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## Maximum axial load carrying capacity of Fibre Reinforced-Polymer (FRP) bar reinforced concrete columns under axial compression

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## Maximum axial load carrying capacity of Fibre Reinforced-Polymer (FRP) bar reinforced concrete columns under axial compression

### Abstract

In this study, a new equation is proposed to compute the maximum axial load carrying capacity of FRP bar reinforced concrete columns under axial compression. The equation proposed in this study was critically compared with the equations proposed in the previous research studies using a wide range of experimental data taken from the available literature. In general, it was found that computing the contribution of the FRP longitudinal bars in concrete columns based on the modulus of elasticity (stiffness) of the FRP bars provides more rational predictions than computing the contribution of the FRP longitudinal bars based on the ultimate tensile strength of the FRP bars. It was also found that using a concrete compressive strength-based empirical equation in estimating the axial strain in the FRP longitudinal bars in concrete columns provides more accurate predictions of the contribution of the longitudinal FRP bars in the axial load sustained by the FRP bar reinforced concrete columns.

### Keywords

(frp), maximum, reinforced, axial, compression, under, columns, concrete, load, carrying, capacity, fibre, reinforced-polymer, bar

### Disciplines

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1 **Maximum Axial Load Carrying Capacity of Fibre Reinforced-Polymer (FRP) Bar**  
2 **Reinforced Concrete Columns under Axial Compression**

3  
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13  
14 **Abstract**

15 In this study, a new equation is proposed to compute the maximum axial load carrying  
16 capacity of FRP bar reinforced concrete columns under axial compression. The equation  
17 proposed in this study was critically compared with the equations proposed in the previous  
18 research studies using a wide range of experimental data taken from the available literature.  
19 In general, it was found that computing the contribution of the FRP longitudinal bars in  
20 concrete columns based on the modulus of elasticity (stiffness) of the FRP bars provides  
21 more rational predictions than computing the contribution of the FRP longitudinal bars  
22 based on the ultimate tensile strength of the FRP bars. It was also found that using a  
23 concrete compressive strength-based empirical equation in estimating the axial strain in the  
24 FRP longitudinal bars in concrete columns provides more accurate predictions of the  
25 contribution of the longitudinal FRP bars in the axial load sustained by the FRP bar  
26 reinforced concrete columns.

27 **Keywords:**

28 Concrete columns; FRP bars; concentric axial load; maximum load carrying capacity

29

## 30 **1 Introduction**

31 The main function of a reinforced concrete column is to sustain axial loads with or without  
32 bending moments. The axial load carrying capacity of steel bar reinforced concrete  
33 columns decreases over the design (service) life of the concrete structures due to the  
34 corrosion of steel bars, especially in coastal regions or in harsh environments. The cost of  
35 rehabilitation and repair of deteriorated concrete structures is significantly high [1]. The  
36 National Association of Corrosion Engineers (NACE) International reported that the  
37 United States of America spends about two billion dollars annually to replace and repair  
38 the piers of the concrete bridges and about one billion dollars annually for maintaining  
39 marine piling systems [2].

40

41 The review of the literature found that Fibre Reinforced-Polymer (FRP) composites can be  
42 used in a wide range of civil/structural applications. The FRP composites have various  
43 structural forms that can be classified into two main classes: 1) External reinforcement  
44 (FRP jacketing) and 2) Internal reinforcement (FRP reinforcing bars) [3-4]. The FRP  
45 composites including FRP bars possess many advantageous characteristics such as the  
46 resistance to the harsh environmental conditions, light weight and high tensile strength [5-  
47 6]. Hence, FRP bars have the potential to replace steel bars and overcome the deterioration  
48 of concrete structures associated with the corrosion of steel reinforcement. However, the  
49 use of FRP bars as reinforcement in compression concrete members is still not  
50 recommended. This is because the ultimate compressive strength of the FRP bar is  
51 considerably lower than its ultimate tensile strength [7]. Chaallal and Benmokrane [8]

52 tested GFRP bars of three different diameters (15.9, 19.1 and 25.4 mm) and observed that  
53 the average compressive strength of the GFRP bars was 77% of the tensile strength.  
54 Kobayashi and Fujisaki [9] reported that the strength of the Aramid-FRP (AFRP), Glass-  
55 FRP (GFRP) and Carbon-FRP (CFRP) bars under axial compression were about 10%, 30-  
56 40% and 30-50% of their tensile strength, respectively. Deitz et al. [10] tested GFRP bars  
57 with a diameter of 15 mm under axial compression and observed that the ultimate  
58 compressive strength of the GFRP bars was approximately equal to 50% of their tensile  
59 strength.

60

61 The acceptance of FRP bars as longitudinal reinforcement in concrete compression  
62 members such as concrete columns requires the development of design guidelines. In this  
63 regard, experimental and analytical research studies were conducted to investigate and to  
64 understand the behaviour of concrete columns reinforced longitudinally with FRP bars.  
65 Generally, the load carrying capacities of the FRP bar reinforced concrete columns under  
66 concentric and eccentric axial loads are lower than the load carrying capacities of steel bar  
67 reinforced concrete columns having same dimensions and reinforced with the same  
68 longitudinal and transverse reinforcements. The reason for this is mainly attributed to the  
69 lower ultimate compressive strength and the lower modulus of elasticity of FRP bars in  
70 compression compared to those of conventional steel bars in compression [2, 11-16].  
71 Alsayed et al. [11] reported that the direct replacement of the longitudinal steel  
72 reinforcement with an equivalent amount of GFRP reinforcement reduced the axial load  
73 carrying capacity of the concrete columns by about 13%, irrespective of the type of the  
74 transverse reinforcement (steel or GFRP). Choo et al. [12] observed through an analytical  
75 study on FRP bar reinforced square concrete columns that ignoring the contribution of the  
76 longitudinal FRP bars in the compression region of the concrete columns may be overly

77 conservative. Tobbi et al. [13] and Afifi et al. [14] reported that GFRP and CFRP  
78 longitudinal bars can contribute up to 10% and 13% of the axial load carrying capacity of  
79 the concrete columns, respectively. Hadhood et al. [15] reported that GFRP longitudinal  
80 bars contributed about 5% of the axial load carrying capacity of GFRP bar reinforced high  
81 strength concrete (HSC) columns. A similar contribution for GFRP bars in HSC columns  
82 was also reported in Hadi et al. [16].

