2=(2,3)}$$

$$\cong < N, T \mid N \cong S_4,\ t_i^{\pi}=t_{\pi(i)},\ t_0^3=1,\ [t_0t_1]^2=(2,3) >$$

A table in Bray[2] lists the homomorphic image as a group isomorphic to $L_3(3)$, the projective special linear group of 3×3 matrices over the field F_3. We will first show that $|G|$=5616 and then the stronger statement that $G \cong L_3(3)$. The action of the control group S_4 = <x,y>, on the symmetric generators is

$$x\sim(0,1,2,3) \text{ and } y\sim(2,3).$$

$\pi(t_0t_1)^2$ translates to $((yx^{-3}y^2x^3)(tt^x)^2$.

Expanding $(t_0t_1)^2$ where $\pi=(3,2)=(2,3)^{-1}$

$$\pi t_0 t_1 t_0 t_1 = 1$$

$$\Rightarrow \quad (2,3)\, t_0 t_1 t_0 = t_1^{-1} \quad \text{(relation 1)}$$

We have found 234 single (right) cosets of N in G using MAGMA. The actions of N and the symmetric generators t_0, t_1, t_2, and t_3 are computed on these cosets. We relabel the 234 cosets sequentially from 1 through 234 (see Appendix D, Table 12).

$x:(N, Nx = N)(Nt_0, Nt_0^x, Nt_0^{x^2}, Nt_0^{x^3})\ldots$

The coset enumeration is summarized in Appendix D, Table 13 from which Cayley Diagram 4 (Appendix H) is obtained.

Proof of Isomorphism

Since $\frac{|N|}{|N|}$ + $\frac{|N|}{|N^{(0)}|}$ + $\frac{|N|}{|N^{(00)}|}$ + $\frac{|N|}{|N^{(01)}|}$ + $\frac{|N|}{|N^{(011)}|}$ + $\frac{|N|}{|N^{(001)}|}$ +

$\frac{|N|}{|N^{(012)}|}$ + $\frac{|N|}{|N^{(0122)}|}$ + $\frac{|N|}{|N^{(0110)}|}$ + $\frac{|N|}{|N^{(0112)}|}$ + $\frac{|N|}{|N^{(00122)}|}$ + $\frac{|N|}{|N^{(01200)}|}$ + $\frac{|N|}{|N^{(0123)}|}$ +

$\frac{|N|}{|N^{(0122)}|}$ + $\frac{|N|}{|N^{(0110)}|}$ + $\frac{|N|}{|N^{(0112)}|}$ + $\frac{|N|}{|N^{(00122)}|}$ + $\frac{|N|}{|N^{(01200)}|}$ + $\frac{|N|}{|N^{(0123)}|}$ + $\frac{|N|}{|N^{(01233)}|}$

+ $\frac{|N|}{|N^{(01221)}|}$ + $\frac{|N|}{|N^{(01223)}|}$ + $\frac{|N|}{|N^{(011233)}|}$ + $\frac{|N|}{|N^{(012332)}|}$

$$=\frac{24}{2}+\frac{24}{2}+\frac{24}{2}+\frac{24}{2}+\frac{24}{8}+\frac{24}{1}+\frac{24}{4}+\frac{24}{1}+\frac{24}{1}+\frac{24}{2}+\frac{24}{1}+\frac{24}{1}+\frac{24}{4}+\frac{24}{1}+\frac{24}{1}+\frac{24}{2}+\frac{24}{1}$$

$+\frac{24}{24}+\frac{24}{6}+\frac{24}{6}+\frac{24}{2}+\frac{24}{2}+\frac{24}{2}+\frac{24}{1}$ =234 (see Appendix D, Table 13),

the maximum possible index of N in G is 234. It follows that the order of the image group G is at most $|N|*234=24*234=5616$. In order to show that $|G|$=5616, we consider G as a subgroup of S_{234} acting on the 234 symbols that we have found. For this purpose we examine the action of the control group N as well as the action of t on the 234 cosets. These permutations are given in Appendix D, Table 14.

It suffices to show that; 1) $|\langle x,y,t\rangle|=5616$, 2) $\langle x,y\rangle \cong S_4$, 3) t has exactly four conjugates under conjugation by N and 4) additional relations hold in $\langle x,y,t\rangle$.

It is verified that $|\langle x,y,t\rangle|=5616$. Since the order of xy is 3, $N=\langle x,y\rangle \cong S_4$.

The action of x and y, on the symmetric generators is given by x: (t_0, t_1, t_2, t_3) and y: (t_2, t_3) where $t_0=t$, $t_1=t_0^x$, $t_1=t_0^{x^2}$, and $t_1=t_0^{x^3}$ (see Appendix D, Table 15). Note that this implies $\langle x,y,t\rangle$ is a homomorphic image of $3^{*4}:S_4$.

Verify relation 1; $(t_0t_1)^2$ = (2,3) by conjugating the four symmetric generators as follows;

$$t_0^{t_0t_1t_0t_1}=t_0\ ,\qquad t_1^{t_0t_1t_0t_1}=t_1\ ,$$

$$t_2^{t_0t_1t_0t_1}=t_3\ ,\qquad t_3^{t_0t_1t_0t_1}=t_2\ ,$$

So $t_0t_1t_0t_1$ acts as the permutation (2,3). Thus

$$G/Ker\phi \cong \langle x,y,t\rangle \Rightarrow |G| \geq |\langle x,y,t\rangle| = 5616.$$

The symmetric generators, generate $G=\langle t_0,t_1,t_2,t_3\rangle$. A well known presentation for $L_3(3)$ is

$L_3(3)\cong \langle a,b\ |a^6,b^2,(ab)^{13},(a^{-1}b^{-1}ab)^6,(ababab^{-1}ab^{-1})^{13}\rangle$ (see Wilson [9]),

and we note that a,b$\in G$ (see Appendix D, Table 16) satisfy this presentation of $L_3(3) \Rightarrow L_3(3) \leq G$. However, $|L_3(3)| = 5616 = |G| \Rightarrow G \cong L_3(3)$.

CHAPTER FIVE

GENERATION OF $U_3(3)$ OVER S_4

The special unitary group $SU_n(q)$ is the set of $n \times n$ matrices with determinant 1 over F_{q^2}. The projective special unitary group $PSU_n(q)$ is the group obtained from $SU_n(q)$ on factoring by the center, scalar unitary matrices. $PSU_n(q)$ is denoted by $U_n(q)$. Moreover, $U_n(q)$ is simple except for $U_2(2)$, $U_2(3)$, and $U_3(2)$. Thus $U_3(3)$ is a simple group.

We consider the infinite semidirect product $3^{*4}:S_4$, of which a symmetric presentation is

$< x,y,t \mid x^4,\ y^2,\ (y*x)^3,\ t^3,\ (t,y),\ (t^x,y) >$

and factor it by the relations,

$((0,1,2,3)t_0)^7=1,$ and $(t_0^{-1}t_1)^2=(2,3)$

$$G \cong \frac{3^{*4}:S_4}{((0,1,2,3)t_0)^7,\ \ (t_0^{-1}t_1)^2=(2,3)}$$

$\cong <N, T \mid N \cong S_4,\ t_i^{\pi}=t_{\pi(i)},\ t_0^3=1,\ ((0,1,2,3)t_0)^7,\ [t_0^{-1}t_1]^2=(2,3)>$

A table in Bray[2] lists the homomorphic image as a group isomorphic to the unitary group $U_3(3)$. $U_3(3)$ is the derived Chevalley group $G_2(2)$.

The action of the control group $S_4 = \langle x,y \rangle$, on the symmetric generators is

$$x \sim (0,1,2,3) \text{ and } y \sim (2,3).$$

$(\pi t_0)^7$ translates to $(yx^{-3}yxyx^2t)^7$.

$\pi^{-1}(t_0^{-1}t_1)^2$ translates to $((yx^{-3}y^2x^3)(t^{-1}t^x)^2$.

Expanding $(\pi t_0)^7$ where $\pi=(0,1,2,3)$, we have

$$\pi t_0\pi t_0\pi t_0\pi t_0\pi t_0\pi t_0\pi t_0 = \pi^7 t_0^{\pi^6} t_0^{\pi^5} t_0^{\pi^4} t_0^{\pi^3} t_0^{\pi^2} t_0^{\pi} t_0 = \pi^7 t_2 t_1 t_0 t_3 t_2 t_1 t_0 = 1$$

$$\Rightarrow (0,3,2,1)\, t_2 t_1 t_0 = t_0^{-1} t_1^{-1} t_2^{-1} t_3^{-1} \quad \text{(relation 1)}$$

Expanding $\pi^{-1}(t_0^{-1}t_1)^2$ where $\pi = (3,2) = (2,3)^{-1}$

$$\pi t_0^{-1} t_1 t_0^{-1} t_1 = 1$$

$$\Rightarrow (2,3)\, t_0^{-1} t_1 = t_1^{-1} t_0 \quad \text{(relation 2)}$$

We have found 252 single (right) cosets of N in G using MAGMA. The actions of N and the symmetric generators t_0, t_1, t_2, and t_3 are computed on these cosets. We relabel the 252 cosets sequentially from 1 through 252 (see Appendix E, Table 17).

$$x:(N, Nx = N)(Nt_0, Nt_0^x, Nt_0^{x^2}, Nt_0^{x^3})\ldots$$

The coset enumeration is summarized in Appendix E, Table 18 from which Cayley Diagram 5 (Appendix H) is obtained.

Proof of Isomorphism

Since $\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(00)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(011)}|}+\frac{|N|}{|N^{(001)}|}+$

$\frac{|N|}{|N^{(0011)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(012)}|}+\frac{|N|}{|N^{(0122)}|}+\frac{|N|}{|N^{(0112)}|}+\frac{|N|}{|N^{(01122)}|}+$

$\frac{|N|}{|N^{(0012)}|}+\frac{|N|}{|N^{(00122)}|}+\frac{|N|}{|N^{(00112)}|}+\frac{|N|}{|N^{(001122)}|}+\frac{|N|}{|N^{(0102)}|}+\frac{|N|}{|N^{(01022)}|}+$

$\frac{|N|}{|N^{(0120)}|}+\frac{|N|}{|N^{(0121)}|}+\frac{|N|}{|N^{(01233)}|}+\frac{|N|}{|N^{(011223)}|}+\frac{|N|}{|N^{(0112233)}|}+\frac{|N|}{|N^{(00120)}|}+$

$\frac{|N|}{|N^{(001233)}|}+\frac{|N|}{|N^{(01213)}|}$

$=\frac{24}{24}+\frac{24}{6}+\frac{24}{6}+\frac{24}{2}+\frac{24}{4}+\frac{24}{4}+\frac{24}{2}+\frac{24}{1}+\frac{24}{1}+\frac{24}{2}+\frac{24}{8}+\frac{24}{2}+\frac{24}{2}+\frac{24}{8}+\frac{24}{2}+$

$\frac{24}{1}+\frac{24}{2}+\frac{24}{2}+\frac{24}{1}+\frac{24}{2}+\frac{24}{4}+\frac{24}{4}+\frac{24}{4}+\frac{24}{2}+\frac{24}{4}+\frac{24}{8}$ =252 (see Appendix E, Table 18), the maximum possible index of N in G is 252. It follows that the order of the image group G is at most $|N|*252=24*252=6048$. In order to show that $|G|$=6048, we consider G as a subgroup of S_{252} acting on the 252 symbols that we have found. For this purpose we examine the action of the control group N as well as the action of t on the 252 cosets. These permutations are given in Appendix E, Table 19.

It suffices to show that; 1) $|\langle x,y,t\rangle| = 6048$, 2) $\langle x,y\rangle \cong S_4$, 3) t has exactly four conjugates under conjugation by N and 4) additional relations hold in $\langle x,y,t\rangle$.

It is verified that $|\langle x,y,t\rangle| = 6048$. Since the order of xy is 3, $N = \langle x,y\rangle \cong S_4$.

The action of x and y, on the symmetric generators is given by x: (t_0, t_1, t_2, t_3) and y: (t_2, t_3) where $t_0 = t$, $t_1 = t_0^x$, $t_1 = t_0^{x^2}$, and $t_3 = t_0^{x^3}$ (see Appendix E, Table 20). Note that this implies $\langle x,y,t\rangle$ is a homomorphic image of $3^{*4} : S_4$.

Verify relation (1), $t_2t_1t_0t_3t_2t_1t_0 = (1,2,3,0)$ by conjugating the four symmetric generators as follows;

$$t_0^{t_2t_1t_0t_3t_2t_1t_0} = t_1, \qquad t_1^{t_2t_1t_0t_3t_2t_1t_0} = t_2,$$

$$t_2^{t_2t_1t_0t_3t_2t_1t_0} = t_3, \qquad t_3^{t_2t_1t_0t_3t_2t_1t_0} = t_0.$$

So $t_2t_1t_0t_3t_2t_1t_0$ acts as the permutation (1,2,3,0).

Verify relation (2), $(t_0^{-1}t_1)^2 = (2,3)$ by conjugating the four symmetric generators as follows;

$$t_0^{t_0t_0t_1t_0t_0t_1} = t_0, \qquad t_1^{t_0t_0t_1t_0t_0t_1} = t_1,$$

$$t_2^{t_0t_0t_1t_0t_0t_1} = t_3, \qquad t_3^{t_0t_0t_1t_0t_0t_1} = t_2.$$

So $t_0t_0t_1t_0t_0t_1$ acts as the permutation (2,3). Thus

$G/_{Ker\phi} \cong \langle x,y,t\rangle \Rightarrow |G| \geq |\langle x,y,t\rangle| = 6048$.

The symmetric generators generate $G = \langle t_0, t_1, t_2, t_3 \rangle$. A well known presentation for $U_3(3)$ is

$U_3(3) \cong \langle a,b \mid a^2, b^6, (ab)^7, a^{-1}(ab^2)^{-3}a(ab^2)^3, b^3(b^{-2}(ab^3a)^{-1}b^2ab^3a)^2 \rangle$ (see Wilson [9]), and we note that $a, b \in G$ (see Appendix E, Table 21) satisfy this presentation of $U_3(3) \Rightarrow U_3(3) \leq G$. However, $|U_3(3)| = 6048 = |G| \Rightarrow G \cong U_3(3)$.

CHAPTER SIX

GENERATION OF G OVER S_5

We consider another infinite semidirect product, still called a progenitor, $2^{*5}:S_5$, of which a symmetric presentation is

$< x,y,t | x^5, y^2, (xy)^4, (x,y)^3, t^2, (t,y), (t^x,y), (t^{xx},y) >$

and factor it by the relations,

$$(t_0t_1)^4=1, \text{ and } ((0,1,2)(3,4)t_0t_2t_1)^2=t_0$$

$$G \cong \frac{2^{*5}:S_5}{(t_0t_1)^4, (0,1,2)(3,4)(t_0t_2t_1)^2=t_0}$$

$\cong < N, T \mid N \cong S_5, t_i^{\pi}=t_{\pi(i)}, t_0^2=1, (t_0t_1)^4,$

$(0,1,2)(3,4)(t_0t_2t_1)^2=t_0 >$.

The isomorphism of this group has not been identified. The action of the control group $S_5 = <x,y>$, on the symmetric generators is

$$x \sim (0,1,2,3,4) \text{ and } y \sim (3,4).$$

Expanding $(t_0 t_1)^4$

$$t_0t_1t_0t_1t_0t_1t_0t_1=1 \Rightarrow t_0t_1t_0t_1 = t_1t_0t_1t_0 \quad \text{(relation 1)}$$

Expanding $\pi(t_0t_2t_1)^2 = t_0$ where $\pi = (0,1,2)(3,4)$

$$\pi t_0t_2t_1t_0t_2t_1 = t_0 \Rightarrow (0,1,2)(3,4)t_0t_2t_1t_0 = t_0t_1t_2 \quad \text{(relation 2)}$$

We have found 980 single (right) cosets of N in G using MAGMA. The actions of N and the symmetric generators t_0, t_1, t_2, t_3, and t_4 are computed on these cosets. We relabel the 980 cosets sequentially from 1 through 980 (see Appendix F, Table 22).

$x : (N, Nx = N)(Nt_0, Nt_0^x, Nt_0^{x^2}, Nt_0^{x^3}, Nt_0^{x^4})\ldots$

The coset enumeration is summarized in Appendix F, Table 23 from which Cayley Diagram 6 (Appendix H) is obtained.

Proof of Isomorphism

Cayley Diagram 6 shows that the maximum possible index of N in G is 980. It follows that the order of the image group G is at most $|N| * 980 = 120 * 980 = 117600$. In order to show that $|G|$=117600, we consider G as a subgroup of S_{980} acting on the 980 symbols that we have found. For this purpose we examine the action of the control group N as well as the action of t on the 980 cosets. These actions can be reproduced in MAGMA.

It suffices to show that; 1) $|\langle x, y, t\rangle| = 117600$, 2) $\langle x, y\rangle \cong S_5$, 3) t has exactly five conjugates under conjugation by N and 4) additional relations hold in $\langle x, y, t\rangle$.

It is verified that $|\langle x,y,t\rangle| = 117600$. Since the order of xy is 4, $N = \langle x,y\rangle \cong S_5$

The action of x and y, on the symmetric generators is given by x: $(t_0, t_1, t_2, t_3, t_4)$ and y: (t_3, t_4) where $t_0 = t$, $t_1 = t_0^x$, $t_2 = t_0^{x^2}$, $t_3 = t_0^{x^3}$ and $t_4 = t_0^{x^4}$. Note that this implies $\langle x,y,t\rangle$ is a homomorphic image of $2^{*5} : S_5$.

Verify relation (1) $(t_0t_1)^4 = 1$ by conjugating the four symmetric generators as follows;

$t_0^{t_0t_1t_0t_1t_0t_1t_0t_1} = t_0$, $t_1^{t_0t_1t_0t_1t_0t_1t_0t_1} = t_1$,

$t_2^{t_0t_1t_0t_1t_0t_1t_0t_1} = t_2$, $t_3^{t_0t_1t_0t_1t_0t_1t_0t_1} = t_3$,

$t_4^{t_0t_1t_0t_1t_0t_1t_0t_1} = t_4$.

So $t_0t_1t_0t_1t_0t_1t_0t_1$ acts as the identity.

Verify relation (2) $((0,1,2)(3,4))t_0t_2t_1t_0t_2t_1t_0 = 1$ by conjugating the four symmetric generators as follows;

$t_0^{t_0t_2t_1t_0t_2t_1t_0} = t_2$, $t_1^{t_0t_2t_1t_0t_2t_1t_0} = t_0$,

$t_2^{t_0t_2t_1t_0t_2t_1t_0} = t_1$, $t_3^{t_0t_2t_1t_0t_2t_1t_0} = t_4$,

$t_4^{t_0t_2t_1t_0t_2t_1t_0} = t_3$.

So $t_0t_2t_1t_0t_2t_1t_0$ acts as the permutation (0,2,1)(3,4). Thus $G/_{Ker\phi} \cong \langle x,y,t\rangle \Rightarrow |G| \geq |\langle x,y,t\rangle| = 117600$.

The symmetric generators generate $G = \langle t_0, t_1, t_2, t_3, t_4 \rangle$.

