Numerical and analytical investigation of load transfer through eccentric columns with different cross sections

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Abstract: Eccentric columns are not a common feature in concrete building structures. However, late changes in floor layouts and constraints due to available spaces may enforce structural designers to use eccentric columns to transfer vertical load. The load paths of the eccentric column connections can create complications. Due to the sudden changes in geometry, additional stresses are developed at the slab-column connection and a discontinuity region is created in which the strain distribution becomes highly nonlinear and unpredictable. The strut and tie method has been extensively used in analysing such complicated structural elements over the years. This method has its own limitations and is based on simplifications including the definition of the load path. This simplified method can under or over predict the design capacities of such columns, which can be inefficient or inadequate for current design requirements. The objective of this study is to comprehensively investigate the load transfer mechanism for vertical loads of eccentric columns using an advanced finite element approach. Clear identification of the load path, effective cross-sectional area for load transfer and alternative design guidelines for column-slab joint are evaluated and the preliminary work is presented in this paper.

Keywords: Eccentric columns, numerical modelling, strut and tie method.

1. Introduction

The essence of structural engineering involves the design of structures to sustain gravity and lateral loads. The identification of the load transfer mechanism, i.e. the load path, through the many elements that compose the structure and into the foundation system is a critical aspect that needs to be catered for during the design phase. The flow of axial forces through a regular building structure occurs mainly through columns and load bearing walls.

Depending on various design requirements, columns can be constructed out of different concrete grades and reinforcement ratios. In addition, columns can take various geometries and be braced or unbraced. Hence, it is clear they can be detailed in a number of ways. Generally, the geometry and sizes of these elements are influenced by architectural layout requirements. In certain circumstances, unsymmetrical column cross sections, shapes and size differences of columns between adjacent floors can cause the load path to deviate from the centroidal axis of the column, thus, causing more complicated load paths and reinforcement arrangements. The complexity of an eccentric load path can cause unexpected stress distributions that can also vary with the design variables. This may lead engineers to over- or under-design concrete columns. A common method used in design is based on the traditional strut and tie approach, where a predefined load path is selected [1, 2]. The accuracy of this method is therefore heavily dependent on the existing guidelines and experience of the designer who defines the load path [3].

The work presented in this paper investigated the ability of the strut and tie method in predicting the load path for an eccentric column. For this purpose, results were compared with the numerical models from ANSYS APDL.

2. Simplified strut and tie model (STM) for eccentric columns

Most concrete design codes provide guidance on how to check for admissible stresses and topology in STMs. For instance, in the case of the Australian standard AS3600 [4], section 7 provides the main definitions. The geometry and shape of struts are determined by the compression field. Shapes are constrained to prismatic, fan or bottle, with the design strength being given by:

$$\phi_{st}\beta_s 0.9 f_c^{'} A_c \tag{1}$$

where ϕ_{st} is a strength reduction factor for design, β_s is the efficiency factor, $f_c^{'}$ is the characteristic compressive strength of concrete cylinders at 28 days and A_c is the smallest cross-sectional area of the concrete strut along its length and normal to the line of action.
The efficiency factor for fanned or bottle shaped struts is given by:

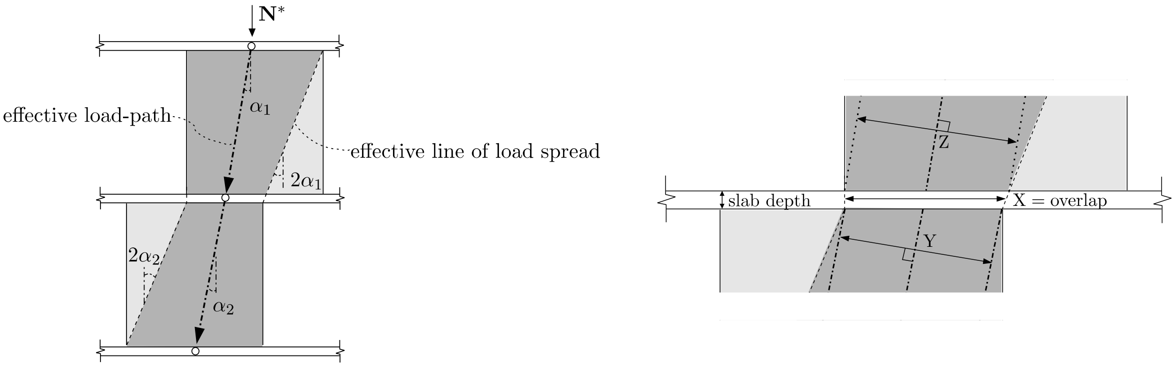
$$\beta_s = \frac{1}{1+0.66\cot^2\theta} \text{ within the limits } 0.3 \leq \beta_s \leq 1.0 \tag{2}$$

where θ is taken as the angle between the strut axis and a tie passing a common node.

The latter equation reflects the significance of the geometry by the efficiency angle and the role which eccentricity and overlap on a column STM is expected to have on the response. In addition, as evident in Eqs. (1) and (2), the strength of the strut and nodal region is a function of the compressive strength of concrete, thus emphasising the role played by both column and slab strengths. Nowadays, due to the emphasis on efficient, effective and economic structures, different grades of concrete may be used for slabs and columns. For example, in high-rise buildings, columns commonly have very high strength concrete, which is needed to withstand the massive compressive stresses caused by the numerous floors above the ground. Less demand is required from slabs, which can be casted using lower grades and less expensive concrete.

The behavior of slab column joints in the presence of different concrete strengths has been documented by Stanislaw and Goldyn [5]. Their experimental study focuses on external column joints with slab overhangs, with the objective of determining the influence of the weaker slab concrete on the load carrying capacity of the column. For this purpose, a column-to-slab concrete strength aspect ratio was defined to determine the actual concrete strength in the connection zone. Typically, this value is limited to 1.4 by ACI code provisions, although researches have suggested this value could be conservative[6] . Hence, determining the appropriate concrete strength of the slab and column interaction zone can be critical, not only for calculating strut and node capacities, but also for bearing capacity checks.

The STM approach can be adopted for designing column-slab connections where complexity is further increased by the eccentricity of columns [7]. The STM topology adopted to carry the forces through the D-region of the column connection is depicted by the node that connects the two compression struts from above and below the slab, as shown in **Figure 1**a. The assumed effective line of load spread is bound by a plane drawn at twice the angle of primary load transfer, α. Concrete outside of this bound is deemed ineffective in contributing to the compression load transfer. This model uses the strut and bearing plane Y and Z respectively – see **Figure 1**b – and the width of the column above and below, as the effective area to determine the principle stresses on both sides of the slab. Although not represented, a horizontal force develops at the slab level due to the change in angle across in the load-path (**Figure 1**a). Naturally, this force has to be taken into account at each affected floor and has to be included for sliding and diaphragm checks.

