

California State University, San Bernardino

CSUSB ScholarWorks

Theses Digitization Project

John M. Pfau Library

2005

Concept attainment in mathematics within content-based instruction for secondary English as a second language

Kwang-Sug Lee

Follow this and additional works at: <https://scholarworks.lib.csusb.edu/etd-project>



Part of the [Bilingual, Multilingual, and Multicultural Education Commons](#), and the [Science and Mathematics Education Commons](#)

Recommended Citation

Lee, Kwang-Sug, "Concept attainment in mathematics within content-based instruction for secondary English as a second language" (2005). *Theses Digitization Project*. 2858.
<https://scholarworks.lib.csusb.edu/etd-project/2858>

This Project is brought to you for free and open access by the John M. Pfau Library at CSUSB ScholarWorks. It has been accepted for inclusion in Theses Digitization Project by an authorized administrator of CSUSB ScholarWorks. For more information, please contact scholarworks@csusb.edu.

CONCEPT ATTAINMENT IN MATHEMATICS WITHIN
CONTENT-BASED INSTRUCTION FOR SECONDARY
ENGLISH AS A SECOND LANGUAGE

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education:
Teaching English to Speakers of Other Languages

by
Kwang-Sug Lee
December 2005

CONCEPT ATTAINMENT IN MATHEMATICS WITHIN
CONTENT-BASED INSTRUCTION FOR SECONDARY
ENGLISH AS A SECOND LANGUAGE

A Project
Presented to the
Faculty of
California State University,
San Bernardino

by
Kwang-Sug Lee
December 2005


Approved by:



Dr. Lynne Díaz-Rico, First Reader

Nov. 30, 2005

Date



Dr. James Mason, Second Reader

© 2005 Kwang-Sug Lee

ABSTRACT

Many English as a second language (ESL) students in the United States face a great challenge in both ESL and content-area classes, particularly mathematics. To help these students overcome their problems, improve their knowledge of ESL and mathematics, and succeed in both areas, ESL programs in the U.S. need to shift their curricular emphasis from language-only to adjunct content based English-as-a-second-language instruction focus to combine ESL and content area instruction.

The main goal of this project is to present a model of concept attainment in mathematics instruction based on adjunct content-based English-as-a-second-language instruction recommended for ESL and mathematics instructors in the U.S. For the purpose of supporting the theoretical framework of this project, this project investigates five key concepts: content-based ESL, concept attainment, use of visuals and manipulatives, Gregorc's learning styles, and adjunct/ content models. By employing this model, teachers can help ESL students enhance their academic progress in both ESL and mathematics.

ACKNOWLEDGEMENTS

I like to express my sincere gratitude to Dr. Lynne T. Diaz-Rico not only for her invaluable guidance in the planning and implementation of this project, but also for the years of education and academic inspiration she provided preparing me for it.

Special thanks are also extended to Dr. James Mason, my second reader, for his support and helpful comments and suggestions.

DEDICATION

This project is dedicated to my parents and to my son, Han-Su Lee, and my daughter, Gy-Su Lee, for their love.

TABLE OF CONTENTS

ABSTRACT	iii
ACKNOWLEDGEMENTS	iv
LIST OF TABLES	ix
LIST OF FIGURES	x
CHAPTER ONE: INTRODUCTION	
Background of the Project	1
The Problems of Language-Based English- as-a-Second-Language Programs	1
The Problems of English-as-a-Second- Language Students in Mathematics Class	2
The Need of Content-Based Instruction for English-as-a-Second-Language Students	3
The Importance of Mathematics as a Content Area	4
Target Teaching Level	5
Purpose of the Project	6
Content of the Project	8
Significance of the Project	9
CHAPTER TWO: REVIEW OF THE LITERATURE	
Content-Based English as a Second Language	12
The Definition of Content-Based Instruction	12
Why Content-Based English-as-a-Second- Language Instruction?	13
The Theoretical Foundations of Content-Based-Instruction	17

Importance of Content Selection in Content-Based Instruction	22
Summary	25
Concept Attainment	27
The Purpose of Concept Attainment	28
The Rationale of Concept Attainment	29
Strategies for Concept Attainment	31
Instructional and Nurturant Effects	33
The Steps of Concept Attainment: A Mathematics Example	34
Summary	36
Using Visuals and Manipulatives to Promote Content Learning and English Acquisition	37
Defining Visuals and Manipulatives	39
Why Use Visuals and Manipulatives in the Classrooms?	41
The Selection of Visual and Manipulative Materials	45
Visual and Manipulative Teaching Strategies	47
Summary	50
Gregorc's Learning Styles	52
The Definition of Learning Styles	52
Approaches to Learning Styles	53
Cognitive Learning Styles	55
Gregorc's Style Delineator	58
Gregorc's Dominant Style Characteristics	61

Summary	65
Adjunct Content Models	67
Defining Adjunct Content Models	69
Why Use Adjunct Content Models	70
The Features of Adjunct Content Models	73
Designing Curriculum of Adjunct Content Models	77
Summary	81
 CHAPTER THREE: THEORETICAL FRAMEWORK	
A Model of Concept Attainment in Mathematics	83
Rationale for the Model	83
Key Concepts of the Model	84
Components of the Model	84
English-as-a-Second-Language Teaching	84
Cognitive Learning Theories	87
A Specific Content Area: Mathematics	88
Concept Attainment in Mathematics	89
Adjunct Content-Based Model	90
 CHAPTER FOUR: CURRICULUM DESIGN	
Lesson Format	94
Lesson Content	96
Concept Attainment Lesson Plan One	96
Concept Attainment Lesson Plan Two	97
Concept Attainment Lesson Plan Three	98
Concept Attainment Lesson Plan Four	98

Concept Attainment Lesson Plan Five	99
Concept Attainment Lesson Plan Six	99
Adjunct Teaching Follow-Up	100
CHAPTER FIVE: ASSESSMENT .	
Assessment of Students	103
The Purpose of Assessment of Students	103
Design of Assessment of Students	103
Curriculum Evaluation	107
The Purpose of Curriculum Evaluation	107
Design of Curriculum Evaluation	108
APPENDIX: INSTRUCTIONAL UNIT – CONCEPT ATTAINMENT	116
REFERENCES	166

LIST OF TABLES

Table 1. Individual Assessment Rubric106
Table 2. Group Assessment Rubric106

LIST OF FIGURES

Figure 1. A Model of Concept Attainment in
Mathematics: Combining English-
as-a-Second-Language and Mathematics
through Adjunct Content-Based
Instruction 86

CHAPTER ONE

INTRODUCTION

Background of the Project

The Problems of Language-Based English-as-a-Second-Language Programs

Many states in the United States have English-as-a-second-language (ESL) programs through the schools in order to help non-native-English-Speaking (NNES) students learn English. However, these ESL programs are not devoted to solving English learners' problems in the mainstream classroom. Most ESL curricula involve language-based courses, in which class time is filled with manipulation of linguistic forms and discussion of correct usage. In addition, these courses are basically devoid of content in mathematics, science, or social studies. Furthermore, these ESL programs are only for part of the day, meaning that students are enrolled in a regular classroom for the majority of the day. Consequently, these ESL programs in the United States do not facilitate English learners' development of English-language ability and academic content knowledge.

Most ESL programs in the United States have many existing language-based courses. In these courses, language skills are taught in isolation from academic

content areas such as social studies, science, mathematics, and literature. As a result, ESL students in the United States face particular challenges to achieving academic success in their mainstream content classes. Consequently, many educators, whether they are ESL teachers or content-area teachers, recognize the need for a teaching method that can facilitate both ESL and academic content in regular classes for English learners in the United States. For this reason, ESL programs in the U.S. need to shift their emphasis from language-only to a content adjunct focus. In addition, it is important for all teachers to have some idea of how to accommodate ESL students in mainstream classes.

The Problems of English-as-a-Second-Language
Students in Mathematics Class

At the turn of the twenty-first century, English learners comprise a substantial portion of the school population in the United States, and their numbers continually increase in mainstream classrooms. In the U.S., this dramatic increase of the ESL population in mainstream classrooms now causes critical issues for both ESL and mainstream content-area education. One of these growing issues is how to help English learners be successful in the mainstream content classes.

Most people assume that English learners do not have problems in mathematics class because mathematics involves numbers and formulae, not words. However, this is not true. English learners face a great challenge in mathematics class. First of all, they do not have the prior knowledge needed for decoding language, making assumptions, drawing conclusions, and processing the cognitive language of mathematics. These results create the need for taking extra steps in order to address language and mathematics simultaneously when English learners learn mathematics. Therefore, one of the many suggested methods is Content-Based Instruction (CBI), which combines ESL and mathematics.

The Need of Content-Based Instruction for English-as-a-Second-Language Students

Content-based instruction (CBI) is a method that integrates ESL instruction with subject-matter instruction. The pedagogical technique of CBI focuses not only on learning ESL, but also using it as a medium to learn academic subjects such as mathematics, science, social studies, and literature. One of many important reasons for the need to develop CBI is because of Krashen's (1985) theory that language acquisition is based on input that is meaningful and understandable to the

learner. In CBI courses, students are asked to look first at the overall meaning of a whole work (top-down approach) before attending to sentence-level language instruction, which focuses mainly on the words and syntactic structures within sentences (bottom-up approach). Therefore, in CBI, students' attention is shifted from learning language itself to learning language through content. As a result, English learners exposed to meaningful subject matter explained in L2 can acquire ESL and content simultaneously.

The Importance of Mathematics as a Content Area

Mathematics has its own particular language features: specialized vocabulary, sentence structures, semantic meanings, and text features. Unlike other content areas such as social studies, science, and literature, mathematics has its own specific concepts, procedures, and applications. In addition, mathematic text is highly dense, requiring up-and-down and left-to-right eye movements and multiple readings. Moreover, mathematics is made up of many symbols such as charts and graphs, and contains a great deal of technical language with precise meanings. When these particular characteristics of mathematics are combined with ESL, mathematics can help English learners improve their cognitive processing skills

for language learning, and provide authentic conceptual and communicative meaning.

In addition, mathematics is an important subject for English learners as well as for mainstream students. In the case of language-based ESL programs, the focus of teaching is always only ESL, so the achievement of English proficiency always ends ESL. However, the significant and final goal for the ESL students who must learn ESL in mainstream settings is to be successful in their content areas, not only in ESL. One of these content areas, mathematics, can especially promote the cognitive processing English learners need for learning all other content areas. Furthermore, instructional activities in mathematics can help English learners acquire both mathematics and ESL simultaneously when they are involved in interactive activities, and they can build a bridge between ESL and mathematics. Therefore, CBI in ESL courses is the best pedagogical teaching model in which English learners can unlock mathematics and language at the same time.

Target Teaching Level

My target teaching levels are grades 9 through 12 in the ESL context, which includes all primary languages. There are several reasons for me to choose these grades as

my target teaching level. First of all, I have a year of experience in teaching mathematics to English learners in grades 9 through 12, and have observed that they face many challenges in mainstream content classes due to their lack of English and content proficiency. The barrier of English causes low motivation and negative attitudes towards both ESL and content areas. As a result, when learning content areas in mainstream classes, they face a great challenge. Secondly, many researchers suggest numerous advantages related to the pedagogical implications of CBI to these grade levels. According to Snow, Met, and Genesee (1989), in K-12 contexts the teaching of English learners has been greatly influenced by the theory and practice of CBI. This gives me an important theoretical framework for selecting these grades as my target grade levels. Through CBI, a focus on content will help my students achieve grade-level standards in school subjects, while they are developing ESL skills.

Purpose of the Project

The purpose of this project is to provide a useful model--the Concept Attainment Model in Mathematics--which implicates the concept attainment teaching method based on adjunct content-based instruction by using visuals and

manipulatives in order to help ESL students be successful for both substantive content areas and ESL skills simultaneously. Traditionally, ESL skills are taught separately from substantive content areas. However, CBI is an alternative model of language education that integrates language and content instruction in the ESL language classroom. The existing CBI models need to be modified to accommodate English learners' varied learning styles, beliefs, and needs. As a result, the goal of this project is to suggest a new adjunct content-based instruction paradigm that employs concept attainment as a teaching method, and that uses visuals and manipulatives as authentic teaching materials in mathematics as a content area.

This project provides not only several theoretical rationales for the shift to the CBI-approach in ESL education but also specific implications for the integration of language and content teaching. These suggestions will be helpful for all ESL teachers who struggle with CBI, but are particularly important for English learners in a mainstream class because they can receive more attention to both content and language.

Content of the Project

This project consists of five chapters: Introduction (Chapter One), Review of the Literature (Chapter Two), Theoretical Framework (Chapter Three), Curriculum Design (Chapter Four), and Assessment (Chapter Five).

Chapter One briefly describes the background of the project, including the description of the target teaching level. In addition, this chapter provides the purpose, the content, and the significance of the project.

Chapter Two consists of review of related literature with five significant theoretical key concepts: content-based instruction (CBI)-ESL, using visuals and manipulatives, concept attainment, Gregorc's learning styles, and adjunct content models. Included in these larger ideas are several subtopics that support a theoretical background more concretely.

Chapter Three proposes the theoretical framework and CBI teaching model that integrates content and ESL in the mainstream class. This chapter describes a specific CBI teaching method and provides the theoretical principles of CBI.

Chapter Four describes a teaching unit of six lessons based on the CBI model and the theoretical framework

introduced in Chapter Three. These lesson plans are presented in the Appendix.

Finally, Chapter Five shows the methods and forms of assessment that have been applied to the six lessons shown in the Appendix. This assessment defines students' achievement of CBI lessons.

Significance of the Project

ESL educators have developed many kinds of ESL teaching methods needed for the second-language acquisition (SLA) field. They have constantly researched the validity, utility, replication, and generalization of these ESL teaching methods. Through these evaluation methods, some methods have survived or have been modified, and others have been eliminated. Among these ESL teaching methods, a useful and valued one is the CBI method.

During my teaching experience in mainstream classrooms in the United States, I have observed that many ESL students from Mexico, China, Japan, and Korea in common face very serious challenges in their mainstream classes. Even though some English learners have much more prior knowledge about subject areas than do English-speaking students, they do not express their content knowledge because of their low level of English

proficiency. For this reason, I feel the need to help these ESL students improve ESL and content areas simultaneously in areas such as mathematics, social studies, science, language arts, and history. Therefore, I strongly believe that CBI is the best teaching method to enable my English learners to overcome these problems of language and content.

In order for ESL teachers to help English learners in a mainstream class to be successful in both content areas and ESL, this project provides one concrete model--Concept Attainment in Mathematics within Content-Based Instruction for Secondary ESL--which makes the CBI lessons decidedly better than any other CBI models. To firmly support the utility, validity, and generalization of this model, the review of the literature related to CBI provides concrete evidence of the success of CBI models, together ways to address current problems in CBI. Moreover, this project surveys new CBI teaching methods, introduces a concept attainment teaching method, and promotes visuals and manipulatives in order to propose sample CBI lessons to ESL teachers.

Furthermore, this project reviews Gregorc's learning styles, which are an important component for the implementation of this model with ESL students.

In sum, the instructional model featured in this project provides meaningful concepts in both language and content, improves metacognitive language and content processing skills, and helps English learners be successful in language and content at the same time.

CHAPTER TWO

REVIEW OF THE LITERATURE

Content-Based English as a Second Language

Content-Based Instruction (CBI) has existed for a long time. In the United States, CBI was first used as a way to deal with the language competency problem in a very efficient way faced by many immigrant students in elementary and secondary schools whose first language was not English. As this CBI has rapidly gained popularity since the 1990s (Dietrich, 2005), it has expanded from the college level to K-12 level, particularly in the domain of English as a second language (ESL).

The Definition of Content-Based Instruction

CBI is "an approach to teaching ESL that attempts to combine language with disciplinary learning, suggesting that teachers can build students' knowledge of grade-level concepts in content areas at the same time students are developing English proficiency" (Schleppegrell, Achugar, & Oteíza, 2004, p. 67); CBI "focuses not only on learning a second language, but using that language as a medium to learn content areas such as mathematics, science, social studies, or other academic subjects" (Reilly, 1988, p. 1). Brinton, Snow, and Wesche (1989) defined CBI as the

integration of content learning with language teaching aims. Additionally, Brown (1994) stated that CBI integrates the learning of some specific subject matter with the learning of ESL. Summarizing the definitions of CBI, CBI can be seen as a method that integrates English-as-a-second-language instruction with subject-matter instruction to help students improve ESL and content knowledge at the same time.

CBI can be applied to various school settings. According to Reilly (1988), even though CBI has been used for many years in adult, professional, and university education programs for foreign students, it is also recognized at the elementary and secondary school level. Grabe and Stroller (1997) described CBI as often connected with ESL academic courses for postsecondary education; however, more recently, it has extended its use into K-12 classrooms, foreign-language classrooms, and immersion programs. For this reason, CBI is now viewed in a wider perspective that integrates ESL and content learning.

Why Content-Based English-as-a-Second-Language Instruction?

There are many English-as-a-second-language (ESL) programs throughout schools and ESL institutes in the United States to help nonnative-speaking students learn

ESL. However, most of these have only focused on English-language education; so although students can enhance their English-language proficiency, they face serious challenges in the academic content areas. To help these English learners improve their ESL and content knowledge simultaneously, CBI has been developed.

More importantly, how English learners can be both functionally and academically literate has been argued for a long time. Kasper (2000) stated the significance of the simultaneous instruction of both language function and academic content-areas as the following:

Being successful in an English-speaking academic environment requires that ESL students be both functionally and academically literate, that they be able to use English to access, understand, articulate, and critically analyze conceptual relationships within, between, and among a wide variety of content areas... content-based instruction (CBI) enables college English as a Second Language (ESL) students to develop and refine these necessary literacy skills. Through planned, purposeful, and academically based activities that target linguistic and critical thinking skills and

engage students in meaningful and authentic language processing, CBI fosters a functional language learning environment that goes beyond simply presenting information in the second language. This functional language-learning environment offers ample opportunities for students to use English to gather, synthesize, and evaluate information... as it teaches them appropriate patterns of academic discourse and sociolinguistic conventions relating to audience and purpose... (p. 3)

In addition, it has been debated whether ESL can be taught from both language form and function. Because language has not only form but also function, it is difficult to determine on which part a teacher should focus. Therefore, although researchers have suggested the relationship between form and function differently, simultaneously teaching both form and function is important. Schleppegrell et al. (2004) reported the importance of simultaneously teaching both language form and function as follows:

A focus on form can be important for students' language development... [and] should be done in ways that are not isolated from the

communicative context, but researchers take different positions on whether the focus on form should be incidental... or through "structured input"..., or should be reactive... rather than proactive, through recognition and awareness of focus lessons... Focus on form approaches are typically not informed by a theory of language that is discourse and meaning based and instead view language acquisition as the accumulation of sets of structures and rules. The alternative view [presented here], based on a functional theory of language, sees how the linguistic features of disciplinary texts construe particular kinds of meanings. This makes a focus on language central to the teaching of disciplinary content. (p. 70)

Therefore, as a method of simultaneously teaching both ESL and academic content-areas, CBI can be a useful solution to promote the knowledge of language form, function, meaning, and academic content areas through providing more opportunities to use language meaningfully and promote the cognitive language skills needed for academic content-area learning.