83

84 Due to the variances in the reported ultimate compressive strength of the FRP bars and  
85 their contribution as longitudinal reinforcement in concrete columns, no theoretical  
86 equation was recommended in the CAN/CSA S806-12 [17] or in ACI 440.1R-15 [18] to  
87 predict the maximum axial load carrying capacity of FRP bar reinforced concrete columns.  
88 Nonetheless, several theoretical equations were proposed in the previous research studies  
89 to predict the maximum axial load carrying capacity of FRP bar reinforced concrete  
90 columns. However, these equations have not been adequately assessed using a wide range  
91 of experimental data.

92

93 In this study, a new equation is proposed to predict the maximum axial load carrying  
94 capacity of concrete columns reinforced longitudinally with FRP bars. The theoretical  
95 equations, proposed in this study and in the previous studies, were critically assessed using  
96 a wide range of experimental data taken from the available literature. The observations  
97 reported in this study can help in establishing guidelines for designing FRP bar reinforced  
98 concrete compression members

99

100

101

## 102 **2 Conceptual assumptions**

103 The analysis of conventional steel bar reinforced concrete members is based on several  
104 basic assumptions, which are essential to compute the load carrying capacity of these  
105 members under different loading conditions. It was reported that these assumptions might  
106 be applicable to be used for GFRP bar reinforced concrete members [12, 15, 19]. Therefore,  
107 the assumptions were presented first and were used to analytically investigate the  
108 behaviour of GFRP bar reinforced circular concrete columns under concentric axial loads.

109 The basic assumptions are:

- 110 1. The maximum strain,  $\varepsilon_c$ , in concrete does not exceed an assumed ultimate concrete  
111 compressive strain,  $\varepsilon_{cu}$ .
- 112 2. A perfect bond exists at the interfaces between the GFRP bars and the surrounding  
113 concrete.
- 114 3. The axial strain in the concrete,  $\varepsilon_c$ , and the axial strain in GFRP reinforcing bars,  $\varepsilon_f$ ,  
115 are equal at any concentric axial load.

116

## 117 **3 Maximum axial load carrying capacity of reinforced concrete columns**

118 This study mainly focuses on the development of a theoretical equation for predicating the  
119 maximum axial load carrying capacity of FRP bar reinforced concrete columns, which  
120 occurs when columns are subjected to pure compression loads (axial loads with zero  
121 eccentricities). The proposed equation for the maximum axial load carrying capacity of  
122 FRP bar reinforced concrete columns can be incorporated in the future design codes for  
123 composite structures. The effects of the combined axial and flexural loads on the behaviour  
124 of FRP bar reinforced concrete columns are considered beyond the scope of this study.

125

126

127 **3.1 Steel bar reinforced concrete columns**

128 The maximum axial load carrying capacity,  $P_o$ , of conventional steel bar reinforced  
129 concrete columns under concentric axial load can be predicted using Eq. 1 [20-21].

130

131 
$$P_o = P_c + P_{bar,st} \quad (1)$$

132

133 
$$P_c = \alpha f'_c (A_g - A_{st}) \quad (2)$$

134

135 
$$P_{bar,st} = f_y A_{st} \quad (3)$$

136

137 Equation 1 represents the summation of the axial loads sustained by the concrete and steel  
138 longitudinal bars. The  $P_c$  represents the contribution of the concrete considering the gross  
139 area of the columns  $A_g$  as shown in Eq. 2. The  $P_{bar,st}$  represents the contribution of the  
140 longitudinal steel bars. The  $f_y$  and  $A_{st}$  are the yield strength and the total cross-sectional  
141 area of the longitudinal steel bars. It is noted that in Eq. 1 - Eq. 3, the effect of shear  
142 reinforcement has not been taken into account for calculating the maximum axial load  
143 carrying capacity of reinforced concrete columns under concentric axial loads [20].

144

145 The compressive strength of plain concrete in full-scale concrete columns tested under  
146 concentric axial loads is generally lower than the compressive strength of standard  
147 concrete cylinders. The differences between the compressive strength of concrete in  
148 columns and the compressive strength of standard concrete cylinders are commonly  
149 attributed to the differences in the shape, size and concrete casting process between  
150 columns and cylinders. In Eq. 2, the parameter  $\alpha$  is a reduction factor that represents the  
151 ratio between the in-place compressive strength of concrete in actual concrete columns to



152 the compressive strength of standard concrete cylinders. Extensive experimental  
153 investigations were carried out on reinforced concrete columns and the parameter  $\alpha$  was  
154 recommended to be taken equal to 0.85 [22]. The recommended value for the parameter  $\alpha$   
155 was considered in ACI 318-14 [20] to determine the contribution of the concrete in the  
156 maximum axial load carrying capacity of conventional steel bar reinforced concrete  
157 columns (Eq. 4).

