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Absorbing Markov Chain Approach to Modelling Disruptions in Supply Chain Networks

By

Supun Perera[a], Michael G H Bell[a], Fumitaka Kurauchi[b] and Dharshana Kasthurirathna[c]

[a] Institute of Transport and Logistics Studies (ITLS), The University of Sydney Business School, Australia

[b] Department of Civil Engineering, Faculty of Engineering and Graduate School of Engineering, Gifu University, Japan

[c] Department of Software Engineering, Faculty of Computing, Sri Lanka Institute of Information and Technology, Sri Lanka

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ABSTRACT:

Recent developments in the area of network science has encouraged researchers to adopt a topological perspective in modelling Supply Chain Networks (SCNs). While topological models can provide macro level insights into the properties of SCN systems, the lack of specificity due to high level of abstraction in these models limit their real-world applicability, especially in relation to assessing the impact on SCNs arising due to individual firm or supply channel level disruptions. In particular, beyond the topological structure, a more comprehensive method should also incorporate the heterogeneity of various components (i.e. firms and inter-firm links) which together form the SCN. To fill the above gap, this work proposes using the idea of absorbing Markov chains to model disruption impacts on SCNs. Since this method does not require path enumeration to identify the number of supply chains which form the SCN, it is deemed more efficient compared to the other traditional methods.

KEY WORDS: ***supply chain network; absorbing markov chain; supply network disruptions***

AUTHORS: **Perera, Bell, Kurauchi and Kasthurirathna**

CONTACT:

INSTITUTE OF TRANSPORT AND LOGISTICS STUDIES (H73)

The Australian Key Centre in Transport and Logistics Management

The University of Sydney NSW 2006 Australia

Telephone: +612 9114 1824

E-mail: business.itlsinfo@sydney.edu.au

Internet: http://sydney.edu.au/business/itls

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I. INTRODUCTION

In today's highly interconnected world, global supply chain networks (SCNs) play a vital role in fuelling international trade and economic growth. Due to this interconnectedness of global businesses, which are no longer isolated by industry or geography, any disruptions to SCNs, such as natural disasters, acts of war and terrorism, and even labour disputes are becoming increasingly complex in nature and global in consequences [1]. These disruptions can ripple through global supply chains, magnifying their original damage. Even relatively minor disturbances, such as labour disputes, ground congestion or air traffic delays can result in disproportionately severe disruptions to local and international trade. Therefore, this 'fragility of interdependence' creates unprecedented risks to global and local economies [2]. Therefore, the design of supply chains that can maintain their function in the face of perturbations is a key goal of today's supply chain management. In particular, new methodologies are required to model the existing SCNs in order to analyse the impact of local disruptions on the overall system.

Traditionally linear supply chains have, in recent years, evolved towards highly complex systems; mainly due to globalization and product specialization (see Figure 1). Therefore, modern supply chains can be viewed as networked populations of autonomous firms. Given the sheer complexity and the large number of variables involved, these SCNs are best viewed holistically through a macroscopic or a topological modelling approach. Such modelling efforts can reveal important insights into the relationship between the topological structure and various functions of SCNs [3, 4, 5].

Due to the difficulty in obtaining large scale datasets on SCNs (supplier-customer relationships), which are often proprietary and confidential, early studies have relied on computer simulations to generate network topologies (through various growth mechanisms) supposedly representative of real world SCNs [3]. Recently however, a number of data driven studies have appeared in literature, which used Bloomberg database to obtain SCN data for publicly listed firms [5, 6, 7]. These network-based models of complex supply chain systems have shown the existence of non–trivial and universal topological footprints, from which valuable system level insights can be obtained [6, 7, 8]. For instance, studies have found that the SCNs tend to be scale free in nature (with distinct hub firms), where the distribution of the number of firm level connections in the SCN (i.e. the degree distribution) follows power-law with the power law exponent between 2 and 3 [6].

