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School of Electrical, Computer and Telecommunications Engineering Faculty of Engineering and Information Sciences

Active suspension control of electric vehicle with in-wheel motors

Xinxin Shao

"This thesis is presented as part of the requirements for the Award of the Degree of Doctor of Philosophy From University of Wollongong"

Jun 2018

Abstract

In-wheel motor (IWM) technology has attracted increasing research interests in recent years due to the numerous advantages it offers. However, the direct attachment of IWMs to the wheels can result in an increase in the vehicle unsprung mass and a significant drop in the suspension ride comfort performance and road holding stability. Other issues such as motor bearing wear motor vibration, air-gap eccentricity and residual unbalanced radial force can adversely influence the motor vibration, passenger comfort and vehicle rollover stability. Active suspension and optimized passive suspension are possible methods deployed to improve the ride comfort and safety of electric vehicles equipped with inwheel motor. The trade-off between ride comfort and handling stability is a major challenge in active suspension design.

This thesis investigates the development of novel active suspension systems for successful implementation of IWM technology in electric cars. Towards such aim, several active suspension methods based on robust H_{∞} control methods are developed to achieve enhanced suspension performance by overcoming the conflicting requirement between ride comfort, suspension deflection and road holding. A novel fault-tolerant H_{∞} controller based on friction compensation is in the presence of system parameter uncertainties, actuator faults, as well as actuator time delay and system friction is proposed. A friction observer-based Takagi-Sugeno (T-S) fuzzy H_{∞} controller is developed for active suspension with sprung mass variation and system friction. This method is validated experimentally on a quarter car test rig. The experimental results demonstrate the effectiveness of proposed control methods in improving vehicle ride performance and road holding capability under different road profiles.

Quarter car suspension model with suspended shaft-less direct-drive motors has the potential to improve the road holding capability and ride performance. Based on the quarter car suspension with dynamic vibration absorber (DVA) model, a multi-objective parameter optimization for active suspension of IWM mounted electric vehicle based on genetic algorithm (GA) is proposed to suppress the sprung mass vibration, motor vibration, motor bearing wear as well as improving ride comfort, suspension deflection and road holding stability. Then a fault-tolerant fuzzy H_{∞} control design approach for active suspension of IWM driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints is proposed. The T-S fuzzy suspension model is used to cope with the possible sprung mass variation. The output

feedback control problem for active suspension system of IWM driven electric vehicles with actuator faults and time delay is further investigated. The suspended motor parameters and vehicle suspension parameters are optimized based on the particle swarm optimization. A robust output feedback H_{∞} controller is designed to guarantee the system's asymptotic stability and simultaneously satisfying the performance constraints. The proposed output feedback controller reveals much better performance than previous work when different actuator thrust losses and time delay occurs.

The road surface roughness is coupled with in-wheel switched reluctance motor air-gap eccentricity and the unbalanced residual vertical force. Coupling effects between road excitation and in wheel switched reluctance motor (SRM) on electric vehicle ride comfort are also analysed in this thesis. A hybrid control method including output feedback controller and SRM controller are designed to suppress SRM vibration and to prolong the SRM lifespan, while at the same time improving vehicle ride comfort. Then a state feedback H_{∞} controller combined with SRM controller is designed for in-wheel SRM driven electric vehicle with DVA structure to enhance vehicle and SRM performance. Simulation results demonstrate the effectiveness of DVA structure based active suspension system with proposed control method its ability to significantly improve the road holding capability and ride performance, as well as motor performance.

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Abbreviations

IWM	In-wheel motor
T-S	Takagi-Sugeno
DVA	Dynamic vibration absorber
GA	Genetic algorithm
SRM	Switched reluctance motor
EVs	Electric vehicles
DYC	Direct yaw moment
AFS	Active front wheel steering
FLC	Fuzzy logic control
HIS	Hydraulically interconnected suspension
ER	Electrorheological
MR	Magnetorheological
SMC	Sliding mode control
PID	Proportional integral derivative
Р	Proportional
PI	Proportional integral
PD	Proportional derivative
ANFIS	Adaptive network-based fuzzy inference system
FTC	Fault-tolerant control
EMS	Electromagnetic suspension systems
LQ	Linear-quadratic
PMA	Linear permanent-magnet actuator
LSRA	Linear switched reluctance actuator
TCS	Traction control systems
ABS	Anti-lock brake systems
ESC	Electronic stability control
CCC	Current chopping control
APC	Angle position control
PWM	Pulse width modulation
PMSM	Permanent magnet synchronous motor
UMP	Unbalanced magnetic pull
SQP	Sequential quadratic programming

LQR	Linear quadratic regulator
4WID-EV	Four-wheel independent drive electric vehicle
4WIS	Four-wheel-independent-steering
RB	Regenerative braking
ABS	Anti-lock braking systems
TPMA	Tubular permanent-magnet actuator
ANFWN	Adaptive neuro fuzzy wavelet networks
КҮР	Kalman-Yakubovich-Popov
LPV	Linear-parameter-varying
I/O	Input-output
BMI	Bilinear matrix inequality
ERL	Exponential Reaching Law
ICE	Internal Combustion Engine
LQG	Linear quadratic gaussian
PDC	Parallel-distributed compensation
LMIs	Linear matrix inequalities
ADM	Advanced-dynamic-damper-motor
PSO	Particle swarm optimization
RMS	Root mean square
MF	Magic-formula
PSD	Power spectral density

1. Introduction

1.1 Background and problem statement

The development of Electric vehicles (EV) has been accelerated in recent years, driven by a number of factors including global warming, heavy dependence on petrol, ever increasing price of fuel, and driving trends. According to the U.S Environmental Protection Agency, electric vehicles convert about 59-62% of the electrical energy from the grid to power at the wheels while conventional gasoline vehicles only convert about 17-21% of the electrical energy [1]. Poor air quality caused by conventional cars in many cities around the word- notably in China - poses a severe health issues for residents. Deployment of EVs could help to reduce the emissions and fine particulate matter (PM2.5) that lead to air pollution, climate change and health problems.

Governments around the world are making efforts to overcome the existing barriers and to accelerate the development of EVs. For example, China has become the most powerful political force driving the globalization of EV technology. According to the "Energy-saving and new-energy vehicle (ENEV) industry development plan (2012–2020)", China will obtain a production capacity of 2 million and cumulative sales of over 5 million battery EVs and plug-in hybrid EVs by 2020 [2]. When compared with the internal combustion engine (ICE) conventional vehicles, EVs have a wide spectrum of benefits such as enhanced comfort, higher efficiency, lower operating costs, as well as reduced greenhouse gas emission and air quality improvement [3]. These technical advantages drive the increasingly development of electric vehicles.

1.1.1 In Wheel motor Drive

From the viewpoint of electrical and control engineering, EVs, driven by electric motors, have three remarkable advantages: 1) motor torque can be generated fast and accurately 2) electric motors can be installed into each wheel; and 3) motor torque can be precisely measured [4]. Based on the vehicle architecture, the propulsion configuration of electric vehicles can be classified as centralized motor driven and in-wheel motor driven layouts. The second configuration, in which the motors are installed on the wheels and referred to as IWM electric vehicle, has lately proved to be an increasingly popular research area in recent years. The "UOT (University of Tokyo) Electric March II", proposed by Yoichi Hori group, is a novel four-wheel motored EV [4]. The in-wheel motor driven layout has a number of benefits such as fast motor response, precise torque generation, simplicity

and ability to generate forward and reverse torques with no adverse effect on driveshaft stiffness [5, 6]. Since the torque of each wheel can be controlled completely and independently, IWM can also improve the performance of traction control systems (TCS), anti-lock brake systems (ABS), and electronic stability control (ESC) [5]. The IWM technology has been attracting an increasing interest of more companies. For example, Protean Electric has designed an innovative in-wheel electric drive system for hybrid and electric vehicles. Eco-move Q-wheel and Nissan Bladeglider concept [7] (as shown in Figure 1-1) are the two best examples for electric IWM.



Figure 1-1 Nissan Leaf and Nissan Bladeglider [7].

1.1.2 Drawbacks of IWM

However, the deployment of IWM in electric vehicles produces new technological challenges. Attaching the electric motor to the wheel can lead to an increase in the unsprung mass. Some studies show that the mass can be increased by $20 \sim 50\%$ [8]. An increase in the unsprung mass leads to an increase in the response of the frequency ranges around 10 Hz, which shows negative effect on suspension ride comfort performance and road holding ability.

Moreover, the motor bearing, carrying the weight of the vehicle body, can easily wear due to heavy loads and the small gap between the motor rotor and the stator. In-wheel motor vibration and heavy load applied on the motor could easily result in loud noise and bearing wear, reducing the life of the motor bearing. In reality, some degree of rotor eccentricity is always presented in in-wheel SRM due to the motor bearing wear, tolerances introduced during the manufacturing process and static friction especially when the rotor is sitting idle. A relative eccentricity between the stator and rotor of 10% is common. The SRM air-gap eccentricity can result in a residual unbalanced radical force, which can greatly deteriorate the motor performance and vehicle ride comfort. This phenomenon is particularly serious to IWM-EV, because the vertical component of SRM residual unbalanced radical force is applied directly on the vehicle wheels and can change the tire load. The same phenomenon also occurs in in-wheel permanent magnet synchronous motor (PMSM) mounted electric vehicle. The uneven magnetic gap of PMSM may lead to unbalanced electromagnetic force, which has negative effect on vehicle ride performance and road-holding ability. These trends clearly show the need of active suspension in electric vehicles, which can contribute to enhanced vehicle ride comfort and dynamic performance.



Figure 1-2 Bridgestone's Dynamic-Damping In-wheel Motor Drive System [9].

Due to the many advantages of in-wheel motors, several companies are working on inwheel motor designs to minimize these adverse effects on IWM to obtain a better vibration isolation performance. Bridgestone proposes the so-called dynamic-dampingin-wheel-motor-driven-system [9], that suspends shaft-less direct-drive motor, isolating it from the unsprung mass. The motor is designed as a vibration absorber that could offset the road vibration input, as shown in Figure 1-2. The system is shown to have the potential to improve ride quality and road-holding performance. Tyre contact force fluctuations in conventional EVs and IMW-EVs with dynamic damping are compared in [10]. Michelin Company proposes active wheel system for battery EV or fuel-cell EV, as shown in Figure 1-3, which consists of an electric motor that drives the wheel, a braking system and an active suspension system [11]. The electric motor is used to drive the vehicle wheel, while the active suspension system is utilised to enhance ride performance, handling stability and passenger safety. The e-Corner proposed by Siemens company is a combination of an active suspension system, an electric motor, as well as an electronic wedge brake-by-wire system mounted in the wheel and hub assembly [12], as shown in Figure 1-4. The use of active suspension in these systems could possibly reduce the adverse effects of increased vehicle unsprung mass. Since passenger health and safety present an ever-increasing demand in the active suspension design. Therefore, research focus on the electric vehicle suspension system related to the comfort, and safety should be investigated. Possible methods to improve the ride comfort and safety of electric vehicle equipped with in-wheel motor are using an active suspension system or optimization of the passive suspension. Active suspension system is necessary for the successful implementation of IWM technology in future vehicles.



Figure 1-3 Michelin Active Wheel System [11].



Figure 1-4 eCorner. (1) Wheel rim (2) Wheel hub motor (3) Electronic wedge brake (4) Active suspension (5) Electronic steering [12].

1.1.3 Active Suspension for IWM

Vehicle suspension system, linking vehicle body and its wheels, is considered to be one of the most important part of vehicle, and contributes to the vehicle ride, handling and safety analysis. The design requirement of suspension system should satisfy conflicting requirements between ride comfort and handling stability. For example, a "hard" suspension has good vehicle handling stability, but it will cause harsh ride. A "soft" suspension will yield a more comfortable ride, but the stability of the vehicle is significantly reduced. Three kinds of vehicle suspension system, such as passive suspension system, semi-active suspension system and active suspension have been also researched to overcome the ride-handling compromise. Over the past decades, research in this area has been focused on controlled suspension systems such as semi-active suspension system, which could make the optimal feedback control according to road conditions and driving conditions. A novel active suspension overcoming the ride-handling compromise for the hybrid and electric vehicle is a trend in the development of future suspensions.

Compared to passive and active hydraulic suspension systems, the active electromagnetic suspension system is more suitable for the electric vehicles since it offers accurate force control, fast dynamic response and simple mechanical construction. Linear electromagnetic actuator and rotary electromagnetic actuator can be applied in the active electromagnetic suspension systems. Mechanical mechanisms such as ball-screw and rack-pinion are often utilised to convert the linear suspension motion into rotary motion. The Bose suspension system, as shown in Figure 1-5, uses a linear electromagnetic motor and an amplifier at each wheel in lieu of a conventional shock-and-spring setup [13]. This kind of suspension system can prevent the vehicle body motion while accelerating, braking and cornering, giving the driver a comfort ride. Ka Wai Eric Cheng et al. propose a novel active electromagnetic suspension system by adopting linear switched reluctance actuator. The proposed linear switched reluctance actuator is simple, low cost, as well as reliable and high performance, which offers a potential alternative to permanent-magnetic actuators [14-16]. Linear permanent-magnet actuator (PMA) is the most popular type of electric actuator applied to active suspension system. B.L.J. Gysene et al. propose an active electromagnetic suspension by incorporating brushless tubular PMA in parallel with a spring [17]. The design, control strategy, power consumption of the direct-drive electromagnetic active suspension are particularly investigated in [17, 18].



Figure 1-5 Bose suspension system [13].

Various control methods such as fuzzy logic control [19, 20], optimal control [21], neural network control, linear quadratic regulator control, sliding control [22], preview control [23] and H_{∞} control [24-27] are proposed for active suspension system to deal with the conflicting requirements between ride comfort, suspension deflection and road holding. Among these control algorithms, H_{∞} control is shown to overcome the trade-off and offer a favourable performance. According to the literature, the H_{∞} control strategy can deal with complexities such as parameter uncertainties, actuator faults, damper time delay and external disturbance. It is assumed that all the components of the suspension system are in normal working conditions when we design the controller. In practice, Failures can occur in actuators, sensors, controllers, or in the system, which could worsen the suspension performance. Fault detection and fault tolerant control methods are utilized for active suspension systems to detect the fault and ensure a better performance of the controlled suspension system [28-30]. Furthermore, the actuator delay occurred due to the pneumatic and hydraulic characteristics of the actuators or the acquisition and transmission of data from sensors to the controller, may damage the control performances and even result in control system instability if not considered in the controller deign.

Various controller design schemes are proposed for active suspension system with actuator delay [31, 32]. However, nonlinear problem of suspension is neglected in most of the previous suspension control researches. Model uncertainties such as suspension sprung, and unstrung mass variation should be taken into consideration in the active

suspension design. Nonlinear spring and piece-wise linear damper dynamic also should be considered. T-S fuzzy approach [33]and linear-parameter-varying technique [34, 35] are two effective methods to deal with the system uncertainties and nonlinearities. External disturbance such as system friction, system uncertainties and external interference have negative effect on control system performance and stability. Control methods based on the disturbance absorber can be utilized to estimate and compensate the influence of external disturbance [36-38].

1.2 Aims, Contributions and Publications of Thesis

1.2.1 Aims

The primary aim of this thesis is to investigate effective methods to provide active suspension in electric cars in the presence of systems friction, system uncertainties, sprung mass variation, and actuator delays and faults. The realised active suspension system improves the vehicle performance and road holding stability under different road conditions. In pursuit of the aim of the project, several objectives are pursued and achieved. Firstly, we have developed friction observer-based controllers for active suspension system to deal with the issues on system friction. Experimental validation of the proposed controllers is conducted on the test rig. As mentioned before, active suspension system or optimization of the passive suspension are two possible methods to improve the ride comfort and safety of electric vehicle equipped with in-wheel motor. Multi-objective parameter optimization is proposed for active suspension of in-wheel motor mounted electric vehicle based on genetic algorithm. Furthermore, several active suspension control methods are developed for active suspension system of in-wheel motor driven electric vehicle vehicles in the presence of sprung mass variation, actuator faults, time delay and control input constraints. The methods developed in this thesis can effectively improve vehicle ride comfort, simultaneously satisfying the performance constraints such as road holding, suspension stroke, dynamic load applied on the bearings and actuator limitation. We have also studied the active suspension control of electric vehicle mounted switched reluctance motor. The coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort is also investigated.

1.2.2 Contributions

The main contributions of this thesis are as follows:

1. A comprehensive review of the literature relating to passive suspension system, controlled suspension and regenerative suspension, outlining the disadvantage of IWM is conducted, and the active suspension methods applied to IWM-EVs in previous work are identified.

2. A quarter-car suspension test rig for the active suspension control research is developed. The experimental rig is used to examine and validate the performance of the proposed control methods and their validity.

3. A fault-tolerant H_{∞} controller based on friction compensation is proposed for active suspension system considering the parameter uncertainties, actuator faults, as well as actuator time delay and system friction. Experimental validation of the proposed control was conducted on the test rig. The proposed control method guarantees asymptotic stability and significantly improves the vehicle ride performance under actuator faults and constant delay. A friction observer-based T-S fuzzy controller is proposed for active suspension considering the sprung mass variation and system friction. The experimental results demonstrate the effectiveness of the proposed controller in improving suspension performance in spite of sprung mass variation and system friction.

4. Multi-objective parameter optimization for active suspension of in-wheel motor mounted electric vehicle based on genetic algorithm is proposed to suppress the sprung mass vibration, motor vibration as well as improving ride comfort, suspension deflection and road holding stability. The quarter-car active suspension with in-wheel motor served as dynamic vibration absorber is developed, and the motor parameters, suspension parameters and active controller are optimized based on the genetic algorithm.

5. A fault-tolerant fuzzy H_{∞} controller for active suspension of in-wheel motor driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints is proposed. The T-S fuzzy suspension model is developed to deal with sprung mass variation. The H_{∞} performance of the proposed controller is derived as linear matrix inequalities (LMIs) which are solved efficiently by means of MATLAB LMI Toolbox. The reliable fuzzy H_{∞} controller achieves a significantly enhanced closed-loop H_{∞} performance and suspension performance.

6. A robust output feedback H_{∞} controller is designed for active suspension system of in-wheel motor driven electric vehicles in the presence of actuator faults and time delays. Simulation results show that the proposed controllers could guarantee the system's asymptotic stability and H_{∞} performance, simultaneously satisfying the

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performance constraints such as road holding, suspension stroke, dynamic load applied on the bearings and actuator limitation.

7. Coupling effects between road excitation and in wheel switched reluctance motor on electric vehicle ride comfort are analysed. To supress SRM vibration and to prolong the SRM lifespan, while at the same time improving vehicle ride comfort, a hybrid control method including output feedback controller and SRM controller is proposed to reduce the sprung mass acceleration and the motor air-gap eccentricity. Furthermore, a state feedback H_{∞} controller is developed for in-wheel switched reluctance motor driven electric vehicle with dynamic vibration absorber to further enhance vehicle ride performance. Combined control methods of CCC and PWM are used to improve SRM performance. Simulation results under bump road excitation and random road excitation demonstrate the effectiveness of DVA structure active suspension system with proposed control method in enhancing suspension and motor performance.

1.2.3 Publications

The outcomes produced in the thesis are disseminated through the following publications:

• Shao, X., Naghdy, F., and Du, H., "Active Suspension Control of Electric Vehicle Driven by Switched Reluctance Motor Based on Vibration Absorbing Structure," *SAE Technical Paper* 2018-01-1401, 2018.

• Qin, Yechen, Chenchen He, Xinxin Shao, Haiping Du, Changle Xiang, and Mingming Dong. "Vibration mitigation for in-wheel switched reluctance motor driven electric vehicle with dynamic vibration absorbing structures." *Journal of Sound and Vibration* 419 (2018): 249-267.

• Shao, Xinxin, Fazel Naghdy, and Haiping Du. "Enhanced ride performance of electric vehicle suspension system based on genetic algorithm optimization." *In Electrical Machines and Systems (ICEMS)*, 2017 20th International Conference on, pp. 1-6. IEEE, 2017.

• Shao, Xinxin, Fazel Naghdy, and Haiping Du. "Reliable fuzzy H_{∞} control for active suspension of in-wheel motor driven electric vehicles with dynamic damping." *Mechanical Systems and Signal Processing* 87 (2017): 365-383.

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• Shao, Xinxin, Haiping Du, and Fazel Naghdy. Enhanced Vehicle Handling and Ride through Anti-Pitch Anti-Roll Hydraulically Interconnected Suspension. No. 2016-01-1561. *SAE Technical Paper*, 2016.

• Shao, Xinxin, Fazel Naghdy, and Haiping Du, Hongyi Li, "Output feedback H_{∞} control for active suspension of in-wheel motor driven electric vehicle with control faults and input delay", *ISA Transactions*, under review.

• Shao, Xinxin, Fazel Naghdy, Haiping Du, Yechen Qin, "Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort and active suspension control". *Journal of Sound and Vibration*, under review.

1.3 Outline of the thesis

This thesis work show novel methods that active control can be applied to suspension to improve conventional and electric vehicle ride comfort and safety. Initially, we propose some control methods for active suspension system of conventional vehicles. Experimental validation of the proposed control is conducted on the test rig. Then the studies are focused on the electric vehicle suspension system. Several optimization algorithms and control methods are proposed for active suspension system of electric vehicle driven by in-wheel motor to improve vehicle ride comfort and safety, as outlined in chapters 4, chapter 5 and chapter 6. Finally, the active suspension control of electric vehicle with mounted switched reluctance motor is investigated. Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort is also investigated. This thesis is organized into eight chapters as follows:

Chapter 1 - Introduction. This chapter provides a brief background and motivation for the current study, outlines the objective and contribution of the project and describes the structure of the thesis.

Chapter 2 - Literature review. A comprehensive review of vehicle suspension systems including passive suspension semi-active suspension and active suspension is conducted. A review of the literature on in-wheel motor electric vehicle suspension regenerative suspension is also covered. Furthermore, an overview of active suspension control algorithms is provided.

Chapter 3 - Experimental validation of robust H_{∞} controller for active suspension with friction estimation. In this chapter, a quarter vehicle experiment test rig setup is described. Then a fault-tolerant H_{∞} controller based on friction compensation is proposed for quarter

vehicle suspension system and experimental validation is conducted. Experimental validation of friction-observer based T-S fuzzy controller for active suspension is also reported in this chapter.

Chapter 4 - Active suspension control using genetic algorithm in in-wheel mounted vehicle. In this chapter, a quarter vehicle suspension model with a suspended motor and active suspension is established. Multi-objective parameter optimization for active suspension of electric vehicle with in-wheel motor based on genetic algorithm is presented. Genetic algorithm is used to optimize the vehicle suspension parameter and optimal control force. Simulation validation and conclusions are provided.

Chapter 5 - Reliable fuzzy H_{∞} control for active suspension of in-wheel motor driven electric vehicle with dynamic damping. An electric vehicle suspension model with an "advanced-dynamic-damper-motor" is developed. A fault-tolerant fuzzy H_{∞} control design approach for active suspension of in-wheel motor driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints is proposed. Simulation validation and conclusions are provided.

Chapter 6 - Output feedback H_{∞} control for active suspension with control faults and input delay in in-wheel motor mounted vehicle. Electric vehicle with suspended motor served as dynamic vibration absorber is developed. Particle swarm optimization is used to optimize the suspended motor parameter. A dynamic feedback controller considering actuator faults and time delay are designed for vehicle suspension system. Simulation validations of the proposed control algorithm are provided.

Chapter 7 - Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort and active suspension control. Firstly, coupling effects of road type and switched reluctance motor on vehicle ride comfort characteristic are illustrated. A hybrid control method including output feedback H_{∞} control and switched reluctance motor control for active suspension and simulation validation are presented. Furthermore, a state feedback H_{∞} controller combined with switched reluctance motor controller is proposed for active suspension with dynamic vibration absorber, and simulation results under random and bump road excitation are also provided.

Chapter 8 - Conclusion and future work. In this chapter, the outcomes of the thesis are critically discussed, and some conclusions are drawn, the future extension of the work is also discussed in this chapter.

2. Literature Review

2.1 Introduction

Electric vehicles (EVs) have various remarkable benefits such as energy source flexibility and environment friendliness when compared to conventional vehicles. They can also produce sufficient driving performance and efficiency using advanced electric motors and battery technologies. The mounting of the electric drives on the wheels, known as "inwheel motor", has attracted increasing research interests recently due to the numerous advantages it offers. However, IWM technology has major drawbacks such as increased unsprung mass, air-gap eccentricity of motor and motor wear, which significantly deteriorate the ride comfort performance and road holding stability.

Suspension systems of conventional vehicles have been extensively researched during the past decades in terms of, control strategy, structural design and dynamic analysis [39]. A review on the effect of IWM on vehicle ride, handling and stability performance have been reported. Dynamic analysis and control strategies of EV suspension system related to the ride, yaw stability and lateral motion stability are also investigated. Active suspension system is essential for successful implementation of IWM technology. Active suspension control methods are utilized to eliminate the increase in vehicle body acceleration and dynamic wheel load caused by the IWM subjected to system uncertainties, suspension nonlinearities and external disturbances.

In this chapter, a comprehensive review of suspension systems of EVs with IWM is initially carried out. The focus is particularly on the methods proposed to improve the impact of IWM on EV ride comfort, handling stability, and energy regeneration. In terms of vehicle ride comfort, issues associated with IWM such as increased unsprung mass, air-gap eccentricity, unbalanced vertical force, motor vibration and motor wear could significantly influence the vehicle performance. Active suspension control and suspension optimization are two effective ways to cope with these issues. In terms of handling stability, direct yaw moment (DYC) and active front wheel steering (AFS) are two effective ways to enhance the handling and stability performance of four-wheel independent drive EVs. Moreover, active suspension can improve handling stability by preventing vehicle roll motion and pitch motion during steering and braking, as well as eliminating road excitation. Hence increasing vehicle ride comfort and safety, have

been explored by many researches. Various control methods such as fuzzy logic control (FLC), optimal control, robust H_{∞} control, as well as sliding control and preview control, which could cope with suspension system nonlinearities, external disturbance and uncertainties, are reviewed. Active suspension systems require a large amount of energy, which limits their implementations in EVs. This has attracted a significant number of studies on regenerative suspension systems in recent years with the aim of harvesting energy from suspension vibration, as well as reducing the vehicle vibration. Furthermore, the regenerative suspension system applied in electric vehicle will be a trend in the future. Different types of regenerative suspension system including hydraulic regenerative suspension, electromagnetic regenerative suspension as well as self-powered magnetorheological damper are also reviewed in this chapter.

2.2 Vehicle suspension system

Vehicle wheels and body are linked using suspension systems, which have three basic elements such as spring and shock absorbers and linkages that connect a vehicle to its wheels. Apart from its basic purpose of isolating the vehicle body from the road roughness and vibration, the suspension prevents the vehicle body from rolling and keeps the tires in contact with the road [40]. Moreover, it also can contribute to the vehicle's road holding/handling for active safety and driving stability, and keep vehicle passengers comfortable. Suspension systems can be classified into passive, semi-active and active suspensions.

2.2.1 Passive suspension

Passive suspension has the traditional springs and dampers, which cannot be changed during different road excitations and manoeuvres. This has many advantages compared to the controlled suspension such as simplicity, high reliability, low cost and zero energy consumption. Vehicle dynamic response is usually assessed under a random road profile described by Gaussian white noise or, bump road profile. Vehicle ride comfort is represented by the vehicle body acceleration as specified in the ISO 2631. The suspension deflection can be utilised to enforce the suspension displacement constraint, which prevents the suspension from reaching its travel limit for preventing ride deterioration and mechanical structural damage. Road holding stability can be generally quantified by tyre deflection. Parameters of a passive suspension system are optimized to suppress vehicle vibration based on genetic algorithm in [41]. Vehicle dynamics are influenced by random road excitation, vehicle velocity and system parameters. Optimum design of a vehicle

passive suspension system under actual random road excitation is presented in [42]. The fundamental problems associated with passive suspension design of heavy road vehicle are studied in [43]. Some performance criteria such as ride, suspension working space, infrastructure damage, rollover stability, and yaw stability are considered when the dynamic properties of heavy vehicle suspension systems are assessed.

Passive anti-roll systems such as anti-roll torsional bar can be used to improve vehicles stability factor, in particular, its anti-roll ability. Spring interconnections between left and right wheels in the form of anti-roll bars has the advantages of improving relatively higher roll stiffness, but they can increase the stiffness in vehicle articulation, which is undesirable in terms of the vehicle road-holding ability and ride comfort performance. Cole [44] suggested that an anti-roll bar applied on the vehicle can result in less road damage than springs alone, and an independent suspension causes less road damage than a rigid axle suspension. A number of studies on alternative passive suspension such as roll-plane interconnected suspension system and pitch-plane interconnected suspension is conducted in the past decades, and the passive interconnected suspension system can optimize the conflicting performance in terms of vehicle ride comfort and handling stability [45].

In an interconnected suspension system, the displacement at one-wheel station can generate forces at other wheel stations, which is realized through mechanical or fluidic. This kind of suspension system has the potential of overcoming the ride comfort-handling stability compromise [46]. Passive interconnected suspension systems have received a significant number of attention recently. Cao et al. provide a comprehensive investigation of interconnected hydro-pneumatic suspensions in [45, 47, 48]. Apart from hydropneumatic suspensions, the hydraulically interconnected suspensions have also attracted many attentions in the past decade. Zhang et al. [46, 49] propose a hydraulically interconnected suspension (HIS) system, which typically consists of a single- or doubleacting hydraulic cylinder at each wheel station, replacing the conventional shock absorber. Modelling and model analysis of tri-axle trucks with passive HIS is proposed by Ding [50]. The Schematic diagram of anti-roll HIS proposed by Zhang et al. [49, 51] is shown in Figure 2-1. From the figure we can see the four cylinders are interconnected by hydraulic circuits, and hydraulic accumulators, damper valves, flexible hoses, and pipelines are comprised in each circuit. Simulation study and experimental validation of an SUV fitted with a HIS under different steering maneuvers is presented in [51, 52],

respectively. The simulation experiment results indicate that this kind of suspension has been also recognized to achieve a desired vehicle dynamic performance at roll motionmode and prevent vehicle roll over.



Figure 2-1 Anti-roll HIS. (a) Schematic diagram of the anti-roll HIS. (b) The assembled anti-roll HIS system [49, 51].

2.2.2 Semi-active suspension system

Compared with passive suspension, the controlled suspension system can make the optimal feedback control in accordance with different road excitations and driving conditions, which can improve vehicle ride comfort and handling stability. Different types of semi-active and active suspensions are discussed and reviewed. Karnopp and Crosby [53] first introduced the concept of semi-active suspension system in the early 1970s by presenting the skyhook algorithm to enhance the vehicle ride performance. The semi-active suspension system is a substitution of the active suspension and has many advantages over it such as energy efficiently, simplicity and cost saving. Usually, in a semi-active suspension system, the actuator is replaced by a variable damper that works in parallel with a spring. The damping force is regulated by modifying the orifice area in the oil-filled damper, thus altering the resistance to fluid flow. The variation of damping can be obtained by introducing mechanisms such as electrorheological (ER) damper and magnetorheological (MR) dampers [54].

2.2.2.1 ER based semi-active suspension

According to the literature [55, 56], semi-active ER suspension system is recognised as

one of the most effective method in improving vehicle ride comfort and handling stability. ER fluid has a quick response feature to an electric field and in consequence a wide bandwidth of control. A continuously variable ER damper, proposed by Choi et al. [55, 56], achieved the desired damping force by applying a skyhook controller. Furthermore, a field test of the semi-active ER suspension system associated with four independent skyhook controllers was conducted to demonstrate its effectiveness [57]. The configuration of the ER shock absorber proposed in [58] is shown in Figure 2-2. Choi et al. [59] proposed a robust sliding mode controller to evaluate control performance of semi-active ER seat suspension system. A sliding mode control (SMC) is propose for ER suspension system based on full vehicle model through hardware-in-the-loop simulation in [60]. Based on this work, another control method known as moving SMC, was proposed to apply for uncertain control system. Sung [61] proposed a fuzzy moving sliding mode controller, and then experimentally realized it on a quarter vehicle model with an ER suspension system. These studies indicate that the ER suspension system associated with proposed control strategy can effectively improve ride comfort of a vehicle.



Figure 2-2 ER damper. (a) Schematic configuration. (b) Photograph of ER damper [58].

2.2.2.2 MR fluid based semi-active suspension

MR fluids have much higher viscosity and therefore yield better strength than ER. The

Figure 2-3 shows the schematic configuration of the MR damper, proposed by S.-B. Choi [62]. A cylindrical MR damper is proposed and manufactured based on Bingham model of a MR fluid in [62]. Several models are proposed to describe the characteristic of the MR damper such as Bouc-Wen hysteresis model, algebraic model, polynomial model, neural network model and LuGre friction model. A semi-active sky-hook control for MR damper suspension system with is presented in [63]. Bouc-Wen model is utilized to characterize the dynamical behaviour of the MR damper. A nonlinear adaptive controller of the semi-active MR damper suspension system with model parameters uncertainties is proposed in [64]. Modified dynamic LuGre friction model is adopted to analyse the performance of MR damper, and experiments are conducted to identify the parameter of MR damper. A new modified algebraic model is presented in [65] to characterize the performance of the MR damper. Algebraic model is preferable for MR damper due to its low computational expenses when compared to differential Bouc-Wen's model which needs high computing intensive. Two different controllers, system controller and damper controller are designed in this study. Furthermore, a polynomial model is proposed in [66] to characterize the dynamical behaviour of the MR damper using experimental data. A static output feedback H_{∞} controller is proposed based on a quarter car suspension model to meet the three main vehicle control objectives of ride comfort, road holding, and suspension deflection.

Various control methods such as sky-hook control, model-following SMC, nonlinear adaptive control, fuzzy-proportional integral derivative (PID) control and H_{∞} control are deployed to construct the semi-active controller for the MR dampers suspension system. A variable stiffness and damping suspension system, which obtains variable stiffness and damping through a MR damper is presented in [67]. Two controllers such as fuzzy logic direct yaw moment controller and variable stiffness and damping on/off controller are proposed for the suspension system. A self-tuning adaptive PID control strategy for vehicle semi-active suspension system is presented in [68], Four basic controller are developed in the paper, such as PID, proportional (P) only, as well as proportional integral (PI) and proportional derivative (PD). Panos [69] proposes a clipped-optimal control method for semi-active suspension system, which can be capable of optimizing the vehicle ride and handling behaviour. A semi-active SMC algorithm for MR damper suspension systems is presented in [70]. The proposed controller doesn't require measurement of the damper force, and it also maintains the sliding mode and achieves

high robustness object to model parameters uncertainties and road disturbances. Neural network control for semi-active MR damper suspension is presented in [71]. The MR damper reveals nonlinear hysteresis between its relative velocity and output force, and the state transition from fluid to semi-solid or solid also present extra nonlinear stiffness of MR damper. A fuzzy-PID control algorithm for semi-active MR damper suspension system is presented in [72], and an adaptive network-based fuzzy inference system (ANFIS) model is used to describe the behaviour of magnetorheological damper. Adaptive semi-active controller for quarter-vehicle MR damper suspension based on Bouc–Wen model is presented in [73]. The controller contains a parameter adaptive method estimating the model parameters uncertainties, an observer estimating the damper hysteresis internal state and an adaptive state-feedback control algorithm guaranteeing the suspension system stabilization.



Figure 2-3 Schematic configuration of the MR damper [62].

2.2.3 Active suspension system

2.2.3.1 Electrohydraulic suspension system

Electrohydraulic systems are typically selected as the actuators to produce the active forces to isolate the vibrations from road profile in an active suspension design. However, highly nonlinear behaviour of electrohydraulic systems makes it hard to construct the ideal control algorithm. The schematic of the electrohydraulic actuator is shown in Figure 2-4. PID controller for nonlinear half-vehicle electrohydraulic suspension system is presented in [74]. An outer loop for suspension travel feedback control and an inner loop for PID hydraulic active force feedback control are used in this paper. A T-S fuzzy model

approach based fuzzy static output feedback control algorithm for electrohydraulic active suspensions is presented in [75]. Haiping Du and Nong Zhang present a T-S fuzzy controller for electrohydraulic active vehicle suspensions taking actuator nonlinear characteristic, control input constraints and sprung mass variation into account in [33]. A H_{∞} controller based on adaptive robust control design approach is proposed for full-car electrohydraulic active suspension to supress road disturbance and increase the ride performance of active suspension systems in [27]. Adaptive PID-sliding-mode faulttolerant control (FTC) method is presented for active suspension system to cope with the control faults and system uncertainties based on full-vehicle model, and an adaptive PID controller is employed for each of the electrohydraulic actuator [76]. A constrained adaptive back-stepping strategy is proposed for half-car active suspension with electrohydraulic to prevent both the pitch and vertical motions, despite the presence of parameters uncertainties and highly nonlinear actuators [77]. Two control loop arrangements including an outer loop for PID suspension parameters control and an inner loop for PID hydraulic actuator force control are proposed in [78] for a half nonlinear active suspension system model.

An active HIS can be built by integrating some active components such as electronic valves and pressure control power unit. The pressure control unit, which shown in Figure 2-5, can estimates the roll angle and controls the circuits' pressures to generate an antiroll moment. Theoretically, the interconnected suspension can de-couple four vehicle body-wheel motion-modes such as roll, bounce, pitch and warp. Lifu Wang [79] proposes the motion-mode energy method, which can decompose vehicle motion into a set of uncoupled vehicle motion-modes. Then Du [80] proposes switched control of vehicle HIS by actively switching the interconnection configuration of suspension in terms of vehicle body motion-modes. Shao [81] presents a fuzzy control of HIS with configuration switching based on the detected dominant vehicle body motion-mode. The results show that the designed controllers can effectively reduce the pitch motion and prevent rollover. Quang Lam [82] propose a fuzzy control method for an active HIS based on a sport utility vehicle, and experimental validation is conducted to verify the effectiveness of proposed controller. An H_{∞} control strategy is employed for a roll-plane active HIS to control vehicle roll motion in [83], and the effectiveness of the proposed H_{∞} control method is demonstrated experimentally on the four-post-test rig.


Figure 2-4 Schematic of the electrohydraulic actuator [27].



Figure 2-5 Photo of the pressure control unit [83].

2.2.3.2 Electromagnetic suspension system

A review of electromagnetic suspension systems (EMS) applied on passenger vehicles is conducted in [84]. The electromagnetic suspension system can be general applied in many applications for example electric vehicle, vibration isolation, as well as energy harvesting and high-speed maglev passenger trains. A great deal of control methods such as SMC [85], linear-quadratic (LQ) control, robust H_{∞} control [86] and fault tolerant control [87] are used for active EMS system in recent years. A fuzzy state feedback control strategy for electromagnetic suspension systems based on T-S fuzzy modelling technique is proposed in [88], in which the fuzzy controller is proposed with nonparallel distributed compensation control law, can be solved using linear matrix inequalities methods. P. K. Sinha [86] proposes a nonlinear state and output feedback controllers for magnetically levitated vehicles with electromagnetic suspension systems. A SMC algorithm is designed for an active EMS in [85], in which the performance of active EMS based on the proposed SMC method is much better than those of the active suspension that utilizing the LQ control. A fault tolerant controller is proposed for EMS systems that exhibit dynamics characteristic of magnetically levitated transport systems in [87].