158

$$159 \quad P_o = 0.85f'_c(A_g - A_s) + f_yA_s \quad (4)$$

160

### 161 **3.2 FRP bar reinforced concrete columns**

162 Different equations were proposed in the previous research studies to predict the maximum  
163 axial load carrying capacity of FRP bar reinforced concrete columns. It is important to note  
164 that the contribution of the concrete, in the analytically computed axial load carrying  
165 capacity of FRP bar reinforced concrete columns, remains similar in all of the proposed  
166 equations. In other words, the differences in the analytically computed values of  $P_o$  for  
167 FRP bar reinforced concrete columns are primarily due to the different concepts adopted in  
168 different proposed equations for calculating the contribution of the FRP longitudinal bars  
169 ( $P_{bar,FRP}$ ).

170

171 The compressive strength of FRP bar is considerably lower than its tensile strength and the  
172 behaviour of FRP bar under compressive loads differs significantly, as mentioned above.  
173 Hence, ACI 440.1R-06 [23] recommends not to reinforce concrete columns longitudinally  
174 with FRP bars and ACI 440.1R-15 [18] provided no recommendations in this regard. The  
175 CAN/CSA S806-12 [17] permits reinforcing concrete columns longitudinally with FRP  
176 bars. However, CAN/CSA S806-12 [17] recommends neglecting the contribution of the

177 FRP longitudinal bars when predicting the maximum axial load carrying capacity of the  
178 FRP bar reinforced concrete columns. Based on the recommendations in the CAN/CSA  
179 S806-12 [17], the maximum axial load carrying capacity of FRP bar reinforced concrete  
180 columns can be predicted using Eq. 5.

181

$$182 \quad P_o = 0.85f'_c(A_g - A_f) \quad (5)$$

183

184 where  $A_f$  represents the total cross-sectional area of GFRP longitudinal bars.

185

186 However, a considerable number of research studies observed that disregarding the  
187 contribution of FRP longitudinal bars in compression, as in Eq. 5, might result in a large  
188 difference between the analytically computed and the experimentally obtained axial load  
189 carrying capacity of the FRP bar reinforced concrete columns [14, 24-25]. Therefore, two  
190 approaches were considered to compute the contribution of FRP longitudinal bars in the  
191 maximum axial load carrying capacity of FRP bar reinforced concrete columns. In the first  
192 approach, the axial load sustained by FRP longitudinal bars is calculated using the tensile  
193 strength of the FRP bars,  $\alpha_f f_{fu} A_f$  (Eq. 6). In the second approach, the axial load sustained  
194 by FRP longitudinal bars is calculated using the axial strain in the FRP bars and the  
195 stiffness (modulus of elasticity) of the FRP bars,  $\varepsilon_f E_f A_f$  (Eq. 7).

196

$$197 \quad P_o = 0.85f'_c(A_g - A_f) + \alpha_f f_{fu} A_f \quad (6)$$

198

$$199 \quad P_o = 0.85f'_c(A_g - A_f) + \varepsilon_f E_f A_f \quad (7)$$

200

201 In Eq. 6, the  $\alpha_f$  is a reduction factor that represents the ratio between the strength of FRP  
202 bar under compression and the strength of the FRP bar under tension. Different values for  
203  $\alpha_f$  were recommended in the previous studies. Alsayed et al. [11] suggested taking  $\alpha_f$   
204 equal to 0.6. Later, Tobbi et al. [13] recommended taking  $\alpha_f$  equal to 0.35 based on  
205 experimental observations reported in Kobayashi and Fujisaki [9]. Also,  $\alpha_f$  was  
206 recommended to be taken equal to 0.35 in Afifi et al. [26] for GFRP bar reinforced circular  
207 concrete columns. However, for CFRP bar reinforced circular concrete columns, Afifi et al.  
208 [14] recommended taking  $\alpha_f$  equal to 0.25.

209

210 In Eq. 7, different values were also suggested for the axial strain in the FRP longitudinal  
211 bars,  $\varepsilon_f$ , at the maximum axial load carrying capacity of the concrete columns. Mohamed  
212 et al. [2] suggested taking  $\varepsilon_f$  equal to 0.002, explaining that this value ( $\varepsilon_f = 0.002$ )  
213 represents the axial strain in the FRP longitudinal bars at the initiation of the micro-cracks  
214 in the plastic stage of the concrete. However, Hadi et al. [25] recommended taking  $\varepsilon_f$  equal  
215 to 0.003, which represents the ultimate strain of the concrete,  $\varepsilon_{cu}$ .

216

217 It is obvious that different research studies proposed different equations based on a limited  
218 number of experimental data. Therefore, there is no consensus in the previous research  
219 studies on a unified equation for predicting the maximum axial load carrying capacity of  
220 FRP bar reinforced concrete columns, which may also be attributed to the variances in the  
221 response of the FRP bars under axial compression.