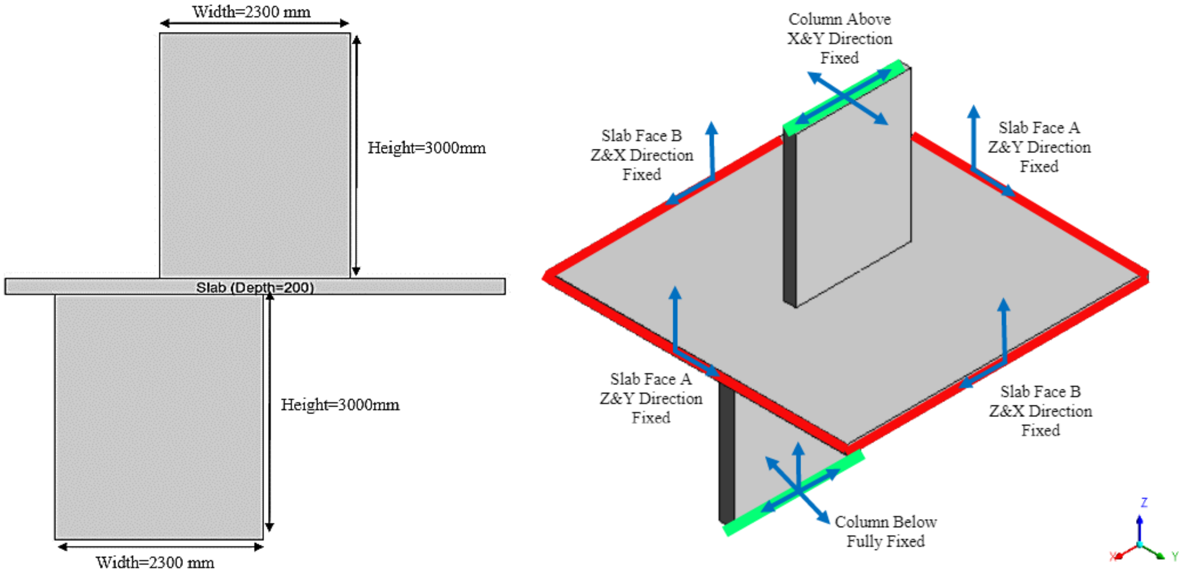


(a) (b)

Figure 1 – STM model: (a) overall geometry; (b) detail with main strut dimensions.

The effective area which is to be accounted for equating the principal compressive stress is measured along the line orthogonal to the effective load-path for the column above and below the transition slab. These measurements are represented by lengths Z and Y in **Figure 1**b and can be derived by geometric analyses of the column and slab interface using the angles represented in **Figure 1**a. A simple calculation sheet representing this fan-shaped STM model was developed to estimate the load transfer forces and stresses at the column faces for variable overlap dimensions ‘X’. In all subsequent analyses, the slab depth has been kept constant at 200 mm with a 3 m slab span in each direction of the 3 m high columns. The angles of load transfer, α_1 and α_2 have been determined using geometric properties. The results gained from the analytical program are to be correlated with the principle stresses and reactions generated by the numerical model at different overlap dimensions to determine the following: i) the suitability of the assumed fan shaped strut transferring the load spread; ii) the compressive stress associated with the failure of the model and its correspondence to the effective strut and bearing strength; and iii) the consequence of performing a linear and a nonlinear analysis on the effective cross sections.

3. Numerical model development



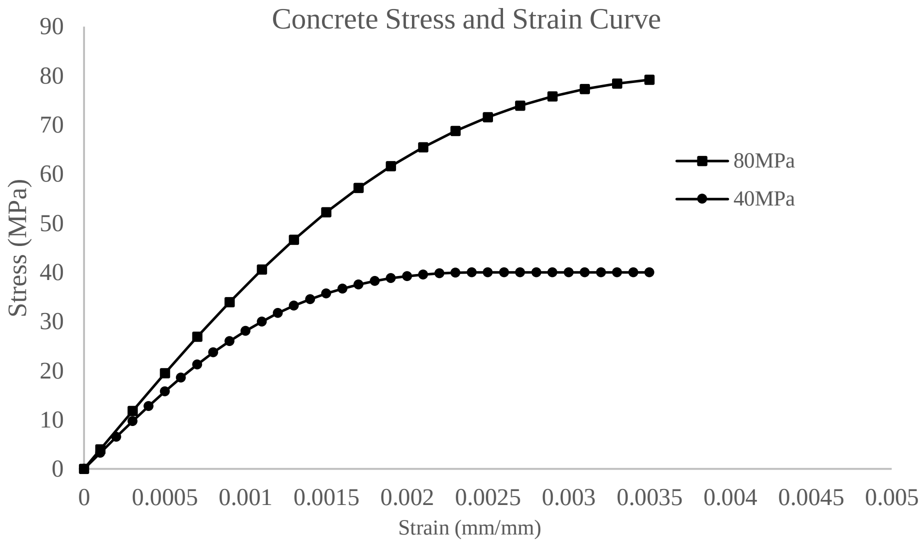
(a) (b)