Linear permanent-magnet actuator (PMA) is the most popular type of electric actuator applied for active suspension system. The dynamic analysis and the experimental results in [89] reveal that PMA is very appropriate for implementing the vehicle active suspension system. B.L.J. Gysene presents an electromagnetic suspension incorporating brushless tubular PMA and spring in [17], as shown in Figure 2-6 (b). Moreover, robust H_{∞} controller for an electromagnetic active suspension system based on a quarter vehicle model validated by both simulation results and experimental work is proposed in [18]. A H_{∞} control method is deployed in this paper to optimize multiple control variables with parameters uncertainties for instance sprung mass, damping stiffness and tire stiffness. Eleven controllers with maximal comfort or best handling are designed by changing frequency dependent weighting filter. Furthermore, experiment setup consisting of a quarter vehicle model and sensors are presented. PMA has some disadvantages for instance high cost, difficult assembly, and maintenance. Linear switched reluctance actuator (LSRA) is a substitution for active electromagnetic suspension system because of high performance, low cost and simple structure. Jiongkang Lin and Ka Wai Eric Cheng propose a novel active EMS based on LSRA [14]. The double-sided LSRA schematic configuration is shown in Figure 2-7, which is consisted of four phases with eight pairs of stator poles and eight translator poles. The simulation and experimental results demonstrate that nonlinear PD control has better performance than linear PD control [14]. Inductance derivative estimation of LSRA based on tracking differentiating method is presented in [15], and direct-force control is utilised to demonstrated the validity of the inductance and its derivative. An active electromagnetic suspension system with LSRA based on adaptive SMC algorithm is proposed in [16], Non-linear quartercar model and dynamic model of LSRA are derived. The computer simulation and test results represent that the proposed adaptive SMC can significantly enhance vehicle ride comfort.



Figure 2-6 (a) Passive suspension system (b) Electromagnetic suspension system [17].



Figure 2-7 (a) Schematic configuration of double-sided LSRA module. (b) 3D model of LSRA [14].

2.3 In-wheel motor electric vehicle suspension system

Propulsion configuration of electric vehicles is either centralized motor driven layout or in-wheel motor driven layout depending on the vehicle architecture. The second configuration, in which the motors are installed on the wheels and referred to as IWM, has lately proved to be a popular research area. The in-wheel motor driven layout has a number of benefits for instance fast motor response, precise torque generation, simplicity and ability to generate forward and reverse torques with no adverse effect on driveshaft stiffness [5, 6]. Moreover, IWM can be deployed to enhance the effectiveness and efficiency of various functions of the vehicle such as driving, turning, stopping, and ride comfort. For example, IWM can improve the performance of traction control systems (TCS), anti-lock brake systems (ABS), and electronic stability control (ESC) [5]. Since ride comfort, handling stability and passenger safety are increasingly becoming critical criteria in vehicle suspension design, an active suspension application is essential for the successful implementation of IWM driven electric vehicles.

2.3.1 Improving vehicle ride comfort

IWM can be used during all the driving conditions such as driving, turning, stopping, and ride comfort. For ride comfort, IWMs can improve vehicle pitch control performance, provide motor active ride control through the vertical component of driving force control, and act as high-frequency dynamic absorber [5]. This is not specific to IWMs and any vehicle with independent control of front and rear axle torque can improve the vehicle pitch motion. However, this study reviewed are primarily in the context of IWMs, though the pitch control performance improvements may be produced by other vehicle configurations. However, IWMs in which the motors are directly attached to the wheels result in an increase in the unsprung mass, which can significantly deteriorate the suspension ride comfort performance and road holding stability [90]. Furthermore, a lager unsprung mass of the electric vehicle may lead to a lower unsprung mass natural frequency which is close to the human sensitive natural frequency in vertical direction. Moreover, the wear of the motor bearing is a problem that should be addressed in active control of IWM EVs [91]. The motor bearing in vehicles with unsprung mass can easily wear because of heavy load and the small gap between the motor rotor and the stator. The air-gap eccentricity present in the in-wheel motor can result in a residual unbalanced radial force, which can adversely influence the motor vibration, passenger comfort and vehicle rollover stability.

The general effect of increased unsprung mass on the sprung mass acceleration, suspension deflection, and tyre deflection based on a quarter-car model is investigated, as shown in Figure 2-8. Increasing the unsprung mass has almost no effect on the natural frequency of sprung mass (around 1-2 Hz). However, an increase in the unsprung mass leads to an increase in the response of the frequency range around 10 Hz, which shows negative effects on the suspension ride comfort performance and road holding ability. Figure 2-9 presents the random response of vehicle body acceleration and tyre dynamic force, illustrating that both vehicle body acceleration and tyre dynamic force of the DVA-

EV are smaller than that of IWM-EV. Ambarish Kulkarni analysed the suspension dynamics of an in-wheel SRM-EV based on a quarter vehicle model simulation and compared it to those of the ICE conventional vehicle [92]. The influence of IWM on the EV ride comfort performance and road holding ability is presented in [93], and possible improvements in ride performance and driver safety of EVs driven by IWMs are also investigated.



Figure 2-8 Frequency response of sprung mass acceleration and tyre deflection.



Figure 2-9 Random response of sprung mass acceleration and tyre dynamic force.

There are a variety of motors that can be deployed in electrical cars such as switched reluctance motor, induction motor, permanent magnet synchronous motor and directcurrent motor. SRM is simple, inexpensive, as well as robust and can offer very good efficiency over a wide load range. However, the residual unbalanced radial force resulted from the rotor eccentricity could significantly deteriorate the vehicle ride comfort and SRM performance, as well as increasing vehicle vibration and acoustic noise. The rotor eccentricity occurs due to the motor bearing wear, motor tolerances and static friction. Furthermore, the residual unbalanced radial force is become larger when the vehicle is under idling, road excitation and unbalanced load conditions. This circumstance is especially severe to IWM-EV, since the vertical component of SRM unbalanced radial force is acted directly on the vehicle wheels and can alter the tyre force [94]. The effects of air-gap eccentricity on SRM vibration are analysed in [95]. The air-gap eccentricity can result in a residual unbalanced radical force, which can influence the motor vibration, passenger comfort and vehicle rollover stability [96]. Figure 2-10 shows the structure of in-wheel SRM. Wei Sun et al. [97] analysed the impact of in-wheel SRM on vehicle dynamic and proposed several control methods to decrease the residual unbalanced force and vehicle starting shock caused by SRM. A filtered-X least mean square controller was proposed for active suspension to suppress the vibration caused by SRM vertical force in [94]. Yanyang Wang et al. [98] analysed the effect of unbalanced vertical forces of inwheel SRM on the ride performance, and lateral dynamic and anti-rollover characteristic of the electric vehicle. Yanyang Wang et al. studied the coupling effects between road surface irregularity and vertical component of SRM unbalanced radial force [96]. The effect of road class on SRM and vehicle performance was not discussed in this paper. In terms of SRM control, the conventional control methods such as current chopping control (CCC), angle position control (APC), pulse width modulation (PWM) control are utilized to control SRM [99, 100]. These can be used to control the chopping current and switching angles. The optimal control method of motoring operation for SRM drives in electric vehicle is proposed in [101] based on the multi-objective optimization model.



Figure 2-10 The structure of in-wheel SRM [97].



Figure 2-11 The structure of in-wheel PMSM [102].

The permanent magnet synchronous motor (PMSM) is increasingly deployed in EV due to its high-power density, excellent torque performance and strong overload capability. IWM magnet gap is also influenced by the road surface roughness excitation. The uneven magnet gap can result in unbalanced magnetic force, which could be applied directly on the vehicle wheels and body, resulting in an increase in the wheel dynamic load and sprung mass acceleration. Figure 2-11 shows the structure of in-wheel PMSM. Di tan et al. [102] analysed the unbalanced electromagnetic force of permanent magnet synchronous motor on electric vehicle rollover characteristic. The effect of roadinfluenced factor on the vehicle rollover was studied. The effect of unbalanced magnetic force on vehicle vertical and lateral coupling dynamic under two operating cases (B level steering single sine input and B level steering angle step input) was analysed in [103]. The vibration analysis of interior PMSM taking air-gap deformations into account is presented in [104]. The non-uniform air-gap results from stator and rotor deformations has great effect on the unbalanced radial magnetic force, theoretical and experimental analysis shows that air-gap deformation is a significant factor to affect the vehicle vibration and noise. In [105], the main cause of the vibration and noise of PMSM are unbalanced magnetic pull (UMP), the air-gap eccentricity, rotor offset and radial clearance also have a great effect on the PMSM. Recently, various advanced control methods such as FLC [106], adaptive PID control [107] and SMC are deployed to effectively control the PMSM system.

Additionally, motor vibration is another problem that should be addressed in the IWM-EVs, and motor performance-related vibration can be reflected on the motor acceleration. The non-uniform gap result from the motor rotor and stator deformation should be controlled to remain small enough to obtain a high torque density and output power. The motor bearings, which carry the weight of a vehicle, can easily wear because of heavy loads and the small gap between the motor rotor and the stator [91]. The dynamic force acted on motor bearing can be utilized to enforce the weight constraint, which should also be reduced to decrease the IWM bearing wear [108]. Therefore, to reduce the motor bearing wear and motor vibration, dynamic force transmitted to the in-wheel motor and motor acceleration are taken as additional controlled outputs besides the traditional optimization objectives such as sprung mass acceleration, suspension deflection and tire deflection.

The parameters of in-wheel motor may not be optimal when designed by experience. Optimization these parameters can also improve ride performance of an electric vehicle driven by in-wheel motors. Sequential Quadratic Programming (SQP) optimization technique is used in [109] to optimize the parameters of dynamic vibration absorber. In order to achieve better vehicle performance and energy efficiency, the mechanical parameters such as vehicle suspension parameters and in-wheel motor parameters and controller should be considered during the optimization of active suspension. In general, there are a number of conflicting control objectives that should be optimized in active suspension control. Pareto optimal solutions can be obtained in solving multi-objective optimization problems. A multi-objective parameters optimization based on GA is proposed for active suspension of an electric vehicle driven by IWM in [110]. However, the active suspension control studies are primarily focused on conventional vehicles and are not applied to active control of IWM-EV suspension systems. Active suspension control of IWM-EV should be deployed to improve vehicle ride quality. In-wheel vibration absorber is used in IWM-EV and various control strategies are proposed to enhance vehicle ride performance in [91, 111]. Wang and Jing [91] propose a finitefrequency state feedback H_{∞} controller for active suspension of IWM-EVs, and demonstrate that the deployment of a dynamic vibration absorber would significantly reduce the force applied to the in-wheel motor bearing. In order to cope with adverse interaction effects between the active suspension and IWM dynamic absorber, a linear quadratic regulator (LQR) control method and a fuzzy PID method are presented for the active suspension damper and the IWM damper, respectively [111]. Unknown faults in components such as sensor and actuator failures can deteriorate the dynamic behaviour of the suspension. Consequently, fault-tolerant fuzzy H_{∞} control design approach for active suspension of in-wheel drive electric vehicles in the presence of actuator faults are proposed in [90, 108]. Dynamic output feedback H_{∞} control methodologies are also developed for active suspension control of IWM-EV with actuator faults and time delay, which will detail introduced in my thesis.

2.3.2 Enhancing vehicle handling stability and control

In a four-wheel independent drive electric vehicle (4WID-EV), the torque of each IWM can be independently controlled. AFS and DYC are two effective ways to enhance the handling and stability of EVs. A fuzzy logic driver-assist stability system for 4WID-EV based on a yaw reference DYC is proposed in [112]. Speed sensor-less fuzzy direct torque control for PMSM driven EV is introduced in [113]. Moreover, the integration control system with active AFS and DYC in 4WID-EV is explored recently. A nonlinear integrated control system which combine AFS and DYC together is proposed for 4WID-EV based on a triple-step nonlinear method [114]. Vehicle lateral motion control for 4WID-EV with the combination of AFS and DYC via in-vehicle networks is studied in [115]. The QP-based torque allocation method, and the message priority scheduling method and generalized PI upper-level controller are used to enhance the lateral performance. A combination of AFS and DYC with good robustness is proposed for 4WID-EVs to deal with in-vehicle network caused time-varying delays issues in [116].

The IWM fault taking place in the 4WID-EV, possibly be resulted from mechanical failures and motor vibration. As a result, the faulty wheel and motor possibly cannot supply enough torque and power. Several fault diagnosis and FLC strategies for 4WID-EV are investigated. An IWM fault diagnosis and FLC method for 4WID-EV is proposed in [117]. A fault-tolerant controller for EV with four-wheel-independent-steering (4WIS) and 4WID is presented in [118] based on a modified SMC method. Active fault tolerant controller for EVs with rear wheel IWMs to improve yaw motion control performance is proposed in [119]. Chukwuma proposes a fault-tolerant IWM design which could achieve a large motor torque, high power density, and maintain a certain level of performance following a failure [120].

Existing research investigations are mostly concentrated upon the ride, handling and safety study of IWM-EVs. However, the IWM technology offers an extra force of regenerative braking (RB) in each wheel during braking. As a result, RB offers an extra degree of freedom in braking controller of the IWM-EV when compared to the conventional vehicles. Therefore, researches related to the control of RB for IWM-EVs

have recently gained attention. A novel predictive-based robust controller is proposed for IWM-EVs fitted with both hydraulic anti-lock braking systems (ABS) and RB systems to follow up the desired wheel slip ratio and prevent wheel locking [121]. A combination of ABS and RB control is proposed to boost the handling and stability performance of the small EVs with two IWMs [122]. Amir Dadashnialehi proposes an intelligent sensor-less ABS for brushless IWM-EVs [123].

2.4 Regenerative suspension system

The regenerative suspension systems with the capability to harvest energy from suspension vibration while decreasing the vehicle oscillate have become increasingly popular with the vehicle electrification. Energy harvested from vehicle suspensions can be worked as an auxiliary power source used for active suspension control first, and then acted as energy regenerating facilities. The regenerative suspension system mainly can be divided into two types based on their operating principles: electrohydraulic regenerative suspension system and electromagnetic regenerative suspension system. Self-sensing or self-powered magnetorheological damper, which could have desired vehicle performance without extra power supply and sensor, is also introduced in this chapter.

2.4.1 Electrohydraulic regenerative suspension system

Electrohydraulic regenerative suspension system has been studied recently. A novel hydraulic pumping regenerative suspension with a combination of a hydraulic actuator and an energy recovery unit is proposed, which can create unidirectional oil fluid to actuate the generator to generate electricity efficiently [124]. An optimal algorithm for hydraulic electromagnetic regenerative shock absorber is discussed in [125], which could maximize the energy-recyclable power. Xu Li et al. [126] propose a hydraulic transmission electromagnetic energy-regenerative active suspension which utilizes both the hydraulic system and the electromagnetic energy regeneration system so that it can enhance vehicle performance and harvest power. A hydraulic-electricity e regenerative suspension system is designed in [127]. a prototype regenerative energy shock absorber combined the advantage of hydraulic system and mechanical transmissions is developed in [128], which can generate electricity by converting the linear motion into rotary motion and save energy.

2.4.2 Electromagnetic regenerative suspension

Electromagnetic regenerative shock absorbers can be can be divided into two types based on different working principles and design. The first is a linear electromagnetic shock absorber, which utilises the relative linear movement between magnets and coils to generate power. The other type is rotary electromagnetic shock absorber, which use rotational permanent magnetic DC or AC generators to convert the linear suspension vibration motion into oscillatory rotation motion. Mechanical mechanisms consisting of ball-screw and rack–pinion are often utilised to convert the linear suspension motion into rotary motion.

2.4.2.1 Linear electromagnetic suspension

Karnopp [129] proposes a permanent magnet linear motor, which can be used to construct mechanical damper with variable damping coefficient for vehicle suspension. Suda [130] proposes a self-powered active suspension system, in which a linear DC motor is served as an actuator and an energy regenerative damper effectively recover the energy through converting kinetic energy to electric energy. A direct-drive electromagnetic active suspension system incorporating a brushless tubular permanent-magnet actuator (TPMA) with a passive spring is designed in [131], which has the advantage of eliminating road disturbances as well as preventing roll and pitch motion during steering and braking. The energy efficiency and power consumption of the direct-drive electromagnetic active suspension system is investigated in [132]. Zuo lei et al. [133] design and experimentally conduct a linear electromagnetic regenerative shock absorber, which can harvest the vibration energy with high efficient without sacrificing ride comfort. The half-scale prototype can harvest 2-8 W of energy at 0.25-0.5 m s⁻¹ RMS suspension velocity.

2.4.2.2 Rotational electromagnetic suspension

Two types of regenerative electromagnetic shock absorber including a linear device and a rotary device are proposed in [134]. The test result shows that the peak power generated by rotary damper is larger than that of the linear damper. An energy harvesting vehicle damper, consisting of multiple independently-controlled generators that are independently controlled by switches, is proposed to achieve the goal that not only recover the energy through converting kinetic energy to electric energy but also to enhance the vehicle performance under different road excitation [135]. The modelling of electromagnetic damper for vehicle suspension is presented in [136], ball screw is used

to transform an axial stroke input into rotary motion. A regenerative shock absorber based on a mechanical motion rectifier is devised in [137], which could significantly improve the energy harvesting effectively and reduce the suspension vibration caused by road excitation. The mechanical motion rectifier is created to convert the linear vibration motion into unidirectional generator rotation motion. Design, modelling, and road tests of a rack–pinion-based electromagnetic regenerative shock absorber is presented in [138]. The regenerative shock absorber mainly include a permanent magnetic generator and a rack–pinion mechanism which has the advantage of recovering the vibration energy with high efficiency.

2.4.3 Self-powered MR damper

Application of MR dampers in energy harvesting has attracted many interest. Different methods are proposed to make self-powered MR dampers to enhance the energy harvesting performance of electromagnetic energy extractors. In the system developed in [139], an energy harvesting linear MR has three main compositions including an MR damper, a power generator and a conditioning electronics unit, which has the potential of recovering vibration energy effectively and shows self-powered and self-sensing abilities. A self-powered, self-sensing MR damper is investigated theoretically and experimentally in [140]. A self-sensing MR damper, which combines energy regenerative, dynamic sensing and MR damping control technologies, is proposed to produce damping force and harvest vibration energy in [141]. An combination of self-sensing magnetorheological damper and the corresponding electronic system are developed in [142], which can not only obtain the dynamic sensing capability but also has large range of controllable damping force.

2.5 Control methods

Many studies are focused on the suspension structure design, dynamic analysis and active control of active suspension system. However, performance requirements, such as ride comfort, road-holding stability, and suspension deflection, are often conflicting expectations. As a results, different control methods such as fuzzy control [19, 20], LQR, neural network method, sliding control, preview control and robust H_{∞} control [143-145] are proposed to cope with the trade-off between these conflicting design requirement, which have been briefly reviewed in this chapter.

2.5.1 Fuzzy logic control

For most active control method, an accurate suspension model should be providing for the controller. FLC method can forecast the behaviour of system with no need of the system mathematical model, which has been used extensively in recent years. Furthermore, recent researches are mainly focus on a hybrid intelligent system in which combine at least two intelligent control technologies together, such as FLC and neural network, FLC and genetic optimization. This kind of system has the potential to learn in a condition with uncertain and imprecise. A fuzzy immune genetic algorithm is proposed for active suspension controller in [19] with the advantage of shortening the decoding time and better fuzzy controller performance. The proposed fuzzy immune genetic strategy can get hold of fuzzy rules and the optimal membership functions the fuzzy controller. An adaptive Mamdani FLC scheme and four adaptive neuro fuzzy wavelet networks (ANFWN) are proposed in [20] for active suspension system to reduce the vehicle vibration and enhance ride performance. The results show that the performance of active suspension with ANFWN-4 is superior to other control scheme based active suspension systems. ANFIS controller, which combine the FLC and neural network together, is proposed for active suspension system with hydraulic actuator in [146], An adaptive FLC design of the active suspension system based on full-vehicle model is investigated in [147], and the GA. Algorithm is used for tuning of the FLC under various traffic conditions. The optimal gain parameters for FLC can be determined by the particle swarm optimization (PSO) in [148]. In [149], FLC is proposed for active suspension. The membership functions are optimized by applying GA.

2.5.2 Optimal control

The conventional fuzzy control cannot guarantee the control precision, and the neural network has high complexity, which is hard to analyse the various performance indexes accurately. Among the active suspension control methods, the optimal is widely used. LQR, as a kind of optimal control, can put forward different objective functions for the different performance need in vehicle active suspension control. Simulated annealing strategy is used to optimize the LQR controller weight matrix with the target function of suspension performance indexes, aiming at improving the vehicle performance and efficiency [150]. Stochastic optimal estimation has been used to control non-stationary response of a 2-DOF vehicle model, and a Kalman filter state estimator was used to estimate the unknown state variables [151]. A novel optimal controller [21], which

combine improved cultural algorithm with fuzzy-PID control algorithm, is presented for the vehicle active suspension to suppress vehicle vertical acceleration. LQR algorithm has two main issues when applied for the active suspension system. [152]. One is that not all the feedback states are available, some of them should be calculated from an estimator. The study conducted in [153] presents a methodology used to solve this problem. The other occurs when there are no zero steady-state of body deflection under external force in the controlled system. ElMadany [154] present an optimal suspension systems based on multivariable integral control to solve this problem. Panos [69] proposes a clippedoptimal control algorithm which can be capable of optimizing the vehicle ride and handling behaviour. The proposed LQ clipped-optimal control law is designed by working out a stochastic Hamilton-Jacobi-Bellman equation. The simulation results reveal that the proposed LQ clipped-optimal control law obtain enhanced handling and ride performance when compared with the standard LQR approach. A novel bang-bang sub-optimal control algorithm is developed for active suspension system in [155] by applying affine quadratic stability, Lyapunov theory and Pontryagin's minimum principle. Simulation implementation is conducted on a virtual car to demonstrate the validity of the proposed controller.

2.5.3 H_{∞} control

According to the literature, the robust H_{∞} control strategy can deal with complexities such as sprung mass uncertainty, actuator faults, damper time delay and time constant uncertainty [24-27]. However, when not all the required suspension system state variables are available, the output-feedback based H_{∞} control algorithm is proposed to address this problem. For example, a dynamic output-feedback delay-dependent multi-object H_{∞} controller is proposed for active suspension system with control delay [156]. An output feedback controller based on a recursive derivative non-singular higher order terminal SMC method is proposed for nonlinear suspension systems in [157]. A robust dynamic output-feedback H_{∞} controller has been is proposed for a full-car active suspension with finite-frequency constraints and actuator faults in [91]. A (Q, S, R)- α -dissipative outputfeedback fuzzy controller is proposed for T–S fuzzy systems with time-varying input delay and output constraints in [31].

In vehicle suspension control methods used to improve vehicle performance, it is assumed that all the control components of the systems are in an ideal working condition. In practice, unknown faults in components such as sensor and actuator failures can deteriorate the dynamic behaviour of the suspension. FTC methods that deal with possible actuator failure have attracted much attention in recent years. FTC based on virtual sensor and virtual actuator for nonlinear system were studied in [158, 159] considering both actuator and sensor faults. In [28], a robust optimal sliding mode controller was proposed to deal with actuator faults and to ensure the overall stability of the full vehicle suspension system. Fault detection filter design for the active suspension system in finite-frequency domain is presented in [160]. The fault detection filter is designed in the middle frequency domain based on the generalized Kalman-Yakubovich-Popov (KYP) lemma. In [161], a fault tolerant control method is proposed for the electromagnetic suspension system subject to failure of the sensors and actuators. Kong et al. [162] proposed a robust non-fragile $H_{\infty}/L_2-L_{\infty}$ static output feedback controller for vehicle active suspension considering actuator time-delays and the controller gain variations. In [29, 30], the non-fragile H_{∞} controller is proposed for the half-vehicle suspension systems in the presence of actuator uncertainty and failure.

The vehicle sprung mass varies with respect to the loading conditions such as the payload and the number of vehicle occupants. Model uncertainty, such as suspension sprung and unsprung mass variations should be taken into consideration in the design of active suspensions. Active suspensions are typical nonlinear uncertain systems in which the spring, damper and actuator have the nonlinear characteristic. A T-S fuzzy approach can be applied to effectively handle the uncertainties and representing complex nonlinear systems [33]. In [163], a sampled-data H_{∞} control for an active suspension system with model uncertainty is proposed. Furthermore, linear-parameter-varying (LPV) technique [34, 35, 164] have attracted an increasing research interest because of its effectiveness in describing nonlinearities and uncertainties. More recently, polytopic parameter uncertainties are used to establish the sprung mass or unsprung mass variation. A gainscheduling sliding mode H_{∞} observer is proposed for the polytopic LPV system with uncertain measurement in [164].

Moreover, in an active suspension system, time-delay issue should receive more attention since it can jeopardize system stability. The actuator delay occurs due to the pneumatic and hydraulic characteristics of the actuators or the acquisition and transmission of data from sensors to the controller, may damage the control performances and even result in control system instability if not considered in the controller deign. Dynamic output feedback fuzzy controller for active suspension system based on T-S fuzzy model with input delay and output constraints are proposed in [31, 32]. An input-output (I/O) feedback linearization algorithm is presented for nonlinear actuator-delay suspension system in [165]. Furthermore, the time-delay suspension system is transformed into an equivalent delay free model based on the finite spectrum assignment. There are two major methods to deal with the input delay problem. One is to design an integrated system model in which the actuator dynamics are established. The other is to take into account the actuator time delay in the controller design process [162, 166]. A new bilinear matrix inequality (BMI) in the presence of a delay-dependent robust H_{∞}/L_2 - L_{∞} output feedback controller for uncertain linear system with input time delay is proposed in [162]. More general actuator delay is considered, and multi-object control constraints are formulated in linear matrix inequalities (LMIs) for active suspension with time delay.

2.5.4 Sliding mode control

The SMC is a variable structure controller with robust characteristic. It can guarantee invariance for external disturbances and parameter variations which have a known bound. In SMC control method, the error state position of the system changes along with sliding surface according to a control law. Before those states reach the sliding surface they are switched between stable and unstable trajectories. Then the error converges to zero when the system stays on the sliding surface. Furthermore, the control system can be guaranteed invariance for external disturbances and matched uncertainties when the closed-loop control system is on the sliding surface [167]. SMC algorithm is proposed to active suspension system to improve the vehicle performance. A new H2/GH2 SMC for active suspension system subject to external disturbances and parameter variations is proposed in [22]. H2/GH2 approach is utilised to design the control gain on the sliding surface. A H_{∞} SMC for an active vehicle suspension system is presented in [168], the road disturbance is unknown but bounded, and the suspension system is divided into two subsystems, a virtual state feedback H_{∞} controller designed based on the first subsystem and a sliding mode H_{∞} controller used to guarantee the state trajectories of the second subsystem to reach the sliding surface.

A SMC for active suspension system which addresses the problem of state delay because of the acquisition and transmission of data from sensors to the controller is proposed in [169]. In this approach, a state predictor is employed to overcome the problem of data acquisition delay. Two controllers such as continuous time control and discrete time control are proposed in this paper. Vaijayanti [170] proposes a disturbance observer based SMC for active suspension systems, The proposed method estimates the effects of the unknown road disturbance, load variation, nonlinearities and uncertainties of the active suspension system. An Exponential Reaching Law (ERL)-SMC of electrohydraulic active suspension is proposed in [171]. Unlike other conventional SMC approach, the proposed ERL-SMC could control both chattering and tracking performance of electrohydraulic active suspension system.

2.5.5 Preview control

Preview control considers the knowledge of road profile ahead of the controlled vehicle to further improve suspension performance. Bender [172] first proposed the optimum linear preview control theory for active vehicle suspension system in 1968. There are two popular preview control methods such as look-ahead preview and wheelbase preview. Look ahead preview method is based on the utilisation of look-ahead sensors, while wheelbase preview is to provide the preview information came from front wheel for the rear vehicle wheel. The semi-active MR damper suspension system with optimal preview control MR shock absorber suspension system based on a full vehicle model under random road condition is proposed by Seong [174]. The optimal preview control algorithm based on the MR suspension is demonstrated to have better performance than conventional optimal control without preview. However, all the state variables cannot be measured in practice for the optimal preview controller design, because the preview sensor signals are likely to be disturbed by the sensor noise.

A optimal preview control theory for active vehicle suspension with look-ahead sensors based on a half car model is presented in [23]. The full state variables are estimated by utilising a method equivalent to the Kalman filter and LQR controller. Elmadany [175] presents an optimal preview control method for vehicle suspension system with integral constraint based on a quarter car model. An optimal preview control of vehicle suspension system considering the time delay between front and rear wheels is proposed in [176]. ElMadany [177] proposes a preview controller for slow-active suspension systems utilising two preview approaches including look-ahead preview and wheelbase preview. However, most of the preview control methods, which use the LQ control method, cannot consider unknown road disturbances applied on the vehicle. A multi-objective control method with wheelbase preview for active vehicle suspension is proposed in [178, 179]. A mixed H_{∞}/GH_2 velocity-dependent controller is used to improve the ride performance

of the active suspension system with wheelbase preview under velocity dependency [179].

2.6 Summary

In this chapter different categories of vehicle suspension systems including passive suspension system, semi-active suspension system and active suspension system had been detailed reviewed. Then a review of the literature on in-wheel motor driven electric vehicle suspension system and methods proposed to improve vehicle ride, handling and stability performance, and control in them was conducted. Moreover, regenerative suspension including electrohydraulic regenerative suspension system and electromagnetic regenerative suspension system were also covered. Finally, the active suspension control methods including FLC, optimal control, robust H_{∞} control, sliding control and preview control which offer superior vehicle performance and energy consumption were discussed. In a summary:

1) In terms of ride comfort, the IWM increase unsprung mass, which has adverse effect on suspension ride performance and road holding ability. The impact of IWM on vehicle performance and motor vibration was discussed. Further researches should focus on the active suspension design which could offset the negative impact of IWM. Active suspension controller and optimization methods need to propose for IWM-EV to improve the ride/handling performance, motor performance.

2) In terms of handling and stability, IWM makes it easier to improve the performance of traction control systems, anti-lock brake systems, and electronic stability control. Michelin active wheel and Siemens eCorner system, which combines active suspension, electric motor, and braking within one unit, have the potential to enhance vehicle handling stability. Analysis related to the above system can be discussed in the future researches. The effect of IWM on electric vehicle steering system and braking system needs further research.

3) The application of a great deal of control methods such as FLC, optimal control, robust H_{∞} control, sliding control and preview control for active suspension of conventional vehicle were reviewed in this paper. Enhanced vehicle performance including ride comfort, handling stability are obtained through active suspension system. Active suspension system such as electrohydraulic suspension system and electromagnetic suspension system that improve handling performance by preventing active roll and pitch motion during steering and braking as well as eliminating road

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excitation could be applied for IWM-EV in the future. Further researches should focus on the active suspension system for IWM-EV based on these control methods.

4) Regeneration suspension such as electrohydraulic regeneration suspension and electromagnetic regeneration suspension can be used in the electric vehicle. Suspension structure design, active control and energy-regeneration of IWM-EV should be the focus of future research.

3. Development of robust H_{∞} controllers for active suspension system with friction compensation and experimental validation

3.1 Introduction

Vehicle suspension systems play a key part in vehicle dynamics as they provide road roughness isolation and rollover prevention while keeping the tires in contact with the road [40]. Furthermore, it also contributes to the vehicle's road holding stability and ride comfort. The design of a suspension system should satisfy conflicting requirements of comfort and handling. There are three kinds of suspension systems, passive, semi-active and active designed to cope with these conflicting requirements. Passive suspension has many advantages such as simplicity, low cost and zero energy consumption. However, passive suspension consists of conventional springs and dampers, which cannot be changed during different road excitations and manoeuvres. Compared with passive suspension, the active suspension systems provide optimal active force according to road conditions and driving conditions, which can improve vehicle ride comfort and handling stability.

Electromagnetic active suspension systems offer many benefits such as increased efficiency, improved dynamic behaviour and stability improvement, which increasingly becoming a more attractive alternative to passive and, semi-active suspension system. The electromagnetic suspension system is a high bandwidth and efficient solution for improving handling and comfort. The Bose suspension system, an application of electromagnetic suspension, can counter the effect of different road profile while maintaining a comfortable ride. A variety of control algorithms such as fuzzy logic control [147], neural network control[20], optimal control[69], sliding mode control[22] and robust H_{∞} control[31] have been proposed for active electromagnetic suspension systems. Fault tolerant control is extensively studied to deal with the unknown faults that occur in the suspension system [158, 159]. Moreover, the problem of actuator delay, if not taken into consideration in the controller design, deteriorated the control and suspension performance.

Many control algorithms are proposed for active suspension with actuator delay [31, 32]. Sprung mass variation due to the vehicle load condition and vehicle parameter uncertainties should also be taken into account in the controller design to further enhance the control performance [33]. Disturbance observer and state observer for active

suspension system are extensively investigated in the previous studies [36, 180]. Ignoring friction in the controller design can deteriorate system performance and control accuracy. Friction estimation for active seat suspension system are proposed in [37, 38]. However, the robust H_{∞} controller based on friction estimation for active suspension system is not explored previously.

In this chapter, the experimental validation of robust H_{∞} control for active suspension is investigated. Vehicle body acceleration-based observer is used to estimate the friction force. Initially, the quarter car vehicle model and quarter car test rig are described in this chapter. Furthermore, a fault-tolerant H_{∞} controller based on friction compensation is proposed for active suspension system considering the parameter uncertainties, actuator faults, as well as actuator time delay and system friction. The effectiveness of proposed fault tolerant controller is demonstrated on the experimental test rig under 3Hz sinusoidal, bump and random road excitation. Moreover, to deal with the sprung mass variation and estimate the accurate friction. A friction observer-based T-S fuzzy controller is proposed for active suspension and validated experimentally on the quarter car test rig.

The reminder of this chapter is structured as follows. In section 3.2, the model of a quarter car vehicle model with active suspension is developed. The quarter car test rig setup partly developed in this work is described in section 3.3. In section 3.4, a fault-tolerant H_{∞} control algorithm for an active suspension system based on friction compensation is proposed. The experimental validation of active suspension with proposed controller is also provided in this section. In section 3.5, a T-S fuzzy control for active suspension based on friction observer is proposed, and the experimental validation of proposed controller is also outlined. Some conclusions are drawn in section 3.6.

3.2 Vehicle suspension system modelling

The quarter-car active suspension model with a spring, a damper and an electromagnetic suspension is deployed in the work presented in this chapter, as shown in Figure 3-1. The quarter car suspension model has two degrees of freedom associated with the vertical motion of the sprung mass and the unsprung mass. Based on Newton's Second Law, the motion equations of vehicle sprung mass can be written as

$$m_{s}\ddot{x}_{s} = k_{s}(x_{u} - x_{s}) + c_{s}(\dot{x}_{u} - \dot{x}_{s}) + f_{a} - f_{r}$$

$$(3-1)$$

$$m_{u}\ddot{x}_{u} = k_{t}(x_{g} - x_{u}) - k_{s}(x_{u} - x_{d}) - c_{s}(\dot{x}_{u} - \dot{x}_{s}) - f_{a} + f_{r}$$

where x_s and x_u denote the vertical displacements of the sprung mass and the unsprung mass. x_g denotes road disturbance, and f_r and f_a denote suspension friction force and actuator force, respectively. The sprung mass and unsprung mass are denoted by m_s and m_u , respectively. The suspension stiffness and damping coefficients are denoted by k_s and c_s , respectively. The tyre stiffness is denoted by k_t .



a) Passive suspension b) Active suspension

Figure 3-1 Quarter vehicle model with passive suspension and active suspension.

Suspension performance objective include ride comfort, suspension defection, and road holding stability. The active suspension control algorithm is deployed to reduce the vehicle vertical acceleration and to produce a better ride performance. Suspension deflection, tyre deflection and maximum actuator force are hard constraints that should be strictly satisfied:

- Ride comfort. The sprung mass acceleration in the vertical direction is used to quantify the vehicle ride comfort performance.
- Suspension deflection. The suspension deflection should not exceed its travel limit determined by the mechanical structure.
- Road-holding stability. To ensure a firm uninterrupted contact of wheels with the road, the tyre deflection should not exceed its travel limit.
- Maximum actuator force. The active control force provided by the active suspension system should be constrained by the maximum actuator force.

Let us define the vehicle state vector as

$$x(t) = \begin{bmatrix} x_s(t) & \dot{x}_s(t) & x_u(t) & \dot{x}_u(t) \end{bmatrix}$$

The dynamic model can be expressed by a state space equation

$$\dot{x}(t) = Ax(t) + B_2(u(t) - f_r(t)) + B_1\omega(t)$$

$$z_1(t) = C_1x(t) + D_1(u(t) - f_r(t))$$
(3-4)

$$z_2(t) = C_2 x(t)$$
 (3-5)

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & \frac{-k_s - k_t}{m_u} & \frac{-c_s}{m_u} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{1}{m_s} & 0 \\ -\frac{1}{m_u} \end{bmatrix}$$
$$C_1 = \begin{bmatrix} \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix}, D_1 = \frac{1}{m_s},$$
$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, u(t) = f_a(t), \omega(t) = \dot{x}_g(t)$$

In order to reduce the power consumption of the actuator, the active control force provided by the active suspension system should be constrained by a threshold, and the actuator saturation nonlinearity is described by

$$u(t) \triangleq \begin{cases} u_{max} & u(t) \ge u_{max} \\ u(t) & u_{max} \ge u(t) \ge -u_{max} \\ -u_{max} & u(t) \le -u_{max} \end{cases}$$
(3-6)

where u_{max} is the maximum actuator control force.

3.3 Fault tolerant H_{∞} controller with friction compensation

3.3.1 Friction estimation

In low relative velocity conditions, the sign of the relative velocity changes frequently in the experiments due to the displacement sensor noise, introducing extra disturbance in the active control, that in turn can lead to an unstable controller output. Therefore, the friction force needs to be estimated to improve controller's performance. In this work, the Coulomb friction model is used to estimate the friction force in the linear motor.

$$\hat{f}_r(t) = \begin{cases} F_r sign(x_s(t) - x_u(t)) & |x_s(t) - x_u(t)| \ge \beta \\ F_r \frac{x_s(t) - x_u(t)}{\beta} & |x_s(t) - x_u(t)| \ge \beta \end{cases}$$

where F_r is a constant friction magnitude, β is a relative velocity threshold.