There exists a large body of literature investigating the robustness of SCNs where the links represent the undirected relationships between firms. These studies have gained insights into SCN robustness based on either or both of the following avenues [3]:

1. Analytically determining the topological metrics of the networks, such as the degree distribution, average path length, clustering coefficient, nestedness and assortativity. These metrics reveal the structural features of the SCN which have direct or indirect robustness implications [6, 7, 8, 10, 11, 12].

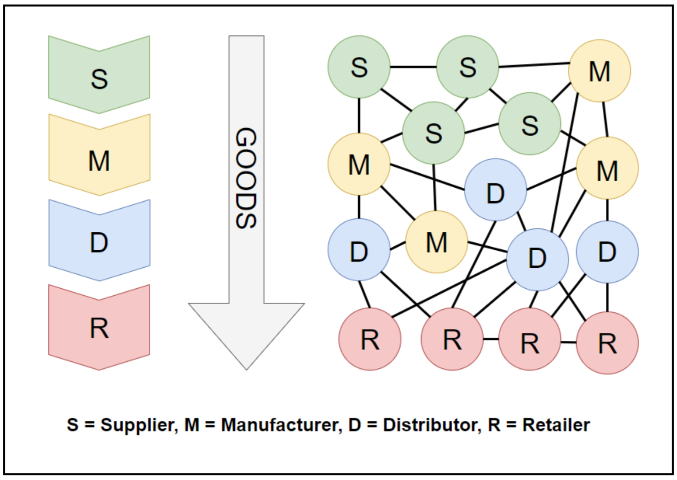

Fig. 1: Evolution of linear supply chains towards complex SCNs

2. Using generic network science based simulation techniques, which involve sequential removal of nodes (randomly or targeted by degree or some other topological attribute) and recording at each step, the size of the largest connected component and/or the average/maximum shortest path length in the largest connected component. By creating profiles of these metrics across the percentage of nodes removed, one can compare the robustness character for various SCN topologies [13, 14, 15].

While the above methods provide general insights into the topological robustness of SCNs, the lack of specificity due to high level of abstraction limits their real world applicability. Additionally, consideration of the topological structure without incorporating the heterogeneity of the singular components (i.e. the nodes and the links) that form the SCN, can only account for a part of the full picture. Therefore, there exists a need for a more specific method for assessing the robustness of SCNs. In particular, this method should consider the topological structure of the SCN while at the same time capturing the heterogeneity of each component.

In addition, the above methodologies do not consider link weights - some SCN exchange relationships may be more important compared to the others, and therefore it is important to represent these links as weighted connections, so that the heterogeneity in capacity and intensity in various connections are captured accurately in the model (weighting of links can be a function of volume, frequency and criticality of flows in a given period [4]). Finally, the approaches observed in the contemporary literature do not account for partial functionality of the nodes – since only full node removals are simulated.

In light of the above, this work seeks to develop a SCN robustness assessment method which is capable of accounting for both the topology of the SCN and the heterogeneity of components. Accordingly, the proposed methodology enables determining the network wide impact of a disruption at a single node and/or a link. A key advantage of the proposed method is that it does not require a-priori path enumeration (which can be computationally expensive) to establish the number of paths within the SCN, since this calculation is implicitly carried out through the proposed matrix operations.

II. Background

A. Absorbing Markov Chains

A Markov process is any stochastic process that satisfies the Markov property. A process is said to satisfy the Markov property if predictions can be made of its future states based solely on its current state, independent of its full history. Since, the future of the process is determined conditional to its present state, its future and past states are independent. As a result, the Markov property is also referred to as memorylessness. This assumption enables one to easily calculate the conditional probabilities for a given system and therefore this concept can be used to model a wide range of real world scenarios [16].