APPENDIX A

TABLES FOR INTRODUCTION

Table 1
The Double Cosets $[w] = NwN$ in $L_2(19)$ where $N = L_2(5)$

Label [w]	Coset stabilizing subgroup $N^{(w)}$	Cosets (Total 57)
[*]	Since N is transitive on T	1
[∞]	$N^{(\infty)} \geq N^{\infty} = \langle x, (1,3)(0,4) \rangle \cong D_{10}$, has orbits $\{\infty\}$ and $\{0,1,2,3,4\}$ on T.	6
[∞0]	$N^{(\infty 0)} \geq N^{\infty 0} = \langle (1,4)(2,3) \rangle \cong C_2$, has orbits $\{0\},\{\infty\}$, $\{1,4\}$ and $\{2,3\}$.	30
[∞00] = [∞]	∞0∞ = ∞01.1∞ ~ 124.1∞ = 12.41∞41.14 = 12.(1,4,,∞)(2,3,0).14 ~ 4314 ~ ∞044~∞0.	
[∞0∞] = [∞0]	and ∞02∞0 = (1,3,4)(2,0,∞) ⇒ ∞02 ~ 0∞	
[∞02] = [∞0]	∞01 = ∞03.31 ~ 0∞31 ~ 0(1,3,∞)(2,0,4)3∞ ~ 43∞. Thus ∞01 ~ 43∞ ~ 1234	
[∞01]	$N^{(\infty 01)} \geq \langle (1,4,\infty)(2,3,0) \rangle \cong C_3$, has orbits $\{1,4,\infty\}$ and $\{2,3,0\}$ on T. Now ∞012 ~ ∞.(1,0,2)(3,4,∞)10 ~ 310.	20
[∞012] = [∞01]		
[∞011] = [∞0]		

APPENDIX B

TABLES FOR GENERATION OF S_6 OVER S_4

Table 2
Coset Array for S6 Over S_4
Label the single cosets in each double coset

```
[*]
cst[  1]: Identity

[0]
cst[  2]:  0
cst[  3]:  1
cst[  5]:  3
cst[  4]:  2
```

$[0^{-1}]$

```
cst[  6]:  00
cst[ 11]:  11
cst[ 21]:  33
cst[ 16]:  22

[01]
cst[ 10]: 10
cst[ 18]: 30
cst[  7]: 01
cst[  9]: 03
cst[ 14]: 20
cst[ 15]: 21
cst[ 17]: 23
cst[  8]: 02
cst[ 12]: 12
cst[ 19]: 31
cst[ 13]: 13
cst[ 20]: 32
```

$[01^{-1}]$

```
cst[ 24]: 100
cst[ 28]: 300
cst[ 26]: 200
cst[ 23]: 033
```

$[0^{-1}1]$

```
cst[ 25]: 110
cst[ 29]: 330
cst[ 27]: 220
cst[ 22]: 003
```

$[01^{-1}0]$

```
cst[ 30]: 0330
```

Table 3

The Double Cosets [w] = NwN in S_6 where N = S_4

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 30)
[*]	N is transitive on {0,1,2,3}.		1
	$N^{(0)} = N^{0} \cong S_3$ has orbits {0},{1,2,3}		
[0]	Nt_0, Nt_1, Nt_2, Nt_3		4
	$00^{-1}=*$		
	$00=0^{-1}$		
$[0^{-1}]$	$Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}, Nt_3^{-1}$		4
	$0^{-1}0=*$		
	$0^{-1}0^{-1}=0$		
	$0010=10$	$\Rightarrow [0^{-1}10]=[01]$	
	$N^{(01)} = N^{01} \cong S_2$ has orbits {0},{1},{2,3}		
[01]	$010=11$	$\Rightarrow [010]=[0^{-1}]$	12
	$0100=110$	$\Rightarrow [010^{-1}]=[0^{-1}1]$	
	$013=(031)01$	$\Rightarrow [012]=[01]$	
	$0133=(013)01$	$\Rightarrow [012^{-1}]=[01]$	
$[01^{-1}]$	$011\sim022\sim033$		4
	$03300=003$	$\Rightarrow [01^{-1}0^{-1}]=[0^{-1}1]$	
	$0332=(0,3,2)0$	$\Rightarrow [01^{-1}2]=[0]$	
	$02233=(0,2,3)03$	$\Rightarrow [01^{-1}2^{-1}]=[01]$	
$[0^{-1}1]$	$001\sim002\sim003$		4
	$0030=033$	$\Rightarrow [0^{-1}10]=[01^{-1}]$	
	$00300=0330$	$\Rightarrow [0^{-1}10^{-1}]=[01^{-1}0]$	
	$0011=10$	$\Rightarrow [0^{-1}1^{-1}]=[01]$	
	$0023=(0,2,3)30$	$\Rightarrow [0^{-1}12]=[01]$	
	$00322=(0,3,2)00$	$\Rightarrow [0^{-1}12^{-1}]=[0^{-1}]$	
$[01^{-1}0]$	**0110=1001**		1
	$0110\sim1221\sim1331\sim2112\sim2332\sim2002\sim 3113\sim3223\sim3003\sim0220\sim0330$		
	$03300=003$	$\Rightarrow [01^{-1}0^{-1}]=[0^{-1}1]$	
	$01101=110$	$\Rightarrow [01^{-1}01]=[0^{-1}0]$	
	$011011=100$	$\Rightarrow [01^{-1}01^{-1}]=[01^{-1}]$	

Table 4
Action of x, y, and t on the Thirty Cosets

x	(2,3,4,5)(6,11,16,21)(10,15,20,9)(24,26,28,23)(18,7,12,17)(25,27,29,22)(14,19,8,13)
y	(5,4)(21,16)(18,14)(28,26)(29,27)(9,8)(15,19)(17,20)(12,13)
t	(1,2,6)(3,10,24)(5,18,28)(11,25,7)(21,29,9)(4,14,26)(16,27,8)(23,30,22)

Table 5

The Symmetric Generators of S_6 Over S_4

t_0	(1, 2, 6)(3, 10, 24)(5, 18, 28)(11, 25, 7)(21, 29, 9)(4, 14, 26)(16, 27, 8)(23, 30, 22)
t_1	(1, 3, 11)(2, 7, 23)(6, 22, 10)(5, 19, 28)(21, 29, 13)(4, 15, 26)(24, 30, 25)(16, 27, 12)
t_2	(1, 4, 16)(2, 8, 23)(6, 22, 14)(3, 12, 24)(5, 20, 28)(11, 25, 15)(21, 29, 17)(26, 30, 27)
t_3	(1, 5, 21)(2, 9, 23)(6, 22, 18)(3, 13, 24)(11, 25, 19)(4, 17, 26)(28, 30, 29)(16, 27, 20)

Table 6

a,b∈G Satisfying Relations for S_6

a	(1, 16, 25)(2, 30, 5)(3, 12, 17)(4, 8)(6, 27, 15, 13, 28, 24)(7, 11, 21)(9, 19, 29, 18, 10, 22)(14, 23)(20, 26)
b	(1, 22)(2, 26)(3, 28)(4, 24)(5, 13)(6, 11)(7, 12)(8, 23)(9, 15)(10, 17)(14, 18)(16, 19)(20, 25)(21, 29)(27, 30)

APPENDIX C

TABLES FOR GENERATION OF $S_6 \times 3$ OVER S_4

Table 7
Coset Array for S6x3 Over S_4
Label the single cosets in each double coset

[*]
cst[1]: Identity

[0]
cst[2]: 0
cst[4]: 1
cst[5]: 3
cst[8]: 2

$[0^{-1}]$
cst[3]: 00
cst[6]: 11
cst[7]: 33
cst[13]: 22

[01]
cst[9]: 10
cst[11]: 30
cst[18]: 20
cst[20]: 21
cst[21]: 03
cst[23]: 01
cst[24]: 23
cst[32]: 31
cst[33]: 13
cst[34]: 32
cst[37]: 02
cst[42]: 12

$[01^{-1}]$
cst[10]: 100
cst[12]: 300
cst[19]: 200
cst[22]: 033

$[0^{-1}1]$
cst[14]: 110
cst[16]: 330
cst[25]: 220
cst[27]: 003

$[0^{-1}1^{-1}]$
cst[15]: 1100
cst[17]: 3300
cst[26]: 2200
cst[28]: 2211
cst[29]: 0033
cst[30]: 0011
cst[31]: 2233
cst[36]: 1122
cst[46]: 3322
cst[47]: 3311
cst[48]: 1133
cst[52]: 0022

[010]
cst[38]: 030
cst[43]: 010
cst[61]: 020
cst[63]: 303

$[010^{-1}]$
cst[39]: 0300
cst[44]: 0100
cst[62]: 0200
cst[64]: 3033

[012]
cst[35]: 210
cst[45]: 230
cst[51]: 120
cst[57]: 320
cst[58]: 310
cst[59]: 130
cst[60]: 103
cst[67]: 302
cst[70]: 013
cst[75]: 032
cst[76]: 023
cst[77]: 203

$[01^{-1}0]$
cst[40]: 0330

$[01^{-1}0^{-1}]$
cst[41]: 03300
cst[65]: 10011
cst[66]: 30033
cst[82]: 20022

$[0^{-1}10]$
cst[49]: 0030
cst[68]: 1101
cst[69]: 3303
cst[86]: 2202

$[0^{-1}10^{-1}]$
cst[50]: 00300

$[0^{-1}1^{-1}0]$
cst[53]: 00330
cst[55]: 00110
cst[71]: 00220
cst[73]: 33003

$[0^{-1}1^{-1}0^{-1}]$
cst[54]: 003300
cst[56]: 001100
cst[72]: 002200
cst[74]: 330033

[0101]
cst[79]: 0303

$[0101^{-1}]$
cst[78]: 01011
cst[80]: 30300
cst[85]: 03033
cst[90]: 02022

$[010^{-1}1^{-1}]$
cst[81]: 030033

[0120]
cst[83]: 0130
cst[84]: 0320
cst[87]: 3023
cst[89]: 0230

$[0^{-1}1^{-1}0^{-1}1^{-1}]$
cst[88]: 00330033

Table 8

The Double Cosets $[w] = NwN$ in $S_6 \times 3$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 90)
[*]	N is transitive on {0,1,2,3}.		1
	$N^{(0)} = N^0 \cong S_3$ has orbits {0},{1,2,3}		
[0]	Nt_0, Nt_1, Nt_2, Nt_3		4
	$00^{-1}=*$		
	$00=0^{-1}$		
$[0^{-1}]$	$Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}, Nt_3^{-1}$		4
	$0^{-1}0=*$		
	$0^{-1}0^{-1}=0$		
	$N^{(01)} = N^{01} \cong S_2$ has orbits {0},{1},{2,3}		
[01]	$0133=(0,1,3)1100^{(0,1)}$	$\Rightarrow [012^{-1}]=[0^{-1}1^{-1}]$	12
$[01^{-1}]$	011~022~033		4
	0332=(0,3,2)0	$\Rightarrow [01^{-1}2]=[0]$	
	02233=(0,2,3)03	$\Rightarrow [01^{-1}2^{-1}]=[01]$	
$[0^{-1}1]$	001~002~003		4
	0023=(0,2,3)0033	$\Rightarrow [0^{-1}12]=[0^{-1}1^{-1}]$	
	00322=(0,3,2)00	$\Rightarrow [0^{-1}12^{-1}]=[0^{-1}]$	
$[0^{-1}1^{-1}]$	00113=(0,1,3)10	$\Rightarrow [0^{-1}1^{-1}2]=[01]$	12
	001133=(0,1,3)103	$\Rightarrow [0^{-1}1^{-1}2^{-1}]=[012]$	
[010]	**010=212=313**		4
	$0103=0100^{(0,3)}$	$\Rightarrow [0102]=[010^{-1}]$	
	01022=21	$\Rightarrow [0102^{-1}]=[01]$	
$[010^{-1}]$	0100~2122~3133		4
	01001=00110	$\Rightarrow [010^{-1}1]=[0^{-1}1^{-1}0]$	
	01002=(0,2,1)21	$\Rightarrow [010^{-1}2]=[01]$	
	010033=(0,3,1)010	$\Rightarrow [010^{-1}2^{-1}]=[010]$	
[012]	012~013		12
	01300=(0,3,1)10011	$\Rightarrow [0120^{-1}]=[01^{-1}0^{-1}]$	
	$0323=0030^{(3,2)}$	$\Rightarrow [0121]=[0^{-1}10]$	
	03233=(0,2,3)30300	$\Rightarrow [0121^{-1}]=[0101^{-1}]$	
	0133=(0,1,3)1100	$\Rightarrow [012^{-1}]=[0^{-1}1^{-1}]$	
	01322=(0,3,2)01	$\Rightarrow [0123^{-1}]=[01]$	

Table 8 (continued)

The Double Cosets [w] = NwN in $S_6 \times 3$ where N = S_4

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 90)
$[01^{-1}0]$	**0110=1001**		
	0110~1221~1331~2112~2332~2002~ 3113~3223~3003~0220~0330		1
	01101=10011	$\Rightarrow [01^{-1}01]=[01^{-1}0^{-1}]$	
	011011=100	$\Rightarrow [01^{-1}01^{-1}]=[01^{-1}]$	
$[01^{-1}0^{-1}]$	01100~02200~03300		4
	011001=(0,3,1)103	$\Rightarrow [01^{-1}0^{-1}1]=[012]$	
	0330033=(0,1,2)3023$^{(1,3,2)}$	$\Rightarrow [01^{-1}0^{-1}1^{-1}]=[0120]$	
	0330022=3023	$\Rightarrow [01^{-1}0^{-1}2^{-1}]=[0120]$	
$[0^{-1}10]$	0010~0020~0030		4
	00303=30300	$\Rightarrow [0^{-1}101]=[0101^{-1}]$	
	001011=(0,1,2)013$^{(3,2)}$	$\Rightarrow [0^{-1}101^{-1}]=[012]$	
	00302=(0,3,2)30300	$\Rightarrow [0^{-1}102]=[0101^{-1}]$	
	002033=032	$\Rightarrow [0^{-1}102^{-1}]=[012]$	
$[0^{-1}10^{-1}]$	**00100=11011**		1
	00100~11211~11311~22122~22322~ 22022~33133~33233~33433~00200~00300		
	001001=110	$\Rightarrow [0^{-1}10^{-1}1]=[0^{-1}1]$	
	0010011=1101	$\Rightarrow [0^{-1}10^{-1}1^{-1}]=[0^{-1}10]$	
$[0^{-1}1^{-1}0]$	00110~22112~33113		4
	003303=030033	$\Rightarrow [0^{-1}1^{-1}01]=[010^{-1}1^{-1}]$	
	0011011=0100	$\Rightarrow [0^{-1}1^{-1}01^{-1}]=[010^{-1}]$	
	001103=(0,3,1)001100	$\Rightarrow [0^{-1}1^{-1}02]=[0^{-1}1^{-1}0^{-1}]$	
	0011022=(0,2,1)2211	$\Rightarrow [0^{-1}1^{-1}02^{-1}]=[0^{-1}1^{-1}]$	
$[0^{-1}1^{-1}0^{-1}]$	**001100=221122=331133**		4
	0011001=(0,1,2)0130$^{(3,2)}$	$\Rightarrow [0^{-1}1^{-1}0^{-1}1]=[0120]$	
	0011002=2211	$\Rightarrow [0^{-1}1^{-1}0^{-1}2]=[0^{-1}1^{-1}]$	
	00110033=00110$^{(0,3)}$	$\Rightarrow [0^{-1}1^{-1}0^{-1}2^{-1}]=[0^{-1}1^{-1}0]$	

Table 8 (continued)

The Double Cosets $[w] = NwN$ in $S_6 \times 3$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 90)
[0101]	**0101=1212=1313=1414=2121=2323**		1
	2020=3131=3232=3030=0202=0303		
	03030=30300	$\Rightarrow [01010] = [0101^{-1}]$	
	030300=303	$\Rightarrow [01010^{-1}] = [010]$	
	02023=03033	$\Rightarrow [01012] = [0101^{-1}]$	
	020233=030	$\Rightarrow [01012^{-1}] = [010]$	
$[0101^{-1}]$	01011=21211=31311		4
	010110=(0,3,1)103	$\Rightarrow [0101^{-1}0] = [012]$	
	0101100=1101	$\Rightarrow [0101^{-1}0^{-1}] = [0^{-1}10]$	
	010112=(0,1,2)120	$\Rightarrow [0101^{-1}2] = [012]$	
	$0101133=1101^{(0,3)}$	$\Rightarrow [0101^{-1}2^{-1}] = [0^{-1}10]$	
$[010^{-1}1^{-1}]$	**010011=101100**		1
	010011~121122~131133~212211~232233~		
	202200~313311~323322~303300~020022~030033		
	0300330=3033	$\Rightarrow [010^{-1}1^{-1}0] = [010^{-1}]$	
	03003300=33003	$\Rightarrow [010^{-1}1^{-1}0^{-1}] = [0^{-1}1^{-1}0]$	
	01001111=00110	$\Rightarrow [010^{-1}1^{-1}1^{-1}] = [0^{-1}1^{-1}0]$	
	$N^{(012)} = N^{012} \cong S_1$ has orbits $\{0\},\{1\},\{2\},\{3\}$		
[0120]	0120~2132~2102~3123~3103~0130		4
	01300=(0,3,1)10011	$\Rightarrow [0120^{-1}] = [01^{-1}0^{-1}]$	
	03203=(0,2,3)00330033	$\Rightarrow [01201] = [0^{-1}1^{-1}0^{-1}1^{-1}]$	
	013011=(0,3,1)001100	$\Rightarrow [01201^{-1}] = [0^{-1}1^{-1}0^{-1}]$	
	012022=(0,1,2)210	$\Rightarrow [01202^{-1}] = [012]$	
	$01302=(0,3,1)10011^{(0,2,3)}$	$\Rightarrow [01203] = [01^{-1}0^{-1}]$	
$[0^{-1}1^{-1}0^{-1}1^{-1}]$	00110011=11221122=11331133=11001100=		1
	22112211=22332233=22002200=33113311=		
	33223322=33003300=00220022=00330033		
	003300330=330033	$\Rightarrow [0^{-1}1^{-1}0^{-1}1^{-1}0] = [0^{-1}1^{-1}0^{-1}]$	
	$0033003300=(0,1,2)3023^{(1,3,2)}$	$\Rightarrow [0^{-1}1^{-1}0^{-1}1^{-1}0^{-1}] = [0120]$	
	$0011001111=(0,1,2)0130^{(3,2)}$	$\Rightarrow [0^{-1}1^{-1}0^{-1}1^{-1}1^{-1}] = [0120]$	
	002200223=003300	$\Rightarrow [0^{-1}1^{-1}0^{-1}1^{-1}2] = [0^{-1}1^{-1}0^{-1}]$	
	$0022002233=(0,3,1)0320^{(1,2)}$	$\Rightarrow [0^{-1}1^{-1}0^{-1}1^{-1}2^{-1}] = [0120]$	

Table 9

Action of x, y, z and t on the Ninety Cosets

x	(1,22)(2,64)(3,50)(4,29)(5,30)(6,7)(8,52)(9,85)(10,12) (11,78)(14,75)(15,39)(16,70)(17,44)(18,90)(20,24)(21,65) (23,66)(25,76)(26,62)(27,74)(28,31)(32,33)(34,42)(35,45) (36,46)(37,82)(38,43)(40,87)(41,80)(47,48)(49,79)(51,57) (53,55)(54,60)(56,67)(58,59)(63,81)(68,69)(72,77)(73,88) (83,84)
y	(1,22)(2,64)(3,50)(4,29)(5,30)(6,7)(8,52)(9,85)(10,12) (11,78)(14,75)(15,39)(16,70)(17,44)(18,90)(20,24)(21,65) (23,66)(25,76)(26,62)(27,74)(28,31)(32,33)(34,42)(35,45) (36,46)(37,82)(38,43)(40,87)(41,80)(47,48)(49,79)(51,57) (53,55)(54, 60)(56, 67)(58, 59)(63, 81)(68, 69)(72, 77) (73, 88)(83,84)
z	(1,88,79)(2,74,80)(3,87,63)(4,56,78)(5,54,85)(6,83,43) (7,84,38)(8,72,90)(9,30,60)(10,55,68)(11,29,67)(12,53,69) (13,89,61)(14,65,44)(15,70,23)(16,66,39)(17,75,21)(18,52,77) (19,71,86)(20,36,35)(22,73,49)(24,46,45)(25,82,62)(26,76,37) (27,41,64)(28,51,42)(31,57,34)(32,48,58)(33,47,59)(40,81,50)
t_0	(1, 2, 3)(4, 9, 10)(5, 11, 12)(6, 14, 15)(7, 16, 17)(8, 18, 19)(13, 25, 26)(20, 35, 36)(21, 38, 39)(22, 40, 41)(23, 43, 44)(24, 45, 46)(27, 49, 50)(28, 42, 51)(29, 53, 54)(30, 55, 56)(31, 34, 57)(32, 58, 48)(33, 59, 47)(37, 61, 62)(52, 71, 72)(60, 68, 78)(63, 79, 80)(64, 73, 81)(65, 70, 83)(66, 75, 84)(67, 69, 85)(74, 87, 88)(76, 89, 82)(77, 86, 90)

Table 10
The Symmetric Generators of $S_6 \times 3$ Over S_4

t_0	(1, 2, 3)(4, 9, 10)(5, 11, 12)(6, 14, 15)(7, 16, 17)(8, 18, 19)(13, 25, 26)(20, 35, 36)(21, 38, 39)(22, 40, 41)(23, 43, 44)(24, 45, 46)(27, 49, 50)(28, 42, 51)(29, 53, 54)(30, 55, 56)(31, 34, 57)(32, 58, 48)(33, 59, 47)(37, 61, 62)(52, 71, 72)(60, 68, 78)(63, 79, 80)(64, 73, 81)(65, 70, 83)(66, 75, 84)(67, 69, 85)(74, 87, 88)(76, 89, 82)(77, 86, 90)
t_1	(1, 4, 6)(2, 23, 22)(3, 27, 30)(5, 32, 12)(7, 16, 47)(8, 20, 19)(9, 63, 64)(10, 40, 65)(11, 67, 29)(13, 25, 28)(14, 68, 50)(15, 73, 74)(17, 21, 75)(18, 77, 52)(24, 45, 46)(26, 37, 76)(31, 34, 57)(33, 38, 39)(35, 86, 90)(36, 71, 72)(41, 60, 87)(42, 61, 62)(43, 79, 78)(44, 55, 81)(48, 53, 54)(49, 80, 70)(51, 89, 82)(56, 83, 88)(58, 69, 85)(59, 84, 66)
t_2	(1, 8, 13)(2, 37, 22)(3, 27, 52)(4, 42, 10)(5, 34, 12)(6, 14, 36)(7, 16, 46)(9, 60, 30)(11, 67, 29)(15, 23, 70)(17, 21, 75)(18, 63, 64)(19, 40, 82)(20, 43, 44)(24, 38, 39)(25, 86, 50)(26, 73, 74)(28, 55, 56)(31, 53, 54)(32, 58, 48)(33, 59, 47)(35, 83, 65)(41, 77, 87)(45, 84, 66)(49, 80, 76)(51, 68, 78)(57, 69, 85)(61, 79, 90)(62, 71, 81)(72, 89, 88)
t_3	(1, 5, 7)(2, 21, 22)(3, 27, 29)(4, 33, 10)(6, 14, 48)(8, 24, 19)(9, 60, 30)(11, 63, 64)(12, 40, 66)(13, 25, 31)(15, 23, 70)(16, 69, 50)(17, 73, 74)(18, 77, 52)(20, 35, 36)(26, 37, 76)(28, 42, 51)(32, 43, 44)(34, 61, 62)(38, 79, 85)(39, 53, 81)(41, 67, 87)(45, 86, 90)(46, 71, 72)(47, 55, 56)(49, 80, 75)(54, 84, 88)(57, 89, 82)(58, 83, 65)(59, 68, 78)