The Theoretical Foundations of Content-Based-Instruction

Even though many research findings have provided the theoretical foundation of CBI, three significant theories support the theoretical development of CBI. These are Krashen's comprehensible input hypothesis, Cummins's two-tiered skill model, and cognitive learning theory (Kasper, 2000). Kasper (2000) reported about these theoretical foundations of CBI as follows:

The linguistic and cognitive theories that provide the foundation for CBI each emphasize the importance of providing multiple opportunities for ESL learners to interact with authentic, contextualized, linguistically challenging materials in a communicative and academic context. These theories view second language acquisition (SLA) as a complex cognitive act, in which prior knowledge and strategy use are both critical to success.

(p. 4)

Based on these theoretical foundations, many various CBI models have been developed in English-as-a-second-language settings to help English learners concurrently

improve the knowledge of both English language and content areas.

Krashen's Comprehensible Input Hypothesis. In his second-language acquisition theory, Krashen (1985) suggested that there are two important factors in second-language acquisition: learning, which refers to "the process of acquiring formal knowledge about language through explicit instruction about language forms and structures" (Kasper, 2000, p. 4) and acquisition, which refers to "the process similar to that which occurs when a first-language is acquired and is used as the medium for learning other things (Kasper, 2000, p. 4). According to Krashen (1982, 1985), because the process of first- and second-language acquisition is similar, second-language instruction should focus on meaning rather than on form. In addition, in describing his comprehensible input hypothesis, Krashen (1982, 1985) suggested that language is best acquired when the learner receives comprehensible input that is slightly above the learner's present proficiency. Therefore, Krashen's comprehensible input hypothesis provides a theoretical foundation for CBI because CBI courses "offer students contextualized language curricula built around meaningful, comprehensible

input through which not only language, but also information, is acquired" (Kasper, 2000, p. 4).

Cummins's Two-Tiered Skill Model. Cummins's (1981) work provides another theoretical foundation of CBI for considering the integration of language and content instruction. Cummins (1981) asserted that becoming proficient in a second language involves a two-phase skill model of acquisition as follows:

When first acquiring a second language, the learner acquires basic interpersonal, or functional, communication skills (BICS), which involve "the ability to converse with others and to articulate needs in the L2, and according to Cummins, can be developed within 1 to 2 years (Kasper, 2000, p. 5). In contrast, cognitive academic language proficiency (CALP), involves "the acquisition of academic literacy skills, that is, the ability to use the L2 both to understand complex, often decontextualized linguistic structures; and to analyze, explore, and deconstruct the concepts presented in academic contexts" (Kasper, 2000, p. 5).

According to Cummins it may take as many as 5 to 7 years to acquire CALP. According to Kasper

(2000), "learners cannot acquire cognitive academic language skills from everyday conversation; developing these cognitive skills requires task-based, experiential learning typified by students' interactions with contexts, tasks, and texts that present them with complex interdisciplinary content." (p. 5)

Thus, Cummins's two-phase model also provided a theoretical foundation by hypothesizing two different kinds of language acquisition, language proficiency used in informal or interpersonal situations and language proficiency used in academic content areas.

Cognitive Learning Theory. Cognitive learning theories provide other important theoretical foundations for CBI. Kasper (2000) reported that "If second-language acquisition is viewed as such, the principles of cognitive learning theory should be applied when developing ESL instructional methodology" (p. 5). Additionally, Anderson (1983, 1985, cited in Kasper, 2000) suggested three learning development stages based on cognitive learning theory as the following:

Learning begins with an instructional or study phase, called the cognitive stage, in which the learner gradually develops a rough mental

representation of task requirements. The learner refines and strengthens this representation in the second associative stage of learning but still consciously attends to rules and sometimes needs outside support when performing the task. In the third stage of learning, the autonomous stage, the task representation is increasingly refined, and the learner is now able to perform the task automatically and autonomously. (p. 5)

Moreover, based on the principles of cognitive learning theory applied to L2 pedagogy, O'Malley and Chamot (1994) suggested a learning model called the cognitive academic language learning approach (CALLA), which depends on a cognitive principle called "scaffolding": "the provision of extensive instructional supports when concepts and skills are being first introduced and the gradual removal of supports when students begin to develop greater proficiency skills, or knowledge" (Kasper, 2000, p. 10).

Therefore, cognitive learning theories provided by these researchers can help students in English-as-a-second-language pedagogy gradually develop the language skills they need for instruction, study, and greater proficiency skills in content areas. Together, the work of

Krashen, Cummins, Anderson, and O'Malley and Chamot laid the foundation of CBI.

Importance of Content Selection in Content-Based Instruction

CBI emphasizes learning content through language. For this reason, the role of content in CBI is as important as language itself. Snow et al. (1989) described the importance of content in CBI as the following:

Content can provide both a motivational and a cognitive basis for language learning. Content provides a primary motivational incentive for language learning insofar as it is interesting and of some value to the learner and therefore worth learning. Language then will be learned because it provides access to content, and language learning may even become incidental to learning about the content... Content also provides a cognitive basis for language learning in that it provides real meaning that is an inherent feature of naturalistic language learning. (p. 202)

Therefore, in CBI, the selection of content is important. However, the teachers often raise the question of how to select content so that instruction will be most

appropriate to student needs. Many useful suggestions about it have been suggested to date.

Stroller and Grabe (1997) suggested the Six-T's approach: Themes, Texts, Topics, Threads, Tasks, and Transitions, to help teachers select the content of lessons for both language and content teaching so students will benefit greatly from integrating language and content instruction. Fujioka-Ito (2000) describes the Six-T's approach as follows:

In this approach, Themes are the major source for curriculum planning. A variety of relevant Texts (all resources and materials) leads to Topic selection. Specific Tasks are designed to teach language knowledge and content information central to the Texts. Transitions and Threads also link throughout the curriculum while creating a sense of coherence. This approach maximizes student involvement in content learning and maintains high level of motivation for learning. (p. 1)

Moreover, Met (1998) suggested some criteria considered in selecting content: (1) the fit with specified language objectives, (2) the fit with students' current language proficiency, and (3) the degree of

cognitive engagement and demand. Kasper (2000) also suggested the selection of content as follows:

Content-based courses may cover a variety of subject areas or may focus on the sustained study of one specific subject area. ESL programs and faculty must decide which Content-based model suits their specific student population and the requirements of their institution, and then design course content appropriately...ESL faculty may also choose to focus on a subject area in which they have expertise, for example, language acquisition... No matter what their major, ESL students all find studying the principles and theories of language acquisition both interesting and relevant, and instruction may be broadened and these principles discussed within the contexts of several other disciplines, such as psychology or sociology.

(p. 18)

O'Malley and Chamot (1994) reported that content should be selected adequately for a given grade level. It should be appropriate for the levels of students and their needs, their experiences, and their prior/current

knowledge. They also proposed using state and local curriculum frameworks as guidelines for content selection.

However, Oxford (2001) reported the selection of content in CBI as follows:

Content-based language instruction is valuable at all levels of proficiency, but the nature of the content might differ by proficiency level. For beginners, the content often involves basic social and interpersonal communication skills, but past the beginning level, the content can become increasingly academic and complex. (p. 3)

Therefore, even though a CBI approach teaches both language and content at the same time, because it focuses much more on content-area instruction than language itself, selecting the content suitable to enhance the effectiveness of CBI is one significant step for the unit of CBI lessons.

Summary

In this section, several notions of CBI have been considered, that is, the definition of CBI, the reasons for using CBI, three important theoretical foundations of CBI, and the importance of content selection in CBI.

CBI is a method that simultaneously integrates ESL with subject-area content in mainstream classes. Using CBI

can help English learners not only build their grade-level concepts in content areas but also develop English proficiency at the same time. In addition, CBI has as a premise of language and content the concurrent integration rather than language being in isolation from content.

Also in this section, to provide theoretical foundations of CBI, three significant theories have been explicated: Krashen's (1982) comprehensible input hypothesis; Cummins's (1981) BICS/CALP distinction, cognitive learning theory. Based on these theories, CBI models have provided English learners with opportunities not only to improve cognitive academic language proficiency but also to provide continually between the ESL classroom and the academic content classroom.

In this section, another significant issue is the importance of content selection in CBI. In CBI, various content-area subjects can be used. However, in CBI, because content and language are combined at the same time, selecting content appropriate to CBI is important. Because CBI may focus much more on the content-area matter than on language itself and provides direct instruction in the special language of the subject matter, it is important for all teachers, regardless of whether they are content teachers or ESL teachers, to pay attention correct

common core and branching into parts derived from the base. He also views the world in curvilinear forms through arch and circles consisting of parts which are "welded" to make a whole. From these "welds" and branches, the AS draws correlations, predicts next steps, and involves himself in their scope and sequence.

(p. 23)

In addition, Gregorc (1982) found that "the AS loves polysyllabic words and words which are used as signs to represent physical objects, processes, and conditions, and are also used to describe generalizations which occur in the world" (p. 25).

Style Characteristics of the Dominant AR Individual.

The world of reality of the dominant Abstract Random (AR) individual is abstract, and ordering ability is random. Gregorc (1982) proposed style characteristics of the dominant AR individual as the following:

The "real" world for the dominant Abstract Random is the abstract, non-physical world of feelings and emotions... The AR lives in a world of feelings and imagination. The dominant Abstract Random's ordering ability is non-linear and multi-dimensional. Events are not perceived

as occurring in a point-by-point progression... To the Abstract Random, an experience is a "happening" with many non-factorable causes joining naturally to form an event... The dominant Abstract Random organizes by putting himself and others into the event, i.e., he uses a human equation in decision-making. (p. 29)

Moreover, Gregorc (1982) reported that "the dominant AR communicates through sound, color, music, symbols, poetry, and gestures, and uses metaphoric language because he thinks in images which cannot be communicated well in a linear or direct manner" (p. 31).

Style Characteristics of the Dominant CR Individual.

The world of reality of the dominant Concrete Random (CR) individual is concrete, and ordering ability is random. Gregorc (1982) stated style characteristics of the dominant CR individual as the following:

The real world for the dominant Concrete Random is the concrete, physical world... The Concrete Random uses intuition to peer into the solid, sensual world to identify its nature and significance... The dominant Concrete Random orders his world of reality in three-dimensional patterns... Events affected by outside variables

can result in a deviation from normal linear progressions to a series of events with interrupted or "skilled" links and the potential of a new, unpredicted event appearing from "out in left field." Several variables can also come together in a confluent manner to form a "stream of consciousness" which, to the CR, has no apparent beginning or end. (p. 35)

Again, Gregorc found that "the CR uses words which may have a present literal meaning and acceptance, but may not always convey what he himself believes the words connote" (p. 37).

In general, all people have the ability to learn within each of the four mediation channels; "no one is a pure type" (Gregorc, 1982b, cited by Ross et al., 2001, p. 400-12). Therefore, the Gregorc's style delineator is a tool that "provides an individual with a key to understand better the subtle and potent qualities of the mind, [their] behavior, the behavior of others and the demands placed upon individuals by their environment" (Gregorc, 1982b, cited by Ross et al., 2001, p. 400-12).

Summary

In this section, many concepts of learning styles have been mentioned such as the definition of learning

styles, cognitive learning styles, approaches of learning styles, Gregorc's style delineator, and four-style characteristics of Gregorc's dominant individual.

Learning styles have been developed from cognitive styles, and as "cognitive styles are specifically related to an educational context, they are usually more generally referred to as learning styles" (Brown, 2000, pp. 113-114). Because these learning styles not only show students' needs, expectations, perspectives, and beliefs, but also help teachers design and modify their courses and curriculum, it is extremely crucial for teachers to recognize their students' learning styles. However, because "it is impossible to say which learning style works best" (Ellis, 2002, p. 508), it is important for teachers not to have a bias or be judgmental about their students' learning styles.

Gregorc's two important mediation abilities are perception and ordering. Perception occurs as concrete and abstract, and ordering emerges as sequential and random. Consequently, these four qualities are shown in four dominant individuals such as CS, AS, AR, and CR. When these Gregorc's four dominant styles are closely related to the pedagogical contexts, they are referred to as Gregorc's learning styles.

Although there are many approaches to learning style, Gregorc's learning style approach is one of the most effective models because "it is designed to help individuals understand and recognize the channels by which they receive and process information efficiently" (Gregorc, 1982, cited by Ross et al., 2001, pp. 400-12). Particularly, in second-language settings, because English learners show different preferred learning styles when learning language, it is more important for ESL teachers to have flexible and positive attitudes towards their students' learning styles. Also, because there are various types of learning style approaches that English learners can use, it is important for teachers to encourage students to use various approaches to learning style.

Adjunct Content Models

In recent years, there has been an increased attention to various types of adjunct content models. These adjunct content models "link a specific language learning course with a content course in which both second language learners and native English speakers are enrolled" (Crandall, 1994, p. 3), and are "fairly restricted vis-à-vis the range of students for which it is best suited" (Brinton et al., 1989, p. 73).

Additionally, from the results of using adjunct content models in writing courses, Goldstein, Campbell, and Cummings (1994) suggested that "many practitioners advocate the use of adjunct-model writing courses as a means of helping students learn content at the same time that they learn to write academic papers for these content courses" (p. 19).

Besides, according to Adamson (1993), "an adjunct course enrolls ESL students for credit in a subject matter content course and an associated ESL course in which the content material is reviewed and the academic skills and background knowledge necessary for success in the course are taught" (cited by Beckett & Gonzalez, 2004, p. 165). The purpose of the adjunct course is "to help students master the subject matter content material, introduce them to L2 academic discourse, and develop skills which they can transfer to other academic areas" (Beckett & Gonzales, 2004, p. 165).

Therefore, in the adjunct courses, because language and content courses are taught separately but are carefully coordinated, the adjunct content courses can "assist students in developing academic coping strategies and cognitive skills which will transfer from one discipline to another" (Brinton et al, 1989, p. 17).

Defining Adjunct Content Models

The adjunct content models have been defined by many researchers. This adjunct content model is one of the main models for content-based second-language instruction.

Brinton et al. (1989) described the adjunct content model as follows:

The third type of content-based instruction is the adjunct model. In adjunct content model, students are enrolled concurrently in two linked courses, a language course and a content course, with the idea being that the two courses share the content base and complement each other in terms of mutually coordinated assignments. Second language learners are sheltered in the language course and integrated in the content course, where both native English and nonnative English-speaking students attend the same lecture. (p. 16)

Otherwise, because in the adjunct content model, language course is combined with the existing content course, the adjunct content model is called a linked content model. According to Kasper (2000), the adjunct, or linked, content models are as follows:

...linked courses are English for academic purposes (EAP) models that involve interdisciplinary collaborations...linked courses pair an ESL course with one specific mainstream course, such as psychology or biology, with students enrolled simultaneously in each. To be most effective, instruction in both components should be completely coordinated, so that the ESL and the content area instructor develop parallel materials and share ideas for course assignments. (p. 12)

Accordingly, one of the three main content-based models, the adjunct content model integrates language course into the existing content courses such as mathematics, science, social studies, and other academic subjects. In this adjunct content course, English is used as the medium of content instruction.

Why Use Adjunct Content Models

Many researchers have proved the effectiveness of employing the adjunct content courses to English learners. Kasper (1994) suggested that "linked courses present academic material in multiple contexts, which allows instruction in the mainstream discipline to be reinforced through the activities in the ESL course, and linguistic

skills to be strengthened through the activities in the mainstream course" (cited by Kasper, 2000, p. 12). In addition, Pally (1997) found that "this sustained content study offers multiple opportunities to review and practice linguistic forms and structures, as it develops critical thinking skills and fosters cognitive academic language proficiency (cited by Kasper, 2000, p. 12). Moreover, about the advantages of adjunct content courses, Kasper (2000) described as follows:

Linked courses offer college ESL students a number of advantages. These courses provide the opportunity to focus on complex and enduring content over the course of a full semester... In addition, adjunct courses enable students to improve their English language skills while earning some degree credits; however, they provide greater flexibility in scheduling and course selection because the entire program is not prearranged. (p. 12)

Besides, the adjunct content courses can help not only language teachers but also English language learners. From the results of pairing an intensive English program ESL writing course with a regular university course, Beckett and Gonzalez (2004) found the merits as follows:

...An adjunct course provides excellent contexts for developing academic strategies because the ESL component of the course is directly related to the students' academic needs, and support for students in revising their notes, preparing for exams, and building a conceptual framework for understanding the reading material and the course dealing with real subject matter...Especially, a content-based adjunct course makes teaching advanced level writing effective by increasing students' academic writing proficiency and empowering them with useful skills necessary for a smooth transition into university credit courses. (pp. 165-166)

However, from the students' point of view in adjunct writing courses, Goldstein et al. (1994) concluded as follows:

...adjunct courses are not always as effective as we might believe or hope, so students perceive them as working well when these courses fit their expectations about what a writing course should be and do, when they are invested in the content of the content course, and when they trust the writing teacher's control of the

content and feel that their writing teacher and content teacher are in sync. (p. 22)

Consequently, even though the adjunct content courses have many advantages such as offering excellent opportunities for English learners to solve the problems they face in content courses, improve knowledge of the content-area subjects as well as their English, and to make the adjunct content instruction successful, the teacher should consider the students' expectations and perspectives when planning the instruction.

The Features of Adjunct Content Models

Unlike theme-based language instruction and sheltered content instruction, adjunct language instruction has its own specific characteristics. First of all, Willis (1998) suggested the features of adjunct content models as follows:

...second language students in a mainstream class, that is, a class taught for target language speakers, are provided with an additional class, a language course that works with the same materials and assignments as that mainstream content class and serves to clarify the material in the content class; thus, the students are both segregated, in the language

course, and integrated, in the content class; and also, linguistic modifications are used in the adjunct model. (pp. 36-60)

There is another significant feature of adjunct content models. Davies (2003) described the feature of adjunct content models as follows:

...the aim of adjunct content instruction is to prepare students for mainstream classes where they will join English L1 learners; adjunct content classes also feature study skills to familiarize the students with listening, note taking and skimming and scanning texts; and they are taught during the summer months before regular college class begin, while others run concurrently with regular lessons. (p. 1)

However, "course content in the adjunct courses parallels that of courses offered during the normal academic year, with some modifications made to facilitate coordination between the two disciplines" (Brinton et al., 1989, p. 59).

The other important strength of adjunct content model is "the participation of a mix of native and nonnative speakers in the content component; further, adjunct instruction appears most appropriate for adults or young

adults with academic goals whose proficiency level is high intermediate to advanced" (Brinton et al., 1989, pp. 73-74).