222

223 In this study, the axial load sustained by FRP longitudinal bars,  $P_{bar,FRP}$ , was predicted  
224 based on the stiffness (modulus of elasticity) of the FRP bars because the modulus of  
225 elasticity of FRP bars in compression is approximately similar to the modulus of elasticity

226 of FRP bars in tension [8, 10]. On the other hand, the compressive strength of the FRP  
 227 longitudinal bars corresponding to their ultimate tensile strength tends to fluctuate  
 228 significantly, depending on the manufactures and the type of the FRP bars (AFRP, GFRP  
 229 and CFRP). The significant fluctuations in the compressive strength of the FRP bars can be  
 230 considered one of the main reasons why different values for the reduction factor  $\alpha_f$  were  
 231 proposed in different research studies. Hence, simply changing the value of the reduction  
 232 factor  $\alpha_f$  in Eq. 6 might not provide reasonable predictions for the maximum axial load  
 233 carrying capacity of FRP bar reinforced concrete columns. The axial strain in the FRP  
 234 longitudinal bars  $\varepsilon_f$  at the maximum axial load carrying capacity of the concrete columns  
 235 was considered to be equal to the concrete axial strain at peak stress  $\varepsilon_{co}$ . The concept  
 236 adopted in this study is consistent with the third assumption in Section 2, which states that  
 237 the axial strain in the concrete and the axial strain in longitudinal FRP reinforcing bars are  
 238 equal at any concentric axial load. Accordingly, the maximum axial load carrying capacity  
 239 of FRP bar reinforced concrete columns can be predicted using Eq. 8:

240

$$241 \quad P_o = 0.85f'_c(A_g - A_f) + \varepsilon_{co}E_fA_f \quad (8)$$

242

243 Based on a considerable number of theoretical and experimental research studies, several  
 244 empirical formulas were proposed in the past few decades for computing the concrete axial  
 245 strain at peak stress,  $\varepsilon_{co}$ . In this study, four of the available, applicable and widely  
 246 accepted formulae (Eq. 9 - Eq. 12) were used to compute  $\varepsilon_{co}$  in Eq.8.

247

$$248 \quad \varepsilon_{co} = 735 (f'_c)^{0.25} \times 10^{-6} \quad (9)$$

249

$$250 \quad \varepsilon_{co} = 780 (f'_c)^{0.25} \times 10^{-6} \quad (10)$$

251

$$252 \quad \varepsilon_{co} = 0.0005 (f'_c)^{0.4} \quad (11)$$

253

$$254 \quad \varepsilon_{co} = 0.0016 \exp(240 f'_c / E_1) \quad (12)$$

255

256 Equation 9 was proposed in Popovics [27] for normal strength concrete with compressive  
257 strength of up to 50 MPa. whereas Eq. 10, proposed in Wee et al. [28], covered concrete  
258 with a compressive strength of up to 125 MPa. Legeron and Paultre [29] proposed Eq. 11  
259 for concrete with compressive strength ranging between 20 and 125 MPa, while Eq. 12,  
260 proposed in Yang et al. [30], is applicable to concrete with compressive strengths ranging  
261 between 10 and 180 MPa. Although Eq. 9 is applicable for normal strength concrete, an  
262 average difference of only 6% was observed between the values of the concrete axial strain,  
263  $\varepsilon_{co}$ , obtained from using Eq. 9 and Eq. 10. But, the values of  $\varepsilon_{co}$  obtained using Eq. 9 and  
264 Eq. 10 were consistently below the values of  $\varepsilon_{co}$  obtained using Eq. 11 and Eq. 12,  
265 especially for concrete having compressive strength greater than 100 MPa.

266

#### 267 **4 Critical assessment of the proposed equations**

268 The equation proposed in this study was critically reviewed using a wide range of  
269 published experimental data (Table 1). The equation proposed in Tobbi et al. [9] was also  
270 examined. Hadi et al. [25] recommended assuming  $\varepsilon_f$  equal to  $\varepsilon_{cu}$ . The equation proposed  
271 in Hadi et al. [25] was also assessed, firstly by taking  $\varepsilon_{cu}$  equal to 0.003 as defined in the  
272 ACI 318-14 [20] and secondly by taking  $\varepsilon_{cu}$  equal to 0.0035 as defined in the CSA A23.3-  
273 14 [21].

274

275 Table 2 presents the ratios between the analytically predicted and the experimentally  
276 obtained axial load carrying capacity ( $P_o/P_{exp.}$ ) for the experimentally tested specimens  
277 presented in Table 1. The analytically predicted axial load carrying capacity,  $P_o$ , was  
278 calculated using either Eq. 6 by taking  $\alpha_f$  equal to 0.35, as recommended in Tobbi et al.  
279 [13] or using Eq. 8, in which the value of  $\varepsilon_{co}$  was either computed using the formulas  
280 presented in the above section (Eq. 9 - Eq. 12) or taken equal to  $\varepsilon_{cu}$  (0.003 or 0.0035 as  
281 defined in the ACI 318-14 [20] and CSA A23.3-14 [21], respectively).

282

283 In Table 2, the accuracy of the equations proposed in this study and in the previous  
284 research studies in predicting the maximum axial load carrying capacity of FRP bar  
285 reinforced concrete columns was examined using four different mathematical  
286 measurements: Mean value ( $\mu$ ); Standard Deviation ( $SD$ ); Coefficient of Variation ( $COV$ )  
287 and the Mean Absolute Percentage Error ( $MAPE$ ). The Mean value ( $\mu$ ) represents the  
288 central value of the discrete set of  $P_o$  values. The Standard Deviation ( $SD$ ) was used to  
289 quantify the level of variation (dispersion) for the values of  $P_o$ . High standard deviation  
290 indicates that the predicted axial load carrying capacities of the FRP bar reinforced  
291 concrete specimens are spread out over a wider range of values (less reliable) and vice  
292 versa. The standard deviation ( $SD$ ) indicates to an absolute term of how much the values of  
293  $P_o$  are spread. The values of  $P_o$  were then assessed as a percentage of how far away they  
294 spread from their mean value using the Coefficient of Variation ( $COV$ ). The lower  
295 the  $COV$ , the lower the dispersion of  $P_o$  is from the mean value. The Mean Absolute  
296 Percentage Error ( $MAPE$ ) is also used as a measurement to examine the accuracy of the  
297 equation proposed in this study and the equations proposed in previous research studies for  
298 the maximum axial load carrying capacities of the FRP bar reinforced concrete columns.  
299 The lower the  $MAPE$ , the better predictions provided by the proposed equation.