Table 11

a,b,c∈G Satisfying Relations for $S_6 \times 3$

a	(1, 60, 61, 75, 72, 58)(2, 36, 69, 64, 28, 84)(3, 37, 24, 85, 65, 55)(4, 83, 49, 52, 31, 39)(5, 44, 68, 87, 26, 46)(6, 73, 18, 34, 66, 78)(7, 80, 20, 53, 41, 42)(8, 48, 79, 30, 89, 17)(9, 13, 21, 90, 32, 88)(10, 63, 76, 45, 54, 14)(11, 82, 33, 50, 23, 71)(12, 27, 51, 38, 74, 35)(15, 86, 29, 62, 47, 40)(16, 56, 43, 22, 77, 57)(19, 67, 25, 59, 81, 70)
b	(1, 27)(2, 40)(3, 73)(6, 23)(7, 21)(9, 55)(10, 60)(11, 53)(12, 67)(13, 37)(15, 83)(17, 84)(18, 71)(19, 77)(22, 63)(26, 89)(29, 69)(30, 68)(38, 75)(41, 88)(43, 70)(49, 87)(50, 80)(52, 86)(61, 76)(64, 79)(74, 81)
c	(1, 88, 79)(2, 74, 80)(3, 87, 63)(4, 56, 78)(5, 54, 85)(6, 83, 43)(7, 84, 38)(8, 72, 90)(9, 30, 60)(10, 55, 68)(11, 29, 67)(12, 53, 69)(13, 89, 61)(14, 65, 44)(15, 70, 23)(16, 66, 39)(17, 75, 21)(18, 52, 77)(19, 71, 86)(20, 36, 35)(22, 73, 49)(24, 46, 45)(25, 82, 62)(26, 76, 37)(27, 41, 64)(28, 51, 42)(31, 57, 34)(32, 48, 58)(33, 47, 59)(40, 81, 50)

APPENDIX D

TABLES FOR GENERATION OF $L_3(3)$ OVER S_4

Table 12

Coset Array for $L_3(3)$ Over S_4

Label the single cosets in each double coset

[*]

cst[1]: [ ]

[0]

cst[2]: 0
cst[4]: 1
cst[5]: 3
cst[8]: 2

[0^{-1}]

cst[3]: 00
cst[6]: 11
cst[7]: 33
cst[13]: 22

[01]

cst[9]: 10
cst[11]: 30
cst[15]: 01
cst[17]: 03
cst[18]: 20
cst[20]: 21
cst[23]: 23
cst[27]: 02
cst[30]: 12
cst[33]: 32
cst[34]: 31
cst[35]: 13

[01^{-1}]

cst[10]: 100
cst[12]: 300
cst[19]: 200
cst[21]: 211
cst[22]: 033
cst[24]: 011
cst[25]: 233
cst[36]: 311
cst[37]: 133
cst[40]: 322
cst[43]: 022
cst[47]: 122

[$0^{-1}1$]

cst[14]: 110
cst[16]: 330
cst[26]: 220
cst[28]: 221
cst[29]: 003
cst[31]: 001
cst[32]: 223
cst[51]: 331
cst[52]: 113
cst[53]: 332
cst[56]: 002
cst[59]: 112

[012]

cst[38]: 210
cst[42]: 301
cst[45]: 230
cst[50]: 103
cst[57]: 120
cst[62]: 320
cst[64]: 310
cst[66]: 130
cst[69]: 201
cst[71]: 203
cst[72]: 321
cst[76]: 102
cst[79]: 012
cst[82]: 123
cst[86]: 302
cst[90]: 032
cst[100]: 231
cst[101]: 013
cst[108]: 031
cst[109]: 213
cst[111]: 021
cst[113]: 023
cst[117]: 312
cst[120]: 132

[012^{-1}]

cst[39]: 2100
cst[46]: 2300
cst[54]: 0311
cst[58]: 1200
cst[60]: 0133
cst[63]: 3200
cst[65]: 3100
cst[67]: 1300
cst[73]: 3211
cst[74]: 1033
cst[83]: 3011
cst[84]: 1233
cst[91]: 0211
cst[93]: 0233
cst[95]: 0122
cst[97]: 1022
cst[102]: 2311
cst[103]: 0322
cst[105]: 3022
cst[110]: 2133
cst[112]: 2033
cst[114]: 2011
cst[142]: 1322
cst[157]: 3122

[$01^{-1}0$]

cst[44]: 0330
cst[48]: 0110
cst[80]: 0220
cst[81]: 3223
cst[87]: 2112
cst[136]: 3113

[$01^{-1}2$]

cst[41]: 2110
cst[49]: 2330
cst[68]: 3110
cst[70]: 1330
cst[75]: 3220
cst[77]: 3221
cst[78]: 1003
cst[85]: 1220
cst[88]: 3001
cst[89]: 1223
cst[115]: 0221
cst[116]: 2003
cst[118]: 2001
cst[119]: 0223
cst[126]: 0331
cst[127]: 2113
cst[128]: 0332

Table 12 (continued)
Coset Array for $L_3(3)$ Over S_4
Label the single cosets in each double coset

cst[129]: 2331
cst[132]: 1002
cst[145]: 0113
cst[148]: 0112
cst[150]: 1332
cst[163]: 3002
cst[189]: 3112

$[0^{-1}12^{-1}]$
cst[55]: 22100
cst[61]: 22300
cst[92]: 33100
cst[94]: 11300
cst[96]: 33200
cst[98]: 33211
cst[99]: 11033
cst[104]: 11200
cst[106]: 33011
cst[107]: 11233
cst[134]: 33022
cst[153]: 11022
cst[155]: 00211
cst[156]: 22033
cst[158]: 22011
cst[159]: 00233
cst[160]: 00311
cst[161]: 22133
cst[164]: 00322
cst[165]: 22311
cst[174]: 00133
cst[176]: 00122
cst[178]: 11322
cst[222]: 33122

$[0120^{-1}]$
cst[130]: 32133
cst[135]: 01200
cst[151]: 30133
cst[154]: 03200
cst[168]: 02300
cst[171]: 01300
cst[177]: 02100
cst[181]: 03100
cst[199]: 30233
cst[202]: 23122
cst[218]: 20122
cst[225]: 31233

[0123]
cst[121]: 3210
cst[124]: 2301
cst[139]: 1230
cst[144]: 2103
cst[169]: 2310
cst[173]: 3201
cst[182]: 2130
cst[185]: 1203
cst[186]: 0123
cst[187]: 3120
cst[190]: 0321
cst[191]: 1320
cst[196]: 3012
cst[198]: 0132
cst[201]: 0231
cst[211]: 3102
cst[212]: 1032
cst[213]: 0312
cst[216]: 0213
cst[221]: 1302
cst[226]: 3021
cst[229]: 1023
cst[230]: 2013
cst[231]: 2031

$[0123^{-1}]$
cst[122]: 02133
cst[137]: 30122
cst[140]: 02311
cst[146]: 20311
cst[170]: 03122
cst[183]: 01322
cst[188]: 01233
cst[192]: 03211
cst[193]: 31022
cst[206]: 30211
cst[207]: 23011
cst[227]: 32011

$[012^{-1}1]$
cst[125]: 23003
cst[133]: 20330
cst[141]: 30110
cst[149]: 31001
cst[162]: 32002
cst[167]: 01331
cst[175]: 30220
cst[180]: 03113
cst[197]: 03223
cst[205]: 02332
cst[210]: 01221
cst[217]: 02112

$[012^{-1}3]$
cst[123]: 30112
cst[138]: 03112
cst[143]: 10332
cst[147]: 02331
cst[172]: 20113
cst[184]: 20331
cst[194]: 03221
cst[195]: 23001
cst[208]: 10223
cst[209]: 32001
cst[214]: 13002
cst[215]: 30221

$[01^{-1}23^{-1}]$
cst[131]: 011233
cst[152]: 033211
cst[166]: 200311
cst[179]: 311022
cst[200]: 033122
cst[203]: 011322
cst[204]: 300122
cst[219]: 022311
cst[220]: 233011
cst[223]: 322011
cst[224]: 300211
cst[228]: 022133

$[0123^{-1}2]$
cst[232]: 032112
cst[233]: 031221
cst[234]: 021331

Table 13

The Double Cosets $[w] = NwN$ in $L_3(3)$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 234)
[*]	N is transitive on {0,1,2,3}.		1
	$N^{(0)} = N^0 \cong S_3$ has orbits {0},{1,2,3}		
[0]	Nt_0, Nt_1, Nt_2, Nt_3 $[00^{-1}] = [*]$ $[00] = [0^{-1}]$		4
$[0^{-1}]$	$Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}, Nt_3^{-1}$ $[0^{-1}0] = [*]$ $[0^{-1}0^{-1}] = [0]$		4
	$N^{(01)} = N^{01} \cong S_2$ has orbits {0},{1},{2,3}		
[01]	$010 = (3,2)00^{(0,1,2,3)}$ $0100 = (3,2)001^{(0,1)}$	$\Rightarrow [010] = [0^{-1}]$ $\Rightarrow [010^{-1}] = [0^{-1}1]$	12
$[01^{-1}]$	$01100 = (3,2)001$ $01122 = (0,3)012^{(0,3)}$	$\Rightarrow [01^{-1}0^{-1}] = [0^{-1}1]$ $\Rightarrow [01^{-1}2^{-1}] = [012]$	12
$[0^{-1}1]$	$0010 = (3,2)011$ $00100 = 0110^{(0,1)}$ $0011 = (3,2)01^{(0,1)}$ $0012 = (0,3,1)0122^{(0,2,1,3)}$	$\Rightarrow [0^{-1}10] = [01^{-1}]$ $\Rightarrow [0^{-1}10^{-1}] = [01^{-1}0]$ $\Rightarrow [0^{-1}1^{-1}] = [01]$ $\Rightarrow [0^{-1}12] = [012^{-1}]$	12
	$N^{(012)} = N^{012} \cong S_1$ has orbits {0},{1},{2},{3}		
[012]	$0120 = (1,3,2)00122^{(0,3,1)}$ $0121 = (0,3)011^{(0,3,1,2)}$ $01211 = (0,3)3221$	$\Rightarrow [0120] = [0^{-1}12^{-1}]$ $\Rightarrow [0121] = [01^{-1}]$ $\Rightarrow [0121^{-1}] = [01^{-1}2]$	24
$[012^{-1}]$	$01220 = (1,3,2)00122^{(0,3,1,2)}$ $012200 = (1,3,2)001^{(0,3,1,2)}$ $012211 = (0,3)0112^{(0,3)}$ $012233 = (0,1)1032$	$\Rightarrow [012^{-1}0] = [0^{-1}12^{-1}]$ $\Rightarrow [012^{-1}0^{-1}] = [0^{-1}1]$ $\Rightarrow [012^{-1}1^{-1}] = [01^{-1}2]$ $\Rightarrow [012^{-1}3^{-1}] = [0123]$	24
$[01^{-1}0]$	$0110 \sim 1001$ $01100 = (3,2)001$ $01101 = 001^{(0,1)}$ $011011 = (3,2)011^{(0,1)}$ $01102 = (0,3,2,1)01233^{(1,2,3)}$ $011022 = (0,1)02331^{(0,2)}$	 $\Rightarrow [01^{-1}0^{-1}] = [0^{-1}1]$ $\Rightarrow [01^{-1}01] = [0^{-1}1]$ $\Rightarrow [01^{-1}01^{-1}] = [01^{-1}]$ $\Rightarrow [01^{-1}02] = [0123^{-1}]$ $\Rightarrow [01^{-1}02^{-1}] = [023^{-1}1]$	6

Table 13 (continued)

The Double Cosets [w] = NwN in $L_3(3)$ where N = S_4

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 234)
$[01^{-1}2]$	$01120=(1,3,2)00122^{(1,3)}$	$\Rightarrow [01^{-1}20]=[0^{-1}12^{-1}]$	24
	$011200=(0,1,3)0123^{(1,2)}$	$\Rightarrow [01^{-1}20^{-1}]=[0123]$	
	$01121=(0,3)0122^{(0,3)}$	$\Rightarrow [01^{-1}21]=[012^{-1}]$	
	$011211=01221^{(1,2)}$	$\Rightarrow [01^{-1}21^{-1}]=[012^{-1}1]$	
	$01122=(0,3)012^{(0,3)}$	$\Rightarrow [01^{-1}2^{-1}]=012$	
	$01123=(0,2,1,3)01200$	$\Rightarrow [01^{-1}23]=[0120^{-1}]$	
$[0^{-1}12^{-1}]$	$001220=(0,1,3)0123^{(1,2,3)}$	$\Rightarrow [0^{-1}12^{-1}0]=[0123]$	24
	$0012200=(1,3,2)0112^{(1,3)}$	$\Rightarrow [0^{-1}12^{-1}0^{-1}]=[01^{-1}2]$	
	$001221=(0,3,2)01200^{(0,1,3)}$	$\Rightarrow [0^{-1}12^{-1}1]=[0120^{-1}]$	
	$0012211=(0,3,2)012^{(0,1,3)}$	$\Rightarrow [0^{-1}12^{-1}1^{-1}]=[012]$	
	$0012222=(0,3,1)0122^{(0,2,1,3)}$	$\Rightarrow [0^{-1}12^{-1}2^{-1}]=[012^{-1}]$	
	$3320011=(0,3)32002^{(0,3,1,2)}$	$\Rightarrow [0^{-1}12^{-1}3^{-1}]=[012^{-1}1]$	
$[0120^{-1}]$	$01200\sim23022$		12
	$0120000=(1,3,2)00122^{(0,3,1)}$	$\Rightarrow [0120^{-1}0^{-1}]=[0^{-1}12^{-1}]$	
	$012001=(0,2,3)233011$	$\Rightarrow [0120^{-1}1]=[01^{-1}23^{-1}]$	
	$2302233=(0,1,2)0112$	$\Rightarrow [0120^{-1}1^{-1}]=[01^{-1}2]$	
	$012002=(0,1,3,2)012$	$\Rightarrow [0120^{-1}2]=[012]$	
	$0120022=(3,2)00122^{(0,1,2)}$	$\Rightarrow [0120^{-1}2^{-1}]=[0^{-1}12^{-1}]$	
	$012003=(0,2,1,3)011233$	$\Rightarrow [0120^{-1}3]=[01^{-1}23^{-1}]$	
	$0120033=(0,2,1,3)0112$	$\Rightarrow [0120^{-1}3^{-1}]=[01^{-1}2]$	
$[0123]$	$01230=(0,1)0112^{(1,2)}$	$\Rightarrow [01230]=[01^{-1}2]$	24
	$012300=(0,2)00122^{(1,3,2)}$	$\Rightarrow [01230^{-1}]=[0^{-1}12^{-1}]$	
	$01232=(0,1)1033$	$\Rightarrow [01232]=[012^{-1}]$	
	$012322=(0,3,1)02331^{(0,1)}$	$\Rightarrow [01232^{-1}]=[023^{-1}1]$	
$[0123^{-1}]$	$01233\sim31200$		12
	$012330=(0,2,3,1)012^{(0,3)}$	$\Rightarrow [0123^{-1}0]=[012]$	
	$0123300=(0,2,3,1)0123^{(0,3)}$	$\Rightarrow [0123^{-1}0^{-1}]=[0123]$	
	$012331=(3,2)10223^{(0,3)}$	$\Rightarrow [0123^{-1}1]=[023^{-1}1]$	
	$0213322=(0,3,2,1)0110^{(1,3,2)}$	$\Rightarrow [0213^{-1}2^{-1}]=[01^{-1}0]$	
	$0123322=(0,2,1)10223^{(0,3)}$	$\Rightarrow [0123^{-1}2^{-1}]=[023^{-1}1]$	

Table 13 (continued)

The Double Cosets $[w] = NwN$ in $L_3(3)$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 234)
$[012^{-1}1]$	01221~32112		12
	$012210=(1,2)00122^{(0,2,1,3)}$	$\Rightarrow [012^{-1}10]=[0^{-1}12^{-1}]$	
	$0122100=(0,2,3,1)011233^{(0,1,2)}$	$\Rightarrow [012^{-1}10^{-1}]=[01^{-1}23^{-1}]$	
	$012211=(0,3)0112^{(0,3)}$	$\Rightarrow [012^{-1}1^{-1}]=[01^{-1}2]$	
	$012212=0112^{(1,2)}$	$\Rightarrow [012^{-1}12]=[01^{-1}2]$	
	$0122122=(0,3)3211$	$\Rightarrow [012^{-1}12^{-1}]=[012^{-1}]$	
$[012^{-1}3]$	01223~03221		12
	$023310=(1,2)0110^{(0,2,3)}$	$\Rightarrow [023^{-1}10]=[01^{-1}0]$	
	$0233100=(3,2)20311^{(1,2)}$	$\Rightarrow [023^{-1}10^{-1}]=[0123^{-1}]$	
	$012231=(0,2,3)0123^{(0,3,2,1)}$	$\Rightarrow [012^{-1}31]=[0123]$	
	$0122311=(0,2)0122^{(1,3)}$	$\Rightarrow [012^{-1}31^{-1}]=[012^{-1}]$	
	$023313=(0,3,1)20311^{(1,2)}$	$\Rightarrow [023^{-1}13]=[0123^{-1}]$	
	$0233133=021331$	$\Rightarrow [023^{-1}13^{-1}]=[0213^{-1}1]$	
	$012233=(0,1)1032$	$\Rightarrow [012^{-1}3^{-1}]=[0123]$	
$[01^{-1}23^{-1}]$	**011233=322100**		12
	$0112330=3221$	$\Rightarrow [01^{-1}23^{-1}0]=[01^{-1}2]$	
	$01123300=(0,2,1,3)32133$	$\Rightarrow [01^{-1}23^{-1}0^{-1}]=[0120^{-1}]$	
$[0123^{-1}2]$	012332~130220~103223~230110~ 203113~312002~321001~021331		3
	$0213310=(0,1,3,2)32001$	$\Rightarrow [0213^{-1}10]=[023^{-1}1]$	
	$02133100=(0,3,2,1)23011^{(1,2)}$	$\Rightarrow [0213^{-1}10^{-1}]=[0123^{-1}]$	
	$0123321=(0,3,2)20113$	$\Rightarrow [0123^{-1}21]=[023^{-1}1]$	
	$0123322=(0,2,1)10223^{(0,3)}$	$\Rightarrow [0123^{-1}2^{-1}]=[023^{-1}1]$	
	$0213313=02331$	$\Rightarrow [0213^{-1}13]=[023^{-1}1]$	
	$02133133=(0,3,1)20311^{(1,2)}$	$\Rightarrow [0213^{-1}13^{-1}]=[0123^{-1}]$	