However, because the adjunct model "involves an interdisciplinary collaboration with courses outside the ESL program, it requires more extensive planning and administrative support and so is subject to problems and concerns that do not occur in self-contained content-based courses" (Kasper, 2000, p. 28). For this reason, Brinton et al. (1989) described the importance of coordination and staffing in the adjunct content courses as follows:

Clearly, the adjunct model requires an extensive network of staff members (e.g., administrative staff, language and content instructors, tutors, counselors) and close coordination among these members in order to achieve its intent. Both English/ESL and content course instructors attend a series of planning meetings before the team begins to determine the shape and specifics of the program. In these meetings, discussion usually focuses on how to best coordinate the English/ESL syllabus with that of the content course, and decisions are made within each discipline about which rhetorical mode to focus

on each week (e.g., definition, compare/contrast). Of particular importance to the English/ESL staff are the criteria by which the content course staff will grade written work, and discussion of this usually centers on how to evaluate structural and stylistic problems. Finally, both groups discuss complementary assignments and coordination of efforts to help improve students study skills.

(p. 59)

On the other hand, there are many difficulties in conducting the adjunct content courses. Kasper (2000) suggested the potential problems in implementing interdisciplinary collaboration among the members related to adjunct content courses as follows:

...Because of administrative and financial concerns, it is often difficult to schedule and offer these interdisciplinary collaborations. Faculty schedules must be set up to allow instructors to meet and to attend each other's classes. Additionally, coordinating instruction often requires that funds be made available for released time as well as for faculty development workshops. For these programs to work most

effectively, both faculty and students must commit the time and the energy necessary to produce a strong interdisciplinary collaboration. (p. 12)

Therefore, first of all, one key characteristic of adjunct content courses is to combine language teaching with mainstream instruction to help English learners overcome their content-area problems, together with enhancing English language proficiency. To make the adjunct content lessons effective, the coordination among the members such as language and content instructors and administrative staff is another specific feature of the adjunct content course. In spite of these characteristics of the adjunct content courses, there exist still some difficulties to be solved such as financial problems.

Designing Curriculum of Adjunct Content Models

In the adjunct content courses, it is not easy for teachers to combine ESL curricula with regular content area curricula at the same time. Despite this difficulty, many researchers and educators have designed the curricula in terms of combining ESL courses with regular content-area courses. Beckett and Gonzalez (2004) suggested the difficulty of the combination of language courses into a regular university course as follows:

Pairing an intensive English program (IEP) ESL writing course with a regular university course is not an easy task. There are many things such as teacher knowledge and logistical issues (i.e., scheduling) to consider. (p. 165)

In the adjunct model, "every effort is made to dovetail the curricula of the language and content courses so that they maximally complement each other" (Brinton et al., 1989, p. 60). Furthermore, adjunct courses "link a language course and content course and are designed to help English learners learn appropriate language and study skills while also mastering academic content" (Kasper, 2000, p. 27).

Particularly, in designing curricula of the adjunct content courses, "content involves the careful planning of both content and language objectives and the selecting, modifying, and organizing of materials and text that support those objectives" (Diaz-Rico & Weed, 2002, p. 120). Diaz-Rico and Weed (2002) described content objectives in designing curriculum of content-based courses as follows:

Planning begins by the teacher first specifying learning goals and identifying competencies students must develop. Standards documents that

spell out what students should know and be able to do are available to provide an overview of the goals. State agencies, district planners, and school officials have developed curricular programs that allow the goals put forth in the documents. The teacher divides these overall goals for the year into units. These units are further divided into specific lessons. Each lesson contains the essential content area objectives...In developing their sequence of content objectives, teachers want to keep two important questions in mind: (1) Have I reviewed the objectives for the year and organized them for thematic flow? and (2) Have I considered the sequence of objectives and rearranged them, if necessary, putting more concrete concepts before more abstract ones (i.e., those that can be taught with hands-on materials, visuals, and demonstrations before those that are difficult to demonstrate or that require more oral and/ or written skills)? (p. 120-121)

Furthermore, together with content objectives in designing curricula of the adjunct content instruction, language objectives are important. Diaz-Rico and Weed

(2002) described language objectives when teachers design curricula of the content courses as follows:

...Again, teachers can refer to the standards document developed by TESOL (1997) as a guide to help them with these goals. The teacher considers the various tasks that language users must be able to perform in the different content areas (e.g., describing in a literature lesson, classifying in a science lesson, justifying in a mathematics lesson, etc.). Importantly, a language objective takes into account not only vocabulary but also the language functions and discourse of the discipline...In reviewing their language objectives, teachers can keep the following questions in mind: (1) What is the concept load of the unit and what are the key concepts to demonstrate and illustrate? (2) What are the structures and discourse of the discipline and are these included in the language objectives? and (3) Are all four language modes included in the planning (listening, speaking, reading, writing)?

(p. 121)

Therefore, because in the adjunct content courses the combination of curricula of language and content course is a difficult task, the teachers should carefully design the curricula of the adjunct content courses through considering many important factors, such as content and language objectives, texts and materials, that influence to make the adjunct content lessons successful,

Summary

In this section, several key concepts related to adjunct content models have been described, such as the definition of adjunct content models, the advantages of using adjunct content courses to English learners, the characteristics of adjunct content models, and designing curricula of adjunct content models.

First of all, the adjunct content model is one of the three content-based models. In addition, the adjunct content class is not developed by itself, but added to an existing content-area class. Also, the aim of the adjunct content course is to combine a language course with a regular academic course. These adjunct content courses provide good opportunities for English learners to develop the academic strategies necessary for learning real academic content.

In addition, the adjunct content models have more complicated characteristics, different from theme-based content models and sheltered content models. For example, in adjunct content model, English learners are simultaneously enrolled in courses in which native-speaking students and non-native-speaking students attend the same class. Additionally, because the techniques of the adjunct content courses are based on the collaboration of a language teacher, a content teacher, and an administrative staff, these members' collaboration is one important factor as to the success the adjunct content courses.

Moreover, to make the goal of adjunct content courses complete, it is essential to design curricula for meeting content objectives and language objectives at the same time. Therefore, before designing the adjunct content courses, many significant factors, such as learners' needs, expectations and objectives, and administrative and financial concerns, should be considered.

Therefore, when all things mentioned above are well established, the adjunct content courses can help English learners master the subject content-area material and then develop skills that they can transfer to academic areas.

CHAPTER THREE
THEORETICAL FRAMEWORK

A Model of Concept Attainment in Mathematics

Rationale for the Model

Through my teaching experience in a United States secondary public school, I have observed that many non-native-English-speaking students face special challenges for achieving academic success as well as English proficiency. Particularly, they face a twofold challenge in mathematics classes. In order to succeed in the U.S. public school and perhaps beyond, these students need to be able to read and understand mathematics texts as well as other varied mathematical concepts such as numbers, formulae, and mathematics words. To help the students be successful in both mathematics and language skills and make learning more efficient in the mainstream mathematics class, I will propose "A Model of Concept Attainment in Mathematics: Combining ESL and Mathematics through Adjunct Content-Based Instruction" in this chapter. This is a model that ESL and mathematics teachers can use in their classes as a content-based pedagogical model.

Key Concepts of the Model

Chapter Two reviewed the literature relevant to content-based instruction (CBI), together with other key concepts, which combine ESL and content area simultaneously, such as mathematics, social studies, science, literature, and the visual and performing arts in mainstream classes. The key concepts displayed in the previous chapter were Krashen's comprehensible input hypothesis, Cummins's two-tiered skill model; cognitive learning theory, content-based ESL, concept attainment, using visuals and manipulatives, Gregorc's learning styles, and adjunct content-based models. These key concepts from the components of this model, together with other factors, such as classroom culture, educational institutions, community interest groups, and state agencies, which can influence this model directly or indirectly (see Figure 1).

Components of the Model

English-as-a-Second-Language Teaching

Krashen's Comprehensible Input Hypothesis. Krashen's comprehensible input hypothesis provides the theoretical framework and background needed for this model. According to Krashen (1982), a second language can be acquired more

successfully when the focus of instruction is on the meaning rather than only on the linguistic forms of the target language. Krashen recommends meaningful activities mixed with linguistic forms; the model in Figure 1 follows this recommendation by using visuals and manipulatives in mathematics. Therefore, this model suggests the idea that students will be most successful when there is a meaningful use of content, such as mathematics, together with language forms.

Using Visuals and Manipulatives. This model recommends the use of authentic materials such as visuals and manipulatives. Even though mathematics is as important as other content areas in the public school, many students are bored and uninterested in mathematics classes. This is because mathematics content features many complicated and abstract concepts and formulae. Thus, to begin with, using these visuals and manipulatives in the mathematics classes can help English learners gain interest and increased instructional motivation in mathematics. In addition, applying concept attainment as a teaching method can help English learners markedly and broadly improve their cognitive language and content-processing skills by using authentic materials.

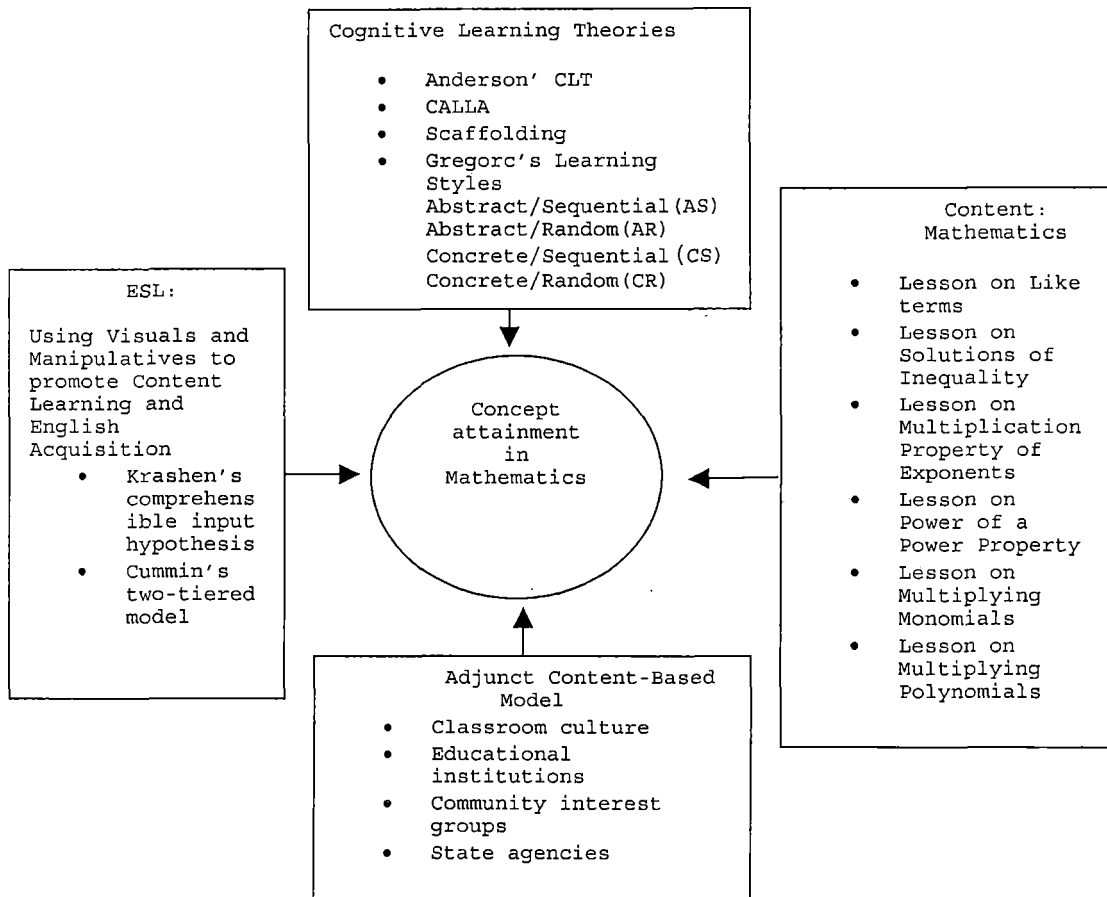


Figure 1. A Model of Concept Attainment in Mathematics: Combining English-as-a-Second-Language and Mathematics through Adjunct Content-Based Instruction

Cummins's Two-Tiered Skill Model. Cummins's (1981) two-tiered skill model also provides an important theoretical framework and background for the model in Figure 1. Cummins (1981) hypothesized two kinds of different tiers of language proficiency: basic interpersonal communication skill (BICS) and cognitive academic language proficiency (CALP). In the first tier of

acquisition, the students can acquire basic language skills and functions for one to two years that allow them to communicate in everyday social contexts that are similar to those of the home. However, in the second tier of acquisition, the students can acquire academic language in more context-reduced situations for five to seven years. Therefore, using this model in the ESL classrooms can help ESL students be successful in Cummins's two tiers of skills: BICS and CALP.

Cognitive Learning Theories

Many researchers have suggested the importance of cognitive learning stages. According to Anderson's (1983, 1985) Cognitive Learning Theory (CLT), learning is developed through a series of stages. In addition, O'Malley and Chamot (1990) applied a learning model, the Cognitive Academic Language Learning Approach (CALLA), to L2 pedagogy. The model relies on a cognitive principle called "scaffolding" (p. 10). These theories also support the model in Figure 3 through using materials drawn from mathematics. This instructional model will maximize English learners' acquisition in both language and content knowledge through a series of cognitive learning stages.

Gregorc's Learning Styles. In this model, students' learning styles are one important component. Because

classrooms are mixed with diverse cultural domains and students have various and different learning styles according to their cultural backgrounds, English learners' learning styles cannot be ignored for this model. These learning styles consist of their personal behaviors and the preferred media used to transmit or receive the information or data from the teacher, as well as reflection of their needs and beliefs. This is why this model includes the need to determine students' learning styles. This model provides Gregorc's four learning styles identified by the Gregorc Style Delineator—concrete/sequential, abstract/random, abstract/sequential, and concrete/random. These categories help teachers understand how to teach English learners with differing learning styles, and how to identify which learning styles are most effective for various students.

A Specific Content Area: Mathematics

One key content area in focus in the model in Figure 3 is mathematics. Using mathematics as a subject area provides many advantages to ESL students. First of all, mathematics can supply many concrete concepts to English learners through concept-attainment lessons. When these mathematical concepts are combined with an adjunct model of ESL, English learners can gain the prior knowledge for

decoding language, making assumptions, drawing conclusions, and processing new ideas that native-English-speaking students acquire and use with no second-language-acquisition challenges. In addition, meaningful activities in mathematics learning, such as using visuals and manipulatives as materials, can help English learners develop their cognitive processing skills, including gaining proficiency in language skills and mathematics simultaneously.

Concept Attainment in Mathematics

The main focus of this model is concept attainment in mathematics. In this model, concept attainment provides one significant teaching model, which is a teaching and learning strategy designed to teach concepts through the presentation of exemplars and non-exemplars. Students form, test, and refine their hypotheses about the concepts of exemplars and non-exemplars that are then presented. Subsequently, students determine the critical attributes of the concepts--the characteristics that make the concept different from all others. Finally, students demonstrate that they have attained the concept by generating their own exemplars and non-exemplars.

Because concept attainment lessons are designed for instruction on specific concepts and on the nature of

concepts, they provide practice in inductive reasoning and opportunities for altering and improving students' concept-building strategies. In addition, especially with abstract concepts, the lessons provide an awareness of alternative perspectives, a sensitivity to logical reasoning in communication, and a tolerance of ambiguity.

Adjunct Content-Based Model

This model incorporates adjunct content-based courses. Students who attend an ESL course offered by the ESL department also attend a content course, such as mathematics, offered by the mathematics department. The key feature of the adjunct model is that students are enrolled concurrently in a language course and a content area. However, teachers need to approach adjunct courses with caution. In the best of circumstances, adjunct courses are a powerful means by which teachers can integrate content and ESL. These adjunct courses can allow teachers to open doors to the academic world for their students, helping them to understand the content and discourse of the communities within which they are learning and to become more effective writers within that community. In addition, adjunct courses can make students' learning motivation of ESL and mathematics instrumental and intrinsic through the collaboration between

non-native-English-speaking students and native-English-speaking students.

Classroom Culture. In this model, classroom culture is considered to be one important component because learning a language includes social and cultural interaction. Because there are non-native speaking and native-speaking students at the same courses in the adjunct courses, the classroom culture is heterogeneous, and the cultural domains overlap. In this case, the classroom culture is shaped by multifaceted socio-cultural forces. However, even though the agendas of the different interest groups get played out, negotiated, and contested, it has a degree of autonomy to develop its own agenda, and each sphere displays diverse cultural tendencies within itself.

Educational Institutions. In this model, the role of educational institutions is important. Based on adjunct content-based models, this instruction needs the positive and active support of the instructional institutions. Through the ongoing attention to the adjunct courses, the instructional institutions support not only the courses but also ESL and content teachers. In addition, they can function as stakeholders. Thus, whether this model can be successful or not depends on the function of educational

institutions including the ESL and content subject teachers.

Community Interest Groups. In this model, the role of community interest groups is also important because the information will be collected from them. They are also often involved depending on whether they can provide information useful in meeting the purposes of the adjunct content-based instruction. In addition, they can determine and support an ESL program in public secondary schools in the mainstream context. These might include community agencies, parents, academics, academic specialists, employers, vocational training specialists, and influential individuals and pressure groups.

State Agencies. In this model, the role of state agencies such as policy makers and department of education officials is also very significant because, particularly, these groups can influence the decision-making of the adjunct course curricula. They have the right to comment on, and have put input into, the curriculum process offered in school.

In sum, this theoretical framework describes a model of concept attainment in mathematics, combining ESL and mathematics through adjunct content-based instruction, that ESL and mathematics teachers can employ in their

classes. To support the theoretical framework of this model, many key concepts have been addressed in this chapter, i.e., Krashen's comprehensible input hypothesis, Cummins's two-tiered skill model, cognitive learning theories, content-based ESL, concept attainment, using visuals and manipulatives, Gregorc's learning styles, and adjunct content models. In addition, to strengthen the theoretical background of adjunct content model, several other important factors have been considered, such as classroom culture, educational institutions, community interest groups, and state agencies. Therefore, all these key concepts discussed in this chapter combine to comprise a theoretical framework to promote English learners' academic success in mathematics.

CHAPTER FOUR
CURRICULUM DESIGN

The curriculum unit presented in the Appendix is designed for mathematics teachers in grades 9 through 12 who are looking for a way to use concept attainment in mathematics classes to help students master key ideas. The title of this unit is Concept Attainment in Mathematics and applies the central concepts explored in Chapter Two and the teaching model formulated in Chapter Three.

The curriculum consists of six lessons. The topics of the curriculum focus on concept attainment in mathematics in order to develop students' language, academic content, and cognitive processing skills.

Lesson Format

Each lesson in the unit of instruction follows a clear concept attainment lesson plan format that provides procedures that are systematic and easy to follow. Each lesson plan includes essential attributes or exemplars, important non-essential attributes or exemplars, instructional goals, performance objectives, presentation, testing, analysis, and evaluation. Every lesson is geared toward the target teaching level of secondary mathematics.

To facilitate the lessons, this unit plan provides several focus sheets, work sheets, and assessment sheets.