3.3.2 Fault tolerant H_{∞} controller design

Remark 3.1 For the active suspension system described above, an ideal control input u(t) obtained by the state-feedback controller and is applied to the suspension system through the actuator generating the actual input $u_f(t)$, which is equal to the ideal control input in the case of a faultless actuator. When actuator faults occur then $u(t) \neq u_f(t)$. Furthermore, the friction force $f_r(t)$ can be approximately estimated as $\hat{f}_r(t)$ based on the Coulomb friction model. Therefore, the fault tolerant H_{∞} controller can be modelled by considering the actuator faults and friction compensation

$$u_f(t) = M_a K x(t-d) + M_a \hat{f}_r(t)$$
(3-8)

where M_a represents the possible fault of the actuator. d is a constant input delay with an upper bound \overline{d} . $M_{amin} \leq M_a \leq M_{amax}$, and M_{amin} and M_{amax} are prescribed lower and upper bounds of the actuator. Three cases are considered to describe three different actuator conditions.

 $1.M_{amin} = M_{amax} = 0$, so that $M_a = 0$, which means that the corresponding actuator $u_f(t)$ has completely failed.

2. $M_{amin} = M_{amax} = 1$, so that we have $M_a = 1$, representing the case of no fault in the actuator.

3. $0 < M_{amin} = M_{amax} < 1$, which indicates that there is a partial fault in the corresponding actuator.

Then we introduce the following scalars that are used in the design of the controller.

$$M_{a0} = (M_{amin} + M_{amax})/2$$

$$L_a = (M_a - M_{a0})/M_{a0}$$
(3-9)

(3-10)

(3-7)

$$J_a = (M_{amax} - M_{amin})/(M_{amin} + M_{amax})$$
(3-11)

Then we have $M_a = M_{a0}(I + L_a)$, and $L_a^T L_a \le J_a^T J_a \le I$

The friction estimation error is defined as

$$e_{f_r}(t) = M_a \hat{f}_r(t) - f_r(t)$$
(3-12)

The vehicle suspension model becomes an uncertain model with actuator input delay when the vehicle inertial parameter uncertainties, actuator fault, friction force, actuator time delay are considered, and can be expressed as

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}_2 M_a K x(t-d) + \bar{B}_\omega \omega(t)$$

$$z_1(t) = \bar{C}_1 x(t) + \bar{D}_1 (u(t) - f_r(t))$$
(3-13)

where

$$\bar{A} = A + \Delta A, \bar{B}_2 = B_2 + \Delta B_2, \bar{B}_\omega = B_\omega + \Delta B_\omega,$$
$$\bar{C}_1 = C_1 + \Delta C_1, \bar{D}_1 = D_1 + \Delta D_1.$$
$$B_\omega = \begin{bmatrix} 0 & 0\\ 0 & -\frac{1}{m_s}\\ 0 & 0\\ \frac{k_t}{m_u} & \frac{1}{m_u} \end{bmatrix}, \quad \omega(t) = \begin{bmatrix} x_g(t)^T & e_{f_r}(t)^T \end{bmatrix}^T$$

The parameter uncertainties considered here are norm-bounded of the form

$$\begin{bmatrix} \Delta A & \Delta B_2 & \Delta B_{\omega} \end{bmatrix} = L_1 F(t) \begin{bmatrix} E_A & E_B & E_{B\omega} \end{bmatrix}$$

$$\begin{bmatrix} \Delta C & \Delta D \end{bmatrix} = L_2 F(t) \begin{bmatrix} E_C & E_D \end{bmatrix}$$
(3-15)

(3-16)

(3-14)

where L_1 , L_2 , E_A , E_B , $E_{B\omega}$, E_C , E_D are known constant real matrices of appropriate dimensions, and F(t) is an unknown matrix function satisfying $F^T(t)F(t) \leq I$.

Lemma 3.1 [181] For a time-varying diagonal matrix $\Phi(t) =$ diag { $\sigma_1(t), \sigma_2(t), \dots, \sigma_p(t)$ } and two matrices *R* and *S* with appropriate dimensions, if $|\Phi(t)| \leq V$, where V > 0 is a known diagonal matrix, then for any scalar $\varepsilon > 0$, we have

$$R\Phi S + S^T \Phi R^T \le \varepsilon R V R^T + \varepsilon^{-1} S^T V S$$

(3-17)

Theorem 3.1 Suppose $\bar{d} > 0$ and $\rho > 0$ are prescribed scalars. Consider the suspension system (3-13) and (3-14), and with input delayed fault tolerant H_{∞} state feedback controller (3-8), the closed-loop system is asymptotically stable and satisfies $||z(t)||_2 \le \gamma ||\omega(t)||_2$ for all ω if there exist matrices $\bar{P} > 0$, $\bar{R} > 0, Y$, \bar{S}_i , i = 1,2,3, and any scalar $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that matrix inequalities are satisfied.

$$\begin{vmatrix} \bar{\psi}_{11} & * & * & * & * & * & * & * & * & * \\ \bar{\psi}_{21} & -\bar{R} & * & * & * & * & * & * & * \\ \bar{\psi}_{31} & 0 & \bar{R} - 2\bar{P} & * & * & * & * & * & * & * \\ \bar{\psi}_{41} & 0 & 0 & -I + \varepsilon_3 L_2 L_2^T & * & * & * & * & * \\ \bar{\psi}_{51} & 0 & \varepsilon_1 \sqrt{\bar{d}} B_2^T & \varepsilon_1 D_1 & -\varepsilon_1 J_a^{-1} & * & * & * \\ \bar{\psi}_{61} & 0 & 0 & 0 & 0 & -\varepsilon_1 J_a^{-1} & * & * & * \\ \bar{\psi}_{61} & 0 & 0 & 0 & E_B & 0 & -\varepsilon_2 I & * \\ \bar{\psi}_{81} & 0 & 0 & 0 & E_D & 0 & 0 & -\varepsilon_3 I \end{vmatrix} < 0$$

(3-18)

$$\begin{bmatrix} -I + \varepsilon_4 J_a & * & * \\ \sqrt{\rho} Y^T M_{a0}{}^T & -u_{max}^2 \bar{P} & * \\ 0 & \sqrt{\rho} M_{a0} Y & -\varepsilon_4 J_a{}^{-1} \end{bmatrix} < 0$$

(3-19)

$$\begin{bmatrix} -I & * \\ \sqrt{\rho}\bar{P}C_2^{\ T} & -z_{max}^2\bar{P} \end{bmatrix} < 0$$

(3-20)

where
$$\bar{\psi}_{11} = \begin{bmatrix} A\bar{P} + A^T\bar{P} + \bar{S_1}^T + \bar{S_1} + \varepsilon_2 L_1 L_1^T & * & * \\ Y^T M_{a0}^T B_2^T - \bar{S_1}^T + \bar{S_2} & -\bar{S_2} - \bar{S_2}^T & * \\ \bar{S_3} + \bar{B_\omega}^T & -\bar{S_3} & -\gamma^2 I \end{bmatrix},$$

 $\bar{\psi}_{21} = \begin{bmatrix} \sqrt{\bar{d}}A\bar{P} & \sqrt{\bar{d}}B_2 M_{a0}Y & \sqrt{\bar{d}}\bar{B}_\omega \end{bmatrix}, \\ \bar{\psi}_{31} = \begin{bmatrix} \bar{S_1}^T & \bar{S_2}^T & \bar{S_3}^T \end{bmatrix},$

$$\bar{\psi}_{41} = \begin{bmatrix} C_1 \bar{P} & D_1 M_{a0} Y & 0 \end{bmatrix}, \quad \bar{\psi}_{51} = \begin{bmatrix} \varepsilon_1 B_2^T & 0 & 0 \end{bmatrix}, \quad \bar{\psi}_{61} = \begin{bmatrix} 0 & M_{a0} Y & 0 \end{bmatrix},$$
$$\bar{\psi}_{71} = \begin{bmatrix} E_A & E_B M_{a0} Y & E_{B\omega} \bar{P} \end{bmatrix}, \quad \bar{\psi}_{81} = \begin{bmatrix} E_C \bar{P} & E_D M_{a0} Y & 0 \end{bmatrix}.$$

Proof: Considering the Lyapunov-Krasovskii function as follows:

$$V(t) = x^{T}(t)Px(t) + \int_{-\bar{d}}^{0} \int_{t+\alpha}^{t} \dot{x}^{T}(s)R\dot{x}(s)dsd\alpha$$
(3-21)

The derivative of V(t) along the solution of system is expressed as

$$\dot{V}(t) = 2x^{T}(t)P\dot{x}(t) + \bar{d}\dot{x}^{T}(t)R\dot{x}(t) - \int_{t-\bar{d}}^{t}\dot{x}^{T}(s)R\dot{x}(s)ds$$
(3-22)

For any appropriately dimensioned matrices \hat{S} , the following equalities hold directly according to the Newton–Leibniz formula, we have

$$2\xi^{T}(t)\hat{S} \times \left(x(t) - x(t - \bar{d}) - \int_{t - \bar{d}}^{t} \dot{x}(s)ds\right) = 0$$
(3-23)

(3-24)

where

$$\xi(t) = \begin{bmatrix} x(t) & x(t-\bar{d}) & \omega(t) \end{bmatrix}^T, \hat{S} = \begin{bmatrix} S_1^T & S_2^T & S_3^T \end{bmatrix}^T$$

Adding the equation in (3-23) into the right-hand side of (3-22) and after some simple calculations, the following inequality is satisfied

$$\begin{split} \dot{V}(t) &\leq \xi^{T}(t) \Big[\Pi + \zeta \bar{d}R\zeta^{T} + \bar{d}\hat{S}R^{-1}\hat{S}^{T} \Big] \xi(t) - \int_{t-\bar{d}}^{t} \Big[\xi^{T}(t)\hat{S} + \dot{x}^{T}(s)R \Big] R^{-1} \Big[\hat{S}\xi(t) + R\dot{x}^{T}(s) \Big] ds &\leq \xi^{T}(t) \Big[\Pi + \zeta \bar{d}R\zeta^{T} + \bar{d}\hat{S}R^{-1}\hat{S}^{T} \Big] \xi(t) \end{split}$$

where

$$\Pi = \begin{bmatrix} P\bar{A} + \bar{A}P + S_1^T + S_1 & * & * \\ K^T M_a^T \bar{B}_2^T P - S_1^T + S_2 & -S_2 - S_2^T & * \\ S_3 + \bar{B}_{\omega}^T P & -S_3 & 0 \end{bmatrix}, \zeta = [\bar{A} \quad \bar{B}_2 M_a K \quad \bar{B}_{\omega}]^T.$$

By Shur complement, $\Pi + \zeta \bar{d}R\zeta^T + \bar{d}\hat{S}R^{-1}\hat{S}^T < 0$ is equivalent to

$$\begin{bmatrix} \Pi & * & * \\ \sqrt{\bar{d}R}\zeta^T & -R & * \\ \sqrt{\bar{d}}\hat{S} & 0 & -R \end{bmatrix} < 0$$

which further leads to $\dot{V}(t) < 0$. Therefore, we can conclude that system (3-13) and (3-14) are robust asymptotically stable with constant time delay with an upper bound \bar{d} . Next, the H_{∞} performance of the system under zero initial condition is established. It is not difficult to achieve

$$z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T \omega(t) + \dot{V}(t) \le \xi^T(t) \Upsilon \xi(t)$$
(3-26)

(3-25)

where
$$\Upsilon = \begin{bmatrix} \Pi & * & * & * \\ \sqrt{\bar{d}}R\zeta^T & -R & * & * \\ \sqrt{\bar{d}}\hat{S} & 0 & -R & * \\ \chi & 0 & 0 & -I \end{bmatrix}, \quad \chi = [\bar{C}_1 \quad \bar{D}_1 M_a K \quad 0],$$

$$\bar{\Pi} = \begin{bmatrix} P\bar{A} + \bar{A}P + S_1^T + S_1 & * & * \\ K^T M_a^T \bar{B}_2^T P - S_1^T + S_2 & -S_2 - S_2^T & * \\ S_3 + \bar{B}_{\omega}^T P & -S_3 & -\gamma^2 I \end{bmatrix}.$$

If $\Upsilon < 0$, we have $z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T \omega(t) + \dot{V}(t) < 0$. Hence $||z_1||_2 < \gamma ||\omega(t)||_2$ is guaranteed for any non-zero $\omega(t) \in L_2[0, \infty)$, and the H_∞ performance is established.

In what follows, the fault tolerant H_{∞} controller is presented for active suspension system with parameter uncertainties and actuator faults.

By using Shur complement, $\Upsilon < 0$ is equivalent to

$$\tilde{Y} + \Lambda L_a \Delta + \Delta^T L_a \Lambda^T + \Sigma_2 F(t) \Sigma_3 + \Sigma_3^{\ T} F(t) \Sigma_2^{\ T} + \Gamma_2 F(t) \Gamma_3 + \Gamma_3^{\ T} F(t) \Gamma_2^{\ T} < 0$$
(3-27)

By applying the Lemma 3.1, we can obtain

$$\tilde{\gamma} + \varepsilon_1 \Lambda J_a \Lambda^T + \varepsilon_1^{-1} \Delta^T J_a \Delta + \varepsilon_2 \Sigma_2 \Sigma_2^T + \varepsilon_2^{-1} \Sigma_3^T \Sigma_3 + \varepsilon_3 \Gamma_2 \Gamma_2^T + \varepsilon_3^{-1} \Gamma_3^T \Gamma_3 < 0$$
(3-28)

which is equivalent to

$$\begin{bmatrix} \tilde{\psi} & * & * & * & * \\ \varepsilon_2 \Sigma_2^T & -\varepsilon_2 I & * & * & * \\ \Sigma_3 & 0 & -\varepsilon_2 I & * & * \\ \varepsilon_2 \Gamma_2^T & 0 & 0 & -\varepsilon_3 I & * \\ \Gamma_3 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0$$

where
$$\tilde{\Psi} = \begin{bmatrix} \tilde{\Upsilon} & * & * \\ \epsilon_1 \Lambda^T & -\epsilon_1 J_a^{-1} & * \\ \Lambda & 0 & -\epsilon_2 J_a^{-1} \end{bmatrix}, \quad \tilde{\Upsilon} = \begin{bmatrix} \tilde{\Pi} & * & * & * \\ \sqrt{d}R \tilde{\zeta}^T & -R & * & * \\ \sqrt{d}\hat{S} & 0 & -R & * \\ \tilde{\chi} & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\Pi} = \begin{bmatrix} PA + AP + S_1^T + S_1 & * & * \\ K^T M_{a0}^T B_2^T P - S_1^T + S_2 & -S_2 - S_2^T & * \\ S_3 + \bar{B}_{\omega}^T P & -S_3 & -\gamma^2 I \end{bmatrix},$$

$$\tilde{\zeta} = \begin{bmatrix} A & B_2 M_{a0} K & \bar{B}_{\omega} \end{bmatrix}^T, \quad \tilde{\chi} = \begin{bmatrix} C_1 & D_1 M_{a0} K & 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} B_2^T & 0 & 0 & 0 & \sqrt{d} B_2^T R & D_1 \end{bmatrix}^T, \quad \Delta = \begin{bmatrix} 0 & M_{a0} K & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} L_1^T P & 0 & 0 & 0 & \sqrt{d} L_1^T R & 0 & 0 \end{bmatrix}^T,$$

$$\Sigma_3 = \begin{bmatrix} E_A & E_B M_{a0} K & E_{B\omega} & 0 & 0 & E_B \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & L_2^T & 0 \end{bmatrix}^T, \quad \Gamma_3 = \begin{bmatrix} E_C & E_D M_{a0} K & 0 & 0 & 0 & E_D \end{bmatrix}$$
By defining

(3-29)

$$\bar{P} = P^{-1}, \bar{R} = R^{-1}, Y = K\bar{P}, \bar{S} = diag\{P^{-1}, P^{-1}, I\} \hat{S}P^{-1},$$
$$T_1 = diag\{P^{-1}, P^{-1}, I, P^{-1}, R^{-1}, I, I, I, I, I, I\}.$$

From $(\bar{R} - \bar{P})\bar{R}^{-1}(\bar{R} - \bar{P})^{-1} \ge 0$, we have $-\bar{P}\bar{R}^{-1}\bar{P} \le \bar{R} - 2\bar{P}$. After replacing $-\bar{P}\bar{R}^{-1}\bar{P}$ with $\bar{R} - 2\bar{P}$ and performing congruence transformation to (3-29) by multiplying full rank matrices T_1^T on the left and T_1 on the right, respectively, the inequalities (3-29) are equivalent to the inequalities (3-18) in Theorem 3.1.

In what follows, we show that the suspension stroke, and maximum control force constraint are guaranteed. The following inequalities hold:

$$\max_{t>0} |z_2(t)_q|^2 \le \max_{t>0} ||x^T(t)C_2^T C_2 x(t)||_2 < \rho \cdot \theta_{max} (P^{-1/2}C_2^T C_2 P^{-1/2})$$
(3-30)

$$\begin{aligned} \max_{t>0} |u(t)|^2 &\leq \max_{t>0} ||x^T(t)[M_{a0}(I+L_a)K]^T[M_{a0}(I+L_a)K]x(t)||_2 \\ &< \rho \cdot \theta_{max} \left(P^{-1/2}[M_{a0}(I+L_a)K]^T[M_{a0}(I+L_a)K]P^{-1/2} \right) \end{aligned}$$
(3-31)

where $\theta_{max}(\cdot)$ represents maximal eigenvalue. The constraints can be guaranteed, if

$$\rho \cdot P^{-1/2} C_2^{\ T} C_2 P^{-1/2} < \{z_{2max}^{\ 2}\}_q I$$

$$(3-32)$$

$$\rho \cdot P^{-1/2} [M_{a0}(I + L_a)K]^T [M_{a0}(I + L_a)K] P^{-1/2} < \{u_{max}^{\ 2}\}_q I$$

$$(3-33)$$

Considering the Schur complement and Lemma 3.1, the inequalities of (3-32) and (3-33) are equivalent to the inequalities (3-19) and (3-20) in Theorem 3.1. Therefore, all the conditions in Theorem 3.1 are satisfied. The proof is completed.

3.4 Takagi-Sugeno fuzzy controller based on friction observer

3.4.1 T-S fuzzy friction observer

The vehicle sprung mass varies with respect to the loading conditions such as the payload and the number of vehicle occupants. The uncertainties associated with the load condition in the sprung mass should be considered during controller design. A T-S fuzzy approach can be applied to handle the uncertainties as the T-S fuzzy model is very effective in representing complex nonlinear systems. A T-S fuzzy H_∞ controller is designed for active suspension to improve suspension performance in the presence of sprung mass variations in this section. The sprung mass $m_s(t)$ varies in a given range of $m_s \in$ $[m_{smin} \quad m_{smax}]$. The uncertain sprung mass $m_s(t)$ is bounded by its minimum value m_{smin} and its maximum value m_{smax} , and can thus be represented by

$$max \frac{1}{m_s} = \frac{1}{m_{smin}} = \widehat{m_s}$$
(3-34)

$$min\frac{1}{m_s} = \frac{1}{m_{smax}} = \widetilde{m}_s$$

$$\frac{1}{m_s} = \mu_1(z(t))\,\widehat{m}_s + \mu_2(z(t))\,\widetilde{m}_s$$

(3-36)

(3-35)

$$\mu_1(z(t)) = \frac{\frac{1}{m_s} - \widetilde{m}_s}{\widehat{m}_s - \widetilde{m}_s}$$

(3-37)

$$\mu_2(z(t)) = \frac{\widehat{m}_s - \frac{1}{m_s}}{\widehat{m}_s - \widecheck{m}_s}$$

$$\sum_{i=1}^2 \mu_i(z(t)) = 1$$
(3-38)

(3-39)

where $z(t) = \frac{1}{m_s}$ is the premise variable. The membership functions $\mu_1(z(t))$ and $\mu_2(z(t))$ are labelled as "Heavy" or "Light".

Hence, the active suspension system with a sprung mass variation is represented by the following fuzzy models.

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(z(t))(A_i x(t) + B_\omega \omega(t) + B_{2i}(u(t) - f_r(t)))$$
(3-40)

$$z_{1}(t) = \sum_{i=1}^{2} \mu_{i}(z(t))(C_{1i}x(t) + D_{1i}(u(t) - f_{r}(t)))$$
(3-41)

This indicates that f_r has a significant influence on the system dynamic, thus f_r needs to be estimated to improve controller's performance.

$$f_r = k_s (x_u - x_s) + c_s (\dot{x}_u - \dot{x}_s) + f_a - m_s \ddot{x}_s$$
(3-42)

A friction observer is defined as

$$\dot{f}_r = -l(f_r - \hat{f}_r)$$
(3-43)

We can obtain the following equations

$$\dot{\hat{f}}_{r} = l \Big(m_{s} \ddot{x}_{s} - (k_{s} (x_{u} - x_{s}) + c_{s} (\dot{x}_{u} - \dot{x}_{s}) + f_{a} - \hat{f}_{r} \Big) = L \Big[Y_{1} - \Big(C_{1} x(t) + D_{1} \big(u - \hat{f}_{r} \big) \Big) \Big]$$
(3-44)

The observer error is defined as:

$$e_f = f_r - \hat{f}_r$$

$$(3-45)$$

$$\dot{e}_f = \dot{f}_r - \dot{f}_r = LD_1e_f$$

$$c_f = j_r \quad j_r = D D_1 c_f \tag{3-46}$$

Considering the T-S fuzzy model and parameter uncertainties, the T-S fuzzy friction observer can be defined as:

$$\dot{f}_{r} = \sum_{i=1}^{2} \mu_{i}(z(t)) \left\{ L_{i} \left[Y_{1} - \left(C_{1i}x(t) + D_{1i}\left(u - \hat{f}_{r}\right) \right) \right] \right\}$$
(3-47)

The differential of estimation error is

$$\dot{e}_{f} = \sum_{i=1}^{2} \mu_{i}(z(t)) \{ L_{i} D_{1i} e_{f} \}$$
(3-48)

At time *t*, it can be shown that $e_f(t) = e^{L_i D_{1i}(t-t_0)} e_f(t_0)$. Therefore, if the $L_i D_{1i} < 0$, the observer error will exponentially converge.

3.4.2 T-S fuzzy H_{∞} controller with friction observer

An estimated state feedback controller u_f can be modelled by considering the friction force and sprung mass uncertainties

$$u_{f}(t) = \sum_{i=1}^{2} \mu_{i}(z(t)) (K_{i}x(t) + \hat{f}_{r})$$
(3-49)

The estimation error is defined as $e_f = f_r - \hat{f}_r$. An augmented state $\bar{x} = [x^T \ e^T]^T$ is defined. Hence, the closed-loop fuzzy system comprising active suspension system, friction observer and controller becomes

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i (z(t)) (\bar{A}_i x(t) + \bar{B}_{1i} w(t))$$
(3-50)

$$z(t) = \sum_{i=1}^{2} \mu_i (z(t)) (\bar{C}_{1i} x(t))$$
(3-51)

where

$$\bar{A}_i = \begin{bmatrix} (A_i + B_{2i}K_j) & -B_{2i} \\ 0 & L_iD_{1i} \end{bmatrix}, \ \bar{B}_{1i} = \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix}, \ \bar{C}_{1i} = \begin{bmatrix} (C_{1i} + D_{1i}K_j) & -D_{1i} \end{bmatrix}.$$

Theorem 3.2 Consider the suspension system (3-50) and (3-51) with T-S fuzzy H_∞ state feedback controller (3-49), the closed-loop system is asymptotically stable and satisfies $||z(t)||_2 \le \gamma ||\omega(t)||_2$ for all ω if there exist matrices $Q_1 > 0$, $P_2 > 0$ \overline{K}_j , G_j such that matrix inequalities are satisfied.

$$\Sigma_{ij} = \begin{bmatrix} sym(A_iQ_1 + B_{2i}\overline{K}_j) & -B_{2i} & B_{1i} & Q_1C_{1i}^T + \overline{K}_jD_{1i}^T \\ & * & G_jD_{1i} + (G_jD_{1i})^T & 0 & -D_{12i}^T \\ & * & * & -\gamma^2 I & 0 \\ & * & * & * & -I \end{bmatrix}$$

$$\Sigma_{ii} < 0, \quad i = 1,2$$
(3-52)

$$\Sigma_{ij} + \Sigma_{ji} < 0, \quad i, j = 1,2$$

$$\begin{bmatrix} -I & \sqrt{\rho}Q_1C_2 \\ * & -z_{max}^2Q_1 \end{bmatrix} < 0$$
(3-53)

$$\begin{bmatrix} * & -z_{max}^2 Q_1 \end{bmatrix} < 0$$

$$\begin{bmatrix} -I & \sqrt{\rho} \overline{K}_j \\ * & -u_{max}^2 Q_1 \end{bmatrix} < 0$$
(3-54)

(3-55)

Proof: Consider the Lyapunov-Krasovskii function as given by:

$$V(x) = \bar{x}^T P \bar{x}$$

$$P = \begin{bmatrix} P_1 & 0\\ 0 & P_2 \end{bmatrix}$$
(3-56)

To establish that system is asymptotically stable with a disturbance attenuation $\gamma > 0$, it is required that the associated Hamiltonian

$$\mathcal{H}(e_f, x, w, t) = z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T \omega(t) + \dot{V}(x)$$

(3-57)

Defining $\xi(t) = \begin{bmatrix} x^T(t) & e_f^T(t) & \omega^T(t) \end{bmatrix}^T$ and the requirement that $\mathcal{H}(e, x, w, t) < 0$ for all $\xi(t) \neq 0$ is obtained

$$\begin{bmatrix} (A_i + B_{2i}K_j)^T P_1 + P_1(A_i + B_{2i}K_j) & -P_1B_{2i} & P_1B_{1i} & (C_{1i} + D_{1i}K_j)^T \\ & * & P_2L_iD_{1i} + (P_2L_iD_{1i})^T & 0 & -(D_{1i})^T \\ & * & * & -\gamma^2I & 0 \\ & * & * & I \end{bmatrix} < 0$$

(3-58)

(3-62)

If $\mathcal{H}(e_f, x, w, t) < 0$, we have $z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T \omega(t) + \dot{V}(t) < 0$. Hence $||z_1||_2 < \gamma ||\omega(t)||_2$ is guaranteed for any nonzero $\omega(t) \in L_2[0, \infty)$, and the H_{∞} performance is established. Defining $Q_1 = P_1^{-1}$, $\overline{K_j} = K_j Q_1$, $G_j = P_2 L_j$ and performing congruence transformation to (3-58) by pre- and post- multiplying diag $\{P_1^{-1} P_1^{-1} I I I\}$, then the above inequality (3-58) is equivalent to the inequalities (3-52) and (3-53) in Theorem 3.2.

In what follows, we show that the suspension stroke, and maximum control force constraint are guaranteed. The following inequalities hold:

$$\begin{split} \max_{t>0} |z_{2}(t)_{q}|^{2} &\leq \max_{t>0} \left\| x^{T}(t)C_{2}^{T}C_{2}x(t) \right\|_{2} < \rho \cdot \theta_{max} \left(P^{-1/2}C_{2}^{T}C_{2}P^{-1/2} \right) \\ \max_{t>0} |u(t)|^{2} &\leq \max_{t>0} \left\| x^{T}(t)K_{i}^{T}K_{i}x(t) \right\|_{2} < \rho \cdot \theta_{max} \left(P^{-1/2}K_{i}^{T}K_{i}P^{-1/2} \right) \end{split}$$
(3-59)
(3-60)

where $\theta_{max}(\cdot)$ represents maximal eigenvalue. The constraints can be guaranteed, if

$$\rho \cdot P^{-1/2} C_2^{\ I} C_2 P^{-1/2} < \{z_{2max}^{\ 2}\}_q I$$

$$\rho \cdot P^{-1/2} K_i^{\ T} K_i P^{-1/2} < \{u_{max}^{\ 2}\}_q I$$
(3-61)

Considering the Schur complement, the inequalities (3-61) and (3-62) are equivalent to the inequalities (3-54) and (3-55) in Theorem 3.2. The proof is completed.

3.5 Experimental validation

3.5.1 Test rig setup description

The effectiveness of electromagnetic suspension system is validated on a 2 degree of freedom quarter car test rig, as shown in Figure 3-2. As shown in this figure, the quarter car test rig consists of an electromagnetic suspension system, a spring, three laser displacement sensors and two accelerometers, a hydraulic system, two real time control boards, a power amplifier and two computers running LABVIEW.



Figure 3-2 2-DOF quarter car test rig.

The hydraulic system is manufactured by the CRAM Corporation, which is used to generate the road excitation in the test rig. The hydraulic system consists of a servo valve, a hydraulic actuator and a hydraulic power unit, as shown in Figure 3-3. The hydraulic power unit compromises a 40L hydraulic tank, 2.2kW single phase electric motor, a hydraulic pump, pressure adjusting valve as well as pressure filtration to achieve the requirements of the servo valve. The hydraulic power unit provides the proper fluid pressure for hydraulic actuator. The hydraulic actuator is controlled by the real time control board with a PID controller.



Figure 3-3 Hydraulic power unit, hydraulic actuator and servo valve.

Three laser displacement sensors and two accelerometers are utilised in the test. The laser displacement sensor (model: ILD 1302-200) is manufactured by the MICRO-EPSILON company. The measuring range is 200mm (60mm to 260mm). The linearity and static resolution is 400um and 40um, respectively. One laser displacement sensor is installed in the test-bed to measure the road excitation, one laser displacement sensor is installed on the sprung mass to test the suspension deflection, and another laser displacement sensor is installed on the unsprung mass to test the unsprung mass displacement. Moreover, two acceleration sensors are installed on sprung mass and bottom of the platform to measure the sprung mass acceleration and road excitation acceleration. The mounting positions of those sensors are shown in the Figure 3-4 and Figure 3-5.







Figure 3-4 Mounting position of laser displacement sensors.


Figure 3-5 Mounting position of acceleration sensors.

The real-time control board (Model: myRio-1900) is manufactured by the National Instrument Corporation, as shown in Figure 3-6. One real-time control board is used to control the hydraulic actuator to provide the demand road excitation based on the PID control method, while the real-time control board is utilised to control the electromagnetic suspension system to provide the active force based on proposed control algorithms. The real-time control boards can calculate the demand voltage based on proposed control algorithm and send it to a power amplifier, which shown in Figure 3-7.



Figure 3-6 Real-time control board.



Figure 3-7 Power amplifier.

Two computers with LABVIEW software are used in the test, as shown in Figure 3-8. One computer is used to generate the road excitation signal and measure the road excitation response, with the tools to display the data graphically. The other computer is used to control the suspension as well as measuring and processing the suspension system signal response.



Figure 3-8 Two computers with LABVIEW software.

The electromagnetic suspension is manufactured by DMKE motor Industrial Corporation, as shown in Figure 3-9. The max load torque is 2000N.m, and the stroke is 100mm. the servo motor power and max speed are 0.75kw and 444mm/s, respectively. The electromagnetic suspension is installed between the sprung mass and unsprung mass, and a motor drive is needed to drive it, as shown in Figure 3-10.



Figure 3-9 Schematic of electromagnetic suspension.



Figure 3-10 Electromagnetic suspension system and the drive.

Steel bars produce the vehicle sprung mass, while the vehicle tyre and wheel hub act as the unsprung mass. Variation of vehicle sprung mass is realized by adding different loads of steel bars on the test rig, as shown in Figure 3-11.



Figure 3-11 Sprung mass variation.

The schematic diagram of the quarter-car suspension system test rig data acquisition system is shown in Figure 3-12. The road excitation is generated by a hydraulic system, which consists of a servo valve, a hydraulic actuator and a hydraulic pump. A laser displacement sensor is installed to measure the road excitation. The road profile displacement is measured by the laser sensor sent to the real time control board with a PID controller. The hydraulic pump provides the proper fluid pressure according to the real time control board. In terms of the active suspension system, unsprung mass displacement and suspension deflection are measured by two laser displacement sensors. Sprung mass acceleration and road excitation acceleration are measured by two accelerometers. Sprung mass displacement can be calculated through unsprung mass

displacement and suspension deflection. The signals produced by sprung mass displacement, sprung mass velocity, unsprung mass displacement, unsprung mass velocity are inputs to the active suspension controller. The signal from the sensors are fed to a real time control board. The control board calculates the demand voltage based on the proposed control algorithm and sends the signal to a power amplifier. Then the voltage output is sent to the suspension system. The entire signal from the control boards are sent to the computer equipped with a National Instruments data acquisition card. The computer run's LabVIEW software to display the time domain characteristics of the suspension system, which we can inspect the data graphically.



Figure 3-12 Schematic of quarter-car suspension system test rig data acquisition layout.

3.5.2 Experimental results of fault tolerant H_{∞} controller

Several road profiles are used to validate the ride performance of active suspension with proposed control algorithm. Firstly, sinusoidal road excitations with frequency of 3 Hz and amplitude of 10 mm is utilised to test the suspension system. The uncontrolled suspension (denotes as uncontrolled) is composed of a quarter-vehicle and an electromagnetic suspension without controller (the control force is zero). The controlled suspension is an electromagnetic suspension with fault-tolerant controller, which is denoted as controlled. The electromagnetic suspension with actuator input faults is

denoted as controlled with actuator fault. Figure 3-13 to Figure 3-16 show vehicle sprung mass acceleration, sprung mass displacement, unsprung mass displacement and active force responses under 3Hz sinusoidal road excitation. From Figure 3-13 we can see that the active suspension significantly reduces the vehicle sprung mass acceleration compared to the passive suspension. The body acceleration of active suspension with about 50% actuator faults is a bit larger than that of active suspension with 0% actuator fault, which indicate that the vehicle ride performance is guaranteed despite actuator fault, actuator time delay and parameter uncertainties. Furthermore, both the sprung mass displacement and unsprung mass displacement of active suspension are decreased compared to those of the passive suspension. When the actuator fault occurs, both the sprung mass displacement and unsprung mass displacement are increased, but still smaller than those of passive suspension.



Figure 3-13 Sprung mass acceleration responses under 3Hz sinusoidal excitations.



Figure 3-14 Sprung mass displacement responses under 3Hz sinusoidal excitations.



Figure 3-15 Unsprung mass displacement responses under 3Hz sinusoidal excitations.



Figure 3-16 Active force responses under 3Hz sinusoidal excitations.

Furthermore, bump road excitation and random road excitation are used to demonstrate the effectiveness of active suspension with proposed control method in experiment test rig, which is given by

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi v_0}{l}t\right) \right), & 0 \le t \le \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases}$$
(3-63)

where *a* is the height of the bump and *l* is the length of the bump. Here we choose a = 0.07m, and l = 0.8m, and the vehicle forward velocity of $v_0 = 0.856m/s$. Figure 3-17 to Figure 3-19 show the dynamic responses such as body acceleration, body displacement

and tyre displacement under bump road excitation. It is observed from Figure 3-17 that sprung mass acceleration of active suspension is reduced when compared to that of passive suspension. When the actuator thrust loss occurs, the sprung mass acceleration is not changed significantly. The sprung mass displacement and unsprung mass acceleration of active suspension are both reduced compared to those of passive suspension. This indicates that the active suspension with proposed controller could improve vehicle performance with the actuator fault, actuator time delay and parameter uncertainties.

To further demonstrate the effectiveness of proposed controller, a random road profile is applied to the test rig. The class A road profile with constant vehicle speed of 10 m/s is used to test the system. The vehicle responses including sprung mass acceleration and unsprung mass displacement under random road excitation are shown in Figure 3-20 to Figure 3-21. From these figures, the body acceleration of active suspension is decreased in comparison with that of passive suspension. The body acceleration is much the same when the actuator fault occurs, which indicate that the vehicle performance is guaranteed. The unsprung mass displacement of active suspension is the same with that of passive suspension, which shows that the tyre deflection is also guaranteed. In the experiments, the suspension deflection is small enough not to exceed its travel limit. It can be observed that the active suspension is able to guarantee a better performance in spite of actuator fault, actuator time delay and parameter uncertainties under different road excitation. In addition, the tyre deflection and maximum control force constraints are guaranteed simultaneously.



Figure 3-17 Sprung mass acceleration responses under bump excitations.



Figure 3-18 Sprung mass displacement responses under bump excitations.



Figure 3-19 Unsprung mass displacement responses under bump excitations.



Figure 3-20 Sprung mass acceleration responses under random excitations.



Figure 3-21 Unsprung mass displacement responses under random excitations.

3.5.3 Experimental results of T-S fuzzy H^{\infty} controller

Sinusoidal road excitation, bump road excitation and random road excitation are used to demonstrate the effectiveness of the active suspension with proposed control algorithm. The uncontrolled suspension (denotes as uncontrolled) is composed of a quarter-vehicle and an electromagnetic suspension without controller (the control force is zero). The controlled suspension is an electromagnetic suspension with T-S fuzzy controller, which is denoted as controlled. Figure 3-22 to Figure 3-25 show the vehicle dynamic responses of uncontrolled suspension and controlled suspension under 3.5 Hz sinusoidal road excitation and sprung mass variation. From Figure 3-23, it can be seen that the body acceleration and sprung mass displacement of controlled suspension is smaller than those of uncontrolled suspension with 250 kg vehicle body load, and the sprung mass acceleration and sprung mass displacement of controlled suspension are also decreased when the vehicle body load is 290 kg, which indicate that the active suspension with the proposed controller could significantly improve the vehicle performance in spite of the sprung mass variation. The unsprung mass displacement of controlled suspension is also reduced compared to the uncontrolled suspension, which indicate that the tyre deflection also reduced in comparison with the uncontrolled suspension. From Figure 3-24 we can see that the suspension deflection of controlled suspension is larger than that of uncontrolled suspension. However, it is much smaller than its travel limits. The estimated friction of controlled suspension is shown in Figure 3-26. It is seen from the figure that the friction force is very large, which should be taken into consideration during the controller design.



Figure 3-22 Sprung mass displacement responses under 3.5Hz sinusoidal excitations. Left: with 250kg load. Right: with 290kg load.



Figure 3-23 Sprung mass acceleration responses under 3.5Hz sinusoidal excitations. Left: with 250kg load. Right: with 290kg load.



Figure 3-24 Suspension deflection responses under 3.5Hz sinusoidal excitations. Left: with 250kg load. Right: with 290kg load.



Figure 3-25 Unsprung mass displacement responses under 3.5Hz sinusoidal excitations. Left: with 250kg load. Right: with 290kg load.



Figure 3-26 Friction estimation under 3.5Hz sinusoidal excitations. Left: with 250kg load. Right: with 290kg load.

Furthermore, Bump road profile is used to validate the better ride performance of active suspension with proposed controller, which is given by

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi\nu_0}{l}t\right) \right), & 0 \le t \le \frac{l}{\nu_0} \\ 0, & t > \frac{l}{\nu_0} \end{cases}$$

(3-64)

where *a* is the height of the bump and *l* is the length of the bump. Here we choose a = 0.07m, and l = 0.8m, and the vehicle forward velocity of $v_0 = 0.856m/s$.