300 Table 2 presents the comparison between the experimental and theoretical axial load  
301 carrying capacity of FRP bar reinforced concrete columns. It was found that Eq. 8, in  
302 which the contribution of the FRP bars is computed based on the stiffness of the FRP bars,  
303 provides more reliable and safer predictions for  $P_o$  compared to Eq. 6, in which the  
304 contribution of the FRP bars is computed using the tensile strength of the FRP bars. This  
305 might be mainly attributed to the fact that the modulus of elasticity of the FRP bars in  
306 tension is approximately equal to the modulus of elasticity of FRP bars in compression,  
307 while there is a large difference between the tensile and the compressive strength of the  
308 FRP bars. It was also found that, in Eq. 8, the use of the formula proposed by Legeron and  
309 Paultre [29] (Eq. 11) in computing  $P_{bar,FRP}$  provides lower discrepant values of  $P_o$ , as  
310 shown in Fig. 1, giving a standard deviation and a coefficient of variation of 0.071 and  
311 7.71, respectively. On the other hand, taking the concrete axial strain at peak stress  $\varepsilon_{co}$   
312 equal to 0.003 when computing  $P_{bar,FRP}$  provided predictions for  $P_o$  with the lowest  
313 percentage of error giving a mean absolute percentage error *MAPE* of 7.542. Furthermore,  
314 taking  $\varepsilon_{co}$  equal to 0.0035 when computing  $P_{bar,FRP}$  provided predictions with the highest  
315 but rather safe mean value  $\mu$  for  $(P_o/P_{exp.})$  of = 0.97, which is very close to the unity, but  
316 with high *SD* and *COV* of 0.082 and 8.47, respectively (Fig. 1).

317

318 Figure 2 shows the relationship between the  $(P_o/P_{exp.})$  for the specimens presented in  
319 Table 2 and the compressive strength of the concrete. In Fig. 2, the  $P_o$  was either obtained  
320 from Eq. 6 assuming  $\alpha_f$  equal to 0.35 as recommended in Tobbi et al [13] or from Eq. 8  
321 taking  $\varepsilon_{co}$  equal to 0.003 or 0.0035 or computed using the formula proposed by Legeron  
322 and Paultre [29].

323

324 Assuming  $\alpha_f$  equal to 0.35 (Eq. 6), as recommended in Tobbi et al [13], over-predicts the  
325 axial load carrying capacity for most of the FRP bar reinforced NSC and HSC columns  
326 presented in Table 1, as shown in Fig 2a. However, using Eq. 8, assuming  $\varepsilon_{co}$  equal to  
327 0.003 for computing the contribution of the FRP bar over-predicts the axial load carrying  
328 capacity of 23% of the FRP bar reinforced specimens. But in general it provides reasonable  
329 predictions with mean value  $\mu$  for  $(P_o/P_{exp.})$  of 0.95. On the other hand, using Eq. 8,  
330 considering the formula proposed in Legeron and Paultre [29] (Eq. 11) for  $\varepsilon_{co}$  in  
331 computing the contribution of the FRP bar over-predicts the axial load carrying capacity of  
332 only 6% of the total number of the specimens presented in Table 2 with a mean value  $\mu$  for  
333  $(P_o/P_{exp.})$  of 0.93, hence, it provides reliable and safe prediction for nearly all the FRP bar  
334 reinforced NSC and HSC column specimens (Fig 2d).

335

## 336 **5 Conclusions**

337 The present study proposes a theoretical equation for predicting the maximum axial load  
338 carrying capacity of FRP bar reinforced concrete columns. In the proposed equation, the  
339 contribution of the FRP longitudinal bars was computed based on the axial strain and the  
340 stiffness (modulus of elasticity) of the FRP longitudinal bars. An empirical equation based  
341 on the concrete compressive strength was developed to compute the axial strain in the FRP  
342 longitudinal bars at the maximum axial load carrying capacity of the FRP bar reinforced  
343 concrete columns. The proposed equation was validated using a large set of experimental  
344 data available in the literature. The equation proposed in this study provided more accurate  
345 and safe predictions of the experimentally tested FRP bar reinforced columns. The  
346 theoretical equation proposed in this study can be easily applied in predicting the axial load  
347 carrying capacity of normal strength and high strength concrete columns reinforced with  
348 different types of FRP bars.



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353

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494 Fig. 1: Experimental versus predicted axial load carrying capacity of FRP bar reinforced  
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497 Fig. 2: The relationship between  $P_o/P_{exp}$  of the FRP bar reinforced concrete column and  
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499  $0.35$ ); b) Eq. 8 ( $\epsilon_{co} = 0.003$ ); c) Eq. 8 ( $\epsilon_{co} = 0.0035$ ) and d) Eq. 8 ( $\epsilon_{co} = 0.005(f'_c)^{0.4}$ ).

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Table 1: Experimental data of FRP bar reinforced concrete columns taken from available previous research studies.