The first stage of each lesson plan is a presentation of essential attributes which are critical to the domain under consideration. In this presentation, the characteristics of exemplars of a category are shown. However, the category may have many other attributes that may not be relevant to the category itself. In this step, teachers do not ask students to focus on an attribute. The second stage presents non-essential attributes, which are attributes not critical to the domain under consideration. The third stage shows instructional goals that describe the general purpose of each lesson. The fourth stage offers performance objectives that refer to a more specific and concrete description of purposes. The fifth stage is comprised of exemplars that are a subset of a collection of data or a data set. The specific positive and negative exemplars are provided in Work Sheets. The sixth stage is a presentation in which students use their thinking skills to discover essential attributes. The seventh stage is testing, in which students list the attributes and name the concept. The eighth stage is analysis, which revises the attributes, defines an attribute range, and eliminates the non-essential

attributes. In this stage, students analyze their own thinking (metacognition). The final stage is evaluation, which includes various activities such as peer and self-evaluation, written tests, and individual or group presentations. Each assessment activity is evaluated with a given number of points. Before the assessment, the teacher explains to the students what they have to do to achieve a high score on the assessment.

Lesson Content

The intent of the unit of instruction is for students to learn various mathematic concepts. Therefore, the lessons all include as a goal attaining mathematic concepts.

Concept Attainment Lesson Plan One

The instructional goal of Lesson One is to attain the concept of Like Terms from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to list terms with the same variables and degree, name the concept, and reflect on the learning process. The first activity presents the clearest exemplar of like terms by using a work sheet. As the second activity, at least two more each of the clearest positive and negative exemplars are provided. After six to eight more exemplars, the

students' thinking is tested by asking students to decide if the next exemplar is positive or negative. The teacher asks students to put "thumbs up" for YES (like terms) and "thumbs down" for NO (unlike terms). Students continue testing the next three to four exemplars and list the attributes publicly; then the teacher has students revise the attributes and name the concept. Subsequently, the teacher has students state the rule, identify additional unlabeled exemplars, and generate exemplars of the concept. Finally, the teacher has students analyze their own thinking (metacognition).

Concept Attainment Lesson Plan Two

The instructional goal of Lesson Two is to determine whether a number is A Solution of Inequality from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to list solutions, name the concept, and reflect on the learning process. In this lesson, students are going to use their thinking skills to discover solutions of an inequality. The positive exemplars for this lesson are members of a list of solutions, and the negative exemplars are members of a list of non-solutions. The teacher asks students not to talk until the end and not to speak out, even when they think they know the key concept. The teacher has students think about exemplars.

Students can change their minds if their ideas are not correct. Students use their thinking skills to discover solutions of an inequality. Finally, students will find at least one example of the solutions of an inequality.

Concept Attainment Lesson Plan Three

The instructional goal of Lesson Three is to Multiply Numbers in Exponential Form from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to simplify exponential expressions, name the concept, and reflect on the learning process. The positive exemplars are members of a list of exponential notations related to multiplication, and the negative exemplars are members of a list of exponential notations related to division. Students use their thinking skills to multiply exponential expressions. Finally, students will find at least one exemplar of multiplying numbers in exponential form.

Concept Attainment Lesson Plan Four

The instructional goal of Lesson Four is to accomplish Raising a Power of a Power from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to accomplish raising a power to a power, name the concept, and reflect on the learning process. The positive exemplars for this lesson are members of a list of exponential notations related to raising a power to a

power, and the negative exemplars are members of a list of exponential notations related to multiplication. Students will use their thinking skills to find a power of a power. At the end of this lesson, students will find at least one exemplar of raising a power to a power in exponential form.

Concept Attainment Lesson Plan Five

The instructional goal of Lesson Five is to Multiply Monomials from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to multiply monomials, name the concept, and reflect on the learning process. The positive exemplars for this lesson are members of a list of monomials, and the negative exemplars are members of a list of non-monomials. In this lesson, students use their thinking skills to multiply monomials. At the end of lesson, students will find at least one exemplar of multiplying monomials.

Concept Attainment Lesson Plan Six

The instructional goal of Lesson Six is to Multiply Polynomials from Algebra 1 for ninth-grade students. The performance objectives of this lesson are to multiply polynomials, name the concept, and reflect on the learning process. In this lesson, students use their thinking skills to multiply polynomials. The positive exemplars for

this lesson are members of a list of polynomials, and the negative exemplars are members of a list of monomials. At the end of this lesson, students will find at least one exemplar of multiplying polynomials.

Adjunct Teaching Follow-Up

In adjunct content-based model, the role of adjunct language teachers is important. First of all, the primary duty of the language teacher is to provide instruction that can improve English learners' language proficiency. In this case, because the adjunct language instruction is conducted through the medium of subject content-area, the language teacher should also be familiar with the content material. For this reason, to be successful in adjunct language lessons, the language teacher should "devote a substantial amount of time to learning the material of the content course, developing language-teaching materials based on the content, and providing feed-back on both the linguistic aspects of the students' work and (to a lesser degree) the quality of the content" (Brinton, Snow, & Wesche, 1989, pp. 64).

In addition, together with the emphasis with the content-area material, another significant responsibility of the language teacher is to meet the specified language objectives. Therefore, unlike the language teachers of

other content-based models such as sheltered model and theme-based model, the adjunct language teacher may have "to take into account a preexisting language syllabus in order to prepare students for a standardized exit exam" (Brinton et al., 1989, p. 64). Simultaneously, the language teacher should also meet students' language objectives within the content-based framework. This may "entail adding or emphasizing certain objectives and/or deemphasizing others which are not as immediately relevant" (Brinton et al., 1989, p. 65).

However, even though the adjunct language teacher has important responsibilities, because the instructional techniques of adjunct model are based on the collaboration with language teacher, content-area teacher, and staff, the members of team always need to work to complement each other, "so that a language teacher who notices gross gaps in a student's content area knowledge would refer that student to the content course teacher or tutor, and vice versa" (Brinton et al., 1989, p. 65).

Therefore, in adjunct content instruction, not only to meet the specific language objectives but also to facilitate the transition of English learners into mainstream content courses, language teachers should use content materials developed by themselves by paying more

time, rather than grammar rules or vocabulary lists, and have the responsibility of collaboration with content teachers and staff members.

In summary, the six lessons use concept attainment activities to help students become more efficient in attaining concepts. Additionally, through these concept attainment lessons, students learn the attributes that define a concept and can distinguish those from other important attributes that do not form the definition. Moreover, because these concept attainment lessons are indirect instructional strategies, students can identify distinguishing characteristics of a given concept and discover a concept for themselves. Therefore, these concept attainment lessons can enhance students' learning, provide a chance to analyze the students' thinking processes and to help them develop more effective strategies, and involve various degrees of student participation and student control, and material of varying complexity.

CHAPTER FIVE

ASSESSMENT

Two kinds of assessments of this curriculum unit will be conducted: the assessment of students and the evaluation of curriculum.

Assessment of Students

The Purpose of Assessment of Students

Assessment is defined as "a process for determining the current level of a learner's performance or knowledge. The results of the assessment are then used to modify or improve the learner's performance or knowledge" (Díaz-Rico & Weed, 2002, p. 180). The assessment purpose of this unit is to evaluate the achievement of students throughout this course.

Design of Assessment of Students

Three different types of tests will be used to assess changes in learning at the beginning stage (pretest, proficiency test), at the intermediate stage (mid-term test, diagnostic test), and at the end of a course (posttest, achievement test). There is no placement test in this course. These will be criterion-referenced tests. I will give students diagnostic information objective-by-objective based on the pretest; diagnostic

progress information objective-by-objective at the midterm; and achievement information objective-by-objective at the end of the course. The result of pretest will be used to modify my lesson plan and instructional materials. The results of the midterm test and posttest will be used to arrive at a final grade for students.

Pretest: Criterion-referenced Proficiency Test. In this lesson, a pretest is important to grasp students' present level of mathematics because this course will focus on developing students' mathematical concepts. Before this course begins, a pretest should be taken to check students' prior knowledge of mathematics. The pretest will be one requirement to take this course. The pretest is an institutionally prepared test designed to measure what prior knowledge about mathematics students have attained.

Mid-term Test: Criterion-referenced Diagnostic Test. To assess students' progress during students' learning process in this course, students who will take this course will take a criterion-referenced diagnostic test as a mid-term exam. This test consists of mathematics questions and concepts. This information will be used to arrive at the final score or grade for a student together with a final

test. If there is poor performance on the diagnostic test, I will identify the cause of the problems by interpreting who or what may have caused the problems: the teacher, the materials, the student, or the course. The test takes 50 minutes.

Posttest: Criterion-referenced Achievement Test. To measure students' achievement, a criterion-referenced achievement test will be administered at the end of the course. The purpose of this posttest is to measure the amount that has been learned. This test will be developed in each skill area to directly reflect the objectives in this course and the types of activities that will take place. Thus, this test could be considered the final examination. To use it as a posttest in each session, I will create two forms of the test for measuring students' achievement. If there is poor performance on the achievement test, further investigation about this course will be needed to find the causes.

Rubric of Assessment of Students. The final assessment tests both the individual and the group. Individual assessment will cover oral communication (30%) and the final production of diagnostic and achievement test (40%). Even for their individual work, students are welcome to help each other through the process of research,

Table 1. Individual Assessment Rubric

Category	Key Points	Measurement
Oral Communication (30%)	How actively were students involved in the group discussion and group presentation?	Teachers observation Oral report rubric
Production of the diagnostic and achievement tests (40%)	How effectively did students understand this course?	Writing rubrics

collaboration, and peer editing. The average points of individual assessment will be added to group points. Therefore, the quality of individual work influences other group members' grades (10%). The group assessment will cover social skills (10%) and group presentations (10%). Rubrics for each assessment are below:

Table 2. Group Assessment Rubric

Category	Key Points	Measurement
Social skills (10%)	How did each group work collaboratively using social skills?	Teacher's observation Self-evaluation
Group presentation (10%)	How did each group effectively present their process and outcome of the project to other groups?	Peer-evaluation
Transfer from average of individual grade (10%)	Average grades of individual members will be added to group points.	

Curriculum Evaluation

The curriculum has involved an examination of the context in which the program occurs, of the goals, syllabus, and structure of a course, and how these can be planned and developed, as well as analysis of the teaching and learning that takes place during the course. This overall and interlinked system of elements (i.e., needs, goals, teachers, learners, syllabuses, materials, and teaching) is known as the second-language curriculum (Richards, 2001).

The purpose of curriculum evaluation is to examine the effects of curriculum. The purpose of course evaluation is to evaluate the effects of a course at significant end points of an educational cycle. In curriculum evaluation there are three types of evaluative methods: formative, illuminative, and summative evaluation.

The Purpose of Curriculum Evaluation

Weir and Roberts (1994) distinguished between two major purposes for language program evaluation: program accountability and program development. Accountability refers to the extent to which those involved in a program are answerable for the quality of their work. Accountability-oriented evaluation usually examines the

effects of a program or project at significant end points of an educational cycle and is usually conducted for the benefit of an external audience or decision maker.

Development-oriented evaluation, by contrast, is designed to improve the quality of a program as it is being implemented. It may involve staff members who are involved in the program as well as others who are not and may have a teacher-development focus (Weir & Roberts, 1994, p. 5).

Design of Curriculum Evaluation

The different purposes for evaluation are referred to as formative, illuminative, and summative evaluation. Among them, formative evaluation will be used to find the problems of the course and to improve the delivery of the program.

Formative Evaluation. The formative evaluation will be carried out as part of the process of course development in order to find out what is working well, and what is not, and what problems need to be addressed. It focuses on ongoing development and improvement of the program. Through this formative evaluation, this course will be evaluated by the information of the following questions: (a) Has enough time been spent on particular objectives?; (b) How well is the textbook being received?; (c) Is the methodology the teacher using appropriate?;

(d) Are the teacher or students having difficulties with any aspect of the course?; (e) Are students enjoying the program? If not, what can be done to improve their motivation?; (f) Are students getting sufficient practice work?; Should the workload be increased or decreased?; and (g) Is the pacing of the material adequate? (Richards, 2001). Information collected during formative evaluation will be used to address problems that have been identified and to improve the delivery of the program.

Illuminative Evaluation. To evaluate this curriculum, another type of evaluation described as illuminative evaluation is used. The purpose of this evaluation is to find out how different aspects of the program work or are being implemented and to provide a deeper understanding of the processes of teaching and learning that occur in the program, without necessarily seeking to change the course in any way as a result. Questions that might be asked within this framework are: (a) How do students carry out group-work tasks? (b) Do all students participate equally in them? (c) What type of error-correction strategies do teachers use? (d) What kinds of decisions do teachers employ while teaching? (e) How do teachers use lesson plans when teaching? (f) What type of teacher-student interaction patterns typically occur in classes? (g) What

reading strategies do students use with different kinds of texts? (h) How do students understand the teacher's intentions during a lesson? and (i) Which students in a class are most or least active? (Richards, 2001).

Summative Evaluation. A third approach to evaluate this curriculum is the type of evaluation with which most teachers and program administrators are familiar and which seeks to make decisions about the worth or value of different aspects of the curriculum. This is known as summative evaluation. Summative evaluation is concerned with determining the effectiveness of a program, its efficiency, and to some extent with its acceptability. It takes place after a program has been implemented and seeks to answer questions such as these: (a) How effective was the course? Did it achieve its aims? (b) What did the students learn? (c) How well was the course received by students and teachers? (d) Did the materials work well? (e) Were the objectives adequate or do they need to be revised? (f) Were the placement and achievement tests adequate? (g) Was the amount of time spent on each unit sufficient? (h) How appropriate were the teaching methods? and (i) What problems were encountered during the course? (Richards, 2001).

In summary, the assessment techniques used to assess the unit lessons are in accordance with the goal and objectives of the concept attainment lesson unit. Each lesson plan includes three types of assessment techniques: pretest; mid-test; and posttest. The pretest, a criterion-referenced proficiency test, will be taken to grasp students' prior knowledge and present level of concepts; the mid-term test, a criterion-referenced diagnostic test, will be taken to assess students' progress during the course; and the posttest, a criterion-referenced achievement test, will be taken to measure the amount that students have learned.

Additionally, to examine the effects of curriculum, curriculum will be also evaluated. To achieve the different purposes for evaluation, three curriculum evaluation methods will be used: formative, illuminative, and summative evaluation. The formative evaluation will be carried out as a part of the process of course to find the problems of curriculum; the illuminative evaluation will be conducted to find out the different aspects working in the program and provide a concrete understanding of the program; and the summative evaluation will be carried out to determine the effectiveness of the program.

Through these evaluations about the unit lessons and curriculum, the lessons can help students improve their learnability and the teacher also enhance his or her teachability. In addition, by evaluating curriculum, the teacher can find the advantages and disadvantages of curriculum, so through modifying or adjusting curriculum, can maximize the effectiveness of curriculum.

The key purpose of this project is to help ESL students in the U.S. get over their challenges in ESL and mathematics in the mainstream classroom and succeed in both ESL and content-area, particularly mathematics. Although many educators have suggested various beneficial teaching and learning methods useful for these ESL students, ESL students still face serious problems in their classrooms. Therefore, to help these ESL students and to be needed for teachers, this project suggests "A Model of Concept Attainment in Mathematics within Content-Based Instruction (CBI) for Secondary English as a Second Language." To support the theoretical framework and backgrounds of this model, five key concepts are investigated: content-based ESL; concept attainment; using visuals and manipulatives; Gregorc's learning styles; and adjunct content-based models.

Content-based ESL is a way to teach ESL and content-area simultaneously by combining them. In this project, it is supported by three important theories: Krashen's comprehensive input hypothesis; Cummins' two-tiered skill model; and cognitive learning theory. Additionally, concept attainment is an indirect instructional strategy that provides one important theoretical framework for unit lessons and curriculum in that it helps students more easily conceptualize complicated and abstract mathematical concepts by using concept attainment activities. Also, using visuals and manipulatives can be useful tools to improve students' metacognitive processing skills. Moreover, because students have their own learning styles, Gregorc's four learning styles such as abstract/sequential (AS), abstract/random (AR), concrete/sequential (CS), and concrete/random (CR), show the crucial characteristics and framework of learning styles. Finally, one of the content-based instruction methods, the adjunct content-based model, is appropriate for this project because the setting of this project is ESL in the U.S., in which ESL students and native-speaking students attend the same classes in adjunct content-based instruction models. Furthermore, to assess the unit lessons and curriculum, this project provides varied

assessment methods such as a pretest, mid-term test, and posttest to assess students' proficiency and achievement through this lesson. Besides, to evaluate curriculum and make it better and more effective, this project suggests three types of curriculum evaluation methods: formative, illuminative, and summative evaluation. Assessing students and curriculum will make this project better.

Many English learners who attend secondary schools in the United States face serious challenges in their content-area class such as mathematics, together with the problems of English language acquisition. For this reason, this project attempts to help these English learners enhance their knowledge both of the English language and mathematics, employ English as a medium of content-area instruction, and eventually succeed in both English and content-area classes.

The main purpose of this project is to provide a model of concept attainment in mathematics using content-based English-as-a-second-language instruction. The theoretical model that this project provides outlines fundamental connections between the sociocultural context of schools, adjunct models of collaboration between ESL and mathematics teachers, and ways that mathematics

teachers can help English learners attain complex concepts
in mathematics.

selection of the content. Therefore, when deciding the content for CBI curricula, teachers should consider several important factors, such as the fit with specified language objectives at students' present language proficiency.

Concept Attainment

Concept attainment is an indirect instructional strategy that uses a structured inquiry process. According to Bruner, Goodnow, and Austin (1967), concept attainment is "the search for and listing of attributes that can be used to distinguish exemplars from non-exemplars of various categories" (p. 233). As concept attainment forces students to distinguish features of a given concept for themselves, it improves their learning. Joyce and Weil (2004) report that "concept attainment requires a student to figure out the attributes of a category that is already formed in another person's mind by comparing and contrasting examples (called exemplars) that contain the characteristics (called attributes) of the concept with examples that do not contain those attributes" (pp. 67-68)

Tennyson and Cocchiarella (1986) have also conducted important research into concept learning and developed many models that can be used to improve instructional

design. Through their research, they have concluded that students develop procedural knowledge (how to attain concepts) with practice; and also, that the more procedural knowledge the students possess, the more effectively they attain and can apply conceptual knowledge. Therefore, in these concept attainment lessons, analyzing thinking appears to be important as a means to facilitate learning the metacognition of concept attainment.

The Purpose of Concept Attainment

The purpose of concept attainment is to improve students' metacognitive thinking skills (Joyce & Weil, 2004). In order to achieve this goal, in concept attainment lessons, many activities are designed to clarify concepts and introduce aspects of content. These make students formulate a concept through using examples. For this reason, concept attainment is well suited to classroom use. Thus, by carefully selecting examples, concept attainment lessons can be applied to all content subjects using concepts. In addition, concept attainment lessons can help students make connections between what they know and what they will be learning, learn how to examine a concept from many perspectives and how to sort out relevant information, and extend their knowledge of a

concept by classifying more than one example of that concept (Saskatoon Public School Division, 2004).