Table 14

Action of x, y and t on the 234 Cosets

x	(2,4,8,5)(3,6,13,7)(9,20,33,17)(10,21,40,22)(11,15,30,23)(12,24,47,25)(14,28,53,29)(16,31,59,32)(18,34,27,35)(19,36,43,37)(26,51,56,52)(38,72,90,50)(39,73,103,74)(41,77,128,78)(42,79,82,45)(44,48,87,81)(46,83,95,84)(49,88,148,89)(54,97,110,63)(55,98,164,99)(57,100,86,101)(58,102,105,60)(61,106,176,107)(62,108,76,109)(64,111,120,71)(65,91,142,112)(66,69,117,113)(67,114,157,93)(68,115,150,116)(70,118,189,119)(75,126,132,127)(80,136)(85,129,163,145)(92,155,178,156)(94,158,222,159)(96,160,153,161)(104,165,134,174)(121,190,212,144)(122,192,146,193)(123,194,143,195)(124,196,186,139)(125,141,210,197)(130,154)(131,152,220,204)(133,149,217,205)(135,151)(137,188,140,207)(138,208,184,209)(147,214,172,215)(162,180)(166,179,228,219)(167,175)(168,177,218,225)(169,226,198,185)(170,206,183,227)(171,181,202,199)(173,213,229,182)(187,201,221,230)(191,231,211,216)(200,223)(203,224)(232,234)
y	(5,8)(7,13)(11,18)(12,19)(16,26)(17,27)(20,34)(21,36)(22,43)(23,33)(25,40)(28,51)(29,56)(30,35)(32,53)(37,47)(38,64)(39,65)(41,68)(42,69)(44,80)(45,62)(46,63)(49,75)(50,76)(52,59)(54,91)(55,92)(57,66)(58,67)(60,95)(61,96)(70,85)(71,86)(72,100)(73,102)(74,97)(77,129)(78,132)(79,101)(82,120)(83,114)(84,142)(87,136)(88,118)(89,150)(90,113)(93,103)(94,104)(98,165)(99,153)(105,112)(106,158)(107,178)(108,111)(109,117)(110,157)(115,126)(116,163)(119,128)(121,169)(122,170)(123,172)(124,173)(125,162)(127,189)(130,202)(131,203)(133,175)(134,156)(135,171)(139,191)(140,192)(141,149)(143,208)(144,211)(145,148)(146,206)(147,194)(151,218)(152,219)(154,168)(155,160)(159,164)(161,222)(166,224)(167,210)(174,176)(177,181)(179,204)(180,217)(182,187)(183,188)(184,215)(185,221)(186,198)(190,201)(195,209)(196,230)(197,205)(199,225)(200,228)(207,227)(212,229)(213,216)(220,223)(226,231)(233,234)
t_0	(1,2,3)(4,9,10)(5,11,12)(6,14,15)(7,16,17)(8,18,19)(13,26,27)(20,38,39)(21,41,42)(22,44,29)(23,45,46)(24,48,31)(25,49,50)(28,54,55)(30,57,58)(32,60,61)(33,62,63)(34,64,65)(35,66,67)(36,68,69)(37,70,71)(40,75,76)(43,80,56)(47,85,86)(51,91,92)(52,93,94)(53,95,96)(59,103,104)(72,121,122)(73,123,124)(74,125,116)(77,130,131)(78,112,133)(79,134,135)(81,137,138)(82,139,140)(83,141,118)(84,43,144)(87,146,147)(88,114,149)(89,151,152)(90,153,154)(97,162,163)(98,166,167)(99,168,113)(100,169,170)(101,156,171)(102,172,173)(105,175,132)(106,177,111)(107,179,180)(108,158,181)(109,182,183)(110,184,185)(115,160,186)(117,187,188)(119,174,190)(120,191,192)(126,155,198)(127,199,200)(128,176,201)(129,202,203)(136,206,194)(142,208,211)(145,159,213)(148,164,216)(150,218,219)(157,215,221)(161,223,197)(165,224,210)(178,204,217)(189,225,228)(193,232,214)(195,227,233)(196,231,229)(205,222,220)(207,234,209)(212,230,226)

Table 15

The Symmetric Generators of $L_3(3)$ Over S_4

t_0	(1, 2, 3)(4, 9, 10)(5, 11, 12)(6, 14, 15)(7, 16, 17)(8, 18, 19)(13, 26, 27)(20, 38, 39)(21, 41, 42)(22, 44, 29)(23, 45, 46)(24, 48, 31)(25, 49, 50)(28, 54, 55)(30, 57, 58)(32, 60, 61)(33, 62, 63)(34, 64, 65)(35, 66, 67)(36, 68, 69)(37, 70, 71)(40, 75, 76)(43, 80, 56)(47, 85, 86)(51, 91, 92)(52, 93, 94)(53, 95, 96)(59, 103, 104)(72, 121, 122)(73, 123, 124)(74, 125, 116)(77, 130, 131)(78, 112, 133)(79, 134, 135)(81, 137, 138)(82, 139, 140)(83, 141, 118)(84, 143, 144)(87, 146, 147)(88, 114, 149)(89, 151, 152)(90, 153, 154)(97, 162, 163)(98, 166, 167)(99, 168, 113)(100, 169, 170)(101, 156, 171)(102, 172, 173)(105, 175, 132)(106, 177, 111)(107, 179, 180)(108, 158, 181)(109, 182, 183)(110, 184, 185)(115, 160, 186)(117, 187, 188)(119, 174, 190)(120, 191, 192)(126, 155, 198)(127, 199, 200)(128, 176, 201)(129, 202, 203)(136, 206, 194)(142, 208, 211)(145, 159, 213)(148, 164, 216)(150, 218, 219)(157, 215, 221)(161, 223, 197)(165, 224, 210)(178, 204, 217)(189, 225, 228)(193, 232, 214)(195, 227, 233)(196, 231, 229)(205, 222, 220)(207, 234, 209)(212, 230, 226)
t_1	(1, 4, 6)(2, 15, 24)(3, 31, 9)(5, 34, 36)(7, 51, 35)(8, 20, 21)(10, 48, 14)(11, 42, 83)(12, 88, 38)(13, 28, 30)(16, 58, 106)(17, 108, 54)(18, 69, 114)(19, 118, 64)(22, 126, 109)(23, 100, 102)(25, 129, 101)(26, 67, 158)(27, 111, 91)(29, 84, 160)(32, 74, 165)(33, 72, 73)(37, 136, 52)(39, 141, 68)(40, 77, 79)(41, 65, 149)(43, 115, 117)(44, 188, 208)(45, 124, 207)(46, 195, 121)(47, 87, 59)(49, 135, 220)(50, 161, 130)(53, 97, 98)(55, 177, 66)(56, 142, 155)(57, 92, 181)(60, 167, 127)(61, 228, 162)(62, 173, 227)(63, 209, 169)(70, 104, 212)(71, 231, 146)(75, 171, 223)(76, 222, 202)(78, 107, 221)(80, 183, 143)(81, 193, 214)(82, 174, 151)(85, 94, 229)(86, 226, 206)(89, 99, 191)(90, 190, 192)(93, 147, 230)(95, 210, 189)(96, 200, 125)(103, 194, 196)(105, 215, 213)(110, 180, 145)(112, 184, 216)(113, 201, 140)(116, 225, 166)(119, 168, 219)(120, 176, 218)(122, 234, 172)(123, 170, 233)(128, 154, 152)(131, 205, 156)(132, 178, 185)(133, 159, 204)(134, 203, 197)(137, 232, 138)(139, 150, 153)(144, 187, 198)(148, 157, 217)(163, 199, 224)(164, 179, 175)(182, 186, 211)

Table 15 (continued)

The Symmetric Generators of $L_3(3)$ Over S_4

t_2	(1, 8, 13)(2, 27, 43)(3, 56, 18)(4, 30, 47)(5, 33, 40)(6, 59, 20)(7, 53, 23)(9, 76, 97)(10, 132, 62)(11, 86, 105)(12, 163, 57)(14, 46, 153)(15, 79, 95)(16, 39, 134)(17, 90, 103)(19, 80, 26)(21, 87, 28)(22, 128, 82)(24, 148, 72)(25, 81, 32)(29, 110, 164)(31, 102, 176)(34, 117, 157)(35, 120, 142)(36, 189, 111)(37, 150, 113)(38, 96, 154)(41, 61, 230)(42, 196, 137)(44, 122, 172)(45, 104, 135)(48, 140, 184)(49, 55, 231)(50, 212, 146)(51, 114, 222)(52, 112, 178)(54, 138, 226)(58, 175, 75)(60, 147, 229)(63, 162, 85)(64, 211, 193)(65, 209, 191)(66, 221, 207)(67, 214, 187)(68, 168, 179)(69, 98, 218)(70, 177, 166)(71, 107, 225)(73, 210, 115)(74, 143, 186)(77, 91, 217)(78, 130, 220)(83, 123, 190)(84, 197, 119)(88, 151, 204)(89, 93, 205)(92, 152, 133)(94, 131, 149)(99, 228, 167)(100, 155, 202)(101, 198, 183)(106, 219, 180)(108, 213, 170)(109, 159, 199)(116, 161, 124)(118, 165, 144)(121, 201, 185)(125, 174, 224)(126, 181, 200)(127, 156, 169)(129, 158, 182)(136, 227, 195)(139, 216, 173)(141, 160, 223)(145, 171, 203)(188, 234, 208)(192, 232, 215)(194, 206, 233)
t_3	(1, 5, 7)(2, 17, 22)(3, 29, 11)(4, 35, 37)(6, 52, 34)(8, 23, 25)(9, 50, 74)(10, 78, 45)(12, 44, 16)(13, 32, 33)(14, 63, 99)(15, 101, 60)(18, 71, 112)(19, 116, 66)(20, 109, 110)(21, 127, 108)(24, 145, 100)(26, 65, 156)(27, 113, 93)(28, 83, 161)(30, 82, 84)(31, 73, 174)(36, 136, 51)(38, 144, 193)(39, 195, 139)(40, 81, 53)(41, 154, 204)(42, 165, 151)(43, 119, 120)(46, 125, 70)(47, 89, 90)(48, 192, 215)(49, 67, 133)(54, 180, 129)(55, 219, 175)(56, 157, 159)(57, 185, 227)(58, 214, 182)(59, 105, 107)(61, 168, 64)(62, 94, 171)(68, 96, 196)(69, 230, 137)(72, 160, 130)(75, 92, 226)(76, 229, 206)(77, 106, 187)(79, 186, 188)(80, 170, 123)(85, 181, 224)(86, 178, 199)(87, 207, 209)(88, 98, 211)(91, 138, 231)(95, 194, 212)(97, 208, 198)(102, 167, 126)(103, 197, 150)(104, 203, 141)(111, 216, 122)(114, 172, 201)(115, 177, 228)(117, 164, 225)(118, 218, 179)(121, 189, 134)(124, 191, 213)(128, 142, 205)(131, 148, 135)(132, 202, 223)(140, 232, 184)(143, 183, 233)(146, 234, 147)(149, 155, 220)(152, 217, 158)(153, 200, 210)(162, 176, 166)(163, 222, 173)(169, 190, 221)

Table 16

a,b∈G Satisfying Relations for $L_3(3)$

a	(1, 25, 40)(2, 93, 142, 4, 84, 103)(3, 159, 178, 6, 107, 164)(5, 32, 23)(7, 81, 13)(8, 33, 53)(9, 206, 183, 15, 188, 146)(10, 223, 203, 24, 131, 220)(11, 199, 20, 34, 225, 18)(12, 173, 195, 36, 121, 172)(14, 70, 115, 31, 126, 85)(16, 139, 111, 51, 201, 57)(17, 89, 113, 35, 119, 82)(19, 123, 169, 21, 209, 124)(22, 205, 47, 37, 197, 43)(26, 66, 190, 28, 108, 191)(27, 120, 128, 30, 90, 150)(29, 109, 157, 52, 71, 105)(38, 68, 61, 69, 88, 165)(39, 189, 122, 114, 163, 227)(41, 64, 98, 118, 42, 96)(44, 216, 222, 136, 185, 134)(45, 141, 72, 100, 149, 62)(46, 210, 228, 102, 162, 224)(48, 184, 215)(49, 175, 226, 129, 217, 187)(50, 194, 229, 101, 208, 186)(54, 91, 152, 67, 58, 219)(55, 138, 92, 158, 214, 106)(56, 112, 86, 59, 110, 117)(60, 234, 97, 74, 233, 95)(63, 166, 125, 73, 200, 167)(65, 207, 116, 83, 170, 127)(75, 182, 180, 77, 231, 133)(76, 198, 143, 79, 212, 147)(78, 160, 202, 145, 94, 135)(80, 156, 221, 87, 161, 213)(99, 177, 176, 174, 181, 153)(104, 148, 130, 155, 132, 171)(137, 151, 154, 193, 168, 218)(140, 232, 192)(144, 179, 196, 230, 204, 211)
b	(1, 22)(2, 29)(3, 44)(4, 74)(5, 11)(6, 99)(8, 112)(9, 116)(10, 125)(13, 156)(14, 113)(15, 168)(16, 17)(18, 78)(19, 133)(20, 193)(21, 204)(23, 66)(24, 196)(26, 101)(27, 171)(28, 129)(30, 227)(31, 231)(32, 93)(35, 45)(36, 215)(38, 214)(39, 232)(40, 123)(41, 178)(42, 217)(43, 226)(46, 67)(47, 224)(48, 229)(49, 50)(51, 100)(52, 60)(53, 120)(54, 203)(55, 202)(56, 212)(57, 195)(58, 233)(59, 150)(61, 94)(62, 63)(64, 65)(68, 157)(69, 221)(70, 71)(72, 77)(73, 75)(76, 124)(79, 106)(80, 230)(81, 159)(82, 182)(83, 175)(85, 165)(86, 210)(87, 147)(88, 149)(89, 199)(91, 170)(92, 169)(95, 192)(96, 191)(98, 208)(102, 220)(103, 219)(104, 218)(105, 141)(109, 139)(111, 134)(115, 160)(117, 189)(118, 132)(119, 206)(121, 131)(122, 130)(126, 201)(127, 151)(128, 198)(135, 177)(136, 174)(137, 145)(138, 213)(140, 183)(142, 166)(143, 144)(148, 164)(152, 200)(153, 154)(155, 176)(158, 181)(162, 163)(167, 211)(172, 222)(173, 205)(179, 180)(184, 185)(187, 228)(188, 225)(190, 194)(197, 223)(207, 234)

APPENDIX E

TABLES FOR GENERATION OF $U_3(3)$ OVER S_4

Table 17
Coset Array for $U_3(3)$ Over S_4
Label the single cosets in each double coset

[*]
cst[1]: [ ]

[0]
cst[2]: 0
cst[4]: 1
cst[5]: 3
cst[8]: 2

[0^{-1}]
cst[3]: 00
cst[6]: 11
cst[7]: 33
cst[13]: 22

[01]
cst[9]: 10
cst[11]: 30
cst[18]: 20
cst[20]: 21
cst[21]: 03
cst[23]: 01
cst[24]: 23
cst[34]: 31
cst[35]: 13
cst[37]: 32
cst[40]: 02
cst[44]: 12

[01^{-1}]
cst[10]: 011
cst[12]: 033
cst[19]: 022
cst[22]: 211
cst[25]: 322
cst[36]: 311

[$0^{-1}1$]
cst[14]: 001
cst[16]: 003
cst[26]: 002
cst[28]: 221
cst[31]: 332
cst[50]: 331

[$0^{-1}2^{-1}$]
cst[15]: 1100
cst[17]: 3300
cst[27]: 2200
cst[29]: 2211
cst[30]: 0033
cst[32]: 0011
cst[33]: 2233
cst[51]: 3311
cst[52]: 1133
cst[55]: 3322
cst[58]: 0022
cst[61]: 1122

[010]
cst[41]: 030
cst[45]: 010
cst[75]: 020
cst[76]: 323
cst[81]: 212
cst[125]: 313

[012]
cst[38]: 210
cst[46]: 230
cst[64]: 310
cst[66]: 130
cst[70]: 320
cst[72]: 321
cst[73]: 103
cst[79]: 120
cst[82]: 301
cst[83]: 123
cst[107]: 021
cst[108]: 203
cst[110]: 201
cst[111]: 023
cst[115]: 031
cst[116]: 213
cst[118]: 032
cst[119]: 231
cst[121]: 102
cst[133]: 013
cst[136]: 012
cst[138]: 132
cst[176]: 302
cst[178]: 312

[012^{-1}]
cst[39]: 3011
cst[47]: 2300
cst[65]: 3100
cst[67]: 2033
cst[71]: 3200
cst[74]: 0122
cst[80]: 3022
cst[84]: 0322
cst[109]: 0211
cst[112]: 0233
cst[117]: 0311
cst[123]: 0133

[$01^{-1}2$]
cst[42]: 0332
cst[48]: 0113
cst[68]: 0223

[$01^{-1}2^{-1}$]
cst[43]: 21100
cst[49]: 32200
cst[69]: 31100
cst[77]: 32211
cst[78]: 01133
cst[85]: 03311
cst[86]: 21133
cst[113]: 02211
cst[114]: 02233
cst[128]: 03322
cst[131]: 01122
cst[142]: 31122

Table 17 (continued)
Coset Array for $U_3(3)$ Over S_4
Label the single cosets in each double coset

[$0^{-1}12$]
cst[53]: 2210
cst[59]: 3320
cst[87]: 3310
cst[93]: 3321
cst[94]: 0013
cst[100]: 0031
cst[101]: 2213
cst[145]: 0021
cst[146]: 0023
cst[153]: 0032
cst[156]: 0012
cst[167]: 3312

[$0^{-1}12^{-1}$]
cst[54]: 00322
cst[60]: 00133
cst[88]: 00233

[$0^{-1}1^{-1}2$]
cst[56]: 33001
cst[62]: 22330
cst[89]: 33110
cst[91]: 22003
cst[95]: 33220
cst[97]: 00112
cst[102]: 33002
cst[104]: 00332
cst[147]: 00221
cst[150]: 00223
cst[158]: 00331
cst[161]: 00113

[$0^{-1}1^{-1}2^{-1}$]
cst[57]: 221100
cst[63]: 223300
cst[90]: 331100
cst[92]: 113300
cst[96]: 332200
cst[98]: 332211
cst[99]: 110033
cst[103]: 112200
cst[105]: 330011
cst[106]: 112233
cst[148]: 002211
cst[149]: 220033
cst[151]: 220011
cst[152]: 002233
cst[159]: 003311
cst[160]: 221133
cst[162]: 003322
cst[163]: 223311
cst[164]: 110022
cst[170]: 001133
cst[171]: 001122
cst[173]: 113322
cst[184]: 331122
cst[216]: 330022

[0102]
cst[126]: 3230
cst[134]: 2120
cst[194]: 3130
cst[196]: 0301
cst[197]: 2123
cst[204]: 3231
cst[205]: 0103
cst[221]: 0201
cst[224]: 0102
cst[231]: 0203
cst[233]: 0302
cst[245]: 3132

[0102^{-1}]
cst[127]: 32300
cst[135]: 21200
cst[175]: 03011
cst[180]: 01033
cst[183]: 02011
cst[188]: 01022
cst[195]: 31300
cst[198]: 21233
cst[203]: 02033
cst[206]: 32311
cst[208]: 03022
cst[241]: 31322

[0120]
cst[124]: 3123
cst[140]: 1321
cst[174]: 0210
cst[177]: 3213
cst[179]: 0230
cst[181]: 1231
cst[182]: 0310
cst[186]: 2312
cst[187]: 0320
cst[192]: 2132
cst[193]: 2012
cst[202]: 0130
cst[207]: 0120
cst[211]: 2032
cst[222]: 3013
cst[225]: 3023
cst[232]: 1031
cst[234]: 1021
cst[236]: 3103
cst[237]: 2102
cst[238]: 1301
cst[239]: 2302
cst[240]: 3203
cst[242]: 1201

[0121]
cst[122]: 1030
cst[132]: 2030
cst[137]: 3010
cst[141]: 2010
cst[190]: 1020
cst[191]: 0323
cst[200]: 3020
cst[201]: 1323
cst[209]: 0212
cst[212]: 3212
cst[243]: 2313
cst[244]: 0313

Table 17 (continued)
Coset Array for $U_3(3)$ Over S_4
Label the single cosets in each double coset

$[0123^{-1}]$
cst[120]: 01233
cst[139]: 03211
cst[185]: 03122
cst[189]: 01322
cst[210]: 02311
cst[223]: 02133

$[01^{-1}2^{-1}3]$
cst[129]: 322001
cst[143]: 211003
cst[199]: 033221
cst[213]: 311002
cst[215]: 011332
cst[246]: 022331

$[01^{-1}2^{-1}3^{-1}]$
cst[130]: 3220011
cst[144]: 2110033
cst[154]: 0113322
cst[168]: 0332211
cst[214]: 3110022
cst[228]: 0223311

$[0^{-1}120]$
cst[157]: 00130
cst[165]: 00230
cst[166]: 00310
cst[172]: 00210
cst[217]: 00320
cst[219]: 00120
cst[220]: 33023
cst[226]: 33123
cst[227]: 22012
cst[235]: 22312
cst[249]: 33013
cst[250]: 33213

$[0^{-1}123^{-1}]$
cst[155]: 332011
cst[169]: 221033
cst[218]: 003211
cst[229]: 331022
cst[230]: 001322
cst[251]: 002311