Research study	Specimen cross-section			FRP longitudinal reinforcement					Concrete
	Specimen	Column shape	Dimensions* (mm)	Type	No. of bars	Diameter (mm)	$f_{fu}$ (MPa)	$E_f$ (MPa)	$f'_c$ (MPa)
Afifi et al. [14]	SP-1	Circular	300	CFRP	6	12.7	1899	140000	42.9
	SP-2	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-3	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-4	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-5	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-6	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-7	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-8	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-9	Circular	300	CFRP	14	12.7	1899	140000	42.9
Afifi et al. [26]	SP-10	Circular	300	GFRP	4	15.9	934	55400	42.9
	SP-11	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-12	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-13	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-14	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-15	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-16	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-17	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-18	Circular	300	GFRP	12	15.9	934	55400	42.9
Mohamed et al. [2]	SP-19	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-20	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-21	Circular	300	GFRP	8	15.9	934	55400	42.9
	SP-22	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-23	Circular	300	CFRP	10	12.7	1899	140000	42.9
	SP-24	Circular	300	CFRP	10	12.7	1899	140000	42.9



Table 1: (Continued)

Research study	Specimen cross-section			FRP longitudinal reinforcement					Concrete
	Specimen	Column shape	Dimensions* (mm)	Type	No. of bars	Diameter (mm)	$f_{fu}$ (MPa)	$E_f$ (MPa)	$f'_c$ (MPa)
Karim et al. [31]	SP-25	Circular	205	GFRP	6	12.7	1600	66000	32
	SP-26	Circular	205	GFRP	6	12.7	1600	66000	32
	SP-27	Circular	205	GFRP	0	0	0	0	32
	SP-28	Circular	205	GFRP	0	0	0	0	32
Hales et al. [32]	SP-29	Circular	305	GFRP	6	16	715	44000	90
Hadhood et al. [15]	SP-30	Circular	305	GFRP	8	15.9	1289	54900	70.2
	SP-31	Circular	305	GFRP	12	15.9	1289	54900	70.2
Hadhood et al. [33]	SP-32	Circular	305	CFRP	8	15.9	1680	141000	35
Hadi et al. [16]	SP-33	Circular	210	GFRP	6	12.7	1548	67800	85
	SP-34	Circular	210	GFRP	6	12.7	1548	67800	85
Tobbi et al. [13]	SP-35	Square	350 x 350	GFRP	8	19.1	728	47600	33
	SP-36	Square	350 x 350	GFRP	12	15.9	751	48200	33
	SP-37	Square	350 x 350	GFRP	4+4	12.7, 15.9	1040, 751	46300, 48200	33
	SP-38	Square	350 x 350	GFRP	8	12.7	1040	46300	33

\* Represents the diameter for circular columns and the length times the width of the square columns

$f_{fu}$ = The ultimate tensile strength of FRP bars

$E_f$ = The modulus of elasticity of FRP bars

$f'_c$ = The compressive strength of the concrete

Table 2: Comparison between the experimental and theoretical axial load carrying capacity of FRP bar reinforced concrete columns available in the previous research studies

Study	Specimen	$P_{exp.}^a$	$P_o/P_{exp.}^b$						
			Eq. (8)					Eq. (6)	
			Popovics [23] <sup>c,e</sup>	Wee et al. [24] <sup>c,e</sup>	Legeron and Paultre [25] <sup>c,e</sup>	Yang et al. [26] <sup>c,e</sup>	ACI 318- 14 [16] <sup>d</sup>	CSA A23.3- 14 [17] <sup>d</sup>	Tobbi et al. [9] <sup>f</sup>
Afifi et al [14]	S-1	2905	0.95	0.95	0.96	0.96	0.99	1.01	1.05
	S-2	3148	0.91	0.92	0.93	0.93	0.97	1.00	1.07
	S-3	2948	0.97	0.98	0.99	0.99	1.04	1.07	1.14
	S-4	3070	0.93	0.94	0.95	0.95	1.00	1.03	1.10
	S-5	3013	0.95	0.96	0.97	0.97	1.02	1.05	1.12
	S-6	2981	0.96	0.97	0.98	0.98	1.03	1.06	1.13
	S-7	3147	0.91	0.92	0.93	0.93	0.97	1.00	1.07
	S-8	2941	0.97	0.98	1.00	0.99	1.04	1.07	1.15
	S-9	3107	0.96	0.97	0.99	0.99	1.05	1.09	1.19
Afifi et al [26]	S-10	2826	0.93	0.93	0.94	0.94	0.95	0.96	0.99
	S-11	2951	0.90	0.91	0.92	0.92	0.94	0.96	1.03
	S-12	2857	0.93	0.94	0.95	0.95	0.97	0.99	1.06
	S-13	2964	0.90	0.91	0.92	0.92	0.94	0.95	1.03
	S-14	2920	0.91	0.92	0.93	0.93	0.95	0.97	1.04
	S-15	2804	0.95	0.96	0.97	0.97	0.99	1.01	1.08
	S-16	3019	0.89	0.89	0.90	0.90	0.92	0.94	1.01
	S-17	2865	0.94	0.94	0.95	0.95	0.97	0.99	1.06
	S-18	2998	0.90	0.92	0.93	0.93	0.96	0.98	1.09
Mohamed et al. [2]	S-19	2840	0.95	0.95	0.96	0.96	0.98	1.00	1.07
	S-20	2871	0.94	0.94	0.95	0.95	0.97	0.98	1.06

Table 2: (Continued)