The Rationale of Concept Attainment

To describe the goal and process of concept attainment, terms such as exemplars and attributes are used. Developed from Bruner's et al. (1967) study of concepts, each term has a special meaning and function in all forms of concept attainment. Joyce and Weil (2004) described the term "exemplars" as the following:

Essentially the exemplars are a subset of a collection of data or a data set. The category is the subset or collection of samples that share one or more characteristics that are missing in the others. It is by comparing the positive exemplars and contrasting them with the negative ones that the concept or category is learned. (p. 66)

In concept attainment, because concept attainment lessons require that positive and negative exemplars be presented to the students, two types of exemplars are employed: positive exemplars and negative exemplars. In this case, negative exemplars are as important as positive exemplars because negative exemplars help the students identify the boundaries of the concept, and since by

comparing two exemplars that contain and do not include certain attributes, students can only identify the characteristics of the attributes precisely, and over time. All exemplars used in concept attainment lessons have their own features called attributes; these are either essential or non-essential. Joyce and Weil (2004) also defined the terms attributes as the following:

...Essential attributes are those important to the domain under consideration, and non-essential attributes are those not relevant to essential attributes. Exemplars of a category have many other attributes that may not be relevant to the category itself. Attribute value is important in concept attainment. This means the degree that an attribute is present in any particular example. When a teacher creates the items of data for instruction, it is wise to begin with exemplars in which the value of the attribute is high, dealing with the more ambiguous ones after the concept has been well established. Multiple attributes are another consideration. Multiple attributes mean that concepts range from cases in which the mere presence of a single attribute is sufficient for

membership in a category to those in which the presence of several attributes is necessary.

(pp. 66-67)

To teach a concept, it is important for the teacher to have to be very clear about its defining attributes and about whether attribute values are a consideration.

Strategies for Concept Attainment

In concept-attainment strategy, students are given a set of terms with positive and negative exemplars of a concept mixed together. Through observing these exemplars, students discuss and list the attributes of each until they develop their hypotheses about the concept. These hypotheses are tested by applying them to other exemplars of the concept. Exemplars could be symbols, words, passages, pictures or objects. Therefore, concept attainment strategies can be employed in all subject areas, and involves students in observing, analyzing, classifying, and inferring (Georgia Department of Education, 2003).

According to Joyce and Weil (2004), to achieve this concept-attainment strategy successfully, three important factors should be considered. First, teachers can construct the concept-attainment exercises in order to study how students think. Second, students can not only

describe how they attain concepts, but they can learn to be more efficient by changing their strategies and learning to use new ones. Finally, by changing the way the teacher presents information and by modifying the model slightly, the teacher can affect how students will process information.

The key to understanding the strategies that students use to attain concepts is to analyze how students approach the information available in the exemplars. Consequently, Joyce and Weil (2004) suggested that there are two kinds of key strategies for analyzing students' approaches in attaining concepts: partistic strategies and holistic strategies. Partistic strategies mean that students concentrate on just certain aspects of the information. On the contrary, holistic strategies mean that students keep all or most of the information in mind. For example, suppose that teachers are teaching concepts for analyzing literary style by comparing passages from novels and short stories. The first set of positive exemplars includes some kind of passage. Then, the students know that this passage will be grouped with the others to come, on the basis of one or more attributes pertaining to style. Some students will concentrate on just one kind of attribute, and others will scan the details of the passage. When comparing this

passage with another positive one, a partist will in some sense appear to have an easier task-just looking to see if the attribute present in the first is also present in the second, and so on. On the other hand, a holist has to keep many attributes in mind and has to eliminate nondefining elements one at a time.

Instructional and Nurturant Effects

The concept attainment can be a useful tool to accomplish various instructional goals. With abstract concepts, concept-attainment activities are designed to teach specific concepts and the nature of concepts, to provide practice in inductive reasoning and opportunities for altering and improving students' concept-building strategies, and to enhance the awareness of alternative perspectives, the sensitivity to logical reasoning in communication, and a tolerance of ambiguity (Joyce & Weil, 2004).

Gagné (1965) discussed a similar approach to concept attainment. However, Merrill and Tennyson (1977) describe a similar approach without extensively analyzing the thinking processes. In addition, McKinney, Warren, Larkins, Ford, and Davis (1983) have reported a series of interesting studies comparing the Merrill/Tennyson approaches with Gagné's and a recitation procedure. Their

work illustrates the complexity of designing studies to meaningfully compare sets of models built on the same premises but differing in details of execution. However, Joyce and Weil (2004) suggested that "the differences in approach and the research to build better models are probably of less importance to teachers than the fact that there are models that do a good job of teaching concepts--ones more powerful than the way concepts have traditionally been taught--and therefore represent useful additions to the teaching/learning repertoire" (pp. 75-76).

The Steps of Concept Attainment: A Mathematics Example

According to Saskatoon Public School Division (2004), the steps of concept attainment lesson are the following: (1) select and define a concept, (2) select the attributes, (3) develop positive and negative exemplars, (4) introduce the process to the students, (5) present the exemplars and list the attributes, (6) develop a concept definition, (7) give additional exemplars, (8) discuss the process with the class, and (9) evaluate.

For example, a teacher chooses a concept to be developed, such as a set of mathematics facts that equal 10. The teacher begins by making a list of positive

exemplars and negative exemplars. These exemplars are put onto sheets of paper or flash cards. Positive exemplars contain attributes of the concept to be taught. For example of this, positive exemplars can be $5+5$, $11-1$, 10×1 , $3+3+4$, $12-2$, $15-5$, $(4 \times 2)+2$, and $9+1$. Negative exemplars chosen are mathematics facts that do not have 10 as the answer, such as $6+6$, $3+3$, $12-4$, 3×3 , 4×4 , $16-5$, 6×2 , $3+4+6$, $2+(2 \times 3)$, and $16-10$. Then, the teacher designates one area of the chalkboard for the positive exemplars and the other area for negative exemplars. The chart could be set up at the front of the room with two columns—one marked YES and the other marked NO. The teacher presents the first card by saying, "This is YES," and places it under the appropriate column. For example, $5+5$ is a YES. Then, the teacher presents the next card by saying, "This is a NO," and also places it under the NO column. For example, $6+6$ is a NO. The teacher repeats this process until there are three examples under each column. Then, the teacher asks students to look at the three examples under the YES column and discuss how they are alike. The teacher also asks "What do they have in common?" For the next three examples under each column, the teacher asks the students to decide if the examples go under YES or NO. At this point, there are six examples

under each column. Many students will have recognized the concept. However, it is important for students not to say it aloud to their classmates. But they can show that they have caught on by giving an example of their own for each column. At this point, the examples are student-generated.

The teacher then asks students whether anyone else has the concept in mind. The students who have not yet recognized the concept are still trying to see the similarities of the YES examples. By now the teacher at least three more examples under each column that are student-generated. Next, the teacher discusses the process with the students. Once they have pointed out that everything under the YES column has an answer of 10, then the teacher prints a new heading at the top of the YES column such as 10 Facts, and prints a new heading for the NO column such as Not-10 Facts (Saskatoon Public School Division, 2004).

Summary

Based on Bruner et al.'s (1967) study and developed by many researchers (Gagné, 1965; Merrill & Tennyson, 1977; McKinney et al., 1983; Joyce & Weil, 2004), concept attainment is a strategy designed to teach concepts by presenting exemplars and non-exemplars to the students. When exemplars and non-exemplars are presented, the

students figure out, evaluate, and refine their hypotheses about the concepts by observing the data presented to them. Then, the students decide the important attributes of the concepts, and demonstrate whether or not they attain the concept by subsequently producing their own exemplars and non-exemplars.

As have been described above, many researchers have shown the evidence of the advantages of applying concept-attainment strategies. They remarkably improve students' metacognitive thinking skills, enhance students' sensitivity to logical reasoning in communication, and strengthen students' tolerance of ambiguity. Logically created and developed, concept-attainment lessons can be a useful instructional means applied to the students of all grade levels and all subject-content areas. However, most importantly, for concept attainment lessons to be successful in classes, it is more important for teachers to clearly recognize all important facts and aspects of concept-attainment activities as presented in this section.

Using Visuals and Manipulatives to Promote Content Learning and English Acquisition

Many kinds of useful teaching tools have been invented by the educators to help both teaching and

learning. Among them, visuals and manipulatives can be instructionally a good and useful tool. According to Canning-Wilson (1999), "using visuals allow for greater cognitive mapping and navigating in a classroom environment, and encourage the learner to predict, infer, and deduce information from a variety of sources" (p. 1).

In addition, Mistretta and Porzio (2000) concluded that "using manipulatives in mathematics classrooms not only helps students understand and explain the mathematical concepts and the related skills that they are expected to master at their grade levels, but also gives teachers valuable insights into students' mastery of these same concepts and skills" (pp. 32-33). However, from the results of the study of using manipulatives in English-as-a-second-language setting, Reimer and Moyer (2005) found that "the virtual manipulatives gave second language learners a way to express their thinking and understanding of patterns through the manipulation of the blocks, and helped the second language learners express their conceptual understanding of the regrouping process when they were unable to verbalize this process" (pp. 5-25).

Therefore, within the learning environment, employing visual and manipulative materials as the teaching and

learning tools is beneficial for both teachers and students because it is effective on facilitating both teaching and learning.

Defining Visuals and Manipulatives

Visuals and Manipulatives have been defined by many researchers. Canning-Wilson (1999) defined visual aids as follows:

In language learning there are two types of visual aids that can be used for learning: top-down visuals and bottom-up visuals. Top-down visuals are used to test ideas against facts or solve specific problems by viewing concept maps or by relating ideas to other facts and ideas, and bottom-up visuals use graphics to help the learner sort, scan, and organize information.
(p. 4).

In addition, from the results of the study of using visual images to improve comprehension for middle school struggling readers, Hibbing and Rankin-Erickson (2003) found that "although classrooms in the United States often have computers, televisions, and VCRs, and school classrooms, media centers, and computer labs are filled with visual images, unfortunately this bombardment of visual images does not necessarily transfer to students'

ability to create mental images that support reading comprehension" (pp. 758-770).

Also, manipulatives have been defined by many researchers. According to Crawford and Brown (2003), there are two types of manipulatives that teachers can use in the classrooms: manufactured manipulatives and digital manipulatives. Crawford and Brown (2003) described these as follows:

Manufactured or teacher-created concrete manipulatives, which are traditionally used within a mathematical classroom environment, are ones such as Cuisenaire rods, color tiles, Unifix cubes, pattern blocks, colored craft sticks, or other related, mass-produced objects. The digital manipulatives, web-based mathematical manipulatives, use the World Wide Web as an innovative medium to expand the learner's conceptual framework of understanding. (pp. 169-180)

Besides, Reimer and Moyer (2005) suggested that there are two types of manipulatives that teachers can use as the useful teaching tools in the classrooms: physical manipulatives and virtual manipulatives. According to Reimer and Moyer (2005), these two are defined as follows:

Physical manipulatives are the traditional manipulatives which have been used in mathematical classroom environment. Virtual manipulatives are essentially replicas of physical manipulatives placed on the World Wide Web in the form of computer applets with additional advantageous features. (pp. 5-25)

In consequence, from these definitions of visuals and manipulatives, regardless of the types of visuals and manipulatives, they have been essentially used as useful teaching and learning tools in the classrooms.

Why Use Visuals and Manipulatives in the Classrooms?

Visuals and manipulatives have been used in the classroom environment for a long time. Particularly, English language teachers have used visuals to help English learners' learning in the classrooms.

Canning-Wilson (1999) described why teachers use visual aids to teach English as follows:

In a teaching environment, visuals are a good and useful tool for teaching purposes because they lead the learner into drawing out language from their own knowledge and personal experiences through exposure to, immersion to

the stimuli presented before them. Also, a visual not only permits strategies to organize knowledge into semantic or associative clusters but also makes the task or situation appear more authentic and prompts the learner to find direct or indirect ways to play with the language and its structures. (pp. 1-3)

In addition, Luckner, Bowen, and Carter (2001) reported that "visual teaching strategies provide nontransient signals that can be used for pre-reading, post-reading, writing, content subjects, assessment, improving social interactions, and behavior management" (pp. 38-44). Moreover, using visuals in the classroom facilitates learning. Especially, when visuals are combined with texts, they "make subjects likely to think about the process of the language more fully and help us as individuals make sense of output and input surrounding us in our daily lives (Canning-Wilson, 1999, p. 5). By examining the development of imagery skills in their participants, Hibbing and Rankin-Erickson (2003) also described the benefits of the use of visual materials as the following:

The strategic use of visual material can enhance reading experiences for reluctant and

low-ability readers and, indeed, can help them become more proficient creators of internal visual imagery that supports comprehension. (pp. 758-770)

Also, many researchers have reported the advantages of employing manipulatives in the classrooms. According to Crawford and Brown (2003), "the primary purpose of the manipulative was to offer a concrete visualization of mathematical concepts that lead towards an understanding of the mathematical concepts as defined by learning objectives" (pp. 169-80). Additionally, Burns (2001a) suggested the merits of using manipulatives within the learning environment as the following: "(1) help make abstract ideas concrete, (2) lift mathematics off textbook pages, (3) build students' confidence by giving them a way to test and confirm their reasoning, (4) are useful tools for solving problems, and (5) make learning mathematics interesting and enjoyable" (cited by Crawford & Brown, 2003, pp. 169-80). Moyer (2001) suggested why manipulatives have become popular as the following:

Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract, have both visual and tactile appeal and can be manipulated by

learners through hands-on experiences... Because students' abstract thinking is closely anchored in their concrete perceptions of the world, actively manipulating these materials allows learners to develop a repertoire of images that can be used in the mental manipulation of abstract concepts. (p. 176)

Especially, together with the information age, web-based mathematical manipulatives have been integrated into the learning environment. Crawford and Brown (2003) suggested the merits of using digital manipulatives within the learning environment as the following:

The focus of web-based manipulatives is to enhance the learner's understanding of advanced theories and levels of understanding. The web-based manipulatives offer the computational abilities that aid in the communication of advanced concepts and theories of the learner. The focus is on the learner and the conceptual framework of understanding that is created due to the appropriate use of digital, web-based mathematical manipulatives. (pp. 169-180)

Consequently, using visual aids and manipulative strategies in the classrooms can help students easily

conceptualize complex and abstract mathematical concepts and maximize a visualization of complicated concepts.

The Selection of Visual and Manipulative Materials

First of all, to help students maximize their learning, it is essential for teachers to select good and useful visual and manipulative materials. Hodgdon (1995) suggested some strategies for choosing visual materials as the following: "(1) choose visuals students will easily recognize, (2) use large-size pictures or photographs with younger students, and (3) use a variety of visual materials, including written words, line drawing pictures, detailed drawings, computer generated pictures, photographs, photocopied pictures, cutouts from magazines, actual labels and wrappers, songs and logos, and coupons and real objects" (cited by Luckner et al., 2001, pp. 38-44). In addition, Luckner et al. (2001) suggested that "examples of visual aids that teachers can use in the classroom to enhance the communication and learning process include the use of a classroom rules chart, job and choice menus, transition time cards and charts, task organizers, daily schedules, and the Internet" (pp. 38-44). Moreover, from the findings of researching the role that visual imagery plays in reading

comprehension, Hibbing and Rankin-Erickson (2003) described visuals as follows:

If students can create their own images on the television screens in their minds as they read, their potential for understanding the text is increased. If students are not able to develop images because they are using all their mental energy to decode the words or their personal experiences, external visual images can be used to develop understanding. Strategic use of external visual images can provide the background knowledge and memory pegs to help students "see" what is happening and unlock confusing text...The use of sketches, illustrations, picture books, and movies provides students with information on which to build their internal images. (pp. 758-770)

Traditionally, teacher-created concrete objects have been used as manipulative materials. However, the information age offers mathematics educators the opportunity to integrate the use of digital manipulatives similar to manufactured and teacher-created ones (Crawford & Brown, 2003). Crawford and Brown (2003) described the

advantages of the use of digital manipulatives within a mathematical classroom environment as follows:

Digital manipulatives can be appropriately and successfully integrated into a mathematical learning environment through the use of web-based materials. The use of digital manipulatives provides an interactive environment with immediate feedback to explore in-depth mathematical theories that would be difficult to stimulate with concrete models. Additionally, younger students are able to see (conceptualize) concepts that would normally be regulated to in-depth abstract mathematical principles. (pp. 169-80)

As a result, in the classrooms, it is important for teachers to select effective visual and manipulative materials as a variety of teaching tools to help students improve their learning and understand difficult and abstract concepts.

Visual and Manipulative Teaching Strategies

Many researchers have suggested visual and manipulative teaching strategies for the successful visual and manipulative lessons within the classroom environment. Luckner et al. (2001) suggested various techniques in

using various visuals within the classroom environment as follows:

The chart of classroom rules can be useful visuals because they can help students learn to manage themselves more independently. Thus, they might include: (1) always try your best, (2) raise your hand when you need help, (3) keep your work and work area neat, (4) respect one another, and (5) pay attention when others are communicating. Classroom jobs and choice menus are also beneficial visuals. Classroom jobs can be posted with words and pictures or photographs, and choice menus can be established using words, photographs, pictures, or logs. Additionally, transition time cards/charts can be good visuals. They can be made of written words and accompanying pictures or photographs. Task organizers can be also useful visuals. These can include a set of pictures in a pocket-size photo album or a chart that lists all the steps to be with accompanying pictures or photographs. (pp. 38-44)

In addition, about using manipulatives, Stein and Bovalino (2001) concluded that "good lessons using

manipulatives do not just happen" (pp. 356-359). For this reason, Stein and Bovalino (2001) described the factors associated with successful manipulatives as follows:

The successful teachers devoted more time to planning the observed lesson and, more generally, to thinking about the role of manipulatives in developing students' mathematical understanding. In particular, these teachers exhibited three notable characteristics. First, they had extensive training in the use of manipulatives. Second, the more successful teachers designed their own lessons. Finally, the teachers who saw the best results had prepared the classroom and the manipulatives for the activity. Overall, the characteristics of the successful teachers can be summed up in two words: training and preparation. (pp. 356-359)

However, using manipulatives within the learning environment comes with specific tasks. Burns (2001b) described several specific "musts" that need to occur to facilitate positive correlation when manipulatives are used. Burns (2001b) suggested these "musts" as follows:

(1) the instructor conducts ongoing dialogue with students about why manipulatives help them learn math, (2) ground rules are set and consistently communicated as students work with manipulatives, (3) students are encouraged to develop a system for using and storing materials in the classroom under the teacher's direction, (4) time given to students for free exploration provides for more "ontime" behavioral applications as necessary, (5) manipulative are a natural for writing assignments, giving students' a writing focus, and (6) parents are given opportunities to gain hands-on experiences using the manipulatives. (pp. 169-180)

Summary

In this section, several key concepts of using visuals and manipulatives within the learning environment have been noted: such as the definition of visuals and manipulatives, the advantages of using visuals and manipulatives in the learning environment, the selection of visual and manipulative materials, and visual and manipulative teaching strategies.