Figure 2 – Three-dimensional model used in the numerical analysis

Figure 2 shows a schematic representation of the three-dimensional model. The numerical analysis was carried out using ANSYS APDL due to its capability of handing crushing and cracking of concrete with SOLID 65 elements [8, 9]. The element is defined with eight nodes having three degrees of freedom each (translations in x, y, and z directions). The element combines a constitutive model based on the smeared crack approach for areas under tensile stresses, whereas a plasticity model can account for the possibility of concrete crushing in compression areas. Each element has eight integration points at which checks for cracking and crushing are performed. The element is linear elastic until reaching either its tensile or compressive strength, upon which cracking or crushing occur, respectively, perpendicular to the relevant principal stress direction with stresses being redistributed locally. For finding a numerical solution and dealing with the nonlinear material behaviour, an iterative solver was adopted.

A parametric study was first carried out on the mesh size and it was found that an element size of 0.15m provides both accurate and time-efficient results. No significant gain was found in adopting smaller elements. The non-linear inelastic concrete model available in ANSYS library was used for concrete with the parameters given below in Table 1. The table also shows the properties for steel bars, simulated using Link 180 elements.

Table 1. Concrete material model parameters used in the analysis

Element Type	Material Properties	
Solid 65	**Multilinear Isotropic**	
	Elastic Modulus	39,600 MPa
	Poisson's ratio	0.2
	Open Shear	0.2
	Closed Shear	1.0
	Ultimate Cracking Stress	3.22 MPa
	Ultimate Crushing Stress	80 MPa
		
Link 180	**Bilinear Isotropic Material**	
	Elastic Modulus	200,000 MPa
	Poisson's ratio	0.3
	Ultimate yield	500 MPa
	Tangent Modulus	2100 MPa

As depicted in Figure 1(b), appropriate boundary conditions need to be considered to simulate the load transfer through offset columns. Braced columns only transfer axial loads and do not transmit any horizontal actions to the slab. Additionally, the bottom of the column below the slab is fully fixed while the top of the column above is only fixed laterally (X and Y directions). The edges of the slabs are restrained in the vertical direction as well as in the direction parallel to the slab face.

Solution inputs such as number of load steps, sub steps and convergence criteria were selected to find the numerical solution. In this case, 1000 load sub steps were used together with a force divergence criterion, which proved to be an efficient way to identify the ultimate design load.

2. Results and discussions

2.1 *Effective load transfer area*

The following section presents a critical comparison between results obtained with STM and finite element models. In order to quantify the magnitude of stresses acting at the cross section, a model with an applied load of 5000 kN and a column overlap of 1250 mm was analysed. The applied load was well below the ultimate load, such that the load path was not influenced by the failure mode of the column and to allow focusing on the behaviour of the connection.

The premise of the analytical model involved designing a portion of the column cross section shaded in grey in Figure 2.1(a). The STM approach assumes that the length of the strut at the slab and column interface is limited by the overlapping dimension of the structure. Based on this assumption, only a portion of slab is effective in transferring the load from the column above to the column below. Figure 2.1 (b) illustrates the plan and cross sectional areas considered effective and ineffective. The cross sections A and B are obtained through perpendicular planes cut immediately above and below the slab, as represented Figures 2(a) and 2(b). The effective length equals the overlap dimension of 1250 mm in Section B, where the load is transferred into the bottom part of the column. In section A, the effective length is equal to A+E.