The bump response of sprung mass acceleration, sprung mass displacement, unsprung mass displacement and friction estimation are shown in Figure 3-27 to Figure 3-30. The bump response of controlled suspension and uncontrolled suspension are compared in the

figures. Figure 3-27 shows that the body acceleration of controlled suspension is reduced when compared to the uncontrolled suspension, which validate the effectiveness of the proposed controller in improving vehicle ride performance. The sprung mass displacement and unsprung mass displacement of the controlled suspension are also smaller in comparison with those of the uncontrolled suspension. The estimated friction is shown in **Error! Reference source not found.**, which we can see that the maximum friction is about 800N.



Figure 3-27 Sprung mass acceleration responses under bump road excitation.



Figure 3-28 Sprung mass displacement responses under bump road excitation.



Figure 3-29 Unsprung mass displacement responses under bump road excitation.



Figure 3-30 Friction estimation under bump road excitation.

To further demonstrate the effectiveness of proposed controller, a random road profile is applied to the test rig. The class A road profile with constant vehicle speed of 10 m/s is used to test the system. Figure 3-31 clearly shows that the sprung mass acceleration of controlled suspension is reduced compared to that of uncontrolled suspension, which show the effectiveness of controlled suspension with proposed controller. Figure 3-32 and Figure 3-33 show that the sprung mass displacement and unsprung mass displacement of controlled suspension are smaller than those of uncontrolled suspension. The estimated friction is shown in Figure 3-34. In conclusion, the active suspension with proposed controller random road excitation.



Figure 3-31 Sprung mass acceleration responses under random road excitation.



Figure 3-32 Sprung mass displacement responses under random road excitation.



Figure 3-33 Unsprung mass displacement responses under random road excitation.



Figure 3-34 Friction estimation under random road excitation.

3.6 Summary

In this chapter, friction observer based robust H_{∞} control for active suspension and experimental validation was investigated. The mathematical model of quarter vehicle model was established. The quarter car suspension test rig and schematic of test rig data acquisition procedure were described. Firstly, a fault-tolerant H_{∞} controller based on friction compensation is proposed for active suspension system. Experimental results demonstrated that the proposed fault-tolerant H_{∞} control method effectively reduced the sprung mass acceleration, sprung mass displacement and unsprung mass displacement in spite of parameter uncertainties, actuator faults and delay, which indicated that the vehicle suspension performance was improved. Furthermore, a friction observer-based T-S fuzzy controller was proposed for active suspension considering the sprung mass variation and system friction. Three types of road profile such as 3.5Hz sinusoidal, bump, random road excitation were used to analyse and compare the responses of uncontrolled suspension and controlled suspension system. The experimental results showed that the proposed friction observer-based T-S fuzzy controller could significantly improve the road holding capability and ride performance under different road profiles.

4. Active suspension control using genetic algorithm in in-wheel motor mounted vehicle

4.1 Introduction

Electric vehicles offer significant driving performance and energy efficiency compared to the conventional internal combustion engine vehicle. The in-wheel motor driven layout has various advantages such as high motor response, precise torque generation, simplicity and efficiency, attracting an increasing research interest in recent years [5]. The deployment of IWM in electrical cars introduces new technological challenges. Installing the motors in the wheel can result in an increase in the unsprung mass, which greatly deteriorates the suspension ride comfort performance and road holding ability. Motor vibration and heavy load applied on the motor could easily result in loud noise and bearing wear, reducing the life of the motor bearing. Hence, the wear of the motor bearing is another problem that should be addressed [109]. Structures with suspended shaft-less direct-drive motors have the potential to improve the road holding capability and ride performance [10]. The motor is designed as a vibration absorber that could offset the road vibration input. Possible improvements in ride comfort and safety of electric vehicles equipped with in-wheel motors can be improved by optimizing the passive suspension system as well as deploying semi-active or active suspension system.

Suspension parameter optimization can effectively improve ride performance and handling stability by searching optimal parameter values to minimise the control objective function, which reflect the performance characteristic. Numerical optimization methods such as particle swarm optimization, sequential quadratic programming algorithms, multi-objective GA optimization methods and evolutionary algorithms have been applied for semi-active suspension or active suspension in conventional vehicles, which could significantly enhance the vehicle ride quality and controller performance [149, 182-184]. The genetic algorithm initializes with random population which evolves through genetic operation such as selection, crossover and mutation. Besides, the GA could find the optimal solution for the multi-objective optimization problem. However, there are few research on multi-objective parameters optimization for active suspension of an electric vehicle with in-wheel motor, which motivated the research of this chapter. In this chapter, Parameter optimization of active suspension in in-wheel motor driven electric vehicle using GA is presented. Parameters of the motor suspension (damping and stiffness

coefficients), vehicle suspension and active controller are optimized based on quarter vehicle model. The optimization process aims to minimize the vertical acceleration of sprung mass and motor, dynamic force transmitted to the motor as well as suspension working space and road holding capability. The optimal parameters are obtained based on GA under different road profiles. The performance of the vehicle with passive suspension, active suspension with unoptimized parameters and optimized parameters are compared. The results show that active suspension with optimized parameters significantly outperforms other suspensions in motor ride performance. The reminder of this chapter is organized as follows.

In Section 4.2, the quarter-car suspension model with a suspended motor and active suspension is established. LQR controller is designed for active suspension in section 4.3. In section 4.4, GA is used to search the optimal control weighting factors and optimize the electric vehicle active suspension parameters. Simulation results are provided in Section 4.5. Finally, conclusions are drawn in Section 4.6.

4.2 System modelling and problem formulation

The quarter-car suspension model with a dynamic absorber attached to the unsprung mass through a spring and a damper is designed, in which the in-wheel motor itself serves as the dynamic absorber, as shown in Figure 4-1. Based on the Newton's Second Law, the motion equations of this active suspension can be written as

$$m_{s}\ddot{x}_{s} = -k_{s}(x_{s} - x_{u}) - c_{s}(\dot{x}_{s} - \dot{x}_{u}) + f_{a}$$

$$m_{d}\ddot{x}_{d} = -k_{d}(x_{d} - x_{u}) - c_{d}(\dot{x}_{d} - \dot{x}_{u})$$
(4-1)

(4-2)

$$m_u \ddot{x}_u = k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) - k_t (x_u - x_g) + c_d (\dot{x}_d - \dot{x}_u) + k_d (x_d - x_u) - f_a$$
(4-3)

where x_s , x_u and x_d denote the vertical displacements of the sprung mass, unsprung mass, and motor mass of the quarter-car. Road disturbance is denoted by x_g . The sprung mass, unsprung mass and motor mass are denoted by m_s , m_u and m_d , respectively. The suspension stiffness and damping coefficients are denoted by k_s and c_s , respectively. The motor stiffness and damping coefficients are denoted by k_d and c_d , respectively, and tyre stiffness is k_t .



Figure 4-1 Quarter-car suspension model [10].

To achieve better vibration isolation performance, the suspended motor parameters, vehicle suspension parameters and active controller are optimized based on GA optimization method. The aim of the optimization is to suppress the sprung mass vibration, motor vibration as well as improving ride comfort, suspension deflection and road holding stability.

- Ride comfort. The sprung mass acceleration in the vertical direction \ddot{x}_s is used to quantify vehicle ride comfort performance.
- Motor vibration. To reduce the bearing wear and vibration of in-wheel motor, \ddot{x}_d should be minimised in the optimization function.
- Suspension deflection. The suspension deflection should not exceed its travel limit due to the mechanical structure.
- Road-holding stability. To ensure a firm uninterrupted contact of wheels with the road, the dynamic tire load should not exceed the static one.
- Dynamic force. The maximum dynamic force applied to the in-wheel motor should not exceed the allowable maximum force.

By defining the vehicle state vector as

$$x(t) = [x_s(t) - x_u(t) \dot{x}_s(t) x_d(t) - x_u(t) \dot{x}_d(t) x_u(t) - x_g(t) \dot{x}_u(t)]$$

The active suspension system can be described by the following state-space equations

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t)$$

(4-4)

$$z(t) = Cx(t) + Du(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -1 \\ \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & 0 & 0 & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & \frac{-k_d}{m_d} & -\frac{c_d}{m_d} & 0 & \frac{c_d}{m_d} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & \frac{k_d}{m_u} & \frac{c_d}{m_u} & -\frac{k_t}{m_u} - \frac{c_s + c_d}{m_u} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} -\frac{1}{m_s} \\ 0 \\ \frac{1}{m_u} \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & 0 & 0 & \frac{c_s}{m_s} \\ 0 & 0 & \frac{-k_d}{m_d} & -\frac{c_d}{m_d} & 0 & \frac{c_d}{m_d} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ k_s & c_s & k_d & c_d & 0 & -(c_s + c_d) \end{bmatrix}, D = \begin{bmatrix} -\frac{1}{m_s} \\ 0 \\ 0 \\ 1 \end{bmatrix}, u(t) = f_a, w(t) = \dot{x}_g(t).$$

4.3 Linear Quadratic Gaussian controller design

LQR control strategy derives the optimal performance of the active suspension based on the quadratic cost function of inputs and outputs. The state vector is assumed to be available in the optimal state feedback law. The LQR controller should minimise all the control quantities such as vehicle body acceleration, motor acceleration, suspension deflection, tyre dynamic force and dynamic force applied to the motor, to improve ride performance and energy efficiency. Therefore, the performance characteristic requirement as well as the controller input limitations can be represented by the performance index J. The genetic optimization of the LQR controller consists of determining the control input u, which minimizes the performance index J. The performance requirements could be written as

$$J = \int_0^\infty [Z^T \Phi Y Z + U^T \Psi U] dt = \int_0^\infty \begin{bmatrix} x \\ u \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt$$
(4-6)

where $Q = C^T \Phi C$, $R = D^T \Phi D + \Psi$, $N = C^T \Phi D$. Φ and Ψ are called weighting factor matrix, which can be written as

$$\Phi = diag(\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4 \quad \phi_5)$$

$$\Psi = \varphi_1$$
(4-7)

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 Φ and Ψ are symmetric and should be optimized based on genetic optimization. The weighting factors determine the importance of each output and input. If φ_1 is much larger than other weighting factors, the control energy is penalized heavily. If ϕ_1 is much larger compared with other weighting factors, it implies that the vehicle ride comfort is the main control objective.

The controller's feedback gain matrix is obtained by solving the Riccati equation.

$$A^{T}S + SA - (SB_{2} + N)R^{-1}(B_{2}^{T}S + N^{T}) + Q = 0$$
(4-9)

where

$$K = R^{-1} (B_2^T S + N^T)$$
(4-10)

Finally, the active suspension force is determined by equation

$$= -KX \tag{4-11}$$

where U is the control force, K is the state feedback gain matrix, and X is the state matrix.

U

4.4 GA optimization

There are multiple control objectives that should be optimized in active suspension control, and these objectives are often in conflict with each other, resulting in a Pareto frontier solution exists. It is necessary to choose the appropriate optimization method to search for optimal solutions, which represents the best solution with respect to all the objective functions. GAs are search routines developed based on principles of natural selection that seeks to minimize (or maximize) a set of objective functions using computational techniques motivated by biological reproduction. In this paper, GA is used to search for the optimal control weighting factors and optimizing the electric vehicle active suspension parameters.

4.4.1 Optimization objective functions

The fitness function is designed to satisfy the optimization objective of vehicle ride comfort and active suspension efficiency. The maximum value and the standard deviation of sprung mass vertical acceleration, suspension deflection and tire dynamic force are used to evaluate the vehicle suspension vibration, which should be minimized during optimization to improve vehicle ride performance. The maximum value and the standard deviation of motor acceleration and dynamic force applied on the motor are used to evaluate the motor vibration and bearing wear. In order to reduce the bearing wear and vibration of suspended motor, the maximum value and the standard deviation of motor acceleration and dynamic force should be minimized in the optimization function. As a result, the optimization objective is to find the optimal value of the following fitness functions.

$$F_{fitness}(x) = [J_1, \tilde{J}_1 J_2, \tilde{J}_2, J_3, \tilde{J}_3, J_4, \tilde{J}_4]$$
(4-12)

$$\begin{cases} J_1 = \min\{\max[(\ddot{x}_s)]\} \\ \tilde{J}_1 = std\{(\ddot{x}_s)\} \end{cases}$$

$$(4-13)$$

$$\begin{cases} J_2 = min\{max[(\ddot{x}_d)]\} \\ \tilde{J}_2 = std\{(\ddot{x}_d)\} \end{cases}$$

(4-14)

(4-15)

$$J_3 = min\{max[(x_s - x_u)]\}$$
$$\tilde{J}_3 = std\{(x_s - x_u)\}$$

$$\begin{aligned} \tilde{J}_4 &= \min\left\{\max\left[\left(k_t(x_u - x_g)\right)\right]\right\}\\ \tilde{J}_4 &= std\{\left(k_t(x_u - x_g)\right)\}\end{aligned}$$

$$\begin{cases} J_{4} = min \left\{ max \left[\begin{pmatrix} k_{s}(x_{u} - x_{s}) + c_{s}(\dot{x}_{u} - \dot{x}_{s}) + \\ k_{d}(x_{u} - x_{d}) + c_{d}(\dot{x}_{u} - \dot{x}_{d}) + f_{a} \end{pmatrix} \right] \right\} \\ \tilde{J}_{4} = std \left\{ \begin{pmatrix} k_{s}(x_{u} - x_{s}) + c_{s}(\dot{x}_{u} - \dot{x}_{s}) + \\ k_{d}(x_{u} - x_{d}) + c_{d}(\dot{x}_{u} - \dot{x}_{d}) + f_{a} \end{pmatrix} \right\}$$
(4-17)

4.4.2 Optimization variables

To obtain better ride performance and reduce vibration, the motor parameters such as motor mass m_d , motor suspension stiffness coefficients k_d and motor suspension damper coefficients c_d should be optimized. It is less practical to modify the vehicle mass. The vehicle suspension stiffness coefficients k_s and damper coefficients c_s could be optimized instead to achieve better vehicle performance. Moreover, controller weighting factors $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \varphi_1$ should be optimized to improve control efficiency. Controller weighting factors $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$ are for sprung mass acceleration, motor acceleration, suspension deflection, tire dynamic force and motor dynamic force, respectively. φ_1 is the active control force. As a result, the optimization variables are subject to the following constraints

$$x = [m_d, k_d, c_d, k_s, c_{s,}\phi_1, \phi_2, \phi_{3,}\phi_{4,}\phi_5, \phi_1]^T$$
(4-18)

Subject to,

$$\begin{split} m_d &\in [10, 50], \ k_d \in [20000 \quad 40000], \\ c_d &\in [1000 \quad 2000], k_s \in [10000 \quad 40000], \\ c_s &\in [500 \quad 1500], \phi_1, \phi_2, \phi_{3,} \phi_{4,} \phi_5, \varphi_1 \in [0 \quad 100]. \end{split}$$

4.4.3 GA optimization

The procedure of multi-objective genetic optimization deployed for in-wheel motor driven electric vehicle is as follows [185]:

- Generate random population.
- Evaluate the fitness value of each individual in the generation.
- Generate new population by crossing over the parent individuals to form new offspring.

• Mutate new off spring at each locus. Finally, place new offspring in the new population.

- Use the newly generated population to run the algorithm.
- If the convergence criterion is satisfied, stop and return the Pareto optimality. Alternatively, the optimization should be run again.
- Find Pareto optimal sets of design variables

$$x^{*} = \left[m_{d}, k_{d}, c_{d}, k_{s}, c_{s}, \phi_{1}, \phi_{2}, \phi_{3}, \phi_{4}, \phi_{5}, \varphi_{1}\right]^{T}$$
(4-19)

To optimize

$$F_{fitness}(x) = [J_1, \tilde{J}_1 J_2, \tilde{J}_2, J_3, \tilde{J}_3, J_4, \tilde{J}_4]$$

Subject to,

$$\begin{split} m_d &\in [10, 50], \ k_d \in [20000 \quad 40000], \\ c_d &\in [1000 \quad 2000], \ k_s \in [10000 \quad 40000], \\ c_s &\in [500 \quad 1500], \ \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \varphi_1 \in [0 \quad 100]. \end{split}$$

4.5 Simulation results

In this section, bump road excitation and random road excitation are used to illustrate the effectiveness of the GA based parameter optimization. The quarter vehicle with in-wheel motor is used. We assume that the maximum suspension deflection $z_{max} = 80$ mm, the maximum control force $u_{max} = 2000$ N and the dynamic force applied on the bearings $F_{max} = 3000$ N. Table 4-1 illustrates the optimized parameters. As the Pareto solution is a set, only one typical solution is used to make the following analysis and comparison. "Passive" refer to the passive suspension. "Unoptimised parameters" represents the active suspension system whose parameters are not optimized. "Optimised parameters" refer to the active suspension system with optimized parameters.

Table 4-1 Passive, unoptimised and optimised parameters

	m_d	k _d	Cd	k_s	C _{s,}	ϕ_1	ϕ_2	$\phi_{3,}$	$\phi_{4,}$	ϕ_5	$arphi_1$
Passive	30	32000	1496	41000	1000	*	*	*	*	*	*
Un- optimized parameters	30	32000	1496	41000	1000	1	1	1	1	1	1
Optimized parameters	38.33	26521	1734	11876	641	1.282	0.386	1.236	0.160	2.079	1.286

4.5.1 Bump road excitation

Bump road profile is used to validate the better ride performance of active suspension with optimized parameters, which is given by

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi\nu_0}{l}t\right) \right), & 0 \le t \le \frac{l}{\nu_0} \\ 0, & t > \frac{l}{\nu_0} \end{cases}$$

where a is the height of the bump and l is the length of the bump. Here we choose a =

(4-21)

0.1*m*, and l = 2m, and the vehicle forward velocity of $v_0 = 30km/h$.

Figure 4-2 shows the sprung mass acceleration of the vehicle suspension system with passive, unoptimised parameters and optimized parameters. It can be seen from this diagram that the sprung mass acceleration of the active suspension with optimised parameters is much smaller than the active suspension with unoptimised parameters and the passive suspension, which indicate that the vehicle ride comfort performance is significantly improved with the GA optimization. Figure 4-3 and Figure 4-4 show the suspension deflection and tyre dynamic force responses under bump manoeuvre. The active suspension with optimised parameters has smaller suspension deflection and tyre dynamic force when compared to those of the passive and the active with unoptimised parameters. It is clear that active suspension performance is greatly improved with GA optimized parameters.



Figure 4-2 Sprung mass acceleration responses in bump manoeuvre.



Figure 4-3 Suspension deflection responses in bump manoeuvre.



Figure 4-4 Tire dynamic force responses in bump manoeuvre.

Figure 4-5 and Figure 4-6 show the motor dynamic responses under the bump road excitation. It can be seen from this diagram that the unoptimised parameters make better ride performance than the passive. The motor acceleration of the active suspension with GA optimized parameters are greatly reduced compared to other two suspensions, which indicate that the motor vibration is greatly reduced. The dynamic force applied on the motor is also significantly reduced according to Figure 4-6. This clearly shows that lower motor vibrations result in lower wear of the motor bearings and increase motor bearing life. With the integrated GA optimised parameters, both the suspension and motor performances are improved. Figure 4-7 shows the actuator force of passive, unoptimised parameters and optimised parameters. The actuator force of active suspension with optimised parameters is slightly larger than that without optimized parameters.



Figure 4-5 Motor acceleration responses in bump manoeuvre.



Figure 4-6 Motor dynamic force responses in bump manoeuvre.



Figure 4-7 Actuator forces in bump manoeuvre.

4.5.2 Random road excitation

Furthermore, random road profile is used to validate the better ride performance of active suspension with optimized parameters. Vehicle dynamic response comparisons between passive suspensions, active suspension with unoptimized parameters and active suspension with optimized parameters are illustrated in this section.

Figure 4-8, Figure 4-9 and Figure 4-10 show sprung mass acceleration, suspension deflection and tyre dynamic force response under random manoeuvre. It can be seen from Figure 4-8 that the sprung mass acceleration of the active suspension with GA optimized parameter is greatly reduced when compared to passive suspension and the active suspension with unoptimised parameters, indicating that the active suspension with optimized parameters has enhanced ride performance. Suspension deflection of the active

suspension with GA optimized parameter is a slightly smaller compared to those of the passive and the active with unoptimized parameters. However, tire dynamic force of the "optimized parameter" is increased slightly compared to those of "passive" and "unoptimized parameters". Since the LQR controller emphasizes the ride comfort, the roadholding stability is sacrificed for ride comfort. However, suspension deflection and tyre dynamic force are guaranteed simultaneously. It is clearly demonstrated that an enhance ride comfort performance can be achieved with the GA optimization.



0.015 - Passive Unoptimization 0.01 Optimization Suspension deflection (m) 0.005 -0.005 -0.01 -0.015 2 0 1 3 4 5 Time (s)

Figure 4-8 Sprung mass acceleration responses in random manoeuvre.

Figure 4-9 Suspension deflection responses in random manoeuvre.



Figure 4-10 Tire dynamic force responses in random manoeuvre.

Figure 4-11 and Figure 4-12 are motor acceleration and motor dynamic force response under random manoeuvre. We can see from Figure 4-11 that the motor acceleration is greatly reduced with the GA optimization, which indicate the motor vibration is greatly reduced. The motor dynamic force of the active suspension with GA optimized parameter is much smaller compare to those of the passive and the active with unoptimised parameter, which indicate that the motor bearing wear is reduced, and the life of the motor bearing is increased. Figure 4-13 shows the actuator forces of three types of suspension systems in random road excitation. The actuator force of the active suspension with optimized parameters is much smaller than that of unoptimised parameters, which indicate that the energy efficiency is improved. Table 4-2 shows the RMS comparison of vehicle dynamic response in random manoeuvre case. We can see that the sprung mass acceleration of active suspension with GA optimization is decreased by 32.2% compared to the passive suspension and decreased by 38.2% in comparison to the active suspension without GA optimization. Motor acceleration of the active suspension with GA optimization is decreased by 47.9% when compared to that of the passive suspension and decreased by 49.6% compared to the active suspension with unoptimised parameters. This indicates that the suspension and motor performance is improved with integrated GA optimization in random road excitation.



Figure 4-11 Motor acceleration responses in random manoeuvre.



Figure 4-12 Motor dynamic force responses in random manoeuvre.



Figure 4-13 Actuator forces in random road manoeuvre.

Suspension type	Sprung mass acceleration (m/s2)	Motor acceleration (m/s2)	Suspension deflection (m)	Tire dynamic force (N)	Motor dynamic force (N)	Actuator force (N)
Passive	0.553	4.248	0.0034	450.16	284.15	0
Unoptimized parameters	0.607	4.390	0.0029	404.01	286.76	145.925
Optimized parameters	0.375	2.213	0.0030	489.79	208.03	102.906

Table 4-2 RMS comparison of vehicle dynamic response in random manoeuvre case

4.6 Summary

In this chapter, GA based parameter optimization of in-wheel motor driven electric vehicle was presented. Quarter-car model with suspended shaft-less direct-drive motors was established first. LQR controller was used to obtain the optimal performance for the active suspension. Furthermore, to increase vehicle performance and energy efficiency, parameters of the motor suspension (damping and stiffness coefficients), vehicle suspension as well as active controller were optimized based on multi-objective GA optimization with a designed fitness function. Two types of road excitation including bump manoeuvre and random manoeuvre were used to analyse and compare the response of optimized active suspension. Simulation results indicated that the active suspension with optimized parameters achieved enhanced ride performance when compared to passive suspension and active suspension with unoptimized parameters. The motor performance was also significantly enhanced. As a result, the proposed integrated GA optimization was proved effective in optimizing vehicle parameters and active controller.

5. Reliable fuzzy H_{∞} control for active suspension of in-wheel motor driven electric vehicles with dynamic damping

5.1 Introduction

Electric vehicles offer many advantages over the Internal Combustion Engine (ICE) vehicles in terms of energy efficiency and environmental friendliness and are regarded as one of the effective methods of decreasing the global CO2 emission. IWM has attracted an increasing research interest in recent years due to its simplicity, efficiency, as well as fast and precise torque generation. However, there are some challenges associated with IWM-EV such as increased unsprung mass, IWM bearing wear and airgap eccentricity, which can significantly deteriorate the suspension ride comfort performance and road holding ability. Bridgestone has developed the so-called dynamic-damping-in-wheelmotor-driven-system [9], which is shown to have the potential to improve ride quality and road-holding performance.

As shown in the literature review, different control methods such as fuzzy control [33, 81], LQR/Linear Quadratic Gaussian (LQG) [186], neural network method [71], linear optimal control [21, 69, 187], robust H_{∞} control [143-145] and adaptive control [27, 188, 189] are proposed for active suspension system in conventional vehicles to improve vehicle ride comfort, handling stability and safety. However, unknown faults in components such as sensor and actuator failures can deteriorate the dynamic behavior of the suspension. FTC method which deals with possible actuator failure has attracted attention in recent years. Kong et al. [162] proposed a robust non-fragile $H_{\infty}/L_2-L_{\infty}$ static output feedback controller for vehicle active suspension considering actuator time-delay and the controller gain variations. In [29, 30], the non-fragile H_{∞} controller was proposed for the half-vehicle suspension systems in the presence of actuator uncertainty and failure. Moreover, the vehicle sprung mass varies with respect to the loading conditions such as the payload and the number of vehicle occupants. Model uncertainty, such as suspension sprung and unsprung mass variations should be taken into consideration in the design of active suspension. A T-S fuzzy approach can be applied to handle the uncertainties as the T-S fuzzy model is very effective in modelling complex nonlinear systems[33, 190]. Moreover, there are two methods, polytope with finite vertices and norm-bounded uncertainty or variation that can be used to describe uncertainties and variations in model dynamics[143, 164]. Furthermore, Linear-parameter-varying technique [34, 35, 164] has

attracted an increasing research interest because of its effectiveness in describing nonlinearities and uncertainties.

Most of the existing control strategies are designed for conventional vehicles and rarely address active control of IWM EV suspension systems. Furthermore, the constraints on actuator control force should be considered in controller design. Despite their importance, issues associated with the actuator nonlinear dynamics, sprung mass uncertainty, and actuator saturation have not been explicitly dealt with in any previous studies on IWM EVs suspensions. In this chapter, a fault-tolerant fuzzy H_{∞} control design approach for active suspension of in-wheel motor driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints is proposed. The controller is designed based on the quarter-car active suspension model with a dynamic-damping-inwheel-motor-driven-system, in which the suspended motor is operated as a dynamic absorber. The T-S fuzzy model is used to model this suspension with possible sprung mass variation. The parallel-distributed compensation (PDC) scheme is deployed to derive a fault-tolerant fuzzy controller for the T-S fuzzy suspension model. In order to reduce the motor wear caused by the dynamic force transmitted to the in-wheel motor, the dynamic force is taken as an additional controlled output besides the traditional optimization objectives such as sprung mass acceleration, suspension deflection and actuator saturation. The H_{∞} performance of the proposed controller is derived as LMIs comprising three equality constraints which are solved efficiently by means of MATLAB LMI Toolbox. The proposed controller is applied to an electric vehicle suspension and its effectiveness is demonstrated through computer simulation.

The remainder of the chapter is structured as follows. In Section 5.2, the EV suspension model with an "advanced-dynamic-damper-motor" is developed. In Section 5.3, the fuzzy H_{∞} controller is designed to improve vehicle performance, ride comfort and suspension deflection. The approach is validated in Section 5.4 and simulation results are provided. Finally, conclusions are drawn in Section 5.5 and future work is discussed.

5.2 System modelling and problem formulation

5.2.1 Effects of increased unsprung mass on ride performance

IWM configuration in which the motors are installed in the wheels results in an increase in the unsprung mass. The general effect of increased unsprung mass on the sprung mass acceleration, suspension deflection, and tyre deflection based on a quarter-car model (as shown in Figure 5-1(a)) is investigated. The vehicle parameters are shown in Table 5-1. The Bode diagram of sprung mass acceleration, suspension deflection, and tyre dynamic force subjected to road disturbance are displayed in Figure 5-3. As shown in Figure 5-3, increasing unsprung mass has almost no effect on the natural frequency of sprung mass (around 1-2 Hz). However, an increase in the unsprung mass leads to an increase in the response of the frequency ranges around 10 Hz, which shows negative effect on suspension ride comfort performance and road holding ability.

5.2.2 Ride performance analysis of different IWM configurations

The quarter-car suspension model with a dynamic absorber attached to the unsprung mass through a spring and a damper is shown in Figure 5-1 (c), in which the motor itself operates as the dynamic absorber. This system is called "advanced-dynamic-dampermotor system" (ADM) [9]. Figure 5-2 shows the structure of a dynamic-damping-inwheel-motor-driven-system which is composed of tyre, flexible coupling, shaftless direct-drive motor and motor suspension [9]. The frequency responses from road disturbance to sprung mass acceleration, suspension deflection, and tyre deflection of a Conventional EV (Conv-EV), In-Wheel Motor Driven EV (IWD-EV) and an EV with an Advanced-Dynamic-Damper-Motor system (ADM-EV) are depicted in Figure 5-4. From the figure we can observe that the ADM-EV decreases the sprung mass acceleration in the range of resonance of unsprung mass. This kind of IWM configuration has the ability of improving vehicle ride comfort performance [10]. Furthermore, the suspension deflection is slightly reduced compared to Conv-EV and IWD-EV, while the tyre deflection of ADM-EV is greatly reduced compared to the IWD-EV. The results show that the ADM-EV performs better than the Conv-EV and IWD-EV, especially in the range of unsprung mass resonance.



Figure 5-1 Suspension model of electric vehicle with dynamic-damper-motor [10].



Figure 5-2 Structure of dynamic-damping-in-wheel-motor-driven-system [4].

m_s	m_u	m_d	k_t	k_s	C_S	k_d	C_d
2401	40.1	20.1	360000	32000	1496	41000	1000
340 kg	40 kg	30 kg	N/m	N/m	N/(m/s)	N/m	N/(m/s)
		30 (2)					
		20 – (a) 10 –					
					ilin .		
		-10					
		-20 -					
		-30 L 0.1	<u> </u>		0	100	
		gp gp (m		<hr/>		1	
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		-20				7	
		-30 - (C)	. /	λ_{n}		
		-40 -					
		-50 -	-	· · · ·	· · · ·		
		-60					
		-70	<u> </u>				
		0.1	Fr	equency (Hz)	U	100	
		•••)	m_=20 — — m	=40 — • – <i>m</i> =	60 <u> </u>		

Table 5-1 Suspension parameter

Figure 5-3 Bode diagrams of vehicle dynamic responses for increasing unsprung mass. (a) Sprung mass acceleration. (b) Suspension deflection. (c) Tyre dynamic force.



Figure 5-4 Comparison of frequency responses between different IWM configurations. (a) Sprung mass acceleration. (b) Suspension deflection. (c) Tyre dynamic force.

5.2.3 In-wheel motor electric vehicle suspension system modelling

The quarter-car active suspension model with a dynamic-damping-in-wheel-motordriven-system is developed, in which the in-wheel motor itself serves as the dynamic absorber, as shown in Figure 5-1 (c) and Figure 4-1. Consider the active suspension system model established in (4-1), (4-2) and (4-3). The active suspension system is described by the following state-space equations

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t)$$
(5-1)

where A, B_1, B_2, u, w are defined in Chapter 4 (4-4).

In order to design a robust $H\infty$ reliable state feedback load-dependent control law for the active suspension system presented, four key suspension performances, ride comfort, dynamic force applied on the wheel motor, suspension deflection and road holding (as defined below), are considered. Furthermore, the active control force provided for the active suspension system should be constrained by the maximum actuator force. Therefore, the vehicle sprung mass acceleration is the main control objective to obtain a

better ride performance while the other three conditions such as suspension deflection, tire dynamic force, actuator force and dynamic force transmitted to the motor are hard constraints that should be strictly satisfied. The following output variables are defined:

$$z_{1}(t) = C_{1}x(t) + D_{1}u(t)$$
(5-2)
$$z_{1}(t) = C_{1}x(t) + D_{2}u(t)$$

$$D_{22}(t) = C_{2}x(t) + D_{2}u(t)$$

$$0 \frac{c_{s}}{m_{s}}, D_{1} = \left[-\frac{1}{m_{s}}\right].$$
(5-3)

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_t & 0 \\ k_s & c_s & k_d & c_d & 0 & -(c_s + c_d) \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T.$$

 $C_1 = \begin{bmatrix} \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & 0 \end{bmatrix} 0$

The uncertainties associated with the load condition in the sprung mass should be considered during controller design. A T-S fuzzy approach can be applied to handle the uncertainties as the T-S fuzzy model is very effective in representing complex nonlinear systems. The sprung mass $m_s(t)$ varies in a given range of $m_s \in [m_{smin} \quad m_{smax}]$. The uncertain sprung mass $m_s(t)$ is bounded by its minimum value m_{smin} and its maximum value m_{smax} , and can thus be represented by

$$max \frac{1}{m_{s}(t)} = \frac{1}{m_{smin}} = \widehat{m}_{s}, \quad min \frac{1}{m_{s}(t)} = \frac{1}{m_{smax}} = \widetilde{m}_{s},$$
$$\frac{1}{m_{s}(t)} = \mu_{1}(z(t)) \ \widehat{m}_{s} + \mu_{2}(z(t)) \ \widetilde{m}_{s}$$
(5-4)

where $z(t) = \frac{1}{m_s(t)}$ is the premise variable. The membership functions $\mu_1(z(t))$ and $\mu_2(z(t))$ can be calculated by

$$\mu_1(z(t)) = \frac{\frac{1}{m_s} - \widetilde{m}_s}{\widehat{m}_s - \widetilde{m}_s}$$
(5-5)

$$\mu_2(z(t)) = \frac{\widehat{m}_s - \frac{1}{m_s}}{\widehat{m}_s - \widecheck{m}_s}$$
$$\sum_{i=1}^{2} \mu_i(z(t)) = 1$$
(5-7)

(5-6)

(5-10)

The membership functions $\mu_1(z(t))$ and $\mu_2(z(t))$ are labelled as "Heavy" or "Light", so the active suspension system with an uncertain sprung mass is represented by the following fuzzy models.

Model Rule 1: If z(t) is heavy. Then

$$\dot{x}(t) = (A_1 x(t) + B_1 w(t) + B_{21} u(t)$$
(5-8)
$$z_1(t) = (C_1 x(t) + D_2 u(t))$$

$$z_1(t) = (L_{11}x(t) + D_{11}u(t)$$
(5-9)

$$z_2(t) = (C_2 x(t) + D_2 u(t))$$

Model Rule 2: If z(t) is light. Then

$$\dot{x}(t) = (A_2 x(t) + B_1 w(t) + B_{22} u(t)$$
(5-11)

$$z_{1}(t) = (C_{12}x(t) + D_{12}u(t)$$
(5-12)

 $z_2(t) = (C_2 x(t) + D_2 u(t)$ (5-13)

Fuzzy blending allows us to infer the overall fuzzy model as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(z(t))(A_i x(t) + B_{1i} w(t) + B_{2i} u(t))$$
(5-14)

$$z_1(t) = \sum_{i=1}^{2} \mu_i(z(t))(C_{1i}x(t) + D_{1i}u(t))$$

(5-15)

$$z_{2}(t) = \sum_{i=1}^{2} \mu_{i}(z(t))(C_{2i}x(t) + D_{2i}u(t))$$
(5-16)

Remark 5.1 [191]: For the active suspension system (5-14), (5-15) and (5-16), an ideal control input u(t) obtained by the state-feedback controller applied to the suspension system through the actuator which generates the actual input $u_f(t)$, which is equal to the ideal control input in the case of a faultless actuator. When actuator faults occur, $u(t) \neq u_f(t)$. Therefore, an estimated state feedback controller $u_f(t)$ can be modelled by considering the actuator fault and sprung mass uncertainties:

$$u_{f}(t) = \sum_{i=1}^{2} \mu_{i}(z(t))(K_{i}x(t)) = \sum_{i=1}^{2} \mu_{i}(z(t))(M_{a}K_{ai}x(t))$$
(5-17)

where K_i and K_{ai} are the estimated and actual corresponding actuator fault-tolerant feedback control gain matrix to be determined. M_a represents the possible fault of the actuator. $M_{an} \le M_a \le M_{am}$, and M_{an} and M_{am} are prescribed lower and upper bounds of the actuator. Three cases are considered to describe three different actuator conditions.

 $1.M_{an} = M_{am} = 0$, so that $M_a = 0$, which means that the corresponding actuator $u_f(t)$ has completely failed.

2. $M_{an} = M_{am} = 1$, so that we have $M_a = 1$, representing the case of no fault in the actuator.

3. $0 < M_{an} = M_{am} < 1$, which indicates that there is a partial fault in the corresponding actuator.

Then we introduce the following scalars which will be used in the design of the controller.

$$M_{a0} = (M_{an} + M_{am})/2$$

$$L_a = (M_a - M_{a0})/M_{a0}$$
(5-18)

(5-19)

$$J_a = (M_{am} - M_{an})/(M_{an} + M_{am})$$
(5-20)

Then we have $M_a = M_{a0}(I + L_a)$, and $L_a^T L_a \le J_a^T J_a \le I$.

The active suspension system with the parameter uncertainties and reliable controller can be written as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i (z(t)) (A_i x(t) + B_{1i} w(t) + B_{2i} M_{a0} (I + L_a) K_{ai} x(t))$$
(5-21)

$$z_{1}(t) = \sum_{i=1}^{2} \mu_{i}(z(t))(C_{1i}x(t) + D_{1i}M_{a0}(I + L_{a})K_{ai}x(t))$$
(5-22)

$$z_{2}(t) = \sum_{i=1}^{2} \mu_{i}(z(t))(C_{2i}x(t) + D_{2i}M_{a0}(I + L_{a})K_{ai}x(t))$$
(5-23)

The fuzzy reliable H ∞ control problem is to propose a controller such that: 1) the closeloop system is asymptotically stable; 2) the H $_\infty$ performance $||z_1||_2 < \gamma ||w||_2$ are guaranteed for all nonzero $w \in L_2[0, \infty)$ in the presence of actuator faults and delay; 3) the active control force should be constrained by the maximum actuator force:

I

$$|u(t)| \le u_{max} \tag{5-24}$$

4) the following control output constraints are guaranteed:

$$|\{z_2(t)\}_q| < \{z_{2max}\}_q, q = 1,2,3$$

(5-25)

where $\{z_{2max}\}_q = \begin{bmatrix} z_{max} & (m_s + m_u + m_d)g & F_{max} \end{bmatrix}^T$.