In the classroom environment, using visual and manipulative materials can be a useful tool. Particularly,

employing visuals in ESL classrooms can help English learners comprehend immediate meaning by effectively recognizing information and concepts by visuals. Particularly, two types of visual aids can be used in language learning, such as top-down visuals and bottom-up visuals. To maximize the effects of students' learning, various visual aids such as pictures, television, drawings, illustrations, picture book, and movies can be used. Moreover, to be successful for using visuals in the classrooms, the lessons of using visuals should be carefully planned according to important strategies and techniques.

Furthermore, in English-as-a-second-language setting, using manipulatives can help English learners express their conceptualized understanding in more meaningful ways. In addition, using manipulatives in teaching mathematics as content-area can also help students develop the understanding of complicated and abstract mathematical concepts. Moreover, traditionally, teacher-manufactured-based manipulatives have been used, but together with the development of technology, digital or virtual manipulatives are more popular. Therefore, using these manipulative materials in the classrooms can improve learning as well as teaching.

Gregorc's Learning Styles

In the English-as-a-second-language setting, there is a wide range of approaches to learning styles. Some learners say that they cannot learn something until they have seen it; otherwise, others seem to need only to hear something once or twice before they know it. However, "learners clearly differ enormously in their preferred approach to English-as-a-second-language learning, but it is impossible to say which learning style works best; quite possibly it is learners who display flexibility who are most successful, but there is no real evidence yet for such a conclusion" (Ellis, 2002, p. 508). Therefore, even though there are many approaches to learning styles, because each approach has its own characteristics, it seems to be difficult to define which learning style is better for second-language acquisition.

The Definition of Learning Styles

Learning styles have been defined by many researchers. Reid (1995) defined learning styles as follows:

Learning style refers to an individual's natural, habitual, and preferred way(s) of absorbing, processing, and retaining new information and skills. These learning styles

persist, regardless of teaching methods and content areas. (p. viii)

However, according to Cornett (1983), "Learning style [is] a consistent pattern of behavior but with a certain range of individual variability...Styles then are overall patterns that give general direction to learning behavior (p. 9). In addition, Smith (1995) defined learning styles as "the perceptual and cognitive skills and strategies which the learner uses to collect, interpret, and store information" (pp. 7-10). Moreover, Ehrman and Oxford (1990) reported that "learning styles indicate preferred or habitual pattern of mental functioning and dealing with new information" (p. 58). Finally, Brown (2000) noted that "when cognitive styles are specifically related to an educational context, where affective and physiological factors are intermingled, they are usually more generally referred to as learning styles" (pp. 113-114).

Therefore, in terms of these various definitions of learning styles, learning styles are the English learners' own clear preferences that are expressed when learning the second language.

Approaches to Learning Styles

There are many approaches for investigating learning styles. A number of researchers have used survey

techniques to collect data of English learners' learning styles. On the basis of such data, Gieve (1991) identified learning styles as having five approaches as follows:

"(1) learners with instrumental motivation together with communicative orientation, (2) learners with no motivation, (3) learners interested in general intellectual development, (4) learners with a strong motivation but with no clear aims, and (5) learners with integrative motivation interested in living abroad" (cited by Ellis, 2002, p. 507). This analysis suggests that "the strength and nature of learners' motivation works as a major dimension of learning style" (Ellis, 2002, p. 507).

However, in general Reid (1995) categorized approaches of learning styles into three types: "cognitive learning styles, sensory learning styles, and personality, or affective/temperament, learning styles" (Reid, 1995, pp. ix-xiii). Reid (1995) described these approaches of learning styles as follows:

Cognitive learning styles are field-independent/field-dependent learning styles, analytic/global learning styles, reflective/impulsive learning styles, and Kolb experiential learning model. In addition, sensory learning styles are perceptual learning

styles, environmental styles, and sociological styles. Finally, affective/temperament learning styles are Myers-Briggs temperament styles, tolerance of ambiguity styles, and right-and-left-hemisphere learners. (pp. ix-xiii)

As a result, although there are many other approaches of learning styles developed by many researchers, these have generally overlapped with each other. A general conclusion concerning these learning styles may be that there is no best learning style approach because the values of learning styles are neutral and English learners have clear preferences for how they go about learning language (Ellis, 2002).

Cognitive Learning Styles

Cognitive learning styles are "information processing habits representing the learner's typical mode of perceiving, thinking, problem solving, remembering, and relating to others". (Ross, Drysdale, & Schulz, 2001, pp. 400-412). However, according to Brown (2000), cognitive style is "the way we learn things in general and the way we attack a problem seem to hinge on a rather amorphous link between personality and cognition, and when cognitive styles are specifically related to an educational context, where affective and physiological

factors are intermingled, they are usually more generally referred to as learning styles (pp. 113-114).

According to Reid (1995), there are four types of cognitive learning styles: field-independent/field-dependent learning styles, analytic/global learning styles, reflective/impulsive learning styles, and Kolb experiential learning model. Reid (1995) described field-independent and field-dependent learner, and analytic/global learning styles as follows:

The field-independent learner learns more effectively step by step, or sequentially, beginning with analyzing facts and proceeding to ideas; however, the field-dependent learner learns more effectively in context, holistically, intuitively, and is especially sensitive to human relationships and interactions. The analytic learner learns more effectively individually, prefers setting own goals, and responds to a sequential, linear, step-by-step presentation of materials, but the global learner learns more effectively through concrete experience, and by interactions with other people.

Moreover, together with these styles discussed above, Reid (1995) explained reflective/impulsive learning styles and the Kolb's experiential learning styles as follows:

The reflective learner learns more effectively when she or he has time to consider options before responding (often more accurate language learners); on the other hand, the impulsive learner learns more effectively when she or he is able to respond immediately and to take risks (often more fluent language learners). The Kolb's experiential learning model consists of concrete experiences plus abstract conceptualization (perception) and reflective observation plus active experimentation (process). The learning model of perception and process is further categorized into four learner types: converger (common sense learner) learns more effectively when she or he is able to perceive abstractly and to process actively; diverger (innovative learner) learns more effectively when she or he is able to perceive concretely and to process reflectively; assimilator (analytic learner) learns more effectively when she or he is able to perceive

abstractly and to process reflectively; and accommodator (dynamic learner) learns more effectively when she or he is able to perceive concretely and to process actively. (p. ix)

Therefore, although there are many types of cognitive learning styles, each approach to cognitive learning style has its own typology. English learners also have their own learning styles and learning preferences depending on various factors such as genders, culture, cognition, majors, and intelligence.

Gregorc's Style Delineator

Gregorc's style delineator is "a self-report tool used to help individuals understand and recognize the channels by which they receive and process information efficiently" (Gregorc, 1982, cited by Ross et al., 2001, pp. 400-412). Although there are many other methods useful for measuring cognitive learning style, the Gregorc style delineator has been used to investigate the relationship between learning style and academic performance in higher education (Drysdale, 1997; O'Brien, 1991; Stewart & Felicetti, 1992). The Gregorc style delineator is designed to "reveal two types of mediation abilities: perception and ordering" (Gregorc, 1982, p. 5).

Perception. Gregorc (1982) explained that "perceptual abilities are the means through which you grasp information. These emerge as two qualities: abstractness and concreteness" (p. 5). According to Gregorc (1982), the characteristics of these two qualities are as follows:

...[Abstractness] enables you to grasp, conceive, and mentally visualize data through the faculty of reason and to emotionally and intuitively register and deal with inner and subjective thoughts, ideas, concepts, feelings, drives, desires, and spiritual experiences. This quality permits you to apprehend and perceive that which is invisible and formless to your physical senses of sight, smell, touch, taste, and hearing. ...[Concreteness] enables you to grasp and mentally register data through the direct use and application of the physical senses. This quality permits you to apprehend that which is visible in the concrete, physical world through your physical senses of sight, smell, touch, taste, and hearing. (p. 5)

Ordering. Gregorc (1982) concluded that "ordering abilities are the ways in which you authoritatively arrange, systematize, reference, and dispose of

information. These emerge as two qualities: sequence and randomness" (p. 5). According to Gregorc (1982), the characteristics of these two qualities are as follows:

... [Sequence] disposes your mind to grasp and organize information in a linear, step-by-step, methodical, predetermined order... This quality enables you to naturally sequence, arrange, and categorize discrete pieces of information. It further encourages you to express yourself in a precise, progressive, and logically systematic manner. ... [Randomness] disposes your mind to grasp and organize information in a nonlinear, galloping, leaping, and multifarious manner... This quality enables you to deal with numerous, diverse, and independent elements of information and activities... This quality encourages you to express yourself in an active, multifaceted and unconventional manner.

(pp. 5-6)

According to Gregorc (1982), "These coupling of these qualities merged to form four distinct transaction ability channels designed as: Concrete/Sequential (CS), Abstract/Sequential (AS), Abstract/Random (AR), and Concrete/Random (CR)" (p. 6).

Gregorc's Dominant Style Characteristics

Gregorc (1982) combined two types of mediational abilities such as perception and ordering to create four mediation channels of mind styles: Concrete Sequential (CS), Abstract Sequential (AS), Abstract Random (AR), and Concrete Random (CR). Gregorc (1982) believes that individuals have the characteristics of each category, but most individuals tend to show a stronger orientation toward specific channels.

Style Characteristics of the Dominant CS Individual.

The world of reality of the dominant CS individual is concrete, and ordering ability is sequential (Gregorc, 1982). Gregorc (1982) described style characteristics of the dominant CS individual as the following:

The real world for the dominant Concrete Sequential is the concrete, physical, objective world... The dominant Concrete Sequential individual receives data from and produces in the concrete world as well...The dominant Concrete Sequential views and approaches experiences in his world of reality in an ordered, sequential, rectilinear, and one-dimensional manner. He expresses concerns about "bottom lines," "cross lines," and

"deadlines." Events are conceived as being joined in a successive and continuous manner like links in a chain. Consequently, he thinks by using a "train of thought" which has a clear beginning and a clear end. (p. 19)

Additionally, according to Gregorc (1982), "the CS uses shuns 'flowery language,' complex sentences, and 'two-dollar words' which, according to him, confuse both the issue and the listener" (p. 21).

Style Characteristics of the Dominant AS Individual.

The world of reality of the dominant Abstract Sequential (AS) individual is abstract, and ordering ability is sequential. Gregorc (1982) explained the style characteristics of the dominant AS individual as the following:

The "real" world for the dominant Abstract Sequential is the abstract nonphysical world of thoughts and mental constructions... The Abstract Sequential lives in a mental world and receives and produces data in this metaphysical, abstract world... The dominant Abstract Sequential's ordering pattern is sequential and can be represented in two-dimensional geometry. He orders in a tree-like manner starting with a

APPENDIX
INSTRUCTIONAL UNIT — CONCEPT ATTAINMENT

CONCEPT ATTAINMENT INSTRUCTIONAL UNIT PLAN

Concept Attainment Lesson One: Like Terms 118

Concept Attainment Lesson Two: Solutions of
Inequalities 126

Concept Attainment Lesson Three: Multiply Numbers
in Exponential Form 134

Concept Attainment Lesson Four: Multiplication
Properties of Exponents: Power of a Power
Property 142

Concept Attainment Lesson Five: Multiplying
Monomials 150

Concept Attainment Lesson Six: Multiplying
Polynomials 158

Concept Attainment Lesson One
Like Terms

Target level: 9th grade

Instructional Goals:

- The students will be able to identify like terms.

Content Objectives:

- The students will list like terms.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning like terms.
- The students practice speaking by aloud saying the positive and negative exemplars learned in like terms

Materials:

Focus Sheet 1-1

- Essential Attributes: Like Terms
- Important Non-Essential Attributes: Unlike Terms

Focus Sheet 1-2

- Identifying Like Terms
- Using Visuals and Manipulatives

Work Sheet 1-1

- Exemplars: List of Like Terms and List of Unlike Terms

Test Sheet 1-1

Task Chain 1: Presentation of the Data and Identification of Concept

1. Labeled examples: compare attributes in positive and negative examples (Focus Sheet 1-1).
 - Students will identify like terms as terms with the same variable raised to the same power ("YES" column) when compared to terms with the different variable raised to the different power ("NO" column).
2. Students generate and test hypothesis (Work Sheet 1-1).

3. Students state a definition according to the essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, yes or no.
 - Yes- Like terms: $5x, 3x$
 - No- Unlike terms: $5, 5m$
2. Teacher confirms hypothesis, names concept, and restates definition according to attributes.
 - Students will confirm the hypothesis by stating that like terms have the same variable raised to the same power.
3. Students generate examples.
 - Yes: $6a, 8a, -4a$
 - No: $1k, 3n, 12g$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses.

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 1-1, which they learned about like terms, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 1-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say like terms, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Assessment Sheet 1-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 1-1

Concept Attainment: Like Terms

Directions: Compare attributes in positive examples and negative examples.

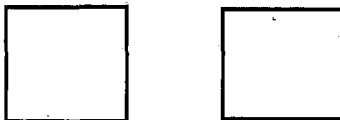
YES	NO
$3x, 5x$	$3x, 3x^0$
$4y, -2y$	$4x, -2y$
$6x^0, -7x^0$	$6y, -7x^0$
$5x^2, 3x^2$	$5x^3, 3x^2$
$-11xy, -8xy$	$9xy, 2x$
$xy, 4yx$	$3xy, 4yz$
$x^3, -x^3$	x^3, x^2
$\frac{1}{x^2}, \frac{3}{x^2}$	$\frac{1}{x^2}, \frac{1}{x^3}$
<p style="text-align: center;">Search Methods:</p> <p>Find terms with the same variable and the same power</p>	

Focus Sheet 1-2

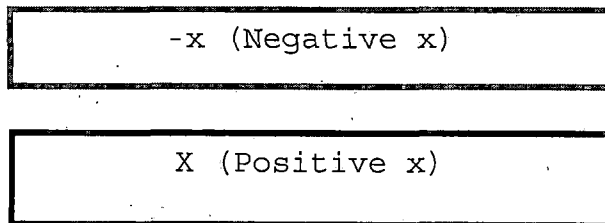
Identifying Like Terms:
Using Visuals and Manipulatives

Like terms can be grouped together because they have the same variable raised to the same power.

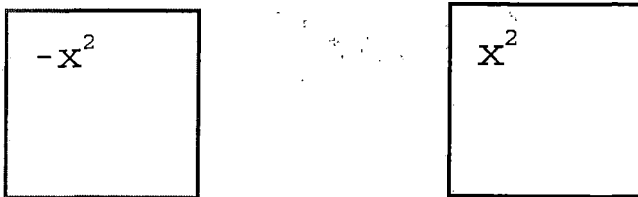
Example 1. (Algebra Tiles: Units)



Example 2. (Algebra Tiles: x)



Example 3. (Algebra Tiles: x^2)



Example 4.

x , $2x$, $-3x$, $5x$, $20x$, $100x$, $250x$, $-300x$

Work Sheet 1-1
 Concept Attainment: Like Terms

Directions: Decide if the two examples are Like Terms (YES) or Unlike Terms (NO).

Examples:

1. $x^3, 4x^3$ 2. $4xy, xy^3$
 3. $-xyz, -2yzx$ 4. $10yx^2, 4xy^2$

YES	NO
Brainstorm Possible Common Attribute	Brainstorm Possible Categories
<p>Why are these items grouped in the YES Column?</p> <p>Why are they grouped in the NO Column?</p> <p>How are the items similar?</p> <p>How are they different?</p> <p>How did the items change?</p>	

Work Sheet 1-2

Concept Attainment: Like Terms
Homework activity for English Learning

Directions: Write the like terms with words and say them aloud.

1. $x^3, 4x^3$

2. $4xy, xy^3$

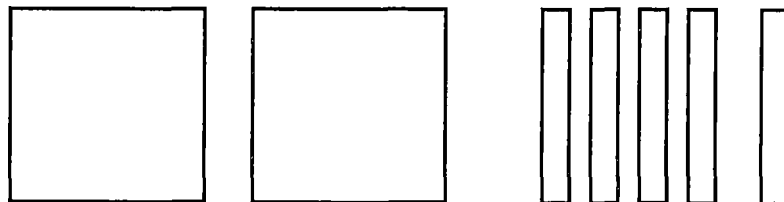
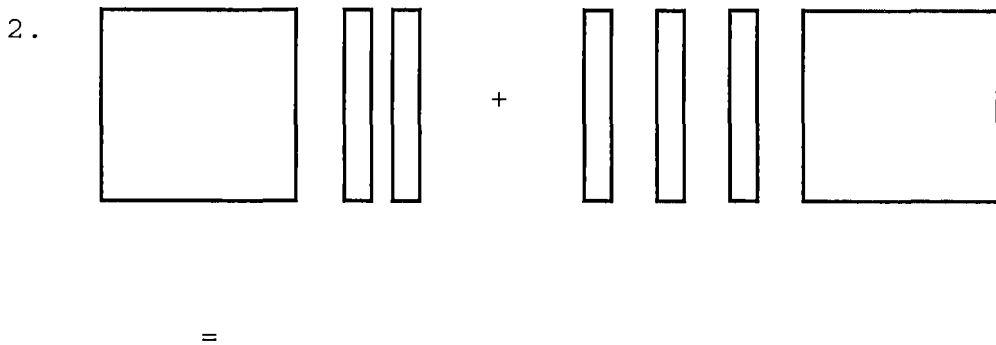
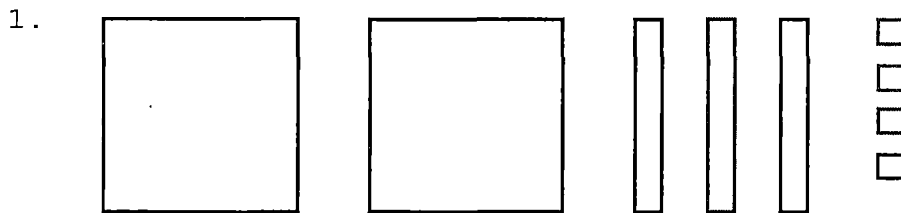
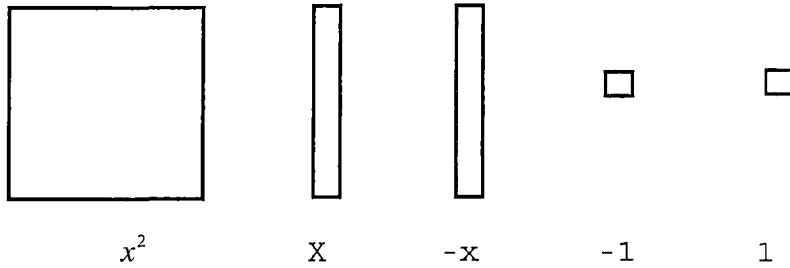
3. $-xyz, -2yzx$

4. $10yx^2, 4xy^2$

Assessment Sheet 1-1

Concept Attainment: Like Terms

Directions: Represent the following situations with an algebraic expression.



Concept Attainment Lesson Two
Solutions of Inequalities

Target level: 9th Grade

Instructional Goals:

- The students will be able to list solutions of inequalities.