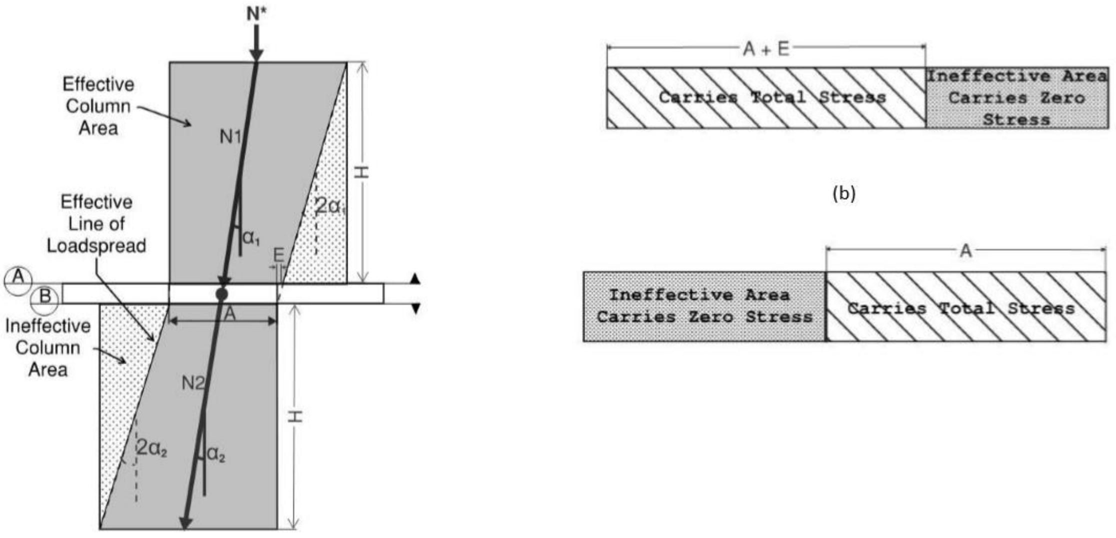


Figure 3 – STM model: (a) Analytical Fan Shaped model; (b) Section A effective cross section column above (c) Section B effective cross section column below.

Table 2 summarises the percentage of assumed analytical effective and ineffective cross sectional areas for above and below columns.

Table 2. Analytically predicted cross sectional effective and ineffective area.

Effective length, mm	Total area mm^2	Effective area %	Ineffective area %
Section A			
1320	575,000	57.4	42.6
Section B			
1250	750,000	50	50

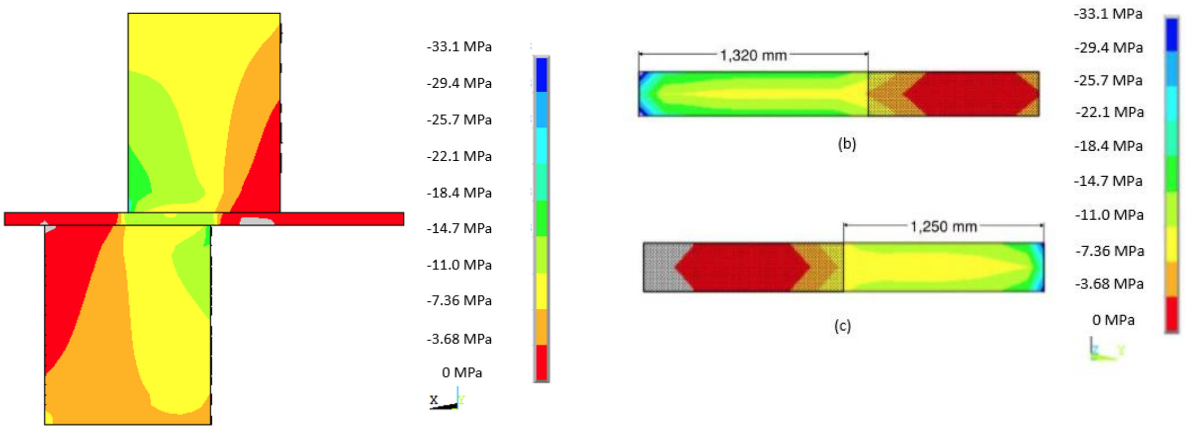


Figure 4 – Numerical stress distribution for 1,250 mm overlap: (a) general overview; (b) section A; (c) Section B.