Mohamed et al. [2]	S-21	2935	0.91	0.92	0.93	0.92	0.95	0.96	1.04
	S-22	2869	1.00	1.01	1.02	1.02	1.07	1.10	1.18
	S-23	2960	0.96	0.97	0.99	0.99	1.03	1.06	1.14
	S-24	3008	0.95	0.96	0.98	0.97	1.02	1.05	1.12
Karim et al. [31]	S-25	1220	0.79	0.80	0.80	0.81	0.84	0.86	1.07
	S-26	1309	0.73	0.74	0.74	0.75	0.79	0.80	1.00
	S-27	1063	0.84	0.84	0.84	0.84	0.84	0.84	0.84
	S-28	1170	0.77	0.77	0.77	0.77	0.77	0.77	0.77
Hales et al. [32]	S-29	7126	0.78	0.79	0.79	0.79	0.79	0.80	0.81
Hadhood et al. [15]	S-31	4709	0.94	0.95	0.96	0.95	0.96	0.97	1.06
	S-32	4716	0.95	0.96	0.97	0.97	0.98	0.99	1.12
Hadhood et al. [33]	S-30	3090	0.82	0.83	0.84	0.84	0.91	0.94	0.99
Hadi et al. [16]	S-33	2721	0.94	0.94	0.96	0.95	0.96	0.97	1.05
	S-34	2398	1.06	1.07	1.09	1.08	1.09	1.10	1.19
Tobbi et al. [13]	S-35	4297	0.87	0.88	0.88	0.89	0.91	0.92	0.97
	S-36	4615	0.81	0.82	0.83	0.83	0.85	0.86	0.91
	S-37	4212	0.88	0.88	0.89	0.89	0.90	0.91	0.94
	S-38	3900	0.94	0.95	0.95	0.95	0.96	0.97	1.02
Mean			0.91	0.92	0.93	0.93	0.95	0.97	1.05
SD			0.068	0.068	0.071	0.070	0.076	0.082	0.096

Table 2: (Continued)

COV (%)	7.39	7.45	7.71	7.55	7.99	8.47	9.17
MAPE	9.642	9.305	8.614	8.612	7.542	7.478	9.692

<sup>a</sup>  $P_{exp}$  is the experimental axial load carrying capacity of FRP bar reinforced concrete columns.

<sup>b</sup>  $P_o$  is the theoretically computed axial load carrying capacity of FRP bar reinforced concrete columns.

<sup>c</sup> Refers to the formula used in computing  $\epsilon_{co}$  (Eq. 9 - Eq. 12)

<sup>d</sup> equal to  $\epsilon_{cu}$  (0.003 or 0.0035 as defined in ACI 318-14 [20] and CSA A23.3-14[21], respectively).

<sup>e</sup> The contribution of the FRP longitudinal bars in  $P_o$  was computed based on the formula defined in the footnote “c” above.

<sup>f</sup> The contribution of the FRP longitudinal bars in  $P_o$  was assumed to be equal to  $0.35f_{fu}A_f$  (Tobbi et al. [13])