5.3 Fuzzy reliable H_{∞} controller design

A robust H_{∞} reliable state-feedback controller for active suspension of ADM-EVs with actuator faults and parameter uncertainties is developed. The structure of the robust H_{∞} reliable state-feedback controller is shown in Figure 5-5. The closed-loop system is asymptotically stable, and it can also ensure a prescribed gain from disturbance to performance output while the suspension stroke, maximum dynamic force applied to the

suspended motor and maximum control force constraints are satisfied. The following remark and lemma are needed to derive the main results.



Figure 5-5 Structure of the fuzzy reliable $H\infty$ controller

Lemma 5.1 For a time-varying diagonal matrix $\Phi(t) = diag\{\sigma_1(t), \sigma_2(t), \dots, \sigma_p(t)\}$ and two matrices *R* and *S* with appropriate dimensions, if $|\Phi(t)| \leq V$, where V > 0 is a known diagonal matrix, then for any scalar $\varepsilon > 0$, we have

$$R\Phi S + S^{T}\Phi R^{T} \le \varepsilon RVR^{T} + \varepsilon^{-1}S^{T}VS$$
(5-26)

Theorem 5.1. Consider the suspension system (5-21), (5-22), and (5-23) with reliable fuzzy H_{∞} state feedback controller (5-17), the closed-loop system is asymptotically stable and satisfies $||z_1(t)||_2 \leq \gamma ||\omega(t)||_2$ for all ω if there exist matrices $Q_j > 0$, \overline{K}_{aj} and any scalar ε_{aij} , ε_{bj} such that matrix inequalities (5-27), (5-28), (5-29), and (5-30) are satisfied.

$$\Sigma_{ij} = \begin{bmatrix} sym(A_iQ_j + B_{2i}M_{a0}\overline{K}_{aj}) & B_{1i} & Q_jC_{1i}^{T} + \overline{K}_{aj}M_{a0}^{T}D_{1i}^{T} & \varepsilon_{aij}B_{2i} & \overline{K}_{aj}M_{a0}^{T} \\ & * & -\gamma^2 I & 0 & 0 & 0 \\ & * & * & -I & \varepsilon_{aij}D_{1i} & 0 \\ & * & * & * & -\varepsilon_{aij}J_a^{-1} & 0 \\ & * & * & * & * & -\varepsilon_{aij}J_a^{-1} \end{bmatrix}$$
$$\overset{\sum_{ii} < 0, \quad i = 1,2$$

(5-27)

 $\Sigma_{ij} + \Sigma_{ji} < 0, \ i, j = 1, 2$

(5-28)

$$\begin{bmatrix} -I & \sqrt{\rho} (Q_j C_{2i} + \overline{K}_{aj} D_{2i}) \\ * & -\{z_{2max}\}_q^2 Q_j \end{bmatrix} < 0$$

$$\begin{bmatrix} -I + \varepsilon_{bj} J_a & \sqrt{\rho} M_{a0} \overline{K}_{aj} & 0 \\ * & -u_{max}^2 Q_j & \sqrt{\rho} \overline{K}_{aj}^{\ T} M_{a0}^{\ T} \\ * & * & -\varepsilon_{bj} J_a^{-1} \end{bmatrix} < 0$$

$$(5-30)$$

Proof: Consider the Lyapunov–Krasovskii function as given by:

$$V(t) = \sum_{j=1}^{2} x^{T}(t) P_{j} x(t)$$
(5-31)

(5-29)

(5-34)

We take the derivative of V(t) along the solution of the system:

$$\dot{V}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \mu_i (z(t)) x^T(t) \{ [A_i + B_{2i} M_{a0} (I + L_a) K_{aj}]^T P_j + P_j [A_i + B_{2i} M_{a0} (I + L_a) K_{aj}] \} x^T(t) + x^T(t) P_j B_{1i} \omega(t) + \omega^T(t) B_{1i}^T P_j x(t)$$
(5-32)

In order to establish that system defined by (5-21), (5-22), and (5-23) is asymptotically stable with a disturbance attenuation $\gamma > 0$, it is required that the associated Hamiltonian

$$\mathcal{H}(e, x, w, t) = z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T \omega(t) + \dot{V}(t)$$
(5-33)

$$\begin{aligned} \mathcal{H} &= \sum_{i=1}^{2} \sum_{j=1}^{2} x^{T}(t) \{ [A_{i} + B_{2i}M_{a0}(I + L_{a})K_{aj}]^{T}P_{j} + P_{j}[A_{i} + B_{2i}M_{a0}(I + L_{a})K_{aj}] \} x(t) \\ &+ x^{T}(t)P_{j}B_{1i}\omega(t) + \omega^{T}(t)B_{1i}^{T}P_{j}x(t) - \gamma^{2}\omega(t)^{T}\omega(t) \\ &+ x^{T}(t)[C_{1i} + D_{1i}M_{a0}(I + L_{a})K_{aj}]^{T}[C_{1i} + D_{1i}M_{a0}(I + L_{a})K_{aj}]x(t) \\ &= \sum_{i=1}^{2} \sum_{j=1}^{2} \xi^{T}(t)\Psi_{ij}\xi(t) \end{aligned}$$

where
$$\Psi_{ij} = \begin{bmatrix} \theta_{11} & P_1 B_{1i} & (C_{1i} + D_{1i} M_{a0} (I + L_a) K_{aj})^T \\ * & -\gamma^2 I & 0 \\ * & * & I \end{bmatrix}$$
,

$$\theta_{11} = (A_i + B_{2i} M_{a0} (I + L_a) K_{aj})^T P_1 + P_1 (A_i + B_{2i} M_{a0} (I + L_a) K_{aj}).$$

Defining $\xi(t) = [x^T(t) \quad \omega^T(t)]^T$ and the requirement that $\mathcal{H}(e, x, w, t) < 0$ for all $\xi(t) \neq 0$ is obtained

$$\Psi_{ij} < 0$$

(5-35)

and it is further equivalent to

$$\begin{bmatrix} (A_{i} + B_{2i}M_{a0}K_{aj})^{T}P_{1} + P_{1}(A_{i} + B_{2i}M_{a0}K_{aj}) & P_{1}B_{1i} & (C_{1i} + D_{1i}M_{a0}K_{aj})^{T} \\ & * & -\gamma^{2}I & 0 \\ & * & I \end{bmatrix} \\ + \begin{bmatrix} P_{1}B_{2i} \\ 0 \\ D_{1i} \end{bmatrix} L_{a}[M_{a0}K_{aj} & 0 & 0] + \left(\begin{bmatrix} P_{1}B_{2i} \\ 0 \\ D_{1i} \end{bmatrix} L_{a}[M_{a0}K_{aj} & 0 & 0] \right)^{T} < 0$$

$$(5-36)$$

By using Lemma 5.1, the inequality is obtained by

$$\begin{bmatrix} sym((A_{i} + B_{2i}M_{a0}K_{j})^{T}P_{1}) & P_{1}B_{1i} & (C_{1i} + D_{1i}M_{a0}K_{j})^{T} & \varepsilon_{aij}P_{1}B_{2i} & (M_{a0}K_{j})^{T} \\ & * & -\gamma^{2}I & 0 & 0 & 0 \\ & * & * & -I & \varepsilon_{aij}D_{1i} & 0 \\ & * & * & * & -\varepsilon_{aij}J_{a}^{-1} & 0 \\ & * & * & * & * & -\varepsilon_{aij}J_{a}^{-1} \end{bmatrix} < 0$$

$$(5-37)$$

Defining $Q_j = P_j^{-1}$, $\overline{K}_{aj} = K_{aj}Q_j$, and performing congruence transformation by preand post-multiplying diag { P_1^{-1} I I I I I}, then the inequality (5-37) is equivalent to the inequalities (5-27) and (5-28) in Theorem 5.1.

In what follows, we show that the suspension stroke, maximum dynamic force applied on the suspended motor, dynamic tyre force and maximum control force constraint are guaranteed. The following inequalities hold:

$$\begin{aligned} \max_{t>0} |z_{2}(t)_{q}|^{2} &\leq \max_{t>0} \left\| x^{T}(t) [C_{2i} + D_{2i}M_{a0}(I + L_{a})K_{aj}]^{T} [C_{2i} \\ &+ D_{2i}M_{a0}(I + L_{a})K_{aj}]x(t) \right\|_{2} \\ &< \rho \\ &\cdot \theta_{max} \left(P_{j}^{-1/2} [C_{2i} + D_{2i}M_{a0}(I + L_{a})K_{aj}]^{T} [C_{2i} \\ &+ D_{2i}M_{a0}(I + L_{a})K_{aj}] P_{j}^{-1/2} \right) \end{aligned}$$
(5-38)

$$\begin{aligned} \max_{t>0} |u(t)|^2 &\leq \max_{t>0} \left\| x^T(t) [M_{a0}(I+L_a)K_{aj}]^T [M_{a0}(I+L_a)K_{aj}]x(t) \right\|_2 \\ &< \rho \cdot \theta_{max} \left(P_j^{-1/2} [M_{a0}(I+L_a)K_{aj}]^T [M_{a0}(I+L_a)K_{aj}] P_j^{-1/2} \right) \end{aligned}$$
(5-39)

where $\theta_{max}(\cdot)$ represents maximal eigenvalue. The constraints can be guaranteed, if

$$\rho \cdot P_j^{-1/2} \left[C_i + D_{2i} M_{a0} (I + L_a) K_{aj} \right]^T \left[C_i + D_{2i} M_{a0} (I + L_a) K_{aj} \right] P_j^{-1/2} < \{ z_{2max}^2 \}_q I$$
(5-40)

$$\rho \cdot P_j^{-1/2} [M_{a0}(I+L_a)K_{aj}]^T [M_{a0}(I+L_a)K_{aj}] P_j^{-1/2} < \{u_{max}^2\}_q I$$
(5-41)

Considering the Schur complement and Lemma 5.1, the inequalities (5-40) and (5-41) are equivalent to the inequalities (5-29) and (5-30) in Theorem 5.1. Therefore, all the conditions in Theorem 5.1 are satisfied. The proof is completed.

Based on the above conditions, the fuzzy reliable H_{∞} controller can be designed with the minimal γ by solving the following convex optimisation problem.

min γ subject to LMIs (5-27), (5-28), (5-29), and (5-30).

The feedback gain matrix for the controller can be given by

$$K_{rf} = M_a \left(\sum_{j=1}^2 \mu_i(z(t))\overline{K}_{aj} \right) \left(\sum_{j=1}^2 \mu_i(z(t))Q_j \right)^{-1}$$
(5-42)

If we assume that there is no actuator fault in the suspension system of the ADM-EV, the fuzzy H_{∞} controller is presented for the active suspension system based on the T-S fuzzy model method and then we have the following corollaries, which can be easily proved following the proof of Theorem 5.1.

Corollary 5.2 Assume ρ is a prescribed positive scalar. Consider system (5-14), (5-15), and (5-16) with the fuzzy controller, and then the closed-loop system is asymptotically stable and satisfies $||z_1(t)||_2 \leq \gamma ||\omega(t)||_2$ for all w if there are matrices $Q_j > 0$ and \overline{K}_{ai} satisfying

$$\begin{bmatrix} sym(A_iQ_j + B_{2i}\overline{K}_{aj}) & B_{1i} & Q_jC_{1i}^T + \overline{K}_{aj}D_{1i}^T \\ & * & -\gamma^2 I & 0 \\ & * & * & -I \end{bmatrix} < 0$$
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$$\begin{bmatrix} -I & \sqrt{\rho} (Q_j C_{2i} + \overline{K}_{aj} D_{2i}) \\ * & -\{z_{2max}\}_q^2 Q_j \end{bmatrix} < 0$$

$$\begin{bmatrix} -I & \sqrt{\rho} \overline{K}_j \\ * & -u_{max}^2 Q_j \end{bmatrix} < 0$$
(5-44)

Moreover, if the fuzzy controller has a feasible solution, the feedback gain matrix for the controller can be given by

$$K_f = \left(\sum_{j=1}^2 \mu_i(z(t))\overline{K}_{aj}\right) \left(\sum_{j=1}^2 \mu_i(z(t))Q_j\right)^{-1}$$
(5-46)

Furthermore, if we assume that there is no variation of sprung mass and actuator faults, the conventional H_{∞} controller is presented by Corollary 5.3.

Corollary 5.3 Assume ρ is a prescribed positive scalar. Consider the system in (5-1), (5-2) and (5-3) with the conventional H_{∞} controller. The closed-loop system is asymptotically stable and satisfies $||z_1(t)||_2 \leq \gamma ||\omega(t)||_2$ for all w if there are matrices Q > 0 and \overline{K}_a satisfying

$$\begin{bmatrix} sym(AQ + B_2\overline{K}_a) & B_1 & QC_1^T + \overline{K}_a D_1^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} -I & \sqrt{\rho}(QC_2 + \overline{K}_a D_2) \\ * & -\{z_{2max}\}_a^2 Q \end{bmatrix} < 0$$
(5-47)

(5-43)

$$\begin{bmatrix} -I & \sqrt{\rho}\overline{K}_a \\ * & -u_{max}^2 Q \end{bmatrix} < 0$$
(5-49)

The feedback gain matrix function for the controller can be given by

$$K_c = \overline{K}_a Q^{-1} \tag{5-50}$$

5.4 Simulation results

First, we consider a conventional H_{∞} controller with a guaranteed H_{∞} performance without considering the sprung mass variation and fault-tolerant control method. The minimum guaranteed closed-loop H_{∞} performance index $\gamma = 8.875$ and the controller gain matrix is given by

$$K_c = 10^4 \times [-1.649 \quad 0.092 \quad -2.448 \quad -0.052 \quad 0.329 \quad 0.113]$$

Next, we apply T-S fuzzy H_{∞} controller K_f for the uncertain active suspension system defined in Corollary 5.2. It is shown that the minimum guaranteed closed-loop H_{∞} performance index is $\gamma = 8.73$, and the fuzzy H_{∞} controller gain matrix is

$$K_f = 10^4 \times [-1.294 \quad 0.237 \quad -2.236 \quad -0.038 \quad -0.230 \quad 0.111]$$

Finally, we propose a reliable fuzzy H_{∞} controller K_{rf} designed for the active suspension in ADM-EVs with actuator faults and uncertain parameters. The minimum guaranteed closed-loop H_{∞} performance index is obtained as $\gamma = 29.4$ and the reliable fuzzy controller gain matrix regarding 0%, 30%, 60%, and 90% actuator thrust loss are

$K_{rf1} = 10^4 \times [-0.386]$	0.994	-2.094	-0.041	-1.540	0.130]
$K_{rf2} = 10^4 \times [-0.270]$	0.696	-1.466	-0.029	-1.077	0.091]
$K_{rf3} = 10^4 \times [-0.154]$	0.398	-0.838	-0.016	-0.616	0.052]
$K_{rf4} = 10^4 \times [-0.386]$	0.993	-2.095	-0.041	-1.540	0.013]

The effectiveness of the proposed reliable fuzzy H_{∞} control method is validated through computer simulation using a bumpy road disturbance and random road excitation. The parameters of the ADM-EV are set as listed in Table 1. We assume that the maximum suspension deflection $z_{max} = 100$ mm, the maximum control force $u_{max} = 2000$ N and the dynamic force applied on the bearings $F_{max} = 3000$ N . Furthermore, the sprung mass is assumed to be within the range 238 kg to 442 kg and $M_{an} = 0.1$, $M_{am} = 1$. The definition and value of each parameter is presented in Table 5-1.

5.4.1 Bump road excitation

In this work, the bumpy road excitation is introduced to analyse the dynamic response

characteristics of the EV suspension, which is given by

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi v_0}{l}t\right) \right), & 0 \le t \le \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases}$$
(5-51)

where *a* is the height of the bump and *l* is the length of the bump. Here we choose a = 0.1m, and l = 2m, and the vehicle forward velocity at $v_0 = 30km/h$.

Figure 5-6 shows the frequency responses for a passive suspension, the active suspension with a conventional H_{∞} controller K_c and an active suspension with the T-S fuzzy controller K_f . It is observed from Figure 5-6 that the two active suspensions perform much better than the passive suspension, and the active suspension with the T-S fuzzy controller performs the best among the three suspensions. Bump responses for the three suspensions are illustrated in Figure 5-7. We can observe from Figure 5-7 (a) that the sprung mass acceleration of active suspensions is greatly reduced when compared with that of passive suspension. This indicates that the suspension performance is significantly improved compared to the passive suspension. The active suspension with T-S fuzzy H_{∞} controller achieves suspension performance that is very similar to the active suspension with conventional H_{∞} controller.

In addition, it can be seen from Figure 5-7 (b-d) that the suspension deflection, tyre dynamic force and maximum control force constraints are guaranteed simultaneously, thus demonstrating the advantage of the T-S fuzzy H_{∞} control method with the sprung mass variation. The in-wheel motor dynamic responses of the passive suspension and active suspension systems with a conventional H_{∞} controller and the T-S fuzzy H_{∞} controller are shown in Figure 5-8. The active suspension systems controlled by the conventional controller and T-S fuzzy controller significantly reduce the dynamic force applied on the motor bearings, which suggests that motor wear could be reduced. Figure 5-8 (a) shows the in-wheel motor acceleration of three suspensions, i.e., passive suspension and active suspension with conventional H_{∞} controller, and active suspension with the T-S fuzzy H_{∞} controller. From the diagram we can see that the in-wheel motor vibrations of the two active controllers are the same and marginally larger than that of passive suspension system. Active suspension has a small adverse influence on the in-wheel motor performance that can be ignored.



Figure 5-6 Frequency responses of the passive suspension and active suspensions.



Figure 5-7 Vehicle dynamic responses under bump road excitation. (a) Vehicle body acceleration response. (b) Suspension deflection. (c) Actuator force. (d) Tire dynamic force.



Figure 5-8 In-wheel motor dynamic responses under bump road excitation. (a) Motor acceleration response. (b) Motor dynamic force.

In order to illustrate the effect of sprung mass variation, Figure 5-9 shows the frequency response and bump response for active suspension with the T-S fuzzy H_{∞} controller and for passive suspension for two values of the sprung mass (238 kg and 442 kg). It is observed from Figure 5-9 (a) that maximum singular values of active suspensions with the T-S fuzzy controllers are much smaller than those of passive suspensions, demonstrating the effectiveness of T-S fuzzy controller in the presence of parameter uncertainty. Furthermore, Figure 5-9 (b) shows that the sprung mass accelerations of active suspensions have lower peaks and shorter settling times than those of passive suspensions. This indicates that the T-S fuzzy controller performs significantly better than the passive suspension despite the sprung mass variation.

To illustrate the effectiveness of the reliable fuzzy H_{∞} controller, frequency analysis and bump analysis of the passive suspension, the active suspension with fuzzy controller and an active suspension with the reliable fuzzy controller are shown with 0%, 30%, 60%, and 90% actuator thrust loss. Figure 5-10 presents the $||T_{zw}||_{\infty}$ of the active suspension with the T-S fuzzy controller K_f and reliable fuzzy H_{∞} controller K_{rf} versus the variation of sprung mass parameter with 0%, 30%, 60%, and 90% actuator thrust loss, respectively. It can be observed from Figure 5-10 that the reliable fuzzy controller K_{rf} always yields a smaller closed-loop H_{∞} norm than the fuzzy controller K_f despite the change of the actuator thrust loss. This shows that the designed reliable fuzzy controller K_{rf} achieves significantly better closed-loop H_{∞} performance than the fuzzy H_{∞} controller K_f when actuator failure occurs. However, when the actuator thrust loss increases, the maximum singular value of both controllers is increased; which indicates a poorer performance of the suspension when actuator thrust loss occurs. Figure 5-11 and Figure 5-12 show the bump responses of the closed-loop systems with the fuzzy controller K_f and reliable fuzzy controller K_{rf} with 0%, 30%, 60%, and 90% actuator thrust loss in frequency and time domains, respectively. The simulation results show that the designed reliable fuzzy controller can provide better performance than the fuzzy controller can in both frequency and time domains, and the reliable fuzzy controller can maintain a satisfactory performance with the variation sprung mass and the partial fault in the actuator.



Figure 5-9 Bump responses of passive suspension and active suspension with T-S fuzzy controller for different sprung masses. (a) Frequency response. (b) Vehicle body acceleration response.



Figure 5-10 $||T_{zw}||_{\infty}$ of active suspensions with T-S fuzzy controller and reliable fuzzy controller versus the uncertain parameter.



Figure 5-11 Frequency responses of active suspensions with T-S fuzzy controller and reliable fuzzy controller.



Figure 5-12 Body acceleration of active suspensions with T-S fuzzy controller and reliable fuzzy controller under bump road excitation.

5.4.2 Random road excitation

A random road excitation is used to demonstrate the effectiveness of the proposed reliable fuzzy H_{∞} control method applied on EV suspension. The disturbance is assumed to be zero-mean white noise with identity power spectral density. Figure 5-13 and Figure 5-14 show the vehicle dynamic responses and in-wheel motor responses of passive suspension, active suspension with conventional H_{∞} controller and T-S fuzzy H_{∞} controller under random road excitation. From Figure 5-13 we can see that active suspensions greatly reduce the vehicle sprung mass acceleration compared to passive suspension. The active suspension performance with T-S fuzzy controller is marginally better than that of conventional H_{∞} controller. Suspension deflection, actuator force and tyre dynamic force are guaranteed simultaneously. The motor acceleration of active suspensions is slightly larger than the passive suspension, which has small adverse influence on the vehicle performance. Dynamic force applied to the in-wheel motor of active suspension is marginally reduced, demonstrating better motor performance than that of passive suspension. The Root Mean Square (RMS) comparison of vehicle dynamic response under random road excitation is shown in Table 5-2. The active suspension with T-S fuzzy controller achieves marginally better suspension performance than the passive suspension, which slightly better than the active suspension conventional H_{∞} controller.



Figure 5-13 Vehicle dynamic responses under random road excitation. (a) Vehicle body acceleration response. (b) Suspension deflection. (c) Actuator force. (d) Tire dynamic force.



Figure 5-14 In-wheel motor dynamic responses under random road excitation. (a) Motor acceleration response. (b) Motor dynamic force.



Figure 5-15 Body acceleration of active suspensions with T-S fuzzy controller and reliable fuzzy controller under random road excitation.

Random responses of passive suspension, active suspension with T-S fuzzy controller and reliable fuzzy controller are shown in Figure 5-15 in the presence of 0%, 30%, 60%, and 90% actuator thrust loss. The body acceleration of active suspension with reliable fuzzy controller is much the same with that of T-S fuzzy H_{∞} controller when there is 0% actuator thrust loss. Table 5-3 shows the RMS comparison of sprung mass acceleration with different actuator thrust losses. We can observe that the proposed reliable fuzzy controller is robust to parameter uncertainties and actuator faults according to Table 5-3. With an increase in the actuator thrust loss, reliable fuzzy controller reveals better performance than the T-S fuzzy controller, which shows that reliable fuzzy controller is able to guarantee a better performance in spite of actuator faults and parameter uncertainties under random road excitation.

	Vehicle performance (RMS)					
Suspension type	\ddot{x}_s (m/s ²)	\ddot{x}_d (m/s ²)	$\frac{x_s(t) - x_u(t)}{z_{max}}$	$\frac{k_t (x_u(t) - x_g(t))}{(m_s + m_u + m_d)g}$	$\frac{F_{dynamic}}{F_{max}}$	$\frac{u}{u_{max}}$
Passive	2.413	17.124	0.167	0.468	0.402	
Conventional H_{∞} controller	0.987	23.938	0.174	0.556	0.294	0.283
T-S fuzzy controller	0.865	23.839	0.177	0.550	0.295	0.278

Table 5-2 RMS comparison of vehicle response under random road excitation.

Table 5-3 RMS comparison of sprung mass acceleration with different actuator thrust loss.

	$\ddot{x}_s(m/s^2)$ (RMS)			
Suspension type	0% actuator thrust loss	30% actuator thrust loss	60% actuator thrust loss	90% actuator thrust loss
Passive	2.413			
T-S fuzzy controller	0.865	1.298	1.783	2.156
Reliable fuzzy controller	0.844	1.054	1.507	2.023

5.5 Summary

In this chapter, to deal with the suspension vibration and in-wheel motor bearing wear, a multi-objective robust H_{∞} reliable fuzzy control for active suspension system of IWM-EV with dynamic damping was proposed. First, the mathematical EV suspension system model was developed, and the variation of sprung mass was represented by a T-S fuzzy model. Then a reliable fuzzy state feedback controller was designed for the T-S fuzzy model to cope with possible actuator faults, sprung mass variation and control input constraints. Finally, the effectiveness of the proposed controller was demonstrated through computer simulation. The results showed that the suspension performance of EV with "advanced-dynamic-damper-motor" was better than a conventional in-wheel motor driven EV. Comparison of the performance of the active suspension and the passive suspension showed that the proposed reliable fuzzy H_{∞} controller achieves a significantly better closed-loop H_{∞} performance than the fuzzy H_{∞} controller does.

6. Output feedback H_{∞} control for active suspension with control faults and input delay in in-wheel motor mounted vehicle

6.1 Introduction

In the previous chapter, a fault-tolerant fuzzy H_{∞} control design approach was proposed for active suspension of in-wheel motor driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints. However, when some of the required state variables of the suspension systems are not measurable, the outputfeedback based control schemes are used. Moreover, actuator delay which occurs in many real control system applications because of pneumatic and hydraulic characteristics of the actuators, may degrade the control performances and cause instability in the resulting control systems if it is not considered in the controller deign. For example, a dynamic output-feedback delay-dependent multi-object H_{∞} controller is proposed for active suspension system with control delay [156]. A (Q, S, R)-α-dissipative output-feedback fuzzy controller is proposed for T-S fuzzy systems with time-varying input delay and output constraints in [31]. Output-feedback fuzzy control for T-S fuzzy suspension systems with input delay and output constraints are proposed in [31, 32]. However, the active suspension control studies reviewed so far are primarily focused on conventional vehicles and are not applied to active control of in-wheel motor electric vehicle suspension systems. Fault-tolerant fuzzy H_{∞} control design approach for active suspension of in-wheel drive electric vehicles in the presence of actuator faults are proposed in [90, 108]. To the best knowledge of authors, there are no studies that analyses the dynamic output feedback $H\infty$ control methodologies for active suspension control of in-wheel motor driven electric vehicle subject to actuator faults and controller time delay.

The output feedback control problem of active suspension system of in-wheel motor driven electric vehicles with actuator faults and time delay is investigated in this chapter. The dynamic damping in-wheel motor driven system, in which the in-wheel motor is designed as a DVA, is developed to improve ride quality and isolate the force transmitted to motor bearings. Furthermore, parameters of vehicle suspension and DVA are optimized based on the particle swarm optimization (PSO) to achieve better suspension performances. As some of the states such as the DVA velocity and unsprung mass velocity are difficult to measure, a robust H_{∞} output feedback controller is developed to deal with the problem of active suspension control with actuator faults and time delay.

The proposed controllers could guarantee the system's asymptotic stability and H_{∞} performance, simultaneously satisfying the performance constraints such as road holding, suspension stroke, dynamic load applied on the bearings and actuator limitation. Finally, the effectiveness of the proposed output feedback controllers is demonstrated based on the quarter vehicle suspension model under bump and random road excitations. The contributions of the approach proposed in this chapter can be outlined as follows: (1) In order to achieve a better vibration isolation performance, a dynamic damping in-wheel motor driven system in which the in-wheel motor serves as a DVA is employed in this paper. (2) Unlike in Ref. [109, 110], the suspended motor parameters, vehicle suspension parameters are optimized based on the particle swarm optimization (PSO). (3) Unlike in Ref. [90, 108], where the authors presented fault-tolerant state feedback $H\infty$ control for active suspension system tacking into account only actuator faults. In this study, a robust output feedback H[∞] controller is designed to guarantee the system's asymptotic stability and $H\infty$ performance in the presence of actuator faults and time delays; simultaneously satisfying the performance constraints. (4) Unlike in Ref. [32, 156, 192], where the authors propose active suspension controller for conventional vehicle. In this work, a multi-objective control for an electric vehicle active suspension is developed with performance requirements such as road holding, suspension stroke, the dynamic load applied on the bearings and actuator limitation; all are considered in the controller design.

The remainder of this chapter is organized as follows. In section 6.2, the electric vehicle suspension model with a DVA is developed. In Section 6.3, particle swarm optimization is used to optimize the vehicle suspension and DVA parameters. In section 6.4, dynamic output feedback controllers considering actuator faults and time delay are designed. The developed algorithms are validated in section 6.5 and simulation results are provided. Finally, conclusions are drawn in Section 6.6 and future work is discussed.

6.2 System modelling

Increased unsprung mass and motor bearing wear are two critical problems in the electric vehicle driven by in-wheel motors. To reduce the motor vibration, the force transmitted to the motor bearing should be isolated. In order to model the system, a quarter car active suspension with a DVA attached to an unsprung mass through a spring and a damper is developed with the in-wheel motor serving as the DVA (shown in Figure 4-1). Based on the active suspension system model established in (4-1), (4-2) and (4-3) with actuator time-varying delay, the motion equations of this active suspension can be written as

$$m_{s}\ddot{x}_{s}(t) = k_{s}(x_{u}(t) - x_{s}(t)) + c_{s}(\dot{x}_{u}(t) - \dot{x}_{s}(t)) + f_{a}(t - d(t))$$

$$m_{d}\ddot{x}_{d} = k_{d}(x_{u}(t) - x_{d}(t)) + c_{d}(\dot{x}_{u}(t) - \dot{x}_{d}(t))$$

$$(6-1)$$

$$(6-2)$$

$$(6-2)$$

$$m_{u}\ddot{x}_{u} = k_{t}(x_{g}(t) - x_{u}(t)) - k_{s}(x_{u}(t) - x_{s}(t)) - c_{s}(\dot{x}_{u}(t) - \dot{x}_{s}(t)) - k_{d}(x_{u}(t) - \dot{x}_{d}(t)) - c_{d}(\dot{x}_{u}(t) - \dot{x}_{d}(t)) - f_{a}(t - d(t))$$
(6-3)

where the definition of each parameter is presented in Chapter 4.

Suspension performance requirements include ride comfort, suspension defection, and road holding stability. The vehicle vertical acceleration is minimised to obtain a better ride performance. The suspension deflection and tire dynamic force are hard constraints that should be strictly satisfied. Based on the performance requirements, the control outputs can be defined by

$$z_{1}(t) = \ddot{x}_{s}$$

$$z_{2}(t) = \begin{bmatrix} x_{s} - x_{u} & k_{t}(x_{u} - x_{g}) \end{bmatrix}$$
(6-4)
(6-5)

Therefore, the active suspension system can be described by the following state-space equations

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t - d(t))$$
(6-6)

$$z_{1}(t) = C_{1}x(t) + D_{1}u(t - d(t))$$

$$z_{2}(t) = C_{2}x(t)$$
(6-7)

(6-8)

where d(t) is a known time-varying delay and satisfies $0 < d(t) \le \overline{d}$ and $\dot{d}(t) \le \mu$. A, B_1, B_2, u, w are defined in Chapter 4 (4-4).

$$C_1 = \begin{bmatrix} \frac{-k_s}{m_s} & \frac{-c_s}{m_s} & 0 & 0 & 0 & \frac{c_s}{m_s} \end{bmatrix}, \ D_1 = \begin{bmatrix} -\frac{1}{m_s} \end{bmatrix}.$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_t & 0 \end{bmatrix}, \ u(t - d(t)) = f_a(t - d(t)).$$

In practice, the state vector of a vehicle suspension system cannot be often measured directly, so it is difficult to control the system by applying the state feedback H_{∞} controller. The output feedback H_{∞} controller can be defined by the following state space equation

$$\dot{\hat{x}}(t) = A_{\kappa}\hat{x}(t) + A_{\kappa d}\hat{x}(t - d(t)) + B_{\kappa}y(t)$$

$$u_{d}(t) = C_{\kappa}\hat{x}(t)$$
(6-9)

(6-10)

where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector of the controller and $A_{\kappa}, B_{\kappa}, C_{\kappa}, D_{\kappa}$ are controller parameter matrices to be determined. The suspension deflection, motor deflection and tyre deflection can be measured by using the laser displacement sensors. However, the motor velocity, sprung mass velocity and unsprung mass velocity are difficult to measure. Therefore, y(t) is the measured output and can be written as

$$y(t) = C_y x(t) \tag{6-11}$$

where $C_y = diag[1 \ 1 \ 1 \ 0 \ 1 \ 0]$.

Unknown faults in actuator failures can deteriorate the dynamic behaviour of the suspension. Considering the actuator faults, the real control force can be modelled as

$$u(t) = \lambda u_d(t) = \left(\lambda_m + N_0 \bar{\lambda}\right) u_d(t)$$
(6-12)

$$\lambda_m = \frac{\lambda_{min} + \lambda_{max}}{2}, \quad \bar{\lambda} = \frac{\lambda_{max} - \lambda_{min}}{2}, \quad N_0 N_0^T \le I.$$

where $u_d(t)$ is the desired actuator force. λ is an unknown parameter due to the actuator faults. Assuming that λ is bounded by its minimum value λ_{min} and its maximum value λ_{max} . N_0 is an unknown parameter.

Substituting (6-9), (6-10) into (6-6), (6-7), (6-8), we can obtain the following closed-loop system

$$\dot{x}(t) = A_{cl}\bar{x}(t) + B_{cl}\bar{x}(t - d(t)) + B_{cl1}w(t)$$

(6-13)

$$\bar{z}_2(t) = C_{cl2}\bar{x}(t)$$

(6-14)

$$\bar{z}_{1}(t) = C_{cl1}\bar{x}(t) + D_{cl1}\bar{x}(t-d(t))$$
(6-15)

 $u(t) = C_u \bar{x}(t)$

(6-16)

where $\bar{x} = [x \ \hat{x}]^T$, $A_{cl} = \bar{A}$, $B_{cl1} = \bar{B}_1$, $B_{cl} = \bar{B} + HN_0E$, $C_{cl1} = \bar{C}_1$, $C_{cl2} = \bar{C}_2$, $D_{cl1} = \bar{D}_1 + D_1N_0E$, $C_u = [0 \ \lambda C_{\kappa}]$.

with

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_{\kappa}C_{y} & A_{\kappa} \end{bmatrix}, \bar{B}_{1} = \begin{bmatrix} B_{1} \\ 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 & B_{2}\lambda_{m}C_{\kappa} \\ 0 & A_{\kappa d} \end{bmatrix}, H = \begin{bmatrix} B_{2} \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 & \bar{\lambda}C_{\kappa} \end{bmatrix}, \bar{C}_{1} = \begin{bmatrix} C_{1} & 0 \end{bmatrix}, \bar{D}_{1} = \begin{bmatrix} 0 & D_{1}\lambda_{m}C_{\kappa} \end{bmatrix}, \bar{C}_{2} = \begin{bmatrix} C_{2} & 0 \end{bmatrix}.$$

The output feedback H ∞ control problem is to propose a controller such that: 1) the closeloop system is asymptotically stable; 2) the H $_\infty$ performance $\|\bar{z}_1\|_2 < \gamma \|w\|_2$ are guaranteed for all nonzero $w \in L_2[0, \infty)$ in the presence of actuator faults and delay; 3) the active control force should be constrained by the maximum actuator force:

$$|u(t)| \le u_{max}$$

(6-17)

4) the following control output constraints are guaranteed:

$$\left|\{\bar{z}_{2}(t)\}_{q}\right| < \{\bar{z}_{2max}\}_{q}, q = 1, 2, 3$$
(6-18)

where $\bar{z}_{2max} = [z_{max} \quad (m_s + m_u + m_d)g]^T$.

6.3 Parameter optimization of suspension and DVA

To achieve better vibration isolation performance, the suspended motor parameters, vehicle suspension parameters are optimized based on the particle swarm optimization. PSO is a population-based search algorithm based on the simulation of the social behaviour of birds within a flock. The earliest attempt to use the concept for social

behaviour simulation was carried out by Kennedy and Eberhart [193]. Unlike genetic algorithm optimization, PSO can generate a high-quality solution with shorter calculation time and more stable convergence characteristic. PSO is initialized with a group of random particles and then searches for optima by updating iterations [194]. In every iteration, each particle is updated by the best value so far (X_{pbest}) and best value in the group (X_{gbest}). The particle tries to modify its position using the current velocity and the distance from X_{pbest} and X_{gbest} . Let X_d^k denote the position of the particle. The position of X_d^k is changed by adding a velocity v_d^k to it

$$X_d^{k+1} = X_d^k + v_d^{k+1}$$
(6-19)

The velocity of the particle is defined as follows:

$$v_d^{k+1} = wv_d^k + c_1 r_1 (X_{pbest} - X_d^k) + c_2 r_2 (X_{pbest} - X_d^k)$$
(6-20)

where X_d^k and v_d^k are the d th particle's position and velocity vector, respectively. c_1 and c_2 are two parameters representing the particle's confidence in itself (cognition) and in the swarm (social behavior), respectively. A relatively high value of c_1 will encourage the particles to move toward their local best experiences, while higher values of c_2 will result in faster convergence to the global best position. X_{pbest} is the personal best position of one particle. X_{gbest} is the position of the best particle of the entire swarm. r_1 , r_2 are random numbers s uniformly distributed in the range (0,1).

The inertia weight w is given by the following equation.

$$w = w_{in} - \frac{w_{in} - w_{fn}}{iter}k$$
(6-21)

where w_{in} , w_{fn} are initial and final weights, *iter* denoting the total number of iterations. *k* is the current iteration number.

In this work, PSO technique is used to search optimal suspension parameters such as motor mass m_d , motor stiffness k_d , motor damping c_d , suspension stiffness k_s and suspension damping c_s . The optimization objective is to suppress the sprung mass vibration, motor vibration as well as improving ride comfort, suspension deflection and road holding stability. The fitness function designed to satisfy the optimization objective

can be written as follows:

$$f_{obj}(x) = p_1 RMS\left(\frac{\ddot{x}_s}{\ddot{x}_{smax}}\right) + p_2 RMS\left(\frac{x_s - x_u}{x_{sumax}}\right) + p_3 RMS\left(\frac{k_t(x_s - x_u)}{(m_s + m_u + m_d)g}\right) + p_4 RMS\left(\frac{F_d}{F_{dmax}}\right)$$

$$(6-22)$$

The procedure of particle swarm optimization for vehicle suspension and DVA is as follows [194]:

- Initialize of algorithmic parameters.
- Evaluate the desired fitness function of each particle.
- Update X_{pbest} and X_{gbest} . Determine the current best value X_{pbest} and best value of all the particle X_{qbest} .
- Update the velocity and position of each particle.
- Iterations are finished, or minimum performance index is reached, stop and return the optimal solution.