[01213]
cst[247]: 03231
cst[248]: 02123
cst[252]: 03132

Table 18

The Double Cosets $[w] = NwN$ in $U_3(3)$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 252)
[*]	N is transitive on {0,1,2,3}.		1
	$N^{(0)} = N^0 \cong S_3$ has orbits {0},{1,2,3}		
[0]	Nt_0, Nt_1, Nt_2, Nt_3 $[00^{-1}] = [*]$ $[00] = [0^{-1}]$		4
$[0^{-1}]$	$Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}, Nt_3^{-1}$ $[0^{-1}0] = [*]$ $[0^{-1}0^{-1}] = [0]$		4
	$N^{(01)} = N^{01} \cong S_2$ has orbits {0},{1},{2,3}		
[01]	0100=(3,2)0011	$\Rightarrow [010^{-1}] = [0^{-1}1^{-1}]$	12
$[01^{-1}]$	011~100 0110=(3,2)1 01100=(3,2)10	 $\Rightarrow [01^{-1}0] = [0]$ $\Rightarrow [01^{-1}0^{-1}] = [01]$	6
$[0^{-1}1]$	001~110 0010=(3,2)1100 00100=(3,2)11	 $\Rightarrow [0^{-1}10] = [0^{-1}1^{-1}]$ $\Rightarrow [0^{-1}10^{-1}] = [00]$	6
$[0^{-1}1^{-1}]$	00110=(3,2)01 001100=(3,2)010	$\Rightarrow [0^{-1}1^{-1}0] = [01]$ $\Rightarrow [0^{-1}1^{-1}0^{-1}] = [010]$	12
[010]	**010=101** 0100=(3,2)0011 0101=(3,2)1100 01011=10	 $\Rightarrow [010^{-1}] = [0^{-1}1^{-1}]$ $\Rightarrow [0101] = [0^{-1}1^{-}]$ $\Rightarrow [0101^{-1}] = [01]$	6
	$N^{(012)} = N^{012} \cong S_1$ has orbits {0},{1},{2},{3}		
[012]	01200=(1,3)03022 02322=(0,1,3)21133 0123=(0,3,2,1)221100	$\Rightarrow [0120^{-1}] = [0102^{-1}]$ $\Rightarrow [0121^{-1}] = [01^{-1}2^{-1}]$ $\Rightarrow [0123] = [0^{-1}1^{-1}2^{-1}]$	24

Table 18 (continued)

The Double Cosets [w] = NwN in $U_3(3)$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 252)
$[012^{-1}]$	0122~3211		12
	012200=(3,2)3123	$\Rightarrow [012^{-1}0^{-1}]=[0120]$	
	01221=(0,3)32	$\Rightarrow [012^{-1}1]=[01]$	
	012211=(0,3)321	$\Rightarrow [012^{-1}1^{-1}]\ [012]$	
	012233=(0,3,1)0210	$\Rightarrow [012^{-1}3^{-1}]=[0120]$	
$[01^{-1}2]$	0112~1002~1003~2331~2330~3221~3220~0113		3
	01130=(0,1,2,3)32200	$\Rightarrow [01^{-1}20]=[01^{-1}2^{-1}]$	
	011300=(0,1,2,3)322	$\Rightarrow [01^{-1}20^{-1}]=[01^{-1}]$	
	01131=(0,2,1)32211	$\Rightarrow [01^{-1}21]=[01^{-1}2^{-1}]$	
	011311=(0,2,1)322	$\Rightarrow [01^{-1}21^{-1}]=[01^{-1}]$	
	011322=(0,2,1,3)011	$\Rightarrow [01^{-1}23^{-1}]=[01^{-1}]$	
$[01^{-1}2^{-1}]$	01122~10022		12
	011220=(1,3,2)302	$\Rightarrow [01^{-1}2^{-1}0]=[012]$	
	0112200=(1,3,2)3020	$\Rightarrow [01^{-1}2^{-1}0^{-1}]=[0212]$	
	011221=(0,3)312	$\Rightarrow [01^{-1}2^{-1}1]=[012]$	
	0221122=(0,3)3212	$\Rightarrow [01^{-1}2^{-1}1^{-1}]=[0121]$	
$[0^{-1}12]$	0012~1102		12
	001200=(1,3,2)330022	$\Rightarrow [0^{-1}120^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	00212=(3,2)22012$^{(1,2)}$	$\Rightarrow [0^{-1}212]=[0^{-1}120]$	
	001211=(0,3)331122	$\Rightarrow [0^{-1}121^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	00132=(0,1,2)3220011$^{(3,2)}$	$\Rightarrow [0^{-1}123]=[01^{-1}2^{-1}3^{-1}]$	
$[0^{-1}12^{-1}]$	00122~11022~11033~22311~22300~33211~33200~00133		3
	001330=(0,1,2,3)332	$\Rightarrow [0^{-1}12^{-1}0]=[0^{-1}1]$	
	0013300=(0,1,2,3)3320	$\Rightarrow [0^{-1}12^{-1}0^{-1}]=[0^{-1}12]$	
	001331=(0,2,1)332	$\Rightarrow [0^{-1}12^{-1}1]=[0^{-1}1]$	
	0013311=(0,2,1)3321	$\Rightarrow [0^{-1}12^{-1}1^{-1}]=[0^{-1}12]$	
	001332=(0,2,1,3)001	$\Rightarrow [0^{-1}12^{-1}3]=[0^{-1}1]$	
$[0^{-1}1^{-1}2]$	00112~33221		12
	001120=(1,3)1321	$\Rightarrow [0^{-1}1^{-1}20]=[0120]$	
	001121=(0,3)332211	$\Rightarrow [0^{-1}1^{-1}21]=[0^{-1}1^{-1}2^{-1}]$	
	0011211=(0,3)3322	$\Rightarrow [0^{-1}1^{-1}21^{-1}]=[0^{-1}1^{-1}]$	
	001123=(0,3,2)2012	$\Rightarrow [0^{-1}1^{-1}23]=[0120]$	

Table 18 (continued)
The Double Cosets [w] = NwN in $U_3(3)$ where N = S_4

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 252)
$[0^{-1}1^{-1}2^{-1}]$	0011220=(1,3,2)0302	⇒ $[0^{-1}1^{-1}2^{-1}0]$ = [0102]	24
	00112200=(0,2,1)1021	⇒ $[0^{-1}1^{-1}2^{-1}0^{-1}]$ = [0120]	
	0022332=(0,1,3)2213	⇒ $[0^{-1}1^{-1}2^{-1}1]$ = $[0^{-1}12]$	
	00221122=(0,3,1)22312	⇒ $[0^{-1}2^{-1}1^{-1}2^{-1}]$ = $[0^{-1}120]$	
	00112233=(0,3,2,1)214	⇒ $[0^{-1}1^{-1}2^{-1}3^{-1}]$ [012]	
[0102]	**0102=1012**		12
	01020=(0,2,1)3023	⇒ [01020] = [0120]	
	010200=(1,3,2)003322	⇒ $[01020^{-1}]$ = $[0^{-1}1^{-1}2^{-1}]$	
	01021=(0,1,2)3123	⇒ [01021] = [0120]	
	010211=(0,3,2)113322	⇒ $[01021^{-1}1]$ = $[0^{-1}1^{-1}2^{-1}]$	
	01023=(0,1)3010	⇒ [01023] = [0212]	
$[0102^{-1}]$	**01022=10122**		12
	010220=(1,3)032	⇒ $[0102^{-1}0]$ = [012]	
	0102200=(1,3)0320	⇒ $[0102^{-1}0^{-1}]$ = [0120]	
	010221=(0,3)132	⇒ $[0102^{-1}1]$ = [012]	
	0102211(0,3)1321	⇒ $[0102^{-1}1^{-1}]$ = [0120]	
	0102233=(0,3,1)00210	⇒ $[0102^{-1}3^{-1}]$ = $[0^{-1}120]$	
[0120]	01200=(1,3)03022	⇒ $[0120^{-1}]$ = $[0102^{-1}]$	24
	01201=(0,2,1)110022	⇒ [01201] = $[0^{-1}1^{-1}2^{-1}]$	
	023022=(1,3,2)2123	⇒ $[01201^{-1}]$ = [0102]	
	01202=(0,2,1)3023	⇒ [01202] = [0120]	
	012022=(1,3,2)1321	⇒ $[01202^{-1}]$ = [0120]	
	01203=(0,2,3)0211	⇒ [01203] = $[012^{-1}]$	
[0121]	**0121=0212**		12
	021200=(1,2)2123	⇒ $[01210^{-1}]$ [0102]	
	02322=(0,1,3)21133	⇒ $[0121^{-1}]$ = $[01^{-1}2^{-1}]$	
	01313=(0,2,1)32211	⇒ [01212] = $[01^{-1}2^{-1}]$	
	012122=021	⇒ $[01212^{-1}]$ = [012]	
	021233=(0,1,2)22312	⇒ $[01213^{-1}]$ = $[0^{-1}120]$	

Table 18 (continued)
The Double Cosets [w] = NwN in $U_3(3)$ where $N = S_4$

Label [w]	Coset stabilizing subgroup $N^{(w)}$		Cosets (Total 252)
$[0123^{-1}]$	01233~10322~23011~32100		6
	012330=(0,2,1,3)321	$\Rightarrow [0123^{-1}0]=[012]$	
	0123300=(0,3,1)112233	$\Rightarrow [0123^{-1}0^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	0123311=(0,2)003322	$\Rightarrow [0123^{-1}1^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	012332=(0,1)103	$\Rightarrow [0123^{-1}2]=[012]$	
	0123322=(0,2,3)330011	$\Rightarrow [0123^{-1}2^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	0123333=(0,3,2,1)221100	$\Rightarrow [0123]=[0^{-1}1^{-1}2^{-1}]$	
$[01^{-1}2^{-1}3]$	011223~100223~100332~011332		6
	$0113320=(0,2,1)33123^{(3,2)}$	$\Rightarrow [01^{-1}2^{-1}30]=[0^{-1}120]$	
	$0113321=(0,1,2,3)33023^{(3,2)}$	$\Rightarrow [01^{-1}2^{-1}31]=[0^{-1}120]$	
	01133211=(0,2,1,3)32300	$\Rightarrow [01^{-1}2^{-1}31^{-1}]=[0102^{-1}]$	
	$01122322=(0,1)01133^{(0,1)}$	$\Rightarrow [01^{-1}2^{-1}32^{-1}]=[01^{-1}2^{-1}]$	
$[01^{-1}2^{-1}3^{-1}]$	0112233~1002233~1003322~0113322		6
	01133220=(0,3,1)332011	$\Rightarrow [01^{-1}2^{-1}3^{-1}0]=[0^{-1}123^{-1}]$	
	$011332200=(0,2,3,1)3321^{(3,2)}$	$\Rightarrow [01^{-1}2^{-1}3^{-1}0^{-1}]=[0^{-1}12]$	
	$01133221=(0,1,2)332011^{(3,2)}$	$\Rightarrow [01^{-1}2^{-1}3^{-1}1]=[0^{-1}123^{-1}]$	
	$011332211=(0,1,2)3320^{(3,2)}$	$\Rightarrow [01^{-1}2^{-1}3^{-1}1^{-1}]=[0^{-1}12]$	
	01122332=(0,2,1,3)01133	$\Rightarrow [01^{-1}2^{-1}3^{-1}2]=[01^{-1}2^{-1}]$	
$[0^{-1}120]$	00120~22102		12
	001200=(1,3,2)330022	$\Rightarrow [0^{-1}120^{-1}]=[0^{-1}1^{-1}2^{-1}]$	
	001201=(0,2,3)3020	$\Rightarrow [0^{-1}1201]=[0212]$	
	001202=(1,3,2)332200	$\Rightarrow [0^{-1}1201]=[0^{-1}1^{-1}2^{-1}]$	
	$0021011=(0,3,1)2210^{(1,2)}$	$\Rightarrow [0^{-1}1202^{-1}]=[0^{-1}12]$	
	001203=(0,2,3)02011	$\Rightarrow [0^{-1}1203]=[0102^{-1}]$	
	$0023011=(0,1,2,3)211003^{(0,3)}$	$\Rightarrow [0^{-1}1203^{-1}]=[01^{-1}2^{-1}3]$	
$[0^{-1}123^{-1}]$	001233~110233~110322~001322		6
	0013220=(0,3,1,2)3231	$\Rightarrow [0^{-1}123^{-1}0]=[0102]$	
	00132211=(0,3,1,2)1323	$\Rightarrow [0^{-1}123^{-1}1^{-1}]=[0121]$	
	$0012332=(0,1)0013^{(0,1)}$	$\Rightarrow [0^{-1}123^{-1}2]=[0^{-1}12]$	
	$00132233=(0,3,2,1)3220011^{(3,2)}$	$\Rightarrow [0^{-1}123^{-1}2^{-1}]=[01^{-1}2^{-1}3^{-1}]$	
	$00132222=(0,1,2)3220011^{(3,2)}$	$\Rightarrow [0^{-1}123]=[01^{-1}2^{-1}3^{-1}]$	
[01213]	01213~13032~10302~23031~20301 and **01213=31210=32120=02123**		3
	021230=(1,2,3)22012	$\Rightarrow [012130]=[0^{-1}120]$	
	0212300=3212	$\Rightarrow [012130^{-1}]=[0121]$	
	$021231=(0,2,1)00130^{(0,3)}$	$\Rightarrow [012131]=[0^{-1}120]$	
	$012132=(0,1,2)00230^{(0,3)}$	$\Rightarrow [012132]=[0^{-1}120]$	
	021233=(0,1,2)22312	$\Rightarrow [01213^{-1}]=[0^{-1}120]$	

Table 19
Action of x, y and t on the 252 Cosets

x	(2,4,8,5)(3,6,13,7)(9,20,37,21)(10,22,25,12)(11,23,44,24)(14,28,31,16)(15,29,55,30)(17,32,61,33)(18,34,40,35)(19,36)(26,50)(27,51,58,52)(38,72,118,73)(39,74,84,47)(41,45,81,76)(42,48)(43,77,128,78)(46,82,136,83)(49,85,131,86)(53,93,153,94)(54,60)(56,97,104,62)(57,98,162,99)(59,100,156,101)(63,105,171,106)(64,107,138,108)(65,109,112,67)(66,110,178,111)(69,113,142,114)(70,115,121,116)(71,117)(75,125)(79,119,176,133)(80,123)(87,145,167,146)(89,147,150,91)(90,148,173,149)(92,151,184,152)(95,158)(96,159,164,160)(102,161)(103,163,216,170)(120,139)(122,141,212,191)(124,179,238,193)(126,196,224,197)(127,175,188,198)(129,199,215,143)(130,168,154,144)(132,137,209,201)(134,204,233,205)(135,206,208,180)(140,211,236,174)(155,218,230,169)(157,172,235,220)(165,166,227,226)(177,187,232,237)(181,239,222,207)(182,234,192,240)(183,241,203,195)(185,189)(186,225,202,242)(190,243,200,244)(194,221,245,231)(210,223)(213,246)(214,228)(217,249,219,250)(229,251)(247,248)
y	(5,8)(7,13)(11,18)(12,19)(16,26)(17,27)(20,34)(21,40)(22,36)(24,37)(28,50)(29,51)(30,58)(33,55)(35,44)(38,64)(39,65)(41,75)(42,68)(43,69)(46,70)(47,71)(52,61)(53,87)(54,88)(56,89)(57,90)(62,95)(63,96)(66,79)(67,80)(72,119)(73,121)(74,123)(78,131)(81,125)(82,110)(83,138)(84,112)(85,113)(86,142)(91,102)(92,103)(94,156)(97,161)(98,163)(99,164)(100,145)(101,167)(104,150)(105,151)(106,173)(107,115)(108,176)(109,117)(111,118)(114,128)(116,178)(120,189)(122,190)(124,192)(132,200)(133,136)(134,194)(135,195)(137,141)(139,210)(140,181)(143,213)(144,214)(146,153)(147,158)(148,159)(149,216)(152,162)(157,219)(160,184)(165,217)(166,172)(168,228)(169,229)(170,171)(174,182)(175,183)(177,186)(179,187)(180,188)(185,223)(193,222)(196,221)(197,245)(198,241)(199,246)(202,207)(203,208)(205,224)(209,244)(211,225)(212,243)(218,251)(227,249)(231,233)(232,234)(235,250)(236,237)(238,242)(239,240)(248,252)
t_0	(1, 2, 3)(4, 9, 10)(5, 11, 12)(6, 14, 15)(7, 16, 17)(8, 18, 19)(13, 26, 27)(20, 38, 39)(21, 41, 30)(22, 42, 43)(23, 45, 32)(24, 46, 47)(25, 48, 49)(28, 53, 54)(29, 56, 57)(31, 59, 60)(33, 62, 63)(34, 64, 65)(35, 66, 67)(36, 68, 69)(37, 70, 71)(40, 75, 58)(44, 79, 80)(50, 87, 88)(51, 89, 90)(52, 91, 92)(55, 95, 96)(61, 102, 103)(72, 106, 120)(73, 122, 114)(74, 104, 124)(76, 126, 127)(77, 129, 130)(78, 108, 132)(81, 134, 135)(82, 137, 113)(83, 98, 139)(84, 97, 140)(85, 110, 141)(86, 143, 144)(93, 154, 155)(94, 157, 149)(99, 146, 165)(100, 166, 151)(101, 168, 169)(105, 145, 172)(107, 174, 175)(109, 158, 177)(111, 179, 180)(112, 161, 181)(115, 182, 183)(116, 184, 185)(117, 147, 186)(118, 187, 188)(119, 173, 189)(121, 190, 128)(123, 150, 192)(125, 194, 195)(131, 176, 200)(133, 202, 203)(136, 207, 208)(138, 163, 210)(142, 213, 214)(148, 196, 193)(152, 205, 211)(153, 217, 164)(156, 219, 216)(159, 221, 222)(160, 223, 178)(162, 224, 225)(167, 228, 229)(170, 231, 232)(171, 233, 234)(191, 230, 204)(197, 209, 218)(198, 199, 235)(201, 247, 220)(206, 215, 226)(212, 248, 227)(236, 239, 242)(237, 240, 238)(241, 246, 250)(243, 252, 249)(244, 251, 245)

Table 20

The Symmetric Generators of $U_3(3)$ Over S_4

t_0	(1, 2, 3)(4, 9, 10)(5, 11, 12)(6, 14, 15)(7, 16, 17)(8, 18, 19)(13, 26, 27)(20, 38, 39)(21, 41, 30)(22, 42, 43)(23, 45, 32)(24, 46, 47)(25, 48, 49)(28, 53, 54)(29, 56, 57)(31, 59, 60)(33, 62, 63)(34, 64, 65)(35, 66, 67)(36, 68, 69)(37, 70, 71)(40, 75, 58)(44, 79, 80)(50, 87, 88)(51, 89, 90)(52, 91, 92)(55, 95, 96)(61, 102, 103)(72, 106, 120)(73, 122, 114)(74, 104, 124)(76, 126, 127)(77, 129, 130)(78, 108, 132)(81, 134, 135)(82, 137, 113)(83, 98, 139)(84, 97, 140)(85, 110, 141)(86, 143, 144)(93, 154, 155)(94, 157, 149)(99, 146, 165)(100, 166, 151)(101, 168, 169)(105, 145, 172)(107, 174, 175)(109, 158, 177)(111, 179, 180)(112, 161, 181)(115, 182, 183)(116, 184, 185)(117, 147, 186)(118, 187, 188)(119, 173, 189)(121, 190, 128)(123, 150, 192)(125, 194, 195)(131, 176, 200)(133, 202, 203)(136, 207, 208)(138, 163, 210)(142, 213, 214)(148, 196, 193)(152, 205, 211)(153, 217, 164)(156, 219, 216)(159, 221, 222)(160, 223, 178)(162, 224, 225)(167, 228, 229)(170, 231, 232)(171, 233, 234)(191, 230, 204)(197, 209, 218)(198, 199, 235)(201, 247, 220)(206, 215, 226)(212, 248, 227)(236, 239, 242)(237, 240, 238)(241, 246, 250)(243, 252, 249)(244, 251, 245)
t_1	(1, 4, 6)(2, 23, 10)(3, 14, 32)(5, 34, 36)(7, 50, 51)(8, 20, 22)(9, 45, 15)(11, 82, 39)(12, 42, 85)(13, 28, 29)(16, 100, 54)(17, 56, 105)(18, 110, 65)(19, 68, 113)(21, 115, 117)(24, 119, 123)(25, 48, 77)(26, 145, 88)(27, 89, 151)(30, 158, 159)(31, 93, 60)(33, 161, 163)(35, 125, 52)(37, 72, 74)(38, 141, 69)(40, 107, 109)(41, 196, 175)(43, 64, 137)(44, 81, 61)(46, 162, 120)(47, 104, 211)(49, 129, 130)(53, 172, 90)(55, 97, 98)(57, 87, 166)(58, 147, 148)(59, 154, 155)(62, 179, 84)(63, 139, 118)(66, 238, 135)(67, 102, 239)(70, 152, 189)(71, 150, 225)(73, 232, 198)(75, 221, 183)(76, 204, 206)(78, 116, 243)(79, 242, 195)(80, 91, 240)(83, 181, 180)(86, 133, 244)(92, 134, 236)(94, 249, 160)(95, 187, 112)(96, 210, 111)(99, 197, 202)(101, 250, 170)(103, 194, 237)(106, 205, 192)(108, 216, 223)(114, 246, 228)(121, 234, 241)(122, 169, 233)(124, 173, 224)(126, 201, 230)(127, 215, 220)(128, 199, 168)(131, 178, 212)(132, 248, 157)(136, 209, 142)(138, 140, 188)(143, 165, 208)(144, 218, 153)(146, 214, 251)(149, 185, 176)(156, 227, 184)(164, 245, 207)(167, 235, 171)(174, 222, 186)(177, 182, 193)(190, 229, 231)(191, 247, 226)(200, 252, 219)(203, 213, 217)