A Markov chain is a type of Markov process, which is often characterised by a transition matrix. The transition matrix defines the probability of an entity moving from one state to another, within the system being modelled. A regular (as opposed to an 'absorbing') Markov chain is characterised by a primitive transition matrix, $\boldsymbol{P}$. Primitivity requires $\boldsymbol{P}(i,i)<1$ for every state i – which implies that in a regular Markov chain, it is impossible for an entity to get "stuck" or "absorbed" in a particular state. In contrast, a Markov chain is said to be absorbing, if it has at least one absorbing state (i.e. a state i with $\boldsymbol{P}(i,i) = 1$) and if it is possible to transition from each non-absorbing state to at least one absorbing state (not necessarily in one step), within the system being modelled. In an absorbing Markov chain, non-absorbing states are referred to as transient states [17].

If an absorbing Markov chain has $\boldsymbol{r}$ absorbing states and $\boldsymbol{t}$ transient states, the transition matrix $\boldsymbol{P}$ will have following canonical form;

$$P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$$

Where $\boldsymbol{Q}$ is a square $(t \ x \ t)$ submatrix which describes the probability of transitioning from each transient state to other transient states, $\boldsymbol{R}$ is a nonzero, rectangular $(t \ x \ r)$ submatrix which describes the probability of transitioning from transient to absorbing states, $\boldsymbol{0}$ is a rectangular $(r \ x \ t)$ matrix of zeros, and $\boldsymbol{I}$ is an identity matrix. The entry $p_{ij}^{(n)}$ of the transition matrix (which, after n steps, is written as $\boldsymbol{P}^{(\mathbf{n})}$) is the probability of being in the state s_j after n steps, when the chain started from state s_i. Therefore, computations can be carried out to investigate the probabilistic future of this chain process [18].

B. Properties of absorbing Markov chains

1) Fundamental matrix

In an absorbing Markov chain, the probability that the process will be absorbed is 1. This implies that as the number of steps, n goes to infinity, the probabilities of transitioning from some transient state to another, as recorded in $\boldsymbol{Q}$ matrix, goes to zero.

$$P^{\infty} = \begin{pmatrix} I & 0 \\ NR & 0 \end{pmatrix}$$

Where the matrix $N = \boldsymbol{I} + \boldsymbol{Q}^1 + \boldsymbol{Q}^2 + \cdots + \boldsymbol{Q}^{\infty} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$ is called the fundamental matrix for the absorbing chain with the transition matrix $\boldsymbol{P}$. Each entry, n_{ij} of $\boldsymbol{N}$ shows the expected number of times the process being modelled will be in the transient state s_j if it started from the transient state s_i.

2) Absorption probabilities and time to absorption

Let t_i be the expected number of steps before the Markov chain process being modelled is absorbed, given that the process starts in state s_i. Then the column vector t, whose i^{th} entry is t_i, is calculated as [16];

$$\boldsymbol{t} = \boldsymbol{Nc} \tag{1}$$

Where c is a column vector with all entries equal to 1.

Furthermore, let $\boldsymbol{B}$ be a rectangular $(t \ x \ r)$ matrix with entries b_{ij}, each of which indicate the probability that an absorbing Markov chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i. Then, the matrix $\boldsymbol{B}$ can be calculated as follows [16];

$$\boldsymbol{B} = \boldsymbol{NR} \tag{2}$$

Where $\boldsymbol{N}$ is the fundamental matrix and $\boldsymbol{R}$ a nonzero rectangular $(t \ x \ r)$ matrix.

C. Canonical example of an absorbing Markov chain

The Drunkard's walk is a classic example used to illustrate absorbing Markov chains (see Figure 2). The Drunkard's walk example describes a situation where a man walks along a four segment stretch of a road which connects his home (at 0) with a bar (at 4). If he is at junctions 1, 2 or 3, then he walks to the left or right with equal probability. Once he reaches a corner, i.e. either his home (0) or the bar (4), he stays there.