Content Objectives:

- The students will list solutions.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning solutions of inequalities.
- The students practice speaking by aloud saying the positive and negative exemplars learned about solutions of inequalities.

Materials:

Focus Sheet 2-1

- YES: Solutions
- NO: No Solutions

Focus Sheet 2-2

- Finding Solutions of the Inequality

Work Sheet 2-1

- Examples: List of Solutions and List of No Solutions

Test Sheet 2-1

Task Chain 1: Presentation of the Data and Identification of Concept

1. Labeled examples: Compare attributes in positive and negative examples (Focus Sheet 2-1).
 - Students will identify solutions ("YES" column) when compared to no solutions ("NO" column).
2. Students generate and test hypothesis (Work Sheet 2-1).

3. Students state a definition according to the essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, as YES or NO.
 - YES: Solutions
 - No: No Solutions
2. Teacher confirms hypothesis, names concept, and restates definition according to attributes.
 - Students will confirm the hypothesis by stating the solutions.
3. Students generate examples.
 - YES: 1, 2, 3 for $x \leq 3$
 - NO: 4, 5, 6 for $x \leq 3$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses.

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 2-1, which they learned about solutions of inequalities, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 2-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say inequalities, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Assessment Sheet 2-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 2-1

Concept Attainment: Solutions of Inequalities

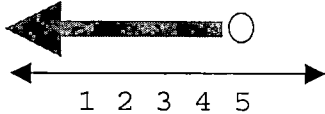
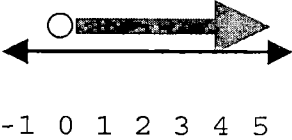
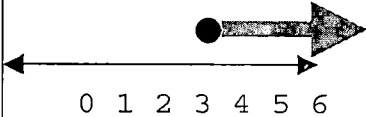
Directions: Compare attributes in positive examples and negative examples.

YES	NO
$x < 2$: 0, -1, -2, -3, -4, -5 $z \leq 1$: 1, 0, -1, -2, -3 $x + 5 \geq 3$ $x + 5 - 5 \geq 3 - 5$: -2, -1, 0, 1, 2 $x \geq -2$ The distance is greater than 8.8: 9, 10, 11, 12	$x < 2$: 2, 3, 4, 5, 6, 7 $z \leq 1$: 2, 3, 4, 5, 6 $x + 5 \geq 3$: -3, -4, -5, -6 The distance is greater than 8.8: 8, 7, 6, 5, 4
<p style="text-align: center;">Search Methods: Graph the solutions on a number line</p>	

Focus Sheet 2-1
 Concept Attainment
 Finding Solutions of Inequalities

An inequality compares two quantities and typically uses one of these symbols:

- <: Is less than
- >: Is greater than
- ≤: Is less than or equal to
- ≥: Is greater than or equal to

Word phrase	Inequality	Sample solutions	Solution set
x is less than 5	$x < 5$	$x = 4$ $x = 2.1$	
a is greater than 0 a is more than 0	$a > 0$	$a = 7$ $a = 25$	
m is greater than or equal to 3 m is at least 3	$m \geq 3$	$m = 17$ $m = 3$	

Work Sheet 2-1

Concept Attainment: Solutions of Inequalities

Directions: Decide if examples are positive (YES) or negative (NO).

Examples:

1. $z < 8$: 7, 6, 5, 4, 3, 2, 1

2. $t \leq -3$: 4, 5, 6, 7, 8, 9

3. $t + 8 \leq 12$: 0, 1, 2, 3, 4

4. $y \geq 20$: 30, 40, 50, 60

YES	NO
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">Brainstorm Possible Common Attribute</div>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">Brainstorm Possible Categories</div>
<p>Why are these items grouped in the YES Column? Why are they grouped in the NO Column? How are the items similar? How are they different? How did the items change?</p>	

Work Sheet 2-2

Concept Attainment: Solutions of Inequalities
Homework Activity for English Learning

Directions: Write the solutions of inequalities with words
and say them aloud.

1. $z < 8$: 7, 6, 5, 4, 3, 2,

2. $t \leq -3$: 4, 5, 6, 7, 8, 9

3. $t + 8 \leq 12$: 0, 1, 2, 3, 4

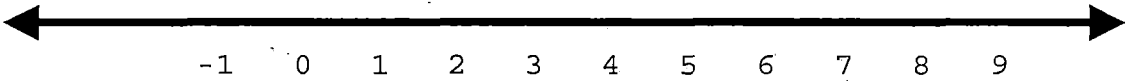
4. $y \geq 20$: 30, 40, 50, 60

Assessment Sheet 2-1

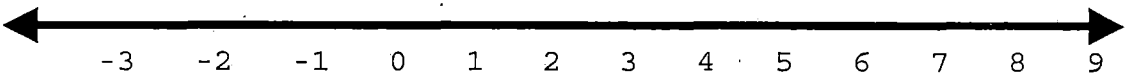
Concept Attainment: Solutions of Inequalities

Directions: Find solutions (using the number line).

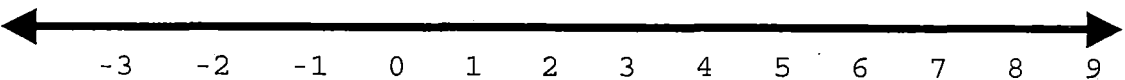
1. $d+4 \leq 6$



2. $x-3 > 2$



3. $h+6 \leq -2$



Concept Attainment Lesson Three
Multiply Numbers in Exponential Form

Target level: 9th Grade

Instructional Goals:

- The students will be able to multiply numbers in exponential form.

Content Objectives:

- The students will simplify exponential expressions.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning multiplying numbers in exponential form.
- The students practice speaking by aloud saying the positive and negative exemplars learned about multiplying numbers in exponential form.

Materials:

Focus Sheet 3-1

- Essential Attributes: List of exponential notation related to multiplication
- Important Non-Essential Attributes: List of exponential notation related to division

Focus Sheet 3-2

- Using multiplication properties of exponents Work Sheet 3-1
- Examples: List of exponential notation related to multiplication and division

Test Sheet 3-1

Task Chain 1: Presentation of the Data and Identification of Concept

1. Labeled examples: compare attributes in positive and negative examples (Focus Sheet 3-1).
 - Students will understand and use the multiplication properties of exponents (yes

- column) when compared to the division properties of exponents (no column).
2. Students generate and test hypothesis (Work Sheet 3-1).
 3. Students state a definition according to the essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, as YES or NO.
 - YES: the multiplication properties of exponents $(3^3)^2$
 - NO: the division properties of exponents $\frac{3^3}{3^2}$
2. Teacher confirms hypothesis, names concept, and restates definition according to attributes.
 - Students will confirm the hypothesis by stating the multiplication properties of exponents
3. Students generate examples.
 - YES: $(p^4)^4 = p^{16}$
 - NO: $\frac{p^4}{p^4} = p^{4-4} = p^0 = 1$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses.

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 3-1, which they learned about multiplying numbers in exponential form, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 3-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say exponential expressions, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Assessment Sheet 3-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 3-1

Concept Attainment: Multiplication Properties of Exponents

Directions: Students compare attributes in positive and negative examples.

YES	NO
$2^2 \times 2^5 = 2^7$ $x^3 \times x^3 = x^{3+3} = x^6$ $(4^3)^2 = 4^3 \times 4^3 = 4^{3+3} = 4^6$	$\frac{5^4}{5^2} = 5^{4-2} = 5^2 = 25$ $\frac{x^5}{x^2} = x^{5-2} = x^3$ $\left(\frac{4}{9}\right)^2 = \frac{4}{9} \times \frac{4}{9} = \frac{4^2}{9^2} = \frac{16}{81}$
<p style="text-align: center;">Search Methods: Add exponents or multiply exponents</p>	

Focus Sheet 3-2

Concept Attainment: Multiplication Properties of
Exponents: Using Visuals and Manipulatives

Product of powers: To multiply powers that have the same base, add the exponents.

$$a^2 \times a^3 = a \times a \times a \times a = a^{2+3} = a^5$$

Multiplying powers with the same base		
Words	Numbers	Algebra
To multiply powers with the same base, keep the base and add the exponents	$3^5 \times 3^8 = 3^{5+8} = 3^{13}$	$b^m \times b^n = b^{m+n}$

Example 1. Multiplying powers with the same base

$$a^{10} \times a^{10} = a^{10+10} = a^{20}$$

Work Sheet 3-1

Concept Attainment: Multiplication Properties of Exponents

Directions: Decide if examples are positive or negative.

Examples:

1. $(n^4)^3 = n^{12}$

2. $\frac{y^2}{x^6} = \frac{y^2}{x^6}$

3. $(3x)^4 = 3^4x^4 = 81x^4$

4. $\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3} = \frac{x^3}{27}$

YES	NO
Brainstorm Possible Common Attribute	Brainstorm Possible Categories
<p>Why are these items grouped in the YES Column? Why are they grouped in the NO Column? How are the items similar? How are they different? How did the items change?</p>	

Work Sheet 3-2

Concept Attainment: Multiplying Numbers in
Exponential Form

Homework Activity for English Learning

Directions: Write multiplying numbers in exponential form
with words and say them aloud.

1. $(n^4)^3 = n^{12}$

2. $\frac{y^2}{x^6} = \frac{y^2}{x^6}$

3. $(3x)^4 = 3^4 x^4 = 81x^4$

4. $\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3} = \frac{x^3}{27}$

Assessment Sheet 3-1

Concept Attainment: Multiplication Properties of Exponents

Directions: Simplify the expression.

1. $x^3 \cdot x^4$

2. $(a^3)^7$

3. $(2d)^3$

4. $8x^2y^{-4}$

5. $(mn)^2$

Concept Attainment Lesson Four:
Multiplication Properties of Exponents:
Power of a Power Property

Target level: 9th Grade

Instructional Goals:

- The students will be able to find a power of a power

Content Objectives:

- The students will simplify exponential expressions.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning power of a power property
- The students practice speaking by aloud saying the positive and negative exemplars learned about power of a power property.

Materials:

Focus Sheet 4-1

- Essential Attributes: List of power of power property
- Important Non-Essential Attributes: List of product of powers property

Focus Sheet 4-2

- Finding a power of a power Work Sheet 4-1
- Examples: List of power of a power property ("YES" column) and list of product of powers property ("NO" column)

Assessment Sheet 4-1

Task Chain 1: Presentation of the data and identification of concept

1. Labeled examples: compare attributes in positive and negative examples (Focus Sheet 4-1).
 - Students will understand and use the power of a power property of exponents ("YES" column) when

compared to the product of powers property of exponents ("NO" column).

2. Students generate and test hypothesis (Work Sheet 4-1).
3. Students state a definition according to the Essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, as YES or NO.
 - YES: the power of a power
 - NO: the product of powers property
2. The teacher confirms hypothesis, names concept, and restates definition according to attributes.
 - Students will confirm the hypothesis by stating the power of a power property that multiplies the exponents.
3. Students generate examples.
 - YES: $(p^4)^4 = p^{4 \times 4} = p^{16}$
 - NO: $a^m \times a^n = a^{m+n}$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses.

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 4-1, which they learned about power of a power property, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 4-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say exponential expressions with power of a power property, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Test Sheet 4-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 4-1

Concept Attainment: Multiplication Properties of
Exponents: Power of a Power Property

Directions: Students compare attributes in positive and
negative examples.

YES	NO
$\left[(-3)^5\right]^2 = (-3)^{5 \times 2} = (-3)^{10}$ $(n^4)^5 = n^{4 \times 5} = n^{20}$ $(4^3)^2 = 4^3 \times 4^3 = 4^{3+3} = 4^6$	$2^2 \times 2^5 = 2^7$ $x^3 \times x^3 = x^{3+3} = x^6$ $n^5 \times n^2 \times n^3 = n^{5+2+3} = n^{10}$
<p>Search Methods: Multiply exponents</p>	

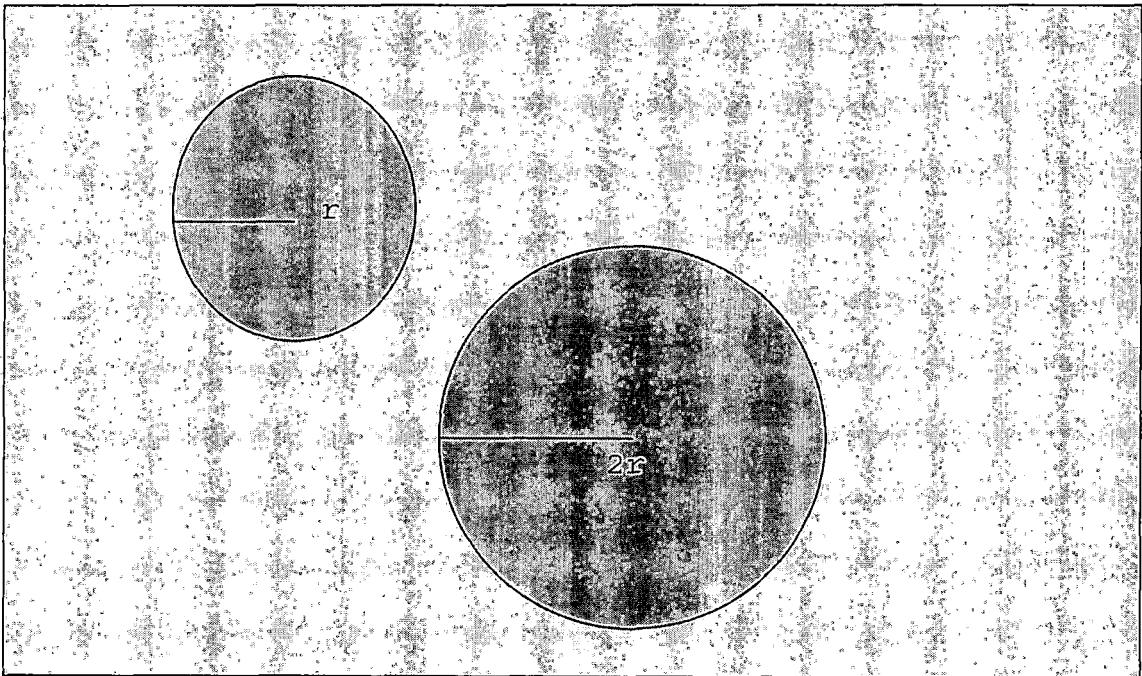
Focus Sheet 4-1

Concept Attainment: Finding Power of a Power Property

Power of a power: To find a power of a power, you multiply the exponents.

Example 1.

Farming: Find the ratio of the area of the larger irrigation circle to the area of the smaller irrigation circle.



The area of a circle can be found using the formula $A = \pi r^2$.

$$\text{Ratio} = \frac{\pi(2r)^2}{\pi r^2} = \frac{\pi \times 2^2 \times r^2}{\pi r^2} = \frac{\pi \times 4 \times r^2}{\pi \times r^2} = \frac{4}{1}$$

The ratio of the areas is 4 to 1.

Work Sheet 4-1

Concept Attainment: Multiplication Properties of
Exponents: Power of a Power Property

Directions: Decide if examples are positive or negative.

Examples:

1. $(n^4)^3 = n^{12}$
2. $x^3 \times x^2 = x^{3+2} = x^5$
3. $(3x)^4 = 3^4 x^4 = 81x^4$
4. $4^{10} \times 4^8 = 4^{10+8} = 4^{18}$

YES	NO
<div style="border: 1px solid black; padding: 5px; width: 80%; margin: 0 auto;">Brainstorm Possible Common Attribute</div>	<div style="border: 1px solid black; padding: 5px; width: 80%; margin: 0 auto;">Brainstorm Possible Categories</div>
<p>Why are these items grouped in the YES Column? Why are they grouped in the NO Column? How are the items similar? How are they different? How did the items change?</p>	

Work Sheet 4-2

Concept Attainment: Power of a Power Property
Homework Activity for English Learning

Directions: Write the exemplars in exponential form with words and say them aloud.

1. $(n^4)^3 = n^{12}$

2. $x^3 \times x^2 = x^{3+2} = x^5$

3. $(3x)^4 = 3^4 x^4 = 81x^4$

4. $4^{10} \times 4^8 = 4^{10+8} = 4^{18}$

Assessment Sheet 4-1

Concept Attainment: Multiplication Properties of
Exponents: Power of a Power Property

Directions: Simplify the expression.

1. $(3^2)^3$

2. $(a^3)^7$

3. $(z^5)^2$

4. $(a^m)^n$

5. $[(-4)^5]^3$

Concept Attainment Lesson Five:
Multiplying Monomials

Target level: 9th Grade

Instructional Goals:

- The students will be able to multiply monomials.

Content Objectives:

- The students will multiply monomials.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning multiplying monomials.
- The students practice speaking by aloud saying the positive and negative exemplars learned about multiplying monomials.

Materials:

Focus Sheet 5-1

- Essential Attributes: products of monomials
- Important Non-Essential Attributes: sum of monomials

Focus Sheet 5-2

- Multiplying monomials: Using visuals and manipulatives

Work Sheet 5-1

- Examples: products of monomials and sum of monomials

Test Sheet 5-1

Task Chain 1: Presentation of the data and identification of concept

1. Labeled examples: compare attributes in positive and negative examples (Focus Sheet 5-1).
 - Students will understand monomials and products of monomials ("YES" column) when compared to the addition of monomials ("NO" column).

2. Students generate and test hypothesis (Work Sheet 5-1).
3. Students state a definition according to the Essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, as YES or NO.
 - YES: Products of monomials
 - NO: sum of monomials
2. Teacher confirms hypothesis, name concept and restates definition according to attributes.
3. Students will confirm the hypothesis by stating products of monomials and understand how to multiply monomials by multiplying coefficients and adding exponents
4. Students generate examples.
 - YES: $\frac{1}{2}h \times b$
 - NO: $\frac{1}{2}h + b$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses.

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 5-1, which they learned about multiplying monomials, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 5-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say monomials, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Assessment Sheet 5-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 5-1

Concept Attainment: Multiplying Monomials

Directions: Students compare attributes in positive and negative examples.

YES	NO
$x^3 \cdot x^3 = x^{3+3} = x^6$ $(3r^2)(5r^4) = 15r^2 \cdot r^4 = 15r^6$ $(3r^2s^3)(5r^4s^5) = 15r^2r^4s^3s^5 = 15r^6s^8$	$x^3 + x^3 = 2x^3$ $(3r^2) + (5r^4) = 3r^2 + 5r^4$ $n^5 + n^2 + n^3 = n^2 + n^3 + n^5$
<p style="text-align: center;">Search Methods: Multiply coefficients and add exponents</p>	

Focus Sheet 5-2

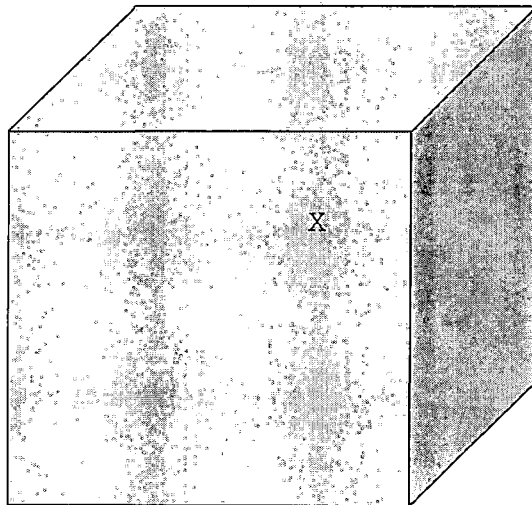
Concept Attainment:

Multiplying Monomials: Using Visuals and Manipulatives

To multiply two powers with the same bases, add the exponents.