Find vector $X_{gbest} = [m_d \ k_d \ c_d \ k_s \ c_s]$

To minimize

$$f_{obj}(x) = p_1 RMS\left(\frac{\dot{x}_s}{\dot{x}_{smax}}\right) + p_2 RMS\left(\frac{x_s - x_u}{x_{sumax}}\right) + p_3 RMS\left(\frac{k_t(x_s - x_u)}{(m_s + m_u + m_d)g}\right) + p_4 RMS\left(\frac{F_d}{F_{dmax}}\right)$$

$$(6-23)$$

Subject to
$$\begin{cases} m_d \in [20 \quad 50] \\ k_d \in [30000 \quad 50000] \\ c_d \in [1000 \quad 3000] \\ k_s \in [30000 \quad 500000] \\ c_s \in [1000 \quad 3000] \end{cases}$$

where p_1, p_2, p_3, p_4 are weighting factors for the four performance indexes. Then the optimal value of the motor parameters and suspension parameters can be obtained.

6.4 Output feedback controller design

In this section, an output feedback controller for the active suspension of an electric vehicle with DVA structure in the presence of actuator faults and delay is developed. The closed-loop system asymptotic stability is verified, and the existence of a gain from disturbance to performance satisfying the suspension deflection, tyre dynamic force, maximum dynamic force applied on the DVA and maximum control force constraints is shown. The following lemma is needed to derive the main results.

Lemma 6.1 [181] For a time-varying diagonal matrix $\Phi(t) = diag\{\sigma_1(t), \sigma_2(t), \dots, \sigma_p(t)\}$ and two matrices R and S with appropriate dimensions, if $|\Phi(t)| \leq V$, where V > 0 is a known diagonal matrix, then for any scalar $\varepsilon > 0$, we have

$$R\Phi S + S^T \Phi R^T \le \varepsilon R V R^T + \varepsilon^{-1} S^T V S$$

(6-24)

Theorem 6.1 Consider the active suspension system in (6-13), (6-14), (6-15) and (6-16) with proposed dynamic output feedback controller in (6-9) and (6-10). For given positive scalars \overline{d} , γ , θ_R and ρ , if there exist matrices $\overline{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, $\overline{S} = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0$, $\overline{R} = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$, X > 0, Y > 0, \hat{A} , \hat{A}_d , \hat{B} , \hat{C} , $\overline{M}_i = \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i3} & M_{i4} \end{bmatrix}$, $\overline{N}_i = \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i3} & N_{i4} \end{bmatrix}$, (i=1,2,3,4) with appropriate dimensions and any positive scalars ε_1 , ε_2 , ε_3 such that the following LMIs hold:

$$\begin{bmatrix} \overline{\Psi}_{11} & \sqrt{d}\overline{M} & \overline{\Psi}_{13} & \overline{\Psi}_{14} & \varepsilon_{1}\overline{\Psi}_{15} & \overline{\Psi}_{16} \\ * & -\overline{R} & 0 & 0 & 0 \\ * & * & -I & 0 & \varepsilon_{1}D_{1} & 0 \\ * & * & * & -\overline{\Psi}_{44} & \varepsilon_{1}\overline{\Psi}_{45} & 0 \\ * & * & * & * & -\varepsilon_{1}I & 0 \\ * & * & * & * & * & -\varepsilon_{1}I \end{bmatrix} < 0$$

(6-25)

$$\begin{bmatrix} \overline{\Psi}_{11} & \sqrt{d}\overline{N} & \overline{\Psi}_{13} & \overline{\Psi}_{14} & \varepsilon_2 \overline{\Psi}_{15} & \overline{\Psi}_{16} \\ * & -\overline{R} & 0 & 0 & 0 \\ * & * & -I & 0 & \varepsilon_2 D_1 & 0 \\ * & * & * & -\overline{\Psi}_{44} & \varepsilon_2 \overline{\Psi}_{45} & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0$$

(6-26)

$$\begin{bmatrix} -\mu_{max}^{2}X & -\mu_{max}^{2}I & \sqrt{\rho}\hat{C}^{T}\lambda_{m}^{T} & \hat{C}^{T}\bar{\lambda}_{m}^{T} \\ * & -\mu_{max}^{2}Y & 0 & 0 \\ * & * & -I + \varepsilon_{2}I & 0 \\ * & * & * & -\varepsilon_{2}I \end{bmatrix} < 0$$

(6-27)

$$\begin{bmatrix} -\{z_{2max}^{2}\}_{q}X & -\{z_{2max}^{2}\}_{q}I & \sqrt{\rho}(X^{T}C_{2}^{T} + \hat{C}^{T}\lambda_{m}^{T}D_{2}^{T}) & 0 & \hat{C}^{T}\bar{\lambda}^{T} \\ * & -\{z_{2max}^{2}\}_{q}Y & \sqrt{\rho}C_{2}^{T} & 0 & 0 \\ * & * & -II & \varepsilon_{1}D_{2} & 0 \\ * & * & * & -\varepsilon_{3}I & 0 \\ * & * & * & -\varepsilon_{3}I & 0 \\ * & * & * & * & -\varepsilon_{3}I \end{bmatrix} < 0$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$$

(6-29)

$$\begin{split} & \text{where } \overline{\Psi}_{11} = \begin{bmatrix} \overline{\Theta}_{11} & \overline{\Theta}_{12} & \overline{\Theta}_{13} & \overline{\Theta}_{14} \\ * & \overline{\Theta}_{22} & \overline{\Theta}_{23} & \overline{\Theta}_{24} \\ * & * & \overline{\Theta}_{33} & \overline{\Theta}_{34} \\ * & * & * & -\gamma^2 I \end{bmatrix}, \\ & \overline{\Theta}_{11} = \begin{bmatrix} AX + XA^T + M_{11} + M_{11}^T + Q_{11} + S_{11} & A + A^T + M_{12} + M_{13}^T + Q_{12} + S_{12} \\ * & Y^T A + A^T Y + BC_Y + C_Y^T B^T + M_{14} + M_{14}^T + Q_{22} + S_{22} \end{bmatrix}, \\ & \overline{\Theta}_{12} = \begin{bmatrix} B_2 \lambda_m \hat{C} + M_{21}^T - M_{11} + N_{11} & B_2 \lambda_m \hat{D}C_Y + M_{23}^T - M_{12} + N_{12} \\ Y^T B_2 \lambda_m \hat{D}C_Y + \hat{A}_d + M_{22}^T - M_{13} + N_{13} & Y^T B_2 \lambda_m \hat{D}C_Y + M_{24}^T - M_{14} + N_{14} \end{bmatrix}, \\ & \overline{\Theta}_{22} = \begin{bmatrix} -(1 - \mu)S_{11} + N_{21}^T + N_{11} - M_{21}^T - M_{21} & -(1 - \mu)S_{12} + N_{23}^T + N_{12} - M_{23}^T - M_{22} \\ -(1 - \mu)S_{22} + N_{24}^T + N_{14} - M_{24}^T - M_{24} \end{bmatrix}, \\ & \overline{\Theta}_{13} = \begin{bmatrix} M_{31}^T - N_{11} & M_{33}^T - N_{12} \\ M_{32}^T - N_{13} & M_{34}^T - N_{14} \end{bmatrix}, \\ & \overline{\Theta}_{23} = \begin{bmatrix} N_{31}^T - M_{31} & -N_{31} \\ N_{32}^T - M_{32}^T - N_{23} & N_{34}^T - M_{34}^T - N_{24} \end{bmatrix}, \\ & \overline{\Theta}_{33} = \begin{bmatrix} -Q_{11} - N_{31}^T - N_{31} & -Q_{12} - N_{33}^T - N_{32} \\ N_{32}^T - N_{23} & N_{34}^T - M_{34}^T - N_{24} \end{bmatrix}, \\ & \overline{\Theta}_{33} = \begin{bmatrix} -Q_{11} - N_{31}^T - N_{31} & -Q_{12} - N_{33}^T - N_{32} \\ N_{22}^T - N_{33}^T - N_{23} & N_{34}^T - M_{34}^T - N_{24} \end{bmatrix}, \\ & \overline{\Theta}_{34} = \begin{bmatrix} -M_{41}^T + M_{41}^T \\ -M_{42}^T + N_{42}^T \end{bmatrix}, \\ & \overline{\Theta}_{34} = \begin{bmatrix} -N_{41}^T \\ -N_{42}^T + N_{42}^T \end{bmatrix}, \\ & \overline{\Theta}_{34} = \begin{bmatrix} -N_{41}^T \\ -N_{42}^T + N_{42}^T \end{bmatrix}, \\ & \overline{\Theta}_{34} = \begin{bmatrix} N_{11} & N_{12} \\ N_{14} \end{bmatrix}, \\ & \overline{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{13} & M_{14} \end{bmatrix}, \\ & \overline{N}_{1} = \begin{bmatrix} N_{11} & N_{12} \\ N_{13} & N_{14} \end{bmatrix}, \\ & \overline{M} = \begin{bmatrix} \overline{M}_{1}^T & \overline{M}_{2}^T & \overline{M}_{3}^T & \overline{M}_{4}^T \end{bmatrix}, \\ & \overline{\Theta}_{26} = \begin{bmatrix} \hat{C}^T \lambda_m^T D_1^T \\ 0 \end{bmatrix}, \\ & \overline{\Theta}_{17} = \begin{bmatrix} \sqrt{d} XA^T & \sqrt{d} A^T \\ \sqrt{d} A^T & \sqrt{d} A^T Y + \sqrt{d} C_Y^T B^T \end{bmatrix} \\ & \overline{\Theta}_{27} = \begin{bmatrix} \sqrt{d} C^T \lambda_m^T B_2^T & \sqrt{d} A_d \\ 0 \end{bmatrix}, \\ & \overline{\Theta}_{47} = \begin{bmatrix} \sqrt{d} C^T \lambda_m^T B_2^T & \sqrt{d} A_d \\ 0 \end{bmatrix}, \\ & \overline{\Theta}_{47} = \begin{bmatrix} \sqrt{d} B_1^T & \sqrt{d} B_1^T Y \end{bmatrix}, \end{aligned}$$

$$\bar{\Psi}_{44} = \begin{bmatrix} \theta_R^2 R_{11} - 2\theta_R X & \theta_R^2 R_{12} - 2\theta_R I \\ * & \theta_R^2 R_{22} - 2\theta_R Y \end{bmatrix}, \ \bar{\Psi}_{15} = \begin{bmatrix} \bar{\theta}_{18}^T & 0 & 0 & 0 \end{bmatrix}^T, \ \bar{\theta}_{18} = \begin{bmatrix} B_2 \\ Y^T B_2 \end{bmatrix},$$
$$\bar{\Psi}_{45} = \begin{bmatrix} \sqrt{\bar{d}} B_2 \\ \sqrt{\bar{d}} Y^T B_2 \end{bmatrix}, \ \bar{\Psi}_{16} = \begin{bmatrix} 0 & \bar{\theta}_{29}^T & 0 & 0 \end{bmatrix}^T, \ \bar{\theta}_{29} = \begin{bmatrix} \hat{C}^T \bar{\lambda}^T \\ 0 \end{bmatrix}.$$

Proof: Considering the Lyapunov-Krasovskii functional as follows:

$$V(t) = \bar{x}^{T}(t)P\bar{x}(t) + \int_{t-\bar{d}}^{t} \bar{x}^{T}(s)Q\bar{x}(s)ds + \int_{t-d(t)}^{t} \bar{x}^{T}(s)S\bar{x}(s)ds + \int_{-\bar{d}}^{0} \int_{t+\alpha}^{t} \dot{x}^{T}(s)R\dot{x}(s)dsd\alpha$$

$$+ \int_{-\bar{d}}^{0} \int_{t+\alpha}^{t} \dot{x}^{T}(s)R\dot{x}(s)dsd\alpha$$
(6-30)

The derivative of V(t) along the solution of system is expressed as

$$\dot{V}(t) \leq 2\bar{x}^{T}(t)P\dot{\bar{x}}(t) + \bar{x}^{T}(t)(Q+S)\bar{x}(t) - \bar{x}^{T}(t-\bar{d})Q\,\bar{x}(t-\bar{d}) - (1-\mu)\bar{x}^{T}(t-d(t))S\,\bar{x}(t-d(t)) + \bar{d}\dot{\bar{x}}^{T}(s)R\dot{\bar{x}}(s) - \int_{t-d(t)}^{t}\dot{\bar{x}}^{T}(s)R\dot{\bar{x}}(s)ds - \int_{t-\bar{d}}^{t-d(t)}\dot{\bar{x}}^{T}(s)R\dot{\bar{x}}(s)ds$$

$$(6-31)$$

For any appropriately dimensioned matrices \widehat{M} and \widehat{N} , the following equalities hold directly according to the Newton–Leibniz formula

$$2\xi^{T}(t)\hat{M} \times \left(\bar{x}(t) - \bar{x}(t - d(t)) - \int_{t - d(t)}^{t} \dot{x}(s)ds\right) = 0$$
(6-32)
$$2\xi^{T}(t)\hat{N} \times \left(\bar{x}(t - d(t)) - \bar{x}(t - \bar{d}) - \int_{t - \bar{d}}^{t - d(t)} \dot{x}(s)ds\right) = 0$$

(6-33)

where

$$\xi(t) = \begin{bmatrix} \bar{x}^{T}(t) & \bar{x}^{T}(t-d(t)) & \bar{x}^{T}(t-\bar{d}) & w^{T}(t) \end{bmatrix}$$
$$\widehat{M} = \begin{bmatrix} M_{1}^{T} & M_{2}^{T} & M_{3}^{T} & M_{4}^{T} \end{bmatrix}^{T}, \ \widehat{N} = \begin{bmatrix} N_{1}^{T} & N_{2}^{T} & N_{3}^{T} & N_{4}^{T} \end{bmatrix}^{T}$$

Adding the two equations in into the right-hand side of (6-31) and after some simple calculations, the following inequality is satisfied

$$\bar{z}_{1}^{T}(t)\bar{z}_{1}(t) - \gamma^{2}w(t)^{T}w(t) + \dot{V}(x) \leq \xi^{T}(t) \left[\Psi + d(t)\widehat{M}R^{-1}\widehat{M}^{T} + \left(\bar{d} - d(t)\widehat{N}R^{-1}\widehat{N}^{T}\right)\right]\xi(t) = \xi^{T}(t) \left[\frac{d(t)}{\bar{d}}\left(\Psi + \bar{d}\widehat{M}R^{-1}\widehat{M}^{T}\right) + \frac{\bar{d}-d(t)}{\bar{d}}\left(\Psi + \bar{d}\widehat{N}R^{-1}\widehat{N}^{T}\right)\right]\xi(t)$$

$$(6-34)$$

where $\Psi = \Phi_{11} + \Phi_{13} \Phi_{13}^{T} + \Phi_{14} R^{-1} \Phi_{14}^{T}$,

$$\Phi_{11} = \begin{bmatrix} \widehat{\Phi}_{11} & PB_{cl} - M_1 + M_2^T + N_1 & M_3^T - N_1 & PB_{cl1} + M_4^T \\ * & \widehat{\Phi}_{22} & N_3^T - M_3^T - N_2 & -M_4^T + N_4^T \\ * & * & -Q - N_3^T - N_3 & -N_4^T \\ * & * & & -\gamma^2 I \end{bmatrix},$$

$$\widehat{\Phi}_{11} = PA_{cl} + PA_{cl}^T + Q + S + M_1^T + M_1, \ \widehat{\Phi}_{22} = -(1-\mu)S - M_2^T - M_2 + N_2^T + N_1,$$

$$\Phi_{13} = \begin{bmatrix} C_{cl1} & D_{cl1} & 0 & 0 \end{bmatrix}^T, \ \Phi_{14} = \begin{bmatrix} \sqrt{\overline{d}}RA_{cl} & \sqrt{\overline{d}}RB_{cl} & 0 & \sqrt{\overline{d}}RB_{cl1} \end{bmatrix}^T.$$

According to inequalities (6-32), (6-33) and Schur complement, we can obtain the following inequality.

$$\Psi + \bar{d}\hat{M}R^{-1}\hat{M}^T < 0$$

$$(6-35)$$

$$\Psi + \hat{N}R^{-1}\hat{N}^T < 0$$

(6-36)

According to the (6-34), (6-35) and (6-36), we can obtain $z_1^T(t)z_1(t) - \gamma^2 \omega(t)^T w(t) + \dot{V}(t) < 0$ for all nonzero $w \in L_2[0, \infty)$. The system (6-13), (6-14) and (6-15) is asymptotically stable for the delay d(t) and the H_{∞} performance is established. In addition, when w(t) = 0, we can have $\dot{V}(t) < 0$, which means the system is asymptotically stable for the delay d(t).

In what follows, we will show that the hard constraints in (6-27) and (6-28) are guaranteed. From the definition of the Lyapunov function in (6-30), we know that $x^{T}(t)Px(t) < \rho$ with $\dot{V} = \gamma^{2}w_{max} + V(0)$. Similarly, the following inequalities hold:

where $\theta_{max}(\cdot)$ represents maximal eigenvalue. The constraints can be guaranteed, if

$$\rho \cdot P^{-\frac{1}{2}} \{C_{cl2}\}_q^T \{C_{cl2}\}_q P^{-\frac{1}{2}} < \{z_{2max}^2\}_q I$$
(6-39)

$$\rho \cdot P^{-\frac{1}{2}} \{C_u\}^T \{C_u\} P^{-\frac{1}{2}} < \{u_{max}{}^2\}_q I$$
(6-40)

By evaluating

$$A_{cl} = \bar{A}, B_{cl1} = \bar{B}_1, B_{cl} = \bar{B} + HN_0E, C_{cl1} = \bar{C}_1, C_{cl2} = \bar{C}_2, D_{cl1} = D_1N_0E.$$

The equations can be written as

$$\Psi + \bar{d}\hat{M}R^{-1}\hat{M}^{T} = \begin{bmatrix} \Psi_{11} & \sqrt{\bar{d}}\hat{M} & \Psi_{13} & \Psi_{14} \\ * & -R & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -R \end{bmatrix} + \begin{bmatrix} \Psi_{15} \\ 0 \\ D_{1} \\ \sqrt{\bar{d}}RH \end{bmatrix} N_{0} \begin{bmatrix} \Psi_{16} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{T} + \begin{bmatrix} \Psi_{16} \\ 0 \\ 0 \\ 0 \end{bmatrix} N_{0}^{T} \begin{bmatrix} \Psi_{15} \\ 0 \\ D_{1} \\ \sqrt{\bar{d}}RH \end{bmatrix}^{T}$$
(6-41)

Then the following LMIs hold:

$$\begin{bmatrix} \Psi_{11} & \sqrt{\bar{d}}\widehat{M} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} \\ * & -R & 0 & 0 & 0 \\ * & * & -I & 0 & \varepsilon_1 D_1 & 0 \\ * & * & * & -R & \varepsilon_1 \sqrt{\bar{d}}RH & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0$$

(6-42)

$$\begin{bmatrix} \Psi_{11} & \sqrt{d}\hat{N} & \Psi_{13} & \Psi_{14} & \Psi_{25} & \Psi_{26} \\ * & -R & 0 & 0 & 0 \\ * & * & -I & 0 & \varepsilon_2 D_1 & 0 \\ * & * & * & -R & \varepsilon_2 \sqrt{d}RH & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0$$

(6-43)

where
$$\Psi_{11} = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} \\ * & * & \Theta_{33} & \Theta_{34} \\ * & * & * & -\gamma^2 I \end{bmatrix}$$
,

$$\begin{split} & \Theta_{11} = P\bar{A} + P\bar{A}^{T} + Q + S + M_{1}^{T} + M_{1}, \ \Theta_{12} = P\bar{B} - M_{1} + M_{2}^{T} + N_{1}, \\ & \Theta_{13} = M_{3}^{T} - N_{1}, \ \Theta_{14} = P\bar{B}_{1} + M_{4}^{T}, \\ & \Theta_{22} = -(1-\mu)S - M_{2}^{T} - M_{2} + N_{2}^{T} + N_{1}, \ \Theta_{23} = N_{3}^{T} - M_{3}^{T} - N_{2}, \ \Theta_{24} = -M_{4}^{T} + N_{4}^{T}, \\ & \Theta_{33} = -Q - N_{3}^{T} - N_{3}, \ \Theta_{34} = -N_{4}^{T}, \\ & \Psi_{13} = [\bar{C}_{1} \quad \bar{D}_{1} \quad 0 \quad 0]^{T}, \ \Psi_{14} = \left[\sqrt{\bar{d}}R\bar{A} \quad \sqrt{\bar{d}}R\bar{B} \quad 0 \quad \sqrt{\bar{d}}R\bar{B}_{1}\right]^{T}, \end{split}$$

$$\Psi_{15} = \begin{bmatrix} \varepsilon_1 H^T P & 0 & 0 & 0 \end{bmatrix}^T, \Psi_{25} = \begin{bmatrix} \varepsilon_2 H^T P & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\Psi_{16} = \begin{bmatrix} 0 & E & 0 & 0 \end{bmatrix}^T, \Psi_{26} = \begin{bmatrix} 0 & E & 0 & 0 \end{bmatrix}^T.$$

There are several nonlinear variables presented in the above equations that cannot be eliminated by the change in the variables, usually deployed in the design of state-feedback controllers. Since the matrix P is nonsingular, we partition matrix P and its inverse as follows

$$P = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}, \ P^{-1} = \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix}.$$

where $X, Y \in \mathbb{R}^{n \times n}$ are symmetric matrices. We can obtain

$$P\begin{bmatrix} X & I\\ M^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y\\ 0 & N^T \end{bmatrix},$$

By defining

$$F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}.$$

then $PF_1 = F_2$, and the following equations can be obtained:

$$\begin{split} F_{1}^{T}P\bar{A}F_{1} &= F_{2}^{T}\bar{A}F_{1} = \begin{bmatrix} AX & A \\ \hat{A} & Y^{T}A + \hat{B}C_{y} \end{bmatrix}, F_{1}^{T}P\bar{B}F_{1} = \begin{bmatrix} B_{2}\lambda_{m}\hat{C} & 0 \\ \hat{A}_{d} & 0 \end{bmatrix}, \\ F_{1}^{T}QF_{1} &= \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}, F_{1}^{T}SF_{1} = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}, F_{1}^{T}RF_{1} = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix}, \\ F_{1}^{T}M_{i}F_{1} &= \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i3} & M_{i4} \end{bmatrix}, F_{1}^{T}N_{i}F_{1} = \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i3} & N_{i4} \end{bmatrix}, \\ F_{1}^{T}P\bar{B}_{1} &= \begin{bmatrix} B_{1} \\ Y^{T}B_{1} \end{bmatrix}, F_{1}^{T}PH = \begin{bmatrix} B_{2} \\ Y^{T}B_{2} \end{bmatrix}, F_{1}^{T}PF_{1} = F_{2}^{T}F_{1} = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, F_{1}^{T}E^{T} = \begin{bmatrix} \hat{C}^{T}\bar{\lambda}^{T} \\ 0 \end{bmatrix}, \\ \bar{C}_{1}F_{1} &= [C_{1}X & C_{1}], \quad \bar{D}_{1}F_{1} = [D_{1}\lambda_{m}\hat{C} & 0]. \end{split}$$

We define the variables

$$\hat{A} = Y^{T}AX + NB_{\kappa}C_{y}X + NA_{\kappa}M^{T}$$

$$\hat{A}_{d} = Y^{T}B_{2}\lambda_{m}C_{\kappa}M^{T} + NA_{\kappa d}M^{T}$$
(6-44)

$$\hat{B} = NB_{\kappa}$$
(6-45)

(6-46)

$$\hat{C} = C_{\kappa} M^T \tag{6-47}$$

Define $\Gamma_1 = diag\{F_1, F_1, F_1, I, F_1, I, (PR^{-1})^T F_1, I, I\}$

Given matrices *X*, *Y* and *M*, *N*, we can determine A_{κ} , $A_{\kappa d}$, B_{κ} , C_{κ} , from \hat{A} , \hat{A}_d , \hat{B} , \hat{C} . Performing congruence transformation by multiplying full rank matrices Γ_1^T , Γ_1^T on the left and Γ_1 , Γ_1 on the right, respectively, the inequalities (6-42) and (6-43) are equivalent to the inequalities (6-25) and (6-26) in Theorem 6.1. This is the end of proof.

Consequently, the output feedback controller parameters are obtained

$$C_{\kappa} = \hat{C}(M^{T})^{-1}$$

$$B_{\kappa} = N^{-1}\hat{B}$$
(6-48)

(6-49)

$$A_{\kappa d} = N^{-1} (\hat{A}_{d} - Y^{T} B_{2} \lambda_{m} C_{\kappa} M^{T}) (M^{T})^{-1}$$

$$A_{\kappa} = N^{-1} [\hat{A} - Y^{T} A X - N B_{\kappa} C_{y} X] (M^{T})^{-1}$$
(6-50)

(6-51)

Remark 6.1 Theorem 6.1 presents an output feedback controller for an active suspension system with control delay and faults. When there are only control faults in the quarter-car model, the fault-tolerant controller is presented for active suspension system of electric vehicle with DVA based on the output feedback H_{∞} control method. The active suspension system can be described by the following state-space equations:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t)$$

$$z_2(t) = C_2 x(t)$$
(6-52)

(6-53)

$$z_1(t) = C_1 x(t) + D_1 u(t)$$
(6-54)

$$y(t) = C_y x(t)$$

(6-55)

where $A, B_1, B_2, C_1, C_2, C_y, D_1$ are defined in (6-6), (6-7) and (6-8).

The output feedback H_{∞} controller can be defined by the following state space equation:

$$\hat{x}(t) = A_{\kappa}\hat{x}(t) + B_{\kappa}y(t)$$

$$(6-56)$$

$$u(t) = \lambda u_{d}(t) = \lambda C_{\kappa}\hat{x}(t) = (\lambda_{m} + N_{0}\bar{\lambda})C_{\kappa}\hat{x}(t)$$

$$(6-57)$$

where A_{κ} , B_{κ} , C_{κ} , λ , λ_m , and $\overline{\lambda}$ are defined in (6-9), (6-10) and (6-12).

Substituting (6-56), (6-57) into (6-52), (6-53) and (6-54), we can obtain the closed-loop system

$$\bar{x}(t) = A_{cl}\bar{x}(t) + B_{cl1}w(t)$$

$$\bar{z}_2(t) = C_{cl2}\bar{x}(t)$$
(6-58)

(6-59)

(6-61)

$$\bar{z}_1(t) = C_{cl1}\bar{x}(t) \tag{6-60}$$

$$u(t) = C_u\bar{x}(t)$$

where $\bar{x} = [x \ \hat{x}]^T$, $A_{cl} = \bar{A} + HN_0E$, $B_{cl1} = \bar{B}$, $C_{cl1} = \bar{C}_1 + D_{12}N_0E$, $C_{cl2} = \bar{C}_2 + D_{22}N_0E$. with

$$\bar{A} = \begin{bmatrix} A & B_2 \lambda_m C_{\kappa} \\ B_{\kappa} C_{y} & A_{\kappa} \end{bmatrix}, \bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, H = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 & \bar{\lambda} C_{\kappa} \end{bmatrix}.$$
$$\bar{C}_1 = \begin{bmatrix} C_1 & D_{12} \lambda_m C_{\kappa} \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} C_2 & D_{22} \lambda_m C_{\kappa} \end{bmatrix}, \ C_u = \begin{bmatrix} 0 & \lambda C_{\kappa} \end{bmatrix}.$$

Employing a similar method to that which is proposed in the previous Theorem 6.1, the following Theorem 6.2 is obtained for the active suspension system with only actuator faults.

Theorem 6.2 Given positive scalars γ and ρ , a dynamic output feedback controller in the form of (6-56) and (6-57) exists, such that the closed-loop system in (6-58), (6-59), (6-60) and (6-61) is asymptotically stable with w(t) = 0, and H_{∞} performance $\|\bar{z}_1\|_2 < \gamma \|w\|_2$ are guaranteed for all nonzero $w \in L_2[0, \infty)$, while the control output constraints in are guaranteed with the disturbance energy under the bound $w_{max} = \frac{\rho}{\gamma^2}$, if there exist symmetric matrices X > 0, Y > 0, positive scalars ε_1 , ε_2 , ε_3 , and $\hat{A}, \hat{B}, \hat{C}$ with

appropriate dimensions satisfying the following LMIs:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ * & -\gamma^{2}I & 0 & 0 & 0 \\ * & * & -I & \varepsilon_{1}D_{12} & 0 \\ * & * & * & -\varepsilon_{1}I & 0 \\ * & * & * & * & -\varepsilon_{1}I \end{bmatrix} < 0$$

$$\begin{bmatrix} -\mu_{max}^{2}X & -\mu_{max}^{2}I & \sqrt{\rho}\hat{C}^{T}\lambda_{m}^{T} & \hat{C}^{T}\bar{\lambda}_{1}^{T} \\ * & -\mu_{max}^{2}Y & 0 & 0 \\ * & * & -I + \varepsilon_{2}I & 0 \\ * & * & & -\varepsilon_{2}I \end{bmatrix} < 0$$

$$\begin{bmatrix} -\{z_{2max}^{2}\}_{q}X & -\{z_{2max}^{2}\}_{q}I & \sqrt{\rho}(X^{T}C_{2}^{T} + \hat{C}^{T}\lambda_{m}^{T}D_{22}^{T}) & 0 & \hat{C}^{T}\bar{\lambda}_{1}^{T} \\ * & -\{z_{2max}^{2}\}_{q}Y & \sqrt{\rho}C_{2}^{T} & 0 & 0 \\ * & * & -II & \varepsilon_{3}D_{22} & 0 \\ * & * & & -II & \varepsilon_{3}D_{22} & 0 \\ * & * & & & -II & \varepsilon_{3}D_{22} & 0 \\ * & * & & & & -\varepsilon_{3}I & 0 \\ * & & & & & & & -\varepsilon_{3}I \end{bmatrix} < 0$$

$$(6-64)$$

 $\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$

(6-65)

where
$$\Lambda_{11} = \begin{bmatrix} AX + XA^T + B_2\lambda_m\hat{C} + (B_2\lambda_m\hat{C})^T & \hat{A}^T + A \\ * & A^TY + Y^TA + \hat{B}C_y + (\hat{B}C_y)^T \end{bmatrix},$$

 $\Lambda_{12} = \begin{bmatrix} B_1 \\ Y^TB_1 \end{bmatrix}, \Lambda_{13} = \begin{bmatrix} (C_1X + D_{12}\lambda_m\hat{C})^T \\ C_1^T \end{bmatrix}, \Lambda_{14} = \begin{bmatrix} \varepsilon_1B_2 \\ \varepsilon_1Y^TB_2 \end{bmatrix}, \Lambda_{15} = \begin{bmatrix} \hat{C}^T\bar{\lambda}^T \\ 0 \end{bmatrix}.$

The Theorem 6.2can be easily proved following the proof of Theorem 6.1. In this case, the output feedback controller parameters are obtained

$$C_{\kappa} = \hat{C} (M^T)^{-1}$$

$$B_{\kappa} = N^{-1} \hat{B}$$
(6-66)

(6-67)

$$A_{\kappa} = N^{-1} [\hat{A} - Y^{T} A X] (M^{T})^{-1} - B_{\kappa} C_{y} X (M^{T})^{-1} - N^{-1} Y^{T} B_{2} \lambda_{m} C_{\kappa}$$
(6-68)

Substitute (6-66), (6-67), (6-68) into (6-56), (6-57), the active suspension control force can be obtained.

6.5 Simulation results

6.5.1 Parameter optimization results

Particle swarm optimization results for suspension and DVA and effectiveness of DVA are illustrated in this section. The parameters of the quarter car model with DVA and without DVA are listed in Table 6-1. Define four performance indexes $p_1 = 0.4$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.2$. The optimized parameters are listed as follows: $m_d = 30$ kg, $k_s = 42288$ N/m, $c_s = 1686$ Ns/m, $k_{a2} = 30542$ N/m, $c_{a2} = 1157$ Ns/m.

Random road excitation is used to demonstrate the effectiveness of DVA configuration. The class B road profile with constant vehicle speed of 40 km/h is used in this section. The "IWM-EV" represent the in-wheel motor driven electric vehicle without DVA, while the "DVA-EV" denote in-wheel motor driven electric vehicle without DVA. Table 6-2 shows the Root Mean Square (RMS) comparison of vehicle dynamic responses in terms of body acceleration, suspension stroke and tyre dynamic load under random road excitation. The DVA-EV has better performance than IWM-EV, which indicate that the DVA-EV structure has the potential to improve ride quality and road-holding performance.

Figure 6-1 shows the random response of sprung mass acceleration and tyre dynamic force, from which we can see that both sprung mass acceleration and tyre dynamic force of the DVA-EV are smaller than that of IWM-EV. Frequency response of the sprung mass acceleration and tyre deflection are shown in Figure 6-2, illustrating that the DVA-EV decreases the sprung mass acceleration around 10 Hz. Furthermore, the suspension deflection of DVA-EV is greatly reduced around 10 HZ compare to the IWM-EV, indicating that the DVA-EV performs better than the IWM-EV, especially in the range of unsprung mass resonance. As a result, this kind of IWM configuration has the ability of improving vehicle ride comfort performance and road holding ability.

Definition	Symbol	Without DVA	With DVA
Sprung mass	m_s	350 kg	350 kg
Mass of tyre, rim of wheel	m_u	50 kg	50 kg
Mass of motor	m_d	30kg	30kg
Stiffness of suspension	k _s	42288 N/m	42288 N/m
Damping of suspension	Cs	1686 Ns/m	1686 Ns/m
Stiffness of tyre	k _t	250000 N/m	250000 N/m
Suspended motor stiffness	k_{a2}	-	30542 N/m
Suspended motor damping	<i>C</i> _{<i>a</i>2}	-	1157 Ns/m

Table 6-1 Optimization results of vehicle suspension parameter.

Table 6-2 RMS comparison of vehicle dynamic responses.

Vehicle type	Body acceleration	Suspension stroke	Tyre dynamic force
IWM-EV	0.1806	0.9382	132.5611
DVA-EV	0.1570	0.8806	98.6614



Figure 6-1 Random response of sprung mass acceleration and tyre dynamic force.



Figure 6-2 Frequency response of sprung mass acceleration and tyre deflection.
6.5.2 Proposed control methods validation

In this section, bump road excitation and random road excitation are used to illustrate the effectiveness of the proposed controller. The quarter vehicle with in-wheel DVA model is used and parameters are listed in Table 6-1. We assume that the maximum suspension deflection $z_{max} = 50$ mm, the maximum control force $u_{max} = 2000$ N and the dynamic force applied on the bearings $F_{max} = 3000$ N. Output feedback controller I is the output feedback controller for the active suspension with only actuator faults while the output feedback controller II is the output feedback controller II is the output feedback controller for the active suspension with actuator faults and time delay.

The dynamic output-feedback controller I for the active suspension systems in (6-58), (6-59), (6-60) and (6-61) with only control faults can be derived by using Theorem 6.2. In addition, we can obtain the minimum guaranteed closed-loop H_{∞} performance index γ is 4.52.

Secondly, we propose the dynamic output-feedback controller for the active suspension systems in (6-13), (6-14) and (6-15) with control delay and faults. In this study, $\rho = 1$, $\theta_R = 1$, upper bounds \bar{d} of actuator delay d(t) is $\bar{d} = 5ms$. We give the dynamic output-feedback controller for the upper bound, and it can be found that the minimum guaranteed closed-loop H_{∞} performance index γ is 6.36.

$$A_{c} = 1 \times 10^{4} \begin{bmatrix} -0.0037 & -0.0002 & 0 & 0 & 0 & 0 \\ -0.0371 & -0.0059 & 0 & 0 & 0 & 0 \\ 0.0919 & -0.0208 & -0.0178 & 0 & 0 & 0 \\ 1.0097 & -0.0488 & 0.0002 & -0.0178 & 0 & 0 \\ -2.0433 & -0.2440 & -0.0005 & -0.0001 & -0.0179 & -0.0001 \\ -4.0055 & -1.0079 & -0.0073 & -0.0052 & -0.0012 & -0.0192 \end{bmatrix},$$

$$A_{d} = \begin{bmatrix} 0.0243 & 0.0021 & 0.0091 & -0.0023 & 0.0003 & -0.0001 \\ -7.6245 & -0.6448 & -2.8443 & 0.7191 & -0.0818 & 0.0253 \\ -17.1847 & -1.4625 & -6.4091 & 1.6194 & -0.1847 & 0.0569 \\ 39.2780 & 3.3321 & 14.6446 & -3.6969 & 0.4227 & -0.1301 \\ 169.6451 & 14.3083 & 63.2485 & -15.9698 & 1.7480 & -0.5525 \\ 154.0348 & 13.0703 & 57.4290 & -14.4991 & 1.6982 & -0.4929 \end{bmatrix}$$

$$B_{c} = 1 \times 10^{6} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0006 & 0 & 0.0004 & 0 & -0.0035 & 0 \\ 0.0015 & -0.0027 & -0.0003 & 0 & 0.0028 & 0 \\ -0.0022 & 0.0006 & -0.0124 & 0 & 0.0148 & 0 \\ 0.2548 & 0.0166 & 0.0137 & 0 & -0.0472 & 0 \\ -0.2908 & -0.0027 & 0.0117 & 0 & -1.2374 & 0 \end{bmatrix},$$

 $C_c = [-971.85 - 82.16 - 362.48 91.64 - 10.43 3.22].$

6.5.2.1 Bump road excitation

The following bump road profile is used to validate effectiveness of proposed:

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi v_0}{l}t\right) \right), & 0 \le t \le \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases}$$
(6-69)

where *a* is the height of the bump and *l* is the length of the bump. Here we choose a = 0.1m, and l = 2m, and the vehicle forward velocity of $v_0 = 30$ km/h.

Figure 6-3 and Figure 6-4 show the bump responses of the active suspension with the fault-tolerant output feedback controller and passive suspension with 30% and 60% actuator thrust loss. In Figure 6-3, with 30% actuator thrust loss, we can see that the body acceleration of the active suspension is much smaller than that of the passive suspension. In addition, the suspension deflection, tyre dynamic force and maximum control force constraints are guaranteed simultaneously. As the actuator thrust loss increases, the active suspension still performs better than the passive suspension. The simulation results show that the proposed output feedback controller provides better performance than the passive suspension with the partial faults in the actuator. Figure 6-5 shows the active forces with 30% and 60% actuator thrust loss. The simulation results show that the proposed output feedback controller performance than the passive suspension with the partial faults in the actuator. Figure 6-5 shows the active forces with and 60% actuator thrust loss. The simulation results show that the proposed output feedback controller performance than the passive suspension with the partial faults in the actuator. Figure 6-5 shows the active forces with and 60% actuator thrust loss. The simulation results show that the proposed output feedback controller performance than the passive suspension with the partial faults in the actuator.



Figure 6-3 Bump response of active suspension with 30% actuator thrust loss



Figure 6-4 Bump response of active suspension with 60% actuator thrust loss.



Figure 6-5 Active force. (a) 30% actuator thrust loss. (b) 60% actuator thrust loss.