The numerical investigations revealed that the complete cross section of the column, including areas deemed ineffective by STM, make a contribution in load transferring. This is evident by the stress distributions obtained from the numerical simulation results (Figure 3(a)) and in contrast to the STM (see Table 2), where a large portion of the cross section is deemed ineffective. As a result, with STM, a significantly smaller portion of concrete column is designed to withstand the vertical load transfer through the column. Furthermore, the cross sections shown in Figure 3(b) show the stress distributions occurring through Sections A and B in the numerical model. The stress contours in the section suggests that the ineffective shaded area of the column is in fact under compressive stress and is partially effective in the load transfer mechanism. This further highlights the differences between predicted analytical effective area of cross section under STM and numerical models. Table 3 summarises the percentage of stress over the assumed ineffective as a fraction of the total stress through the cross section.

Table 3. Numerical prediction of cross sectional effective and ineffective areas.

Effective length, mm	Total area mm^2	Effective area %	Ineffective area %
Section A			
1,320	575,000	81	19
Section B			
1,250	750,000	86	14

Although in practice, engineering models are commonly simplified to aid engineers in design and produce fast results, an optimum and efficient design is equally important. Highly conservative approaches can impact the cost and decrease a designer's competiveness. Based on the numerical findings, certain modifications could be proposed for the STM approach.

2.2 Improvements to the model

One of the objectives of this study was to contribute to improve the analytical technique by assessing the stress distribution in columns and columns-slab joints. The use of a fan shape strut can be seen as an accurate enough model for determining the stress distribution through the cross section, although the numerical analysis indicated the original STM approach to be over-conservative.

It is observed that the load transfer angle varies from 45^0-55^0 when the overlap length changes from 1,250 to 1,800 mm. This is the angle measured between the load path and the top and bottom faces of the slab.

A slight variation of this angle can cause a significant variation in the stress, thus design tends to move towards being overestimated.

A new modification for the STM approach is suggested by incorporating an effective area of stress transfer through the slab. Based on the minimum angle criterion, an angle of 45^0 is proposed as shown in the figure 3. This approach increases the effective area of load transfer assumed at both faces of the slab. The percentage of increment in the effective area is 30% for the adopted geometry, which represents a significant increment when considering the amount of load transfer in the columns and the optimisation that can be achieved in the design.

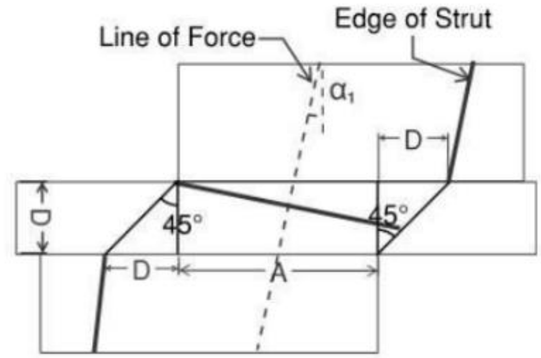


Figure 5 – Modified STM model.

To quantify the improvement that can be achieved with the proposed changes in the original STM model, an analysis was performed. The modified STM approach with an increased effective strut revealed closely matching results with the numerical model, compared to the original STM approach. For each column, three different overlaps were analysed at Section A and Section B. Results are shown in Figures 4 and 5, where it can be seen that by increasing the effective length of the strut in the modified STM model, the overestimation of stresses in the compressive strut can be reduced. This optimised STM also appears to be relevant for the column above, where the overestimation in principle stresses could be reduced by more than 20%.

The proposed modified STM has also implications for required concrete strength in columns. It was found that a small increment in the effective length can produce the same strut capacity as the original model for a 10% reduction in concrete strength – see Figure 6. This is a very interesting finding for cost effective and optimised design of buildings with complex eccentric column arrangements.