Table 2.0 (continued)
The Symmetric Generators of $U_3(3)$ Over S_4

t_2	(1, 8, 13)(2, 40, 19)(3, 26, 58)(4, 44, 22)(5, 37, 25)(6, 28, 61)(7, 31, 55)(9, 121, 71)(10, 48,131)(11, 176, 80)(12, 42, 128)(14, 156, 60)(15, 95, 164)(16, 153, 54)(17, 102, 216)(18, 75, 27)(20, 81, 29)(21, 118, 84)(23, 136, 74)(24, 76, 33)(30, 104, 162)(32, 97, 171)(34, 178, 109)(35, 138, 112)(36, 68, 142)(38, 237, 127)(39, 62, 236)(41, 233, 208)(43, 70, 200)(45, 224, 188)(46, 239, 135)(47, 56, 238)(49, 79, 190)(50, 167, 88)(51, 147, 184)(52, 150, 173)(53, 219, 96)(57, 126, 242)(59, 217, 103)(63, 134, 240)(64, 170, 210)(65, 161, 222)(66, 159, 223)(67, 158, 232)(69, 213, 214)(72, 212, 113)(73, 105, 120)(77, 107, 209)(78, 215, 154)(82, 99, 139)(83, 201, 114)(85, 199, 168)(86, 111, 191)(87, 228, 229)(89, 182, 123)(90, 189, 133)(91, 202, 117)(92, 185, 115)(93, 235, 148)(94, 130, 230)(98, 145, 227)(100, 144, 218)(101, 226, 152)(106, 146, 220)(108, 211, 198)(110, 193, 206)(116, 192, 203)(119, 186, 183)(122, 248, 165)(124, 187, 234)(125, 245, 241)(129, 166, 180)(132, 169, 196)(137, 247, 172)(140, 207, 225)(141, 155, 205)(143, 157, 175)(149, 197, 179)(151, 204, 174)(160, 231, 181)(163, 221, 177)(194, 243, 251)(195, 246, 249)(244, 252, 250)
t_3	(1, 5, 7)(2, 21, 12)(3, 16, 30)(4, 35, 36)(6, 50, 52)(8, 24, 25)(9, 73, 47)(10, 48, 78)(11, 41, 17)(13, 31, 33)(14, 94, 60)(15, 62, 99)(18, 108, 67)(19, 68, 114)(20, 116, 117)(22, 42, 86)(23, 133, 123)(26, 146, 88)(27, 91, 149)(28, 101, 54)(29, 158, 160)(32, 161, 170)(34, 125, 51)(37, 76, 55)(38, 171, 139)(39, 97, 193)(40, 111, 112)(43, 143, 144)(44, 83, 84)(45, 205, 180)(46, 132, 69)(49, 66, 122)(53, 168, 169)(56, 174, 74)(57, 120, 136)(58, 150, 152)(59, 165, 92)(61, 104, 106)(63, 87, 157)(64, 236, 127)(65, 95, 237)(70, 240, 195)(71, 89, 242)(72, 177, 175)(75, 231, 203)(77, 115, 244)(79, 148, 185)(80, 147, 234)(81, 197, 198)(82, 222, 206)(85, 119, 243)(90, 126, 238)(93, 250, 159)(96, 194, 239)(98, 196, 186)(100, 249, 163)(102, 207, 109)(103, 223, 107)(105, 204, 182)(110, 164, 210)(113, 246, 228)(118, 191, 142)(121, 151, 189)(124, 208, 178)(128, 138, 201)(129, 172, 188)(130, 230, 156)(131, 215, 154)(134, 212, 218)(135, 199, 227)(137, 155, 224)(140, 184, 233)(141, 247, 166)(145, 214, 251)(153, 220, 173)(162, 167, 226)(176, 225, 241)(179, 232, 192)(181, 202, 211)(183, 213, 219)(187, 216, 245)(190, 252, 217)(200, 229, 221)(209, 248, 235)

Table 21

a,b∈G Satisfying Relations for $U_3(3)$

a	(5,8)(7,13)(11,18)(12,19)(16,26)(17,27)(20,34)(21,40)(22,36)(24,37)(28,50)(29,51)(30,58)(33,55)(35,44)(38,64)(39,65)(41,75)(42,68)(43,69)(46,70)(47,71)(52,61)(53,87)(54,88)(56,89)(57,90)(62,95)(63,96)(66,79)(67,80)(72,119)(73,121)(74,123)(78,131)(81,125)(82,110)(83,138)(84,112)(85,113)(86,142)(91,102)(92,103)(94,156)(97,161)(98,163)(99,164)(100,145)(101,167)(104,150)(105,151)(106,173)(107,115)(108,176)(109,117)(111,118)(114,128)(116,178)(120,189)(122,190)(124,192)(132,200)(133,136)(134,194)(135,195)(137,141)(139,210)(140,181)(143,213)(144,214)(146,153)(147,158)(148,159)(149,216)(152,162)(157,219)(160,184)(165,217)(166,172)(168,228)(169,229)(170,171)(174,182)(175,183)(177,186)(179,187)(180,188)(185,223)(193,222)(196,221)(197,245)(198,241)(199,246)(202,207)(203,208)(205,224)(209,244)(211,225)(212,243)(218,251)(227,249)(231,233)(232,234)(235,250)(236,237)(238,242)(239,240)(248,252)
b	(1,233,214,16,139,250)(2,169,119)(3,122,96,11,47,181)(4,102,115,48,56,246)(5,104,116)(6,176,216,62,129,241)(7,118,103,21,215,177)(8,248,133,68,218,125)(9,67,93,18,132,204)(10,239,190,46,180,173)(12,211,27,41,127,87)(13,175,164,91,74,243)(14,185,34,26,196,161)(15,149,225,59,99,191)(17,63,49)(19,157,71,66,60,202)(20,79,232,97,38,195)(22,159,58,143,188,194)(23,213,100,95,82,189)(24,83,101)(25,105,200,108,206,182)(28,134,249)(29,228,128,120,172,252)(30,220,43)(31,197,35)(32,217,240,126,166,112)(33,130,234,146,226,193)(36,153,80,73,210,184)(37,135,94,150,209,52)(39,242,231,98,170,55)(40,208,205,89,109,51)(42,158,50)(44,198,222,147,212,244)(45,203,113,70,238,230)(53,92,72)(54,236,229,168,123,142)(57,114,111,124,78,76)(61,117,131,223,171,251)(64,88,85,75,165,247)(65,145,219,179,224,154)(69,144,140)(77,121,162,155,110,183)(81,160,167,199,174,245)(84,141,221,148,163,156)(86,138,227)(90,107,207,192,136,106)(137,152,151,187,237,201)(178,235,186)

APPENDIX F

TABLES FOR GENERATION OF G OVER S_5

Table 22
Coset Array for G over S_5
Label the single cosets in each double coset

[*]
cst[1]: [ ]

[0]
cst[2]: 0
cst[3]: 1
cst[4]: 4
cst[5]: 2
cst[7]: 3

[01]
cst[6]: 10
cst[8]: 40
cst[9]: 20
cst[10]: 21
cst[11]: 04
cst[12]: 30
cst[13]: 01
cst[14]: 34
cst[15]: 31
cst[16]: 14
cst[17]: 32
cst[19]: 03
cst[21]: 43
cst[22]: 41
cst[23]: 24
cst[24]: 12
cst[27]: 23
cst[28]: 42
cst[30]: 13
cst[39]: 02

[010]
cst[20]: 040
cst[25]: 010
cst[35]: 030
cst[36]: 101
cst[37]: 434
cst[43]: 121
cst[44]: 404
cst[61]: 141
cst[62]: 424
cst[63]: 212
cst[65]: 343
cst[67]: 323
cst[70]: 020
cst[77]: 232
cst[79]: 303
cst[110]: 202
cst[111]: 131
cst[114]: 313
cst[123]: 414
cst[133]: 242

[012]
cst[18]: 210
cst[26]: 340
cst[29]: 310
cst[31]: 140
cst[32]: 320
cst[33]: 321
cst[34]: 104
cst[38]: 430
cst[40]: 410
cst[41]: 240
cst[42]: 120
cst[45]: 401
cst[46]: 234
cst[47]: 230
cst[48]: 420
cst[49]: 421
cst[50]: 204
cst[51]: 130
cst[52]: 201
cst[53]: 034
cst[54]: 431
cst[55]: 214
cst[56]: 432
cst[58]: 103
cst[60]: 043
cst[68]: 041
cst[69]: 324
cst[71]: 021
cst[72]: 304
cst[73]: 301
cst[74]: 134
cst[75]: 231
cst[76]: 014
cst[81]: 012
cst[83]: 243
cst[85]: 123
cst[86]: 341
cst[87]: 124
cst[88]: 031
cst[89]: 314
cst[90]: 032
cst[92]: 203
cst[94]: 143
cst[95]: 241
cst[96]: 024
cst[97]: 312
cst[99]: 423
cst[100]: 042
cst[102]: 213
cst[104]: 342
cst[121]: 102
cst[124]: 132
cst[125]: 403
cst[127]: 412
cst[130]: 023
cst[132]: 013
cst[144]: 402
cst[147]: 142
cst[148]: 413
cst[156]: 302

[0101]
cst[64]: 0101
cst[80]: 0404
cst[116]: 2121
cst[137]: 0303
cst[138]: 4343
cst[179]: 0202
cst[188]: 3232
cst[220]: 4141
cst[221]: 4242
cst[281]: 3131

[0102]
cst[66]: 4340
cst[78]: 1210
cst[112]: 1410
cst[113]: 4240
cst[115]: 2120
cst[117]: 3430
cst[118]: 0401
cst[119]: 3234

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

cst[120]: 3230
cst[134]: 2320
cst[135]: 2321
cst[136]: 0104
cst[180]: 1310
cst[181]: 2021
cst[182]: 0304
cst[183]: 0301
cst[184]: 3134
cst[185]: 3130
cst[186]: 3231
cst[187]: 1014
cst[190]: 4041
cst[191]: 2324
cst[192]: 1012
cst[194]: 4243
cst[196]: 2123
cst[197]: 4341
cst[198]: 2124
cst[201]: 4140
cst[211]: 2420
cst[212]: 3431
cst[213]: 1214
cst[214]: 3432
cst[215]: 2421
cst[217]: 0103
cst[219]: 4043
cst[282]: 0204
cst[283]: 3132
cst[284]: 0403
cst[286]: 1412
cst[288]: 4143
cst[290]: 2023
cst[291]: 4241
cst[292]: 2024
cst[293]: 4342
cst[295]: 1013
cst[298]: 0102
cst[299]: 3031
cst[300]: 2423
cst[302]: 1213
cst[308]: 0402
cst[314]: 0201
cst[315]: 3034
cst[329]: 1314
cst[330]: 4042
cst[333]: 3032
cst[416]: 0203
cst[418]: 4142
cst[422]: 1312
cst[429]: 0302
cst[431]: 1413

[0121]
cst[59]: 1040
cst[82]: 4010
cst[93]: 2040
cst[98]: 2010
cst[107]: 1030
cst[108]: 2101
cst[109]: 0434
cst[126]: 3040
cst[128]: 3010
cst[139]: 0121
cst[140]: 3404
cst[152]: 2030
cst[153]: 3101
cst[154]: 1434
cst[159]: 3121
cst[160]: 1404
cst[172]: 2141
cst[173]: 0424
cst[174]: 3212
cst[176]: 0343
cst[178]: 4323
cst[199]: 1020
cst[203]: 4030
cst[204]: 4101
cst[205]: 2434
cst[207]: 4121
cst[208]: 2404
cst[223]: 1232
cst[225]: 4303
cst[227]: 2343
cst[230]: 4020
cst[241]: 3141
cst[242]: 1424
cst[243]: 4212
cst[245]: 1343
cst[247]: 0323
cst[249]: 3020
cst[252]: 4232
cst[254]: 1303
cst[269]: 3202
cst[270]: 2131
cst[273]: 4313
cst[279]: 3424
cst[311]: 0414
cst[317]: 0141
cst[318]: 0212
cst[321]: 1323
cst[322]: 0232
cst[324]: 2303
cst[342]: 1242
cst[347]: 3242
cst[351]: 0131
cst[352]: 3414
cst[362]: 4202
cst[363]: 4131
cst[366]: 0313
cst[373]: 2414
cst[379]: 0242
cst[412]: 2313
cst[457]: 1202

[0123]
cst[57]: 3210
cst[84]: 4321
cst[91]: 4210
cst[101]: 4310
cst[103]: 4123
cst[105]: 4320
cst[106]: 0123
cst[122]: 4231
cst[129]: 4312
cst[131]: 2310
cst[141]: 3421
cst[142]: 0432
cst[143]: 3214
cst[145]: 3410
cst[146]: 4213
cst[149]: 4132
cst[150]: 0321
cst[151]: 0132
cst[155]: 3412
cst[157]: 2410
cst[158]: 3120
cst[161]: 4230
cst[162]: 0421
cst[163]: 0231
cst[164]: 3124
cst[165]: 0234
cst[166]: 3012
cst[167]: 3420

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

cst[168]: 0431
cst[169]: 0124
cst[171]: 4012
cst[200]: 0342
cst[202]: 2314
cst[206]: 4120
cst[209]: 0423
cst[210]: 0213
cst[228]: 4032
cst[231]: 4021
cst[232]: 0324
cst[233]: 0134
cst[234]: 2413
cst[235]: 4130
cst[236]: 0243
cst[237]: 3021
cst[239]: 0142
cst[248]: 4023
cst[250]: 0312
cst[251]: 2014
cst[256]: 0341
cst[257]: 3014
cst[259]: 0241
cst[261]: 2013
cst[263]: 4031
cst[264]: 4013
cst[312]: 0412
cst[316]: 0214
cst[328]: 0143
cst[355]: 3024
cst[356]: 0314
cst[380]: 0413

[01012]
cst[175]: 43432
cst[189]: 21210
cst[222]: 43430
cst[226]: 21213
cst[244]: 42423
cst[255]: 32321
cst[275]: 04043
cst[296]: 32320
cst[297]: 01014
cst[319]: 32324
cst[325]: 31312
cst[339]: 41410
cst[340]: 42420
cst[341]: 04041
cst[346]: 21214
cst[348]: 01012
cst[368]: 03034
cst[378]: 42421
cst[394]: 41413
cst[415]: 31310
cst[433]: 43431
cst[434]: 01013
cst[466]: 41412
cst[467]: 02021
cst[486]: 03031
cst[487]: 31314
cst[543]: 03032
cst[565]: 02024
cst[620]: 04042
cst[651]: 02023

[01020]
cst[193]: 04010
cst[218]: 01040
cst[285]: 03040
cst[287]: 03010
cst[303]: 10121
cst[304]: 43404
cst[336]: 01030
cst[337]: 12101
cst[338]: 40434
cst[407]: 14101
cst[417]: 02040
cst[420]: 04030
cst[421]: 42434
cst[424]: 14121
cst[425]: 42404
cst[435]: 01020
cst[439]: 21232
cst[440]: 20212
cst[441]: 34303
cst[443]: 32343
cst[448]: 04020
cst[455]: 02010
cst[480]: 12141
cst[481]: 40424
cst[482]: 23212
cst[484]: 30343
cst[485]: 34323
cst[496]: 41404
cst[572]: 13101
cst[597]: 02030
cst[598]: 41434
cst[600]: 10141
cst[601]: 43424
cst[603]: 31323
cst[604]: 20232
cst[605]: 13121
cst[607]: 32303
cst[609]: 31343
cst[614]: 03020
cst[621]: 12131
cst[622]: 40414
cst[626]: 21242
cst[629]: 23242
cst[633]: 10131
cst[634]: 43414
cst[664]: 23202
cst[666]: 30323
cst[667]: 20242
cst[668]: 34313
cst[669]: 24212
cst[672]: 24232
cst[683]: 31303
cst[743]: 21202
cst[759]: 13141
cst[760]: 41424
cst[763]: 32313
cst[773]: 14131
cst[774]: 42414
cst[777]: 30313
cst[811]: 24202

[01023]
cst[170]: 23214
cst[195]: 32340
cst[216]: 23210
cst[229]: 32341
cst[238]: 21234
cst[240]: 32314
cst[258]: 21243
cst[260]: 31324
cst[262]: 34321
cst[265]: 31342
cst[266]: 24213
cst[267]: 34320
cst[268]: 12103
cst[289]: 31340
cst[294]: 32310

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

cst[301]: 23240
cst[305]: 42430
cst[306]: 43401
cst[307]: 21230
cst[309]: 43410
cst[310]: 21240
cst[313]: 42431
cst[326]: 23241
cst[327]: 13124
cst[331]: 34310
cst[332]: 12140
cst[334]: 24210
cst[335]: 12104
cst[350]: 43402
cst[353]: 24231
cst[354]: 34312
cst[357]: 43421
cst[358]: 10123
cst[359]: 42413
cst[360]: 43420
cst[361]: 21203
cst[374]: 12134
cst[381]: 41432
cst[382]: 32304
cst[383]: 10132
cst[384]: 41423
cst[385]: 20213
cst[386]: 40432
cst[387]: 42403
cst[388]: 20231
cst[389]: 30324
cst[390]: 13102
cst[391]: 40431
cst[393]: 01042
cst[419]: 31320
cst[423]: 14120
cst[426]: 41430
cst[427]: 42401
cst[428]: 20234
cst[430]: 42410
cst[432]: 21204
cst[436]: 24230
cst[437]: 34301
cst[438]: 12130
cst[444]: 03041
cst[445]: 04012
cst[447]: 10124
cst[449]: 04021
cst[450]: 32301
cst[451]: 10134
cst[453]: 12143
cst[454]: 03042
cst[456]: 14123
cst[468]: 34302
cst[469]: 02013
cst[470]: 43412
cst[471]: 13140
cst[472]: 40421
cst[473]: 23204
cst[474]: 23201
cst[475]: 01034
cst[476]: 30321
cst[477]: 13104
cst[479]: 01043
cst[502]: 04013
cst[506]: 30342
cst[507]: 13120
cst[508]: 40423
cst[509]: 14132
cst[510]: 04032
cst[512]: 03024
cst[513]: 31302
cst[514]: 04031
cst[516]: 10142
cst[534]: 01023
cst[545]: 13142
cst[546]: 02043
cst[549]: 02034
cst[550]: 30312
cst[551]: 20214
cst[552]: 14102
cst[553]: 03014
cst[555]: 20241
cst[558]: 02041
cst[559]: 30341
cst[563]: 01032
cst[564]: 02031
cst[595]: 40413
cst[599]: 41420
cst[610]: 03012
cst[612]: 20243
cst[615]: 03021
cst[616]: 31304
cst[617]: 14130
cst[619]: 10143
cst[623]: 40412
cst[625]: 01024
cst[637]: 04023
cst[638]: 14103
cst[653]: 02014
cst[654]: 41402
cst[655]: 24201
cst[657]: 24203
cst[694]: 41403
cst[716]: 30314

[01210]
cst[177]: 04340
cst[224]: 01210
cst[272]: 04240
cst[276]: 03430
cst[277]: 10401
cst[278]: 43234
cst[344]: 12321
cst[345]: 40104
cst[372]: 03230
cst[398]: 10301
cst[399]: 43134
cst[403]: 43034
cst[404]: 14041
cst[405]: 42324
cst[406]: 21012
cst[408]: 34243
cst[410]: 32123
cst[442]: 40304
cst[452]: 04140
cst[458]: 01410
cst[459]: 02120
cst[463]: 02320
cst[483]: 10201
cst[488]: 41014
cst[492]: 23432
cst[493]: 12421
cst[495]: 30103
cst[497]: 34043
cst[499]: 12021
cst[504]: 01310
cst[522]: 03130
cst[531]: 14341
cst[532]: 42124
cst[544]: 02420
cst[571]: 21412
cst[573]: 34143
cst[575]: 32023
cst[582]: 32423
cst[583]: 20102

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

cst[584]: 13031
cst[586]: 31213
cst[588]: 23132
cst[608]: 40204
cst[628]: 30403
cst[639]: 13231
cst[644]: 13431
cst[645]: 41214
cst[671]: 21312
cst[674]: 31013
cst[679]: 24342
cst[680]: 23032
cst[684]: 24042
cst[704]: 14241
cst[705]: 42024
cst[710]: 20402
cst[721]: 41314
cst[744]: 24142
cst[770]: 30203
cst[812]: 20302
cst[835]: 31413