For the Drunkard's walk example presented in Figure 2, the transition matrix can be written as follows;

$$\boldsymbol{P} = \begin{array}{c} \\ \mathbf{0} \\ \mathbf{4} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbf{0} & \mathbf{4} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 & 0 \end{pmatrix} \end{array}$$

The fundamental matrix, $\boldsymbol{N}$ can then be calculated as;

$$\boldsymbol{N} = (\boldsymbol{I}-\boldsymbol{Q})^{-1} = \begin{array}{c c} & \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{pmatrix} 1.5 & 1 & 0.5 \\ 1 & 2 & 1 \\ 0.5 & 1 & 1.5 \end{pmatrix} \end{array}$$

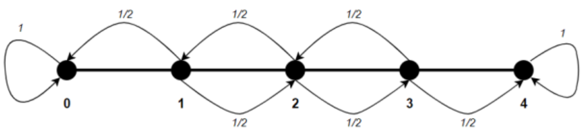

Fig. 2: Illustration of Drunkard's walk example

The entries in the above matrix indicate the number of times the Drunkard will be at each junction (column-wise) if he started from a particular junction (row-wise).

The expected number of steps before absorption is then established as [16];

$$\boldsymbol{t} = \boldsymbol{N}\boldsymbol{c} = \begin{array}{c c} & \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{pmatrix} 1.5 & 1 & 0.5 \\ 1 & 2 & 1 \\ 0.5 & 1 & 1.5 \end{pmatrix} \end{array} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix}$$

The entries in the above column vector shows the number of steps the Drunkard is expected to take prior to entering his home or the bar, given the junction he started from (row-wise).

Finally, the absorption probabilities can be calculated as [16];

$$\boldsymbol{B} = \boldsymbol{N}\boldsymbol{R} = \begin{array}{c c} & \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{pmatrix} 1.5 & 1 & 0.5 \\ 1 & 2 & 1 \\ 0.5 & 1 & 1.5 \end{pmatrix} \end{array} \begin{pmatrix} 0.5 & 0 \\ 0 & 0 \\ 0 & 0.5 \end{pmatrix} = \begin{array}{c c} & \begin{array}{cc} \mathbf{0} & \mathbf{4} \end{array} \\ \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{pmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \\ 0.25 & 0.75 \end{pmatrix} \end{array}$$

The entries in the above matrix indicate the probability of the Drunkard ending up at either his home (0) or the bar (4), given he starts his journey at a particular junction (row-wise).

III. Methodology

The following section outlines the methodology proposed to evaluate the impact of localised disruptions on SCNs, along with a worked example.

Figure 3 illustrates a sample four- tier SCN which is structurally similar to that of a directed material flow SCN – as discussed in [6]. The top tier (nodes 1 and 2) represent the suppliers, the second tier (nodes 3 and 4) represent the manufacturers, the third tier (nodes 5 and 6) represents the distributors and the fourth tier (node 7) represents a single retailer in this case. The pseudo origins are shown in Yellow squares (with the value added by each firm 1 to 6, to the final product delivered to the consumers by the SCN shown within brackets in Red colour) adjacent to each value adding node (note that the retailer is not considered in this case to add any value to the SCN). Node 14 is a pseudo node used for illustration purposes to indicate the absorption point of the Markov process (which is the consumers in the case of a SCN). The weights shown beside each SCN link indicate the proportion of the value transferred to the downstream firms by each upstream firm (at each level these weights add up to 1). The values added by each firm and the link weights have been randomly allocated in this example for illustration purposes.