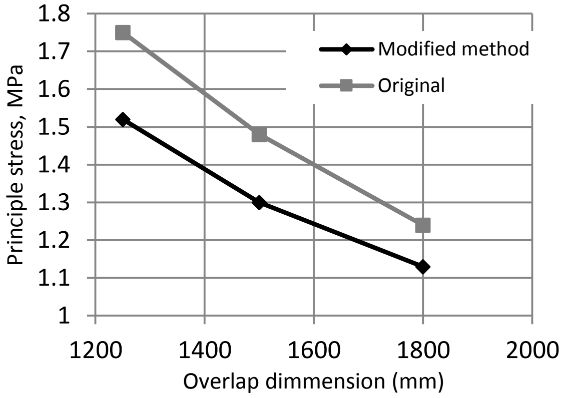


Figure 4: Change in overestimated stress - section A

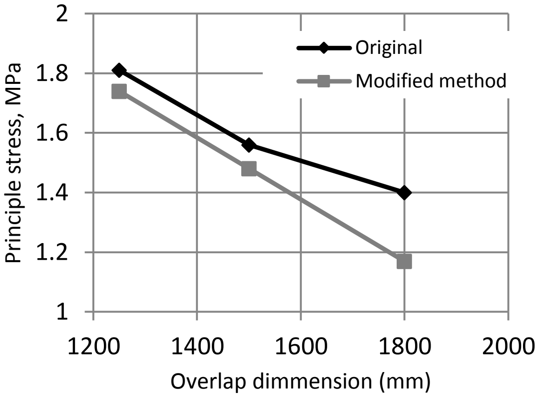


Figure 5: Change in overestimated stress - section B

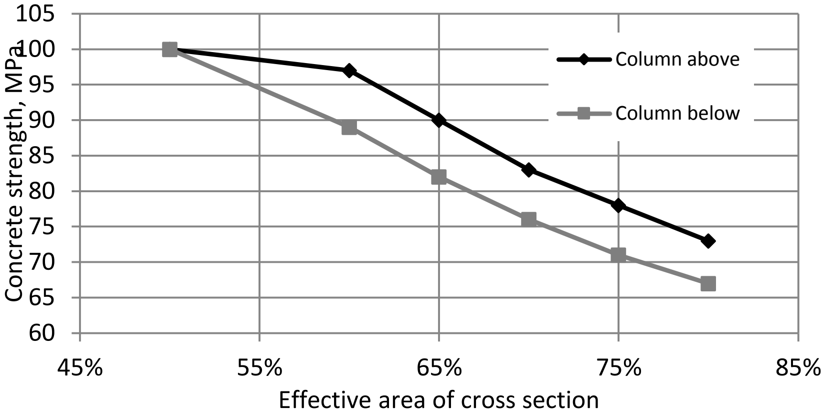


Figure 6: Design strength versus effective cross sectional area

3. Conclusions

This paper presented a preliminary numerical and analytical investigation aiming at improving the current design procedure for eccentric columns. The investigation discovered STM to significantly overestimate the stress through the compressive strut by assuming a large portion of the column to be ineffective in the load transfer path. As column overlaps increased from 1,250 to 1,800 mm, it was found that the effective line of load spread of struts became less conservative, thereby reducing the level of overestimation. Nearly 20% of the load can be carried by the ineffective area close to the slab face. Consequently, STM results are significantly conservative and this can lead to the use of higher than needed concrete grades. Increasing the effective area by 20% could see a 25% reduction of the concrete strength required for the same ultimate capacity.

Further analysing the stress distributions given in the numerical analysis revealed that the slab plays a more substantial part in the load transfer mechanism than predicted. As such, it became evident that the load from the column above spreads into the slab at approximately a 45° angle before distributing into the column below. Therefore, the numerical analysis indicated that the critical effective length of the strut could be increased by the depth of the slab in the analytical model. This optimisation led to a 10% reduction in the concrete strength required to achieve the same strut capacity.

4. Acknowledgements

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