Figure 6-6 and Figure 6-7 show the bump response of active suspension with two different output feedback controllers with 30% and 60% actuator thrust loss. Figure 6-8 show the active forces of two output feedback controllers with 30% and 60% actuator thrust loss. As observed in these diagrams, the proposed dynamic output feedback controller II achieves the better suspension and motor performance than those of the passive suspension and output feedback controller I when actuator fault and delay occur. This clearly demonstrates the effectiveness of the output feedback controller II in improving the ride performance with control delay. Moreover, with actuator time delay, output feedback controller I achieves worst performance among three types of suspensions. This is because the output feedback controller I become instable when actuator delay occurs in the system. With an increase in the actuator thrust loss, the output feedback controller II reveals better performance than the output feedback controller I and passive suspension, which shows that the proposed output feedback controller II can guarantee a better performance in spite of actuator faults and time delay under bump road excitation.



Figure 6-6 Bump response of active suspensions with 30% actuator thrust loss and 5ms time delay.



Figure 6-7 Bump response of active suspensions with 60% actuator thrust loss and 5ms time delay.



Figure 6-8 Active force. (a) 30% actuator thrust loss. (b) 60% actuator thrust loss.

6.5.2.2 Random road excitation

The random road excitation is used to demonstrate the effectiveness of the proposed control system. The power spectral density (PSD) of the random road excitation can be expressed by the following equation:

$$G_q(n) = G_q(n_0)(\frac{n}{n_0})^{-\omega}$$
(6-70)

The PSD of the road velocity can be expressed by the following equation:

$$G_{\dot{q}}(n) = (2\pi n)^2 G_q(n)$$

(6-71)

The random road excitation is determined by feeding a white noise through a linear firstorder filter:

$$\dot{x}_{g} + 2\pi f_{0} x_{g} = 2\pi \sqrt{G_{q}(n_{0})v} w(t)$$
(6-72)

where *n* is the spatial frequency and n_0 is the reference spatial frequency. $G_q(n_0)$ is PSD for the reference spatial frequency. ω is frequency index, usually $\omega = 2$. *v* is the vehicle speed, and w(t) stands for the white noise disturbance of the road. The class B road profile with constant vehicle speed of 30 km/h is used to test the system. The RMS comparison of the vehicle dynamic response under random road excitation is shown in Table 6-3. The active suspension with the fault-tolerant controller achieves marginally better suspension performance than the passive suspension in the presence of 30% and 60% actuator thrust loss. Suspension deflection, actuator force maximum force applied to the motor bearing and tyre dynamic force are guaranteed simultaneously. The RMS comparison of the passive suspension, active suspensions with output feedback controller I and output feedback controller II with different actuator faults and delays under random road excitation are shown in Table 6-4. The performance of output feedback controller I worsen with the actuator delay. The active suspension with the output feedback controller II reveals much better performance than that with output feedback controller I and passive suspension; which shows that output feedback controller II can guarantee a better performance in spite of actuator faults and delays under random road excitation.

Suspension types	Body acceleration (m/s ²)	Suspension deflection (m)	Relative dynamic tyre load	Motor dynamic force (N)	Active force (N)
Passive	0.1617	8.9964e-4	0.0263	75.0124	
Controller I with 30% actuator thrust loss	0.1179	7.2677e-04	0.0270	67.8389	22.5692
Controller I with 60% actuator thrust loss	0.1294	7.1433e-04	0.0265	69.2669	16.5594

Table 6-3 The RMS comparison of vehicle dynamic responses under random road excitation

Table 6-4 The RMS comparison of vehicle dynamic responses with different actuator faults and delays under random road excitation

Suspension types	Body acceleration (m/s ²)	Suspension deflection (m)	Relative dynamic tyre load	Motor dynamic force (N)	Active force (N)
Passive	0.1617	8.9964e-04	0.0282	75.0124	
Controller I with 30% actuator thrust loss	0.2539	0.0016	0.0312	174.7807	102.1168
Controller II with 30% actuator thrust loss	0.1301	7.0113e-04	0.0293	68.5019	21.1690
Controller I with 60% actuator thrust loss	0.2204	0.0013	0.0299	114.5852	53.0063
Controller II with 60% actuator thrust loss	0.1369	7.2105e-04	0.0285	69.3987	15.2151

6.6 Summary

In this chapter, the problem of output feedback H_{∞} control for active suspensions deployed in in-wheel motor driven electric vehicles with actuator faults and time delay was investigated. A quarter car active suspension with DVA was established and it was demonstrated to improve ride performance and road holding ability around 10Hz. Parameters of the vehicle suspension and DVA were optimized based on the PSO. In

order to achieve a better ride comfort and to reduce the force applied on the in-wheel motor bearing, a robust H_{∞} dynamic output feedback controller was derived such that the close-loop system was asymptotic stability and simultaneously satisfied the constraint performances such as road holding, suspension stroke, dynamic load applied to the bearings and actuator limitation. Finally, the simulation results demonstrated that the proposed controller offered better suspension performance in spite of actuator faults and time delay. Meanwhile, the proposed fault-tolerant output feedback H_{∞} controller achieved a significantly better vehicle and motor performance than those of the passive suspension for different actuator thrust losses. With different actuator thrust losses and time delay, the proposed output feedback controller II revealed much better performance than the output feedback controller I and the passive suspension.

7. Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort and active suspension control

7.1 Introduction

A novel approach considered for deployment in electric vehicles is the IWM as it provides faster motor response, generates precise torque, and can produce both forward and reverse torques [5]. A variety of motors, such as induction motors, permanent magnet synchronous motors and switched reluctance motors, can be deployed in electric vehicles. The SRM have inherent advantages, such as simplicity, low cost [195], robustness and high efficiency, making them a preferred option for deployment in electric vehicles [196]. However, the air-gap eccentricity present in the IWM can result in a residual unbalanced radial force, which can adversely influence the motor vibration, passenger comfort and vehicle rollover stability.

In the analysis of ride comfort and rollover characteristics, it was assumed that the magnet gap eccentricity remains constant [97, 98]. The in-wheel motor magnet gap is affected by the road excitation but there is few study reported in the literature on the coupling effect between road excitation, eccentricity and unbalanced vertical forces. In reality, it is observed that road excitation could result in SRM stator and rotor vibration, which will aggravate the air-gap eccentricity. The SRM air-gap eccentricity induces a residual unbalanced radial force that is a major drawback associated with SRM vibration. The SRM unbalanced vertical force is directly applied to the wheels, undermining the comfort of the electric vehicle ride. In this chapter, the coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort is numerically analysed. By conducting numerical simulations based on the developed quarter-car active suspension model and SRM model, it is shown that the road roughness is highly coupled with SRM air-gap eccentricity and unbalanced residual vertical force. The SRM air-gap eccentricity is influenced by the road excitation and becomes time-varying such that a residual unbalanced radial force is induced; which is one of the major causes of SRM vibration.

An active suspension system could significantly improve vehicle ride comfort, handling stability and safety in a conventional vehicle [157, 197, 198]. It can produce controllable forces to suppress the vehicle vibration. In IWM electric vehicle, wheel motion controls

should be given more attention as individual wheel motors can lead to a larger unsprung mass. Based on the previous literature review, a combined control system consisting of active suspension control and SRM control for in-wheel motor driven electric vehicle has not been investigated. In this chapter, In order to supress SRM vibration and to prolong the SRM lifespan, while at the same time improving vehicle ride comfort, an output feedback controller is designed to reduce both the sprung mass acceleration and the motor air-gap eccentricity. Moreover, an SRM controller is adapted by using the combined CCC and PWM to further improve the SRM performance. A comparison of passive suspension and suspensions with different control methods on vehicle and SRM dynamic response is illustrated.

Active suspension control for in-wheel SRM driven electric vehicle with DVA based on robust H_{∞} control method is also presented in this chapter. Structures with suspended shaft-less direct drive motors have the potential to improve the road holding capability and ride performance. The quarter car active suspension model equipped with in-wheel SRM is established, in which the SRM stator serves as a dynamic vibration absorber. The SRM air-gap eccentricity is influenced by the road excitation and becomes time-varying such that a residual unbalanced radial force is induced. This is one of the major causes of SRM vibration. CCC and PWM are adapted to suppress motor vibration. Moreover, a state feedback H_{∞} controller is developed for the active suspension with DVA to further enhance vehicle ride performance. A comparison of passive suspension with conventional SRM, passive suspension with DVA, active suspension with DVA on vehicle suspension and SRM dynamic responses are presented. Simulation results under bump road excitation and random road excitation demonstrate the effectiveness of DVA structure active suspension system with proposed control method in enhancing suspension and motor performance.

The remainder of this chapter is structured as follows. In Section 7.2, the development of the conventional quarter car vehicle model, active suspension model with DVA structure, tyre model and SRM model is explained. In Section 7.3, effects of road type and SRM on vehicle ride comfort characteristic are analysed. A hybrid control method based on output feedback H_{∞} control and SRM control for conventional SRM-EV and corresponding simulation results are illustrated in Section 7.4. In section 7.5, a state feedback H_{∞} control ler is proposed for active suspension with DVA, Combined control methods of CCC and PWM are used to improve SRM performance. Simulation results under random

and bump road excitation are presented. Finally, conclusions are drawn in Section 7.6 and future work is discussed.

7.2 In-wheel motor driven electric vehicle modelling

The in-wheel SRM driven electric vehicle consists of three sub-models: a quarter-car model used to investigate the effect of in-wheel SRM on the sprung mass acceleration, suspension deflection, and tyre deflection, the longitudinal dynamic model that provides the longitudinal force to calculate the vehicle speed, and the SRM model that provides the desired torque for the electric vehicle. As shown in Figure 7-1, the in-wheel SRM model influence both the vehicle vertical dynamics and longitudinal dynamics.

7.2.1 Vehicle modelling

A conventional vehicle with passive suspension and an electric vehicle driven by in-wheel SRM with active suspension are developed in this paper, as shown in Figure 7-2. The quarter vehicle passive suspension model has two degrees of freedom associated with the vertical motion of the sprung mass and the unsprung mass. Based on Newton's Second Law, the motion equations of passive suspension system can be written as

$$m_{s1}\ddot{x}_{s1} = k_{s1}(x_{u1} - x_{s1}) + c_{s1}(\dot{x}_{u1} - \dot{x}_{s1})$$
(7-1)

$$m_{u1}\ddot{x}_{u1} = k_{t1}(x_{g1} - x_{u1}) - k_{s1}(x_{u1} - x_{d1}) - c_{s1}(\dot{x}_{u1} - \dot{x}_{s1})$$
(7-2)

In order to analyse the effect of SRM on vehicle ride comfort characteristics, quarter car passive suspension models with conventional SRM and active suspension model with DVA structure are introduced, as shown in Figure 7-2. Firstly, the quarter-car passive suspension model with three degrees of freedom of vertical motion of unsprung mass, SRM stator, and sprung mass is established. The SRM stator is fastened to the shaft, while the SRM rotor is fastened to the hub. The SRM rotor and wheel hub are installed on the shaft through SRM bearing and hub bearing. Therefore, the vehicle wheel mass can be divided into two parts: the total mass of the tyre, the rim and the SRM rotor, denoted by m_u , and the mass of SRM stator and housing, denoted by m_d . They are linked with each other through SRM bearing and hub bearing, of which the stiffness is denote by k_d . The air-gap between the SRM rotor and stator are produced due to the rotor eccentricity, which will generate an unbalanced radical force. The vertical component of unbalanced

vertical force directly act on the masses m_u and m_d , which is denoted by f_d . The air-gap eccentricity is not constant, which is effected by road excitations in the driving conditions. Based on the Newton's Second Law, the motion equations of this passive suspension can be written as

$$m_{s}\ddot{x}_{s} = k_{s}(x_{d} - x_{s}) + c_{s}(\dot{x}_{d} - \dot{x}_{s})$$

$$m_{d}\ddot{x}_{d} = -k_{s}(x_{d} - x_{s}) - c_{s}(\dot{x}_{d} - \dot{x}_{s}) + k_{d}(x_{u} - x_{d}) + f_{d}$$

$$m_{u}\ddot{x}_{u} = k_{t}(x_{g} - x_{u}) - k_{d}(x_{u} - x_{d}) - f_{d}$$
(7-3)
(7-3)
(7-4)

(7-5)

(7-9)

The quarter car active suspension with a DVA attached to unsprung mass through a spring and a damper is developed with the in-wheel SRM serving as the DVA. This system is called "advanced-dynamic-damper-motor". It is demonstrated that this kind of IWM configuration has the ability of improving vehicle ride comfort performance. Based on the Newton's Second Law, the motion equations of this active suspension can be written as

$$m_{s}\ddot{x}_{s} = k_{s2}(x_{u2} - x_{s}) + c_{s2}(\dot{x}_{u2} - \dot{x}_{s}) + f_{a}$$

$$(7-6)$$

$$m_{d2}\ddot{x}_{d2} = k_{a2}(x_{u2} - x_{d2}) + c_{a2}(\dot{x}_{u2} - \dot{x}_{d2}) - k_{r}(x_{d2} - x_{r}) + f_{r}$$

$$(7-7)$$

$$m_r \ddot{x}_r = k_r (x_{d2} - x_r) - f_r$$
(7-8)

$$m_{u2}\ddot{x}_{u2} = k_{t2}(x_g - x_{u2}) - k_{s2}(x_{u2} - x_s) - c_{s2}(\dot{x}_{u2} - \dot{x}_s) - k_{a2}(x_{u2} - x_{d2}) - c_{a2}(\dot{x}_{u2} - \dot{x}_{d2}) - f_r$$

where x_{s1} , x_{u1} and x_{g1} denote the vertical displacements of the sprung mass, the unsprung mass and road disturbance. x_s , x_u and x_d denote the vertical displacements of the sprung mass, unsprung mass, motor stator mass of quarter-car with conventional SRM structure. x_{u2} , x_{d2} and x_r denote the vertical displacement of unsprung mass, motor stator and motor rotor of quarter-car with DVA. Road disturbance is denoted by x_g . The value of each parameter is presented in Table 7-1.



Figure 7-1 In-wheel SRM driven electric vehicle suspension model



- c_{s2} : Suspension damping f_d : Unbalanced vertical force k_{t2}: Tyre stiffness
 - k_t : Tyre stiffness
- k_r : Bearing stiffness f_r : Unbalanced vertical force
- k_{a2} : Suspended motor stiffness

 c_{a2} : Suspended motor damping

Figure 7-2 Quarter vehicle suspension model. (1) Electric vehicle with DVA-SRM. (2) Electric vehicle with conventional in-wheel SRM. (3) Conventional vehicle.

7.2.2 Tyre modelling

The Magic-Formula (MF) tyre model proposed by Pacejka [199] is used in this work to model tyre-road interaction forces and moments in vehicle dynamics. The tyre parameters used in this paper are presented in Table 7-1. The input for the MF tyre model consists of the wheel load (F_z) , and the longitudinal slip κ . The output is the longitudinal force F_x that is the contact point between the tyre and the road, as shown in Figure 7-3.

The longitudinal tire force depends on the slip ratio, normal load on the tire and friction coefficient of the tire-road interface. The longitudinal tire force equation can be described as:

$$F_{x} = D_{x} \sin[C_{x} \arctan\{B_{x}\kappa_{x} - E_{x}(B_{x}\kappa_{x} - \arctan(B_{x}\kappa_{x}))\}] + SV_{x}$$
(7-10)

where B_x , C_x , D_x , E_x are empirical parameters described as

$$C_x = a_0 \tag{7-11}$$

$$D_x = a_1 F_z^2 + a_2 F_z \tag{7-12}$$

$$B_{\chi} = \frac{(a_3 F_z^2 + a_4 F_z) e^{-a_5 F_z}}{C_{\chi} D_{\chi}}$$
(7-13)

$$E_x = a_6 F_z^2 + a_7 F_z + a_8 aga{7-14}$$

Longitudinal slip can be defined as:

$$\kappa_x = \frac{\omega R_e - \nu}{\omega R_e} \tag{7-15}$$

7.2.3 Longitudinal dynamic model

Consider a vehicle moving on a flat road, the external longitudinal forces acting on the vehicle include aerodynamic drag forces, longitudinal tire forces and rolling resistance forces. A force balance along the vehicle longitudinal axis yields

$$m_t \dot{v} = F_x - R_x - F_{aero} \tag{7-16}$$

where v is the longitudinal vehicle velocity. F_x is longitudinal tire force, F_{aero} and R_x are the equivalent longitudinal aerodynamic drag force and rolling resistance force, which are represented as:

$$F_{areo} = \frac{1}{2} \rho C_d A_f v^2$$

$$R_x = \mu F_z$$
(7-17)

$$F_z = m_t g - k_t (x_g - x_u)$$
(7-19)

(7-18)

where ρ is the mass density of air, C_d is the aerodynamic drag coefficient, A_f is the frontal area of the vehicle. μ is the rolling resistance coefficient.

The driving equation of the wheel can be written as

$$I_{\omega}\dot{\omega} = T_t - F_x R_e \tag{7-20}$$

where I_{ω} is the rotational inertia of the wheel, ω is the wheel angular speed, T_t is SRM torque, which will discuss later. R_e is the radius of the wheel. The longitudinal dynamic model of the quarter car is presented in Figure 7-3. The typical variation of longitudinal tire force as a function of the slip ratio is shown in Figure 7-4. As shown, the longitudinal tyre force is found to be proportional to the slip ratio during normal driving.



Figure 7-3 Longitudinal dynamic model.



Table 7-1 Vehicle suspension parameter values

Conventional vehicle symbol	Value	Electric vehicle symbol	Value	Tyre symbol	Value
m_{s1}	337.5 kg	m _s	337.5 kg	a_0	1.65
m_{u1}	50 kg	m _u	50 kg	a_1	-67N ⁻¹
k_{s1}	22500 N/m	m_d	35 kg	a_2	1098
c_{s1}	1450 Ns/m	I_{ω}	1.2 Kgm ²	a_3	6 N/%/N ²
k_{t1}	250000 N/m	R_e	0.269 m	a_4	176 N/%
		k_s	22500 N/m	a_5	0.0886 N ⁻¹
		C _s	1450 Ns/m	a_6	-0.0615
		k _d	7000000 N/m	a_7	-0.0615
		k_t	250000 N/m	a_8	0.675

7.2.4 Switched reluctance motor modelling

In this work, an 8/6 four phase in-wheel SRM with exterior rotor is used, as shown in Figure 7-5. The SRM drive system includes the SRM, the converter and the associated control system.

7.2.4.1 Electromagnetic equation

We use the Fourier series method to obtain the flux linkage and inductance [200, 201]. Three points on the inductance-current profile are used. These are inductance at the aligned position, unaligned position, and a point in the middle of the two. The inductance $L(\theta, i)$ corresponding to rotor angular position and winding current by means of the Fourier series can be written as

$$L(\theta, i) = L_0(i) + L_1(i)\cos(N_r\theta + \varphi_1) + L_2(i)\cos(2N_r\theta + \varphi_2)$$
(7-21)

where N_r is the number of rotor poles.

Three coefficients L_0 , L_1 and L_2 can be derived as a function of the aligned position inductance L_a , the unaligned position inductance L_u and the inductance at the midway from the aligned position L_m

$$L_{0}(i) = \frac{1}{2} \left[\frac{1}{2} (L_{a} + L_{u}) + L_{m} \right]$$

$$L_{1}(i) = \frac{1}{2} (L_{a} - L_{u})$$
(7-23)

$$L_2(i) = \frac{1}{2} \left[\frac{1}{2} (L_a + L_u) - L_m \right]$$
(7-24)

where

$$L_{a}(i) = L(\theta = 0^{\circ}) = \sum_{n=0}^{N} a_{n} i^{n}$$
(7-25)

$$L_m(i) = L\left(\theta = \frac{\pi}{2N_r}\right) = \sum_{n=0}^N b_n i^n$$

$$L_u(i) = L\left(\theta = \frac{\pi}{N_r}\right)$$
(7-26)

(7-27)

where a_n and b_n are the polynomial fitting coefficients.

Then the inductance of phase k can be derived from

$$L_{k}(\theta, i_{k}) = \frac{1}{2} [\cos^{2}(N_{r}\theta) - \cos(N_{r}\theta)] \sum_{n=0}^{N} a_{n}i_{k}^{n} + \sin^{2}(N_{r}\theta) \sum_{n=0}^{N} b_{n}i_{k}^{n} + \frac{1}{2} L_{u} [\cos^{2}(N_{r}\theta) + \cos(N_{r}\theta)]$$
(7-28)

Since the inductance is the partial differential of flux linkage to current, the flux linkage and torque of phase k are presented as the following equations

$$\psi_{k}(\theta, i_{k}) = \int_{0}^{i_{k}} L_{k}(\theta, i_{k}) di_{k}$$

$$(7-29)$$

$$T_{k} = \int_{0}^{i_{k}} \frac{\partial \psi(\theta, i_{k})}{\partial \theta} di_{k}$$

(7-30)

The total torque of the SRM is

$$T_t = \sum_{k=1}^4 T_k \tag{7-31}$$

7.2.4.2 Voltage equation

Base on Faraday's law, the voltage across the terminals of one phase winding can be written as

$$U_{k} = R_{k}i_{k} + \frac{d\psi}{dt} = R_{k}i_{k} + \frac{\partial\psi_{k}}{\partial i_{k}}\frac{di_{k}}{dt} + \frac{\partial\psi_{k}}{\partial\theta}\frac{d\theta}{dt} = R_{k}i_{k} + L_{k}(\theta, i_{k})\frac{di_{k}}{dt} + \frac{\partial\psi_{k}(\theta, i_{k})}{\partial\theta}\omega$$
(7-32)

where R_k is the phase winding resistance, i_k is the current of phase k. Then the phase current is

$$i_{k} = \int \frac{U_{k} - R_{k}i_{k} - \frac{\partial \Psi_{k}(\theta, i_{k})}{\partial \theta}}{L_{k}(\theta, i_{k})} dt$$

(7-33)

7.2.4.3 Unbalanced residual radial force

In the magnetic circuit of the SRM, the rotating part must assume a minimum reluctance position at the instance of excitation. The radial electromagnetic forces are produced to generate the reluctance torque. The radial force between stator and rotor poles of phase k [97] can be simplified as

$$F_{k} = -\frac{1}{2} i_{k}^{2} \frac{L_{k}(\theta, i_{k})}{l_{g}}$$
(7-34)

The geometric centre of the rotor should coincide with that of the stator core for the ideal operating condition. However, there is relative airgap eccentricity during actual working conditions. The relative eccentricity is defined as

$$\varepsilon = \frac{n_e}{l_g} \tag{7-35}$$

where l_g is the radial airgap length and h_e is the airgap eccentricity in the radial direction. Because of the airgap eccentricity, the radial forces between stator and rotor poles in the opposite direction cannot balance each other. As a result, an unbalanced residual radial force is produced. The unbalanced vertical force is directly applied to the vehicle body and wheels and has an adverse effect on the vehicle vertical vibration and ride comfort. The unbalanced residual radial forces in the opposite poles can be written as

$$F_{d} = -\frac{1}{2}i_{k}^{2}\frac{L_{k}(\theta,i_{k})}{l_{g}+h_{e}} + \frac{1}{2}i_{k}^{2}\frac{L_{k}(\theta,i_{k})}{l_{g}}$$
(7-36)

The vertical component of the unbalanced radial force is the SRM vertical force, and the total unbalanced vertical force is obtained by

$$f_d = \sum_{k=1}^4 F_d \cos\theta \tag{7-37}$$

Hence, the unbalanced vertical force is directly influenced by the airgap eccentricity. If h_e is zero, the unbalanced vertical force becomes zero. Since the airgap is small, only a small airgap eccentricity could induce large unbalanced vertical forces. The airgap eccentricity is highly coupled with the road excitation. Road types and vehicle speed have an influence on the SRM airgap eccentricity, which could result in an unbalanced vertical force.



Figure 7-5 Structure of 8/6 four phase SRM and SRM vertical force.

7.3 Effect of road condition on vehicle and SRM performance

The road surface roughness is coupled with SRM air-gap eccentricity and the unbalanced residual vertical force. Different road conditions result in different degrees of SRM stator and rotor vibration that will aggravate the motor and vehicle vibration to some extent. Therefore, it is necessary to analyse the effect of different road conditions as well the coupling effect between the road and the SRM on vehicle ride performance. The road surface and vehicle speed are two important factors in vehicle ride comfort analysis. Therefore, the effect of road surface and vehicle speed on vehicle ride performance is investigated in this section. The vehicle parameter values used in this section are presented in Table 7-1.

7.3.1 Stochastic road modelling

The stochastic road surface is a commonly used road excitation that affects vehicle vertical vibration dynamics. In order to analyse the coupling effect of road excitation and SRM on an electric vehicle's dynamic response under stochastic road excitation, the stochastic road surface is modelled and included. The road surface roughness is determined by feeding a white noise through a linear first-order filter:

$$\dot{z_g} + 2\pi f_0 z_g = 2\pi \sqrt{G_q(n_0)} v w$$
(7-38)

The PSD of the stochastic road excitation can be expressed as the following equation:

$$G_q(n) = G_q(n_0) (\frac{n}{n_0})^{-\omega}$$
(7-39)

where *n* is the spatial frequency and n_0 is the reference spatial frequency. $G_q(n_0)$ is PSD for the reference spatial frequency, determined by road class. ω is frequency index, usually $\omega = 2$. *v* is the vehicle speed, and *w* stands for the white noise disturbance of the road. Figure 7-6 illustrates the PSDs of stochastic roads for different road surface types and different forward velocities, which are utilised to analyse the effect on vehicle and SRM dynamic responses.



Figure 7-6 PSD of road. (a) Different road types. (b) Different forward velocities.

7.3.2 Factor of road surface type

In order to analyse the effect of different road surface types on vehicle vibration characteristics and SRM dynamic responses, three road grades (Class A, Class B and Class C) are established in this section, as shown in Figure 7-6-(a). Road roughness coefficients $are16 \times 10^{-6}$, 64×10^{-6} and 256×10^{-6} m³, respectively. Vehicle speed is kept as 60 km/h during the simulation. SRM airgap eccentricity and unbalanced vertical force are influenced mainly by the roughness of the road. The airgap eccentricity is not constant because it is affected by the road excitation. Moreover, the unbalanced residual radial force increases sharply with the airgap eccentricity. That is, the SRM eccentricity has a significant effect on the unbalanced residual radial force. Since the airgap is small, a small change of eccentricity will cause a significant change of unbalanced residual radial force.

The airgap eccentricity and SRM vertical force responses to three different road surface types in both time domain and frequency domain are shown in Figure 7-7 and Figure 7-8, respectively. It can be seen from the time plots that the SRM airgap eccentricity and vertical force are greatly increased when the road grade is increased from A to C. The maximum values of SRM airgap eccentricity and vertical force under Class C random road excitation are about 2×10^{-4} m and 2000 N, while the maximum values of SRM airgap eccentricity and vertical force under Class B random road excitation are about 1×10^{-4} m and 2000 N, while the maximum values of SRM airgap eccentricity and vertical force under Class B random road excitation are about 1×10^{-4} m and 800 N. When it comes to Class A, the maximum values of SRM airgap eccentricity and vertical force are 8×10^{-5} m and 500 N, respectively. The natural frequencies of sprung mass, unsprung mass and SRM stator are about 1.5 Hz, 10Hz and 100 Hz, respectively. The PSD of airgap eccentricity and vertical force are also increased when the road profile is increased from A to C at the resonance frequencies of sprung mass, unsprung mass and SRM stator, which indicate that the road surface has a

significant negative impact on the SRM airgap eccentricity and unbalanced vertical force. Figure 7-9 and Figure 7-10 illustrate the sprung mass acceleration and vehicle tyre deflection and responses to stochastic roads for different road grades. As observed in the diagrams, both of sprung mass acceleration tyre and vehicle tyre deflection increase with the road grade in both time domain and frequency domain, indicating that the road grade has an adverse effect on the vehicle ride comfort characteristics.



Figure 7-7 Airgap eccentricity responses to different road types. (a): Time domain. (b): Frequency domain.



Figure 7-8 SRM vertical force responses to different road types. (a): Time domain. (b): Frequency domain.



Figure 7-9 Body acceleration responses to different road types. (a): Time domain. (b): Frequency domain.



Figure 7-10 Tyre deflection responses to different road types. (a): Time domain. (b): Frequency domain.

7.3.3 Factor of vehicle speed

To analyse the effect of vehicle speed on SRM and vehicle dynamic responses, three vehicle speeds are set to 40 km/h, 60 km/h and 100 km/h, as shown in Figure 7-6-(b). Class C road in which road roughness coefficient is 64×10^{-6} m3 is used. Accordingly, SRM airgap eccentricity and unbalanced vertical force of different vehicle speeds can be obtained, as shown in Figure 7-11and Figure 7-12. As shown in the diagrams, the SRM airgap eccentricity and unbalanced vertical force increase with an increase in vehicle speed. The maximum vertical force can reach up to 1000 N when the vehicle speed is 100 km/h. The PSD of SRM airgap eccentricity and unbalance vertical force are increased when the vehicle speed increase from 40 km/h to 100 km/h. The results indicate that the vehicle speed has great effect on SMR dynamic response. The motor performance deteriorates when the vehicle speed increases.

Figure 7-13 and Figure 7-14 show the vehicle dynamic responses for different vehicle speeds in both time domain and frequency domain. As shown in the diagrams, the vehicle sprung mass acceleration and tyre deflection are slightly increased with an increase in the vehicle speed. The PSD of vehicle body acceleration and tyre deflection with 100 km/h vehicle speed are larger when compared to those with 40 km/h at nearly 1.5 Hz and 10 Hz. This is because the sprung mass resonance frequency and unsprung mass resonance frequency are about 1.5 Hz and 10 Hz, respectively. Moreover, the increased vehicle speed results in an increase of road excitation. The vehicle speed has adverse impact on vehicle ride performance and road holding stability.



Figure 7-11 Airgap eccentricity responses to different vehicle speeds. (a): Time domain. (b): Frequency domain.



Figure 7-12 Vertical force responses to different vehicle speeds. (a): Time domain. (b): Frequency domain.



Figure 7-13 Body acceleration responses to different vehicle speeds. (a): Time domain. (b): Frequency domain.



Figure 7-14 Tyre deflection responses to different vehicle speeds. (a): Time domain. (b): Frequency domain.

7.3.4 Road and SRM coupling effect on vehicle dynamic responses

To analyse the coupling effect of the SRM and road surface on vehicle dynamic response, two types of vehicle such as conventional vehicle and electric vehicle with SRM (see Figure 7-2) with different road types are compared. Thus, four scenarios such as Class C without SRM, Class B without SRM, Class C with SRM and Class B with SRM (as shown in Figure 7-15) are considered. Vehicle speed is set at 60 km/h. PSDs of vehicle dynamic responses are obtained in terms of sprung mass acceleration and tyre deflection (Figure 7-15). As illustrated, an increase in the road grade can have a significant adverse effect on vehicle sprung mass acceleration and tyre deflection than a conventional vehicle without SRM at the resonance frequency of unsprung mass. It is because the electric vehicle with SRM has larger unsprung mass than that of conventional vehicle. The results indicate that the coupling of road grade and SRM not only affects vehicle ride comfort but also influences vehicle handling stability.

Discussion now focusses on these two types of vehicles (conventional vehicle and electric vehicle, as shown in Figure 7-2) with different vehicle speeds to compare the effect of vehicle speed and SRM coupling on vehicle vibration. Four cases, 80 km/h without SRM, 40 km/h without SRM, 80 km/h with SRM and 40 km/h with SRM, are used (as shown in Figure 7-16), and Class B road profile is used in four cases. For the four cases, vehicle dynamic responses in frequency domain are presented as shown in Figure 7-16. An electric vehicle with SRM has larger sprung mass acceleration and tyre dynamic force than those of an electric vehicle without SRM at the resonance frequency of unsprung mass, while sprung mass acceleration and tyre dynamic force of electric vehicle with SRM are much similar with those of without SRM at the resonance frequency of sprung

mass, which show that the increased unsprung mass greatly deteriorates the suspension ride comfort performance. When the bearing stiffer is very small, increasing the stiffness can reduce the body acceleration. In practice, the bearing stiffness should be large enough to support the heavy load. The bearing stiffness is 7000000 N/m in this thesis, which is very large. It has hardly no effect on the body acceleration if we increase the bearing stiffness. The body acceleration can be reduced effectively by utilizing the active suspension system.



Figure 7-15 PSDs of vehicle dynamic responses to stochastic road. (a): Body acceleration. (b): Tyre deflection.



Figure 7-16 PSDs of vehicle dynamic responses to stochastic road. (a): Body acceleration. (b): Tyre deflection.

7.4 Controller design of SRM driven EV

In the previous section, the effects of road condition on vehicle ride performance were discussed. To reduce the SRM airgap eccentricity and suppress the vibration of the SRM, an output feedback H_{∞} controller is proposed in this section for in-wheel motor driven electric vehicles to improve vehicle performance.

7.4.1 Fault tolerant H_{∞} controller design

If the actuator faults are not considered in the controller, they could significantly deteriorate the vehicle performance. A fault tolerant H_{∞} control strategy is developed for active suspension of in-wheel SRM mounted electric vehicle in the presence of actuator faults and control constraints. Suspension performance objectives include ride comfort, suspension defection, and road holding stability. The vehicle vertical acceleration is minimised to obtain a better ride performance. The following conditions are hard constraints that should be strictly satisfied:

• Ride comfort: The sprung mass acceleration in the vertical direction is used to quantify vehicle ride comfort performance.

• Suspension deflection: The suspension deflection should not exceed its travel limit determined by the mechanical structure.

• Road-holding stability: To ensure a firm uninterrupted contact of wheels with the road, the dynamic tyre load should not exceed the static one.

• Maximum actuator force: The active control force provided by the active suspension system should be constrained by the maximum actuator force.

Based on the dynamic model of a quarter-car active suspension described in Eq. (7-3), Eq.(7-4), Eq.(7-5), and defining the vehicle state vector as:

$$x(t) = [x_s(t) - x_d(t) \quad \dot{x}_s(t) \quad x_d(t) - x_u(t) \quad \dot{x}_d(t) \quad x_u(t) - x_g(t) \quad \dot{x}_u(t)]$$

The active suspension system can be described by the following state-space equations:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t)$$

(7-40)

$$z_{1}(t) = C_{1}x(t) + D_{1}u(t)$$

$$z_{2}(t) = C_{2}x(t)$$
(7-41)

$$(7-42)$$

The state vector of a vehicle suspension system cannot fully be measured in practice. Hence, it is difficult to control the system by applying the state feedback H_{∞} controller. The suspension deflection, tyre deflection and sprung mass displacement can be measured in the test rig by using the laser displacement sensors. The sprung mass velocity and unsprung mass velocity can be calculated by differentiating the sprung mass displacement and unsprung mass displacement. However, the airgap eccentricity and SRM velocity are difficult to measure in practice. Therefore, the measurement output is described by the following equations.

$$y(t) = C_y x(t)$$

The output feedback H_{∞} controller can be defined by the following state space equation:

$$\dot{\hat{x}}(t) = A_{\kappa}\hat{x}(t) + B_{\kappa}y(t)$$

$$u_{d}(t) = C_{\kappa}\hat{x}(t)$$
(7-44)

(7-43)

(7-45)

where $\hat{x} \in \mathbb{R}^n$ is the state vector of the controller and $A_{\kappa}, B_{\kappa}, C_{\kappa}$ are controller parameter matrices to be determined.

Considering the actuator faults, the real control force can be modelled as

$$u(t) = \lambda u_d(t) = \left(\lambda_m + N_0 \bar{\lambda}\right) u_d(t)$$

$$(7-46)$$

$$\frac{\lambda_{max}}{\bar{\lambda}} - \frac{\lambda_{max} - \lambda_{min}}{\bar{\lambda}} = N_0 N_0^T \leq I$$

 $\lambda_m = \frac{\lambda_{min} + \lambda_{max}}{2}, \quad \bar{\lambda} = \frac{\lambda_{max} - \lambda_{min}}{2}, \quad N_0 N_0^T \leq I.$

where $u_d(t)$ is the desired actuator force, and λ is an unknown parameter, assuming that λ is bounded by its minimum value λ_{min} and its maximum value λ_{max} , and N_0 is an unknown parameter.

Substituting Eqs. (7-44), (7-45) and (7-46) into Eqs. (7-40), (7-41) and (7-42), we can obtain the closed-loop system

$$\dot{\bar{x}}(t) = A_{cl}\bar{x}(t) + B_{cl1}w(t)$$
(7-47)

$$\bar{z}_1(t) = C_{cl1}\bar{x}(t) \tag{7-48}$$

$$\bar{z}_2(t) = C_{cl2}\bar{x}(t)$$
 (7-49)

$$u(t) = C_u \bar{x}(t)$$

<i>.</i> –		
(7	501	
()	-307	

(7-51)

where
$$\bar{x} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$
, $A_{cl} = \begin{bmatrix} A & B_2 \lambda C_{\kappa} \\ B_{\kappa} C_{y} & A_{\kappa} \end{bmatrix}$, $B_{cl1} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, $C_{cl1} = \begin{bmatrix} C_1 & D_1 \lambda C_{\kappa} \end{bmatrix}$,
 $C_{cl2} = \begin{bmatrix} C_2 & 0 \end{bmatrix}$, $\bar{C}_1 = \begin{bmatrix} C_1 & D_1 \lambda_m C_{\kappa} \end{bmatrix}$, $\bar{C}_2 = \begin{bmatrix} C_2 & 0 \end{bmatrix}$, $C_u = \begin{bmatrix} 0 & \lambda C_{\kappa} \end{bmatrix}$.
 $\bar{A} = \begin{bmatrix} A & B_2 \lambda_m C_{\kappa} \\ B_{\kappa} C_{y} & A_{\kappa} \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, $H = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$, $E = \begin{bmatrix} 0 & \bar{\lambda} C_{\kappa} \end{bmatrix}$.
 $A_{cl} = \bar{A} + HN_0E$, $B_{cl1} = \bar{B}$, $C_{cl1} = \bar{C}_1 + D_1 N_0E$, $C_{cl2} = \bar{C}_2$.

A fault tolerant H_{∞} controller for active suspension of in-wheel SRM mounted electric vehicle in the presence of actuator faults and control constraints is developed, and the following Theorem 7.1 is obtained.