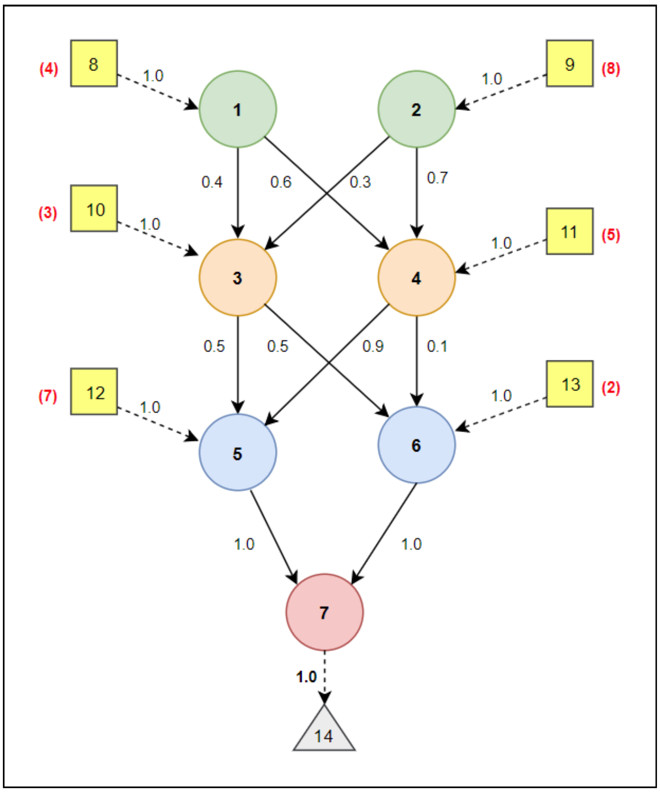

Fig. 3: Sample SCN considered for application of the method

For this example SCN, one could conveniently develop the transition matrix $\boldsymbol{P}$ and then the fundamental matrix $\boldsymbol{N}$. These two matrices ($\boldsymbol{P}$ and $\boldsymbol{N}$) are shown at the top and the bottom of figure 4, respectively. Note that the $\boldsymbol{Q}$ matrix is highlighted in Grey colour, within the $\boldsymbol{P}$ matrix shown in figure 4.

The fundamental matrix $\boldsymbol{N}$, which is equal to $(\mathbf{I}\text{-}\mathbf{Q})^{-1}$, can be further subdivided by origin nodes and transition nodes, as follows;

$$\boldsymbol{N} = (\mathbf{I}-\mathbf{Q})^{-1} = \begin{pmatrix} \mathbf{I} & \mathbf{Q}_1 \\ \mathbf{0} & \mathbf{Q}_2 \end{pmatrix}$$

The matrices $\boldsymbol{Q_1}$ and $\boldsymbol{Q_2}$ are shown in the $\boldsymbol{N}$ matrix (bottom of figure 4) in light Grey and light Blue highlights, respectively.

Given the origin value flows as a row vector $\boldsymbol{g}$, which in this case is, $\boldsymbol{g} = [4\ \ 8\ \ 3\ \ 5\ \ 7\ \ 2]$, the number of visits for each transition node $\boldsymbol{v}$, can be calculated as follows;

$$v = g\mathbf{Q}_1(\mathbf{I} - \mathbf{Q}_2)^{-1}$$

The $\boldsymbol{v}$ matrix result for the example SCN considered here is shown in figure 5. Each number in this row vector corresponds to the value transferred through each transition node from 1 to 7 in the SCN.

	14	8	9	10	11	12	13	1	2	3	4	5	6	7
14	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	0.4	0.6	0	0	0
2	0	0	0	0	0	0	0	0	0	0.3	0.7	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0
4	0	0	0	0	0	0	0	0	0	0	0	0.9	0.1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0

	8	9	10	11	12	13	1	2	3	4	5	6	7
8	1	0	0	0	0	0	1	0	0.4	0.6	0.74	0.26	1
9	0	1	0	0	0	0	0	1	0.3	0.7	0.78	0.22	1
10	0	0	1	0	0	0	0	0	1	0	0.5	0.5	1
11	0	0	0	1	0	0	0	0	0	1	0.9	0.1	1
12	0	0	0	0	1	0	0	0	0	0	1	0	1
13	0	0	0	0	0	1	0	0	0	0	0	1	1
1	0	0	0	0	0	0	1	0	0.4	0.6	0.74	0.26	1
2	0	0	0	0	0	0	0	1	0.3	0.7	0.78	0.22	1
3	0	0	0	0	0	0	0	0	1	0	0.5	0.5	1
4	0	0	0	0	0	0	0	0	0	1	0.9	0.1	1
5	0	0	0	0	0	0	0	0	0	0	1	0	1
6	0	0	0	0	0	0	0	0	0	0	0	1	1
7	0	0	0	0	0	0	0	0	0	0	0	0	1