[01213]
cst[246]: 14340
cst[253]: 31210
cst[271]: 21410
cst[274]: 32120
cst[280]: 43230
cst[320]: 24340
cst[323]: 41210
cst[343]: 12320
cst[349]: 23430
cst[364]: 31410
cst[365]: 14240
cst[367]: 42120
cst[369]: 13430
cst[370]: 20401
cst[371]: 03234
cst[375]: 42320
cst[376]: 42321
cst[377]: 20104
cst[395]: 21310
cst[396]: 32021
cst[397]: 10304
cst[400]: 43130
cst[401]: 43231
cst[402]: 21014
cst[411]: 34240
cst[413]: 04341
cst[414]: 32124
cst[460]: 30401
cst[461]: 13234
cst[462]: 13230
cst[464]: 02321
cst[465]: 30104
cst[489]: 12420
cst[490]: 23431
cst[491]: 01214
cst[498]: 32420
cst[500]: 34041
cst[501]: 12324
cst[505]: 34140
cst[511]: 01312
cst[517]: 41310
cst[518]: 42021
cst[519]: 20304
cst[520]: 20301
cst[521]: 03134
cst[523]: 03231
cst[524]: 31014
cst[525]: 24041
cst[526]: 02324
cst[527]: 31012
cst[529]: 04243
cst[530]: 42123
cst[533]: 24140
cst[535]: 03431
cst[536]: 31214
cst[537]: 03432
cst[538]: 32421
cst[539]: 41312
cst[540]: 20103
cst[542]: 14043
cst[548]: 01213
cst[556]: 02123
cst[560]: 04143
cst[566]: 10204
cst[567]: 43132
cst[568]: 41012
cst[569]: 10403
cst[576]: 04241
cst[577]: 32024
cst[578]: 04342
cst[579]: 34241
cst[580]: 41213
cst[581]: 21013
cst[591]: 40301
cst[592]: 23134
cst[593]: 23130
cst[594]: 10402
cst[596]: 34142
cst[613]: 02423
cst[632]: 01412
cst[636]: 01413
cst[641]: 14243
cst[642]: 24341
cst[643]: 02124
cst[646]: 13432
cst[647]: 02421
cst[648]: 40103
cst[650]: 24043
cst[656]: 04142
cst[660]: 03132
cst[675]: 23031
cst[676]: 01314
cst[677]: 34042
cst[678]: 21413
cst[685]: 43031
cst[686]: 21314
cst[687]: 40102
cst[688]: 12423
cst[689]: 14342
cst[692]: 40201
cst[693]: 23034
cst[698]: 30204
cst[699]: 20403
cst[701]: 31412
cst[703]: 42023
cst[706]: 41013
cst[707]: 30102
cst[713]: 30201
cst[714]: 13034
cst[715]: 14042
cst[717]: 43032
cst[733]: 10203
cst[740]: 10302
cst[741]: 40302
cst[754]: 24143
cst[755]: 12023
cst[756]: 12024
cst[758]: 40203
cst[792]: 30402
cst[795]: 13032

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

[010203]
cst[528]: 212423
cst[541]: 242123
cst[570]: 141312
cst[602]: 424340
cst[606]: 141210
cst[627]: 212320
cst[630]: 323430
cst[640]: 313432
cst[649]: 343132
cst[665]: 121410
cst[670]: 232120
cst[673]: 343230
cst[700]: 242321
cst[702]: 212324
cst[708]: 323034
cst[709]: 101312
cst[718]: 232124
cst[719]: 303234
cst[720]: 131012
cst[729]: 303432
cst[734]: 141213
cst[735]: 131412
cst[736]: 202423
cst[737]: 030201
cst[753]: 414342
cst[757]: 121413
cst[761]: 414340
cst[762]: 434240
cst[764]: 030401
cst[765]: 313234
cst[766]: 313230
cst[767]: 131210
cst[768]: 202321
cst[769]: 030104
cst[771]: 313430
cst[776]: 121315
cst[779]: 212420
cst[780]: 323431
cst[781]: 101214
cst[782]: 232420
cst[783]: 434041
cst[786]: 434140
cst[791]: 424043
cst[796]: 434142
cst[797]: 404243
cst[798]: 232021
cst[813]: 010304
cst[814]: 343130
cst[815]: 242120
cst[816]: 343231
cst[817]: 121014
cst[818]: 242320
cst[819]: 404341
cst[823]: 424143
cst[827]: 202123
cst[828]: 414243
cst[832]: 232421
cst[833]: 101213
cst[836]: 323134
cst[837]: 414043
cst[838]: 101412
cst[839]: 040201
cst[843]: 121013
cst[844]: 141012
cst[845]: 242021
cst[846]: 020401
cst[852]: 404342
cst[859]: 131214
cst[860]: 202324
cst[861]: 030102
cst[862]: 242023
cst[863]: 020301
cst[864]: 313034
cst[866]: 424140
cst[868]: 121314
cst[869]: 424341
cst[879]: 020403
cst[880]: 303231
cst[881]: 232024
cst[882]: 010302
cst[883]: 434241
cst[884]: 131410
cst[885]: 414240
cst[886]: 040301
cst[887]: 323130
cst[889]: 202124
cst[890]: 020104
cst[891]: 202421
cst[892]: 040103
cst[894]: 424041
cst[896]: 141310
cst[899]: 010204
cst[902]: 323031
cst[903]: 101314
cst[904]: 434042
cst[905]: 343031
cst[906]: 040102
cst[914]: 040203
cst[915]: 303132
cst[919]: 404143
cst[921]: 010403
cst[923]: 404241
cst[924]: 131014
cst[925]: 303431
cst[926]: 010402
cst[929]: 030204
cst[930]: 313032
cst[932]: 020304
cst[934]: 343032
cst[935]: 212023
cst[939]: 101413
cst[946]: 020103
cst[948]: 303134
cst[951]: 010203
cst[952]: 030402
cst[956]: 414042
cst[958]: 040302
cst[960]: 212024
cst[962]: 141013
cst[969]: 404142

[010232]
cst[392]: 121030
cst[409]: 434101
cst[446]: 434010
cst[478]: 121040
cst[494]: 121404
cst[503]: 434020
cst[515]: 212030
cst[547]: 323040
cst[554]: 424030
cst[557]: 131020
cst[561]: 232141
cst[562]: 010424
cst[574]: 434202
cst[585]: 424101
cst[587]: 343101
cst[589]: 040212
cst[590]: 323404
cst[611]: 424010
cst[618]: 212040
cst[624]: 343010
cst[631]: 040121
cst[635]: 323010
cst[652]: 343020

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

cst[658]: 232040
cst[659]: 232010
cst[661]: 131040
cst[662]: 232101
cst[663]: 010434
cst[681]: 121303
cst[682]: 010343
cst[690]: 040131
cst[691]: 323414
cst[695]: 313020
cst[696]: 323141
cst[697]: 101424
cst[711]: 141202
cst[712]: 424303
cst[722]: 212434
cst[723]: 141020
cst[724]: 030141
cst[725]: 313424
cst[726]: 242131
cst[727]: 020414
cst[728]: 343202
cst[730]: 010323
cst[732]: 404313
cst[738]: 212303
cst[739]: 040313
cst[742]: 323101
cst[745]: 030212
cst[746]: 313040
cst[747]: 313404
cst[748]: 404212
cst[749]: 232404
cst[750]: 101323
cst[752]: 212343
cst[772]: 030121
cst[775]: 101434
cst[778]: 404121
cst[784]: 101232
cst[787]: 434121
cst[788]: 212404
cst[789]: 141030
cst[790]: 131404
cst[793]: 131202
cst[794]: 414303
cst[799]: 404131
cst[800]: 232414
cst[801]: 414020
cst[802]: 242010
cst[803]: 242030
cst[804]: 121434
cst[805]: 343121
cst[806]: 242101
cst[807]: 020434
cst[808]: 343212
cst[810]: 404323
cst[820]: 010242
cst[822]: 404232
cst[824]: 101242
cst[826]: 424313
cst[829]: 414030
cst[830]: 424131
cst[831]: 202414
cst[834]: 141303
cst[840]: 202313
cst[841]: 030414
cst[842]: 313242
cst[847]: 414202
cst[848]: 202131
cst[850]: 414323
cst[851]: 303242
cst[853]: 020313
cst[857]: 303414
cst[858]: 303424
cst[870]: 434212
cst[871]: 242303
cst[872]: 141323
cst[874]: 202434
cst[875]: 202343
cst[876]: 303212
cst[877]: 121343
cst[878]: 303141
cst[895]: 141232
cst[897]: 101343
cst[898]: 040323
cst[900]: 010232
cst[901]: 303121
cst[907]: 202141
cst[909]: 040232
cst[910]: 030424
cst[911]: 020343
cst[912]: 242313
cst[913]: 020141
cst[916]: 020131
cst[917]: 131424
cst[931]: 313202
cst[933]: 030242
cst[938]: 414232
cst[953]: 131242

[0102030]
cst[865]: 0302010
cst[888]: 0304010
cst[893]: 0301040
cst[922]: 0103040
cst[936]: 0402010
cst[940]: 0204010
cst[947]: 0301020
cst[949]: 0203010
cst[955]: 0204030
cst[957]: 0103020
cst[959]: 0403010
cst[961]: 0102030
cst[963]: 0201040
cst[964]: 0401030
cst[965]: 0304020
cst[966]: 0401020
cst[967]: 0102040
cst[972]: 0402030
cst[974]: 0104030
cst[975]: 0302040
cst[976]: 0104020
cst[978]: 0203040
cst[979]: 0201030
cst[980]: 0403020

[0102320]
cst[731]: 0104240
cst[751]: 0402120
cst[785]: 0401210
cst[809]: 0104340
cst[821]: 0103430
cst[825]: 0401310
cst[849]: 0301410
cst[854]: 0204140
cst[855]: 0103230
cst[856]: 0301210
cst[867]: 0403130
cst[873]: 0302120
cst[908]: 0203130
cst[918]: 0204340
cst[920]: 0302420
cst[927]: 0102420
cst[928]: 0203430
cst[937]: 0304140
cst[941]: 0304240
cst[942]: 0201310

Table 22 (continued)
Coset Array for G over S_5
Label the single cosets in each double coset

```
cst[ 943]: 4341214
cst[ 944]: 0201410
cst[ 945]: 4243134
cst[ 950]: 0403230
cst[ 954]: 4342124
cst[ 968]: 0102320
cst[ 970]: 4241314
cst[ 971]: 0402320
cst[ 973]: 4143234
cst[ 977]: 4142324
```

Table 23

The Double Cosets [w] = NwN in G where N = S_5

Label [w]	Coset stabilizing subgroup $N^{(w)}$	Cosets (Total 980)
[*]	N is transitive on {0,1,2,3,4}.	1
	$N^{(0)} = N^0 \cong S_4$ has orbits {0},{1,2,3,4}	
[0]	Nt_0, Nt_1, Nt_2, Nt_3, Nt_4	5
	$N^{(01)} = N^{01} \cong S_3$ has orbits {0},{1},{2,3,4}	
[01]		20
[010]		20
[012]	0120=(012)(43)(12)021 ⇒ [0120]=[012]	60
	$N^{(010)} = N^{010} \cong S_3$ has orbits {0},{1},{2,3,4}	
[0101]	0101~1010 01010=(01)(43)101 ⇒ [01010]=[010]	10
[0102]	01021=(021)(43)(02)2120 ⇒ [01021]=[0102]	60
	$N^{(012)} = N^{012} \cong S_2$ has orbits {0},{1},{2},{3,4}	
[0121]	01212=(12)(43)(04132)43430 ⇒ [01212]=[01012]	60
	$N^{(0123)} = N^{0123} \cong S_1$ has orbits {0},{1},{2},{3},{4}	
[0123]	0123~2104 01230=(02431)(02134)23214 ⇒ [01230]=[01023] 01231=(04)(123)(04)(23)4132 ⇒ [01231]=[0123] 01232=(04)(23)(142)04013 ⇒ [01232]=[01023] 01234=(03142)(02)210 ⇒ [01234]=[012]	60
	$N^{(0101)} \geq \langle S_3, (01)\rangle$ has orbits {0,1},{2,3,4}	
[01012]	01012~10102 ⇒ $(0,1) \in N^{(012)}$ ⇒ $N^{(012)} \geq \{(0,1),(3,4)\}$ 010120=(012)(43)(012)12021 ⇒ [010120]=[01210] 010123=(01)(43)(023)(14)2434 ⇒ [010123]=[0121]	30
	$N^{(0102)}$ has orbits {0},{1},{2},{3,4}	
[01020]	010201=(012)(43)(02)21202 ⇒ [010201]=[01020] 010202=(012)(43)(12)02120 ⇒ [010202]=[01210]	60

Table 23 (continued)

The Double Cosets [w]= NwN in G where N = S_5

Label [w]	Coset stabilizing subgroup $N^{(w)}$	Cosets	(Total 980)
[01023]	010230=(1243)(13)(42)03041	⇒ [010230]=[01023]	120
	010231=(02)(143)(01)(23)10302	⇒ [010231]=[01213]	
	010234=(01)(43)(124)0243	⇒ [010234]=[0123]	
	$N^{(0121)}$ has orbits {0},{1},{2},{3,4}		
[01210]	012101=(012)(43)(12)02010	⇒ [012101]=[01020]	60
	012102=(021)(43))(021)20201	⇒ [012102]=[01012]	
	012103=(12)(43)(42)010434	⇒ [012103]=[010232]	
[01213]	012130=(024)(13)(01)(23)10132	⇒ [012130]=[01023]	120
	012131=(014)(134)030204	⇒ [012131]=[010203]	
	012132=(04)(132)(04)(13)43231	⇒ [012132]=[01213]	
	012134=(0143)(13)03231	⇒ [012134]=[01213]	
	$N^{(01020)}$ has orbits {0},{1},{2},{3,4}		
[010203]	0102031=(0314)(03241)303424	⇒ [0102031]=[010232]	120
	0102032=(032)(14)(03)(14)343230	⇒ [0102032]=[010203]	
	0102034=(034)(143)04241	⇒ [0102034]=[01213]	
	$N^{(01023)}$ has orbits {0},{1},{2},{3},{4}		
[010232]			
	0102321=(02)(143)(02)(43)212040	⇒ [0102321]=[010232]	120
	0102323=(14)(23)(432)01410	⇒ [0102323]=[01210]	
	0102324=(0124)(01423)141310	⇒ [0102324]=[010203]	
	$N^{(010203)}$ has orbits {0},{1},{2},{3,4}		
[0102030]			
	0102030~1310141~2024212~3431323~4243404		24
	01020301=(04)(12)(01342)131014	⇒ [01020301]=[010203]	
	01020302=(04)(23)(02431)202421	⇒ [01020302]=[010203]	
	01020303=(043)(03214)343132	⇒ [01020303]=[010203]	
	01020304=(014)(04123)424340	⇒ [01020304]=[010203]	
	$N^{(010232)}$ has orbits {0},{1},{2},{3,4}		
[0102320]			
	0102320~1213031~2320102~3031213		30
	01023201=(03)(42)(0123)121303	⇒ [01023201]=[010232]	
	01023202=(04231)(02)(13)232010	⇒ [01023202]=[010232]	
	01023203=(14)(23)(0321)303121	⇒ [01023203]=[010232]	
	01023204=(02)(143)(13)0302120	⇒ [01023204]=[0102320]	

APPENDIX G

MAGMA SCRIPTS

```
/* cst34S4.h for 3*4:S(4)                                              */
/* cst[] holds single cosets making up double cosets of Cayley graph */
G := K;
f, G1, K := CosetAction(G, sub<G|x,y>);
ltrs := Degree(G1);

ts := [ (t^(x^i)) @ f : i in [1 .. 4] ];

cst := [null : i in [1 .. ltrs]] where null is [Integers() | ];
xcst := cst;
/*******************************************************************/
/*                        functions/procedures                       */
/* prodim := function(pt, Q, I) Image of pt under permutations Q[I]  */
/* magma_zero := function(rep,tf) Convert coset rep 0 to magma zero  */
/* csti   := function()                  initialize cst              */
/* cstq   := function(cst, detail)  query cst detail=true/false      */
/* cstx   := function(cst, xcst) compare cst with xcst:=previous cst */
/* dc_i   := function(lbl,cst,ts)                                    */
/* dc_ij  := function(lbl,cst,ts)                                    */
/* dc_ijk := function(lbl,cst,ts)                                    */
/* dcpath := function(lbl,cst,ts)                                    */
/* getpath := function(cst,ts,coset_index)                           */
/*******************************************************************/
prodim := function(pt, Q, I)    // Image of pt under permutations Q[I]
  v := pt;
  for i in I do
     v := v^(Q[i]);
  end for;
 return v;
end function;
/*******************************************************************/
magma_zero := function(rep,tf)   //Convert double coset representative
0 to magma zero
   zero := 4;   // for S(4)
   if tf then
      for i := 1 to #rep do
         if rep[i] eq 0 then rep[i] := zero; end if;
      end for;
   else
      for i := 1 to #rep do
         if rep[i] eq zero then rep[i] := 0; end if;
      end for;
   end if;
   return rep;
end function;
/*******************************************************************/
csti:= function()         // initialize cst
   cst := [null : i in [1 .. ltrs]] where null is [Integers() | ];
   return cst;
end function;
```

```
/*******************************************************************/
cstq := function(cst, detail)   // query cst   detail = true/false
   cnt := 0;
   for i := 1 to #cst do
      if not IsEmpty(cst[i]) then
         cnt := cnt + 1;
         if detail then
            Sprintf("%4o: %o",i,cst[i]);
         end if;
      end if;
   end for;
   return cnt;
end function;
/*******************************************************************/
cstx := function(cst, xcst)   // compare cst with  xcst:= previous cst
   cnt := 0;
   indx := 1;
   cst_rep  := [];
   xcst_rep := [];
if cstq(xcst,false) ne 0  then
   for i := 1 to #xcst do
      if xcst[i] ne [] then
         xcst_cmp := xcst[i];  // keep the lowest coset representative
         xcst_cmp := magma_zero(xcst_cmp, false);
         if xcst_rep eq [] then
            xcst_rep := xcst_cmp;
            indx := i;
         elif xcst_cmp lt xcst_rep then
            xcst_rep := xcst_cmp;
            indx := i;
         end if;

         if cst[i] ne [] then
            cnt := cnt + 1;
            cst_cmp  := cst[i]; // keep the lowest coset representative
            cst_cmp := magma_zero(cst_cmp, false);
            if cst_rep eq [] then
               cst_rep := cst_cmp;
            elif cst_cmp lt cst_rep then
               cst_rep := cst_cmp;
            end if;
         end if;
      end if;
   end for;
   if (cnt gt 0) or (xcst[1] ne []) then  // check [*] also
      txt := Sprintf(" cst[%4o]: %-30o= %-40o",indx,cst_rep,xcst_rep);
   else  txt :=
  Sprintf("cst[%4o]:%-20oAdd %ocosets",indx,xcst_rep,cstq(xcst,false));
   end if;
   cmd := Sprintf("%4o:%o ",cstq(cst,false),txt);
   Write("rpt_cst",cmd);
end if; // xcst empty?
 return cnt;
end function;
```

```
/*********************************************************************/
dc_i := function(lbl,cst,ts)
   for i := 1 to 4 do
      lbl1 := [];
      for indx := 1 to #lbl do
         if lbl[indx] eq 0 then Append(~lbl1,i);
         end if;
      end for;
      cst[prodim(1, ts, lbl1)] := lbl1;
   end for;
   return cst;
end function;
/*********************************************************************/
dc_ij := function(lbl,cst,ts)
   for i := 1 to 4 do
      for j in {1, 2, 3, 4} diff {i} do
         lbl1 := [];
         for indx := 1 to #lbl do
            if lbl[indx] eq 0 then Append(~lbl1,i);
               elif lbl[indx] eq 1 then Append(~lbl1,j);
            end if;
         end for;
         cst[prodim(1, ts, lbl1)] := lbl1;
      end for;
   end for;
   return cst;
end function;
/*********************************************************************/
dc_ijk := function(lbl,cst,ts)
   for i := 1 to 4 do
      for j in {1, 2, 3, 4} diff {i} do
         for k in {1, 2, 3, 4} diff {i,j} do
            lbl1 := [];
            for indx := 1 to #lbl do
               if lbl[indx] eq 0 then Append(~lbl1,i);
                  elif lbl[indx] eq 1 then Append(~lbl1,j);
                     elif lbl[indx] eq 2 then Append(~lbl1,k);
               end if;
            end for;
            cst[prodim(1, ts, lbl1)] := lbl1;
         end for;
      end for;
   end for;
   return cst;
end function;
```

```
/*******************************************************************/
dc_ijkl := function(lbl,cst,ts)
   for i := 1 to 4 do
      for j in {1, 2, 3, 4} diff {i} do
         for k in {1, 2, 3, 4} diff {i,j} do
            for l in {1,2,3,4} diff {i,j,k} do
               lbl1 := [];
               for indx := 1 to #lbl do
                  if lbl[indx] eq 0 then Append(~lbl1,i);
                     elif lbl[indx] eq 1 then Append(~lbl1,j);
                        elif lbl[indx] eq 2 then Append(~lbl1,k);
                           elif lbl[indx] eq 3 then Append(~lbl1,l);
                  end if;
               end for;
               cst[prodim(1, ts, lbl1)] := lbl1;
            end for;
         end for;
      end for;
   end for;
   return cst;
end function;
/*******************************************************************/
dcpath := function(lbl,cst,ts)
//                        Syntax check for double coset label
if lbl[1] ne 0 then
cmd := Sprintf("Syntax error: Double coset %o must start with 0",lbl);;
   Write("rpt_err",cmd);
   return cst;
end if;
dctst := 1;
for indx := 2 to (#lbl) do
   if lbl[indx] gt dctst then
      cmd := Sprintf("Syntax error: Double coset %o must use %o
                                before using %o",lbl, dctst, dctst+1);
      Write("rpt_err",cmd);
      return cst;
   elif lbl[indx] eq dctst then
      dctst := dctst + 1;
   end if;
end for;

casex := #Seqset(lbl);
if casex eq 1 then cst := dc_i(lbl, cst,ts);
   elif casex eq 2 then cst := dc_ij(lbl, cst,ts);
      elif casex eq 3 then cst := dc_ijk(lbl, cst,ts);
         elif casex eq 4 then cst := dc_ijkl(lbl, cst,ts);
            else "Write a procedure for this double coset.";
end if;
return cst;
end function;
```