Theorem 7.1: Given positive scalars γ and ρ , a dynamic output feedback controller exists such that the closed-loop system in Eqs (7-47), (7-48) and (7-49) is asymptotically stable and satisfies $\|\bar{z}_1(t)\|_2 \leq \gamma \|w(t)\|_2$ for all w. If there exist symmetric matrices X, Y and $\hat{A}, \hat{B}, \hat{C}$ with appropriate dimensions and any positive scalars ε_1 , ε_2 , ε_3 , then the following LMIs hold:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -I & \varepsilon_1 D_{12} & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0$$

$$\begin{bmatrix} -\mu_{max}^{2}X & -\mu_{max}^{2}I & \sqrt{\rho}\hat{C}^{T}\lambda_{m}^{T} & \hat{C}^{T}\bar{\lambda}_{l}^{T} \\ * & -\mu_{max}^{2}Y & 0 & 0 \\ * & * & -I + \varepsilon_{2}I & 0 \\ * & * & * & -\varepsilon_{2}I \end{bmatrix} < 0$$

$$(7-52)$$

$$Z_{2max}^{2}X - Z_{2max}^{2}I & \sqrt{\rho}X^{T}C_{2}^{T} & 0 & \hat{C}^{T}\bar{\lambda}_{l}^{T}$$

$$\begin{bmatrix} -z_{2max}^{2}X & -z_{2max}^{2}I & \sqrt{\rho}X^{T}C_{2}^{T} & 0 & \hat{C}^{T}\bar{\lambda}^{T} \\ * & -z_{2max}^{2}Y & \sqrt{\rho}C_{2}^{T} & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\epsilon_{3}I & 0 \\ * & * & * & * & -\epsilon_{3}I \end{bmatrix} < 0$$

$$(7-53)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$$

(7-54)

(7-57)

$$\Lambda_{11} = \begin{bmatrix} AX + XA^{T} + B_{2}\lambda_{m}\hat{C} + (B_{2}\lambda_{m}\hat{C})^{T} & \hat{A}^{T} + A \\ * & A^{T}Y + Y^{T}A + \hat{B}C_{y} + (\hat{B}C_{y})^{T} \end{bmatrix},$$
$$\Lambda_{12} = \begin{bmatrix} B_{1} \\ Y^{T}B_{1} \end{bmatrix}, \Lambda_{13} = \begin{bmatrix} (C_{1}X + D_{1}\lambda_{m}\hat{C})^{T} \\ C_{1}^{T} \end{bmatrix}, \Lambda_{14} = \begin{bmatrix} \varepsilon_{1}B_{2} \\ \varepsilon_{1}Y^{T}B_{2} \end{bmatrix}, \Lambda_{15} = \begin{bmatrix} \hat{C}^{T}\bar{\lambda}^{T} \\ 0 \end{bmatrix}.$$

Proof: Consider the Lyapunov function as given by:

$$V(x) = \bar{x}^T P \bar{x}$$
(7-55)

In order to establish that system in Eqs (7-47), (7-48) and (7-49) is asymptotically stable with a disturbance attenuation $\gamma > 0$, it is required that the associated Hamiltonian

$$\mathcal{H}(x, w, t) = z_{1}^{T}(t)z_{1}(t) - \gamma^{2}\omega(t)^{T}\omega(t) + \dot{V}(x)$$
(7-56)
$$\mathcal{H}(x, w, t) = \zeta^{T}(t) \begin{bmatrix} A_{cl}^{T}P + PA_{cl} & PB_{cl1} & C_{cl1}^{T} \\ * & -\gamma^{2}I & 0 \\ * & * & -I \end{bmatrix} \zeta(t)$$

where $\zeta(t) = [x(t) \ w(t)]$ and the requirement that $\mathcal{H}(x, w, t) < 0$ for all $\zeta(t) \neq 0$ is obtained

$$\begin{bmatrix} A_{cl}{}^{T}P + PA_{cl} & PB_{cl1} & C_{cl1}{}^{T} \\ * & -\gamma^{2}I & 0 \\ * & * & -I \end{bmatrix} < 0$$
(7-58)

and it is further equivalent to

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} & P\bar{B} & \bar{C}_1^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \begin{bmatrix} PH \\ 0 \\ D_1 \end{bmatrix} N_0 \begin{bmatrix} E & 0 & 0 \end{bmatrix} + \left(\begin{bmatrix} PH \\ 0 \\ D_1 \end{bmatrix} N_0 \begin{bmatrix} E & 0 & 0 \end{bmatrix} \right)^T < 0$$
(7-59)

Based on the lemma 3.1 in [90], the Eq. (7-59) is obtained by

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} & P\bar{B} & \bar{C_1}^T & \varepsilon_1 P H & E^T \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -I & \varepsilon_1 D_1 & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0$$

(7-60)

By using the variable substitution method proposed in [156], We partition matrix P and its inverse as follows

$$P = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}, \ P^{-1} = \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix}.$$

where $X, Y \in \mathbb{R}^{n \times n}$ are symmetric matrices. We can obtain Eq.(7-54) by the equality $PP^{-1} = I$.

We also can obtain

$$P\begin{bmatrix} X & I\\ M^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y\\ 0 & N^T \end{bmatrix},$$

If we define

$$F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}.$$

then $PF_1 = F_2$, and we can obtain

$$\begin{split} F_1^T P \bar{A} F_1 &= F_2^T \bar{A} F_1 = \begin{bmatrix} AX + B_2 \lambda_m C_\kappa M^T & A \\ Y^T A X + N B_\kappa C_y X + Y^T B_2 \lambda_m C_\kappa M^T + N A_\kappa M^T & Y^T A + N B_\kappa C_y \end{bmatrix}, \\ F_1^T P \bar{B} &= \begin{bmatrix} B_1 \\ Y^T B_1 \end{bmatrix}, F_1^T P H = \begin{bmatrix} B_2 \\ Y^T B_2 \end{bmatrix}, F_1^T P F_1 = F_2^T F_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, F_1^T E^T = \begin{bmatrix} M C_\kappa^T \bar{\lambda}^T \\ 0 \end{bmatrix}, \end{split}$$

$$\bar{C}_1 F_1 = \begin{bmatrix} C_1 X + D_1 \lambda_m C_\kappa M^T & C_1 \end{bmatrix}.$$

We define the variables

$$\hat{A} = Y^T A X + N B_{\kappa} C_y X + Y^T B_2 \lambda_m C_{\kappa} M^T + N A_{\kappa} M^T$$
(7-61)

$$\hat{B} = NB_{\kappa}$$

$$(7-62)$$

$$\hat{C} = C_{\kappa}M^{T}$$

(7-63)

According to Eqs (7-64) and (7-65), Eqs (7-52) and (7-53) can be obtained. From the above analysis, we obtain Theorem 7.1. Then the output feedback controller parameters are obtained

$$C_{\kappa} = \hat{C} (M^T)^{-1}$$

$$B_{\kappa} = N^{-1} \hat{B}$$
(7-66)

(7-67)

$$A_{\kappa} = N^{-1} [\hat{A} - Y^{T} A X] (M^{T})^{-1} - B_{\kappa} C_{y} X (M^{T})^{-1} - N^{-1} Y^{T} B_{2} \lambda_{m} C_{\kappa}$$

7.4.2 SRM controller design

Several control methods such as CCC, PWM, and APC can be utilized for controlling the chopping current and switching angles of an SRM. In this work, a combined control method using PWM and CCC is proposed as the SRM controller. For the voltage PWM control, the supply voltage is 240 Vdc, and the turn-on and turn-off angles are 5.5° and 25°, respectively. The frequency of the PWM signal is 2 kHz. In terms of PWM, the difference between reference speed and actual speed will be used as the feedback signal to determine the PWM duty cycle via a PID controller. In terms of CCC, sprung mass acceleration is selected as the feedback signal to calculate the threshold chopping current via a PID controller. The PWM duty cycle and threshold chopping current are combined to ensure the designed current through the switched signal controller. The control voltage of the SRM is obtained by using the combined control methods of CCC and PWM, as shown in Figure 7-17.



Figure 7-17 Control diagram for in-wheel SRM driven electric vehicle

In summary, a hybrid control system consisting of fault tolerant H_{∞} suspension control and SRM control for an in-wheel motor driven electric vehicle is applied. An output feedback H_{∞} suspension control is used to enhance suspension ride comfort and reduce SRM airgap eccentricity, while an SRM controller is used to control motor speed and reduce residual forces to further assist ride comfort improvement.

7.4.3 Simulation results

Stochastic road excitation and bump road excitation are used to analyse dynamic

responses characteristic of vehicle suspension and in-wheel SRM. We assume that the maximum suspension deflection is 100 mm; the maximum control force is 2000 N. A hybrid control system consisting of fault tolerant H_{∞} controller and SRM controller for in-wheel SRM driven electric vehicle is applied. Two types of electric vehicle, an in-wheel SRM driven electric vehicle with passive suspension (refer to Passive), an in-wheel SRM driven electric vehicle with proposed hybrid control method (refer to Active), are used to illustrate the result of vehicle vibration dynamic response and SRM characteristic. The vehicle parameter values used in this section are presented in Table 7-1.

7.4.3.1 Stochastic road excitation

The stochastic road surface is used to demonstrate the effectiveness of the proposed control system first. The class B road profile with constant vehicle speed of 60 km/h is used to test the system. Figure 7-18 shows the PSDs of sprung mass acceleration and tyre deflection under stochastic road excitation, from which we can see that sprung mass acceleration and tyre deflection of "Active" are much smaller than that of "Passive" at the resonance frequency of sprung mass. The result means that the ride comfort is improved with the hybrid control system. The PSD of tyre deflection with the hybrid control system is increased at the resonance frequency of unsprung mass. The frequency responses of SRM vertical force and airgap eccentricity are shown in Figure 7-19, from which we can see that SRM vertical force and airgap eccentricity of electric vehicle with the hybrid control system are reduced compared to those of passive one at round 100 Hz. This indicate that SRM performance is dramatically increased at the resonance frequency of SRM stator.



Figure 7-18 PSDs of vehicle dynamic responses to stochastic road. (a) Sprung mass acceleration. (b) Tyre deflection.



Figure 7-19 PSDs of SRM dynamic responses to stochastic road. (a) SRM vertical force. (b) Airgap eccentricity.

Figure 7-20 illustrates the time domain responses of sprung mass acceleration and tyre deflection. The sprung mass acceleration of electric vehicle with the hybrid control system is reduced compared to that of electric vehicle with passive suspension, indicating that the vehicle ride comfort is significantly improved. The tyre deflection of "Active" is slightly increased when compared to that of "Passive". The active suspension controller emphasizes the ride comfort, and the road-holding stability is sacrificed for vehicle ride comfort. The tyre deflection is a hard constraint and should not exceed its travel limits. Figure 7-21 shows the SRM dynamic responses under the stochastic road excitation in terms of airgap eccentricity and unbalanced vertical force. Both the airgap eccentricity and unbalanced vertical force of the "Active" are reduced when compared to those of "Passive", which indicate that the hybrid control system could effectively improve SRM dynamic performance and prolong SRM lifespan.

Table 7-2 shows RMS comparison of vehicle dynamic response and the SRM dynamic response under the stochastic road. When the hybrid control system is applied, the RMS of vehicle sprung mass acceleration (that reflects vehicle performance), airgap eccentricity and unbalanced vertical force are decreased by 23.3%, 7.1%, and 36.4% compared to those of passive suspension. Moreover, the RMS of tyre deflection is increased by 8.3% compared to the passive suspension. However, the values of tyre dynamic force are under tight constraints. When 60% actuator loss occurs in the system, the sprung mass acceleration, airgap eccentricity and unbalanced vertical force of "Active" are slightly increased, but they are still smaller than those of "Passive". Based on the above analysis, it can be suggested that the proposed hybrid control system produces enhanced vehicle dynamic performance and SRM performance in the presence of actuator loss.



Figure 7-20 Vehicle dynamic responses to stochastic road. (a) Sprung mass acceleration. (b) Tyre deflection.



Figure 7-21 SRM dynamic responses to stochastic road. (a) SRM vertical force. (b) Airgap eccentricity.

rable 7-2 KWS comparison of venicle dynamic response and SKW dynamic response	Table 7-2	2 RMS	comparison	of vehicle d	lynamic res	ponse and S	RM dyr	namic resp	onse
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Suspension types	Sprung mass acceleration (m/s ²)	Suspension deflection (m)	Tyre deflection(m)	Airgap eccentricity (m)	Vertical force (N)
Passive	0.3585	0.0024	0.0011	2.6208e-5	152.4576
Active	0.2750	0.0024	0.0012	2.4349e-5	96.9287
Active with 60% actuator faults	0.2956	0.0019	0.0011	2.5350e-05	98.3633

7.4.3.2 Bump road excitation

Bump road profile is also used to validate the better ride performance of proposed hybrid control system, which is given by

$$x(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi v_0}{l}t\right) \right), & 0 \le t \le \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases}$$
(7-69)

where *a* is the height of the bump and *l* is the length of the bump. Here we choose a = 0.05 m, and l = 2 m, and the constant vehicle forward velocity of $v_0 = 36$ km/h. Figure 7-22 shows the sprung mass acceleration and tyre defection response under the bump road excitation. It can be seen from Figure 7-22 that the sprung mass acceleration of hybrid control system is much smaller than that of the passive suspension system, which indicates that the electric vehicle with hybrid control system has better ride performance than the electric vehicle with passive suspension. We can obtain the tyre dynamic force through tyre deflection. The tyre dynamic force is less than its static one, which guarantees the vehicle road holding stability. SRM airgap eccentricity and unbalanced vertical force of "Active" are reduced compared to those of "Passive", indicating that the SRM performance is improved. With the hybrid control system, both the suspension and SRM performance are improved.



Figure 7-22 Vehicle dynamic responses to bump road excitation. (a) Sprung mass acceleration. (b) Tyre deflection.



Figure 7-23 SRM dynamic responses to bump road excitation. (a) SRM vertical force. (b) Airgap eccentricity.

7.5 Active suspension control of DVA-SRM driven EV

7.5.1 State feedback H_{∞} controller design

The active suspension controller is designed based on the quarter-car model with the DVA-SRM. The control objective is to suppress the sprung mass vibration as well as improving ride comfort, suspension deflection and road holding stability. The vehicle vertical acceleration is minimised to obtain a better ride performance and the latter three conditions such as suspension deflection, road-holding stability and maximum actuator force are hard constraints that should be strictly satisfied. By defining the vehicle state vector as

$$\tilde{x}(t) = [x_s(t) - x_{u2}(t) \ \dot{x}_s(t) \ x_{d2}(t) - x_{u2}(t) \ \dot{x}_{d2}(t) \ x_{u2}(t) - x_g(t) \ \dot{x}_{u2}(t) \ x_r - x_{d2} \ \dot{x}_r]$$

The active suspension system in (7-6), (7-7) and (7-8) can be described by the following state-space equations

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}_1\tilde{w}(t) + \tilde{B}_2\tilde{u}(t)$$

$$(7-70)$$

$$\tilde{z}_1(t) = \tilde{C}_1\tilde{x}(t) + \tilde{D}_1\tilde{u}(t)$$

$$(7-71)$$

$$\tilde{z}_2(t) = \tilde{C}_2 \tilde{x}(t) \tag{7-72}$$

where

A robust H_{∞} state feedback controller is proposed for active suspension of suspended SRM driven electric vehicle. The closed-loop system is asymptotically stable, and it can also ensure a prescribed gain from disturbance to performance output while the suspension stroke, tyre dynamic force and maximum control force constraints are satisfied. Then the following Theorem 7.2 is obtained.

Theorem 7.2 [90]. Consider the active suspension of electric vehicle with DVA structure SRM (7-70), (7-71) and (7-72), a robust H_{∞} state feedback control gain can be found such that the closed-loop system is asymptotically stable and satisfies $\|\tilde{z}_1\|_2 \leq \gamma \|\tilde{w}\|_2$ for all w, if there exists matrices P > 0 and M satisfying following LMIs (7-73), (7-74) and (7-75). Moreover, the feedback gain matrix function for the controller can be given by $K = MP^{-1}$.

$$\begin{bmatrix} sym(\tilde{A} + \tilde{B}_{2}M) & \tilde{B}_{1} & P\tilde{C}_{1}^{T} + M^{T}\tilde{D}_{1}^{T} \\ * & -\gamma^{2}I & 0 \\ * & * & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} -I & \sqrt{\rho}(P\tilde{C}_{2} + M\tilde{D}_{2}) \\ * & -z_{2max}^{2}P \end{bmatrix} < 0$$

$$\begin{bmatrix} -I & \sqrt{\rho}M \\ * & -u_{max}^{2}P \end{bmatrix} < 0$$

$$(7-74)$$

(7-75)

Finally, the robust H_{∞} control method developed for the active suspension of SRM driven electric vehicle is constructed as

$$\tilde{u}(t) = K\tilde{x}(t)$$
(7-76)

7.5.2 SRM controller design

A combined control method using PWM and CCC is used for SRM in this work [97]. In term of PWM, the difference between reference speed and actual speed is used as the feedback signal to determine the PWM duty cycle via a PID controller. In terms of CCC, sprung mass acceleration is selected as the feedback signal to calculate the threshold chopping duty via a PID controller. The control voltage of the SRM is obtained by using

the combined control methods of CCC and PWM, as shown in Figure 7-17. A switched signal controller is used to combine control signal from PWM and that from CCC tother to generate a unique control voltage.

7.5.3 Simulation results

Stochastic road excitation and bump road excitation are used to analyse dynamic responses characteristic of vehicle suspension and SRM. We assume that the maximum suspension deflection is 100 mm; the maximum control force is 2000 N. The vehicle parameter values used in this section are presented in Table 7-3. As a result, the robust H_{∞} controller for active suspension of DVA-SRM electric vehicle is considered. The controller gain matrix is given by

 $K = 1 \times 10^4 [-0.678 \quad 0.322 \quad 0.208 \quad 0.007 \quad -2.483 \quad 0.028 \quad -0.164 \quad 0.005].$

DVA-SRM Symbol	Value	Conventional SRM Symbol	Value
m_s	340 kg	m_s	340kg
m_{u2}	40 kg	m_u	60kg
m_{d2}	30kg	m_d	30kg
m_r	20 kg	k_s	32000 N/m
k_{s2}	32000 N/m	Cs	1496 Ns/m
<i>C</i> _{<i>s</i>2}	1496 Ns/m	k_d	7000000 N/m
k_{t2}	360000 N/m	k_t	360000 N/m
k_r	7000000 N/m		
k_{a2}	41000 N/m		
C _{a2}	1000 N/m		

Table 7-3 Vehicle suspension parameter values.

A stochastic road excitation is used to validate the better ride performance of active suspension with DVA-SRM first. The class B road profile with constant vehicle speed of 60 km/h is used to test the system. Figure 7-24, Figure 7-25 and Figure 7-26 illustrate the vehicle suspension dynamic responses including sprung mass acceleration, suspension deflection and tyre deflection. "Passive-conventional" refers to the passive suspension of conventional SRM driven electric vehicle. "Passive-DVA" and "Active-DVA" represent the passive suspension and active suspension of DVA structure electric vehicle. The diagrams show that the sprung mass acceleration, suspension deflection and tyre deflection and the sprung mass acceleration. The DVA structure makes better ride performance than the conventional one, which indicates

that this kind of in-wheel motor configuration has the ability of improving vehicle ride comfort performance. Furthermore, the sprung mass acceleration, suspension deflection and tyre deflection of the active-DVA are greatly reduced compared to those of the passive-DVA, demonstrating the effectiveness of robust H_{∞} controller in improving vehicle ride performance. Figure 7-27, Figure 7-28, and Figure 7-29 show the SRM dynamic responses under the stochastic road excitation in terms of airgap eccentricity and unbalanced vertical force. According to these diagrams, SRM airgap eccentricity, unbalanced vertical force and motor acceleration of DVA structure electric vehicles are much smaller than those of conventional SRM electric vehicle. The SRM performance of passive-DVA and active-DVA are similar. The airgap eccentricity and motor acceleration of active-DVA is slightly larger than those of passive-DVA. It can be concluded that the DVA structure can significantly reduce the SRM airgap. Since the unbalanced vertical force is directly influenced by the airgap eccentricity, airgap eccentricity reduction will not only reduce SRM stator vibration but will also result in a decrease of unbalanced vertical force, which can effectively improve SRM dynamic performance and prolong SRM lifespan.



Figure 7-24 Sprung mass acceleration responses to stochastic road.



Figure 7-25 Suspension deflection responses to stochastic road.



Figure 7-26 Tyre deflection responses to stochastic road.

Three different types of suspension in vehicle dynamic responses and SRM dynamic responses are compared in Table 7-4 . For the passive suspension of DVA structure electric vehicle, the RMS of sprung mass acceleration (that reflects vehicle performance) is decreased by 21% compared to passive suspension of conventional SRM, and the suspension deflection and tyre deflection are decreased by 10% and 34.9% compared to those of conventional SRM. Furthermore, air-gap eccentricity, vertical force and motor acceleration of passive-DVA are decreased by 85%, 88% and 81% when compared to those of passive-conventional systems. It can be concluded that the DVA configuration has the potential to improve the road holding capability and ride performance, as well as motor performance. Furthermore, the motor airgap eccentricity, vertical force and motor acceleration of active-DVA are not much changed compared to those of passive-DVA. However, the sprung mass acceleration, suspension deflection and tyre deflection of active suspension are reduced by 39.8%, 11% and 2%, respectively when compared to those of passive suspension. This indicates that the active suspension with robust H_{∞} controller can effectively improve vehicle performance.



Figure 7-27 SRM airgap eccentricity responses to stochastic road.



Figure 7-28 SRM vertical force responses to stochastic road.



Figure 7-29 SRM motor acceleration responses to stochastic road.

Table 7-4 RMS	comparison of	vehicle dynamic	response and	SRM dynamic response.
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Suspension types	Passive suspension- conventional DVA		Active suspension- DVA
Sprung mass acceleration (m/s ²)	0.356	0.281	0.169
Suspension deflection (m)	0.0020	0.0018	0.0016
Tyre deflection (m)	9.856e-4	6.417e-4	6.281e-4
Airgap eccentricity (m)	2.841e-5	4.135e-6	4.458e-6
Vertical force (N)	92.071	10.583	10.953
Motor acceleration (m/s ²)	8.154	1.588	1.643

Then bump road profile in Equation (7-69) is used to validate the better ride performance of active suspension with robust H_∞ controller. Here we choose a = 0.05 m, and l = 2 m, and the constant vehicle forward velocity of $v_0 = 60$ km/h. Figure 7-30, Figure 7-31 and Figure 7-32 show sprung mass acceleration, suspension deflection and tyre deflection

responses under the bump road excitation. It can be seen from Figure 7-30 that the sprung mass acceleration of active suspension with robust H_{∞} controller is much smaller than that of passive suspension, which indicates that active suspension has better ride performance than the passive suspension. Suspension deflection and tyre deflection are also greatly reduced when compared to those of the passive suspension, which demonstrates the effectiveness of robust H_{∞} controller. SRM airgap eccentricity and unbalanced vertical force responses to bump road are illustrated in Figure 7-33, Figure 7-34 and Figure 7-35, respectively. The active suspension with robust H_{∞} controller reduces the SRM airgap eccentricity and unbalanced vertical force, which significantly improves SRM performance and prolongs the lifespan of the SRM. The SRM stator vertical acceleration of active suspension is reduced compared to that of passive suspension according to Figure 7-35, indicating that the motor vibration is reduced. With the robust H_{∞} controller, both the suspension and motor performance are improved.



Figure 7-30 Sprung mass acceleration responses to bump road.



Figure 7-31 Suspension deflection responses to bump road.



Figure 7-32 Tyre deflection responses to bump road.



Figure 7-33 SRM airgap eccentricity responses to bump road.



Figure 7-34 SRM vertical force responses to bump road.



7.6 Summary

The coupling effect between road excitation and an in-wheel SRM and its impact on vehicle ride comfort characteristics was studied. The conventional quarter-car active suspension model, quarter car model with DVA, switched reluctance motor model and MF tyre model were developed. SRM airgap eccentricity and unbalanced vertical forces were greatly affected by the road roughness. The coupling effect of road and SRM had an adverse effect on the vehicle ride comfort characteristics. In order to reduce the residual unbalanced vertical force and to suppress motor vibration, combined control methods of CCC and PWM were deployed to suppress motor vibration. An output feedback H_{∞} suspension control was proposed for in-wheel SRM driven electric vehicles. Simulation results demonstrated that the proposed control method effectively reduced the SRM airgap eccentricity, residual unbalanced radial forces and produced a better vehicle ride.

Furthermore, a robust H_{∞} controller was used to obtain better performance for the active suspension. Two types of road excitation including bump manoeuvre and random manoeuvre were used to analyse and compare the responses of different types of suspension. Simulation results indicated that the DVA configuration could significantly improve the road holding capability and ride performance, as well as motor performance. The airgap eccentricity and residual unbalanced vertical force were greatly reduced with the help of DVA. Furthermore, the active suspension-DVA with robust H_{∞} controller achieved enhanced ride performance when compared to passive suspension-DVA. With the robust H_{∞} controller, both the suspension and motor performance were improved.

8. Conclusion and future work

8.1 Overview

In this thesis, the effects of in-wheel motor on electric vehicle ride performance were investigated. Several control strategies were developed for electric vehicle active suspension systems with disturbance, uncertainties, as well as actuator faults and delay. The primary aim was to reduce sprung mass acceleration, suspension deflection, tyre dynamic force and simultaneously achieve enhanced ride performance and road holding stability. Experimental validation was conducted on the quarter car suspension test rig to verify the performance of the proposed control methods under different road excitations. The major achievements of the thesis are listed as follows:

The fault tolerant H∞ controller based on friction estimation proposed in Chapter
 3 could effectively guarantee a better vehicle suspension performance in spite of system
 friction, actuator fault and time delay under different road excitation.

(2) The experimental results indicated that the proposed friction observer-based T-S fuzzy $H\infty$ controller could significantly improve vehicle ride performance in the presence of sprung mass variations and system friction under different road profiles. The friction in the controller was also estimated, which could further enhance the control performance.

(3) Increased unsprung mass in EV with IWM configuration had negative effect on suspension ride comfort performance and road holding ability.

(4) The dynamic-damping-in-wheel-motor-driven-system in which the in-wheel motor itself served as the dynamic absorber was demonstrated to have the ability of improving vehicle ride comfort performance.

(5) The multi-objective genetic algorithm was proved effective in optimizing vehicle suspension parameters, motor parameters and active controller in IWM driven EV. The active suspension with optimized parameters achieved enhanced suspension and motor performance than those of passive suspension and active suspension with unoptimized parameters.

(6) For active suspension of IWM-EVs, the robust T-S fuzzy $H\infty$ reliable statefeedback controller proposed in Chapter 5 achieved a significantly better ride performance than conventional $H\infty$ controller and fuzzy $H\infty$ controller do when different actuator thrust losses occur.

(7) Particle swarm optimization was proved effective in optimizing vehicle suspension parameters and motor parameters in IWM driven EV.

(8) The dynamic output feedback controller proposed for active suspension of IWM-EVs in Chapter 6 was demonstrated to guarantee the system's asymptotic stability and $H\infty$ performance in the presence of actuator faults and time delays; simultaneously satisfying the performance constraints such as such as road holding, suspension stroke, the dynamic load applied on the bearings and actuator limitation.

(9) The road grade and vehicle speed were shown to have adverse impact on SRM dynamic responses, vehicle ride performance and road holding stability SRM driven EV.

(10) For SRM driven electric vehicle, the proposed hybrid control method based on output feedback $H\infty$ control and SRM control in Chapter 7 was indicated to produce enhanced vehicle dynamic performance and SRM performance in the presence of actuator loss.

(11) DVA configuration in SRM driven EV could significantly improve the road holding capability and ride performance, as well as motor performance.

(12) With hybrid control system consist of robust $H\infty$ controller and SRM controller, the active suspension-DVA was shown to achieve enhanced ride performance when compared to passive suspension-DVA.

8.2 Comprehensive literature review

Three kinds of vehicle suspension systems, including passive suspension system, semiactive suspension system and active suspension system, were studied in Chapter 2. Then a comprehensive review of the literature on in-wheel motor mounted electric vehicle suspension system and methods proposed to improve vehicle ride, handling and stability performance, and control in them was conducted. The impacts of IWM on vehicle ride performance and motor vibration was discussed. Moreover, regenerative suspension including electrohydraulic regenerative suspension system and electromagnetic regenerative suspension system that could be used in the electric vehicle were also covered. Finally, the active suspension control methods including FLC, optimal control, robust H_{∞} control, sliding control and preview control which offer superior vehicle performance and energy consumption were discussed. Enhanced vehicle performance including ride comfort, handling stability could be obtained through active suspension system.

8.3 Friction observer based robust H_{∞} control for active suspension and experimental validation

Friction observer based robust H_{∞} control for active suspension was conducted in Chapter 3. Firstly, a fault-tolerant H_{∞} controller based on friction compensation was proposed for active suspension system considering the system parameter uncertainties, actuator faults, as well as actuator time delay and system friction. The quarter car test rig was setup to analyse the dynamic responses of controlled suspension system and controlled suspension system under different road profiles. The dynamic responses comparison of uncontrolled suspension system and controlled suspension system were conducted in the quarter car test rig. Experimental results showed that the proposed fault-tolerant H_{∞} control method effectively reduced the sprung mass acceleration, sprung mass displacement and unsprung mass displacement under different road profiles such as 3Hz sinusoidal road excitation, bump road excitation and random road excitation, which demonstrated the effectiveness of proposed controller in improving vehicle performance. This indicated that the active suspension with proposed controller could improve vehicle performance with the actuator faults, actuator time delay and parameter uncertainties.

Secondly, a friction observer-based T-S fuzzy controller for active suspension considering the sprung mass variation and system friction were developed. T-S fuzzy approach was used to model the active suspension system with sprung mass variation, while vehicle body acceleration-based observer was used to estimate the system friction force. Three types of road profiles such as 3.5Hz sinusoidal, bump, random road excitations were used to analyse and compare the responses of uncontrolled suspension and controlled suspension system in the quarter car test rig. The experimental results showed that sprung mass acceleration and sprung mass displacement of controlled suspension was smaller than those of uncontrolled suspension in spite of sprung mass variation, which indicated that the proposed friction observer-based T-S fuzzy controller could significantly improve the road holding capability and ride performance under different road profiles.

8.4 Active suspension control in in-wheel motor mounted electric vehicle

Genetic algorithm-based parameter optimization of in-wheel motor driven electric vehicle was presented in Chapter 4. Quarter-car model with suspended shaft-less directdrive motors was established first. LQR controller was used to obtain the optimal performance for the active suspension. Furthermore, to increase vehicle performance and its energy efficiency, parameters of the motor suspension (damping and stiffness coefficients), vehicle suspension as well as active controller were optimized based on multi-objective GA optimization with a designed fitness function. Two types of road excitation including bump manoeuvre and random manoeuvre were used to analyse and compare the response of optimized active suspension. The active suspension with optimized parameters achieved enhanced ride performance when compared to passive suspension and active suspension with un-optimized parameters. The motor performance was also significantly enhanced. As a result, the proposed integrated GA optimization was proved effective in optimizing vehicle parameters and active controller.

In Chapter 5, to deal with the suspension vibration and in-wheel motor bearing wear, a multi-objective robust H_{∞} reliable fuzzy control for active suspension system of IWM EV with dynamic damping was proposed. The variation of sprung mass was represented by a T-S fuzzy model. A reliable fuzzy state feedback controller was designed for the T-S fuzzy active suspension model to cope with possible actuator faults, sprung mass variation and control input constraints. The suspension performance of EV with "advanced-dynamic-damper-motor" was better than a conventional in-wheel motor driven EV. Comparison of the performance of the active suspension and the passive suspension showed that the proposed reliable fuzzy H_{∞} controller offered the best suspension performance. Meanwhile, for different actuator thrust losses, the proposed reliable fuzzy H_{∞} controller achieved a significantly better closed-loop H_{∞} performance than the fuzzy H_{∞} controller does.

In Chapter 6, the problem of output feedback H_{∞} control for active suspensions deployed in in-wheel motor driven electric vehicles with actuator faults and time delay was investigated. A quarter car active suspension with DVA was established and it was demonstrated to improve ride performance and road holding ability around 10Hz. Parameters of the vehicle suspension and DVA were optimized based on the particle swarm optimization. A robust H_{∞} dynamic output feedback controller was derived such that the close-loop system was asymptotic stabile and simultaneously satisfied the constraint performances such as road holding, suspension stroke, dynamic load applied to the bearings and actuator limitation. The proposed controller offered the best suspension performance in spite of actuator faults and time delay. Meanwhile, the proposed fault-tolerant output feedback H_{∞} controller achieved a significantly better vehicle and motor performance than those of the passive suspension for different actuator thrust losses. With different actuator thrust losses and time delay, the proposed output feedback controller II revealed much better performance than the output feedback controller I and the passive suspension.

8.5 Coupling effect between road excitation and an in-wheel switched reluctance motor on vehicle ride comfort and active suspension control

The coupling effect between road excitation and an in-wheel SRM and its impact on vehicle ride comfort characteristics was investigated in Chapter 7. Firstly, the conventional quarter-car active suspension model and quarter vehicle active suspension model with DVA were developed. Switched reluctance motor was modelled using an analytical Fourier fitting method. SRM air-gap eccentricity and unbalanced vertical forces were greatly affected by the road roughness. The road grade, vehicle speed and coupling effect of road surface and SRM had adverse effects on the vehicle ride comfort characteristics. Furthermore, a hybrid control system was proposed for in-wheel motor driven electric vehicles. Combined control methods of CCC and PWM were deployed to suppress motor vibration and reduce the residual unbalanced vertical force. An output feedback H_{∞} suspension control was proposed to reduce sprung mass acceleration and improve vehicle performance. Simulation results demonstrated that the proposed control method effectively reduced the SRM air-gap eccentricity, residual unbalanced radial forces and produced a better vehicle ride.

Furthermore, based on the quarter vehicle active suspension model with SRM served as DVA, combined control methods of CCC and PWM were deployed to suppress motor vibration, a state feedback H_{∞} controller was used to obtain better performance for the active suspension. Two types of road excitation including bump manoeuvre and random manoeuvre were used to analyse and compare the responses of different types of suspension. Simulation results indicated that the DVA configuration could significantly improve the road holding capability and ride performance, as well as motor performance. The air-gap eccentricity and residual unbalanced vertical force were greatly reduced with the help of DVA. Furthermore, the robust H_{∞} controller based active suspension with DVA configuration achieved enhanced ride performance when compared to with DVA configuration. With the robust H_{∞} controller, both the suspension and motor performance were improved.

8.6 Future work

The work conducted in this thesis can be extended in the following directions:

(1)In Chapter 3, all the state variables are known when the controllers are designed, and all the state variable are available for measurement in the experiments. Not all the state variable such as unsprung mass velocity and tyre deflection are measured. Therefore, a state observer can be constructed to estimate the state vector in the future work. A faulttolerant H_{∞} controller based on friction compensation and state observer could be proposed and tested in the quarter vehicle test rig. T-S fuzzy controller based on disturbance observer and state observer for active suspension considering the sprung mass variation and system friction could also be studies in the future. In the future research, disturbance observer-based sliding mode control strategy could be proposed for the active suspension control considering the effect of road disturbance, nonlinearities and suspension system uncertainties, system friction. Experimental validation can be conducted on the quarter car test rig to demonstrate the effectiveness of proposed controller to reduce the sprung mass acceleration and stimulatingly improving vehicle ride performance. Furthermore, the power harvesting performance of the active electromagnetic suspension need to be investigated in the future. The trade-off among energy harvesting, ride comfort, and road handling of regenerative suspension system should be investigated theoretically and experimentally in the future work.

(2) Genetic algorithm based multi-objective parameter optimization of in-wheel motor driven electric vehicle was presented in Chapter 4. The partial swam optimizationbased parameter optimization of in-wheel motor mounted electric vehicle can be conducted in the future work. Experimental validation of the proposed optimization algorithm will be conducted. There is a need to further expand and improve the proposed reliable fuzzy H_{∞} control method for in-wheel motor driven electric vehicle presented in Chapter 5. A dynamic fault diagnosis observers and adaptive robust fault tolerant controller could be designed for the active suspension to estimate the possible multiple types of faults. Moreover, to deal with the issues of state estimation, system uncertainties and road disturbance, reliable fuzzy controller based on disturbance observer and state observer for active suspension of IWM-EVs. In addition to the proposed output feedback H_{∞} control for active suspension of IWM-EVs with control faults and input delay developed in Chapter 6, finite-frequency H_{∞} control of active suspensions in IWM-EVs with control faults and delay could also be studied. Experimental validation of developed controllers for active suspension system of IWM-EVs could be conducted on the test rig.

(3) The coupling effect between road excitation and an in-wheel SRM and its impact on vehicle ride comfort characteristics was investigated in Chapter 7. In the future work, a nonlinear complicated full vehicle model that includes the longitudinal, lateral and vertical motions integrated in-wheel motor model and suspension can be developed. The coupling effects of road condition and unbalanced vertical forces on electric vehicle rollover characteristics and yaw moments needs to be investigated. The impact of longitudinal components of unbalanced radial forces on vehicle vertical vibration and rollover stability should be also considered. The effect of in-wheel SRM on electric vehicle steering system and braking system needs further research. Furthermore, regeneration suspension such as electrohydraulic regeneration suspension and electromagnetic regeneration suspension can be applied in the electric vehicle. Suspension structure design, active control strategies and energy-regeneration of IWM-EV should be the focus of future research. Experimental validation of different control methods applied on the active suspension of SRM mounted electric vehicle should be conducted under different road excitations.

(4) For IWM driven electric vehicle, attaching the IWM to the wheel can lead to the increase in the unsprung mass, which influences the vehicle vertical ride behavior. vehicle parameters such as sprung mass, unsprung mass, IWM mass, suspension stiffness and damping coefficients, as well as motor stiffness and damping coefficients are very critical parameters for vehicle and motor performance in the IWM driven electric vehicle. A general practice in understanding the importance of vehicle parameters and their variations in the dynamics of vehicle system can be conducted by using sensitivity analysis method. It would have been easier to appreciate the impact and effort of the vehicle parameters to the vehicle comfort performance. Detailed analysis about the effect of vehicle and motor parameters on vehicle ride comfort, handling stability and controller performance will be discussed by using a sensitivity analysis in the future work. Moreover, the friction occurred in the controller has an effect on the controller performance, and the effort of friction to controller performance by using sensitivity analysis will be the subject of future work.

(5) The in-wheel motor drive technology has many benefits such as fast motor response, precise torque generation, simplicity and as well as the ability to implement X-by-Wire chassis control system (ABS, TCS or ESP), etc. However, the IWM technology

has several challenges such as increased unsprung mass, cost, motor controller complexity and lack of durability. Cost versus benefit is extremely crucial in the adaption of the in-wheel motor drive technology in the automotive industry. Although two in-wheel motors in the IWM driven electric vehicle are somewhat more expensive than one central motor and drivetrain in the conventional electric vehicle, the IWM drive system is more simple and efficient because it has a lower mass and doesn't suffer from frictional losses in the transmission. The detailed analysis on benefit, challenge and cost of IWM technology in automobile industry will be the subject of future work.

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