Fig. 4: The transition (*P*)matrix (top) and the fundamental (*N*) matrix (bottom) for the example SCN

Now, in order to model the disruptions, one could manipulate the entries of the transition matrix $\boldsymbol{P}$ as desired (the disrupted case transition matrix is denoted as $\boldsymbol{P}'$). For instance a disruption of a link can be modeled by updating the records of the $\boldsymbol{Q}$ matrix (disrupted case denoted as $\boldsymbol{Q}'$) as shown below;

$$P' = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{R} & \mathbf{Q}' \end{pmatrix}$$

With the above manipulation, one could re-calculate the fundamental matrix (denoted as $\mathbf{N}'$ for the disrupted scenario) and the $\boldsymbol{v}$ matrix (denoted as $\boldsymbol{v}'$ for the disrupted scenario) as below;

$$N' = (\mathbf{I} - \mathbf{Q}')^{-1} = \begin{pmatrix} \mathbf{I} & \mathbf{Q}'_1 \\ \mathbf{0} & \mathbf{Q}'_2 \end{pmatrix}$$

$$v' = g\mathbf{Q}_1(\mathbf{I} - \mathbf{Q}_2{}')^{-1}$$

Note that in the row vector $\boldsymbol{g}$, which includes the origin flow values, should remain constant during both pre and post disruption analyses.

Table I illustrates the results obtained for the sample SCN considered here – i.e. the value transferred through each SCN node as links are independently removed. Figure 6 illustrates this result graphically. From these results, it is clear that the values transferred through nodes 1 and 2 are unaffected by removal of any link, as these two nodes are at the top tier of (i.e. they are raw material suppliers) and are the starting points of the SCN.

Node ID	1	2	3	4	5	6	7
Value delivered	4	8	7	13	22.2	6.8	29

Fig. 5: Value trasnferred through each transition node from 1 to 7 in the SCN

TABLE I. IMPACT ON VALUE TRANSFER WITHIN THE SCN DUE TO LINK LEVEL DISRUPTIONS

	Value transferred across each supply node							Value transferred to the consumer (node 14)
Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
Initial (fully functional SCN) scenario	4	8	7	13	22.2	6.8	29	29
Disrupted link								
1-3	4	8	5.4	13	21.4	6	27.4	27.4
1-4	4	8	7	10.6	20.04	6.56	26.6	26.6
2-3	4	8	4.6	13	21	5.6	26.6	26.6
2-4	4	8	7	7.4	17.16	6.24	23.4	23.4
3-5	4	8	7	13	18.7	6.8	25.5	25.5
3-6	4	8	7	13	22.2	3.3	25.5	25.5
4-5	4	8	7	13	10.5	6.8	17.3	17.3
4-6	4	8	7	13	22.2	5.5	27.7	27.7
5-7	4	8	7	13	22.2	6.8	6.8	6.8
6-7	4	8	7	13	22.2	6.8	22.2	22.2

From table I and figure 6, it is evident that the largest impact to the SCN arises when link 5-7 is disrupted. This result is intuitive since the criticality of links increases as they get closer to the consumer level (due to cumulative values carried on from upstream firms). In particular, when this link is disrupted, approximately 76.6% of the SCN value will not be able to reach the consumers. However, in contrast, when link 6-7, which is at the same level as link 5-7 is disrupted, only 23.4% of the SCN will be lost by the consumer base.