To multiply two monomials, multiply the coefficients and add the exponents of the variables that are the same.

Example. Find the area of the base of the cube.



Solution: $x \cdot x = x^2$

Work Sheet 5-1

Concept Attainment: Multiplying Monomials

Directions: Decide if examples are positive or negative.

Examples:

1. $(n^4)(n^8) = n^{12}$
2. $x^3 \times x^2 = x^{3+2} = x^5$
3. $(3x)(4z) = 12xz$
4. $(7x^3) + (4x^3) = 11x^3$

YES	NO
Brainstorm Possible Common Attribute	Brainstorm Possible Categories
Why are these items grouped in the YES Column? Why are they grouped in the NO Column? How are the items similar? How are they different? How did the items change?	

Work Sheet 5-2

Concept Attainment: Multiplying Monomials

Homework Activity for English Learning

Directions: Write the exemplars about multiplying monomials with words and say them aloud.

1. $(n^4)(n^8) = n^{12}$

2. $x^3 \times x^2 = x^{3+2} = x^5$

3. $(3x)(4z) = 12xz$

4. $(7x^3) + (4x^3) = 11x^3$

Assessment Sheet 5-1

Concept Attainment: Multiplying Monomials

Directions: Multiply.

1. $(3^2)(x^2)$

2. $(a^3)(2x^2a^2)$

3. $5m(3m^4)$

4. $(-s^4t^3)(st)$

5. $(-gh^3)(-3g^2h^5)$

Concept Attainment Lesson Six:
Multiplying Polynomials

Target level: 9th Grade

Instructional Goals:

- The students will be able to multiply polynomials.

Content Objectives:

- The students will multiply polynomials.
- The students will name the concept.
- The students will reflect on the learning process.

English Language Objectives:

- The students write the positive and negative exemplars in English to improve their writing proficiency about words, phrases, and sentences as a homework activity through learning multiplying polynomials.
- The students practice speaking by aloud saying the positive and negative exemplars learned about multiplying polynomials.

Materials:

Focus Sheet 6-1

- Essential Attributes: products of polynomials
- Important Non-Essential Attributes: sum of polynomials

Focus Sheet 6-2

- Multiplying polynomials by using visuals and manipulatives

Work Sheet 6-1

- Examples: products of polynomials and sum of polynomials

Test Sheet 6-1

Task Chain 1: Presentation of the data and identification of concept

1. Labeled examples: compare attributes in positive and negative examples (Focus Sheet 6-1).
 - Students will understand polynomials and products of polynomials ("YES" column) when compared to the sum of polynomials ("NO" column).

2. Students generate and test hypothesis (Work Sheet 6-1).
3. Students state a definition according to the Essential attributes.

Task Chain 2: Testing attainment of concept

1. Identify additional unlabeled examples, as YES or NO.
 - YES: Products of polynomials
 - NO: Sum of polynomials
2. The teacher confirms hypothesis, names concept, and restates definition according to attributes.
 - Students will confirm the hypothesis by stating products of polynomials and understand how to multiply polynomials by distributing first, multiplying coefficients and, adding exponents.
3. Students generate examples.
 - YES: $(x+y)(z+w)$
 - NO: $\frac{1}{2}(h+b)+(b_1+b_2)$

Task Chain 3: Analysis of thinking

1. Students describe thoughts.
2. Students discuss role of hypothesis and attributes.
3. Students discuss type and number of hypotheses

Task Chain 4: Homework Activity: Students' English Language Learning

1. Students define the exemplars provided in Work Sheet 6-1, which they learned about multiplying polynomials, as words, phrases, or sentence by using English texts or materials.
2. Students practice speaking on their own the positive and negative exemplars provided in Work Sheet 6-1 by saying the positive and negative exemplars aloud.

Adjunct Teaching Follow-Up:

1. The adjunct teacher will practice how to say polynomials, mathematical expressions, and formulae in English.
2. The adjunct teacher will read problems aloud in English to model pronunciation, followed by students' practice.

Assessment:

1. Students will take Assessment Sheet 6-1.

Evaluation Criteria:

90 - 100 pts.	Excellent	A
80 - 89 pts.	Good	B
70 - 79 pts.	Fair	C
60 - 69 pts.	Acceptable	D
Below 59 pts.	Inadequate	F

Focus Sheet 6.1

Concept Attainment: Multiplying Polynomials

Directions: Students compare attributes in positive and negative examples.

YES	NO
$\begin{aligned} &(x+2)(x+3) \\ &= x(x+3)+2(x+3) \\ &= x(x)+x(3)+2(x)+2(3) \\ &= x^2+3x+2x+6 \\ &= x^2+5x+6 \end{aligned}$ $\begin{aligned} &(x-2)(5+3x-x^2) \\ &= x(5+3x-x^2)-2(5+3x-x^2) \\ &= 5x+3x^2-x^3-10-6x+2x^2 \\ &= -x^3+5x^2-x-10 \end{aligned}$	$\begin{aligned} &(5x^3-2x+x^2+7)+(3x^2+7-4x) \\ &= 5x^3+x^2+3x^2-2x-4x+7+7 \\ &= 5x^3+4x^2-6x+14 \end{aligned}$ $\begin{aligned} &(2x^2+x-5) \\ &+ (x^2+x+6) \\ &= 3x^2+2x+1 \end{aligned}$
<p style="text-align: center;">Search Methods:</p> <ul style="list-style-type: none"> • Distribute • Multiply • Combine like terms 	

Focus Sheet 6.1

Concept Attainment: Multiplying Polynomials:
Using visuals and manipulatives

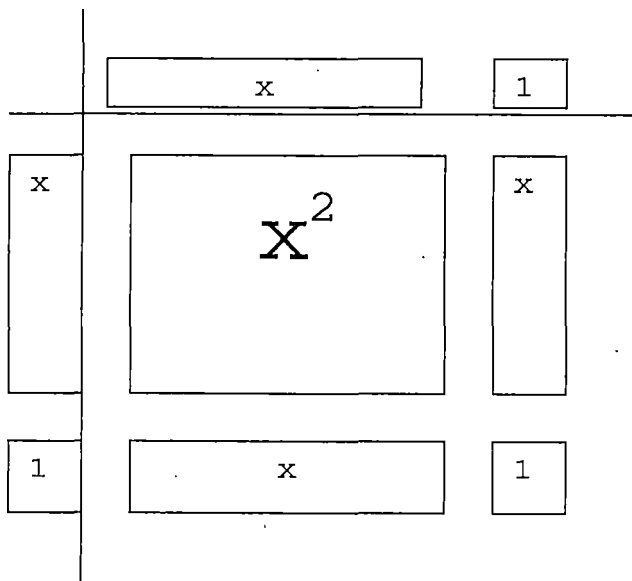
Multiply polynomials.

Example 1. $2(x+3)$

x			
x			

$$2(X + 3) = 2x + 6$$

Example 2. $(x + 1)(x + 1)$



Work Sheet 6-1

Concept Attainment: Multiplying Polynomials

Directions: Decide if examples are positive or negative.

Examples:

1. $3(2x-3) = 3(2x) - 3(3) = 6x - 9$

2. $2 + (x^3 + x^2) = 2 + x^3 + x^2$

$$(3x+1)(4z-1) = 3x(4z-1) + 1(4z-1)$$

$$= 3x(4z) + 3x(-1) + 1(4z) + 1(-1)$$

3. $= 12z - 3x + 4z - 1$
 $= 12z + 4z - 3x - 1$
 $= 16z - 3x - 1$

4. $(7x^3 + 1) + (4x^3 + 2) = 11x^3 + 3$

YES	NO
Brainstorm Possible Common Attribute	Brainstorm Possible Categories
Why are these items grouped in the YES Column? Why are they grouped in the NO Column? How are the items similar? How are they different? How did the items change?	

Work Sheet 6-2

Concept Attainment: Multiplying Polynomials

Homework Activity for English Learning

Directions: Write the exemplars about multiplying polynomials with words and say them aloud.

1. $3(2x-3) = 3(2x) - 3(3) = 6x - 9$

2. $2 + (x^3 + x^2) = 2 + x^3 + x^2$

3. $(3x+1)(4z-1) = 3x(4z-1) + 1(4z-1)$
 $= 3x(4z) + 3x(-1) + 1(4z) + 1(-1)$
 $= 12z - 3x + 4z - 1$
 $= 12z + 4z - 3x - 1$
 $= 16z - 3x - 1$

4. $(7x^3 + 1) + (4x^3 + 2) = 11x^3 + 3$

Assessment Sheet 6-1

Concept Attainment: Multiplying Polynomials

Directions: Multiply.

1. $(3^2 - y)(x^2)$

2. $(a^3 + b)(2x^2 + b)$

3. $(5m + 1)(3m^4 + m^2 + 1)$

REFERENCES

- Adamson, H. D. (1993). Academic competence theory and classroom practice: Preparing ESL students for content courses. London: Longman.
- Anderson, J. (1983). The architecture of cognition. Cambridge, MA: Harvard University Press.
- Anderson, J. (1985). Cognitive psychology and its implications (2nd ed.). New York: Freeman.
- Beckett, G. H., & Gonzalez, V. (2004). Content-based ESL writing curriculum: A language socialization model. NABE Journal of Research and Practice, 2(1), 161-175.
- Brinton, D. M., Snow, M. A., & Wesche, M. B. (1989). Content-based second language instruction. New York: Newbury.
- Brown, H. D. (1994). Teaching by principles: An interactive approach to language pedagogy. Englewood Cliffs, NJ: Prentice Hall Regents.
- Brown, H. D. (2000). Principles of language learning and teaching. White Plains, NY: Longman.
- Bruner, J. S., Goodnow, J. J., & Austin, G. A. (1967). A study of thinking. New York: Science Editions.
- Burns, M. (2001a). A letter to parents. Retrieved February 9, 2001, from <http://www.teacher.scholastic.com/lessonrepro/lessonplans/instructor/letter.htm>
- Burns, M. (2001b). 7 musts for using manipulatives. Retrieved February 9, 2001, from <http://www.teacher.scholastic.com/lessonrepro/lessonplans/instructor/musts.htm>
- Canning-Wilson, C. (1999). Using pictures in EFL and ESL classrooms. Paper presented at the Current Trends in English Language Testing Conference at Abu Dhabi, United Arab Emirates.
- Cornett, C. E. (1983). What you should know about teaching and learning styles. Bloomington, IN: Phi Delta Kappa Education Foundation.

- Crandall, J. (1994). Content-centered language learning. ERIC Clearinghouse on Languages and Linguistics. Washington DC.
- Crawford, C., & Brown, E. (2003). Integrating internet-based mathematical manipulatives within a learning environment. The Journal of Computers in Mathematics and Science Teaching, 22 (2), 169-80.
- Cummins, J. (1981). The role of primary language development in promoting educational success for language minority students. In J. Cummins (Ed.), Schooling and language minority students: A theoretical framework (pp. 1-50). Los Angeles: Evaluation, Dissemination, and Assessment Center.
- Davies, S. (2003). Content-based instruction in EFL contexts. The Internet TESL Journal, IX (2), Retrieved October 10, 2005, from <http://www.iteslj.org/Articles/Davies-CBI.html>
- Díaz-Rico, L. T., & Weed, K. Z. (2002). The crosscultural language, and academic development handbook (2nd ed.). Boston, MA: Allyn & Bacon.
- Dietrich, M. S. (2005). Integrating content into the language classroom. In R. Jourdenais & S. Springer (Eds.), Content, tasks and projects in the language classroom: 2004 conference proceedings (pp. 47-60). Monterey, CA: Monterey Institute of International Studies.
- Drysdale, M. T. B. (1997). Correlates of university persistence: A retrospective study. Calgary, ALB: Unpublished master's thesis, University of Calgary.
- Ehrman M., & Oxford, R. (1990). Adult language learning styles and strategies in an intensive training setting. The Modern Language Journal, 74(3), 311-324.
- Ellis, R. (2002). The study of second language acquisition. Oxford: Oxford University Press.
- Fujioka-Ito, N. (2000). Developing a project in content-based instruction extended from a reading passage. StudyAbroad.com Country Home Pages, Retrieved October 11, 2005, from <http://www.japan.studyabroad.com>

- Gagné, R. (1965). The learning of concepts. School Review, 75, 187-196.
- Georgia Department of Education. (2003). Concept attainment. Retrieved October 13, 2005, From <http://www.windarooss.qld.edu.au>
- Gieve, S. (1991). Goals and preferred language styles of Japanese English majors' In Proceedings of the Conference on Second Language Research in Japan, Tokyo: International University of Japan.
- Goldstein, L., Campbell, C., & Cummings, M. C. (1994). Smiling through the turbulence: The flight attendant syndrome and other issues of writing instructor status in the adjunct model. The CATESOL Journal, Spring, 19-29.
- Grabe, W., & Stoller, F. L. (1997). Content-based instruction: Research foundations. In M. A. Snow & D. M. Brinton (Eds), The content-based classroom: Perspectives on integrating language and content (pp. 5-21). White Plains, NY: Longman.
- Gregorc, A. F. (1982). An adult's guide to style. Maynard, MA: Gabriel Systems.
- Gregorc, A. (1982b). An adult's guide to style. Columbia, CT: Gregorc Associates, Inc.
- Hibbing, A., & Rankin-Erickson, L. J. (2003). A picture is worth a thousand words: using visual images to improve comprehension for middle school struggling readers. The Reading Teacher 56, 8, 758-70.
- Hodgdon, L. (1995). Visual strategies for improving communication. Vol. 1. Practical support for school and home. Troy, MI: Quirk Roberts.
- Joyce, B., & Weil, M. (2004). Models of teaching. Boston, NY: Pearson.
- Kasper, L. F. (1994). Improved reading performance for ESL students through academic course pairing. Journal of Reading, 37, 376-384.
- Kasper, L. F. (2000). Content-based college ESL instruction. Mahwah, NJ: Lawrence Erlbaum Associate.

- Krashen, S. (1982). Principles and practice in second language acquisition. Oxford: Pergamon.
- Krashen, S. (1985). Input hypothesis: Issues and implications. New York: Longman.
- Luckner, J., Bowen, S., & Carter, K. (2001). Visual teaching strategies. Teaching Exceptional Children 33,3, 38-44.
- McKinney, C., Warren, A., Larkins, G., Ford, M. J., & Davis, J. C. III. (1983). The effectiveness of three methods of teaching social studies concepts to fourth-grade students: An aptitude-treatment interaction study. American Educational Research Journal, 20, 663-670.
- Merrill, M. D., & Tennyson, R. D. (1977). Concept teaching: An instructional design guide. Englewood Cliffs, NJ.: Educational Technology.
- Met, M. (1998). Curriculum decision-making in content-based language teaching. In J. Cenoz & F. Genesee (Eds.), Beyond bilingualism: Multilingualism and multilingual education (pp. 35-63). Philadelphia, PA: Multilingual Matters.
- Mistretta, R., & Porzio, J. (2000). Using manipulatives to show what we know. Teaching Children Mathematics, 7(1), 32-3.
- Moyer, P.S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. Educational Studies in Mathematics, 47, 175-197.
- O'Brien, T. P. (1991). Relationships among selected characteristics of college students and cognitive style preferences. College Student Journal, 25, 631-636.
- O'Malley, J. M., & Chamot, A. U. (1990). Learning strategies in second language acquisition. Cambridge: Cambridge University Press.
- O'Malley, J. M., & Chamot, A. U. (1994). The CALLA handbook: Implementing the cognitive academic language learning approach. Reading, MA: Addison-Wesley.

- Oxford, R. (2001). Integrated skills in the ESL/EFL classroom. ERIC Clearinghouse on Languages and Linguistics, Retrieved October 14, 2005, from <http://www.cal.org/resources/digest/0105oxford.html>
- Pally, M. (1997). Critical thinking in ESL: An argument for sustained content. Journal of Second Language Writing, 6(3), 293-311.
- Reid, J. (1995). Learning styles in the ESL/EFL classroom. Boston: Heinle & Heinle.
- Reilly, T. (1988). ESL through content area instruction. ERIC Clearinghouse on Language and Linguistics. Washington, DC.
- Reimer, K., & Moyer, P. S. (2005). Third-grade learn about fractions using virtual manipulatives: A classroom study. The Journal of computers in Mathematics and Science Teaching, 24(1), 5-25.
- Richards, J. C. (2001). Curriculum development in language teaching. New York, NY: Cambridge University Press.
- Ross, J. L., Drysdale, T. B., & Schulz, R. A. (2001). Cognitive learning styles and academic performance in two postsecondary computer application courses. Journal of Research on Computing in Education, 33(4), 400-412.
- Saskatoon Public School Division. (2004). Instructional strategies online: concept attainment. Retrieved October 15, 2005, From <http://olc.spsd.sk.ca>
- Schleppegrell, M. J., Achugar, M., & Oteíza, T. (2004). The grammar of history: Enhancing content-based instruction through a financial focus on language. TESOL Quarterly, 38 (1), 67-93.
- Smith, G. B. (1995). More about learning styles of the gifted. Our Gifted Children 3, 6, 7-10.
- Snow, M. A., Met, M., & Genesee, F. (1989). A conceptual framework for the integration of language and content in second/foreign language instruction. TESOL Quarterly, 23, 201-217.

- Stein, M. K., & Bovalino, J. W. (2001). Manipulatives: One piece of the puzzle. Mathematics Teaching in the Middle School, 6 (6), 356-359.
- Stewart, K., & Felicetti, L. (1992). Learning styles of marketing majors. Educational Research Quarterly, 15, 15-23.
- Stoller, F. L., & Grabe, W. (1997). The six-T's approach to content-based instruction. In M. A. Snow & D. M. Brinton (Eds.) The content-based classroom: Perspectives on integrating language and content (pp. 78-94). New York: Longman.
- Teachers of English to Speakers of Other Languages. (1997). ESL standards for pre-k-12 students. Alexandria, VA: Author. Retrieved October 16, 2005, From <http://www.tesol.edu/assoc/k12standards/it/01.html>
- Tennyson, R. D., & Cocchiarella, M. (1986). An empirically based instructional design theory for teaching concepts. Review of Educational Research, 56, 40-71.
- Weir, C., & Roberts, J. (1994). Evaluation in ELT. Oxford: Blackwell.
- Willis, A. S. (1998). Feasibility of content-based instruction in Japanese foreign language courses: Some questions to ask. Studies in International Relations, 19 (1), 39-60.