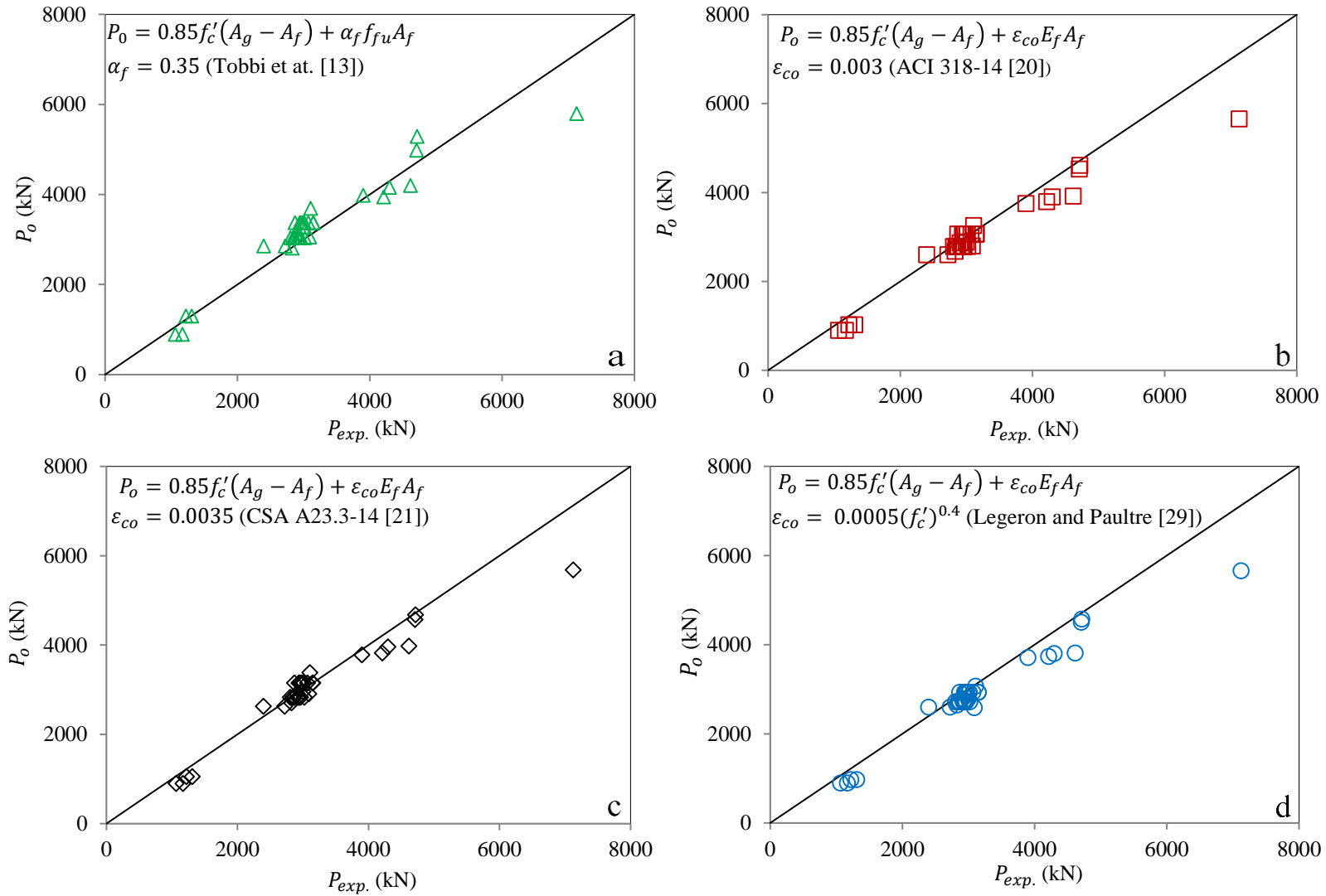


Fig. 1: Experimental versus predicted axial load carrying capacity of FRP bar reinforced concrete columns obtained using: a) Eq. 6 ( $\alpha_f = 0.35$ ); b) Eq. 8 ( $\varepsilon_{co} = 0.003$ ); c) Eq. 8 ( $\varepsilon_{co} = 0.0035$ ) and d) Eq. 8 ( $\varepsilon_{co} = 0.0005(f'_c)^{0.4}$ ).

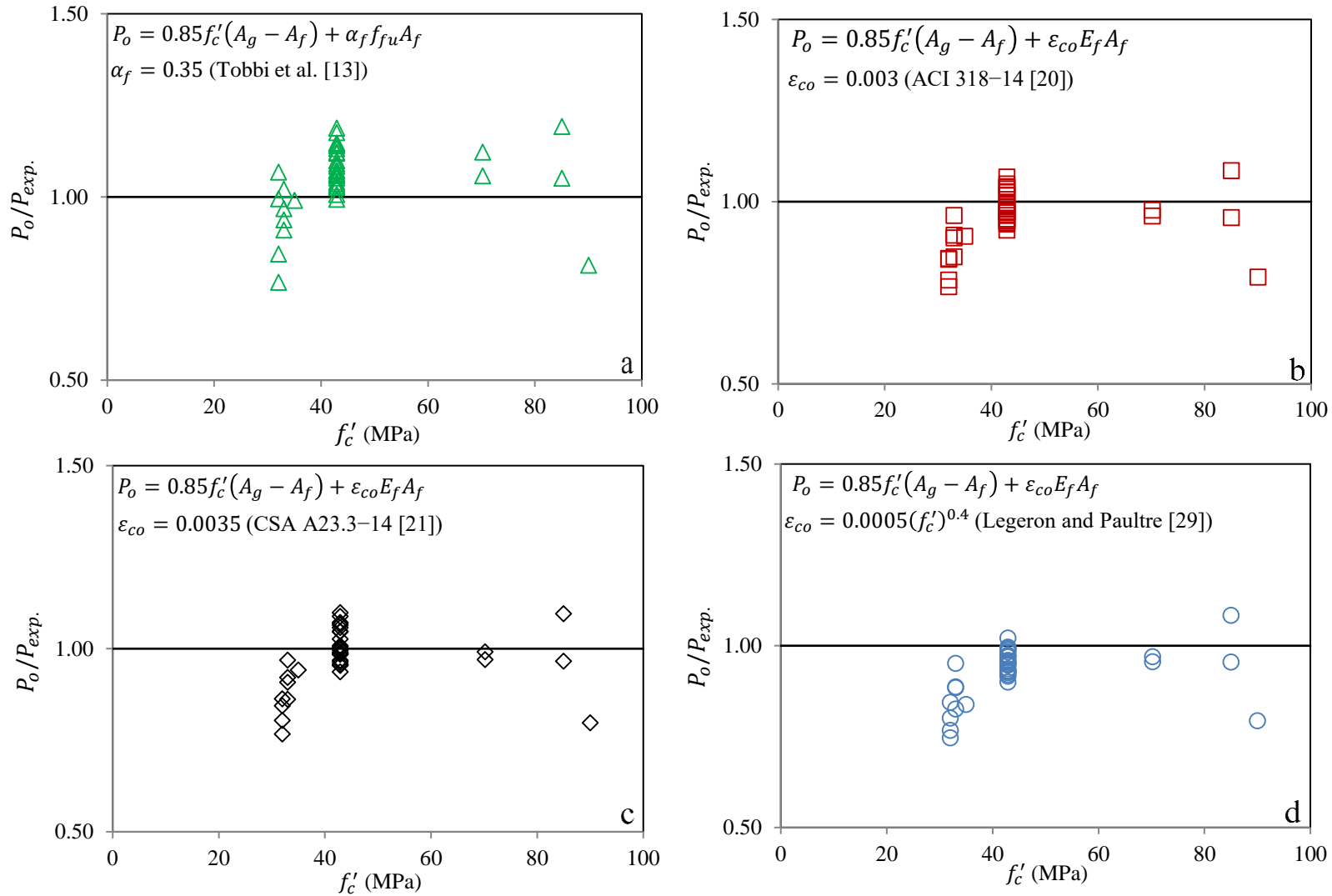


Fig. 2: The relationship between  $P_o/P_{exp.}$  of the FRP bar reinforced concrete column and the compressive strength of the concrete  $f'_c$ . Note:  $P_o$  were obtained using: a) Eq. 6 ( $\alpha_f = 0.35$ ); b) Eq. 8 ( $\epsilon_{co} = 0.003$ ); c) Eq. 8 ( $\epsilon_{co} = 0.0035$ ) and d) Eq. 8 ( $\epsilon_{co} = 0.0005(f'_c)^{0.4}$ ).