Figure 7 presents the link criticality rankings obtained from the analysis. It is interesting to note that for the considered SCN, link 4-5 is deemed more critical than the link 6-7 (which is located further downstream). Indeed, the level of value delivered through various SCN paths are a function of the link weights, which have been randomly allocated in the example scenario considered here.

Finally, it is noted that this methodology is also able to model the disruptions to nodes. In the case of nodes, a disrupted node will be reflected as zeros along its respective rows and columns in the $\boldsymbol{Q}'$ matrix (i.e. all links to and from this node are removed). Furthermore, this analysis can consider partial functionality of nodes and/or links within the SCN. In order to model such situations, one would update the reductions in respective weights in the $\boldsymbol{Q}^{l}$ matrix.

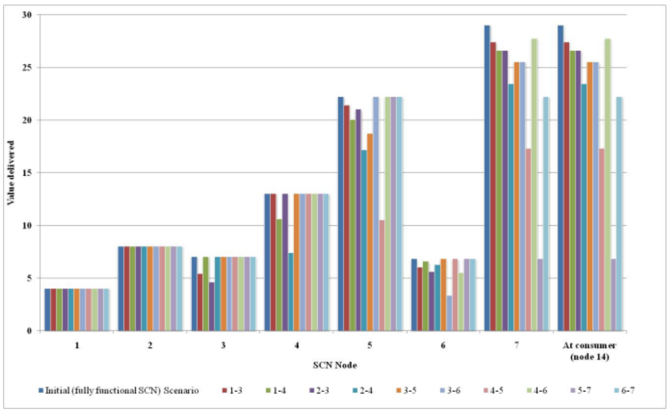

Fig. 6: Value transferred through each SCN node as links are independently removed

Link ID	1-3	1-4	2-3	2-4	3-5	3-6	4-5	4-6	5-7	6-7
Criticality rank	7	6	6	4	5	5	2	8	1	3

Fig. 7: Link criticality ranks obtained from the analysis

IV. Discussion

Reference [19] defines the function of a supply chain as to ‘transfer information, products and finance amongst suppliers of raw materials, manufacturers, distributors, retailers, and consumers’. Therefore, an efficient supply chain permits the goods to be produced and delivered in the right amounts, at the right time, to the right locations efficiently and reliably [20]. In line with the above, a robust supply chain should respond quickly and effectively to a given perturbation such as the failure of an individual component within the overall system due to an unforeseen circumstance.

Robustness of SCNs have been modeled in literature using network science metrics or simulations which sequentially remove nodes (randomly or targeted by degree or some other topological attribute) and record at each time step, the size of the largest connected component and/or the average shortest path length in the largest connected component [3]. Such topology based methods assume homogeneity in SCN components, in terms of importance. Additionally, the high levels of abstraction in these models limit their real world applicability.

This work has presented a novel methodology to quantify (by ranking the links based on their criticality) the robustness of material flow SCNs. The advantage of the proposed absorbing Markov chain approach is that it does not require computationally expensive path enumeration to be carried out for a given SCN. Rather, it relies on matrix operations for calculation of the value flow along the links of the given SCN.

In summary, while the traditional network science methods can only model the full node and/or link removal scenarios, the proposed method can also account for partial functionality of disrupted components within the SCN – which is likely to be encountered in real world scenarios.

V. Conclusions

This work has applied the concept of absorbing Markov chains to model the disruption impacts on SCNs. The proposed method is computationally efficient compared to other traditional methods available in the area of network science. The proposed model incorporates information beyond the topology of the SCN as is a useful tool for decision making. In particular, practitioners could use this model to rank the links in their SCNs based on their criticality – which is determined on the basis of the value lost by the consumers of the SCN if the subject link is not fully functional. Determination of criticality of the links can provide practitioners with valuable insights on allocating resources to various components of the SCN. Future work on this topic should investigate the generalisability of the proposed model based on a collection of network topologies.

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