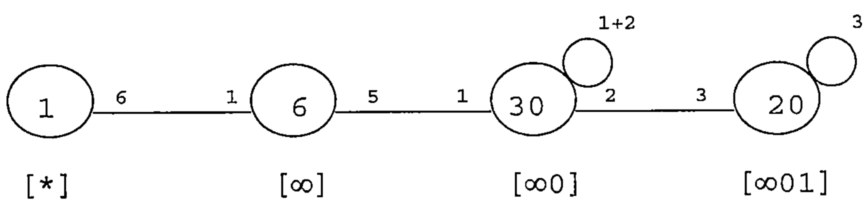
```
/*****************************************************************/
getpath := function(cst,ts,coset_index)
   coset_index := coset_index -1;  //because cst[1] eq [] which is [*]
   System("rm -f rpt_err");
   System("rm -f rpt_cst");

paths:=[ [0],[0,0],[1],[1,1],[2],[2,2],[3],[3,3] ];//perm of dble coset
   dc_lbl := [ [] ];            //Start testing paths from [*] dbl coset
   indx := 0;
   txt := "\nDone!";
   cst_cnt := cstq(cst,false);
   cst_index := [ [ ] ];      // the double cosets

   while cst_cnt lt coset_index do
      indx := indx + 1;
      if indx gt #dc_lbl then
txt := "\nError: No more dble cosets to check. But cst is not filled.";
         break;
      else
         cmd := Sprintf("%o\n%o", "_"^80, dc_lbl[indx]);
         Write("rpt_cst",cmd);
         for i in paths do
            tstdc := dc_lbl[indx];
            for j in i do
               Append(~tstdc,j);
            end for;

            xcst := csti();
   xcst := dcpath(tstdc,xcst,ts);  //Test new dbl coset for equivalence
   if (cstq(xcst,false) ne 0) and (xcst[1] eq []) then //check [*] also
               cnt  := cstx(cst,xcst);
               if cnt eq 0 then
         cst  := dcpath(tstdc,cst,ts);    //Add new dbl coset to cst
         Append(~dc_lbl,tstdc);    //Must test all paths from dbl coset

                  new_row := true;
               for j := 1 to #xcst do          //Identify double cosets
                     if not IsEmpty(xcst[j]) then
                        if new_row then
                           Append(~cst_index, [j]);
                           new_row := false;
                        else
                           Append(~cst_index[#cst_index],j);
                        end if;
                     end if;
                  end for;

               end if;
            end if;
         end for;
      end if;
      cst_cnt := cstq(cst,false);
   end while;
```

```
   cmd := Sprintf("\n%o\n", "_"^80);
   Write("rpt_cst",cmd);
  cmd := Sprintf("cst[] is filled with 1 + %o entries, equal to the
                              index of %o",cstq(cst,false),1+coset_index);
   Write("rpt_cst",cmd);
cmd :="All further double cosets yield relations pointing backward in
the Cayley graph.";
   Write("rpt_cst",cmd);

   indx := indx + 1;
   while indx le #dc_lbl do
      cmd := Sprintf("%o\n%o", "_"^80, dc_lbl[indx]);
      Write("rpt_cst",cmd);
      for i in paths do
         tstdc := dc_lbl[indx];
         for j in i do
            Append(~tstdc,j);
         end for;

         xcst := csti();
    xcst := dcpath(tstdc,xcst,ts); //Test new dbl coset for equivalence
    if (cstq(xcst,false) ne 0) and (xcst[1] eq []) then //check[*] also
            cnt  := cstx(cst,xcst);
         end if;
      end for;
      indx := indx + 1;
   end while;

   print txt;
   return cst,cst_index;
end function;
/**********************************************************************/
```

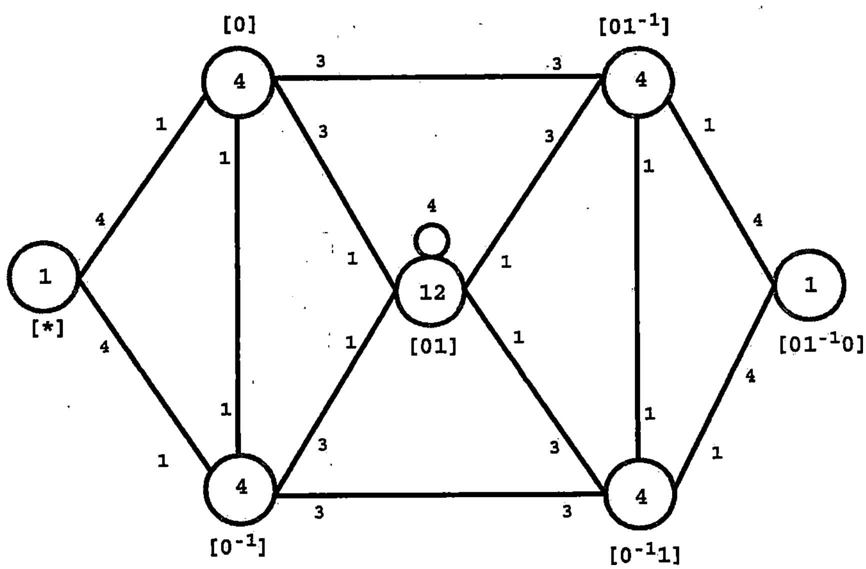
APPENDIX H

CAYLEY DIAGRAMS

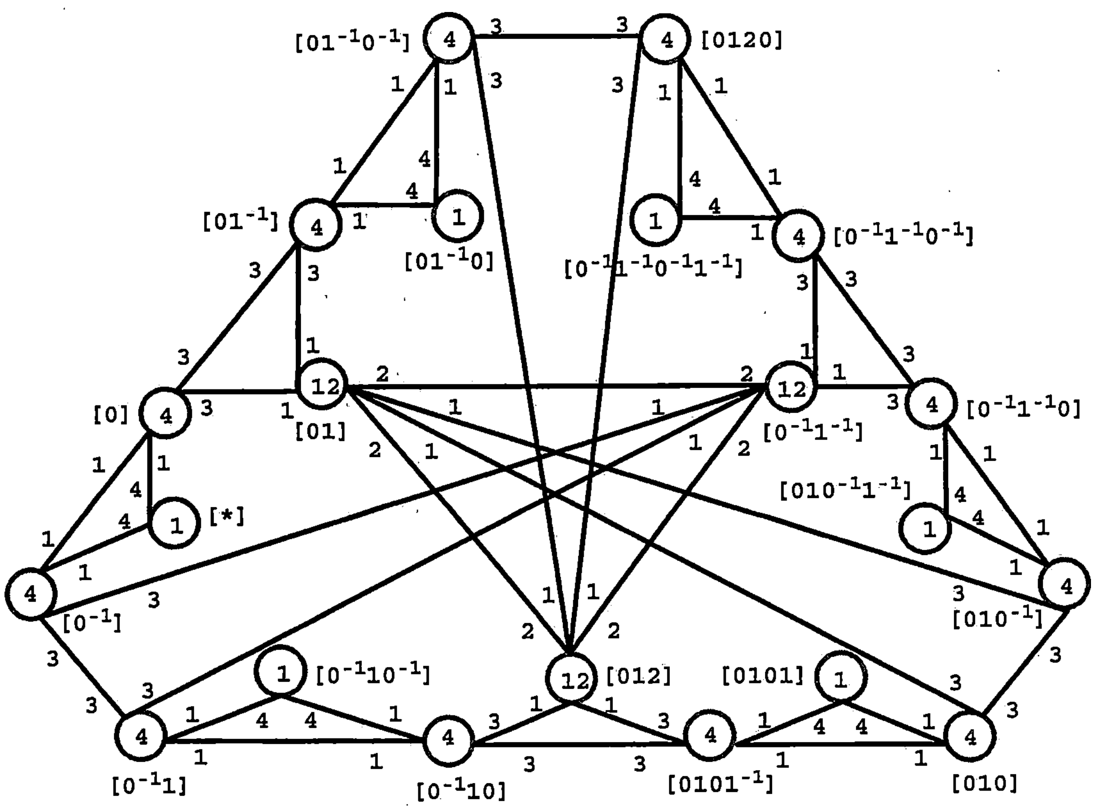
Cayley Diagram 1
$L_2(19)$ Over $L_2(5)$



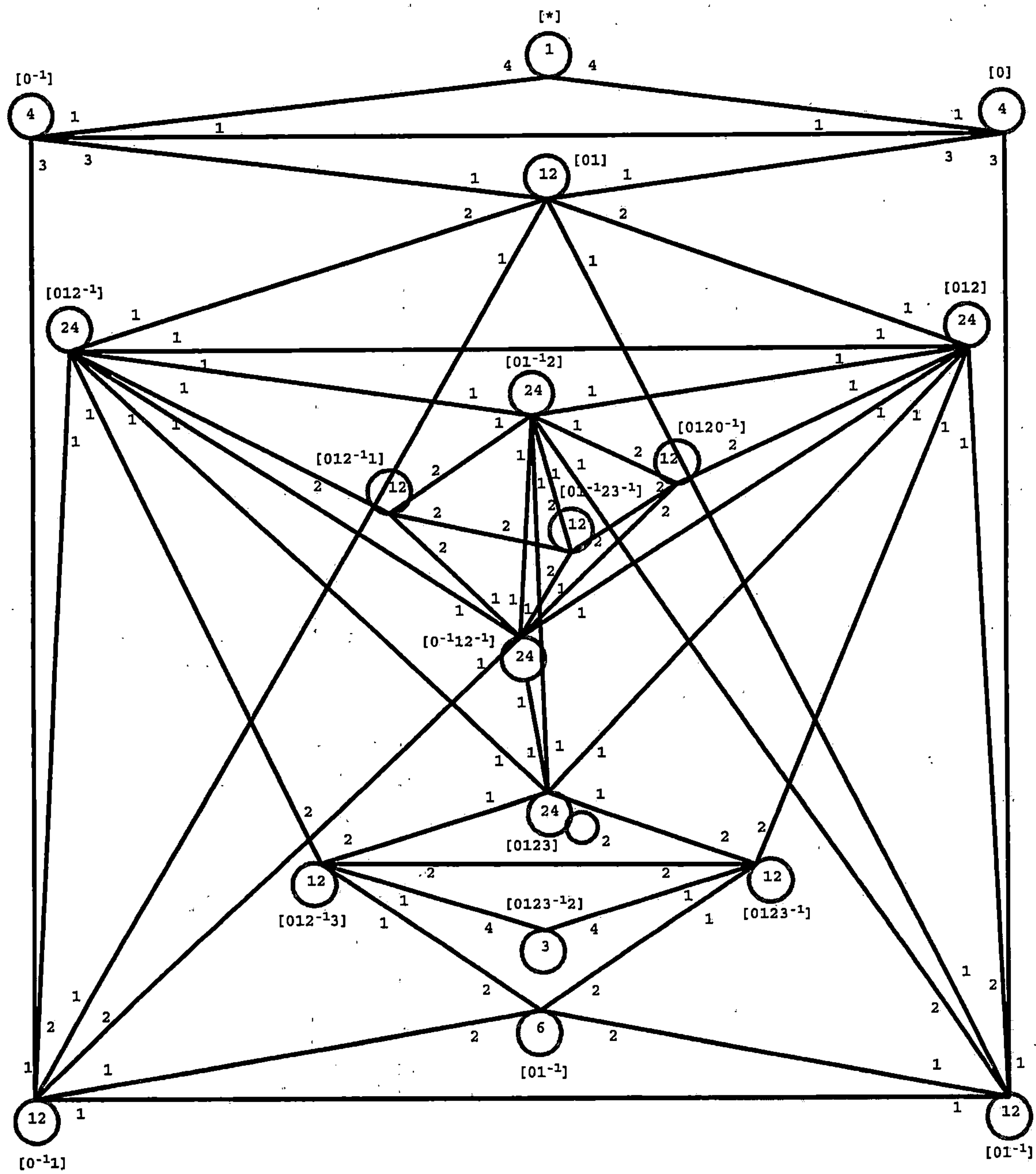
Cayley Diagram 2
S_6 Over S_4



Cayley Diagram 3
$S_6 \times 3$ Over S_4

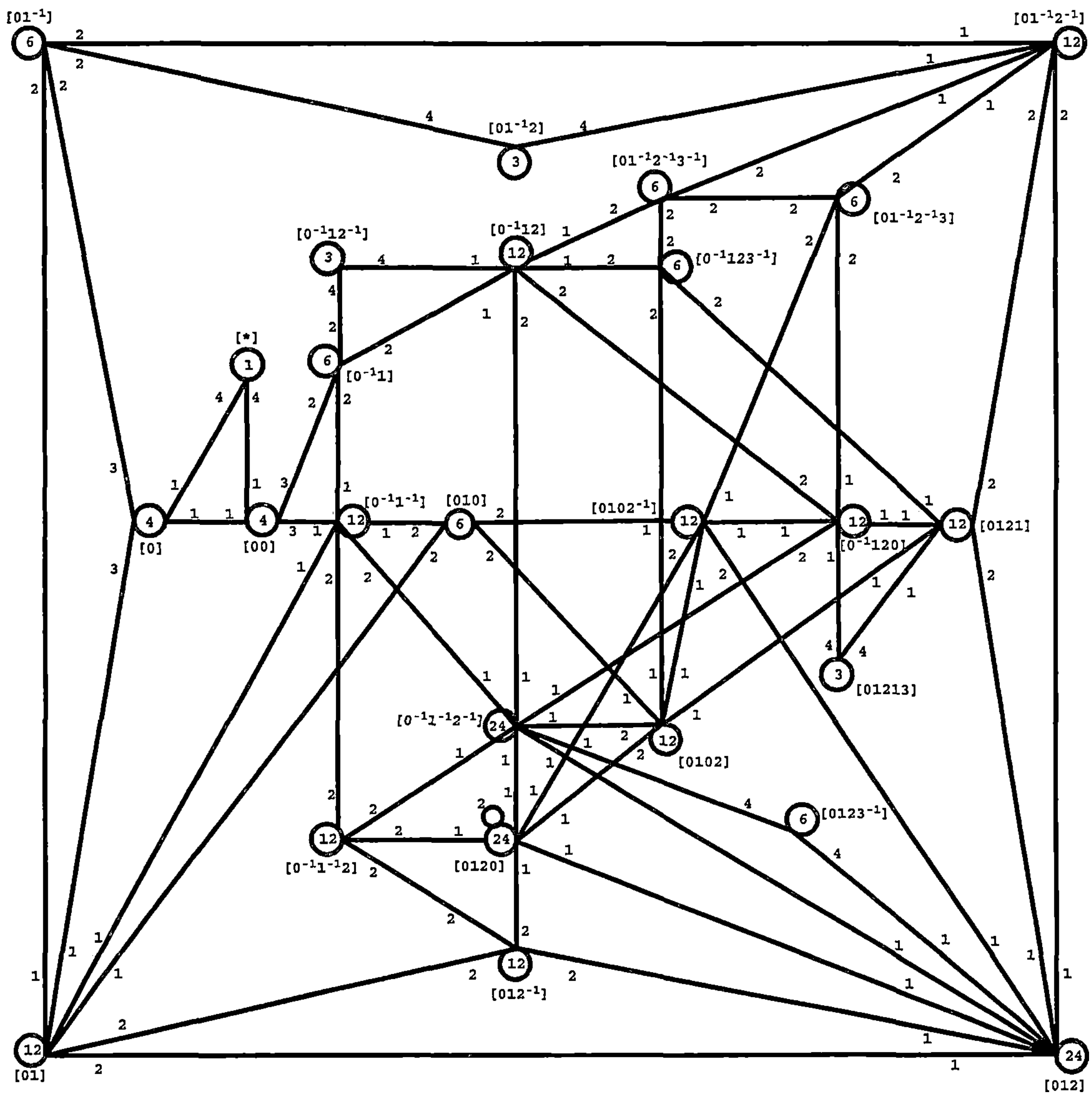


Cayley Diagram 4
$L_3(3)$ Over S_4



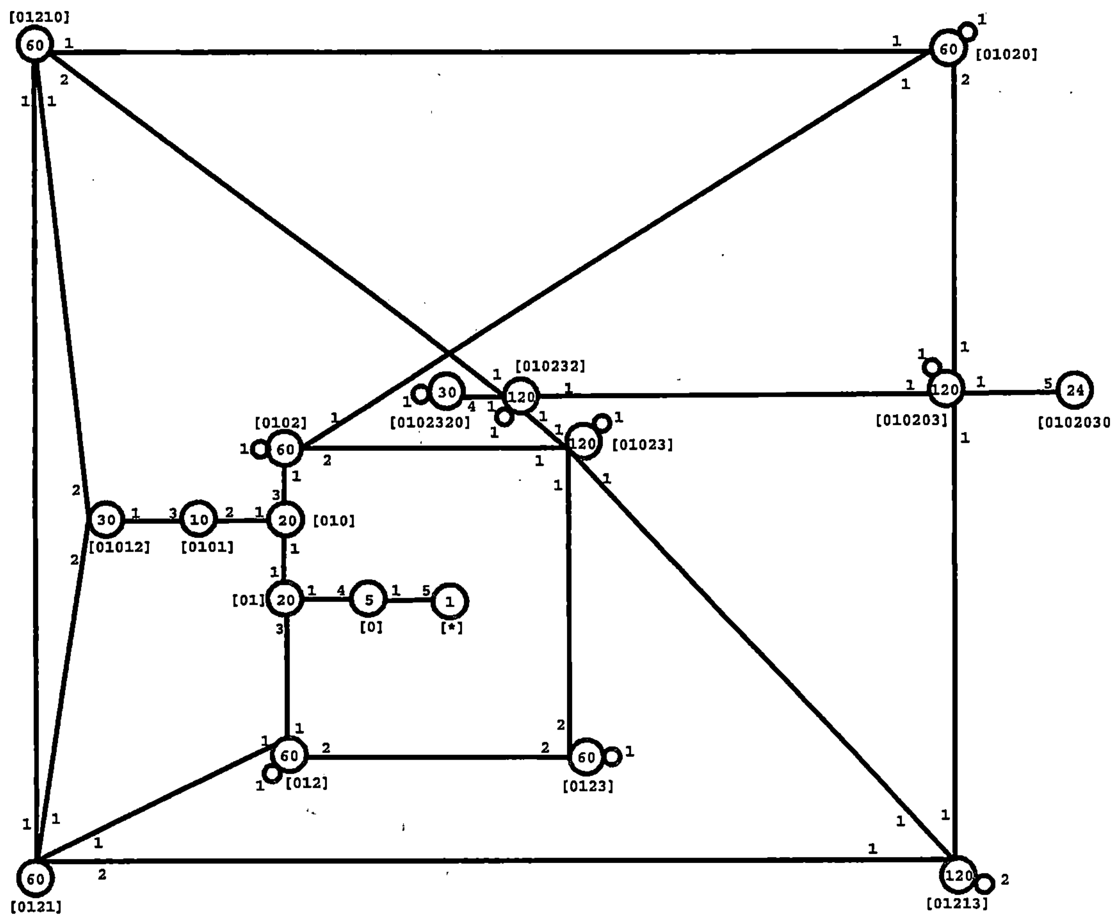
Cayley Diagram 5

$U_3(3)$ Over S_4



Cayley Diagram 6

A Group G Over S_5



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